CS 201
Computational Physics
Assignment 5

Group 17
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Winter Semester 2017-18

1 RC circuit with DC source

1.1 Problem Statement

RC circuit with DC source (charging and discharging of capacitor) (figure given at the end)

1.2 Assumptions

1. There is no resistance in the wires.
2. The power source, capacitors have no resistance.

1.3 Mathematical Model

1.3.1 (a) Charging of the Capacitor :

Initial Conditions :

1. R = 1 Ω
2. C = 0.2 F
3. t = 2.0s
4. $Q_o = 1$
5. $V_o = 0$
6. dt = 0.05s

Let there be a circuit with a resistor, capacitor and a power source with the capacitor initially discharged.

Using KVL in the loop we get :

$$V_o = \frac{Q(t)}{C} + I(t)R \tag{1.3.1}$$

as, $I = \frac{dQ}{dt}$

$$V_o = \frac{Q}{C} + \frac{dQ}{dt}R \tag{1.3.2}$$

Hence we will use the following solution :

$$Q_{t+1} = Q_t + \frac{V_oC - Q_t}{RC}dt \tag{1.3.3}$$

$$I_{t+1} = I_t - \frac{I_t}{RC}dt \tag{1.3.4}$$

and for V we have :

$$V_{t+1} = V_t + \frac{V_o - V_t}{RC}dt \tag{1.3.5}$$

1.3.2 (b) Discharging of the capacitor :

In this case, we assume that the capacitor is charged to a charge Q_o, and a voltage V_o. We have $Q_o = V_o C$.

In the circuit, from KVL, we have :

$$-\frac{Q}{C} = \frac{dQ}{dt}R \tag{1.3.6}$$

and hence we have the solution :

$$Q_{t+1} = Q_t - \frac{Q_t}{RC}dt \tag{1.3.7}$$

and for current :

$$I_{t+1} = I_t - \frac{I_t}{RC}dt \tag{1.3.8}$$

as Q = VC

$$V_{t+1} = V_t - \frac{V}{RC}dt \tag{1.3.9}$$

1.4 Analytical Solution

We have :

$$V_o = \frac{Q}{C} + \frac{dQ}{dt}R \tag{1.4.1}$$

on simplifying we get :

$$\frac{dt}{RC} = \frac{dQ}{V_o C - Q} \tag{1.4.2}$$

On integrating we get :

$$\int_0^Q \frac{dQ}{V_o C - Q} = \int_0^t \frac{Q}{RC} \tag{1.4.3}$$

$$-ln_e(V_o C - Q)]_0^Q = \frac{t}{RC} \tag{1.4.4}$$

On taking the exponential :

$$\frac{V_oC - Q}{V_0C} = e^{-\frac{t}{RC}} \tag{1.4.5}$$

Hence on simplyfying :

$$Q(t) = V_oC(1 - e^{-\frac{t}{RC}}) \tag{1.4.6}$$

as $V = \frac{Q}{C}$,

$$V(t) = V_o(1 - e^{-\frac{t}{RC}}) \tag{1.4.7}$$

1.4.1 (b) Discharging of a capacitor

We have :

$$-\frac{Q}{C} = \frac{dQ}{dt}R \tag{1.4.8}$$

On rearranging :

$$\frac{dQ}{Q} = -\frac{dt}{RC} \tag{1.4.9}$$

On integrating :

$$\int_{Q_o}^{Q} \frac{dQ}{Q} = \int_0^t \frac{dt}{RC} \tag{1.4.10}$$

Therefore we get :

$$ln_e Q]_0^t = -\frac{t}{RC}]_0^t \tag{1.4.11}$$

$$ln_e \frac{Q}{Q_o} = -\frac{t}{RC} \tag{1.4.12}$$

Therefore final equations we have :

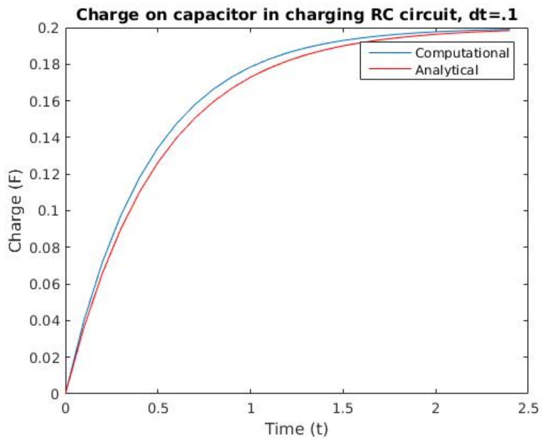
$$Q = Q_o e^{-\frac{t}{RC}} \tag{1.4.13}$$

