

Ordinary equivariant homology and cohomology
(taking into account fixed point under subgroups)

Bredon cohomology

The orbit category \mathcal{O}_G

G can be a discrete group

$$\text{Obj}(\mathcal{O}_G) = \{ \text{orbits } G/H \mid H \subseteq G \}$$

morphisms = G -equivariant maps

(= H -conjugacies)

Coefficient system: A functor $\mathcal{O}_G^{\text{op}} \rightarrow \mathbf{Ab}$. } an abelian category, enough projectives

Given a G -space X , we can get a chain complex of coefficient systems

$$(C_G(X))_n : G/H \longmapsto C_n(X^H)$$

fixed points

singular chains

$$\{ G/H \times \Delta^n \xrightarrow{\quad} X \}$$

G -equivariant

Given a G -coefficient system $\mathcal{H} : \mathcal{O}_G^{\text{op}} \rightarrow \mathbf{Ab}$,

$$H^n(X; \mathcal{H}) := H^n \text{Hom}_{\mathcal{O}_G} (C_G(X), \mathcal{H})$$

in the abelian category of coefficient systems
cochain complex of ab. groups

projective coefficient systems

If G is a finite group, we can consider the special case when H is a Mackey function. $H^n(X; G)$ already defined.

different system

But now we also have homology.

$$g: \mathcal{D}_G \rightarrow \mathcal{A}_G$$

Chin

⑦ $\varphi: G/H \rightarrow G/K$

$$\mathcal{H}(G/H) \otimes C_m(X^K)$$

$$H_n(X; \mathbb{Z}) := H_n(\mathbb{Z} \otimes_{\mathbb{Z}} C_n(X))$$

$\mathcal{O}_G^T \leftarrow \left(\bigoplus_{H \subseteq G} \mathcal{L}(H) \otimes \underbrace{C_G(X)_m(G/H)}_{C_m(X^H)} \right)$

2 given n ;

Preview: $H_n(\mathbb{Z}; \mathbb{Z}), H^*(\mathbb{Z}; \mathbb{Z})$ } are represented by
 simple numbers & cohomology \mathbb{Z} -spectra $H\mathbb{Z}$

single homology & cohomology
w. coeffs. in a Mackey functor

Maths Junction

Note: X : G -CW-complex
set of n -cells = G -set

$$\{C_c^{\text{all}}(X)_n = \{I_n = \text{set of } n\text{-cells}\} \quad H_c^{\sim}(X; \mathbb{Z})$$

$$C_G(X)_m(G/H) = I_m^H = H_G^m(X; \mathbb{Z})_m$$

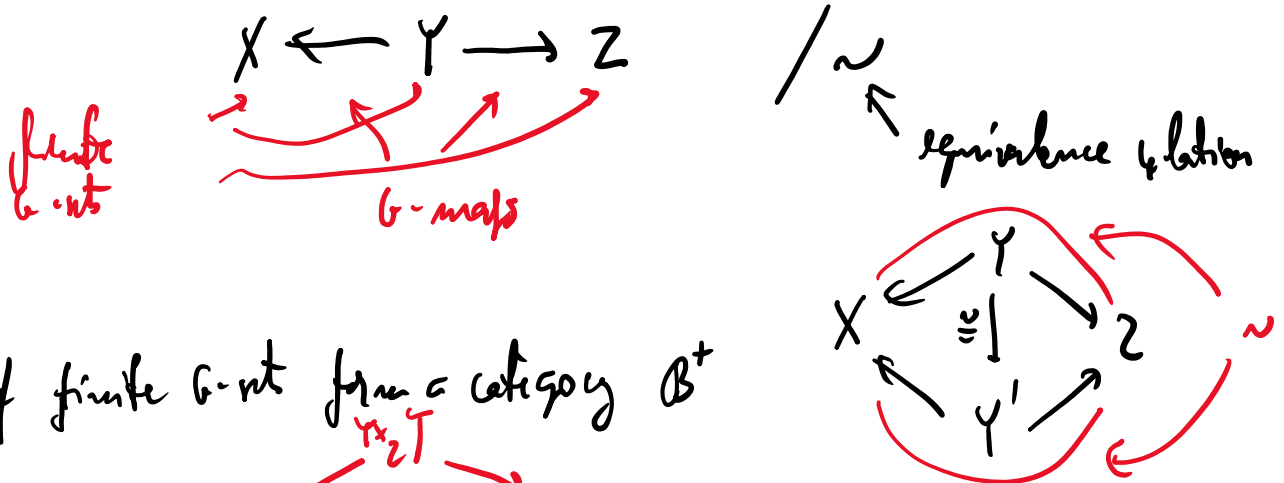
6. front d

Nachherkunft \Rightarrow same in homology

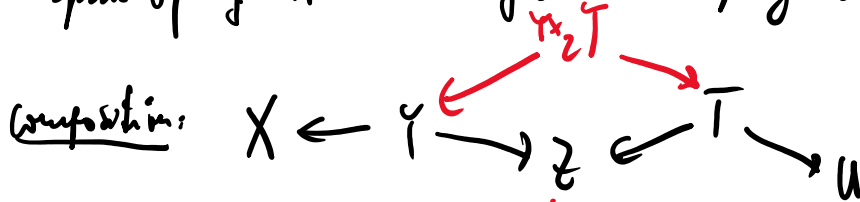
generalised (co)homology:
Adiyah-Kervatich spectral sequence

Back to Mackey functors (G finite group)

A different description: A span of G -sets is a diagram of G -sets of the form



Spans of finite G -sets form a category \mathcal{B}^+



$$X \leftarrow Y_1 \rightarrow Z \oplus X \leftarrow Y_2 \rightarrow Z$$

Operations on spans: $\perp, (X)$.

almost an abelian category (no inverse) $X \leftarrow Y, \perp Y_2 \rightarrow Z$
 morphisms can formally introduce an inverse

an abelian category:
 The Burnside category

$$\mathcal{B}^+ \rightsquigarrow \mathcal{B}$$

Π spans (X, Z)

commutative monoid

Apply K (unimodular abelian group)

A Nachezⁿ functor = An additive functor $B \rightarrow A$
 $B^+ \rightarrow A^+$
 $X \leftarrow Y \rightarrow Z$
 \uparrow
 equivalent
 data

$$G/H \xrightarrow{j} G/K \quad \xrightarrow{\quad}$$

$$M(G/K) \xrightarrow{r} M(G/H) \quad \xleftarrow{\quad}$$

$$G/K \xleftarrow{j} G/H \xrightarrow{\text{Id}} G/H$$

$$M(G/H) \xrightarrow{c} M(G/K)$$

$$G/H \xleftarrow{\text{Id}} G/H \xrightarrow{j} G/K$$

$$: M(X) \xrightarrow{r} M(Y) \xrightarrow{c} M(Z)$$

The Burnside category point of view explains how Mackey functors form a tensor category

M, N Mackey functors $M, N: \mathcal{O} \longrightarrow \mathbf{Ab}$

$M \otimes N$

$\mathcal{O} \boxtimes \mathcal{O} \xrightarrow{M \boxtimes N} \mathbf{Ab}$

