**Modeling of Counterparty Credit Risk in Czech Interest Rate Swaps**

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**Abstract:** According to the Basel Committee's estimate three fourths of the counterparty credit risk losses during the financial crisis originate from Credit valuation adjustment's losses and not from actual defaults. Therefore Third Basel Accord (EU, 2013a) and (EU, 2013b) has instructed banks to calculate capital requirement for risk of Credit valuation adjustment (CVA) from 2015. Banks are trying to model CVA to hold the prescribed standards and also reached the lowest possible impact on their profit. In this paper, we try to model CVA by using methods that are in compliance with the prescribed standards and also achieve the smallest possible impact on the bank's earnings. To do so, the data set of interest rate swaps from the year 2015 is used. The interest rate term structure is simulated by Hull-White one-factor model and Monte Carlo methods. Than the probability of default for each counterparty is constructed. Safe level of CVA is reached in spite of calculated CVA achieves a lower level than CVA previously used by bank. This allows a reduction of capital requirements for banks. All calculations are computed in Matlab environment which contains packages of financial mathematics and stats with the function for the Hull-White model and the function for the estimation of probability of default which are necessary for the computation of the CVA.

**Keywords:** counterparty credit risk, credit valuation adjustment, probability of default, interest rate swaps, yield curve, Hull-White model, Monte Carlo simulations, credit exposure

**Introduction**

The current situation in the banking market pressures banks to look for new opportunities to generate income. Common methods of making a profit are not as profitable as they were in the past. We can observe not only a negative interbank offered rates, but also competitive fight for clients which causes strong pressure to decrease bank fees and almost unprofitable lending. The banks, therefore, search for new possibilities how to decrease costs such as loan loss provision and credit valuation adjustment. One of the possibilities could be development of a new approach to CVA modeling respecting regulatory standards and simultaneously achieving maximal profit.

A good introduction to pricing counter party credit risk can be found in a paper by Michael Pykhtin and Steven Zhu (2007). This paper discusses approaches to CVA calculation. Canabarro and Duffie (2003) deal with measuring counterparty risk. In their article are defined basic terms and models of counterparty exposures. A detailed review of counterparty credit risk modeling is given in (Jon Gregory, 2010). This book interestingly explains the rise of counterparty risk during the financial crisis. The quantification of credit exposure is presented as well as risk mitigation methods.

Under usual approaches CVA is measured at the counterparty level. Nevertheless, it can be sometimes required to determine contributions of individual trades to the CVA at the counterparty level. Pykhtin and Rosen (2010) thoroughly analyze the problem of allocating CVA to the individual trades. They explain how this problem can be simplified to calculating contributions of the trades to the expected exposure of the each counterparty where the expected exposure is conditioned by the default of counterparty.

A measure of credit quality of a counterparty is default probability. The counterparty's probability of default is typically derived from credit default swaps (CDS). Arora, Gandhi and Longstaff (2012) examine the credit default swaps market and his relevance in counterparty credit risk pricing.

The counterparty credit exposures may be correlated with the credit quality of a counterparty. If this correlation is negative then it is called wrong way risk. In actual fact, risk from correlation occurs always, however it is usually ignored to simplify modeling of exposure. Nevertheless, there exist cases when wrong way risk is too significant to be ignored. This case may be commodity trades with a producer of that commodity. Hull and White (2012) introduced one of the first models of wrong way risk in CVA calculations.

The international accounting standards IFRS 13 and SFAS 157 require banks to report the value of their derivative portfolio net of the credit valuation adjustment. The accounting standards were set up in response to the financial crisis. A purpose of the standards are that the value of derivatives have to be adjusted with their counterparty risk. As a consequence, all banks are under an obligation to calculate CVA on a monthly basis.

The banks often use primitive parametric models, which are very conservatively set due to risk vigilance. We suppose that a more sophisticated model would bring lower CVA as well as lower capital requirement for a bank.

