In[ ]:=
```
J_m_[x_] = BesselJ[m, x]
```

Out[ ]=

$$J_m(x)$$

In[ ]:=
```
j_m_,n_ = BesselJZero[m, n]
```

Out[ ]=

$$j_{m,n}$$

In[ ]:=
```
IP[u_, v_] := Integrate[x u v, {x, 0, 1}]
```

Bessel function of order zero

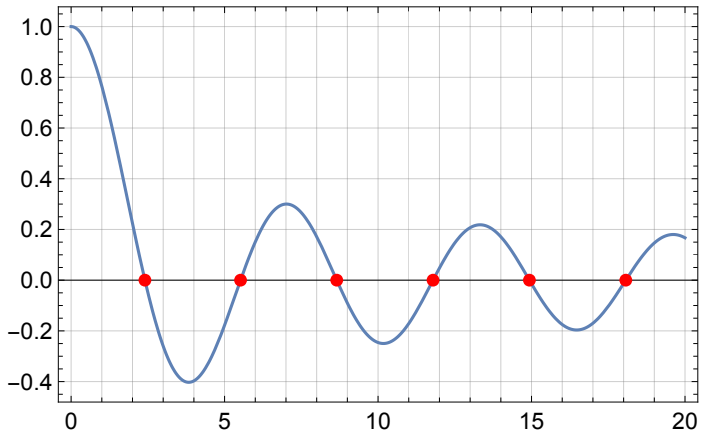
In[ ]:=
```
Series[J_0[x], {x, 0, 10}]
```

Out[ ]=

$$1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \frac{x^8}{147\,456} - \frac{x^{10}}{14\,745\,600} + O(x^{11})$$

In[ ]:=
```
Plot[J_0[x], {x, 0, 20}, GridLines → {Table[i, {i, 0, 20}], Automatic},
 Epilog → {PointSize[0.02], Red, Table[Point[{BesselJZero[0, n], 0}], {n, 1, 6}]}]
```

Out[ ]=



In[ ]:=
```
Table[{n, j_0,n, N[j_0,n]}, {n, 1, 6}]
```

Out[ ]=

$$\begin{pmatrix} 1 & j_{0,1} & 2.40482\,55576\,9577 \\ 2 & j_{0,2} & 5.52007\,81102\,8631 \\ 3 & j_{0,3} & 8.65372\,79129\,1101 \\ 4 & j_{0,4} & 11.79153\,44390\,143 \\ 5 & j_{0,5} & 14.93091\,77084\,878 \\ 6 & j_{0,6} & 18.07106\,39679\,109 \end{pmatrix}$$

In[ ]:=
```
IP[J_0[j_0,n x], J_0[j_0,n x]]
```

Out[ ]=

$$\frac{1}{2}\left(J_0(j_{0,n})^2 + J_1(j_{0,n})^2\right)$$

Bessel function of different orders

In[ ]:=
```
Series[J1[x], {x, 0, 10}]
```

Out[ ]=

$$\frac{x}{2}-\frac{x^3}{16}+\frac{x^5}{384}-\frac{x^7}{18\,432}+\frac{x^9}{1\,474\,560}+O(x^{11})$$

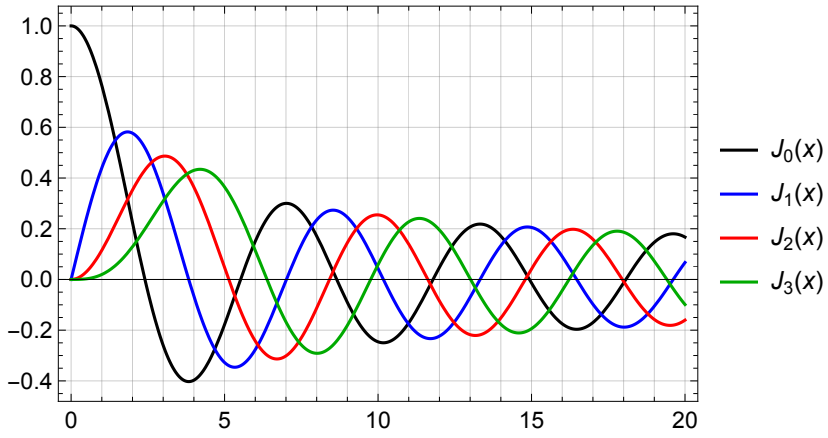
In[ ]:=
```
Series[J2[x], {x, 0, 10}]
```

Out[ ]=

$$\frac{x^2}{8}-\frac{x^4}{96}+\frac{x^6}{3072}-\frac{x^8}{184\,320}+\frac{x^{10}}{17\,694\,720}+O(x^{11})$$

