Vectors and matrices with Mathematica

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You can find more on this topic at

`http://reference.wolfram.com/language/tutorial/LinearAlgebra.html`

Vectors

- ### Write vectors componentwise in curly braces, separated by commas

```
In[173]:= a = {1, 2, 3}
Out[173]= {1, 2, 3}
```

```
In[174]:= b = {-1, 0, 2}
Out[174]= {-1, 0, 2}
```

- ### Vector operations

Addition / subtraction

```
In[175]:= a + b
Out[175]= {0, 2, 5}
```

```
In[176]:= a - b
Out[176]= {2, 2, 1}
```

Multiplication by scalars

```
In[177]:= 2 a
Out[177]= {2, 4, 6}
```

```
In[178]:= 2 a + 3 b
Out[178]= {-1, 4, 12}
```

- ### Dot product

Use period, or Dot function

```
In[179]:= a.b
Out[179]= 5
```

In[180]:=
```
Dot[a, b]
```
Out[180]= 5

- ## Elementwise multiplication

Ordinary multiplication notation does elementwise product

In[181]:=
```
a b
```
Out[181]= $\{-1, 0, 6\}$

Be careful not to do this when you mean to compute a dot product!

- ## Cross product

The Cross function does cross products

In[182]:=
```
Cross[a, b]
```
Out[182]= $\{4, -5, 2\}$

- ## Vectors can be defined symbolically

In[183]:=
```
p = {x, y}
```
Out[183]= $\{\{-9.53660\,17378\,022, 6.75490\,29926\,0373, 1.00171\,33373\,4028, -4.62094\,92329\,7804, 3.87456\,18803\,7764, -1.29870\,34704\,7836, 1.26094\,58860\,0506, -5.96829\,80913\,8979\}, y\}$

In[184]:=
```
q = {s, t}
```
Out[184]= $\{s, t\}$

In[185]:=
```
p.q
```
••• Dot: Nonrectangular tensor encountered.

Out[185]= $\{\{-9.53660\,17378\,022, 6.75490\,29926\,0373, 1.00171\,33373\,4028, -4.62094\,92329\,7804, 3.87456\,18803\,7764, -1.29870\,34704\,7836, 1.26094\,58860\,0506, -5.96829\,80913\,8979\}, y\}.\{s, t\}$

In[186]:=
```
p q
```
Out[186]= $\{\{-9.53660\,17378\,022\,s, 6.75490\,29926\,0373\,s, 1.00171\,33373\,4028\,s, -4.62094\,92329\,7804\,s, 3.87456\,18803\,7764\,s, -1.29870\,34704\,7836\,s, 1.26094\,58860\,0506\,s, -5.96829\,80913\,8979\,s\}, t\,y\}$

In[187]:=
```
h
```
Out[187]= h

In[188]:=
```
p + q
```
Out[188]= $\{\{s - 9.53660\,17378\,022, s + 6.75490\,29926\,0373, s + 1.00171\,33373\,4028, s - 4.62094\,92329\,7804, s + 3.87456\,18803\,7764, s - 1.29870\,34704\,7836, s + 1.26094\,58860\,0506, s - 5.96829\,80913\,8979\}, t + y\}$

Matrices

Forming matrices

Use curly braces

In[189]:=
```
B = {{1, 2}, {3, 4}, {5, 6}}
```

Out[189]=
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

In[190]:=
```
Transpose[B]
```

Out[190]=
$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

Matrix-vector multiplication

Use the dot operator

In[191]:=
```
{{1, 2}, {3, 4}, {5, 6}}.{x, y} // MatrixForm
```

Dot: Nonrectangular tensor encountered.

Out[191]//MatrixForm=
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}.\{\{-9.53660\,17378\,022,\ 6.75490\,29926\,0373,\ 1.00171\,33373\,4028,\ -4.62094\,92329\,7804,\ 3.87456\,18803\,7764,\ -1.29870\,34704\,7836,\ 1.26094\,58860\,0506,\ -5.96829\,80913\,8979\},\ y\}$$

No dot --> elementwise (identical to “dot-star” in matlab)

In[192]:=
```
{{1, 2}, {3, 4}, {5, 6}} {x, y, z} // MatrixForm
```

Thread: Objects of unequal length in
{1, 2} {−9.5366017378022, 6.75490299260373, 1.00171333734028, −4.62094923297804, 3.87456188037764, −1.29870347047836, 1.26094588600506, −5.96829809138979} cannot be combined.

