

# Deep Learning

## 13. Recurrent Neural Networks

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# So far...

① Perceptron, MLP, Gradient Descent (Backpropagation)

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- ③ 'Feedforward Neural networks'

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- ③ Processing of successive i/p is independent of each other

# Consider 'auto-completion' task

Q deep|

deep — Search with Google

🕒 **kuldeep birdar**

Q deep**pika padukone**

Q deep**thi sunaina**

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① Successive i/p are not independent



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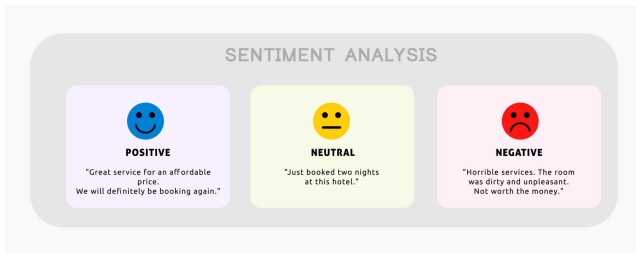
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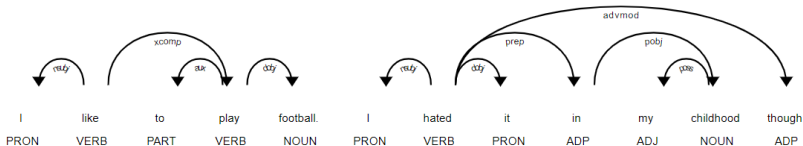
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- ③ Same underlying task at different 'time instances'
- ④ **Sequence Learning Problems**

# Sequence Learning Tasks: Example



Sentiment Analysis (Source)

# Sequence Learning Tasks: Example



POS-Tagging (Source:Kaggle)

# Sequence Learning Tasks: Example



Action Recognition (Source)

# Sequence Learning Tasks: Example

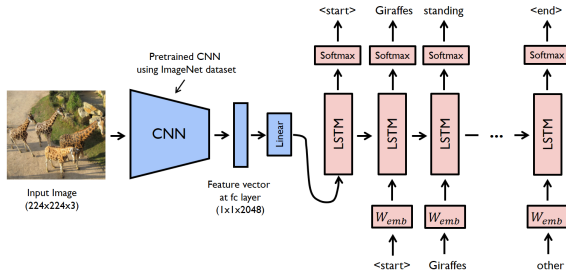
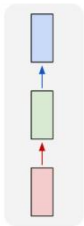


Image Captioning(Source)

# Sequence Learning Tasks: Variations

one to one

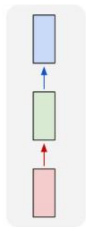


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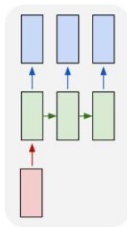


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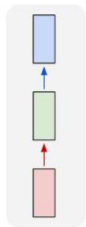
one to many



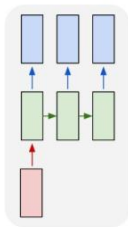
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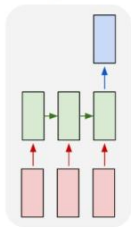
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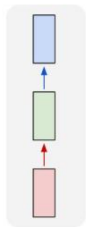
many to one



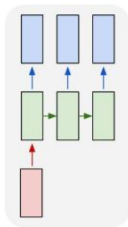
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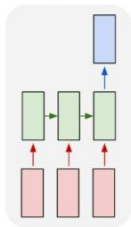
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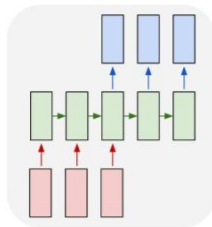
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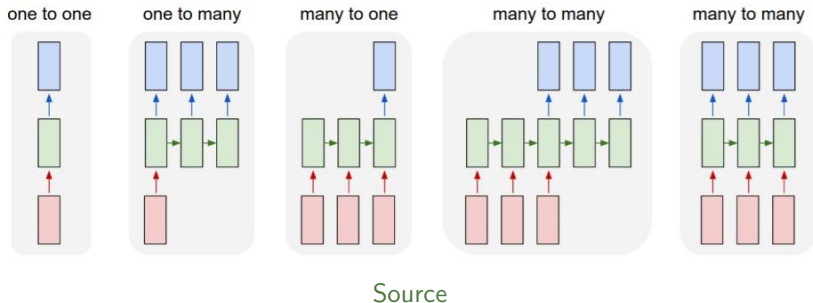


