Complex Analysis

Homework 1: 1.1) 16, 30, 42

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16) Write the number $\frac{(4+5i)+2i^3}{(2+i)^2}$ in the form $a+bi$.

$$\begin{aligned}
\frac{(4+5i)+2i^3}{(2+i)^2} &= \frac{(4+5i)-2i}{4+4i-1} \\
&= \frac{4+3i}{3+4i} \\
&= \frac{(4+3i)(3-4i)}{(3+4i)(3-4i)} \\
&= \frac{12-16i+9i+12}{9+16} \\
&= \frac{24-7i}{25} \\
&= \frac{24}{25} - \frac{7}{25}i
\end{aligned}$$

30) Let $z = x + yi$. Express $\mathrm{Im}(\bar{z}^2 + z^2)$ in terms of x and y.

$$\begin{aligned}
\bar{z}^2 + z^2 &= (x - yi)^2 + (x + yi)^2 \\
&= x^2 - 2xyi - y^2 + x^2 + 2xyi - y^2 \\
&= 2(x^2 - y^2) + 0i \\
\implies \mathrm{Im}(\bar{z}^2 + z^2) &= 0
\end{aligned}$$

42) Solve the equation $\frac{z}{1+\bar{z}} = 3+4i$ for $z = a+bi$.
Note that $z = a+bi$ and $\bar{z} = a-bi$. Plugging in, we have

$$\begin{aligned}
\frac{a+bi}{1+a-bi} &= 3+4i \\
\Longrightarrow \ a+bi &= (3+4i)((1+a)-bi) \\
&= 3+3a-3bi+4i+4ai+4b \\
&= (3a+4b+3)+(4a-3b+4)i
\end{aligned}$$

By the definition of equality of complex numbers, we see that the above equality implies

$$a = 3a+4b+3 \qquad \text{and} \qquad b = 4a-3b+4$$

We see the second equation yields the value of b in terms of a:

$$\begin{aligned}
b &= 4a-3b+4 \\
\Longrightarrow 4b &= 4a+4 \\
\Longrightarrow b &= a+1
\end{aligned}$$

and plugging in for b in the first equation we can solve for a:

$$\begin{aligned}
a &= 3a+4b+3 = 3a+(4a+4)+3 \\
\Longrightarrow \ -6a &= 7 \\
\Longrightarrow a &= -\frac{7}{6}
\end{aligned}$$

Thus, simply plugging in the value of a to get b we have

$$z = -\frac{7}{6} - \frac{1}{6}i$$