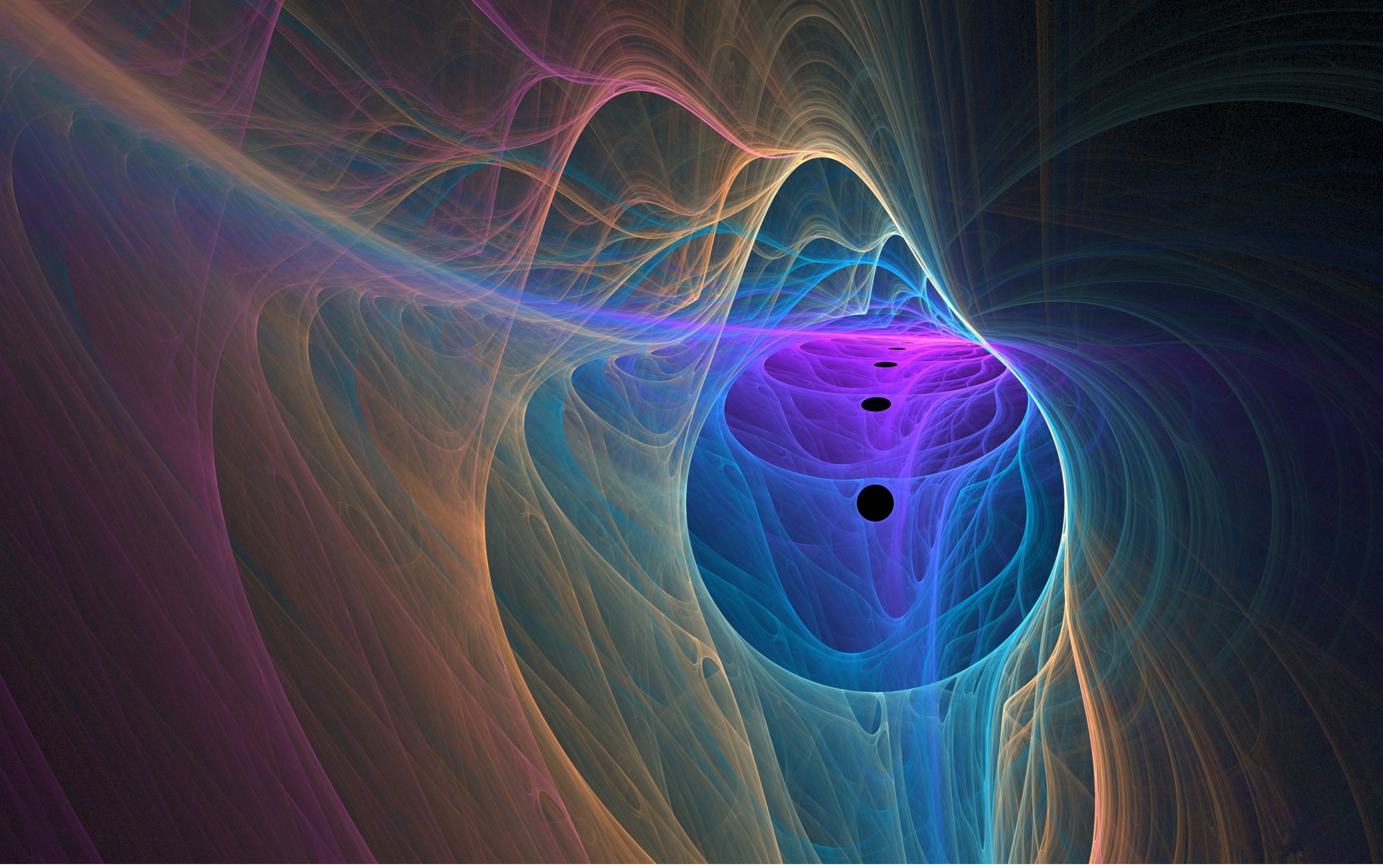

Model arrangement of Facility Location Problems: Coverage Models and Hierarchical Extensions

