**Modeling of Counterparty Credit Risk in Interest Rate Swaps**

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**Abstract:** According to the Basel Committee's estimate three fourths of the counterparty credit risk losses during the financial crisis originate from Credit valuation adjustment's losses and not from actual defaults. Therefore Third Basel Accord has instructed banks to calculate capital requirement for risk of Credit valuation adjustment (CVA) from 2015. Banks are trying to model CVA to hold the prescribed standards and also reached the lowest possible impact on their profit. In this paper, we try to model CVA by using methods that are in compliance with the prescribed standards and also achieve the smallest possible impact on the bank's earnings. To do so, the data set of interest rate swaps from the year 2015 is used. The interest rate term structure is simulated by Hull-White one-factor model and Monte Carlo methods. Than the probability of default for each counterparty is constructed. Safe level of CVA is reached in spite of calculated CVA achieves a lower level than CVA previously used by bank. This allows a reduction of capital requirements for banks. All calculations are computed in Matlab environment which contains packages of financial mathematics and stats with the function for the Hull-White model and the function for the estimation of probability of default which are necessary for the computation of the CVA.

**Keywords:** counterparty credit risk, credit valuation adjustment, probability of default, interest rate swaps, yield curve, Hull-White model, Monte Carlo simulations, credit exposure

**Introduction**

The current situation in the banking market pressures banks to look for new opportunities to generate income. Common methods of making a profit are not as profitable as they were in the past. We can observe not only a negative interbank offered rates, but also competitive fight for clients, which causes strong pressure to decrease bank fees and almost unprofitable lending. The banks, therefore, search for new possibilities how to decrease costs such as loan loss provision and credit valuation adjustment. One of the possibilities could be development of a new approach to CVA modeling respecting regulatory standards and simultaneously achieving maximal profit.

A good introduction to pricing counter party credit risk can be found in a paper by Michael Pykhtin and Steven Zhu (2007). This paper discusses approaches to CVA calculation. Canabarro and Duffie (2003) deal with measuring counterparty risk. In their article are defined basic terms and models of counterparty exposures. A detailed review of counterparty credit risk modeling is given in (Jon Gregory, 2010). This book interestingly explains the rise of counterparty risk during the financial crisis. The quantification of credit exposure is presented as well as risk mitigation methods.

Under usual approaches CVA is measured at the counterparty level. Nevertheless, it can be sometimes required to determine contributions of individual trades to the CVA at the counterparty level. Pykhtin and Rosen (2010) thoroughly analyze the problem of allocating CVA to the individual trades. They explain how this problem can be simplified to calculating contributions of the trades to the expected exposure of the each counterparty where the expected exposure is conditioned by the default of counterparty.

A measure of credit quality of a counterparty is default probability. The counterparty's probability of default is typically derived from credit default swaps (CDS). Arora, Gandhi and Longstaff (2012) examine the credit default swaps market and his relevance in counterparty credit risk pricing.

The counterparty credit exposures may be correlated with the credit quality of a counterparty. If this correlation is negative then it is called wrong way risk. In actual fact, risk from correlation occurs always, however it is usually ignored to simplify modeling of exposure. Nevertheless, there exist cases when wrong way risk is too significant to be ignored. This case may be commodity trades with a producer of that commodity. Hull and White (2012) introduced one of the first models wrong way risk in CVA calculations.

The international accounting standards IFRS 13 and SFAS 157 require banks to report the value of their derivative portfolio net of the credit valuation adjustment. The accounting standards were set up in response to the financial crisis. A purpose of the standards are that the value of derivatives have to be adjusted with their counterparty risk. As a consequence, all banks are under an obligation to calculate CVA on a monthly basis.

The banks often use primitive parametric models, which are very conservatively set due to risk vigilance. We suppose that a more sophisticated model would bring lower CVA as well as lower capital requirement for a bank.

Sem napíšeme co v té práci děláme.

Pro výpočet CVA potřebujeme určit expected loss. EL=PD x LGD x EAD.

LGD bereme regulatory LGD

K odhadu PD jsme vyuzili internal rating models.

CVA budeme počítat pro IRS.

Pro oceňování IRS je nutné modelovat interest rate, protože se od ní odvozuje cena IRS.

IRS je derivát interest rate.

K tomu použili H-W model.

M-C simulace k oceneni IRS.

