Max points: 50 **Open notes and books**

- This exam contains **13 questions** in all (**3 pages**)
- **Make suitable assumptions where appropriate and indicate them clearly.**

Fuel/air	LFL mole % (ϕ)	RFL mole % (ϕ)
methane/air	5 (0.50)	14.9 (1.67)
methanol/air	6.7 (0.51)	26.0 (4.0)
ethane/air	3 (0.52)	12.4 (2.4)
butane/air	1.8 (0.57)	8.4 (2.8)
Jet-A/air	0.7	4.8

Table 1: Lean and Rich flammability limits for some common fuel/air mixtures

1. (2 points) The quenching diameter for a stoichiometric mixture of propane and air at atmospheric pressure and temperature in a cylindrical metal tube is 2 mm.
   (i) If the air was replaced with pure O_2, do you expect the flame to be able to propagate through the 2 mm diameter metal tube.
   (ii) If unburnt reactant mixture is at 2 atm and 500 K, would you expect a flame to be able to propagate through a 2 mm diameter metal tube?

2. (4 points) State True/False. Support your answer with an argument or counter-example as appropriate.
   (i) Thickness of a premixed flame decreases with increase in pressure.
   (ii) Extinction of a non-premixed flame burning ethylene as a fuel is easier than one burning methane as the fuel at similar conditions.
   (iii) Quenching diameter for a premixed flame scales like the flame thickness.
   (iv) A stationary premixed flame is never stretched.

3. (4 points) A flammability test is conducted on a fuel sample, presumed to be 100% propane. The experiment yields a value of LFL=2% propane. It is however known that the actual LFL of pure propane is 2.1%. It is suspected tested sample could be due to contaminated with ethane (LFL = 3%) or butane (LFL = 1.8%). Reason out which one of the two it is and calculate the amount of impurity (in mole% of the total fuel).

4. (3 points) Mixtures of methane in two different oxygen-inert mixtures are considered, while the oxygen to inert ratio was held fixed at 0.21/0.79. The inerts considered are Argon and CO_2. Which of these two mixtures are expected to have a higher flame speed? Explain your answer based on expressions derived from the phenomenological analysis presented in the lecture.

5. (4 points) How does the burning intensity at the tip of a Bunsen flame burning lean propane-air compare to the other parts of the flame? With a simple CV energy balance, explain your answer.

6. (5 points) Consider the Stefan-Maxwell relation for diffusion velocity in the absence of pressure, body forces, and second order effects, given by,

$$\nabla X_i = \sum_{j=1}^{N} \frac{X_i X_j}{\mathcal{D}_{i,j}} \left(\vec{V_j} - \vec{V_i} \right)$$

   Show that when all the binary diffusion coefficients $\mathcal{D}_{i,j}$ are the same, equal to $\mathcal{D}$ (say), the above relation simplifies to the Fick's law of diffusion, $\vec{V_i} = -\mathcal{D} \ln \nabla Y_i$.

7. (3 points) Sketch the regimes of turbulent premixed combustion and elaborate on their characteristics.

8. (3 points) Minimum ignition energy:

   (i) How does minimum ignition energy of a premixed mixture of fuel and air vary with pressure?

   (ii) How many more times ignition energy is required to achieve ignition of a fuel/air mixture at sea level ($p = 101325$ Pa) versus at a higher altitude, where the ambient pressure is $p = 47166$ Pa.

   (iii) Discuss the implications of your calculations for high altitude relight in aircraft gas turbine engines.

   Take overall reaction order to be $n = 1.8$. The effect of change in unburnt gas temperature on minimum ignition energy can be assumed to be minimal.

9. (3 points) For a one-step irreversible reaction, $\nu_F \mathrm{F} + \nu_O \mathrm{O} \rightarrow \nu_P \mathrm{P}$, the fuel mass balance and energy conservation equations are given by,

$$\frac{\partial}{\partial t}(\rho Y_F) + \frac{\partial}{\partial x_i}(\rho u_i Y_F) = \frac{\partial}{\partial x_i}\left(\rho D_F \frac{\partial Y_F}{\partial x_i}\right) + \dot{\omega}_F \tag{1}$$

$$\frac{\partial}{\partial t}(\rho c_P T) + \frac{\partial}{\partial x_i}(\rho u_i c_P T) = \frac{\partial}{\partial x_i}\left(\lambda \frac{\partial T}{\partial x_i}\right) + \dot{\omega}_T \tag{2}$$

   In the narrow reaction region, the diffusion terms are more dominant than the convection and temporal terms. Obtain an expression for a local conserved scalar based on Y_F and T valid in this region, and write down its balance equation, showing that it is not produced or consumed (no source term). You may assume that c_P is a constant. Note: $\dot{\omega}_T = -q\dot{\omega}_F$ (q: heat of combustion: J/kg of fuel), $\mathbf{Le_F} = \dfrac{\lambda/(\rho \mathbf{c_P})}{\mathbf{D_F}} \neq \mathbf{1}$, is a constant.

