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U19CS076

Maths Tutorial - 2

Q1] $\gcd(306, 657) = ?$

Let assume $\gcd(306, 657) = t$

Euclidian Algo

$657 = 306 \times 2 + 45$ — ①

$306 = 45 \times 6 + 36$ — ②

$45 = \cancel{45}\ 36 \times 1 + 9$ — ③

$36 = 9 \times 4 + 0$ — ④

by eq ④ $\gcd(657, 306) = t = \gcd(306, 657) =$

$\gcd(272, 1479)$

let assume $\gcd(1479, 272) = t$

Euclidian Algo :-

$1479 = 272 \times 5 + 119$ — ①

$272 = 119 \times 2 + 34$ — ②

$119 = 34 \times 3 + 17$ — ③

$34 = 17 \times 2 + 0$ — ④

by eqⁿ ④

$\gcd(1479, 272) = \gcd(272, 1479) = t =$

Q2]a) $\gcd(56, 72) = 56x\ 72y$

Euclidian Algo :-

$72 = 56 \times 1 + 16$

$56 = 16 \times 3 + 8$

$16 = 8 \times 2 + 0$

Here $\gcd(56, 72) = 8$

$8 = 56 - 3 \times (16)$

$= 56 - 3(72 - 56)$

$= 56 - 3 \times 72 + 3 \times 56$

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$\cancel{56}$

$gcd(56,72) = 4\times56 - 3\times72$
$= 56x + 72y$

$x = 4, y = -3$

b) $gcd(1769, 2378) = 1769 + 2378y$

Euclidian Algo :-

$2378 = 1769 \times 1 + 609$
$\cancel{16}\ 1769 = 609 \times 2 + 551$
$609 = \cancel{551\times} \ 551 \times 1 + 58$
$551 = 58 \times 9 + 29$
$58 = 29 \times 2 + 0$

$gcd(1769, 2378) = 29$

$29 = 551 - 58 \times 9$
$= 551 - 9(609 - 551)$
$= 551 - 9 \times 609 + 551 \times 9$
$= 10 \times 551 - 9 * 609$
$= 10 \times 551 - 9 * 609$
$= 10(1769 - 609 \times 2) - 9 \times 609$
$= 10 * 1769 - 20 * 609 - 609 * 9$
$= 10 \times 1769 - \cancel{20}\ 29 \times 609$
$= 10 * 1769 - 29(2378 - 1769)$

$gcd(1769, 2378) = 29 = 39 \times 1769 - 29 * 2378$
$gcd(1769, 2378) = 1769x + 2378$

$x = 39, y = -21$

3] Given that :- $gcd(a,b) = d$
Euclidian algo $\rightarrow d = ax + by$ — (1)

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Now dividing d both sides

$$\frac{d}{d} = x\left(\frac{a}{b}\right) + y\left(b/a\right)$$

$$1 = x\left(\frac{a}{b}\right) + y\left(b/d\right) \quad \text{— (2)}$$

Now from eq (2). we can say

$$\gcd\left(\frac{a}{d}, b/d\right) = 1$$

Q4] a) It is given that $GCD(a,b) = 1$

Let $GCD(a-b, a+b) = d$

$\Rightarrow$ d divides $a-b$ and $a+d$

There exists integers m and n such that

$$a+b = m*d \quad \ldots \text{(1)}$$

$$a-b = n*d \quad \ldots \text{(2)}$$

adding and subtracting eq (1), (2)

$$2a = (m+n)*d \quad \ldots \text{(3)}$$

$$2b = (m-n)*d \quad \ldots \text{(4)}$$

Since $\gcd(a,b) = 1$ given

$2 * GCD(a,b) = 2$

$\gcd(2a, 2b) = 2$ since ~~gcd (ka, kb) = k~~

$\gcd(ka, kb) = k \gcd(a,b)$

From eq (3), (4)

$\gcd((m+n) \times d, (m-n) \times d) = 2$

$d \times GCD((m+n), (m-n)) = 2$

d * some integer $= 2$

$\Rightarrow$ d divides 2

$d \leq 2$ if x divides y, then $|x| \leq |y|$

$d = 1$ or 2 since gcd is always a +ve integer.

