$\gamma := 1.47$ $\quad M_m := 28.01\ \frac{gram}{mol}$ $\quad R_s := \frac{R_m}{M_m} = 296.8394 \cdot \frac{1}{K}\frac{J}{kg}$ $\quad R_s = 296.8394\ \frac{Gy}{K}$

$A_3 := 100000\ micron^2 = 0.1\ mm^2$ $\quad A_4 := 100 \cdot \pi\ micron^2$ $\quad A_5 := 400 \cdot \pi\ micron^2 = 1.2566 \cdot 10^{-9}\ m^2$

$T_0 := 300\ K$ $\quad p_0 := 1.5\ bar$ $\quad A_2 := A_4$ $\quad A_1 := A_5$

$L_{c.1} := 40\ micron$ $\quad L_{c.2} := 20\ micron$ $\quad L_{c.3} := 40\ micron$ $\quad L_{c.4} := 20\ micron$ $\quad L_{c.5} := 40\ micron$

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Function Definitions:

Variable Definitions:

Isentropic Massflow:

$$massflow(\gamma, M, A, T_t, p_t) := A \cdot p_t \cdot \sqrt{\frac{\gamma}{R_s \cdot T_t}} \cdot M \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-\frac{\gamma+1}{2 \cdot (\gamma - 1)}}$$

$$molecules\_per\_sec(mdot, M_m) := mdot \cdot \frac{N_A}{M_m}$$

Dynamic Viscosity using sutherlands formular:

$\mu_{ref} := 1.716 \cdot 10^{-5}\ \frac{N\,s}{m^2}$ $\quad T_{ref} := 276\ K$ $\quad S_\mu := 111\ K$

$$p_{crit.ratio} := \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} = 0.5168$$

$$sutherland(T) := \mu_{ref} \cdot \left(\frac{T}{T_{ref}}\right)^{\frac{3}{2}} \cdot \frac{T_{ref} + S_\mu}{T + S_\mu}$$

$$T_{crit.ratio} := \frac{2}{\gamma + 1} = 0.8097$$

Knudsen number:

$$knudsen(p, T, L_c) := \frac{sutherland(T) \cdot R_s}{p \cdot L_c} \cdot \sqrt{\frac{\pi \cdot M_m \cdot T}{2\ k\ N_A}}$$

Reynolds number:

$$reynolds(p, T, L_c, Ma, \gamma) := \frac{Ma}{knudsen(p, T, L_c)} \cdot \sqrt{\frac{\gamma \cdot \pi}{2}}$$

Isentropic Relations:

$$temp\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-1}$$

$$pressure\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{-\frac{\gamma}{\gamma - 1}}$$

$$density\_to\_total(\gamma, M) := \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right.$$

Machnumber solvers:

$$solve\_machnumber\_sub(A_{ratio}, \gamma) := solve\left(\frac{1}{M} \cdot \left(\left(\frac{2}{\gamma + 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)\right)^{\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} - A_{ratio} = 0, M, 0, 1\right)$$

$$solve\_machnumber\_super(A_{ratio}, \gamma) := solve\left(\frac{1}{M} \cdot \left(\left(\frac{2}{\gamma + 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)\right)^{\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} - A_{ratio} = 0, M, 1, 12\right)$$

$$\left(terminal\_velocity(R, T_0, \gamma) := \sqrt{2 \cdot R \cdot \frac{\gamma}{\gamma - 1} \cdot T_0}\right)$$

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## Chapter 3.3: one-dimensional isentropic case

Position 3 (middle of Reactor):

$M_{3.1} := solve\_machnumber\_sub(318, \gamma) = ▮$

From external Solver:

$M_{3.1} := 0.0018$

$M_{3.2} := solve\_machnumber\_super(318, \gamma) = 10.543$

$m_{3.1} := massflow(\gamma, M_{3.1}, A_3, T_0, p_0) = 1.097 \cdot 10^{-7}\ \frac{kg}{s}$

$m_{3.2} := massflow(\gamma, M_{3.2}, A_3, T_0, p_0) = 1.1006 \cdot 10^{-7}\ \frac{kg}{s}$

$p_{3.1.to.total} := pressure\_to\_total(\gamma, M_{3.1}) = 1$

$p_{3.2.to.total} := pressure\_to\_total(\gamma, M_{3.2}) = 3.2891 \cdot 10^{-5}$

$T_{3.1.to.total} := temp\_to\_total(\gamma, M_{3.1}) = 1$

$T_{3.2.to.total} := temp\_to\_total(\gamma, M_{3.2}) = 0.0369$

Position 2 & 4 (nozzle throats):

