

Kennedy Robinson

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In [1]: %pylab nbagg
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sys
from matplotlib.pyplot import *
matplotlib inline

Populating the interactive namespace from numpy and matplotlib
Populating the interactive namespace from numpy and matplotlib

In [2]: meansignal_sum6 = [14237.225,18259.944375, 11346.766781,12437.151875,18981.198438,973918.71563,38672.12,28002.898125,2
6925.4925,31458.516888,64302.200313,21200.245313]

sigma_sum6 = [1323.9876353,1704.8505011,1036.0358449,1163.236844,1810.0083419,93877.922605,3714.435517,2628.0533828,26
14.5975534,2956.4096675,6240.2201715,1981.7102169,]

mean_sigm1_sgm12 = np.array([38309.077813,38317.936563,25857.128625,27230.224688,39779.210313,1813899.9688,76105.50187
5,56027.665928,53907.312589,62334.325938,122499.140688,483584.384063])

sigma_sum12 = np.array([2402.4656588,2480.0498959,2279.0372509,2262.2192534,2451.6150603,71550.124238,3399.0839217,282
9.1542822,2860.0233785,3136.5192019,5006.584389,2604.803448])

meansignal_sum15 = [3530.7127093,3574.3418731,3514.979962,3413.5694601,3450.6277625,54708.325154,3894.2418978,3719.984326,3
729.620887,3952.6294976,4850.2823737,3636.0136737]

meansignal_sum18 = [42929.10125,52591.13625,39176.395625,39439.311563,54062.520938,2133897.5313,97227.083438,73177.932
813,70906.1975,80705.614063,151019.77188,55407.938125]

sigma_sum18 = [5024.0190788,5059.5356173,5043.0863788,4887.347919,4832.4252067,40795.624416,5108.9485567,5154.35407,51
30.7586996,5331.2258265,5618.410005,5100.2832238]

In [3]: plot(log10(sigma_sum6),log10(meansignal_sum6),'o','color='blue')
xlabel('log(noise)')
ylabel('log(signal)')
plot(log10(sigma_sum12),log10(meansignal_sum12),'o','color='orange')
plot(log10(sigma_sum18),log10(meansignal_sum18),'o','color='red')
title('log(signal) vs. log(noise)')

Out[3]: Text(0.5,1,'log(signal) vs. log(noise)')

log(signal) vs. log(noise)

In [4]: xlabel('log(noise)')
ylabel('log(signal)')
%plot(log10(meansignal_sum12),log10(sigma_sum12),'o','color='orange')
title('Aperture 12 log(signal) Sum vs. log(noise) Sum')
plot(log10(sigma_sum12),log10(meansignal_sum12),'o','color='orange')

Out[4]: [
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In [6]: msky = [15.0466244,15.0117606,15.2864994,15.0934313,14.4303997,17.0697806,15.3793803,15.8551184,15.1112006,15.8740031,
15.8160219,15.5764997]
print('The sky signal in DU/pix is', msky)
scale_sky_sigma_DU = np.array([25.66206,25.55962,25.65346,25.04702,24.54859,26.19321,25.86418,26.0384,25.95094,26.1274
6,26.01312,26.10219])
msky_noise = scale_sky_sigma_DU*0.189225
print('The sky noise in DU/pix is',msky_noise)
plt.plot(log10(msky_noise),log10(msky),'o','color='blue')
title('log(sky_signal) As A Function of log(sky_noise)')
xlabel('log(sky_noise)')
ylabel('log(sky_signal)')

The sky signal in DU/pix is [15.0466244, 15.0117606, 15.2864994, 15.0934313, 14.4303997, 17.0697806, 15.3793803, 15.8
551184, 15.1112006, 15.8740031, 15.8160219, 15.5764997]
The sky noise in DU/pix is [4.8559033, 4.83651909, 4.85427597, 4.73952236, 4.64520694, 4.95641016
4.89414946, 4.92711624, 4.91056662, 4.94396862, 4.92232663, 4.9388217 ]

