CS 201
Computational Physics
Assignment 2

Group 17
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Winter Semester 2017-18

1 Free Fall

1.1 Problem statement

Computationally analyze the motion of freely falling body using Euler's method as discussed during the lecture. Consider realistic initial conditions (height, initial velocity etc.) Compare your result with analytical solution and study the effect of discretization (time step) on computational result. Plot the results showing the velocity of the body and the distance travelled by it at different instant of time. The above problem is not realistic from Earth's viewpoint, use the code to analyze the motion of falling body on the moon (there is hardly any atmosphere, so in reality also we can neglect the effect of atmosphere, however initial conditions will be different).

1.2 Assumptions

1. Effect of atmosphere, drag force/air resistance is neglected.
2. Variation of gravitational acceleration with height is neglected.
3. Point objects are considered.

1.3 Mathematical Model

Inital condiitons: $x_o = 100m$.
g(earth) = 9.8
g(moon) = 1.622

Mathematical equations:

$$\frac{dx}{dt} = v \tag{1}$$

$$\frac{dv}{dt} = g \tag{2}$$

Finite difference form:

$$x_{i+1} = x_i + v\Delta t \tag{3}$$

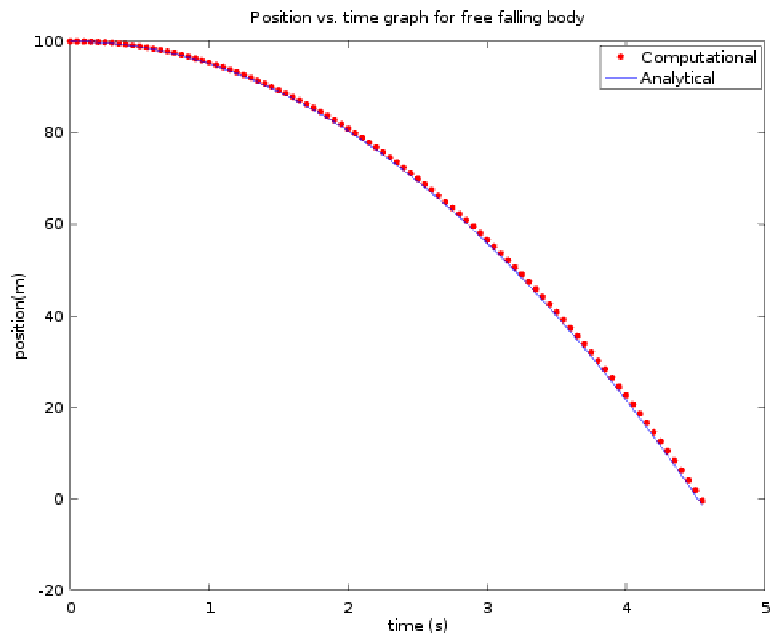
$$v_{i+1} = v_i + g\Delta t \tag{4}$$

1.4 Analytical Solution

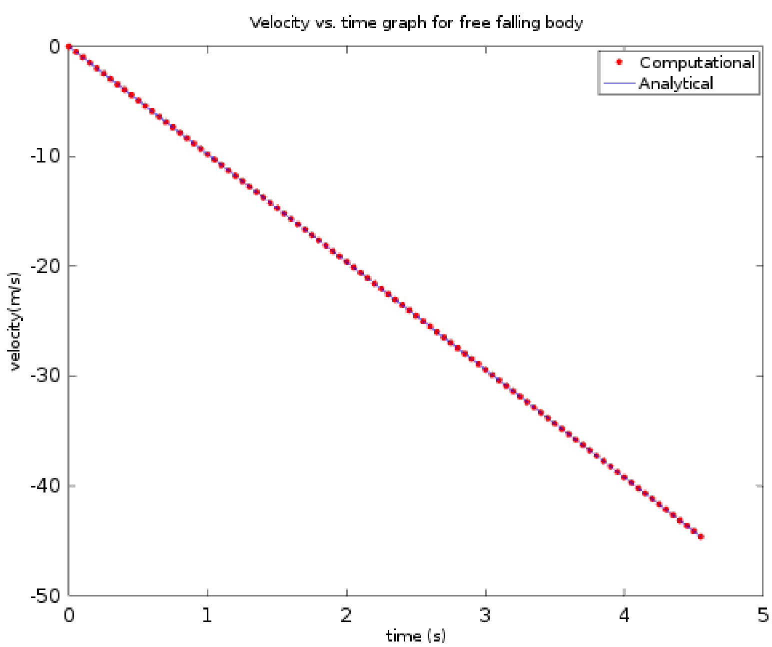
$$x = x_o + v_o t + \frac{1}{2}gt^2 \tag{5}$$

$$v = v_o + gt \tag{6}$$

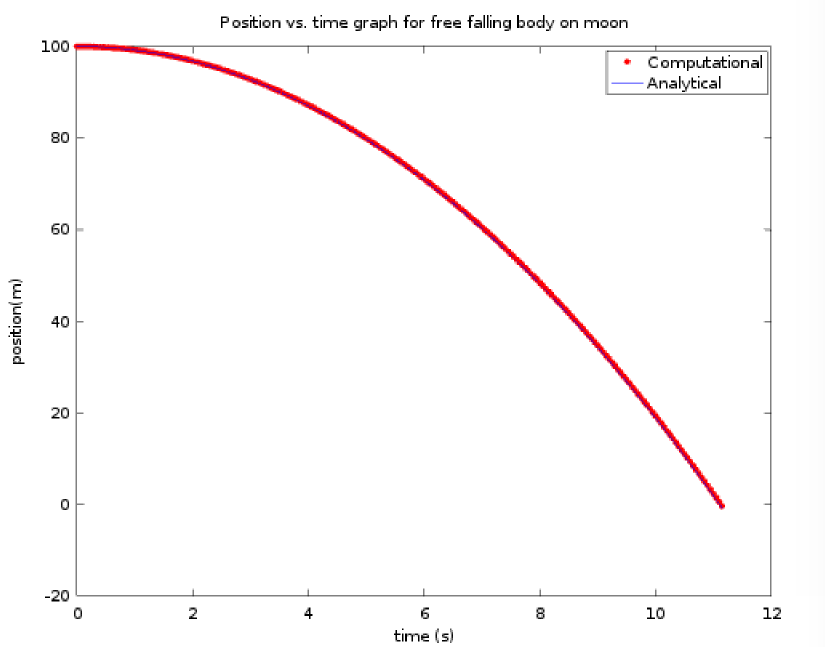
1.5 Result



Position vs. time graph for free falling body
Computational
Analytical
position(m)
time (s)
100
80
60
40
20
0
-20
0
1
2
3
4
5



Velocity vs. time graph for free falling body
Computational
Analytical
velocity(m/s)
time (s)
0
-10
-20
-30
-40
-50
0
1
2
3
4
5



1.6 Observations and Interpretations

1. The difference between the computational and analytical models is very less, and hence the computational model is fairly accurate.

