THE UNIVERSITY of TULSA

PE 7813 – GEOMECHANICS

SIGNIFICANCE OF ANISOTROPY IN SHALE

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Chapter 1: Introduction

Anisotropy in shale is becoming an increasingly important subject of research. The oil industry largely uses isotropic simplification due to constraints in measuring the elastic constants associated with anisotropic models. However, this simplification can have severe consequences for drilling and completion of a well drilled in Shale. This is because shale formations have laminated structures, which result in significant differences in mechanical properties along orientations parallel to and perpendicular to bedding planes. This results in a dominant anisotropic structure. Due to the described inherent laminar structure of Shale, it is common convention to model shale as Transverse Isotropic material having its axis of symmetry in the vertical direction.

The main objective of this study is to underscore the importance of including anisotropy in shale modelling. A background on the constitutive modelling of anisotropy in shale has been presented. The model was applied to a case study and it was found that the failure to consider the effect of anisotropic behavior of shale produces largely erroneous results. These results demonstrate the importance of considering anisotropy in estimation of in-situ stresses and consecutively in wellbore stability analysis.

Chapter 2: Literature Review

Considering the effect of anisotropy in shale is a relatively new topic in the industry. Hence, the literature referred for this study is based on recent studies. Our main focus was to develop a good understanding of the theoretical modelling of anisotropy in shale as well as to learn how we can implement the available models to field data.

Safdar et al [2] pointed out the importance of shale anisotropy in estimating in-situ stresses and well bore stability analysis in Horn River Basin. They reported that there has been a significant increase in the well instability problem in shale. They suggested that this is mainly because of the failure to incorporate anisotropy is shale formations. They supported their theory by showing that the horizontal stresses were significantly higher when anisotropy was taken into account as compared to isotropic simplification case.

For being able to model anisotropy, we need to determine five independent elastic constants. This is more complicated as compared to the isotropic simplification case where we need only two independent constants. Till date, the industry has been using the simplified isotropic model due to constraints in measuring all five constants required in anisotropic modeling. However, there have been advances in petrophysical methods to measure the stiffness coefficients. Ostadhassan el al [5], showed how advanced sonic data can be used to measure the stiffness coefficient through a case study on Bakken formation. Kier et al [4] presented some technical advancements that can be applied to correct sonic logs for shale anisotropy for more accurate results.

A number of papers [1],[2],[6] were referred to study the effect of anisotropy on completions design, fracturing design and wellbore stability model. Different models were presented to model the hoop stresses in horizontal well by taking anisotropy into account.

Chapter 3: Background on Isotropy and Anisotropy

Based on the symmetry of the material, three different types of isotropy is considered in rock mechanics:

- **Completely Isotropic:** When the material properties are same in all three perpendicular directions
- **Transversely Isotropic:** When the material properties are same along the plane but different along the direction perpendicular to the plane
- **Orthorhombic symmetry:** When the material properties are different in all three different directions

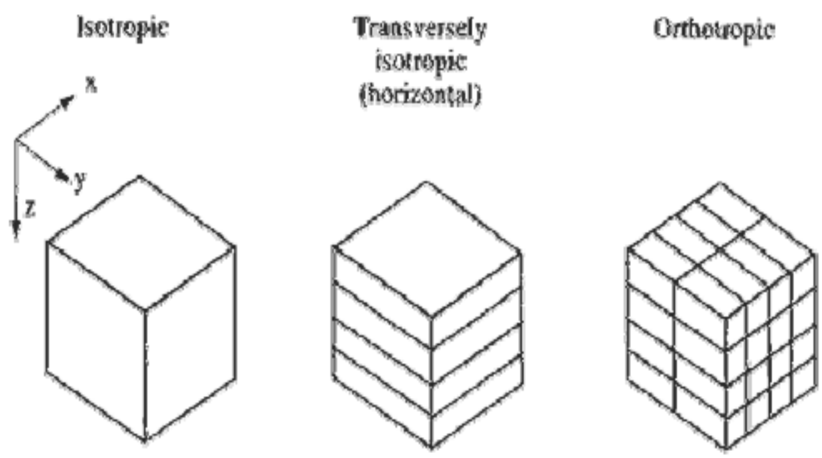

Figure 1: Different types of isotropy in rock mechanics

<u>Shale Anisotropy:</u>

Shale formation is composed of fine grained sedimentary rocks that contain substantial amount of clay minerals. Shale is a layered material composed of platy type of grains which, when compacted, distorts and elongates in one or more direction. However, the grains are aligned in parallel to sub-parallel orientation. Therefore shale is strongly anisotropic.