In[ ]:=
```
Plot[{J0[x], J1[x], J2[x], J3[x]}, {x, 0, 20},
 GridLines → {Table[i, {i, 0, 20}], Automatic},
 PlotStyle → {Black, Blue, Red, Darker[Green]}, PlotLegends → "Expressions"]
```

Out[ ]=



In[ ]:=
```
Table[{n, j1,n, N[j1,n]}, {n, 1, 6}]
```

Out[ ]=

$$\begin{pmatrix} 1 & j_{1,1} & 3.83170\,59702\,0751 \\ 2 & j_{1,2} & 7.01558\,66698\,1562 \\ 3 & j_{1,3} & 10.17346\,81350\,627 \\ 4 & j_{1,4} & 13.32369\,19363\,142 \\ 5 & j_{1,5} & 16.47063\,00508\,776 \\ 6 & j_{1,6} & 19.61585\,85104\,682 \end{pmatrix}$$

In[ ]:=
```
Table[{n, j2,n, N[j2,n]}, {n, 1, 6}]
```

Out[ ]=

$$\begin{pmatrix} 1 & j_{2,1} & 5.13562\,23018\,4068 \\ 2 & j_{2,2} & 8.41724\,41403\,9983 \\ 3 & j_{2,3} & 11.61984\,11721\,491 \\ 4 & j_{2,4} & 14.79595\,17823\,513 \\ 5 & j_{2,5} & 17.95981\,94949\,878 \\ 6 & j_{2,6} & 21.11699\,70530\,218 \end{pmatrix}$$

In[ ]:=
```
Assuming[{m ∈ Integers, n ∈ Integers}, IP[Jm[jm,n x], Jm[jm,n x]]]
```

Out[ ]=

$$\frac{1}{2}\left(J_m(j_{m,n})^2 - \frac{2\,m\,J_{m+1}(j_{m,n})\,J_m(j_{m,n})}{j_{m,n}} + J_{m+1}(j_{m,n})^2\right) \text{ if } m > -1$$

Standing waves on a disk

In[ ]:=
```
vm_,n_[r_, θ_] = Jm[jm,n r] Cos[m θ]
```

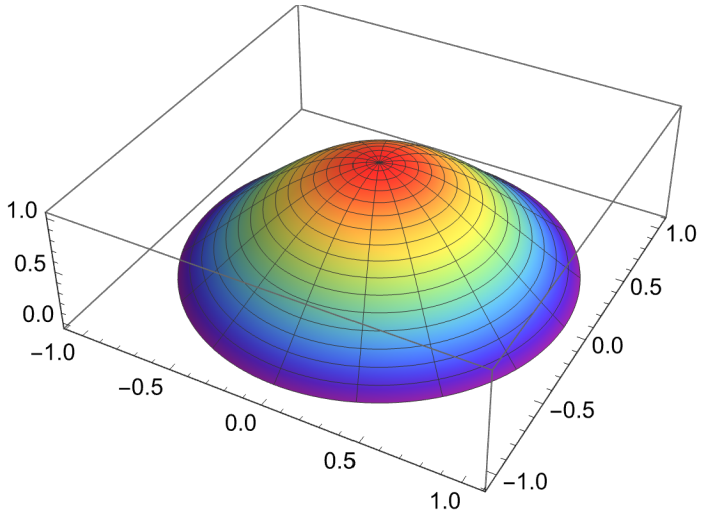
Out[ ]=

$$\cos(\theta\, m)\, J_m(r\, j_{m,n})$$

In[ ]:=
```
showMode[m_, n_] := Module[{k = N[jm,n]},
  RevolutionPlot3D[Jm[k r] Cos[m θ], {r, 0, 1}, {θ, 0, 2 Pi}, PlotPoints → 128]]
```

