Krithikha Balamurugan

U19CS076

Math Tutorial 4

1. Given

μ_a = population mean = 10 mm

$\bar{x}$ = Sample mean = 9.52 mm

s = 0.6 mm

x = 10

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{4.52 - 10}{0.6} \times \sqrt{10}$$

$$= \frac{-0.48}{0.6} \times \sqrt{10}$$

$$= -0.8\sqrt{10}$$

$$= -2.5298$$

2. $N = 9$ (< 30 so it is small sample)

Step 1 : Null hypothesis (H_0) ; $\mu_0 = 65$

Alternate hypothesis (H_0): $\mu \neq 65$

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(Two tailed test)

Degree of freedom :- $n = 1 - 9 - 1 = 8$

Critical value $(f(x)) = 2.262$

step 3 :

x	$x - \bar{x}$	$(x - \bar{x})^2$
63	-3.77	14.2129
63	-3.77	14.2129
64	-2.77	7.6729
65	-1.77	3.1329
66	-0.77	0.5929
69	2.33	5.4289
70	3.33	11.0889
70	3.33	11.0889
~~71~~	4.33	18.7489
~~601~~		
$\Sigma = 601$		86.1801

$$\bar{x} = \frac{601}{9} = 66.77$$

$$s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}} = 8.2821$$

$$t_{(x)} = \frac{66.77 - 65}{3.2821} \times \sqrt{9}$$

$$= 1.6178$$

Since $|t_{\omega}| < t_{x}$, x_0 is accepted

Mean height of univ is 65 inches.

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(3). Step:- Set up null and alternative happiness H_0 and H_1

$H_0 = \bar{x} = \mu \quad H_1 = \bar{x} \neq \mu$

Step 2:

Let the level of significance be 5% and 1%. With $n-1 = 16-1 = 15$ degrees of freedom

$t_{0.05}$ for $(16-1)$ i.e $15\ df = 2.131$

$t_{0.01}$ for $(16-1)$ ie $15\ df = 2.947$

Step 3 Test statics

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

where $\bar{x} = \frac{1}{x} = \sum_{i=1}^{n} x_i$ and $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

we are given $n = 16$ $\bar{x} = 53$, $\mu = 56$ and

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 = 150$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{16-1}(150) = 10$$

$$s = \sqrt{10}$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{53 - 56}{\sqrt{10}/16}$$

$$= \frac{-48}{\sqrt{10}}$$

Step 4: Calculated value of $t = \frac{48}{\sqrt{10}}$

Table value of $t = 2.131 \rightarrow$ Rejected

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Hence we conclude that the assumption of mean of 56 for the population is not reasonable

95 % confidence limit

$\bar{x} \pm t_{0.05}\left(\frac{s}{\sqrt{n}}\right) = 53 \pm 2.131 \times \frac{\sqrt{10}}{16}$

$= 53 \pm 0.42$

$= (52.58, 53.42)$

99 % confidence limit

$\bar{x} \pm t_{0.01}\left(\frac{s}{\sqrt{n}}\right) = 53 \pm 2.947 \frac{\sqrt{10}}{16}$

$= 53 \pm 0.58$

$= (52.42, 53.58)$

4) $n_1 = 8$ and $n_2 = 7$ (< 30 it is small sample)

Step 1

Null hypothesis $(H_0) = \mu_1 = \mu_2$ (i.e difference between sample is not significantly)

Alternative hypothesis $(H_0) = \mu_1 < \mu_2$ (i.e. difference between sample is significantly)

(Two failed test)

Step 2 $los = 5\%$ (Two failed test)

Degree of freedom $= n_1 + n_2 - 2$

$= 8 + 7 - 2 = 13$

$\Rightarrow$ Critical value $(t_\alpha) = 2.16$

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Step 3

Sampled	$x_{1t} - \bar{x}$	$(x_{1t} - \bar{x})^2$	S2	$(x_{2t} - \bar{x}_2)$	$(x_{2t} - \bar{x}_2)^2$
9	-2.75	7.5625	10	-0.42	0.1764
11	-0.75	0.5625	12	1.58	2.4964
13	1.25	1.5625	10	-0.42	0.1764
11	-0.75	0.5625	14	3.58	12.8164
15	3.25	10.5625	9	-1.42	2.0164
9	-2.75	7.5625	8	-2.42	5.8564
12	0.25	0.0625	10	-0.42	0.1763
14	2.25	5.0625			
34			73		

