Cosmology

Learners' Space

Contents

Introduction

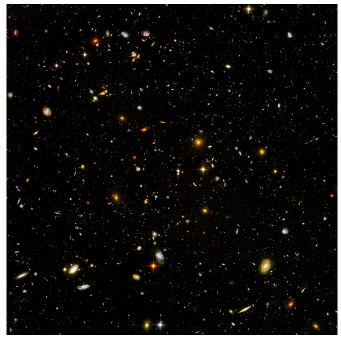
Cosmology is the study of the universe - its components, processes, and the governing physics. Cosmology asks big questions like - How did the universe form? How did it evolve? What are the components of the universe and how have they shaped its current condition? What will be the fate of the universe ? Cosmology as a science is an attempt to find answers to these big questions. Cosmology tells us how everything around us came to being as it is today. In this module we are going to discuss only a small slice of the entire field of cosmology. I would urge the reader to read the entire module in one go for better pedagogical clarity. Let's start...

Why does the universe expand ?

2.1 The Cosmological Principle

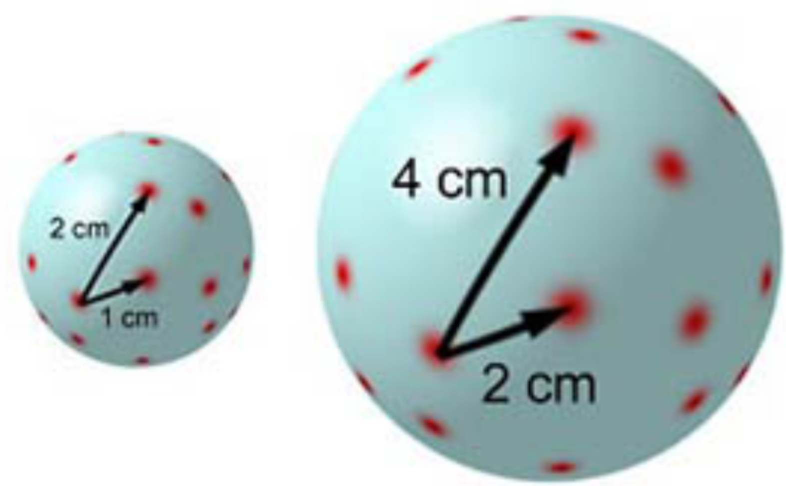
Our location in the universe is not special (also called homogeneity) and wherever we look the universe ‘roughly’ looks the same in all directions (also called isotropy). This is also called ’Invariant under rotation and invariant under translation’ respectively (because physicists love complicated words).
The cosmological principle is one of the most important fact that would lead us to understand why the universe expands. This single principle allows us to derive the changing nature of the universe. Now the thing is we won’t be going into the general relativity-based derivation of the upcoming equations. We can easily derive them using classical physics equations coupled with the implications of this cosmological principle. That is why we would first look into this principle and derive all the other terms without using general relativity(and the accompanying tensor calculus). If the reader wants to explore the more difficult route then that can also be done by following the methods mentioned ahead in the module.

Following are a few examples of these homogeneity and isotropy. Suppose we are a 2D organism on a flat infinite plane then wherever we go or in whichever direction we look in is the exact same. Think of another example - The curved surface of a cylinder. Now if we are on the curved surface every point is the same, so the cylinder is homogeneous, however, there are 2 distinct directions. One is along the surface of the cylinder(the rolling part) and one is in the vertical direction(along the axis).I think the reader will apprehend that this cylinder is homogeneous but not isotropic.
As depressing as it sounds, the cosmological principle. leads us to solve many mysteries of the universe. We are going to use homogeneity and isotropy to determine how the universe changes with time. Now the principle is only valid if we think on the scales of superclusters. If we observe on our galaxial scale there are obvious non-uniformities which become negligible on cosmic scales. Just like mountains on earth seem pretty huge but if we observe from the space ,even the Himalayas - the tallest mountains feel like mere bumps on the crusts. E.g. surface of a sphere



2.2 Scaling

There are basically two domains wherein we can observe our universe - space and time. In order to maintain homogeneity and isotropy we can think of different models of our universe. A static universe - The space is constant in the time domain which means it neither contracts nor expands. A contracting or expanding universe - As the name suggests the space will just stretch or contract but the isotropy and homogeneity are still maintained



As the figure shows the space is still isotropic and homogeneous on a larger region even though the distances have increased. Thus the universe can be 'scaled' i.e. contracted or expanded and yet we can keep cosmological principle intact.

2.3 Scale Factor

Thus the space between two objects (i.e. distance)

$$R_{proper} = a(t)\, R_{comoving}$$

where $a(t)$ is the scale factor.
$R_{comoving}$ is the distance between any 2 points in the universe at the present time and R_{proper} is the distance between 2 points in the universe in the rest frame.