The aim of the paper is modeling CVA of Czech interest rate swaps so that the regulatory standards are observed but the better profit is achieved. To do so we use advanced mathematical methods. First we use Monte Carlo simulations to create possible scenarios of interest rates in the market. The simulations are executed using one of the best known interest rate evolution models, Hull-White one factor model. For each interest rate simulation we create the yield surface. For each scenario the IRS are priced at each future simulation date. Than the discounted expected exposure for each counterparty is computed. Next the probability of default for each client is modeled and CVA for each counterparty is computed. Thanks to our methods, we obtain CVA that allows a reduction of capital requirements for banks. All computations and simulations in the paper are executed in MATLAB.

**Methodology**

In this section, we developed the basic methodology to compute CVA and describe the basic terms.

**Components of credit valuation adjustment and terminology**

The basic concepts and notation for counterparty credit risk and CVA will be shown in this section. Counterparty credit risk (CCR) is the risk that the counterparty defaults before the final settlement of a transaction's cash flows. CVA can be explained as the difference between the portfolio's risk-free value and the portfolio's true value taking into account the possibility of default of the counterparty. In the next definition CVA is calculated as expectation of credit loss. *The credit valuation adjustment* is defined as

(1)

where is recovery rate, is the discounted expected exposure at time and is probability of default.

In what follows, we specify the components of CVA. *Recovery rate* is the value of unity less *Loss given default (LGD)*, i.e. . The LGD is the percentage amount of the exposure expected to be lost if the counterparty defaults.

The counterparty *credit exposure* of the bank to a counterparty at time (hereafter simply exposure) is defined as the economic loss, incurred on all outstanding transactions with the counterparty if the counterparty defaults at . Denote the value of the -th instrument in the portfolio at time by . The value of the counterparty portfolio is given by

(2)

When netting is not allowed, the exposure E(t) is given by

(3)

For a counterparty portfolio with a netting agreement, the exposure is

(4)

*Exposure at default (EAD)* is the total value which a bank is exposed to a counterparty at the time of default. For simplification in the follow equations we denote EAD as , where is time of the default. The EAD may be seen as a random variable. Therefore, an *expected exposure at default* is given by the mean value of the EAD and it is denoted as .

Discounting is a financial mechanism in which a future value is being recalculated to the present value. The *discount factor*, , is the factor by which a future cash flow must be multiplied in order to obtain the present value. Consider the discount factor at time , defined as

(5)

where is risk-free rate of return, is the value of risk free asset at time and is Euler's number. Hence, the *discounted expected exposure* at time conditional on the counterparty default at time is given by

(6)

Next component of the equation (1) is *Probability of Default*, which describes the creditworthiness of a counterparty. It provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations. There are many alternatives for estimating the probability of default. One of them is based on market value of the CDS as it has been mentioned in the introduction. Another option is to use estimation of the PD provided by external ratings agencies (such as S&P, Fitch or Moody's). A frequently used approach taken by many banks is to use internal rating models for estimating PD based on historical default experience. An output of the model is PD of the counterparty during one year.

Our paper is focused on the CVA for *interest rate swaps (IRS)*. Hence, in the next section a concise explanation of the IRS will be given.

**Interest Rate Swaps**

IRS is an agreement between two traders as defined in (Ševčovič et al., 2011). Consider *plain vanilla IRS*. Under this contract a party A commits to paying a party B the fixed interest rate from the defined amount so called *principal amount* or *notional value*. A party B is committed to pay to the party A a floating interest rate. The party A is called *the payer* and the party B is called *the receiver.*

The *fixed rate* of the IRS is determined on the beginning of the swap contract and it is unchanged during the life of the contract. On the other side, the *floating rat*e may vary during the time and is often dependent to a reference rate that gives the floating rate at every period of time. In our analysis we use *Prague Inter Bank Offered Rate* (PRIBOR) as a reference rate.

