Error Analysis Computational Homework 4

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In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import random as r
from tqdm import tqdm
import math
```

In [2]:
```
N=5
T=1
h=0.5
j=0.1/N
```

In [3]:
```
def H(p,phi):
    return 0.5*p**2 + (0.5*T*phi**2)/j -(N)*np.log(2*np.cosh(h/T+phi))
def phidot(p):
    return (p)
def pdot(phi):
    return(-phi*T/j + N*np.tanh(h/T+phi))
```

In [4]:
```
def leapfrog(p,phi,n):
    epsilon=1/n ## if the length if trajectory is 1: $p^2+ phi^2=1$
    P,Phi=p,phi
    Phi=phi+epsilon*0.5*phidot(p)
    for i in range(n):
        Phi= Phi + epsilon*phidot(P)
        P=P+epsilon*pdot(Phi)

    Phi=Phi-0.5*epsilon*phidot(P)
    return P,Phi
```

In [5]:
```
def deltaH (p,phi,P,Phi):
    Prob= np.min([1,np.exp(H(p,phi)-H(P,Phi))])
#         print(np.exp(H(p,phi)-H(P,Phi)))
    if(Prob<1):
        #print(Prob)
        a=np.random.uniform(0,1)
        if(a<Prob):
            Prob=1

    if(Prob==1):
        return Prob,P,Phi
    else:
        return Prob,p,phi
```

In [6]:
```
def autocor(marr):

    gamma = np.zeros(50)
    mavg = np.mean(marr)
    for tau in tqdm(range(50)):
        count = 0
        for k in range(len(marr)):

            for l in range(len(marr)):

                if(abs(k-l) == tau):
                    count+=1
                    gamma[tau] += (marr[k] - mavg)*(marr[l] - mavg)

        gamma[tau] *= 1/count

    Ctau = gamma/gamma[0]
    return Ctau
```

In [7]:
```
def binning(marr,b):
    Mavg=[]
    for i in range (0,len(marr),b):
        mavg=0
        for j in range(b):
            mavg+=marr[i+j]
        mavg=mavg/b
        Mavg.append(mavg)
    return(Mavg)
```

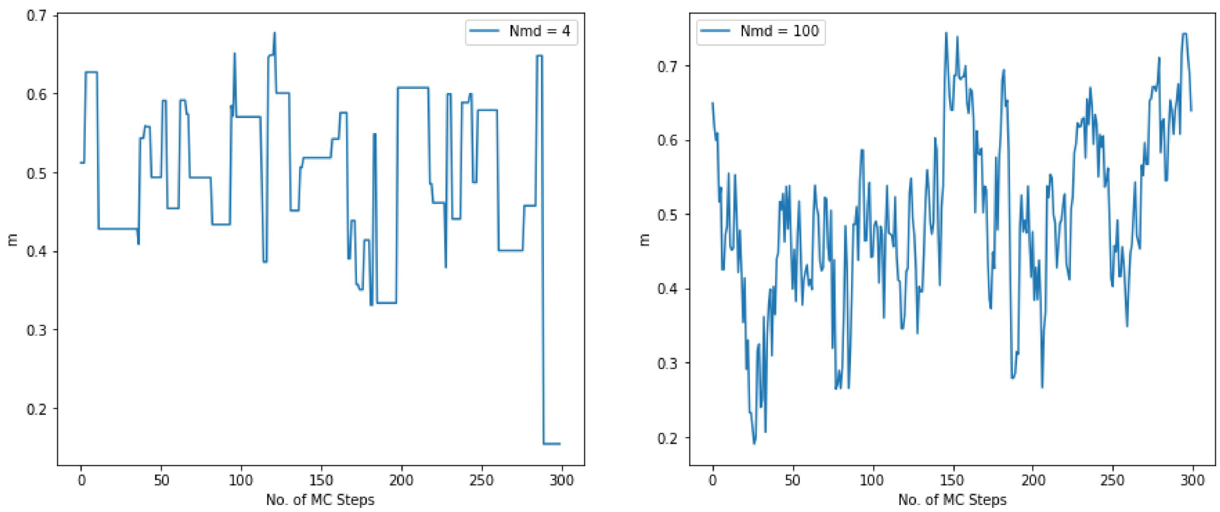
In [8]:
```
## Bootstrap error
def bootstrap(marr,Nbs):
    means = []
    for j in range(Nbs):
        Marr = []
        for i in range(len(marr)):
            Marr.append(np.random.choice(marr))
        means.append(np.mean(Marr))

