

Burnside category: $\mathcal{B} = \mathcal{B}_G$ is an abelian category

Maeky functors = Additive functors $\mathcal{B} \rightarrow \mathbf{Ab}$.

Obj \mathcal{B} = finite G -sets $\oplus = \coprod$

$$\text{Mor}_{\mathcal{B}}(G/H, G/J) = \mathbb{Z} \{ G/H \xleftarrow{G/K} G/J \}$$

To describe the tensor product of Maeky functors: \square . (Dress)

$$\text{Obj } \mathcal{B} \boxtimes \mathcal{B} = \coprod_{H_1, H_2 \subseteq G} \begin{pmatrix} G/H_1 \\ G/H_2 \end{pmatrix}$$

$$\text{Mor}_{\mathcal{B} \boxtimes \mathcal{B}} \left(\begin{pmatrix} G/H_1 \\ G/H_2 \end{pmatrix}, \begin{pmatrix} G/J_1 \\ G/J_2 \end{pmatrix} \right) = \mathbb{Z} \left\{ \begin{pmatrix} G/H_1 \xleftarrow{G/K_1} G/J_1 \\ G/H_2 \xleftarrow{G/K_2} G/J_2 \end{pmatrix} \right\}$$

We have a functor $\chi: \mathcal{B} \boxtimes \mathcal{B} \rightarrow \mathcal{B}$

$$\begin{pmatrix} G/H_1 \\ G/H_2 \end{pmatrix} \rightarrow G/H_1 * G/H_2$$

For two Maeky functors $M, N: \mathcal{B} \rightarrow \mathbf{Ab}$

$$M \otimes N: \mathcal{B} \boxtimes \mathcal{B} \rightarrow \mathbf{Ab}$$

$$\begin{pmatrix} G/H_1 \\ G/H_2 \end{pmatrix} \mapsto M(G/H_1) \otimes N(G/H_2)$$

We define $M \square N = \chi_{\#}(M \otimes N)$

additive
pushforward along the functor $\chi: \mathcal{B} \boxtimes \mathcal{B} \rightarrow \mathcal{B}$.
left Kan extension = left adjoint to pullback χ^* .

□ make Mackey functors into a tensor category

(coefficients of "ordinary" G -equivariant (\mathbb{C}) homology theories)

What is the unit of □?

Answer: The Burnside Mackey functor:

$$\mathcal{A} = \mathcal{A}_G \quad \mathcal{A}(G/H) = \mathbb{A}(H)$$

$K \cong$ class of finite H -sets, $H \leq G$

restriction in \mathcal{A} : passing to a subgroup

corestriction: $K \times_H ?$

Green functor: commutative (associative unital) ring in the symmetric tensor category of Mackey functors.

Can talk about modules over a green functor.

In particular, Mackey functors = Modules over \mathcal{A} .

Another application of the Burnside category: Constructing enough projectives in G - Mackey functors.

- ① α (the Burnside ring Mackey functor) is projective: free on one generator in isotypy G (G/G)
 $* \in A(G)$
 $[G/G]$

enough projective

Pushforward functor: $H \hookrightarrow G$
 $\text{Ind}_G^H: \mathcal{B}_H \xrightarrow{G \times_H ?} \mathcal{B}_G$

- ② $\text{Ind}_G^H \alpha_H$ is projective: free on one generator in isotypy H (G/H)

Example: \mathbb{Z}/p

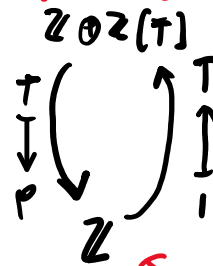
convention:

integers \mathbb{Z}/p
($\mathbb{Z}/p/\mathbb{Z}/p$)

strategy 0
($\mathbb{Z}/p/\mathbb{Z}/p$)

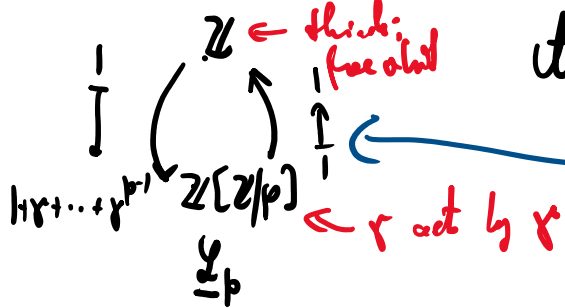
normal stuff

free object



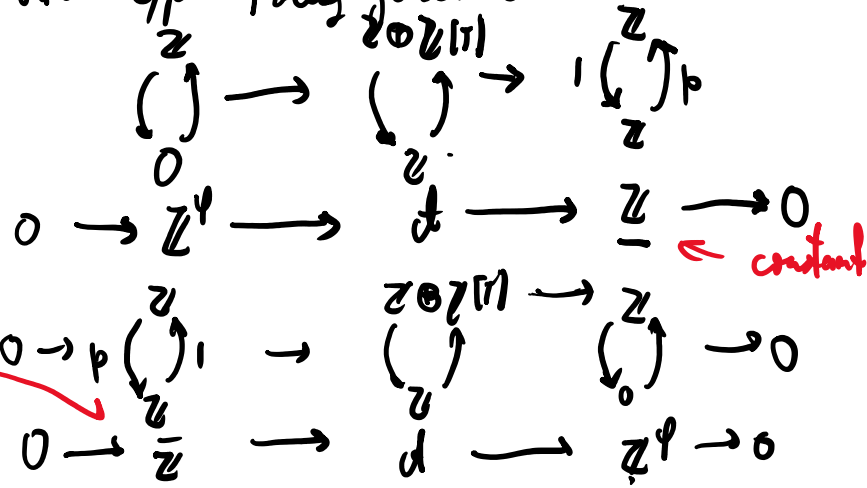
trivial \mathbb{Z}/p -action

$\mathbb{Z}/p = d_{\{0\}}$
 \mathbb{Z}



enough projections in \mathbb{Z}/p -Mod category

Examples of other \mathbb{Z}/p -Mod functors:



coconstant

constant