2. The time taken for the body to fall from the same height is more on the moon. This is due to the fact that gravitational acceleration on the moon is lower than that on Earth.

2 Horizontal motion

2.1 Problem Statement

Write down the equation for position of an object moving horizontally with a constant velocity "v". Assume v=50 m/s, use the Euler method (finite difference) to solve the equation as a function of time. Compare your computational result with the exact solution. Compare the result for different values of the time-step.

2.2 Assumptions

1. Velocity is constant.

2.3 Mathematical Model

Mathematical equations:

$$\frac{dx}{dt} = v \tag{7}$$

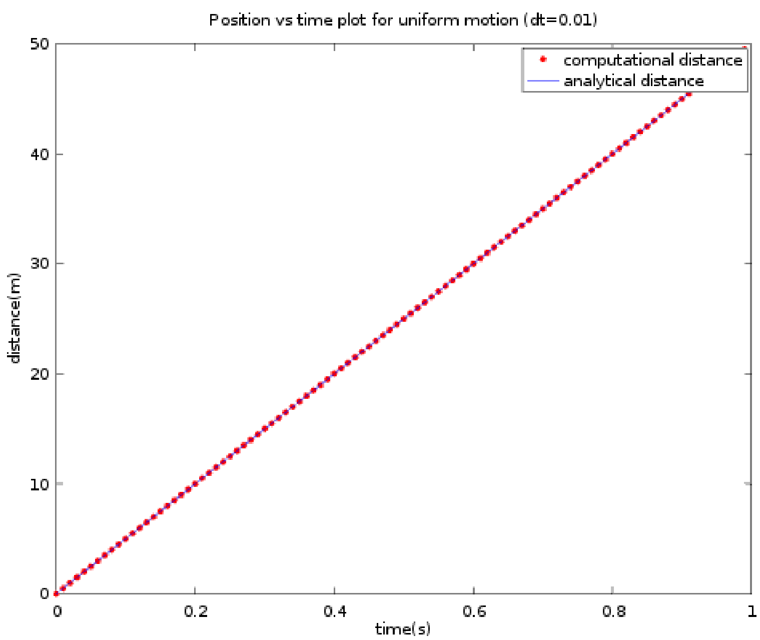
Finite difference form:

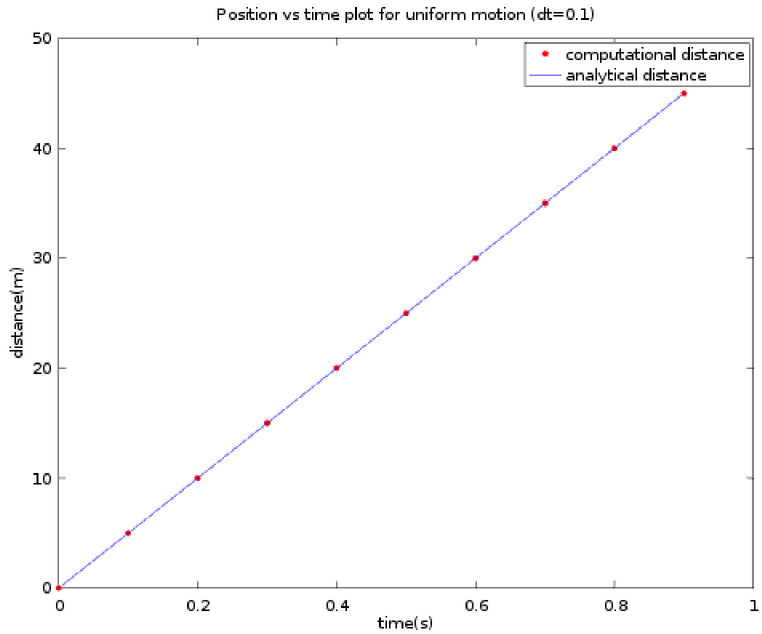
$$x_{i+1} = x_i + v\Delta t \tag{8}$$

2.4 Analytical Solution

$$x = x_o + v_o t + \frac{1}{2} g t^2 \tag{9}$$

2.5 Results





2.6 Observations and Interpretations

1. The analytical and computational solutions coincide, thus the model is fairly accurate.

3 Effect of buoyancy

3.1 Problem Statement

(a) Add the effect of atmosphere to problem 1 (still neglecting viscosity and drag). Suppose the falling object is a sphere of radius "r", computationally study the effect of buoyancy on the motion of the object. Net force needs to be modeled properly (as discussed during lecture); choose proper density of air. Study the effect of "r" and "mass". You can assume constant "g".
(b) Also computationally investigate the motion of the same object traveling through a liquid (say water), and compare the motion with the case of air. Use computational data and plots to explain your answer (motion as a function of time).

3.2 Assumptions

1. Variation of gravitational acceleration with height is neglected.
2. Point objects are considered.
3. Viscous drag is not considered.

