Complex Analysis

Homework 4: 1.5) 12, 20, 46a

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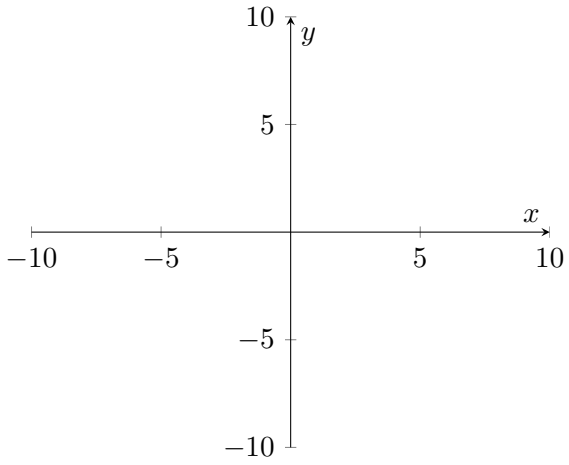
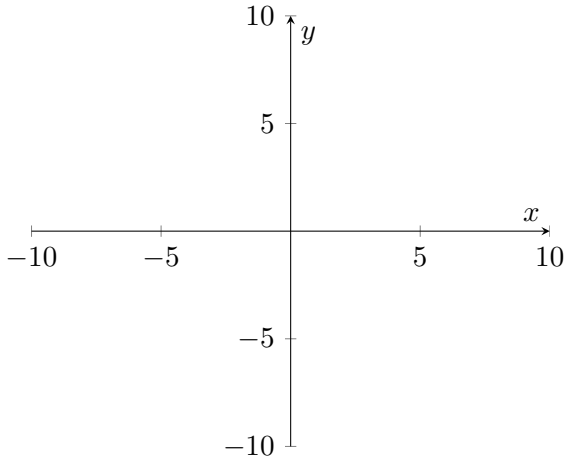
12) Sketch the graph of the given equation in the complex plane: $\arg(z) = \pi/4$

This is simply the positive portion of the line $y = x$, as this is the set of z which have an angle of $\pi/4$ above the x-axis.



20) Sketch the set S of points in the complex plane satisfying $\mathrm{Im}(z) < \mathrm{Re}(z)$. Determine whether the set is open, closed, a domain or connected.



This set is open, and connected, but not closed, therefore it is a domain.

46a) Suppose S_1 and S_2 are open sets in the complex plane. Is the union $S_1 \cup S_2$ an open set?

Yes. Let $z \in S_1 \cup S_2$. Then either $z \in S_1$ or $z \in S_2$. Without loss of generality, assume $z \in S_1$. Then z is an interior point, since S_1 is an open set, and thus there exists a neighbor hood of z entirely contained in S_1. But since all points in S_1 are in $S_1 \cup S_2$, such a neighborhood also lies entirely in $S_1 \cup S_2$, thus z is an interior point of $S_1 \cup S_2$. This implies that every point in $S_1 \cup S_2$ is an interior point, thus $S_1 \cup S_2$ is an open set.