Vectors and matrices with Mathematica

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You can find more on this topic at

`http://reference.wolfram.com/language/tutorial/LinearAlgebra.html`

Vectors

- ### Write vectors componentwise in curly braces, separated by commas

```
In[69]:= a = {1, 2, 3}
Out[69]= {1, 2, 3}
```

```
In[70]:= b = {-1, 0, 2}
Out[70]= {-1, 0, 2}
```

- ### Vector operations

Addition / subtraction

```
In[71]:= a + b
Out[71]= {0, 2, 5}
```

```
In[72]:= a - b
Out[72]= {2, 2, 1}
```

Multiplication by scalars

```
In[73]:= 2 a
Out[73]= {2, 4, 6}
```

```
In[74]:= 2 a + 3 b
Out[74]= {-1, 4, 12}
```

- ### Dot product

Use period, or Dot function

```
In[75]:= a.b
Out[75]= 5
```

In[76]:=
```
Dot[a, b]
```
Out[76]= 5

- ### Elementwise multiplication

Ordinary multiplication notation does elementwise product

In[77]:=
```
a b
```
Out[77]= $\{-1, 0, 6\}$

Be careful not to do this when you mean to compute a dot product!

- ### Cross product

The Cross function does cross products

In[78]:=
```
Cross[a, b]
```
Out[78]= $\{4, -5, 2\}$

- ### Vectors can be defined symbolically

In[79]:=
```
Remove[x, y]
```

In[80]:=
```
p = {x, y}
```
Out[80]= $\{x, y\}$

In[81]:=
```
q = {s, t}
```
Out[81]= $\{s, t\}$

In[82]:=
```
p.q
```
Out[82]= $s\,x + t\,y$

In[83]:=
```
p q
```
Out[83]= $\{s\,x, t\,y\}$

In[84]:=
```
h
```
Out[84]= h

In[85]:=
```
p + q
```
Out[85]= $\{s + x, t + y\}$

Matrices

- ### Forming matrices

Use curly braces

In[86]:=
```
B = {{1, 2}, {3, 4}, {5, 6}}
```
Out[86]=

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

In[87]:=
```
Transpose[B]
```
Out[87]=

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

- ## Matrix-vector multiplication

Use the dot operator

In[88]:=
```
{{1, 2}, {3, 4}, {5, 6}}.{x, y} // MatrixForm
```
Out[88]//MatrixForm=

$$\begin{pmatrix} x + 2\,y \\ 3\,x + 4\,y \\ 5\,x + 6\,y \end{pmatrix}$$

No dot --> elementwise (identical to “dot-star” in matlab)

In[89]:=
```
{{1, 2}, {3, 4}, {5, 6}} {x, y, z} // MatrixForm
```
Out[89]//MatrixForm=

$$\begin{pmatrix} x & 2\,x \\ 3\,y & 4\,y \\ 5\,z & 6\,z \end{pmatrix}$$

- ## Matrix-matrix multiplication

In[90]:=
```
AA = {{1, 2}, {3, 4}, {5, 6}}
```
Out[90]=

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

In[91]:=
```
BB = {{10, 20}, {30, 40}}
```
Out[91]=

$$\begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix}$$

You can multiply a 3×2 times a 2×2 matrix

In[92]:=
```
AA.BB
```
Out[92]=

$$\begin{pmatrix} 70 & 100 \\ 150 & 220 \\ 230 & 340 \end{pmatrix}$$

A 2×2 can’t be multiplied into a 3×2 matrix.

In[93]:=
```
BB.AA
```

Dot: Tensors $\begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ have incompatible shapes.

Out[93]= $\begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix} . \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$

Null space

If the null space is trivial, NullSpace returns an empty list. Be sure to remember that the zero vector is *always* a member of the null space.

