U19CS076

Linear Algebra and Statistical Analysis

Tutorial - 2

Q1] Test for consistensy and solve them

$x - 2y + 3t = 2$
$2x + y + z + t = -4$
$4x - 3y + z + 7t = 8$

Augmented matrix. (A|B)

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 2 \\ 2 & 1 & 1 & 1 & -4 \\ 4 & -3 & 1 & 7 & 8 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 2 \\ 0 & 5 & 1 & -5 & -8 \\ 0 & 5 & 1 & -5 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_3$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 & -8 \\ 0 & 5 & 1 & -5 & 0 \end{array}\right]$$

$\rho(A|B) \neq \rho(A)$:- It is incositistent

$\Rightarrow$ NO Solution

Q2] Solve

(i) Augmented matrix $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 3 & 4 & 4 & 0 \\ 7 & 10 & 12 & 0 \end{array}\right]$

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$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 7R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & -4 & -9 & 0 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$f(A|B) = f(A)$
$\Rightarrow 3$ unknowns
(Trivial solution

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 2y + 3z = 0$
$-2y - 5z = 0$
$z = 0$
$-2y = 0$
$y = 0$

$0 + 0 + x = 0$
$x = 0$

$x, y, z = 0$

ii] Augmented matrix

$$\left[\begin{array}{cccc|c} 4 & 2 & 1 & 3 & 0 \\ 6 & 3 & 4 & 7 & 0 \\ 2 & 1 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \to 4R_2 - 6R_1$, $R_3 \to 4R_3 - 2R_1$

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$$\left[\begin{array}{cccc|c} 4 & 2 & 1 & 3 & 0 \\ 0 & 0 & 10 & 10 & 0 \\ 0 & 0 & -2 & -2 & 0 \end{array}\right]$$

$R_3 \to 5R_3 + R_2$

$$\left[\begin{array}{cccc|c} 4 & 2 & 1 & 3 & 0 \\ 0 & 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A|B) = \rho(A) = 2 \to$ but 4 unknowns

$\Rightarrow$ Non trivial solution

$$4x + 2y + z + 3w = 0$$
$$10(z+w) = 0$$
$$4x + 2y + 2z + \boxed{z+w} = 0$$

$$2x + y + z = 0$$
$$y = -2x - w, \quad z = -w$$

3. For what value of k the eqn have a solution. Solve completely of each case.

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

The augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & K \\ 4 & 1 & 10 & K^2 \end{array}\right]$$

$R_2 \to R_2 - 2R_1, \; R_3 \to R_3 - 4R_1$

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$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & k-2 \\ 0 & -3 & 6 & k^2-4 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & k^2-3k+2 \end{array}\right]$$

① If $k^2 - 3k + 2 = 2 \to$ infinite solution
then $f(A/B) = f(A) < n = 3$

$k^2 - 3k + 2$
$\Rightarrow k = 1, k = 2$
$x + y + z = 1$
$-y + 2z = k - 2$

$k = 2$	$k = 1$
$2z - y = 0$	$2z - y = -1$
$z = y/2$	$y = 2z + 1$
$x = 1 - 2z - z$	$x = 1 - z - 2z - 1$
$= 1 - 3z$	$x = -3z$

② If $k^2 - 3k + 2 \neq 0$ $f(A/B) \neq f(A)$
$\Rightarrow$ NO Solution

Q4] Test for consistency of solve

Augmented matrix $\to$ $\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 7 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$

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$R_3 \to R_3 - R_1$, $R_2 \to 2R_2 - 3R_1$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 23 & -27 & 11 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 23 & -27 & 11 \\ 0 & -24 & 0 & 5 \end{array}\right]$$

$\rho(A|B) = 3 = \rho(A)$

$n = 3$ (unique solution)