3.3 Mathematical Model

Initial condiitons: $g = 9.8m/s^2$
$\rho_a = 1.225kg/m^3$
$\rho_{water} = 1000kg/m^3$

(a) Buoyant force (air) = $\rho_a g V_{object}$
(b) Buoyant force (water) = $\rho_{water} g V_{object}$

The net force on the object is due to gravity and opposing buoyant force.

$$\frac{dx}{dt} = v \tag{10}$$

$$\frac{dv}{dt} = g - \frac{4\pi r^3 \rho g}{3m} \tag{11}$$

Finite Difference form:

$$x_{i+1} = x_i + v\Delta t \tag{12}$$

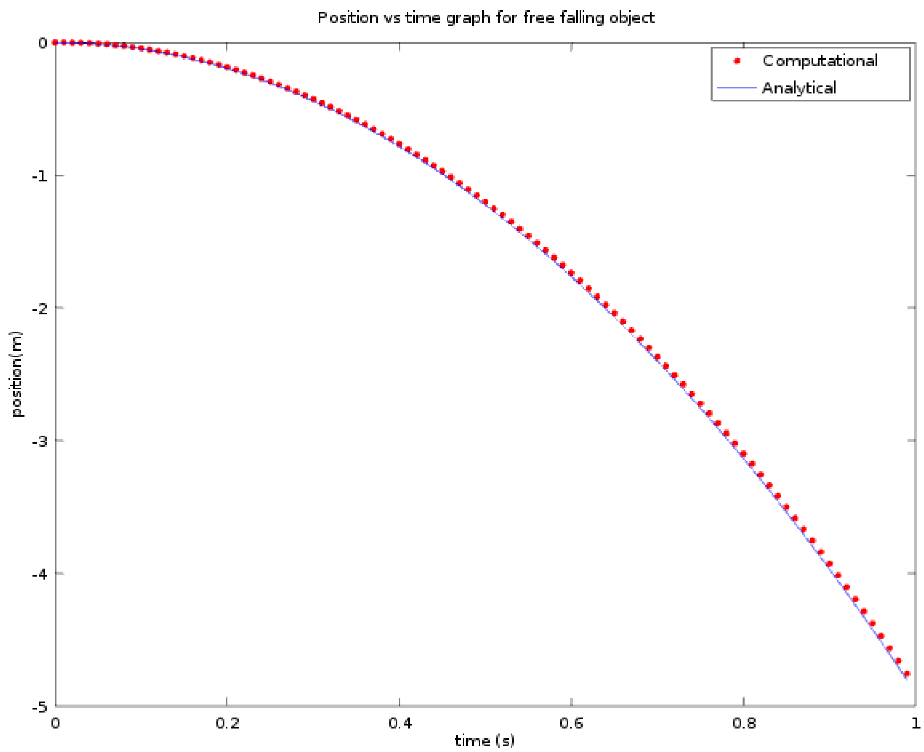
$$v_{i+1} = v_i + (g - \frac{4\pi r^3 \rho g}{3m})\Delta t \tag{13}$$

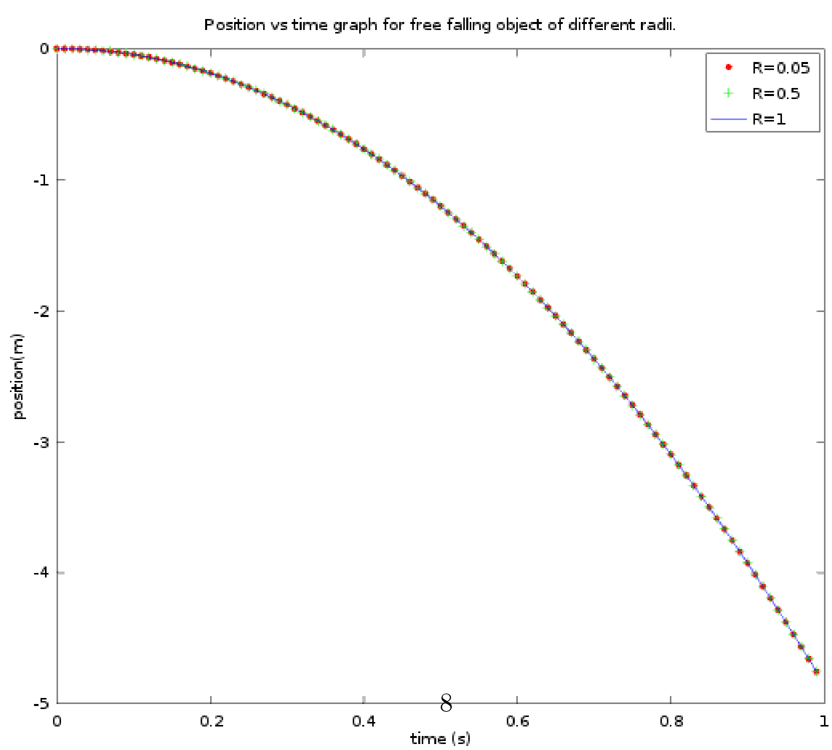
3.4 Analytical Solution

$$x = x_o + v_o t + \frac{1}{2}(g - \frac{4\pi r^3 \rho g}{3m})t^2 \tag{14}$$

$$v = v_o + (g - \frac{4\pi r^3 \rho g}{3m})t \tag{15}$$

3.5 Results


Position vs time graph for free falling object
Computational
Analytical
position(m)
time (s)
0
-1
-2
-3
-4
-5
0
0.2
0.4
0.6
0.8
1


Position vs time graph for free falling object of different radii.
R=0.05
R=0.5
R=1
position(m)
time (s)
0
-1
-2
-3
-4
-5
0
0.2
0.4
0.6
0.8
1

3.6 Observations and Interpretations

1. The velocity attained by the object at any point of time is less than that in problem 1.
2. The motion of the object is not dependent on its radius or mass.
3. As density of water is larger than density of air, buoyant force in water is larger and hence motion in water is slower than in air.

4 Effect of viscous drag

4.1 Problem Statement

Now add the effect of viscous drag to the problem 3(b) assuming a small sphere is falling through the liquid with low speed. Model the system using viscous force given by Stokes law as discussed during the class. Choose proper coefficient of viscosity (look at the unit), and analyze the phenomena of terminal velocity.

4.2 Assumptions

1. Variation of gravitational acceleration with height is neglected.
2. Point objects are considered.

4.3 Mathematical Model

Initial condiitons: $g = 9.8m/s^2$
$\rho_{liquid} = 1000kg/m^3$
$\eta(water) = 0.00089Pa.s$

Buoyant force (liquid) = $\rho_{liq} g V_{object}$
Viscous drag = $6\pi\eta r v$
The net force on the object is due to gravity and opposing buoyant force and viscous drag.

