Variable Definitions:

$\gamma := 1.47$ $\quad M_m := 28.01\,\frac{\text{gram}}{\text{mol}}$ $\quad R_s := \frac{R_m}{M_m} = 296.8394 \cdot \frac{1}{K}\,\frac{J}{kg}$

$A_3 := 100000\,\text{micron}^2 = 0.1\,\text{mm}^2$ $\quad A_4 := 100 \cdot \pi\,\text{micron}^2$ $\quad A_5 := 400 \cdot \pi\,\text{micron}^2$

$T_t := 500\,K$ $\quad p_t := 1.5\,\text{bar}$ $\quad A_2 := A_4$ $\quad A_1 := A_5$

$L_{c.1} := 40\,\text{micron}$ $\quad L_{c.2} := 40\,\text{micron}$ $\quad L_{c.3} := 40\,\text{micron}$ $\quad L_{c.4} := 20\,\text{micron}$ $\quad L_{c.5} := 40\,\text{micron}$

Function Definitions:

Isentropic Massflow:

$$massflow(\gamma, M, A, T_t, p_t) := A \cdot p_t \cdot \sqrt{\frac{\gamma}{R_s \cdot T_t}} \cdot M \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-\frac{\gamma + 1}{2 \cdot (\gamma - 1)}}$$

Dynamic Viscosity using sutherlands formular:

$\mu_{ref} := 1.716 \cdot 10^{-5}\,\frac{N\,s}{m^2}$ $\quad T_{ref} := 276\,K$ $\quad S_\mu := 111\,K$

$$p_{crit.ratio} := \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} = 0.5168$$

$$sutherland(T) := \mu_{ref} \cdot \left(\frac{T}{T_{ref}}\right)^{\frac{3}{2}} \cdot \frac{T_{ref} + S_\mu}{T + S_\mu}$$

$$T_{crit.ratio} := \frac{2}{\gamma + 1} = 0.8097$$

Knudsen number:

$$knudsen(p, T, L_c) := \frac{sutherland(T) \cdot R_s}{p \cdot L_c} \cdot \sqrt{\frac{\pi \cdot M_m \cdot T}{2\,k\,N_A}}$$

Reynolds number:

$$reynolds(p, T, L_c, Ma, \gamma) := \frac{knudsen(p, T, L_c)}{Ma} \cdot \sqrt{\frac{2}{\gamma \cdot \pi}}$$

Isentropic Relations:

$$temp\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-1}$$

$$pressure\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-\frac{\gamma}{\gamma - 1}}$$

$$density\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-\frac{1}{\gamma - 1}}$$

Machnumber solvers:

$$solve\_machnumber\_sub(A_{ratio}, \gamma) := \text{solve}\left(\frac{1}{M} \cdot \left(\left(\frac{2}{\gamma + 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)\right)^{\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} - A_{ratio} = 0, M, 0, 1\right)$$

$$solve\_machnumber\_super(A_{ratio}, \gamma) := \text{solve}\left(\frac{1}{M} \cdot \left(\left(\frac{2}{\gamma + 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)\right)^{\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} - A_{ratio} = 0, M, 1, 12\right)$$

Chapter 3.3: one-dimensional isentropic case

Position 3 (middle of Reactor):

$M_{3.1} := solve\_machnumber\_sub(318, \gamma) = \blacksquare$

From external Solver:

$M_{3.1} := 0.0018$

$M_{3.2} := solve\_machnumber\_super(318, \gamma) = 10.543$

$m_{3.1} := massflow(\gamma, M_{3.1}, A_3, T_t, p_t) = 8.4972 \cdot 10^{-8}\,\frac{kg}{s}$

$m_{3.2} := massflow(\gamma, M_{3.2}, A_3, T_t, p_t) = 8.5253 \cdot 10^{-8}\,\frac{kg}{s}$

$p_{3.1.to.total} := pressure\_to\_total(\gamma, M_{3.1}) = 1$

$T_{3.1.to.total} := temp\_to\_total(\gamma, M_{3.1}) = 1$

$p_{3.2.to.total} := pressure\_to\_total(\gamma, M_{3.2}) = 3.2891 \cdot 10^{-5}$