$$\bar{x}_1 = \frac{\Sigma x_1 t}{x_1} = \frac{94}{8} = 11.75$$

$$\bar{x}_2 = \frac{\Sigma x_2 t}{n_2} = \frac{73}{7} = 10.42$$

$$S_p = \sqrt{\frac{(x_{1t} - \bar{x}_1)^2 + \Sigma (x_{2t} - \bar{x}_1)^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{33.5 + 23.7148}{13}} = \sqrt{\frac{57.214}{13}}$$

$$= 2.0978$$

$$SE = SP \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 2.0978 \sqrt{\frac{1}{8} + \frac{1}{7}}$$

$$= 2.0978 \sqrt{\frac{15}{56}}$$

$$= 1.0857$$

Step4 → Test states

$$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{11.75 - 10.42}{1.0857}$$

$$= \frac{1.33}{1.0857}$$

$$\Rightarrow = 1.2250$$

Step5

Since $t_{cal} < t_{\alpha}$ H_0 is accepted
Different b/w sample meant not significant

(5). $n_1 = 9$, $n_2 = 9$ ($4 < 30$ small sample)

Step 1 Null hypothesis (H_0 : $= \mu_1 = \mu_2$)
Alternative hypothesis $H_a = \mu_1 < \mu_2$
Two failed test

Step2 LOS = 5%
Degree of freedom $= n_1 + n_2 - 2 = 16$
→ Critical value $t_{\alpha} = 2.31$

Step3

Before	$x_{1t} - \bar{x}_1$	$(x_{1t} - \bar{x}_1)^2$	After	$x_{2t} - \bar{x}_2$	$(x_2 + \bar{x}_2)^2$
10	-0.33	0.1089	12	0.89	0.7921
15	4.67	21.8089	17	5.89	34.6921
9	-1.33	1.7689	8	-3.11	9.6721
3	-7.33	53.7289	5	-6.11	37.3321
7	-3.33	11.0889	6	-5.11	26.1121
12	1.67	2.7889	11	-0.11	0.0121
16	5.67	32.1489	18	6.89	47.4721
17	6.67	44.4889	20	8.89	79.0321
4	-6.33	40.0689	3	-8.11	65.7721
93			100		

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$\bar{x}_1 = \frac{\Sigma x_1 t}{n_1} = 10.33$

$\bar{x}_2 = \frac{\Sigma x_2 t}{n_2} = \frac{100}{9} = 11.11$

$Sp = \sqrt{\frac{208.001 + 300.8889}{16}} = 5.639$

$SE = Sp\sqrt{\frac{1}{9}+\frac{1}{9}} = 5.639 \times \frac{\sqrt{2}}{3}$

$SE = 2.6581$

Step 4 Test statics

$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{10.33 - 11.11}{2.6581}$

$|t_{cal}| = 0.2934$

Step 5

Since $|t_{cal}| < t_\alpha$ H_0 is accepted course was not effective in improving performance.

Q6. Step1 Set up Null and alternative hypothesis H_0 and H_1

$H_0 : \bar{x} = \mu$, $H_1 : \bar{x} \neq \mu$

Step2

Let the live of significance be 5%. with $n-1 = 20-1 = 19$ degree of freedom

$t_{0.05} = \cancel{20}\ 2.09$

Step3 Test

$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{42-45}{5/\sqrt{20}}$

$= \frac{-3 \times \sqrt{20}}{5} = -2.68$

$|t| = 2.68$

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Step 4 $t_{cal} = 2.68$, $t_\alpha = 2.09$ (Rejected)

Hence we concluded that assumption of mean 45 is not resonablle.

Q7. $n_1 = 8$ and $n_2 = 7$ (< 30 small sample)

Step 1 Null $H_0 = \mu_1 = \mu_2$

Alternative hypothesis $H_0 = \mu_1 < \mu_2$

Step 2 LDS = 5% (two failed test)

Degree of freedom $= n_1 + n_2 - 2 = 13$

Critical value $(t_2) = 1.77$

Step 3

$\bar{x}_1 = 1234$ hrs $\bar{x}_2 = 1036$ hrs

$$sp = \sqrt{\frac{s_1^2 n_1 + s_2^2 n_2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{36^2 \times 8 + 40^2 \times 7}{13}}$$

$$= \sqrt{\frac{10368 + 11200}{13}} = \sqrt{\frac{21568}{13}} = 40.7317$$

$$SE = sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 40.731\sqrt{\frac{1}{8} + \frac{1}{7}}$$

$$= 40.7317\sqrt{\frac{15}{56}} = 21.0806$$

Step 4 : Test statics

$$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{234 - 1036}{21.0806}$$

$$= 9.3925$$

Step 5 $t_{cal} > t_\alpha$ H_0 is rejected

H_1 is accepted

Type I is superior to Type O

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(8). Eye colour in Sons

Father Eye Color		Not light	light	Total
	Not light	230	148	378
	light	251	471	622
	Total	381	619	1000

