Math 521 HW1

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1 Theory

1. Change of basis

   Given standard vectors defining $\mathcal{B}_1$ are $e^{(1)} = \begin{pmatrix}1 & 0\end{pmatrix}^T$ and $e^{(2)} = \begin{pmatrix}0 & 1\end{pmatrix}^T$ or $E = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$
   Basis vectors defining $\mathcal{B}_2$ are $v^{(1)} = \begin{pmatrix}1 & 1\end{pmatrix}^T$ and $v^{(2)} = \begin{pmatrix}-1 & 1\end{pmatrix}^T$ or $V = \begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$
   Given $u_{\mathcal{B}1} = \begin{pmatrix}1 & 1\end{pmatrix}^T$
   Since both $\mathcal{B}_1$ and $\mathcal{B}_1$ span R^2, any vector in R^2 can be represented as

   $u = x_1 e^{(1)} + x_2 e^{(2)}$ and
   $u = y_1 v^{(1)} + y_2 v^{(2)}$ or

   $E\mathrm{u}_{\mathcal{B}1} = V\mathrm{u}_{\mathcal{B}2}$

   $=> \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\mathrm{u}_{\mathcal{B}2}$
   $\mathrm{u}_{\mathcal{B}2} = \begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}^{-1}\begin{pmatrix}1\\ 1\end{pmatrix} = \begin{pmatrix}0.5 & 0.5\\ -0.5 & 0.5\end{pmatrix}$