$$\frac{dx}{dt} = v \tag{16}$$

$$\frac{dv}{dt} = g - \frac{4\pi r^3 \rho g}{3m} - 6\pi\eta r v \tag{17}$$

Finite Difference form:

$$x_{i+1} = x_i + v\Delta t \tag{18}$$

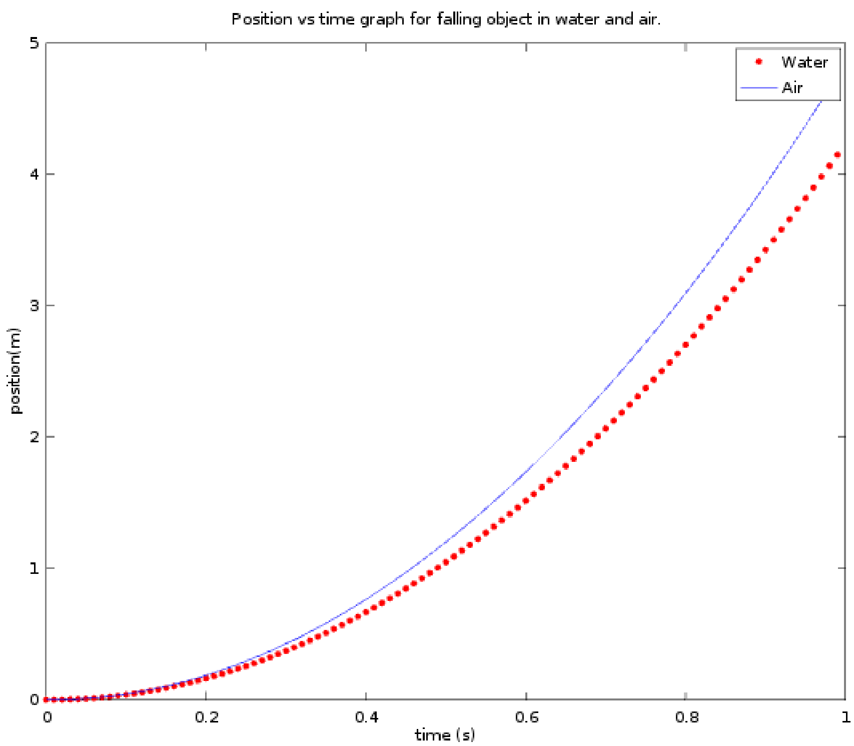
$$v_{i+1} = v_i + (g - \frac{4\pi r^3 \rho g}{3m} - 6\pi\eta r v_i)\Delta t \tag{19}$$

4.4 Analytical Solution

On integrating and solving further,

$$v = \frac{3gm - V\rho g}{6\pi\eta r}(1 - e^{\frac{6\pi\eta rt}{3m}}) \tag{20}$$

4.5 Results



4.6 Observations and Interpretations

The object attains a constant terminal velocity after some time, when gravity is balanced by the opposing forces. The opposing viscous drag is proportional to the velocity, and hence as velocity increases, viscous force increases and reaches a point where the net force is zero.

5 Variation of g with height

5.1 Problem Statement

Modify the program (problem 3) and include the variation of "g" with height.

Use the program to computationally investigate the motion of a body dropped from a height of 20 KM (assume constant air density). How will you use the above program to investigate free fall in a deep mine (by taking proper initial conditions Google).

5.2 Assumptions

1. Variation of gravitational acceleration with height is considered.
2. Point objects are considered.
3. Viscous drag is not considered.
4. Air density is constant.

5.3 Mathematical Model

Initial condiitons: $g_o = 9.8 m/s^2$
$Radius(earth) = 64,000 km$

Variation of g with height h(from Earth's surface) : $g(h) = \frac{g_o}{(1+\frac{h}{R})^2}$
Buoyant force (air) = $\rho_a g(h) V_{object}$

$$\frac{dx}{dt} = v \tag{21}$$

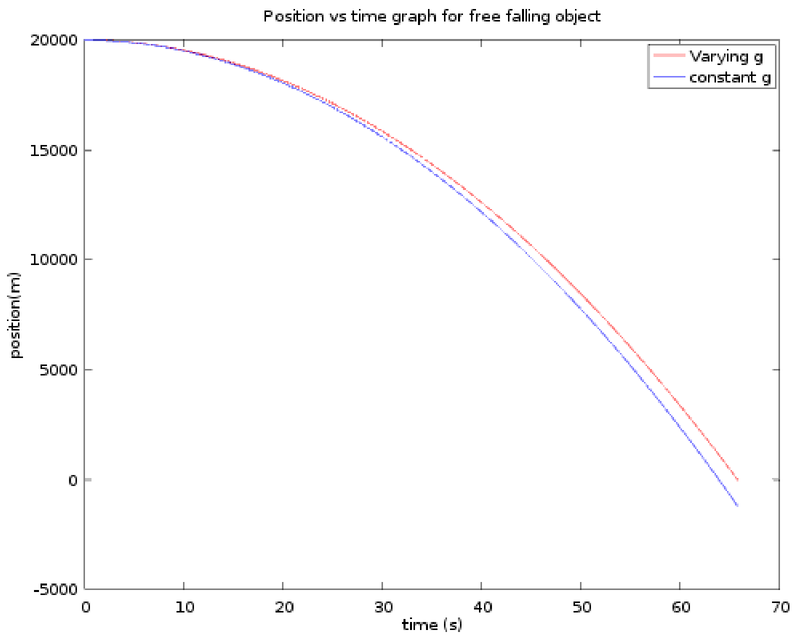
$$\frac{dv}{dt} = g(x) - \frac{4\pi r^3 \rho g(x)}{3mass} \tag{22}$$

Finite Difference form:

$$x_{i+1} = x_i + v\Delta t \tag{23}$$

$$v_{i+1} = v_i + (g(x) - \frac{4\pi r^3 \rho g(x)}{3mass})\Delta t \tag{24}$$

5.4 Results



5.5 Observations and Interpretations

From the graph we can conclude that the time taken for an object dropped from a height to reach Earth is less when we consider gravitational acceleration to be constant. when we consider the variation, at a height away from the surface, value of g is less and hence the object starts with lesser gravitational pull, and takes more time.

6 Motion in vaccum

6.1 Problem Statement

A stone is thrown vertically upwards from the ground with some initial velocity in vacuum (choose a proper realistic velocity). Track the complete motion till it comes down to the ground (computationally). What is the velocity when it strikes the ground, compare with analytical result?