Since shale is a layered material it has similar properties along the plane and different properties along the direction perpendicular to the plane. Therefore, shale is modeled as a transversely isotropic (TI) material.

Chapter 4: Constitutive Modeling

Elastic properties of a material are given by Hooke's law which states that strain produced in a material is directly proportional to the applied stress

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{1}$$

Where σ_{ij} and ϵ_{kl} are second order components of the stress tensor and strain tensor respectively. C_{ijkl} denote the components of the fourth order elastic stiffness tensor.

According to the Voigt notation, there are only six independent stress components and six independent strain components. Therefore, stiffness tensor can be expressed by 36 independent coefficients.

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \tag{2}$$

Based on the symmetry of the material, the fourth order stiffness tensor can be simplified further. For an orthotropic material, the stiffness tensor has only nine independent coefficients shown below:

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \tag{3}$$

For an isotropic material, there are only two independent stiffness coefficients. The stiffness tensor is shown below:

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \tag{4}$$

For transversely isotropic material, there are five independent coefficients shown below:

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{11} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \tag{5}$$

The coefficients are related by:

$$C_{11} - C_{12} - 2C_{66} = 0 \tag{6}$$

The dynamic elastic constants can be determined from the stiffness coefficients using the following equations:

$$E' = C_{33} - 2\frac{{C_{13}}^2}{C_{11} + C_{12}} \tag{7}$$

$$E = \frac{(C_{11} - C_{12})(C_{11}C_{33} - 2{C_{13}}^2 + C_{12}C_{33})}{C_{11}C_{33} - {C_{13}}^2} \tag{8}$$

$$v' = \frac{C_{13}}{C_{11} + C_{12}} \tag{9}$$

$$v = \frac{(C_{33}C_{12} - {C_{13}}^2)}{C_{11}C_{33} - {C_{13}}^2} \tag{10}$$

Where, E and E’ denote the Young’s modulus in parallel and perpendicular direction respectively. v and v’ denote the Poisson’s ratio in parallel and perpendicular direction respectively.

Chapter 5: Measurement of Stiffness Coefficients

Advanced sonic data can be used to determine the stiffness coefficients for anisotropic shale formation. There are two type of waves commonly used in petrophysical study:

- **Longitudinal Wave Velocity:** The wave velocity direction is parallel to the particle motion
- **Shear Wave Velocity:** The wave velocity direction is perpendicular to the particle motion

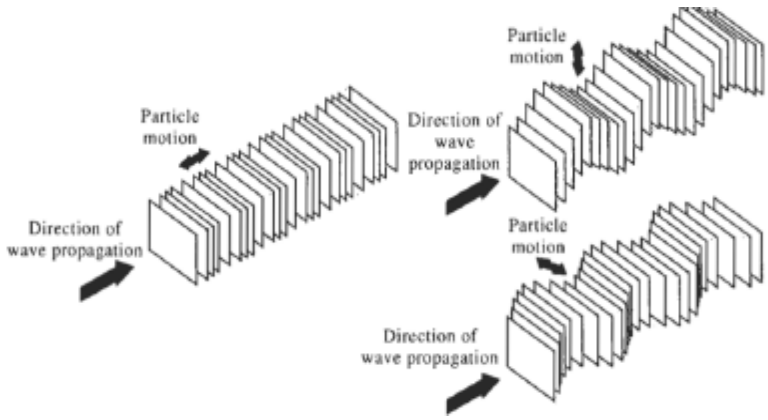

Figure 2: Longitudinal and shear wave velocity

Considering the wave propagation in a plane containing the symmetry axis, the velocities of the three modes are given by:

