Vienna University of Technology

Faculty of Physics

Laboratory III

Laboratory Report

Elasticity Modulus

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Contents

1 Elasticity Modulus of Tension Rods

1.1 Fundamentals

The elasticity modulus (Young's modulus) E describes the relationship between stress and strain in the elastic deformation region. For a tension rod under axial load:

$$E = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{\Delta L}{L_0}} = \frac{\frac{F}{A}}{\frac{2U_0}{\Delta U}} \tag{1}$$

where F is the applied force, A is the cross-sectional area, ΔL is the elongation, and L_0 is the original length. The measurement uses strain gauges with a bridge circuit configuration.

1.2 Setup

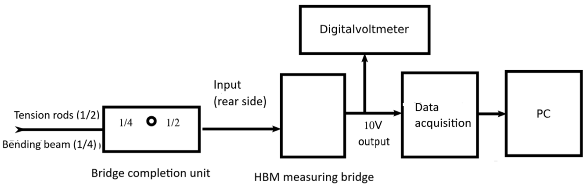


Figure 1: Measurement Setup. [1]

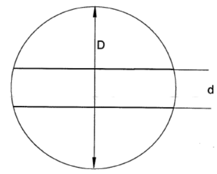


Figure 2: Geometry of the Tension Rods. [1]

Equipment:

- Bridge extension device
- Tension rods (Aluminum, Steel, Brass, Plexiglas)
- Strain gauge measurement system
- Calibrated weights
- Digital multimeter

K-factor: $k = 2.03 \pm 1\%$ (pre-calibrated)

Material Specifications:

Material	Durchmesser D[mm]	Breite d[mm]
Aluminium	14.85	3.7
Steel	14.85	3.7
Brass	14.85	3.7
Plexiglas	14.85	3.7

Table 1: Geomteric material specifications

1.3 Procedure

1. Switch the bridge extension device to 1/2 position
2. Connect the first tension rod to the bridge extension
3. Wait 5 minutes for thermal equilibration
4. Bridge Adjustment:
   - Set the bridge voltage to 5V using switch S5. Turn A1 to "ON" and set A2 to 1. Ensure the calibration switch S6 is at 0, and set the filter S7 to 10 Hz. Set S4 to full bridge and leave it in this position for the entire experiment.
   - Next, adjust the measurement range with S3 until instrument M1 no longer shows full-scale deflection. Then perform a zero adjustment using S1, S2, and P1, gradually switching back to the most sensitive range via S3.
   - The capacitance is balanced by adjusting P2 until the reading on instrument M2 shows a minimum value.
5. Bridge Calibration:
   - Set S3 to the most sensitive range (0.05). The zero point should remain unchanged (thermal equilibrium); otherwise, correct with P3.

- Set calibration switch S6 to +0.05. Instrument M1 should now show full-scale deflection (100), and the digital voltmeter should read exactly 10V (±0.05V). If not, fine-tune with P3.
- For verification, reduce the bridge voltage to 2.5V. The pointer on M1 should indicate 50. Finally, return S6 to 0.

6. For each material, perform 4 measurement series:
   - Series with 5V and 2.5V excitation voltage, and
   - Series with 2 kg and 5 kg loading
7. Take 5 measurements per series
8. Record all voltage readings and corresponding loads
9. Repeat for all four materials