Out[192]//MatrixForm=
$$\begin{pmatrix} \{1, 2\}\,\{-9.53660\,17378\,022,\ 6.75490\,29926\,0373,\ 1.00171\,33373\,4028,\ -4.62094\,92329\,7804,\ 3.87456\,18803\,7764,\ -1.29 \\ \{3\,y, 4\,y\} \\ \{5\,z, 6\,z\} \end{pmatrix}$$

Matrix-matrix multiplication

In[193]:=
```
AA = {{1, 2}, {3, 4}, {5, 6}}
```

Out[193]=
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

In[194]:=
```
BB = {{10, 20}, {30, 40}}
```

Out[194]=
$$\begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix}$$

You can multiply a 3×2 times a 2×2 matrix

In[195]:=
```
AA.BB
```
Out[195]=
$$\begin{pmatrix} 70 & 100 \\ 150 & 220 \\ 230 & 340 \end{pmatrix}$$

A 2×2 can't be multiplied into a 3×2 matrix.

In[196]:=
```
BB.AA
```
Dot: Tensors $\begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ have incompatible shapes.

Out[196]=
$$\begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix}.\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

- ### Null space

If the null space is trivial, NullSpace returns an empty list. Be sure to remember that the zero vector is *always* a member of the null space.

In[197]:=
```
AA = {{1, 1}, {1, 2}};
```
In[198]:=
```
NullSpace[AA]
```
Out[198]= {}

In[199]:=
```
BB = {{1, 2, 3}, {0, 0, 0}, {0, 0, 0}}
```
Out[199]=
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[200]:=
```
vNull = Transpose[NullSpace[BB]]
```
Out[200]=
$$\begin{pmatrix} -3 & -2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In[201]:=
```
BB.vNull
```
Out[201]=
$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[202]:=

Solving linear systems

Solving $A x = b$

• The LinearSolve function

In[203]:=
```
AA = {{1, 2}, {2, 3}}
```
Out[203]=
$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

In[204]:=
```
b = {4, 5}
```
Out[204]= $\{4, 5\}$

Warning: Mathematica makes no distinction between row vectors and column vectors

Solve AA.x = b; expect solution {-2,3}

In[205]:=
```
LinearSolve[AA, b]
```
Out[205]= $\{-2, 3\}$

Solution is as expected

In[206]:=
```
Solve[{x1 + 2 x2 == 4, 2 x1 + 3 x2 == 5}, {x1, x2}]
```
Out[206]=
$$\{\{\{-9.53660\,17378\,022, 6.75490\,29926\,0373, 1.00171\,33373\,4028, -4.62094\,92329\,7804, 3.87456\,18803\,7764, -1.29870\,34704\,7836, 1.26094\,58860\,0506, -5.96829\,80913\,8979\}_1 \to -2, \{-9.53660\,17378\,022, 6.75490\,29926\,0373, 1.00171\,33373\,4028, -4.62094\,92329\,7804, 3.87456\,18803\,7764, -1.29870\,34704\,7836, 1.26094\,58860\,0506, -5.96829\,80913\,8979\}_2 \to 3\}\}$$

Another example:

In[207]:=
```
AA = Table[i j / (1 + i + j), {i, 1, 5}, {j, 1, 5}]
```
Out[207]=
$$\begin{pmatrix} \frac{1}{3} & \frac{1}{2} & \frac{3}{5} & \frac{2}{3} & \frac{5}{7} \\ \frac{1}{2} & \frac{4}{5} & 1 & \frac{8}{7} & \frac{5}{4} \\ \frac{3}{5} & 1 & \frac{9}{7} & \frac{3}{2} & \frac{5}{3} \\ \frac{2}{3} & \frac{8}{7} & \frac{3}{2} & \frac{16}{9} & 2 \\ \frac{5}{7} & \frac{5}{4} & \frac{5}{3} & 2 & \frac{25}{11} \end{pmatrix}$$

In[208]:=
```
b = Table[i^2, {i, 1, 5}]
```
Out[208]= $\{1, 4, 9, 16, 25\}$

In[209]:=
```
LinearSolve[AA, b]
```
Out[209]= $\{3465, -17\,920, 39\,060, -37\,800, 13\,398\}$

You can solve systems with irrational numbers. It gets ugly.

