U19CS076

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Tutorial - 3

1] (1,6), (2, 4), (3,3), (4,5), (5,4), (6, 2)

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 3 \\ 5 \\ 4 \\ 2 \end{bmatrix} \quad ; (mx + b = y)$$

$$\therefore Ax^* = y$$
$$AA^T x^* = A^T y$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} x^* = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 3 \\ 5 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 91 & 21 \\ 21 & 6 \end{bmatrix} \begin{bmatrix} m^* \\ b^* \end{bmatrix} = \begin{bmatrix} 75 \\ 24 \end{bmatrix}$$

$$\therefore 91m^* + 21b^* = 75 \quad \times 3$$
$$21m^* + 6b^* = 24 \quad \times 13$$

$$\therefore 273m^* + 63b^* = 225$$
$$273m^* + 78b^* = 312$$

$$15b^* = 87$$
$$b^* = 87/15$$

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$$21m^* + 8\left(\frac{87}{15}\right) = 24$$

$$105m^* + 174 = 120$$

$$\boxed{m^* = -18/35}$$

$\therefore y = mx + b$

i) $x = 4$ $\quad x=4$

$$\Rightarrow y = \left(-18/35\right)x + \left(87/15\right)$$

$$x = 3.743$$

ii) $y = 4$

$\therefore y = mx + b$

$$4 = \left(-18/35\right)x + \left(87/15\right)$$

$$\Rightarrow 18/35\, x = 27/15$$

$$x = \frac{27 \times 35}{15 \times 18}$$

$$x = 7/2 = 3.5$$

for (1,6) $\quad y = mx + b$

$$y = \left(-18/35\right)1 + \left(87/15\right)$$

$$y^* = 5.2857$$

for (2,4) $\quad y = \left(-18/35\right)2 + \left(87/15\right)$

$$y^* = 4.7714$$

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For (3,3) $\hat{y} = (-18/35)3 + (87/15)$

$\therefore \hat{y} = 4.25714$

For (4,5) $\hat{y} = (-18/35)4 + (87/15)$

$\hat{y} = 3.74286$

For (5,4) $\hat{y} = (-18/35)5 + (87/15)$

$\therefore \hat{y} = 3.2286$

For (6,2)

$\hat{y} = (-18/35)6 + (87/15)$

$\hat{y} = 2.7143$

$\Rightarrow$

x	y	$\hat{y}$	$\hat{y}-y$	$(\hat{y}-y)^2$
1	6	5.28	-0.72	0.5184
2	4	4.77	0.77	0.5929
3	3	4.28	1.28	1.6384
4	5	3.74	-1.26	1.5876
5	4	3.23	-0.77	0.5929
6	2	2.71	0.71	0.5041

Standard Error of estimate $= \sqrt{\dfrac{\Sigma(\hat{y}-y)^2}{n-2}}$

$= \sqrt{\dfrac{5.4343}{\cancel{4}\ 6}} = \cancel{2.341}$ 0.9461

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Q2]

x	y	u	v	u^2	v^2	uv
56	147	9	19	81	361	171
42	125	-5	-3	25	9	15
72	160	25	32	625	1024	800
36	118	-11	-10	121	100	110
63	149	16	21	256	441	336
47	128	0	0	0	0	0
55	150	8	22	64	484	176
49	145	2	27	4	729	54
38	115	-9	-13	81	169	117
42	140	-5	12	25	144	-60
68	152	21	24	441	576	504
60	165	13	27	169	729	351
		64	158	1892	4766	2574

N = 8

$$r = \frac{\Sigma uv - \frac{1}{N}\Sigma u \Sigma v}{\sqrt{\left[\Sigma u^2 - \frac{1}{N}(\Sigma u)^2\right]\left\{\Sigma v^2 - \frac{1}{N}(\Sigma v)^2\right\}}}$$

$$= \frac{2574 - 1/8\,(64)(158)}{\sqrt{(1892) - \frac{1}{8}(64)^2\left[4766 - \frac{1}{8}(158)^2\right]}}$$

$$= \frac{1310}{\sqrt{(1380)(3120.5)}} = \frac{1310}{2075.16}$$

~~r = 0.631277~~ $r = 0.8964$

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Q3] pH 7.15 7.20 7.18 7.19 7.21 7.20 7.16 7.18

