1. It is known that thermal diffusivity and reaction rate per unit volume are given by,

$$\alpha = \lambda/(\rho_u c_P) \propto p^{-1}$$
$$\dot{\omega}''' \propto T_b^{-n} p^n \exp\left(-\frac{E_a}{R_u T_b}\right),$$

where n is the reaction order.

   (i) Write down the expression for laminar flame speed (S_u°) upto a proportionality constant

   (ii) The flame speed for some hydrocarbon/air mixture is 30 cm/s and its adiabatic flame temperature is 1600 K. With the addition of inert gas, the flame temperature drops to 1450 K. What is the corresponding flame speed? The activation energy of such hydrocarbon reactions is generally assumed to be 160 kJ/mol. Since the reaction is of second-order, the addition of the inert can be considered to have no other effect on any property of the system. (Ans: 23.7 cm/s)

   (iii) If the thickness of the hydrocarbon/air flame is 1 mm, what is the thickness upon addition of inert? Is there any use for adding inerts to hydrocarbon/air mixtures in the context of premixed flames? (Ans: 1.86 mm)

2. A laminar flame is propagating at constant velocity through a stoichiometric methane-air mixture that fills a long cylindrical tube. For an unburned mixture at 300 K and 1 atm, the flame speed is measured to be 40 cm/s with a flame thickness of 1 mm with a burned gas temperature of 2230 K. Data show that for the same unburned gas mixture at 300 K and 2 atm, the flame speed is 28 cm/s and the burned gas temperature is 2240 K. The activation energy of such hydrocarbon reactions is generally assumed to be 160 kJ/mol.

   (i) Estimate the overall reaction order for methane oxidation. (Ans: 0.91)

   (ii) Estimate the flame thickness for the unburned mixture at 300 K and 2 atm. (Ans: 0.7139 mm)

3. A mixture at 1 bar, 298 K contains hydrogen and oxygen in the molar ratio of 0.8 and 0.2, respectively. The species velocities of hydrogen and oxygen are 20 m/s and 10 m/s. Find the mole-averaged and mass-averaged mixture velocities and the diffusion velocities of the two species. (Ans: 18 m/s, 12 m/s, $V_{H_2} = 8$ m/s, $V_{O_2} = -2$ m/s)

4. A binary mixture of A and B, has a mixture density of 1.2 kg/m^3. The mass fraction of A varies as $Y_A(x) = 0.8 - 2x$ with distance x. If the binary diffusion coefficient is 5×10^{-5} m^2/s, find the diffusion flux values of A and B. If the mixture has a (mass-averaged) bulk velocity of 0.01 m/s, find the net fluxes of A and B at $x = 0.1$ m. (Ans: $7.32 \times 10^{-3} kg/m^2 s$, $4.68 \times 10^{-3} kg/m^2$s)

5. Consider the Stefan-Maxwell relation for diffusion velocity in the absence of pressure, body forces, and second order effects, given by,

$$\nabla X_i = \sum_{j=1}^{N} \frac{X_i X_j}{\mathcal{D}_{i,j}} \left(\vec{V_j} - \vec{V_i}\right)$$

Show that when all the binary diffusion coefficients $\mathcal{D}_{i,j}$ are the same, equal to $\mathcal{D}$ (say), the above relation simplifies to the Fick's law of diffusion, $\vec{V_i} = -\mathcal{D} \ln \nabla Y_i$. (Check CKL $5.2.4.1)

6. The momentum balance equation for a 1D steady premixed flame can be used to compute the pressure field after all the other unknowns have been solved for. This equation is given by, $\dfrac{dp}{dx} = -\rho v \dfrac{dv}{dx}$.

   (a) Integrate the momentum equation between $-\infty$ and $+\infty$ to obtain an expression for the pressure jump across the flame front

   (b) For a stoichiometric methane-air flame with a speed of 40 cm/s and temperature $T_b/T_u \approx 7$, calculate the pressure jump across the flame. (Ans: -1.5 Pa)

7. (Class exercise) The energy equation in terms of sensible enthalpy for a 1D premixed adiabatic unstretched steady flame is given by

$$\frac{d}{dx}\left(\sum_i \rho_i v h_i^s\right) + \frac{d}{dx}\left(\sum_i \rho_i V_i h_i^s\right) = \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) - \sum_i \omega_i''' h_i^\circ \tag{1}$$

The exercise guides you to simplify the energy equation discussed for a 1D premixed adiabatic unstretched steady flame discussed in the lecture in terms of sensible enthalpy and arrive at the equation in terms of temperature. Note that $\rho_i = \rho Y_i$ and the mixture specific heat at constant pressure, $c_p = \sum c_{p,i} Y_i$.

