Deep Learning

18 Variational Autoencoder

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Autoencdoers

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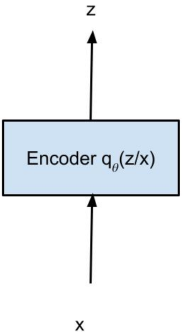
2. We attempted to project the data into the latent space and model it via a probability distribution
3. This wasn't satisfying

Variational Autoencoders

1. ‘Regularized’ autoencoder to enforce latent space ‘organization’

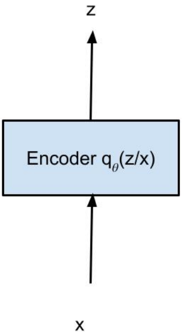
Variational Autoencoders

1. Key idea is to make both Encoder and Decoder stochastic
   - instead of encoding an i/p as a single point, we encode it as a distribution over the latent space



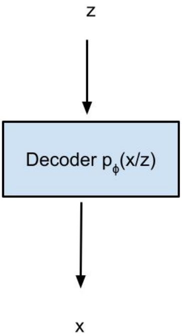
Variational Autoencoders

1. Key idea is to make both Encoder and Decoder stochastic
   - instead of encoding an i/p as a single point, we encode it as a distribution over the latent space
2. Latent variable z is drawn from a probability distribution for the given input x



Variational Autoencoders

1. Then, the reconstruction is chosen probabilistically from the sampled z



VAE Encoder

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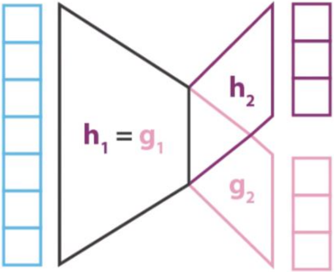
3. NN implementation of the encoder gives (for every input x) a vector mean and a diagonal covariance

VAE Encoder



x

$$\mu_x = g(x) = g_2(g_1(x))$$

$$\sigma_x = h(x) = h_2(h_1(x))$$

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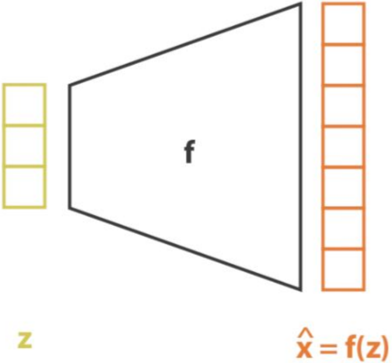
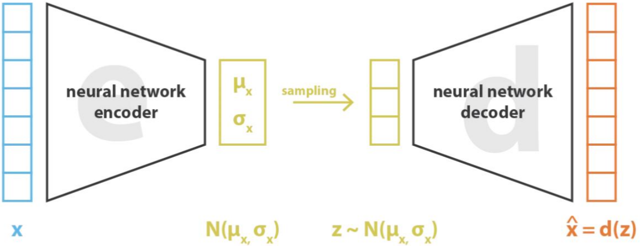
2. $p_\phi(x/z)$ gives mean and variance for each pixel in the output
3. Reconstruction of x is via sampling (with some assumptions, the data sample can be output)

VAE Decoder



VAE Forward pass



VAE loss function

1. Loss for AE: l_2 distance between the input and its reconstruction

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1. Loss for AE: l_2 distance between the input and its reconstruction
2. In case of VAE: we need to learn parameters of two probability distributions
3. For each input x_i we maximize expected value of returning x_i (or, minimize the NLL)