Out[6]: Text(0.5,1,'log(sky_signal)')

log(sky signal) As A Function of log(sky noise)
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```
In [7]: flux_signal = np.array([12497.86969,31523.27656,18937.92719,20397.97969,33247.38844,1806262.25,69144.51813,48851.05375
,47066.47313,55150.62,115339.7281,36533.30938])
flux_noise_div2 = flux_signal/2
flux_noise = np.array([1012.456767,1308.940778,773.1262321,909.5965475,1458.691654,71843.47919,2910.03176,2010.339623,
2031.431518,2266.368145,4684.620777,1562.233958])
flux_noise_div2 = flux_noise/2

print('The flux signal per second is:',flux_signal_div2)
print('The flux noise per second is:', flux_noise_div2)

s_ = n = flux_signal_div2/flux_noise_div2
print('The signal to noise is"', s_ , n )

The flux signal per second is: [ 11748.93485 15761.63828 9468.963595 10198.989845 16623.69422
90313.125 34572.259065 24425.526875 23533.236565 27575.31
57669.86405 18266.65469 ]
The flux noise per second is: [ 506.3283835 654.470389 386.56311605 454.79827375
729.345827 35921.739595 1455.01588 1005.1698115
1015.715759 1133.1840725 2342.3108885 781.11879 ]
The signal to noise is: [ 23.20017779 24.08304263 24.48525886 22.42530465 22.79260894 25.14163109
23.76073474 24.29990097 23.16911632 24.33436074 24.62093168 23.38529975]
```

```
In [8]: plt.plot(log10(flux_signal_div2),log10(flux_noise_div2),'o','color='blue')
xlabel('log(flux_signal)')
ylabel('log(flux_noise)')
title('log(Flux_noise) As A Function Of log(flux_signal)')

Out[8]: Text(0.5,1,'log(Flux_noise) As A Function Of log(flux_signal)')

log(flux signal) As A Function Of log(flux signal)
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What power law should relate alpha and S? How do your observations compare with this expectation? Are there discrepancies at the low S end, explain.

The noise should be related to signal by N = sqrt(S). The signal versus noise plots for the Sum and flux correspond to the expected relation, with minor discrepancies at low signal, appearing to be in the region of flat field problem. The signal versus noise plot however varies greatly from the expected N = sqrt(S), demonstrating a fluctuations in the background noise.

Note: S = f t, where f is "flux" (counts per second). Although we haven't demonstrated it, it's fairly obvious that alpha should depend on S, but not on f or t individually. Can you think of detectors and/or observing circumstances where this elementary assumption might be violated?

This assumption would be violated if the

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In [9]: #sigma_flux^2 = sigma_sum^2 + sigma_msky*area^2
f = np.array([55150.6606,69144.5181,48851.0538,20397.9797,31523.2766])
sig_flux = f/2
print('The flux signal is:',sig_flux)
noise_sum = np.array([3136.519202,3399.083922,2829.154282,2262.219253,2480.049886])
noise_msky = np.array([5.023076635,4.972461072,5.005955173,4.815358822,4.913907651])
AREA_ID = np.array([452.5569406,452.6124031,452.6372563,452.6558938,452.6338156])
noise_sum_squared = noise_sum**2
noise_sky_squared = noise_msky**2
noise_sky_sq_area = noise_sky_squared*AREA_ID
sigma_flux_squared = noise_sum_squared + noise_sky_sq_area
noise_flux = np.sqrt(sigma_flux_squared)
print('The noise of the flux is:', noise_flux)
Sig_to_noise = sig_flux/noise_flux
print('The signal to noise ratio is S/N:', Sig_to_noise)

