MATH 695 10/7/2022 Comment: Using K-theory to unstruct element in stable homotopy groups of spheres: 2 1- A. compathycotron

X E Tanti (N)

52N - 52N bt CN QETTanti (N) 1 based maps of dig al, can got i'll to figue 0 by adeling in This L'Ail mot based, but can move base proint to the base proint: finite group die Manti (5°) (complex) J-homomorphism. Gives the heet one can do at primes p>2. At p=2, use Ko instead, of K separat one construction for infinite orthogonal group O. Theorem (OoH): DE 0 ~ D real sprhors alsh & sp/a D0 = 0/u 2 Sp/a ~ W/0 20/U ~ U/Sp E compact form) SUND~BOXX Q U/Sp~BSpX2 2 BSp x 2 ~ Sp

Atiyah: K. theory and reality Take the proof of Both periodicity we did and add complex compagation. (All the Apr we dod presure it!) deduce real Boss pervodrants via considerations with real primors.

So we get finer information from J: IT KO -1 The S inod8 II. KO Adams: On the groups J(X) I-IV. egned to demonwertors Of Bernoulli numbers,

Bach to a more general discussion. Wegenber What does algebraic topology see? homotopy lymalence of general spaces is trofine a relation. (harnology, harnotopy groups will never tell the Canton set Jean the discute set) Smaller goal: The set of connected components ?? When we can fluid i's KoX = I pash - components }

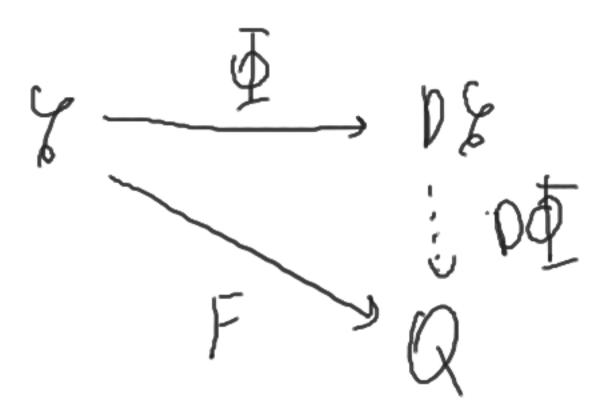
Définistion: A (continuous) map f: X -> Y is called a weak equivalence if Mof: Mo X = Mo Y and for all m > 1 and all x e X $T_{n}f:T_{n}(X,x) \stackrel{\sim}{=} T_{n}(Y,f(x)).$

Algebruic topology works in a castegory where weak estagory) equivalences are made into isomorphisms (and derived castagory)

Also uxful! "formal shelton" A may f: X -> Y is called a D-equivalence it mf: MoX -> To Y vs outs and an n-equivalence for n > 0 if $\Pi_0 f: \Pi_0 X \xrightarrow{\sim} \Pi_0 Y$ and for h > 1 , x e X The f: The (X,x) - The (Y,f(x))

vs an scomethorn for be < n and order for be z M.

We have a certegoy & with some lass of moylvans É (usually we assume & forme a mocentique, mobiles all = and satisfies the 2/3 property: in h=got of two out of the maps figh are in E, is vithe third). A derived cartegory (if one brists) is a cartegory DE Jøgether with a function \$: 6 -> DE which sends et & into an i'monoghum and is universal will that property:



Frends moghernes in É virolo vormoghernes Shan Il DI whole makes the dog. Anothy commute.

Noto: I must be a bijection or objects.

Abternative 2-categorical definition which is immundant to equivelene of categories.

Why might a deward witigog not evont? (set theory)

(MW) (4) Prove that a 1-equivalence. $U(1) \subset U(m)$ is