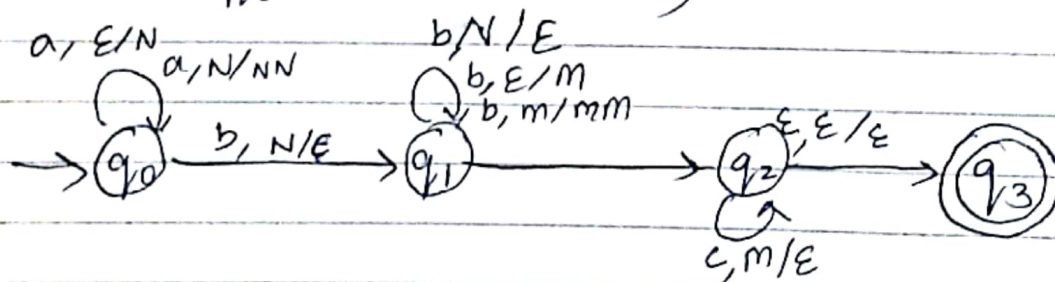## Tutorial - 9

Automata and Formal Languages

Q1] Design push down automata for $\{a^n b^n + m c^m \mid n, m >= 1\}$ and gave 6 specification for PDA

$$M = \{ \{q_0, q_1, q_2, q_3\}, \{a, b, c\}, \{N, m\}, \delta, q_0, \varepsilon, q_3 \}$$

Where $\delta =$

$\delta(q_0, a, \varepsilon) = (q_0, N)$

$\delta(q_0, a, N) = (q_0, NN)$

$\delta(q_0, b, N) = (q_1, \varepsilon)$

$\delta(q_1, b, N) = (q_1, \varepsilon)$

$\delta(q_1, b, \varepsilon) = (q_1, m)$

$\delta(q_1, b, m) = (q_1, mm)$

$\delta(q_1, c, m) = (q_2, \varepsilon)$
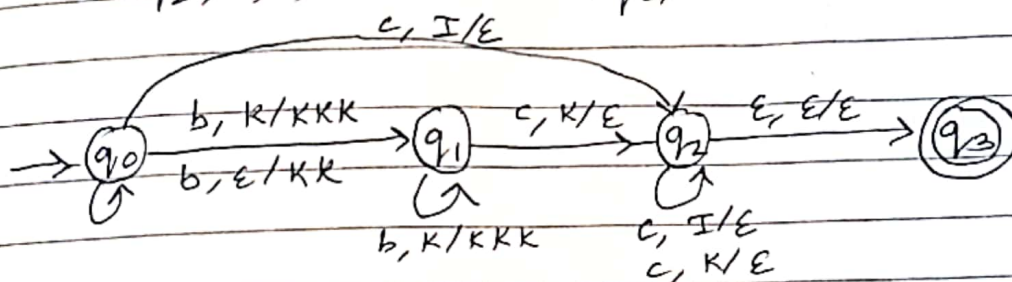
$\delta(q_2, c, m) = (q_2, \varepsilon)$

Q2] Find a push down automata that recognizes the following languages and give 6 tuple specification for PDA

$$L = \{ a^l b^* c^{i+2k} \mid i, k >= 0 \}$$

Ans $\quad M = (\{q_0, q_1, q_2, q_3\}, \{a, b, c\}, \{I, k\}, \delta, q_0, q_3$

where S is defined by :-

$$\delta(q_0, a, \varepsilon) = (q_0, I)$$
$$\delta(q_0, a, I) = (q_0, II)$$
$$\delta(q_0, b, \varepsilon) = (q_1, KK)$$
$$\delta(q_0, b, I) = (q_1, KKI)$$
$$\delta(q_0, b, K) = (q_1, KKK)$$
$$\delta(q_0, c, I) = (q_2, \varepsilon)$$
$$\delta(q_1, c, K) = (q_2, \varepsilon)$$
$$\delta(q_2, c, I) = (q_2, \varepsilon)$$
$$\delta(q_2, c, K) = (q_2, \varepsilon)$$
$$\delta(q_2, \varepsilon, \varepsilon) = (q_3, \varepsilon)$$

Q3] Find the language

$$S \rightarrow aABB \,/\, aAA$$
$$A \rightarrow aBB \,/\, a$$
$$B \rightarrow bBB \,/\, A$$

Ans $S \rightarrow \{aaa, \ aabaaa, \ aaaboa, \ \ldots\}$

S is set of all strings of a and b such that strings starts with 2 a's and ends with 2a's and length of string is even and also string includes 'aaa'.