This scale factor tells us whether the space is expanding or contracting with time. Now, we can never know the actual size of the universe.That's why, instead of using radius increase or decrease, we use a scale factor. If the universe was a sphere it would be very easy to just find its expansion from its radius but we don't know its radius! That's why scale factor is a clever trick to articulate the changing nature of the universe.

$$\text{If } a(t) > 1 \rightarrow \text{distance increases} \rightarrow \text{universe expands}$$
$$\text{If } a(t) < 1 \rightarrow \text{distance decreases} \rightarrow \text{universe contracts}$$
$$\text{If } a(t) = 1 \rightarrow \text{distance constant} \rightarrow \text{static universe}$$

2.3.1 Historical sidenote

In 1929 Edwin Hubble actually found out that the universe is expanding. He observed Cepheid variable stars which change their brightness (more formally luminosity) at a fixed time period. Henrietta Leavitt in the early 1900s formulated that a cepheid variable with a larger absolute magnitude has a larger time period. We can find out the absolute magnitude(brightness of a star if it is at 10 parsec) from the period. We can measure the apparent magnitude of the star and from these 2 we get the distance of the cepheid from us by the formula $m - M = 5log(D/10) \ldots$ ($m \rightarrow$apparent magnitude $M \rightarrow$absolute magnitude $d \rightarrow$ distance in parsecs) Edwin Hubble found out that the universe is in fact expanding according to this relation and the red shift observed in starlight. He stated that the further the star from us the greater the red-shift. The relation between velocity and distance called as Hubble's law is in section 2.5 below.

2.4 What is a metric ?

We can know more about homogeneous & isotropic surfaces from a 'metric'. A metric is an equation which tells us the distance between any 2 points on the surface. Or more specifically it is the equation that remains invariant throughout that surface (here referring to the manifold of spacetime)

E.g. What is a homogeneous and isotropic surface in 2-D $\rightarrow$ An infinite plane. Now in a 2D plane, the equation of distance between any 2 points is constant.

$$ds^2 = dx^2 + dy^2$$

Oh this seems familiar, doesn't it? This equation is what we call a metric.

Now let's see what a metric looks like in 3D. In 3D what is an example of a homogeneous and isotropic surface? $\rightarrow$ A sphere. A metric for a sphere would be

$$ds^2 = dx^2 + dy^2 + dz^2$$

You know what, let's write this metric in the spherical coordinate systems and you'll know in a while why we are doing so. So let us write (x,y,z) in terms of (R,θ,ϕ).

The equations turns out to be

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2\theta \, d\phi^2$$

(Just put in $x = R\sin\theta\cos\phi$, y=Rsin $\theta \sin\phi$ and z = Rcos ϕ then differentiate them individually and subsitute)

Ok now let us extrapolate what we know into 4 dimensions. Now what will the metric look like

$$ds^2 = dx^2 + dy^2 + dz^2 \pm c\,dt^2$$

(where dt represents time and c is the speed of light)

Now we have actually represented 'space-time' - the 3 spatial dimensions x,y,z, and t to represent the time dimension. This equation is similar to the four-vector in special relativity. The space will expand and contract according to the sign on dt and what will the sign depend on?

→ Yes you guessed it yet again - The scale factor

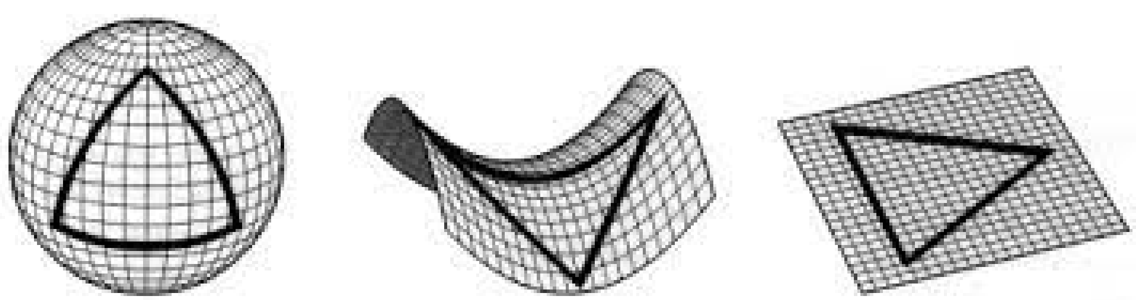
Now again if we want to represent this in terms of spherical coordinates in 4 dimensions (yes I know this are low-key getting crazy, but bear for a few paragraphs and hopefully this should become clear)

$$ds^2 = d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta\, d\phi^2) \pm cdt^2 \text{ ... In a flat universe}$$

$$ds^2 = d\chi^2 + R^2\sin^2(\chi/R)(d\theta^2 + \sin^2\theta\, d\phi^2) \pm cdt^2 \text{ ... In a Positively curved universe}$$

$$ds^2 = d\chi^2 + R^2\sinh^2(\chi/R(d\theta^2 + \sin^2\theta\, d\phi^2) \pm cdt^2 \text{ ... In a negatively curved universe}$$

But what are flat, negatively and positively curved universes?



Positive Curvature Negative Curvature Flat Curvature

Our universe can have one of these three shapes depending on which metric it follows! The universe can be closed like a sphere or in a saddle shape or completely flat.