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1 Coverage Models

Definition 1.1 (Notation)**.** The following notation is used throughout this document:

- i: index of demand nodes, j: index of facility nodes
- I: set of demand nodes, J: set of potential facility sites
- S: service distance or time standard
- p: number of facilities to locate
- d_{ij}: distance or travel time from demand node i to facility j
- a_i: demand at node i (e.g., population)
- w_j: cost of establishing a facility at site j
- $N_i = \{j \in J : d_{ij} \leq S\}$: set of facilities that can cover demand node i
- $M_j = \{i \in I : d_{ij} \leq S\}$: set of demand nodes covered by facility j
- $x_j = \begin{cases} 1, & \text{if a facility is located at site } j \\ 0, & \text{otherwise} \end{cases}$
- $y_i = \begin{cases} 1, & \text{if demand node } i \text{ is covered} \\ 0, & \text{otherwise} \end{cases}$
- X_j: number of servers at facility site j (can be ≥ 0)

1.1 Location Set Covering Problem (LSCP)

Definition 1.2 (LSCP)**.** The Location Set Covering Problem minimizes the number of facilities required to cover all demand nodes:

$$\text{Minimize } Z = \sum_{j \in J} x_j$$

$$\text{subject to: } \begin{cases} \sum_{j \in N_i} x_j \geq 1, & \forall i \in I \\ x_j \in \{0, 1\}, & \forall j \in J \end{cases}$$

Remark 1.1. LSCP assumes complete coverage of all demand nodes, which can be unrealistic in resource-constrained settings.

1.2 Maximum Covering Location Problem (MCLP)

Definition 1.3 (MCLP)**.** The Maximum Covering Location Problem maximizes covered demand given a fixed number of facilities p:

$$\text{Maximize } Z = \sum_{i \in I} a_i y_i$$

$$\text{subject to: } \begin{cases} y_i \leq \sum_{j \in N_i} x_j, & \forall i \in I \\ \sum_{j \in J} x_j \leq p \\ x_j \in \{0,1\}, & \forall j \in J \\ y_i \in \{0,1\}, & \forall i \in I \end{cases}$$

Remark 1.2. The first constraint ensures that a demand node is only counted as covered if at least one facility within the coverage standard is established. The binary constraints on x_j and y_i are essential for the model's formulation.

1.3 Probabilistic Extensions: Key Concepts

Definition 1.4 (Busy Rate and Reliability)**.** In probabilistic coverage models, we consider:

- q: global busy rate (probability a server is busy)
- q_i: local busy rate for demand node i
- α: reliability level (minimum probability that at least one server is available)
- H_k: marginal increase in expected coverage when adding a k-th server

1.4 Maximum Expected Covering Location Problem (MEXCLP)

Definition 1.5 (Expected Coverage Derivation)**.** Assuming independent and identically distributed server busy probabilities q, the probability that at least one of k servers is available for a demand node is:

$$P_k = 1 - q^k$$

The increase in expected coverage when adding the k-th server to a demand node is:

$$H_k = P_k - P_{k-1} = (1 - q^k) - (1 - q^{k-1}) = q^{k-1}(1 - q)$$

Thus, the expected coverage contributed by the k^{th} server covering demand node i is $a_i H_k$.

Definition 1.6 (MEXCLP)**.** MEXCLP maximizes expected coverage given p servers with independent busy probability q:

$$\text{Maximize } Z = \sum_{i \in I} \sum_{k=1}^{n_i} a_i q^{k-1}(1 - q) z_{ik}$$

$$\text{subject to: } \begin{cases} \sum_{k=1}^{n_i} z_{ik} \leq \sum_{j \in N_i} X_j, & \forall i \in I \\ \sum_{j \in J} X_j \leq p \\ X_j \in \mathbb{N}_0, & \forall j \in J \\ z_{ik} \in \{0,1\}, & \forall i \in I, k = 1, \ldots, n_i \end{cases}$$

where X_j is the number of servers at facility j, $z_{ik} = 1$ if demand node i is covered by at least k servers, and n_i is the maximum possible number of servers covering node i.