$$V_p(\theta) = (2p)^{\frac{-1}{2}}\sqrt{c_{11}sin^2\theta + c_{33}cos^2\theta \ + c_{44} + K(\theta)} \tag{11}$$

$$V_{SV}(\theta) = (2p)^{\frac{-1}{2}}\sqrt{c_{11}sin^2\theta + c_{33}cos^2\theta \ + c_{44} - K(\theta)} \tag{12}$$

$$V_{SH}(\theta) = \sqrt{\frac{c_{66}sin^2\theta + c_{44}cos^2\theta}{\rho}} \tag{13}$$

$$\mathrm{K}(\theta) = \sqrt{((c_{11}-c_{44})sin^2\theta + (c_{33} - c_{44})cos^2\theta)^2 \ + (c_{13} + c_{44})^2 sin^2 2\theta} \tag{14}$$

Where, Vp and Vs denote the longitudinal and shear velocity respectively and θ is the direction between wave velocity and the axis of symmetry. Solving equations (11), (12), (13) and (14) gives the stiffness coefficients.

Chapter 6: Computations and Results

Mechanical Properties Determination

The starting point of the computations conducted in the project was from determination of the stiffness coefficients required to model anisotropy in linear elastic deformation. As seen from the previous section, 5 independent stiffness coefficients were required in order to determine the mechanical properties of VTI (Vertical Transverse Isotropic) shale. Based on a cased study conducted in Bakken Shale formation by University of North Dakota [5] the values of stiffness coefficient for the Upper Bakken formation was obtained and calculated Table 1.

C13	0.93	Mpsi
C12	0.93	Mpsi
C11	4.17	Mpsi
C33	2.59	Mpsi
C66	1.62	Mpsi

Table 1: Values of calculated stiffness coefficients

Based on the values of stiffness coefficient, the poission's ratio and Young's modulus in vertical and horizontal direction is calculated using equations (15) to (18).

$$E(vertical) = C33 - 2 * \frac{C13^2}{C11 + C12} \tag{15}$$

$$E(horizontal) = \frac{(C11 - C12) * (C11C33 - 2C13^2 + C12C13)}{C11C33 - C13^2} \tag{16}$$

$$\nu(vertical) = \frac{C13}{C11 + C12} \tag{17}$$

$$\nu(horizontal) = \frac{C33C12 - C13^2}{C33C11 - C13^2} \quad (18)$$

The mechanical properties calculated for Upper Bakken formation is summarized in Table 2.

Ev	2.25	Mpsi
Eh	3.74	Mpsi
υ (Vertical)	0.18	
υ (Horizontal)	0.16	

Table 2: Calculated values of dynamics elastic constants

The Anisotropy Index, which is defined as the ratio of the young's modulus in horizontal direction to young's modulus in vertical direction, was calculated to be 1.67 for the Upper Bakken Shale.

Model Assumptions

It is important to outline the simplifying assumptions made during Geomechanical calculations conducted in the project.

- Uniaxial strain model is considered.
- Tectonic stresses and other regional stresses arising due to uplifts and geological structural complexities in the region are neglected for computation of the horizontal stresses. The horizontal stress is purely due to the gravitational effect arising due to overburden in the region.
- Normal overburden gradient is considered. (1 Psi/ft).
- Pore Pressure is calculated as 500 Psi over normal pore pressure gradient of 0.45 Psi/ft.
- The formation is considered to be homogenous.

- The effect of viscoelastic deformation (creep - change in strain W.R.T time) and temperature effects on deformation is not considered. Elastic deformation of the rock is considered.

In-situ Stresses

The determination of in-situ stresses is an important step for any Geomechanical Earth Model. The horizontal stress arises due gravitational loading. The vertical normal stress is assumed to be equal to the weight of the overlying rock and can be computed by integrating the bulk density log data.

$$\sigma(vertical) \;= g\int_0^z \rho b\,(z)\,dz \tag{19}$$

Where ρb is the bulk density of the overlying grains.

Hook's law is used in computation of the horizontal in-situ stress for isotropic medium and is given by equation 20.