6.2 Assumptions

1. Variation of g with height is ignored.
2. Air drag and buoyancy are ignored.

6.3 Mathematical Model

Initial velocity = 50 m/s
Mathematical equations:

$$\frac{dx}{dt} = v \tag{25}$$

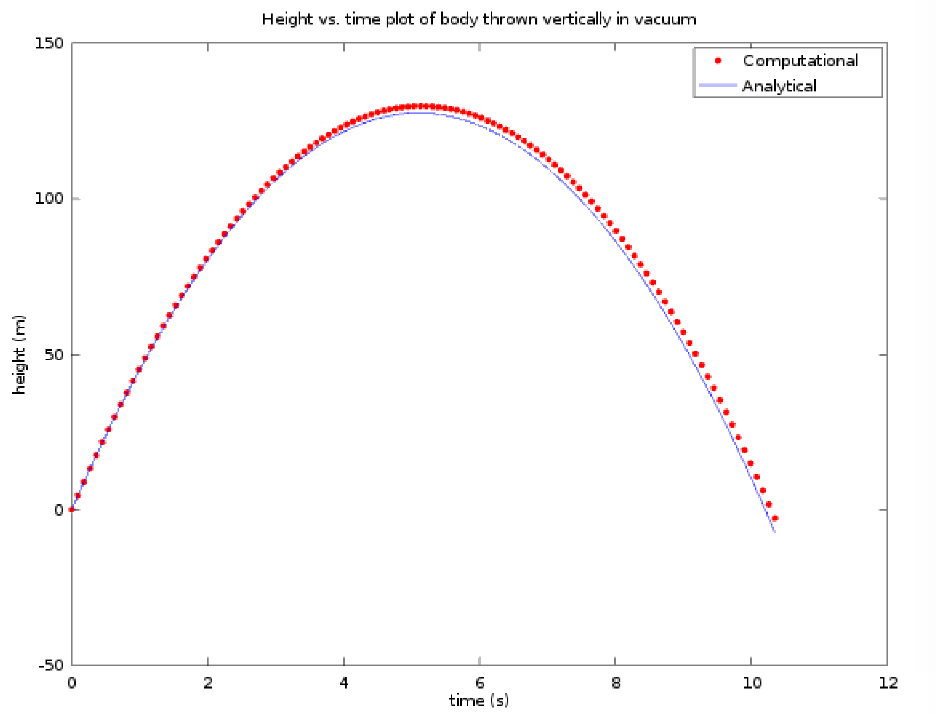
$$\frac{dv}{dt} = g \tag{26}$$

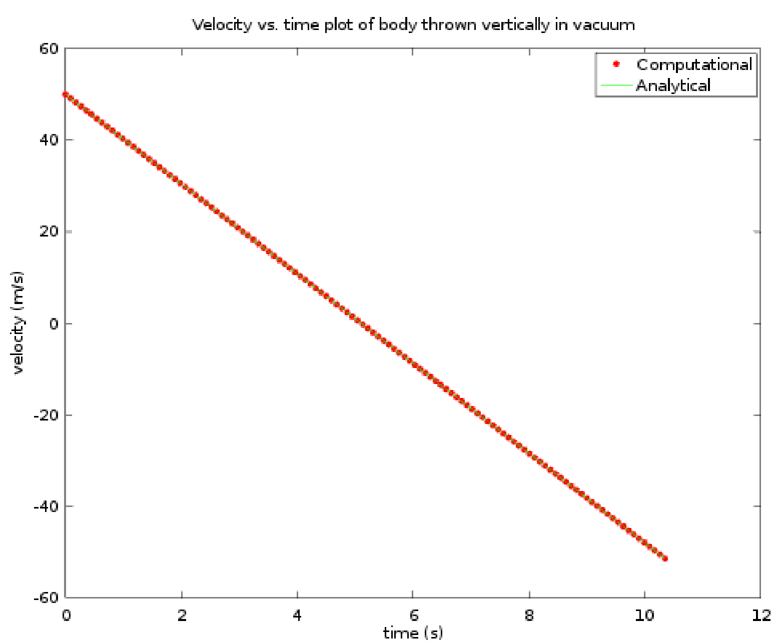
6.4 Analytical Solution

$$x = x_o + v_o t + \frac{1}{2} g t^2 \tag{27}$$

$$v = v_o + gt \tag{28}$$

6.5 Results


Height vs. time plot of body thrown vertically in vacuum
Computational
Analytical
height (m)
time (s)
150
100
50
0
-50
0
2
4
6
8
10
12


Velocity vs. time plot of body thrown vertically in vacuum
Computational
Analytical
velocity (m/s)
time (s)
60
40
20
0
-20
-40
-60
0
2
4
6
8
10
12

6.6 Observations and Interpretations

The computational and analytical solutions for height are approximately same. The velocity when the body strikes the ground is -50 m/s which matches the analytical solution.

7 Balloon

7.1 Problem Statement

Computationally study the motion of a balloon filled with Helium (use realistic data from Google). Also study the same for 3-4 different gases of your choice. Will the balloon rise up or fall down? Vary the size of the balloon (5 diff size) to study its effect on velocity and distance travelled as a function of time. All conclusions should be based on the plots from your computational data.

7.2 Assumptions

1. Balloon is perfectly spherical in shape.
2. Air resistance is neglected.
3. Air density is assumed to be constant throughout.
4. Gravity is assumed to be constant throughout.

7.3 Mathematical Model

Initial Conditions:
g = 9.8
$\rho_{air} = 1.29 kg/m^3$
$\rho_{gas} = 0.1664(He), 0.0899(H), 1.250(N_2), 1.429(O_2), 1.977(CO_2)$

7.4 Mathematical Equations :

$$\frac{dx}{dt} = v \tag{29}$$

$$\frac{dv}{dt} = a = a_{buoyant} - g \tag{30}$$

here $a_{buoyant}$ is the acceleration due to buoyancy :

