

Homework 3

1. Suppose that we have a portfolio of m loans. The loss of the portfolio in a year may be modeled as follows:

$$L = \Pr \left\{ \sum_{i=1}^m L_i \cdot 1_{\{X_i \geq \delta_i\}} \right\},$$

where L_i is the loss given default and X_i is a default indicator for $i = 1, \dots, m$. For simplicity, we assume that $L_i, i = 1, \dots, m$ are constants.

The critical issue for modeling the loan portfolio loss is the modeling of the default dependence of the loads. Under the t -copula model, we let

$$X_i = \frac{\rho Z_0 + \sqrt{1 - \rho^2} Z_i}{\sqrt{\chi_d^2/d}},$$

where Z_0, Z_1, \dots, Z_m are independent standard normal random variables, χ_d^2 is an independent chi-squared random variable with d degrees of freedom.

Let $m = 100$, $L_i = 1$ and $\delta_i = 1.7$ for all $i = 1, 2, \dots, m$. Furthermore, we let $d = 5$ and $\rho = 0.5$. Develop at least **three** variance-reduction techniques to estimate $\Pr\{L \geq 10\}$, and compare their variance-reduction ratios to a crude Monte Carlo estimator.