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Source

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# Recurrent Neural Networks (RNN)

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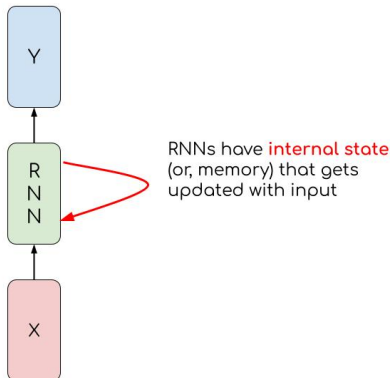
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# Recurrent Neural Networks (RNN)

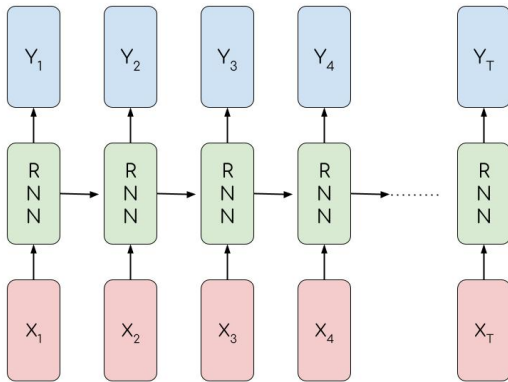
- ① NNs designed to solve sequence learning tasks
- ② Characteristics
  - ① Model the dependence among the i/p
  - ② Handle variable length of i/p
  - ③ Same function applied at all time instances



# RNNs: internal state



# RNNs: unfolding



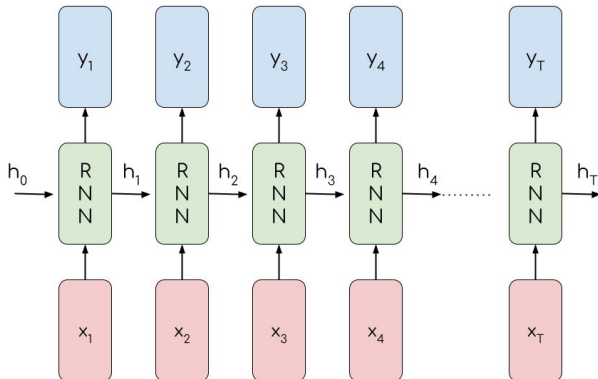
- ① Apply the same transformation at every time step  $\rightarrow$  'Recurrent' NNs

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- ② i/p sequence  $x_t \in \mathbb{R}^D$
- ③ Initial recurrent state  $h_0 \in \mathbb{R}^Q$
- ④ RNN model computes sequence of recurrent states iteratively  
$$h_t = \phi(x_t, h_{t-1}; w)$$

# RNNs



# Elmon RNN (1990)

- ① Start with  $h_0 = 0$



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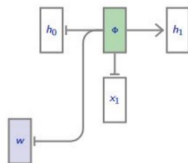
- ① Start with  $h_0 = 0$
- ②  $h_t = \tanh(W_{xh} \cdot x_t + W_{hh} \cdot h_{t-1} + b_h)$

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- ① Start with  $h_0 = 0$
- ②  $h_t = \tanh(W_{xh} \cdot x_t + W_{hh} \cdot h_{t-1} + b_h)$
- ③  $y_t = \text{softmax}(W_{hy} \cdot h_t + b_y)$