1.5 Probabilistic Location Set Covering Problem (PLSCP)

Definition 1.7 (Local Busy Rate Derivation)**.** The local busy rate for demand node i is estimated as:

$$q_i = \frac{\text{total service time required in } M_i}{\text{total service capacity available to } i}$$

More precisely, if $\bar{t}$ is the average service time (hours) and f_k is the request rate at node k (requests/day):

$$q_i = \frac{\bar{t}\sum_{k\in M_i} f_k}{24\sum_{j\in N_i} X_j} = \frac{\rho_i}{\sum_{j\in N_i} X_j}$$

where $\rho_i = \frac{\bar{t}\sum_{k\in M_i} f_k}{24}$ is the utilization rate.

Theorem 1.1 (Reliability Constraint Derivation)**.** *The probability that at least one server is available for demand node i is:*

$$P_i = 1 - q_i^{\sum_{j\in N_i} X_j} = 1 - \left(\frac{\rho_i}{\sum_{j\in N_i} X_j}\right)^{\sum_{j\in N_i} X_j}$$

Requiring $P_i \geq \alpha$ leads to:

$$1 - \left(\frac{\rho_i}{\sum_{j\in N_i} X_j}\right)^{\sum_{j\in N_i} X_j} \geq \alpha$$

This is equivalent to finding the smallest integer b_i such that:

$$1 - \left(\frac{\rho_i}{b_i}\right)^{b_i} \geq \alpha$$

Thus, we require $\sum_{j\in N_i} X_j \geq b_i$.

Definition 1.8 (PLSCP)**.** PLSCP minimizes the number of servers required to achieve a reliability level α:

$$\text{Minimize } Z = \sum_{j\in J} X_j$$

$$\text{subject to: } \begin{cases} \sum\limits_{j\in N_i} X_j \geq b_i, & \forall i \in I \\ X_j \in \mathbb{N}_0, & \forall j \in J \end{cases}$$

where b_i is the smallest integer satisfying $1 - (\rho_i/b_i)^{b_i} \geq \alpha$.

1.6 Maximum Availability Location Problem (MALP)

Definition 1.9 (MALP)**.** MALP maximizes demand covered with reliability α given p servers:

$$\text{Maximize } Z = \sum_{i\in I} a_i z_{ib_i}$$

$$\text{subject to:} \begin{cases} \sum_{k=1}^{b_i} z_{ik} \leq \sum_{j \in N_i} X_j, & \forall i \in I \\ z_{ik} \leq z_{i(k-1)}, & \forall i \in I, k = 2, \ldots, b_i \\ \sum_{j \in J} X_j \leq p & \\ X_j \in \mathbb{N}_0, & \forall j \in J \\ z_{ik} \in \{0, 1\}, & \forall i \in I, k = 1, \ldots, b_i \end{cases}$$

where b_i is as defined in PLSCP, and $z_{ik} = 1$ if demand node i is covered by at least k servers.

Remark 1.3. The constraints $z_{ik} \leq z_{i(k-1)}$ ensure logical consistency: if a node is covered by at least k servers, it must also be covered by at least $k-1$ servers.

