Deep Learning

20 Generative Adversarial Network (GAN)

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1. Work by Ian Goodfellow et al. (NeurIPS 2014)

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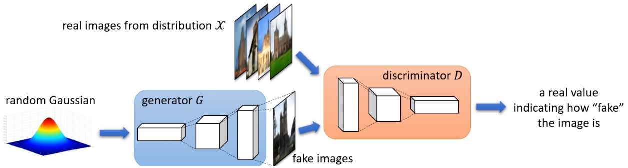
3. Draws samples that are similar to the train data (but not exactly them)

Method



Credit: Microsoft research blog

1. Introduce a latent variable (z) with a simple prior (p_z)

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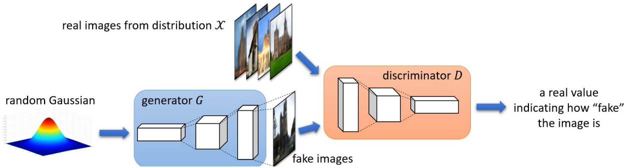

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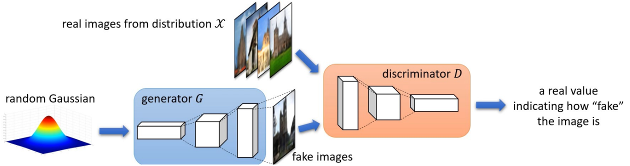

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2. Draw $z \sim p_z$, i/p to the generator (G) $\rightarrow \hat{x} \sim p_G$
3. Machinery to ensure $p_G \approx p_{\text{data}}$

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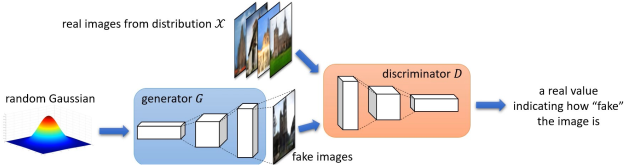

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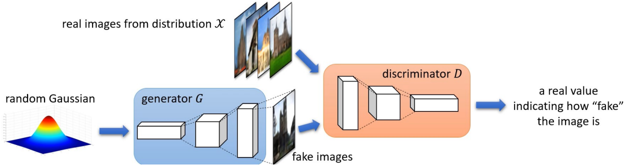
1. Employ a classifier to differentiate between **real** samples $x \sim p_{\text{data}}$ (label 1) and **generated**(fake) ones $\hat{x} \sim p_G$ (label 0)

$p_G \approx p_{\textbf{data}}$



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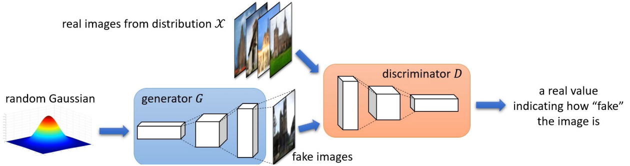

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2. Referred to as the **Discriminator (D)**
3. Train the G such that D misclassifies generated samples $\hat{x}$ into class 1 (can't differentiate b/w $x \sim p_{\text{data}}$ and $\hat{x} \sim p_G$)

Training Objective

$$\min_G \max_D \left(\mathbb{E}_{x \sim p_{\text{data}}}[log D(x)] + \color{red}{\mathbb{E}_{z \sim p_z}[log(1 - D(G(z)))]} \right)$$

1. minmax optimization (or, zero-sum game)

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Training Objective

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भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

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1. minmax optimization (or, zero-sum game)
2. With a sigmoid o/p neuron, $D(\cdot) \rightarrow$ probability that the i/p is real
3. Expectation in practice is average over a batch of samples

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5. Which would be ≈ 0 for a confident $D \rightarrow$ (no gradients to train G!)

