$\gamma := 1.47 \qquad M_m := 28.01\ \frac{gram}{mol} \qquad R_s := \frac{R_m}{M_m} = 296.8394 \cdot \frac{1}{K}\ \frac{J}{kg}$

$A_3 := 100000\ micron^2 = 0.1\ mm^2 \qquad A_4 := 100 \cdot \pi\ micron^2 \qquad A_5 := 400 \cdot \pi\ micron^2$

$T_t := 500\ K \qquad p_t := 1.5\ bar$

Isentropic Massflow:

$$massflow(\gamma, M, A, T_t, p_t) := A \cdot p_t \cdot \sqrt{\frac{\gamma}{R_s \cdot T_t}} \cdot M \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-\frac{\gamma+1}{2 \cdot (\gamma - 1)}}$$

Dynamic Viscosity using sutherlands formular:

$\mu_r := 1.716 \cdot 10^{-5}\ \frac{N\ s}{m^2} \qquad T_r := 276\ K \qquad S_\mu := 111\ K$

$$sutherland(T) := \mu_r \cdot \left(\frac{T}{T_r}\right)^{\frac{3}{2}} \cdot \frac{T_r + S_\mu}{T + S_\mu}$$

Knudsen number:

$$knudsen(p, T, L_c) := \frac{sutherland(T) \cdot R_s}{p \cdot L_c} \cdot \sqrt{\frac{\pi \cdot M_m \cdot T}{2\ k\ N_A}}$$

Reynolds number:

$$reynolds(p, T, L_c) := \frac{p \cdot L_c}{R_s \cdot T \cdot sutherland(T)}$$

Isentropic Relations:

$$temp\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-1} \qquad pressure\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-\frac{\gamma}{\gamma - 1}} \qquad density\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right.$$

$$solve\_machnumber\_sub(A_{ratio}, \gamma) := \mathrm{solve}\left(\frac{1}{M} \cdot \left(\left(\frac{2}{\gamma + 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)\right)^{\frac{\gamma+1}{2 \cdot (\gamma - 1)}} - A_{ratio} = 0, M, 0, 1\right)$$

$$solve\_machnumber\_super(A_{ratio}, \gamma) := \mathrm{solve}\left(\frac{1}{M} \cdot \left(\left(\frac{2}{\gamma + 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)\right)^{\frac{\gamma+1}{2 \cdot (\gamma - 1)}} - A_{ratio} = 0, M, 1, 12\right)$$

Position 3 (middle of Reactor):
Table isentropic 1D flow:

$M_{3.1} := solve\_machnumber\_sub(318, \gamma) = \blacksquare$

From external Solver:

$M_{3.1} := 0.0018$

$M_{3.2} := solve\_machnumber\_super(318, \gamma) = 10.543$

$m_{3.1} := massflow(\gamma, M_{3.1}, A_3, T_t, p_t) = 8.4972 \cdot 10^{-8}\ \frac{kg}{s}$

$m_{3.2} := massflow(\gamma, M_{3.2}, A_3, T_t, p_t) = 8.5253 \cdot 10^{-8}\ \frac{kg}{s}$

$p_{3.1.to.total} := pressure\_to\_total(\gamma, M_{3.1}) = 1$

$p_{3.2.to.total} := pressure\_to\_total(\gamma, M_{3.2}) = 3.2891 \cdot 10^{-5}$

$T_{3.1.to.total} := temp\_to\_total(\gamma, M_{3.1}) = 1$

$T_{3.2.to.total} := temp\_to\_total(\gamma, M_{3.2}) = 0.0369$

Position 2 & 4 (nozzle throats):

$m_{2.4} := massflow(\gamma, 1, A_4, T_t, p_t) = 8.517 \cdot 10^{-8}\ \frac{kg}{s}$

$p_{2.4.to.total} := pressure\_to\_total(\gamma, 1) = 0.5168$

$\rho_{2.4.to.total} := density\_to\_total(\gamma, 1) = 0.6382$

$T_{2.4.to.total} := temp\_to\_total(\gamma, 1) = 0.8097$

Position 5 (outlet nozzle exit plane):

$M_{5.1} := solve\_machnumber\_sub(4, \gamma) = 0.1455$

$m_{5.1} := massflow(\gamma, M_{5.1}, A_5, T_t, p_t) = 8.5169 \cdot 10^{-8}\ \frac{kg}{s}$