2.4.1 What is χ ?

Still I haven't told you what χ is. χ is just like a radial coordinate. In a flat universe χ is the Radial distance R, in a positively curved universe χ will vary and as it is inside sin(χ/R) it will adjust the radial distance between -R and R just like in a sphere. Similarly in a negatively curved universe χ is inside hyperbolic sine. I know this analogy is going a bit haywire so I suggest the interested people to watch this video for a greater depth in understanding.

What is χ ?

χ is just another coordinate used to make sure that the space is curved in a particular manner. Now the distance can change according to time (remember the scale factor). We can describe the distance in a general format as follows :

$$dl^2 = a^2(t)[d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

$$\begin{array}{lcrl} f(\chi) & \rightarrow & R\sin(\chi/R) & \ldots \text{ Positive curvature} \\ f(\chi) & \rightarrow & R & \ldots \text{ Flat universe} \\ f(\chi) & \rightarrow & R\sinh(\chi/R) & \ldots \text{ Negative curvature} \end{array}$$

This is called the **Friedmann-Robertson-Lemaitre-Walker** metric and this is also a solution to the **Friedmann equations** of the general theory of relativity. The reader may want to derive the metric from that route but it is certainly beyond the scope of this course.

Derivation for the FRLW metric

2.5 Hubble's Law

Now velocity of expansion of the universe just as Edwin Hubble measured it can be given as

$$v = \frac{dR_{proper}}{dt} = \frac{d[a(t)R_i]}{dt} = \frac{R_i\, da}{dt} + \frac{a\, dR_i}{dt}$$

The second term will be zero because it is due to the galaxies moving w.r.t. Each other and not the expansion of the universe. The galaxies move due to gravitational force and as we know it is the weakest force in the universe, trust me on this, the $\frac{dR_i}{dt}$ is quite low. We can write $\frac{da}{dt}$ as a' and R_{proper} as aR_i. By the way, what is this R_{proper} ? In cosmology, co-moving distance and proper distance (or physical distance) are two closely related distance measures used to define distances between objects. Co-moving distance factors out the expansion of the universe, giving a distance that does not change in time due to the expansion of space (though this may change due to other, local factors, such as the motion of a galaxy within a cluster). Proper distance roughly corresponds to where a distant object would be at a specific moment of cosmological time, which can change over time due to the expansion of the universe. The first term in the equation corresponds to the velocity due to this proper distance while the term 2 is the comoving velocity between individual galaxies.

$$v = (a'/a)R_{proper}$$

This is what we call Hubble's law. As mentioned above, the further apart the galaxies, which means greater the proper distance between them; the greater is the velocity of them moving apart (due to expansion of the universe). $\frac{a'}{a}$ is called Hubble's parameter and is roughly 70km/s/Mpc. This value is not a constant, it changes with time because a(t) itself changes with time. Because cosmic time-scales are extremely huge, the constant Hubble measured will still be roughly the same for few million years. This was measured by Edwin Hubble back in the day and this proved the expansion of the universe. Thus we finally found out that the universe is expanding and is described by the equation

$$v = HR_{proper}$$

2.6 Eras of the Universe

Now we come back to the scale factor. From the scale factor we can determine what was the behaviour of the universe at different time frames – how fast was the universe expanding & when did it slow down

The scale factor can be obtained by solving Friedmann equations for the universe. The scale factor had different values in different time frames – We call these 'Eras' of the universe. By the way, time frames given below are not absolute. We don't know where the switch between eras occurred. The years are just an approximation consistent with observations.

Radiation dominated era – (initial 47000 years)

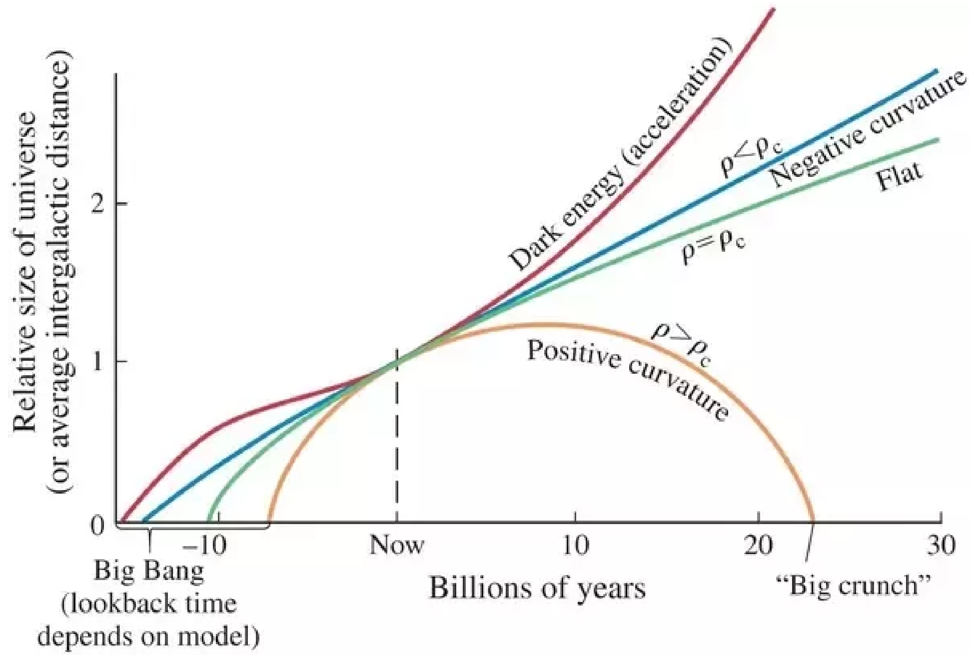
→ In the early stages of the universe, the universe was made of only photons and neutrinos i.e. radiation.
In this stage $a(t) \propto t^{\frac{1}{2}}$

