CREDIT RISK MODELING OF FINANCIAL DERIVATIVES

KAFKOVÁ SILVIE AND KŘIVÁNKOVÁ LENKA

Abstract. According to the Basel Committee's estimate three fourths of the counterparty credit risk losses during the financial crisis originate from Credit valuation adjustment's losses and not from actual defaults. Therefore Third Basel Accord has instructed banks to calculate capital requirement for risk of Credit valuation adjustment (CVA) from 2015. Banks are trying to model CVA to hold the prescribed standards and also reached the lowest possible impact on their profit. In this paper, we try to model CVA

Keywords: credit valuation adjustment, probability of default, interest rate swaps, Hull-White model, Monte Carlo simulations, credit exposure

1. Introduction

As a motivation to why one would need a risk measure to capture shifts in credit spread, the Basel Committee estimate that about three fourths of the CCR losses during the financial crisis originate from CVA losses and not actual defaults.

Od roku 2015 předepisuje Basel III bankám povinnost počítat kapitalový požadavek pro riziko Credit Valuation Adjustment (CVA). Současná situace na bankovním trhu vyvíjí na banky velký tlak hledat nové možnosti jak dosáhnout zisku. Běžné způsoby tvorby zisku v současnosti selhávají. Na trhu můžeme pozorovat nejen nízké až záporné úrokové sazby, ale i konkurenční boj o klienty, který přináší neustálý tlak na snižování bankovních poplatků a poskytování téměř nerentabilní klientských úvěrů. Banky proto hledají nové možnosti, jak snižovat náklady jako jsou například opravné položky k úvěrům klientů nebo k finančním derivátům. Jednou z možností může být hledání způsobu jak CVA modelovat tak, aby dodržely předepsané standardy a zároveň dosáhly co nejmenšího dopadu na jejich zisk.

Banky často používají triviální parametrické modely, které jsou ale nastaveny velmi konzervativně kvůli rizikové obezřetnosti. Předpokládáme, že sofistikovanější model by mohl přinést nižší CVA a tudíž i nižší kapitálový požadavek pro banku.

2. Methods of credit valuation adjustment computation

In this section, we developed the basic methodology to compute CVA and describe the basic terms.

2.1. **Components of credit valuation adjustment and terminology.** The basic concepts and notation for counterparty credit risk and CVA will be shown in this section. Counterparty credit risk (CCR) is the risk that the counterparty defaults before the final settlement of a transaction's cash flows. CVA can be explained as the difference between the portfolio's risk-free value and the portfolio's true value taking into account the possibility of default of the counterparty. In the next definition CVA is calculated as expectation of credit loss.

Definition 1. *The credit valuation adjustment* is defined as

$$CVA = (1 - R) \int_0^T \hat{e_d}(t) \,\mathrm{d}PD(t), \tag{1}$$

where R is recovery rate, $\hat{e_d}(t)$ is the discounted expected exposure at time t and $PD(t)$ is probability of default.

In what follows, we specify the components of CVA. Recovery rate is the value of unity less *Loss given default (LGD)*, i.e. $R = 1 - \ LGD$. The LGD is the percentage amount of the exposure expected to be lost if the counterparty defaults.

The counterparty credit exposure $E(t)$ of the bank to a counterparty at time t (hereafter simply exposure) is defined as the economic loss, incurred on all outstanding transactions with the counterparty if the counterparty defaults at t. Denote the value of the i-th instrument in the portfolio at time t by $V_i(t)$. The value of the counterparty portfolio is given by

$$V(t) = \sum_{i=1}^{N} V_i(t). \tag{2}$$

When netting is not allowed, the exposure E(t) is given by

$$E(t) = \sum_{i=1}^{N} \max\{V_i(t), 0\}. \tag{3}$$

For a counterparty portfolio with a netting agreement, the exposure is

$$E(t) = \max\{V(t), 0\}. \tag{4}$$

Discounting is a financial mechanism in which a future value is being recalculated to the present value. The discount factor, $D(t)$, is the factor by which a future cash flow must be multiplied in order to obtain the present value. Consider the discount factor at time t, defined as

$$D(t) = \frac{B_0}{B_t} = e^{-rt}, \tag{5}$$

where r is risk-free rate of return and B_t is the value of risk free asset at time t. Hence, the discounted expected exposure at time t conditional on the counterparty default at time t is given by

$$\hat{e}_d(t) = \mathrm{E}[D(t)E(t)]. \tag{6}$$

Next component of the equation (1) is *Probability of Default*, $PD(t)$ which describes the creditworthiness of a counterparty. It provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations. There are many alternatives for estimating the probability of default. The frequently used approach, taken by many banks, is to use external ratings agencies (such as S&P, Fitch or Moody's) for estimating PD from historical default experience.

