Evolutionary Game Theory

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In this paper we will discuss Evolutionary Game Theory and how it can explain interactions between members of the same species. We will use the fundamental Hawk-Dove model for analysis. We will also briefly mention how the topic can be applied to other situations.

Evolutionary game theory (EGT) is classical game theory's biological counterpart. It is used to explain and model the natural selection and species evolution. Classical game theory is the study of mathematical models of strategic interactions between rational decision makers in a situation with set rules and outcomes. The individuals make decisions and their payoff is generated based on their choices. All the players in the game will simultaneously reason how the other players react and their strategies will be influenced by that.[1] In evolutionary game theory, the decision makers are biological agents whose behaviors are entirely determined by their genetic traits. Those agents interact with other different organisms to generate fitness. The genetically determined characteristics will be their strategies in the game and their fitness in the current force of natural selection will be their payoff, and their payoff will not only depend on their genetic traits, but also on the strategies of the other biological agents (opponents) they interact with as well. [1]

In evolutionary game theory, there are two major concepts: evolutionary stable strategies (ESS) and replicator dynamics. ESS is a strategy that when adopted by a group of agents, will not be replaced by other agents choosing a different strategy.[3] Given that initial group of agents choose strategy s and another small group of agents (opponents) with population share choose a different strategy *s'*, s is called evolutionary stable if

$$u(s, \epsilon s' + (1-\epsilon)s) > u(s', \epsilon s' + (1-\epsilon)s) \tag{1}$$

where u (*s,s'*) denotes the payoff (fitness) of strategy s given that the opponent chooses strategy s' [2]

More explanation of ESS will be included in the later example of the hawk-dove game.

For replicator dynamic, the concept is that population of the species is divided into groups where agents (replicators) among the same group will have the same strategy. Agents from multiple groups will interact with each other and their payoff will be generated at the end of the game. Next, the reproduction rate of agents is based on their payoff from the previous step. Groups with higher payoff than the average payoff will have more offspring and have their population grown in the future while group will lower payoff will change their strategies over time.

1 Replicator Equation

The replicator equation is a non-linear and non-innovative differential equation. Non-innovative means that the equation does not account for any mutations that could occur and create new strategies within the population. We recognize that accounting for these is more realistic but for the sake of simplicity we are forgoing it.

$$\dot{x}_i = x_i[f_i(x) - \phi(x)],\ \phi(x) = \sum_{j=1}^{n} x_j f_j(x) \tag{2}$$

Equation 1 is the general formula for a replicator equation.X is a vector representing the population distribution where x_i is the percentage of agents who have strategy i. $f_i(x)$ returns the expected fitness of that strategy whereas $\phi(x)$ returns the expected fitness of the entire population. So if the expected fitness for the strategy is more than the population average than the growth rate will be positive, or negative if it is less than. Using this same notion and assuming that the fitness is reflected in the payoff matrix we can rewrite the equation as following

$$\dot{x}_i = x_i[(Ax)_i - x^T Ax] \tag{3}$$

where A is the payoff matrix.[4]

2 Hawk-Dove Game

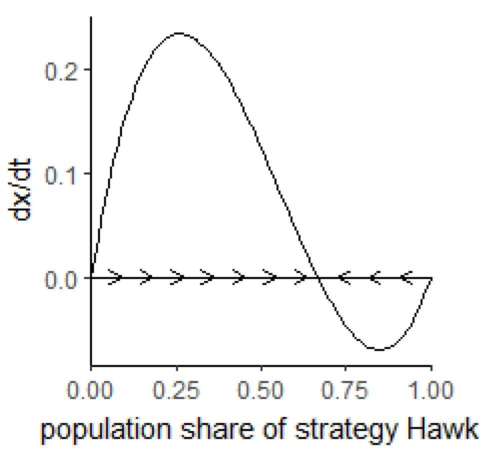
The Hawk-Dove game is the first proposed evolutionary game model by John Maynard Smith. The Hawk strategy is aggressive. It will always choose to fight no matter the circumstances. The dove, however, is docile and try to share the resource at stake. But if contested, it will run away. The game takes in 2 parameters: The value of the resource (V), and the cost of losing a confrontation over this resource (C). V could be representing a mate, food, territory, or anything of that sorts. Whereas C represents injury, fatigue, loss of time, or something related. The general payoff matrix is as follows.

