CS 201
Computational Physics
Assignment 7

Group 17
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1 Double Pendulum

1.1 Problem Statement

Derive the Euler-Lagrange equations for double pendulum and computationally investigate the dynamics of the double pendulum.

1.2 Assumptions

1. The pendulums oscillate only in 2D.
2. No damping factor is present.
3. String is mass less and remains taut.

1.3 Mathematical Model

1. l_1 : length of the string of first pendulum.
2. l_2 : length of the string of second pendulum.
3. m_1 : mass of first pendulum.
4. m_2 : mass of second pendulum.
5. θ_1 : Angle made by pendulum 1 with the y axis.
6. θ_2 : Angle made by pendulum 2 with the y axis.

Displacements of both pendulums are given by:

$$x_1 = l_1 sin(\theta_1) \tag{1.3.1}$$

$$y_1 = -l_1 cos(\theta_1) \tag{1.3.2}$$

$$x_2 = l_1 sin(\theta_1) + l_2 sin(\theta_2) \tag{1.3.3}$$

$$y_2 = -l_1 cos(\theta_1) - l_2 cos(\theta_2) \tag{1.3.4}$$

Total Kinetic Energy of the system, T :

$$T = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 [l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 cos(\theta_1 - \theta_2)] \tag{1.3.5}$$

Total Potential Energy, V :

$$V = -(m_1 + m_2) g l_1 cos(\theta_1) - m_2 g l_2 cos(\theta_2) \tag{1.3.6}$$

$$L = T - V \tag{1.3.7}$$

Applying Euler-Lagrange equations,

$$\frac{\delta L}{\delta \theta_1} - \frac{d}{dt}\frac{\delta L}{\delta \dot{\theta_1}} \tag{1.3.8}$$

$$\frac{\delta L}{\delta \theta_2} - \frac{d}{dt}\frac{\delta L}{\delta \dot{\theta_2}} \tag{1.3.9}$$

Solving the above equations,

$$\ddot{\theta_1} = \frac{-m_2 l_2 \ddot{\theta_2} cos(\theta_1 - \theta_2) - m_2 l_2 \dot{\theta_2}^2 sin(\theta_1 - \theta_2) - (m_1 + m_2) g sin(\theta_1)}{(m_1 + m_2) l_1} \tag{1.3.10}$$

$$\ddot{\theta_2} = \frac{-l_1 \ddot{\theta_1} cos(\theta_1 - \theta_2) + l_1 \dot{\theta_1}^2 sin(\theta_1 = \theta_2) - g sin(\theta_2)}{l_2} \tag{1.3.11}$$

These are a pair of second order dependent ODEs. To solve them, we do the following:
$a = (m_1 + m_2) l_1$
$b = m_2 l_2 cos(\theta_1 - \theta_2)$
$c = m_2 l_1 cos(\theta_1 - \theta_2)$
$d = m_2 l_2$
$e = -m_2 l_2 \dot{\theta_2}^2 sin(\theta_1 - \theta_2) - g(m_1 + m_2) sin(\theta_1)$
$e = m_2 l_1 \dot{\theta_1}^2 sin(\theta_1 - \theta_2) - g m_2 sin(\theta_2)$

Then,

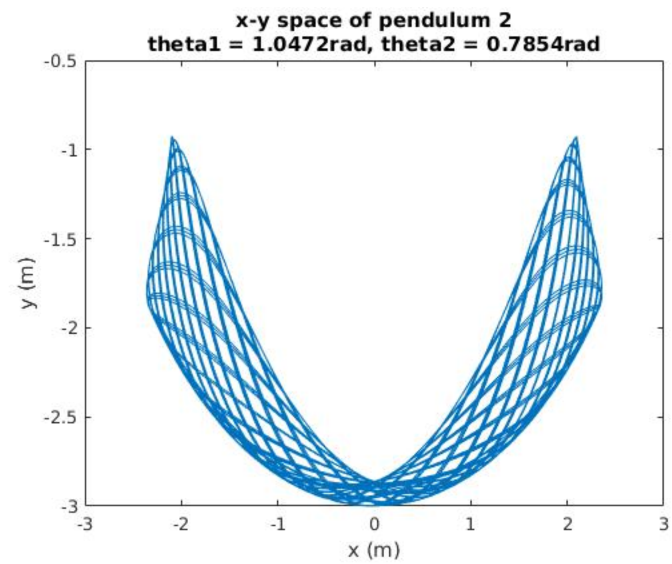
$$\ddot{\theta_1} = \frac{de - bf}{ad - bc} \tag{1.3.12}$$

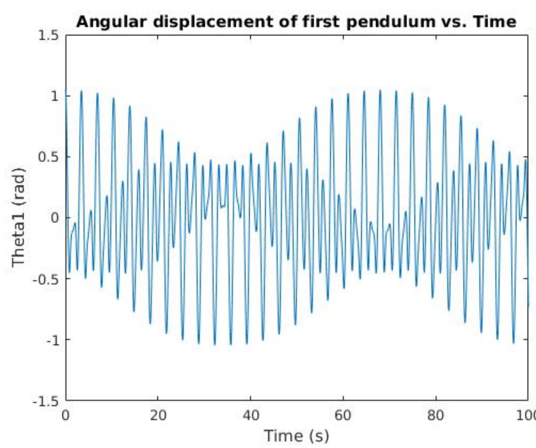
$$\ddot{\theta_2} = \frac{af - ce}{ad - bc} \tag{1.3.13}$$

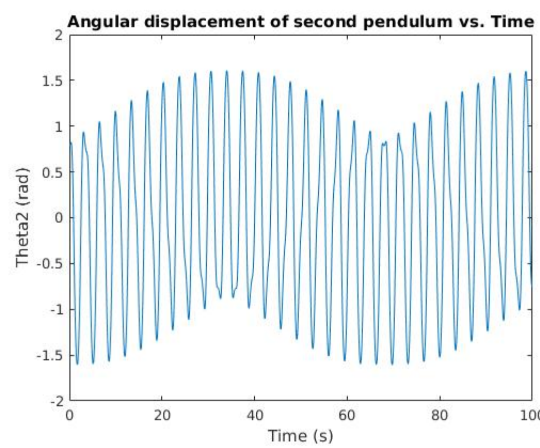
These equations are now solved using an ODE solver.

1.4 Results and Observations

For all the following cases, initial angular velocity of both pendulums is 0.
For small angles of initial inclination: **Case 1:** $\theta_1 = 60^o, \theta_2 = 45^o$ **-**

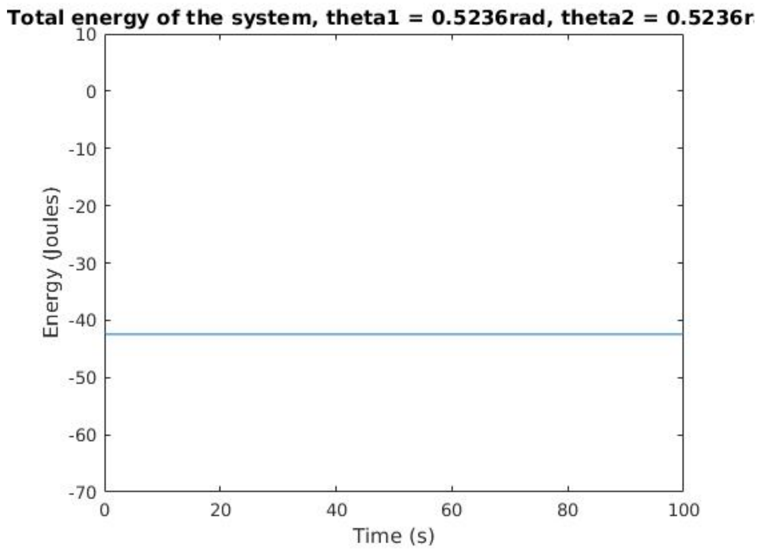




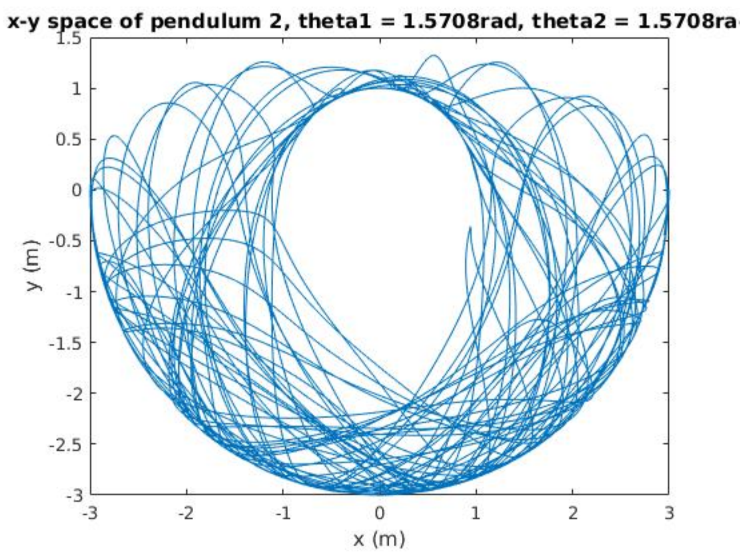


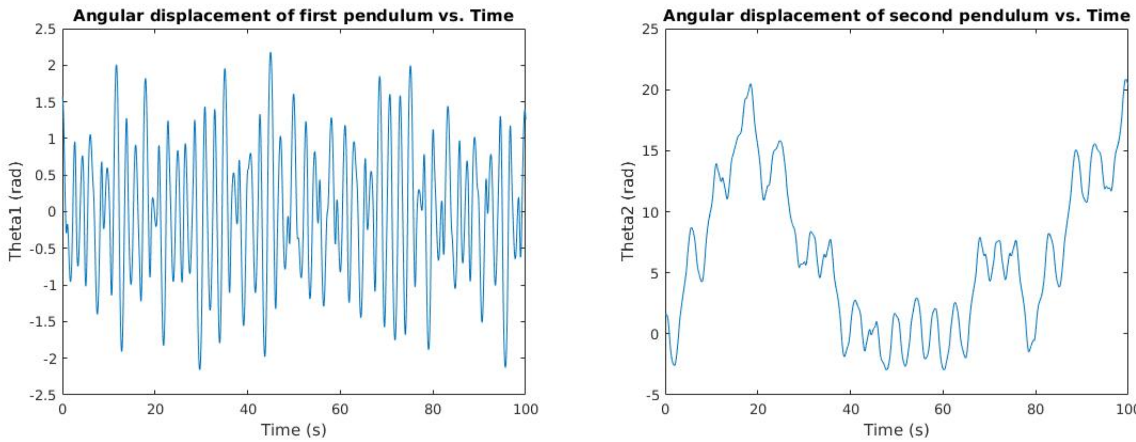
The motion of the second pendulum is not very chaotic. The angular displacements of both pendulums follow a consistent pattern.

To check the accuracy of our solution, we plot the total energy of the system. If the result is accurate, it should be constant with time (by conservation of energy).



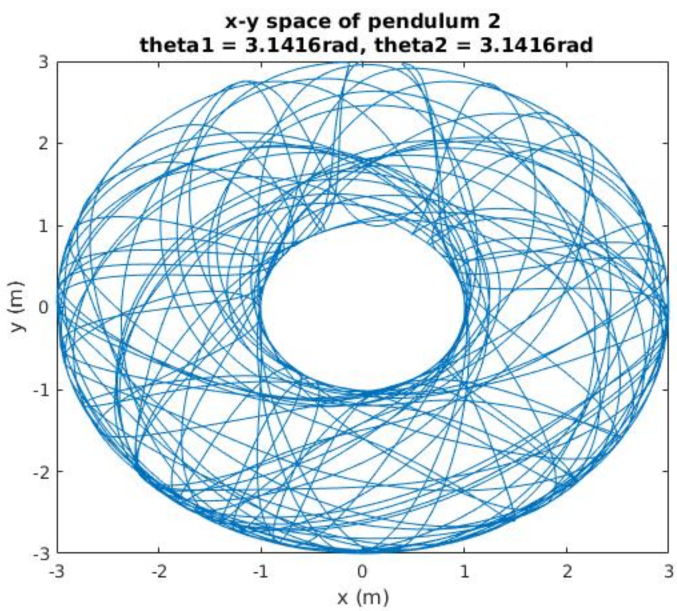
Case 2: $\theta_1 = 90^o, \theta_2 = 90^o$ -