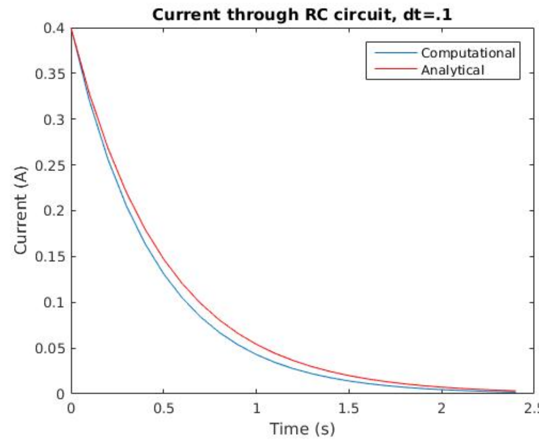
and on dividing by C :

$$V = V_o e^{-\frac{t}{RC}} \tag{1.4.14}$$

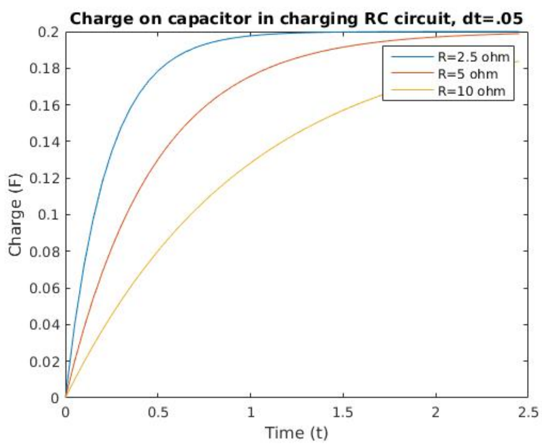
1.5 Results

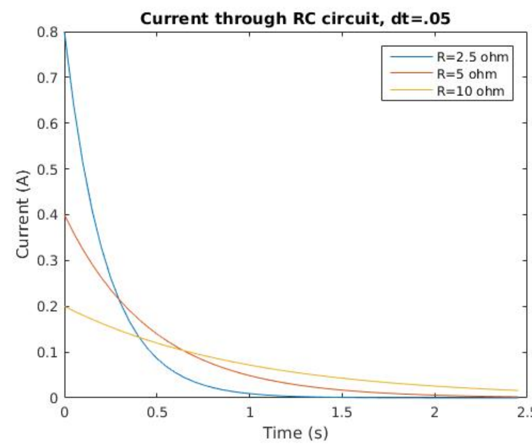
Graphs for charging RC circuit:



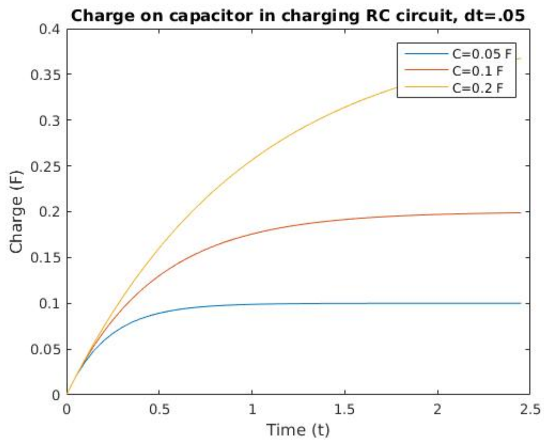


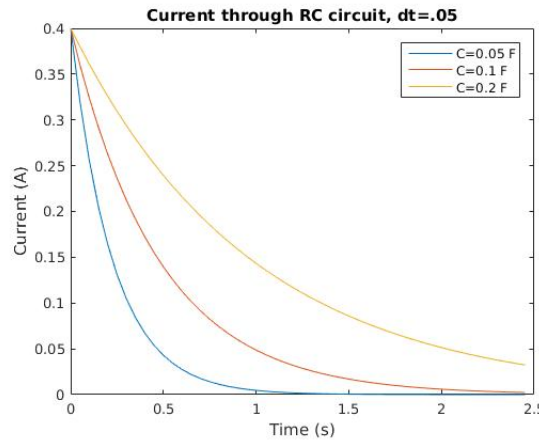
Variation of charge and current with R:



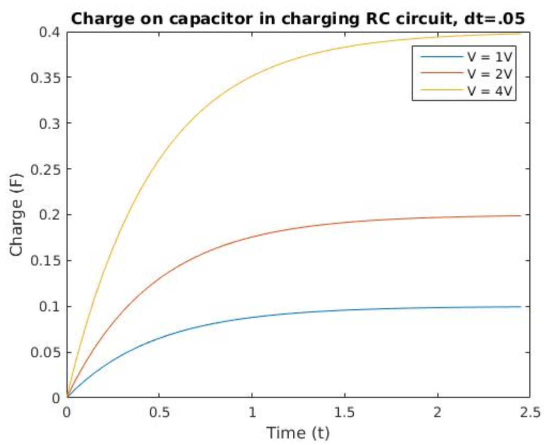


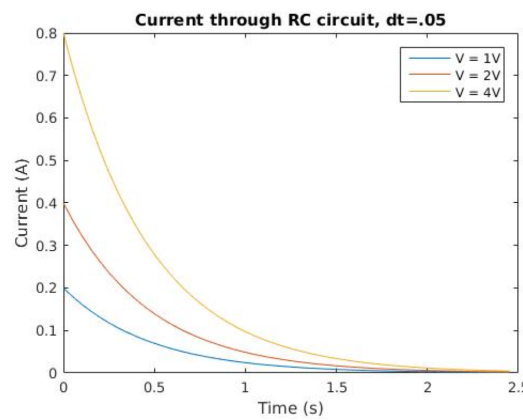
Variation of charge and current with C:



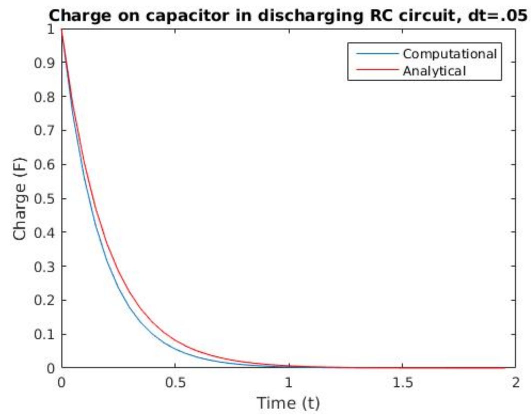


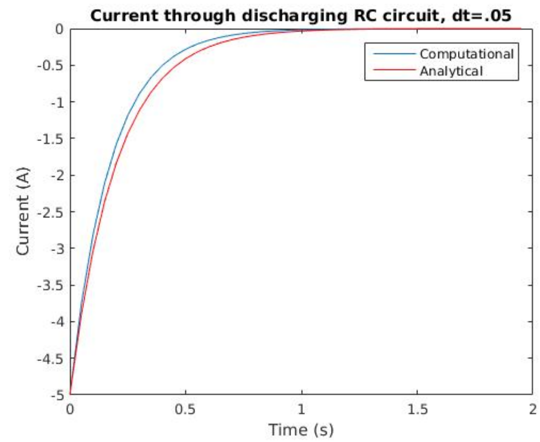
Variation of charge and current with V:



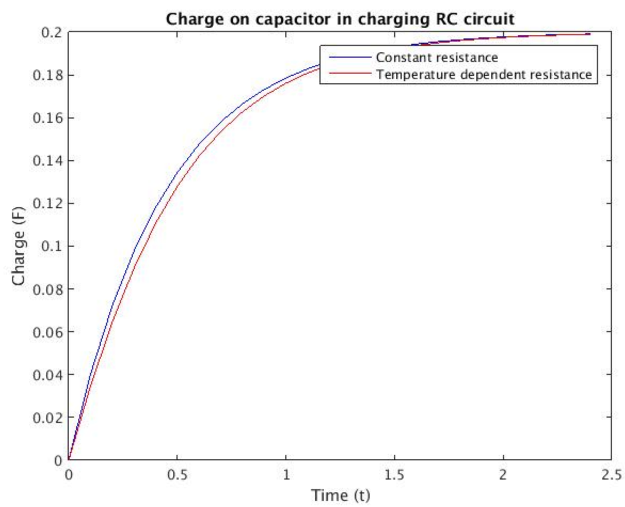


Graphs for discharging RC circuit:





Graph for temperature dependent resistance:



1.6 Observations and Interpretations

1. As $\frac{dQ}{dt}$ is inversely proportional to R, as R increases, the rate, $\frac{dQ}{dt}$ decreases and hence the time taken to reach terminal value increases.

2. Similarly, as $\frac{dQ}{dt}$ is also inversely proportional to C, higher the value of C, more the time taken to reach the terminal value.

3. As V increases, the rate of change increases and hence the time taken to achieve terminal value will decrease.

4. If it is a temperature dependent resistance, time taken for charging increases.

2 RL circuit with DC source

2.1 Problem Statement

(a) Study the behavior of V_R and V_L across R and L respectively. R=10 ohm, L=5 Henry, V=1 V, Time of simulation = 2.5 s, time-step=.1 s (b) Gradually change V, R or L only and study the behavior of i(t), di(t)/dt, V_R and V_L (c) in the problems of RL and RC circuit, note that their behavior seems to be analogous to that of falling body problem. Can you confirm this statement?