2 Hierarchical Location Models

Definition 2.1 (Hierarchical Medical System)**.** A two-tier system consisting of:

- Clinics: provide primary/basic services
- Hospitals: provide advanced/specialized services and support clinics
- Two referral patterns:
  - Top-down (clinic to hospital): R1 model
  - Bottom-up (hospital to clinic): R2 model

2.1 Top-Down Referral Model (R1)

Definition 2.2 (Additional Notation for Hierarchical Models)**.** We consider:

- $c_j = \begin{cases} 1, & \text{if a clinic is located at site } j \\ 0, & \text{otherwise} \end{cases}$
- $h_j = \begin{cases} 1, & \text{if a hospital is located at site } j \\ 0, & \text{otherwise} \end{cases}$
- P_c: number of clinics to locate
- P_h: number of hospitals to locate
- $V_i = \{j \in J : d_{ij} \leq S\}$: set of hospital sites that can provide clinic services to node i
- $U_i = \{j \in J : d_{ij} \leq S\}$: set of hospital sites that can provide hospital services to node i
- $q_j = \begin{cases} 1, & \text{if clinic at site } j \text{ is not covered by any hospital} \\ 0, & \text{otherwise} \end{cases}$

Definition 2.3 (R1 Model)**.** R1 has two objectives: maximize clinic coverage and minimize clinics without hospital access:

$$\text{Maximize } Z = \left(\sum_{i \in I} a_i y_i, -\sum_{j \in J} q_j \right)$$

$$\text{subject to:} \begin{cases} \sum_{j\in N_i} c_j + \sum_{j\in V_i} h_j \geq y_i, & \forall i \in I \\ \sum_{k\in M_j} h_k - c_j + q_j \geq 0, & \forall j \in J \\ \sum_j h_j = P_h, \quad \sum_j c_j = P_c \\ c_j, h_j, y_i, q_j \in \{0,1\}, & \forall i \in I, j \in J \end{cases}$$

Remark 2.1. The constraint $\sum_{k\in M_j} h_k - c_j + q_j \geq 0$ ensures $q_j = 1$ when $c_j = 1$ and $\sum_{k\in M_j} h_k = 0$ (clinic j has no hospital covering it).

2.2 Bottom-Up Referral Model (R2)

Definition 2.4 (R2 Model)**.** R2 maximizes both clinic coverage and hospital accessibility for referred patients:

$$\text{Maximize } Z = \left[\sum_{i\in I} a_i y_i, \sum_{i\in I}\left(\sum_{j\in N_i} \alpha_i a_i x_{ij}\right)\right]$$

$$\text{subject to:} \begin{cases} \sum_{j\in N_i} c_j + \sum_{j\in V_i} h_j \geq y_i, & \forall i \in I \\ x_{ij} \leq c_j + h_j, & \forall i \in I, \forall j \in N_i \\ x_{ij} \leq h_j, & \forall i \in I, \forall j \in V_i \setminus N_i \\ x_{ij} \leq \sum_{k\in M_j} h_k, & \forall i \in I, \forall j \in N_i \\ \sum_{j\in N_i} x_{ij} \leq 1, & \forall i \in I \\ \sum_j h_j = P_h, \quad \sum_j c_j = P_c \\ c_j, h_j, x_{ij}, y_i \in \{0,1\}, & \forall i \in I, j \in J \end{cases}$$

where α_i is the referral rate from clinic to hospital for demand at node i.

Remark 2.2. $x_{ij} = 1$ indicates demand at i is served by clinic j and that clinic has hospital support for referrals.

3 Time Satisfaction Based Coverage Models

Definition 3.1 (Time Satisfaction Function)**.** Let $F(t_{ij})$ denote the time satisfaction function, where t_{ij} is the travel time from demand node i to facility j. This function represents customer satisfaction as a function of travel time, typically ranging from 0 to 1. The function is defined with two time thresholds:

- L_i: Lower time threshold (maximum acceptable time for full satisfaction)
- U_i: Upper time threshold (time beyond which satisfaction is zero)
- For all $i \in I$, we assume $0 < L_i < U_i$

3.1 Linear Time Satisfaction Function

Definition 3.2 (Linear Function)**.** The linear time satisfaction function is the simplest form, where satisfaction decreases linearly from 1 to 0 as travel time increases from L_i to U_i:

$$F(t_{ij}) = \begin{cases} 1, & t_{ij} \leq L_i \\ \frac{U_i - t_{ij}}{U_i - L_i}, & L_i < t_{ij} \leq U_i \\ 0, & t_{ij} > U_i \end{cases} \quad \forall i \in I, \forall j \in J$$

Remark 3.1. This function assumes that customer dissatisfaction increases at a constant rate as travel time increases beyond L_i.