3. Results

The numerical illustration of the calculation of CVA will be introduced in this section. We work with data set of interest rate swaps from the year 2015. An *interest rate swap (IRS)* is an agreement between two parties to exchange a interest rate cash flows on specified intervals and over a certain period of time. These data come from a bank operating in the Czech Republic. The data contains information about five vanilla interest rate swaps and each of them is associated with different counterparty. Asi by to chtělo více informací o datech.

Table 1. Variables in a data set

Notation	Name of Variable
ID	Counterparty ID
principal	Principal of swap
maturity	Maturity date of swap
LegRateReceiving	Interest rate received by bank
LegRatePaying	Interest rate paid by bank
period	Period of paying

3.1. **Initial yield curve.** In order to evaluate the swaps, is necessary to describe a discount curve. The initial discount curve can be obtain by a linear interpolation of the risk free interest rates of different maturities. The initial rates were found at the web page of PATRIA (http://www.patria.cz/kurzy/CZK/5Y/irs/graf.html) and we can see them in the Table 2.The interest rate shorter than one year were extracted from PRIBOR rates on October 30th, 2015. The Picture 1 shows the initial yield curve at settlement date which was built from the initial rates in the Table 2.

3.2. **Interest rate simulation.** The interest rate simulation is based on historical data of the three month PRIBOR rate, from 2. 1. 2014 to 24. 11. 2015. The development of three month PRIBOR rate can be observed in the figure 2.

The well known interest rate evolution model, Hull-White one-factor model, will be used for modeling of the interest rate term structure. The model was first published by John Hull and Alan White in [9] and generalized later in [10]. The requirement for more accurate fit to the currently-observed yield curve, led Hull and White to the introduction of a time variable parameter in the Vasicek model. The model assumes that short rates have a normal distribution, and also that the short rates exhibit the mean reversion character. The Hull-White model extends the Vasicek and Cox-Ingersoll-Ross (CIR) models.

We can define this model by equation

$$\mathrm{d}r(t) = [\theta(t) - \alpha r]\,\mathrm{d}t + \sigma\,\mathrm{d}W, \tag{7}$$

Table 2. Initial rates

Maturity (years)	**Rate (%)**
0.25	0.14
0.5	0.16
1	0.21
2	0.25
3	0.28
4	0.33
5	0.40
6	0.47
7	0.56
8	0.65
9	0.75
10	0.83
12	0.99
15	1.16
20	1.27

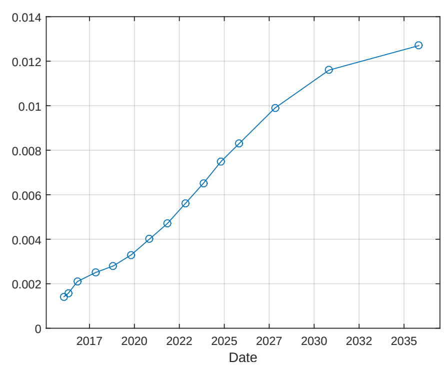

Figure 1. Yield Curve

where

dr**:** The change in the interest rate after a small change in time, dt.
α **:** Constant reversion speed.
σ **:** Volatility of the interest rate.
dW**:** A Wiener process.
$\theta(t)$**:** Drift function defined as

$$\theta(t) = \frac{\partial F(0,t)}{\partial t} + \alpha F(0,t) + \frac{\sigma^2}{2\alpha}(1 - \mathrm{e}^{-2\alpha t}), \tag{8}$$

where
$F(0,t)$**:** Instantaneous forward rate at time t.