The valuation of the IRS is tied to the evolution of the interest rate. Therefore, in the next section we briefly refer to the interest rate evolution model, Hull-White model.

**Hull-White one-factor model**

The well-known interest rate evolution model, Hull-White one-factor model, will be used for modeling of the interest rate term structure. The model was first published by John Hull and Alan White (1990) and generalized later in (Hull and White, 2001). A general overview of the model can be found in (Brigo, Capponi and Pallavicini, 2014). The requirement for more accurate fit to the currently-observed yield curve, led Hull and White to the introduction of a time variable parameter in the Vasicek model. The model assumes that short rates have a normal distribution, and also that the short rates exhibit the mean reversion character. The Hull-White model extends the Vasicek and Cox-Ingersoll-Ross (CIR) models.

We can define this model by equation

(7)

where is the change in the interest rate after a small change in time,. is constant reversion speed, is volatility of the interest rate, is a Wiener process and is a drift function defined as

(8)

where is an instantaneous forward rate at time . Instantaneous forward rate is defined as

(9)

where is price of a zero coupon bond at time with maturity at time .

The constants and are extracted from the historical three month PRIBOR rates. We use equation (7) to simulate the short interest rates. We can expand the entire interest rate curve from the short rate using

, (10)

where

The main advantage of the Hull-White model is that it can be fitted exactly to the initial term structure of interest rates.

**Results**

The numerical illustration of the calculation of CVA will be introduced in this section. We work with data set of interest rate swaps from the year 2015. These data come from a bank operating in the Czech Republic. The data contains information about five vanilla interest rate swaps and each of them is associated with different counterparty. In these swap trades the bank is receiver. We can observe the variables in a data set in the Tab. I.

Table I: Variables in a data set

|  |  |
| --- | --- |
| Notation | Name of Variable |
| ID | Counterparty ID |
| principal | Principal amount of swap |
| maturity | Maturity date of swap |
| LegRateReceiving | Interest rate received by bank |
| LegRatePaying | Interest rate paid by bank |
| period | Period of paying |

Source: Author’s compilation according to our data set from the cooperating bank

We start our analysis with creating a discount curve which is important for evaluating the swaps.

**Initial yield curve**

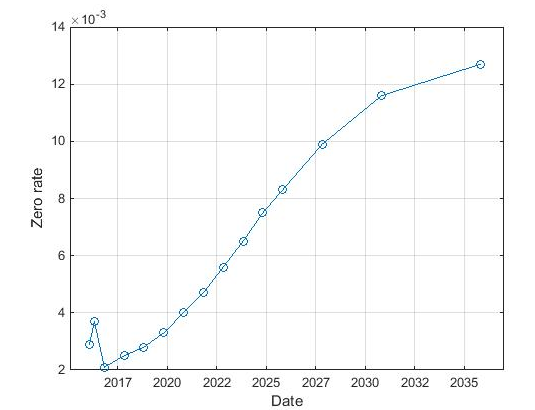
In order to evaluate the swaps, is necessary to describe a discount curve. The initial discount curve we obtain from web page PATRIA and terminal Bloomberg. We use as a discount rate IRS rates from one year IRS to twenty years IRS. The short rates (less than annual rate) were obtained from PRIBOR 3m and 6m. The initial rates can be seen in the Tab. II. The Fig. 1 shows the initial yield curve at settlement date which was built from the initial rates in the Tab. II.

