B.Tech Mini Project Report
MODELS OF OPINION SPREAD

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In this work, we investigate modifications to Sznajd's model of opinion dynamics in two dimensions. Namely, we observe the effect of social media campaigns on the process of opinion formation in a population of voters. Our results suggest that increasing the influence of social media decreases the minimum threshold of initial opinion density required to achieve a phase transition. After crossing a certain probability of media influence, only one opinion will always prevail. We observe by how much is this effect reversed on adding an opposing media campaign. Further, we simulate two variants of the Sznajd's model to observe the effect of a voter choosing a certain side when there is conflict in his neighbourhood. The phase transition in the community's overall opinion is no longer observed and there is no clear majority for most values of initial opinion density.

INTRODUCTION

Complex behaviors of human crowds have been studied by socio-physicists for a long time, using models that take inspiration from statistical physics. These models have been used to study different social phenomenon with the hypothesis that laws on the microscopic scale can explain phenomena on the macroscopic scale. These models also assume that individuals, in many cases, act as particles without free will [1]. By modeling behavior of individuals, one can infer emergent patterns shown by the crowd as a whole. Due to the law of large numbers, averaging over repeated simulations will result in results close to the expected value [1]. Opinion dynamics is popularly studied example of sociophysics, which models local interactions of individuals to observe how their opinions changes in a community. The Sznajd model[2] simulates opinion dynamics with rules based on *social validation*, i.e., individuals make decisions based on what others around them are doing. In the following work, we simulate the Snazjd model in one dimension and two dimensions. We also simulate the effect of social media on opinion dynamics, including the effect of two opposing media campaigns. Further, we simulate two modifications to the two dimensional Sznajd model and observe the results on how the opinion dynamics change as compared to the original model.

I. SZNAJD'S 1D MODEL

Sznajd's model in one dimension is represented by a chain of N Ising spins ($S_i; i = 1, 2, ..., N$) where each spin represents the opinion of a single voter. Each spin can take one of two possible values, i.e., up (+1) or down (−1). Let an up spin denote choice A and down spin denote choice B. This convention is followed throughout this paper. Thus, a voter's opinion is either in favor of A or B. Starting from some initial distribution of spins, the model is evolved in time using the following set of rules for updating spins. At each time step t:

1. A site S_i is randomly chosen. Each site on the lattice has an equal probability of being chosen.
2. If $S_iS_{i+1} = 1$, then S_{i-1} and S_{i+2} take the direction of this pair, i.e., if a pair of voters share the same opinion, then their nearest neighbours agree with them.
3. If $S_iS_{i+1} = -1$, i.e., then S_{i-1} takes the direction of S_{i+1} and S_{i+2} takes the direction of S_i, i.e., if a pair of neighbours disagrees, then each of their nearest neighbours also disagrees with them.

Simulating these rules leads to 3 possible steady states:

1. 100% of the spins are up spins, implying all the voters agree and are in favor of choice A (consensus).
2. 100% of the spins are down spins, implying all the voters agree and are in favor of choice B (consensus).
3. 50% of the spins are up, and 50% are down. The spins alternate between up and down, implying that every voter disagrees with their immediate neighbours (stalemate).

Simulation

Sznajd's Model in one dimension was simulated for a lattice chain with $N = 64$. Averaging was done over 500 simulations. Initially, C_u fraction of the voters have an up spin, and the rest have a down spin. The assignment of these spins to the sites were done randomly. The simulation was run until the system reached a steady state. The final magnetization of the Ising spin system, that represents the net opinion of the community of voters, was calculated as:

$$M = \frac{1}{N} \sum S_i \qquad (1)$$

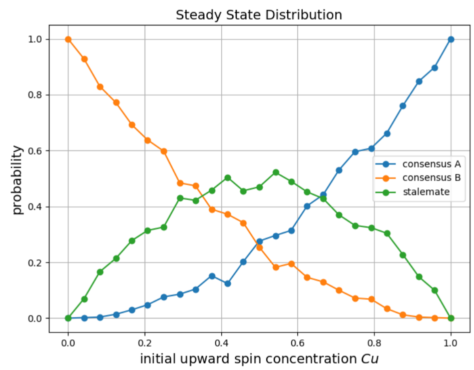

FIG. 1. Probability distribution of the 3 different steady states vs. the initial upward spin concentration.

As seen in Fig. 1, at $C_u = 0.5$, probability of consensus towards choice A is 25%, consensus towards choice B is 25% and a stalemate will occur 50% of the time.

