CS 201
Computational Physics
Assignment 3

Group 17
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1 Projectile Motion: Cannon Shell/ Missile Problem

1.1 (a) Variation in g

1.1.1 Problem Statement

Investigate (computationally) the cannon-shell trajectories ignoring both air drag and the effect of air density. Compare your result with exact solutions. Acceleration due to gravity depends on altitude; include this effect in your computational model by making some rational assumption.

1.1.2 Assumptions

1. Air drag and change in air density with altitude are ignored.
2. Tagret is at same height as cannon.

1.1.3 Mathematical Model

Initial condiitons: $g_o = 9.8m/s^2$
$Radius(earth) = 64,000km$

Variation of g with height h(from Earth's surface) : $g(h) = \frac{g_o}{(1+\frac{h}{R})^2}$
Initial firing velocity = 750 m/s
Angle that initial velocity makes with horizontal x axis = θ

$$\frac{dx}{dt} = vcos(\theta) = v_x \tag{1.1.1}$$

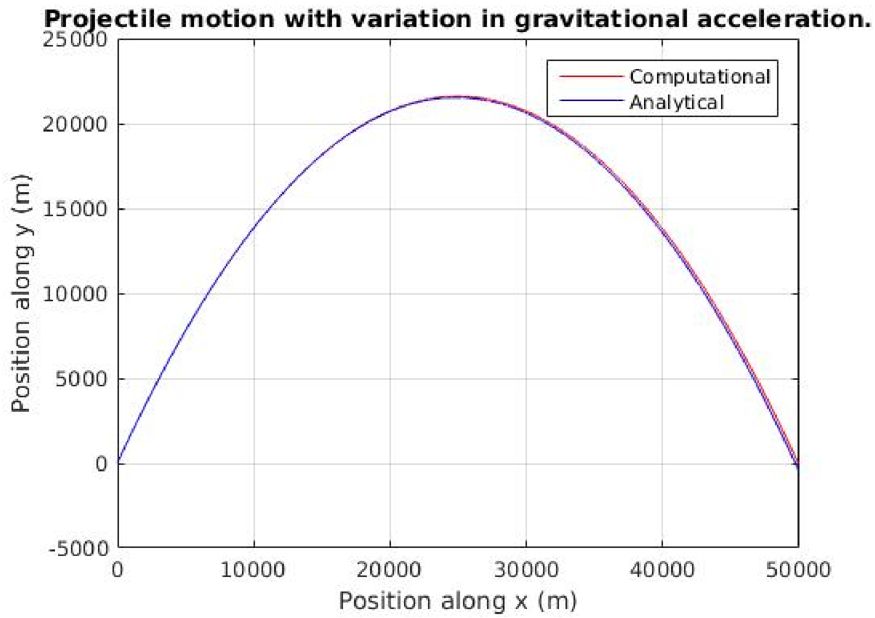
$$\frac{dy}{dt} = vsin(\theta) = v_y \tag{1.1.2}$$

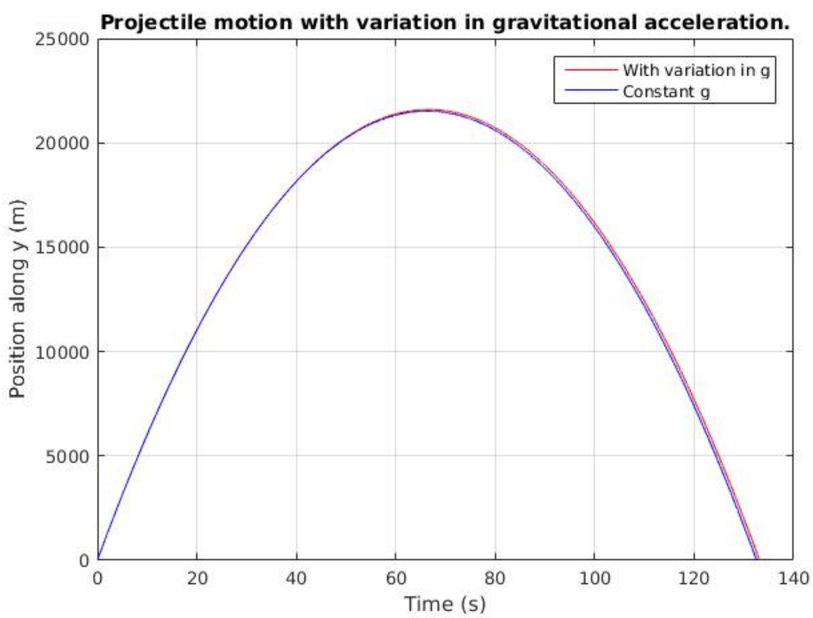
$$\frac{dv_x}{dt} = 0 \tag{1.1.3}$$

$$\frac{dv_y}{dt} = g \tag{1.1.4}$$

1.1.4 Analytical Solution

1.1.5 Results





1.1.6 Observations and Interpretations

The time taken for the projectile to reach the ground is more when variation gravitational acceleration is considered, as compared to when g is constant.