$2x - 3y + 7z = 5$

$23y - 27z = 11$

$-24y = 5 \quad \to \quad \boxed{y = -5/24}$

$$z = \frac{11 - 23(-5/24)}{-27} = -\frac{379}{648}$$

$\boxed{z = -379/648}$

$$x = \frac{5 - 7\left(\frac{-379}{648}\right) + 3\left(\frac{-5}{24}\right)}{2}$$

$$\boxed{x = \frac{343}{81}}$$

$$\Rightarrow x = \frac{343}{81}, \; y = \frac{-5}{24}, \; z = \frac{-379}{648}$$

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ii)

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \\ 3 & -5 & 5 & 2 \\ 3 & 9 & -1 & 4 \end{array}\right] \rightarrow \text{Augmented matrix}$$

$R_2 \rightarrow R_2 - 2R_1$; $R_3 \rightarrow R_3 - 3R_1$; $R_4 = R_4 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & -11 & 2 & -7 \\ 0 & 3 & -4 & -5 \end{array}\right]$$

$R_3 \rightarrow R_3 - 11R_2$; $R_4 \rightarrow R_4 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -0 & 0 \end{array}\right]$$

$f(A|B) = f(A) = 3 \rightarrow$ No. of unknown $= 3$

$z = 2$

$-y = -1 \rightarrow y = 1$

$x + 2y + z = 3$

$x = 3 - 2 - 2$

$x = -1$

$\Rightarrow x = -1, y = 1, z = 2$

Q5] Augmented matrix → (for finding values of a & b)

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 3 \\ 1 & 2 & 2 & b \\ 1 & 5 & 3 & 9 \end{array}\right]$$

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$R_3 \to R_3 - 3R_1$, $R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 3 \\ -1 & 2-2a & 0 & b-6 \\ -2 & 5-3a & 0 & 0 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 3 \\ -1 & 2-2a & 0 & b-6 \\ 0 & 1+a & 0 & -2(b-6) \end{array}\right]$$

The equation is consistent only when,

1. $a+1 \neq 0$
   $a \neq -1$] – For unique solutions

→ The equations are consistent when $a \neq -1$, $b =$ any value

Q6] Find value of λ for which equations are consistent.
Find ~~valu~~ ratio $x:y:z$ where λ has smallest of values. What happens when λ has the greatest values.

Writing the given equations in determinant form

$$\begin{bmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & \lambda-1 \end{bmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 0 & -\lambda+3 & \lambda-3 \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{bmatrix} = 0$$

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$R_2 \to R_2 - \lambda_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 0 & -\lambda+3 & \lambda-3 \\ \lambda-1 & 5\lambda-5 & 6 \\ 2 & 4\lambda-2 & 2\lambda \end{bmatrix} = 0$$

$C_2 \to C_2 + C_3$

$$\begin{bmatrix} 0 & 0 & \lambda-3 \\ \lambda-1 & 5\lambda+1 & 6 \\ 2 & 6\lambda-2 & 2\lambda \end{bmatrix} = 0$$

$\Delta = 0$ [For consistency $\to$ true]

$(\lambda-3)[(\lambda-1)(6\lambda-2) - (5\lambda+1)2] = 0$

$(\lambda-3)[6\lambda^2 - 2\lambda - 6\lambda + 2 - (10\lambda+2)] = 0$

$(\lambda-3)[6\lambda^2 - 18\lambda] = 0$

$\hookrightarrow \lambda = 3$ $\quad \hookrightarrow 6\lambda(\lambda-3) = 0$

$\lambda = 3, 0$

$\Rightarrow \lambda = 3, \lambda = 0$

For minimum value of λ, $\lambda = 0$

$-x + y = 0$, $\boxed{x = y}$

$-x - 2y + 3z = 0 \to -x - 2x + 3z = 0$

$= \boxed{x = z}$

$2x + y - 3z = 0$ $\Rightarrow 2y + y - 3z = 0$

$\Rightarrow \boxed{z = y}$

$\Rightarrow \boxed{x : y : z = 1:1:1}$

$\hookrightarrow$ Inconsistent NO Solution

For maximum value of λ, $\lambda = 3$

$2x + 10y + 6z = 0$, $2x + 10y + 6z = 0$

$2x + 10y + 6z = 0$

For all eq. are same