PART A : THEORY

1. Prove:

$$(\text{o}\ ((\text{curry map})\ f)\ ((\text{curry map})\ g)) = ((\text{curry map})\ (\text{o}\ f\ g))$$

Applying the function (both LHS & RHS) to list xs:

$$((\text{o}\ ((\text{curry map})\ f)\ ((\text{curry map})\ g))\ \ xs) = (((\text{curry map})\ (\text{o}\ f\ g))\ \ xs)$$

PART A : BASE CASE { xs ⟶ '() }

Assume xs = '()

$$((\text{o}\ ((\text{curry map})\ f)\ ((\text{curry map})\ g))\ xs)$$

= { by assumption xs = '() }

$$((\text{o}\ ((\text{curry map})\ f)\ ((\text{curry map})\ g))\ \ '())$$

= { by apply-compose law }

$$(((\text{curry map})\ f)\ (((\text{curry map})\ g))\ '()))$$

= { by apply-curry law }

$$(((\text{curry map})\ f)\ (\ \text{map}\ g\ '()\ ))$$

= { by map-nil law }

$$(((\text{curry map})\ f\ )\ '()\ )$$

= { by apply-curry law }

(map f '())

= { by map-nil law }

'()

= { by map-nil law from right to left }

(map (o f g) '())

= { by apply-curry law from right to left }

(((curry map) (o f g)) '())

= { by assumption xs = '() }

(((curry map) (o f g)) xs)

PART B: INDUCTIVE HYPOTHESIS

Given xs = (cons a as)

Then inductive hypothesis:

((o ((curry map) f) ((curry map) g)) as)
= (((curry map) (o f g)) as)

PART C : INDUCTIVE CASE $\{ xs \mapsto$ (cons a as)$\}$

((o ((curry map) f) ((curry map) g)) xs)

= { by assumption xs = (cons a as) }

((o ((curry map) f) ((curry map) g)) (cons a as))

= { by apply-compose law }

(((curry map) f) (((curry map) g) (cons a as)))

= { by apply-curry law }

(((curry map) f) (map g (cons a as)))

= { by apply-curry law }

(map f (map g (cons a as)))

= { by map-cons law }

(map f (cons (g a) (map g as)))

= { by map-cons law }

(cons (f (g a)) (map f (map g as)))

= { by apply-curry law from right to left }

(cons (f (g a)) (map f (((curry map) g) as)))

= { by apply-curry law from right to left }

(cons (f (g a)) (((curry map) f) (((curry map) g) as)))

= { by apply-compose law from right to left }

(cons (f (g a)) ((o ((curry map) f) ((curry map) g)) as))

= { by inductive hypothesis }

(cons (f (g a)) (((curry map) (o f g)) as))

= { by apply-compose law }

(cons ((o f g) a) (((curry map) (o f g)) as))

= { by apply-curry law }

(cons ((o f g) a) (map (o f g) as))

= { by map-cons law from right to left }

(map (o f g) (cons a as))

= { by apply-curry law from right to left }

(((curry map) (o f g)) (cons a as))

= { by assumption , xs = (cons a as) }

(((curry map) (o f g)) xs)

PART D: CONCLUSION

Thus from the above inductive proof, we can conclude:

(o ((curry map) f) ((curry map) g))
= ((curry map) (o f g))