The flux signal is: [27575.3303 34572.25905 24425.5269 10198.98985 15761.6383 ]
The noise of the flux is: [3138.33894026 3400.72970369 2831.15821779 2264.53791952 2482.25239535]
The signal to noise ratio is S/N: [ 8.78660044 10.16612964 8.62739735 4.50379409 6.34973234]
```

Calculate the signal-to-noise ratio for your final (sky subtracted) counts for each star. Plot both log (S) vs. log (S), and log (N) vs. log (S). Here, the noise N is the same as sigma, and S is sky-subtracted. What relation between S/N and S would be expected to hold for observations limited by photon statistics in the star light? Plot this relation on your graphs, and comment on how well it fits. Most importantly, comment on, and explain, any discrepancies that (may) exist at the bright and faint end of your plot.

The S/N should be proportional to sqrt(S). The graphs have identical shapes, however, the S/N vs. S has half the S/N of the sqrt(S) vs. S plot. There is a kink at higher signals in the S/N vs. S plot. This is likely due to scintillation noise.

```
In [10]: plt.plot(log10(sig_flux),log10(Sig_to_noise),'o','color='blue') #S/N vs. S
xlabel('signal')
ylabel('S/N')
title('S/N As A Function Of Signal')
sig_flux = np.array([27575.3303, 34572.25905,24425.5269, 10198.98985,15761.6383])
sqrt_sig_flux = np.sqrt(sig_flux)
plt.plot(log10(sig_flux),log10(sqrt_sig_flux),'o','color='orange') #sqrt(S) vs. S

Out[10]: [
```

```
In [11]: #This Rmag was calculated using the IDs : V-(V-R)
Rmag = [11.136,10.8672,11.231,12.084,11.61]
flux_ap12_ID8 = [55150.6606,69144.5181,48851.0538,20397.9797,31523.2766]
log_flux = (log10(flux_ap12_ID8))
logFlux = [4.74155072,4.83975775,4.68887394,4.30958716,4.49863135]
```

```
In [12]: plt.plot(Rmag,log10(flux_ap12_ID8),'o')
xlabel('Rmag')
ylabel('log(flux)')
title('log(flux) Vs. R magnitude')
plt.show()

log(flux) Vs. R magnitude
```

```
In [13]: from statistics import mean
import numpy as np
xs = np.array([11.136,10.8672,11.231,12.084,11.61, 3.0958716,4.49863135])
ys = np.array([4.74155072,4.83975775,4.68887394,4.30958716,4.49863135], dtype=np.float64)
def best_fit_slope_and_intercept(xs,ys):
    m = ((mean(xs)*mean(ys)) - (mean(xs)*ys)) /
    ((mean(xs)*mean(xs)) - mean(xs**2))
    b = mean(ys) - m*mean(xs)
    return m, b
m, b = best_fit_slope_and_intercept(xs,ys)
print('The slope of the graph is:',m)
print('The y-intercept is:', b)
m = -2.5*(log10(flux_ap12_ID8))
print('The magnitudes are:', m)
R = -0.44*m + b
print('The brightness is:',R, 'mag arcsec^-2')

The slope of the graph is: -0.4467079068818384
The y-intercept is: 9.701735596910137
The magnitudes are: [-11.8538768 -12.09939439 -11.72218484 -10.77396789 -11.24657838]
The brightness is: [11.79224221 11.79113402 11.79979807 11.793726 11.7226546 11.85250998
11.80269198 11.81724566 11.79428008 11.81781433 11.8166662 11.80877609] Mag Arcsec-2
```

Calculate the sky brightness (mag arcsec^-2) during your observations. Compare with the sky brightness at a "good" site (~22.5 mag arcsec^-2 in V). Explain the differences. How does the limit photometric observations of faint stars in Vectors?

If the sky is too bright, fainter stars will be more difficult to observe as the signal from the sky would dominate the signal from the faint star. Therefore, the difference between the sky brightness and a good site is 22.5 - 11.8 = 10.7. Therefore the star is brighter by 2.5\*10.7 magnitudes.