In[94]:=
```
AA = {{1, 1}, {1, 2}};
```

In[95]:=
```
NullSpace[AA]
```

Out[95]= {}

Let's look at an example with a nontrivial null space

In[96]:=
```
BB = {{1, 2, 3}, {0, 0, 0}, {0, 0, 0}}
```

Out[96]= $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

The result is returned a list of basis vectors for the null space, which looks like a matrix with the basis vectors in its rows.

In[97]:=
```
NullSpace[BB]
```

Out[97]= $\begin{pmatrix} -3 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}$

It's usually better to put the basis vectors into columns; we can do that using the transpose

In[98]:=
```
vNull = Transpose[NullSpace[BB]]
```

Out[98]= $\begin{pmatrix} -3 & -2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$

Sure enough, the matrix times the basis vectors of the null space is zero.

In[99]:=
```
BB.vNull
```

Out[99]= $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

In[100]:=

Solving linear systems

Solving $A\,x = b$

The LinearSolve function

In[101]:=
```
AA = {{1, 2}, {2, 3}}
```

Out[101]= $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

In[102]:=
```
b = {4, 5}
```

Out[102]= $\{4, 5\}$

Warning: Mathematica makes no distinction between row vectors and column vectors

Solve AA.x = b; expect solution {-2,3}

In[103]:=
```
LinearSolve[AA, b]
```

Out[103]= $\{-2, 3\}$

Solution is as expected

In[104]:=
```
Solve[{x1 + 2 x2 == 4, 2 x1 + 3 x2 == 5}, {x1, x2}]
```

Out[104]= $\{\{x_1 \to -2, x_2 \to 3\}\}$

Another example:

In[105]:=
```
AA = Table[i j / (1 + i + j), {i, 1, 5}, {j, 1, 5}]
```

Out[105]= $\begin{pmatrix} \frac{1}{3} & \frac{1}{2} & \frac{3}{5} & \frac{2}{3} & \frac{5}{7} \\ \frac{1}{2} & \frac{4}{5} & 1 & \frac{8}{7} & \frac{5}{4} \\ \frac{3}{5} & 1 & \frac{9}{7} & \frac{3}{2} & \frac{5}{3} \\ \frac{2}{3} & \frac{8}{7} & \frac{3}{2} & \frac{16}{9} & 2 \\ \frac{5}{7} & \frac{5}{4} & \frac{5}{3} & 2 & \frac{25}{11} \end{pmatrix}$

In[106]:=
```
b = Table[i^2, {i, 1, 5}]
```

Out[106]= $\{1, 4, 9, 16, 25\}$

In[107]:=
```
LinearSolve[AA, b]
```

Out[107]= $\{3465, -17\,920, 39\,060, -37\,800, 13\,398\}$

You can solve systems with irrational numbers. It gets ugly.

In[108]:=
```
AA = {{E, Pi, Sqrt[2]}, {1, 2, 3}, {(Sqrt[5] - 1) / 2, Sqrt[Pi], E^2}}
```

Out[108]= $\begin{pmatrix} e & \pi & \sqrt{2} \\ 1 & 2 & 3 \\ \frac{1}{2}\left(\sqrt{5} - 1\right) & \sqrt{\pi} & e^2 \end{pmatrix}$

In[109]:=
```
b = {Pi / 2, Sqrt[E], 4}
```

Out[109]= $\left\{\frac{\pi}{2}, \sqrt{e}, 4\right\}$

In[110]:=
```
LinearSolve[AA, b]
```

Out[110]=
$$\left\{\frac{-16\sqrt{2}+24\pi+2e^2\pi-2e^{5/2}\pi-3\pi^{3/2}+2\sqrt{2e\pi}}{2\sqrt{2}-2\sqrt{10}+4e^3-6e\sqrt{\pi}-3\pi+3\sqrt{5}\pi-2e^2\pi+2\sqrt{2\pi}},\right.$$
$$-\frac{16\sqrt{2}-48e+4e^{7/2}+2\sqrt{2e}-2\sqrt{10e}-3\pi+3\sqrt{5}\pi-2e^2\pi}{2\left(-2\sqrt{2}+2\sqrt{10}-4e^3+6e\sqrt{\pi}+3\pi-3\sqrt{5}\pi+2e^2\pi-2\sqrt{2\pi}\right)},$$
$$\left.\frac{-16e+2e^{3/2}\sqrt{\pi}+7\pi+\sqrt{5}\pi+\sqrt{e}\pi-\sqrt{5e}\pi-\pi^{3/2}}{-2\sqrt{2}+2\sqrt{10}-4e^3+6e\sqrt{\pi}+3\pi-3\sqrt{5}\pi+2e^2\pi-2\sqrt{2\pi}}\right\}$$

