The lt3graph package*†

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Development of this package is organized at github.com/mhelvens/latex-lt3graph. I am happy to receive feedback there!

1 Introduction

This package provides a data-structure for use in the LATEX3 programming environment. It allows you to represent a *directed graph*, which contains *vertices* (nodes), and *edges* (arrows) to connect them. One such a graph is defined below:

```
\texplSyntaxOn
\\graph_new:N \l_my_graph \\
\\graph_put_vertex:Nn \l_my_graph \{v\}
\\graph_put_vertex:Nn \l_my_graph \{x\}
\\graph_put_vertex:Nn \l_my_graph \{z\}
\\graph_put_vertex:Nn \l_my_graph \{z\}
\\graph_put_edge:Nnn \l_my_graph \{z\}
\\graph_put_edge:Nnn \l_my_graph \{v\} \{w\}
\\graph_put_edge:Nnn \l_my_graph \{w\} \{z\}
\\graph_put_edge:Nnn \l_my_graph \{w\} \{z\}
\\graph_put_edge:Nnn \l_my_graph \{w\} \{z\}
\\graph_put_edge:Nnn \l_my_graph \{y\} \{z\}
\\graph_put_edge:Nnn \l_my_graph \{y\} \{z\}
\\graph_put_edge:Nnn \l_my_graph \{z\} \{z\}
```

Each vertex is identified by a key, which, to this library, is a string: a list of characters with category code 12 and spaces with category code 10. An edge is then declared between two vertices by referring to their keys.

We could then, for example, use TikZ to draw this graph:

^{*}This document corresponds to $lt3graph\ v0.1.4$, dated 2014/08/31.

 $^{^{\}dagger}$ The prefix 1t3 indicates that this package is a user-contributed exp13 library, in contrast to packages prefixed with 13, which are officially supported by the LATEX3 team.

¹ Mathematically speaking, a directed graph is a tuple (V, E) with a set of vertices V and a set of edges $E \subseteq V \times V$ connecting those vertices.

Just to be clear, this library is *not about drawing* graphs. It does not, inherently, understand any TikZ. It is about *representing* graphs. This allows us do perform analysis on their structure. We could, for example, determine if there is a cycle in the graph:

```
\ExplSyntaxOn
\graph_if_cyclic:NTF \l_my_graph {Yep} {Nope}
\ExplSyntaxOff
Nope
```

Indeed, there are no cycles in this graph. We can also list its vertices in topological order:

```
\ExplSyntaxOn
     \clist_new:N \LinearClist
     \graph_map_topological_order_inline:Nn \l_my_graph
          { \clist_put_right:Nn \LinearClist {\texttt{#1}} }
\ExplSyntaxOff
Visiting dependencies first: \(\LinearClist \\)
Visiting dependencies first: v, w, y, z, x
```

There is a great deal more that can be done with graphs (some of which is even implemented in this library). A common use-case will be to attach data to vertices and/or edges. You could accomplish this with a property map from 13prop, but this library has already done that for you! Every vertex and every edge can store arbitrary token lists.²

In the next example we store the *degree* (the number of edges, both incoming and outgoing) of each vertex inside that vertex as data. We then query all vertices directly reachable from \mathbf{w} and print their information in the output stream:

²This makes the mathematical representation of our graphs actually a 4-tuple (V, E, v, e), where $v: V \to TL$ is a function that maps every vertex to a token list and $e: E \to TL$ is a function that maps every edge (i.e., pair of vertices) to a token list.

```
\ExplSyntaxOn
    \cs_generate_variant:Nn \graph_put_vertex:Nnn {Nnf}
    \graph_map_vertices_inline:Nn \l_my_graph {
        \graph_put_vertex:Nnf \l_my_graph {#1}
        { \graph_get_degree:Nn \l_my_graph {#1} }
}
\ExplSyntaxOff
```

It's just an additional parameter on the \graph_put_vertex function. Edges can store data in the same way:

```
\ExplSyntaxOn
   \graph_map_edges_inline:Nn \l_my_graph {
      \graph_put_edge:Nnnn \l_my_graph {#1} {#2}
      { \int_eval:n{##1 * ##2} }
}
\ExplSyntaxOff
```

The values ##1 and ##2 represent the data stored in, respectively, vertices #1 and #2. This is a feature of \graph_put_edge:Nnnn added for your convenience.

We can show the resulting graph in a table, which is handy for debugging:

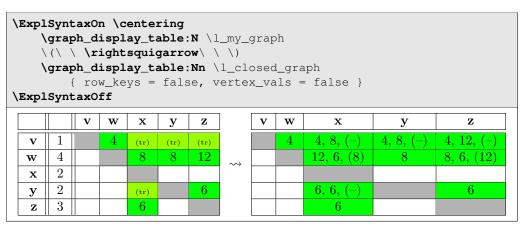
```
\ExplSyntaxOn \centering
     \graph_display_table:N \l_my_graph
\ExplSyntaxOff
                                                                \mathbf{z}
                                    1
                                                   (tr)
                                                         (tr)
                                                                (tr)
                                    4
                                                                12
                              \mathbf{w}
                                    2
                                    2
                                                                6
                               \mathbf{y}
                                                    (tr)
                                    3
                                                    6
```

The **green** cells represent edges directly connecting two vertices. The (tr) cells don't have edges, but indicate that there is a sequence of edges connecting two vertices transitively.

Two vertices can have at most two arrows connecting them: one for each direction. If you want to represent a *multidigraph* (or *quiver*; I'm not making this up), you could consider storing a (pointer to a) list at each edge.

Finally, we demonstrate some transformation functions. The first generates the transitive closure of a graph:

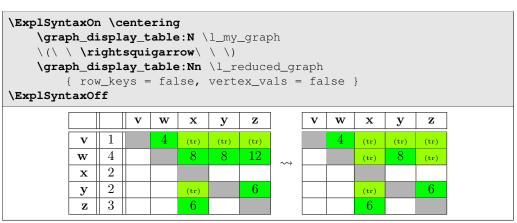
```
\ExplSyntaxOn
   \graph_new:N \l_closed_graph
   \cs_new:Nn \__closure_combiner:nnn { #1,~#2,~(#3) }
   \graph_set_transitive_closure:NNNn
    \l_closed_graph \l_my_graph
   \__closure_combiner:nnn {--}
\ExplSyntaxOff
```



There is a simpler version (\graph_set_transitive_closure:NN) that sets the values of the new edges to the empty token-list. The demonstrated version takes an expandable function to determine the new value, which has access to the values of the two edges being combined (as #1 and #2), as well as the value of the possibly already existing transitive edge (as #3). If there was no transitive edge there already, the value passed as #3 is the fourth argument of the transformation function; in this case --.