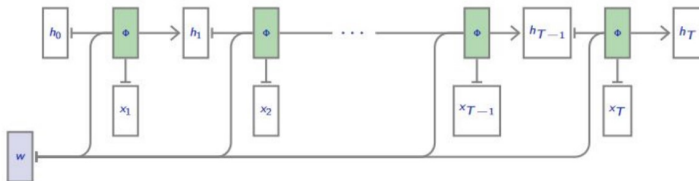
# RNNs as computational graph

- 1 Use the same set of parameters at each time step



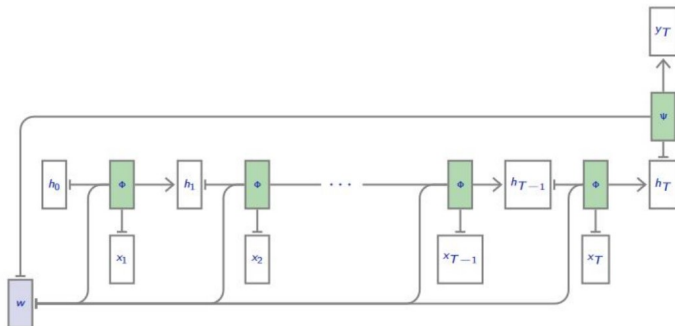
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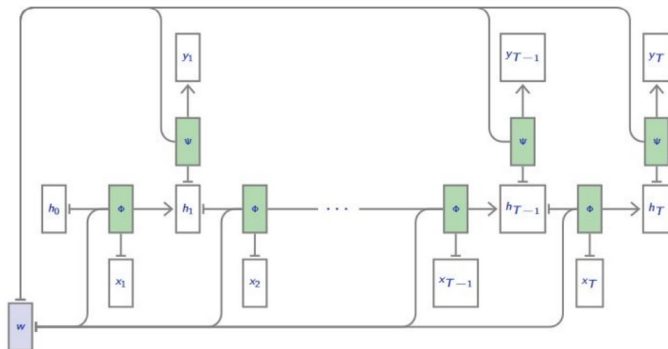
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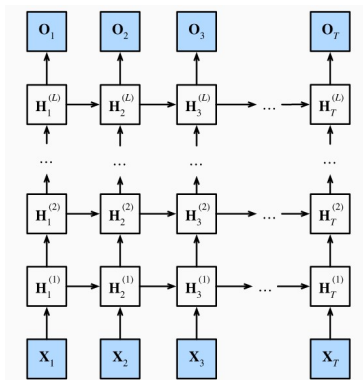
# RNNs as computational graph

- 1 Use the same set of parameters at each time step
- 2 Flexible to realize different variants (with some tricks!)



# Multi-layered RNNs

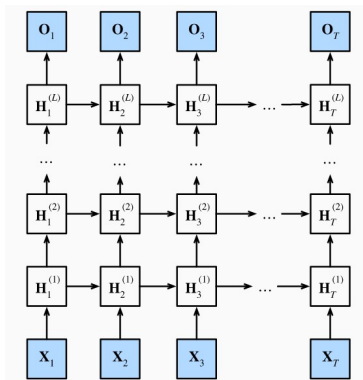
- 1 Stack multiple RNNs between i/p and o/p layers



Source

# Multi-layered RNNs

- 1 Stack multiple RNNs between i/p and o/p layers
- 2  $H_t^{(l)} = W_{xh}^{(l)} \cdot H_t^{(l-1)} + W_{hh}^{(l)} \cdot H_{t-1}^{(l)} + b_h^{(l)}$

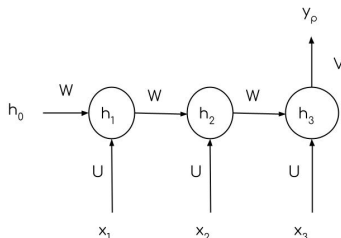


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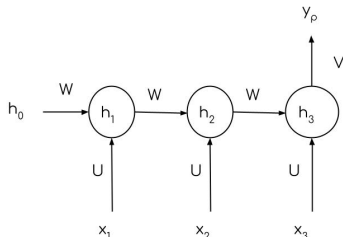
# Backpropagation Through Time (BPTT)

- 1 Consider a many-to-one variant RNN (e.g. sentiment analysis)



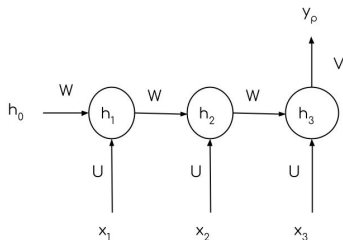
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- 1 Consider a many-to-one variant RNN (e.g. sentiment analysis)
- 2 Let's separate the parameters into  $U$ ,  $V$ , and  $W$