In[210]:=
```
AA = {{E, Pi, Sqrt[2]}, {1, 2, 3}, {(Sqrt[5] - 1) / 2, Sqrt[Pi], E^2}}
```
Out[210]=
$$\begin{pmatrix} e & \pi & \sqrt{2} \\ 1 & 2 & 3 \\ \frac{1}{2}\left(\sqrt{5}-1\right) & \sqrt{\pi} & e^2 \end{pmatrix}$$

In[211]:=
```
b = {Pi/2, Sqrt[E], 4}
```

Out[211]= $\left\{\frac{\pi}{2}, \sqrt{e}, 4\right\}$

In[212]:=
```
LinearSolve[AA, b]
```

Out[212]=
$$\left\{\frac{-16\sqrt{2}+24\pi+2e^2\pi-2e^{5/2}\pi-3\pi^{3/2}+2\sqrt{2e\pi}}{2\sqrt{2}-2\sqrt{10}+4e^3-6e\sqrt{\pi}-3\pi+3\sqrt{5}\pi-2e^2\pi+2\sqrt{2\pi}},\right.$$
$$-\frac{16\sqrt{2}-48e+4e^{7/2}+2\sqrt{2e}-2\sqrt{10e}-3\pi+3\sqrt{5}\pi-2e^2\pi}{2\left(-2\sqrt{2}+2\sqrt{10}-4e^3+6e\sqrt{\pi}+3\pi-3\sqrt{5}\pi+2e^2\pi-2\sqrt{2\pi}\right)},$$
$$\left.\frac{-16e+2e^{3/2}\sqrt{\pi}+7\pi+\sqrt{5}\pi+\sqrt{e}\pi-\sqrt{5e}\pi-\pi^{3/2}}{-2\sqrt{2}+2\sqrt{10}-4e^3+6e\sqrt{\pi}+3\pi-3\sqrt{5}\pi+2e^2\pi-2\sqrt{2\pi}}\right\}$$

It's usually best to do such problems numerically

In[213]:=
```
LinearSolve[AA, N[b]]
```

Out[213]= {0.782065540326523, −0.438253061439899, 0.581053951084468}

• Numerical solution of $A\,\boldsymbol{x} = \boldsymbol{b}$

In[214]:=
```
A = RandomReal[{-10, 10}, {12, 12}]
```

Out[214]=
$$\begin{pmatrix}
-6.227826575150 92 & -4.338280615852 78 & -8.287135689140 24 & 6.087772326260 78 & -8.967632947793 42 & -3.7 \\
0.146069524462366 & 5.307712854169 18 & 5.910589990696 62 & 3.289871401129 17 & 9.906214974945 17 & 9.4 \\
7.134412404987 42 & -1.023030427343 78 & 5.479239708247 02 & 4.103498237043 75 & 7.167404469517 32 & -4. \\
-3.243017925648 99 & -6.136844355514 42 & -1.388732820079 89 & 4.078286532527 83 & -2.691024545643 16 & -9. \\
-3.258868172932 98 & 5.929753824997 84 & 9.646551332812 36 & 2.00753775554418 & -1.582967102901 99 & 7.3 \\
3.155826101465 18 & 3.116738779851 51 & 2.958111805275 68 & 9.091231488721 51 & -4.906102312534 78 & 8.3 \\
-4.145599371861 52 & -6.706775138773 22 & 3.728837029994 77 & -3.456919661302 29 & -4.455815455893 54 & 1.4 \\
3.856755983573 03 & -1.560043562737 23 & 2.23089180960 93 & -8.256519182609 19 & -5.060395759407 08 & 7.8 \\
-4.518083300398 73 & 6.485020637087 13 & -5.270430045811 62 & 3.541676876670 78 & 8.208022717822 57 & -1.1 \\
2.121731089929 49 & 8.796298598167 31 & 3.038557960991 77 & -4.088808613420 27 & 3.570110628455 45 & 9.7 \\
5.296364133013 97 & 9.232441976433 68 & 5.816450074653 32 & 8.495193970511 87 & -6.404532032557 33 & -0.2 \\
-4.336430935377 24 & -8.486535323224 54 & -6.397560329212 46 & 0.104049208945337 & 3.77075103448 39 & 4.2
\end{pmatrix}$$

In[215]:=
```
xAns = RandomReal[{-10, 10}, {12}]
```

Out[215]= {−3.077180900832 55, −6.593579723476 79, −0.843305645600012, 0.0317569753477187, 1.832058156624 36, 8.690977656302 69, 1.40379120725 83, −8.013796469132 26, −4.759236324008 53, −7.235564607337 84, −6.987908522680 55, −5.081819308634 12}

Multiply $A\,\boldsymbol{x}$ to produce a RHS $\boldsymbol{b}$

In[216]:=
```
b = A.xAns
```

Out[216]= {42.244287483812 1, 57.519207672703 1, 37.082721695477 9, −177.181094900911, 42.832591715435, 145.579950700766, −3.739744083223 35, 107.936907178 42, 22.393423542818 2, 260.188167509394, −143.787059903095, 35.371190858269}