Matter-dominated era – (47000 years to 9.8 billion years)

→ In later times, the universe became more matter dominated as compared to radiation. The first protons and electrons formed, combined to form atoms and later form molecules. In this stage we find the first stars and galaxies. Even our galaxy was born during this period.
In this era $a(t) \propto t^{\frac{2}{3}}$

Dark-energy dominated era – (9.8 billion years to the present day)

→ The (most probably) final era of the universe where dark-energy starts to dominate. Dark-energy would start to pull the universe more and more away. The matter in the universe will slowly (extremely slowly) start to drift apart from each other. It is speculated that in the final stages the expansion would cause the universe to freeze (motions of particles cease and all the blackholes evaporate), also called the heat death of the universe also called the 'Big Freeze'. This will still take billions and billions of years so we don't need to worry about that ;)
$a(t) \propto \exp(H_o t)$ Where H_o is the Hubble's constant.



The Y axis actually represents a(t), larger the scale factor larger is the increase in distance between galaxies.

2.6.1 Deriving the scale factors for each era

let's consider a small volume $\delta x\,\delta y\,\delta z$ with mass density μ. The volume of this region is $a^3\,\delta x\,\delta y\,\delta z$ so we define the density as $\rho = \dfrac{\mu}{a^3}$ We will assume that we are at rest at the centre of the universe for further derivations. According to Newton's theorem and gravitational law.

Newton's theorem - For an isotropic system, the gravitational force on a particular object due to all the other objects is equal to the force due to a sphere with all the objects' mass centred at the origin and the object at the radius.

$F = -\frac{GMm}{D^2}$...(1) F=mA ...(2) $A = a''(t)D$...(3) Equating and substituting ρ as mass in the above equations (Readers may try out for themselves) we get

$$\frac{a''}{a} = \frac{-4\pi G\rho}{3}$$

Now we know that escape velocity is just $2GM/R$. So at the critical condition we can have a relation of a and a' in terms of density and G – Friedmann equation

$$\left(\frac{a'}{a}\right)^2 = 8\pi G\frac{\rho}{3}$$

As ρ is inversely proportional to a^3 so Hubble constant is inversely proportional to $a^{\frac{3}{2}}$. So the Hubble constant doesn't change sign, neither does it go to zero. So if we solve the differential equation then

$$a(t) = ct^{\frac{2}{3}}$$

In the Newtonian universe space-time is flat and infinite. If we in a critical state universe i.e. energy is 0, the universe would follow a Newtonian model. But if we consider that the energy isn't 0 instead a constant E (which can of course be positive or negative) ,the Friedmann equation becomes

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{C}{a^2}$$

Where C is a constant representing E/m and some more terms. We can write density as $\frac{\nu}{a^3}$

$$(\tfrac{a'}{a})^2 = 8\pi G\tfrac{\nu}{3a^3} + \tfrac{C}{a^2}$$

In this equation on the RHS if a is small enough then the second term is negligible as compared to term 1 – we've already discussed this case. On the other hand when a is very big term 2 overweighs term 1. When a is large (and C>0) enough the expansion overweighs the gravitational pull and a(t)' = C i.e. a(t)=Ct. This universe is called the **matter dominated universe**

When we look at the universe with relativity in mind, the mass density is written in the form E, via E=mc^2. So mass density = Energy density/c^2 We will now derive equations for a radiation dominated universe i.e. a universe filled with radiation i.e. photons. E=hc/λ where lambda is the wavelength of the photon.
So if we consider a box in the grid of galaxies with volume V=a^3 the energy would be E=hc/a where lambda is a, because of resonance inside the box(just like particle in a box from quantum mechanics). So the density would be proportional to 1/a. So the equation now changes to (the power of a changes to 4 now)

$$(\tfrac{a'}{a})^2 = 8\pi G\tfrac{\nu}{3a^4} + \tfrac{C}{a^2}$$

In the radiation dominated universe a(t) =(const.)$t^{\frac{1}{2}}$ So in our universe initially we had a **radiation dominated universe** and later on we have a matter dominated universe. But apart from these 2 terms in the equation we have a third term which was discovered later called the dark energy term.