# Backpropagation Through Time (BPTT)

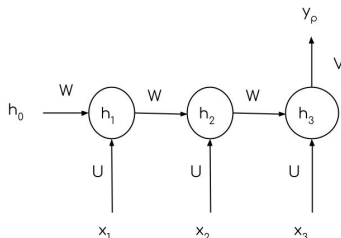
- ① Let's now perform SGD  
(assume loss  $L$  is  
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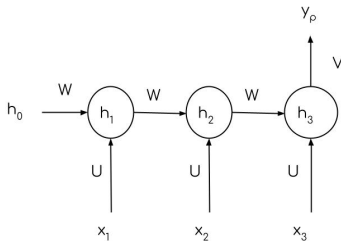
① Let's now perform SGD  
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②  $\rightarrow$  we need to compute  
 $\frac{\partial L}{\partial V}$ ,  $\frac{\partial L}{\partial W}$ , and  $\frac{\partial L}{\partial U}$



# Backpropagation Through Time (BPTT)

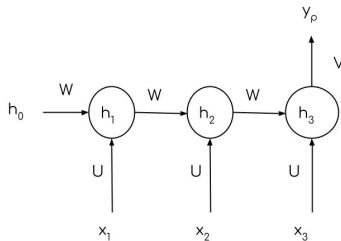
$$\textcircled{1} \quad \frac{\partial L}{\partial V} = \frac{\partial L}{\partial y_p} \frac{\partial y_p}{\partial V} = \frac{\partial L}{\partial y_p} \cdot \frac{\partial y_p}{\partial z_3} \cdot \frac{\partial z_3}{\partial V}$$



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$$\textcircled{2} \quad y_p = \text{softmax}(z_3) \text{ and}$$
$$z_3 = V \cdot h_3 + b_y$$

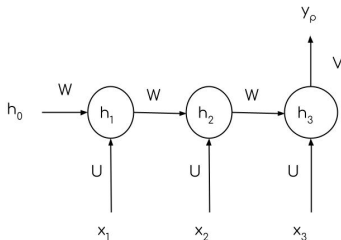


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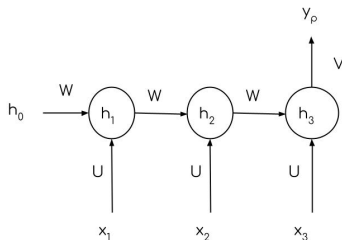
$$\textcircled{2} \quad y_p = \text{softmax}(z_3) \text{ and } z_3 = V \cdot h_3 + b_y$$

$\textcircled{3}$  Since we know that  $h_3, b_y$  are independent of  $V$ , we can compute  $\frac{\partial L}{\partial V}$  in a single step



# Backpropagation Through Time (BPTT)

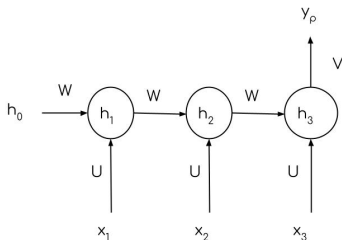
① Let's now consider  $\frac{\partial L}{\partial W}$



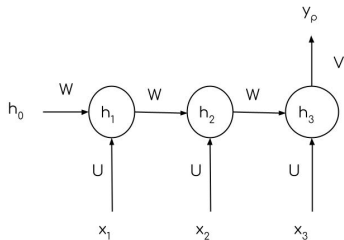


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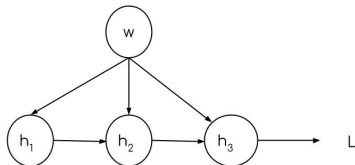
- ① Let's now consider  $\frac{\partial L}{\partial W}$
- ② There are multiple ' $W$ 's in the computational graph!