It is seen that for isotropic formation, mechanical properties of the formation is not considered for calculation of in-situ stresses.

$$\sigma(horizontal) - \alpha P \;= \frac{\upsilon}{1-\nu}(\sigma(vertical) - \alpha P) + \frac{E}{1-\upsilon^2}\varepsilon H \;+\; \frac{E\upsilon}{1-\upsilon^2}\varepsilon h \tag{20}$$

Applying the uniaxial strain model and modifying the hook's law for anisotropic consideration, the equation for horizontal stress is reduced to equation 21.

$$\sigma(horizontal) - \;\alpha P \;= \frac{Eh}{Ev}\frac{\upsilon(vert)}{1-\upsilon(horz)}(\sigma(vertical) - \;\alpha P) \tag{21}$$

The value of in-situ horizontal stress is plotted with respect to the anisotropy index. It is observed that in-situ horizontal stress increases linearly with the increase in anisotropy index of the formation.

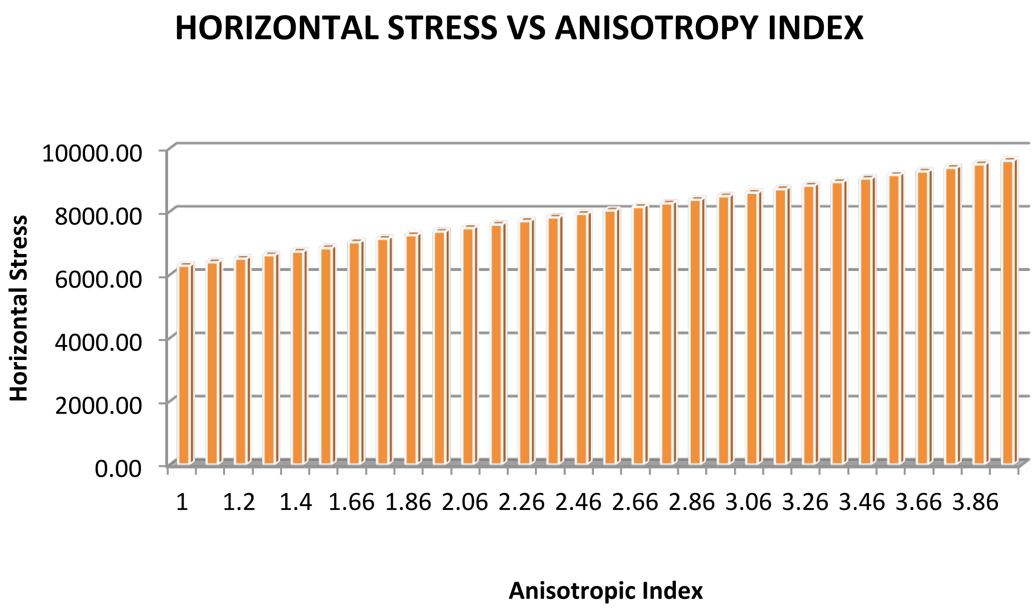

Figure 3: Horizontal stress vs anisotropic index

Wellbore Stresses in Anisotropic Formation

When a wellbore is drilled, the rock stresses in the vicinity of the wellbore are redistributed as the support originally offered by the drilled out rock is partially replaced by the hydraulic pressure of the drilling mud. The redistributed stresses (i.e. wellbore stresses) are normally referred to as the hoop stress, σ_θ, which acts circumferentially around the wellbore wall, the radial stress, σ_r, and the axial stress, σ_a, which acts parallel to the wellbore axis. Since drilling mud pressure is uniform in all directions, it cannot balance the earth stresses, which may not be equal in all directions. Consequently, rock surrounding the wellbore is strained and may fail if the redistributed stresses exceed the rock strength, either in tension or compression, resulting in instability. If hoop stress at the wellbore wall reduces to zero or to a negative value, tensile fractures (drilling induced fractures) are created along a plane perpendicular to the minimum stress, which can cause lost circulation. Alternatively, the formation can fail in compression or shear when shear stress exceeds the formation strength. In this case, the formation caves in or spalls off, creating breakouts or sometimes, hole pack off. Therefore, wellbore stresses and rock strength are key parameters in wellbore stability analysis.