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b) It is given that $GCD(a,b) = 1$ — (A)
Let $gcd(a+b, a^2-ab+b^2) = d$.
So $d \mid a+b$ — (1)
and $d \mid a^2 - ab + b^2$ — (2).

$a^2 - ab + b^2$ can be written as
$(a+b)^2 - 3ab$
$d \mid (a+b)^2 - 3ab$ — (3)
by eqn (1), $d \mid (a+b)^2$ and d should divide 3ad.
$d \mid -3ab$
by eqn (A) $GCD(a,b) = 1$ implies $d \nmid ab$,
thus $d \mid 3$
So the $gcd(a+b, a^2-ab+b^2) = 1$ or $\underline{\underline{3}}$

(5). It is given that $gcd(a,b) = 1$ — (A)
Let $gcd(a+b, ab) = d$
then $d \mid ab \Rightarrow$ (1)
$d \mid a+b \rightarrow$ (2).
From eqn (2) it shows that $d \mid a$ and $d \mid b$ at same time but from eq (A) $gcd(a,b) = 1$
So it is contribution so $d = 1$
then let $gcd(a+b, ab) = 1$

(6). Let $lcm(143, 227) = t$
then $t = \dfrac{143 \times 227}{GCD(143, 227)}$

$GCD(143, 227) \Rightarrow$
Euclidian :- $227 = 143 \times 1 + 84$
$143 = 84 \times 1 + 59$
$84 = 59 \times 1 + 25$
$59 = 25 \times 2 + 9$

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$25 = 9 \times 2 + 7$
$9 = 7 \times 1 + 2$
$7 = 2 \times 3 + 1$
$2 = 1 \times 2 + 0$

$GCD(143, 227) = 1$.

$lcm = t = \dfrac{143 \times 227}{1} = 32,461$

Let $lcm(306, 657) = x$

$\Rightarrow x = \dfrac{306 \times 657}{GCD(306, 657)}$

$GCD(306, 657)$

Euclidian :-

$657 = 306 \times 2 + 45$
$306 = 45 \times 6 + 36$
$45 = 36 \times 1 + 9$
$36 = 9 \times 4 + 0$

$GCD(306, 657) = 9$

$lcm \rightarrow x = \dfrac{306 \times 657}{9}$

$= 22,338$

7 a) $6x + 51y = 22$

$ax + by = c$

Diophantine eqn has solution when $d | c$ where

$d = GCD(a, b)$

$GCD(6, 51)$:-

$51 = 6 \times 8 + 3$
$6 = 3 \times 2 + 0$

$GCD(6, 51) = \underline{\underline{3}}$

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But 3 not divisible for 22
so eqn not solvable.

b) $33x + 14y = 115$
$ax + by = c$
Diophentine eqn has soln where $d|c$ where
$d = GCD(a,b)$
$GCD(33,14)$:-
~~GCD(6,5)~~ $33 = 14 \times 2 + 5$
$14 = 5 \times 2 + 4$
$5 = 4 \times 1 + 1$
$4 = 1 \times 4 + 0$
$GCD(33,14) = 1$
Here 1 divides 115 so eqn is solvable
$1 | 115$

(c) $14x + 35y = 93$
$ax + by = c$
Diaphentine eqn has soln when $d|c$ where
$d = gcd(a,b)$

$gcd(14,35)$
$35 = 14 \times 2 + 7$
$14 = 7 \times 2 + 0$
$GCD(35,14) = 7$
But 7 not divides 93 so eq. not solvable.

