

Foundations of Machine Learning

AI2000 and AI5000

FoML-15

Linear Discriminant Functions

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July-Nov 2025



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So far in FoML

- Intro to ML and Probability refresher
- MLE, MAP, and fully Bayesian treatment
- Linear Regression with basis functions (regularization, model selection)
- Bias-Variance Decomposition (Bayesian Regression)
- Decision Theory - three broad classification strategies
- Probabilistic Generative Models - Continuous & discrete data

Discriminant Functions



Discriminant

- Function that takes an input and assigns one of the classes as output
- Restrict to '*Linear Discriminants*'
 - Decision surfaces are hyperplanes

Discriminant Functions - two classes

- Input $x \in \mathbb{R}^D$
- Targets $t \in \{C_1, C_2\}$
- Discriminant Function

$$y(\mathbf{x}) = f(\tilde{\mathbf{w}}^T \phi(\mathbf{x}))$$

$$\phi(\mathbf{x}) = (\phi_0(\mathbf{x}), \dots, \phi_{M-1}(\mathbf{x}))^T$$

Linear Decision boundaries

$$y(\mathbf{x}) = \text{const}$$

Generalized Linear Models

because of the 'nonlinear'
activation function f



Discriminant Functions - two classes

- Simplest discriminant function $y(\mathbf{x}, \tilde{\mathbf{w}}) = \mathbf{w}^T \mathbf{x} + w_0$
- Decision boundary $y(\mathbf{x}, \tilde{\mathbf{w}}) = 0$

$$\underline{x} \in \mathbb{R}^D$$

$$y(\underline{x}) > 0 \rightarrow c_1$$

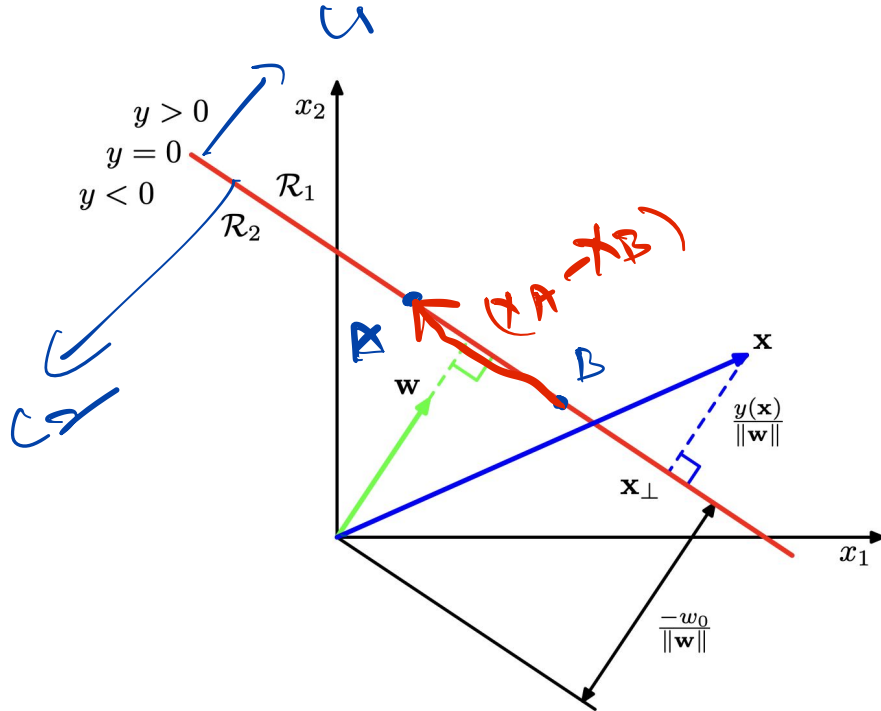
$$y(\underline{x}) < 0 \rightarrow c_2$$

