MATPMD4

Assignment 1 (Stochastic Processes)

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Please follow these instructions carefully; marks will be lost for submission that do not follow these instructions.

- Submit a single pdf file called MATPMD4_Assignment1_XXXX.pdf for marking, where XXXX is your student id. You may submit a number of other files as evidence of your calculations but everything you wish to be marked MUST appear in the named file (for example, DO NOT refer to other files for parts of questions, i.e. see file xxx for part (ii), it will not be marked).
- You may wish to do your assignment in a jupyter notebook, if you do so you must convert it to a pdf for submission, you may wish to also submit your .ipynb file.
- Completed assignments should be uploaded to canvas by the due date.

Date of Issue: January 25$^{\text{th}}$ 2023
Report Due by: 16:00 March 3$^{\text{rd}}$ 2023

Before you start, we will define some probabilities. Take your 7-digit student number, append a random digit to the end to make it an 8 digit number. Define the following probabilities

- p_1: take the first 2 digits of your number and divide by 100.
- p_2: take the second 2 digits of your number and divide by 100.
- p_3: take the third 2 digits of your number and divide by 100.
- p_4: take the fourth 2 digits of your number and divide by 100.

For example, if your student id is 3123456, then adding a random digit (say 3) gives 31234563 and so $p_1 = 0.31$, $p_2 = 0.23$, $p_3 = 0.45$, $p_4 = 0.63$.

1. State your student id and the values of p_1, p_2, p_3, p_4.

2. Successive generations of a particular flower have different coloured stamen (red, violet, blue and yellow). It is noted that

   - the probabilities of having red, violet, and blue stamens are 24%, 32%, 26% in the next generation if the current one is red,
   - 31% 26%, 24% of having a red, violet, or yellow flower if the current generation is violet.
   - if the current generation is blue the chance of the next being violet, red, and yellow are 25%, 20%, 25%
   - if the current generation is yellow, the probabilities of observing red, violet, blue and yellow are 0.18 0.23 0.27 0.32

   (a) Draw the state space diagram of this system

   (b) What are the 2 and 4 step transition matrices? (you may use python to calculate this but you need to show the code snippet, I would also suggest that you arrange your transition matrix in the order red, violet, blue, and yellow)

   (c) Assuming, in a garden of flowers, the distribution of colours is red (33%), violet (24%), blue (3%), and yellow (40%), calculate the distribution of colours after 5 generations? (you may use python to calculate this but you need to show the code snippet)

   (d) What colour distribution will I get in the long term? (you may use python to do the calculations but you need to show the code snippet)

[12]

3. For the transition probability matrix

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0.2 & 0.1 & 0.15 & 0 & 0.55 \\ 0 & 1 & 0 & 0 & 0 \\ 0.35 & 0.2 & 0.2 & 0.1 & 0.15 \\ 0 & 0 & 0 & 1 & 0 \\ 0.25 & 0.2 & 0.15 & 0.25 & 0.15 \end{pmatrix} \end{array}$$

   (a) Rewrite $\mathbf{P}$ in the canonical form, clearly identifying $\mathbf{R}$ and $\mathbf{Q}$.

   (b) For each state, i, calculate the mean number of times that the process is in a transient state j, given it started in i.

   (c) For each state i, find the mean number of transitions before the process hits an absorbing state, given that the process starts in a transient state i.

   (d) For each state i, find the probability of ending in each of the absorbing states.

[13]

4. Simulate a discrete random walk in $\mathbb{Z}^2$ starting at the origin and moving north with probability p_1 (see the instructions at the top of this document), south with $p = p_2$, east with $p = p_3$, and west with $p = p_4$.

   For your random walk,

(a) Plot 5 realisations of your walk with 2500 steps on the same plot (it may be wise to use different colours for each random walk).

(b) Calculate the expected length of the walk after 2500 steps (i.e. the distance travelled from the starting point). [You will need to run many walks to get an accurate estimate, experiment with the number of walks until you get a number that gives a sufficiently stable expectation value. You can also choose whatever distance measure you wish]

Please state clearly the choices you have made and and values used. **[25]**

5. A generalised random walk is given by

$$d\mathbf{S} = a(\mathbf{x}, t)dt + b(\mathbf{x}, t)dW$$

where $a(\mathbf{x}, t), b(\mathbf{x}, t)$ are given functions of space and time and dW is a *Weiner* process i.e. $\delta W = \epsilon\sqrt{\delta t}$ and ϵ is a random number normally distributed with a mean of 0 and a variance of 1.

Simulate this random walk in 1 dimension with your own choice of $a(\mathbf{x}, t)$ and $b(\mathbf{x}, t)$ (you may choose to keep these constant, e.g. a=1). Estimate the expected value for your walk after 2500 steps. **[25]**

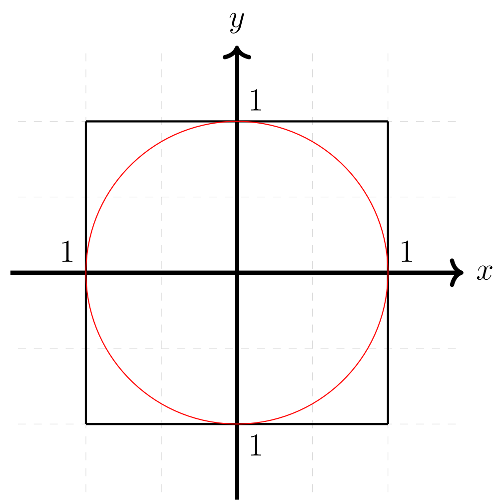
6. In the lectures we estimated the value of π using de Buffon's needle. Here we will use an alternative approach. In the diagram below, we have a circle of radius 1.0, enclosed by a 2×2 square (in some arbitrary units). The area of the circle is $\pi r^2 = \pi$ and the area of the square is 4 (in units squared). If we divide the area of the circle, by the area of the square we get $\frac{\pi}{4}$. For a very large number of points, we have

$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi}{4} = \frac{\text{num points in circle}}{\text{num points in square}}$$

If you sequentially select points within the square at random, the state of the point being within the circle constitutes a Markov chain. Generate 5 Markov Chains by randomly picking points and calculating the ratio of those that fall within the circle to the total number of points selected, this is $\frac{\pi}{4}$ (you may prefer to multiply the ratio by 4 so it approaches π).

(a) Generate 5 chains, calculate the value of π after a large number of steps (you are free to choose how long this should be and you should state the burn-in for each chain).

(b) Calculate the within and between chain correlations.



[25]