2.2 Assumptions

1. There is no resistance in the wires.
2. The power source and inductors are ideal.

2.3 Mathematical Model

Initial conditions:
$R = 10\Omega, L = 5H, V = 1V$
Time of simulation = 2.5s, dt = 0.1s

Volatge across an inductor $V_L = -L\frac{dI}{dt}$
Using Kirchoff's Voltage Law,

$$V = IR + L\frac{dI}{dt} \tag{2.3.1}$$

$$\frac{dI}{dt} = \frac{V}{L} - \frac{IR}{L} \tag{2.3.2}$$

Using Euler's method,

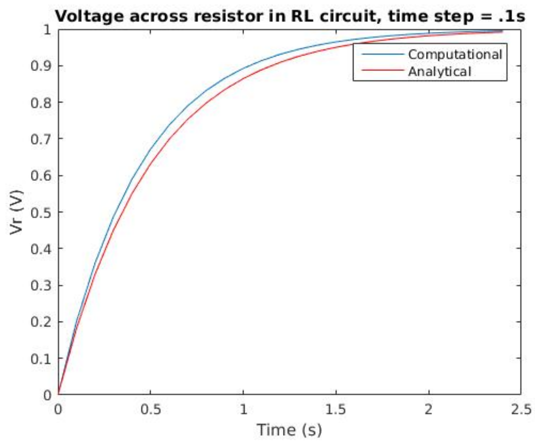
$$I_{i+1} = I_i + \frac{V - IR}{L}\Delta t \tag{2.3.3}$$

