<u>CS105 – HW02 – OPSEM</u>

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PART A

1. *Translating opsem into English.* Restate each of the following formal statements using informal English:

   a. Either $x \in \text{dom}\,\rho$ or $x \in \text{dom}\,\xi$.

   → Either x is a formal parameter or a global variable

   b. $\langle e, \xi, \phi, \rho\rangle \Downarrow \langle v, \xi, \phi, \rho\rangle$.

   → expression e evaluates to value v, without changing any global variables, formal parameters, or function definitions

2. Translating English into opsem. Take each of the following informal statements and restate it using the formalism of operational semantics.

   a. If global variable *x* is not defined, it is possible that expression *e* will evaluate successfully.

   → if $x \notin \text{dom } \xi$, $\exists \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$

   b. If *e* evaluates successfully, no global variables' values are changed.

   → if $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$ then, $\xi = \xi'$ and ρ *may not equal** ρ'

   c. If *e* evaluates successfully, no new global variables are defined.

   → if $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$, then ξ *may not equal* ξ' and ρ *may not equal* ρ'

* may not equal : using may not equal as the expression e is unknown & can possibly change the values of given environments

3. Analyzing inference rules. The following notation suggests an alternative to the WhileEnd rule of Impcore. Either explain why this rule is effectively the same as the original rule, or explain why it's different.

a.

```
                    ⟨ e1,ξ,φ,ρ⟩  ⇓ ⟨ 0,ξ′,φ,ρ′⟩
---------------------------------------- (WhileEnd')
⟨ while(e1, e2), ξ, φ, ρ⟩  ⇓ ⟨ 0, ξ′, φ, ρ′⟩
```

→ The above inference rule is effectively the same as the original 'While End' since they both represent the result of evaluating a while loop that terminates immediately, because the value of the condition expression e1 is equal to 0.

In the original, the premise says if e1 evaluates to v1 where v1 = 0, then we can conclude that the while loop ends as the expressions e1 and e2 evaluate to 0. In the alternative one, the premise directly just says if e1 evaluates to 0, then we can conclude that the while loop ends as the expressions e1 and e2 evaluate to 0.

b.

```
          x ∈ dom ρ   ⟨ e,ξ,φ,ρ⟩  ⇓ ⟨ v,ξ′,φ,ρ′⟩
------------------------------------------------ (FormalAssign')
          ⟨ set(x, e), ξ, φ, ρ⟩  ⇓ ⟨ v, ξ′, φ, ρ′⟩
```

→ The above inference rule is not the same as the original 'Formal Assign'. The difference between the two lies in the conclusion judgement. On evaluating ⟨ set(x, e), ξ, φ, ρ⟩ , the original rule changes the ρ environment to ρ′{ x -> v} , whereas the one above only changes the ρ environment to ρ′.

Code example to show the difference between two rules:

```
( define test ()

    (set x 6)

    x )
```

The original Formal Assign rule would ensure that the output is 6, since the value of 6 is updated in environment ρ. However, with the above new rule, it may cause an error or output any other random value since value 6 was not updated in environment ρ.

PART B

4. Attached below is parts a through c ; part d is submitted as an impcore file.

a) Rules to give impcore AWK-like semantics :

$$\frac{x \notin \text{dom } P \qquad x \notin \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, \rho \rangle} \quad (\text{AWK} - \text{VAR})$$

$$\frac{x \notin \text{dom } P \qquad x \notin \text{dom } \xi \qquad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, \rho' \rangle} \quad (\text{AWK-ASSIGN})$$

b) Rules to give impcore Icon-like semantics :

$$\frac{x \notin \text{dom } P \qquad x \notin \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho\{x \mapsto 0\} \rangle} \quad (\text{ICON} - \text{VAR})$$

$$\frac{x \notin \text{dom } P \qquad x \notin \text{dom } \xi \qquad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho'\{x \mapsto v\} \rangle} \quad (\text{ICON - ASSIGN})$$

c) I prefer the ICON implementation more than the AWK implementation because setting an undefined variable to 0 in the global scope can cause a host of issues, including:

- masking programming errors such as in the case one meant to initialize a variable but did not
- debugging issues ; if we have too many unbound variables, it might lead to unexpected behaviour and tracing the errors might be hard.
- lack of explicit documentation ; without binding the variables we lose out on readibility of code making it harder for anyone to revisit it.

PART C

5. Use the operational semantics to prove that if you evaluate (begin (set x 3) x) in an environment where $\rho(x) = 99$, then the result of the evaluation is 3

e_1 :

$$\dfrac{x \in \operatorname{dom} \rho \qquad \dfrac{}{\langle \mathrm{LIT}(3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}\ (\text{LITERAL})}{\langle \mathrm{SET}(x\ 3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle}\ (\text{FORMAL ASSIGN})$$

e_2 :

$$\dfrac{x \in \operatorname{dom} \rho}{\langle \mathrm{VAR}(x), \xi, \phi, \rho\{x \mapsto 3\} \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle}\ (\text{FORMAL VAR})$$

$$\dfrac{}{\langle \mathrm{BEGIN}(\mathrm{SET}(x, 3), \mathrm{VAR}(x)), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle}\ (\text{BEGIN})$$

(due to lack of space, e1 and e2 are stacked instead of being side by side, hence they are labelled as expression 1 and expression 2)

→ From the above derivation, we can see that the premises lead to (begin (set x 3) x) being set to 3 as can be seen from the conclusion of the evaluation judgement.

