CREDIT RISK MODELING OF FINANCIAL DERIVATIVES

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Abstract.

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1. Introduction

As a motivation to why one would need a risk measure to capture shifts in credit spread, the Basel Committee estimate that about three fourths of the CCR losses during the financial crisis originate from CVA losses and not actual defaults.

2. Methods of credit valuation adjustment computation

In this section, we developed the basic methodology to compute CVA and describe the basic terms.

2.1. Components of credit valuation adjustment and terminology.

The basic concepts and notation for counterparty credit risk and CVA will be shown in this section. Counterparty credit risk (CCR) is the risk that the counterparty defaults before the final settlement of a transaction's cash flows. CVA can be explained as the difference between the portfolio's risk-free value and the portfolio's true value taking into account the possibility of default of the counterparty. In the next definition CVA is calculated as expectation of credit loss.

Definition 1. *The credit valuation adjustment* is defined as

$$CVA = (1 - R) \int_0^T \hat{e_d}(t) \, \mathrm{d}PD(t), \tag{1}$$

where R is recovery rate, $\hat{e_d}(t)$ is the discounted expected exposure at time t and $PD(t)$ is probability of default.

In what follows, we specify the components of CVA. Recovery rate is the value of unity less *Loss given default (LGD)*, i.e. $R = 1 - LGD$. The LGD is the percentage amount of the exposure expected to be lost if the counterparty defaults.

The counterparty credit exposure $E(t)$ of the bank to a counterparty at time t (hereafter simply exposure) is defined as the economic loss, incurred on all outstanding transactions with the counterparty if the counterparty defaults at t. Denote the value of the i-th instrument in the portfolio at time t by $V_i(t)$. The value of the counterparty portfolio is given by

$$V(t) = \sum_{i=1}^{N} V_i(t). \tag{2}$$

When netting is not allowed, the exposure E(t) is given by

$$E(t) = \sum_{i=1}^{N} \max\{V_i(t), 0\}. \tag{3}$$

For a counterparty portfolio with a netting agreement, the exposure is

$$E(t) = \max\{V(t), 0\}. \tag{4}$$

Discounting is a financial mechanism in which a future value is being recalculated to the present value. The discount factor, $D(t)$, is the factor by which a future cash flow must be multiplied in order to obtain the present value. Consider the discount factor at time t, defined as

$$D(t) = \frac{B_0}{B_t} = e^{-rt}, \tag{5}$$

where r is risk-free rate of return and B_t is the value of risk free asset at time t. Hence, the discounted expected exposure at time t conditional on the counterparty default at time t is given by

$$\hat{e}_d(t) = \mathrm{E}[D(t)E(t)]. \tag{6}$$

Next component of the equation (1) is *Probability of Default*, $PD(t)$ which describes the creditworthiness of a counterparty. It provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations. The PD udává v procentech s jakou pravděpodobností dojde k selhání klienta během jednoho roku. There are many alternatives for estimating the probability of default. The frequently used approach, taken by many banks, is to use external ratings agencies (such as S&P, Fitch or Moody's) for estimating PD from historical default experience.

3. Results

The numerical illustration of the calculation of CVA will be introduced in this section. We work with data set of interest rate swaps from the year 2015. These data come from a bank operating in the Czech Republic. The data contains information about five vanilla interest rate swaps and each of them is associated with different counterparty. Asi by to chtělo více informací o datech.

3.1. **Initial yield curve.** In order to evaluate the swaps, is necessary to describe a discount curve. The initial discount curve can be obtain by a linear interpolation of the risk free interest rates of different maturities. The initial rates were found at the web page of PATRIA (http://www.patria.cz/kurzy/CZK/5Y/irs/graf.html) and we can see them in the table 1. dát sem odkaz na stránku, Leni? The interest rate shorter than one year were extracted from PRIBOR rates on October 30th, 2015.

