Abstract Algebra

Homework 5: 8.5, 8.11, 8.24, 1

Kenny Roffo

Due March 6

8.5) Write down all the elements of S_4, and indicate which ones are 1in A_4. Check your results against Theorem 8.5.

$S_4 = \{(1234), (1243), (1324), (1342), (1423), (1432), (123), (132), (124), (142), (134), (143),$

$(234), (243), (12), (13), (14), (23), (24), (34), (12)(34), (13)(24), (14)(23), e\}$

$A_4 = \{(123), (132), (124), (142), (134), (143), (234), (243), (12)(34), (13)(24), (14)(23), e\}$

8.11a) Give an example of two elements x and y in S_9 such that $o(x) = o(y) = 5$ and $o(xy) = 9$:

Consider $x = (12345)$, $y = (56789)$. Then $o(x) = o(y) = 5$, and $xy = (12345)(56789) = (123456789)$. Then $o(xy) = 9$.

8.11b) What is the largest order that an element of S_9 can have?

The largest order of an element of S_9 would be of the form (4-cycle)(5-cycle) together which has order 20.

8.24) If H and K are subgroups of a group G then HK denotes the set of all elements of G that can be written in the form hk, with $h \in H$ and $k \in K$. Find subgroups H and K of S_3 such that HK is not a subgroup of S_3:

Consider $H = \{(12), e\} \leq S_3$, $K = \{(13), e\} \leq S_3$. Then $HK = \{e, (12), (13), (12)(13)\} = \{e, (12), (13), (132)\}$. Note that $(132)^{-1} = (123)$, but (123) is not an element of HK. Thus HK is not closed under inverses, so HK is not a subgroup of S_3.

1) For all subgroups H of S_3, find $N_{S_3}(H)$.

$H_1 = \{e\}$ is a normal subgroup of S_3 with normalizer $N_{S_3}(H_1) = S_3$ since e commutes with everything in the group (this implies ${}^eS_3 = S_3$).

$H_2 = \{(12), e\}$ is a subgroup of S_3 with normalizer $N_{S_3}(H_2) = H_2$ since (12) is its own inverse and e is always in the normalizer of a group.

$H_3 = \{(13), e\}$ is a subgroup of S_3 with normalizer $N_{S_3}(H_3) = H_3$ since (13) is its own inverse and e is always in the normalizer of a group.

$H_4 = \{(23), e\}$ is a subgroup of S_3 with normalizer $N_{S_3}(H_4) = H_4$ since (23) is its own inverse and e is always in the normalizer of a group.

$H_5 = \{(123), (132), e\}$ is a normal subgroup of S_3 with normalizer $N_{S_3}(H_5) = S_3$ since (123) is the inverse of (132) and e is always in the normalizer of a group.

S_3 is a normal subgroup of S_3 with normalizer $N_{S_3}(S_3) = S_3$ since $N_G(G) = G$ for all groups G.