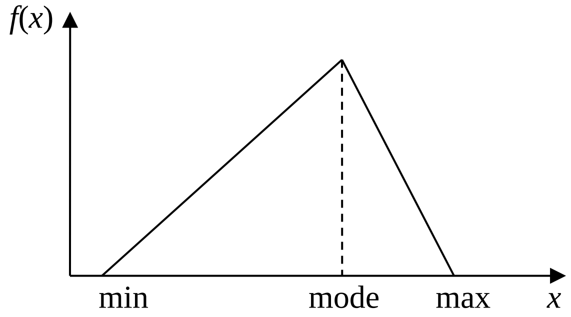
Homework 2

1. Modify the code of the reliability example on Page 10 of Note 1 such that the lifetime of a component follows an exponential distribution with mean 3.5 days and the repair time follows an exponential distribution with mean 2.5 days. You job is to estimate the expected time to failure and the probability that the system fails within 5 days. Using 1000 replications, and report a point estimate and a 95% confidence interval for each quantity.

2. Let the density of a distribution be
$$f(x) = \begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}.$$
Design an algorithm that generates random observations from the distribution using the inverse transform method. Write a program in either Python or Matlab, generate 1000 replications, and plot the histogram.

3. The density of a triangular distribution is defined by three critical values, min, max and mode (see the following figure).



Design an algorithm that generates random observations from the distribution using the acceptance-rejection method. Set min=0, max=5 and mode =3. Write a computer program in either Python or Matlab, generate 1000 replications, and plot the histogram.

4. Consider the simulation optimization problem,
$$\min_{x \in S} g(x)$$
where $g(x)$ = E[$G(x)$] and $G(x)$ is the output of a simulation replication conducted at x. Let x^* be a global optimal solution.

Grid search is often used to find a global optimal solution to the problem. It first chooses m grid points, $x_1, x_2, \ldots, x_m$, in S. It then takes r i.i.d. observations from each of the m grid points and calculates the sample means, $\overline{G}(x_1), \overline{G}(x_2), \ldots, \overline{G}(x_m)$. Let
$$\hat{x}_m^* = \arg\min\{\overline{G}(x_1), \overline{G}(x_2), \ldots, \overline{G}(x_m)\} \text{ and } x_m^* = \arg\min\{g(x_1), g(x_2), \ldots, g(x_m)\}.$$

Suppose that the grid points are chosen such that $g(x_m^*) \to g(x^*)$ as $m \to \infty$. We further assume that $\sup_{x \in S} \text{Var}[G(x)] = \sigma^2 < \infty$. Then what relationship r and m need to satisfy to ensure that $g(\hat{x}_m^*) \to g(x^*)$ almost surely as $m \to \infty$? (*Hint: You may need to use:* $|\min\{a_i\} - \min\{b_i\}| \leq \max\{|a_i - b_i|\}$ *and* $\Pr\{A \text{ or } B\} \leq \Pr\{A\} + \Pr\{B\}$.)