1.2 (b) Air drag and air density variation

1.2.1 Problem statement

Investigate the trajectory of the canon shell including both air drag (proportional to square of velocity) and reduced air density at high altitudes. Perform your calculation for different firing angles; and determine the value of the angle that gives the maximum range.

$$F_{drag} = -Bv^2 \tag{1.2.1}$$

Density of atmosphere varies as follows:

$$\rho = \rho_o e^{\frac{-y}{y_o}} \tag{1.2.2}$$

y is the altitude; $y_o = 1000$ m.
Drag force with air resistance:

$$F*_{drag} = \frac{\rho}{\rho_o} F_{drag}(y = 0) \tag{1.2.3}$$

Take initial speed=750 m/s;
B/m= 4E-5 m

1.2.2 Assumptions

1. Tagret is at same height as cannon.

1.2.3 Mathematical Model

$$F_{d_x} = F_d cos(\theta) = F_d \frac{v_x}{v} \tag{1.2.4}$$

$$F_{d_y} = F_d sin(\theta) = F_d \frac{v_y}{v} \tag{1.2.5}$$

$$F_{d_x} = -Bv_x v e^{-\frac{y}{y_o}} \tag{1.2.6}$$

$$F_{d_y} = -Bv_y v e^{-\frac{y}{y_o}} \tag{1.2.7}$$

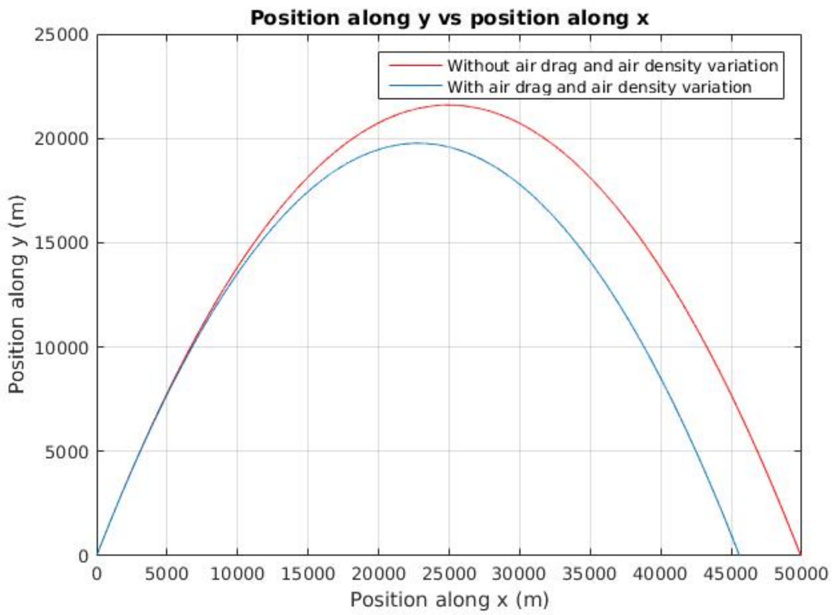
Therefore, the earlier derivative equations change to,

