

Trees

Tree is a non-linear hierarchical data structure.

Root

Parent

child

siblings

Ancestor

Descendant

Degree

Height

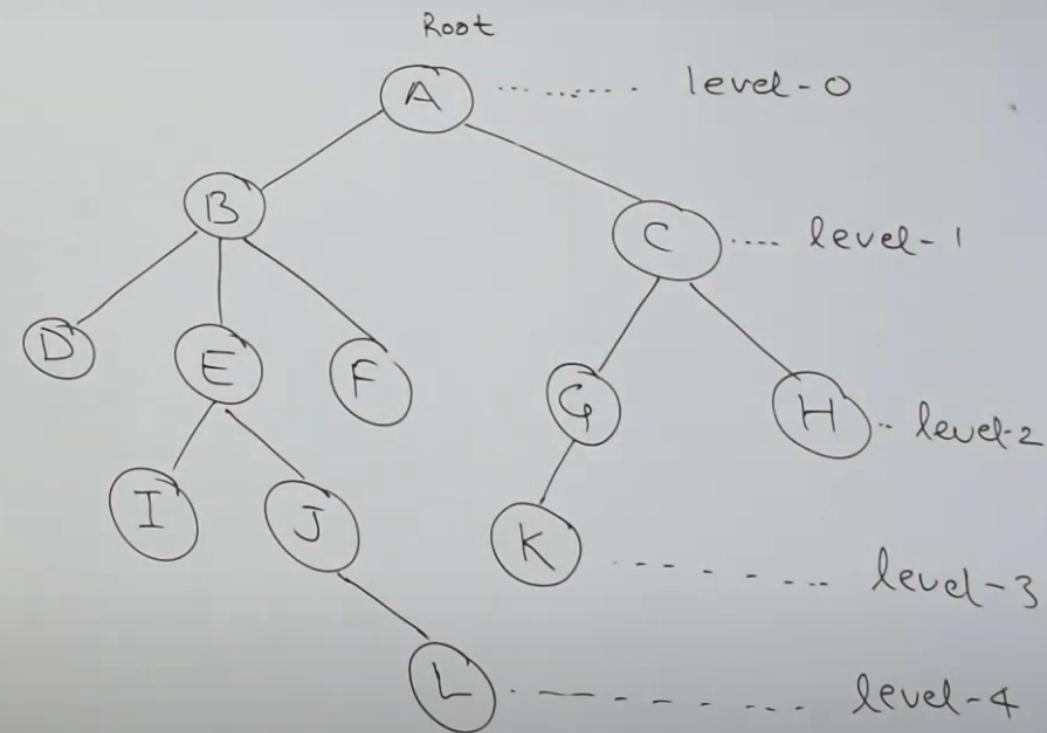
Depth

Internal Node

Leaf Node

subtree

Forest



$$n = 2^{h+1} - 1$$

$$n+1 = 2^{h+1}$$

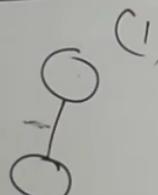
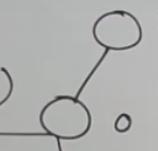
$$h+1 = \log_2(n+1)$$

$$\boxed{h = \log_2(n+1) - 1}$$

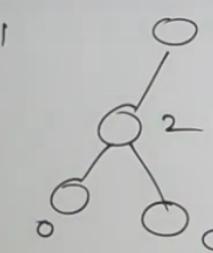
$$h=1$$

$$n=2$$

$$\boxed{h=n-1}$$

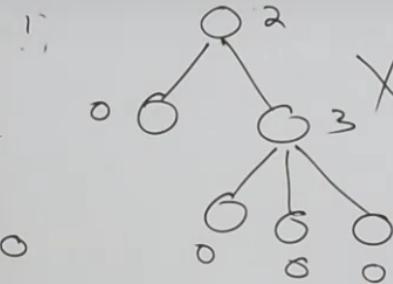


(1)



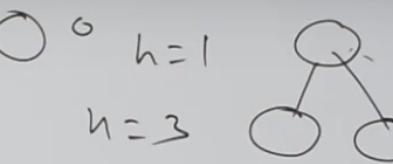
$$h=2$$

$$n=3$$



$$h=3$$

$$n=4$$



$$h=1$$

$$n=3$$

$$h=2$$

$$n=7$$

$$\frac{a(r^n - 1)}{r-1} \quad r \geq 1$$

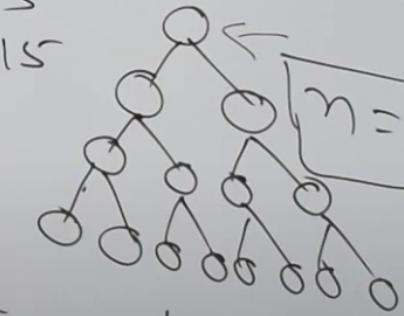
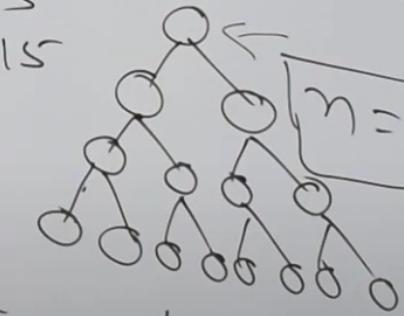
$$r = \frac{2}{1} < 2$$

$$a = 1$$

$$\frac{1(2^{h+1} - 1)}{2-1}$$

$$h=3$$

$$n=15$$



$$\log_2(n+1) - 1 \leq h \leq n-1$$

$$\underline{\underline{O(\log_2 n)}} \leq O(n)$$

$$\boxed{1 + 2 + 4 + 8 + \dots + 2^{h+1}}$$

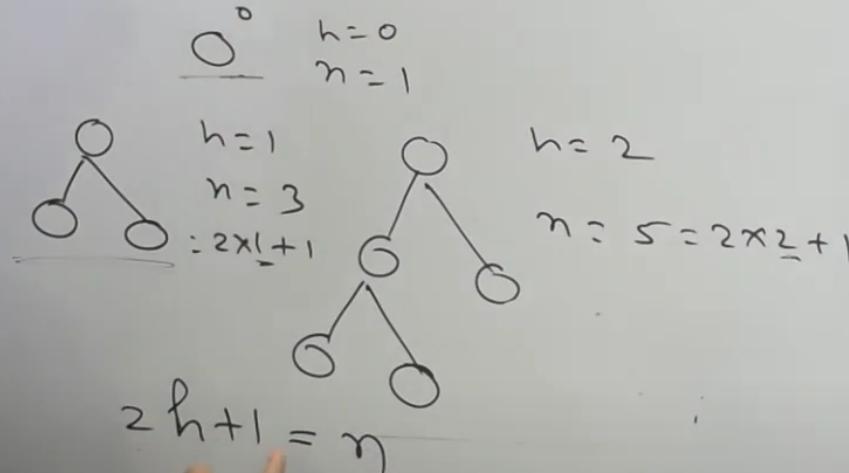
$$\boxed{1 + 2 + 2^2 + 2^3 + \dots + 2^{h+1}}$$



1. Full Binary Tree / Strict B.T. ($0, 2$)

2. Complete Binary Tree

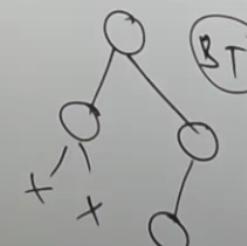
- * All leaf nodes are at same level.
- * Depth of all leaf nodes are same.



$2^h + 1 = n$

$2^h + 1 = n$

	B.T	FBT	CBT
min nodes	$n = 2^h + 1$	$n = 2^{h+1} - 1$	$n = 2^{h+1} - 1$
max nodes	$n = 2^h - 1$	$n = 2^h - 1$	$n = 2^h - 1$
min height	$h = \log(n+1) - 1$	$h = \log(n+1) - 1$	$h = \log(n+1) - 1$
max height	$h = h - 1$	$h = \frac{n-1}{2}$	$h = \log(n+1) - 1$