$m_{2.4} := massflow\left(\gamma, 1, A_4, T_0, p_0\right) = 1.0995 \cdot 10^{-7}\ \frac{kg}{s}$

$m_{1D\_isen} := molecules\_per\_sec\left(m_{2.4}, M_m\right) = 2.364 \cdot 10^{18}\ \frac{1}{s}$

$p_{2.4.to.total} := pressure\_to\_total(\gamma, 1) = 0.5168$

$\rho_{2.4.to.total} := density\_to\_total(\gamma, 1) = 0.6382$

$T_{2.4.to.total} := temp\_to\_total(\gamma, 1) = 0.8097$

Position 5 (outlet nozzle exit plane):

$M_{5.1} := solve\_machnumber\_sub(4, \gamma) = 0.1455$

$M_{5.2} := solve\_machnumber\_super(4, \gamma) = 3.063$

$m_{5.1} := massflow\left(\gamma, M_{5.1}, A_5, T_0, p_0\right) = 1.0995 \cdot 10^{-7}\ \frac{kg}{s}$

$m_{5.2} := massflow\left(\gamma, M_{5.2}, A_5, T_0, p_0\right) = 1.0995 \cdot 10^{-7}\ \frac{kg}{s}$

$p_{5.1.to.total} := pressure\_to\_total\left(\gamma, M_{5.1}\right) = 0.9846$

$p_{5.2.to.total} := pressure\_to\_total\left(\gamma, M_{5.2}\right) = 0.0262$

$\rho_{5.1.to.total} := density\_to\_total\left(\gamma, M_{5.1}\right) = 0.9895$

$\rho_{5.2.to.total} := density\_to\_total\left(\gamma, M_{5.2}\right) = 0.0839$

$T_{5.1.to.total} := temp\_to\_total\left(\gamma, M_{5.1}\right) = 0.9951$

$T_{5.2.to.total} := temp\_to\_total\left(\gamma, M_{5.2}\right) = 0.312$

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## Chapter 3.3: one-dimensional isentropic knudsen and reynolds numbers

knudsen numbers:

$Kn_1 := knudsen\left(p_0, T_0, L_{c.1}\right) = 0.0011$

$Kn_{2.4} := knudsen\left(p_0 \cdot p_{2.4.to.total}, T_0 \cdot T_{2.4.to.total}, L_{c.2}\right) = 0.0034$

$Kn_{3.1} := knudsen\left(p_0 \cdot p_{3.1.to.total}, T_0 \cdot T_{3.1.to.total}, L_{c.3}\right) = 0.0011$

$Kn_{3.2} := knudsen\left(p_0 \cdot p_{3.2.to.total}, T_0 \cdot T_{3.2.to.total}, L_{c.3}\right) = 0.1589$

$Kn_{5.1} := knudsen\left(p_0 \cdot p_{5.1.to.total}, T_0 \cdot T_{5.1.to.total}, L_{c.5}\right) = 0.0012$

$Kn_{5.2} := knudsen\left(p_0 \cdot p_{5.2.to.total}, T_0 \cdot T_{5.2.to.total}, L_{c.5}\right) = 0.0085$

reynolds numbers:

$Re_1 := reynolds\left(p_0, T_0, L_{c.1}, 0, \gamma\right) = 0$

$Re_{2.4} := reynolds\left(p_0 \cdot p_{2.4.to.total}, T_0 \cdot T_{2.4.to.total}, L_{c.2}, 1\right.$

$Re_{3.1} := reynolds\left(p_0 \cdot p_{3.1.to.total}, T_0 \cdot T_{3.1.to.total}, L_{c.3}, M_{3.1}, \gamma\right) = 2.3964$

$Re_{3.2} := reynolds\left(p_0 \cdot p_{3.2.to.total}, T_0 \cdot T_{3.2.to.total}, L_{c.3}, M\right.$

$Re_{5.1} := reynolds\left(p_0 \cdot p_{5.1.to.total}, T_0 \cdot T_{5.1.to.total}, L_{c.5}, M_{5.1}, \gamma\right) = 191.8725$

$Re_{5.2} := reynolds\left(p_0 \cdot p_{5.2.to.total}, T_0 \cdot T_{5.2.to.total}, L_{c.5}, M\right.$

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## Chapter 3.5: Disconneted Reservoirs (Isentropic):

$$M_{D.iso} := \left(1 + \frac{\gamma - 1}{2}\right)^{-\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} = 0.5743$$