$$a_{buoyant} = \frac{\rho_(air) * V * g}{m} \tag{31}$$

Here m is the mass of the balloon , V is the volume of the balloon. Here

$$V = 4 * \pi * r^3 \tag{32}$$

$$m = \rho_{gas} * V \tag{33}$$

7.4.1 Finite Difference Form :

$$v_{i+1} = v_i + a * \Delta * t \tag{34}$$

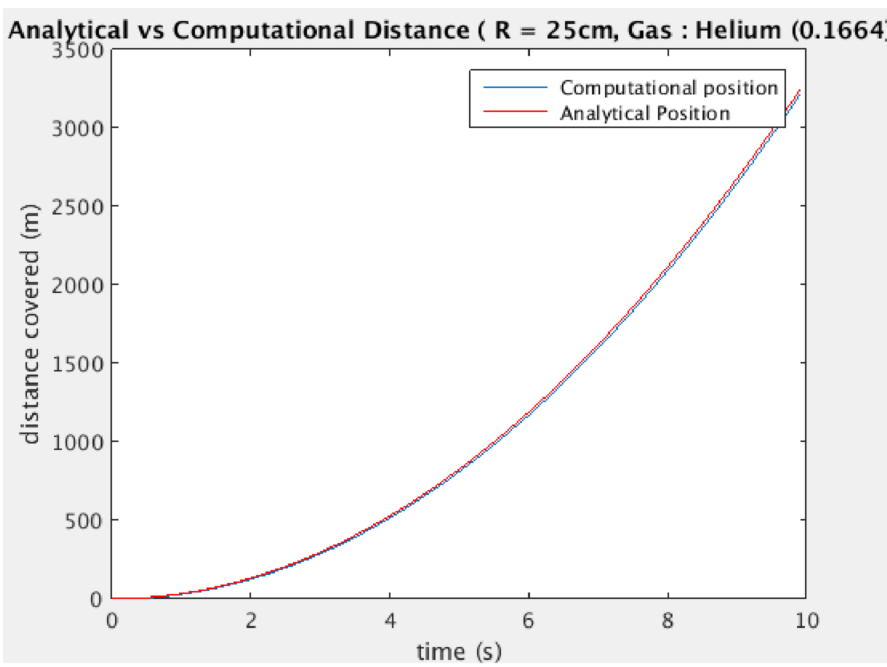
$$x_{i+1} = x_i + v_i * \Delta * t \tag{35}$$

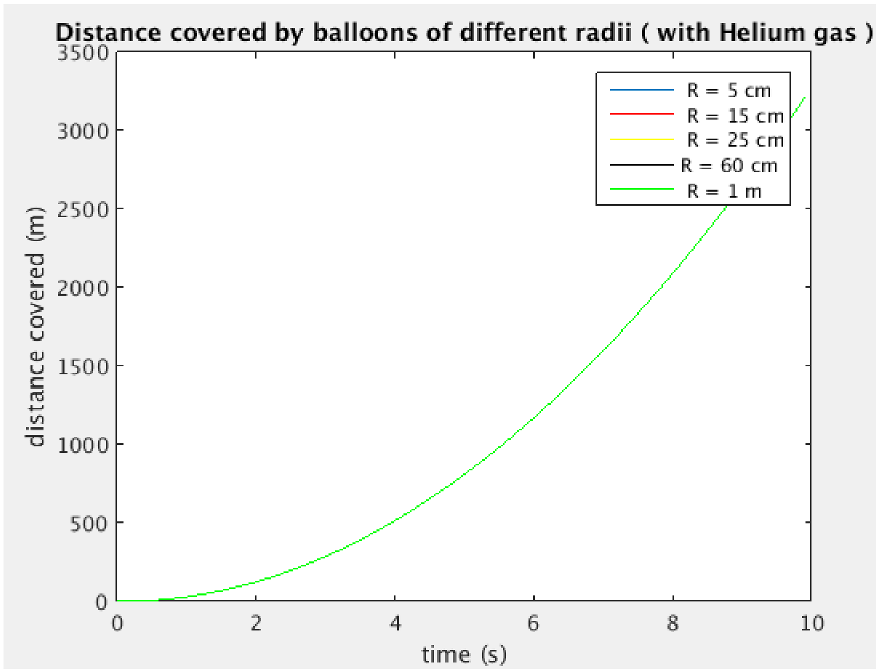
7.5 Analytical Solution :