**Materials and Methods**

In this section, we developed the basic methodology to compute CVA and describe the basic terms.

**Components of credit valuation adjustment and terminology**

The basic concepts and notation for counterparty credit risk and CVA will be shown in this section. Counterparty credit risk (CCR) is the risk that the counterparty defaults before the final settlement of a transaction's cash flows. CVA can be explained as the difference between the portfolio's risk-free value and the portfolio's true value taking into account the possibility of default of the counterparty. In the next definition CVA is calculated as expectation of credit loss. *The credit valuation adjustment* is defined as

(1)

where is recovery rate, is the discounted expected exposure at time and is probability of default.

In what follows, we specify the components of CVA. *Recovery rate* is the value of unity less *Loss given default (LGD)*, i.e. . The LGD is the percentage amount of the exposure expected to be lost if the counterparty defaults.

The counterparty *credit exposure* of the bank to a counterparty at time (hereafter simply exposure) is defined as the economic loss, incurred on all outstanding transactions with the counterparty if the counterparty defaults at . Denote the value of the -th instrument in the portfolio at time by . The value of the counterparty portfolio is given by

(2)

When netting is not allowed, the exposure E(t) is given by

(3)

For a counterparty portfolio with a netting agreement, the exposure is

(4)

*Exposure at default (EAD)* is the total value which a bank is exposed to a counterparty at the time of default. For simplification in the follow equations we denote EAD as , where is time of the default. The EAD may be seen as a random variable. Therefore, an *expected exposure at default* is given by the mean value of the EAD and it is denoted as .

Discounting is a financial mechanism in which a future value is being recalculated to the present value. The *discount factor*, , is the factor by which a future cash flow must be multiplied in order to obtain the present value. Consider the discount factor at time , defined as

(5)

where is risk-free rate of return, is the value of risk free asset at time and is Euler's number. Hence, the *discounted expected exposure* at time conditional on the counterparty default at time is given by

(6)

Next component of the equation (1) is *Probability of Default*, which describes the creditworthiness of a counterparty. It provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations. There are many alternatives for estimating the probability of default. One of them is based on market value of the CDS as it has been mentioned in the introduction. Another option is to use estimation of the PD provided by external ratings agencies (such as S&P, Fitch or Moody's). A frequently used approach taken by many banks is to use internal rating models for estimating PD based on historical default experience. An output of the model is PD of the counterparty during one year.

Our paper is focused on the CVA for *interest rate swaps (IRS)*. Hence, in the next section a concise explanation of the IRS will be given.

**Interest Rate Swaps**

IRS is an agreement between two traders as defined in (Ševčovič et all., 2011). We consider plain vanilla IRS. Under this contract a party A commits to paying a party B the fixed interest rate from the defined amount so called *principal amount* or *notional value*. A party B is committed to pay to the party A a floating interest rate. The party A is called *the payer* and the party B is called *the receiver.*

The *fixed rate* of the IRS is determined on the beginning of the swap contract and it is unchanged during the life of the contract. On the other side, the *floating rat*e may vary during the time and is often dependent to a reference rate that gives the floating rate at every period of time. In our analysis we use *Prague Inter Bank Offered Rate* (PRIBOR) as a reference rate.

The valuation of the IRS is tied to the evolution of the interest rate. Therefore, in the next section we briefly refer to the interest rate evolution model, Hull-White model.

**Hull-White one-factor model**

The well-known interest rate evolution model, Hull-White one-factor model, will be used for modeling of the interest rate term structure. The model was first published by John Hull and Alan White (1990) and generalized later in (Hull and White, 2001). A general overview of the model can be found in (Brigo, Capponi and Pallavicini, 2014). The requirement for more accurate fit to the currently-observed yield curve, led Hull and White to the introduction of a time variable parameter in the Vasicek model. The model assumes that short rates have a normal distribution, and also that the short rates exhibit the mean reversion character. The Hull-White model extends the Vasicek and Cox-Ingersoll-Ross (CIR) models.

We can define this model by equation

(7)

where is the change in the interest rate after a small change in time,. is constant reversion speed, is volatility of the interest rate, is a Wiener process and is a drift function defined as

(8)

where is an instantaneous forward rate at time . Instantaneous forward rate is defined as

(9)

where is price of a zero coupon bond at time with maturity at time .