$$\text{Mean } (\bar{x}) = \frac{\Sigma pH}{n} = \frac{57.47}{8} = 7.18$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
7.15	−0.03	0.0009
7.20	0.02	0.0004
7.18	0	0
7.19	0.01	0.0001
7.21	0.03	0.0009
7.20	0.02	0.0004
7.16	− 0.02	0.0004
7.18	0	0

$$S.D = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}} = \sqrt{\frac{0.0031}{8}}$$

$$= \underline{0.01968}$$

$$\text{Variance} = (SD)^2 = \underline{0.0003875}$$

Q4]

x	$x - \bar{x}$	$(x - \bar{x})^2$
116.8	−2	4
171.4	2.6	6.76
169.1	0.3	0.09
178.5	9.7	94.09
168	−0.8	0.64
157.9	−10.9	118.81
170.1	1.3	1.69

$$\bar{x} = \frac{\Sigma x}{n} = \frac{1181.8}{7} = 168.8$$

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Sample variance

$$S^2 = \frac{\Sigma(X-\bar{X})^2}{n-1} = \frac{226.08}{7-1} = 37.7$$

Sample standard deviation

$$S = \sqrt{S} = \sqrt{37.7} = 6.1$$

(5) If $X_i \sim \text{geometric}(\theta)$, then

$$P_{X_i}(x;\theta) = (1-\theta)^{x-1}\theta$$

Thus the likelihood function is given by

$$L(x_1, x_2, \ldots x_n; \theta) = P_{X_1 X_2 \ldots X_n}(x_1, x_2 \ldots x_n; \theta)$$

$$= P_{X_1}(x_1;\theta)\, P_{X_2}(x_2;\theta) \ldots P_{X_n}(x_n;\theta)$$

$$= (1-\theta)\left[\sum_{i=1}^{n} x_i - n\right]\theta^n$$

Then the log likelihood function is given by

$$\ln L(x_1, x_2 \ldots x_n; \theta) = \left(\sum_{i=1}^{n} x_i - n\right)\ln(1-\theta) + n\ln\theta$$

$$\frac{d \ln L(x_1, x_2 \ldots x_n; \theta)}{d\theta} = \left[\sum_{i=1}^{n} x_i - n\right] \cdot \frac{-1}{1-\theta} + \frac{n}{\theta}$$

By setting the derivative to zero, we can check that the maximum value of θ is given by

$$\hat{\theta}_{ML} = \frac{n}{\sum_{i=1}^{n} x_i}$$

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MLE can be written as $\hat{\theta}_{ML} = \frac{n}{\sum_{i=1}^{n} X_i}$

(6). If $X_i \sim$ uniform $(0, \theta)$ then

$$\cancel{f} \; f_X(x) = \begin{cases} 1/\theta & 0 \le x \le \theta \\ 0 & \text{otherwise} \end{cases}$$

The likelihood function is given by

$$L(x_1, x_2 \cdots x_n; \theta) = f_{X_1 X_2 \cdots X_n}(x_1, x_2 \cdots x_n; \theta)$$

$$= f_{X_1}(x_i; \theta) \, f_{X_2}(x_2; \theta) \cdots$$

$$\cdots f_{X_n}(x_n; \theta)$$

$$= \begin{cases} 1/\theta^n & 0 \le x_1, x_2, \cdots x_n \le \theta \\ 0 & \text{otherwise} \end{cases}$$

Note that $1/\theta^n$ is decresing function of θ.