The second transformation function generates the transitive reduction:

```
\ExplSyntaxOn
\graph_new:N \l_reduced_graph
\graph_set_transitive_reduction:NN
\l_reduced_graph \l_my_graph
\ExplSyntaxOff
```



2 API Documentation

Sorry! There is no full API documentation yet. But in the meantime, much of the API is integrated in the examples of the previous section, and everything is documented (however sparsely) in the implementation below.

3 Implementation

We now show and explain the entire implementation from lt3graph.sty.

3.1 Package Info

```
1 \NeedsTeXFormat{LaTeX2e}
2 \RequirePackage{expl3}
3 \ProvidesExplPackage{lt3graph}{2014/08/31}{0.1.4}
4 {a LaTeX3 datastructure for representing directed graphs with data}
```

3.2 Required Packages

These are the packages we'll need:

```
5 \RequirePackage{13keys2e}
6 \RequirePackage{xparse}
7 \RequirePackage{withargs}
```

3.3 Additions to LaTeX3 Fundamentals

These are three macros for working with 'set literals' in an expandable context. They use internal macros from 13prop... Something I'm really not supposed to do.

3.4 Data Access

These functions generate the multi-part csnames under which all graph data is stored:

The following functions generate multi-part keys to use in property maps:

```
26 \cs_new:Nn \__graph_key:n { key (#1) }
27 \cs_new:Nn \__graph_key:nn { key (#1) (#2) }
28 \cs_new:Nn \__graph_key:nnn { key (#1) (#2) (#3) }
29 \cs_new:Nn \__graph_key:nnnn { key (#1) (#2) (#3) (#4) }
```

```
30 \cs_new:Nn \__graph_key:nnnnn { key (#1) (#2) (#3) (#4) (#5) }
```

A quick way to iterate through property maps holding graph data:

3.5 Storing data through pointers

The following function embodies a LATEX3 design pattern for representing non-null pointers. This allows data to be 'protected' behind a macro redirection. Any number of expandable operations can be applied to the pointer indiscriminately without altering the data, even when using :x, :o or :f expansion. Expansion using :v dereferences the pointer and returns the data exactly as it was passed through #2. Expansion using :c returns a control sequence through which the data can be modified.

```
\cs_new_protected:Nn \__graph_ptr_new:Nn {
    \withargs [\uniquecsname] {
      \tl_set:Nn #1 {##1}
       \tl_new:c
                      {##1}
      \tl_set:cn
                     {##1} {#2}
41
    }
42
43 }
  \cs_new_protected:Nn \__graph_ptr_gnew:Nn {
44
    \withargs [\uniquecsname] {
45
      \tl_gset:Nn #1 {##1}
46
                       {##1}
47
      \tl_new:c
                       {##1} {#2}
      \tl_gset:cn
48
    }
49
50 }
```

3.6 Creating and initializing graphs

Globally create a new graph:

```
\cs_new_protected:Nn \graph_new:N {
    \graph_if_exist:NTF #1 {
      % TODO: error
54
      \tl_new:N #1
55
      \tl_set:Nf #1 { \tl_trim_spaces:f {\str_tail:n{#1}} }
56
      \int_new:c {\__graph_tl:nnn{graph}{#1}{vertex-count}}
      \__graph_for_each_prop_datatype:n
58
        { \prop_new:c {\__graph_tl:nnn{graph}{#1}{##1}} }
59
60
61 }
62 \cs_generate_variant:Nn \tl_trim_spaces:n {f}
```

Remove all data from a graph:

```
63 \cs_new_protected:Nn \graph_clear:N
64 {\_graph_clear:Nn #1 { } }
65 \cs_new_protected:Nn \graph_gclear:N
66 {\_graph_clear:Nn #1 {g} }
67 \cs_new_protected:Nn \__graph_clear:Nn {
68 \_graph_for_each_prop_datatype:n
69 { \use:c{prop_#2clear:c} {\_graph_tl:nnn{graph}{#1}{##1}} }
70 }
```

Create a new graph if it doesn't already exist, then remove all data from it:

Set all data in graph #1 equal to that in graph #2:

```
80 \cs_new_protected:Nn \graph_set_eq:NN
  { \__graph_set_eq:NNn #1 #2 { } }
82 \cs_new_protected:Nn \graph_gset_eq:NN
   { \__graph_set_eq:NNn #1 #2 {g} }
84 \cs_new_protected:Nn \__graph_set_eq:NNn {
    \use:c{graph_#3clear:N} #1
    \__graph_for_each_prop_datatype:n
86
87
88
        \use:c{prop_#3set_eq:cc}
          {\_graph_tl:nnn{graph}{#1}{##1}}
89
          {\__graph_tl:nnn{graph}{#2}{##1}}
      }
91
92 }
```

An expandable test of whether a graph exists. It does not actually test whether the command sequence contains a graph and is essentially the same as $\cs_{if}=\cc_{$

```
93 \cs_set_eq:NN \graph_if_exist:Np \cs_if_exist:Np
94 \cs_set_eq:NN \graph_if_exist:NT \cs_if_exist:NT
95 \cs_set_eq:NN \graph_if_exist:NF \cs_if_exist:NF
96 \cs_set_eq:NN \graph_if_exist:NTF \cs_if_exist:NTF
```

3.7 Manipulating graphs

Put a new vertex inside a graph:

```
97 \cs_new_protected:Nn \graph_put_vertex:Nn
98 { \__graph_put_vertex:Nnnn #1 {#2} {} {} }
99 \cs_new_protected:Nn \graph_gput_vertex:Nn
100 { \__graph_put_vertex:Nnnn #1 {#2} {} {g} }
101 \cs_new_protected:Nn \graph_put_vertex:Nnn
```

```
{ \__graph_put_vertex:Nnnn #1 {#2} {#3} { } }
103 \cs_new_protected:Nn \graph_gput_vertex:Nnn
    { \__graph_put_vertex:Nnnn #1 {#2} {#3} {g} }
105 \cs_new_protected:Nn \__graph_put_vertex:Nnnn
106
      %%% create pointer to value
108
      \use:c{__graph_ptr_#4new:Nn} \l__graph_vertex_data_tl {#3}
      %%% add the vertex
      \use:c{prop_#4put:cnV} {\__graph_tl:nnn{graph}{#1}{vertices}}
          {#2} \l__graph_vertex_data_tl
      %%% increment the vertex counter
      %
      \use:c{int_#4incr:c} {\__graph_tl:nnn{graph}{#1}{vertex-count}}
      \graph_get_vertex:NnNT #1 {#2} \l_tmpa_tl {
        %%% initialize degree to 0
        \label{lem:con} $$ \sup_{g^2} {\pi^2} {1:nnn{g^2}, $4put:cnn} $$ {\__graph_tl:nnn{g^2}, $4put:cnn} $$ {0}$ $$
124
      }
    }
126
127 \tl_new:N \l__graph_vertex_data_tl
```