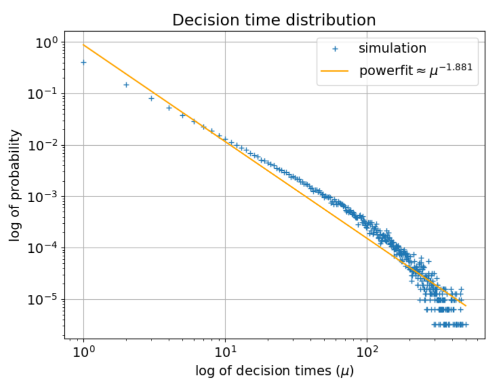

FIG. 2. Distribution of decision times follows a power law with an exponent ~ -1.8.

The decision time, defined as the duration of time between two changes of opinion by a voter, is seen to follow a power law (Fig 2). This implies that most voters change their opinion repeatedly after very small intervals of time. However, a few voters stay for a long time without changing their opinion.

II. SZNAJD'S 2D MODEL

Sznajd's model in one dimension can be extended to two dimensions and is represented by a NxN lattice of N^2 Ising spins. The rules for updating the spins (as described by Stauffer[3]) are modified as follows. At each time step t:

1. A 2x2 matrix on the bigger NxN matrix is selected randomly.
2. If all 4 selected sites align in the same direction, then their eight nearest neighbours all take this spin.
3. If all 4 selected sites do not align, then no change takes place.

In a social context, this means that a group of voters with consensus among them can persuade their neighbors to agree with them. If this group has a conflict of opinion in it, then its neighbors are not affected.

A. Simulation

Sznajd's Model in two dimensions was simulated for a lattice with $N = 24$. Averaging was done over 300 simulations. The initial state was determined the same way as in one dimension, using parameter C_u. Simulation leads to 2 possible steady states, full consensus towards choice A or choice B. As seen in Fig. 3, the probability of a stalemate is 0, for any value of C_u. For $C_u < 0.5$, the system will always reach a consensus of all spins down and for $C_u > 0.5$, it will always reach a consensus of all spins up. Hence, $C_u = 0.5$ is the critical point for the phase transition of the system.

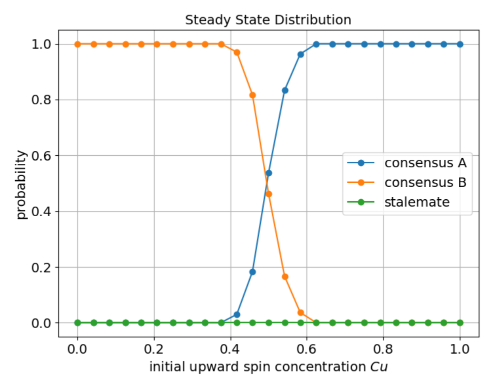

FIG. 3. Probability distribution of the 3 different steady states vs. the initial upward spin concentration.

III. SZNAJD'S 2D MODEL WITH EFFECT OF SOCIAL MEDIA

In reality, apart from the opinion of one's neighbors, social media also affects voters' opinions. To incorporate this effect in the mode, an external field is applied to the lattice of Ising spins. Here, the external field is applied uniformly throughout the duration of simulation and is considered to be in favour of choice A. The 3rd step of the rules is modified such that if all 4 selected sites do not align, then each of the eight neighbors change their opinion under the influence of external field and take the +1 spin with probability p or do not change their spin with probability $1-p$. In other words, if the collective opinion of a group of voters is not strong enough to persuade its neighbors, then social media can change the neighbors opinion with a probability p.

A. Simulation

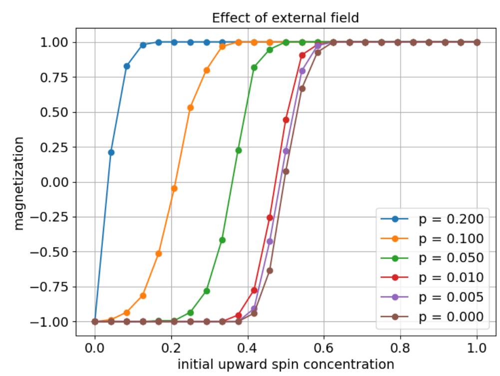

FIG. 4. Magnetization versus the initial upward spin concentration for different values of media influence probability p.