$p_{ratio} := pressure\_to\_total\left(\gamma, M_{D.iso}\right) = 0.7918$

$T_{ratio} := temp\_to\_total\left(\gamma, M_{D.iso}\right) = 0.9281$

$\rho_{ratio} := density\_to\_total\left(\gamma, M_{D.iso}\right) = ($

$m_{D.iso} := massflow\left(\gamma, M_{D.iso}, A_2, T_0, p_0\right) = 9.037 \cdot 10^{-8}\ \frac{kg}{s}$

$q_{m.D.isen} := molecules\_per\_sec\left(m_{D.iso}, M_m\right) = 1.9429 \cdot 10^{18}\ Hz$

## Chapter 3.5: Disconneted Reservoirs (Isothermal):

$$f := \left( -\frac{1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma - 1)^2} + \frac{2}{\gamma - 1} \cdot \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}} \right) = 0.3076$$

$$M_{D.term} := \sqrt{f} = 0.5546$$

$$p_{ratio} := pressure\_to\_total\left(\gamma, M_{D.term}\right) = 0.8039 \qquad T_{ratio} := temp\_to\_total\left(\gamma, M_{D.term}\right) = 0.9326$$

$$\rho_{ratio} := density\_to\_total\left(\gamma, M_{D.term}\right) = 0.862$$

$$m_{D.term} := massflow\left(\gamma, M_{D.term}, A_2, T_0, p_0\right) = 8.8392 \cdot 10^{-8} \; \frac{\mathrm{kg}}{\mathrm{s}}$$

$$q_{m.D.iso} := molecules\_per\_sec\left(m_{D.term}, M_m\right) = 1.9004 \cdot 10^{18} \; \mathrm{Hz}$$

Chapter 3.5: Formulation with leak (isentopic connection):

$$m_{L.isen}\left(A, \gamma, T_r, p_r\right) := A \cdot p_r \cdot \sqrt{\frac{\gamma}{R_s \cdot T_r}} \cdot \left( 1 - \left( 1 + \frac{\gamma - 1}{2} \right)^{-\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} \right)$$

$$p_r := p_{crit.ratio} \cdot p_0 = 0.7752 \; \mathrm{bar} \qquad T_r := T_{crit.ratio} \cdot T_0 = 242.915 \; \mathrm{K}$$

$$m_{L.isen} := m_{L.isen}\left(A_4, \gamma, T_r, p_r\right) = 4.6808 \cdot 10^{-8} \; \frac{\mathrm{kg}}{\mathrm{s}}$$

$$q_{m.L.isen} := molecules\_per\_sec\left(m_{L.isen}, M_m\right) = 1.0064 \cdot 10^{18} \; \mathrm{Hz}$$

Chapter 3.5: Formulation with leak (isothermal connection):

$$m_{L.iso}\left(A, \gamma, T, p_r\right) := A \cdot p_r \cdot \sqrt{\frac{\gamma}{R_s \cdot T}} \cdot \left( 1 + \frac{\gamma - 1}{2} \right)^{-\frac{\gamma + 1}{2 \cdot (\gamma - 1)}} \cdot \left( \left( 1 + \frac{\gamma - 1}{2} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right)$$

$$m_{L.isot} := m_{L.iso}\left(A_4, \gamma, T_0, p_r\right) = 5.3133 \cdot 10^{-8} \; \frac{\mathrm{kg}}{\mathrm{s}}$$

$$q_{m.L.iso} := molecules\_per\_sec\left(m_{L.isot}, M_m\right) = 1.1424 \cdot 10^{18} \; \mathrm{Hz}$$

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Chapter 3.6: Maximum turning angle

$$\theta_{max} := \frac{\pi}{2} \cdot \left( \sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1 \right) - \sqrt{\frac{\gamma + 1}{\gamma - 1}} \cdot \mathrm{atan}\left( \sqrt{\frac{\gamma - 1}{\gamma + 1} \cdot \left( {M_{5.2}}^2 - 1 \right)} \right) + \mathrm{atan}\left( \sqrt{{M_{5.2}}^2 - 1} \right) = 68.912 \; \mathrm{deg}$$

Terminal velocity:

$$v_{infty} := terminal\_velocity\left(R_s, T_0, \gamma\right) = 746.3562 \; \frac{\mathrm{m}}{\mathrm{s}}$$

$$T_{max} := 600 \; \mathrm{K}$$

$$v_{infty} := terminal\_velocity\left(R_s, T_{max}, \gamma\right) = 1055.5071 \; \frac{\mathrm{m}}{\mathrm{s}}$$

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