In an analysis of the mud weight in vertical well for anisotropic formation W.R.T anisotropic index was computed. The near well bore stresses were calculated with the previously calculated far-field stresses. The stresses in the horizontal direction are the same as the material is modeled as Transversely Isotropic. The strength of the rock in tension was considered zero. Mohr-Coulomb's criterion was considered for failure in compression. It was assumed that the UCS of the rock is known through a vertical core sample. The cohesive strength of the rock was assumed to be 1182 Psi and internal friction angle is 42.4°. It is observed that more the mud window is increased with increase in anisotropy. The minimum mud weight is also increased but the rate of increase is less compared to the rate of increase of maximum mud weight.

In a separate analysis, maximum and minimum mud weights were calculated W.R.T wellbore inclination for different anisotropy index. It is observed that the effect of anisotropy results in increase in maximum mud weight but the maximum mud weight is reduced as inclination of the wellbore is increased narrowing the drilling mud window from vertical to horizontal.

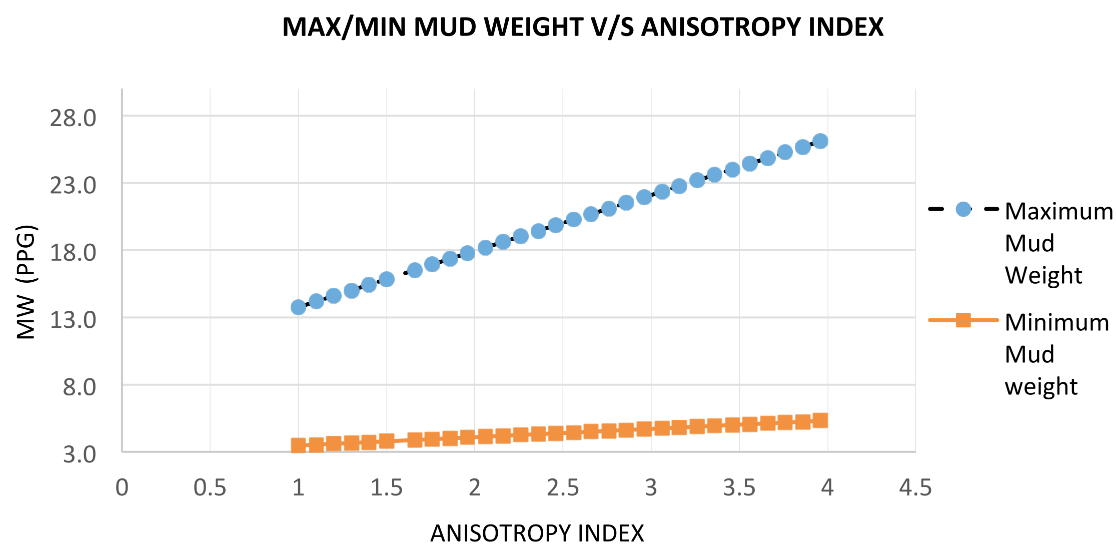

Figure 4: Max/min mud weight V/S anisotropy index

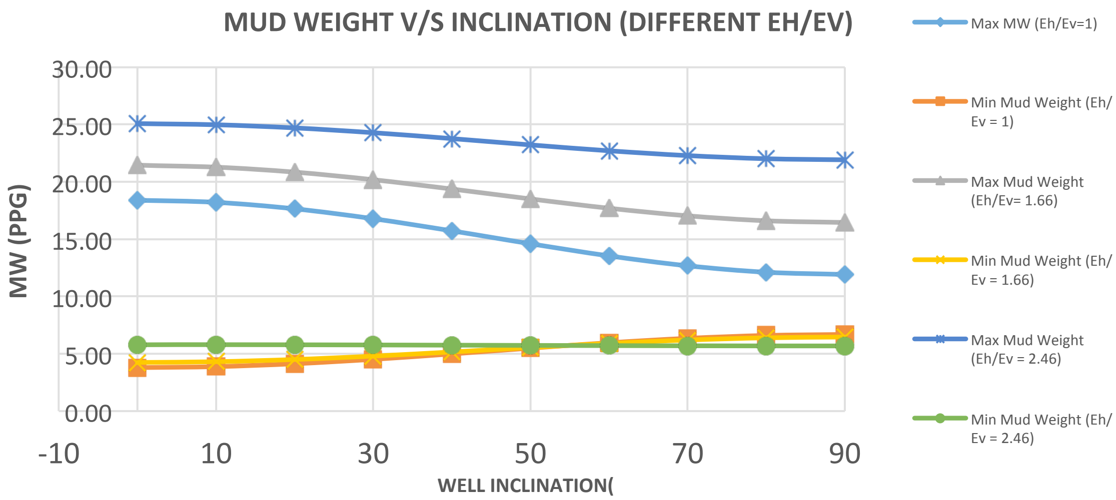

Figure 5: Mud weight vs inclination for different anisotropy index

However, it should be noted that in an anisotropic formation, the value of UCS would change with respect to orientation, which is not considered in our calculation due to unavailability of data. This can give false values of minimum mud weight as a function of inclination. Additionally, from the available literature it is concluded that the mud window is narrowed as the anisotropy is increased. The reason for this divergence in the results is due to the fact that the near well bore hoop stress calculation does not include material properties, which are important factor for anisotropic modeling.

Horizontal Well Analysis

Hoop stress around a horizontal wellbore in anisotropic formation is modeled with the implementation of modified Kirsch's equation. For an isotropic material and vertical wellbore, the hoop stress is computed by equation (22)