Leaf nodes = 2^h
internal nodes = $2^h - 1$

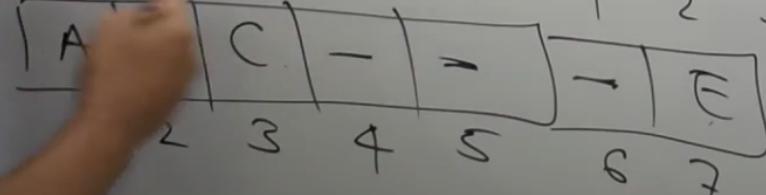
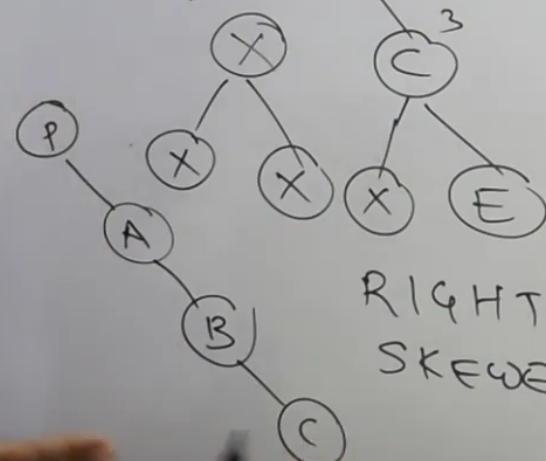
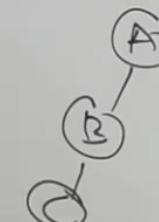
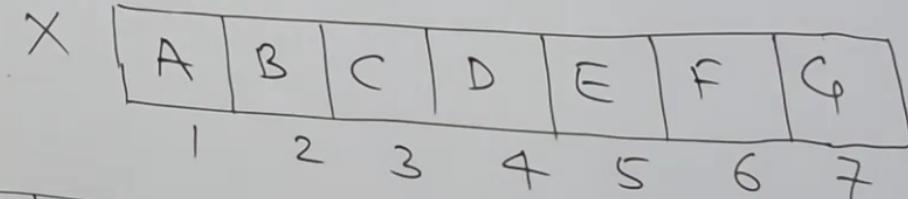
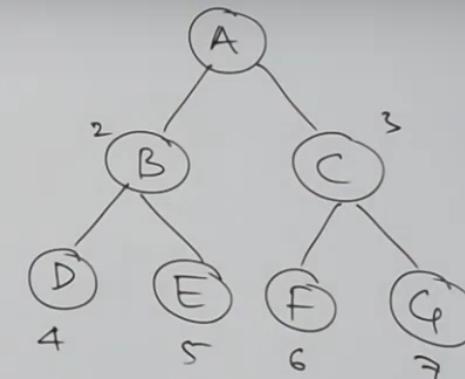


Array Representation of Binary Tree.

$$3 = 7$$

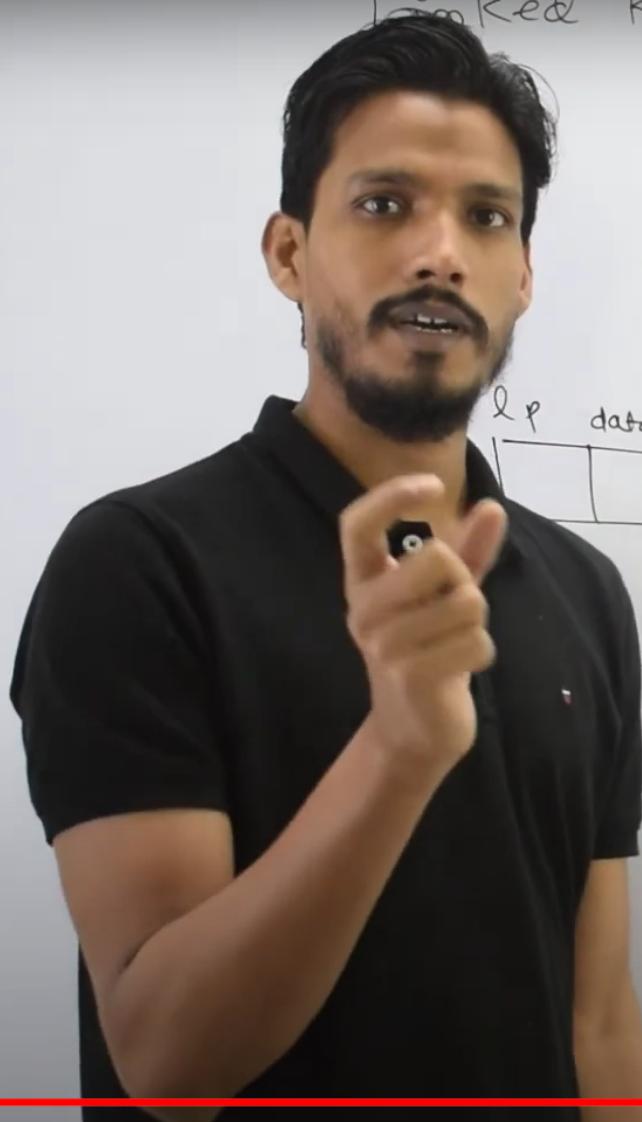
$$4 = 15$$

$$n = \frac{2^n - 1}{2}$$

 $\stackrel{0}{l}$ $Lchild = 2^l$ $Rchild = 2^l + 1$ $Parent = \left\lfloor \frac{l}{2} \right\rfloor$ HeapACBT
CBT

Linked Representation of
Binary Tree.

Press Esc to exit full screen



struct node

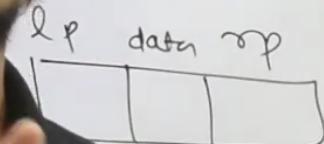
{

struct node *lp;

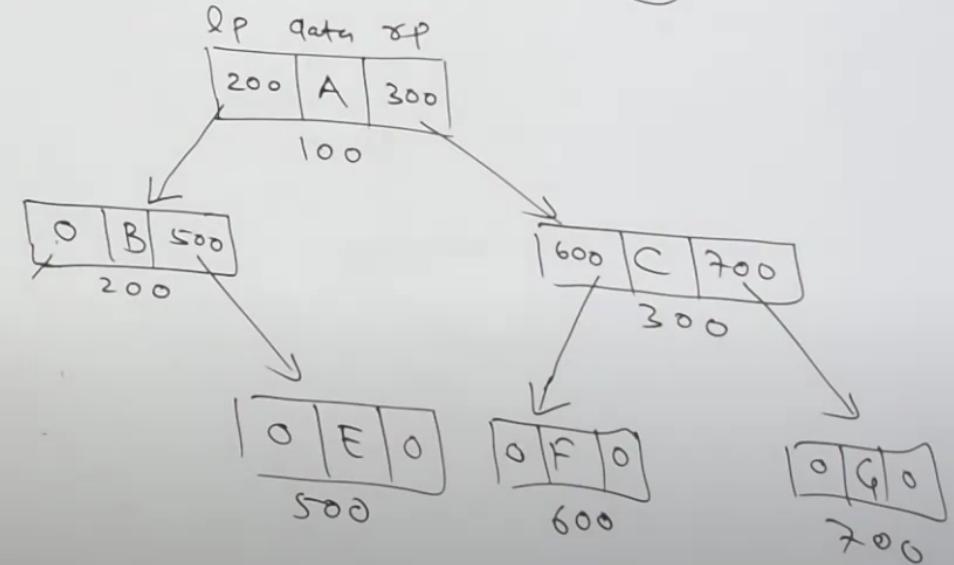
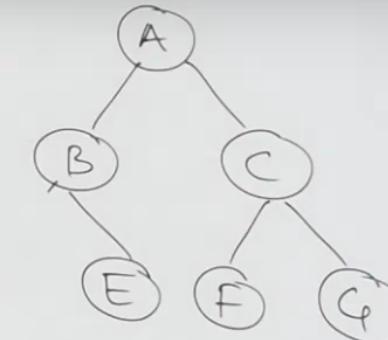
int data;

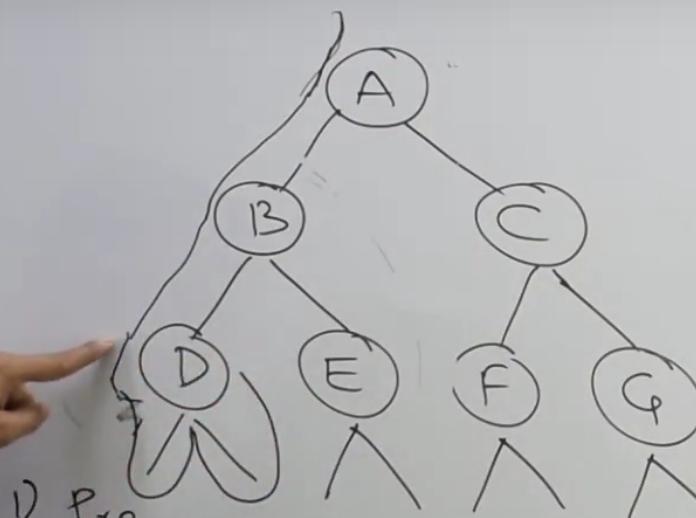
struct node *rp;

};