$$\frac{dv_x}{dt} = -\frac{Bv_x v e^{-\frac{y}{y_o}}}{m} \tag{1.2.8}$$

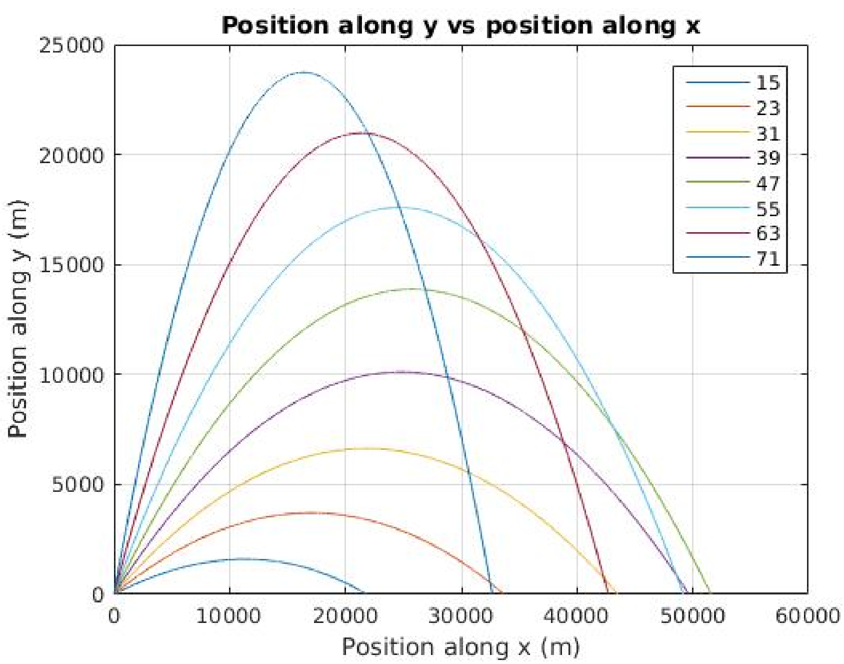
$$\frac{dv_y}{dt} = -g - \frac{Bv_y v e^{-\frac{y}{y_o}}}{m} \tag{1.2.9}$$

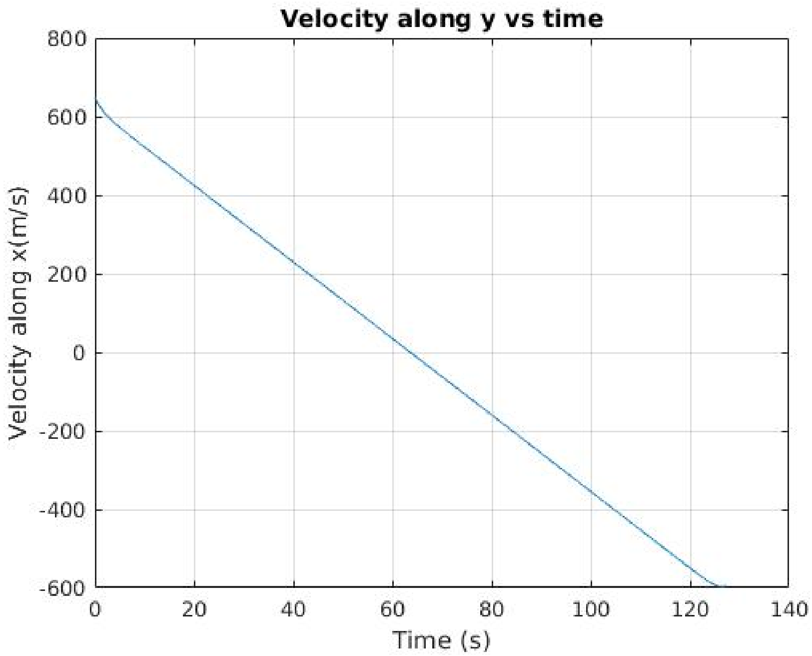
On solving these ODEs using MATLAB solvers, we get the following results:

1.2.4 Results

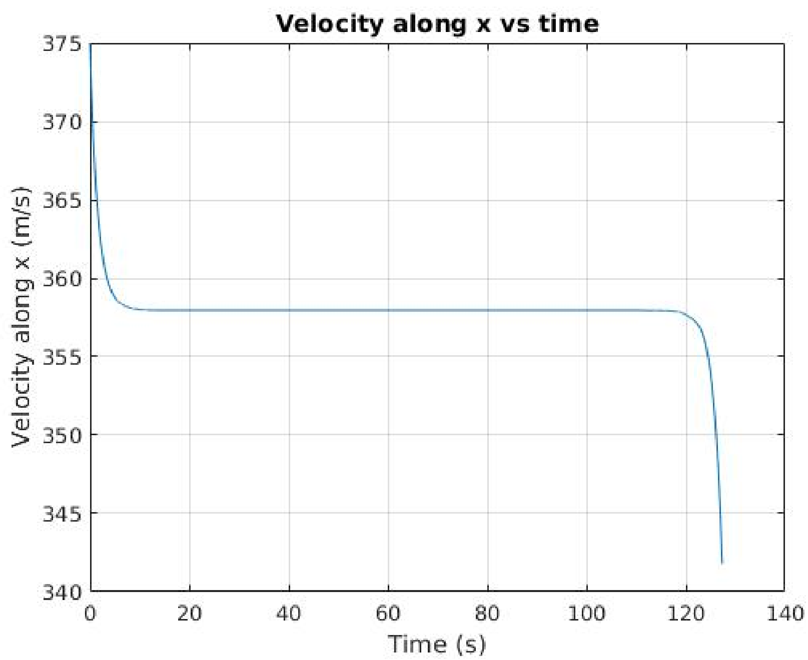


The following graph shows the trajectory of the projectile for different angles denoted in degrees.