    Me = np.mean(means)
    std = np.std(means)

    return std
```

In [9]:
```
phyarr=[]
prob_arr = []
marr=[]
phi=1.
for i in range(12800):
    p=np.random.normal(0,1)
    pf,phif=leapfrog(p,phi,100) ## leapfrog convergence is good enough for n=100
    prob,P_f,Phi_f=deltaH(p,phi,pf,phif)
    p,phi=P_f,Phi_f
    if(i>=6400):
        phyarr.append(phi)
        marr.append(np.tanh(h/T+phyarr[i-6400]))
        prob_arr.append(prob)
```

In [10]:
```
phyarr1=[]
marr1=[]
phi=1.
for i in range(12800):
    p=np.random.normal(0,1)
    pf,phif=leapfrog(p,phi,4) ## nmd is given to be 4
    prob,P_f,Phi_f=deltaH(p,phi,pf,phif)
    p,phi=P_f,Phi_f
    if(i>=6400):   ## thermalization
        phyarr1.append(phi)
        marr1.append(np.tanh(h/T+phyarr1[i-6400]))
```

In [11]:
```
plt.figure(figsize=(15,6))
plt.subplot(121)
plt.ylabel('m')
plt.xlabel('No. of MC Steps')
plt.plot(marr1[0:300],label = 'Nmd = 4')
plt.legend()
plt.subplot(122)
plt.ylabel('m')
plt.xlabel('No. of MC Steps')
plt.plot(marr[0:300],label = 'Nmd = 100')
#res = np.array(marr1)-np.array(marr)
#plt.plot(res[7700:],label = 'Residual')
plt.legend()
```

Out[11]:
```
<matplotlib.legend.Legend at 0x1d336286e20>
```



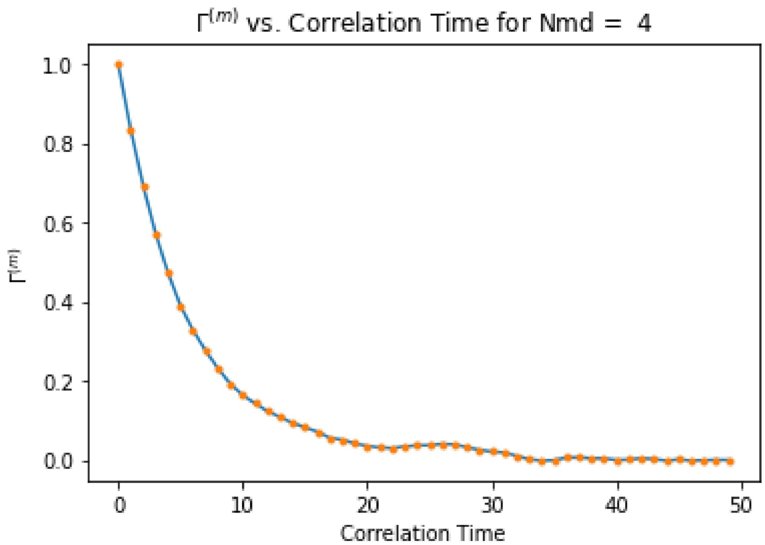
The Two trajectories are different in the sense that for $N_{md} = 4$ the graph is stable i.e. we see many straight lines in the graph which indicates that many of the proposed moves are rejected by our montecarlo algorithm i.e. the system remains in the same state for multiple moves which is not the case where $N_{md} = 100$ most of the moves are accepted and thats why our state changes rapidly.

In [12]:
```
Ctau4 = autocor(marr1)
Ctau100 = autocor(marr)
```

