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ML HW #7

<u>Exercise 8.2</u>: Consider the data below and a 'hyperplane' (b, w) that separates the data.

$$X = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 2 & 0 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix} \quad w = \begin{bmatrix} 1.2 \\ -3.2 \end{bmatrix} \quad b = -0.5$$

(a) Compute $p = \min_{n=1,\dots,N} y_n(w^T x_n + b)$

$n=1: (-1)[(1.2)(0) + (-3.2)(0) - 0.5] = 0.5$

$n=2: (-1)[(1.2)(2) + (-3.2)(2) - 0.5] = 4.5$

$n=3: (+1)[(1.2)(2) + (-3.2)(0) - 0.5] = 1.9$

$$p = \min(0.5, 4.5, 1.9) = \boxed{0.5}$$

(b) compute weights $\frac{1}{p}(b, w)$ and show that they satisfy (8.2)

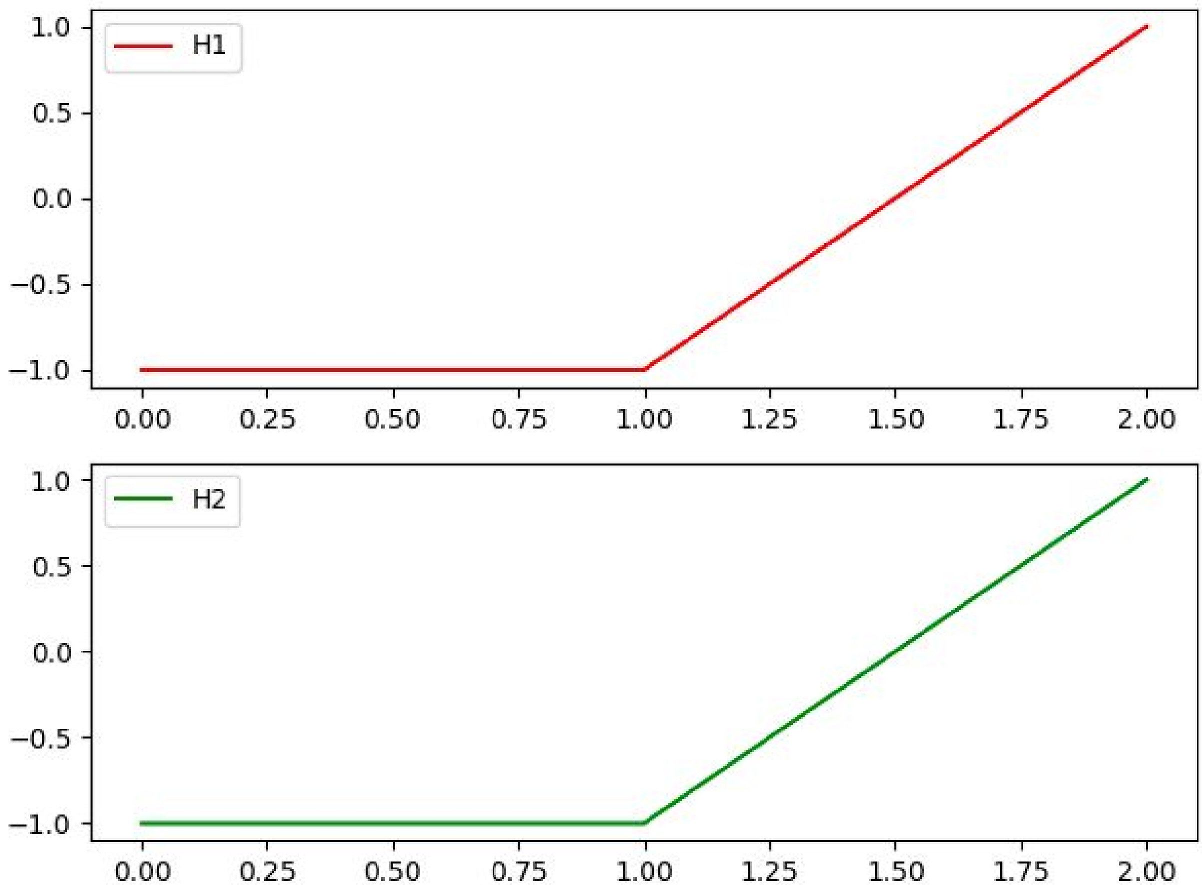
$$\frac{1}{0.5}\left(-0.5, \begin{bmatrix} 1.2 \\ -3.2 \end{bmatrix}\right) = \left(-1, \begin{bmatrix} 2.4 \\ -6.4 \end{bmatrix}\right)$$

verification: $\min_{n=1,\dots,N} y_n(w^T x_n + b) = 1$

$n=1: (-1)[(2.4)(0) + (-6.4)(0) + (-1)] = 1$
$n=2: (-1)[(2.4)(2) + (-6.4)(2) + (-1)] = 9$
$n=3: (1)[(2.4)(2) + (-6.4)(0) + (-1)] = 3.8$

$\min_{n=1,\dots,N} y_n(w^T x_n + b) = 1$ (✓)

(c) Plot both hyperplanes to show they are the same separator:



2. $X = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$ Find: max. decision hyperplane, i.e.) b and w

Inequalities: $\quad -b \geq 1 \quad (i)$

$-(w_1 + w_2 + b) \geq 1 \quad (ii)$

$w_1 + b \geq 1 \quad (iii)$

$w_1 \geq 1 - b \quad (iii)$

$-b \geq 1 \quad (i) \quad \Rightarrow w_1 \geq 2$

$b \leq -1$

$-w_1 - w_2 - b \geq 1 \Rightarrow -w_1 - b \geq 1 + w_2 \quad (ii)$

$\Rightarrow w_1 + b \leq -1 - w_2$

$(iii) \quad 1 \leq w_1 + b \leq -1 - w_2 \quad (ii)$

$\Rightarrow 1 \leq -1 - w_2 \Rightarrow 2 \leq -w_2 \Rightarrow -2 \geq w_2$

$\frac{1}{2}(w_1^2 + w_2^2) \geq 1$ w/ $w_1 \geq 2$ and $w_2 \leq -2$

$(b, w) = \left(-1, \begin{bmatrix} 2 \\ -2 \end{bmatrix}\right)$

$g(x) = \text{sign}(2x_1 - 2x_2 - 1)$

Margin: $\frac{1}{\sqrt{2^2 + (-2)^2}} = \frac{1}{\sqrt{8}}$