

MATH 425

8/29/2022

Note Title

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Notes are posted on my web page:
www.math.lsa.umich.edu/~ikriz

Igor Kriz

Text: Sheldon Ross : A first course in probability
for this section, ed. 8 or newer

Requirements : 3 Midterm tests , no final
100 pts each In person, in class
tentative dates : Oct 5
Nov 2
Dec 9

HW (total : 100 pts) score = percentage
max. points on the class : 400

HW assigned in each class

typed each Friday on Gradescope

Due by the beginning of next class ^{after it has been posted on gradescope}
due also on Gradescope (usually next Monday 10 AM)

this coming week by Wed 10 AM.

Invitation code: 7G-EJ35

Office hours MWF 11AM - 12PM or by
appointment

Office EH 3846.

A general picture of probability theory

- randomness ?? We do not know what randomness is or if it exists.

- instead of randomness, we discuss measure.

We have a set of outcomes also called a sample space S . An event E is just some subset of S .

In the beginning, we assume S is finite.
To each outcome, we assign a probability.
The probability is a number in $[0, 1]$
(between 0 and 1, included) $P(s) \in [0, 1]$

The probabilities add up to 1.

↕
is an
element of

↙ sum

$$\sum_{s \in S} P(s) = 1$$

↖ over all events s in S .

The probability of an event E is

$$P(E) = \sum_{s \in E} P(s).$$

In many cases, the probabilities of outcomes are determined by symmetries.

Example: Casting a standard cubic die.

Outcomes:

$$S = \{1, 2, 3, 4, 5, 6\}$$

↙ ↘
a set of the elements
inside the braces

A fair die is symmetrical
equal likelihood of outcomes:

$$P(1) = P(2) = P(3) = P(4) = P(5) \\ = P(6)$$

Therefore, $P(i) = \frac{1}{6}$

for all $i \in \{1, \dots, 6\}$.

Example: Suppose I cast two fair dice and I record the two numbers that come up (without remembering which die was which).

Outcomes: $\{1,1\}, \{2,2\}, \{3,3\}, \{4,4\}, \{5,5\}, \{6,6\}$

different numbers: $\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{1,6\}$
 $\{2,3\}, \{2,4\}, \{2,5\}, \{2,6\}$
 $\{3,4\}, \{3,5\}, \{3,6\}$

$$\binom{6}{2} = 15$$

← soon!

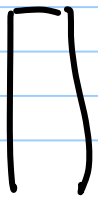
$\{4,5\}$, $\{4,6\}$

$\{5,6\}$

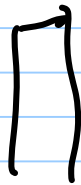
total of 20 outcomes. Not equally likely.

We can actually record number which
die was which.

number
on die 1



number
on die 2



ordered pairs of numbers
which come up are symmetrical

$$P \begin{matrix} dx_1 \\ (1, 1) \end{matrix} = P(5, 6) = P(6, 5)$$

\vdots

$6 \cdot 6 = 36$ outcomes equally likely

$$P(i, j) = \frac{1}{36}$$

From this point of view, unordered pairs are events.

$$P\{i, i\} = P(i, i) = \frac{1}{36}$$

If $i \neq j$

$$P\{i, j\} = P(i, j) + P(j, i) = \frac{1}{36} + \frac{1}{36} = \frac{1}{18}$$

$$\begin{array}{c} \swarrow \text{different} \quad \nwarrow \text{same} \\ 15 \cdot \frac{1}{18} + 6 \cdot \frac{1}{36} = \frac{30 + 6}{36} = 1 \quad \checkmark \end{array}$$

We have encountered several concepts of probability.

① mathematical model

(different sample spaces can describe the same situation, sometimes one is better than the other)

② counting principles, independent events, etc.

HW:

① Suppose we cast two fair standard dice. Compute the probability that the sum of the numbers that come up is $2, 3, 4, 5, 6, 7, \dots, 12$.
(each of them)

② Suppose we flip three fair coins. Calculate the probability

that heads comes up exactly twice.