Tutorial 1 G= set of all integers, a.b = a-b Pa,beG => a,bez === a = a for 4 a E G Closusie

 $\begin{array}{l}
Af & a,b,c \in G \\
a \cdot (b \cdot c) = a \cdot (bc) = abc \\
(a \cdot b) \cdot c = (ab) \cdot c = abc
\end{array}$ 

ax · (ay · az) = ax · (ay+z) (ax · ay) · az = (axiy) · az = ascty+? as For tax EG ax. de = aax =ax a is identify element For Yax EG ac a = ae N = -x > not possible a = a = a = a 3C+7-3C=7-7=0G= set of all rational numbers with odd denom, a.b = a+b For a.b & G a.b = a+b Denomination will be product of 2 old

numbers - always odd EG1 => Closed > Aff a. (b.c) = a. (b+c) (a.b).c = (a+b+c).c Associative (Denom = and G = odd) Sf. ata EG a.e. e.a = a ate = eta = a e= 0 > e= Vold number e E G Identify exists If ta 6 Gr a.a'= a'.a=e  $a + a^{-1} = a^{-1} + a = e = 0$   $a^{-1} = -a$   $a^{-1} \in G$ Inverse exists It is a group 2. 4 G is abelian group, ta, b ∈ G, a.b = b.a For n=1 = (a.6) (as) = a.6/ Assume  $(a \cdot b)^n = (a^n \cdot b^n)$ then  $(a \cdot b)^{n+1} = a^{n+1} \cdot b^{n+1}$ 

Given  $(ab)^2 = a^2b^2$  $(ab)(a \cdot b) = a^2 \cdot b^2$   $a \cdot b \cdot a \cdot b = a \cdot a \cdot b \cdot b$  [2eft & sight cancellation]b. a = a.b (9) a) If Group G is having 3 elements Order = 3,  $a,b \in G$  with  $a \neq b$   $b = \mathbf{a} \cdot \mathbf{e} = \mathbf{e} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$   $b = \mathbf{b} \cdot \mathbf{e} = \mathbf{e} \cdot \mathbf{b} = \mathbf{e} \cdot \mathbf{b} = \mathbf{e} \cdot \mathbf{b}$   $f = b = \mathbf{e} \cdot \mathbf{b} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{e} = \mathbf{e} \cdot \mathbf{b} = \mathbf{e} \cdot \mathbf{b}$   $f = \mathbf{b} \cdot \mathbf{a} = \mathbf{a} \cdot \mathbf{b}$ When a and b are not identity  $a.b \neq a \quad (if \ a.b = a \Rightarrow b = e)$   $a.b \neq b$ Say Sa, b, le 3 a · b = e & b · a = e be in a · b = b · a G is abelian for O(G) = 3

het O(G) = 4 for some a, b EG either a/b = eIf  $a = e \Rightarrow a \cdot b = b = a$   $b = e \Rightarrow a \cdot b = a = b \cdot a$ If neither a & b arde a b = b Let 3rd element be c G > { a, b, c, e } So either a.b=e ( Since order of Gr is prime Gr is a cyclic group since every cyclic group is abelien

11) If G is a group of even order a \times e group order a group orde Assume no element is present with  $a^2 = e$  except a = e for  $a \in G$ .  $\Rightarrow a^2 \neq e$   $a \cdot a \neq e$ .