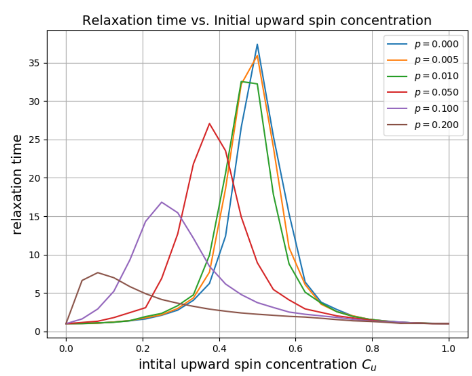

FIG. 5. Relaxation time versus the initial upward spin concentration for different values of media influence probability p.

Fig. 4 shows that on adding an external field (characterised by probability p), the critical value of C_u that leads to phase transition from all down spins to all up spins changes. On increasing p, the critical value decrease, implying that a smaller initial concentration of up spins is sufficient to reach a consensus. For $p >= 0.1$, the system does not undergo a phase transition and always reaches a consensus of up spins due to the overpowering external field. Social media in favour of choice A also decreases the time taken to reach consensus (Fig. 5). Decision times no longer follow a power law (Fig 6).

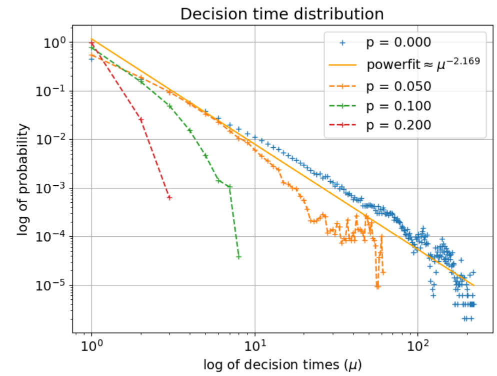

FIG. 6. Decision time distribution for different values of media influence probability p.

An external social media influence reaches a steady state faster and more voters change their opinion after short duration of time.

IV. MODIFICATION TO SZNAJD'S 2D MODEL

It is unlikely that if a group has a conflict of opinion in it, then its neighbors will not be affected. We propose two modifications to the rules described in Section. II.

1. *Modification 1*

While the eight neighbors change their spin to +1 with a probability p, with probability $1-p$, instead of keeping their spins unchanged, each neighbor takes the spin of its neighbor's neighbor.

Simulation show that introducing this modification leads to the disappearance of a sharp phase transition in the value of magnetization (Fig 7). Further, the system does not always reach a consensus for small values of $p < 0.005$. This is shown in Fig 8. Apart from full consensus and stalemate, the system can reach another steady state (Fig 10) under these modified rules. Increasing the effect of the external field to even a small value ($p = 0.005$) polarized the opinions strongly and the only steady states possible are consensus (Fig 9). For larger values of $p > 0.005$ the external field overpowers the internal conflicts and always leads to a state of full consensus of all up spins.

2. *Modification 2*

While the eight neighbors change their spin to +1 with a probability p, with probability $1-p$, instead of keeping their spins unchanged, each neighbor takes the spin of its neighbor. This modification, like Fig. 7, breaks the phase transition (as seen in Fig 11) but a configuration

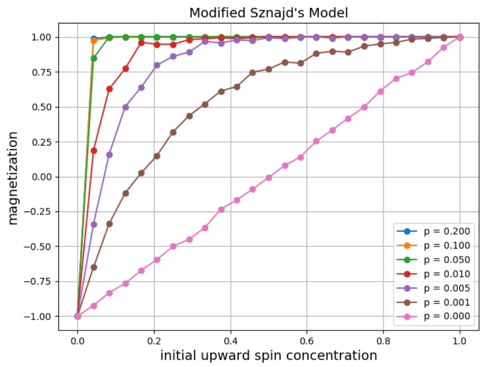

FIG. 7. Magnetization versus the initial upward spin concentration for different values of probability p for the first modification.

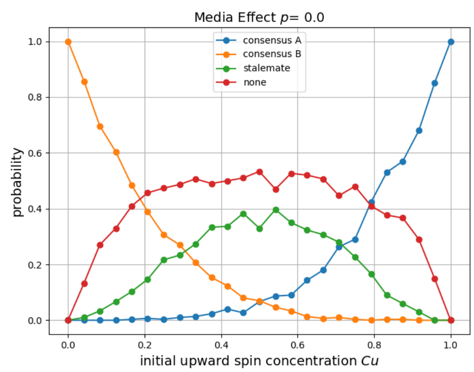

FIG. 8. Probability distribution of the different steady states vs. the initial upward spin concentration for $p = 0$.