Velocity along y vs time
Velocity along x(m/s)
Time (s)
800
600
400
200
0
-200
-400
-600
0
20
40
60
80
100
120
140



Velocity along x vs time
Velocity along x (m/s)
Time (s)
375
370
365
360
355
350
345
340
0
20
40
60
80
100
120
140

1.2.5 Observations and Interpretations

1. The maximum range of the projectile is lower due to air drag and variation in air density, as now the x component of velocity also faces a retarding force.

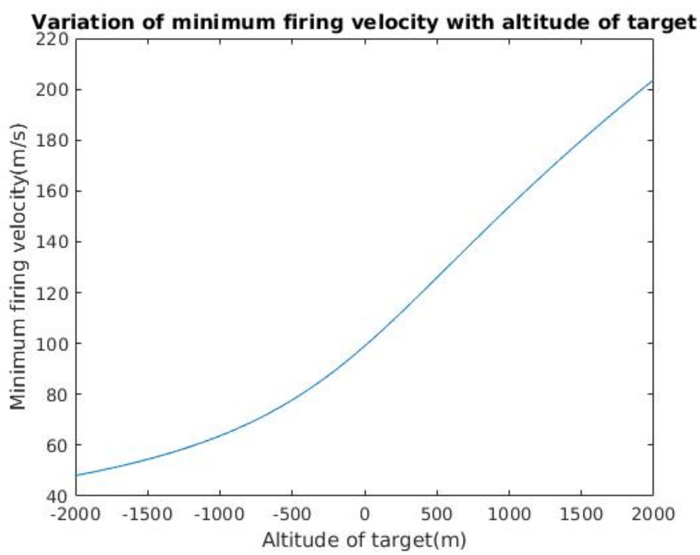
2. The maximum range first increases as firing angle increases. It attains a maximum value at angle = 47^o. As the angle increases after this, the maximum range starts to decrease again.

1.3 (c) Target at different altitude

1.3.1 Problem Statement

Generalize the program so that it can deal with situations where the target is at a different altitude (higher or lower) than the canon. Investigate for both the cases. How the minimum firing velocity to hit a target varies as the altitude of the target varies.

1.3.2 Result



2 Cycling Problem

2.1 Problem Statement :

1. Rewrite the bicycle problem/code as discussed in the class. Investigate the effect of rider's power, mass and frontal area on the ultimate velocity. Generally for a rider in the middle of a group the effective frontal area is

about 30 less than the rider at the front. How much less energy does a rider in the group expend than one at the front (assuming both moving at 12.5 m/s).

2. Run your code (case (a) discussed during class) with initial v=0; observe the output and give possible explanation. Explain why it is important to give a non-zero initial velocity.

3. As discussed in the class, we have assumed that the bicyclist maintains a constant power. What about the assumption when the bicycle has a very small velocity? (instantaneous power=product of force and velocity).

4. At low velocities it is more realistic to assume, that the rider is able to exert a constant force. That means for small "v" there is a constant force, which means eqn is dv/dt=F0/m

   Modify your matlab code to include this term for small velocities, that means we have 2 regimes and 2 eqns one for small velocities and one for larger velocities. Make your code work automatically for both the regimes and crossover from small to large v occur when the power reaches P(=F0v) . Take F0=P/v where v=5m/s.

2.2 Mathematical Model :

2.2.1 Assumptions :

1. Air density remains constant throughout.
2. The area against which we calculate the air resistance is the area of the body of cyclist.
3. The cyclist is continuously supplying power. In reality, rate of change of applied power would become zero after some time.

2.2.2 Initial Conditions :

1. Mass of rider = 62.5 kg.
2. Mass of cycle = 7 kg
3. Air Density = 1.225 kg/m^3.
4. Velocity (constant for some problems) = 12.5 m/s
5. Area of Contact = 0.48 m^2

2.3 Part 1:

2.3.1 Equations :

For part 1.a :

Here, we have to study the change of velocity w.r.t change in power i.e. $\frac{dv}{dP}$

In order to compute $\frac{dv}{dP}$, we can first compute $\frac{dP}{dt}$ and $\frac{dv}{dt}$ and then divide the second one by the first one. Here we take in consideration the following initial conditions :

1. $v_{ini} = 5ms^{-1}$
2. The rate of change of Power is constant

Hence we have the following equations :

$$\frac{dP}{dt} = k \tag{2.3.1}$$

(k is a constant)

$$\frac{dv}{dt} = \frac{P}{mass * v} - \frac{\rho * a * v^2}{2} \tag{2.3.2}$$

For part 1.b :

Here, we have to find the relation between Power applied and Area in contact with velocity being constant. That means we are supposed to find $\frac{dP}{da}$.