Put a new edge inside a graph:

```
128 \cs_new_protected:Nn \graph_put_edge:Nnn
    { \_graph_put_edge:Nnnnn #1 {#2} {#3} {} { } }
130 \cs_new_protected:Nn \graph_gput_edge:Nnn
    { \_graph_put_edge: Nnnnn #1 {#2} {#3} {} {g} }
132 \cs_new_protected:Nn \graph_put_edge:Nnnn
    { \__graph_put_edge:Nnnnn #1 {#2} {#3} {#4} { } }
134 \cs_new_protected:Nn \graph_gput_edge:Nnnn
    { \__graph_put_edge:Nnnnn #1 {#2} {#3} {#4} {g} }
\cs_new_protected:Nn \__graph_put_edge:Nnnnn
       \graph_get_vertex:NnNTF #1 {#2} \l__graph_from_value_tl {
138
         \graph_get_vertex:NnNTF #1 {#3} \l__graph_to_value_tl {
139
           \graph_get_edge:NnnNF #1 {#2} {#3} \l_tmpa_tl {
140
             %%% increment outgoing degree of vertex #2
             \use:c{prop_#5put:cnf} {\__graph_tl:nnn{graph}{#1}{outdegree}} {#2}
               {\int_eval:n {
                   \prop_get:cn {\qraph_tl:nnn{graph}{\#1}{outdegree}} {\#2} + 1 \\
147
             %%% increment incoming degree of vertex #3
148
149
             \label{limin_graph} $$\sup_{graph_tl:nnn\{graph\}{\#1}{indegree}} {\#3}$
               {\int_eval:n {
                   \prop_get:cn {\__graph_tl:nnn{graph}{#1}{indegree}} {#3} + 1
```

```
}}
           }
154
           %%% actually add the edge
156
           \withargs: VVn \l__graph_from_value_tl \l__graph_to_value_tl {
             \use:c{prop_#5put:cox}
               { \__graph_tl:nnn{graph}{#1}{edge-froms}
               { \__graph_key:nn{#2}{#3}
                                                            }
                                                            }
               { \tl_to_str:n{#2}
162
             \use:c{prop_#5put:cox}
163
               { \__graph_tl:nnn{graph}{#1}{edge-tos}
164
               { \__graph_key:nn{#2}{#3}
               { \tl_to_str:n{#3}
                                                            }
166
             \use:c{__graph_ptr_#5new:Nn} \l__graph_edge_data_tl {#4}
             \use:c{prop_#5put:coV}
               { \__graph_tl:nnn{graph}{#1}{edge-values}
               { \__graph_key:nn{#2}{#3}
               \l__graph_edge_data_tl
             \use:c{prop_#5put:cox}
               { \__graph_tl:nnn{graph}{#1}{edge-triples} }
               { \__graph_key:nn{#2}{#3}
               { \\tl_to_str:n{#2}}
                 {\tl_to_str:n{#3}}
                 {\l_graph_edge_data_tl}
                                                            }
179
           % TODO: Error ('to' vertex doesn't exist)
180
         }
181
         % TODO: Error ('from' vertex doesn't exist)
183
      }
184
    }
185
\cs_generate_variant:Nn \prop_gput:Nnn {cox, coV, cnf}
187 \cs_generate_variant:Nn \prop_put:Nnn {cox, coV, cnf}
188 \cs_generate_variant:Nn \withargs:nnn {VVn}
189 \tl_new:N \l__graph_edge_data_tl
190 \tl_new:N \l__graph_from_value_tl
191 \tl_new:N \l__graph_to_value_tl
```

Remove a vertex from a graph, automatically removing any connected edges:

Remove an edge from the graph:

```
222 \cs_new_protected:Nn \graph_remove_edge:Nnn
    { \__graph_remove_edge:Nnnn #1 {#2} {#3} { } }
224 \cs_new_protected:Nn \graph_gremove_edge:Nnn
    { \_graph_remove_edge:Nnnn #1 {#2} {#3} {g} }
226 \cs_new_protected:Nn \__graph_remove_edge:Nnnn {
    \graph_get_edge:NnnNT #1 {#2} {#3} \l__graph_edge_data_tl {
      %%% decrement outdegree of vertex #2
228
      \use:c{prop_#4put:cnf} {\__graph_tl:nnn{graph}{#1}{outdegree}} {#2}
        {\int_eval:n {
            234
      %%% decrement indegree of vertex #3
235
236
      \use:c{prop_#4put:cnf} {\__graph_tl:nnn{graph}{#1}{indegree}} {#3}
238
        {\int_eval:n {
            \prop_get:cn {\qraph_tl:nnn{graph}{\#1}{indegree}} {\#3} - 1
240
241
      %%% actually remove edge
242
243
      \use:c{prop_#4remove:co}
244
        { \__graph_tl:nnn{graph}{#1}{edge-froms}
245
        { \__graph_key:nn{#2}{#3}
246
      \use:c{prop_#4remove:co}
247
        { \__graph_tl:nnn{graph}{#1}{edge-tos}}
                                                   }
248
        { \__graph_key:nn{#2}{#3}
249
      \use:c{prop_#4remove:co}
        { \__graph_tl:nnn{graph}{#1}{edge-values}
251
        { \__graph_key:nn{#2}{#3}
252
      \use:c{prop_#4remove:co}
        { \__graph_tl:nnn{graph}{#1}{edge-triples} }
```

```
255 { \__graph_key:nn{#2}{#3} }
256 }
257 }
258 \cs_generate_variant:Nn \prop_remove:Nn {co}
259 \cs_generate_variant:Nn \prop_gremove:Nn {co}
260 \cs_generate_variant:Nn \prop_put:Nnn {cnf}
261 \cs_generate_variant:Nn \prop_gput:Nnn {cnf}
262 %\tl_new:N \l__graph_edge_data_tl % reusing from other function
```

Add all edges from graph #2 to graph #1, but only between nodes already present in #1:

```
263 \cs_new_protected:Nn \graph_put_edges_from:NN
264 { \__graph_gput_edges_from:NNn #1 #2 { } }
265 \cs_new_protected:Nn \graph_gput_edges_from:NN
266 { \__graph_gput_edges_from:NNn #1 #2 {g} }
267 \cs_new_protected:Nn \__graph_gput_edges_from:NNn
268 {
269 \graph_map_edges_inline:Nn #2 {
270 \graph_if_vertex_exist:NnT #1 {##1} {
271 \graph_if_vertex_exist:NnT #1 {##2} {
272 \use:c{graph_#3put_edge:Nnnn} #1 {##1} {##2} {##3}
273 }
274 }
275 }
276 }
```

3.8 Recovering values from graphs with branching

Test whether a vertex #2 exists. If so, its value is stored in #3 and T is left in the input stream. If it doesn't, F is left in the input stream.