3.2 Convex/Concave Time Satisfaction Function

Definition 3.3 (Convex/Concave Function)**.** The convex/concave time satisfaction function allows for different sensitivity patterns through a shape parameter k_i:

$$F(t_{ij}) = \begin{cases} 1 - \left[\frac{t_{ij} - L_i}{U_i - L_i}\right]^{k_i}, & t_{ij} \leq L_i \\ 0, & L_i < t_{ij} \leq U_i \\ 0, & t_{ij} > U_i \end{cases} \quad \forall i \in I, \forall j \in J$$

where $k_i > 0$ is the time sensitivity coefficient for demand node i.

Remark 3.2. The shape of the function depends on k_i:

- When $k_i < 1$, the function is concave (sensitivity decreases with time)
- When $k_i = 1$, the function becomes linear
- When $k_i > 1$, the function is convex (sensitivity increases with time)

An alternative symmetric form passing through point $\left(\frac{L_i+U_i}{2}, \frac{1}{2}\right)$ can be defined.

3.3 Cosine Distributed Time Satisfaction Function

Definition 3.4 (Cosine Function)**.** The cosine distributed time satisfaction function uses a cosine curve segment from $\pi/2$ to $3\pi/2$:

$$F(t_{ij}) = \begin{cases} 1, & t_{ij} \leq L_i \\ \frac{1}{2} + \frac{1}{2}\cos\left[\frac{\pi}{U_i - L_i}\left(t_{ij} - \frac{U_i + L_i}{2}\right) + \frac{\pi}{2}\right], & L_i < t_{ij} \leq U_i \\ 0, & t_{ij} > U_i \end{cases} \quad \forall i \in I, \forall j \in J$$

Remark 3.3. This function has smaller satisfaction changes near the thresholds L_i and U_i, and larger changes in the middle portion of the curve.

3.4 Descending Exponential Sigmoid Time Satisfaction Function

Definition 3.5 (Exponential Sigmoid Function)**.** The descending exponential sigmoid function models rapid initial satisfaction decrease followed by slower decline:

$$F(t_{ij}) = \begin{cases} 1, & t_{ij} \leq L_i \\ \frac{2e^{-\beta_i(t_{ij} - L_i)}}{1 + e^{-\beta_i(t_{ij} - L_i)}}, & t_{ij} > L_i \end{cases} \quad \forall i \in I, \forall j \in J$$

where $\beta_i \in \mathbb{R}^+$ is the time sensitivity coefficient for demand node i.

Remark 3.4.

- Customer satisfaction drops rapidly just beyond L_i
- As travel time further increases, satisfaction decline becomes less sensitive

- Larger β_i values indicate higher time sensitivity
- The upper threshold U_i is implicitly defined by β_i

3.5 Descending Semi-Cauchy Distributed Time Satisfaction Function

Definition 3.6 (Semi-Cauchy Function)**.** The descending semi-Cauchy distributed time satisfaction function has similar characteristics to the exponential sigmoid:

$$F(t_{ij}) = \begin{cases} 1, & t_{ij} \leq L_i \\ \frac{1}{1+\beta_i(t_{ij}-L_i)^2}, & t_{ij} > L_i \end{cases} \quad \forall i \in I, \forall j \in J$$

where $\beta_i \in \mathbb{R}^+$ is the time sensitivity coefficient for demand node i.

