Work Report on Radial Basis Function Neural Networks

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1 Derivation and Implementation

1.1 Derivation of Equations

We have completed the derivation of Models M1 - M5 as described in the paper 'On the development and performance evaluation of improved Radial Basis Function Neural Networks' using Gassuian fn.

1.2 Implementation of Equations

Implementation of the M1 and M2 is done with accuracy.

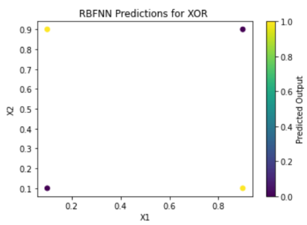
1.3 Model 1

Python implementation of Model 1 is shown with output. Below is the explanation of the implementation:

1. A Radial Basis Function Neural Network (RBFNN) class is defined.
2. Gaussian radial basis functions are used for activation.
3. Predictions are made based on the trained RBFNN.
4. The XOR dataset is used for demonstration.
5. In the predictions generated by the model, instances with values close to 0 were classified as belonging to class 0, while values diverging from 0 were categorized as class 1.
6. Mean Squared Error (MSE) is calculated to evaluate model performance.
7. A scatter plot visualizes the RBFNN's predictions for the XOR dataset.

```
import numpy as np
import matplotlib.pyplot as plt

class RBFNN:
    def __init__(self, sigma):
        self.sigma = sigma
        self.centers = np.array([[0, 0], [0, 1], [1, 0], [1, 1]])
        self.weights = None

    def _gaussian(self, x, c):
        return np.exp(-np.linalg.norm(x - c) ** 2 / (2 * self.sigma ** 2))

    def _calculate_activation(self, X):
        activations = np.zeros((X.shape[0], self.centers.shape[0]))
        for i, center in enumerate(self.centers):
            for j, x in enumerate(X):
                activations[j, i] = self._gaussian(x, center)
        return activations

    def fit(self, X, y):
        # Calculate activations
        activations = self._calculate_activation(X)

        # Solve for weights
        self.weights = np.linalg.pinv(activations.T @ activations) @ activations.T @ y

    def predict(self, X):
        activations = self._calculate_activation(X)
        return activations @ self.weights


# Example usage:
if __name__ == "__main__":
    # Define XOR dataset
    X = np.array([[0.1, 0.1], [0.1, 0.9], [0.9, 0.1], [0.9, 0.9]])
    y = np.array([0, 1, 1, 0])

    # Initialize and train RBFNN
    rbfnn = RBFNN(sigma=0.1)
    rbfnn.fit(X, y)

    # Predict
    predictions = rbfnn.predict(X)
    print("Predictions:", predictions)

    # Calculate mean squared error
    mse = np.mean((predictions - y) ** 2)
    print("Mean Squared Error:", mse)

    # Plot the results
    plt.scatter(X[:, 0], X[:, 1], c=predictions, cmap='viridis')
    plt.colorbar(label='Predicted Output')
    plt.xlabel('X1')
    plt.ylabel('X2')
    plt.title('RBFNN Predictions for XOR ')
    plt.show()
```

```
Predictions: [ 3.39868340e-17  1.00000000e+00  1.00000000e+00 -1.30570525e-17]
Mean Squared Error: 2.498330116506209e-32
```



1.4 Model 2

The implementation of Model 2 is described further:

1. Data points with noise are generated for training the RBFNet.
2. The K-means clustering algorithm is used to find cluster centers and variances from the data points.
3. The RBFNet class is initialized with hyperparameters such as the number of clusters, learning rate, and epochs.