Leaf





1) Pre : A B D

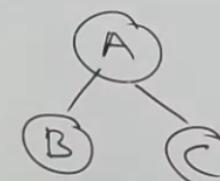
2) In : D B E

3) Post

Preorder : Root, Left, Right

Inorder : Left, Root, Right

Postorder : Left, Right, Root



Pre: A B C

In: B A C

Post: B C A

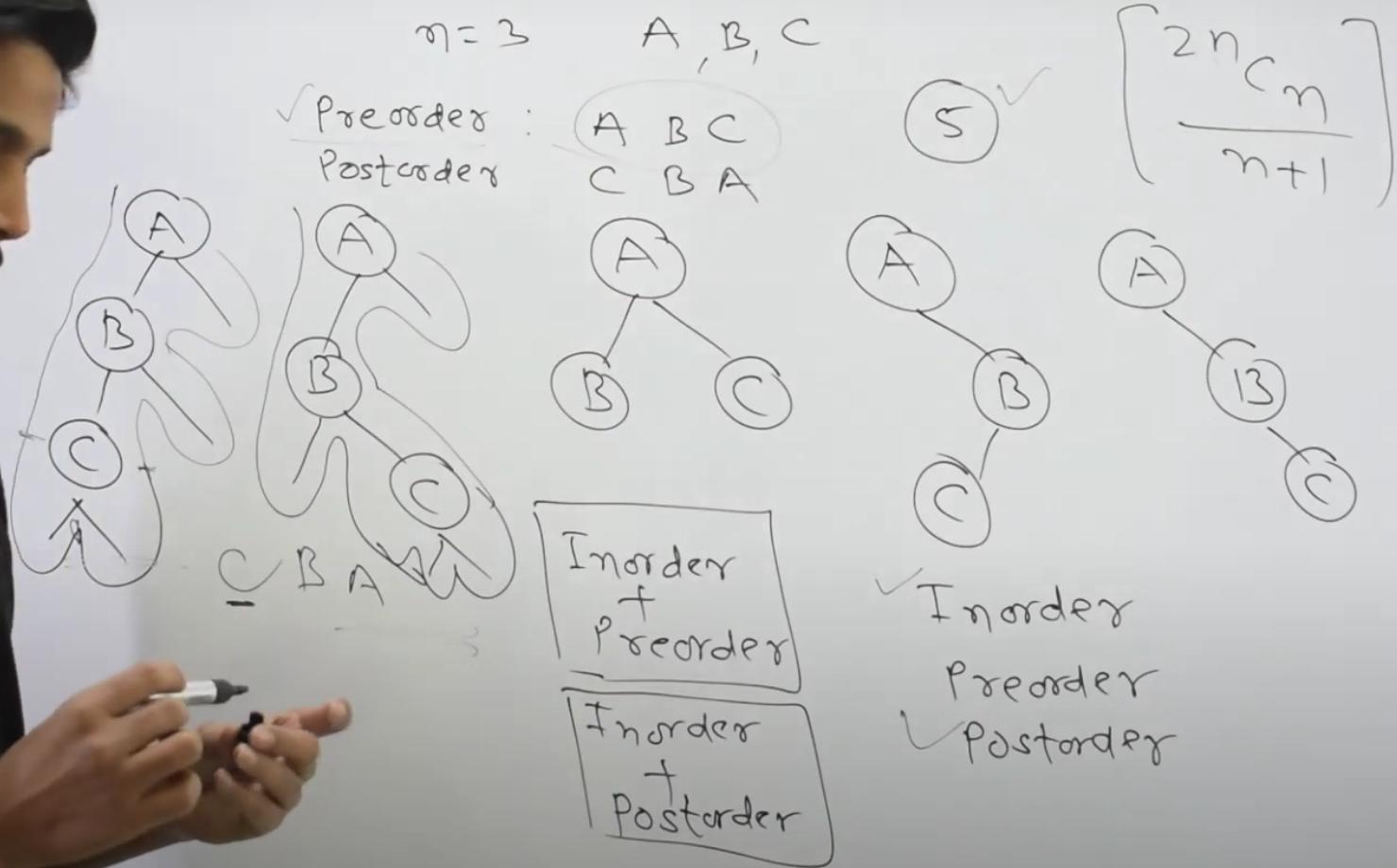
Pre: A(B D E)(C F G)

Inorder: (D B E) A (F C G)

Post: (D E B) (F G C) A



Binary Tree Construction.

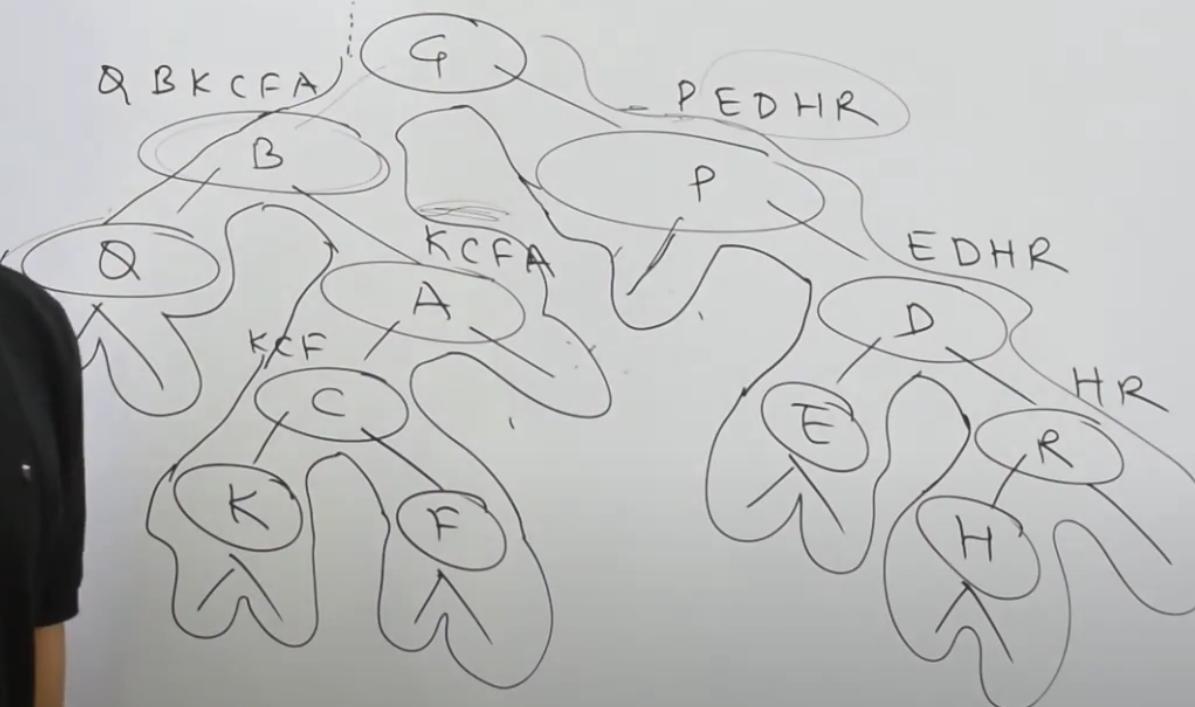


Construct Binary Tree from Preorder and Inorder traversal | Data Structure



(Left child) →
(Root)

Inorder: Q, B, K, C, F, A, G, P, E, D, H, R
Preorder: G, B, Q, A, C, K, E, P, D, E, R, H