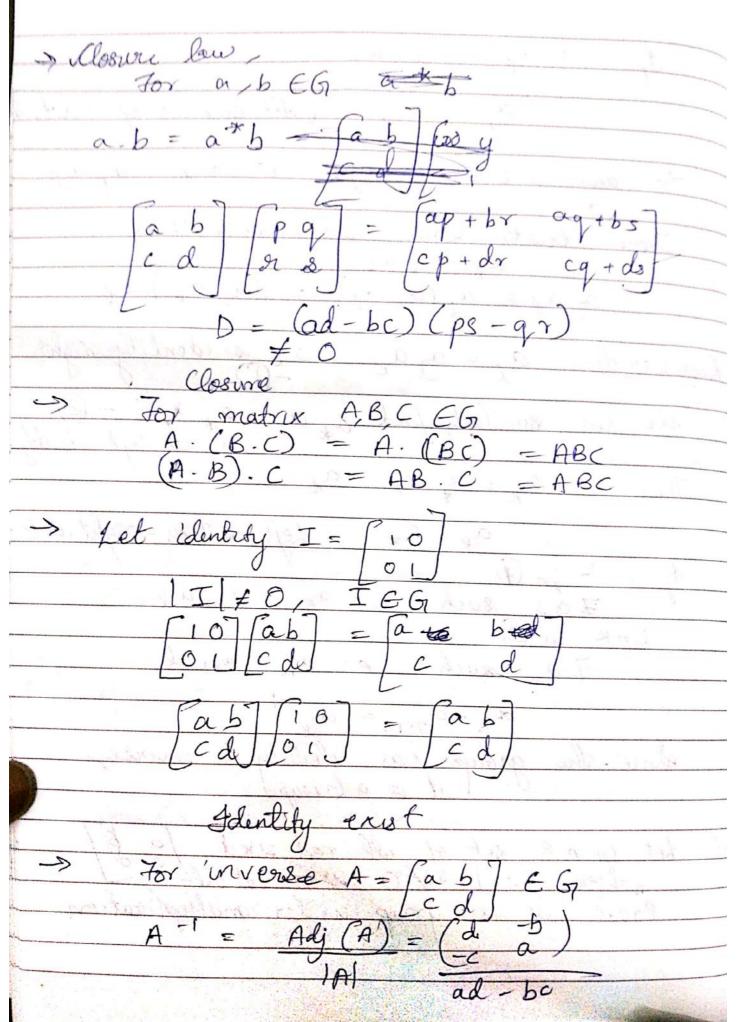
Sor others For every non identity element a there exist and in a group as not mutually disjoint subset of order 2. We assume count of possible subsets = some tre integer n as G is finite group.  $O(G) = 2\eta + 1$ O (G1) = 2n+11.

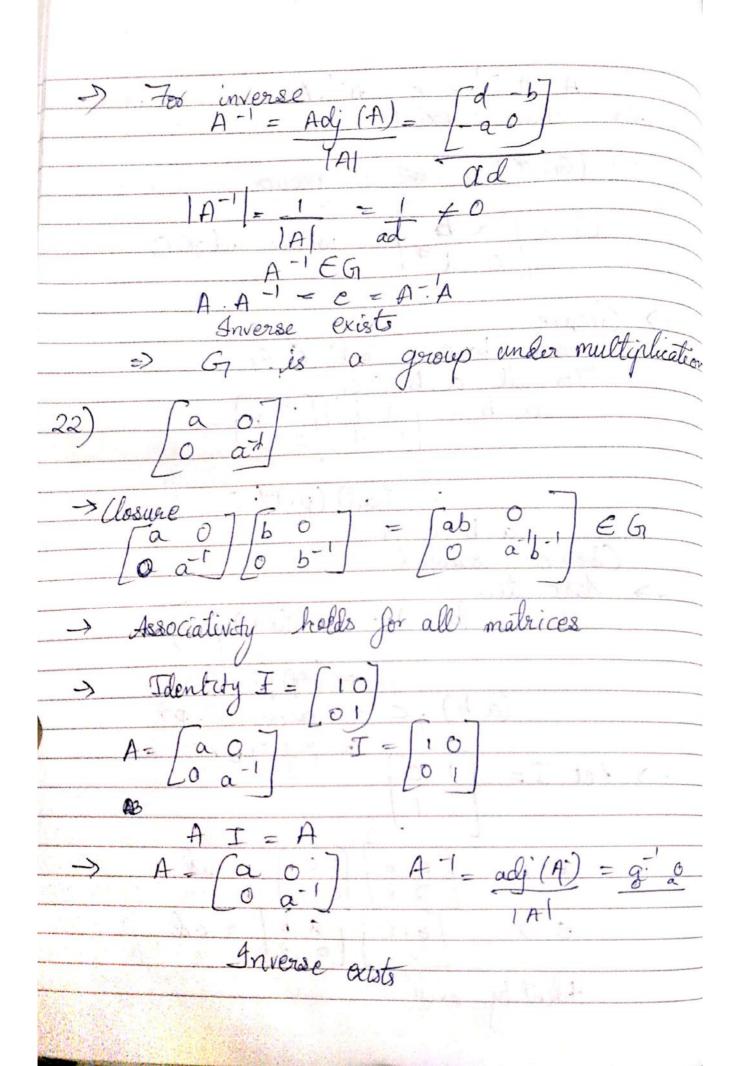
Dorder of G1 is odd  $\rightarrow$  against the g