```
100%|██████████████████████████████████████████████████████████| 50/50 [04:00<00:00,  4.81s/it]
100%|██████████████████████████████████████████████████████████| 50/50 [04:09<00:00,  4.98s/it]
```

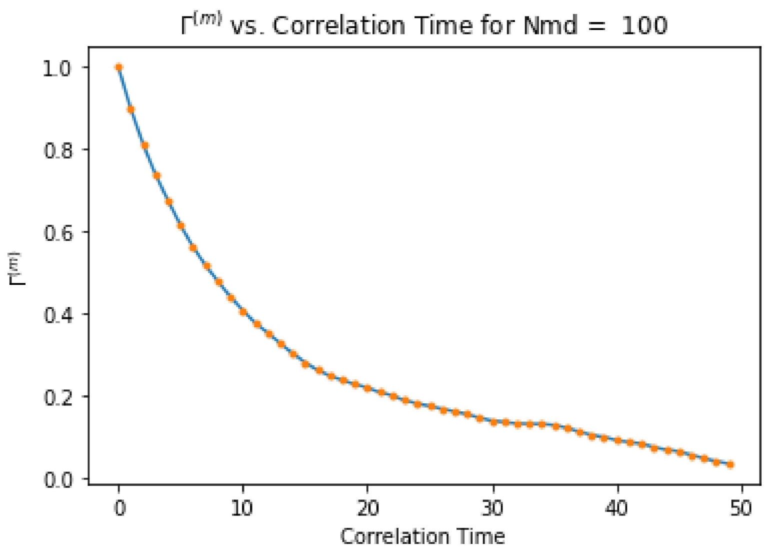
In [13]:
```
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.title('$\Gamma^{(m)}$ vs. Correlation Time for Nmd =  4')
plt.plot(Ctau4)
plt.plot(Ctau4,'.')
```

Out[13]:
```
[<matplotlib.lines.Line2D at 0x1d336363f70>]
```



In [14]:
```
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.title('$\Gamma^{(m)}$ vs. Correlation Time for Nmd =  100')
plt.plot(Ctau100)
plt.plot(Ctau100,'.')
```

Out[14]:
```
[<matplotlib.lines.Line2D at 0x1d33645e370>]
```

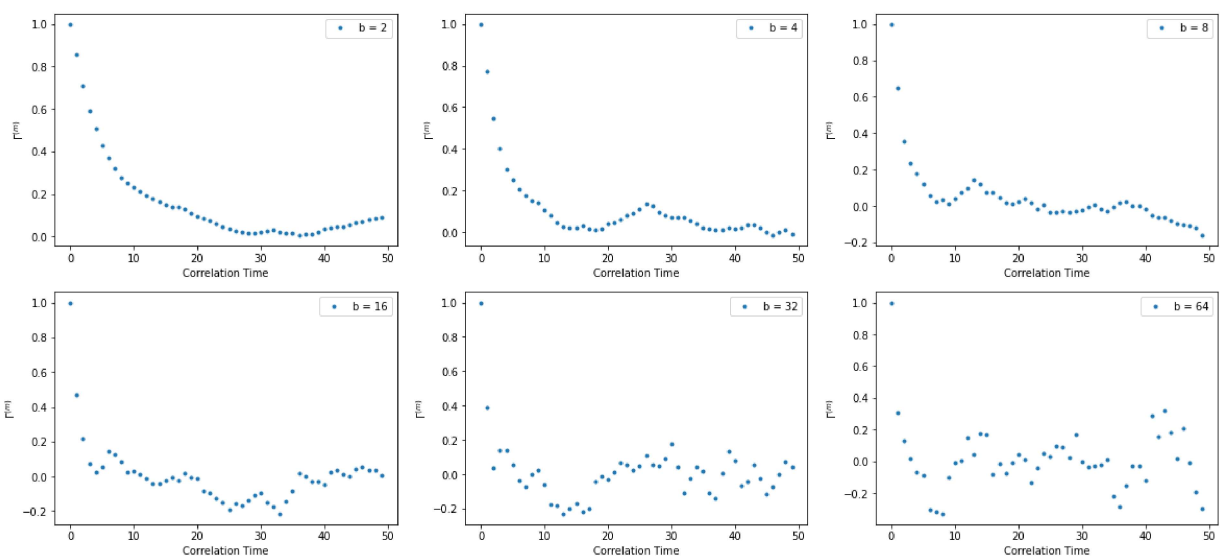


In [15]:
```
Cbinar=[]
Mbinar = []

for i in range(1,7):
    Mbin=binning(marr,2**i)
    Mbinar.append(Mbin)
    Cbin=autocor(Mbin)
    Cbinar.append(Cbin)
```