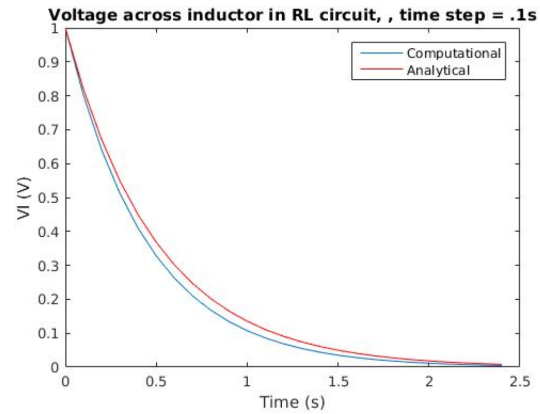
2.4 Analytical Solution

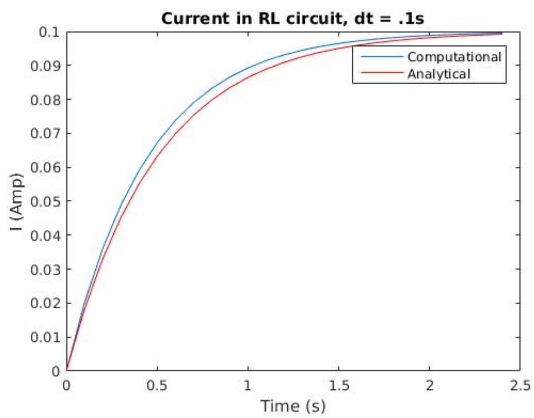
Solving equation 2.3.2,

$$I(t) = \frac{V}{R}(1 - e^{\frac{-Rt}{L}}) \tag{2.4.1}$$

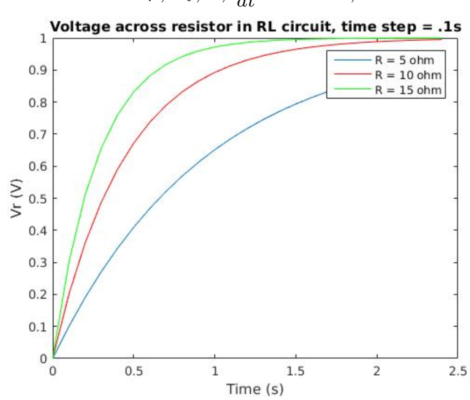
2.5 Results

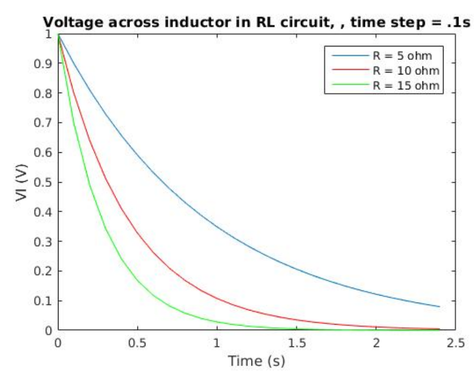


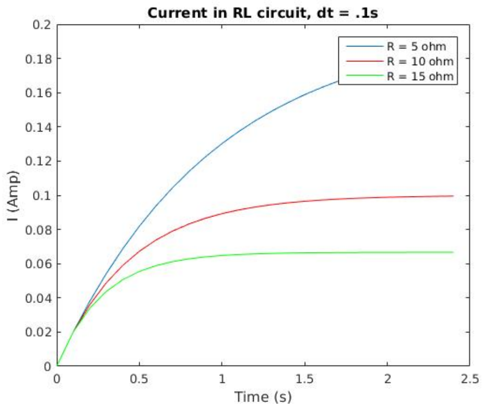


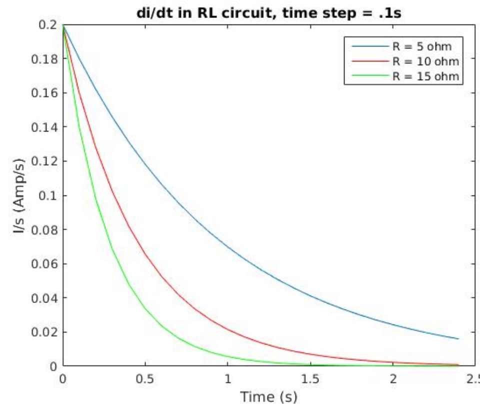


Variation of $V_r, V_l, I, \frac{dI}{dt}$ with R,

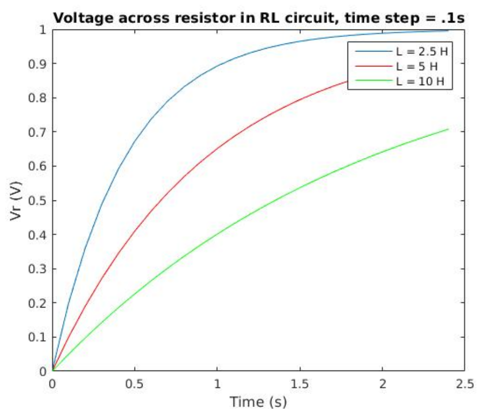


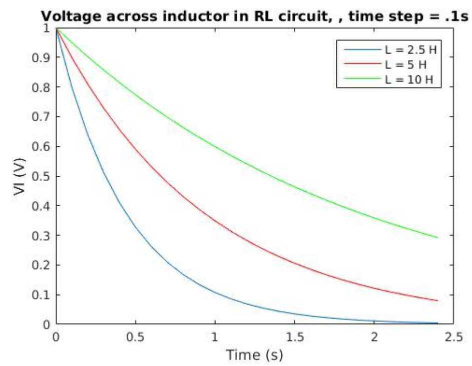


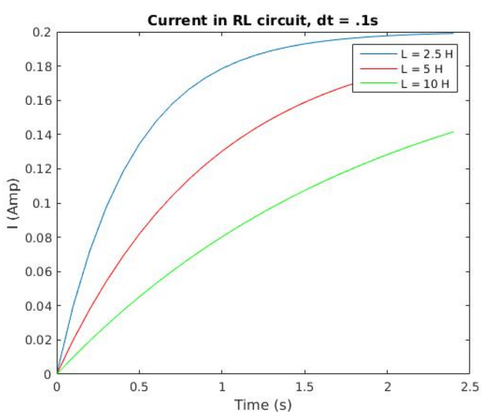


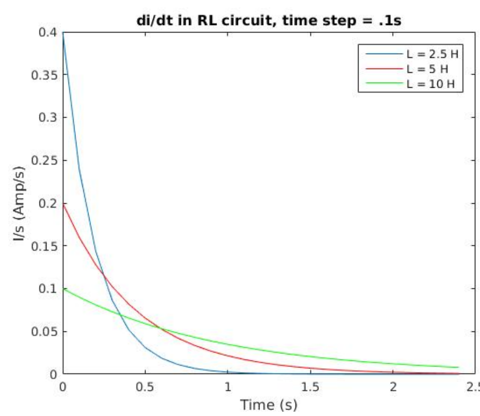


Variation of $V_r, V_l, I, \frac{dI}{dt}$ with L,

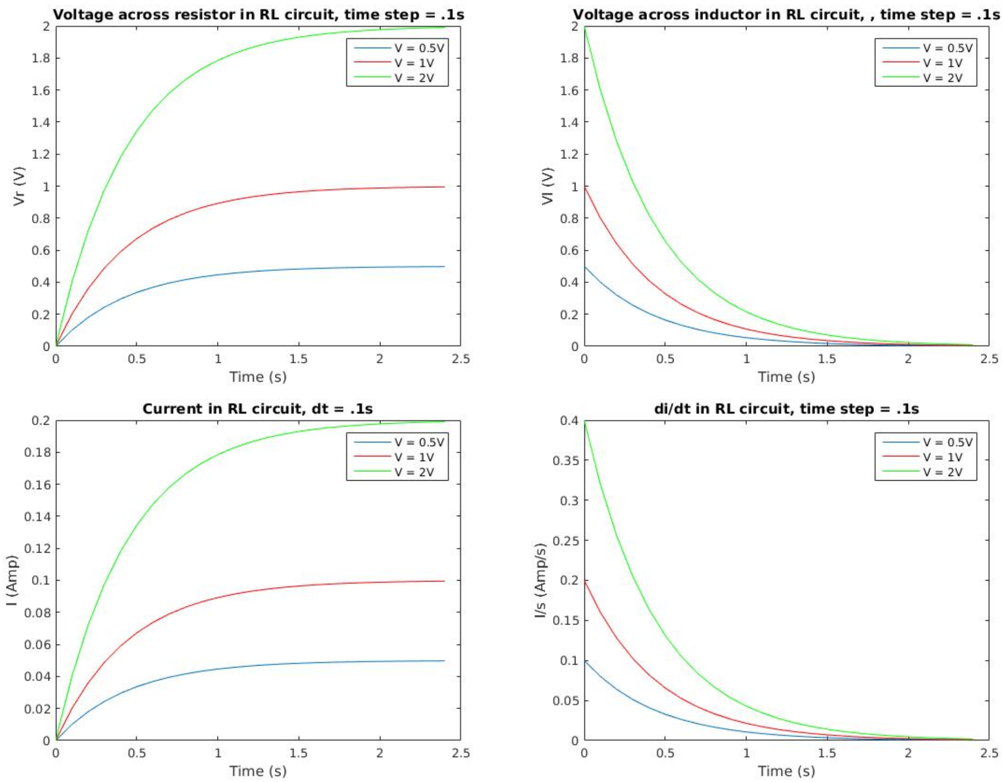








Variation of $V_r, V_l, I, \frac{dI}{dt}$ with V,



2.6 Observations and Interpretations

1. The voltage across R increases exponentially and saturates to a constant value. The voltage across L decreases exponentially and saturates to zero. The current in the circuit increases exponentially. This is because, initially at t=0, the inductor behaves as an open circuit and current is zero. As current starts increasing, the value of $\frac{dI}{dt}$ decreases (eq. 1.3.2) and current reaches a terminal value ($= \frac{V}{R}$) after a long time.

2. As R increases, in eq. 1.3.2, the value of $\frac{dI}{dt}$ decreases faster and current reaches a terminal value sooner. This terminal value ($= \frac{V}{R}$) decreases as it is inversely proportional to V.

3. As L increases, the value of $\frac{dI}{dt}$ decreases, and hence the current in the circuit increases at slower rate and takes longer to reach a terminal value.

4. As V increases, the value of $\frac{dI}{dt}$ decreases slowly, and hence the current reaches its terminal value a bit sooner. This terminal value ($= \frac{V}{R}$) increases as it is linearly proportional to V.