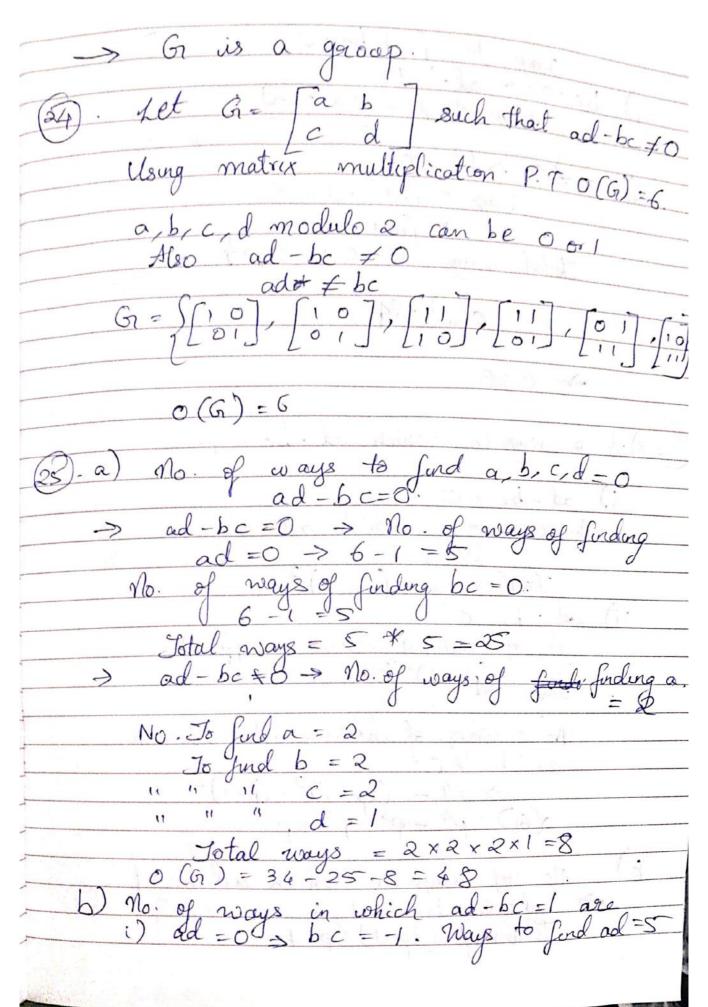
Assumption wrong

There must be an element  $a \in G$ ,  $a \neq e$ such that  $a^2 = e$  when o(G) is even (4) & Since G is associative -> Semi-group. S= {a,,a,a, a, a, -- an }-0  $3! = \{a_1 a_2, a_3 a_2 \dots a_n a_l\}$  5' belongs to  $S \rightarrow Closure prop.$   $S' \subseteq S$ 