4. The fit method trains the RBFNet using K-means clustered centers and variances.

5. The predict method makes predictions using the trained RBFNet.

6. The Radial Basis Function (RBF) calculates similarity between data points and cluster centers.

7. Expected and predicted data points are plotted for visualization.

8. Prediction accuracy and Mean Squared Error (MSE) are calculated to evaluate model performance.

```python
import matplotlib.pyplot as plt
import numpy as np
import math
import random
import statistics as stat

def generate_data_points(num_data_points):
    """Generate random data points with noise"""
    y_list = []
    desired_y_list = []
    x_list = []

    for i in range(num_data_points):
        x = np.random.uniform(0.0, 1.0)
        x_list.append(x)
        y = 0.5 + 0.4 * math.sin(3 * math.pi * x)
        noise = np.random.uniform(-0.1, 0.1)
        y_noise = y + noise
        y_list.append(y_noise)
        desired_y_list.append(y)
    return x_list, y_list, desired_y_list

def kmeans(data, num_clusters):
    """K-means clustering algorithm"""
    clusters_x = np.random.choice(np.squeeze(data[0]), size=num_clusters)
    clusters_y = np.random.choice(np.squeeze(data[1]), size=num_clusters)
    clusters = np.array([clusters_x, clusters_y])
    prev_clusters = clusters.copy()

    # Initialize variance
    variance = np.zeros(num_clusters)

    # Initialize distance matrix
    dp, num_clusters = (len(data[0]), num_clusters)
    distance = np.array([[[0.0, 0.0, 0.0] for i in range(dp)] for j in range(num_clusters)])

    # Iteratively update clusters until convergence
    converged = False
    while not converged:
        for i in range(num_clusters):
            cluster = [clusters[0][i], clusters[1][i]]
            for j in range(len(data[0])):
                dp = [data[0][j], data[1][j]]
                squared_distance = (cluster[0] - dp[0])**2 + (cluster[1] - dp[1])**2
                distance[i][j][0] = squared_distance
                distance[i][j][1] = dp[0]
                distance[i][j][2] = dp[1]
```

```python
        distanceT = distance.transpose(1,0,2)
        current_cluster_index = 0
        smallest_data_point_x = 0
        smallest_data_point_y = 0
        smallestDistance = 1000
        clusters.fill(0)
        num_dp_belongs_to_each_cluster = [1 for i in range(num_clusters)]
        cluster_dp_x = [[] for i in range(num_clusters)]
        cluster_dp_y = [[] for i in range(num_clusters)]

        for i in range(len(distanceT)):
            for j in range(len(distanceT[i])):
                dis = distanceT[i][j][0]
                if dis < smallestDistance:
                    smallestDistance = dis
                    smallest_data_point_x = distanceT[i][j][1]
                    smallest_data_point_y = distanceT[i][j][2]
                    current_cluster_index = j
            smallestDistance = 1000
            num_dp_belongs_to_each_cluster[current_cluster_index] += 1
            clusters[0][current_cluster_index] += smallest_data_point_x
            cluster_dp_x[current_cluster_index].append(smallest_data_point_x)
            clusters[1][current_cluster_index] += smallest_data_point_y
            cluster_dp_y[current_cluster_index].append(smallest_data_point_y)

        for i in range(num_clusters):
            clusters[0][i] = clusters[0][i] / num_dp_belongs_to_each_cluster[i]
            clusters[1][i] = clusters[1][i] / num_dp_belongs_to_each_cluster[i]

        converged = np.linalg.norm(clusters - prev_clusters) < 1e-6
        prev_clusters = clusters.copy()

    clusters = clusters.transpose()
    clustersWithNoPoints = []
    for i in range(num_clusters):
        dp_for_cluster = num_dp_belongs_to_each_cluster[i]
        if dp_for_cluster < 2:
            clustersWithNoPoints.append(i)
            continue
        else:
            distance_dp_to_cluster = []
            for j in range(len(cluster_dp_x[i])):
                cluster_x = clusters[i][0]
                clsuter_y = clusters[i][1]
                dp_x = cluster_dp_x[i][j]
                dp_y = cluster_dp_y[i][j]
                delta_x_square = (cluster_x - dp_x)**2
                delta_y_square = (clsuter_y - dp_y)**2
```