$T_{3.2.to.total} := temp\_to\_total(\gamma, M_{3.2}) = 0.0369$

Position 2 & 4 (nozzle throats):

$m_{2.4} := massflow(\gamma, 1, A_4, T_t, p_t) = 8.517 \cdot 10^{-8} \frac{kg}{s}$

$p_{2.4.to.total} := pressure\_to\_total(\gamma, 1) = 0.5168$

$\rho_{2.4.to.total} := density\_to\_total(\gamma, 1) = 0.6382$

$T_{2.4.to.total} := temp\_to\_total(\gamma, 1) = 0.8097$

Position 5 (outlet nozzle exit plane):

$M_{5.1} := solve\_machnumber\_sub(4, \gamma) = 0.1455$

$m_{5.1} := massflow(\gamma, M_{5.1}, A_5, T_t, p_t) = 8.5169 \cdot 10^{-8} \frac{kg}{s}$

$p_{5.1.to.total} := pressure\_to\_total(\gamma, M_{5.1}) = 0.9846$

$\rho_{5.1.to.total} := density\_to\_total(\gamma, M_{5.1}) = 0.9895$

$T_{5.1.to.total} := temp\_to\_total(\gamma, M_{5.1}) = 0.9951$

$M_{5.2} := solve\_machnumber\_super(4, \gamma) = 3.063$

$m_{5.2} := massflow(\gamma, M_{5.2}, A_5, T_t, p_t) = 8.517 \cdot 10^{-8} \frac{kg}{s}$

$p_{5.2.to.total} := pressure\_to\_total(\gamma, M_{5.2}) = 0.0262$

$\rho_{5.2.to.total} := density\_to\_total(\gamma, M_{5.2}) = 0.0839$

$T_{5.2.to.total} := temp\_to\_total(\gamma, M_{5.2}) = 0.312$

Chapter 3.3: one-dimensional isentropic knudsen and reynolds numbers

knudsen numbers:

$Kn_1 := knudsen(p_t, T_t, L_{c.1}) = 0.0021$

$Kn_{2.4} := knudsen(p_t \cdot p_{2.4.to.total}, T_t \cdot T_{2.4.to.total}, L_{c.2}) = 0.0032$

$Kn_{3.1} := knudsen(p_t \cdot p_{3.1.to.total}, T_t \cdot T_{3.1.to.total}, L_{c.3}) = 0.0021$

$Kn_{3.2} := knudsen(p_t \cdot p_{3.2.to.total}, T_t \cdot T_{3.2.to.total}, L_{c.3}) = 0.4161$

$Kn_{5.1} := knudsen(p_t \cdot p_{5.1.to.total}, T_t \cdot T_{5.1.to.total}, L_{c.5}) = 0.0022$

$Kn_{5.2} := knudsen(p_t \cdot p_{5.2.to.total}, T_t \cdot T_{5.2.to.total}, L_{c.5}) = 0.0181$

reynolds numbers:

$Re_1 := reynolds(p_t, T_t, L_{c.1}, 0.001, \gamma) = 1.4035$

$Re_{2.4} := reynolds(p_t, T_t, L_{c.2}, 1, \gamma) = 0.0014$

$Re_{3.1} := reynolds(p_t, T_t, L_{c.3}, M_{3.1}, \gamma) = 0.7797$

$Re_{3.2} := reynolds(p_t, T_t, L_{c.3}, M_{3.2}, \gamma) = 0.0001$

$Re_{5.1} := reynolds(p_t, T_t, L_{c.5}, M_{5.1}, \gamma) = 0.0096$

$Re_{5.1} := reynolds(p_t, T_t, L_{c.5}, M_{5.2}, \gamma) = 0.0005$

Chapter 3.5: Disconneted Reservoirs (Isentropic):

$$M_{D.iso} := \left(1 + \frac{\gamma - 1}{2}\right)^{-\frac{\gamma+1}{2 \cdot (\gamma - 1)}} = 0.5743$$