a & a Call elements of S distinct a is identify right



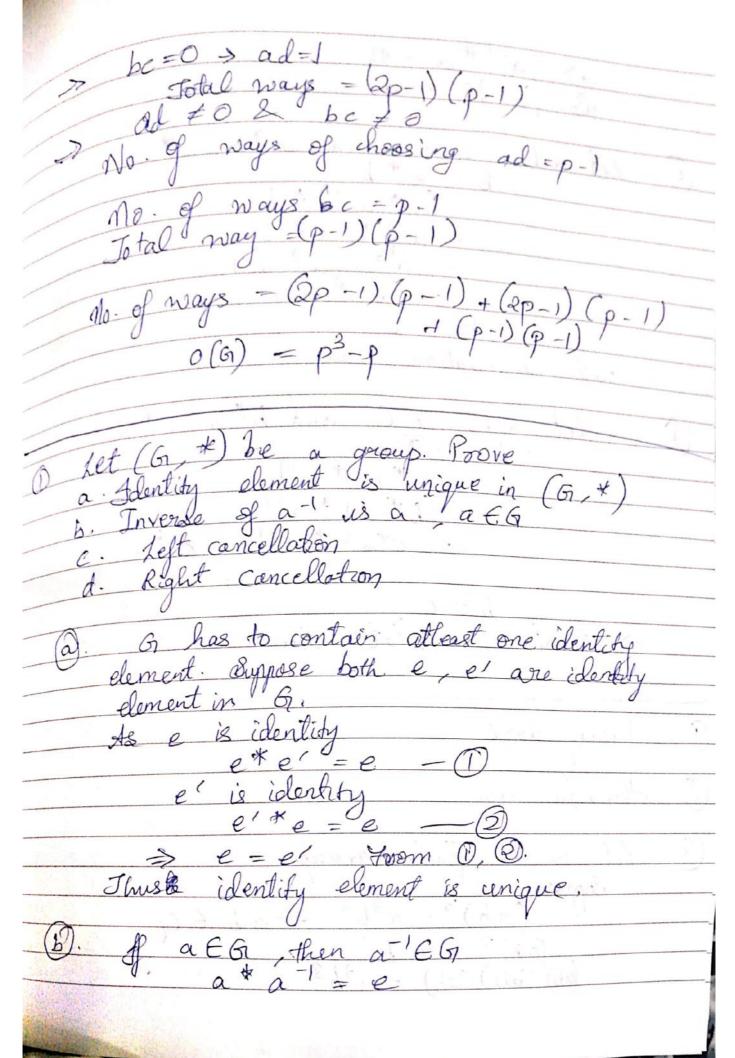


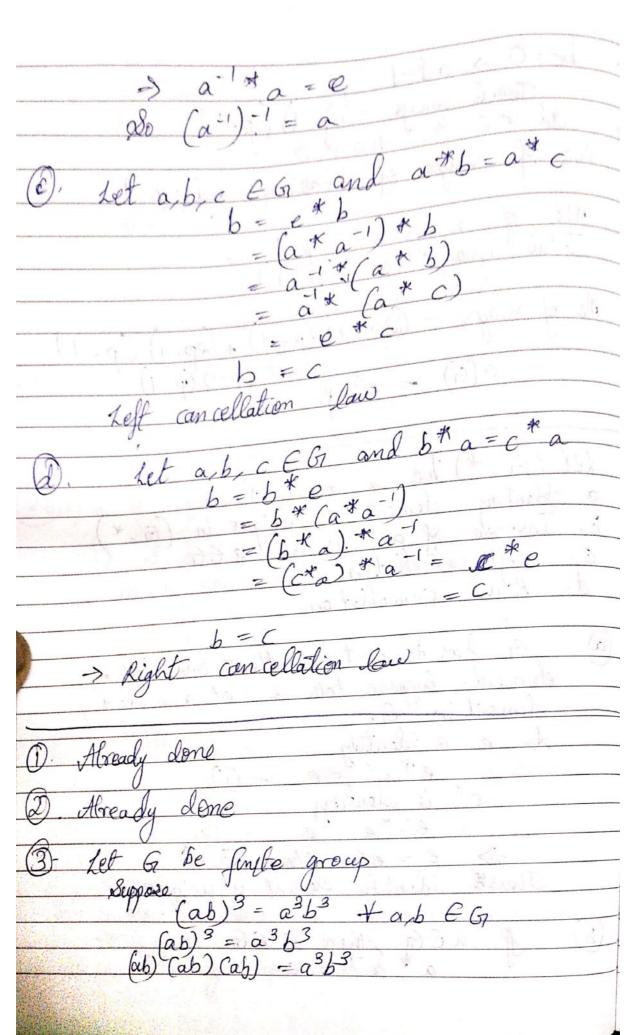


ii) be=0 -> ad=-1

Ways to find ad=2 be in plays.

(ii)  $ad \neq 0 \Rightarrow bc \neq 0$ (iii)  $ad \neq 0 \Rightarrow bc \neq 0$ (iv)  $ad \neq 0 \Rightarrow$ a) No. of ways in which ad -bc of ways in which ad=0  $\Rightarrow 2p-1$   $= 5 \cdot 2p-1$   $= 6 \cdot 2p-1$ 





 $(ba)(ba) = a^2b^2$  (Cancellation)  $(ba)^2 = a^2b^2$  $(ba)^3 = b^2 a^3$   $ba (ba)^2 = b^3 a^3$ And every element of g can be uniquely represented as cube  $a^3b^2 = b^2a^3$  $\begin{array}{ccc}
a^2b^2 & b^2a^2 \\
(ba) & (ba) & = b^2a^2 \\
ab & = ba & Proved.
\end{array}$ P. Tany subgrop of a cyclic group is a cyclic group is a Jet Gr = [a] be cyclic grp

H is a subgrp of Gr.

H = Se 3 acyclic group.

H is also a cyclic group. If His proper subgroup, Hontains one element am other than in Am EH het m be least +ve integer, & am E H

tet am EH = division algorithm

there exists turn interess Shere exists two integers q'? n = mq + r n - mq = rSince ameris (am) 2 En Since

Now least +ve But since Every ydic grow of Order reas (CO, A)

9.8) IF aga bange, P.T. O(a)/m. Driven that am = e. a etc. => a has finible order = 7 K = 0 (a). By the division alger than there exists unique integers r= a, 8 m = Kq +x Now, e = am = a kar ar a kar ar = (ak) ar = eaar = ar But since K is the smallest Positive integer m = Katr = Katu a. 7) If Gr has