Q4] Find the language :-

$$S \rightarrow aAa$$
$$A \rightarrow Sb \,/\, bcc \,/\, DaA$$

$$C \rightarrow abb \mid DD$$
$$E \rightarrow aC$$
$$D \rightarrow aDA$$

Ans E and D states, are unnecessarily as E state is unreachable and D is ambiguous.

Reduced grammar is :-

$S \rightarrow aAa$, $A \rightarrow sb \mid bcc$, $C \rightarrow abb$

Language of S : {abababbabba, ab abba bbba...}

S is set of strings starting and ending with such that

$$S = ab \, (abb)^2 \, b^* a$$

Q5] Find a pushdown automata with 2 states that accepts language :-

A) $L = \{a^n b^{2n} \mid n >= 1\}$

Ans $M = \{\{q_0, q_1\}, \{a, b\}, \{N\}, \delta, q_0, \varepsilon, q_1\}$
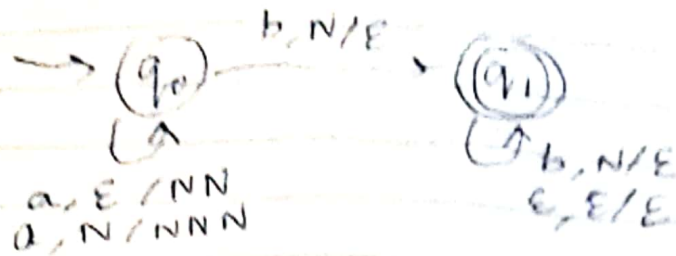
Where δ is defined are :-

$$\delta(q_0, a, \varepsilon) = (q_0, NN)$$
$$\delta(q_0, b, N) = (q_1, \varepsilon)$$
$$\delta(q_1, b, N) = (q_1, \varepsilon)$$
$$\delta(q_1, \varepsilon, \varepsilon) = (q_1, \varepsilon)$$
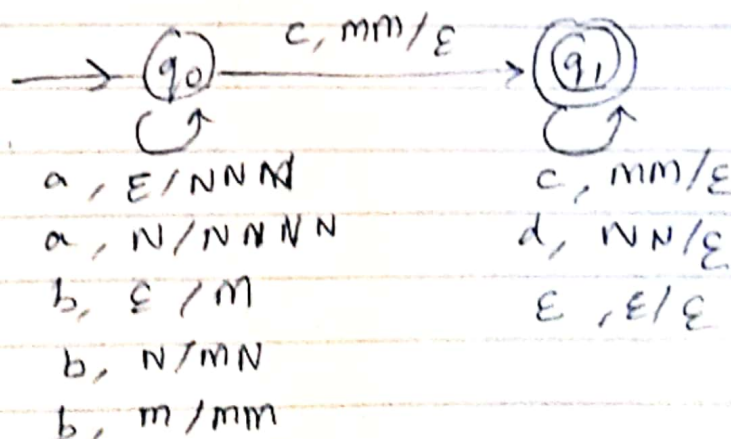$$\delta(q_0, a, N) = (q_0, NNN)$$

$$\longrightarrow \boxed{q_0} \xrightarrow{\ b,N/\varepsilon\ } \boxed{\boxed{q_1}}$$

$\boxed{q_0}$ loop:
$a, \varepsilon / NN$
$a, N / NNN$

$\boxed{\boxed{q_1}}$ loop:
$b, N/\varepsilon$
$\varepsilon, \varepsilon/\varepsilon$

B] $L = \{ a^{2n} \ b^{2m} \ c^{m} \ d^{2n} \mid n >= 0, m > 0 \}$

$M = \{ \{q_0, q_1\}, \{a, b, c, d\}, \{N, m\}, \delta, q_0, \{q_1\} \}$

where δ is defined by :-

$\delta (q_0, a, \varepsilon) = (q_0, NN)$

$\delta (q_0, a, N) = (q_0, NNNN)$

$\delta (q_0, b, N) = (q_0, mN)$

$\delta (q_0, b, m) = (q_0, m)$

$\delta (q_0, b, \varepsilon) = (q_1, \varepsilon)$

$\delta (q_0, c, mm) = (q_1, \varepsilon)$

$\delta (q_1, c, mm) = (q_1, \varepsilon)$

$\delta (q_1, d, mN) = (q_1, \varepsilon)$

$\delta (q_1, \varepsilon, \varepsilon) = (q_1, \varepsilon)$

$$\longrightarrow \boxed{q_0} \xrightarrow{\ c, mm/\varepsilon\ } \boxed{\boxed{q_1}}$$