1.4 Measurement values

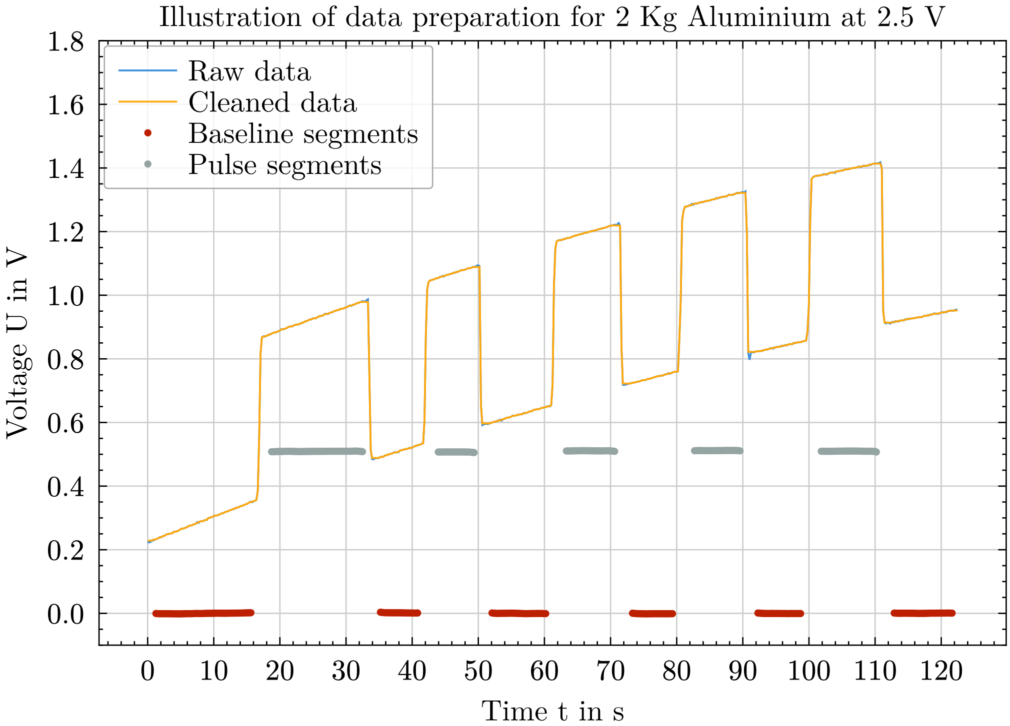


Figure 3: After applying a rolling average onto the raw data, it is segmented into baseline, pulses, rising and falling slopes. Afterwards a quadratic function is fitted onto the baseline dataset and subtracted from the segmented dataset. This results in clear baseline and pulse datasets which can be averaged over.

1.5 Data Analysis

1. Calculate cross-sectional area: $A = D^2\left(\arccos\left(\frac{d}{D}\right) - \frac{d}{D}\sqrt{1-\left(\frac{d}{D}\right)^2}\right)$
2. Calculate stress: $\sigma = \frac{F}{A}$
3. Calculate strain from voltage readings: $\varepsilon = \frac{U_0}{k \cdot U_b}$, but since the k-factor is already accounted for in the calibration , but since the k-factor is already accounted for in the calibration $\varepsilon = \frac{U_0}{U_b}$ is used.
4. Plot stress vs. strain for each material
5. Determine EE from the slope of the linear region
6. Perform error propagation analysis considering individual measurement uncertainties (weight, cross-section, measurement, drift)-> but no data to it…so only mean deviation?

A calibration error with this setup can be approximated to ±0,1V with the measuring Voltage of 2,5V and 5V. Therefore the relative error is 0,04%.

7. Compare experimental values with literature values
   - Aluminum: E = 70 GPa
   - Steel: E = 210 GPa
   - Brass: E = 78-123 GPa
   - Plexiglas: E = 3 GPa

Weight (kg)	Voltage (V)	Material	Pulse Height (V)	Strain	Stress (MPa)	E-Modulus (GPa)
2	2.5	Alu	0.4888	4.9	0.36	73.82292
2	5	Alu	1.0104	5.1	0.36	71.42713
5	2.5	Alu	1.2627	12.6	0.9	71.44378
5	5	Alu	2.5241	12.6	0.9	71.48303
2	2.5	Glas	4.3173	43.2	0.18	4.06407
2	5	Glas	2.1536	10.8	0.18	16.29468
5	2.5	Glas	5.5678	55.7	0.44	7.87824
5	5	Glas	5.7349	28.7	0.44	15.29735
2	2.5	Messing	0.1307	1.3	0.42	318.57334
2	5	Messing	0.2718	1.4	0.42	306.42657
5	2.5	Messing	0.7991	8	1.04	130.27601
5	5	Messing	0.8748	4.4	1.04	238.01466
2	2.5	Stahl	0.3826	3.8	0.6	155.94209
2	5	Stahl	0.5509	2.8	0.6	216.6264
5	2.5	Stahl	0.682	6.8	1.49	218.72494
5	5	Stahl	0.7365	3.7	1.49	405.09967

Table 2:

1.6 Error Analysis

A calibration error with this setup can be approximated to ±0.1V with the measuring Voltage of 2.5V and 5V. Therrefore the relative error is 0.04%.

2 Bending Beam Analysis

2.1 Fundamentals

The strain gauge measures strain ε at the surface of the beam:

$$\varepsilon = \frac{\sigma}{E} = \frac{M \cdot \frac{h}{2}}{I \cdot E} \tag{2}$$

Where the bending moment M at distance x from the free end:

$$M = F \cdot (L - x) \tag{3}$$

Therefore:

$$\varepsilon = \frac{F \cdot (L - x) \cdot \frac{h}{2}}{I \cdot E} \tag{4}$$

The strain gauge output voltage is proportional to strain:

$$U_A = \gamma \cdot \varepsilon = \gamma \cdot \frac{F \cdot (L - x) \cdot \frac{h}{2}}{I \cdot E} \tag{5}$$

