U19CS076

Krithikha Balamurugan

Assignment.

1. PT, If $f:(G_1, *_1) \to (G_2, *_2)$ is homomorphism from G_1 to G_2 then $f(e_1) = e_2$, $f(a^{-1}) = (f(a))^{-1}$, $\forall a \in G_1$. Also, Ker f is normal sub group of G_1.

$\Rightarrow$ Now since f is homomorphism we know that

$$f(a *_1 b) = f(a) *_2 f(b) \qquad a, b \in G_1$$

$f(a), f(b) \in G_2$

Let $a = e_1$ & $b = a$

$$f(e_1 *_1 a) = f(e_1) *_2 f(a)$$

$$f(a) = f(e_1) *_2 f(a) \quad -(1) \quad (e_1 *_1 a = a)$$

Now let $b = e_1$

$$f(a *_1 e_1) = f(a) *_2 f(e_1)$$

$$f(a) = f(a) *_2 f(e_1) \quad -(2)$$

From (1) & (2),

$$f(a) *_2 f(e_1) = f(e_1) *_2 f(a) = f(a)$$

$\therefore$ $f(e_1)$ is identity of G_2

e_2 is identity of G_2

Now

$$f(a^{-1} *_1 a) = f(a^{-1}) *_2 f(a)$$

$$f(e_1) = f(a^{-1}) *_2 f(a) \quad -(1)$$

$$f(a *_1 a^{-1}) = f(a) *_2 f(a^{-1})$$

$$f(e_1) = f(a) *_2 f(a^{-1}) \quad -(2)$$

From (1) & (2),

$$f(a) *_2 f(a^{-1}) = f(a^{-1}) *_2 f(a) = f(e) = e_2$$

$$\therefore \quad f(a^{-1}) = [f(a)]^{-1}.$$

U19CS076

For ker f

$$f(e_1) = e_2$$

$\therefore$ ker $f \neq \phi$

We know that (e_1) is Normal Sub group of G_1 if $kerf = \{G_1\}$ then the statement is TRUE.

If Ker $\neq \{e_1\}$

For $f(n *_1 y^{-1})$

[$\forall$ $n, y \in kerf$ $n *_1 y^{-1} \in kerf \Rightarrow$ Kerf is sub. group of G_1]

A Sub group is normal when

$$g^{-1} h g \in H \quad \forall h \in H \; \& \; g \in G$$

$$f(g^{-1} *_1 h *_1 g) = f(g^{-1}) *_2 f(h) *_2 f(g) \qquad \begin{array}{c} n \in kerf \\ f(h) = e_2 \end{array}$$

$$= f(g^{-1}) *_2 e_2 *_2 f(g)$$

$$f(g^{-1}) *_2 f(g)$$

$$= e_2 \quad \text{(proved above)}$$

$\therefore f(g^{-1} *_1 h *_1 g) \in kerf$

Hence Ker f is normal subgroup of G_1

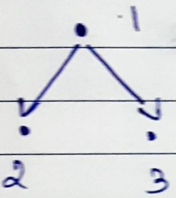
U19CS076

2. Define Digraphs, what is the minimum No of edges in a strongly connected digraph having n vertices? What shape does such digraph have? Why.

Digraphs stands for directed graphs; it consists of vertex and arrows going from one vertex to another.

Mathematically

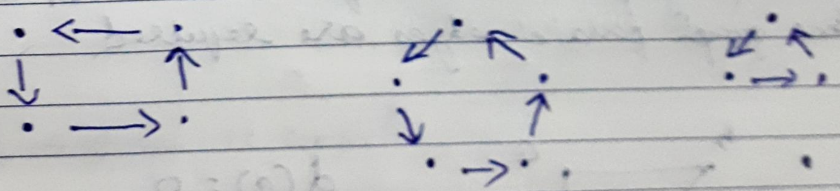
A digraph is an ordered pair of sets $G = (V, A)$ when V is a set of vertices & A is a set of ordered pairs (edge) of vertices of V.



