Solution of three-tank hydraulic flow

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See comments in the SimpleDESystem notebook for some information on typesetting and syntax.

$$\begin{aligned} & \text{In}[\bullet] \coloneqq & \mathsf{A} = \{\{-3, 1, 1\}, \{1, -3, 1\}, \{1, 1, -3\}\} \\ & \text{Out}[\bullet] = \\ & \begin{pmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{pmatrix} \\ & \text{In}[\bullet] \coloneqq & \mathsf{u}_0 = \{3, 7, 8\} \\ & \text{Out}[\bullet] = \\ & \{3, 7, 8\} \\ & \text{In}[\bullet] \coloneqq & \mathsf{A} = \mathsf{DiagonalMatrix}[\mathsf{Eigenvalues}[\mathsf{A}]] \\ & \text{Out}[\bullet] = \\ & \begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\ & \text{In}[\bullet] \coloneqq & \mathsf{V} = \mathsf{Transpose}[\mathsf{Eigenvectors}[\mathsf{A}]] \\ & \text{Out}[\bullet] = \\ & \begin{pmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \\ & \text{In}[\bullet] \coloneqq & \mathsf{c} = \mathsf{LinearSolve}[\mathsf{V}, \mathsf{u}_0] \\ & \text{Out}[\bullet] = \\ & \{2, 1, 6\} \\ & \text{In}[\bullet] \coloneqq & \mathsf{decoupledSoln}[\mathsf{t}_{-}] = \mathsf{MatrixExp}[\mathsf{A} \, \mathsf{t}] \cdot \mathsf{c} \\ & \text{Out}[\bullet] = \\ & \{2 \, e^{-4t}, \, e^{-4t}, \, 6 \, e^{-t}\} \\ & \text{In}[\bullet] \coloneqq & \mathsf{uSoln}[\mathsf{t}_{-}] = \mathsf{V} \cdot \mathsf{decoupledSoln}[\mathsf{t}] \\ & \text{Out}[\bullet] \coloneqq & \{6 \, e^{-t} - 3 \, e^{-4t}, \, e^{-4t} + 6 \, e^{-t}, \, 2 \, e^{-4t} + 6 \, e^{-t}\} \end{aligned}$$