```
In [14]: sky = np.array([79.517105, 79.33286, 80.784775, 79.764465, 76.26054, 90.20891, 81.27563, 83.789764, 79.85937, 83.88
9565, 83.58315, 82.317345])
b = 9.701735596910137
mag = -2.5*log10(sky)
R = -0.44*mag + b
print(m)
print('The sky brightness is',R, 'Mag Arcsec^-2')

[-11.8538768 -12.09939439 -11.72218484 -10.77396789 -11.24657838]
The sky brightness is [11.79224221 11.79113402 11.79979807 11.793726 11.7226546 11.85250998
11.80269198 11.81724566 11.79428008 11.81781433 11.8166662 11.80877609] Mag Arcsec-2
```

```
In [15]: import array
#scaling the sky signal to correspond to the area of star aperture
msky = [15.0466244,15.0117606,15.2864994,15.0934313,14.4303997,17.0697806,15.3793803,15.8551184,15.1112006,15.8740031,
15.8160219,15.5764997]
msky_array = array.array('f',msky)
scale = float(0.189225)
m = (msky_array/1021)
sca = np.divide(msa,scale)
print('The scaled msky values are', sca)
scale_sky_sigma_DU = [25.66206,25.55962,25.65346,25.04702,24.54859,26.19321,25.86418,26.0384,25.95094,26.12746,26.0131
2,26.10026]
object_sigma = [2341.378,2419.296,2219.008,2203.653,2393.252,70426.26,3331.845,2766.179,2796.649,3069.102,4918.332,254
2.863]

The scaled msky values are: [79.517105 79.33286 80.784775 79.764465 76.26054 90.20891 81.27563
83.789764 79.85937 83.889565 83.58315 82.317345]
```

```
In [16]: #sigma_flux^2 = sigma_sum^2 + sigma_msky*area^2
f = np.array([55150.6606,69144.5181,48851.0538,20397.9797,31523.2766])
sig_flux = f/2
print('The flux signal is:',sig_flux)
noise_sum = np.array([3136.519202,3399.083922,2829.154282,2262.219253,2480.049886])
noise_msky = np.array([5.023076635,4.972461072,5.005955173,4.815358822,4.913907651])
AREA_ID = np.array([452.5569406,452.6124031,452.6372563,452.6558938,452.6338156])
noise_sum_squared = noise_sum**2
noise_sky_squared = noise_msky**2
noise_sky_sq_area = noise_sky_squared*AREA_ID
sigma_flux_squared = noise_sum_squared + noise_sky_sq_area
noise_flux = np.sqrt(sigma_flux_squared)
print('The noise of the flux is:', noise_flux)
Sig_to_noise = sig_flux/noise_flux
print('The signal to noise ratio is S/N:', Sig_to_noise)

The flux signal is: [27575.3303 34572.25905 24425.5269 10198.98985 15761.6383 ]
The noise of the flux is: [3138.33894026 3400.72970369 2831.15821779 2264.53791952 2482.25239535]
The signal to noise ratio is S/N: [ 8.78660044 10.16612964 8.62739735 4.50379409 6.34973234]
```

```
In [17]: plt.plot(f,Sig_to_noise,'o','color='blue')
xlabel('signal')
ylabel('S/N')
title('S/N As A Function Of Signal')

Out[17]: Text(0.5,1,'S/N As A Function Of Signal')

S/N As A Function Of Signal
```

```
In [18]: # S/N = ((flux/2 * gain)**2) / (V*(flux/2 * gain)**2 + (msky/2 * gain * area)**2 + ((sigma_singleR0 * gain)**2 * area))
# a
sky = np.array([15.0466244,15.0117606,15.2864994,15.0934313,14.4303997,17.0697806,15.3793803,15.8551184,15.1112006,15.8740031,
15.8160219,15.5764997])
msky_signal_div2 = msky_signal_ida/2
single_readout = float(1.353511648496958)

In [19]: AREA_ID = np.array([452.5569406,452.6124031,452.6372563,452.6558938,452.6338156])