As, the velocity is constant, we can say that,

$$\frac{dV}{dt} = 0 \tag{2.3.3}$$

Hence,

$$\frac{P}{mass * v} = \frac{\rho * a * v^2}{2} \tag{2.3.4}$$

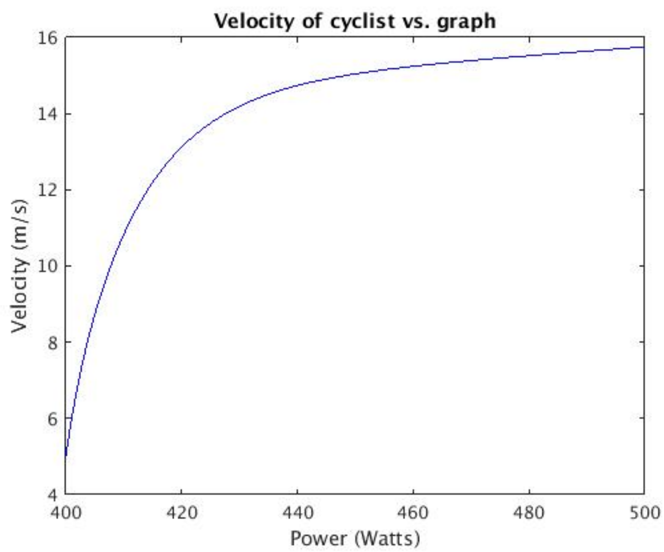
Now, on differentiating w.r.t. a,

$$\frac{dP}{da} * \frac{1}{mass * v} = \frac{\rho * v^2}{2} \tag{2.3.5}$$

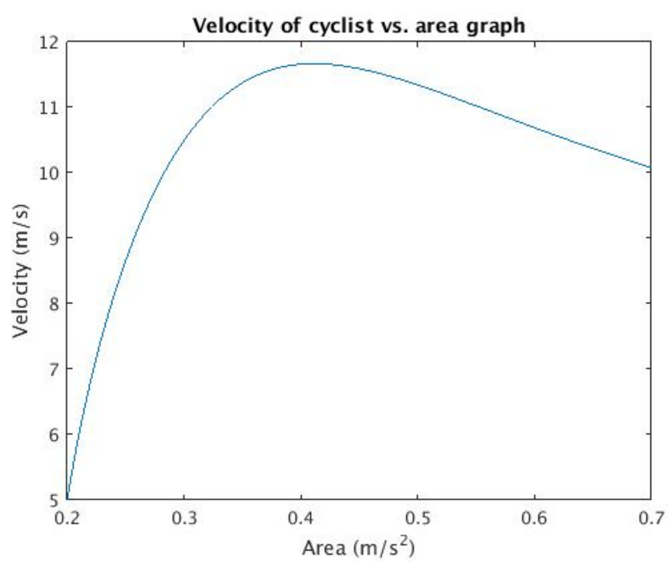
Hence,

$$\frac{dP}{da} = \frac{mass * \rho * v^3}{2} \tag{2.3.6}$$

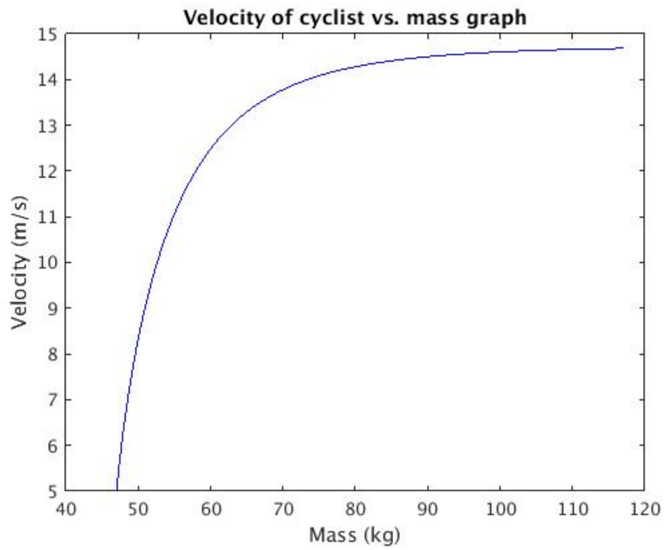
2.3.2 Result



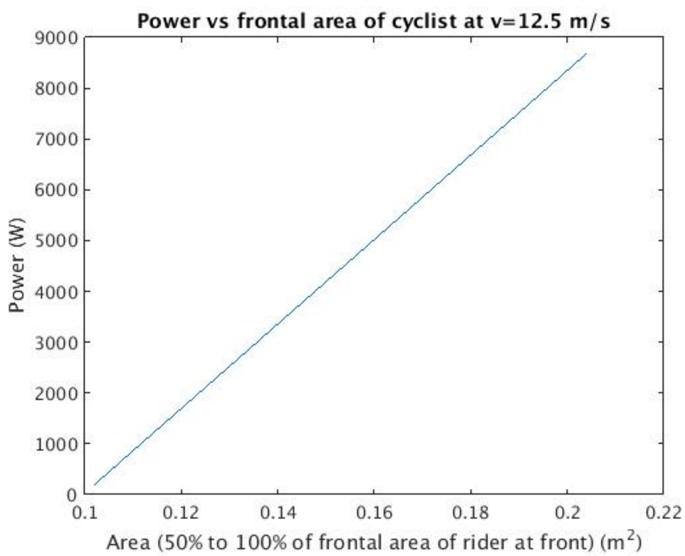
Velocity of cyclist vs. graph
Velocity (m/s)
Power (Watts)
16
14
12
10
8
6
4
400
420
440
460
480
500



Velocity of cyclist vs. area graph
Velocity (m/s)
Area (m/s^2)
12
11
10
9
8
7
6
5
0.2
0.3
0.4
0.5
0.6
0.7



(b)



2.3.3 Observations and Interpretations

1. (a) Initially power is high and velocity is low. So, the first term in eq. 2.3.2 dominates over the second term (which varies as v^2. As velocity increases, the second term starts to increase rapidly and thus the net acceleration decreases, but remains positive as P is also increasing with a constant rate.

2. If we start with initial power = 0, the second term dominates and velocity first decreases from its initial velocity. At the point of time when power

increases enough to overpower the second term, velocity starts to increase and the situation continues as above.