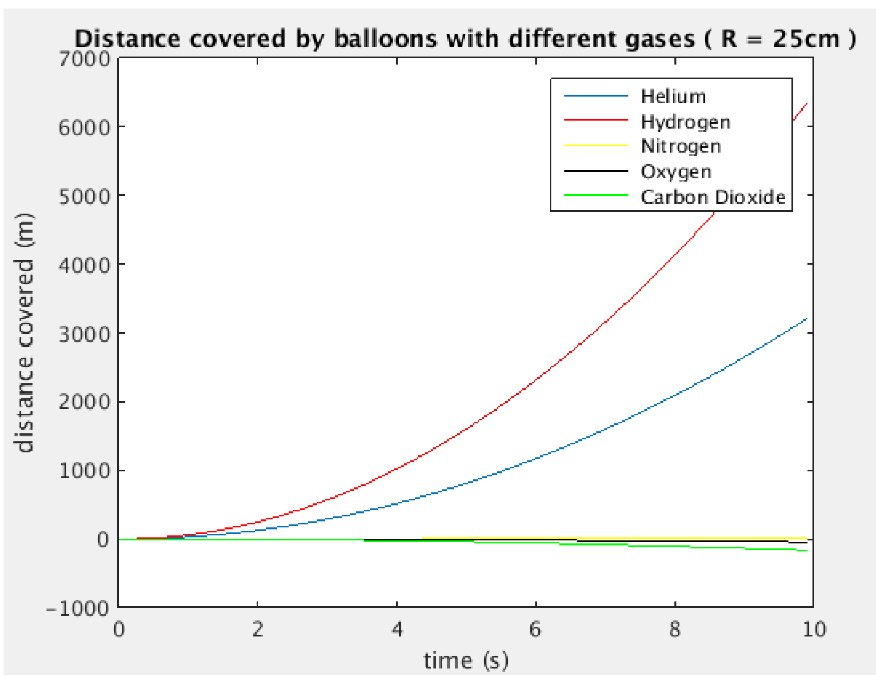
$$v = a * t \tag{36}$$

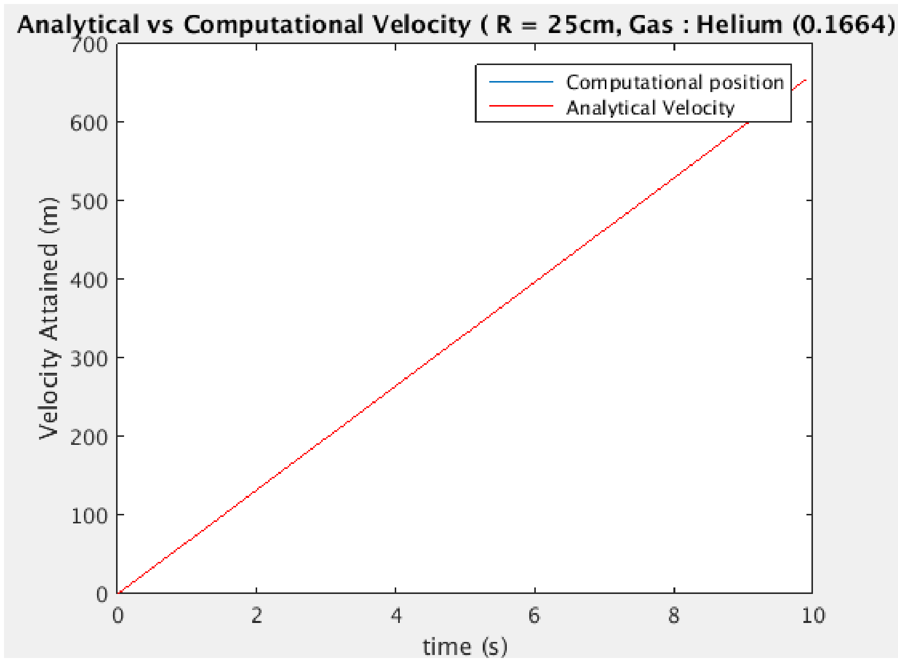
$$x = 0.5 * a * t^2 \tag{37}$$

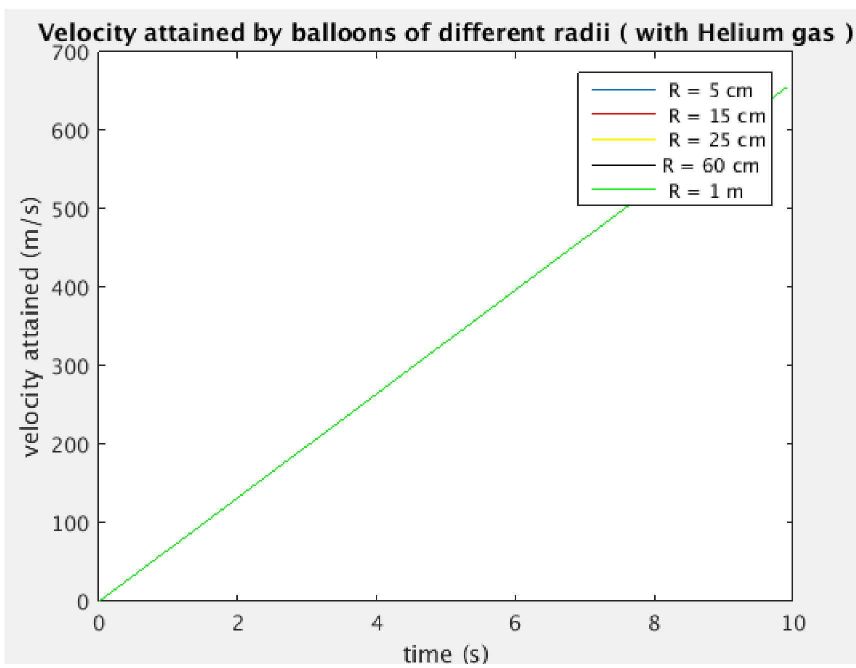
7.6 Result :

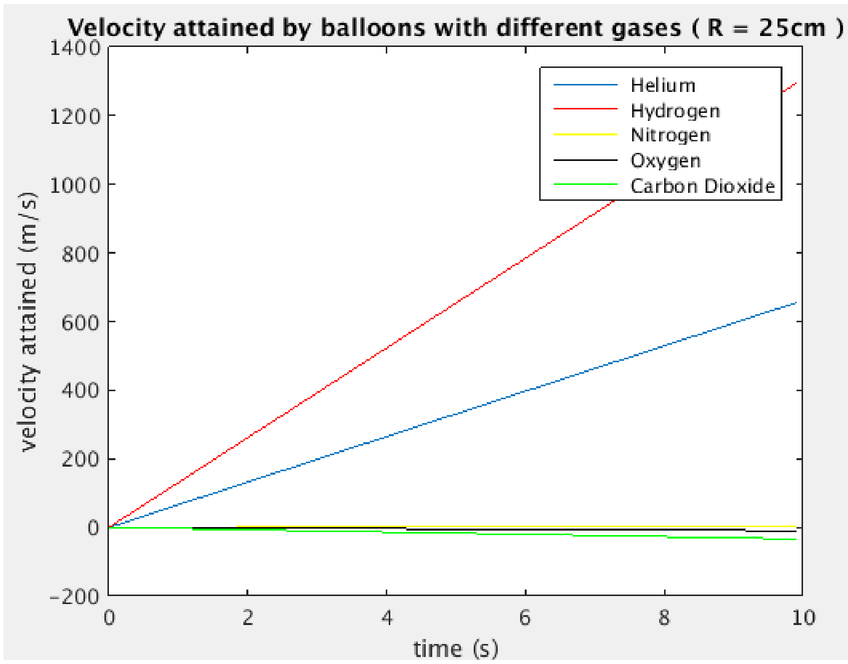



Distance covered by balloons of different radii (with Helium gas)
R = 5 cm
R = 15 cm
R = 25 cm
R = 60 cm
R = 1 m
distance covered (m)
time (s)
0
500
1000
1500
2000
2500
3000
3500
0
2
4
6
8
10


Distance covered by balloons with different gases (R = 25cm)
Helium
Hydrogen
Nitrogen
Oxygen
Carbon Dioxide
distance covered (m)
time (s)
-1000
0
1000
2000
3000
4000
5000
6000
7000
0
2
4
6
8
10


Analytical vs Computational Velocity (R = 25cm, Gas : Helium (0.1664)
Computational position
Analytical Velocity
Velocity Attained (m)
time (s)
0
100
200
300
400
500
600
700
0
2
4
6
8
10


Velocity attained by balloons of different radii (with Helium gas)
R = 5 cm
R = 15 cm
R = 25 cm
R = 60 cm
R = 1 m
velocity attained (m/s)
time (s)
0
100
200
300
400
500
600
700
0
2
4
6
8
10



7.7 Observations and Interpretations :

So we infer the following results :

1. The motion of the balloon is independent to that of it's radius.
2. Denser the gas, slower the balloon moves and lesser the distance it covers.