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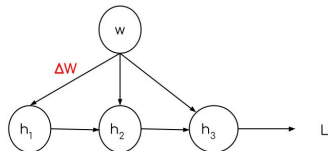


- 1 For ease of understanding



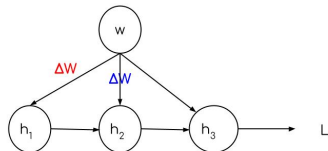
# Backpropagation Through Time (BPTT)

- ①  $\Delta w$  change in  $W \rightarrow$   
 $\left( \frac{\partial h_1}{\partial W} \cdot \Delta w \right)$  change in  $h_1$



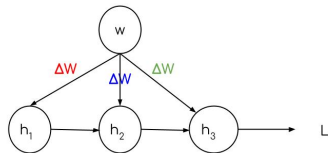
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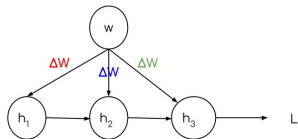
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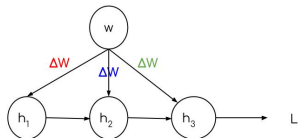
# Backpropagation Through Time (BPTT)

①  $\Delta L =$   
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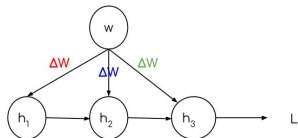
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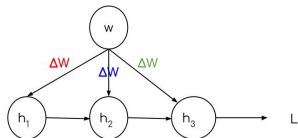
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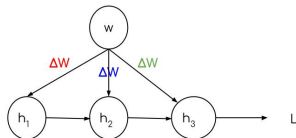
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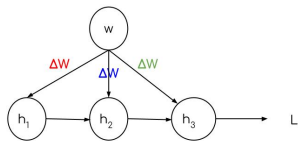


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$$① \quad \frac{\partial L}{\partial W} = \frac{\partial L}{\partial h_1} \frac{\partial h_1}{\partial W} + \frac{\partial L}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L}{\partial h_3} \frac{\partial h_3}{\partial W}$$

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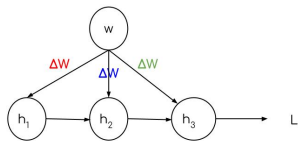
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$$① \quad \frac{\partial L}{\partial W} = \frac{\partial L}{\partial h_1} \frac{\partial h_1}{\partial W} + \frac{\partial L}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L}{\partial h_3} \frac{\partial h_3}{\partial W}$$

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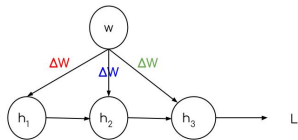
$$\textcircled{2} \quad \frac{\partial L}{\partial h_2} = \frac{\partial L}{\partial h_3} \frac{\partial h_3}{\partial h_2}$$

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$\textcircled{5}$

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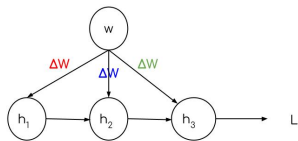
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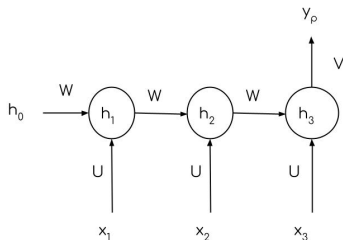
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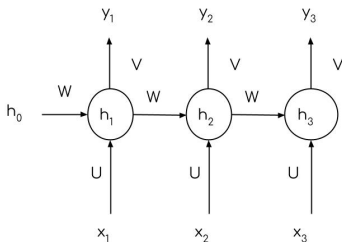
# Backpropagation Through Time (BPTT)

① Similarly  $\frac{\partial L}{\partial U}$



# Backpropagation Through Time (BPTT)

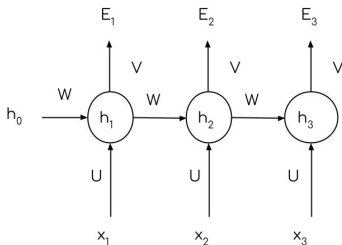
- 1 Consider a many-to-many variant RNN (e.g. PoS tagging)





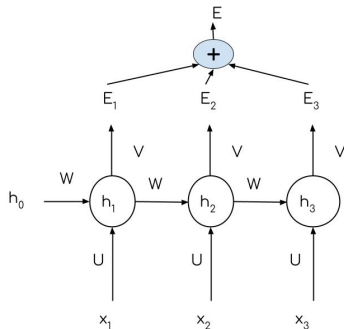
# Backpropagation Through Time (BPTT)