3. If we increase area at initial small velocity, keeping power and mass constant, the drag force increases.

4. (b) When the velocity is constant, the power required to maintain is directly proportional to the area of the cyclist with the product of air density, cyclist's mass and cube of the velocity being the proportionality constant. Hence, when the cyclist is in the middle, he just has to use 0.7 times the energy than the person in the front.

2.4 Part 2

As we can see from above, the expression of acceleration involves the velocity in denominator, if we take initial velocity to be zero, it will tend to infinity hence not rendering the equation useful and hence, we cannot use that equation.
The output is an empty graph, with value of velocity = NaN

2.5 Part 3

In case of a very small velocity, as the second term in the eq. 2.3.2 of acceleration contains the square of velocity along with the other two terms being very small as well (area is of the order of $10^{-2}m^2$ and ρ is typically $1\ kg/m^3$), the term altogether becomes really small. Hence, it can be neglected as a whole.

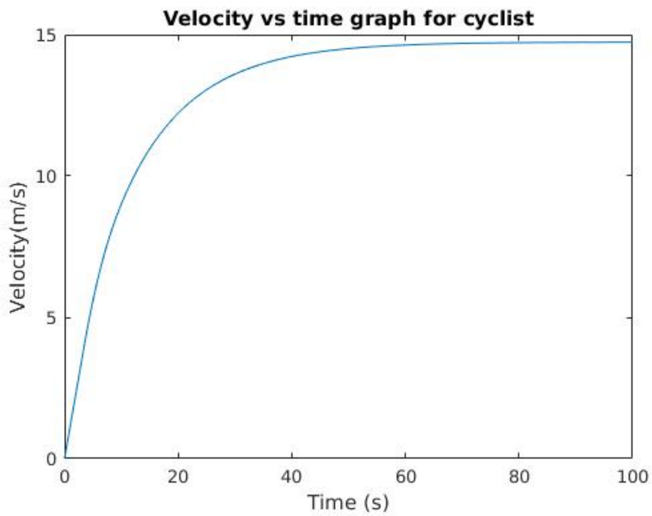
Also, as power is proportional to velocity, it won't increase significantly at very small velocities and on assuming it to be nearly constant, we can say that the Force applied by the cyclist is also nearly constant as due to very small velocity, the change in velocity is nearly insignificant.