(8) a) $56x + 72y = 40.$
$ax + by = c$
$c = 40$, $a = 56$, $b = 72$
$gcd(56,72) \Rightarrow 8$
$8 | 40$ eqn solvable

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$\gcd(56, 72) \Rightarrow 8 = \cancel{16}\,56 - 16 \times 3$

$= \cancel{16}\,56 - 3(72 - 56)$

$= 56 - 3 \times 72 + 3 \times 56$

$8 = 4 \times 56 - 3 \times 72$

Multiplying by 5 both sides

$40 = 5 \times 4 \times 56 - 5 \times 3 \times 72$

$= 20 \times 56 - 15 \times 72$

$x = x_0 + (b/d)t, \quad y = y_0 - (a/d)t$

$x = 20 + (72/8)t, \quad y = -15 - \left(\frac{56}{8}\right)t$

$x = 20 + 9t \qquad y = -15 - 7t \qquad t \in \mathbb{Z}$

(b) ~~2y x~~ $24x + 138y = 18$

$ax + by = c$

$c = 18, \quad a = 24, \quad b = 138$

$\gcd(24, 138) \Rightarrow 138 = 24 \times 5 + 18$

$24 = 18 \times 1 + 6$

$18 = 6 \times 3 + 0$

$6 \mid 18$ so eqn has solution

$\gcd(24, 138) = 6 \Rightarrow 24 - 18$

$= 24 - (138 - 24 \times 5)$

$= 24 + 24 \times 5 - 138 \times 1$

$6 = 6 \times 24 - 138 \times 1$

Multiplying by 3 both the sides

$18 = 18 \times 24 - 3 \times 138$

$x = 20 + (b/d)t \qquad y = y_0 - (a/d)t$

$x_0 = 18 \qquad y_0 = -3$

$x = 18 + \left(\frac{138}{6}\right)t \qquad y = -3 - 4t$

$x = 18 + 23t, \quad y = -3 - 4t \qquad t \in \mathbb{Z}$

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(c) $221x + 35y = 11$

$ax + by = c$ $\quad c = 11$

$a = 221,\ b = 35$

$\gcd(221, 35) \Rightarrow 221 = 35 \times 6 + 11$

$35 = 11 \times 3 + 2$

$11 = 5 \times 2 + 1$

$2 = 2 \times 1 + 0$

$\gcd(221, 35) = 1$

$1 \mid 11 \Rightarrow$ so eqn has solution

$1 = 11 - 5 \times 2$

$= 11 - 5(35 - 11 \times 3)$

$= 11 - 5 \times 35 + 15 \times 11$

$= 16 \times 11 - 35 \times 5$

$= 16(221 - 35 \times 6) - 5 \times 35$

$1 = 16 \times 221 - 101 \times 35$

Multiplying 11 both sides

$11 = 176 \times 221 - 1111 \times 35$

$x = x_0 + (b/d)t$ $\quad y = y_0 - (a/d)t$

$= 176 + (35/1)t$ $\quad = -1111 - \left(\frac{221}{1}\right)t$

$x = 176 + 35t$ $\quad y = -1111 - 221t$

$t \in Z$

(9). a) $18x + 5y = 48$

$ax + by = c$

$c = 48 \quad a = 18 \quad b = 5$

$\gcd(18, 5) \Rightarrow 18 = 5 \times 3 + 3$

$5 = 3 \times 1 + 2$

$3 = 2 \times 1 + 1$

$\gcd(18, 5) = 1$

$1 \mid 48$ the eqn has solution

$1 = 3 - 2 \times 1$
$= 3 - (5 - 3)$
$= 3 - 5 + 3$
$= 3 \times 2 + 5$
$= 2(18 - 3 \times 5) - 5$
$= 18 \times 2 - 7 \times 5$

Multiplying 48 both sides

$48 = 96 \times 18 - 336 \times 5$

$x = x_0 + (b/d)t$ $y = y_0 - (a/d)t$
$x = 96 + (5/1)t$ $y = -336 - (18/1)t$

$x = 96 + 5t$, $y = -336 - 18t$

For positive value

$x > 0$ $y > 0$
$96 + 5t > 0$ $-336 - 18t > 0$
$5t > -96$ $336 + 18t < 0$
$t > -96/5$ $18t < -336$
$t < \frac{-336}{18}$

