Kenny Roffo **Quiz 3 Formula Sheet**

Newton's Divided Differences:
$P(x) = f[x_1] + f[x_1x_2](x - x_1) + f[x_1x_2x_3](x - x_1)(x - x_2) + ...$

$$f[x_k] = f(x_k)$$
$$f[x_kx_{k+1}] = \frac{f[x_{k+1}] - f[x_k]}{x_{k+1} - x_k}$$
$$f[x_kx_{k+1}x_{k+2}] = \frac{f[x_{k+1}x_{k+2}] - f[x_kx_{k+1}]}{x_{k+2} - x_k}.....$$

Interpolation Error Formula: $f(x) - P(x) = \frac{(x-x_1)(x-x_2)...(x-x_n)}{n!} f^{(n)}(c)$
Here c must be between the largest and smallest x_n.

Chebyshev Nodes: The choices for n points on the interval $[-1, 1]$ to interpolate a function with the minimized maximum error are given by $x_i = \cos\left(\frac{(2i-1)\pi}{2n}\right)$ for $i = 1, 2, ..., n$. The minimum error is then $\frac{1}{2^{n-1}}$

Continuous Least Squares
$E = ||f(x) - P(x)||^2 = \int_a^b (f(x) - P(x))^2 \, \mathrm{d}x$ $\quad$ $< f, g >= \int_{-1}^1 f(x)g(x) \, \mathrm{d}x$
Beginning with $\{1, x, x^2, ...\}$ the Gram-Schmidt process yields $\{1, x, x^2 - \frac{1}{3}, x^3 - \frac{3}{5}, x^4 - \frac{6}{7}x^2 + \frac{3}{35}, ...\}$ for the Legendre Polynomials. (This is on $[-1, 1]$ only!); The solution to the least square error problem is $P(x) = a_0l_0(x) + a_1l_1(x) + ...$ where $a_i = \frac{<f,l_i>}{<l_i,l_i>}$

Discrete Least Squares
Use general forms of polynomials $y = a + bx$ for linear regression, $y = a + bx + cx^2$ for quadratic regression, etc. Make a system of equations by plugging in x and y for each point. This system forms a matrix equation: $A\overline{x} = b$ This system can be solved using $A^TA\overline{x} = A^Tb$. The solution can be found using the augmented matrix $[A^TA|A^Tb]$.