Test whether an edge #2-#3 exists. If so, its value is stored in #4 and T is left in the input stream. If it doesn't, F is left in the input stream.

```
\prg_new_protected_conditional:Nnn \graph_get_edge:NnnN
     {T, F, TF}
     {
286
       \prop_get:coNTF
287
         { \__graph_tl:nnn{graph}{#1}{edge-values} }
288
         {\ \ \ }graph_key:nn{#2}{#3}
         #4
290
         { \tl_set:Nv #4 {#4} \prg_return_true:
291
                                \prg_return_false: }
         {
292
293
    }
```

3.9 Graph Conditionals

An expandable test for the existence of a vertex:

```
\prg_new_conditional:Nnn \graph_if_vertex_exist:Nn
    {p, T, F, TF}
295
     {
296
       \prop_if_in:cnTF
297
         { \__graph_tl:nnn {graph} {#1} {vertices} }
298
         { #2 }
299
         { \prg_return_true: }
301
         { \prg_return_false: }
    }
302
```

An expandable test for the existence of an edge:

Test whether graph #1 contains a cycle reachable from vertex #2:

```
312 \cs_new:Npn \graph_if_vertex_can_reach_cycle_p:Nn #1#2
    { \_graph_if_vertex_can_reach_cycle_p:Nnn #1 {#2} {\_graph_empty_set} }
314 \cs_new:Npn \graph_if_vertex_can_reach_cycle:NnTF #1#2
    { \_graph_if_vertex_can_reach_cycle:NnnTF #1 {#2} {\_graph_empty_set} }
316 \cs_new:Npn \graph_if_vertex_can_reach_cycle:NnT #1#2
    { \_graph_if_vertex_can_reach_cycle:NnnT #1 {#2} {\_graph_empty_set} }
318 \cs_new:Npn \graph_if_vertex_can_reach_cycle:NnF #1#2
    321 \prg_new_conditional:Nnn \__graph_if_vertex_can_reach_cycle:Nnn
    {p, T, F, TF}
    % #1: graph id
    % #2: vertex id
    % #3: visited vertices in 'prop literal' format (internal 13prop)
326
      \graph_map_outgoing_edges_tokens:Nnn #1 {#2}
        { \__graph_if_vertex_can_reach_cycle:Nnnnn #1 {#3} }
      \prg_return_false:
330
332 \cs_new:Nn \__graph_if_vertex_can_reach_cycle:Nnnnn
    % #1: graph id
334
    % #2: visited vertices in 'prop literal' format (internal 13prop)
335
    % #3: start vertex (not used)
    % #4: current vertex
337
    % #5: edge value (behind ptr, not used)
338
```

Test whether graph #1 contains any cycles:

```
348 \prg_new_conditional:Nnn \graph_if_cyclic:N
    {p, T, F, TF}
349
    % #1: graph id
351
       \graph_map_vertices_tokens:Nn #1
352
         { \__graph_if_cyclic:Nnn #1 }
353
       \prg_return_false:
354
    }
355
357 \cs_new:Nn \__graph_if_cyclic:Nnn
    % #1: graph id
    % #2: vertex id
359
    % #3: vertex value (not used)
360
361
362
       \bool_if:nT
         { \graph_if_vertex_can_reach_cycle_p: Nn #1 {#2} }
363
         { \prop_map_break:n {\use_i:nn \prg_return_true:} }
364
365
    }
```

Test whether graph #1 contains any cycles:

```
366 % \prg_new_protected_conditional:Nnn \graph_get_cycle:NN
367 %
       {T, F, TF}
368 %
       % #1: graph id
369 %
       \% #2: 13seq variable to put the cycle description in
370 %
371 %
         \seq_clear:N #2
372 %
         \__graph_get_cycle:NNTF #1 #2
373 %
           {\prg_return_true: }
374 %
           {\prg_return_false:}
375 %
376 %
% \prg_new_protected_conditional:Nnn \__graph_get_cycle:NN
       {T, F, TF}
378 %
379 %
       % #1: graph id
       % #2: 13seq variable
380 %
381 %
382 %
         \graph_map_successors_inline:Nnn #1 {} {
383 %
           \sq_if_in:NnTF #2 {##1} {
             % TODO
384 %
385 %
           }{
             % TODO
386 %
```

```
387 % }
388 % }
389 % }
390 %
```

Assume that graph #1 is acyclic and test whether a path exists from #2 to #3:

```
\prg_new_conditional:Nnn \graph_acyclic_if_path_exist:Nnn
    {p, T, F, TF}
    % #1: graph id
393
    % #2: start vertex
394
    % #3: end vertex
395
396
       \graph_map_outgoing_edges_tokens:Nnn #1 {#2}
397
         { \__graph_acyclic_if_path_exist:Nnnnn #1 {#3} }
398
       \prg_return_false:
400
402 \cs_new:Nn \__graph_acyclic_if_path_exist:Nnnnn
    % #1: graph id
403
    % #2: end vertex
404
    % #3: start vertex (not used)
405
    % #4: possible end vertex
406
    % #5: edge value (behind ptr, do not use)
407
408
       \bool_if:nT
409
410
         {
           \str_if_eq_p:nn {#4} {#2} ||
411
           \graph_acyclic_if_path_exist_p:Nnn #1 {#4} {#2}
412
413
         { \prop_map_break:n {\use_i:nn \prg_return_true:} }
414
    }
```

3.10 Querying Information

Get the number of vertices in the graph:

```
416 \cs_new:Nn \graph_vertex_count:N {
417 \int_use:c {\__graph_tl:nnn{graph}{#1}{vertex-count}}
418 }
```

Get the number of edges leading out of vertex #2:

```
419 \cs_new:Nn \graph_get_outdegree:Nn {
420 \prop_get:cn {\__graph_tl:nnn{graph}{#1}{outdegree}} {#2}
421 }
```

Get the number of edges leading into vertex #2:

```
422 \cs_new:Nn \graph_get_indegree:Nn {
423 \prop_get:cn {\__graph_tl:nnn{graph}{#1}{indegree}} {#2}
424 }
```

Get the number of edges connected to vertex #2:

```
425 \cs_new:Nn \graph_get_degree:Nn {
426 \int_eval:n{ \graph_get_outdegree:Nn #1 {#2} +
427 \graph_get_indegree:Nn #1 {#2} }
428 }
```

3.11 Mapping Graphs

Applies the tokens #2 to all vertex name/value pairs in the graph. The tokens are supplied with two arguments as trailing brace groups.