This dark energy component is not dependent on a(t) instead it is a constant vacuum energy all over the universe. Now we consider radiation pressure in each universe.
In matter dominated universe – P = 0 (the galaxies don't move so fast relative to the ambient background to exert pressures)
In radiation dominated universe – P = ρ/3 (derived from momentum and energy density)(ρ is now the energy density. please don't get confused) Now P=$\omega\rho$ where the parameter ω changes for every type of the universe. If we consider a box in the universe then the

$$dE = -PdV \ \dots \ (1)$$
Now consider $E = \rho V$
Now $\rho dV + V d\rho = -PdV \dots$ (from 1)
$$V d\rho = -(P + \rho)dV$$
Exploiting the fact that $P = \omega \cdot \rho$
$$V d\rho = -(\omega + 1)\rho dV$$
Solving the differential equation we get

$$log\rho = -(\omega + 1)logV$$
$$\rho = \frac{1}{V^{\omega+1}}$$

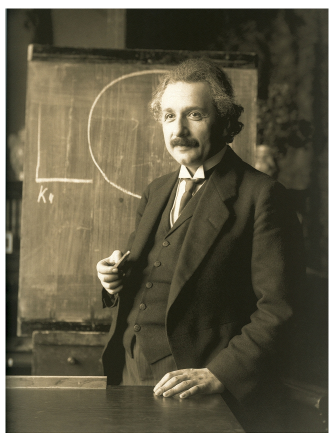
Now V = a^3 , plug that in the Friedmann equation When ω = -1 is the power of a is zero thus ρ doesn't depend on a i.e. the condition of Dark energy – constant energy density. Thus dark energy is in a way negative pressure (as energy density is positive). It corresponds to a tension that opposes contraction i.e. something that causes expansion!

$$\left(\frac{a'}{a}\right)^2 = 8G\pi\frac{\rho}{3}$$

Einstein called 8G$\pi\rho = \Lambda$ . . . Cosmological Constant
If there is dark energy then $a = ce^{\frac{t\sqrt{\Lambda}}{3}}$

Then it increases exponentially with time i.e. the universe expands exponentially. This expansion is not strong enough to rip apart everything in the universe – for instance, our galaxy won't be pulled apart but our supercluster would be, on a large enough time-scale.

The reader would gain much more insight into the topic if the reader understands the Einstein Equations. This equation involves a bit of knowledge of Tensors. This route to deriving the equations was hinted at, in the beginning of the module. Also further details can be found in the book mentioned on the last page(which the reader may check out if curious). I have given a video link where the following equation is explained and derived step by step. Interested people may check it out...



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

The derivation

Cosmic Microwave Background Radiation

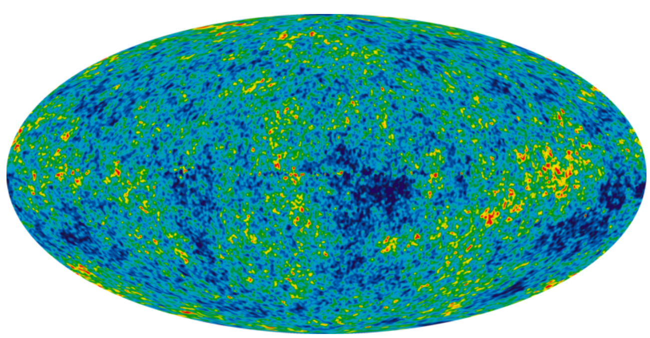
We can reasonably say that the universe is electrically neutral as a whole because there are no large enough lumps of uniform mono-charge anywhere in the galaxy. Let us assume a similar box universe as described above. We define the temperature in today's thermodynamics as a form of energy, which means it is directly proportional to energy with one of the proportionality constants being the Boltzmann Constant. We know the theoretical intensity of light's formula in terms of Temperature and frequency and light. We write the Planck's formula as follows

$$I = \frac{2h\nu^3}{c^3(e^{\frac{h\nu}{kT}}-1)}$$

From the binomial approximation we get

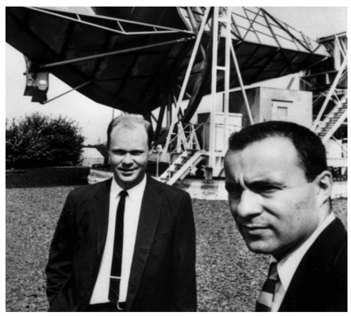
$$\frac{hc}{kT} = \lambda$$

The dominant wavelength of radiation (in terms of energy as well as number of photons) in our universe is the millimetre microwave wavelength – The Cosmic Microwave Background Radiation. The distribution of wavelengths follow this blackbody radiation equation with peak wavelength at roughly 1 mm.