5. The RC and RL circuits are analogous to the falling body problem. In every case, the rates of change of charge and current are proportional to negative of charge and current respectively. Hence, as the time passes, the charge itself increases and thus decreases the rate of change of charge. After a time, it reaches a constant, terminal value. The same happens with current.
   In the case of a falling body, the retarding forces are proportional to negative of the velocity. So, as the body falls and velocity increases, the rate of change of velocity decreases and it reaches a terminal velocity after a time due to these opposing forces.

3 LC Circuit

3.1 Problem Statement

Oscillating Circuit

3.2 Assumptions

1. The wires have no resistance.
2. The components have no resistance of their own.

3.3 Mathematical Model

Initial Conditions :

1. C = 1F
2. L = 1H
3. $Q_o = 1$C
4. $I_o = 0$
5. dt = 0.1 s
6. $t_{max} = 20$s

In an LC circuit, there is oscillation of between the two components. As it involves a two degree differential equation, along with the variable itself that is

$$\frac{Q(t)}{C} + L\frac{dI_L}{dt} = 0 \tag{3.3.1}$$

as $I = \frac{dQ}{dt}$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \tag{3.3.2}$$

We used an ODE solver to solve these equations

3.4 Analytical Solution

as we have :

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \tag{3.4.1}$$

as for current :

$$I = I_L = I_C \tag{3.4.2}$$

We have :

$$I_C = C\frac{dV_C}{dt} \tag{3.4.3}$$

On rearranging we get :

$$\frac{d^2}{dt^2}I(t) + \frac{1}{LC}I(t) = 0 \tag{3.4.4}$$

Angular frequency for resonance would be :

$$X_L = X_C \tag{3.4.5}$$

i.e.