Applies the function #2 to all vertex name/value pairs in the graph. The function is supplied with two arguments as trailing brace groups.

Applies the inline function #2 to all vertex name/value pairs in the graph. The inline function is supplied with two arguments: '#1' for the name, '#2' for the value.

```
442 \cs_new_protected:Nn \graph_map_vertices_inline:Nn {
443 \withargs (c) [\uniquecsname] [#2] {
444 \cs_set:Npn ##1 ####1###2 {##2}
445 \graph_map_vertices_function:NN #1 ##1
446 }
447 }
```

Applies the tokens #2 to all edge from/to/value triples in the graph. The tokens are supplied with three arguments as trailing brace groups.

```
448 \cs_new:\n \graph_map_edges_tokens:\n {
449 \prop_map_tokens:cn
450 {\__graph_tl:nnn\{graph}\{\frac{#1}\}\{edge-triples\}\}
451 {\__graph_map_edges_tokens_aux:nnn \{\frac{#2}\}\}
452 }
453 \cs_new:\n \__graph_map_edges_tokens_aux:nnn
454 {\__graph_map_edges_tokens_aux:nnn \{\frac{#1}\}\{\frac{#3}\}\{\frac{#4}\}\}
455 \cs_new:\n \__graph_map_edges_tokens_aux:nnnn
456 {\frac{#1}\{\frac{#3}\}\{\frac{#4}\}\}
457 \cs_generate_variant:\n \__graph_map_edges_tokens_aux:nnnn \{\frac{nnnv}\}
```

Applies the function #2 to all edge from/to/value triples in the graph. The function is supplied with three arguments as trailing brace groups.

```
458 \cs_new:\n \graph_map_edges_function:\N\ {
459 \prop_map_tokens:cn
460 {\_graph_tl:nnn{graph}{#1}{edge-triples}}
461 {\_graph_map_edges_function_aux:\nnn #2}
462 }
463 \cs_new:\n\_graph_map_edges_function_aux:\nnn
464 {\_graph_map_edges_function_aux:\nnn #1 #3}
465 \cs_new:\n\_graph_map_edges_function_aux:\nnn
466 {#1 {#2} {#3} {#4}}
467 \cs_generate_variant:\n\_graph_map_edges_function_aux:\nnn {\nnv}
```

Applies the tokens #2 to all edge from/to/value triples in the graph. The tokens are supplied with three arguments: '#1' for the 'from' vertex, '#2' for the 'to' vertex and '#3' for the edge value.

```
468 \cs_new_protected:Nn \graph_map_edges_inline:Nn {
469 \withargs (c) [\uniquecsname] [#2] {
470 \cs_set:Npn ##1 ####1###2###3 {##2}
471 \graph_map_edges_function:NN #1 ##1
472 }
473 }
```

Applies the tokens #3 to the from/to/value triples for the edges going 'to' vertex #2. The tokens are supplied with three arguments as trailing brace groups.

```
474 \cs_new: Nn \graph_map_incoming_edges_tokens: Nnn {
    % #1: graph
    % #2: base vertex
    % #3: tokens to execute
     \prop_map_tokens:cn
       { \__graph_tl:nnn{graph}{#1}{edge-triples} }
       { \_graph_map_incoming_edges_tokens_aux:nnnn {#2} {#3} }
480
481 }
482 \cs_new:Nn \__graph_map_incoming_edges_tokens_aux:nnnn
    % #1: base vertex
483
    % #2: tokens to execute
484
    % #3: edge key
485
    % #4: edge-triple {from}{to}{value}
    { \__graph_map_incoming_edges_tokens_aux:nnnnv {#1} {#2} #4 }
488 \cs_new:Nn \__graph_map_incoming_edges_tokens_aux:nnnnn
    % #1: base vertex
    % #2: tokens to execute
    % #3: edge 'from' vertex
    % #4: edge 'to' vertex
    % #5: edge value
    { \str_if_eq:nnT {#1} {#4} { #2 {#3} {#4} {#5} } }
495 \cs_generate_variant:Nn \__graph_map_incoming_edges_tokens_aux:nnnnn {nnnnv}
```

Applies the function #3 to the from/to/value triples for the edges going 'to' vertex #2. The function is supplied with three arguments as trailing brace groups.

```
496 \cs_new:Nn \graph_map_incoming_edges_function:NnN {
    % #1: graph
    % #2: base vertex
498
    % #3: function to execute
499
    \prop_map_tokens:cn
500
      { \__graph_tl:nnn{graph}{#1}{edge-triples} }
501
       { \__graph_map_incoming_edges_function_aux:nNnn {#2} #3 }
502
503 }
504 \cs_new:Nn \__graph_map_incoming_edges_function_aux:nNnn
    % #1: base vertex
    % #2: function to execute
    % #3: edge key
    % #4: edge-triple {from}{to}{value}
    { \__graph_map_incoming_edges_function_aux:nNnnv {#1} #2 #4 }
510 \cs_new:Nn \__graph_map_incoming_edges_function_aux:nNnnn
    % #1: base vertex
    % #2: function to execute
    % #3: edge 'from' vertex
    % #4: edge 'to' vertex
514
515
   % #5: edge value
    { \str_if_eq:nnT {#1} {#4} { #2 {#3} {#4} {#5} } }
516
517 \cs_generate_variant:Nn \__graph_map_incoming_edges_function_aux:nNnnn {nNnnv}
```

Applies the inline function #3 to the from/to/value triples for the edges going 'to' vertex #2. The inline function is supplied with three arguments: '#1' for the 'from' vertex, '#2' is equal to the #2 supplied to this function and '#3' contains the edge value.