3.6 Discrete Time Satisfaction Function

Definition 3.7 (Discrete Step Function)**.** The discrete time satisfaction function uses a stepwise approach with multiple satisfaction levels:

$$F(t_{ij}) = \begin{cases} 1, & t_{ij} \leq L_i \\ S_i^r, & t_i^{r-1} < t_{ij} \leq t_i^r, \quad r \in [2, R] \\ 0, & t_{ij} > U_i \end{cases} \quad \forall i \in I, \forall j \in J$$

where:

- t_i^r: Time thresholds with $L_i = t_i^1 < t_i^2 < \cdots < t_i^R = U_i$
- S_i^r: Satisfaction levels with $1 = S_i^1 > S_i^2 > \cdots > S_i^R = 0$
- R: Number of discrete levels

Remark 3.5. Discrete functions are practical for implementation and align with human perception patterns where satisfaction changes in steps rather than continuously.

3.7 Selection and Application Considerations

1. **Model Flexibility**: Continuous functions offer smooth transitions but may be computationally more complex
2. **Customer Psychology**: Different functions capture different customer sensitivity patterns
3. **Data Availability**: Choice may depend on available empirical data on customer time preferences
4. **Computational Efficiency**: Discrete functions are often easier to implement in optimization models
5. **Realism**: The function should reflect actual customer behavior in the specific context

3.8 Time Satisfaction Based Maximum Covering Location Problem (TSBMCLP)

Definition 3.8 (TSBMCLP)**.** TSBMCLP maximizes total satisfaction-weighted coverage given p facilities:

$$\text{Maximize } Z = \sum_{i \in I} \sum_{j \in J} a_i F(t_{ij}) y_{ij}$$

$$\text{subject to:} \begin{cases} y_{ij} \leq x_j, & \forall i \in I, \forall j \in J \\ \sum\limits_{j \in J} x_j \leq p & \\ \sum\limits_{j \in J} y_{ij} = 1, & \forall i \in I \\ x_j \in \{0,1\}, & \forall j \in J \\ y_{ij} \in \{0,1\}, & \forall i \in I, \forall j \in J \end{cases}$$

where $y_{ij} = 1$ indicates demand node i is assigned to facility j (the most satisfactory one).

Remark 3.6. Unlike MCLP, TSBMCLP allows each demand node to be assigned to only one facility (the most satisfactory one), and uses satisfaction-weighted demand rather than simple coverage.

3.9 Time Satisfaction Based Location Set Covering Problem (TSBLSCP)

Definition 3.9 (TSBLSCP)**.** TSBLSCP minimizes total facility cost while ensuring that at least a proportion β of total demand is covered with satisfaction level at least α_i:

$$\text{Minimize } Z = \sum_{j \in J} w_j x_j$$

$$\text{subject to:} \begin{cases} \sum\limits_{i \in I} a_i y_i \geq \beta \sum\limits_{i \in I} a_i & \\ y_i \leq \sum\limits_{j \in J} c_{ij} x_j, & \forall i \in I \\ x_j \in \{0,1\}, & \forall j \in J \\ y_i \in \{0,1\}, & \forall i \in I \end{cases}$$

where $c_{ij} = 1$ if $\alpha_i \leq F(t_{ij})$ (i.e., facility j covers demand node i at the required satisfaction level α_i), and 0 otherwise.

3.10 Time Satisfaction Based R1 Model (TSB-R1)

Definition 3.10 (TSB-R1 Model)**.** TSB-R1 maximizes clinic coverage satisfaction and minimizes clinic dissatisfaction with hospital access:

$$\text{Maximize} \quad Z = \left(\sum_{i \in I} \sum_{j \in J} a_i F(t_{ij}) y_{ij}, - \sum_{j \in J_1} q_j \right)$$

$$\text{subject to:} \begin{cases} y_{ij} \leq c_j + h_j \leq 1 & \forall i \in I \quad \forall j \in J \\ \sum_{k \in M_j} h_k - c_j + q_j \geq 0 & \forall j \in J_1 \\ \sum_{j \in J_1} c_j = P_c \quad \sum_k h_k = P_h & \\ \sum_{j \in J} y_{ij} = 1 \quad \sum_{k \in J_2} q_{jk} = 1 & \forall i \in I \quad \forall j \in J \\ c_j, h_k, y_{ij}, q_{jk} \in \{0,1\} & \end{cases}$$