(a)

$$\text{Show that } \frac{d}{dx}\left(\sum_i \rho v Y_i h_i^s\right) = \rho v \left(\sum_i h_i^s \frac{dY_i}{dx} + c_p \frac{dT}{dx}\right) \tag{2}$$

(b)

$$\text{Show that } \frac{d}{dx}\left(\sum_i \rho V_i Y_i h_i^s\right) = \sum_i h_i^s \frac{d}{dx}\left(\rho Y_i V_i\right) + \sum_i \rho Y_i V_i c_{p,i} \frac{dT}{dx} \tag{3}$$

(c) Using relations (2) and (3), arrive at the energy equation in terms of temperature as:

$$\rho v c_p \frac{dT}{dx} + \sum_i \rho Y_i V_i c_{p,i} \frac{dT}{dx} = \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) - \sum_i \omega_i''' h_i \tag{4}$$

8. The objective of this problem is to design a flame arrestor for use in the feed system of a waste incinerator in a chemical processing plant. The feed system delivers a mixture of gases from a process reactor that includes O_2 and hydrocarbons to the incinerator. The flame arrestor comprises a perforated copper plate installed in the feed pipe. Your design should include a specification of the number (per area) and diameter of the circular holes (equally spaced in a square array), and the hole center-line spacing. Pressure drop in the feed system is a concern and should not exceed one dynamic head ($1/2\rho v^2$), where the pressure drop is given by $\frac{\Delta p}{1/2\rho v^2} = \left(\frac{A_{\text{pipe}}}{A_{\text{holes}}} - 1\right)^2$. The nominal composition of the feed stream is: 18% O_2, 76% N_2 and 6% methane. The feed stream temperature and pressure are $25°C$ and 1 atm, respectively. The laminar flame thickness for the prescribed mixture composition is 1 mm.

9. Calculate the flammability limit of a fuel mixture comprising of 40% butane ($X_F|_{\text{LFL}} = 0.018$), 30% propane ($X_F|_{\text{LFL}} = 0.022$), and 30% iso-butane ($X_F|_{\text{LFL}} = 0.018$), which is typical of the composition of LPG. According to your calculations, is a mixture containing 3% volume LPG in air (i.e. 97% is air) flammable? If yes, calculate the minimum amount of CO_2 that the mixture must be diluted with to render it non-flammable. Report the composition (in mole %) of the diluted non-flammable mixture. (0.0190, Yes, {fuel: 0.02946, CO_2: 0.0177, air: 0.9528 })

10. Obtain the expression for minimum ignition energy derived in class in terms of S_u. For typical average property values given here at 1 atm and 300 K for hydrocarbon/air mixtures, obtain the minimum ignition energy: $\lambda = 0.04$ W/mK, $c_p = 1200$ J/kg K, $T_b = 2200$ K, $T_u = 300$ K, ρ_u=1.2 kg/m^3, $S_u = 40$ cm/s. To be certain of the ignition, report the ignition energy required ramped up by a factor of safety of 100.

11. Watch this video (VLC player works): Dollar bill trick. How do you explain the non-flaming of the dollar bill? What connection does this have to the ignition of solid fuels? Explain.

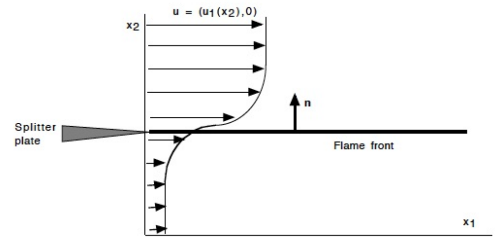
12. Stretch rate (κ) is central to study the influence of flow non-uniformity, flame curvature, and flame/flow unsteadiness on premixed flame propagation. In terms of the unburned gas velocity $\bar{u}$, and local flame speed S_u, and the normal vector $\vec{n}$ pointing into the fresh gases, for a stationary flame, the stretch rate is given by

$\kappa = \nabla_t \cdot \vec{u} + S_u \nabla \cdot \vec{n}$. Calculate the stretch rate for the following stationary flames and comment on them. It is that given that

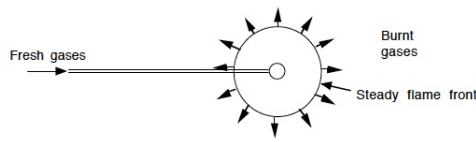
$$\nabla_t \cdot \vec{f} = (1 - n_1^2)\frac{\partial f_1}{\partial x_1} - n_1 n_2 \frac{\partial f_1}{\partial x_2} - n_2 n_1 \frac{\partial f_2}{\partial x_1} + (1 - n_2^2)\frac{\partial f_2}{\partial x_2} \text{ in Cartesian coordinates } (x_1, x_2)$$

$$\nabla_t \cdot \vec{f} = \nabla \cdot \vec{f} - n_r^2 \frac{1}{r}\frac{\partial f_r}{\partial r}, \nabla \cdot \vec{f} = \frac{1}{r}\frac{\partial (r f_r)}{\partial r} \text{ in cylindrical coordinates}(r, \phi, \text{axisymmetric}).$$

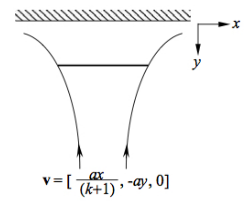
Note: $S_u = -\vec{u} \cdot \vec{n}$ for stationary flames.



(a) Planar flame in a shear flow, stabilized behind a splitter plate



(b) Cylindrical steady flame



(c) Forward stagnation flame

Based on your observations, answer the following, by providing an example or a counter-example

(i) Is a curved flame always subjected to stretch?

(ii) If in (b) above, the flame was moving outward (like the spherical flame in the experiment to measure flame speeds), will the flame be subjected to stretch?

(iii) Can a flat flame ever be stretched?

(iv) Can a flat flame be unstretched despite presence of velocity gradients in flow?

13. (Class Exercise) Derive an expression for the term featuring in momentum transport by convection: $\overline{\rho u v}$ in terms of (a) Reynolds averaged (mean and fluctuations) velocities and (b) Favre averaged (mean and fluctuations) velocities. Discuss which formulation is advantageous when dealing with varying density flows as encountered in turbulent combustion.