The constants and are extracted from the historical three month PRIBOR rates. We use equation (7) to simulate the short interest rates. We can expand the entire interest rate curve from the short rate using

(10)

**Results**

The numerical illustration of the calculation of CVA will be introduced in this section. We work with data set of interest rate swaps from the year 2015. These data come from a bank operating in the Czech Republic. The data contains information about five vanilla interest rate swaps and each of them is associated with different counterparty. In these swap trades the bank is receiver. Jsi si jistá? V datech se píše PAY, ale nejsem si jistá co to znamená. We can observe the variables in a data set in the Tab. I.

Table I: Variables in a data set

|  |  |
| --- | --- |
| Notation | Name of Variable |
| ID | Counterparty ID |
| principal | Principal amount of swap |
| maturity | Maturity date of swap |
| LegRateReceiving | Interest rate received by bank |
| LegRatePaying | Interest rate paid by bank |
| period | Period of paying |

Source: Author’s compilation according to our data set from the cooperating bank

We start our analysis with creating a discount curve which is important for evaluating the swaps.

**Initial yield curve**

In order to evaluate the swaps, is necessary to describe a discount curve. The initial discount curve can be obtain by a linear interpolation of the risk free interest rates of different maturities. The initial rates were found at the web page of PATRIA (URL: http://www.patria.cz/kurzy/CZK/5Y/irs/graf.html) and we can see them in the Tab. II. The interest rate shorter than one year were extracted from PRIBOR rates on October 30th, 2015. The Fig. 1 shows the initial yield curve at settlement date which was built from the initial rates in the Tab. II.

Table II: Initial rates

|  |  |
| --- | --- |
| Maturity (years) | Rate (%) |
| 0.25 | 0.14 |
| 0.5 | 0.16 |
| 1 | 0.21 |
| 2 | 0.25 |
| 3 | 0.28 |
| 4 | 0.33 |
| 5 | 0.40 |
| 6 | 0.47 |
| 7 | 0.56 |
| 8 | 0.65 |
| 9 | 0.75 |
| 10 | 0.83 |
| 12 | 0.99 |
| 15 | 1.16 |
| 20 | 1.27 |

Source: toto je divny, protoze 3m PRIBOR byl v data.xlsx i na https://www.cnb.cz/cs/financni\_trhy/penezni\_trh/pribor/denni.jsp 0,29 a 6m PRIBOR 0,36

My jsme to udělaly tak, že jsme vzali PATRIA IRS (viz data.xlsx list zero\_rates) pro 1Y-20Y a pro 0,5 a 0,25 jsme provedli lineární extrapolaci. Možná tam napsat něco jako, že IRS jsou deriváty interest rate a proto z nich můžeme odvodit interest rate, ale nevim uplne jak.

Ještě bychom se mohli odvolat na to, že se s PRIBORem údajně v té době manipulovalo, tak raději bereme ty IRS viz: <https://cs.wikipedia.org/wiki/Prague_InterBank_Offered_Rate>

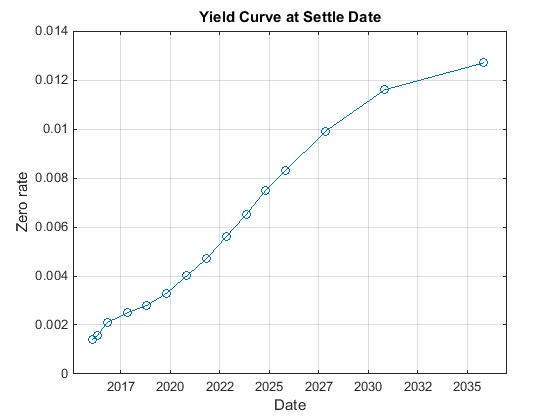


Figure 1: Yield Curve

**Interest rate simulation**

Tady musíme napsat, proč to vůbec děláme a proč zrovna 3M PRIBOR a ne jiný? Je to tím, že ty IRS jsou založeny na 3-M PRIBORU?

The interest rate simulation is based on historical data of the three month PRIBOR rate, from 2. 1. 2014 to 24. 11. 2015. The development of three month PRIBOR rate can be observed in the Fig. 2.