TABLE 1. Initial rates

Maturity (years)	**Rate (%)**
0.25	0.14
0.5	0.16
1	0.21
2	0.25
3	0.28
4	0.33
5	0.40
6	0.47
7	0.56
8	0.65
9	0.75
10	0.83
12	0.99
15	1.16
20	1.27

The picture 1 shows the initial yield curve at settlement date which was built from the initial rates in the table 1.

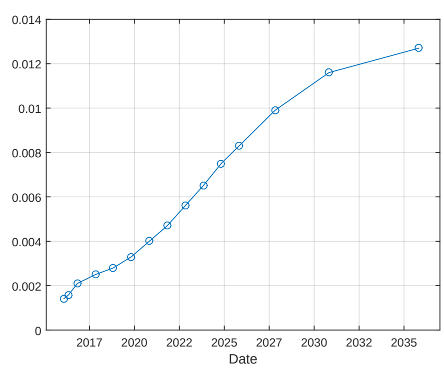

FIGURE 1. Yield Curve

3.2. **Interest rate simulation.** The interest rate simulation is based on historical data of the three month PRIBOR rate, from 2. 1. 2014 to 24. 11. 2015. The development of three month PRIBOR rate can be observed in the figure 2.

The well known interest rate evolution model, Hull-White one-factor model, will be used for modeling of the interest rate term structure. We can define this model by equation

$$\mathrm{d}r(t) = [\theta(t) - \alpha r]\,\mathrm{d}t + \sigma\,\mathrm{d}W, \tag{7}$$

where

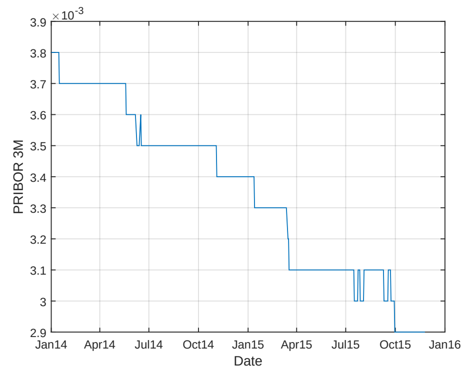

FIGURE 2. The three month PRIBOR rate

dr**:** The change in the interest rate after a small change in time, dt.
α **:** Mean reversion rate.
σ **:** Volatility of the interest rate.
dW**:** A Wiener process.
$\theta(t)$**:** Drift function defined as

$$\theta(t) = F_t'(0,t) + \alpha F(0,t) + \frac{\sigma^2}{2\alpha}(1 - \mathrm{e}^{-2\alpha t}), \tag{8}$$

$F(0,t)$**:** Instantaneous forward rate at time t.
$F_t'(0,t)$**:** Partial derivative of F with respect to time t.

The constants α and σ are extracted from the historical three month PRIBOR rates. We use equation (7) to simulated the short interest rates. We can expanded the entire interest rate curve from the short rate using

$$\begin{aligned} R(t,T) &= -\frac{1}{T-t}\ln A(t,T) + \frac{1}{T-t}B(t,T)r(t) \\ \ln A(t,T) &= \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4\alpha^3}\sigma^2(\mathrm{e}^{alphaT} - \mathrm{e}^{-\alpha t})^2(\mathrm{e}^{2\alpha t} - 1) \\ B(t,T) &= \frac{1 - \mathrm{e}^{-\alpha(T-t)}}{\alpha}. \end{aligned} \tag{9}$$

The large number of simulations was made. The example of yield surfaces obtained in one interest rate simulation is posible to observe in the Figure 3. For each scenarios the swaps are priced at each future simulation date.