- 1 Consider a many-to-many variant RNN (e.g. PoS tagging)
- 2 Full sequence is one training example (although there is an error computed at each time step)



# Backpropagation Through Time (BPTT)

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- ④ Leads to Vanishing Gradient problem!
- ⑤ No impact of earlier time steps at later times (**difficult to learn long-term dependencies!**)

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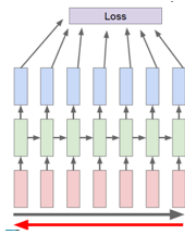
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  - Gradient clipping

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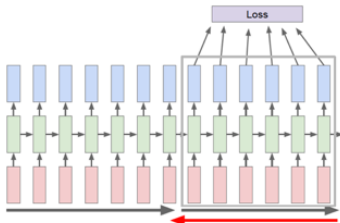


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- ④ Better initialization, Regularization, short time sequences (Truncation)

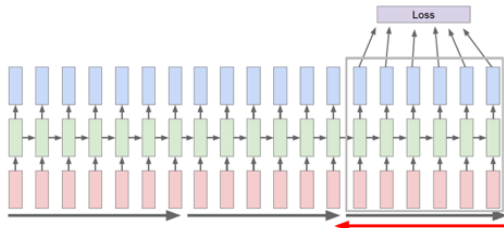
# Backpropagation Through Time (BPTT)



(a)



(b)



(c)

Truncated BPTT (CS231n)

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## ① Architectural modifications to RNNs

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- GRU (Cho et al. 2014)



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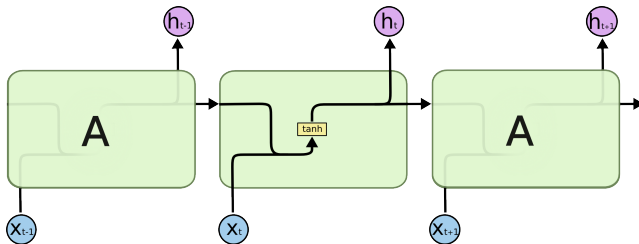
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  - Gates are dynamically computed based on the context

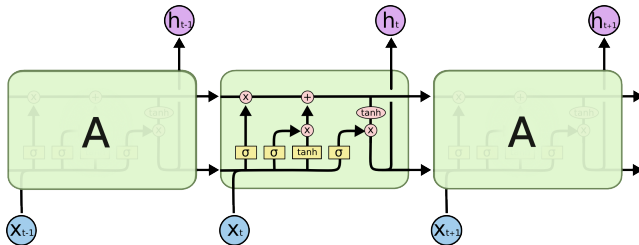
# LSTM



RNNs are chain of repeating moduels. Basic RNN (Colah's blog)

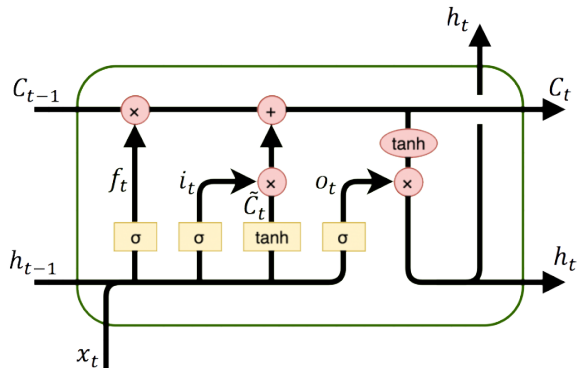


# LSTM



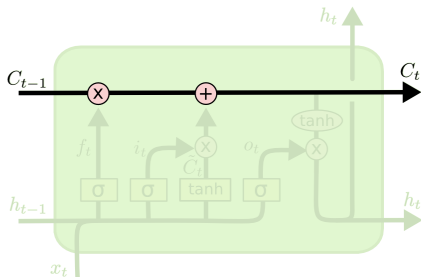
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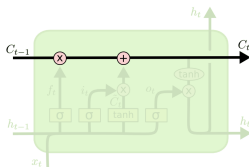
# LSTM: the cell state