8 Parachute

8.1 Problem Statement

Parachute problem: frictional force on the object increases as the objects moves faster (as we learned today in the class). Role of parachute is to produce the frictional force in the form of air drag. Consider the most simple form, so the equation for velocity :

$$\frac{dv}{dt} = a - bv \tag{38}$$

where a (from applied force), b (from friction) are constants. Use Euler's method to solve for "v" as a function of time. Choose a=10 and b=1. What is the terminal velocity in this case.

8.2 Assumptions

1. There is no other factor that affects the net force and hence acceleration the body.

8.3 Mathematical Model :

Initial Conditions :// a = 10 b = 2

8.3.1 Mathematical Equations :

$$\frac{dx}{dt} = v \tag{39}$$

$$\frac{dv}{dt} = a - b * v \tag{40}$$

On solving further:

$$\int_0^v \frac{dv}{a - bv} = \int_0^t dt \tag{41}$$

$$\log_e \frac{a - bv}{a} = t \tag{42}$$

Hence on further solving

$$v = \frac{a(1 - \exp -bt)}{b} \tag{43}$$

8.3.2 Finite Difference Form :

Here, we use Euler-Cromer method to find the change in acceleration.
$v_0 = 0$
Hence,
$a_0 = a$

$$a_{i+1} = a - bv_{i+1} \tag{44}$$

$$v_{i+1} = v_i + a_i * \Delta t \tag{45}$$

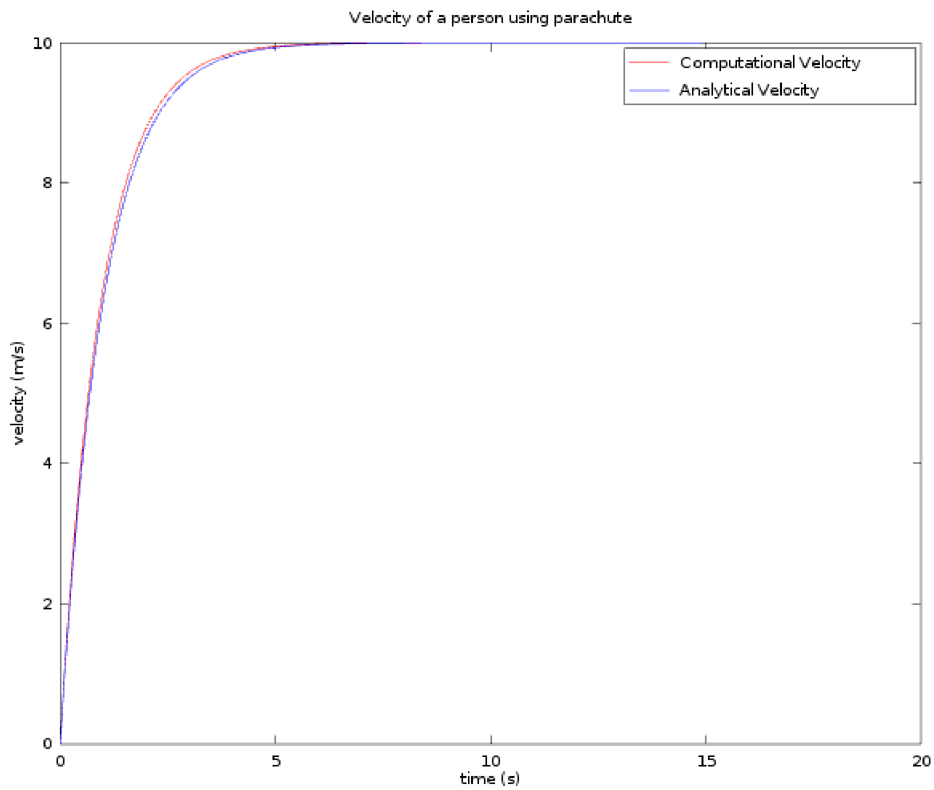
$$x_{i+1} = x_i + v_i * \Delta t \tag{46}$$

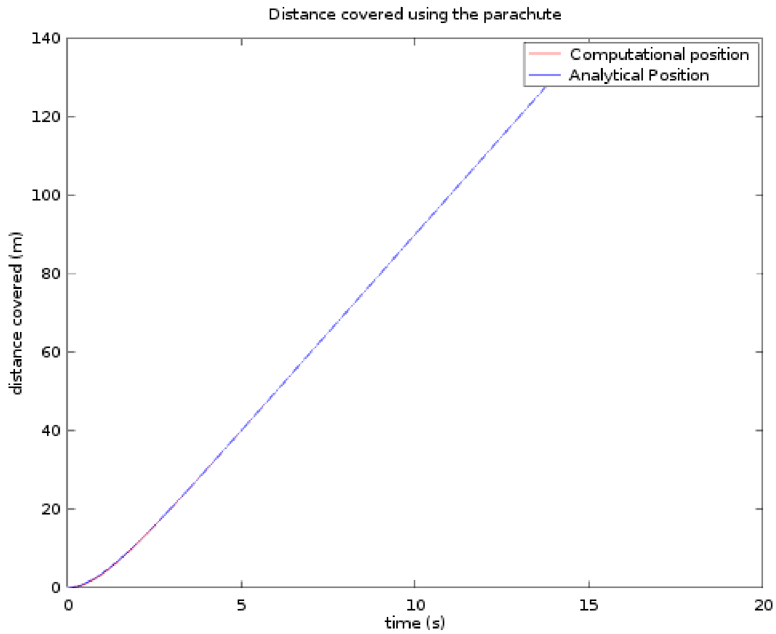
8.3.3 Analytical Solution :

$$v = \frac{a(1 - e^{-bt})}{b} \tag{47}$$

$$x = \frac{at}{b} + \frac{ae^{-bt}}{b^2} - \frac{a}{b^2} \tag{48}$$

8.4 Result :


Velocity of a person using parachute
Computational Velocity
Analytical Velocity
velocity (m/s)
time (s)
0
2
4
6
8
10
0
5
10
15
20



8.5 Observation and Inferences :

The following can be interpreted :

1. The acceleration of a body in midair decreases gradually until it ultimately becomes zero.

2. When the net force on the body is zero, the body moves with a constant velocity also known as the terminal velocity.

3. In this case, terminal velocity is 10 m/s and in general case, the terminal velocity is when :

$$\frac{dv}{dt} = 0 \tag{49}$$

   Hence,

$$a - vb = 0; \tag{50}$$

   Thus,

$$v = \frac{a}{b} \tag{51}$$