$$L\omega = \frac{1}{\omega C} \tag{3.4.6}$$

Hence,

$$\omega_o = \frac{1}{\sqrt{LC}} \tag{3.4.7}$$

Hence, applying it in the previous equation :

$$\frac{d^2}{dt^2}I(t) + \omega^2 I(t) = 0 \tag{3.4.8}$$

The solution is of the form :

$$I(t) = Ae^{+j\omega t} + Be^{-j\omega t} \tag{3.4.9}$$

On applying the following preconditions :

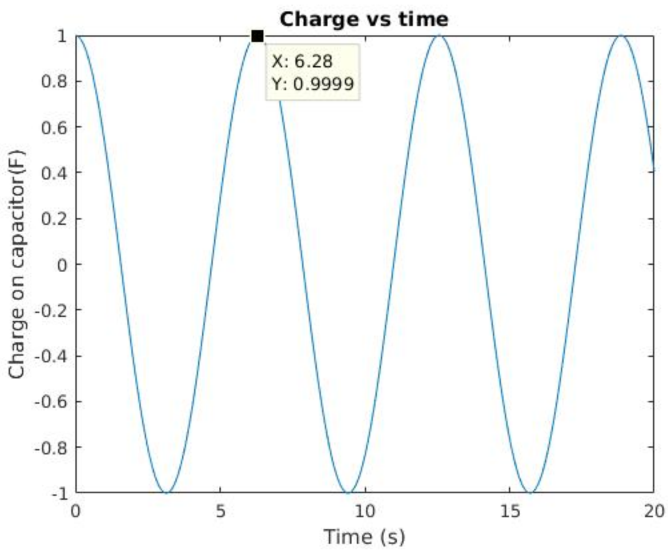
$$I(t) = I_o cos(\omega t + \phi) \tag{3.4.10}$$

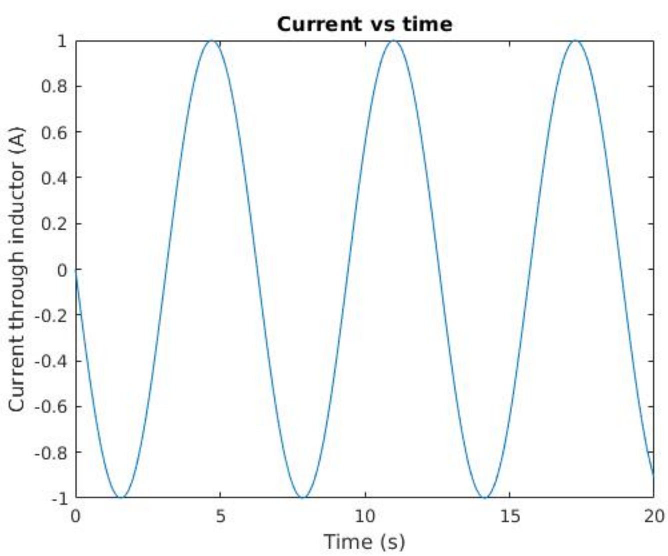
and Voltage across the inductor, V_L :

$$V_L = L\frac{dI}{dt} = -\omega_o L I_o sin(\omega_o t + \phi) \tag{3.4.11}$$

3.5 Results

Graphs for $L = 1H, C = 1F$





3.6 Observations and Interpretations

As marked in the charge vs. time graph, the time period of oscillations is 6.28s, and hence frequency is 1 rad/s. This matches with the circuit's natural frequency $\omega_o = \frac{1}{\sqrt{LC}}$. For $L = 1H, C = 1F, \omega = 1rad/s$.

4 Driven RLC circuit

4.1 Problem Statement

Computationally investigate the values of charge and current at different instants obtained for the following parameters: C= 1F, L=1 H, R=.2 ohm, initial charge=0C, initial current=0 A, amplitude and freq. od driver = 1.0, 6 Hz Time-step=.1 s, Max. time=15 s If the driving voltage is sinusoidal in nature, the problem can be solved analytically. For an external EMF E=E0 sin(w t) The current in the circuit is given by i=i0 sin(wt + phi) Phi-phase Verify and compare these results with your computational implementation. Investigate the resonance behavior and effect of driving freq. This is analogous to 1-D harmonic oscillator problem.