$\boxed{q_0}$ loop:
$a, \varepsilon / NNN$
$a, N / NNNN$
$b, \varepsilon / m$
$b, N / mN$
$b, m / mm$

$\boxed{\boxed{q_1}}$ loop:
$c, mm/\varepsilon$
$d, NN/\varepsilon$
$\varepsilon, \varepsilon/\varepsilon$

2] Consider a push-down automata

$M = \{ \{q_1, q_2, q_3\}, \{0, 1, 6\}, \{R, G, B\}, \delta, q_1, R, \phi \}$

where δ is defined as :-

$$\delta (q_1, 0, R) = (q_1, BR)$$
$$\delta (q_1, 0, B) = (q_1, BB)$$
$$\delta (q_1, 0, G) = (q_1, BG)$$
$$\delta (q_1, 6, R) = (q_2, R)$$
$$\delta (q_1, 6, B) = (q_2, B)$$
$$\delta (q_1, 6, G) = (q_2, G)$$
$$\delta (q_2, 0, B) = (q_2, \varepsilon)$$
$$\delta (q_2, \varepsilon, R) = (q_2, \varepsilon)$$
$$\delta (q_1, 1, R) = (q_1, GR)$$
$$\delta (q_1, 1, B) = (q_1, GB)$$
$$\delta (q_1, 1, G) = (q_1, GG)$$
$$\delta (q_2, 1, G) = (q_2, \varepsilon)$$

Ans   Given an execution trace PDA M showing input string 001C100 is in N(m)

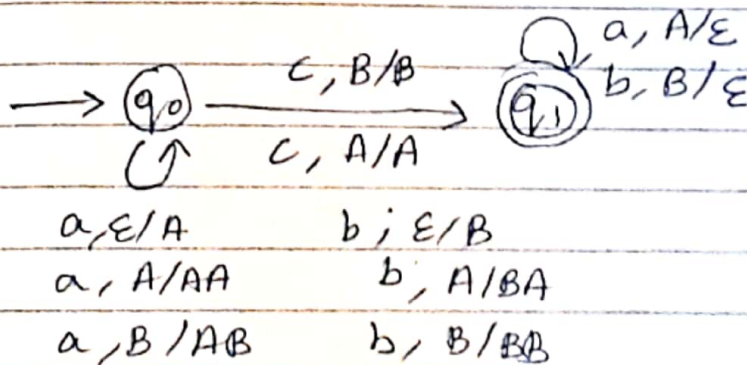| Input symbol	Stack	state	Next-State
Initial	R	q_1	q_1
	BR	q_1	q_1
0	BBR	q_1	q_1
0	GBBR	q_1	q_1
1	GBBR	q_1	q_2
C	BBR	q_2	q_2
1	BR	q_2	q_2
0	R	q_2	q_2
0	ε	q_2	δ_1
ε			

Input symbol   Stack

Hence `001C100` is in N(m)

57] Design a PDA for palindrom with middle symbol c and trace string abcba

$M = \{ \{q_0, q_1\}, \{a, b, c\}, \{A, B\}, \delta, q_0, \varepsilon, q_1\}$

where δ is defined as

$\delta(q_0, a, \varepsilon)$	=	(q_0, A)
$\delta(q_0, b, \varepsilon)$	=	(q_0, B)
$\delta(q_0, a, A)$	=	(q_0, AA)
$\delta(q_0, b, A)$	=	(q_0, BA)
$\delta(q_0, a, A)$	=	(q_1, ε)
$\delta(q_0, a, B)$	=	(q_0, AB)
$\delta(q_0, b, B)$	=	(q_0, BB)
$\delta(q_0, c, A)$	=	(q_1, A)
$\delta(q_0, c, B)$	=	(q_1, B)
$\delta(q_1, b, B)$	=	(q_1, ε)