Applies the tokens #3 to the from/to/value triples for the edges going 'from' vertex #2. The tokens are supplied with three arguments as trailing brace groups.

```
527 \cs_new:Nn \graph_map_outgoing_edges_tokens:Nnn {
    % #1: graph
528
529
    % #2: base vertex
    % #3: tokens to execute
530
    \prop_map_tokens:cn
       { \__graph_tl:nnn{graph}{#1}{edge-triples} }
532
533
       { \__graph_map_outgoing_edges_tokens_aux:nnnn {#2} {#3} }
534 }
535 \cs_new:Nn \__graph_map_outgoing_edges_tokens_aux:nnnn
    % #1: base vertex
536
    % #2: tokens to execute
537
    % #3: edge key (not used)
538
    % #4: edge-triple {from}{to}{value}
```

Applies the function #3 to the from/to/value triples for the edges going 'from' vertex #2. The function is supplied with three arguments as trailing brace groups.

```
549 \cs_new: Nn \graph_map_outgoing_edges_function: NnN {
    % #1: graph
    % #2: base vertex
551
    % #3: function to execute
    \prop_map_tokens:cn
      { \__graph_tl:nnn{graph}{#1}{edge-triples} }
       { \__graph_map_outgoing_edges_function_aux:nNnn {#2} #3 }
556 }
557 \cs_new:Nn \__graph_map_outgoing_edges_function_aux:nNnn
    % #1: base vertex
    % #2: function to execute
    % #3: edge key
    % #4: edge-triple {from}{to}{value}
    { \__graph_map_outgoing_edges_function_aux:nNnnv {#1} #2 #4 }
563 \cs_new:Nn \__graph_map_outgoing_edges_function_aux:nNnnn
    % #1: base vertex
    % #2: function to execute
    % #3: edge 'from' vertex
   % #4: edge 'to' vertex
    % #5: edge value
    { \str_if_eq:nnT {#1} {#3} { #2 {#3} {#4} {#5} } }
570 \cs_generate_variant:Nn \__graph_map_outgoing_edges_function_aux:nNnnn {nNnnv}
```

Applies the inline function #3 to the from/to/value triples for the edges going 'from' vertex #2. The inline function is supplied with three arguments: '#1' is equal to the #2 supplied to this function, '#2' contains the 'to' vertex and '#3' contains the edge value.

Applies the tokens #3 to the key/value pairs of the vertices reachable from vertex #2 in one step. The tokens are supplied with two arguments as trailing brace groups.

```
580 \cs_new:Nn \graph_map_successors_tokens:Nnn {
    % #1: graph
    % #2: base vertex
582
    % #3: tokens to execute
583
    \prop_map_tokens:cn
584
      { \__graph_tl:nnn{graph}{#1}{edge-triples} }
585
      { \ \ \ } graph_map_successors_tokens_aux:Nnnnn #1 {#2} {#3} }
586
587 }
588 \cs_new:Nn \__graph_map_successors_tokens_aux:Nnnnn {
    % #1: the graph
    % #2: base vertex
    % #3: tokens to execute
    % #4: edge key (not used)
    % #5: edge-triple {from}{to}{value}
    \__graph_map_successors_tokens_aux:Nnnnnn #1 {#2} {#3} #5
594
595 }
596 \cs_new:Nn \__graph_map_successors_tokens_aux:Nnnnnn {
    % #1: the graph
597
    % #2: base vertex
598
    % #3: tokens to execute
599
   % #4: edge 'from' vertex
600
    % #5: edge 'to' vertex
601
   % #6: ptr to edge value (not used)
   \str_if_eq:nnT {#2} {#4} {
604
      \__graph_map_successors_tokens_aux:nnv
          605
   }
606
607 }
608 \cs_new:Nn \__graph_map_successors_tokens_aux:nnn {
    % #1: tokens to execute
609
    % #2: successor key
    % #3: successor value
611
    #1 {#2} {#3}
613 }
614 \cs_generate_variant:Nn \__graph_map_successors_tokens_aux:nnn {nnv}
```

Applies the function #3 to the key/value pairs of the vertices reachable from vertex #2 in one step. The function is supplied with two arguments as trailing brace groups.

```
615 \cs_new:Nn \graph_map_successors_function:NnN {
   % #1: graph
    % #2: base vertex
617
    % #3: function to execute
618
    \prop_map_tokens:cn
619
       { \_graph_tl:nnn{graph}{#1}{edge-triples} }
620
       { \__graph_map_successors_function_aux:NnNnn #1 {#2} #3 }
621
622 }
623 \cs_new:Nn \__graph_map_successors_function_aux:NnNnn {
624
    % #1: the graph
    % #2: base vertex
625
    % #3: function to execute
    % #4: edge key (not used)
627
    % #5: edge-triple {from}{to}{value}
```

```
\__graph_map_successors_function_aux:NnNnnn #1 {#2} #3 #5
630 }
631 \cs_new:Nn \__graph_map_successors_function_aux:NnNnnn {
     % #1: the graph
632
     % #2: base vertex
633
     % #3: function to execute
634
     % #4: edge 'from' vertex
635
     % #5: edge 'to' vertex
636
     % #6: ptr to edge value (not used)
     \str_if_eq:nnT {#2} {#4} {
638
       \__graph_map_successors_function_aux:Nnv
639
           #3 {#5} {\prop_get:cn{\__graph_tl:nnn{graph}{#1}{vertices}}{#5}}
640
    }
641
642 }
   \cs_new:Nn \__graph_map_successors_function_aux:Nnn {
     % #1: function to execute
     % #2: successor key
     % #3: successor value
647
     #1 {#2} {#3}
648 }
649 \cs_generate_variant:Nn \__graph_map_successors_function_aux:Nnn {Nnv}
```

Applies the inline function #3 to the key/value pairs of the vertices reachable from vertex #2 in one step. The inline function is supplied with two arguments: '#1' is the key, and '#2' is the value of the successor vertex.