$t > -96/5$ $t < -56/3$

$-56/3 > t > -96/5$
$t = -19$

$x = 96 + 5 \times -19$, $y = -336 - 18 \times -19$
$x = 1$ $= -336 + 342$
$y = 6$

(b). $123x + 360y = 99$
$ax + by = c$
$c = 99$ $a = 123$ $b = 360$
$\gcd(123, 360) \Rightarrow 360 = 123 \times 2 + 114$

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$123 = 114 \times 1 + 9$

$114 = 9 \times 12 + 6$

$9 = 6 \times 1 + 3$

$6 = 3 \times 2 + 0$

gcd $(123, 360) = 3$

$3 | 99$ so eqn has solution

$3 = 9 - 6$

$= 9 - (114 - 9 \times 12)$

$= 13 \times 9 - 114$

$= 13(123 - 114) - 114$

$= 13 \times 123 - 14 \times 114$

$= 13 \times 123 - 14(360 - 123 \times 2)$

$3 = 41 \times 123 - 14 \times 360$

Multiplying 33 both the sides

$99 = 1353 \times 123 - 462 \times 360$

$x = x_0 + (b/d)t, \quad y = y_0 - (a/d)t$

$= 1353 + \left(\frac{360}{3}\right)t, \quad y = -412 - \left(\frac{123}{3}\right)t$

$= 1353 + 120t, \quad y = -412 - 41t$

For positive value integer

$x > 0$ $\qquad$ $y > 0$

$1353 + 120t > 0$ $\qquad$ $-462 - 41t > 0$

$t < \frac{-1353}{120}$ $\qquad$ $t < \frac{-462}{41}$

$\frac{-462}{41} > t > \frac{-1353}{120}$

No possible values of x and y

(C). $158x - 57y = 7$

$ax + by = c$

$c = 7, \quad a = 158, \quad b = -57$

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$\gcd(158, -57) \Rightarrow$

$$158 = 2 \times 57 + 44$$
$$57 = 1 \times 44 + 13$$
$$44 = 3 \times 13 + 5$$
$$13 = 2 \times 5 + 3$$
$$5 = 3 \times 1 + 2$$
$$3 = 2 \times 1 + 1$$
$$2 = 2 \times 1 + 0$$

$\gcd(158, -57) = 1$

$1 \mid 7 \Rightarrow$ The eqn has solution

$$1 = 3 - 2$$
$$= 3 - (5 - 3)$$
$$= 2 \cdot 3 - 5$$
$$= 2(13 - 2 \cdot 5) - 5$$
$$= 2 \cdot 13 - 5 \cdot 5$$
$$= 2 \cdot 13 - 5 \cdot (44 - 3 \cdot 13)$$
$$= 2 \cdot 13 - 5 \cdot 44 + 15 \cdot 13$$
$$= 17 \cdot 13 - 5 \cdot 44$$
$$= 17(57 - 44) - 5 \cdot 44$$
$$= 17(57) - 22 \cdot 44$$
$$= 17(57) - 22(158 - 2(57))$$
$$1 = 158(-22) + 57(61)$$
$$= -22(158) + (-57)(-61)$$

Multiplying 7 both sides

$$7 = (-154)(158) + (-57)(-427)$$

$x = x_0 + (b/d)t$ $\qquad$ $y = y_0 - (a/d)t$

$x_0 = -154$ $\qquad$ $y_0 = -427$

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$x = -154 - \left(\frac{57}{1}\right)t \quad y = -427 - \left(\frac{158}{1}\right)t$

$x = -154 - 57t, \quad y = -427 - 158t$

$x > 0$ $\qquad$ $y > 0$

$t < \frac{-154}{57}$ $\qquad$ $t < -\frac{427}{158}$

$t < -2.7018$ $\qquad$ $t < -2.7025$

$\Rightarrow t < -2.7025$

The soln (in +ve integers) at eqn
$158x - 57y = 7$ is given by

$x = -154 - 57t, \; y = -427 - 158t$
for $t \in \{\ldots, -6, -5, -4, -3\}$

(10). Let x be number of adults
and y be number of children

$x(1.80) + y(0.75) = 90$
$180x + 75y = 9000$
$ax + by = c$
$a = 180, \; b = 75, \; c = 9000$

