Introduction to Complex Networks, SC435
Assignment 1 - SIR Model on a Network

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I. MONTE CARLO SIMULATIONS

For a given set of parameters:

1. β : probability of infection

2. α : rate of recovery

an SIR model can be simulated on a network using the following rules:

1. At every time step t, every infected node infects each of its susceptible neighbours with probability β.

2. At every time step t, every infected node recovers with probability α.

A. Sample simulation

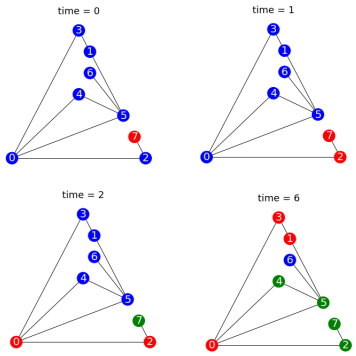

FIG. 1. Initial few states of the network with parameters, β = 0.5, α = 0.1

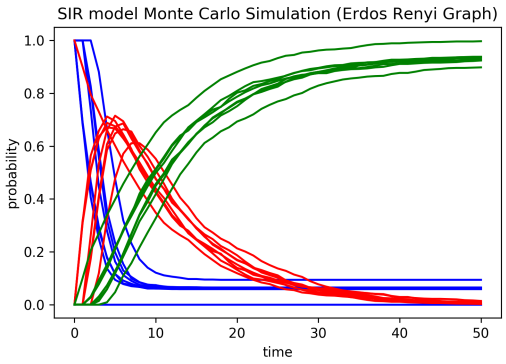

FIG. 2. Probability distributions of each node being in all 3 compartments : Susceptible (blue), Infected (red) and Recovered (green). $\beta = 0.3$, $\alpha = 0.1$

II. DIFFERENTIAL EQUATIONS

The probability of each node being susceptible, infected or recovered at each time step can be calculated using the structure of the network. Let:

1. $s_i(t)$: probability of node i being susceptible at time t

2. $x_i(t)$: probability of node i being infected at time t

3. $r_i(t)$: probability of node i being recovered at time t

where, $s_i(t) + x_i(t) + r_i(t) = 1$ for any node i at any time t.

$$s_i(t+\delta t) = s_i(t) - s_i(t)\beta \sum_j A_{ij} x_j(t)\delta t \tag{1}$$

$$x_i(t+\delta t) = x_i(t) + s_i(t)\beta \sum_j A_{ij} x_j(t)\delta t - x_i(t)\alpha\delta t \tag{2}$$

$$r_i(t+\delta t) = r_i(t) + x_i(t)\alpha\delta t \tag{3}$$

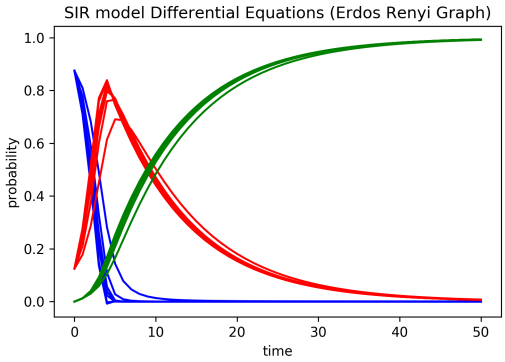

FIG. 3. Probability distributions of each node being in all 3 compartments : Susceptible (blue), Infected (red) and Recovered (green). $\beta = 0.3$, $\alpha = 0.1$

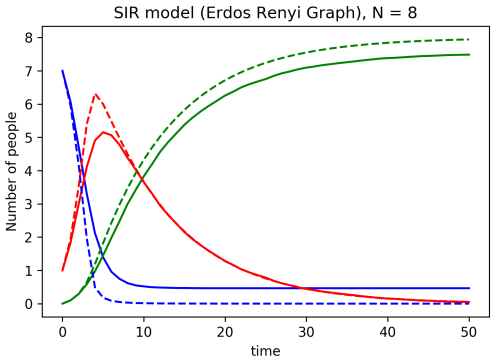

FIG. 4. Number of people that are Susceptible (blue), Infected (red) and Recovered (green). The dotted lines represent the solution via differential equations and the solid lines represent the Monte Carlo simulation results.