$p_{5.1.to.total} := pressure\_to\_total(\gamma, M_{5.1}) = 0.9846$

$\rho_{5.1.to.total} := density\_to\_total(\gamma, M_{5.1}) = 0.9895$

$T_{5.1.to.total} := temp\_to\_total(\gamma, M_{5.1}) = 0.9951$

$M_{5.2} := solve\_machnumber\_super(4, \gamma) = 3.063$

$m_{5.2} := massflow(\gamma, M_{5.2}, A_5, T_t, p_t) = 8.517 \cdot 10^{-8}\ \frac{kg}{s}$

$p_{5.2.to.total} := pressure\_to\_total(\gamma, M_{5.2}) = 0.0262$

$\rho_{5.2.to.total} := density\_to\_total(\gamma, M_{5.2}) = 0.0839$

$T_{5.2.to.total} := temp\_to\_total(\gamma, M_{5.2}) = 0.312$

$L_{c.1} := 40\ micron$ $L_{c.2} := 40\ micron$ $L_{c.3} := 40\ micron$ $L_{c.4} := 20\ micron$ $L_{c.5} := 40\ micron$

$Kn_1 := knudsen(p_t, T_t, L_{c.1}) = 0.0021$

$Kn_{2.4} := knudsen(p_t \cdot p_{2.4.to.total}, T_t \cdot T_{2.4.to.total}, L_{c.2}) = 0.0032$

$Kn_3 := knudsen(p_t \cdot p_{3.1.to.total}, T_t \cdot T_{3.1.to.total}, L_{c.3}) = 0.0021$

$Kn_5 := knudsen(p_t \cdot p_{5.2.to.total}, T_t \cdot T_{5.2.to.total}, L_{c.5}) = 0.0181$

$Re_1 := reynolds(p_t, T_t, L_{c.1}) = 1.5254\ \frac{s}{m}$

$Kn_{2.4} := knudsen(p_t \cdot p_{2.4.to.total}, T_t \cdot T_{2.4.to.total}, L_{c.2}) = 0.0032$

$Kn_3 := knudsen(p_t \cdot p_{3.1.to.total}, T_t \cdot T_{3.1.to.total}, L_{c.3}) = 0.0021$

$Kn_5 := knudsen(p_t \cdot p_{5.2.to.total}, T_t \cdot T_{5.2.to.total}, L_{c.5}) = 0.0181$

Disconeted Reservoirs (Isentropic):

$$M_0 := \left(1 + \frac{\gamma - 1}{2}\right)^{-\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} = 0.5743$$

$p_{ratio} := pressure\_to\_total(\gamma, M_0) = 0.7918$ $T_{ratio} := temp\_to\_total(\gamma, M_0) = 0.9281$ $\rho_{ratio} := density\_to\_total(\gamma, M_0) = 0.85$

Disconeted Reservoirs (Isothermal):

$$f := \left(-\frac{1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma - 1)^2} + \frac{2}{\gamma - 1} \cdot \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{\gamma - 1}}}\right) = 0.3076 \qquad M_0 := \sqrt{f} = 0.5546$$

$p_{ratio} := pressure\_to\_total(\gamma, M_0) = 0.8039$

Formulation with leak (isentopic connection):

$$m_{L.isen}\left(A, \gamma, T_r, p_r\right) := A \cdot p_r \cdot \sqrt{\frac{\gamma}{R_s \cdot T_r}} \cdot \left(1 - \left(1 + \frac{\gamma - 1}{2}\right)^{-\frac{\gamma + 1}{2 \cdot (\gamma - 1)}}\right)$$

$$m_{L.isen}\left(A_4, \gamma, T_t, p_t\right) = 6.3135 \cdot 10^{-8} \ \frac{\text{kg}}{\text{s}}$$

Formulation with leak (isothermal connection):

$$m_{L.iso}\left(A, \gamma, T_r, p_r\right) := A \cdot p_r \cdot \sqrt{\frac{\gamma}{R_s \cdot T_r}} \cdot \left(1 + \frac{\gamma - 1}{2}\right)^{-\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} \cdot \left(\left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma}{\gamma - 1}} - 1\right)$$

$$m_{L.iso}\left(A_4, \gamma, T_t, p_t\right) = 7.9642 \cdot 10^{-8} \ \frac{\text{kg}}{\text{s}}$$