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V19CS076

Tutorial

Q1] Find the rank of following matrices

① $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 5 & 5 \end{bmatrix}$

$R_2 = R_2 - R_1$
$R_3 = R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$$

$R_3 = R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 2$$

② $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

$R_2 = R_2 - 3R_1$
$R_3 = R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$R_3 = R_3 \times 3 + R_2$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \rho(A) = 2$$

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(3) $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$R_3 = R_3 - 3R_2$ $\quad$ $R_4 = R_4 - 6R_2$

$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$

$R_2 \to R_2 - R_1/2$

$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -5/2 & -3/2 & -7/2 \\ 0 & 4 & 9 & 10 \\ 0 & 4 & 9 & 10 \end{bmatrix}$ $\cancel{\rho = (3)}$

$R_4 \to R_4 - R_3$

$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -5/2 & -3/2 & -7/2 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\rho(A) = 3$

(4) $A = \begin{bmatrix} 2 & -1 & 0 & 5 \\ 0 & 3 & 1 & 4 \end{bmatrix}$ $\rho(A) = 2$

Cannot be reduced furthur

$\rho(A) = 2$

(5)

Q2] Reduce to triangular form

$$A = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$, $R_3 \to R_3 + 5/3 R_1$

$$A = \begin{bmatrix} 3 & -4 & -5 \\ 0 & -11 & -11 \\ 0 & -11/3 & -22/3 \end{bmatrix}$$

$R_3 \to R_3 - R_2/3$

$$A = \begin{bmatrix} 3 & -4 & -5 \\ 0 & -11 & -11 \\ 0 & 0 & -11/3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 \\ 0 & -11 & -11 \\ 0 & 0 & -11/3 \end{bmatrix}$$

$R_1 = R_1 - 4/11 R_2$, $R_2 = R_2 - 3R_1$

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & -11 & 0 \\ 0 & 0 & -11/3 \end{bmatrix}$$

$R_1 \to R_1 + 3/11 R_3$

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11/3 \end{bmatrix}$$

$\Rightarrow$ The matrix is converted to upper triangular matrix

Q3] Given PAQ form

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

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$A = I_3 \, A \, I_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 = R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 = C_2 - C_1, \quad C_3 = C_3 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 = C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3/2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1/2 & 1/2 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$I_3 = P \quad A \quad Q$

Rank of $A \Rightarrow \rho(A) = 3$

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Q4

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_3 = R_3 + 5R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

~~$R_1 \to R_1 - 2R_2$~~ $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -5 & 2 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix} A$$

$R_3 \to R_3/2$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -5 & 2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} A$$

$R_1 = R_1 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} A$$

$$I = A^{-1}A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

5) $$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Cofactor of matrix

$A_{11} = \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = 1$ $\quad A_{13} = \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -2$

$A_{21} = -\begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -1$

$A_{12} = -\begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = -2$

$A_{22} = \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3$ $\quad A_{23} = -\begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} = 3$

$A_{31} = \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = 0$ $\quad A_{31} = -\begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = -4$

$A_{33} = \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = -3$

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Cofactor matrix $= \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$

$Adj(A) =$ Transpose of Cofactor matrix $= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -1 \\ -2 & 3 & 3 \end{bmatrix}$ — (1)

ii) $TP = A^3 = A^{-1}$

$A = \begin{bmatrix} 3 & -2 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-6+0 & -9+9-4 & 12-12+4 \\ 6-6+0 & -6+9-4 & 8-12+4 \\ 0-2+0 & 3-1 & 0-4+1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & -2 & 3 \end{bmatrix}$

$A^3 = A \cdot A^2$

$= \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ ← (2)

$\Rightarrow$ From (1) & (2) $A^{-1} = A^3$ Hence proved