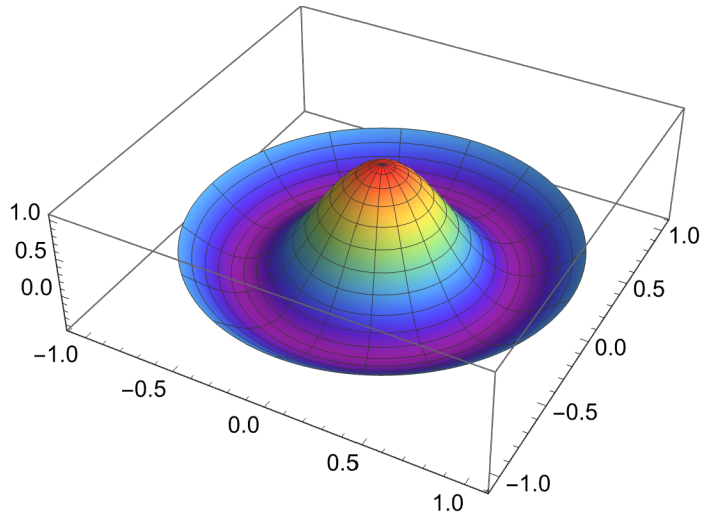
In[ ]:=
```
showMode[0, 1]
```

Out[ ]=



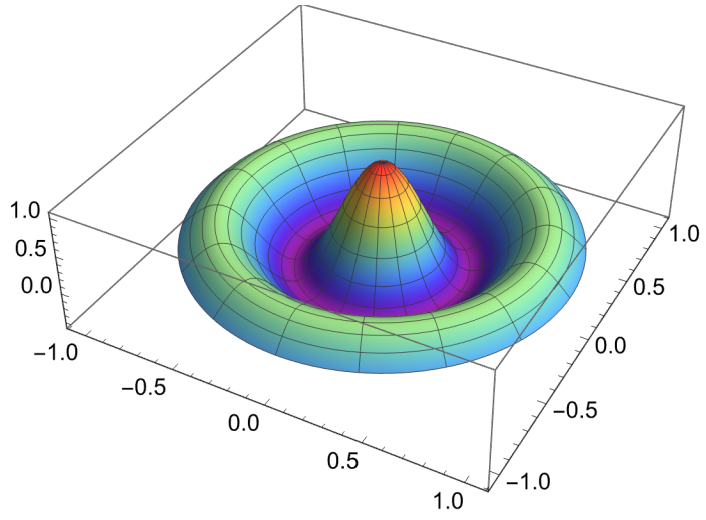
In[ ]:=
```
showMode[0, 2]
```

Out[ ]=



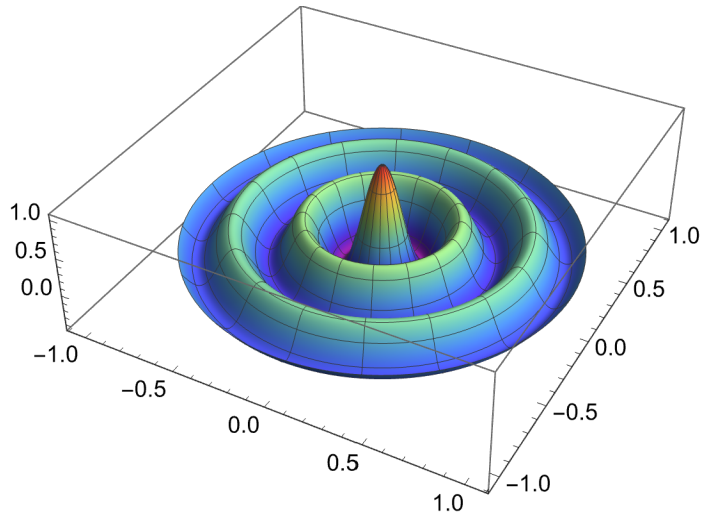
In[ ]:=
```
showMode[0, 3]
```

Out[ ]=



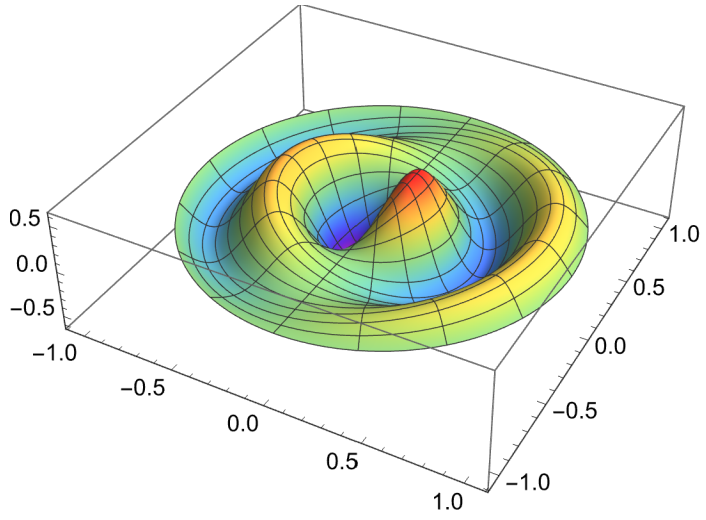
In[ ]:=
```
showMode[0, 6]
```

Out[ ]=



In[ ]:=
```
showMode[1, 3]
```

Out[ ]=



In[ ]:= showMode[2, 5]

Out[ ]=