	Hawk	Dove
Hawk	(V-C)/2	V
Dove	0	V/2

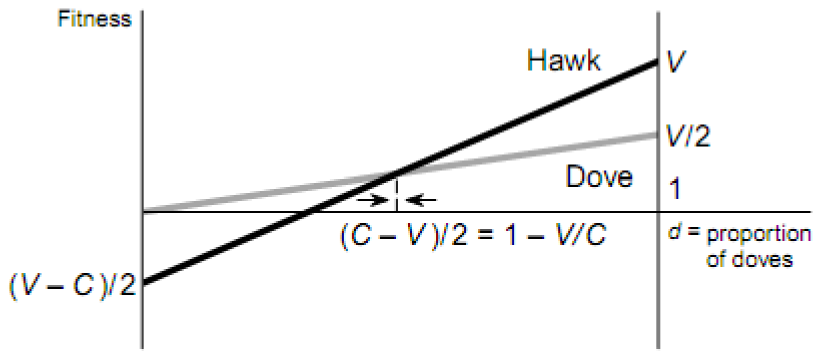
The first possible case for the hawk dove game is when the value of the resource is less than the cost of losing the confrontation over said resource. This is the most reasonable assumption when examining an evolutionary contest because fights over a resource typically end with the loser of the contest dying or suffering a substantial loss. To show this we will let $V = 4$ and $C = 6$. With these values the payoff matrix comes out to be.

	Hawk	Dove
Hawk	-1	4
Dove	0	2

We now plug these values into the standard replicator equations given before. Now using this replicator equation we can plot the dynamics of the hawks in the population by using an Evolutionary Games library in RStudio.



The phase diagram shows where the population share of hawks will tend to and at what rate it will do so under the conditions specified. Also using the ESS method in the library we can find out exactly what proportions of each strategy will be present at the equilibrium point. In this case there will be mixed strategy with a $\frac{2}{3}$ share of hawks and a $\frac{1}{3}$ share of doves. We also know by setting the average payoff of each strategy equal to each other (definition of ESS) and solving for hawk population share we get that it will always be $\frac{v}{c}$ while V ¡ C. As V and C are further apart the ESS will contain more doves and as they get closer together more hawks will be present.



games1.png

This graph shows the average payoff for each population of strategy users as the proportions of each change. The intersection point of these lines is the ESS. All the way to the right is where the population only contains doves. The average payoff here is $\frac{v}{2}$ because all participants will share resources evenly with one another. The average fitness here is higher than at the evolutionary stable point. This goes to show that the ESS is not a globally optimal solution, in this scenario individual gain supersedes overall group fitness, and ends up causing every member to be slightly worse off than they could possibly be.

The second possible case in this game is when the value of the resource is greater than the cost of losing. This scenario is used for modelling contests that do not involve much danger for the participants such as ritualistic mating competitions where the winner gets to reproduce which is highly valued but the loser is not harmed. Now, we switch the values of V and C to model this, so V

= 6 and $C = 4$. With these new values the payoff matrix becomes.

	Hawk	Dove
Hawk	1	6
Dove	0	3

Before the hawk averaged a loss of 1 when interacting with another hawk now it will average a gain of 1. So we can see that no matter what the hawk stands to gain something whereas the dove still has a chance of getting nothing. Now common sense tells us that the hawk strategy has become a pure evolutionary stable strategy because there is no point in playing dove when the cost of losing is so low relative the value of winning. Examining the population dynamics of hawks with these conditions supports this showing that from any initial conditions the population will quickly become entirely hawks.