Now we define a phenomenon called Decoupling – The phenomenon where matter could finally form due to cooling down of the universe due to sufficient expansion. Photon decoupling is closely related to recombination, which occurred around 378,000 years after the Big Bang (at a redshift of z = 1100), when the universe was a hot opaque plasma. During recombination, free electrons became bound to protons (hydrogen nuclei) to form neutral hydrogen atoms. Because direct recombinations to the ground state (lowest energy) of hydrogen are very inefficient, these hydrogen atoms generally form with the electrons in a high energy state, and the electrons quickly transition to their low energy state by emitting photons. Because the neutral hydrogen that formed was transparent to light, those photons which were not captured by other hydrogen atoms were able, for the first time in the history of the universe, to travel long distances. They can still be detected today, although they now appear as radio waves, and form the cosmic microwave background ("CMB"). They reveal crucial clues about how the universe formed. This was the first time the universe became transparent and this means this the oldest time in the age of the universe from which we can get any observable light which is what we call the CMB – **The afterglow of the Big-Bang**.

The story of the discovery of Cosmic Microwave Background Radiation is very interesting. The story of the discovery of cosmic microwave background radiation begins with two radio astronomers, Arno Penzias and Robert Wilson, working at Bell Telephone Laboratories in New Jersey, USA, during the 1960s.



Penzias and Wilson were using a large horn-shaped antenna to study radio waves bouncing off Echo balloon satellites. They noticed a persistent background noise that they couldn't explain. At first, they thought it might be due to pigeon droppings inside the horn or some other equipment malfunction. They worked diligently to remove any possible sources of interference but couldn't eliminate the noise completely. Frustrated and unable to account for the constant signal, they sought help from colleagues and other experts. One of their colleagues, Bernard Burke, suggested that the mysterious noise might be related to the radiation left over from the Big Bang. Around the same time, theoretical physicists Robert Dicke and Jim Peebles at Princeton University were independently working on the concept of the cosmic microwave background radiation. They had theorized that if the universe indeed began with a Big Bang, there should be a faint glow of radiation permeating all of space. Penzias and Wilson learned about Dicke and Peebles' work and realized that their unexplained noise might be precisely what the theorists had predicted. They contacted Dicke, and together they compared notes and conducted further tests. In 1965, Penzias, Wilson, Dicke, and Peebles jointly published a seminal paper presenting their findings. They confirmed that the background noise was, indeed, cosmic microwave background radiation, the afterglow of the Big Bang, which had been predicted by the Big Bang theory. The discovery of cosmic microwave background radiation was a crucial piece of evidence supporting the Big Bang theory and revolutionized our understanding of the universe's origins. Penzias and Wilson were honored for their groundbreaking work and were awarded the Nobel Prize in Physics in 1978 for their pivotal discovery.

Why do we need dark-matter ?

Now we would move on to understand how we speculated about the existence of dark matter in the first place. Scientists made some weird observations while studying something known as Galaxy Rotation Curves.

