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Tutorial - 3

Relations

(1). If R is relation from $A = \{1, 2, 3, 4\}$ to $B = \{2, 3, 4, 5\}$. List elements in R defined by aRb if a & b both are odd. Also, write the domain, range of R.

$$R = \{(1,3), (1,5), (3,3), (3,5)\}$$

Domain $= \{1, 3\}$

Range $= \{3, 5\}$

(2) If R is a relation from $A = \{1, 2, 3\}$ to $B = \{4, 5\}$ given by $R = \{(1,4), (2,4), (1,5), (3,5)\}$. Find R^{-1}

if $R = (a, b)$ from $A \rightarrow B$
then $R^{-1} = (b, a)$ from $B \rightarrow A$

$$R^{-1} = \{(4,1), (4,2), (5,1), (5,3)\}$$

(3). Give an example of a relation that is both symmetric and anti-symmentric

Since for symmetric relation →
For any $(x, y) \in R$, $(y, x) \in R$ to be TRUE
For anti symmetric relation →
For $(x, y) \in R \rightarrow (y, x) \notin R$ unless $x = y$

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⇒ Equality relation $x = y$ on set of real numbers is both symmetric and antisymmetric

→ $A = \{x : x \in R\}$

~~$R = \{(x,y) : (x=y)\}$~~

$R = aRa = \{(x, x) : \forall x \in A\}$

is an example of both symmetric and anti-symmetric function

(4). Give an example of ~~fur~~ relation that is neither symmetric nor anti symmetric

A relation can contain an element (a, b) while it doesnt contain (b, a)

→ Not symmetric

and it can contain (c, d) while it also contains (d, c)

→ Not antisymmetric

Eg: $R = \{(1, 2), (2, 1), (3, 5)\}$

is neither symmetric ~~or~~ nor anti symmetric

(5). Give an example of relation that is reflexive and symmetric but not transitive

Example: $R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$ on $P = \{1, 2, 3\}$

Since $(x, x) \in R, \forall x \in P$ → reflexive

$(x, y) \in R \Rightarrow (y, x) \in R \; \forall x, y \in P$ → symmetry

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Since $(3,2) \in R$
$(2,1) \in R$

$\Rightarrow$ Since $(3,1) \notin R$ it is <u>not transitive</u>

6. Give an example of relation that is reflexive and transitive but not symmetric.

$R = \{(5,5), (6,6), (7,7), (5,6), (6,7)\}$
on $S = \{5,6,7\}$

R is reflexive as $(s,s) \in R \ \forall s \in S$

R is transitive

If $(5,5) \in R$, $(5,6) \in R$
$\Rightarrow (5,6) \in R$

$(6,6) \in R$, $(6,7) \in R$
$\Rightarrow (6,7) \in R$

$\Rightarrow$ So R is transitive as
if $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R \ \forall (a,b,c) \in S$

But R is not symmetric
$(5,6) \in R$ but $(6,5) \notin R$
So not symmetric

7. Give an example of relation that is symmetric and transitive but not reflexive

~~$R = \{(2,3), (3,4), (2,4)\}$~~

$R = \{(2,3), (3,2), (2,2)\}$
on $A = \{2,3\}$

Since $(3,3) \notin R$ it is not reflexive

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but R is symmetric
as $(2,3) \in R$ so $(3,2) \in R$
and R is transitive
as $(2,3) \in R$ and $(3,2) \in R$
$\Rightarrow (2,2) \in R$

(8). If $R_1 = \{(1,2), (2,3), (3,4)\}$ &
$R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1)\}$
be the relations from $\{1,2,3\}$ to $\{1,2,3,4\}$ then find

i) $R_1 \cup R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,4)\}$

ii) $R_1 \cap R_2 = \{(1,2), (2,3)\}$

iii) $R_1 - R_2 = \{(3,4)\}$

iv) $R_2 - R_1 = \{(1,1), (2,1), (2,2), (3,1)\}$

(9). If $R = \{(x, x^2)\}$ & $S = \{(x, 2x)\}$ where x is not negative integer find.

