Solving a differential equation

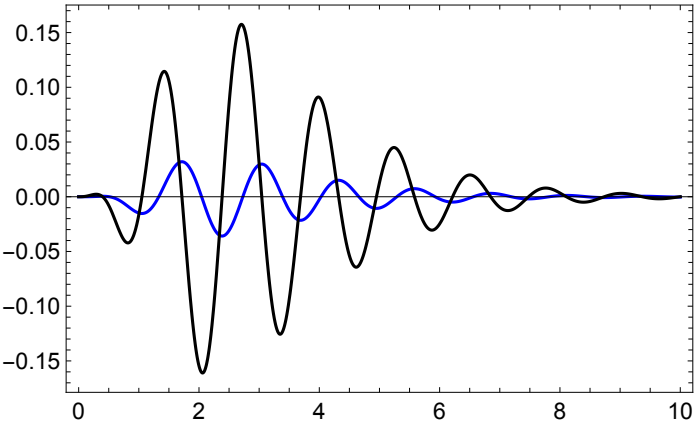
You learned how to do this using undetermined coefficients, variation of parameters, or Laplace transforms.

In[10]:=
```
ySoln[x_] = y[x] /. DSolve[{y''[x] + 2 y'[x] + 10 y[x] == x^2 Cos[5 x] Exp[-x],
      y[0] == 0, y'[0] == 0}, y[x], x][[1]] // FullSimplify
```

Out[10]=
$$\frac{1}{512} e^{-x} \left(\left(21 - 32 x^2\right) \cos(5 x) + 40 x \sin(5 x) - 21 \cos(3 x)\right)$$

In[12]:=
```
Plot[{ySoln[x], ySoln'[x]}, {x, 0, 10}, PlotRange → All, PlotStyle → {Blue, Black}]
```

Out[12]=



Solving linear systems

In[13]:=
```
A = {
   {2, -1, 0, 0, 0},
   {-1, 2, -1, 0, 0},
   {0, -1, 2, -1, 0},
   {0, 0, -1, 2, -1},
   {0, 0, 0, -1, 2}
  }
```

Out[13]=
$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

In[14]:=
```
b = {1, 2, 3, 4, 5}
```

Out[14]=
$$\{1, 2, 3, 4, 5\}$$

In[15]:=
```
LinearSolve[A, b]
```

Out[15]=
$$\left\{\frac{35}{6}, \frac{32}{3}, \frac{27}{2}, \frac{40}{3}, \frac{55}{6}\right\}$$

In[18]:=
```
B = RandomReal[{-1, 1}, {10, 10}]
```

Out[18]=
$$\begin{pmatrix} 0.01912\,94935\,63041\,9 & 0.42458\,96898\,44071 & -0.18092\,35680\,9172 & -0.46039\,87374\,99277 & 0.41532\,41055\,90274 \\ -0.43160\,49173\,81921 & 0.86509\,88327\,67469 & -0.51533\,78476\,88586 & 0.78746\,57932\,73818 & -0.27878\,74680\,64031 \\ -0.96746\,56641\,45901 & -0.13616\,69322\,05986 & -0.76158\,12636\,63769 & -0.16776\,75469\,23135 & 0.24272\,64492\,17943 \\ -0.38943\,18988\,91357 & -0.36481\,26947\,9691 & -0.51502\,41356\,9462 & -0.55298\,72425\,76496 & -0.22833\,65954\,93027 \\ -0.60078\,88589\,66566 & -0.29143\,34334\,43444 & -0.61659\,50766\,59185 & 0.54180\,14325\,50917 & 0.27949\,32000\,83026 \\ -0.51927\,55337\,89392 & 0.63625\,13382\,01858 & -0.68493\,40566\,83386 & -0.18136\,45408\,89575 & -0.75281\,14619\,79563 \\ -0.26984\,72375\,72571 & 0.67822\,49876\,05532 & 0.69720\,51690\,0387 & 0.64745\,02706\,14208 & 0.03574\,11594\,40330\,1 \\ 0.85261\,72509\,54195 & 0.53255\,70731\,56394 & -0.53264\,94025\,44978 & -0.80218\,22148\,98634 & -0.91185\,96242\,3432 \\ 0.98422\,37267\,23386 & -0.17809\,17368\,31074 & 0.80581\,07544\,82661 & -0.87680\,63235\,09168 & 0.62635\,10743\,40454 \\ 0.72861\,97628\,05054 & 0.34093\,18098\,08625 & 0.93675\,04080\,51036 & -0.29419\,58140\,53457 & 0.73980\,40321\,3892 \end{pmatrix}$$

