Tutorial 4

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Q.1. (a.) $25x = 15 \pmod{29}$

$\gcd(25, 29) = 1$ and $1/15$.

$\therefore$ has a unique solution.

$25x = 15 \pmod{29}$

$-4x = 15 \pmod{29}$

$-28x = 105 \pmod{29}$

$-28x = 18 \pmod{29}$

$x = 18 \pmod{29}$.

(b.) $5x = 2 \pmod{26}$.

$\gcd(5, 25) = 1$ and $1/2$

$\therefore$ has a unique solution

$5x = 2 \pmod{26}$

$25x = 10 \pmod{26}$

$-x = 10 \pmod{26}$

$x = -10 \pmod{26}$

$x = 10 \pmod{26}$

(c) $34x = 60 \pmod{98}$

$\gcd(34, 98) = 2$ $\quad 2 | 60$.

$34x \equiv 60 \pmod{98}$ $\therefore$ has unique solution

$17x \equiv 30 \pmod{49}$

$51x = 90 \pmod{49}$

$2x = 41 \pmod{49}$

$2x \equiv -8 \pmod{49}$

$50x \equiv -200 \pmod{49}$

$x \equiv 45 \pmod{49}$ $\quad x_0 = 45$

$x = x_0 + \frac{n}{d} t$ $\quad t = 0, 1$

$x = 45 + 49t$ $\quad x = 45 \quad t = 0$

$x = 94 \quad t = 1$

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(d) $140x \equiv 133 \pmod{301}$

$\gcd(140, 301) = 7$ $\quad 7/133$

$\therefore$ Unique solution.

$140x \equiv 133 \pmod{301}$

$20x \equiv 19 \pmod{43}$

$40x \equiv 38 \pmod{43}$

$-3x \equiv 38 \pmod{43}$

$-3x \equiv -5 \pmod{43}$

$3x \equiv 5 \pmod{43}$

$42x \equiv 70 \pmod{43}$

$-x \equiv -16 \pmod{43}$

$x \equiv 16 \pmod{43}$. $\quad x_0 = 16$.

$x = x_0 + \frac{n}{d}t$.

$= 16 + \frac{301}{7}t \Rightarrow x = 16 + 43t$.

When: $t = 0$ $\quad x = 16$

$t = 1$ $\quad x = 59$

$t = 2$ $\quad x = 102$

$t = 3$ $\quad x = 145$

$t = 4$ $\quad x = 188$

$t = 5$ $\quad x = 231$

$t = 6$. $\quad x = 274$.

Q2 (a) $4x - 51y = 9$

$4x = 9 - 51y$

$4x \equiv 9 \pmod{51}$

$\gcd(4, 51) = 1$ $\quad 1/9$.

$\therefore$ has unique soln.

$4x \equiv 9 \pmod{51}$

$52x \equiv 117 \pmod{51}$.

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$x = 15 \pmod{51}$

then,

$x = x_0 + \frac{n}{d} t \Rightarrow x = \frac{15 + 51t}{1}$ — ①

$4x - 51y = 9$.

$\gcd(51, 4) = 1$ $\; 1/9$ $\therefore$ unique solution.

$51y = 9 - 4x$

$= 9 \pmod 4$

$-y = 1 \pmod 4$

$y = -1 \pmod 4$

$y = 3 \pmod 4$

$y = y_0 + \frac{n}{d} s = 3 + \frac{4}{1} s \Rightarrow y = 3 + 4s$

$\therefore 4x + 51y = 9$

$4(15 + 51t) + 51(3 + 4s) = 9$.

$= 60 + 204t + 153 + 204s = 9$.

$204t + 204s = -204$

$t + s = -1$,

$\therefore y = 3 + 4(-1 - t)$

$\boxed{y = -1 - 4t}$

b) $5x - 53y = 17$

$5x = 17 + 53y$

$5x \equiv 17 \pmod{53}$

$\gcd(5, 53) = 1$ $\; 1/17$ $\therefore$ has unique solu

$5x \equiv 17 \pmod{53}$

$50x \equiv 170 \pmod{53}$

$-3x \equiv 11 \pmod{53}$

$-54x \equiv -198 \pmod{53}$

$54x \equiv -198 \pmod{53}$

$x \equiv -198 \pmod{53}$

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$-x \equiv -198 \pmod{53}$

$-x \equiv -14 \pmod{53}$

$-x \equiv 14 \pmod{53}$

$x = x_0 + \frac{n}{d}t \Rightarrow x = 14 + \frac{53}{1}t$

$-53y = 17 - 5x$

$-53y \equiv 17 \pmod 5$

$\gcd(53, 5) = 1 \quad 1/17$ ∴ Unique soln.