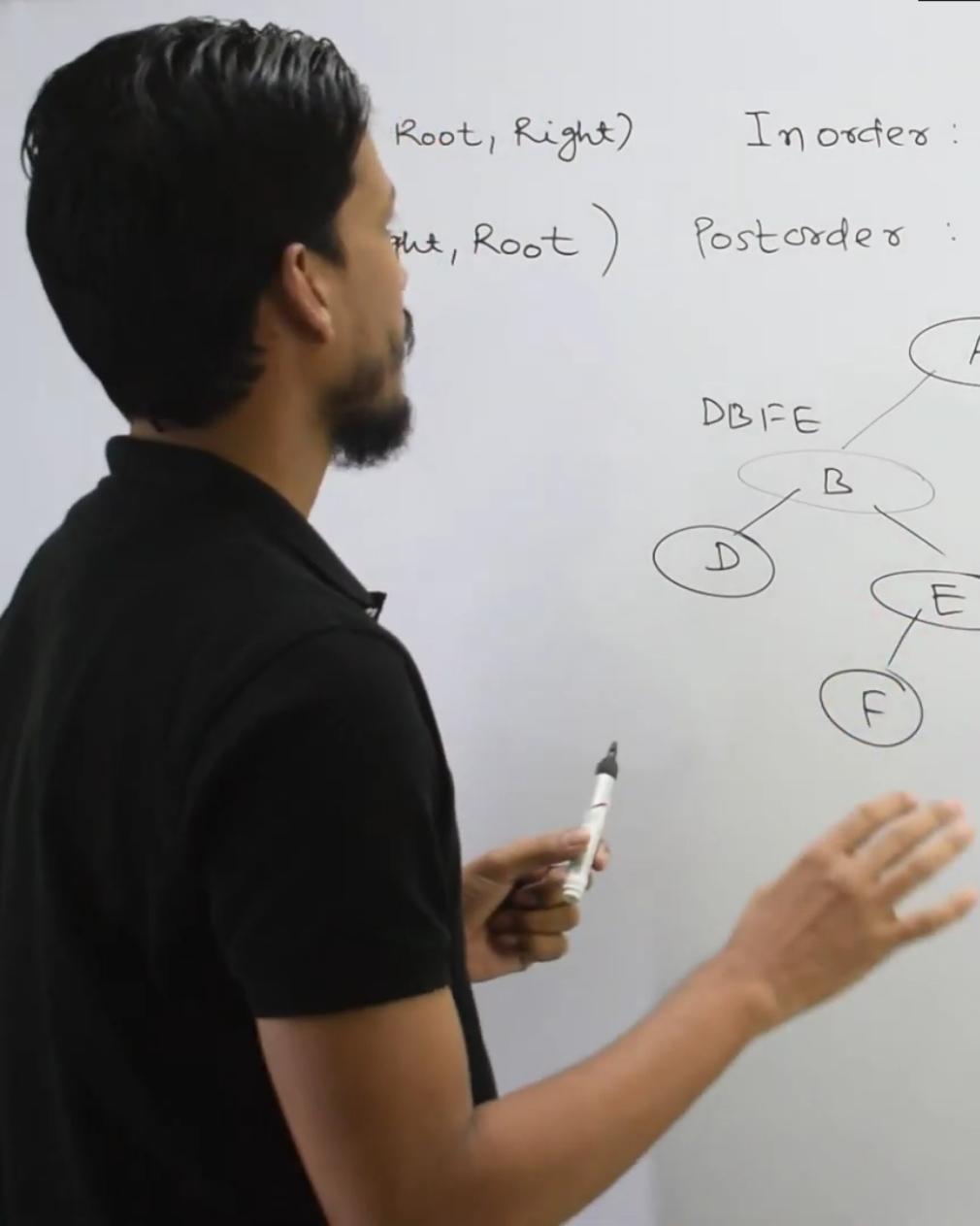
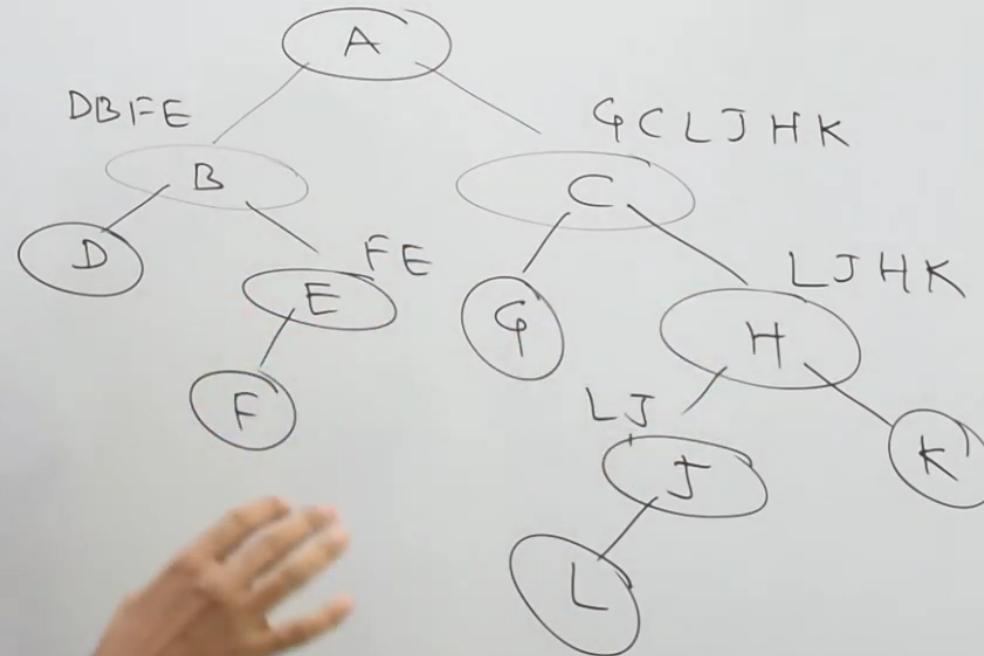
Root, Right)

Right, Root)

Inorder : (D B, F E) A (G C L J K H I K C A)

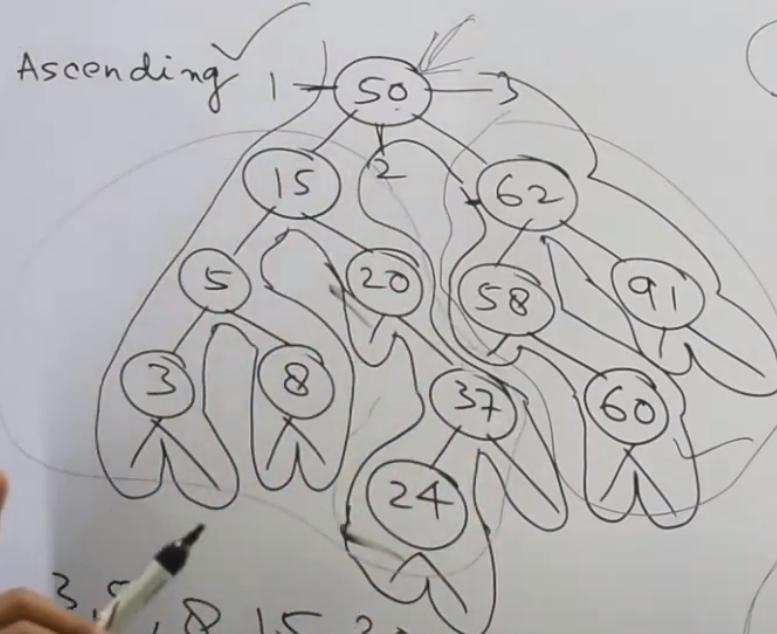
Postorder :

D F E B G L J K H I K C A



Binary Search Tree

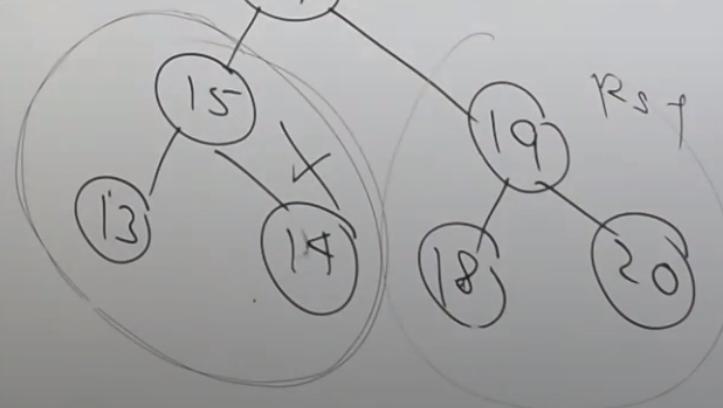
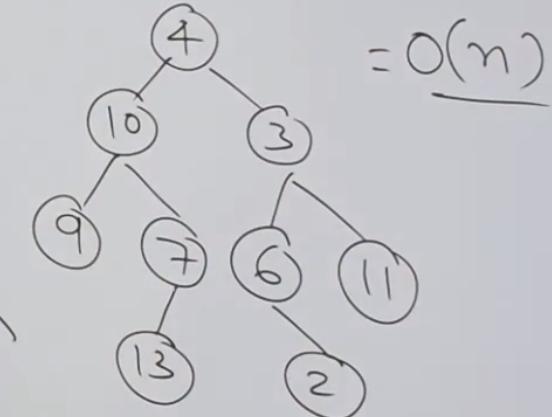
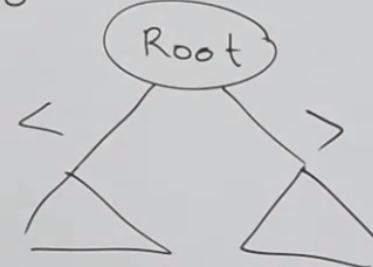
50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24



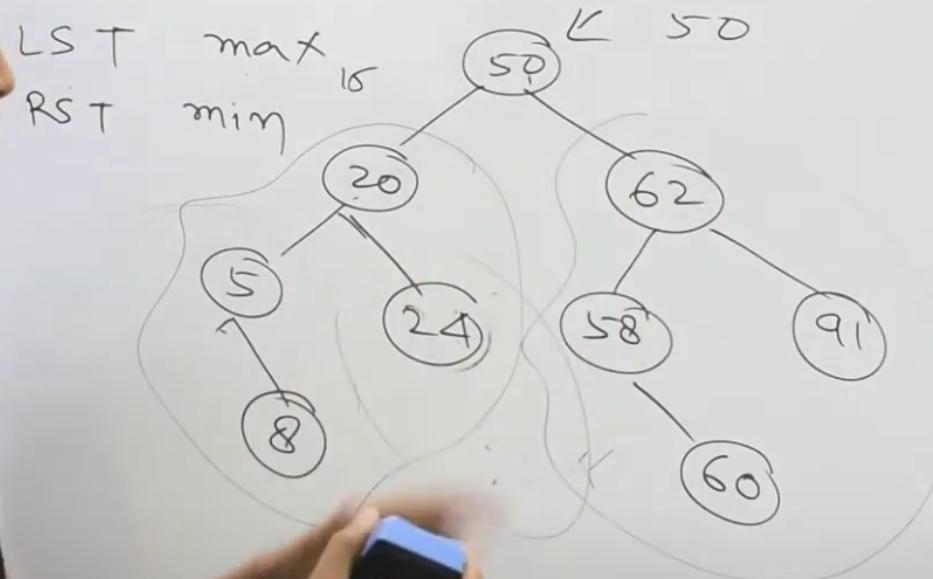
3, 5, 8, 15, 20, 24, 32, 50
58, 60, 62, 91

Pre ✓ Post ✓

Ino -



Binary Search Tree

, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24

- 1) Leaf Node
- 2) Internal Node
- 1) one child
- 2) Two child .