i) $R \cap S = \{(0,0), (2,4)\}$

ii) $R \cup S = \{(0,0), (1,1), (1,2), (2,4), (3,6), (3,9), (4,8), (4,16) \ldots\}$
$\rightarrow a (R \cup S) b$: $a \in x$
$b \in x^2 \cup 2x$

iii) $R - S = \{(1,1), (3,9), (4,16) \ldots\}$
$= R - \{(0,0), (2,4)\}$

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iv) $S - R = \{(1,2),(3,6),(4,8)\cdots\}$

$= S - \{(0,0),(2,4)\}$

(10) If relations R_1, R_2, R_3, R_4, R_5 is defined on set of real numbers as given below

i) $R_1 = \{(a,b) \mid a \geq b\}$
ii) $R_2 = \{(a,b) \mid a < b\}$
iii) $R_3 = \{(a,b) \mid a \leq b\}$
iv) $R_4 = \{(a,b) \mid a = b\}$
v) $R_5 = \{(a,b) \mid a \neq b\}$

then find

a) $R_2 \cup R_5$ b) $R_3 \cap R_5$ c) $R_2 - R_1$ d) ~~$R_1 - R_2$~~ $R_1 \cdot R_2$
e) $R_2 \cdot R_3$ ~~f) $R_1(R_2)$~~

a) $R_2 \cup R_5 = \{(a,b) \mid a < b\} \cup \{(a,b) \mid a \neq b\}$
$= \{(a,b) \mid a < b\}$

$= R_5$

b) $R_3 \cap R_5 = \{(a,b) \mid a \leq b\} \cap \{(a,b) \mid a \neq b\}$
$= \{(a,b) \mid a < b\}$

c) $R_2 - R_1 = R_2$ as $R_2 \cap R_1 = \phi$

d) ~~$R_1 R_2$~~ $R_1 \cdot (R_2) = \{(a,b) \mid a \geq b\} \cdot \{(a,b) \mid a < b\}$
$= R$

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e) $R_2 \cdot R_3 = \{(a,b) \mid a<b\} \cdot \{(a,b) \mid a \leq b\}$
$= R_3$

f) Q not visible

(11). If R, S, T be relations on set $A = \{0,1,2,3\}$ defined by
$R = \{(a,b) \mid a+b=3\}$, $S = \{(a,b) \mid 3$ is divisible by $a+b\}$
$T = \{(a,b) \mid \max(a,b) = 3\}$

i) R.T   ii) T.R   c) S.S

$R = \{(0,3), (1,2), (2,1), (3,0)\}$

$S = \{(0,1), (1,2), (2,1), (1,0), (0,3), (3,0)\}$

$T = \{(0,3), (1,3), (2,3), (3,2), (3,1), (3,0)\}$

i) $R.T = \{(0,0), (0,1), (0,2), (1,3), (2,3), (3,3)\}$

ii) $T.R = \{(0,0), (1,0), (2,0), (3,1), (3,2), (3,3)\}$

iii) $S.S = \{(0,0), (1,1), (2,2), (3,1), (0,2), (1,3), (3,3)\}$

(12) Determine whether relation R on the set of all integers is reflexive, symmetric, anti-symmetric or transitive where aRb iff

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~~a) a ≠ b~~

(13) - Verify the following are equivalence relation or not

i) R on set of real numbers
aRb iff $a-b$ is integer

✓ Reflexive $a-a=0, \forall a \in R$
and $0 \in Z$

✓ Symmetric $a-b = -(b-a) \; \forall (a,b) \in R$
both $\in Z$

✓ Transitive $(a,b) \in R, (b,c) \in R$
$a-b \in Z \quad b-c \in Z$
$(a-c) \in Z$

$\Rightarrow$ Equivalence relation

(14) If R is relation on set of integers such that $(a,b) \in R$ iff $b = a^n$ for some +ve integer n
Show R is partial order relation.