3.11 Time Satisfaction Based R2 Model (TSB-R2)

Definition 3.11 (TSB-R2 Model)**.** TSB-R2 maximizes clinic coverage satisfaction and hospital accessibility satisfaction for referred patients:

$$\text{Maximize } Z = \left(\sum_{i \in I} \sum_{j \in J_1} a_i F(t_{ij}) y_{ij}, \sum_{i \in I} \sum_{j \in J_1} \sum_{k \in J_2} \alpha_i a_i \left[F(t_{ij}) y_{ij} + G(t_{jk}) y^*_{jk} \right] \right)$$

$$\text{subject to: } \begin{cases} c_j + h_j \geq y_{ij}, & \forall i \in I, \forall j \in J \\ y_{ij} \leq c_j + h_j, & \forall i \in I, \forall j \in J \\ y^*_{jk} \leq h_k, & \forall j \in J, \forall k \in J_2 \\ c_j + h_j \leq 1, & \forall j \in J \\ \sum\limits_{j \in J_1} c_j = P_c, \quad \sum\limits_{k \in J_2} h_k = P_h & \\ \sum\limits_{j \in J} y_{ij} = 1, \quad \sum\limits_{k \in J_2} y^*_{jk} = 1, & \forall i \in I, \forall j \in J \\ c_j, h_k, y_{ij}, y^*_{jk} \in \{0, 1\}, & \forall i \in I, \forall j \in J, \forall k \in J_2 \end{cases}$$

where J_1 and J_2 are clinic and hospital candidate sites respectively, α_i is the referral rate from clinic to hospital for demand at node i, $y_{ij} = 1$ if demand at i is served by clinic j, and $y^*_{jk} = 1$ if clinic j refers patients to hospital k.

4 Integrated Models

4.1 MALP-R1 Integration

Definition 4.1 (MALP-R1 Integrated Model)**.** This model combines probabilistic reliability from MALP with the hierarchical structure from R1, incorporating time satisfaction:

$$\text{Maximize } Z = \left(\sum_{i \in I} a_i z_{ib_i}, -\sum_{j \in J} q_j \right)$$

$$\text{subject to: } \begin{cases} \sum\limits_{k=1}^{b_i} z_{ik} \leq \sum\limits_{j \in N_i} X_j, & \forall i \in I \\ z_{ik} \leq z_{i(k-1)}, & \forall i \in I, k = 2, \ldots, b_i \\ X_j = c_j + h_j, & \forall j \in J \\ \sum\limits_{k \in M_j} h_k - c_j + q_j \geq 0, & \forall j \in J \\ \sum\limits_j X_j \leq P, \quad \sum\limits_j c_j = P_c, \quad \sum\limits_j h_j = P_h & \\ X_j \in \mathbb{N}_0, & \forall j \in J \\ c_j, h_j, q_j, z_{ik} \in \{0, 1\}, & \forall i \in I, j \in J, k = 1, \ldots, b_i \end{cases}$$

Remark 4.1. This integration allows multiple servers per facility while maintaining hierarchical referral requirements and probabilistic reliability constraints. The binary variables c_j and h_j determine whether a facility is a clinic, hospital, or both, while z_{ik} tracks coverage reliability.