Solve the system

In[217]:=
```
xNum = LinearSolve[A, b]
```
Out[217]= {−3.07718090083254, −6.59357972347679, −0.84330564560017, 0.031756975347714 5, 1.83205815662435, 8.690977656302 69, 1.403791207258 31, −8.013796469132 25, −4.759236324008 53, −7.235564607337 84, −6.987908522680 54, −5.081819308634 12}

In most cases the solution will be very accurate

In[218]:=
```
err = Norm[xNum - xAns]
```
Out[218]= $1.534035027517 34\times10^{-14}$

Even for a big matrix the error in solving a system can be small.

(When forming large objects, put semicolons at the end of the line to suppress output)

In[219]:=
```
ABig = RandomReal[{-1, 1}, {5000, 5000}];
```
In[220]:=
```
xBig = RandomReal[{-1, 1}, {5000}];
```
In[221]:=
```
bBig = ABig.xBig;
```
The solution is fast

In[222]:=
```
{time, xNum} = AbsoluteTiming[ LinearSolve[ABig, bBig]];
```
In[223]:=
```
time
```
Out[223]= 0.597809

The error is reasonably small

In[224]:=
```
err = Norm[xBig - xNum]
```
Out[224]= $6.918123264194 31\times10^{-11}$

However, for pathological cases the error can be very bad. The Hilbert matrix is a classic example of a matrix that's tricky to solve accurately.

In[225]:=
```
H = HilbertMatrix[12]
```
Out[225]=

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} \\ \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} \\ \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} \\ \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} \\ \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} \\ \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} \end{pmatrix}$$

In[226]:=
```
b = H.xAns
```

Out[226]= {−7.94451 83011 0368, −5.21589 91617 2844, −4.07093 83261 8493, −3.40861 09743 0506, −2.96480 33743 5966, −2.64084 79407 5198, −2.39083 28286 8319, −2.19024 02222 8809, −2.02465 57402 5852, −1.88497 81189 6026, −1.76513 53406 3949, −1.66089 52973 7212}

In[227]:=
```
xNum = LinearSolve[H, b]
```

LinearSolve: Result for LinearSolve of badly conditioned matrix (≪1≫) may contain significant numerical errors.

Out[227]= {−3.07718 08054 2956, −6.59359 17078 5994, −0.84293 18327 57012, 0.02670 16997 97703 5, 1.86886 76178 6499, 8.53023 42870 5088, 1.84917 54332 8808, −8.81591 85911 9668, −3.82320 00957 7936, −7.91819 08422 0397, −6.70519 96530 0592, −5.13257 30717 9621}

The error is huge!

In[228]:=
```
err = Norm[xNum - xAns]
```

Out[228]= 1.51447 30822 7644

• Inverse matrices

You can compute symbolic inverses

In[229]:=
```
AInv = Inverse[AA]
```

Out[229]=

$$\begin{pmatrix} \frac{2e^2-3\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\sqrt{2\pi}-e^2\pi}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{3\pi-2\sqrt{2}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \\ \frac{-\frac{3}{2}+\frac{3\sqrt{5}}{2}-e^2}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\frac{1}{\sqrt{2}}-\sqrt{\frac{5}{2}}+e^3}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\sqrt{2}-3e}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \\ \frac{1-\sqrt{5}+\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{-e\sqrt{\pi}-\frac{\pi}{2}+\frac{\sqrt{5}\pi}{2}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{2e-\pi}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \end{pmatrix}$$

Solve $A\,x = b$ for x by computing $x = A^{-1}\,b$.

In[230]:=
```
AInv . b
```

Dot: Tensors (≪1≫) and
{−7.94451830110368, −5.21589916172844, −4.07093832618493, −3.40861097430506, −2.96480337435966, −≪18≫, −≪18≫, −≪19≫, −2.02465574025852, −1.88497811896026, −1.76513534063949, −1.66089529737212}
have incompatible shapes.