It’s usually best to do such problems numerically

In[111]:=
```
LinearSolve[AA, N[b]]
```

Out[111]= {0.78206 55403 26523, −0.43825 30614 39899, 0.58105 39510 84468}

• Numerical solution of $A\,\boldsymbol{x} = \boldsymbol{b}$

In[112]:=
```
A = RandomReal[{-10, 10}, {12, 12}]
```

Out[112]=
$$\begin{pmatrix}
-1.34372\,62677\,5443 & 2.89067\,62547\,4937 & -9.41271\,51215\,9376 & -4.50619\,21987\,1491 & -8.00386\,53846\,7917 \\
-2.36168\,30230\,0114 & 6.69696\,55444\,2878 & -1.30593\,87555\,85 & 1.49247\,90405\,1853 & 5.76277\,01796\,2013 \\
-6.14008\,01473\,9593 & -8.19964\,42286\,9736 & -2.06240\,04241\,7062 & 5.97965\,04051\,5953 & 5.24176\,64645\,3157 \\
-3.35600\,03886\,7499 & 7.50184\,46550\,7626 & 4.66220\,12885\,402 & 0.26109\,05238\,51863 & -7.12402\,88148\,3439 \\
-2.06811\,21983\,6309 & 2.99235\,30212\,9841 & 9.86318\,84603\,6531 & 4.17478\,20494\,7599 & 6.62043\,39204\,107 \\
-4.60969\,91745\,3867 & 2.55923\,39928\,9253 & 3.56055\,08949\,5233 & -9.83364\,28317\,5849 & 0.81436\,80321\,0865 \\
6.88867\,94023\,6507 & -7.65893\,62700\,8978 & -2.90158\,85324\,1057 & 3.30858\,67331\,6889 & 3.79630\,16832\,3344 \\
-5.48566\,01390\,2177 & 3.95189\,48236\,2962 & -8.19058\,99014\,529 & 8.15169\,95234\,5717 & 1.76808\,81720\,3494 \\
-7.55378\,25684\,0836 & 6.30689\,66235\,9898 & -3.78620\,64825\,0433 & -1.55624\,93544\,517 & 0.70031\,16979\,18327 \\
2.35354\,65815\,5476 & -4.46314\,61122\,5816 & 0.02712\,17973\,26902\,5 & -0.19246\,97889\,62026 & -8.74969\,80412\,8707 \\
-9.67550\,33520\,3912 & 8.57047\,34811\,1266 & -6.11686\,55456\,2294 & 7.07301\,89696\,5726 & 8.72029\,07910\,1267 \\
-1.00392\,10651\,1547 & -6.66403\,67988\,7003 & 7.67299\,32247\,8333 & 0.04322\,23595\,04838\,4 & 9.47777\,67655\,5461
\end{pmatrix}$$

In[113]:=
```
xAns = RandomReal[{-10, 10}, {12}]
```

Out[113]= {7.58258 29447 661, 5.71539 61935 3119, 4.98248 92923 5825, 3.33237 84218 9978, 4.16966 02432 4643, −8.01997 18407 9859, −4.94315 98354 1537, 7.42365 53784 5707, 3.23057 81416 1994, 4.67032 05176 367, −8.94065 79349 8743, −0.15510 83495 74782}