In [20]: sigma_fluxes_ida= np.array([2266.368145,2910.03176,2010.339623,909.5965475,1308.940778])
sigma_fluxes_div2 = sigma_fluxes_ida/2
print(sigma_fluxes_div2)
print(noise_flux)

[1133.1840725 1455.01588 1005.1698115 454.79827375 654.470389 ]
[3138.33894026 3400.72970369 2831.15821779 2264.53791952 2482.25239535]
```

```
In [21]: # For S/N = 10
t10_pos = (B + np.sqrt((B**2)-(4*A*C)))/(2*A)
print(t10_pos)

# For S/N = 20
f1 = float(0.000000005)
single_readout = float(1.353511648496958)
sky_div2 = msky_DU_Pix/2
a = f1*g
b = sky_div2*g*AREA_ID
c = (((single_readout*g)**2)*AREA_ID)
A = a**2
B = -(400)*(a + b)
C = -(1000)*c
t20_pos = (B + np.sqrt((B**2)-(4*A*C)))/(2*A)
print(t20_pos)

# For S/N = 50
f1 = float(0.000000005)
single_readout = float(1.353511648496958)
sky_div2 = msky_DU_Pix/2
a = f1*g
b = sky_div2*g*AREA_ID
c = (((single_readout*g)**2)*AREA_ID)
A = a**2
B = -(2500)*(a + b)
C = -(10000)*c
t50_pos = (B + np.sqrt((B**2)-(4*A*C)))/(2*A)
print(t50_pos)

# For S/N = 100
f1 = float(0.000000005)
single_readout = float(1.353511648496958)
sky_div2 = msky_DU_Pix/2
a = f1*g
b = sky_div2*g*AREA_ID
c = (((single_readout*g)**2)*AREA_ID)
A = a**2
B = -(10000)*(a + b)
C = -(10000)*c
t100_pos = (B + np.sqrt((B**2)-(4*A*C)))/(2*A)
print(t100_pos)

[0. 0. 0. 0. 0.]
[0. 0. 0. 0. 0.]
[0. 0. 0. 0. 0.]
[0. 0. 0. 0. 0.]
```

```
In [22]: #faintest star that can be observed in 10 minutes with S/N = 100
sn = 100
t = 600
f = f/sn
m = -2.5*log10(f)
print(R)
R = 9.701735596910137
print(R)