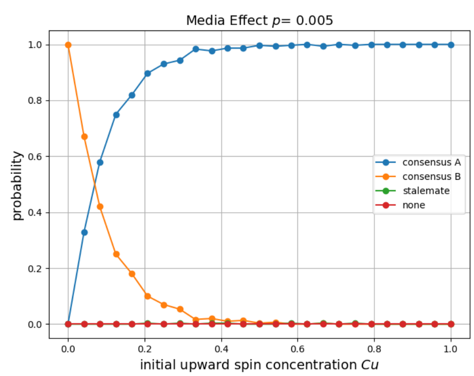

FIG. 9. Probability distribution of the different steady states vs. the initial upward spin concentration for $p = 0.005$.

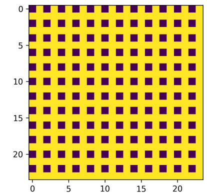

FIG. 10. A steady state configuration that is neither consensus nor stalemate. Each color represents one of the two possible opinions.

such as Fig 10 is not a possible steady state. The only possible states are full consensus for up spins and full consensus for down spins (Fig 12 and Fig 13). The value of magnetization increases linearly with increasing C_u for small values of external medial field. At larger values, the field becomes overpowering and always takes the system to a consensus of all up spins (Fig 11).

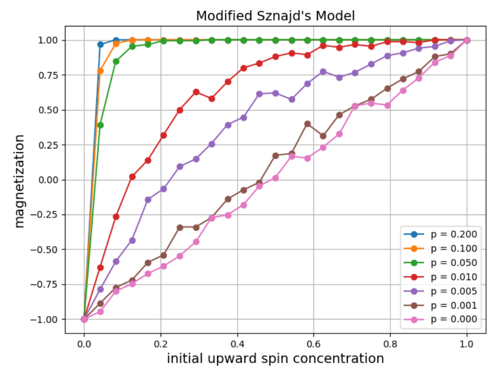

FIG. 11. Magnetization versus the initial upward spin concentration for different values of probability p for the second modification.

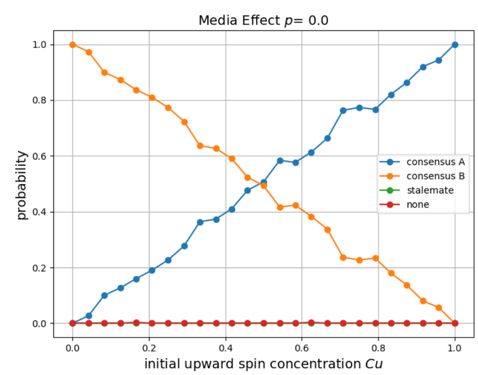

FIG. 12. Probability distribution of the different steady states vs. the initial upward spin concentration for $p = 0$.

Thus, it is observed that both the modifications break the phase transition observed in the original model. This implies that, if there is no media influence, the community of voters does not reach a clear consensus unless the initial state of the system is already a consensus. If social media is added to the system with influence probability

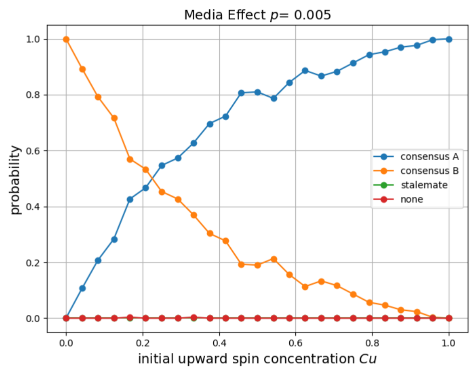

FIG. 13. Probability distribution of the different steady states vs. the initial upward spin concentration for $p = 0.005$.

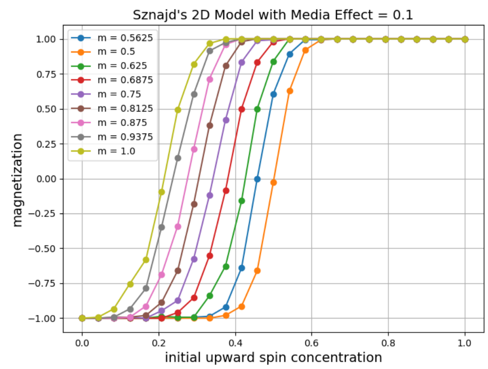

FIG. 14. Magnetization versus the initial upward spin concentration for $p = 0.1$ and different fractions (m) of social media in favor of choice A.

p, for $p > 0.01$ the system will always reach a consensus in favor the side the media supported. This is value is lower than that in the original model, implying that now, it is easier to reach a consensus even with a weaker media influence.