$$\sigma'(\theta) = (\sigma'_H + \sigma'_h) - 2(\sigma'_H - \sigma'_h)cos2\theta - (P_w - P_p) \quad (22)$$

Where σ' (θ) is the effective hoop stress and σ'_H, σ'_h are the maximum and minimum horizontal effective stresses respectively. P_w is the wellbore pressure and P_p is the pore pressure of the formation. It is clear that equation (22) does not include any material property and can give erroneous result if used for stress computation in an anisotropic model. For anisotropic material, Jagger and Cook (2003) provided equation 23 for estimation of hoop stress.

$$\sigma'(\theta) = a\sigma'_{\theta,a} + b\sigma'_{\theta,b} + c\sigma'_{\theta,P_w} \tag{23}$$

Where $\sigma'_{\theta,a}$ and $\sigma'_{\theta,b}$ are the effective hoop stresses resulting from the applied orthogonal far-field stresses, and σ'_{θ,P_w} is the hoop stress resulting from the applied wellbore over pressure. The coefficient a, b and c are a function of angular orientation around the wellbore, as well as the anisotropic material properties. Suarez-River et al [1] gave an approximation for determination of the constants given by equations (24-27).

$$a(0°) = 0.3577 \ln\left(\frac{Eh}{Ev}\right) - 0.9751 \tag{24}$$

$$a(90°) = -0.3577 \ln\left(\frac{Eh}{Ev}\right) + 2.9751 \tag{25}$$

$$b(0°) = 0.7219 \ln\left(\frac{Eh}{Ev}\right) - 2.9529 \tag{26}$$

$$b(90°) = -0.7219 \ln\left(\frac{Eh}{Ev}\right) - 0.9529 \tag{27}$$

Regardless of the orientation, c is always approximated as -1. For out computation the overbalance was not considered in the wellbore hence σ'_{θ,P_w} was zero. These approximations of constants are based on the fact that the influence of out of plane poission's ratio is small and can be neglected. The resulting constants depend exclusively on the ratio of Young's moduli. Figure 6 provides evaluation of elastic hoop stress around a horizontal wellbore as a function of anisotropy index. As can be seen from the graph, the top and bottom of the horizontal wellbore is more prone to fracturing. As the anisotropy index is increased, rock exhibits high tension on the high and low side of the borehole. In an area of high anisotropy, the wellbore may have drilling induced fractures even in underbalanced conditions.