Expected count for each cell

Cell 1 :- $\frac{378 \times 381}{1000} = 144.018$

Cell 2 : $\frac{378 \times 619}{1000} = 233.982$

Cell 3 : $\frac{381 \times 622}{1000} = 236.982$

Cell 4 : $\frac{622 \times 619}{1000} = 385.018$

$$\chi^2_{cal} = \Sigma \frac{(obser - exp)^2}{exp}$$

$$= \frac{(230-144.018)^2}{144.018} + \frac{(148-233.982)^2}{233.982} + \frac{(251-236.982)^2}{236.982} + \frac{(471-385.018)^2}{385.018}$$

$$\chi^2 = 51.33 + 31.59 + 0.82 + 19.20$$

$$= 102.94$$

$$\chi^2_{cal} > \chi^2_{tab}$$

Ho rejected.

(9). Table of leaves white / red

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	Flat leaves	Curved leaves	Total
9). White	99	30	135
Red	20	5	25
Total	119	41	160

$$\text{Cell } 1 : \frac{135 \times 119}{160} = 100.40$$

$$\text{Cell } 2 = \frac{135 \times 41}{160} = 34.59$$

$$\text{Cell } 3 = \frac{135 \times 119}{160} = 18.59$$

$$\text{Cell } 4 = \frac{25 \times 41}{160} = 6.40$$

$$X^2_{cal} = \frac{(99-100.40)^2}{100.40} + \frac{(36-34.59)^2}{34.59} + \frac{(20-18.59)^2}{18.59} + \frac{(5-6.40)^2}{6.40}$$

$$X^2_{cal} = 0.0195 + 0.0576 + 0.1069 + 0.3062$$

$$= 0.4900$$

$X^2_{cal} > X^2_{tab}$ H_0 is rejected

10. Null hypothesis

H_0 : The given data this is binomial distribution

$p = q = \frac{1}{2}$, $n = 5$, $N = 320$

Expected frequencies

No. of heads	$P(X=x) = 5C_x p^x q^{n-x}$	Expected frequency
0	1/32	10
1	5/32	50
2	10/32	100
3	10/32	100

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4 / 5	5/32	50
	1/32	10
		320

Computed of chsure values

Observed freq	Expect freq.	$(D-E)^2$	$\frac{(D-E)^2}{E}$
6	10	16	1.6
27	50	529	10.58
72	100	784	7.84
112	100	144	1.44
71	50	4.41	8.52
32	10	484	48.4
			78.68

$\chi^2_{cal} = 78.68$

$\chi^2_{cal} > \chi^2_{tab}$, Ho is rejected

(ii). Null hypothesis
The given data fits the poisson distribution
Level of significance
$\alpha = 0.05$
Computation of expected freq.

$$m = \frac{6\times1 + 5\times2 + 5\times4 + 7\times3 + 30\times2 + 72\times1}{275 + 72 + 30 + 7 + 5 + 2 + 1}$$

$$= \frac{6 + 10 + 2 + 21 + 60 + 72}{392} = \frac{181}{392}$$

$$= 0.482$$

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$$P(0) = e^{-0.482} \frac{(0.482)^0}{0!}$$

$$= 0.617$$

$$f(0) = NP(0) = 392 \times 0.617$$

$$= 241.864$$

The other expected freq. will be obtained by using recurrence formula.

$$f(x+1) = \frac{m}{y+1} \times f(x)$$

Putting $y = 0, 1, 2$ we obtain the following frequenc

$$f(1) = 0.482 \times 241.862 = 116.578$$

$$f(2) = \frac{0.482}{2} \times 116.578 = 28.095$$

$$f(3) = \frac{0.482}{3} \times 28.095 = 4.513$$

$$f(4) = \frac{0.482}{4} \times 4.513 = 0.543$$

$$f(5) = \frac{0.482}{5} \times 0.543 = 0.052$$

$$f(6) = \frac{0.482}{6} \times 0.052 = 0.004$$

x	0	1	2	3	4	5	6	Total
f	241	116	28	7	1	0	0	312

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Observed freq	Expected freq	$(O-E)^2$	$\frac{(O-E)^2}{E}$
275	241	1156	4.796
72	116	1136	16.689
30	28	4	0.142
7	4	9	2.25
5	1	16	16
2	1	4	1
1	1	0	0
			40.877

$$X^2_{cal} = 40.877$$

$$X^2_{cal} > X^2_{tab}$$

P.D. is not good fit.