10. (4 points) Consider a laminar non-premixed flame: gaseous fuel issues out of a nozzle, quiescent air from the ambient serves as the oxidizer, establishing a conical flame. Follow the steps below to arrive at the height of the flame.

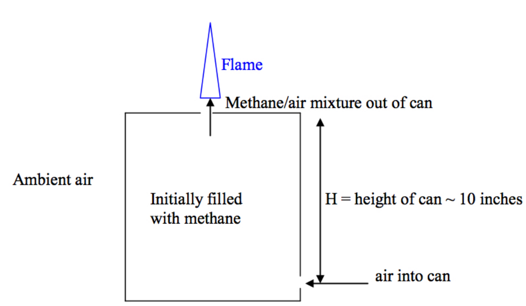
    (a) Oxidizer arrives at the flame by diffusion. For a nozzle diameter d and diffusion coefficient D, write down an expression for the time taken for diffusion of oxidizer to the flame tip, t_D.

    (b) During the time t_D, the fuel issuing at a mass flow rate $\dot{m}_F$, reaches the flame tip, whose height is h. Write a relation for h in terms of these variables.

    (c) If this flame is established at an ambient pressure of 1 atm versus 0.1 atm, how does the height of the flame vary?

    (d) Given that $D \sim T^{1.75}$, if the ambient temperature is 250 K versus 300 K, how does the height of the flame vary?

11. (6 points) Experiment: A can of volume V has two holes punched as shown: bottom hole at location 1 has diameter d_1 and top hole at location 2 has a diameter d_2 ($d_2 < d_1$). The holes are separated by a vertical distance H. The can is initially filled with pure methane at 298 K and 1 atm. A spark is applied at the top igniting a diffusion flame. Driven by buoyancy effects, air is sucked in through the bottom hole (density: ρ_{air}), and the mixture exits through the top hole, where it is consumed by the flame.



After some time, the exiting stream is enriched in air (flame becomes partially premixed), and then rich enough in air, that a premixed flame can be established. The flow velocities in the set-up are so low that once a premixed flame is established, it immediately flashes back into the can. Assuming that the contents of the can are well-mixed at all times, follow the steps below to obtain the time of flashback, t_{FB}:

(i) Let the velocity of gas at station '1' be v_1 and that of air at station '2' be v_2. Relate pressures, kinetic, and potential energies using Bernoulli equation within the can, assuming an inviscid flow. Simplify this expression further noting that $p_1 - p_2 = \rho_{\text{air}} g H$ throughout the problem.

(ii) Assuming that the gases in the can are incompressible (rate of volume inflow = rate of volume outflow), eliminate v_1 to obtain an expression for v_2 in terms of the known quantities.

(iii) Write down an expression for the number of moles of methane per unit volume in the can at any instant of time.

(iv) If the outlet stream can be assumed to have the same number of moles of methane per unit volume as in the can, obtain an expression for rate of change of moles of methane in the can at any instant of time in terms of the problem parameters. Thereby, obtain a differential equation for $\dfrac{dX_{\text{CH}_4}}{dt}$ within the can.

(v) When numerically integrated, a table of X_{CH_4} versus time can be generated. What quantity of interest to a premixed flame can be determined if the time at which flashback occurs (t_{FB}) is known from this experiment?

12. (4 points) Burke-Schumann solution:

(i) Calculate the point in z space where a pure CH_4-air non-premixed flame will be located, under the infinitely fast irreversible chemistry assumption.

(ii) If the fuel was diluted to 10% methane and rest inert, where will the flame be located? Explain the shift in the flame location in the z space.

(iii) For (i), calculate the stoichiometric scalar dissipation rate (χ_{st}) for a 1D steady strained counterflow flame, given $a = 30\,\text{s}^{-1}$.

(iv) If the stoichiometric scalar dissipation rate at extinction is $29\, s^{-1}$, show the χ_{st} obtained in question (iii) on a representative S-curve for ignition-extinction behavior.

13. (5 points) Turbulent combustion

(i) For a turbulent air flow issuing out of a 1-cm cylindrical tube at a mean velocity of $\bar{u} = 100\,\text{m}/s$, the turbulence intensity is 10% ($= u_0'/\bar{u}$). Then, the turbulent Reynolds number (integral scale), $Re_o =$ ____________________. Assume a kinematic viscosity for air = 0.1 cm^2/s.

(ii) Exact solutions of flow variables in turbulent combustion are computationally intensive because ____

(iii) In terms of Favre mean and fluctuation velocities, the term featuring in species transport by convection: $\overline{\rho u_i Y_k}$ is ____
Further, a model for the unclosed term, is given by ____

(iv) The flamelet regime applies for turbulent premixed combustion when (in terms of velocity and/or length scales) ____