A partial order relation - Reflexive, antisymmetric
Transitive

For $n=1$ $(a,b) \to b = a$ when $n=1$

At $n=1$ $(a,b) \in R \Rightarrow (b,a) \in R$ when $a=b$ i.e $n=1$

At $n \neq 1$ $(a,b) \in R \Rightarrow (b,a) \notin R$

$\Rightarrow$ Anti - symmetric

$b = a^n$ $\quad \cancel{b = a^x}$ $\quad b = a^x$

$\cancel{a^n = a^x}$ $\quad a^n = a^x$

But since $x \in Z$, $\Rightarrow x = 1/n$ not possible

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✓ So x is anti-symmetric,

✓ Reflexive $(a,a) \in R$ as $a = a^1$ $\underline{n=1}$

$a,b)$

✓ Transitive $(a,b) \in R$, $(b,c) \in R$

$b = a^n$, $c = b^n$

$\Rightarrow (a^n)^n = c$

$a^{n^2} = c$

$\Rightarrow (a,c) \in R$

→ Reflexive, anti-symmetric, transitive

$\Rightarrow$ Partial order relation

15. Draw Diagraph for relation R on A

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

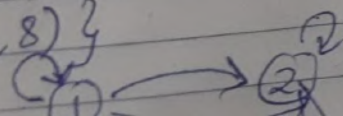
Let xRy when y is divisible by x.

Is R equivalence? Is R partial ordering?

$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8),$
$(2,2), (2,4), (2,6), (2,8),$
$(3,3), (3,6)$
$(4,4)$ $(4,8)$
$(5,5)$
$(6,6)$
$(7,7)$
$(8,8)\}$