V. EFFECT OF NON-UNIFORM SOCIAL MEDIA

In Section III, we assumed that the external field representing social media favored a particular choice (in our case, choice A). Realistically, both A and B would influence voters by social media. Step 3 in section III is modified using a non-uniform external field as follows: neighbors take the $+1$ spin with probability mp, take -1 with probability $(1-m)p$ and do not change their spin with probability $1-p$. m represents the fraction of external field that is in favor of choice A.

A. Simulation

Decreasing the fraction of external social media effect in favour of choice A, increases the value of critical initial upwards spin concentration (Fig 14). This implies that if choice A has on opposing social media effect, then it takes a larger number of initial upward spins to reach consensus for choice A.

Fig 15 shows that a lower fraction (m) of the external field effect in favor choice A increases the time taken for the system to reach a steady state. In other words, if there are two conflicting social media campaigns, it takes a longer time for voters to reach a consensus, and the time taken is maximum when the campaigns are of equal strength ($m = 0.5$) the initial opinions are equally split amongst the voters ($C_u = 0.5$).

Conflicting social media campaigns also leads to distribution times following a power law with a few voters changing their opinion after long intervals of time (Fig 16).

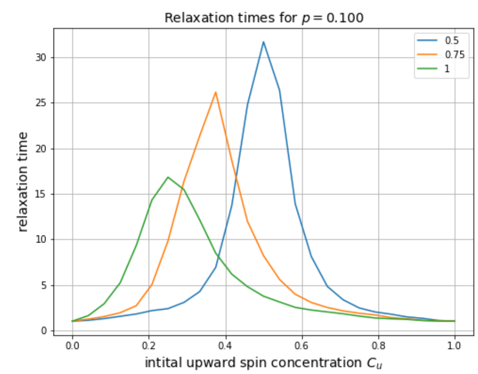

FIG. 15. Relaxation time versus the initial upward spin concentration for $p = 0.1$ and different fractions (m) of social media in favor of choice A.

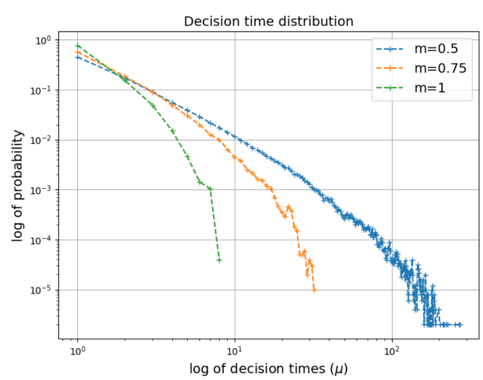

FIG. 16. Decision time distribution for $p = 0.1$ and different fractions (m) of social media in favor of choice A.

VI. CONCLUSION

In this work, we simulated Sznajd's model of opinion dynamics in one dimension and verified the results obtained in [2]. Using Stauffer's generalization [3], the model was extended to two dimensions, and a phase transition between the two types of consensus was observed at a critical value of initial density of opinions. Further, we modeled the effect of social media using an external field[1] and found that this critical value of initial density decreases as we increase the strength of social media influence. For an influence < 0.1, there is no phase transition, and opinion supported by the media will prevail irrespective of the initial opinion density. Social media influence also decreases the time taken to reach consensus. We then modeled two modifications to Sznajd's model in two dimensions by incorporating the idea that if an individual encounters conflict in his neighbourhood, then they will take either the neighbor's neighbor's opinion or the neighbor's opinion itself. The former gives rise to a new steady state configuration of stalemate. These modifications both showed that the phase transition observed in Sznajd's model earlier vanishes. Without social media influence, the net opinion varies linearly with change in initial density. With social influence, the net opinion changes to the side favored by the media even at very small values of initial density. Lastly, we modeled the effect of opposing social media campaigns and observed that the minimum initial density of opinions required for a consensus increases when an opposing social media influence is introduced.

[1] K. Nazeri, arXiv preprint arXiv:1805.08310 (2018).
[2] K. Sznajd-Weron and J. Sznajd, International Journal of Modern Physics C **11**, 1157 (2000).
[3] D. Stauffer, A. O. Sousa, and S. M. De Oliveira, International Journal of Modern Physics C **11**, 1239 (2000).