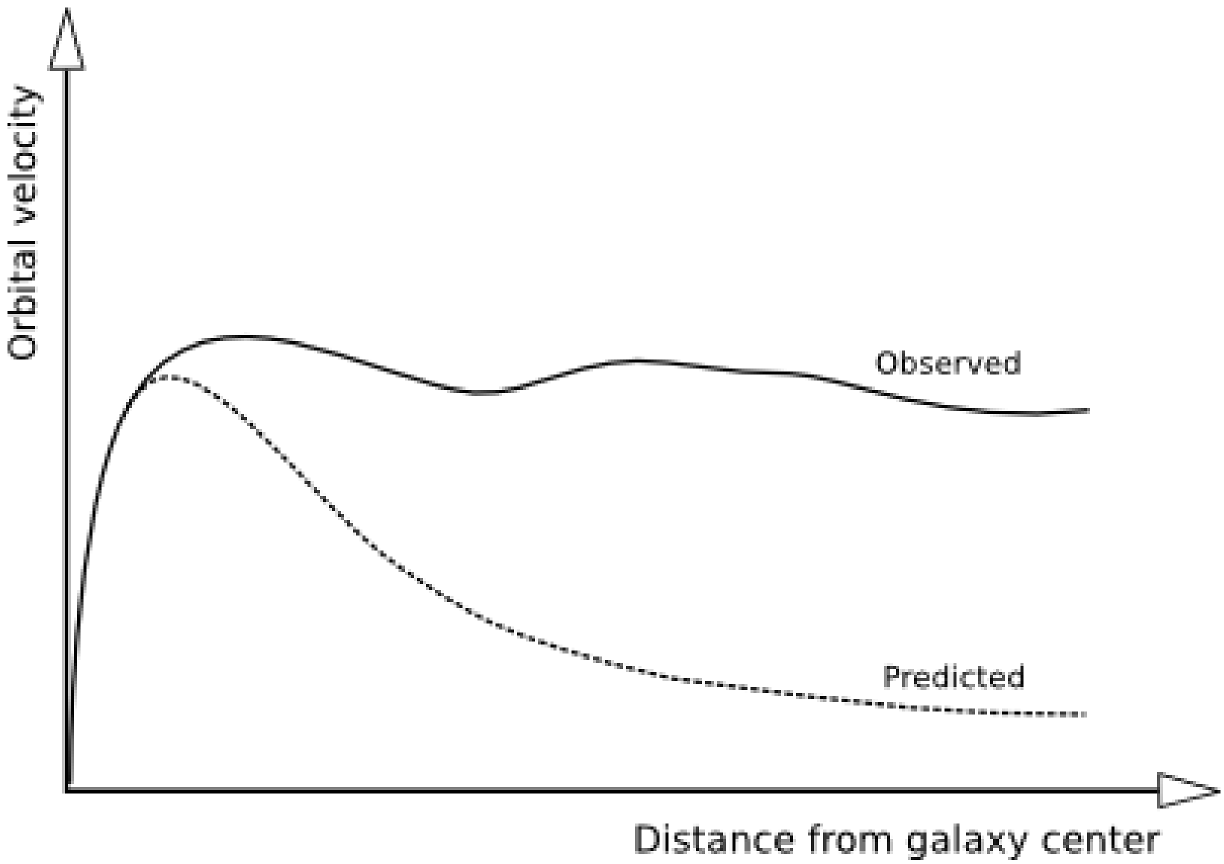
4.1 Galaxy Rotation Curves

A galaxy rotation curve is a plot of orbital velocities of visible stars in the galaxy and the distance from the galaxy's centre. We would measure the velocities of these stars from their redshifts as compared to the centre of the galaxy.

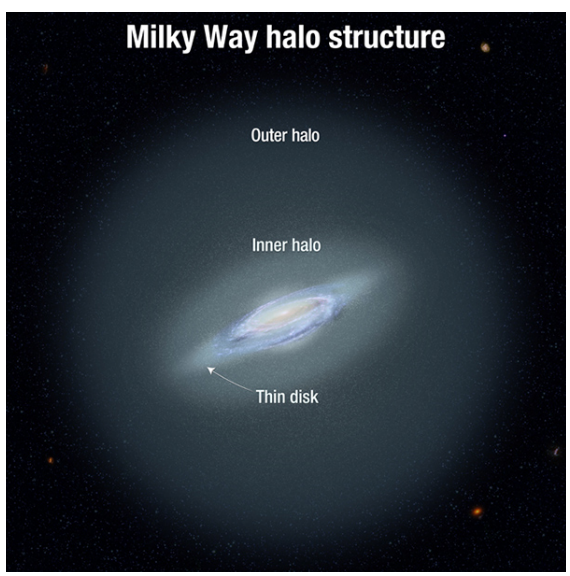
4.1.1 Historical Sidenote

In the 1930's Jan Oort, while observing the motion of stars in the neighborhood of a galaxy, noticed a strange behaviour. He observed that the stars had way too much velocity. Way more than expected by the galaxy's gravitational force acting as the centripetal force. He observed that the mass of the galaxy was 3 times the estimated amount ! All of this mass was not emitting any light, and that is where we had our first encounter with dark matter! Matter that exhibits a significant gravitational pull but emits no light.

Normally when we plot a rotation curve, the velocity of stars along the periphery of the galaxy should be lower as compared to stars near the centre(F=$\frac{v^2}{r}$) – r increases then v decreases). But actual data told a completely different story.



In observations we found out that the orbital velocity remained almost constant even when we moved further away from the galaxy's centre. This can only be explained if mass is not just concentrated at the centre of the galaxy. Instead it was distributed along the galaxy's periphery. It was theorised that 'dark matter' was distributed along the outermost parts of the galaxy also known as the galaxy 'halo'.



On an average the halo extends 50 kpC (kilo parsec) around the galaxy. Fritz Zwicky observed a similar phenomenon in superclusters of galaxies. This proved the ubiquity of dark matter.

Navarro, Frenk & White in a 1996 paper gave a relation for the observed rotation curve. The following relation was obtained by curve-fitting the observed halo-data (as a result it has no derivation as such). $\rho(r)$ is the mass density of matter (light and dark combined) arranged w.r.t. r

$$\rho(r) = \frac{\rho}{\frac{r}{R_s}(1 + \frac{r}{R_s})^2}$$

Where ρ is the density at the center, r is the radius from the center & R_s is the scale radius. and Rs differ for each galaxy

Scale radius 'R_s' is used to describe quantities that decline exponentially, such as the surface brightness of spiral galaxies. In this case, it is defined as the radius at which the brightness of the galaxy has fallen off by a factor of e (≈2.71828) from the center. The brightness is given by:

$$b = b_0 e^{\frac{-r}{r_0}}$$

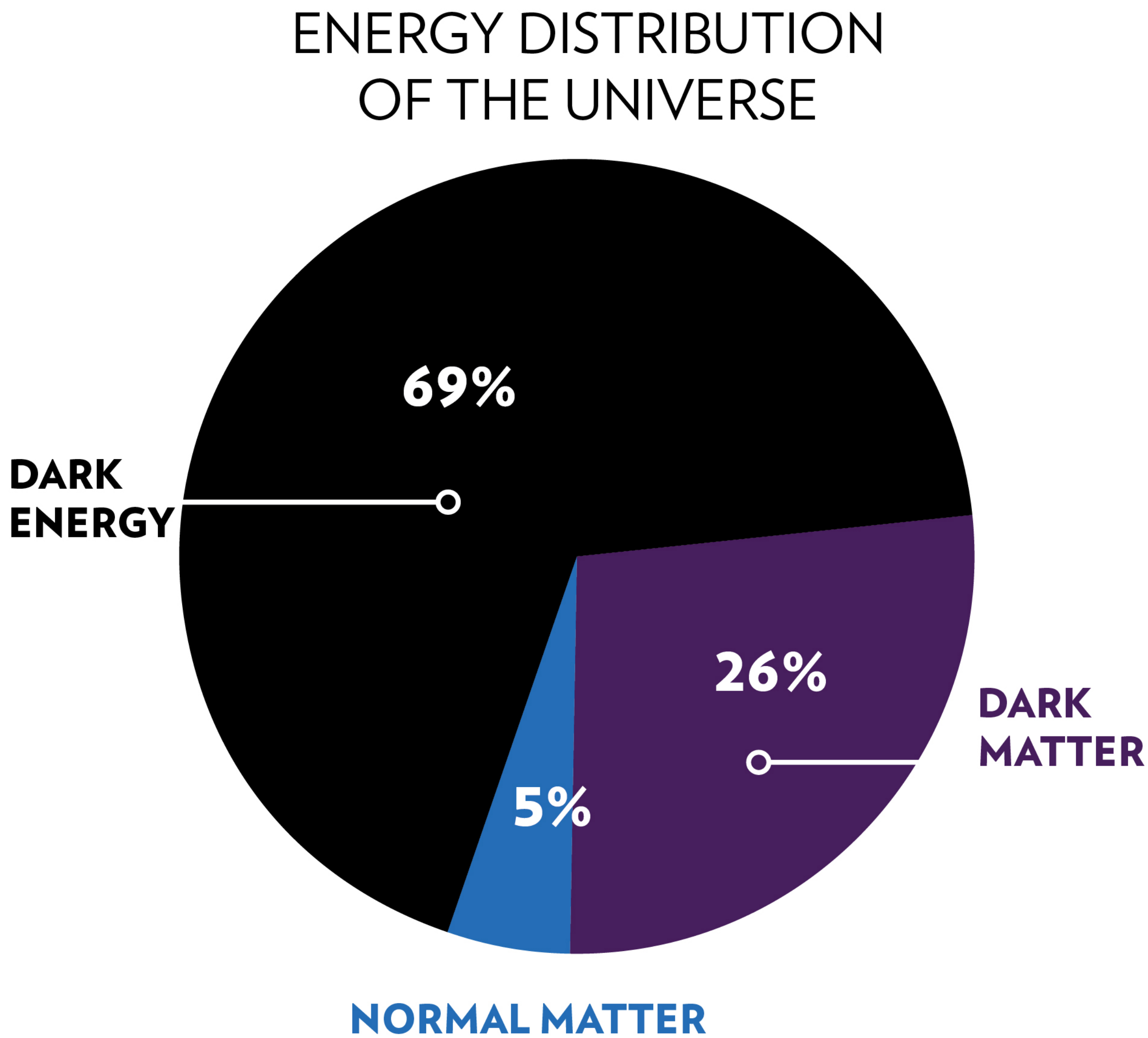
where b_0 is the brightness in the center of the galaxy and r_0 is the scale length

In spiral galaxies, we find a rotation curve given by

$$v(r) = (r\frac{d\phi}{dr})^{\frac{1}{2}}$$

4.1.2 Energy distribution of the Universe

In all the distribution of energy (and mass consequently, from E=mc^2) is observed to be as follows



Hope that I have convinced you how dark energy and dark matter came along in our observations and were later incorporated into cosmology. Finally, we have reached the end of our module. Thanks for hanging around for so long. Hope you learned some key concepts of cosmology and this module motivates you to delve deeper into the mysteries of our universe... Mind you. this module was just the tip of an iceberg, cosmology is a vast field and there are still a huge number of domains to explore within it...

If the reader wishes to learn these topics at an even greater depth then I highly recommend the following cosmology lecture playlist delivered by **Prof. Leonard Susskind** in 2009 at Stanford Univ. The Lectures' Playlist
The reader may also refer to the book **Introduction to General Relativity - Spacetime and Geometry** by **Sean M. Carroll** which contributed greatly to the creation of a parent document to this module.
Another great read on Cosmology would be **Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity** by **Steven Weinberg**, one of the pioneers of the study of the early universe. The above book is on the theoretical physics aspects of cosmology. Weinberg's **The First Three Minutes** is an easier take on the physics of the early universe which can be given a read however the book is in no way a textbook of cosmology.

Image credits (not in any particular order):

- Cosmic Microwave Background
- Rotation curve
- Edwin Hubble
- Hubble deep field
- Energy distribution
- Dr Albert Einstein
- Milky way Halo
- Artist's rendered image of the universe
- Penzias and Wilson
- Curvatures of the universes
- Scale factor change over the eras of the universe