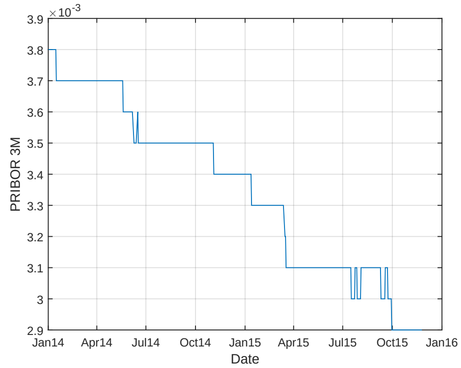

FIGURE 2. The three month PRIBOR rate

Instantaneous forward rate $F(0,t)$ is defined as

$$F(0,t) = \frac{\partial \ln P(0,t)}{\partial t} \tag{9}$$

where $P(t,T)$ is price of a zero coupon bond at time t with maturity at time T.

The constants α and σ are extracted from the historical three month PRIBOR rates. We use equation (7) to simulated the short interest rates. We can expand the entire interest rate curve from the short rate using

$$\begin{aligned} P(t,T) &= A(t,T)\,\mathrm{e}^{-B(t,T)r(t)} \\ \ln A(t,T) &= \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4\alpha^3}\sigma^2(\mathrm{e}^{-\alpha T} - \mathrm{e}^{-\alpha t})^2(\mathrm{e}^{2\alpha t} - 1) \\ B(t,T) &= \frac{1-\mathrm{e}^{-\alpha(T-t)}}{\alpha}. \end{aligned} \tag{10}$$

The large number of simulations was made. The example of yield surfaces obtained in one interest rate simulation is possible to observe in the Figure 3. For each scenarios

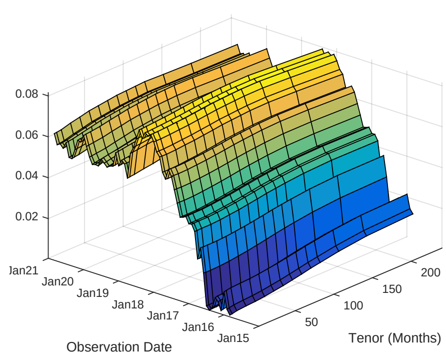

FIGURE 3. One Possible Scenario of Yield Curve Evolution

the swaps are priced at each future simulation date.

The main advantage of the Hull-White model is that it can be fitted exactly to the initial term structure of interest rates.

3.3. **Probability of default.** We assume that the wait time for default of a counterparty is a random variable X with exponential distribution. Then the cumulative distribution function of this variable can be define as

$$F(x) = P(X \leq x) = \begin{cases} 1 - \mathrm{e}^{-\lambda x}, & x \geq 0 \\ 0, & x \leq 0 \end{cases} \tag{11}$$

where $\lambda > 0$ is the parameter of the distribution. Probability of default of the counterparty during one year we denote PD. Then

$$PD = P(X \leq 1) = 1 - \mathrm{e}^{-\lambda}. \tag{12}$$

If we use value of PD from odkud bereme PD, Leni? λ could be estimate from (12)

$$\begin{aligned} PD &= 1 - \mathrm{e}^{-\lambda} \\ \lambda &= -\ln(1 - PD). \end{aligned} \tag{13}$$

Then we can construct probability curve for each counterparty. It is possible to observed them in the Figure 4.

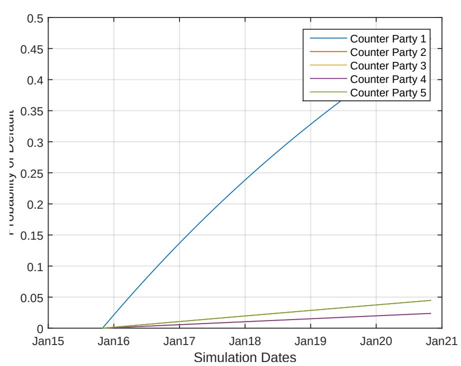

FIGURE 4. Default Probability Curves

3.4. **Computation of CVA.** Let consider the exposure is independent of default. Then total exposure of all contracts can be computed on the bases of equation (4). The bank has estimated recovery rate $R = 40\%$. Leni, jaká je vaše recovery rate?. Now we have everything for the CVA calculation according to the equation (1). The resulting values of CVA for each counterparty can be observed in the Table 3.

Counterparty	**CVA (in CZK)**
1	1 056 075.83
2	265 290.14
3	369.19
4	1 045.54
5	103 319.55

Table 3. Resulting CVA

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