3.3. **Probability of default.** We assume that the wait time for default of a counterparty is a random variable X with exponential distribution. Then the cumulative distribution function of this variable can be define as

$$F(x) = P(X \le x) = \begin{cases} 1 - \mathrm{e}^{-\lambda x}, & x \ge 0 \\ 0, & x \le 0 \end{cases} \tag{10}$$

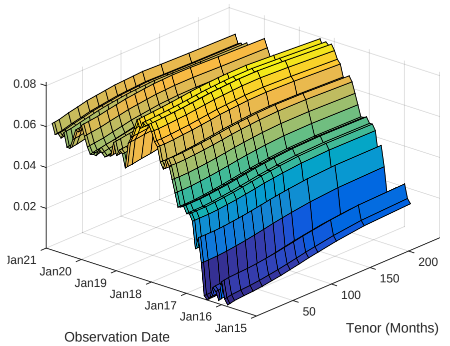

FIGURE 3. One Possible Scenario of Yield Curve Evolution

where $\lambda > 0$ is the parameter of the distribution. Probability of default of the counterparty during one year we denote PD. Then

$$PD = P(X \leq 1) = 1 - \mathrm{e}^{-\lambda}. \tag{11}$$

If we use value of PD from odkud bereme PD, Leni? λ could be estimate from (11)

$$\begin{aligned} PD &= 1 - \mathrm{e}^{-\lambda} \\ \lambda &= -\ln(1 - PD). \end{aligned} \tag{12}$$

Then we can construct probabilty curve for each counterparty. It is posible to observed them in the Figure 4.

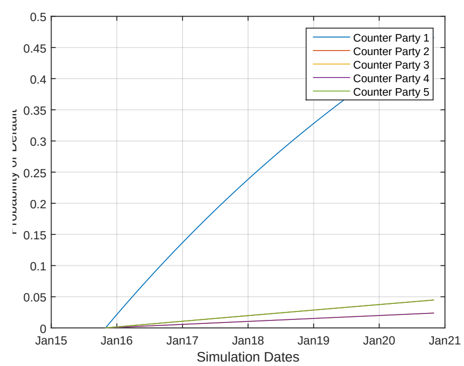

FIGURE 4. Default Probability Curves

3.4. **Computation of CVA.** Let consider the exposure is independent of default. Then total exposure of all contracts can be computed on the bases of equation (4). The bank has estimated recovery rate $R = 40\%$. Leni, jaká je vaše recovery rate?. Now we have everything for the CVA calculation according to the equation (1). The resulting values of CVA for each counterparty can be observed in the Table 2.

Counterparty	**CVA (in CZK)**
1	1 056 075.83
2	265 290.14
3	369.19
4	1 045.54
5	103 319.55

Table 2. Resulting CVA

References

[1] N. Arora, P. Gandhi, and F. A. Longstaff. Counterparty credit risk and the credit default swap market. *Journal of Financial Economics*, 103(2):280–293, 2012.

[2] J.-P. Bouchaud and M. Potters. *Theory of financial risk and derivative pricing: from statistical physics to risk management*. Cambridge university press, 2003.

[3] D. Brigo, A. Capponi, and A. Pallavicini. Arbitrage-free bilateral counterparty risk valuation under collateralization and application to credit default swaps. *Mathematical Finance*, 24(1):125–146, 2014.

[4] E. Canabarro and D. Duffie. Measuring and marking counterparty risk. *Asset/Liability Management for Financial Institutions, Institutional Investor Books*, 2003.

[5] D. Duffie. *Intertemporal asset pricing theory*, volume 1. Elsevier, 2003.

[6] A. Etheridge. *A course in financial calculus*. Cambridge University Press, 2002.

[7] J. Gregory. *Counterparty credit risk: the new challenge for global financial markets*, volume 470. John Wiley & Sons, 2010.

[8] J. Hull. *Options, futures, and other derivatives (8. ed., global ed.)*. Boston [u.a.] : Pearson, 2012.

[9] J. Hull and A. White. Cva and wrong-way risk. *Financial Analysts Journal*, 68(5):58–69, 2012.

[10] J. Jílek. Finanční rizika. 1. vyd. praha: Grada, 2000. 635 s. Technical report, ISBN 80-7169-579-3, 2000.

[11] M. Pykhtin and D. Rosen. Pricing counterparty risk at the trade level and cva allocations. 2010.