Table II: Initial rates

|  |  |
| --- | --- |
| Maturity (years) | Rate (%) |
| 0.25 | 0.29 |
| 0.5 | 0.37 |
| 1 | 0.21 |
| 2 | 0.25 |
| 3 | 0.28 |
| 4 | 0.33 |
| 5 | 0.40 |
| 6 | 0.47 |
| 7 | 0.56 |
| 8 | 0.65 |
| 9 | 0.75 |
| 10 | 0.83 |
| 12 | 0.99 |
| 15 | 1.16 |
| 20 | 1.27 |

Source: Author’s compilation according to Bloomberg and https://www.patria.cz/kurzy/historie/sazby.html, [cit. 2015-10-30]

Figure 1: Yield Curve

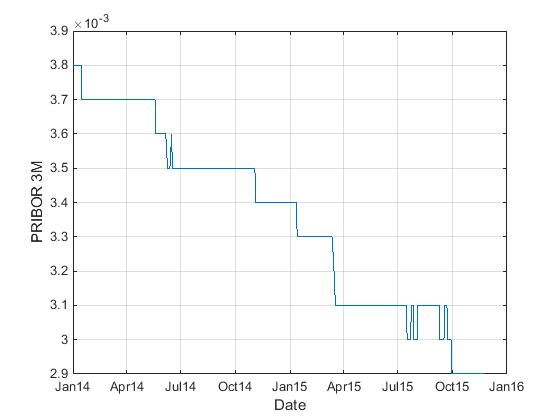


Source: Author’s elaboration according to Tab. II

Further, we need simulate the evolution of interest rate for the valuation of swaps.

**Interest rate simulation**

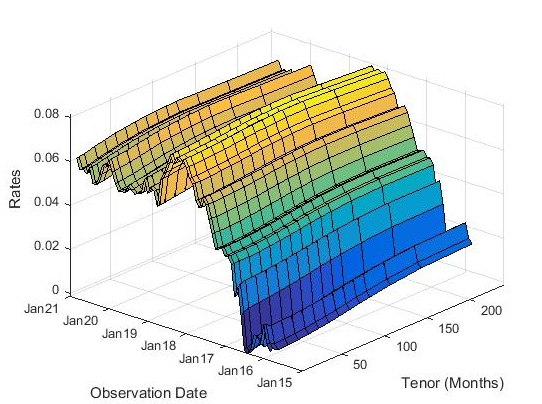
In our data set, the floating rate of the swap is indexed to the three month PRIBOR reference rate. Therefore, the interest rate simulation is based on historical data of the three month PRIBOR rate, from 2. 1. 2014 to 24. 11. 2015. The development of three month PRIBOR rate can be observed in the Fig. 2.

Figure 2: The three month PRIBOR rate

Source: Author’s compilation according to https://www.cnb.cz/cs/financni\_trhy/penezni\_trh/pribor/rok\_form.jsp [cit. 2015-11-30]

We use Monte Carlo method for the pricing swaps. For more information about this method, see e.g. (Hammersley, 2013) or (Rubinstein and Kroese, 2011). One thousand simulations were made. Each of simulation was modeled by Hull-White model, which is defined by equation (7). We need estimate the parameters and . As described above, the parameters of this model were estimated from three month PRIBOR. It is and . The example of yield surfaces obtained in one interest rate simulation is possible to observe in the Fig. 3. For each scenario the swaps are priced at each future simulation date.

Figure 3: One Possible Scenario of Yield Curve Evolution



Source: Author’s elaboration

In the next text, the probability of default of each client is determined.

**Probability of default**

We assume that the wait time for default of the counterparty is a random variable with exponential distribution. Then the cumulative distribution function of this variable can be define as

(11)

where is the parameter of the distribution. Probability of default of the counterparty during one year we denote . Then

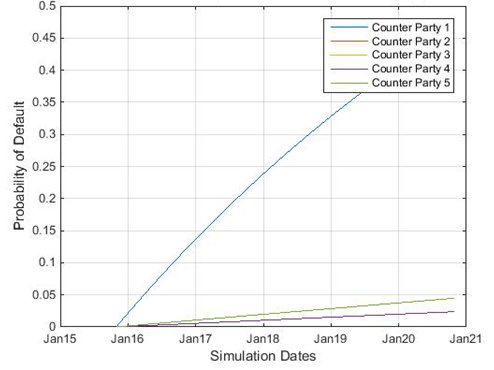
(12)

If we use value of *PD* from internal rating model of bank, which has provide the data, could be estimate from (12)

(13)

Then we can construct probability curve for each counterparties. It is possible to observe them in the Fig. 4.