For G,

$A = \{(1,2)\ (1,3)\}$

Minimum no. of edges in strongly connected digraph having n vertices would be 'n' as we can make a cycle from them



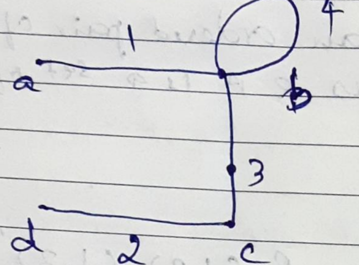
Polygons would be form with no pendent or isolated vertices

U19CS076

3. Draw a graph having the given properties or explain why no such graph exist,

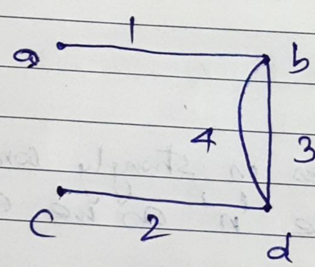
(i) Graph with vertices of degree 1, 1, 2 & 3
(ii) Graph with vertices of degree 1, 1, 3 & 3
(iii) Simple graph with 4 vertices of degree 1, 1, 3 & 3
(iv) Graph with 5 vertices of degree 0, 1, 2, 2, 3.

(i)



d(a) = 1
d(d) = 1
d(c) = 2
d(b) = 3

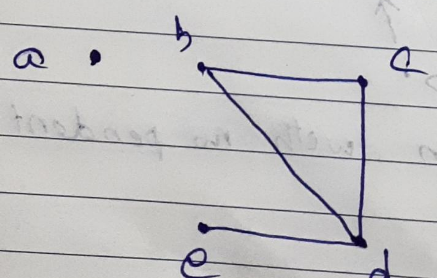
(ii)



d(a) = 1
d(c) = 1
d(b) = 3
d(d) = 3

(iii) A simple graph can not be formed as in order to for three graph parallel edges are required.

(iv)



d(a) = 0
d(b) = 2
d(c) = 2
d(d) = 3
d(e) = 1

U19CS076

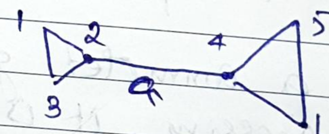
4 P.T edge connectivity of graph G, can not exceed the smallest degree in G & vertex connectivity of G can never exceed the edge connectivity of G.

Edge connectivity be $\lambda(G)$ & vertex connectivity be $k(G)$

To prove $k(G) \le \lambda(G) \le \delta(G)$

Now suppose v_1 is the vertex with degree $\delta(G)$ in order to isolate this vertex & make it disconnected edges incident on it should be remove (at max) then $\lambda(G) = \delta(G)$

But there might be case where it can be done with lesser amount. eg.



Here the graph can be disconnected by remov. 'a' then for $\lambda(G) = 1$ but $\delta(G) = 2$.

$$\lambda(G) \le \delta(G) \quad - (1)$$

If a graph is complete then

$$k(G) = \lambda(G) = |G| - 1$$

if $\lambda(G) \le 1$

then $\lambda(G) = k(G)$

Now suppose G is not complete. $\lambda(G) = k \ge 2$.

Set of edges disconnected = $\{x_1y_1, x_2y_2 \ldots x_ky_k\}$

If $G - \{x_1, x_2, x_3 \ldots x_k\}$ are disconnected then $k(G) = k$.

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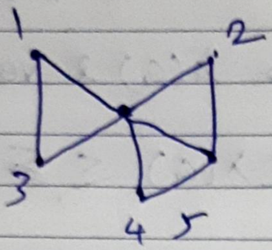
this is maximum condition, but in some cases like



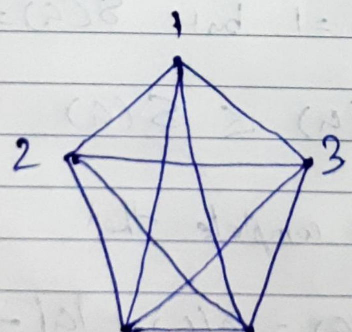
$\lambda(G) = 2$
But $k(G) = 1$
$\therefore$ removin vertex 6.

thus $k(G) \leq \lambda(G) \leq \delta(G)$ hence proved.