a, ε/A   b ; ε/B
a, A/AA   b, A/BA
a, B/AB   b, B/BB

Tracing abcba

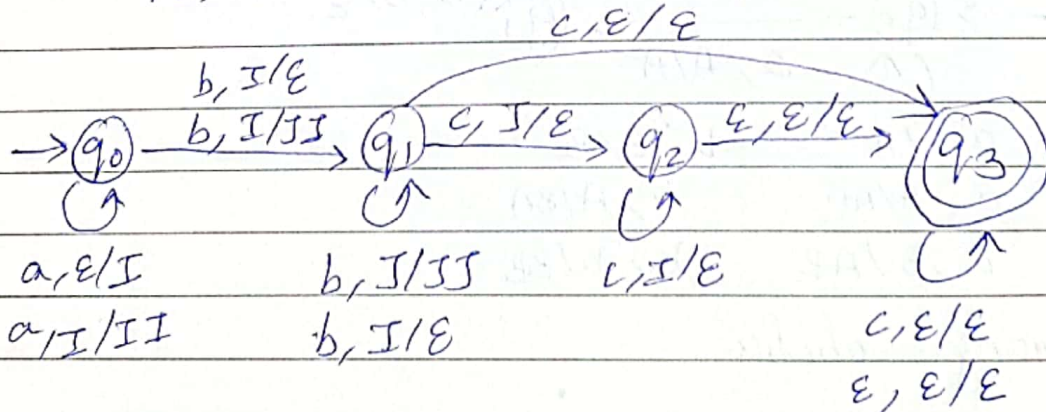| Input symbol	stack	state	next-state	
a	A	q_0	q_0	Hence
b	BA	q_0	q_0	abcba
c	BA	q_0	q_1	is
b	A	q_1	q_1	accepted
a	ε	q_1	q_1	

Q8. Design a PDA for $L = \{a^i b^j c^k \mid i, j, k \geq 0$ and $i = j$ or $j = k\}$

Ans $M = \{\{q_0, q_1, q_2, q_3\} \{a, b, c\}, \{I, J\}, \delta, q_0, \varepsilon, q_3\}$

only non deterministic PDA possible

δ is defined as :-

$\delta(q_0, a, \varepsilon) = (q_0, I)$
$\delta(q_0, a, I) = (q_0, II)$
$\delta(q_0, b, I) = \{(q_1, JI), (q_1, \varepsilon)\}$
$\delta(q_1, b, I) = (q_1, \varepsilon)$
$\delta(q_1, b, J) = (q_1, JJ)$
$\delta(q_1, c, \varepsilon) = (q_3, \varepsilon)$
$\delta(q_1, c, J) = (q_2, \varepsilon)$
$\delta(q_2, c, J) = (q_2, \varepsilon)$
$\delta(q_3, c, \varepsilon) = (q_2, \varepsilon)$
$\delta(q_2, \varepsilon, \varepsilon) = (q_3, \varepsilon)$

$c, \varepsilon / \varepsilon$

$b, I/\varepsilon$
$b, I/JI$ $\quad$ $c, J/\varepsilon$ $\quad$ $\varepsilon, \varepsilon/\varepsilon$

$\rightarrow q_0 \xrightarrow{b, I/JI} q_1 \xrightarrow{c, J/\varepsilon} q_2 \xrightarrow{\varepsilon, \varepsilon/\varepsilon} q_3$

$a, \varepsilon/I$ $\qquad$ $b, J/JJ$ $\qquad$ $c, J/\varepsilon$
$a, I/II$ $\qquad$ $b, I/\varepsilon$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $c, \varepsilon/\varepsilon$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\varepsilon, \varepsilon/\varepsilon$

Q9] Construct a PDA that accepts
$L = \{WWR \mid W = (a+b)^* \}$

Ans $\qquad$ as $L = WWR$
$[N = (a+b)^*$
$WW = (a+b)^* (a+b)^* = (a+b)^*]$

where δ is defined as

$$\delta(q_0, a, R) = (q_0, R)$$
$$\delta(q_0, b, R) = (q_0, R)$$
$$\delta(q_0, R, R) = (q_1, \varepsilon)$$
$$\delta(q_1, \varepsilon, \varepsilon) = (q_1, \varepsilon)$$

$0, R/R$

$b, R/R$

$\rightarrow (q_0) \xrightarrow{\ R,\ R/\varepsilon\ } (q_1)$

$\varepsilon, \varepsilon/\varepsilon$