-1.9453781259591092
10.55770197332145
```

What is the faintest star that can be observed in 10 minutes of total observing time for both Victoria and a dark site with at least 1% accuracy (S/N > 10)? The above calculations depend on both the star aperture and sky annulus sizes. Explain why.

The faintest star that can be observed in 10 minutes has a brightness of 10.56. The above calculations depend on the star aperture as the aperture of an optical system is what determines the angle of the incoming rays in the image plane. The size of the aperture alters the amount of overall light that will reach the lens, ultimately affecting the brightness of the image. A larger aperture will allow more photons to pass and hence create a brighter image. Therefore the above calculations depend on aperture size as the exposure time needed to reach a certain brightness would be smaller if the aperture is larger. The above calculations depend on sky annulus as its area is used to compute the background data, hence, allowing the signal from the background to be eliminated from the incoming photons during the exposure.

From first principles, show that the observed count rate for one of your stars "agrees" with what would have been predicted. Hint: V = 0 corresponds to ~1000 photons cm^-2 s^-1 A^-1 incident on the top of the atmosphere at 5500A, to an accuracy of +/-5%

SO R = 0 corresponds to 1000 photons/2secs^-1/p(0.435)/2cm^-2 s^-1 A^-1

=1000 photons / 1783.40361 cm^-2 s^-1 A^-1 = 0.560725566 ph/cm^-2 s^-1 A^-1

For very bright stars and high signal-to-noise ratios, another source of noise appears: "scintillation noise" -- a form of noise due to short timescale atmospheric fluctuations. Explain what this noise would do to your S/N vs. S plot.

Scintillation noise would create a kink in the S/N vs. S plot at higher signal levels as scintillation noise only appears for brighter stars.

Can you see any evidence of this in your data? If not, why not?

There appears to be scintillation noise present roughly around a signal of 5500 and a S/N between 8 and 9.

Why is scintillation noise not observed for fainter stars?

Scintillation noise is not observed for fainter stars as there would not be enough photons absorbed by the detector to create a speckles.

Explain why errors due to not flat fielding, or to non-photometric observing conditions, appear in the S/N-S plot in a similar fashion to scintillation noise.

Flat fielding removes the noise created by pixel to pixel variations throughout the given array as well as the noise created by dust and scratches. Therefore, if flat fielding is not done the image will appear grainy. Non-photometric conditions would consist of cloudy skies with a transparency over 2%. This would mean the incoming photons would be blocked by the elements present in the unclear sky, resulting in blotchy image. Scintillation noise is a second order effect caused by higher altitude turbulence. Scintillation noise creates a propagation in the curvature of the wavefront resulting in speckles. Therefore, all 3 errors mentioned created "speckle" like images and will therefore appear in the S/N vs. S plot in a similar fashion.

In the case where many stars exist on a single CCD frame, and where there is at least one bright, non-saturated, uncrowded star on the frame, it is better to use the brighter star as a reference for the point spread function, and then use least squares (or maximum likelihood) fitting to obtain the brightness of other stars relative to this star. Even for uncrowded fields, this results in significantly better photometry, especially for faint stars. Why?

In non-coherent imaging systems such as a telescope, the image intensity formation is linear meaning that if two objects are observed at the same time, the resulting image will be equal to the sum of the two independent objects. It is easier to determine the magnitude of a brighter star, therefore, taking the brighter star as the template for the point spread function would mean that the remaining intensity would correspond to the fainter objects in the image. The method of least squares approximates the solution by minimizing the sum of the squares of the remainders left over in the results of every calculation. Therefore this method would be ideal for finding the brightness of faint stars as the intensity would be determined through the remainders.

Assume that measurements of the brightness of a star are completely dominated by background noise (sky or readout). Further assume that the sky level is well measured. For a 2D Gaussian star profile, how does the S/N of a light measurement depend on star aperture size? (Here you will take account of the fact that apertures of different sizes collect different amounts of brightness from a star) Plot S/N vs. aperture size and comment. Explain why there is a maximum S/N. Derive the value of this maximum S/N, and the aperture radius at which it is achieved. Compare this "optimal" radius to the star aperture radius that you ended up using, and comment

In a background dominated system S/N = (SO/((ph)^2)). Therefore if sky or readout noise dominates, the signal to noise of a brightness measurement will decrease with larger aperture radii.

There will be a maximum signal to noise as there will be a maximum aperture.

```
In [23]: AREA_ID = np.array([452.5569406,452.6124031,452.6372563,452.6558938,452.6338156])
Sig_to_noise = np.array([8.78660044, 10.16612964, 8.62739735, 4.50379409, 6.34973234])

plt.plot(AREA_ID,Sig_to_noise,'o','color='blue')
xlabel('Area')
ylabel('S/N')
title('S/N As A Function Of Area')

Out[23]: Text(0.5,1,'S/N As A Function Of Area')

S/N As A Function Of Area
```

In the case of photoelectric photometry (or, for that matter, any kind of photometry using a single element detector), one must alternate between measurements of star and sky. Consider a fixed total amount of time available to observe a star (and its associated sky). Suppose that it is the ratio of star flux to sky flux that is the limiting factor. What is the optimum amount of time to divide the observing time between the object and sky so as to achieve maximum S/N. (Hints: here S/N is the signal-to-noise ratio of the final object brightness -- i.e., the S/N of (O+S)-S where O is object and S is sky brightness. Write down a general expression for S/N. Maximize.)

You should observe Star first, then sky near it. If the sky is bright the sky can be observed less frequently.