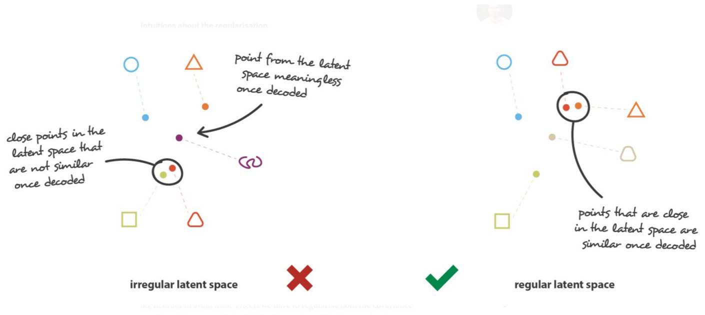
$$-\mathbb{E}_{z \sim q_\theta(z/x_i)}[log\, p_\phi(x_i/z)]$$

VAE loss function

$$-\mathbb{E}_{z \sim q_\theta(z/x_i)}[log\, p_\phi(x_i/z)]$$

1. Problem: Input images may be memorized in the latent space
   - $\rightarrow$ similar inputs may get different representations in z space
   - $\rightarrow$ close points in the latent space should not give two completely different contents once decoded

VAE loss function



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1. Continuity and Completeness: We prefer continuous latent representations to give meaningful parameterization (e.g. smooth transition between i/ps)

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1. Continuity and Completeness: We prefer continuous latent representations to give meaningful parameterization (e.g. smooth transition between i/ps)
2. Solution: Force $q_\theta(z/x_i)$ to be close to a standard distribution (e.g. Gaussian)

VAE loss function

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$$l_i(\theta, \phi) = -\mathbb{E}_{z \sim q_\theta(z/x_i)}[log\, p_\phi(x_i/z)] + \mathbb{KL}(q_\theta(z/x_i)||p(z))$$

1. First term promotes recovery, sencond term keeps encoding continuous (beats memorization)

VAE loss function

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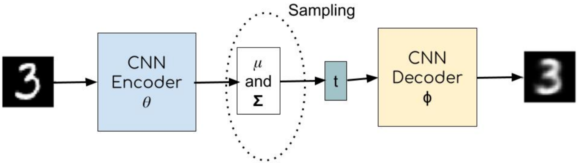
1. Problem: Differentiating over θ and ϕ



VAE loss function

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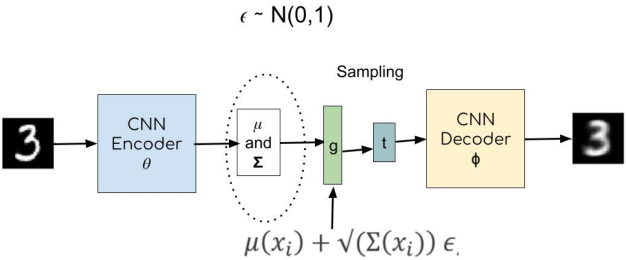
1. Reparameterization: Draw samples from N(0,1) $\rightarrow$ doesn't depend on parameters



Generation with VAE

- Sample z from the prior p(z)

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- Sample z from the prior p(z)
- Run z through the decoder (ϕ) $\rightarrow$ distribution over data
- Sample from that distribution to generate the sample x

Generation with VAE

Figure credits: Wojceich

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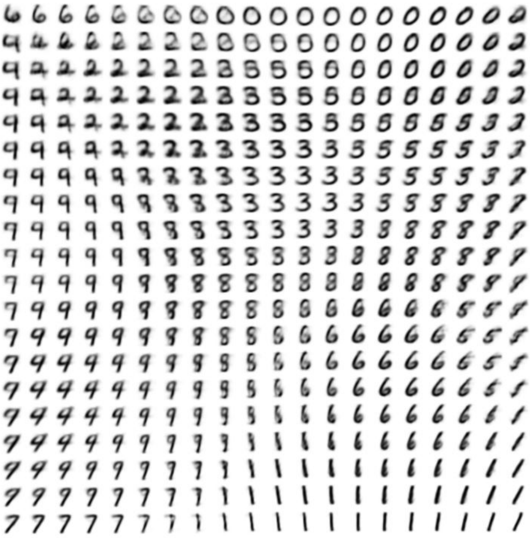

Figure credits: Kingma et al.

Edit/Manipulate samples with VAE