Applies the tokens #2 to all vertex name/value pairs in topological order. The tokens are supplied with two arguments as trailing brace groups. Assumes that the graph is acyclic (for now).

```
659 \cs_new_protected:Nn \graph_map_topological_order_tokens:Nn {
660
    %% Fill \l__graph_source_vertices with source-nodes and count indegrees
661
662
     \prop_gclear_new:c {1__graph_source_vertices_(\int_use:N\g__graph_nesting_depth_int)_prop}
663
     \prop_gclear_new:c {l__graph_tmp_indeg_(\int_use:N\g__graph_nesting_depth_int)_prop}
664
     \graph_map_vertices_inline:Nn #1 {
665
       \prop_put:cnf {l__graph_tmp_indeg_(\int_use:N\g__graph_nesting_depth_int)_prop} {##1}
666
           { \graph_get_indegree: Nn #1 {##1} }
667
       \int_compare:nT {\graph_get_indegree:Nn #1 {##1} = 0} {
668
         \prop_put:cnn {1__graph_source_vertices_(\int_use:N\g__graph_nesting_depth_int)_prop}
669
    } }
670
671
    %%% Main loop
672
```

```
\bool_until_do:nn {\prop_if_empty_p:c {l__graph_source_vertices_(\int_use:N\g__graph_nesting)}
              \hfill 
676
                    _graph_prop_any_key_pop:cN
                       {l__graph_source_vertices_(\int_use:N\g__graph_nesting_depth_int)_prop}
                       \l_graph_topo_key_tl
              \graph_get_vertex:NVNT #1 \l__graph_topo_key_tl \l__graph_topo_val_tl {
681
                   %%% Deduct one from the counter of all affected nodes
682
                   %%% and add all now-empty vertices to source_vertices
683
                   %
                   \graph_map_outgoing_edges_inline:NVn #1 \l__graph_topo_key_tl {
                       \prop_put:cnf {1__graph_tmp_indeg_(\int_use:N\g__graph_nesting_depth_int)_prop} {##2
                            {\int_eval:n {\prop_get:cn {l__graph_tmp_indeg_(\int_use:N\g__graph_hesting_depth_
                       \int_compare:nT {\prop_get:cn {l__graph_tmp_indeg_(\int_use:N\g__graph_nesting_depth
                            \prop_put:cnn {l__graph_source_vertices_(\int_use:N\g__graph_nesting_depth_int)_pr
                   } }
690
                   %%% Run the mapping funtion on the key and value from that vertex
                   %/// and manage the nesting depth counter
                   %
694
                   \int_gincr:N \g__graph_nesting_depth_int
695
                   \withargs:VVn \l__graph_topo_key_tl \l__graph_topo_val_tl
696
                       { #2 {##1} {##2} }
697
698
                   \int_gdecr:N \g__graph_nesting_depth_int
699
700 } }
701 \cs_new_protected:Nn \__graph_prop_any_key_pop:NN {
          \prop_map_inline:Nn #1 {
703
              \tl_set:Nn #2 {##1}
704
              \prop_remove:Nn #1 {##1}
               \prop_map_break:n {\use_none:nnn}
706
          \tl_set:Nn #2 {\q_no_value} % TODO: test
708 }
709 \cs_generate_variant:Nn \__graph_prop_any_key_pop:NN
                                                                                                                                           {cN}
710 \cs_generate_variant:Nn \withargs:nnn
                                                                                                                                           \{VVn\}
711 \cs_generate_variant:Nn \graph_map_outgoing_edges_inline:Nnn {NVn}
712 \cs_generate_variant:Nn \prop_put:Nnn
                                                                                                                                           {cnf}
713 \cs_generate_variant:Nn \graph_get_vertex:NnNT
                                                                                                                                           {NVNT}
714 \tl_new:N \l__graph_topo_key_tl
715 \tl_new:N \l__graph_topo_val_tl
716 \int_new:N \g__graph_nesting_depth_int
```

Applies the function #2 to all vertex name/value pairs in topological order. The function is supplied with two arguments as trailing brace groups. Assumes that the graph is acyclic (for now).

```
717 \cs_new:Nn \graph_map_topological_order_function:NN {
718 \graph_map_topological_order_tokens:Nn #1 {#2}
719 }
```

Applies the inline function #2 to all vertex name/value pairs in topological order. The inline function is supplied with two arguments: '#1' for the name and '#2' for the value. Assumes that the graph is acyclic (for now).

```
720 \cs_new_protected:Nn \graph_map_topological_order_inline:Nn {
721 \withargs (c) [\uniquecsname] [#2] {
722 \cs_set:Npn ##1 ####1###2 {##2}
723 \graph_map_topological_order_function:NN #1 ##1
724 } }
```

3.12 Transforming Graphs

Set graph #1 to the transitive closure of graph #2.

```
725 \cs_new_protected:Nn \graph_set_transitive_closure:NN {
    \__graph_set_transitive_closure:NNNnn #1 #2 \use_none:nnn {} { }
726
727 }
  \cs_new_protected:Nn \graph_gset_transitive_closure:NN {
     \__graph_set_transitive_closure:NNNnn #1 #2 \use_none:nnn {} {g}
729
730 }
   \cs_new_protected:Nn \graph_set_transitive_closure:NNNn {
     \__graph_set_transitive_closure:NNNnn #1 #2 #3 {#4} { }
732
733 }
   \cs_new_protected:Nn \graph_gset_transitive_closure:NNNn {
     \__graph_set_transitive_closure:NNNnn #1 #2 #3 {#4} {g}
735
736 }
737 \cs_new_protected:Nn \__graph_set_transitive_closure:NNNnn {
738
    % #1: target
    % #2: source
739
    % #3: combination function with argspec :nnn
741
    % #4: default 'old' value
    \use:c{graph_#5set_eq:NN} #1 #2
742
744
     \cs_set:Nn \__graph_edge_combinator:nnn {
       \exp_not:n { #3 {##1} {##2} {##3} } }
745
     \cs_generate_variant:Nn \__graph_edge_combinator:nnn {VVV}
746
747
     \graph_map_vertices_inline:Nn #2 {
748
749
       \graph_map_vertices_inline:Nn #2 {
750
         \graph_get_edge:NnnNT #2 {##1} {###1}
             \l__graph_edge_value_i_tl {
           \graph_map_vertices_inline:Nn #2 {
             \graph_get_edge:NnnNT #2 {####1} {#######1}
                 \l__graph_edge_value_ii_tl {
754
               \graph_get_edge:NnnNF #1 {##1} {#######1}
                   \l__graph_edge_value_old_tl {
756
                 \tl_set:Nn \l__graph_edge_value_old_tl {#4}
               }
758
               \exp_args:NNx \tl_set:No \l__graph_edge_value_new_tl {
760
                 \__graph_edge_combinator:VVV
761
                   \l_graph_edge_value_i_tl
762
                   \l_graph_edge_value_ii_tl
763
                   \l_graph_edge_value_old_tl
764
```

Assume that graph #2 contains no cycles, and set graph #1 to its transitive reduction.

```
775 \cs_new_protected:Nn \graph_set_transitive_reduction:NN {
    \__graph_set_transitive_reduction:NNn #1 #2 { } }
  \cs_new_protected:Nn \graph_gset_transitive_reduction:NN {
    \__graph_set_transitive_reduction:NNn #1 #2 {g} }
779 \cs_new_protected:Nn \__graph_set_transitive_reduction:NNn {
    % #1: target
    % #2: source
781
    \use:c{graph_#3set_eq:NN} #1 #2
782
    \graph_map_vertices_inline:Nn #2 {
       \graph_map_vertices_inline:Nn #2 {
784
         \graph_get_edge:NnnNT #2 {##1} {###1} \l_tmpa_tl {
785
           \graph_map_vertices_inline:Nn #2 {
786
             \graph_get_edge:NnnNT #2 {####1} {#######1} \l_tmpa_tl {
787
               \use:c{graph_#3remove_edge:Nnn} #1 {##1} {######1}
    789
790 }
```

3.13 Displaying Graphs

We define some additional functions that can display the graph in table-form. This is the option-less version, which delegates to the full version:

```
791 \cs_new_protected:Nn \graph_display_table:N {
792 \graph_display_table:Nn #1 {} }
```