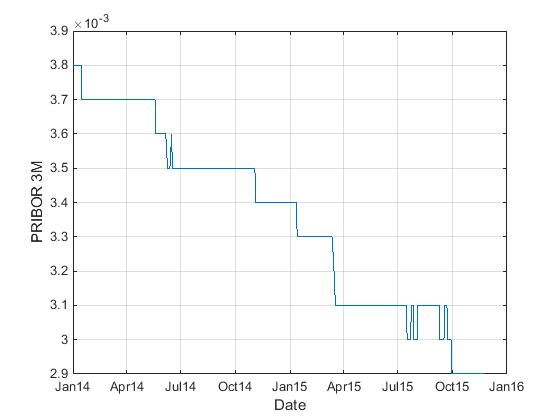


Figure 2: The three month PRIBOR rate

The large number of simulations was made. The example of yield surfaces obtained in one interest rate simulation is possible to observe in the Fig. 3. For each scenarios the swaps are priced at each future simulation date.

The main advantage of the Hull-White model is that it can be fitted exactly to the initial term structure of interest rates.

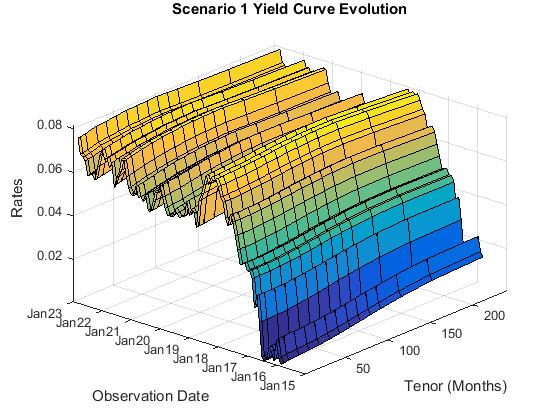


Figure 3: One Possible Scenario of Yield Curve Evolution

**Probability of default**

We assume that the wait time for default of a counterparty is a random variable with exponential distribution. Then the cumulative distribution function of this variable can be define as

(11)

where is the parameter of the distribution. Probability of default of the counterparty during one year we denote . Then

(12)

If we use value of PD from internal rating model of bank, which has provide the data, could be estimate from (12)

(13)

Then we can construct probability curve for each counterparty. It is possible to observe them in the Fig. 4.

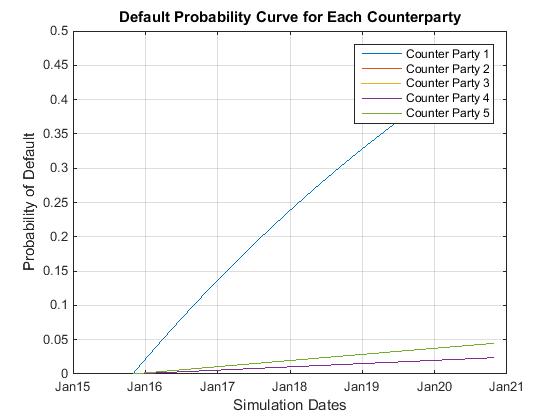


Figure 4: Default Probability Curves

**Computation of CVA**

Let consider the exposure is independent of default. Then total exposure of all contracts can be computed on the bases of equation (4). For estimate of the recovery rate the bank uses LGD given by regulator. According to European Banking Authority (EBA) the regulatory loss given default is 55% (URL:https://www.eba.europa.eu/regulation-and-policy/single-rulebook/interactive-single-rulebook/-/interactive-single-rulebook/article-id/1598).

**Přepočítat výsledky na LGD 45% místo R=40%!!! Tedy R=55%!!!**

Now we have everything for the CVA calculation according to the equation (1). The resulting values of CVA for each counterparty can be observed in the Tab. III.

|  |  |
| --- | --- |
| Counterparty | CVA (in CZK) |
|  | 1 056 075.83 |
|  | 265 290.14 |
|  | 369.19 |
|  | 1 045.54 |
|  | 103 319.55 |

Table III: Resulting CVA

**Discussion**

Diskutování výsledků, srovnávání s jinými výsledky v literatuře. Případně informace o tom, kam dál směřovat výzkum.

Srovnání oceňování IRS pomocí M-C simulace vs. Parciální diferenciální rovnice (Ševčovič). Místo H-W modelu jiné modely jako CIR.

Srovnání na to co vyslo bance a vyzdvihnutí, že náš model vychází líp!!!! Dát tam srovnávací tabulku.

**Conclusion**

Shrnutí hlavních výsledků z kapitoly results, ale už zcela bez diskuze.

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