```
100%|| 50/50 [01:01<00:00,  1.24s/it]
100%|| 50/50 [00:15<00:00,  3.18it/s]
100%|| 50/50 [00:04<00:00, 12.39it/s]
100%|| 50/50 [00:01<00:00, 47.03it/s]
100%|| 50/50 [00:00<00:00, 206.56it/s]
100%|| 50/50 [00:00<00:00, 847.34it/s]
```

In [16]:
```
plt.figure(figsize = (20,9))
plt.subplot(231)
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.plot(Cbinar[0],'.',label = 'b = 2')
plt.legend()
plt.subplot(232)
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.plot(Cbinar[1],'.',label = 'b = 4')
plt.legend()
plt.subplot(233)
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.plot(Cbinar[2],'.',label = 'b = 8')
plt.legend()
plt.subplot(234)
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.plot(Cbinar[3],'.',label = 'b = 16')
plt.legend()
plt.subplot(235)
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.plot(Cbinar[4],'.',label = 'b = 32')
plt.legend()
plt.subplot(236)
plt.xlabel('Correlation Time')
plt.ylabel('$\Gamma^{(m)}$')
plt.plot(Cbinar[5],'.',label = 'b = 64')
plt.legend()
```

Out[16]: <matplotlib.legend.Legend at 0x1d336781c70>

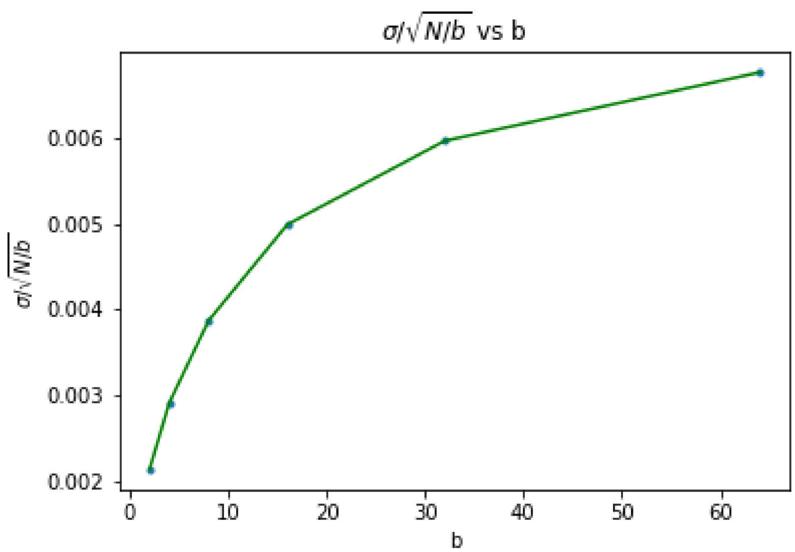


The autocorrelation between the initial consecutive points decrease as we increase the bin size similar to what we expected whereas when we keep on increasing the bin size, we see increasing fluctuation in the autocorrelation function which we didn't expect and we are not sure why that happens.

In [17]:
```
sigmaar = []
barr = []
for i in range(6):
    barr.append(2**(i+1))
    Mbinar[i] = np.array(Mbinar[i])
    sigmaar.append(np.std(Mbinar[i]))
```

In [22]:
```
sigmaar = np.array(sigmaar)
plt.xlabel('b')
plt.ylabel('$\sigma/\sqrt{N/b}$')
plt.title('$\sigma/\sqrt{N/b}$ vs b')
plt.plot(barr,sigmaar/np.sqrt(6400/np.array(barr)),'.')
plt.plot(barr,sigmaar/np.sqrt(6400/np.array(barr)),color = 'green')
```

Out[22]:
```
[<matplotlib.lines.Line2D at 0x1d33690f0d0>]
```

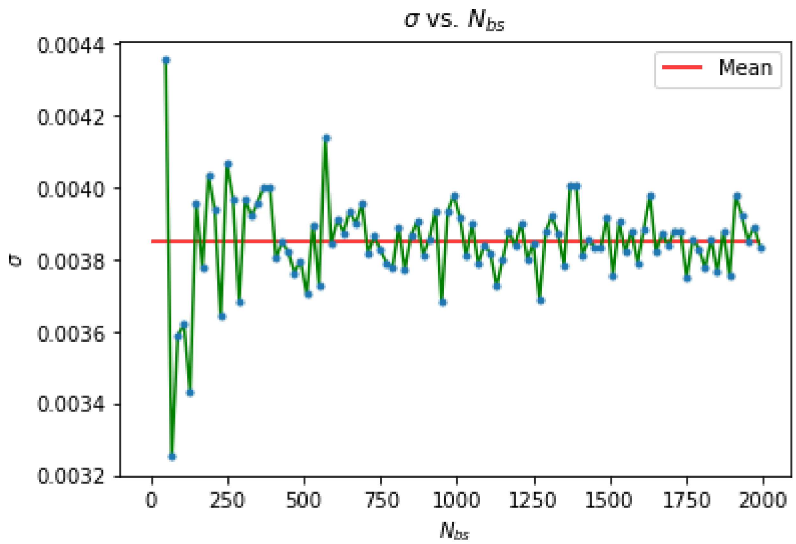


In [19]:
```
stdarr = []
Nbsarr = []

for i in tqdm(range(50,2000,20)):
    Nbsarr.append(i)
    stdarr.append(bootstrap(Mbinar[2],i))
```

```
100%|██████████████████████████████| 98/98 [05:03<00:00,  3.09s/it]
```

In [20]:
```
plt.plot(Nbsarr,stdarr,color = 'green')
plt.plot(Nbsarr,stdarr,'.')
plt.xlabel('$N_{bs}$')
plt.ylabel('$\sigma$')
plt.title('$\sigma$ vs. $N_{bs}$')
plt.hlines(np.mean(stdarr),xmin=0,xmax = Nbsarr[-1],color = 'r',label = 'Mean')
plt.legend()
```

Out[20]: <matplotlib.legend.Legend at 0x1d3368bf430>



In [21]:
```
print('Stable Value of the Error =  ',np.mean(stdarr))
print('Naive Standard Error =  ',sigmaar[2]/np.sqrt(6400/np.array(barr[2])))
```

```
Stable Value of the Error =   0.003849751340526135
Naive Standard Error =   0.003870652146633497
```

The stable value of the bootstrap error agrees very well with the Naive standard error that we computed for b = 8 earlier.

In [ ]: