CS 201
Computational Physics
Assignment 4

Group 17
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Winter Semester 2017-18

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1.1 Problem Statement

Charge Particle trajectories under Lorentz force:
Write a general MATLAB code to reproduce charge particle motions under Lorentz force in the following cases :

1. Static and Uniform E field.
2. Static and Uniform B field
3. Static and uniform E,B (E x B drift).
4. Static and uniform B with gravitational acceleration for different masses.

1.2 Assumptions

1. There is no other force acting like friction or drag.

1.3 Mathematical Model

1.3.1 Equations :

The force experienced by a charged particle in an electric field is :

$$\vec{F}_E = q\vec{E} \tag{1.3.1}$$

The force experienced by a charged particle in a magnetic field is :

$$\vec{F}_B = q(\vec{v}X\vec{B}) \tag{1.3.2}$$

When, magnetic force acts, the force is perpendicular to the velocity of the particle at every moment, and hence the particle moves in circular motion . The radius of the circle is given by :

$$\frac{mv^2}{r} = qvB \tag{1.3.3}$$

On dividing by v and qB, we get :

$$r = \frac{mv}{qB} \tag{1.3.4}$$

1.4 Analytical Solution

1.4.1 Case 1. Static and Uniform $\vec{E}$

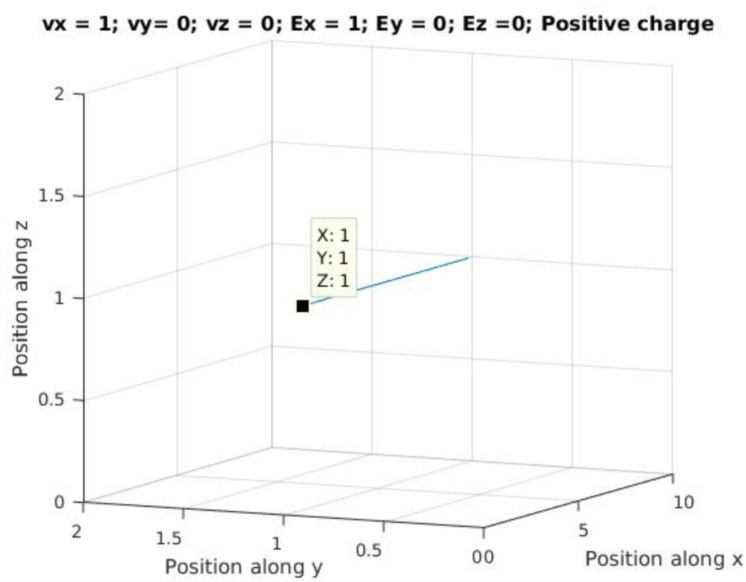
In this case, the charged particle would move in a straight line in the direction of/opposite the electric field, depending upon the charge of the particle.

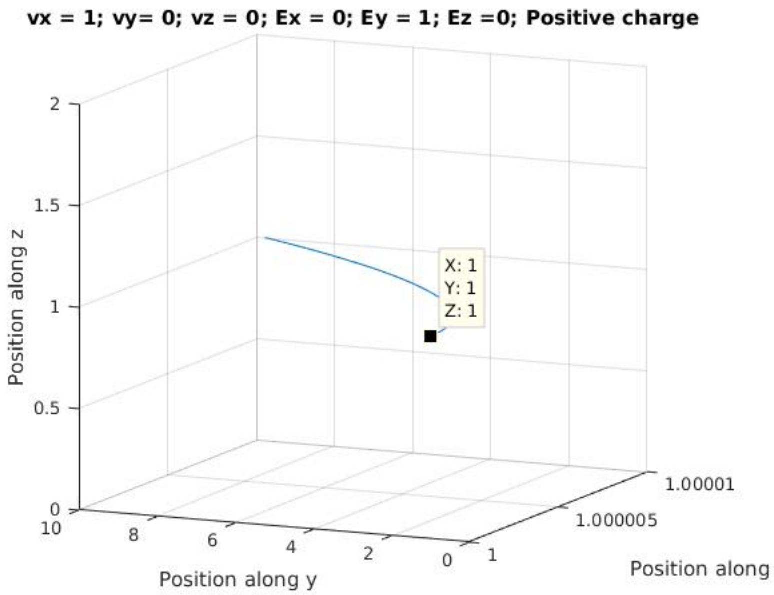
The Force will be :

$$\vec{F} = q(E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}) \tag{1.4.1}$$

The acceleration is given by :

$$\vec{a} = \frac{q\vec{E}}{m} \tag{1.4.2}$$

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Y: 1
Z: 1
Position along z
Position along y
Position along
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1
0.5
0
10
8
6
4
2
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1
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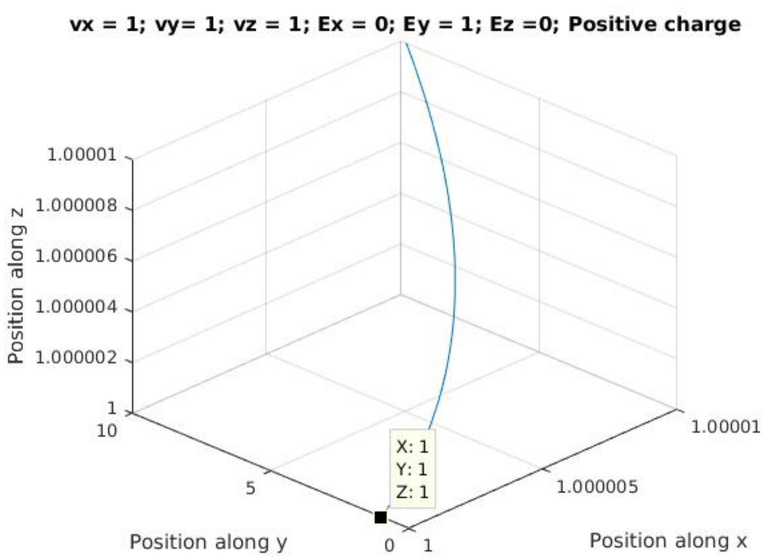
1.00001


vx = 1; vy= 1; vz = 1; Ex = 0; Ey = 1; Ez =0; Positive charge
X: 1
Y: 1
Z: 1
Position along z
Position along y
Position along x
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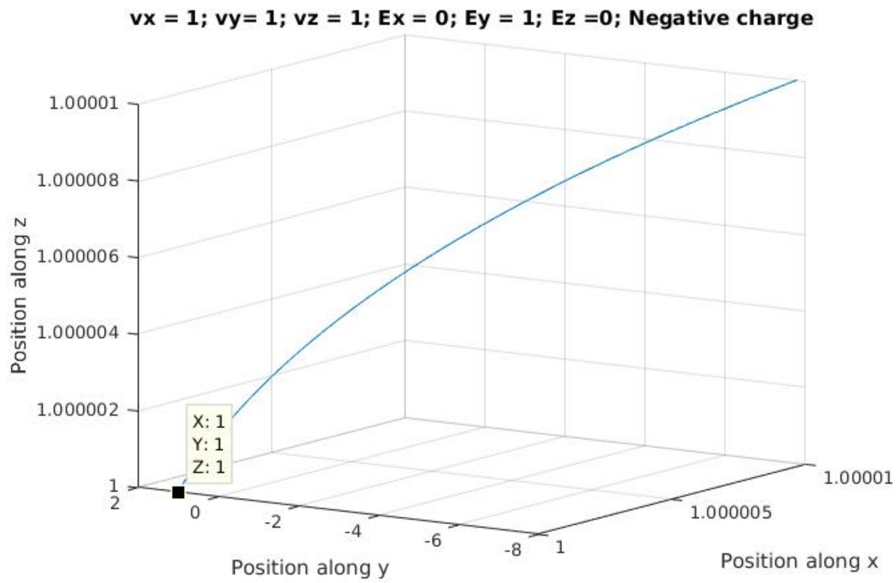
1.000002
1
10
5
0
1
1.000005
1.00001



1. If the object is at rest, it will follow a straight line along /opposite the direction of the field depending on the charge of the particle.
2. If the body has an initial motion, if it is in a perpendicular direction, it will describe a parabolic motion.
3. If the body has initial motion opposite to the field,with perpendicular components, it will describe a projectile trajectory.

1.4.2 Case 2. Static and Unifrom $\vec{B}$

In this case, the particle will go in circular motion, if it only has velocity perpendicular to that of magnetic field and if there is a velocity parallel to the magnetic field, the particle will descirbe a helical motion with a constant pitch.

Here, radius of the helix would be :

$$r = \frac{mv_{per}}{qB} \tag{1.4.3}$$

here, v_{per}, is the component of velocity, perpendicular to the magnetic field.

For the pitch of the helix :

$$pitch, p = v_{||}t \tag{1.4.4}$$

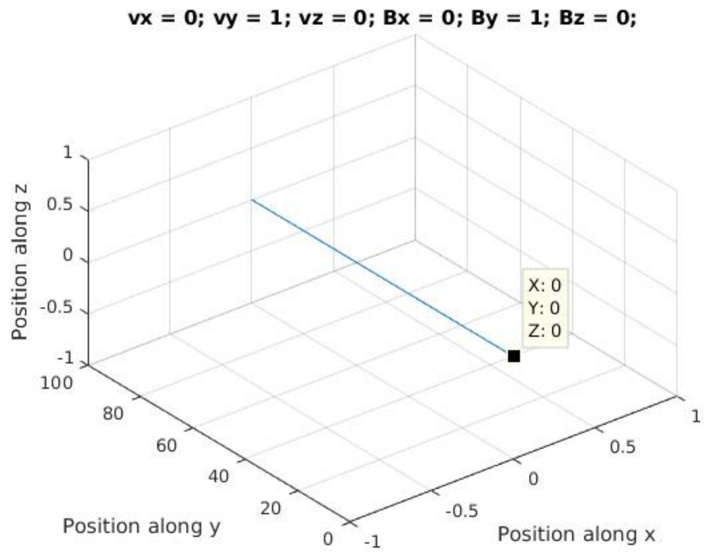
here, $v_{||}$, is the component of velocity, perpendicular to the magnetic field.

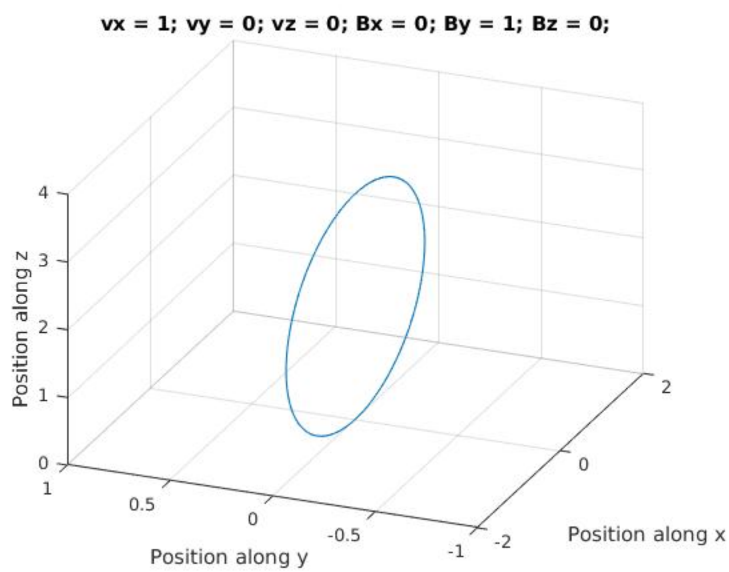
Here, time t is the time taken to complete one full revolution :

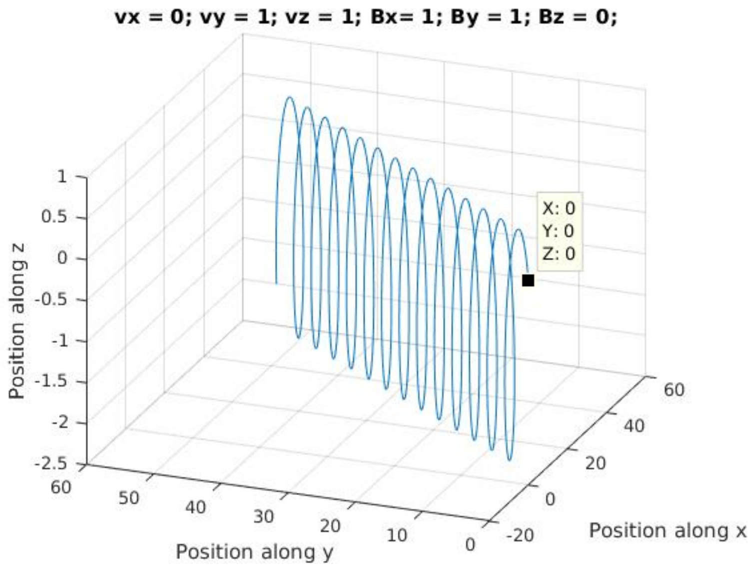
$$t = \frac{2\pi r}{v} \tag{1.4.5}$$

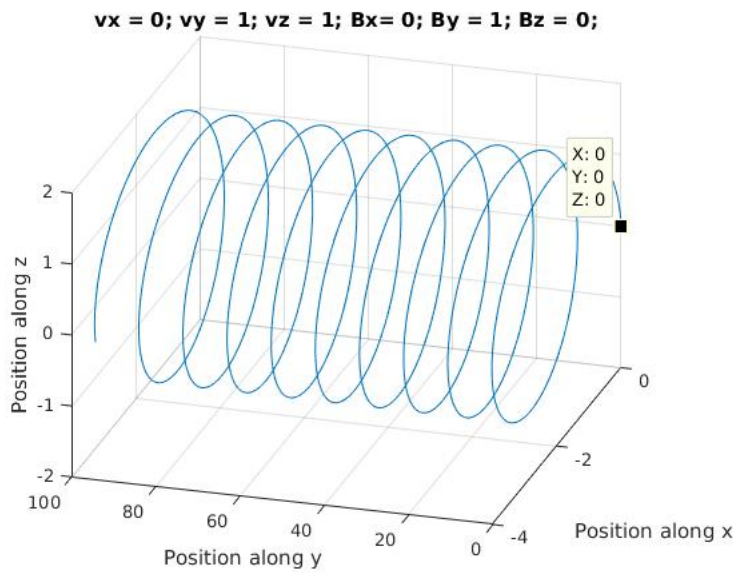
Hence

$$p = \frac{2\pi m}{qB} \tag{1.4.6}$$









In this case, the particle will describe a circular motion perpendicular to the magnetic field. If there is a component parallel to $\vec{B}$, we'd get helical motion with constant pitch.

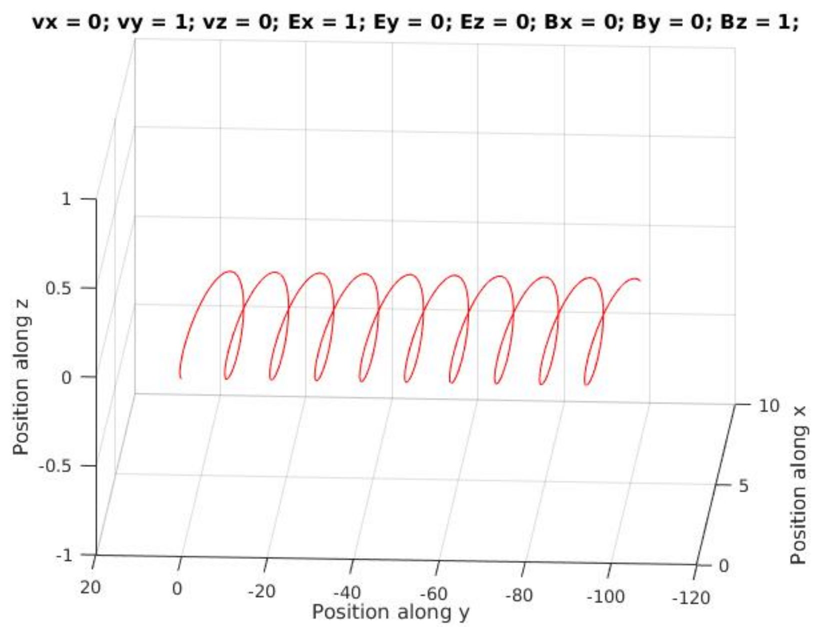
1.4.3 Case 3. Static and Uniform $\vec{E}, \vec{B}$

Here,

$$F = q(\vec{E} + \vec{v} X \vec{B}) \tag{1.4.7}$$

Also we'd have a drift velocity due to the electric field. the drift velocity v_d is given by :

$$v_d = \frac{\vec{E} X \vec{B}}{B^2} \tag{1.4.8}$$



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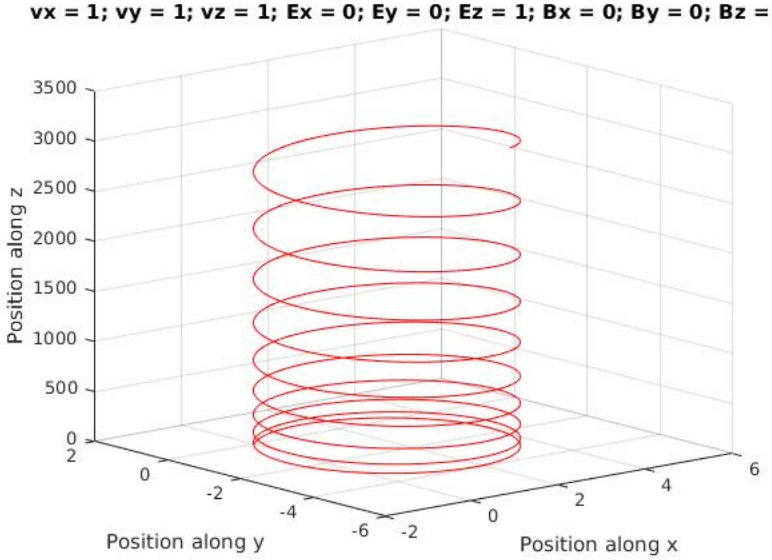

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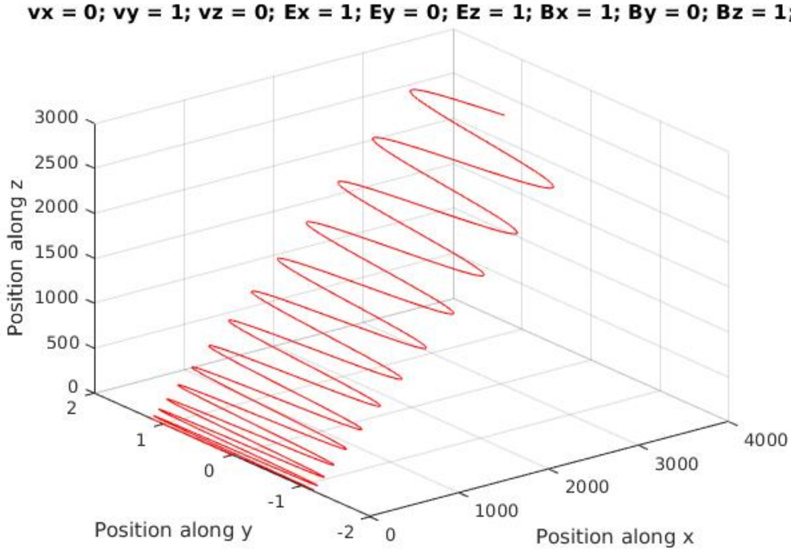

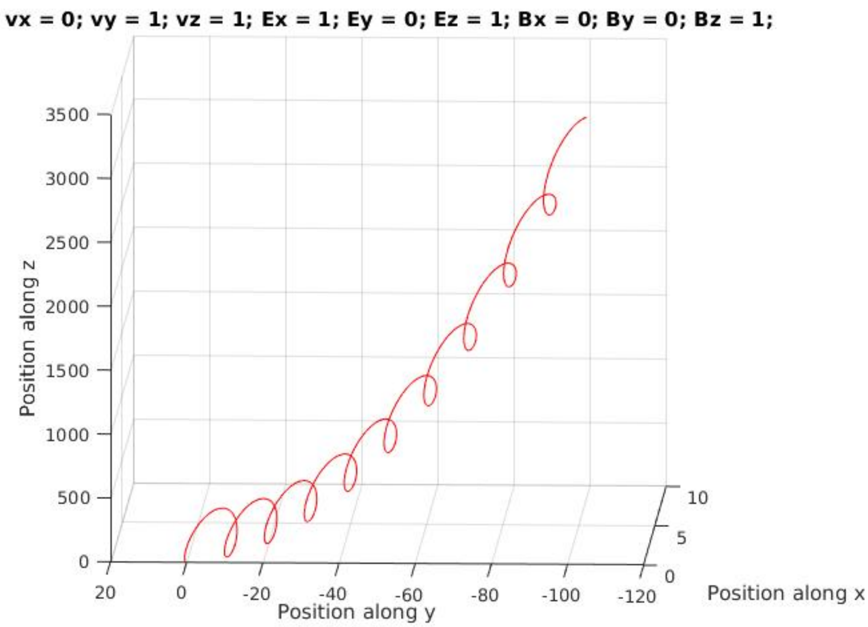
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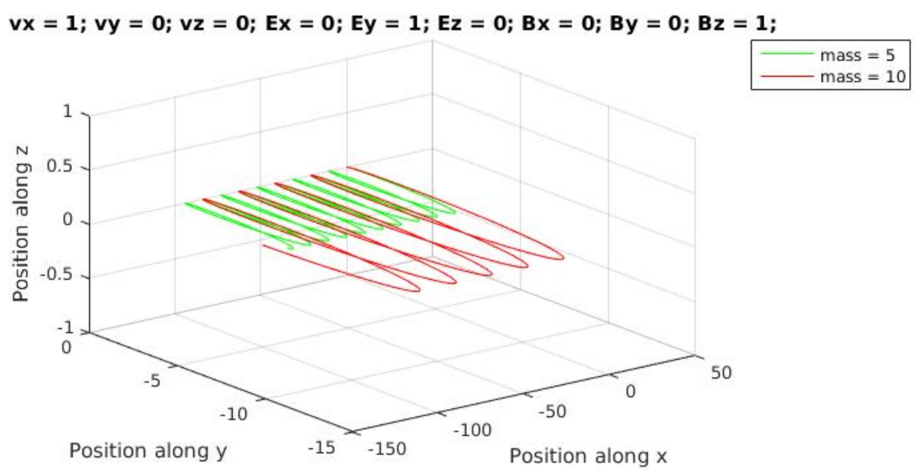
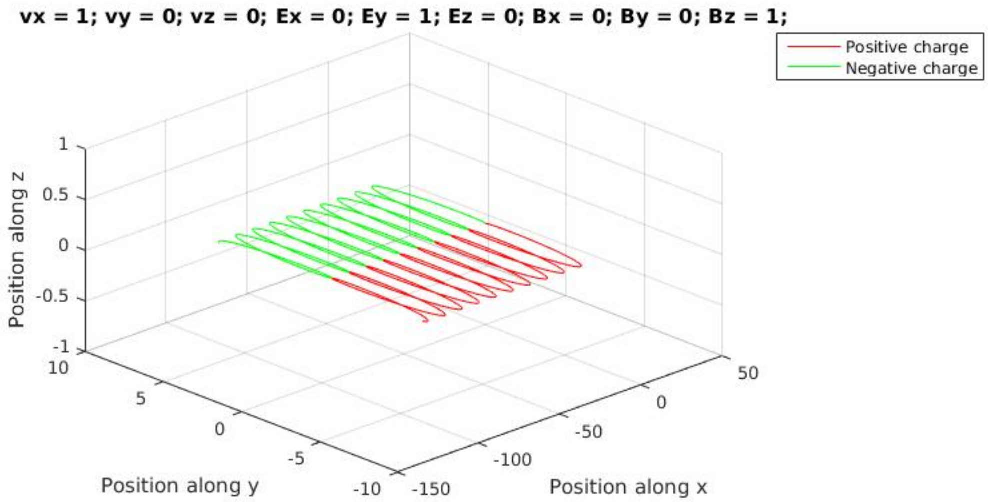
Position along z
Position along y
Position along x


vx = 1; vy = 0; vz = 0; Ex = 0; Ey = 1; Ez = 0; Bx = 0; By = 0; Bz = 1;
mass = 5
mass = 10
Position along z
Position along y
Position along x



Here, depending on the direction of $\vec{E}$, three cases arise :

1. If $\vec{E}$ is parallel to $\vec{B}$, We will again get helical motion but with varying pitch which increases with every revolution.
2. If, there is $\vec{E}$ perpendicular to $\vec{B}$, we will get a distorted circle, with a smaller curve when velocity and $\vec{E}$ are in opposite directions and a longer curve when they both are in same direction and this pattern will continue in the direction of initial velocity in a periodic manner as opposed to a constant circle in the previous case.

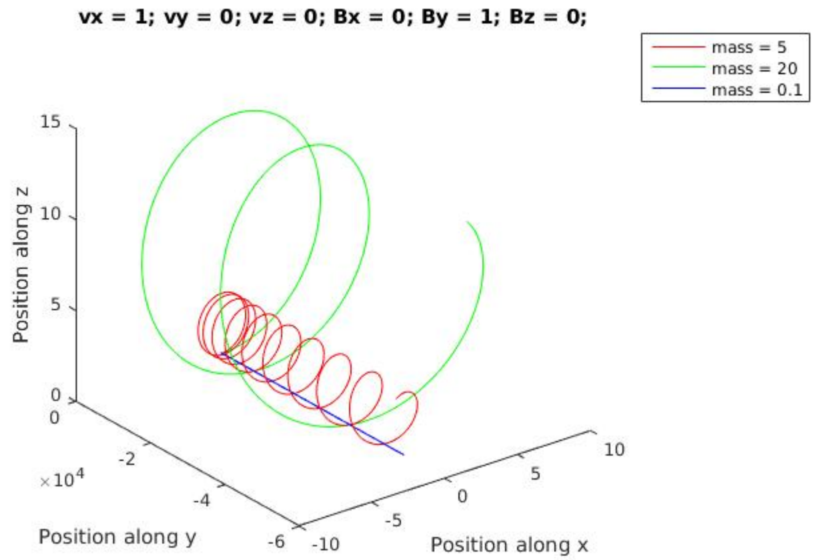
1.4.4 Case 4: Static and uniform B, and under gravitational force (for different mass)

The motion in this case would be similar to that in Case 3 but, now the forces would depend on both mass and charge. In contrast to the previous case, as the direction of force due to $\vec{E}$ depended on the particle, now it would be independent of charge and along $\vec{g}$.

$$F_{net} = mg + q(\vec{v}X\vec{B}) \tag{1.4.9}$$

Also, we'd have a drift velocity in this case,that is given by :

$$v_d = \frac{m}{q}\frac{\vec{g}X\vec{B}}{B^2} \tag{1.4.10}$$



In this case, we will have similar trajectories to that of Case 3, just that the direction will not depend on the charge.

1.4.5 Case 5 : Static and non-uniform (in the direction perpendicular to B) B field (grad B drift)

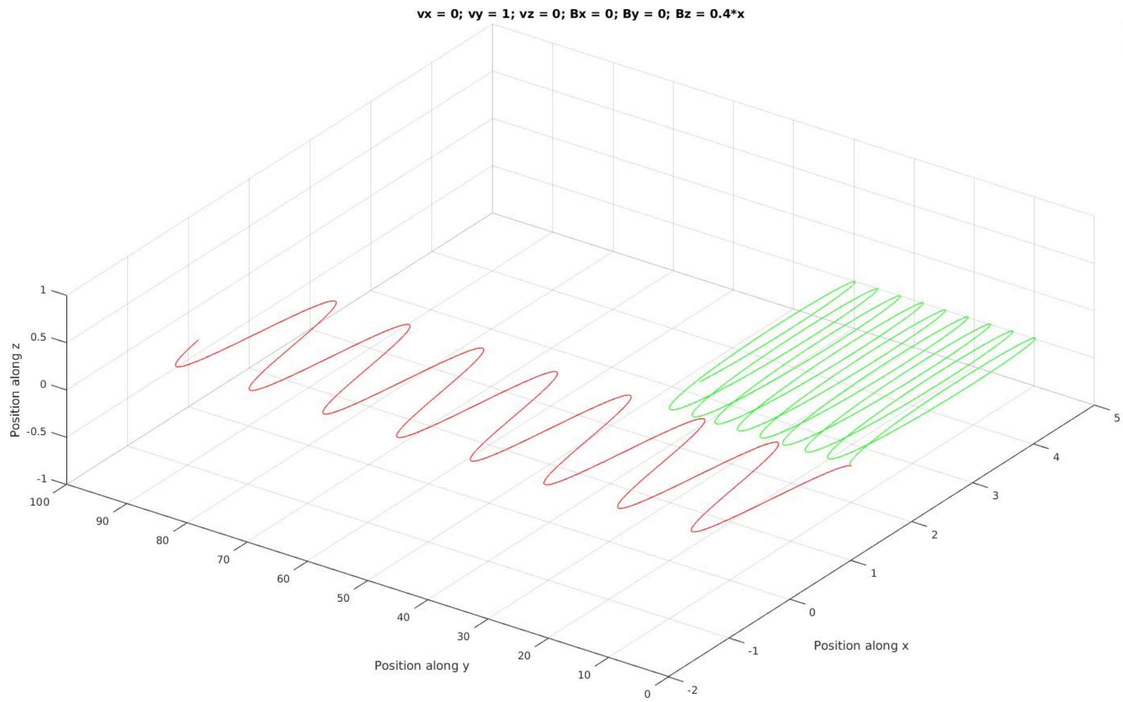
Here,

$$\vec{B} = B_o r \hat{B} \tag{1.4.11}$$

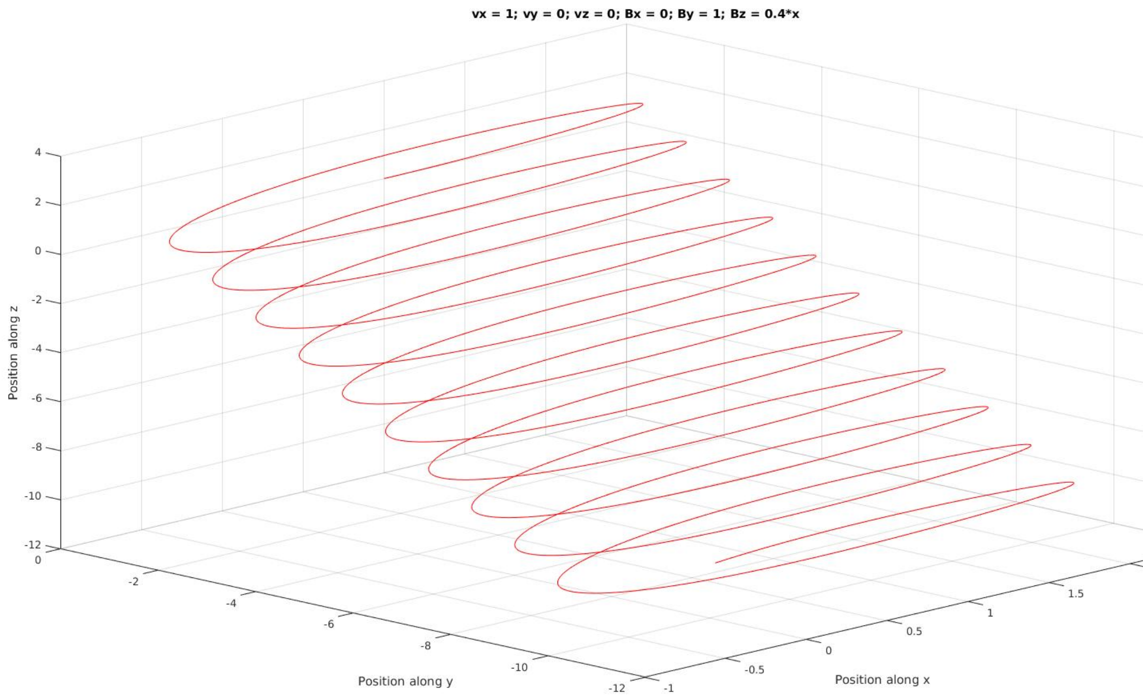
here, r is the position of object on x-y plane.

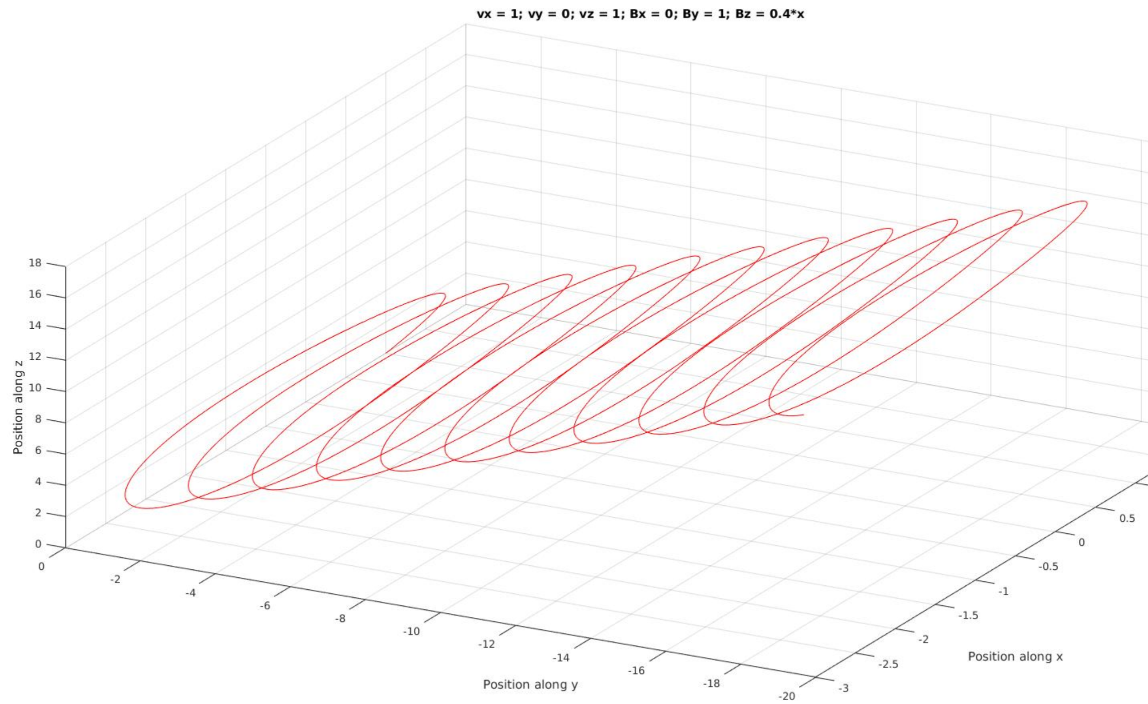
So we now have,

$$\vec{F} = q(\vec{v} X \vec{B_o} r) \tag{1.4.12}$$



B is along z direction and gradient of B is along x, hence the drift velocity is along positive y direction. In the case of positive charge, the force due to B field causes the particle to move in +x direction.
In the case of negative charge, the motion due to B field is in -x direction but drift velocity is still along +y, hence the overall motion is also along +y.


vx = 1; vy = 0; vz = 0; Bx = 0; By = 1; Bz = 0.4*x
Position along z
Position along y
Position along x
4
2
0
-2
-4
-6
-8
-10
-12
0
-2
-4
-6
-8
-10
-12
-1
-0.5
0
0.5
1
1.5



2

2.1 Problem Statement

Compute with your matlab code, the cyclotron frequency and thecyclotron radius for – an electron in the Earth's ionosphere at 300 km altitude, where the magnetic flux density B .00005 Tesla, consideringthat the electron moves at the thermal velocity (kT/m), with T=1000 K,where "k" is Boltzmann's constant. Plot a graph to show themotion/results and compare your results with analytical calculations. What will be the effect of gravitational drift velocity "vg" in this case?

2.2 Assumptions

1. No other force like the EMF due to other electrons is in place

2.3 Mathematical Model

Inside a cyclotron, there is a constant magnetic field perpendicular to the motion of a charged particle and a time varying electric field along the direction of charged particle.

Due to the magnetic field, the charged particle experiences a circular motion and due to the electric field, the velocity of the particle increases and thus increasing the radius of the circle. Hence the particle starts describing a spiral motion and when it reaches the exit, it is free of the magnetic field with a very high velocity.

In order to find how frequently the electric field should be varied, we need to find the time taken to complete one revolution.

As the time period is free of the velocity of the charged particle, we have :

$$T = \frac{2\pi m}{qB} \tag{2.3.1}$$

As the electric field needs to be varied twice every revolution, the time period will be half that of T.

Thus the cyclotron frequency we get would be :

$$f_{cyl} = \frac{1}{T} = \frac{qB}{\pi m} \tag{2.3.2}$$

The gravitational drift velocity of a chargeg particle is given by :

$$v_g = \frac{m}{q} \frac{\vec{g} X \vec{B}}{B^2} \tag{2.3.3}$$

As we can see, the drift velocity will be perpendicular to $\vec{g}$ as well as $\vec{B}$. As this velocity has a constant direction, it will distort the circular motion of the particle and when the velocity of the particle and drift will be in same direction, the particle will further move away whereas when they would be in opposite direction, it's curve would be smaller and hence we would get a flattened helix motion of the particle.

2.4 Analytical Solution

Here, from given,

1. q $= 1.6 * 10^{-19} C$
2. m $= 9.106 * 10^{-31} kg$

3. Magnetic flux density, B = $0.00005T$
4. Boltzmann's Constant, k = $1.0648 * 10^{-23} m^2 kg s^{-2} K^{-1}$

   So, we have,

$$f_{cyl} = \frac{1.6 * 5}{3.1415 * 9.106} * 10^{29} \tag{2.4.1}$$

$$f_{cyl} = 2.8 * 10^{29} Hz \tag{2.4.2}$$

Similarly,
velocity of electron,

$$v_{e^-} = \sqrt{\frac{kT}{m}} \tag{2.4.3}$$

$$v_{e^-} = \sqrt{\frac{1.0648 * 1000}{9.106}} = 3.419 * 10^4 ms^{-1} \tag{2.4.4}$$

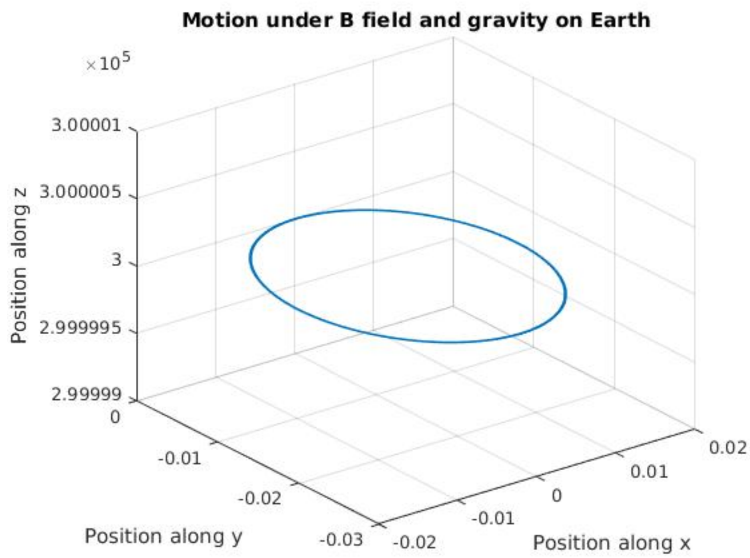
gravitational drift velocity will be :

$$v_g = \frac{m}{q} \frac{\vec{g} X \vec{B}}{B^2} \tag{2.4.5}$$

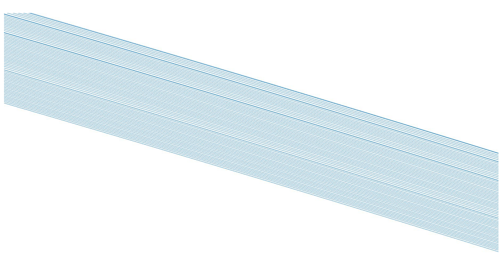
$$v_g = \frac{9.106 * 9.8 * 5}{1.6 * 25} * 10^{-7} = 1.115 * 10^{-6} ms^{-1} \tag{2.4.6}$$

As we can see, the drift velocity is nearly negligible compared to the gravitational drift, it will have very less effect on the electron and hence it's path will be circular with a negligible drift.

2.5 Results



The motion is circular with extremely small drift velocity along +x, which is visible only on zooming in on the graph as shown:



3

3.1 Problem Statement

Modify the program to describe the motion of a charged particle in an oscillating electric field. Consider a uniform alternating electric field in
x-direction
E=$E_{x_0} sin(\omega t)$
Angular freq Omega= 2πf
Amplitude E_{x_0}
Plot the trajectory of the particle in time. What happens when the particle has

non-zero initial velocity in x or y direction.

3.2 Assumptions

1. No other force acts on the particle like gravitational force.
2. drift is neglected.

3.3 Mathematical Model

Here we will use the same equations as previous question, but with small differences.

We had,

$$F_B = q(\vec{v}X\vec{B}) \tag{3.3.1}$$

so, we get

$$a_B = \frac{q(\vec{v}X\vec{B})}{m} \tag{3.3.2}$$

Now, since, the electric field is acting along x direction,

$$a_x = E_{x_o}sin(\omega t) + q(v_yB_z - v_zB_y) \tag{3.3.3}$$

that is :

$$\frac{dv}{dt} = E_{x_o}sin(\omega t) + \frac{q(v_yB_z - v_zB_y)}{m} \tag{3.3.4}$$

Integrating both sides, we get :

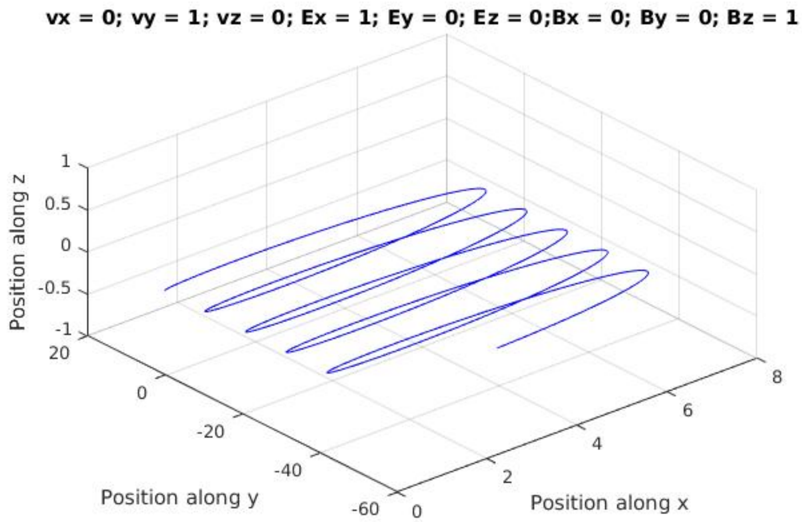
$$v = \frac{q(v_yB_z - v_zB_y)}{m}t - \frac{E_{x_o}cos(\omega t)}{\omega} \tag{3.3.5}$$

On further integrating we get

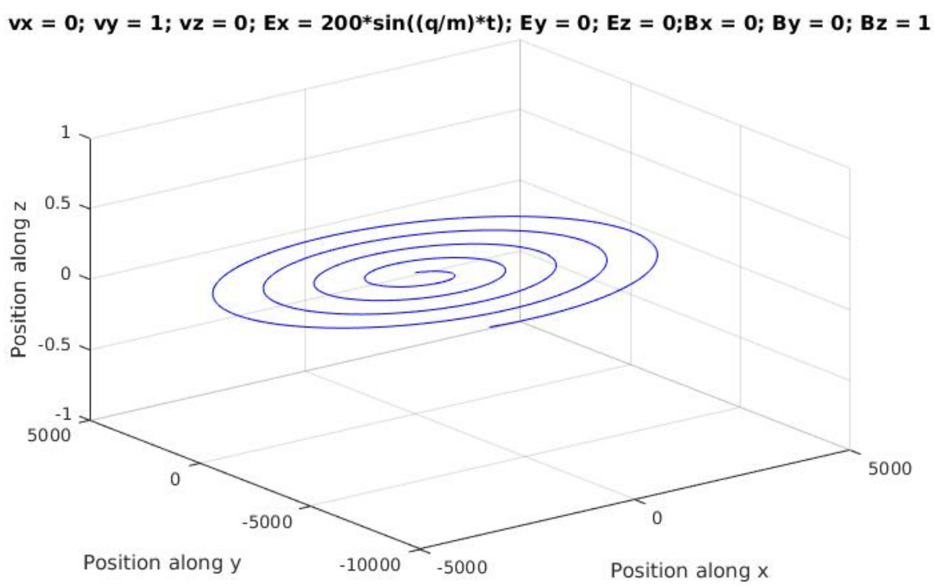
$$x = \frac{q(v_yB_z - v_zB_y)}{m}t^2 - \frac{E_{x_o}sin(\omega t)}{\omega^2} \tag{3.3.6}$$

