Distributed Systems
CS 380D

Vijay Chidambaram

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Types of knowledge

- Common knowledge:
  - known by everyone in group
  - each node can assume others know this
- Distributed knowledge:
  - known by some members of group
  - a node **cannot** assume others know this
- Simultaneous actions requires common knowledge

Common Knowledge

- Impossible to obtain if communication is over unreliable channels
- Demonstrated in the Coordinated Attack Problem
- Internal Common Knowledge:
    - assume something is common knowledge
    - hope no node encounters state that disproves assumption

Muddy Children Puzzle

- n children play, k get muddy
- Each can observe all others, don't know their own state
- Dad says "at least one of you is muddy"
- Dad asks each of them: "do you know if you are muddy"?
- claim: After k-1 rounds, all children will answer yes

Muddy Children Puzzle

- Children get information from:
  - Observation of other children
  - Hearing what other children say
  - Inferences based on previous rounds
- Common knowledge: father says at start “at least one of you is muddy”

Proof by induction

- k = 1:
  - Muddy child observes all others are clean
  - But father said someone is muddy
  - Hence child realizes they are muddy, answers yes
  - Once other children hear muddy child answer yes, they also answer yes
- k = 2:
  - Each muddy child observes one other muddy child
  - in first round, k = 1, all answer no as they are unsure of their own state
  - Muddy child realizes they are muddy, since other muddy child answered no in first round (hence other child must see someone muddy)
  - In second round, all answer "yes"

Proof by induction

- k = 3
  - Say muddy children are a, b, c
  - if a is clean, b and c would have answered yes in second round
  - Hence a is not clean; b and c do similar reasoning
  - All answer yes on third round

Does father need to provide common knowledge?

- One might think no: for $k > 1$, seems like children get the information from direct observation
  - However, it is not common knowledge
  - For $k = 2$, muddy child a observes muddy child b. But does not know if b observes a, and therefore knows $k >= 1$

Does father need to provide common knowledge?

- Showing it does not work for k = 2:
  - Muddy children are A and B
  - In first round, even if A had seen all clean kids, they would have still answered “no” (because they do not know k >= 1)
  - In second round, A and B realizing they are muddy depends on muddy child saying yes in round 1
  - A saying “no” in round 1 does not provide B with any information
  - B still thinks k = 1 or k = 2

Does father need to provide common knowledge?

- Valid sequence if k = 1 from B's viewpoint:
  - A is only muddy child
  - A does not realize k > = 1, cannot decide between k = 0 and k = 1
  - A says "no" in first round
- B still cannot decide between k = 1 or k = 2 (both can happen with prior seq)

Common knowledge

- k >= 1 is distributed knowledge, not common knowledge
- This case clearly shows the difference between the two

Hierarchy of States of Knowledge

- Agent's knowledge depends on:
  - Starting knowledge
  - Observed history since start
- If agent i knows P then $K_i(P)$
- Agents know only true things

Hierarchy of States of Knowledge

- D(G, P) = group G has distributed knowledge of P (union of knowledge of G members = P)
- S(G, P) = someone in G knows P
- E(G, P) = everyone in G knows P
- E(G, K, P) = E(E(E.. E(G,P))))) k times
- E(E(G,P)) = everyone in G knows that everyone in G knows P
- Common knowledge: E(G, K, P) for all K >= 1

Muddy Children Puzzle

- m = “at least one child is muddy”
- Without father speaking,
  - E(G, K–1, m) = true
  - E(G, K+1, m) = false

Muddy Children Puzzle

- E(G, m) = true
- In k = 2,
  - E(G, 1, m) = true, everyone knows m
  - E(G, 2, m) = false, everyone does not know everyone knows m
  - Specifically, with muddy children A and B, A does not know B knows K >= 1
- Fathers statement makes E(G, 2, m) = true

Knowledge in distributed systems

- Communication in a distributed systems seeks to move up the hierarchy of knowledge:
  - changing S(G, P) = E(G, P) = C(G, P)
- Fact discovery:
  - Changing D to S to E to C
  - Example: finding deadlock in a set of distributed locks
- Fact publication:
  - Changing S to C
  - Example: new protocol for communication

Common Knowledge

- How does one establish it?
  - By being part of a community
    - Membership procedure imparts common knowledge
    - Example: community of licensed drivers knows what signs mean
  - By being co-present at knowledge creation
    - Example: children being in same room as father when he makes announcement

Coordinated Attack Problem

- General A sends time in message to General B
- A will not attack without ack from B
- B sends ack to A
- But B will not attack without ack from A
- A sends Ack(Ack(A)) to B
- A will not attack without ack of this message
- And so it goes.. A and B cannot agree with finite messages
- Can use induction to prove no set of K messages is enough

Coordinated Attack Problem

- Generals A and B need common knowledge of the attack time
- After A sends the first message to B, E(G) = true, but E(E(G)) is not
- After A sends the ack to B's ack, E(E(G)) = true, but E(3, G) = false
- Coordinated attack requires C(G) = E(k, G) = true for any k