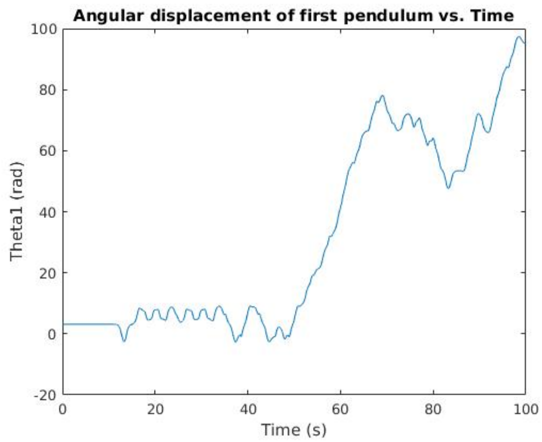


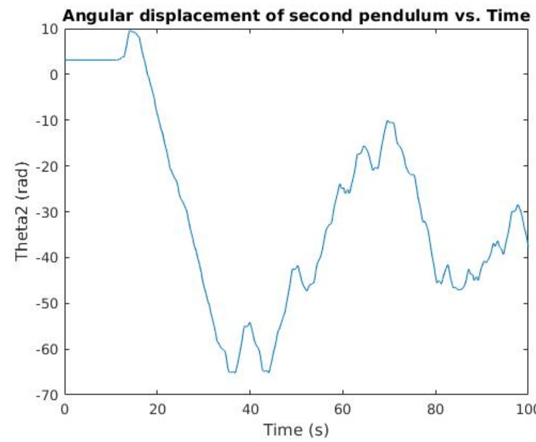


The motion of the second pendulum is more chaotic and it starts overturning and completing full rotations around its pivot . The angular displacements also vary erratically.

Case 3: $\theta_1 = 180^o, \theta_2 = 180^o$ -

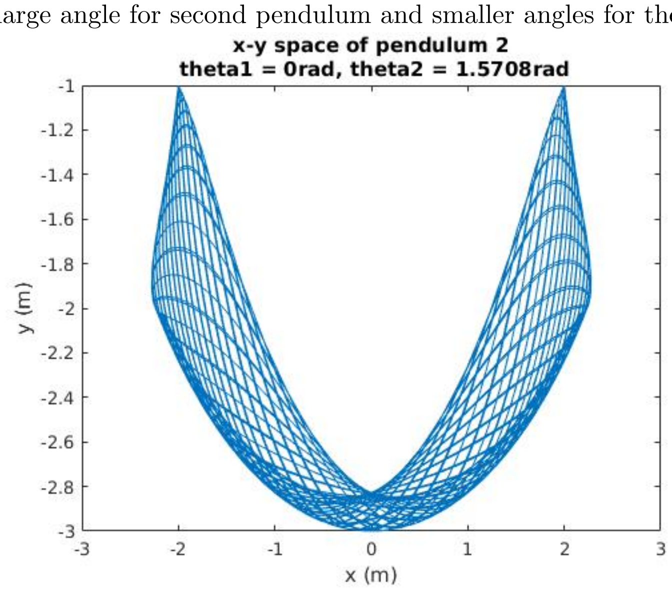






For large initial angular displacements for both pendulums, the motion is chaotic. The second pendulum overturns and crosses rotation by 90^o multiple times. The angular displacements vary very erratically.

Case 4: $\theta_1 = 0^o, \theta_2 = 90^o$ **-**
(large angle for second pendulum and smaller angles for the first pendulum):



The motion of the second pendulum is not chaotic, and its angular displacement follows a consistent pattern.
Thus,
When both initial angles are small, the motion of the second pendulum is not chaotic. For large initial angle for second pendulum (90^o), and small angles

for first pendulum, motion remains non chaotic. For larger angles than these two cases, the motion is chaotic. On increasing the mass and decreasing length, motion becomes more chaotic.