L R Root

L Root Right

Postorder : 10, 9, 23, 22, 27, 25, 15, 50, 95, 60, 40, 29

Inorder : 9, 10, 15, 22, 23, 25, 27, 29, 40, 50, 60, 95

Premoder:

40, 50, 60, 95

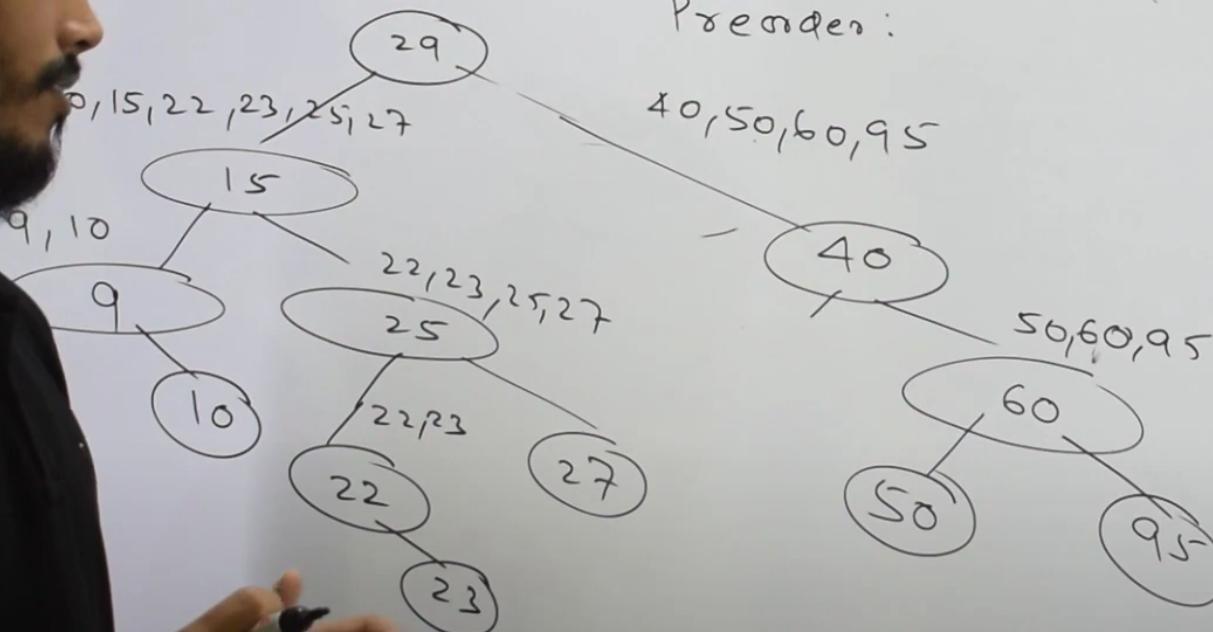
40

50, 60, 95

60

50

95



$n=0$

Empty BST

 $n=1$

1

$$\frac{2^n C_n}{(n+1)}$$

 $n=2$

2

How many different BST can be constructed using 'n' distinct keys?

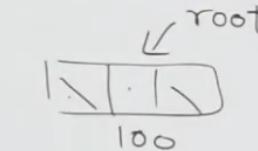
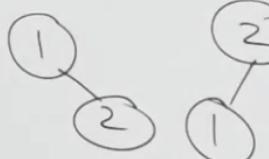
 $n=3$

5

 $n=4$

14

1, 2

`root = null`

1, 2, 3



AVL Tree

Empty Tree

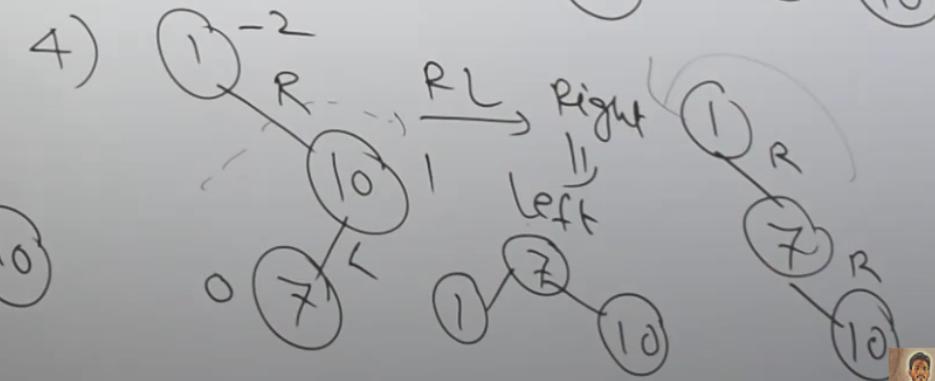
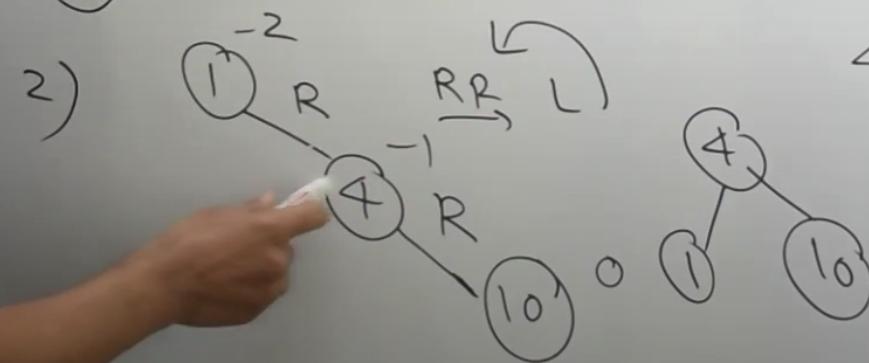
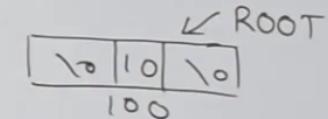
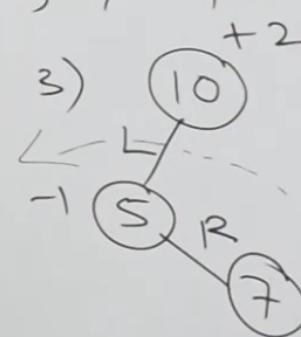
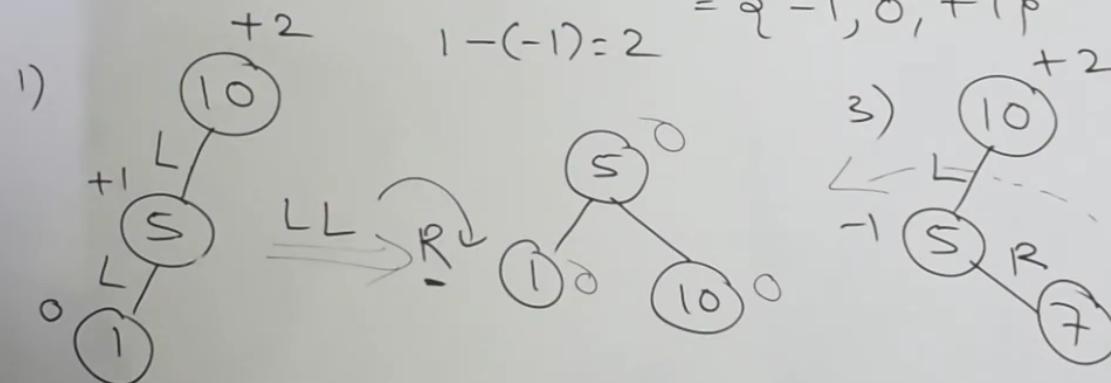
$$1. H(\emptyset) = -1$$