$-53y \equiv 17 \pmod 5$

$24y \equiv 2 \pmod 5$

$y \equiv 1 \pmod 5$

$y = 1 + 5s$

∴ $5(14 + 53t) - 53(1+5s) = 17$

$70 + 265t = 53 - 265s = 17$

$s = t$

Thus $\Rightarrow y = 1 + 5t$

(Q.3) a : $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$,

$x \equiv 15 \pmod{31}$

Soln:- $n_1 = 11 \quad n_2 = 29 \quad n_3 = 31$

$\gcd(11, 29) = \gcd(29, 31) = \gcd(31, 11) = 1$

$n = 11, 29, 31 = 9889$

$n_1 = 29, 31 = 899$

$n_2 = 11, 31 = 341$

$n_3 = 11, 29 = 319$

$n_1 x \equiv 1 \pmod{11}$

$899x \equiv 1 \pmod{11}$

$-3x \equiv 1 \pmod{11}$

$-12x \equiv 4 \pmod{11}$

$-x \equiv 4 \pmod{11}$

$n_2 x \equiv 1 \pmod{29}$

$341 x \equiv 1 \pmod{29}$

$-7x \equiv 1 \pmod{29}$

$-28x \equiv 4 \pmod{29}$

$x \equiv 4 \pmod{29}$

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$x \equiv 7 \pmod{11}$ $\quad x^{-} = 4.$

$x_1 \equiv 7.$

$n_3 x \equiv 1 \pmod{31}$

$319x \equiv 1 \pmod{31}$

$-27x \equiv 3 \pmod{31}$

$-4x \equiv 3 \pmod{31}$

$-32x \equiv 24 \pmod{31}$

$-x \equiv 24 \pmod{31}$

$x \equiv 7 \pmod{31}$ $\quad x_3 = 7$

Thus a solution of the system is given by:

$\bar{x} = a_1 n_1 x_1 + a_2 n_2 x_2 + a_3 n_3 x_3$

$= 5 \times 7 \times 899 + 14 \times 341 \times 4 + 15 \times 319 \times 7.$

$= 84056.$

We get a unique solution modulo 9889:

$\bar{x} = 84056 \equiv 4944 \pmod{9889}$

b) $2x \equiv 1 \pmod 5$, $3x \equiv 9 \pmod 6$, $4x \equiv 1 \pmod 7$,

$5x \equiv 9 \pmod{11}$

$n_1 = 5, \quad n_2 = 9, \quad n_3 = 7, \quad n_4 = 11$

$n = 5 \times 9 \times 7 \times 11 = 770$

$\gcd(5,2) = \gcd(2,7) = \gcd(7,11) =$

$\gcd(5,7) = \gcd(2,11) = 1. \quad \gcd(5,11) =$

$N_1 = 154 \quad N_2 = 385 \quad N_3 = 110, \quad N_4 = 70.$

$N_1 x \equiv 1 \pmod 5$

$154x \equiv 1 \pmod 5$

$-x \equiv 1 \pmod 5$

$x \equiv -1 \pmod 5$

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$x \equiv 4 \pmod 5$

$x_1 \equiv 4$

$N_2 x \equiv 1 \pmod 6$

$385 x \equiv 1 \pmod 6$

$x \equiv 1 \pmod 6$ $\qquad x_2 = 1$

$N_3 x \equiv 1 \pmod 7$

$110 x \equiv 1 \pmod 7$

$5x \equiv 1 \pmod 7$

$20 x \equiv 4 \pmod 7$

$-x \equiv 4 \pmod 7$

$-x \equiv -3 \pmod 7$

$x \equiv 3 \pmod 7$ $\qquad x^3 = 3.$

$N_4 x \equiv 1 \pmod{11}$

$70 x \equiv 1 \pmod{11}$

$4x \equiv 1 \pmod{11}$

$12 x \equiv 3 \pmod{11}$

$x \equiv 3 \pmod{11}$ $\qquad x_4 = 11/3.$

$\bar{x} = a_1 N_1 x + a_2 N_2 x_2 + a_3 N_3 x_3 + a_4 N_4 x_4 +$

$3 \times 154 \times 4 + 3 \times 385 \times 1 + 2 \times 110 \times 3 + 4 \times 70 \times 3$

We get a unique soln Modulo 770: $\bar{x} = 4503 = 653 \pmod{770}$.