$p_{ratio} := pressure\_to\_total(\gamma, M_{D.iso}) = 0.7918$

$T_{ratio} := temp\_to\_total(\gamma, M_{D.iso}) = 0.9281$

$\rho_{ratio} := density\_to\_total(\gamma, M_{D.iso}) = 0.8531$

$m_{D.term} := massflow(\gamma, M_{D.iso}, A_2, T_t, p_t) = 7 \cdot 10^{-8} \frac{kg}{s}$

Chapter 3.5: Disconneted Reservoirs (Isothermal):

$$f := \left(-\frac{1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \frac{2}{\gamma-1} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \right) = 0.3076 \qquad M_{D.term} := \sqrt{f} = 0.5546$$

$$p_{ratio} := pressure\_to\_total\left(\gamma, M_{D.term}\right) = 0.8039 \qquad T_{ratio} := temp\_to\_total\left(\gamma, M_{D.term}\right) = 0.9326$$

$$\rho_{ratio} := density\_to\_total\left(\gamma, M_{D.term}\right) = 0.862$$

$$m_{D.term} := massflow\left(\gamma, M_{D.term}, A_2, T_t, p_t\right) = 6.8468 \cdot 10^{-8}\ \frac{\text{kg}}{\text{s}}$$

Chapter 3.5: Formulation with leak (isentopic connection):

$$m_{L.isen}\left(A, \gamma, T_r, p_r\right) := A \cdot p_r \cdot \sqrt{\frac{\gamma}{R_s \cdot T_r}} \cdot \left(1 - \left(1 + \frac{\gamma-1}{2}\right)^{-\frac{\gamma+1}{2\cdot(\gamma-1)}}\right)$$

$$p_r := p_{crit.ratio} \cdot p_t = 0.7752\ \text{bar} \qquad T_r := T_{crit.ratio} \cdot T_t = 404.8583\ \text{K}$$

$$m_{L.isen}\left(A_4, \gamma, T_r, p_r\right) = 3.6258 \cdot 10^{-8}\ \frac{\text{kg}}{\text{s}}$$

Chapter 3.5: Formulation with leak (isothermal connection):

$$m_{L.iso}\left(A, \gamma, T, p_r\right) := A \cdot p_r \cdot \sqrt{\frac{\gamma}{R_s \cdot T}} \cdot \left(1 + \frac{\gamma-1}{2}\right)^{-\frac{\gamma+1}{2\cdot(\gamma-1)}} \cdot \left(\left(1 + \frac{\gamma-1}{2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)$$

$$m_{L.iso}\left(A_4, \gamma, T_r, p_r\right) = 4.5737 \cdot 10^{-8}\ \frac{\text{kg}}{\text{s}}$$

Chapter 3.2: expected knudsen number

Test:

$$\alpha := 0.06\ \frac{\text{Pa}}{\text{K}^{\frac{3}{2}}} \qquad Kn := knudsen\left(1\ \text{bar}, 300\ \text{K}, 20\ \text{micron}\right) = 0.0034$$

$$p := 1\ \text{bar}$$

$$p := \alpha \cdot \frac{T_t^{\frac{3}{2}}}{0.1} = 0.0671\ \text{bar} \qquad a := \frac{p \cdot Kn}{T_t^{\frac{3}{2}}} = 0.0021\ \frac{\text{Pa}}{\text{K}^{\frac{3}{2}}}$$

Chapter 3.6: Maximum turning angle

$$\theta_{max} := \frac{\pi}{2} \cdot \left(\sqrt{\frac{\gamma+1}{\gamma-1}} - 1\right) - \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot atan\left(\sqrt{\frac{\gamma-1}{\gamma+1} \cdot \left(M_{5.2}^{\ 2} - 1\right)}\right) + atan\left(\sqrt{M_{5.2}^{\ 2} - 1}\right) = 68.912\ \text{deg}$$