The full version has a second argument accepting options that determine table formatting. We first define those options. Please note that with the standard options, the xcolor package is required with the table option, because of our use of the \cellcolor command.

```
\keys_define:nn {lt3graph-display} {
    row_keys .bool_set:N = \l__graph_display_row_keys_bool,
794
    row_keys .initial:n
                            = {true},
795
    row_keys .default:n
                           = {true},
796
797
    vertex_vals .bool_set:N = \l__graph_display_vertex_vals_bool,
798
                               = {true},
799
    vertex_vals .initial:n
    vertex_vals .default:n
                               = {true},
800
801
    row_keys_format
                           .tl_set:N = \l__graph_format_row_keys_tl,
802
    row_keys_format
                           .initial:n = \textbf,
```

```
row_keys_format
                           .value_required:,
804
805
    col_keys_format
                           .tl_set:N = \l__graph_format_col_keys_tl,
806
                           .initial:n = \textbf,
    col_keys_format
807
    col_keys_format
                           .value_required:,
808
809
    vertex_vals_format
                           .tl_set:N = \l__graph_format_vertex_vals_tl,
810
    vertex_vals_format
                           .initial:n = \use:n,
811
    vertex_vals_format
                           .value_required:,
812
813
                           .tl_set:N = \l__graph_format_edge_vals_tl,
814
    edge_vals_format
    edge_vals_format
                           .initial:n = \use:n,
815
    edge_vals_format
                           .value_required:,
816
    edge_diagonal_format .tl_set:N = \l__graph_format_edge_diagonal_tl,
     edge_diagonal_format .initial:n = \cellcolor{black!30!white},
     edge_diagonal_format .value_required:,
                           .tl_set:N = \l__graph_format_edge_direct_tl,
822
    edge_direct_format
                           .initial:n = \cellcolor{green},
823
    edge_direct_format
    edge_direct_format
824
                           .value_required:,
826
    \verb|edge_transitive_format.tl_set:N| = \label{eq:ll_graph_format_edge_transitive_tl},
    edge_transitive_format .initial:n = \cellcolor{green!40!yellow}\tiny(tr),
827
828
    edge_transitive_format .value_required:,
829
    edge_none_format
                           .tl_set:N = \l__graph_format_edge_none_tl,
830
    edge_none_format
                           .initial:n = \{\},
831
    edge_none_format
                           .value_required:
832
833 }
```

Now we define the function itself. It displays a table showing the structure and content of graph #1. If argument #2 is passed, its options are applied to format the output.

```
834 \cs_new_protected:Nn \graph_display_table:Nn {
835 \group_begin:
```

We process those options passed with #2:

```
%keys_set:nn {lt3graph-display} {#2}
```

We populate the top row of the table:

```
\tl_put_right:Nn \l__graph_table_content_tl {\hline}
     \seq_clear:N \l__graph_row_seq
838
    \bool_if:NT \l__graph_display_row_keys_bool
839
         { \seq_put_right: Nn \l__graph_row_seq {}
840
           \tl_put_right:Nn \l__graph_table_colspec_tl {|r|} }
841
842
     \bool_if:NT \l__graph_display_vertex_vals_bool
843
         { \seq_put_right: Nn \l_graph_row_seq {}
           \tl_put_right:Nn \l__graph_table_colspec_tl {|c|} }
844
     \graph_map_vertices_inline:Nn #1 {
845
       \tl_put_right:Nn \l__graph_table_colspec_tl {|c}
846
       \seq_put_right:Nn \l__graph_row_seq
847
```

We populate the remaining rows:

```
\text{graph_map_vertices_inline:Nn #1 {} \text{seq_clear:N \l__graph_row_seq} \text{bool_if:NT \l__graph_display_row_keys_bool {} \text{seq_put_right:Nn \l__graph_row_seq} \text{\l__graph_format_row_keys_t1 \left\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\right\rig
```

We start building the vertex cell value. First we distinguish between a direct connection, a transitive connection, and no connection, and format accordingly:

```
\graph_get_edge:NnnNTF #1 {##1} \l_tmpa_tl {
           \quark_if_no_value:VF \l_tmpa_tl {
865
            \tl_set_eq:NN \l__graph_cell_content_tl \l_tmpa_tl
866
             \tl_set:Nf \l__graph_cell_content_tl
867
                 { \exp_args:NV \l__graph_format_edge_direct_tl
869
                                \l_graph_cell_content_tl } }
        }{\graph_acyclic_if_path_exist:NnnTF #1 {##1} {###1} {
870
           \tl_set_eq:NN \l__graph_cell_content_tl
              \l__graph_format_edge_transitive_tl
        }{
           \tl_set_eq:NN \l__graph_cell_content_tl
               \l_graph_format_edge_none_tl
        }}
```

Secondary formatting comes from cells on the diagonal, i.e., a key compared to itself:

```
\str_if_eq:nnT {##1} {####1} {

***Total transfer to the strain of the strain of
```

Tertiary formatting is applied to all vertex value cells:

We can now add the cell to the row sequence:

```
\seq_put_right:NV \l__graph_row_seq \l__graph_cell_content_tl

885 }
```

We are finished with this row; go on to the next iteration:

Finally, we print the table itself:

```
890 \withargs:VVn \l__graph_table_colspec_tl \l__graph_table_content_tl
891 { \begin{tabular}{##1}##2\end{tabular} }

892 \group_end:
893 }
```

Now follow the local variants and variables used in the function:

```
\cs_generate_variant:Nn \quark_if_no_value:nF {VF}

ss \cs_generate_variant:Nn \withargs:nnn {VVn}

tl_new:N \l_graph_table_colspec_tl

tl_new:N \l_graph_table_content_tl

ktl_new:N \l_graph_cell_content_tl

ss \tool_new:N \l_graph_table_skipfirst_bool

seq_new:N \l_graph_row_seq
```

Change History

```
0.0.1
                                            0.1.1
                                               General: fixed a similar bug in
   General: initial version . . . . . . . . . . . . . 1
                                                   \graph_(g)put_edges_from .... 1
0.0.8
                                            0.1.2
   General: a great many untracked
                                               General: allowing \graph_map_topo-
      changes .....
                                                   logical_order_... to be nested .
0.0.9
                                            0.1.3
   General: creation of the documentation 1
                                               General: fixed a bug in \graph_re-
0.1.0
                                                   move_vertex and added a \graph_-
   General: fixed a bug in
                                                   vertex_count:N function .....
                                            0.1.4
       \graph_(g)put_vertex and
                                               General: no longer loading individual
       \graph_(g)put_edge, which caused
      their global versions not to work
                                                  l3kernel packages, which leads to an
      properly .....
                                                   error for recent versions of expl3 . 1
```

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