Training Strategy

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**

  for k steps **do**

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
  - Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right) \right].$$

  end for

- Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

Idea of convergence

1. Adversarial components $\rightarrow$ nontrivial convergence for the training

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1. Adversarial components $\rightarrow$ nontrivial convergence for the training
2. In other words, objective is not to push the loss/objective towards 0

Optimality

$$\text{min}_G \ \text{max}_D \left(\mathbb{E}_{x \sim p_{\text{data}}}[logD(x)] + \mathbb{E}_{z \sim p_z}[log(1 - D(G(z)))] \right)$$

$$\rightarrow \text{min}_G \ \text{max}_D \int_x \left(p_{\text{data}}(x) \cdot logD(x) + p_G(x) \cdot log(1 - D(G(x))) \right) dx$$

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let $y = D(x)$, $a = p_{\text{data}}$, and $b = p_G$

$\rightarrow f(y) = a \cdot \log y + b \cdot \log(1 - y)$

f exhibits local maximum at $y = \frac{a}{a+b}$

Optimal discriminator $D^*_G(x) = \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)}$

Optimality

$$\min_G \int_X \left(p_{\text{data}}(x) \cdot log D_G^*(x) + p_G(x) \cdot log(1 - D_G^*(G(x))) \right) dx$$

$$\min_G \int_X \left(p_{\text{data}}(x) \cdot \left[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)} \right] + p_G(x) \cdot log(1 - \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)}) \right) dx$$

$$\min_G \int_X \left(p_{\text{data}}(x) \cdot \left[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)} \right] + p_G(x) \cdot log(\frac{p_G(x)}{p_{\text{data}}(x) + P_G(x)}) \right) dx$$

$$\min_G \left(\mathbb{E}_{x \sim p_{\text{data}}} \left[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)} \right] + \mathbb{E}_{x \sim p_G} \cdot log(\frac{p_{\mathsf{G}}(x)}{p_{\text{data}}(x) + P_G(x)}) \right)$$

Optimality

$$\min_G \left(\mathbb{E}_{x\sim p_{\text{data}}} \left[\log \frac{2*p_{\text{data}}(x)}{2*(p_{\text{data}}(x)+P_G(x))} \right] + \mathbb{E}_{x\sim p_G} \cdot log(\frac{2*p_G(x)}{2*(p_{\text{data}}(x)+P_G(x)}))\right)$$

$$\min_G \left(\mathbb{E}_{x\sim p_{\text{data}}} \left[\log \frac{2*p_{\text{data}}(x)}{(p_{\text{data}}(x)+P_G(x))} \right] + \mathbb{E}_{x\sim p_G} \cdot log(\frac{2*p_G(x)}{(p_{\text{data}}(x)+P_G(x)}) - \log 4)\right)$$

$$\min_G \left(\mathbf{KL}(\mathbf{p}_{\text{data}}(\mathbf{x}), \frac{\mathbf{p}_{\text{data}}(\mathbf{x})+\mathbf{P_G}(\mathbf{x})}{\mathbf{2}}) + \mathbf{KL}(\mathbf{p}_{\text{G}}(\mathbf{x}), \frac{(\mathbf{p}_{\text{data}}(\mathbf{x})+\mathbf{P_G}(\mathbf{x})}{\mathbf{2}}) - \log \mathbf{4})\right)$$

$$\min_G \left(2 * \mathbf{JSD}(\mathbf{p}_{\text{data}}, \mathbf{p_G}) - \log \mathbf{4} \right)$$

$\rightarrow$ minimized when $p_{\text{data}} = p_G$

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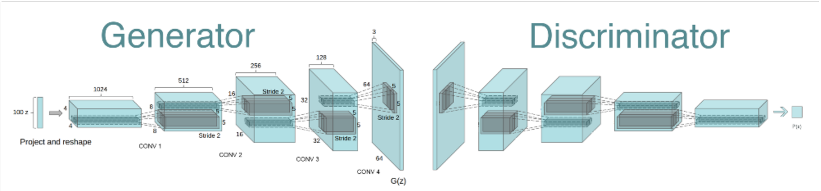
2. $p_{\text{data}} = p_G$ (Optimal Generator for any D)
3. $D_G^*(x) = \frac{1}{2}$

Deep Convolutional GAN (DC-GAN)