Solving for the weight G = F/g:

$$G = \frac{2 \cdot I \cdot E \cdot U_A}{\gamma \cdot g \cdot (L - x) \cdot h} \tag{6}$$

2.2 Setup

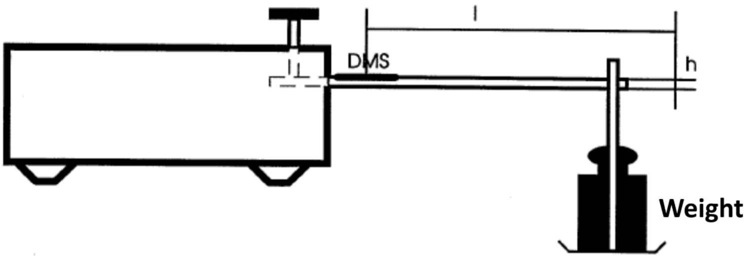


Figure 4: Cantilever beam setup. [1]

Equipment:
- Cantilever beam setup
- Strain gauge measurement system
- Calibrated weights (2g, 5g, 10g, 20g, 50g, 100g, 200g, 500g)
- Two unknown weights
- Ruler for length measurements
- Caliper for cross-section measurements

2.3 Procedure

1. Set the bridge extension device to 1/4 bridge configuration.
2. Connect the bending beam to the bridge extension device and wait at least 5 minutes before proceeding.
3. Measure the dimensions of the bending beam using a caliper: length (15,55±0,25 mm), width (2,05±0,05 mm), and height (5±0,05 mm).

4. Check and, if necessary, adjust the zero point and calibration:
   - Zero point: adjust using P1 and P2
   - Calibration: adjust using P3
5. Perform measurements at both 5V and 2.5V bridge voltage.
6. Determine the bridge output voltage (U_A) for different masses (weights), starting from 500g downward until no significant difference can be measured.

2.4 Measurement Values

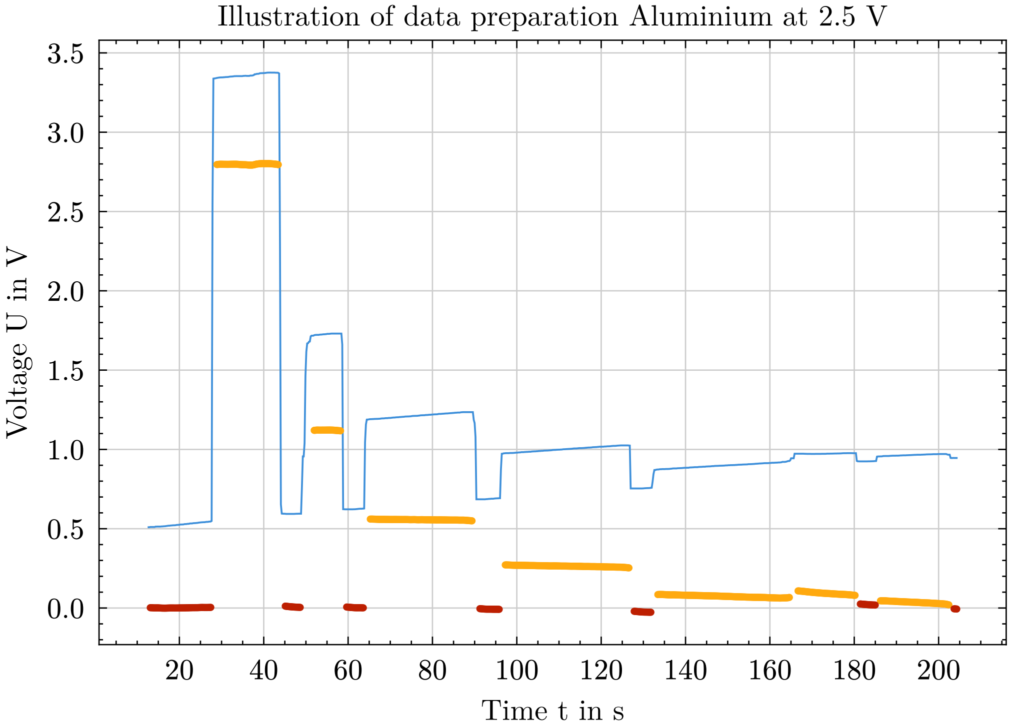
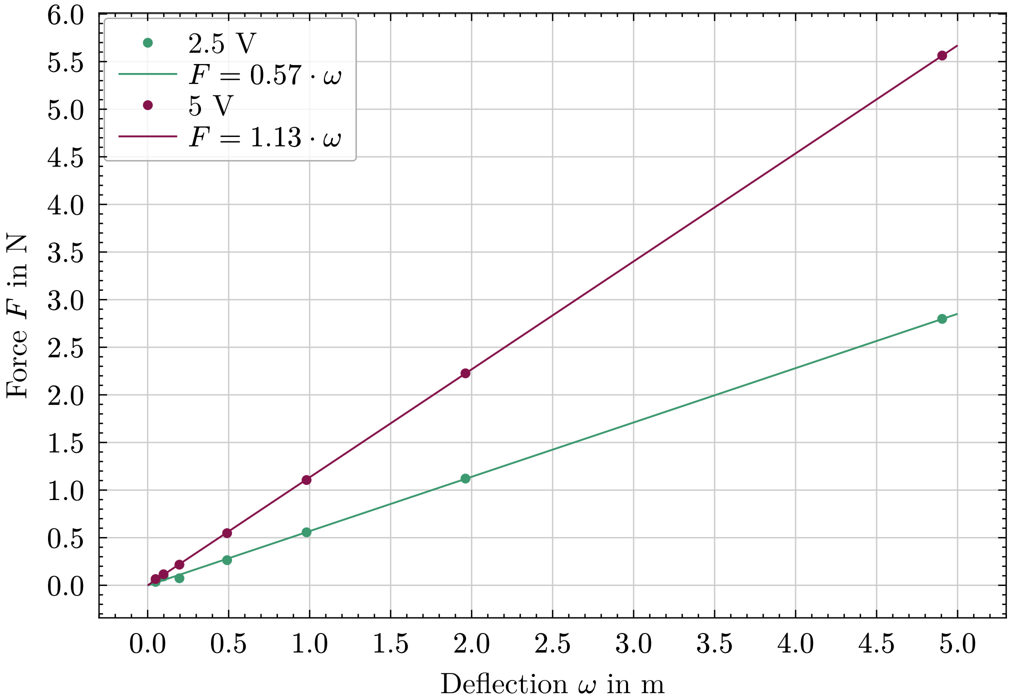