Figure 4: Default Probability Curves



Source: Author’s elaboration

The counterparties 3 and 4 have the same rating and the counterparties 2 and 5 have the same rating too. Therefore we can see that their default probability curves are identical.

**Computation of CVA**

Let consider the exposure is independent of default. Then total exposure of all contracts can be computed on the bases of equation (4). For estimate of the recovery rate the bank uses LGD given by regulator. According to European Banking Authority (EBA) the regulatory loss given default is 45% (URL:https://www.eba.europa.eu/regulation-and-policy/single-rulebook/interactive-single-rulebook/-/interactive-single-rulebook/article-id/1598).

Now we have everything for the CVA calculation according to the equation (1). The resulting values of CVA for each counterparty can be observed in the Tab. III.

Table III: Resulting CVA

|  |  |
| --- | --- |
| Counterparty | CVA (in CZK) |
|  | 800 505.82 |
|  | 199 337.6 |
|  | 276.59 |
|  | 783.73 |
|  | 78 501.06 |

Source: Author’s elaboration

**Discussion**

In the following part we turn one's attention to a confrontation of the achieved results with computation of CVA by the bank which provided date for our calculation. We are drawing a comparison calculation of CVA which was presented in this paper with approach which is being used in the bank.

Základní princip výpočtu CVA spočívá v odhadu očekávané ztráty (EL) v průběhu života derivátu. Přitom platí . Rozdělením období do maturity, zdiskontováním a sečtením EL v každém z těchto období získáme CVA. Pro výpočet CVA používáme rozdělení zbývajícího času do maturity derivátu na časová okna o délce 1 roku. Čas do maturity se zaokrouhluje na celé roky nahoru. Tedy:

Basic principle of bank’s calculation CVA….

where is number of years to maturity of financial derivative, is probability of default during -th year. Hodnota LGD je nastavena pro všechny protistrany na hodnotu 45%. Banka uvažuje konstantní EAD po celou dobu trvání obchodu, což je hlavní rozdíl mezi jejím a naším přístupem.V každém časovém okamžiku se EAD odhaduje jako aktuální tržní hodnota finančního derivátu.

Probability of default pro jednotlivá roční okna výpočtu se počítají následovně. Pro i-tý rok je PD dáno vzorcem

where is the average probability of default during one year which is given by internal rating of the counterparty. Tato metoda určení PD je diskrétní narozdíl od námi používaného spojitého přístupu.

We can observe the comparison of our results of CVA with CVA used by bank in the Tab. IV. We can see that our results of CVA achieve significantly lower values than CVA used by bank.

Table IV: Comparison of our results with CVA used by bank

|  |  |  |
| --- | --- | --- |
| Counterparty | Our results of CVA (in CZK) | CVA uses by bank (in CZK) |
| 1 | 800 505.82 | 1 646 700.37 |
| 2 | 199 337.6 | 309 836.82 |
| 3 | 276.59 | 384.89 |
| 4 | 783.73 | 1095.12 |
| 5 | 78 501.06 | 121 149.12 |

Source: Author’s compilation according to our results and data set from the cooperating bank

*Diskutování výsledků, srovnávání s jinými výsledky v literatuře. Případně informace o tom, kam dál směřovat výzkum.*

*Srovnání oceňování IRS pomocí M-C simulace vs. Parciální diferenciální rovnice (Ševčovič). Místo H-W modelu jiné modely jako CIR.*

*Srovnání na to co vyslo bance a vyzdvihnutí, že náš model vychází líp!!!! Dát tam srovnávací tabulku.*

**Conclusion**

*Shrnutí hlavních výsledků z kapitoly results, ale už zcela bez diskuze.*

It has been presented a method of Counterparty credit risk valuation using Credit valuation adjustment. It has been summarized regulatory requirements for the risk control of the CCR. The components of the CVA have been specified. It has been demonstrated CVA calculation for five various interest rate swaps. The approach to CVA calculation presented in this article proved to be more profitable than caluclation used in cooperating bank.