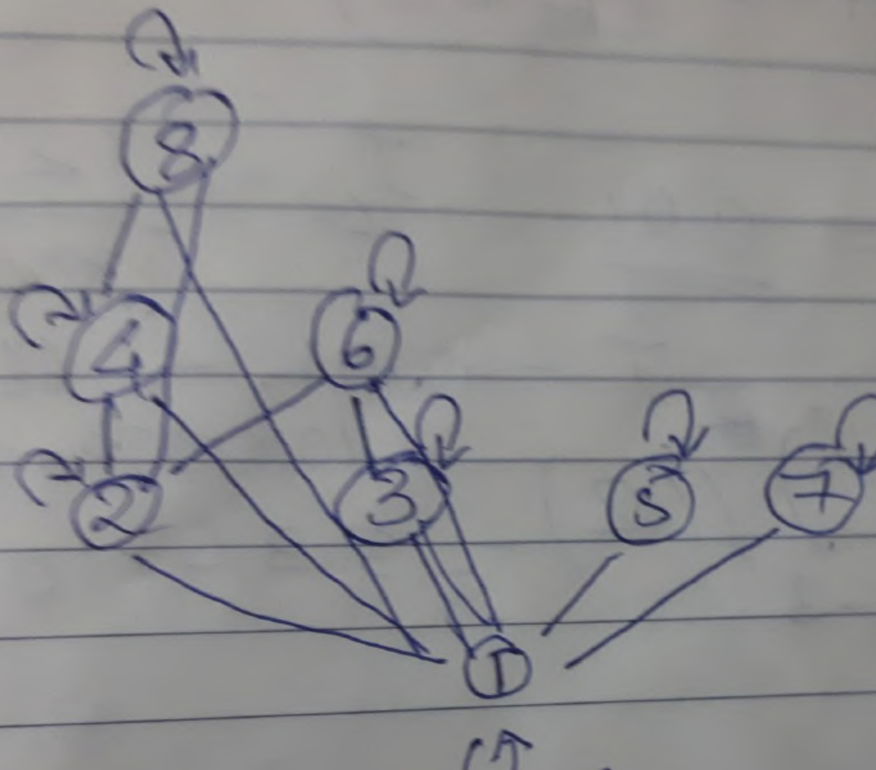
R is not symmetric

R is anti symmetric



R is not equivalence

R is not equivalence
R is anti symmetric



Digraph

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12. a) $a \neq b$

Not Reflexive

as $(a, a) \notin R$

Symmetric

$\to (a, b) \in R \Rightarrow (b, a) \in R$

as $a \neq b$ then $b \neq a$

Transitive

If $(a, b) \in R$, $(b, c) \in R$

$a \neq b$, $b \neq c$

$\Rightarrow a \neq c$

$(a, c) \in R$

b) $a \cdot b \geq 0$

$(a, a) \in R \quad \forall a \in R$

If a is +ve $\quad a \cdot a = a^2 \geq 0$

a is −ve $\quad (-a) \cdot (-a) = +a^2 \geq 0$

Reflexive

Symmetric

If $(a, b) \in R \Rightarrow a \cdot b \geq 0$

$= b \cdot a \geq a \Rightarrow (b, a) \in R$

Transitive

If $(a, b) \in R$, $(b, c) \in R$

$ab \geq 0 \qquad bc \geq 0$

$\Rightarrow ac \geq 0$

$(a, c) \in R$

c) $ab \geq 1$

as $(0, 0) \notin R$

Not reflexive

Symmetric $(a, b) \in R \Rightarrow ab \geq 1 \Rightarrow ba \geq 1$

$\Rightarrow (b, a) \in R$

Transitive $(a, b) \in R \Rightarrow ab \geq 1$, $(b, c) \in R \Rightarrow bc \geq 1$

$\Rightarrow ac \geq 1 \Rightarrow (a, c) \in R$

d) a is a multiple of b

$a = \lambda b$

✓ Reflexive as $(a, a) \in R \quad \forall a \in R$

$a = 1a$

✗ Not Symmetric ✓ Antisymmetric

if $(a,b) \in R \Rightarrow a = \lambda b$

so $(b,a) \notin R$ unless $a=b$

✓ Transitivity

if $(a,b) \in R \rightarrow a = \lambda b$

$(b,c) \in R \rightarrow b = \lambda c$

$\Rightarrow a = \lambda(\lambda c)$

$\Rightarrow$ is divisible

$(a,c) \in R$

e) $|a-b| = 1$

✗ Not reflexive as $a - a = 0$

If $(a,b) \in R \Rightarrow a-b=1 \Rightarrow b-a=-1$

$\Rightarrow |b-a| = 1 \Rightarrow (b,a) \in R$

✓ Symmetric

If $(a,b) \in R, (b,c) \in R \Rightarrow |a-b| = 1, |b-c| = 1 \Rightarrow a-(1+c) = 0$

$a - c = -1$

$|a-c| = 1$

✓ Transitive

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f) $a = b^2$

$(a,a) \notin R \ \forall a \in Z$

✗ $\Rightarrow$ Not reflexive

if $(a,b) \in R \Rightarrow a = b^2$

$b = a^{1/2} \Rightarrow (b,a) \notin R$

✗ $\Rightarrow$ Not symmetric

if $(a,b) \in R, (b,c) \in R \Rightarrow a = b^2, b = c^2$

$a = (c^2)^2 = c^4$

$\Rightarrow (a,c) \notin R$

✗ $\Rightarrow$ Not transitive

g) $a \geq b$

$(a,a) \in R$ as $a = a \ \forall a \in Z$

✓ $\Rightarrow$ Reflexive

if $(a,b) \in R \Rightarrow a \geq b \Rightarrow (b,a) \notin R$ unless $a = b$

✗ Not symmetric ✓ ANTI-SYMMETRIC

if $(a,b) \in R$, $(b,c) \in R$

$a \geq b$ $b \geq c$

$\Rightarrow a \geq c \Rightarrow (a,c) \in R$

✓ Transitive