Tutorial-4

① $T(n) = 3T(\frac{n}{2}) + n^2$

$a=3, \quad b=2, \quad f(n) = n^2$

Using master's theorem,

$c = \log_b a$

$c = \log_2 3 = 1.58 \; \cancel{\approx 2}$

$\Rightarrow n^c = n^{1.58} \; \cancel{\approx n^2}$

$\Rightarrow \boxed{TC = \theta(n^2)}$

② $T(n) = 4T(\frac{n}{2}) + n^2$

$a=4, \quad b=2, \quad f(n) = n^2$

$c = \log_b a$

$c = \log_2 4 = 2$

$\Rightarrow n^c = n^2$

$\Rightarrow \boxed{TC = \theta(n^2 \log n)} \quad \{\because f(n) = n^c\}$

③ $T(n) = T(\frac{n}{2}) + 2^n$

$a=1, \quad b=2, \quad f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$\Rightarrow n^c = n^0 = 1$

$\because f(n) > n^c$

$\Rightarrow \boxed{TC = \theta(2^n)}$

(4) $T(n) = 2^n T(\frac{n}{2}) + n^n$

$\because$ a is not constant

$\therefore$ Master's theorem is not applicable here.

(5) $T(n) = 16T(\frac{n}{4}) + n$

$a = 16, \quad b = 4, \quad f(n) = n$

$c = \log_b a = \log_4 16 = 2$

$\Rightarrow n^c = n^2$

$\because n^c > f(n)$

$\therefore \boxed{TC = O(n^2)}$

(6) $T(n) = 2T(\frac{n}{2}) + n \log n$

$a = 2, \quad b = 2, \quad f(n) = n \log n$

$c = \log_b a = \log_2 2 = 1$

$\Rightarrow n^c = n^1$

$\because f(n) > n^c$

$\Rightarrow \boxed{TC = O(n \log n)}$

(7) $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$

$a = 2, \quad b = 2, \quad f(n) = n/\log n$

$c = \log_2 2 = 1$

$\Rightarrow n^c = n$

$\because (n^c - f(n))$ is non polynomial difference

$\therefore$ Master's theorem is not applicable.

(8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.51}$

$c = \log_b a = \log_4 2$

$c = \log_4 4^{1/2} = \frac{1}{2}\log_4 4 = \frac{1}{2} = 0.5$

$\Rightarrow n^c = n^{0.5}$

$\Rightarrow \boxed{TC = \Theta(n^{0.51})}$

(9) $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

$\because \quad a = 0.5 < 1$

$\therefore$ Master's theorem is not applicable.

(10) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a = 16, \quad b = 4, \quad f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

$\Rightarrow n^c = n^2$

$\because \quad f(n) > n^c$

$\Rightarrow \boxed{TC = \Theta(n!)}$

(11) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4, \quad b = 2, \quad f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\Rightarrow n^c = n^2$

$\because \quad n^c > f(n)$

$\therefore \boxed{TC = \Theta(n^2)}$

(12) $T(n) = \sqrt{n}\, T\left(\frac{n}{2}\right) + \log n$

$a = \sqrt{n}$ is not constant

$\therefore$ Master's theorem is not applicable.

(13) $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3, \quad b = 2, \quad f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

$\because n^c > f(n)$

$\Rightarrow \boxed{TC = \Theta(n^{1.58})}$

(14) $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3, \quad b = 3, \quad f(n) = \sqrt{n}$

$c = \log_b a = \log_3 3 = 1$

$\Rightarrow n^c = n$

$\because n^c > f(n)$

$\Rightarrow \boxed{TC = \Theta(n)}$

(15) $T(n) = 4T\left(\frac{n}{2}\right) + cn$

$a = 4, \quad b = 2, \quad f(n) = cn$

$c = \log_b a = \log_2 4 = 2$

$\Rightarrow n^c = n^2$

$\because n^c > f(n)$

$\Rightarrow \boxed{TC = \Theta(n^2)}$

(16) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a=3, \quad b=4, \quad f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.79$

$\Rightarrow n^c = n^{0.79}$

$\because f(n) > n^c$

$\therefore \boxed{TC = \Theta(n\log n)}$

(17) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a=3, \quad b=3, \quad f(n) = n/2$

$c = \log_b a = \log_3 3 = 1$

$\Rightarrow n^c = n$

$\because n^c \approx f(n)$

$\Rightarrow \boxed{TC = \Theta(n\log n)}$

(18) $T(n) = 6T\left(\frac{n}{3}\right) + n^2\log n$

$a=6, \quad b=3, \quad f(n) = n^2\log n$

$c = \log_b a = \log_3 6$

$\because 1 < \log_3 6 < 2$

$\Rightarrow n < n^c < n^2$ which is less than $f(n)$

$\Rightarrow \boxed{TC = \Theta(n^2\log n)}$

(19) $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a=4, \quad b=2, \quad f(n) = n/\log n$

$c = \log_b a = \log_2 4 = 2$

$\Rightarrow n^c = n^2 \quad \Rightarrow \boxed{TC = \Theta(n^2)}$

(20) $T(n) = 64\, T\left(\frac{n}{8}\right) - n^2 \log n$

$\because$ $f(n) = -n^2 \log n$ is negative

$\therefore$ Master's theorem is not applicable.

(21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$c = \log_b a = \log_3 7$

$\because$ $1 < \log_3 7 < 2$

$\therefore$ $n < n^c < n^2$ smaller than $f(n)$

$\therefore$ $\boxed{TC = \Theta(n^2)}$

(22) $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

$a = 1, \quad b = 2, \quad f(n) = 2n - n\cos n$

$c = \log_b a = \log_2 1 = 0$

$\Rightarrow n^c = n^0 = 1$

$\because$ $f(n) > n^c$

$\therefore$ ~~it~~ must be in case 3

for that regularity condition is,

$af(n/b) \leq cf(n)$ where $c < 1$

$\Rightarrow 2 \cdot \frac{n}{2} - \frac{n}{2} \cos \frac{n}{2} \leq c(2n - n\cos n)$

$n - n/2 \cos n/2 \leq c(2n - n\cos n)$

if $n = 2\pi k$ where k is odd and large

$2\pi k - \pi k \cos(\pi k) \leq c(4\pi k - 2\pi k \cos(2\pi k))$

$\because$ $\cos \pi = -1$

& $\cos 2\pi k = 1$ $\{$ if k is odd $\}$

$\therefore \quad 2\pi k + \pi k \leq c(4\pi k - 2\pi k)$

$\Rightarrow \quad 3\pi k \leq c(2\pi k)$

$\Rightarrow \quad \boxed{c \geq 3/2}$

$\therefore$ $c < 1$ is not possible and hence regularity condition is not met.

$\therefore$ Master's theorem is not applicable.