6. Show that expression if (x x 0) is “observationally equivalent” to expression x

Specifically, use the operational semantics to prove that if there exist environments ξ, ϕ, and ρ (and ξ', ρ', ξ'', and ρ'') such that

$\langle \mathrm{IF}(\mathrm{VAR}(x), \mathrm{VAR}(x), \mathrm{LITERAL}(0)), \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi', \phi, \rho' \rangle$

and

$\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle v_2, \xi'', \phi, \rho'' \rangle$

then $v_1 = v_2$.

→ The IF expression has two rules, IF TRUE and IF FALSE. The VAR expression also has two rules FORMAL VAR and GLOBAL VAR.

Using the IF TRUE derivation first:

$$\dfrac{\dfrac{x \in \mathrm{dom}\ \rho}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ (\text{FORMAL VAR}) \quad \rho(x) \neq 0 \quad \dfrac{x \in \mathrm{dom}\ \rho}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ (\text{FORMAL VAR})}{\langle \mathrm{IF}(\mathrm{VAR}(x), \mathrm{VAR}(x), \mathrm{LIT}(0)), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ (\text{IF TRUE})$$

$$\dfrac{\dfrac{x \notin \mathrm{dom}\ \rho \quad x \in \mathrm{dom}\ \xi}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi(x), \xi, \phi, \rho \rangle}\ (\text{GLOBAL VAR}) \quad \xi(x) \neq 0 \quad \dfrac{x \notin \mathrm{dom}\ \rho \quad x \in \mathrm{dom}\ \xi}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi(x), \xi, \phi, \rho \rangle}\ (\text{GLOBAL VAR})}{\langle \mathrm{IF}(\mathrm{VAR}(x), \mathrm{VAR}(x), \mathrm{LIT}(0)), \xi, \phi, \rho \rangle \Downarrow \langle \xi(x), \xi, \phi, \rho \rangle}\ (\text{IF TRUE})$$

We can see that given the LHS of the evaluation judgement , it evaluates to ρ (x) using the FORMAL VAR rule and evaluates to ξ(x) using the GLOBAL VAR rule respectively. We can see that neither the IF rule nor the VAR rules changes either environments - ρ, φ, ξ.

i. If x ∈ dom ρ → we get the value of x from ρ : from the question we know that VAR(x) evaluates to value v1
ii. If x ∈ dom ξ → we get the value of x from ξ : from the question we know that VAR(x) evaluates to value v1

From the question, we also know that the IF expression evaluates to v2.

Now, since the environments remain the same, and both the VAR expression and IF expression evaluate to either ρ (x) or ξ(x), we see that the values are coming from the same scope.

Thus, in the case that IF is true, we can say :

v1 = v2 = ρ (x) or ξ(x)

Using the IF FALSE derivation now:

$$\frac{\dfrac{x \in \mathrm{dom}\ \rho}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle}\ (\text{FORMAL VAR}) \quad \rho(x) = 0 \quad \dfrac{}{\langle \mathrm{LIT}(0), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho \rangle}\ (\text{LITERAL})}{\langle \mathrm{IF}(\mathrm{VAR}(x), \mathrm{VAR}(x), \mathrm{LIT}(0)), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho \rangle}\ (\text{IF FALSE})$$

$$\frac{\dfrac{x \notin \mathrm{dom}\ \rho \quad x \in \mathrm{dom}\ \xi}{\langle \mathrm{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi(x), \xi, \phi, \rho \rangle}\ (\text{GLOBAL VAR}) \quad \xi(x) = 0 \quad \dfrac{}{\langle \mathrm{LIT}(0), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho \rangle}\ (\text{LITERAL})}{\langle \mathrm{IF}(\mathrm{VAR}(x), \mathrm{VAR}(x), \mathrm{LIT}(0)), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho \rangle}\ (\text{IF FALSE})$$

We can see that given the LHS of the evaluation judgement , it evaluates to 0 using either the FORMAL VAR rule or the GLOBAL VAR rule. We can see that neither the IF rule nor the VAR rules changes either environments - ρ, φ, ξ.

i. If x ∈ dom ρ → we get the value of x from ρ : from the question we know that VAR(x) evaluates to value v1
ii. If x ∈ dom ξ → we get the value of x from ξ : from the question we know that VAR(x) evaluates to value v1

But here, for the IF FALSE rule to be applicable ρ (x) or ξ(x) must be equal to 0.

From the question, we also know that the IF expression evaluates to v2.

Now, since the environments remain the same, and the VAR expression evaluate ρ (x) or ξ(x) whose value MUST be 0 and the IF expression evaluates to 0 , we see that the values are coming from the same scope.

Thus, in the case that IF is false, we can say :

$$v1 = v2 = \rho(x) \text{ or } \xi(x) = 0$$