Q1. What is the time complexity of below code and how?

```
void fun (int n)
  {
     int j=1, i=0;
     while (i<n) {
        i+=j;
        j++;
     }
  }
```

↳

$j = 1$ $\quad i = 1$

$j = 2$ $\quad i = 1 + 2$

$j = 3$ $\quad i = 1 + 2 + 3$

m - level.

for (i)

$\therefore\ 1 + 2 + 3 + \ldots + < n$

$\therefore\ 1 + 2 + 3 + m < n$

$\therefore\ \frac{m(m+1)}{2} < n$

$m \approx \sqrt{n}$

By summation method

$\Rightarrow \sum_{i=1}^{m} 1 \Rightarrow 1 + 1 + \ldots + \sqrt{n}$ times

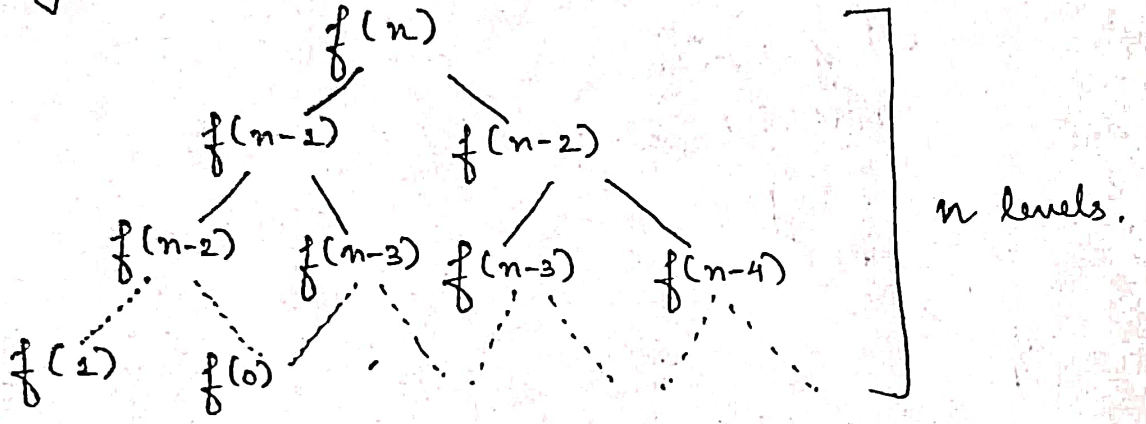
$\boxed{T(n) = \sqrt{n}}$ → Ans

Q2. Write recurrence relation for function that prints Fibanacci series. Solve it to get the time complexity. What will be the space complexity and why?

→ For Fibonacci Series

$f(n) = f(n-1) + f(n-2)$ $\qquad f(0) = 0$, $f(1) = 1$

By forming a tree




∵ At every function call we get 2 function calls

∴ for n levels

We have $= 2 \times 2 \ldots n$ times

∴ $\boxed{T(n) = 2^n}$

**MAXIMUM SPACE**

Considering Recursive Stack :

No. of calls maximum $= n$

For each call we have space complexity $O(1)$

∴ $\boxed{T(n) = O(n)}$

Without considering Recursive Stack :

each call we have time complexity $O(1)$

∴ $\boxed{T(n) = O(1)}$

Q3.

Write programs which have complexity:

$n(\log n)$, $n^3$, $\log(\log n)$

1) <u>n log n</u> → <u>Quick Sort</u>

```
void quicksort (int arr[], int low, int high)
{
    if (low < high)
    {
        int pi = partition (arr, low, high);
        quicksort ( arr, low, pi-1);
        quicksort ( arr, pi+1, high);
    }
}

int partition (int arr[], int low, int high)
{
    int pivot = arr[high];
    int i = (low-1);
    for ( int j= low ; j<= high -1; j++)
    {
        if (arr[i] < pivot)
        {
            i++ ;
            swap ( & arr[i], & arr[j]);
        }
    }
    swap (& arr [i+1], & arr [high]);
    return (i+1);
}
```

2) <u>$n^3$</u> → <u>Multiplication of 2 square matrix</u>

```
for (i=0; i<r1; i++) {
    for(j=0; j<c2; j++)
        for(k=0; k<c1; k++)
        {
            res[i][j]+ = a[i][k] * b[k][j];
        }
```

3) **log( log n)**

```
for ( i=2 ; i<n; i = i * i)
    {
        count ++;
    }
```

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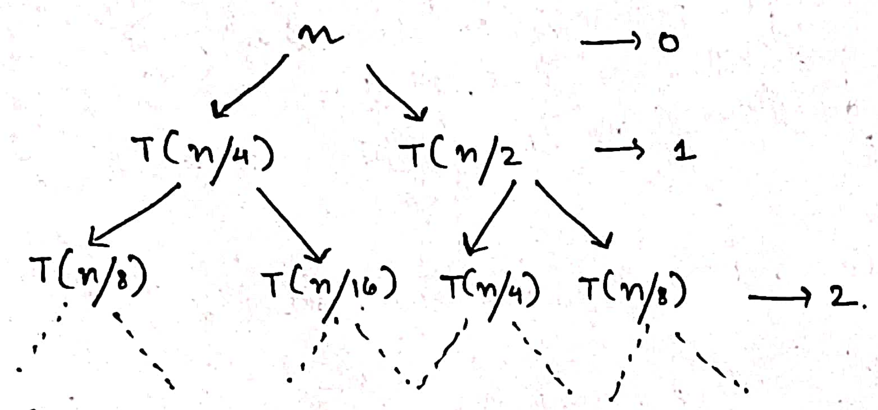
**Q4.** Solve the following recurrence relation