4.2 Assumptions

1. There is no resistance in the wires.
2. All components are ideal.

4.3 Mathematical Model

$C = 1F, L = 1H, R = 0.2\Omega$
$Q_o = 0C, I_o = 0A$
Driver voltage source amplitude $V_o = 1V$, frequency $f = 6Hz$
Time step = 0.1s, Max time = 15s

Using Kirchoff's Voltage Law,

$$V_o sin(\omega t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} \tag{4.3.1}$$

Thus,

$$\frac{di}{dt} = \frac{V_o sin(\omega t)}{L} - \frac{R}{L}\frac{dq}{dt} - \frac{q}{LC} \tag{4.3.2}$$

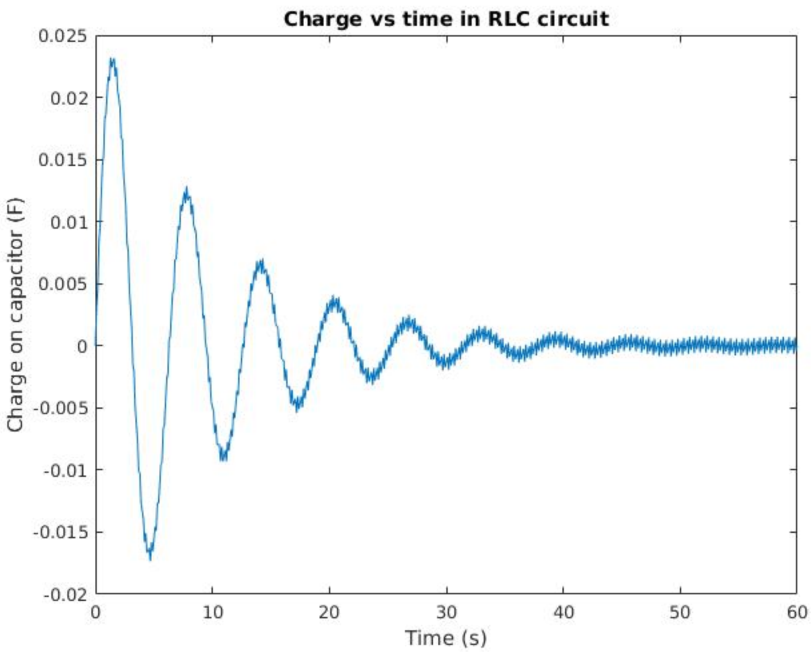
We can solve for solutions for charge and current using the ODE solver, ode45.

4.4 Analytical Solution

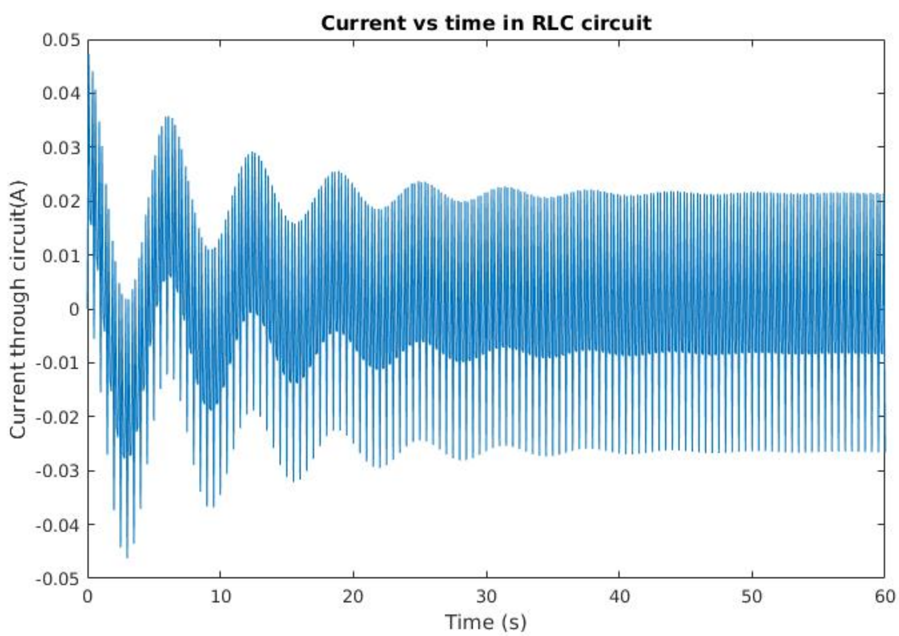
The analytical steady state solution is of the form:

$$i(t) = i_o sin(\omega t + \phi) \tag{4.4.1}$$

4.5 Results



Charge vs time in RLC circuit
Charge on capacitor (F)
Time (s)
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
0
10
20
30
40
50
60



Current vs time in RLC circuit
Current through circuit(A)
Time (s)
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
10
20
30
40
50
60

4.6 Observations and Interpretations

This is analogous to the case of overdamped oscillations. The effect of the external driving force takes over and the charge and current after a long time, oscillate at the driving oscillator's frequency (6 Hz in our case), and not at the circuit's natural frequency ($f_o = \frac{1}{\sqrt{LC}}$). This steady state current is given by equation 4.4.1.