Toy Design Problem

Foundations of Computer Science - Final Assignment

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The states of the following machines represent which direction the levers of the toy are facing, in order from left to right. This means a state of LRL represents the Levers 1 and 3 being oriented left, and Lever 2 being oriented right. Since some of these physical states can be achieved in different manners, if a state transition results in a final state it is labeled in the same manner, but with an f appended to the state name. The transitions themselves are labeled 1 or 0 depending on the tube through which the ball entered the toy. When these transitions occur, the levers' orientations are changed, thus the state of the toy is changed.

1 Finite State Machine

I have designed a 5-tuple in the following manner:

$M(Q, \Sigma, q_0, \delta, F)$
$Q = \{LLL, RLL, LRR, LRL, RRR, RRL, RLR, LRLf, LLRf, RRLf, LLLf, RLRf, RLLf\}$
$\Sigma = \{0, 1\}$
$q_0 = LLL$
$F = \{LRLf, LLRf, RRLf, LLLf, RLRf, RLLf\}$
$\delta : Q \times \Sigma \to Q$

δ	0	1
LLL	RLL	LRR
RLL	LRL	RRR
LRR	RRR	$LRLf$
LRL	RRL	$LLRf$
RRR	$LLRf$	$RRLf$
RRL	$LLLf$	$RLRf$
RLR	LRR	$RLLf$
$LRLf$	RRL	$LLRf$
$LLRf$	RLR	$LLLf$
$RRLf$	$LLLf$	$RLRf$
$LLLf$	RLL	LRR
$RLRf$	LRR	$RLLf$
$RLLf$	LRL	RRR

To show this DFA is minimal we start by partitioning the states into the final and non-final states:

$$\Pi_0 : {}^{A}[LLL\ RLL\ LRR\ LRL\ RRR\ RRL\ RLR]\ {}^{B}[LRLf\ LLRf\ RRLf\ LLLf\ RLRf\ RLLf]$$

From here we look at which sets each element maps to. For example, when the inputs are 0 and 1, the outputs are elements of the sets A and A for LLL, A and B for LLR, A and A for RLL, etc. Since LLL and RLL map to the same sets for both inputs, they will stay together. However, LRR will branch off into a new set along with any other states which transition to something in the set A for input 0, and B for input 1. Note that if elements of A have the same mappings as elements of B they do not become members of the same set at the next level of the process since they came from different sets. This means the next step gives:

$$\Pi_1 : {}^{C}[LLL\ RLL]\ {}^{D}[LRR\ LRL\ RLR]\ {}^{E}[RRR\ RRL]\ {}^{F}[LRLf\ LLRf\ RLRf]\ {}^{G}[RRLf]\ {}^{H}[LLLf\ RLLf]$$

We execute this process again until we find no change. Note that the G set contains one element. This is called a singleton, and is as simplified as it can get. If after this process we only have singletons, then the machine is already minimized.

$$\Pi_2 : {}^{I}[LLL]\ {}^{J}[RLL]\ {}^{K}[LRR\ LRL]\ {}^{L}[RLR]\ {}^{M}[RRR]\ {}^{N}[RRL]$$
$$ {}^{O}[LRLf]\ {}^{P}[LLRf\ RLRf]\ {}^{Q}[RRLf]\ {}^{R}[LLLf]\ {}^{S}[RLLf]$$

$$\Pi_3 : {}^{T}[LLL]\ {}^{U}[RLL]\ {}^{V}[LRR]\ {}^{W}[LRL]\ {}^{X}[RLR]\ {}^{Y}[RRR]\ {}^{Z}[RRL]$$
$$ {}^{\alpha}[LRLf]\ {}^{\beta}[LLRf]\ {}^{\gamma}[RLRf]\ {}^{\phi}[RRLf]\ {}^{\sigma}[LLLf]\ {}^{\zeta}[RLLf]$$

We see the partition has been reduced to only singleton's, so it cannot be reduced further, and also the DFA is minimized.

2 Mealy Machine

The states and transitions in this Mealy machine are the same as for the DFA, however there are no final states. Instead, there is an output function. The possible outputs are A and B, corresponding to the ball exiting through either of these tubes.

I have designed a 6-tuple in the following manner:

$$M(Q, \Sigma, \Delta, q_0, \delta, \epsilon)$$
$$Q = \{LLL, RLL, LRR, LRL, RRR, RRL, LLR, RLR\}$$
$$\Delta = \{A, B\}\Sigma \qquad = \{0, 1\}$$
$$q_0 = LLL$$
$$\delta : Q \times \Sigma \to Q$$

δ, ϵ	0	1
LLL	RLL, A	LRR, A
RLL	LRL, A	RRR, A
LRR	RRR, A	LRL, B
LRL	RRL, A	LLR, B
RRR	LLR, B	RRL, B
RRL	LLL, B	RLR, B
LLR	RLR, A	LLL, B
RLR	LRR, A	RLL, B

Now we must ensure the Mealy machine is minimized. We start with the trivial partition where everything is in one set:

$$\Pi_0 : \ [LLL\ RLL\ LRR\ LRL\ RRR\ RRL\ LLR\ RLR]$$

Now we split this up based on the output from the different states (for example all states which output an A and B for inputs 0 and 1 respectively will be grouped together).

$$\Pi_1 : \ {}^{A}[LLL\ RLL]\ {}^{B}[LRR\ LRL\ LLR\ RLR]\ {}^{C}[RRR\ RRL]$$

Finally, we continue in the same manner as for a DFA which was discussed in the DFA section.

$$\Pi_2 : \ {}^{D}[LLL]\ {}^{E}[RLL]\ {}^{F}[LRR\ LRL]\ {}^{G}[LLR]\ {}^{H}[RLR]\ {}^{I}[RRR]\ {}^{J}[RRL]$$

$$\Pi_3 : \ {}^{K}[LLL]\ {}^{L}[RLL]\ {}^{M}[LRR]\ {}^{N}[LRL]\ {}^{O}[LLR]\ {}^{P}[RLR]\ {}^{Q}[RRR]\ {}^{R}[RRL]$$

Every set is a singleton, thus the machine is indeed minimized.