$$T(n) = T(n/4) + T(n/2) + cn^2$$

→




**At level**

$0 \rightarrow Cn^2$

$1 \rightarrow \frac{n^2}{4^2} + \frac{n^2}{2^2} = \frac{C5n^2}{16}$

$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$

⋮

max level $= \frac{n}{2^k} = 1$

$= K = \log_2 n$

$$T(n) = c\left(n^2 + (5/16)\, n^2 + (5/16)^2 n^2 + \dots + (5/16)^{\log n} n^2\right)$$

$$T(n) = Cn^2\left[1 + \left(\frac{5}{16}\right) + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^{\log n}\right]$$

$$T(n) = Cn^2 \times 1 \times \left(\frac{1 - (5/16)^{\log n}}{1 - (5/16)}\right)$$

$$T(n) = Cn^2 \times \frac{11}{5} \times \left(1 - \left(\frac{5}{16}\right)^{\log n}\right)$$

$$T(n) = O(n^2 c)$$

$\underline{O(Cn^2)}$ Ans.

Q5. What is the time complexity of following fun( ) ?

```
int fun (int n) {
  for (int i=1; i<=n; i++) {
    for (int j=1; j<n; j+=i) {
      // Some O(1) task
} }}
```

→ for

| i | j |
|---|---|
| 1 | 1 |
| 2 | 1 + 3 + 5 |
| 3 | 1 + 4 + 7 |
| ⋮ | 1 + 5 + 9 |
| n | |

$j = (n-1)/i$ - times

$$\sum_{i=1}^{n} \frac{(n-1)}{i}$$

$$\therefore T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-1)}{3} + \dots + \frac{(n-1)}{n}$$

$$T(n) = n\left[1 + 1/2 + 1/3 + \dots + 1/n\right] - 1 \times \left[1 + 1/2 + 1/3 + \dots + 1/n\right]$$

$$= n \log n - \log n$$

$\underline{T(n) = O(n \log n)}$ → Ans.

Q6. What should be time complexity of

```
for (int i=2; i<=n; i=pow(i,k))
{
    // Some O(1)
}
```

where k is a constant

→ for

| i |
|---|
| $2^1$ |
| $2^k$ |
| $2^{k^2}$ |
| $2^{k^3}$ |
| ⋮ |
| $2^{k^m}$ |

where

$$2^{k^m} <= n$$

$$k^m = \log_2 n$$

$$m = \log_k \log_2 n$$

$$\therefore \sum_{i=1}^{m} 1$$

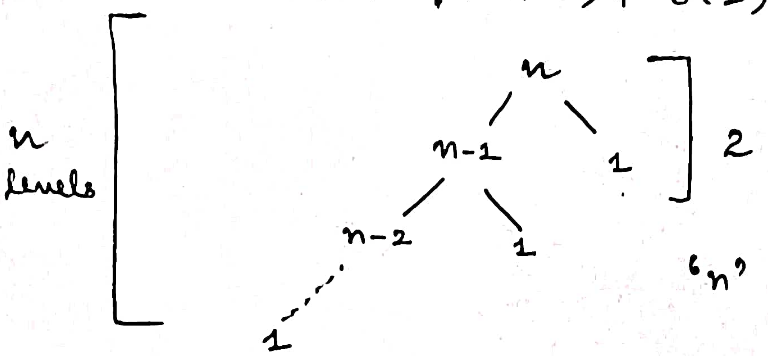
$1 + 1 + 1 \dots$ m times

$\underline{T(n) = O(\log_k \log n)}$ → Ans.

Q7. Write a recurrence relation when quick sort repeatedly divides array into 2 parts of 99% and 1%. Derive time complexity in this case. Show the recurrence tree while deriving time complexity & find difference in heights of both extreme parts. What do you understand by this analysis?

→ Given algorithm divides array in 99% and 1% part

$\because T(n) = T(n-1) + O(1)$




'n' work is done at each level

$T(n) = (T(n-1) + T(n-2) + \dots + T(1) + O(1)) \times n$

$= n \times n$

$\therefore \boxed{T(n) = O(n^2)}$

lowest height $= 2$

highest height $= n$

$\boxed{\therefore \text{difference} = n-2}$ $n > 1$

The given algorithm produces linear result

Q8) Arrange following in increasing order of rate of growth :

7

a) $n, n!, \log n, \log\log n, \text{root}(n), \log(n!), n\log n, \log^2(n), 2^n, 2^{2^n}, 4^n, n^2, 100$

→ $100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

b) $2(2^n), 4n, 2n, 1, \log(n), \log(\log(n)), \sqrt{\log(n)}, \log 2n, 2\log(n), n, \log(n!), n!, n2, n\log(n)$

→ $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

c) $8^{2n}, \log_2(n), n\log_6(n), n\log_2(n), \log(n!), n!, \log_8(n), 96, 8n2, 7n3, 5n$

→ $96 < \log_8 n < \log 2n < 5n < n\log_6(n) < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$