ROOT == NULL

1. Self Balancing BST.

2. Balancing factor = $H(LST) - H(RST)$ 2. $H(\text{Leaf}) = 0$

$$= \{-1, 0, +1\}$$

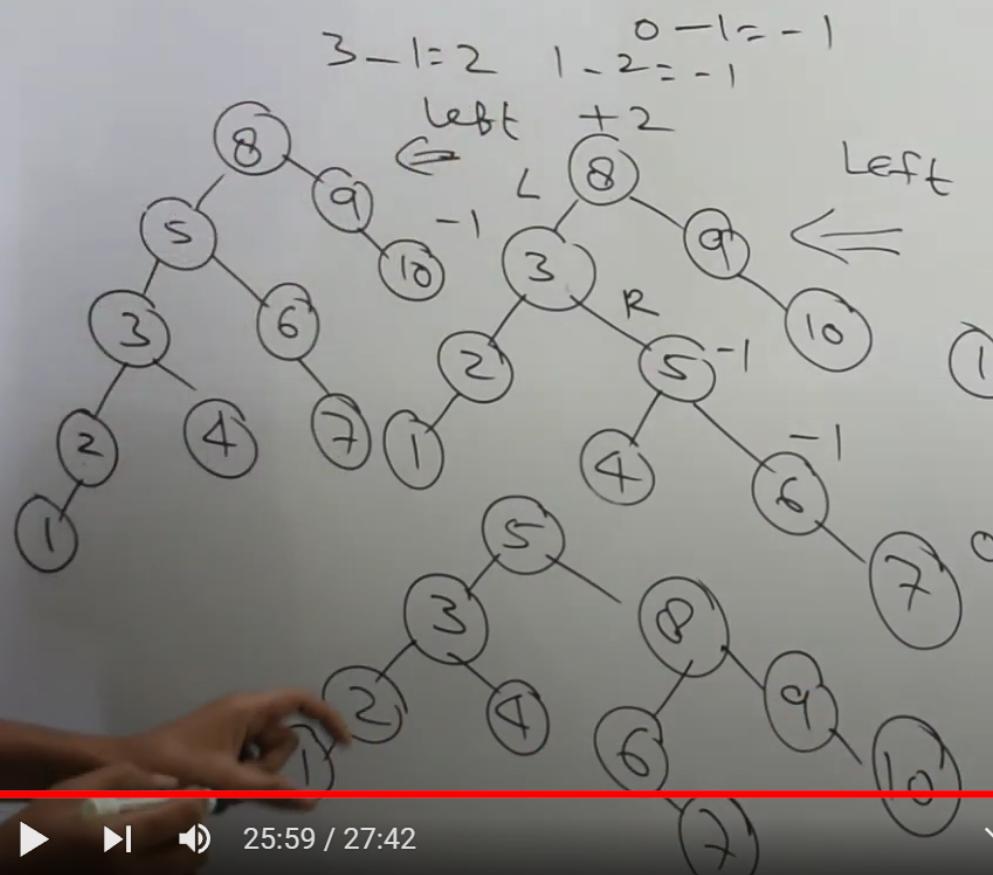


AVL Tree

$$\text{Balance factor} = H(LST) - H(RST)$$

$$= \{-1, 0, +1\}$$

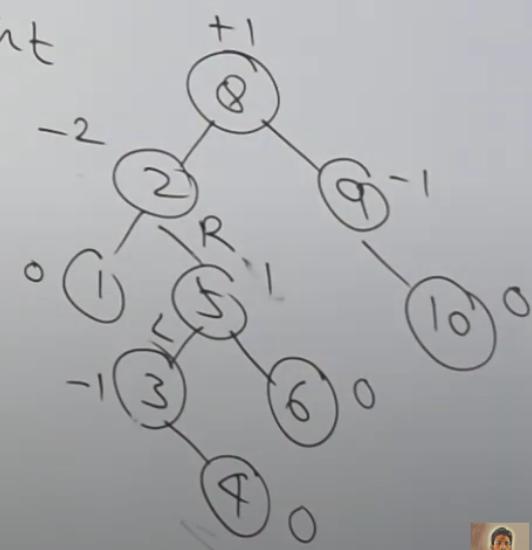
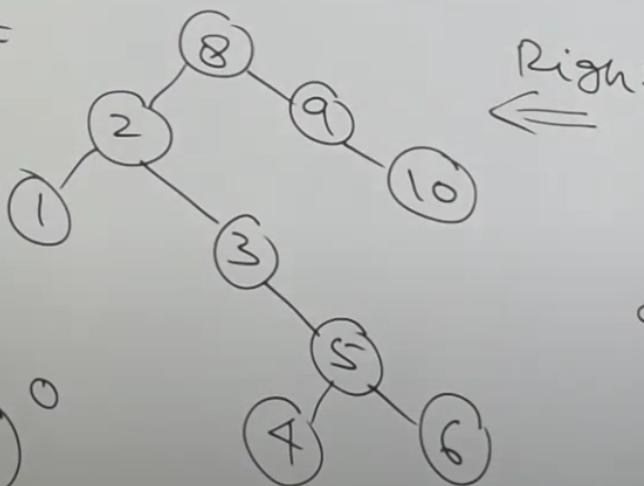
8, 9, 10, 2, 1, 5, 3, 6, 4, 7



$$-1 - 0 = -1$$

$$0 - 2 = -2$$

LL → Right
 RR → Left
 LR → Left ⇒ Right
 RL → Right ⇒ Left



Empty Tree

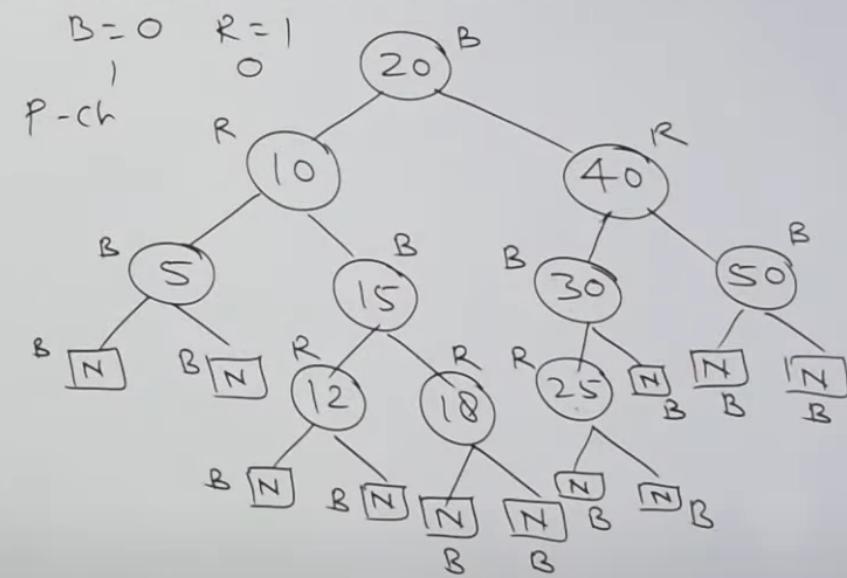
$$H(\emptyset) = -1$$

$$H(\text{Leaf}) = 0$$

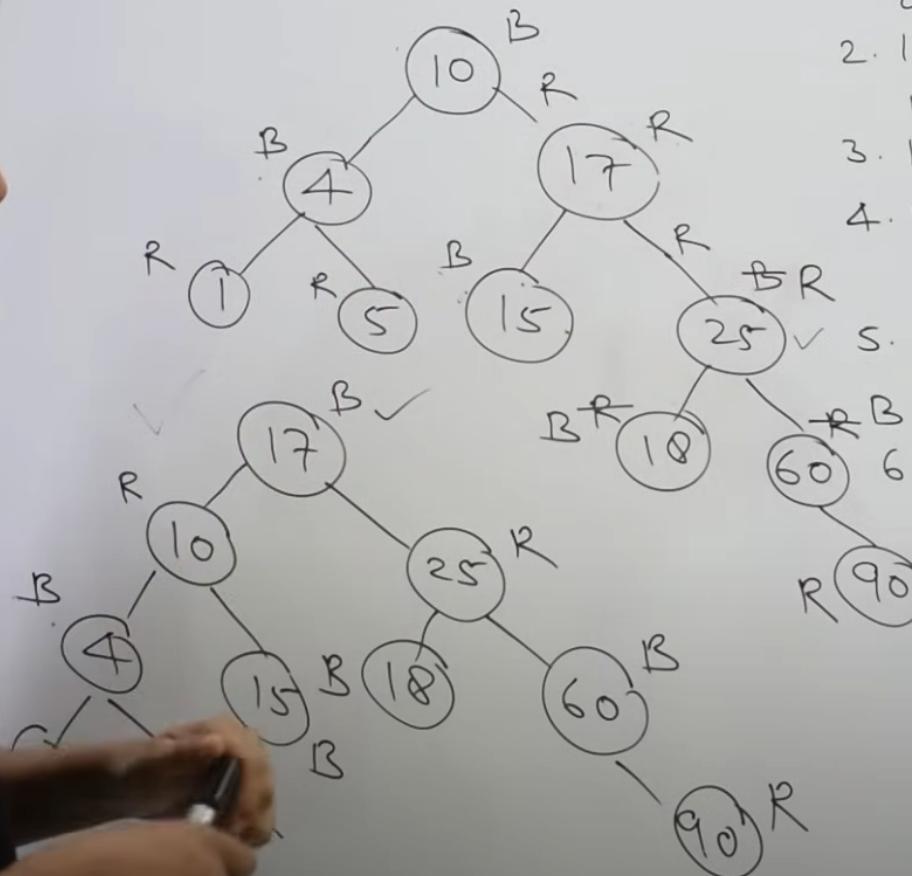
RB Tree is a Binary Search Tree with one extra bit of storage per node.