5 Queueing Theory Models for Facility Location(NOT FINISHED YET)

5.1 Descriptive Models Overview

Definition 5.1 (Descriptive Models)**.** Descriptive models evaluate the performance of existing or proposed facility layouts. Key characteristics include:

- Primary use: **System performance evaluation**
- Common methods: **Queueing models** and **simulation**
- Models are typically **nonlinear** with high computational complexity
- Provide detailed system performance metrics

5.2 Hypercube Queueing Model

Definition 5.2 (Hypercube Model State Representation)**.** We consider:

- Each server state: busy (1) or idle (0)
- System state vector: $\mathbf{b} = (b_1, b_2, ..., b_p)$ for p servers
- Total states: 2^p (vertices of a p-dimensional hypercube)
- Assumptions:
  - Call arrivals: Poisson distribution with mean λ_i
  - Service times: Exponential distribution with mean $1/\mu$
  - System type: $M/M/p$ queue
  - Each demand node has a priority dispatch list

Theorem 5.1 (Steady-State Probability Calculation)**.** *The steady-state probability P_k for state k satisfies:*

$$P_k(\lambda_k + w_k\mu) = \sum_{j=k-(1 \ server \ busy)} P_j\lambda_k + \sum_{j=k+(1 \ server \ busy)} P_j\mu$$

where w_k is the number of busy servers in state k.
The solution can be obtained iteratively:

$$P_k^n(\lambda_k + w_k\mu) = \sum_{j=k-(1 \ server \ busy)} P_j^{n-1}\lambda_k + \sum_{j=k+(1 \ server \ busy)} P_j^{n-1}\mu$$

with normalization condition: $\sum_k P_k = 1$.

Remark 5.1. The hypercube model computes detailed performance metrics but has computational complexity $O(2^p)$, making it impractical for large systems.

5.3 Approximate Hypercube Model

Definition 5.3 (Larson's Approximation)**.** The approximation avoids solving 2^p equations by directly computing server utilizations ρ_j through p nonlinear equations. Key formulas:

- Probability that exactly k servers are busy:

$$P_k = Q[p, \rho, k-1]\rho^{k-1}(1-\rho)$$

- Correction factor accounting for server interdependence:

$$Q[p, \rho, k-1] = \frac{\sum_{l=k-1}^{p-1} \frac{(p-k-2)(p-l)}{(l-k+1)!} \frac{p^k \rho^{l-k+1}}{p!}}{(1-\rho)\left[\sum_{i=0}^{p-1} \frac{p^i \rho^i}{i!}\right] + \frac{p^p \rho^p}{p!}}$$

- Server dispatch rate:

$$R_j^T = \sum_k \sum_{i \in G_j^k} \lambda_i P[\text{first } k-1 \text{ preferred servers busy, server } j \text{ idle}]$$

6 Model Classification Summary

Table 1: Classification of Coverage Models by Objective Function

Objective	**Models**
Minimize cost	LSCP, PLSCP, QPLSCP, TSBLSCP
Maximize covered population	MCLP, TSBMCLP, R1, R2
Maximize probability/expectation	MEXCLP
Maximize accessibility	MALP I, MALP II, R2 (clinic-to-hospital)
Minimize uncovered clinics	R1

Table 2: Classification of Coverage Models by Constraints

Constraint Type	**Models**
Fixed number of facilities p	MCLP, TSBMCLP, MEXCLP, MALP, R1, R2
Multiple servers per facility ($X_j \geq 1$)	MEXCLP, PLSCP, QPLSCP, MALP
Variable local busy rates q_i	PLSCP, QPLSCP, MALP
Hierarchical facility location	R1, R2, TSB-R1, TSB-R2
Time satisfaction functions	TSBMCLP, TSBLSCP, TSB-R1, TSB-R2

7 Conclusion

This document provides a comprehensive overview of facility location models, covering:

- Basic coverage models (LSCP, MCLP)
- Probabilistic extensions (MEXCLP, PLSCP, MALP)
- Hierarchical models (R1, R2)
- Time satisfaction based models (TSBMCLP, TSBLSCP, TSB-R1, TSB-R2)
- Integrated models (MALP-R1)
- Queueing theory based descriptive models

Future research directions include integrating time satisfaction functions with hierarchical probabilistic models, developing efficient solution algorithms for complex integrated models, and applying these models to real-world healthcare facility location problems.