Cell state in LSTM (Colah's blog)

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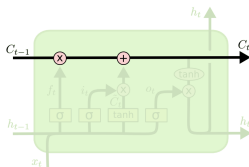
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# LSTM: the cell state

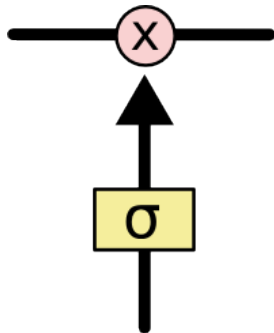
- 1 info can flow through unchanged
- 2 gates can add/remove information to cell state



Cell state in LSTM (Colah's blog)

# LSTM: the gates

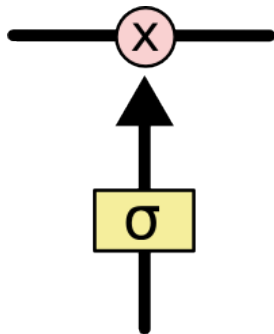
- ① sigmoid neural nets (o/p numbers in  $[0, 1]$ )



Cell state in LSTM (Colah's blog)

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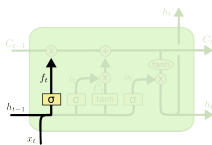
- ① sigmoid neural nets (o/p numbers in  $[0, 1]$ )
- ② Pointwise multiplication operation



Cell state in LSTM (Colah's blog)

# LSTM: the forget gate

- 1 Decide what to throw away from cell state (e.g. forgetting the gender of old subject in light of a new one)



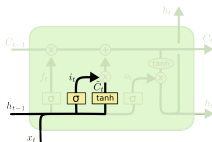
$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

Forget gate in LSTM (Colah's blog)



# LSTM: the input gate

- 1 Next is to decide what new to store in cell state (e.g. add the gender of a new subject)



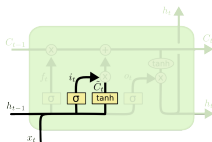
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

Input gate in LSTM (Colah's blog)

# LSTM: the input gate

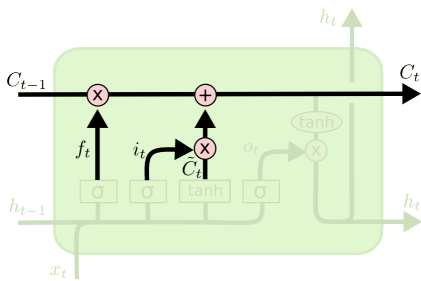
- ① Next is to decide what new to store in cell state (e.g. add the gender of a new subject)
- ② Done in two steps
  - input gate decides what to update
  - A tanh layer creates a candidate cell state



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Input gate in LSTM (Colah's blog)

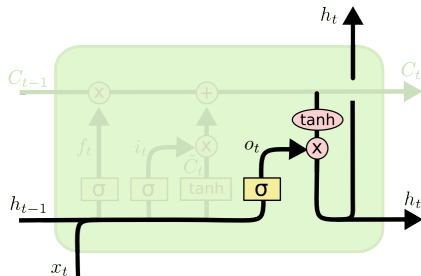
# LSTM: the cell state update



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Cell state update in LSTM (Colah's blog)

# LSTM: the output



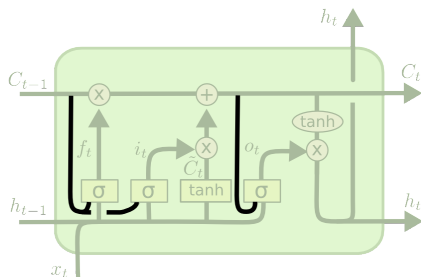
$$o_t = \sigma(W_o[h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

Output computation in LSTM (Colah's blog)

e.g. may be a verb that is coming next in case of a language model

# LSTM variants



$$f_t = \sigma (W_f \cdot [C_{t-1}, h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma (W_i \cdot [C_{t-1}, h_{t-1}, x_t] + b_i)$$

$$o_t = \sigma (W_o \cdot [C_t, h_{t-1}, x_t] + b_o)$$

Variant with gates looking into the Cell state in LSTM by Ger et al. (Colah's blog)