Radford et al. NeurIPS 2016

1. Combined the developments of CNNs with the generative modeling

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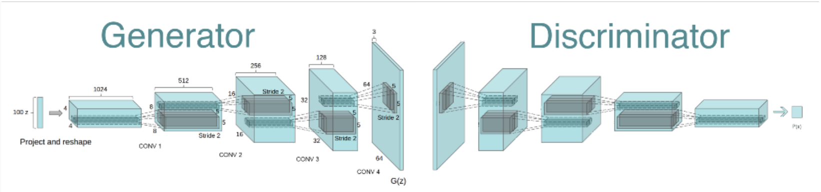

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1. Combined the developments of CNNs with the generative modeling
2. Demonstrated some of the best practices for stable training of deep GAN architectures

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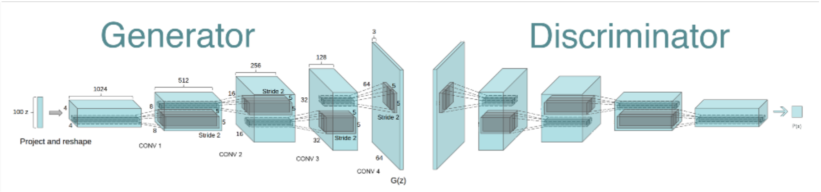

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1. Strided convolution in place of spatial pooling (learn spatial downsampling)

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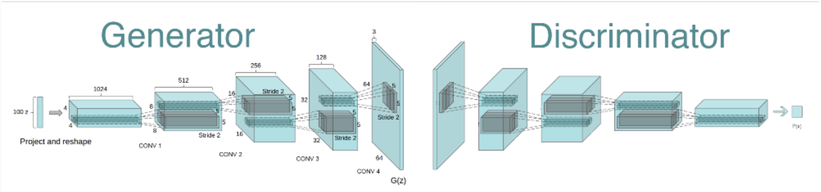

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2. No dense layers

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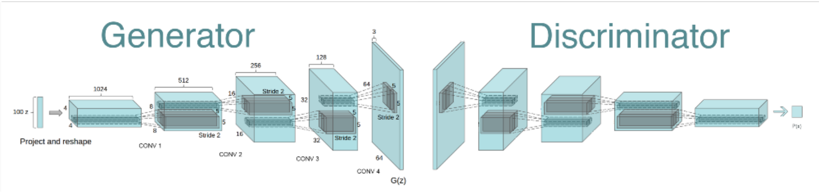

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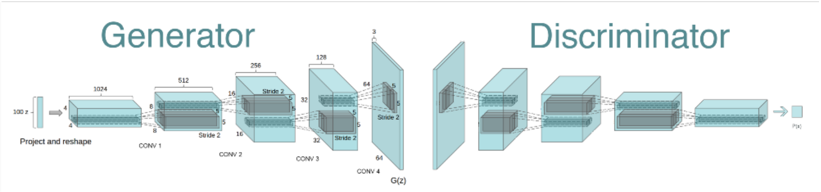

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4. ReLU (tanh for the o/p layer) for G and Leaky-ReLU (sigmoid for the o/p layer) for D

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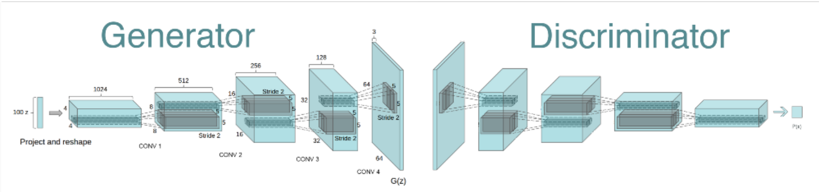

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1. Smooth interpolation in the latent space and Vector arithmetic

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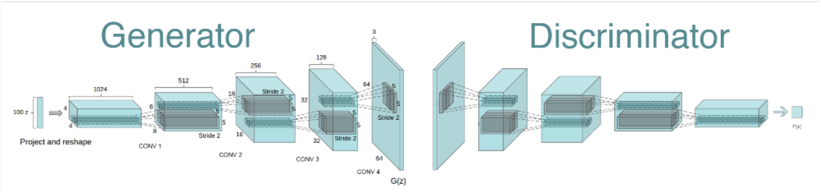

Radford et al. NeurIPS 2016

1. Smooth interpolation in the latent space and Vector arithmetic
2. Unsupervised feature learning (via the Discriminator)