1. Every node is either Red or Black.
2. The root is black. No R-R
3. Every Leaf (NIL) is black.
4. If a node is Red, then both its children are black.
5. For each node, all paths from node to descendant leaves contain the same no. of black nodes.

lp	Color	Key	rp
----	-------	-----	----



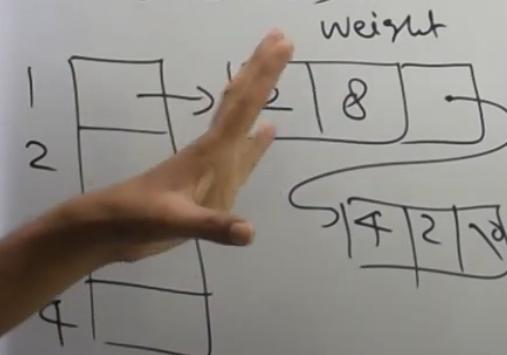
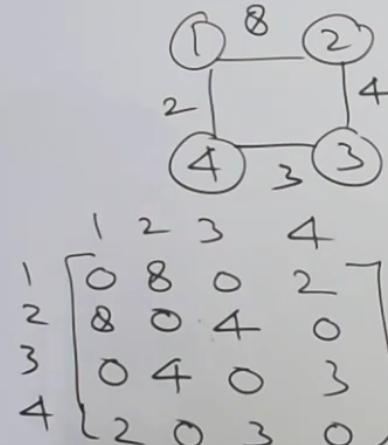
10, 18, 5, 4, 15, 17, 25, 60, 1, 90



1. If tree is empty then create newnode as root node with color Black.
2. If tree is not empty then insert newnode with color Red.
3. If the parent of newnode is Black then exit.
4. If the parent of newnode is Red then check the color of parent's sibling of newnode.
5. If it's color is Black or NULL then make suitable rotation & Recolor it.
6. If it's color is Red then Recolor.

1. Root is always Black.
2. No two adjacent Red nodes
3. No. of Black nodes in every path are same.

Representation of Graph.

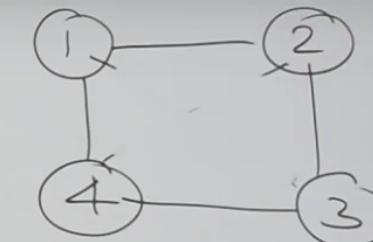
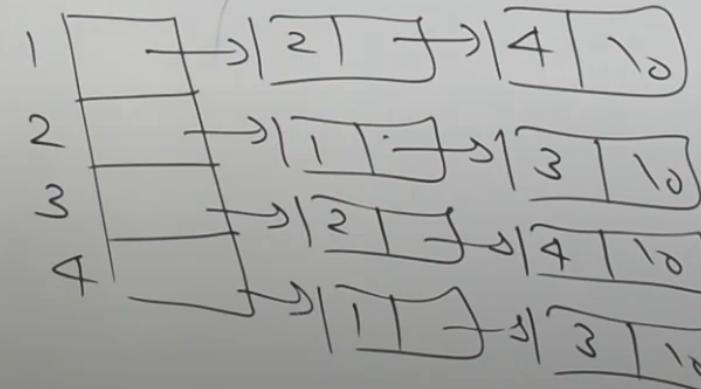


1. Adj matrix (2D Array)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

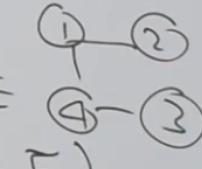
$$\begin{aligned} V \times V \\ = O(V^2) \\ 4 \times 4 \end{aligned}$$

2. Adj List



$$E = \frac{V(V-1)}{2}$$

$$\text{Dense } E = O(V^2)$$



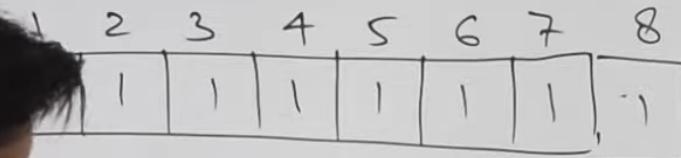
$$V + 2E = O(V+E)$$

Sparse

$$E = O(V)$$

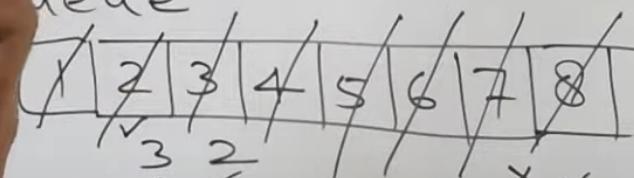
$$V-1$$

Breadth First Search (BFS)



1 2 3 4 5 6 7 8

queue



{2, 3} u=2 {1, 4, 5} u=4 - {2, 8}

$$\frac{3, 2}{4=3}$$

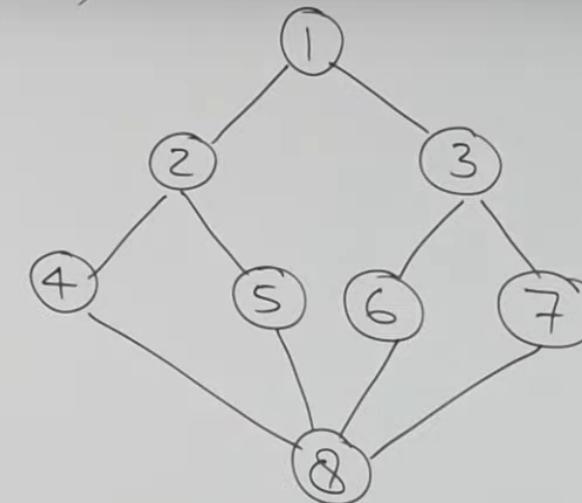
{1, 4, 5} u=4 - {2, 8}

{1, 6, 7} u=5 = {2, 8}

1 7 6

4=6

{3, 8} {3, 8}



Visited Explored

x ✓

x ✓

x x

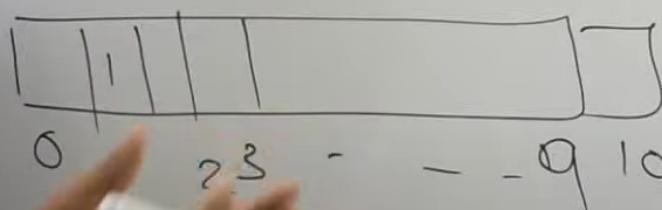


DAT

Record



(i - - 10)



USA $\Rightarrow O(n)$

SA $\Rightarrow O(\log n)$

LL $\Rightarrow O(n)$

B.T $\Rightarrow O(n)$

BST $\Rightarrow O(n)$

BBST $\Rightarrow O(\log n)$

O(1)



Hashing

Taking a set of very large no. and then mapping into a set of small no

HASH Fⁿ

CRT

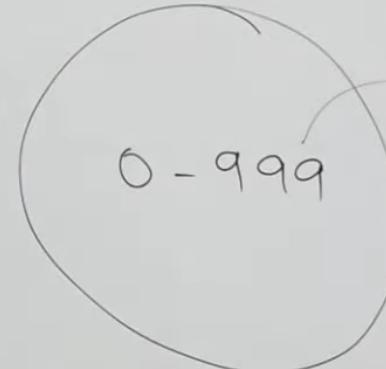
- 1) chaining $146 \% 10 = 6$
- 2) open Add $133 \% 10 = 3$
- 3) Linear $990 \% 10 = 0$
- 4) Quadratic $369 \% 10 = 9$
- 5) Double Hashing $233 \% 10 = 3$
- 6) Double Hashing $990 \% 10 = 0$

990	N	N	133	N	N	146	N	N	369
0	1	2	3	4	5	6	7	8	9



Collision

O(1)



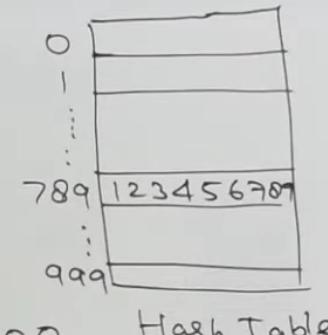
Types of Hash Function

Division Modulo

$$M = 1000$$

$$\text{Key} = 123456789$$

$$\begin{aligned}
 HF(\text{Key}) &= \text{Key mod } M \\
 &= 123456789 \bmod 1000 \\
 &= 789
 \end{aligned}$$

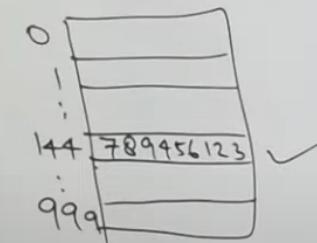


3) Digit Extraction

$$M = 1000$$

$$\text{Key} = \underline{789} \underline{456} \underline{123}$$

$$\begin{aligned}
 &= 789 + 456 + 123 = \underline{\underline{1368}} \\
 &= \underline{\underline{136}} + \underline{\underline{8}} \\
 &= \underline{\underline{144}}
 \end{aligned}$$

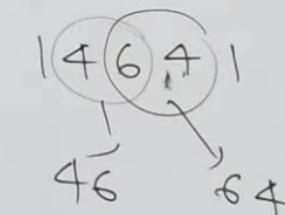


2) Mid Square

$$M = 100$$

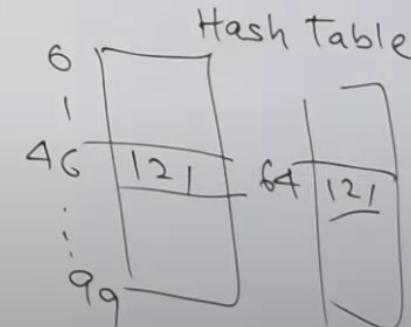
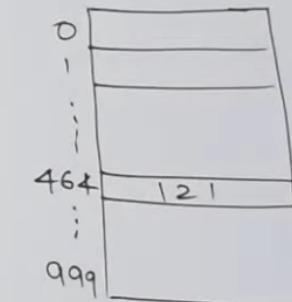
$$\text{Key} = 121$$

$$(121)^2 = 14641$$



1) Square

2) Mid



CHAINING (OPEN HASHING)

