1 2.6

the notation used is $\langle v|w\rangle = (|v\rangle, |w\rangle)$

$$(\sum_i \lambda_i |w_i\rangle, |v\rangle) = (\sum_i \lambda_i |w_i\rangle)^\dagger |v\rangle = \sum_i \lambda_i^* \langle w_i|v\rangle = \sum_i \lambda_i^*(|w_i\rangle, |v\rangle)$$

2 2.7

–if $\langle w|v\rangle = 0$,then $|w\rangle$ and $|v\rangle$ are orthogonal

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 1 - 1 = 0$$

hence they are orthogonal!
–Normalized form of $\begin{pmatrix} a \\ b \end{pmatrix}$ is $\frac{1}{\sqrt{a^2+b^2}} \begin{pmatrix} a \\ b \end{pmatrix}$ so, normalised form of the give matrices are :

$$|w\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|v\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

3 2.9

If $|i\rangle$ are orthonormal basis that spans the vector space V. so,they will obey completeness relation. $\sum_i |i\rangle \langle i| = I$ also, any operator A: V$\longrightarrow$W such that $|v_i\rangle$ and $|w_i\rangle$ are orthonormal basis for the vector spaces V and W respectively. so we can write A as: $A = IAI$

$A = \sum_i |v_i\rangle \langle v_i| \, A \sum_j |w_j\rangle \langle w_j|$
A $= \sum_{i,j} \langle v_i| \, A \, |w_j\rangle \, |v_i\rangle \langle w_j|$

so, if the basis are $|0\rangle$ and $|1\rangle$ then pauli matrices can be represented as the outer product as follows:

$X = \langle 0| \, X \, |0\rangle \, |0\rangle \langle 0| + \langle 1| \, X \, |0\rangle \, |1\rangle \langle 0| + \langle 0| \, X \, |1\rangle \, |0\rangle \langle 1| +$
$\langle 1| \, X \, |1\rangle \, |1\rangle \langle 1|$
$X = |0\rangle \langle 1| + |1\rangle \langle 0|$

similarly for Y:
Y$|0\rangle = i \, |1\rangle$ Y$|1\rangle = -i \, |0\rangle$
$Y = \langle 0| \, Y \, |0\rangle \, |0\rangle \langle 0| + \langle 1| \, Y \, |0\rangle \, |1\rangle \langle 0| + \langle 0| \, Y \, |1\rangle \, |0\rangle \langle 1| +$
$\langle 1| \, Y \, |1\rangle \, |1\rangle \langle 1|$
$Y = -i \, |0\rangle \langle 1| + i \, |1\rangle \langle 0|$

similarly for Z: Z$|0\rangle = |0\rangle$ Z$|1\rangle = - |1\rangle$
$Z = \langle 0| \, Z \, |0\rangle \, |0\rangle \langle 0| + \langle 1| \, Z \, |0\rangle \, |1\rangle \langle 0| + \langle 0| \, Z \, |1\rangle \, |0\rangle \langle 1| +$
$\langle 1| \, Z \, |1\rangle \, |1\rangle \langle 1|$
$X = |0\rangle \langle 0| - |1\rangle \langle 1|$

4 2.10

given operator $A = (|v_j\rangle \langle v_k|)$
and since $|v_i\rangle$ forms the orthonormal basis for vector space V then, $\sum_i |v_i\rangle \langle v_i| = I$
A with respect to basis $|v_i\rangle$ can be written as :

$A = \sum_i \; |v_i\rangle \langle v_i| \, (|v_j\rangle \langle v_k|) \sum_l |v_l\rangle \langle v_l|$

$A = \sum_{i,l} (\langle v_i | v_j \rangle)(\langle v_k | v_l \rangle \, (|v_i\rangle \langle v_l|)$
since, the basis in that spans vector space V are orthogonal, hence, $A = \sum_{i,l} \delta_{ij}\delta_{kl} \, |v_i\rangle \langle v_l|$

5 2.12

$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ using $det|A - \lambda I| = 0$
$\begin{vmatrix} 1-\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} = 0$
$(1-\lambda)^2 = 0$
$\lambda = 1, 1$ the eigenvalues are comming out to be degenerate. one of the eigenvector can be : $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ but there no other "eigenvector" for the given matrix A which is orthogonal to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$! so, this matrix is not diagonalizable.

6 2.13

$(\langle v|)^\dagger = |v\rangle$
$(|w\rangle)^\dagger = \langle w|$
$(|w\rangle \langle v|)^\dagger = \langle v|^\dagger \, |w\rangle^\dagger = |v\rangle \langle w|$

7 (2.15)

A $= \sum_i a_i \, |i\rangle \langle i|$ here $|i\rangle$ is the eigenvalues and a_i is the corresponding eigenvalues.
$A^\dagger = \sum_i (a_i)^* \, |i\rangle \langle i|$

$(A^\dagger)^\dagger = \sum_i (a_i) \left|i\right\rangle \left\langle i\right|$
$(A^\dagger)^\dagger = A$

8 2.16

projector operator P $= \sum_i \left|i\right\rangle \left\langle i\right|$ and each vector $\left|i\right\rangle$ is orthogonal to each other.
$P^2 = P.P = \sum_i \left|i\right\rangle \left\langle i\right| \sum_j \left|j\right\rangle \left\langle j\right|$
$= \sum_{i,j} \left|i\right\rangle \left\langle i|j\right\rangle \left\langle j\right|$
$= \sum_{i,j} \left|i\right\rangle \left\langle j\right| \delta_{ij}$
$= \sum_i \left|i\right\rangle \left\langle i\right| = P$

9 2.17

Any normal matrix follows the property $A^\dagger A = AA^\dagger$
$A^\dagger A = AA^\dagger$
$A^{-1}A^\dagger A = A^{-1}AA^\dagger$
$A^{-1}A^\dagger A = A^\dagger$..............(1)
for A to be hermetian it follows from eq (1) that:
$A^{-1}A^\dagger = I \implies A^\dagger = A$

if A $= \sum_i a_i \left|i\right\rangle \left\langle i\right|$, where $\left|i\right\rangle$ are the eigenvectors of the A and a_i are the eigenvalues then, $A^\dagger = A \implies \sum_i a_i^* \left|i\right\rangle \left\langle i\right| = \sum_i a_i \left|i\right\rangle \left\langle i\right| \implies a_i^* = a_i$, and this condition is only possible if a is real !

10 2.18

if U is the unitary operator, $|v\rangle$ is the normalised eigenvector and a is its eigenvalue then,
$U|v\rangle = a|v\rangle$
$\langle v| U^\dagger = a^* \langle v|$
$\langle v| U^\dagger U |v\rangle = a^* a \langle v|v\rangle$
$\implies 1 = a^* a$...............(1) since, $U^\dagger U = I$
so the modulus $|a| = |\sqrt{aa^*}| = 1$
since, any complex number can be written as $re^{i\theta}$ following equation (1), we get :
$|r^2 e^{i\theta} e^{-i\theta}| = |r^2| = 1 \implies |r| = 1$

So, eigenvalues of U is a = $e^{i\theta}$, where θ is real