Out[230]=

$$\begin{pmatrix} \frac{2e^2-3\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\sqrt{2\pi}-e^2\pi}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{3\pi-2\sqrt{2}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \\ \frac{-\frac{3}{2}+\frac{3\sqrt{5}}{2}-e^2}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\frac{1}{\sqrt{2}}-\sqrt{\frac{5}{2}}+e^3}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{\sqrt{2}-3e}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \\ \frac{1-\sqrt{5}+\sqrt{\pi}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{-e\sqrt{\pi}-\frac{\pi}{2}+\frac{\sqrt{5}\pi}{2}}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} & \frac{2e-\pi}{\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\pi}{2}-e^2\pi+\sqrt{2\pi}} \end{pmatrix}.$$

{−7.94451 83011 0368, −5.21589 91617 2844, −4.07093 83261 8493, −3.40861 09743 0506, −2.96480 33743 5966, −2.64084 79407 5198, −2.39083 28286 8319, −2.19024 02222 8809, −2.02465 57402 5852, −1.88497 81189 6026, −1.76513 53406 3949, −1.66089 52973 7212}

You probably want to simplify the result

In[231]:= `FullSimplify[AInv.b]`

Dot: Tensors (≪1≫) and
{−7.94451830110368, −5.21589916172844, −4.07093832618493, −3.40861097430506, −2.96480337435966, −≪18≫, −≪18≫, −≪19≫, −2.02465574025852, −1.88497811896026, −1.76513534063949, −1.66089529737212}
have incompatible shapes.

Out[231]=
$$\left(\begin{array}{ccc}
\frac{4e^2-6\sqrt{\pi}}{4e^3+2\sqrt{2}(1-\sqrt{5}+\sqrt{\pi})-6e\sqrt{\pi}+3(\sqrt{5}-1)\pi-2e^2\pi} & \frac{2(\sqrt{2\pi}-e^2\pi)}{4e^3+2\sqrt{2}(1-\sqrt{5}+\sqrt{\pi})-6e\sqrt{\pi}+3(\sqrt{5}-1)\pi-2e^2\pi} & \frac{6\pi-4\sqrt{2}}{4e^3+2\sqrt{2}(1-\sqrt{5}+\sqrt{\pi})-6e\sqrt{\pi}+3(\sqrt{5}-1)} \\
\frac{3-3\sqrt{5}+2e^2}{-4e^3+2\sqrt{2}(-1+\sqrt{5}-\sqrt{\pi})+6e\sqrt{\pi}-3(\sqrt{5}-1)\pi+2e^2\pi} & \frac{\sqrt{2}(\sqrt{5}-1)-2e^3}{-4e^3+2\sqrt{2}(-1+\sqrt{5}-\sqrt{\pi})+6e\sqrt{\pi}-3(\sqrt{5}-1)\pi+2e^2\pi} & \frac{6e-2\sqrt{2}}{-4e^3+2\sqrt{2}(-1+\sqrt{5}-\sqrt{\pi})+6e\sqrt{\pi}-3(\sqrt{5}-1} \\
\frac{2-2\sqrt{5}+2\sqrt{\pi}}{4e^3+2\sqrt{2}(1-\sqrt{5}+\sqrt{\pi})-6e\sqrt{\pi}+3(\sqrt{5}-1)\pi-2e^2\pi} & \frac{(\sqrt{5}-1)\pi-2e\sqrt{\pi}}{4e^3+2\sqrt{2}(1-\sqrt{5}+\sqrt{\pi})-6e\sqrt{\pi}+3(\sqrt{5}-1)\pi-2e^2\pi} & \frac{4e-2\pi}{4e^3+2\sqrt{2}(1-\sqrt{5}+\sqrt{\pi})-6e\sqrt{\pi}+3(\sqrt{5}-1)}
\end{array}\right.$$

−2.96480 33743 5966, −2.64084 79407 5198, −2.39083 28286 8319, −2.19024 02222 8809,
−2.02465 57402 5852, −1.88497 81189 6026, −1.76513 53406 3949, −1.66089 52973 7212}

Determinants

The Det[ ] function computes the determinant of a square matrix

In[232]:= `Remove[a, b, c, d]`

In[233]:= `Det[{{a, b}, {c, d}}]`

Out[233]= $a\,d-b\,c$

In[234]:= `Det[{{a, b, c}, {d, e, f}, {h, i, j}}]`

Out[234]= $a\,e\,j-a\,f\,i-b\,d\,j+b\,f\,h+c\,d\,i-c\,e\,h$

In[235]:= `Det[AA]`

Out[235]= $$\sqrt{2}-\sqrt{10}+2e^3-3e\sqrt{\pi}-\frac{3\pi}{2}+\frac{3\sqrt{5}\,\pi}{2}-e^2\pi+\sqrt{2\pi}$$

In[236]:= `Det[{{-3 - λ, 1}, {1, -3 - λ}}]`

Out[236]= $\lambda^2+6\lambda+8$