We can see that as the cost of losing becomes less and less compared to the value of the resource sharing, and not fighting becomes a much less viable option. This is because fighting is less dangerous and now more hawks are present so a dove will be less likely to ever interact with another member willing to share anything. Once C drops enough to where it is equal to V an ESS does not exist, because there cannot be a strategy resistant to being overtaken. Strategy user proportions will fluctuate back and forth. Circumstances like this are similar to games like rock-paper-scissors, where no strategy is truly better than another. When C drops below V like in the second case the game essentially becomes a Prisoner's Dilemma game where it's always in the player's best interest to defect which is equivalent to playing hawk in this game.

This fairly simple game model can be extended to include any extra strategies that can devised to model other species and contests. There are various other strategies that can be modified and interchanged. The bully strategy is always playing hawk until greeted by another hawk, in which the bully will become a dove and run away. The retaliator strategy is always acting dove until threatened by a hawk, then they would fight back. The bourgeois strategy is acting hawk on one's own territory and acting dove on someone else's. These new strategies all can be pitted against each other in a payoff matrix to examine how they interact. For example a retaliator always stands to win the entire resource without a fight when against a bully. The third strategy we are going to append to the model and analyze is referred to as the assessor strategy, which we will call owl for simplicity. An owl will assess an opponent's resource holding potential prior to engagement. Resource holding potential can refer to a number of characteristics such as size, weapons, or sexual traits, depending on type of contest and species involved. To model this we must first make a few assumptions. All members of the contest must equally value the resource, for any given player the opponents is equally likely to have more or less resource holding potential, and lastly that assessing a situation is costless and accurate. This last assumption is reasonable because animals are very likely well versed in what traits determine resource holding potential. For instance, male deer seeing each others body and antler size, or lions engaging in a roaring contest prior to battle. The extended payoff matrix is as follows.

	Hawk	Dove	Owl
Hawk	$\frac{v-c}{2}$	v	$\frac{v-c}{2}$
Dove	0	$\frac{v}{2}$	$\frac{v}{4}$
Owl	$\frac{v}{2}$	$\frac{3v}{4}$	$\frac{v}{2}$

Interactions between hawks and doves remain identical to before. Now, when an owl meets a hawk it will average a payoff of $\frac{v}{2}$ because it will either fight when it knows it will win or get nothing. When an owl meets a dove it will average $\frac{3v}{4}$ because it will either win the full resource or share it. When an owl meets another owl it is the same as meeting a hawk. When a dove meets an owl it averages $\frac{v}{4}$ because it either shares the resource or loses it entirely. Lastly when a hawk meets an owl it's identical to it meeting another hawk. From examining how the population tends under these circumstances it is found that the ESS is a pure strategy of using owl. No matter what values of V and C or what initial conditions are the population will always converge to entirely owls. This demonstrates how using a responsive strategy is basically always more useful than committing to one tactic entirely.

3 Applications and Shortcomings

Animals usually do not battle to the death in real life situations. Instead they engage in a sort of ritual fight. This can be explained by assessor strategies. Both animals test each other so that they minimize the cost.

Evolutionary game theory can also be applied to other topics, although with mixed success.

3.1 Cancer

When applying it to cancer, we interpret it as a game between 2 agents, the malignant cells and the healthy cells. The malignant cells are aggressive like hawks and healthy cells are more like doves. They play for biological energy. The problem, however, is that the fitness numbers are purely theoretical and it is hard to describe the cell behavior mathematically. Also, if we consider the high mutation rate of malignant cells, then theoretically the normal cells will never win, which is not true in real life.[5]

3.2 Economics

When applied to economics we can think of the different strategies as economic behavior. For example, whether someone is bearish or bullish. Or how they approach risk. These different strategies "play" the market and there will be an outcome that will redistribute market resources among these players. The issue is that the replicator dynamics are not easily describable. They do not follow a standard growth model like biological reproduction follows. The market is also unpredictable which is hard to account for in an evolutionary game model.[6]

3.3 Traffic

Traffic flow can be described as a contest between drivers (agents) who have certain traits like being an aggressive, slow, or fast driver, who are competing for a resource that is space on the road. This theoretical model is interesting and could play a part in the automated driving revolution. However, traffic flow has already been described using fluid dynamics and it is unclear whether an evolutionary game model can prove to be more effective than that. I will be excited to see how this new topic of discussion will become a basis for models in the future.[7]

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