Multiply $A\,\boldsymbol{x}$ to produce a RHS $\boldsymbol{b}$

In[114]:=
```
b = A.xAns
```

Out[114]= {−78.49186 48054 42, 23.28747 43074 561, 2.03918 06320 4008, 49.33283 71181 868, 81.25889 16595 625, 48.16002 06775 677, 28.95982 26981 529, −50.92160 76135 332, 34.57814 14422 957, 149.81962 01988 4, −10.70323 57981 998, 122.33634 73263 76}

Solve the system

In[115]:=
```
xNum = LinearSolve[A, b]
```

Out[115]= {7.58258 29447 6611, 5.71539 61935 3121, 4.98248 92923 5825, 3.33237 84218 9978, 4.16966 02432 4643, −8.01997 18407 9859, −4.94315 98354 1536, 7.42365 53784 5708, 3.23057 81416 1993, 4.67032 05176 367, −8.94065 79349 8742, −0.15510 83495 74784}

In most cases the solution will be very accurate

In[116]:=
```
err = Norm[xNum - xAns]
```
Out[116]= $2.50821963135958\times10^{-14}$

Even for a big matrix the error in solving a system can be small.

(When forming large objects, put semicolons at the end of the line to suppress output)

In[117]:=
```
ABig = RandomReal[{-1, 1}, {5000, 5000}];
```
In[118]:=
```
xBig = RandomReal[{-1, 1}, {5000}];
```
In[119]:=
```
bBig = ABig.xBig;
```
The solution is fast

In[120]:=
```
{time, xNum} = AbsoluteTiming[ LinearSolve[ABig, bBig]];
```
In[121]:=
```
time
```
Out[121]= 0.588444

The error is reasonably small

In[122]:=
```
err = Norm[xBig - xNum]
```
Out[122]= $1.311160766859 07\times10^{-10}$

However, for pathological cases the error can be very bad. The Hilbert matrix is a classic example of a matrix that's tricky to solve accurately.

In[123]:=
```
H = HilbertMatrix[12]
```
Out[123]=

$$\begin{pmatrix}
1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \\
\frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} \\
\frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} \\
\frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} \\
\frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} \\
\frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} \\
\frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} \\
\frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} \\
\frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} \\
\frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} \\
\frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} \\
\frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23}
\end{pmatrix}$$

In[124]:=
```
b = H.xAns
```
Out[124]= {12.6535391156 56, 7.35535644069861, 5.27862599636049, 4.13536653246269, 3.40433448782165, 2.89451766910101, 2.51797421522879, 2.22820665524493, 1.99820294345535, 1.81115411306826, 1.65603493486689, 1.52530559431433}

In[125]:=
```
xNum = LinearSolve[H, b]
```

LinearSolve: Result for LinearSolve of badly conditioned matrix (≪1≫) may contain significant numerical errors.

Out[125]= {7.582582767408 25, 5.715419350312 23, 4.981744069496 39, 3.342720810144 65, 4.092689710035 81, −7.677500820266 31, −5.907584221807 08, 9.185388032701 15, 1.148781825673 41, 6.205556826469 31, −9.582863117375 15, −0.038771907520 4665}

The error is huge!

In[126]:=
```
err = Norm[xNum - xAns]
```

Out[126]= 3.357666275101 52

- ## Inverse matrices

You can compute symbolic inverses

In[127]:=
```
AInv = Inverse[AA]
```

Out[127]=
$$\begin{pmatrix} \frac{2e^2-3\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\sqrt{2\pi}-e^2\pi}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{3\pi-2\sqrt{2}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \\ \frac{-\frac{3}{2}+\frac{3\sqrt{5}}{2}-e^2}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\frac{1}{\sqrt{2}}-\sqrt{\frac{5}{2}}+e^3}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\sqrt{2}-3e}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \\ \frac{1-\sqrt{5}+\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{-e\sqrt{\pi}-\frac{\pi}{2}+\frac{\sqrt{5}\pi}{2}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{2e-\pi}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \end{pmatrix}$$

Solve $A\,x = b$ for x by computing $x = A^{-1}\,b$.