(Q-4) Since $2/a$ is equivalent to $4 \nmid a+2$ and $6/a+4$ is equivalent to $3 \nmid a+1$, so the given Problem reduces in tending the Smallest

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integer $a > 2$ such that $3/a$, $4/a+2$, $5/a+3$.
Thus, we have to find the least value of $a > 2$ such that $a \equiv 2 \pmod 6$. since 4 & 6 are not co-prime, so Chinese Remainder Theorem is not applicable. However we observe that $a = 62$ serves as a solution. This is the smallest solution because $a \equiv 2$ & 6 implies that if $2 < a < 62$. then Possible value for a are 8, 14, 20, 26, 32, 38, 44, 50 & 56. But $8 \equiv 2$ and 5, $14 \equiv 2 \pmod 5$, $20 \equiv 2 \pmod 5$, $26 \equiv 2 \pmod 5$, $32 \equiv 2 \pmod 5$, $38 \equiv 2 \pmod 5$, $44 \equiv 2 \pmod 5$, $50 \equiv 2 \pmod 5$ and $56 \equiv 2 \pmod 5$. Thus, the desired least value of $a = 62$.

(Q. 5.) The question Provides us The following equations.

$x \equiv 3 \pmod{17}$
$x \equiv 10 \pmod{16}$
$x \equiv 0 \pmod{15}$

$\gcd(15, 16) = \gcd(15, 17)$
$\gcd = (17, 16) = 1$

$n = 17 . 16 . 15 = 4080$

$N_1 = 240$
$N_3 = 272$
$N_2 = 255$

$N_1 x \equiv 1 \pmod{n_1}$
$240 x \equiv 1 \pmod{17}$
$2x \equiv 1 \pmod{17}$
$18x \equiv 9 \pmod{17}$
$8x \equiv 9 \pmod{17}$

$x_1 = 9$.

$N_2 x \equiv 1 \pmod{n_2}$
$255 x \equiv 1 \pmod{16}$
$-x \equiv 1 \pmod{16}$
$x \equiv 15 \pmod{16}$ $x_2 = 15$

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$N_3 x \equiv 1 \pmod{n_3}$

$272x \equiv 1 \pmod{15}$

$2x \equiv 1 \pmod{15}$

$16x \equiv 8 \pmod{15}$

$x \equiv 8 \pmod{15}$ $x_3 = 8$.

Thus, a solution at the system is given by:

$\bar{x} = a_1 n_1 x_1 + a_2 n_2 x_2 + a_3 n_3 x_3$

$= 9.3.24 + 15.10.255 + 0.$

$= 44 + 30$

$\bar{x} = 3930 \pmod{4080}$.

So 3930 coins were stolen.

(q.b) Equations

$x \equiv 1 \pmod{9}$

$x \equiv 10 \pmod{11}$

$x \equiv 0 \pmod{3}$

$1 < x < 1200$, $gcd(9,11) = gcd(11,13)$ $gcd(13,9) = 1$

Chinese remainder theorem $n = 17, 16, 15 = 4080$.

$N_1 = 240$; $N_2 = 255$, $N_3 = 272$.

$N_1 x \equiv 1 \pmod{n_1}$

$143x \equiv 1 \pmod{n_1}$

$8x \equiv 1 \pmod{9}$

$-x \equiv 1 \pmod{9}$

$-x \equiv 1 \pmod{9}$ $= 7$ $x = 8 \pmod{9}$ $x_1 = 8$

$x_1 = 1$

$N_2 x \equiv 1 \pmod{n_2}$

$117x \equiv 1 \pmod{11}$

$-4x \equiv 1 \pmod{11}$

$-12x \equiv 3 \pmod{11}$

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$12x \equiv -3 \ (\text{Mod } 11)$

$x \equiv -3 \ (\text{Mod } 11)$

$x \equiv 8 \ (\text{Mod } 11)$. $x_2 \equiv 8$.

$n_3 x \equiv 1 \ (\text{Mod } n_3)$

$99x \equiv 1 \ (\text{Mod } 13)$

$-5x \equiv 1 \ (\text{Mod } 13)$

$-15x \equiv 3 \ (\text{Mod } 13)$

$-2x \equiv 3 \ (\text{Mod } 13)$

$-12x \equiv 18 \ (\text{Mod } 13)$

$x \equiv 5 \ (\text{Mod } 13)$ $x_3 \equiv 5$.

