Complex Analysis

Homework 11: 3.2) 22, 32, 38

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22) Show that the function $f(z) = |z|$ is nowhere differentiable.

Let $z = x + iy$ be any point in the complex plain, and let $\Delta z = \Delta x + i\Delta y$.
Then

$$\begin{aligned} f(z+\Delta z) - f(z) &= |z + \Delta z| - |z| \\ &= |x + iy + \Delta x + i\Delta y| - |x + iy| \\ &= |x + \Delta x + i(y + \Delta y)| - |x + iy| \\ &= \sqrt{(x+\Delta x)^2 + (y+\Delta y)^2} - \sqrt{x^2+y^2} \\ &= \sqrt{x^2 + 2x\Delta x + \Delta x^2 + y^2 + 2y\Delta y + \Delta y^2} - \sqrt{x^2+y^2} \end{aligned}$$

Having a *lot* of trouble on this one.

32) Prove that $\frac{\mathrm{d}}{\mathrm{d}z}[f(z) + g(z)] = f'(z) + g'(z)$.

Let $z = x + iy$ be a complex number and let f and g be functions differentiable at z. Then $lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z} = f'(z)$ and $lim_{\Delta z \to 0} \frac{g(z+\Delta z) - g(z)}{\Delta z} = g'(z)$ exist. We see

$$\begin{aligned} lim_{\Delta z \to 0} \frac{f(z+\Delta z) + g(z+\Delta z) - f(z) - g(z)}{\Delta z} &= lim_{\Delta z \to 0} \left(\frac{f(z+\Delta z) - f(z)}{\Delta z} + \frac{g(z+\Delta z) - g(z)}{\Delta z} \right) \\ &= lim_{\Delta z \to 0} \left(\frac{f(z+\Delta z) - f(z)}{\Delta z} \right) \\ &+ lim_{\Delta z \to 0} \left(\frac{g(z+\Delta z) - g(z)}{\Delta z} \right) \\ &= f'(z) + g'(z) \end{aligned}$$

Therefore, since f'(z) and g'(z) exist, the limit exists, and thus the derivative of the sum of functions is the sum of their derivatives (at a point z).

38 a) Let $f(z) = z^2$. Write down the real and imaginary parts of f and f'. What do you observe?

$\text{Re}(f) = x^2 - y^2$ and $\text{Im}(f) = 2ixy$. Also, $\text{Re}(f') = 2x$ and $\text{Im}(f') = 2iy$. I notice that the real and imaginary parts of f' are the derivative with respect to x of the real and imaginary parts of f.

38 b) Repeat part **a** for $f(z) = 3iz + 2$.

$$3iz + 2 = 3i(x + iy) + 2 = (-3y + 2) + i3x$$

$\text{Re}(f) = -3y + 2$ and $\text{Im}(f) = i3x$. Also, $\text{Re}(f') = 0$ and $\text{Im}(f') = 3i$. I notice the same thing as in part **a**.

38 c) Make a conjecture about the relationship between real and imaginary parts of f versus f'.

The real and imaginary parts of f' are the derivatives with respect to the x of the real and imaginary parts of f respectively.