Q1) $T(n) = 3T(n/2) + n^2$

$\rightarrow$ $T(n) = aT(n/b) + f(n^2)$

$\rightarrow$ $a \geq 1,\ b \geq 1$

On campairing

$a = 3,\ b = 2,\ f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n^2)$

Q2) $T(n) = 4T(n/2) + n^2$

$\rightarrow$ $a \geq 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

Q3) $T(n) = T(n/2) + 2^n$

$\rightarrow$

$a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

Q4) $T(n) = 2^n T(n/2) + n^n$

$\rightarrow$ $a = 2^n$

$b = 2,\ f(n) = n^n$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \ngtr n^n$

$f(n) = n^c$

$T(n) = \theta(n^2 \log_2 n)$

Q5) $T(n) = 16T(n/4) + n$

$\rightarrow$ $a = 16,\ b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2\log_4 4$

$= 2$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

Q6) $T(n) = 2T(n/2) + n \log n$

$\rightarrow$ $a = 2,\ b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$T(n) = \theta(n \log n)$

Q7) $T(n) = 2T(n/2) + n/\log n$

$\rightarrow$ $a=2,\ b=2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

Q8) $T(n) = 2T(n/4) + n^{0.51}$

$\rightarrow$ $a=2,\ b=4,\ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$\therefore n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

Q9) $T(n) = 0.5T(n/2) + 1/n$

$\rightarrow$ $a=0.5,\ b=2$

$a \geq 1$ but here a is 0.5 so we cannot apply Master's Theorem.

Q10) $T(n) = 16T(n/4) + n!$

$\rightarrow$ $a=16,\ b=4,\ f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

Q11) $4T(n/2) + \log n$

$\rightarrow$ $a=4,\ b=2,\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\therefore \log n < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$= \theta(n^2)$

Q12) $T(n) = \text{sqrt}(n)T(n/2) + \log n$

$\rightarrow$ $a = \sqrt{n},\ b=2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$T(n) = \theta(f(n))$

$= \theta(\log(n))$

Q13) $T(n) = 3T(n/2) + n$

$\rightarrow$ $a=3;\ b=2;\ f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5489}$

$n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.5849})$

Q14) $T(n) = 3T(n/3) + \text{sqrt}(n)$

$\rightarrow$ $a=3,\ b=3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$f(n) < n^c$

$T(n) = \theta(n)$

Q15) $T(n) = 4T(n/2) + n$

$\longrightarrow$ $a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$f(n) = \theta(n^2)$

Q16) $T(n) = 3T(n/4) + n\log n$

$\longrightarrow$ $a = 3, b = 4, f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \theta(n\log n)$

Q17) $T(n) = 3T(n/3) + n/2$

$\longrightarrow$ $a = 3; b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \theta(n)$

Q18) $T(n) = 6T(n/3) + n^2\log n$

$\longrightarrow$ $a = 6; b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2\log n$

$\therefore T(n) = \theta(n^2\log n)$

Q19) $T(n) = 4T(n/2) + n/\log n$

$\longrightarrow$ $a = 4, b = 2, f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \theta(n^2)$

Q20) $T(n) = 64T(n/8) - n^2\log n$

$\longrightarrow$ $a = 64 \quad b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$n^c = n^2$

$\therefore n^2\log n > n^2$

$T(n) = \theta(n^2\log n)$

Q21) $T(n) = 7T(n/3) + n^2$

$\longrightarrow$ $a = 7; b = 3; f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \theta(n^2)$

Q22) $T(n) = T(n/2) + n(2 - \cos n)$

$\longrightarrow$ $a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \theta(n(2 - \cos n))$