no non trivail subgroups. S.T. G must be finite of Prime order. Let there be group or of order of a) = or Since it has no non-brivial sub-growns there only, a and Ee) are its subgroups where e is identity, Now, by lagranges theorem, O(G1) = KO(H) = 7 order of group or is divisible by order of sub groups H. sink the Subgrous are Grand (e) their orders are and 1. So, n has only two factors = n and 1 this roleans nis a prime number.

. G must be Printe of Prime order.
The second of th
in a sure while a think wood as a
Q.8). Let Go be snowl that the intestation of all  its sub snowl, which are different from (e)  is a sub-growl and different from identing  Prove that every element in Go has finite  Let ME e ] 7 H < br. H = K & {e} }.  Let Mi = < a i > are sub-growls of Go.  Since K < N  C. Sub-growl of cyclic growl)
C. Sus grow of Cycle Distance
= for some integer o
Since < an > < a' >  n = ik i H 'Vn'
As a is a fixed given Positive no it
As a war division sing &
has only finitely, many divisions. Since & K x e, we only have finitely many
Hi 7 e i.e.: There, exists a j such
that Hj = <ai>= e i.e. a)=e.</ai>
That my that the same of the s
Special Succession of the second seco
water in the eff our ist he is the
(1)0.7 sildi manesth chesto. Vi won
ing! she's it is no great for which is
the state of the s
21. 3 3 3 1 600 10 3 100 2 100 10 July 0 July 0 10 11 11 11 11 11 11 11 11 11 11 11 1
the state of the s
The state of the s

Scanned with CamScanner