Thus to maximize it we need to choose the smallest possible value for θ for $i = 1, 2 \cdots n$ we need to have $\theta \ge x_i$. Thus the smallest ~~pr~~ possible value for θ is

$$\hat{\theta}_{ML} = \max(X_1, X_2 \cdots X_n)$$

Therefore MLF can be written as

$$\hat{\theta}_{ML} = \max(X_1, X_2, \cdots X_n)$$

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⑦. Example 1

$n_1 = 40$

$\bar{x}_1 = 647$ hrs

$s_1 = 31$ hrs

Example 2

$n_2 = 45$

$\bar{x}_2 = 742$ hrs

$s_2 = 29$ hrs

ii) 95% confidence level

$\bar{x}_1 - \bar{x}_2 = 647 - 742 = -95$

$\alpha = 1 - 95/100 = 0.05$

$\alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$$\text{interval} = (\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= -95 \pm 1.96 \sqrt{\frac{(31)^2}{40} + \frac{(29)^2}{45}}$$

$$= -95 \pm 1.96 \times 6.535$$

$$= -95 \pm 12.8086$$

$$\Rightarrow (-107.8086, -82.1914)$$

ii) 99% confidence level

$\bar{x}_1 - \bar{x}_2 = -95$

$\alpha = 1 - 99/100 = 0.01$

$\alpha/2 = 0.005$

$Z_{\alpha/2} = 2.57$

$$\text{Interval} = (\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2} \sqrt{\frac{x_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= -95 \pm 2.57 \sqrt{\frac{31^2}{40} + \frac{29^3}{45}}$$

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$= -95 \pm 2.57 \sqrt{24.025 + 18.688}$

$= -95 \pm 2.57 \times 6.535$

$= -95 \pm 16.794$

Interval : $(-111.794, -78.205)$

(8). We need to find a 95% ($= 100(1-\alpha)\%$) confidence interval for μ_0.

$\alpha = 1 - 95/100$

$\alpha = 0.05$

$\alpha/2 = 0.025$

In order to find required confidence interval, we will be use following theorem.

If $\bar{d}$ and s_d are the mean and standard deviation, respectively of the normally distributed differences of n random pairs of measurement a

$100(1-\alpha)\%$

confidence interval for $\mu_0 = \mu_1 - \mu_2$ is

$$\bar{d} - t_{\alpha/2} \frac{s_d}{\sqrt{n}} \le \mu_d < \bar{d} + t_{\alpha/2} \frac{s_d}{\sqrt{n}} \quad \text{--- (1)}$$

From given data we have

$d_1 = 38 - 45 = -7$

$d_2 = 23 - 25 = -2$

$d_3 = 35 - 31 = 4$

$d_4 = 41 - 38 = 3$

$d_5 = 44 - 50 = -6$

$d_6 = 29 - 33 = -4$

$d_7 = 37 - 36 = 1$

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$d8 = 31 - 40 = -9$

$d9 = 38 - 43 = -5$

Hence, $\bar{d} = \frac{1}{n} \Sigma d_i$

$= \frac{1}{9}(-7 - 2 + 4 + 3 - 6 - 4 + 1 - 9 - 5)$

$= \frac{-25}{9}$

$= -2.778$

$s_d = \sqrt{\frac{1}{n-1} \Sigma (d_i - \bar{d})^2}$

$= \sqrt{\frac{1}{9-1} \left((-7+2.7778)^2 + (-2+2.7778)^2 + \cdots \right)}$

$= \sqrt{20.9444}$

$= 4.5765$

$t_{\alpha/2} = t_{0.002} = 2.31$

$\bar{d} - t_{\alpha/2} \frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{\alpha/2} \frac{s_d}{\sqrt{n}}$

$-2.7778 - 2.31\left(\frac{4.5765}{\sqrt{9}}\right) < \mu_d < -2.7778 + 2.31\left(\frac{4.5765}{\sqrt{9}}\right)$

$-4.3217 < \mu_d < 0.7461$

9. Sample $(n_1) = 10$

$\bar{x} = 230$ calories

$s = 15$ calories

99% confidence interval for of σ^2 (variance)

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Given data :-

$x^2_{\alpha/2} = 21.66$, $x^2_{1-\alpha/2} = 2.088$ a 2 9d.of)

$\alpha = 1 - 99/1000$

$\alpha = 0.001$

$\alpha/2 = 0.005$, $x^2_{\alpha/2} = 21.666$

$x^2_{1-\alpha/2} = 2.088$

Interval

$$\frac{(n-1)s^2}{x^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{x^2_{1-\alpha/2}}$$

$$\frac{(10-1)225}{21.666} \le \sigma^2 \le \frac{(10-1)225}{2.088}$$

$$\boxed{93.464 \le \sigma^2 \le 969.33}$$