$$\phi_i(\underline{x}) = x_i \quad i = 1 \text{ to } D$$

$$\phi_0(\underline{x}) = 1$$



Discriminant Functions - two classes



W determines the orientation of the decision boundary

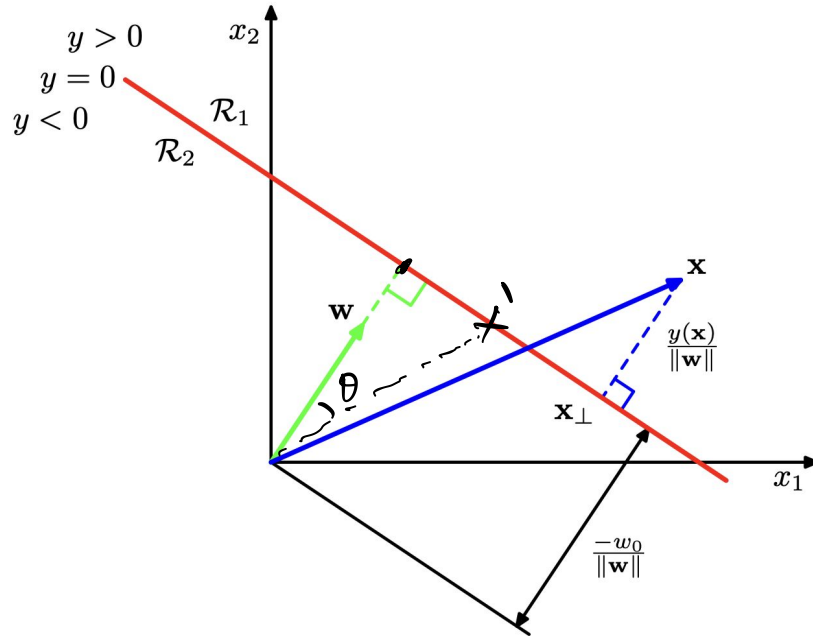
x_A & x_B lie on Boundary

$$w^T x_A = 0 \quad w^T x_B = 0$$

$$w^T (x_A - x_B) = 0$$

$$\underline{w^T x = 0}$$

Discriminant Functions - two classes



Normal distance from origin

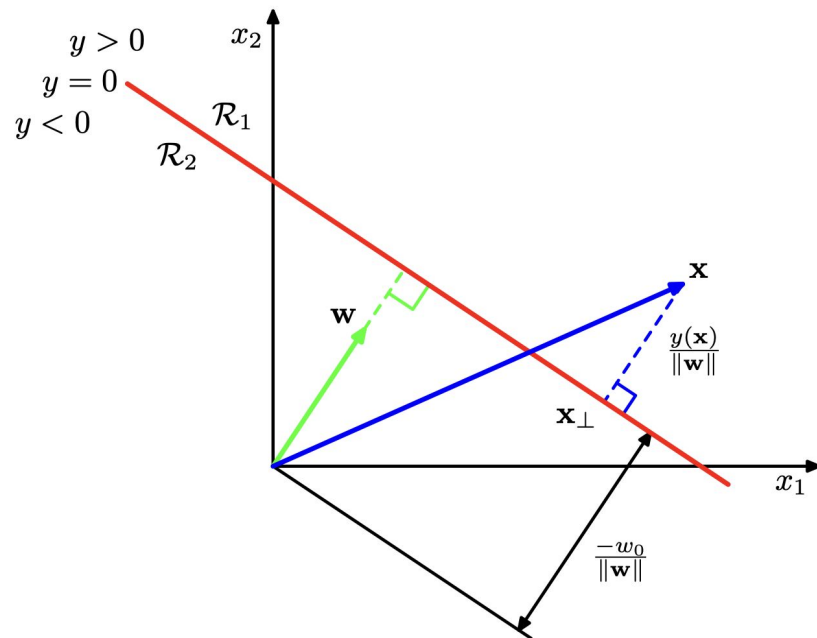
$$\frac{w^T x'}{\|w\|} = \frac{-w_0}{\|w\|}$$

w_0 shifts the boundary away from origin

offset from origin



Discriminant Functions - two classes

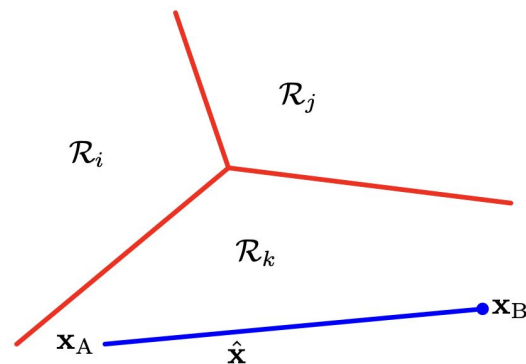


$y(x)$ gives the signed perpendicular distance from the boundary



Discriminant Functions: Multiple Classes

- K-class discriminant $y_k(\mathbf{x}) = \mathbf{w}_k^T \mathbf{x} + w_{k0}$
- Class assignment
 - to C_k if
- Decision boundary:
- Decision regions (for GLM) are convex



Next

Least Squares for Classification