3.4 Analytical Solution

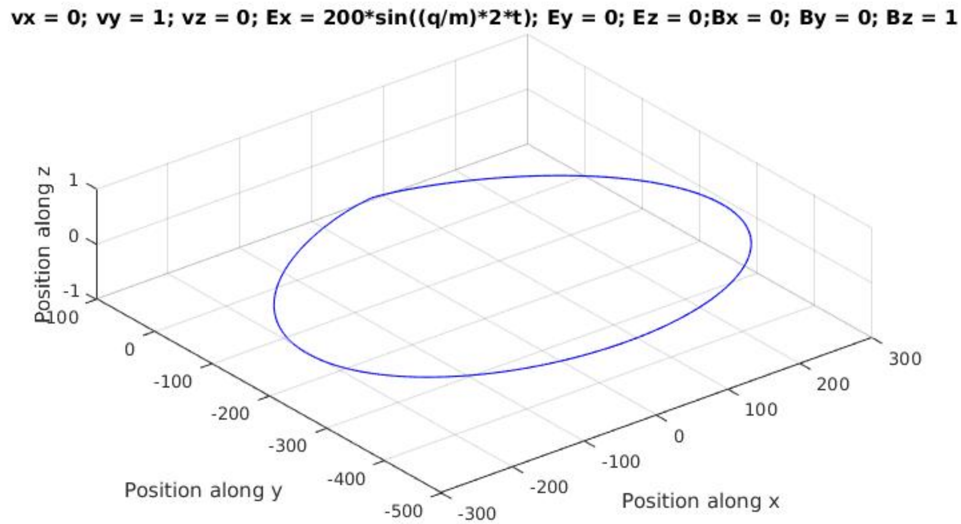
3.5 Results and Observations



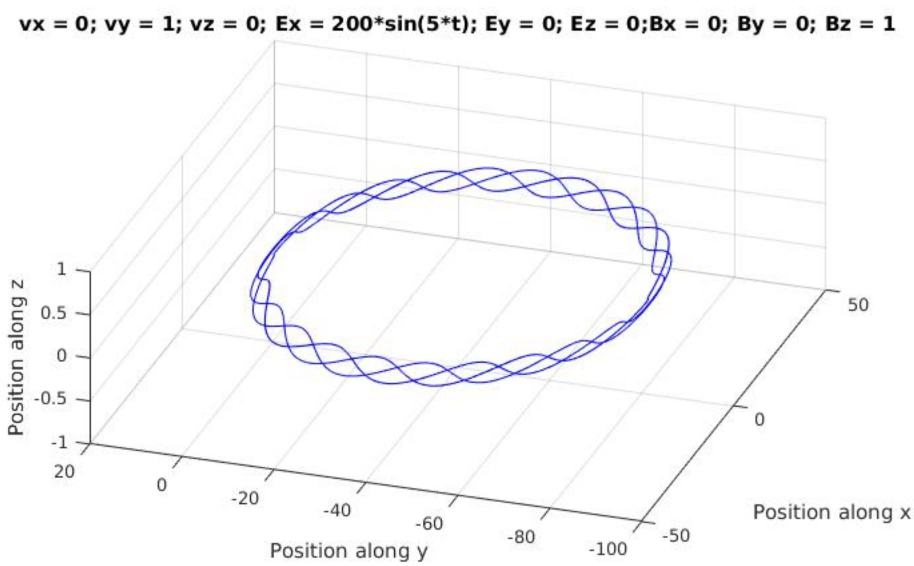
When the E field is constant, the motion is helical.



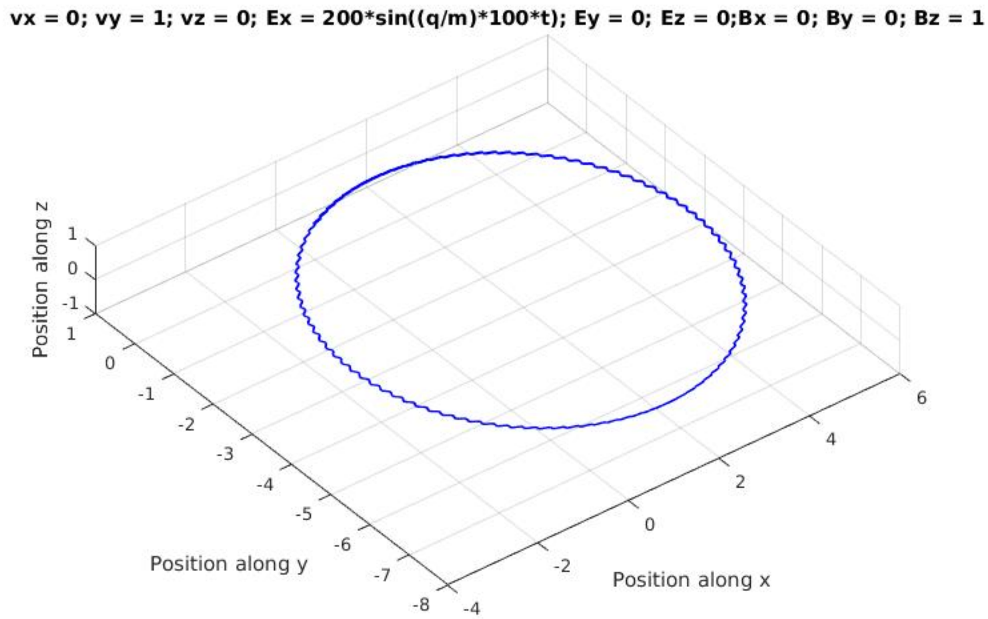
When frequency of E field is the same as frequency of the charged particle, the motion is spiral. In the first half of the time period, the E field is in positive x direction, and on completing half of the circular motion, it switches direction and the particle is further accelerated in that new direction, hence completing a spiral motion.



The frequency of the E field is twice the particle's freq. Hence, in one time period, the particle completes one circular motion and when the E field switches directions, the same path is overlapped in the opposite direction.



When the freq. of the E field exceeds the freq. of particle substantially, there are multiple higher frequency oscillations before one circular motion is completed, as visible in the graph.



When the E field's freq. is even higher, the particle vibrates at very high frequency while completing the bigger circular motion (due to B field).