$\gcd(180, 75) \Rightarrow 180 = 75 \times 2 + 30$
$75 = 30 \times 2 + 15$
$30 = 15 \times 2 + 0$

$\gcd(180, 75) = 15$
$15 = 75 - 30 \times 2$
$= 75 - 2(180 - 95 \times 2)$
$15 = 75 \times 5 - 180 \times 2$
$15 = 5 \times 75 - 2 \times 180$
$15 = 5 \times 75 - 2 \times 180$

Multiplying 600 both sides

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$9000 = 3000 \times 75 - 1200 \times 180$

$x = x_0 + (b/d)t, \quad y = y_0 - (a/d)t$

$x = -1200 + 5t, \quad y = 3000 - 12t$

Children and adults are positive in numbers

$x \geq 0 \qquad y \geq 0$

$-1200 + 5t \geq 0 \qquad 3000 - 12t \geq 0$

$5t \geq 1200 \qquad 3000 \geq 12t$

$t \geq 240 \qquad 250 \geq t$

$240 \leq t \leq 250$

Since more adults than children attend

$x \geq y$

$-1200 + 5t \geq 3000 - 12t$

$17t > 4200$

$t > \frac{4200}{12}$

$t > 247.06$

$t = 248, 249, 250$

$x = -1200 + 5t, \quad y = 3000 - 12t$

$x = 40$	$y = 24$	$t = 248$
$x = 45$	$y = 12$	$t = 249$
$x = 50$	$y = 0$	$t = 250$

(11). Let x be number of nines and y be the number of sixes such that sum is 1260

equations :- $ax + by = 126$ (x = no. of nines y = no. sixes)

$6x + 9y = 114$ (x = no. of sixes y = no. nines)

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both the eqn must be true simultaneously
Observing both the eqts these system can have only one soln.

Solving eqns $9x + 6y = 126 \quad \times 6$
$6x + 9y = 164 \quad \times \cancel{3}9$

$54x + 36y = 1756$
$54x + 81y = 1026$

$-45y = -270$
$y = 6$
$\Rightarrow x = 10$

So originally, there were 10 nines, 6 sixes

(12) Let x denote number of men,
y denote number of woman
z denote number of children

$x + y + z = 100$
$3x + 2y + 0.5z = 100 \quad — (1)$

From $x + y + z = 100$ we have that $z = 100 - x - y$
After substituting last result in eq (1) and multiplying it by 2 we get

$5x + 3y = 100 \quad — (2)$
$ax + by = c$
$a = 5, \quad b = 3, \quad c = 100$
$\gcd(5, 3) \Rightarrow 5 = 3 \times 1 + 2$
$3 = 2 \times 1 + 1$
$2 = 1 \times 2 + 0$
$\gcd(5, 3) = 1$

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$gcd(5, 3) = 1$

$1 = 3 - 2 \times 1$

$= 3 - (5-3)$

$= 2 \times 3 - 5$

Multiplying by 100 both sides

$100 = 200 \times 3 - 100 \times 5$

$x_0 = -\cancel{1}00$, $y_0 = 200$

$x = -100 + 3t$, $y = 200 - 5t$

$x \geq 0$ $\quad$ $y \geq 0$

$-100 + 3t > 0$ $\quad$ $200 - 5t > 0$

$t \geq 33.33$ $\quad$ $t \leq 40$

$t = \{34, 35, 36, 37, 38, 39, 40\}$

$x = -100 - 3t$, $y = 200 - 5t$, $z = 100 - x - y$

t	34	35	36	37	38	39	40
x	2	5	8	11	14	17	20
y	30	25	20	15	10	5	0
z	68	70	72	74	76	78	80