2.6 Part 4

Here, for velocities below 5 m/s, we assume a constant force being applied on the system.

$$\frac{dv}{dt} = \frac{F_0}{mass} \tag{2.6.1}$$

2.6.1 Result



2.6.2 Observations and Interpretations

Till the total power is less than $F0 * v$(at that instant, the graph is linear as the force is constant. At $v = 5m/s$, the air drag comes into play and hence net force changes. The drag force keeps increasing and at one point of time, equals the cyclist's applied force. Then the velocity saturates, as seen in the figure.

3 Sliding Block

3.1 Problem Statement

Investigate(computationally) the motion of a block sliding without friction down a fixed inclined plane with different initial parameters. Derive the analytic solution for displacement, velocity, and acceleration. Compare the computational results with analytic solutions for the case when the angle of the inclined plane is 30 degree - to check the accuracy of the computational model. (Example 2.1 ;Marion and Thornton).

3.2 Assumptions

1. The plane is frictionless.
2. The direction down the inclination of the plane is +ve x-axis.

3.3 Mathematical Model

Let x denote the displacement down the inclined plane. Initial conditions: $x_o = 0, v_{x_o} = 0$

Angle of inclination $= \theta = 30^o$

The net force on the body is due to gravity and normal reaction only (we are considering frictionless plane)
Along the plane,

$$\frac{d^2x}{dt^2} = gsin(\theta) \tag{3.3.1}$$

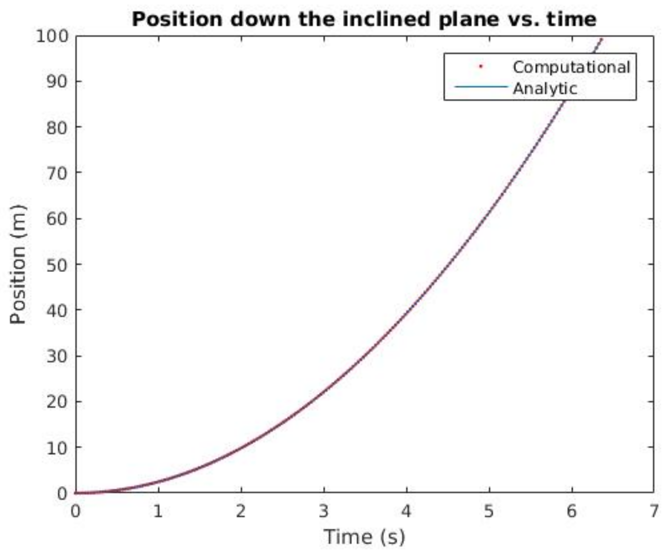
$$\frac{dx}{dt} = v_x \tag{3.3.2}$$

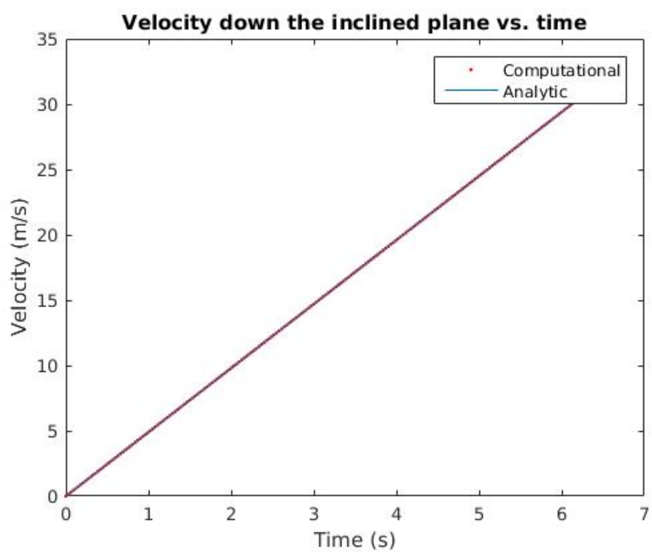
3.4 Analytical Solution

$$x = v_{x_o}t + \frac{1}{2}gt^2 \tag{3.4.1}$$

$$v = v_{x_o} + gsin(\theta)t \tag{3.4.2}$$

3.5 Results





3.6 Observations and Interpretations

The analytical and computational results are nearly the sname, hence the model is fairly accurate.

