Complex Analysis

Homework 1: 1.1) 16, 30, 42

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16) Write the number $\frac{(4+5i)+2i^3}{(2+i)^2}$ in the form $a+bi$.

$$\begin{aligned}\frac{(4+5i)+2i^3}{(2+i)^2} &= \frac{(4+5i)-2i}{4+4i-1}\\ &= \frac{4+3i}{3+4i}\\ &= \frac{(4+3i)(3-4i)}{(3+4i)(3-4i)}\\ &= \frac{12-16i+9i+12}{9+16}\\ &= \frac{24-7i}{25}\\ &= \frac{24}{25}-\frac{7}{25}i\end{aligned}$$

30) Let $z = x+yi$. Express $\mathrm{Im}(\bar{z}^2+z^2)$ in terms of x and y.

$$\begin{aligned}\bar{z}^2+z^2 &= (x-yi)^2+(x+yi)^2\\ &= x^2-2xyi-y^2+x^2+2xyi-y^2\\ &= 2(x^2-y^2)+0i\\ \implies \mathrm{Im}(\bar{z}^2+z^2) &= 0\end{aligned}$$

42) Solve the equation $\frac{z}{1+\bar{z}} = 3+4i$ for $z = a+bi$.
Note that $z = a+bi$ and $\bar{z} = a - bi$. Plugging in, we have

$$\begin{aligned} \frac{a+bi}{1+a-bi} &= 3+4i \\ \implies a+bi &= (3+4i)((1+a)-bi) \\ &= 3+3a-3bi+4i+4ai+4b \\ &= (3a+4b+3)+(4a-3b+4)i \end{aligned}$$

By the definition of equality of complex numbers, we see that the above equality implies

$$a = 3a+4b+3 \qquad \text{and} \qquad b = 4a-3b+4$$

We see the first equation yields the value of b in terms of a:

$$\begin{aligned} b &= 4a-3b+4 \\ \implies 4b &= 4a+4 \\ \implies b &= a+1 \end{aligned}$$

and plugging in for b in the first equation we can solve for a:

$$\begin{aligned} a &= 3a+4b+3 = 3a+(4a+4)+3 \\ \implies -6a &= 7 \\ \implies a &= -\frac{6}{7} \end{aligned}$$

Thus, simply plugging in the value of a to get b we have

$$z = -\frac{6}{7} - \frac{1}{6}i$$