2 Doubts

- M-6 Model derivation:

```python
                distance_dp_to_cluster.append(math.sqrt(delta_x_square + delta_y_square))
            if len(distance_dp_to_cluster) < 2:
                variance[i] = 0
            else:
                variance[i] = stat.variance(distance_dp_to_cluster)

    if len(clustersWithNoPoints) > 0:
        avg_variance_all_other_clusters = []
        for i in range(num_clusters):
            if i not in clustersWithNoPoints:
                avg_variance_all_other_clusters.append(variance[i])
        variance[clustersWithNoPoints] = np.mean(avg_variance_all_other_clusters)

    all_same_variance = np.mean(variance)
    all_same_variance = np.array([all_same_variance for i in range(len(variance))])

    return clusters, variance

class RBFNet(object):
    """Implementation of a Radial Basis Function Network"""
    def __init__(self, k=3, lr=0.01, epochs=100):
        """Initialize the RBFNet with hyperparameters and random weights and biases"""
        self.k = k
        self.lr = lr
        self.epochs = epochs
        self.w = np.random.randn(k)
        self.b = np.random.randn(1)

    def fit(self, X, y):
        """Train the RBFNet"""
        self.centers, self.variance = kmeans(X, self.k)
        X = X.transpose()
        for epoch in range(self.epochs):
            for i in range(X.shape[0]):
                # Forward pass
                a = np.array([rbf(X[i], center, variance) for center, variance in zip(self.centers, self.variance)])
                F = a.T.dot(self.w) + self.b
                loss = (y[i] - F).flatten() ** 2

                # Backward pass
                error = -(y[i] - F).flatten()

                # Online update
                self.w = self.w - self.lr * a * error
                self.b = self.b - self.lr * error

    def predict(self, X):
        """Make predictions"""
        y_pred = []
        X = X.transpose()
```

```python
        X = X.transpose()
        for i in range(len(X)):
            a = np.array([rbf(X[i], center, variance) for center, variance in zip(self.centers, self.variance)])
            F = a.T.dot(self.w) + self.b
            y_pred.append(F)
        return y_pred

def rbf(x, centers, variance):
    """Radial Basis Function"""
    return np.exp(-np.linalg.norm(centers - x) ** 2)

# Generate random data points
x_list, y_list, desired_y_list = generate_data_points(75)
data = np.array([x_list, y_list])

# Create and train the RBFNet
rbf_net = RBFNet(lr=0.01, k=8, epochs=100)
rbf_net.fit(data, desired_y_list)

# Make predictions
y_pred = rbf_net.predict(data)

# Plot the result
plt.plot(data[0], y_list, 'ro', label='Expected')
plt.plot(data[0], y_pred, 'co', label='Predicted')

# Calculate the accuracy and mse
num_correct_prediction_points = 0
for i in range(len(y_list)):
    if abs(y_pred[i] - y_list[i]) < 1e-1:
        num_correct_prediction_points += 1
print('Prediction accuracy of points: ', num_correct_prediction_points/len(y_list))



plt.legend()
plt.tight_layout()
plt.show()

# Convert lists to NumPy arrays
y_pred_array = np.array(y_pred)
desired_y_array = np.array(desired_y_list)

# Calculate Mean Squared Error
mse = np.mean((y_pred_array - desired_y_array) ** 2)
print('Mean Squared Error:', mse)
```

  – What are the key assumptions made in the derivation of the M-6 model and how to perform the derivation?

3 Future Work

1. Discussion on FLANN

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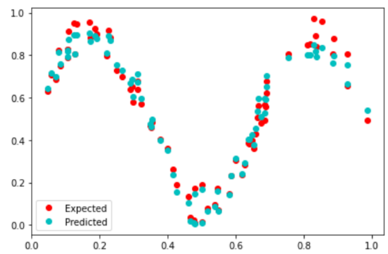
2. Discuss about exploring the use of CNN for feature extraction from images and feeding these features into an RBFNN for further processing or classification.

   - Implement and evaluate the performance of CNN-based feature extraction followed by RBFNN processing on different image datasets.
   - Investigate the impact of various CNN architectures and hyperparameters on the quality of extracted features.
   - Explore the effectiveness of RBFNNs in handling extracted features for tasks such as classification or regression.

3. Training separate CNN and RBFNN models independently and combining their predictions.

   - Develop and train standalone CNN and RBFNN models on relevant datasets.

Prediction accuracy of points: 0.9733333333333334



Mean Squared Error: 0.15263163469602117

- Investigate methods for combining predictions from CNN and RBFNN models, such as ensemble techniques or model stacking.
- Evaluate the performance improvement achieved by combining predictions compared to individual models.