Keys: 25, 97, 80, 65, 38, 40, 59, 75, 17, 99, 35, 23

$$\checkmark M = 10.$$

$$h(\text{key}) = \text{key} \bmod M.$$

$$10, 20, 30, 40, 50, 60 \quad 25 \bmod 10 = 5$$

$$65 \bmod 10 = 5$$

worst

Avg UNIFORM In $O(1)$ (4)

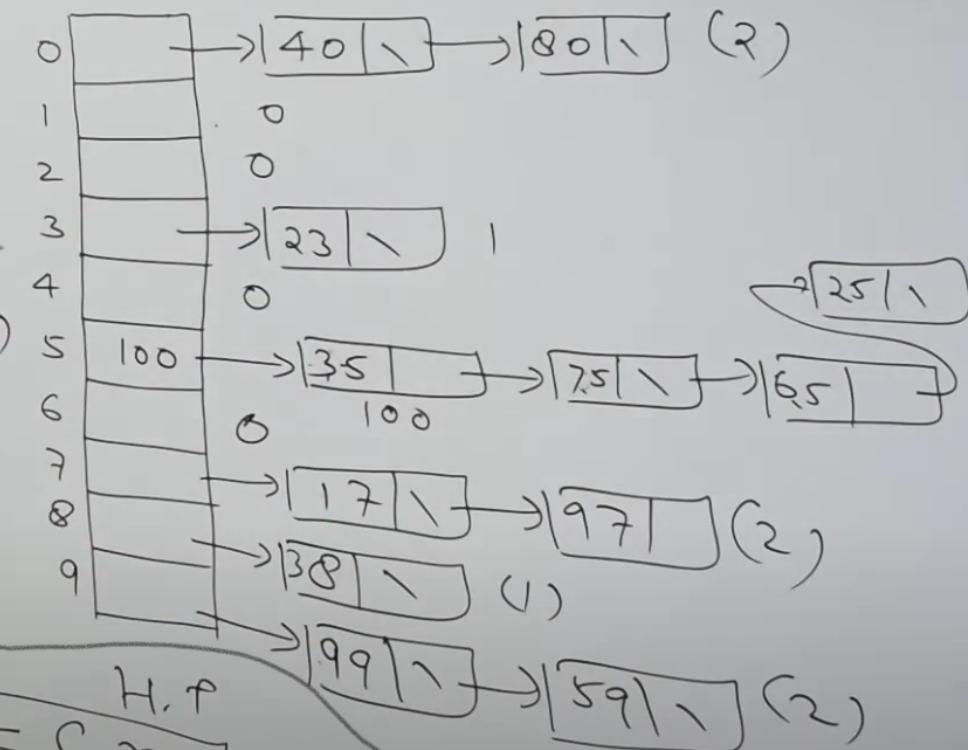
Search $\Rightarrow O(n)$

Del $\Rightarrow O(n)$

$$\frac{100}{10}, 10$$

$$\text{Load factor } \alpha = \frac{n}{m}$$

$$\begin{aligned} \text{Search} &= C + \frac{n}{m} = C + \frac{n}{10} \\ &= C + \frac{Cn}{10} \\ &= O(1) \end{aligned}$$



Linear Probing (Open Addressing)

Keys: 56, 79, 41, 44, 21, 43, 11, 29 $21 \bmod 10 = 1$

$$\checkmark H'(k) = k \bmod M, M=10 \quad \begin{array}{l} 56 \bmod 10 = 6 \\ 79 \bmod 10 = 9 \end{array}$$

$$H(k, i) = (H'(k) + i) \bmod M \quad H(56, 0) = (6+0) \bmod 10 = 6$$

$$\text{for } i=0, 1, 2, \dots, M-1 \quad [H(21, 0) = (1+0) \bmod 10 = 1]$$

$$H(29, 1) = (9+1) \bmod 10 = 0 \quad H(21, 1) = (1+1) \bmod 10 = 2$$

$$4 \left\{ \begin{array}{l} H(11, 0) = (1+0) \bmod 10 = 1 \\ H(11, 1) = (1+1) \bmod 10 = 2 \\ H(11, 2) = (1+2) \bmod 10 = 3 \\ H(11, 3) = (1+3) \bmod 10 = 4 \\ H(11, 4) = (1+4) \bmod 10 = 5 \end{array} \right.$$

0	
1	41
2	21
3	43
4	44
5	11
6	56
7	
8	
9	79

Linear Probing (Open Addressing)

Press Esc to exit full screen

79, 41, 44, 21, 43, 11, 29

$$H(k) = k \bmod M, M=10$$

$$(k+i) \bmod M$$

 $\dots M-1$

$O(n)$
↑
Worst
Search

Primary Clustering } 6
 $31 \div 10 = 1$

0	29
1	41
2	21
3	43
4	44
5	11
6	56
7	
8	
9	79



Linear Probing (Open Addressing)

Keys: 56, 31, 44, 21, 43, 11, 29

Modulo operation
Mod M, M=10

$$H(K, i) = (H(K) + i) \bmod M$$

for $i=0, 1, 2, \dots$

Primary clustering
31

44

Rehashing
21

0	29
1	41
2	21
3	43
4	\$
5	11
6	56
7	
8	
9	79



Quadratic Probing

Press Esc to exit full screen

$$k_1 = 29 \Rightarrow 1, 5, 1, 9, 9, 1, 5, 1, 9$$

$$k_2 = 19 \Rightarrow 1$$

0	58
1	49
2	
3	
4	
5	69
6	
7	
8	18
9	89

49, 58, 69, 29

M=10SC

LB	QP
PC	PC X
SC	SC

$$h(k) = k \bmod M \quad c_1 = c_2 = 1 \quad c_2 \neq 0$$

$$(h(k) + c_1 \cdot i + c_2 \cdot i^2) \bmod M$$

.....(M-1)

$$h(49, 1) = (9+1+1) \bmod 10 = 1$$

$$h(49, 0) \bmod 10 = 9$$

$$h(58, 1) = (8+1+1) \bmod 10 = 0$$

$$h(58, 0) \bmod 10 = 8$$

$$h(69, 1) = (9+1+1) \bmod 10 = 1$$

$$h(69, 0) \bmod 10 = 9$$

$$h(69, 2) = (9+2+4) \bmod 10 = 5$$

$$h(29, 0) \bmod 10 = 9$$

$$h(29, 1) = (9+1+1) \bmod 10 = 1$$

$$h(29, 4) \bmod 10 = 1$$

$$h(29, 2) = (9+2+4) \bmod 10 = 5$$

$$h(29, 3) \bmod 10 = 9$$

$$h(29, 3) = (9+3+9) \bmod 10 = 1$$

$$h(29, 4) = (9+4+16) \bmod 10 = 9$$

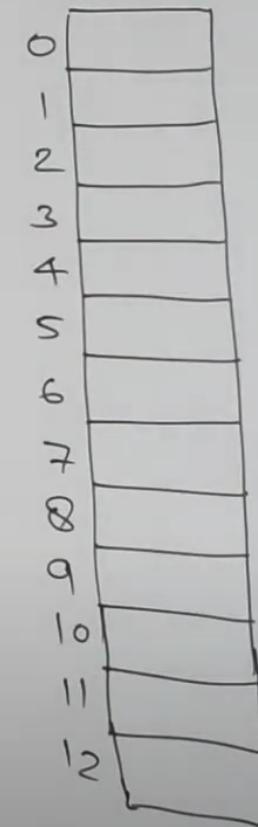
Double Hashing

79, 69, 98, 72, 14, 50 $\boxed{M=13}$

$$h_1(k) = k \bmod M^{\Rightarrow 13}, \quad h_2(k) = 1 + k \bmod (M-2)^{\Rightarrow 11}$$

$$h(k, i) = (h_1(k) + i \cdot h_2(k)) \bmod M$$

$$i = 0, 1, 2, \dots, (M-1)$$



Double Hashing

$s \bmod 13 = 1$

79, 69, 98, 72, 14, 50 $\boxed{M=13}$

$$h_1(k) = k \bmod M^{\frac{1}{13}}, \quad h_2(k) = 1 + k \bmod$$

$$h(k, i) = (h_1(k) + i h_2(k)) \bmod M$$

$$i = 0, 1, 2, \dots, (M-1)$$

LB	QP	DH	$h(72, 1) = (7 + (1 + 72 \bmod 11)^6) \bmod 13 \rightarrow 72$ $= (7 + 7) \bmod 13 = 1$
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PC ✓
SC ✓

$$\begin{aligned} h(72, 2) &= (7 + 2(1 + 72 \bmod 11)^6) \bmod 13 \\ &= (7 + 2(7))^6 \bmod 13 \end{aligned}$$

$$h(14, 1) = 7 + 14 = 1$$

$$(1 + 4) \bmod 13 = 5$$

0	
1	79
2	
3	
4	69
5	14
6	
7	98
8	72
9	
10	
11	
12	