5. Define planar graph, region are K_5, $K_{3,3}$ planar graphs? why.

When a connected graph can be drawn without any edges crossing, it is called Plannar Graph.
When a planar graph is drawn, it divides the plane into regions also known as faces.

K_5



Euler's Formula.

$v - e + f = 2$

$v = 5$, $e = 10$
For it to be planar
$v - e + f = 2$
$5 - 10 + f = 2 \Rightarrow \boxed{f = 7}$

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Now each face is surrounded by atleast 3 edges.

thus

$3f \le B$ & $B = 2e$ (each edge is required by 2 faces)

$\therefore$ $3f \le 2e$

$3(7) \le 2(10)$

$21 \le 20$ NOT POSSIBLE

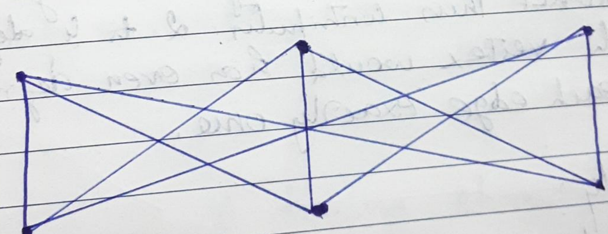
thus

$f \ne 7$

& then $v - e + f \ne 2$ for K_5

$\therefore$ K_5 is not PLANER.

$K_{3,3}$



$B - 9 + f = 2$

$f = 5$

Now each edge is surrounded by atleast 4 edges

thus

$4f \le B$

$20 \le 18$

NOT POSSIBLE

thus $K_{3,3}$ is not PLANAR.

U19CS076

6 Define Euler graph & ST a graph G is Euler graph iff every vertex in G has even degree; what happens if there are exactly two odd vertices

Euler graph:

A graph is said to be Euler graph when there exist a closed walk which visits, Every edge of the graph exactly once.

Let G(V, E) be a graph (Euler)

if trail $T \equiv [v_0 v_1 \ldots v_{k-1} v_k]$

Now for each $v \in V$
the trail enters v through on edge and departs v from another thus contributing 2 to the degree of v each time
Then each vertex would have even degree as T passes through each edge exactly once.

Conversely:

If every vertices of a graph has even degree if we start from a vertex v & then there is alway and even no of options to propogate. thus if we choose to propogate forward from one edge there is always an edge from where we can propogate in thus a losed walk is always form.

Now as we move to next vertex, there would now be odd number of edge to move forward from as one of the edge has already been used.

Thus, if we choose to move forward with one edge and come back to some nodes after few traversal, there is always an edge to move forward from there thus no dead ends occurs.

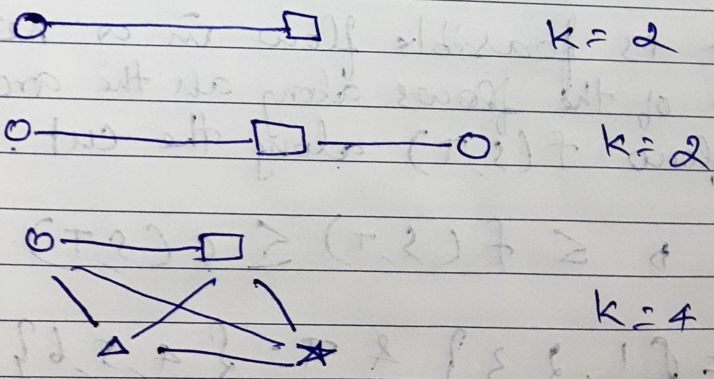
A closed walk covering all edges is formed which is Euler graph.

If there are 2 odd degree vertices then also Euler graph would not be formed.

7 What is k-chromatic graph? Explain with example. Also find chromatic number of complete graph k_n.

Chromatic number (k) is the smallest number of colours needed to colour the vertices of G. So that no two adjacent vertices show the same colour.

A graph G, having chromatic number k is called a k chromatic graph.



Symbols of vertices acts as colours.