In[19]:=
```
c = RandomReal[{-1, 1}, {10}]
```

Out[19]=
{0.72028 66183 94532, −0.77725 63955 15219, 0.98636 58480 03012, −0.72402 26303 51553, −0.66734 20123 12819, 0.07122 00590 41145 6, 0.80346 39411 94462, −0.73145 86122 44181, −0.42309 01395 46273, −0.05179 46994 14045 5}

In[20]:=
```
LinearSolve[B, c]
```

Out[20]=
{−8.12513 31754 305, 2.96390 78021 5032, 18.50940 64836 347, −3.39650 12789 8684, −7.24297 32635 6559, −9.02753 67231 4851, −2.16829 10926 6269, 4.55605 52460 9266, −2.12236 72096 1578, −5.12202 33203 3847}

Finding eigenvalues

In[22]:=
```
Eigenvalues[A]
```

Out[22]=
$$\left\{2+\sqrt{3}, 3, 2, 1, 2-\sqrt{3}\right\}$$

In[21]:=
```
Eigenvalues[B]
```

Out[21]=
{1.14787 56875 4886 + 1.33798 27912 5406 i, 1.14787 56875 4886 − 1.33798 27912 5406 i, −0.09390 28865 51515 1 + 1.75105 28434 7483 i, −0.09390 28865 51515 1 − 1.75105 28434 7483 i, −1.43677 34050 378 + 0.48665 40324 66812 i, −1.43677 34050 378 − 0.48665 40324 66812 i, 0.93107 06427 93684 + 0. i, 0.50799 97380 86135 + 0. i, 0.11344 99132 46648 + 0.02906 94492 35812 2 i, 0.11344 99132 46648 − 0.02906 94492 35812 2 i}

Solving a system of differential equations

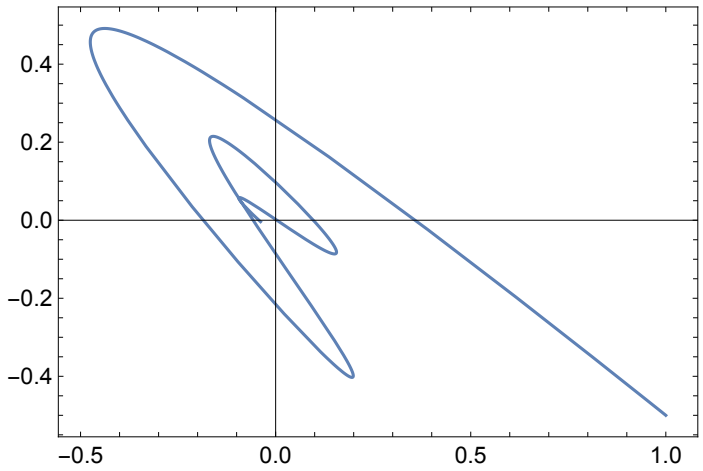
Equations for two coupled oscillators with damping (two connected spring-mass systems or LRC circuits)

In[34]:=
```
{uSoln[t_], vSoln[t_]} = {u[t], v[t]} /. DSolve[
      {u''[t] == -2 u[t] + v[t] - 1/2 u'[t],
       v''[t] == u[t] - 2 v[t] - 1/2 v'[t],
       u[0] == 1, v[0] == -1/2, u'[0] == 0, v'[0] == 0}, {u[t], v[t]}, t][[1]] //
  FullSimplify
```

