Two-Particle Systems

Dylan Mcintyre and Kenny Roffo

SUNY Oswego

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- To use the Schrödinger equation we must use know the Hamiltonian:

$$H = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\mathbf{r}_1, \mathbf{r}_2, t)$$

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- Here, ψ satisfies the time-independent Schrödinger wave equation:

$$-\frac{\hbar^2}{2m_1}\nabla_1^2\psi - \frac{\hbar^2}{2m_2}\nabla_2^2\psi + V\psi = E\psi$$

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- We represent this by a wave function which doesn't assume either particle is in a particular state:

$$\psi \pm (\mathbf{r}_1, \mathbf{r}_2) = A \left[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) \pm \psi_b(\mathbf{r}_1)\psi_a(\mathbf{r}_2) \right]$$

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- As a consequence, we see we cannot have two Fermions cannot occupy the same state else $\psi_a = \psi_b$ and

$$\psi_-(\mathbf{r}_1, \mathbf{r}_2) = A \left[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) - \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)\right] = 0$$

- This result is the famous *Pauli Exclusion Principle*

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Thank you!