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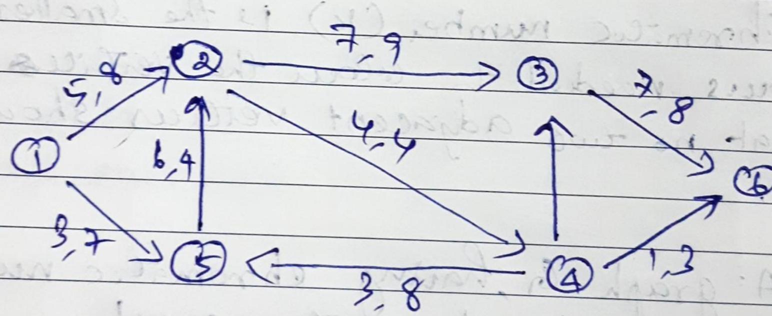
For a complete graph k_n

Chromatic number would be 'n' as there would (n-1) vertices connected to each vertex thus it should be different from (n-1) vertex colours and since the other vertices are also adjacent to each other thus (n-1) different colours + 1 of the vertex itself makes.

n different colours.

8. What is feasible flow? Find out in the capacity flow given below and hence verify $f(a) = f(S,T) - f(T,S)$, for (S,T) is cut.



If it is feasible flow in a network, the sum of the flows along all the arcs in cut(S,T) is the flow $f(S,T)$ along the cut.

$$0 \leq f(S,T) \leq c(S,T)$$

If $S = \{1, 2, 3\}$ & $T = \{4, 5, 6\}$

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$f(G) = f(S,T) - f(T,S)$
$= (3+4+7) - (0+6) = 8 <$ Capacity $(7+2+1+5)$
$= 19.$

If $S = \{1, 4, 5\}$ & T is its complement

$f(G) = (5+0+1+6) - 4 = 8 <$ Capacity $(6+4+5+9)$
$= 26.$

If f is feasible flow in a capacitated network the arc (i, j) is saturated if $f(i,j) = c(i,j)$ it is given as $f(i,y) = 0$ & is for positive if $f(i,j)$ is positive

k less than $c(i,j)$

9. Explain Hoare's logic for program verification in brief.

Hoare's logic is a formal system with a set of logical rules for reasoning rigorously about the correctness of computer programs.

Central feature of Hoare logic is the Hoare triple. It describes how the execution of a piece of code changes the state of computation.

U19CS076

It is in the form

$\{P\} \; C \; \{Q\}$

Assertion and 'C' is command.

P is precondition and Q is post condition.

Using Hoare's logic only partial correctness can be proven, while algebra needs to be proven separatively.

Intuitive reading of Hoare Triple is whenever P holds of the state before the execution of C then Q will hold afterwards or C does not terminates. In the later case, there is no "after", so Q can be any statement at all, Indeed one can choose Q to be false to express that C does not terminate.

10 Explain brief:
Linear recurrance relation, inclusion and exclusion for counting.

Linear Recurrence Relation is an equation that defines the n^{th} term in a sequence in terms of k previous terms in a sequence. The recurrence relation is in the form

U19CS076

$$x_n = c_1 x_{n-1} + c_2 x_{n-2} + \cdots + c_k x_{n-k}.$$

where C_i is some constant coefficient.

Principle of Inclusion and Exclusion for counting.

It is a counting technique that computes the number of elements that satisfy at least one of several properties while guaranteeing that elements satisfying more than one property ~~are~~ not counted twice.

Idea is summing the number of elements that satisfy atleast one of two categories & substracting the over lap to prevent double counting.

eg,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

11. Explain in brief - first counting principle se
and circular permutations

First counting principle

Suppose some event E can occur in
ways a second F can occur in n ways
Suppose both events cannot occur sim
Then E or F can occur in m+n ways

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Second Counting Principle

Suppose there is an event E which can occur in m ways and independent of this event, there is a second event F which can occur n ways. Then combinati[on of] E and F can occur in mn ways.

Circular permutation

Circular permutation is total number of ways in which n distinct objects can be arranged around a fix circle.

It is of two types

1) Clockwise and anticlockwise orders are different
2) Clockwise and anticlockwise orders are same.

Formula

Case 1 :- $P_n = (n-1)!$

Case 2 :- $P_n = \frac{(n-1)!}{2}$.