Figure 5: After applying a rolling average onto the raw data, it is segmented into baseline, pulses, rising and falling slopes. Afterwards a quadratic function is fitted onto the baseline dataset and subtracted from the segmented dataset. This results in clear baseline and pulse datasets which can be averaged over.



Beam Geometry:

- Length L = 15.55±0,25 mm
- Width b = 2.05±0,05 mm
- Height h = 5±0,05 mm
- Point of attack x = 13.5±0,25 mm
- Second moment of area $I = \frac{bh^3}{12} = I = \frac{bh^3}{12}$

Parameter	2.5V Dataset	5V Dataset	Units
Slope k	0.571	1.134	N/V
Intercept	−0.0016	−0.0007	N
Sensitivity γ	52112.539382	26215.685994	$V \cdot m^{-1}$
Effective length (L-x)	2.05	2.05	mm
Second moment I	21.354	21.354	mm^4

Table 3: Values for known weights

2.5 Unknown Weight Determination

Using the 5V calibration curve: $F = 1.13 \cdot U_A$ −0.0007

Voltage	Deflection (V)	Predicted Force (N)	Predicted Mass (g)
2.5V	3.4047	3.861	393.5
	2.8329	3.212	327.4
5V	6.7661	7.673	782.1
	5.6027	6.353	647.6

Table 4: Values for unknown weights

2.6 Strain Verification: $\varepsilon \propto \frac{h}{2}$

From beam theory, the maximum strain occurs at the surface (distance h/2 from neutral axis):

$$\varepsilon_{\max} = \frac{M \cdot \frac{h}{2}}{I} = \frac{F \cdot (L - x) \cdot \frac{h}{2}}{I} \tag{7}$$

For your beam geometry:
- $\frac{h}{2} = 2.5$ mm
- $I = 21.354\text{mm}^4$
- Effective length $(L - x) = 2.05$ mm

The strain is directly proportional to h/2, confirming the theoretical relationship.

2.7 Sensitivity Analysis

The relationship $G = \gamma \cdot U_A$ yields:

- 2.5V sensitivity: $\gamma = 52112.539382\text{Vm}^{-1}$
- 5V sensitivity: $\gamma = 26215.685994\text{Vm}^{-1}$

The ratio of sensitivities is 0.5, which should be close to 2.0 if the system is linear.

2.8 Error Sources

1. **Geometric tolerances**: ±0.25 mm on length, ±0.05 mm on height
2. **Loading position uncertainty**: ±0.25 mm affects moment arm
3. **Strain gauge placement**: Must be at maximum strain location
4. **Temperature effects**: Affects E-modulus and gauge sensitivity
5. **Linearity assumption**: Valid only for small deflections

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Bibliography

[1] J. F. H. Müller, “Elastizität fester Körper,” 2010.