Out[34]=
$$\left\{\frac{1}{2820}e^{-t/4}\left(47\sqrt{15}\sin\left(\frac{\sqrt{15}\,t}{4}\right)+45\sqrt{47}\sin\left(\frac{\sqrt{47}\,t}{4}\right)+705\cos\left(\frac{\sqrt{15}\,t}{4}\right)+2115\cos\left(\frac{\sqrt{47}\,t}{4}\right)\right),\right.$$
$$\left.\frac{e^{-t/4}\left(47\sqrt{15}\sin\left(\frac{\sqrt{15}\,t}{4}\right)+705\cos\left(\frac{\sqrt{15}\,t}{4}\right)-45\left(\sqrt{47}\sin\left(\frac{\sqrt{47}\,t}{4}\right)+47\cos\left(\frac{\sqrt{47}\,t}{4}\right)\right)\right)}{2820}\right\}$$

In[35]:=
```
ParametricPlot[{uSoln[t], vSoln[t]}, {t, 0, 10}, PlotRange → All]
```

Out[35]=



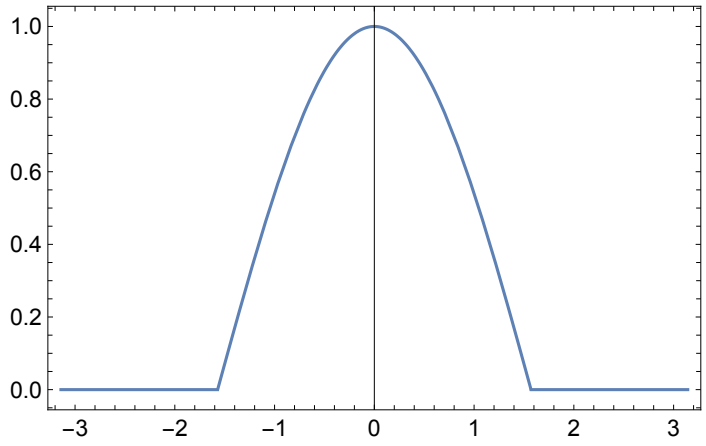
Fourier series for rectified cosine

In[36]:=
```
f[t_] = Piecewise[{{Cos[t], Abs[t] < Pi/2}}]
```

Out[36]=
$$\begin{cases}\cos(t) & |t|<\frac{\pi}{2}\\ 0 & \text{True}\end{cases}$$

In[37]:=
```
Plot[f[t], {t, -Pi, Pi}]
```

Out[37]=



In[38]:=
```
A_n_ = 1 / Pi Integrate[f[t] Cos[n t], {t, -Pi, Pi}]
```

Out[38]=
$$-\frac{2\cos\left(\frac{\pi n}{2}\right)}{\pi\left(n^2-1\right)}$$

In[40]:=
```
A1 = Limit[An, n → 1]
```

Out[40]=
$$\frac{1}{2}$$

In[41]:=
```
fSum[M_, t_] := A0 / 2 + Sum[An Cos[n t], {n, 1, M}]
```

In[45]:=
```
f4[t_] = fSum[4, t]
```

Out[45]=
$$\frac{\cos(t)}{2}+\frac{2\cos(2t)}{3\pi}-\frac{2\cos(4t)}{15\pi}+\frac{1}{\pi}$$

In[42]:=
```
f8[t_] = fSum[8, t]
```

Out[42]=
$$\frac{\cos(t)}{2}+\frac{2\cos(2t)}{3\pi}-\frac{2\cos(4t)}{15\pi}+\frac{2\cos(6t)}{35\pi}-\frac{2\cos(8t)}{63\pi}+\frac{1}{\pi}$$

In[46]:=
```
Plot[{f[t], f4[t], f8[t]}, {t, -Pi, Pi}, PlotStyle → {Red, Black, Blue}]
```

Out[46]=