4 Sliding Block with friction

4.1 Problem Statement :

Introduce the effect of static friction and kinetic friction into the previous problem. Take coefficient of static friction = 0.4 and coefficient of kinetic friction = 0.3 and computationally analyze the motion for different initial angles. Report your computational observations and how the results compare with theoretical solutions (Example 2.2-2.3; Marion and Thornton).

4.2 Assumptions :

1. Coefficient of friction is constant throughout the slope.
2. There is no air drag or viscous force due to air.
3. There is no thermal expansion due to friction.

4.3 Initial Values :

1. Gravitational Acceleration, $g = 9.8m/s^2$
2. Static Friction, $\mu_s = 0.4$
3. Kinetic Friction, $\mu_k = 0.3$
4. Mass, m = 5 kg
5. Angle of inclination = 30^o

4.4 Mathematical Equations

The acceleration of the block due to gravity will be downwards and hence it will experience a frictional force upwards in the directional force which will depend on the Normal Force, N due to the slope, which will be equal and opposite to that of the force due to the body perpendicular force.

$$N = mgcos(\theta) \tag{4.4.1}$$

Hence in the direction of the slope,

$$F_{net} = mgsin(\theta) - \mu mgcos(\theta) \tag{4.4.2}$$

Therefor, on dividing by m,

$$a_{net} = gsin(\theta) - \mu gcos(\theta) \tag{4.4.3}$$

Initially, the block is at rest. So at the instant it just starts moving, for that time interval dt, the friction that would be acting would be static friction and next instant on wards, it would be replaced by kinetic friction.So,

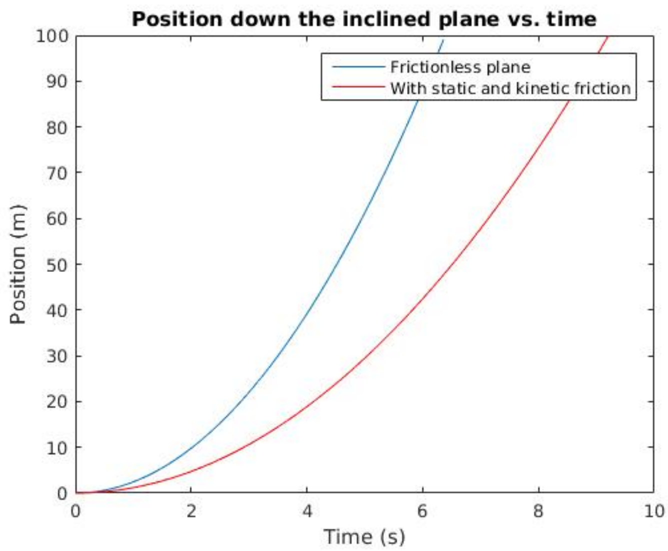
at t = 0,

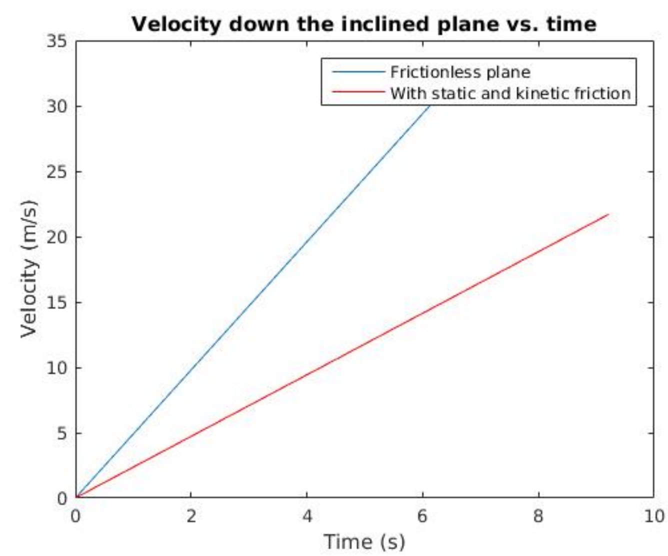
$$v(t) = (gsin(\theta) - \mu_s gcos(\theta))dt \tag{4.4.4}$$

from the next instant,

$$v(t) = (gsin(\theta) - \mu_k gcos(\theta))dt \tag{4.4.5}$$

4.5 Results





4.6 Observations and Interpretations

Hence from the above graphs, we can infer the following :

1. Friction remains constant irrespective of the velocity of the block.
2. Velocity and acceleration of the block is independent of it's mass.
3. More the friction, more the time taken by the block to reach the bottom of the slope.
4. Lower the inclination of the block, more the time taken by the block to reach the ground.