Thus a solution of the system is given by:-

$x = a_1 n_1 x_1 + a_2 n_2 x_2 + a_3 n_3 x_3$,

$= 1 \cdot 143 \cdot 8 + 2 \cdot 255 \cdot 8 + 6 \cdot 49 \cdot 5$

$= 5986$.

$= 838 \ (\text{Mod } 1287)$

(Q.7) We want to solve the following system.

$x \equiv 1 \ (\text{Mod } 2)$

$x \equiv 2 \ (\text{Mod } 3)$

$x \equiv 5 \ (\text{Mod } 6)$

$x \equiv 5 \ (\text{Mod } 12)$

gcd $(3, 6) = 1$ then multiply eq (2) by (2)

$2x \equiv 4 \ (\text{Mod } 6)$ — (5)

Now subtract eq. 3 from eq. 5

$x \equiv -1 \equiv 5 \ (\text{Mod } 6)$ — (6)

gcd $6, 12 = 1$ then multiply eq. (6) by (2)

$2x \equiv 10 \ (\text{mod } 12)$ — (7).

Now subtract eqn. (4) from eq. (7)

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$x \equiv 5 \pmod{12}$

since, gcd $(2,12) \neq 1$ then multiply eqn. ① by 6

$6x \equiv 6 \pmod{12}$ — ⑨

subtract ean. ⑧ from ean. ⑨

$5x \equiv 1 \pmod{12}$

$35x \equiv 7 \pmod{12}$

$-x \equiv 7 \pmod{12}$

$x \equiv 5 \pmod{12}$

Thus, $x = 5 + 12t \quad x > 12$ for some $t \in \mathbb{Z}$

$x = 5 + 2$

$x = 17.$

(Q. 9) We want to find a solution of the system.

$3x + 4y \equiv 5 \pmod{13}$

$2x + 5y \equiv 7 \pmod{13}$

Multiply eq ① by ② and multiply eq. ② by 3.

$6x + 8y \equiv 10 \bmod 13$ — ③.

$6x + 15y \equiv 21 \pmod{13}$ — ④

Now subtract eq. 4 from eq. 3.

$7y \equiv 11 \pmod{13}$

$14y \equiv 22 \pmod{13}$

$y \equiv 4 \pmod{13}$ — ⑤

substitute eq. 5 into eq. ①.

$3x + 4 \cdot 9 \equiv 5 \pmod{13}$

$3x + 36 \equiv 5 \pmod{13}$

$3x - 3 \equiv 5 \pmod{13}$

$3x \equiv 8 \pmod{13}$

$12x \equiv 32 \pmod{13}$

$-x \equiv 6 \pmod{13}$

$x \equiv 7 \pmod{13}$

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The solution of system is :

$$y \equiv 9 \pmod{13}$$
$$x \equiv 7 \pmod{13}$$

(Q..8) If t_n be n^{th} triangular number then

$$t_1^2 + t_2^2 + \ldots + t_n^2 = \frac{t_n}{30}(3n^3 + 12n^2 + 13n + 2).$$

$$3n^3 + 12n^2 + 13n + 2 \equiv 0 \pmod{2, 3, 5}.$$

If we solve this congrevence eqn. we find the values of n that t_n divides $t_1^2 + t_2^2 + \ldots + t_n^2$

For first modulo 2, we can write the congrivence $n^3 + n \equiv 0 \pmod 2$. This always holds.

For modulo 3 the congrsence can be written as $n + 2 \equiv 0 \pmod 3$. This holds precisely if $n \equiv 1 \pmod 3$. Work modulo 5, the congrevence can be written. work modulo 5 the congrevence can be written

$3n^3 - 3n^2 + 3n - 3 \equiv 0 \pmod 5$, then as $(n-1)(n^2+1) \equiv 0 \pmod 5$. This has the soln $n \equiv 1 \pmod 5$, $n \equiv 2 \pmod 5$, $n_3 \equiv \pmod 5$.

So, the conditions come down to $n \equiv 1 \pmod 5$ and $n \equiv 1, 2,$ or 3 modulo 5.

The solutions are therefore $n \equiv 1$, $n \equiv 7$ and $n \equiv 13 \pmod{25}$.

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