In[128]:=
```
b = {Sqrt[2], Pi/8, Sqrt[Pi]}
```

Out[128]= $\left\{\sqrt{2}, \frac{\pi}{8}, \sqrt{\pi}\right\}$

In[129]:= `AInv . b`

Out[129]=
$$\left\{\frac{\sqrt{2}\left(2e^2-3\sqrt{\pi}\right)}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi}+\frac{\sqrt{\pi}\left(3\pi-2\sqrt{2}\right)}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi}+\frac{\pi\left(\sqrt{2}\pi-e^2\pi\right)}{8\left(\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi\right)},\right.$$
$$\frac{\sqrt{2}\left(-\frac{3}{2}+\frac{3\sqrt{5}}{2}-e^2\right)}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi}+\frac{\left(\sqrt{2}-3e\right)\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi}+\frac{\left(\frac{1}{\sqrt{2}}-\sqrt{\frac{5}{2}}+e^3\right)\pi}{8\left(\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi\right)},$$
$$\left.\frac{\sqrt{2}\left(1-\sqrt{5}+\sqrt{\pi}\right)}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi}+\frac{(2e-\pi)\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi}+\frac{\pi\left(-e\sqrt{\pi}-\frac{\pi}{2}+\frac{\sqrt{5}\pi}{2}\right)}{8\left(\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2}\pi\right)}\right\}$$

You probably want to simplify the result

In[130]:= `FullSimplify[AInv.b]`

Out[130]=
$$\left\{\frac{-\left(24+\sqrt{2}\right)\pi^{3/2}+40\sqrt{2}\pi+e^2\left(\pi^2-16\sqrt{2}\right)}{4\left(-4e^3+2\sqrt{2}\left(-1+\sqrt{5}-\sqrt{\pi}\right)+6e\sqrt{\pi}-3\left(\sqrt{5}-1\right)\pi+2e^2\pi\right)},\right.$$
$$\frac{8\sqrt{2}\left(3-3\sqrt{5}+2e^2\right)-16\left(\sqrt{2}-3e\right)\sqrt{\pi}+\sqrt{2}\left(\sqrt{5}-1\right)\pi-2e^3\pi}{8\left(-4e^3+2\sqrt{2}\left(-1+\sqrt{5}-\sqrt{\pi}\right)+6e\sqrt{\pi}-3\left(\sqrt{5}-1\right)\pi+2e^2\pi\right)},$$
$$\left.\frac{16\sqrt{2}\left(\sqrt{5}-1\right)-16\left(\sqrt{2}+2e\right)\sqrt{\pi}+2(8+e)\pi^{3/2}-\left(\sqrt{5}-1\right)\pi^2}{8\left(-4e^3+2\sqrt{2}\left(-1+\sqrt{5}-\sqrt{\pi}\right)+6e\sqrt{\pi}-3\left(\sqrt{5}-1\right)\pi+2e^2\pi\right)}\right\}$$

Determinants

The Det[ ] function computes the determinant of a square matrix

In[131]:=
```
Remove[a, b, c, d]
```

In[132]:=
```
Det[{{a, b}, {c, d}}]
```

Out[132]= $a\,d - b\,c$

In[133]:=
```
Det[{{a, b, c}, {d, e, f}, {h, i, j}}]
```

Out[133]= $a\,e\,j - a\,f\,i - b\,d\,j + b\,f\,h + c\,d\,i - c\,e\,h$

In[134]:=
```
Det[AA]
```

Out[134]= $\sqrt{2} - \sqrt{10} + 2\,e^3 - 3\,e\,\sqrt{\pi} - \frac{3\,\pi}{2} + \frac{3\,\sqrt{5}\,\pi}{2} - e^2\,\pi + \sqrt{2\,\pi}$

In[135]:=
```
Det[{{-3 - λ, 1}, {1, -3 - λ}}]
```

Out[135]= $\lambda^2 + 6\,\lambda + 8$