**References**

ARORA, N., GANDHI, P. and LONGSTAFF, F. A. 2012. Counterparty credit risk and the credit default swap market. *Journal of Financial Economics*, 103(2):280-293.

BOUCHAUD, J.-P. and POTTERS, M. 2003. *Theory of financial risk and derivative pricing: from statistical physics to risk management*. edition. Cambridge: Cambridge University Press.

BRIGO, D., CAPPONI, A. and PALLAVICINI, A. 2014. Arbitrage-free bilateral counterparty risk valuation under collateralization and application to credit default swaps. *Mathematical Finance*, 24(1):125--146.

CANABARRO, E. and DUFFIE, D. 2003. Measuring and marking counterparty risk. *Asset/Liability Management for Financial Institutions, Institutional Investor Books*. Euromoney Institutional Investor PLC. [Online]. Available at: http://www.darrellduffie.com/uploads/surveys/duffiecanabarro2004.pdf. [Accessed: 2016, November 10]

DUFFIE, D. 2003. Intertemporal asset pricing theory. *Handbook of the Economics of Finance*, *1(2)*: 639-742.

ETHERIDGE, A. 2002. *A course in financial calculus*. New York: Cambridge University Press.

EU. 2013a. Directive 2013/36/EU of the European Parliament and of the Council of 26 June 2013 on access to the activity of credit institutions and the prudential supervision of credit institutions and investment firms, amending Directive 2002/87/EC and repealing Directives 2006/48/EC and 2006/49/EC Text with EEA relevance. *European Commission.*

EU. 2013b. Regulation (EU) No 575/2013 of the European Parliament and of the Council of 26 June 2013 on prudential requirements for credit institutions and investment firms and amending Regulation (EU) No 648/2012 Text with EEA relevance. *European Commission.*

GREGORY, J. 2010. *Counterparty credit risk: the new challenge for global financial markets*, West Sussex: John Wiley & Sons.

HAMMERSLEY, J. 2013. *Monte carlo methods*. London: Chapman and Hall.

HULL, J. 2012 *Options, futures, and other derivatives (8. ed., global ed.)*. Boston [u.a.] : Pearson.

HULL, J. and WHITE, A. 1990. Pricing interest-rate-derivative securities. *Review of financial studies*, 3(4): 573-592.

HULL, J. and WHITE, A. 2001. The general hull-white model and supercalibration. *Financial Analysts Journal*, 57(6): 34-43.

HULL, J. and WHITE, A. 2012. Cva and wrong-way risk. *Financial Analysts Journal*, 68(5): 58-69.

IASB. 2011. *IFRS 13 Fair Value Measurement*. Available at: http://www.ifrs.org/IFRSs/Pages/IFRS.aspx

PYKHTIN, M. and ROSEN, D. 2010. Pricing counterparty risk at the trade level and CVA allocations. [Online]. FEDS Working Paper No. 10. Available at: http://ssrn.com/abstract=1782063

PYKHTIN, M. and ZHU, S. H. 2007. A guide to modeling counterparty credit risk. *GARP Risk Review.* Available at: www.javaquant.net/papers/CounterPartyRisk.pdf

RUBINSTEIN, R. Y. and KROESE, D. P.2011. *Simulation and the Monte Carlo method.* Hoboken: John Wiley & Sons, 2011.

ŠEVCOVIC, D., STEHLIKOVÁ, B.; MIKULA, K. 2011 *Analytical and numerical methods for pricing financial derivatives.* New York : Nova Science Pub. Inc.