Math 521 HW4

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1 Theory

1.1 Problem 1

Given $C_x u = \lambda_x u$ and $C_s v = \lambda_s v$ where the matrices are related as $C_x = C_s + \alpha I$
Substituting C_x in the first eigenvalue problem we have
$C_s u + \alpha I u = \lambda_x u$
$=> C_s u = (\lambda_x - \alpha)u$
Hence the eigenvalues of C_s are $(\lambda_x - \alpha)$ and eigenvectors are u. Hence $u = v$ and $\lambda_s = \lambda_x - \alpha$

1.2 Problem 2

For $x \in R^m$ and x is a non-zero vector.Then $x^T AA^T x = (A^T x)^T (A^T x) \geq 0$.
Hence the matrix AA^T is positive semi-definite, which means the eigenvalues of this matrix are nonnegative.
For a given eigenvector $\lambda, AA^T v = \lambda v$. therefore

$$Mv = (\alpha^2 I + AA^T)v = \alpha^2 Iv + AA^T v = \alpha^2 v + \lambda v = (\alpha^2 + \lambda)v$$

Since $\lambda \geq 0$ and $\alpha^2 > 0$, the eigenvalues of matrix M are positive nonzero. Since non-singular matrices can only have non-zero eigenvalues and we just proved that M falls in that category, then M must be non-singular.

1.3 Problem 3

Here M is the total number of distinct classes, n_i is the number of data points in class i, m_i is the class mean of the ith class, and m is the mean across all n data points.

$S_B = S_T - S_W$

$= \sum_{i=1}^{M} \sum_{x \in D_i} (x_i - \mathbf{m})(x_i - \mathbf{m})^T - \sum_{i=1}^{M} \sum_{x \in D_i} (x - \mathbf{m}_i)(x - \mathbf{m}_i)^T$

$= \sum_{i=1}^{M} \sum_{x \in D_i} [(x_i - \mathbf{m})(x_i - \mathbf{m})^T - (x - \mathbf{m}_i)(x - \mathbf{m}_i)^T]$

$$= \sum_{i=1}^{M} \sum_{x \in D_i} [x.x^T - mx^T - xm^T + mm^T - xx^T + m_i x^T + x m_i^T - m_i.m_i^T]$$

$$= \sum_{i=1}^{M} [-n_i m m_i^T - n_i m_i m^T + n_i m m^T + n_i m_i m_i^T + n_i m_i m_i^T - n_i m_i m_i^T]$$

$$= \sum_{i=1}^{M} n_i (m_i - m)(m_i - m)^T$$