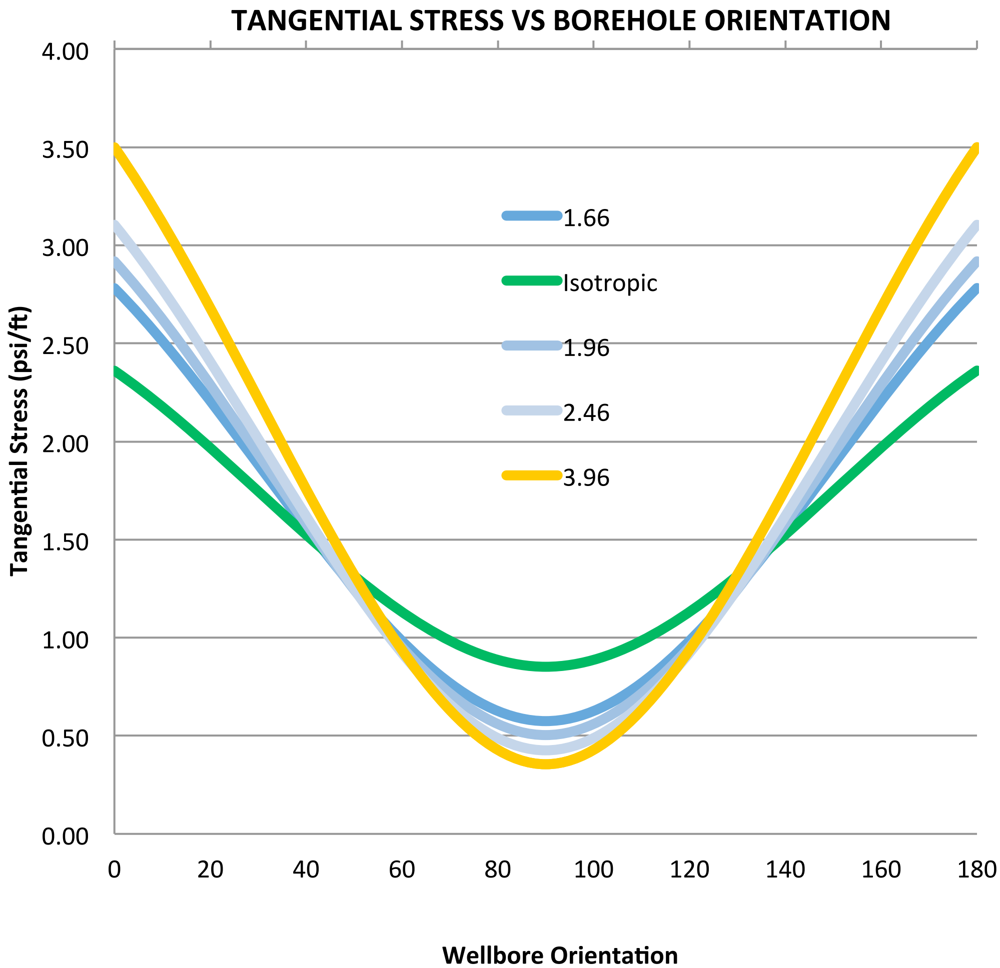

Figure 6: Tangential stress vs borehole inclination for different anisotropy

Chapter 7: Conclusions

Shale formations are strongly anisotropic due to their inherent laminated bedding structure. This results in significant differences in mechanical properties along the orientation parallel and perpendicular to the bedding plane. Failure to consider the effect of anisotropic behavior of shale can result in underestimation of in-situ stress condition. Anisotropic earth models have been available but were rarely used for petroleum related rock mechanics application till last decade. Isotropic models were most common due to their inherent simplicity and limited data requirement, however due to advancement in sonic logging tools, it is relatively easy to obtain the stiffness coefficients required to model VTI materials with a reasonable degree of accuracy.

Results suggest that observation of pervasive tensile fractures along the high and low side of a horizontal wellbore can be considered as a strong indication of anisotropic rock behavior. Anisotropy plays a significant role in completion optimization in terms of fracture initiation and propagation design. The mud window optimization during drilling operation in order to prevent wellbore collapse issues and/or losses while drilling issues can be done only when anisotropic model is used. The mud weights are highly sensitive to the degree of anisotropy in the rock however rock strength should be known by core analysis before failure criterion is applied, as UCS values are also sensitive to the direction in anisotropic formation.

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