2012 TAMS Tournament

Geometry (math with shapes)

11.6.2012

This is a 10 question, 50 minute test. Questions 1-3 are worth 3 points; 4-6 are worth 5 points; 7-10 are worth 10 points. There is no guessing penalty. Give all answers as a closed form expression.

1. Concerntric circles have diameters 1, 2, and 4. The smallest circle is red, the region between the smallest circle and the medium circle is blue, and the region between the medium and the largest circle is green. What is the ratio of green to blue if the smallest circle is doubled in size?

2. A square of area 40 is inscribed in a semicircle. What is the area of the semicircle?

3. What is the area of the smallest square that can contain a 2 by 3 rectangle and a 3 by 4 rectangle are contained in a square without overlap, given that the sides of the rectangles are paralell or perpendicular to the sides of the square?

4. A circle of radius 2 is centered at O. Square OABC has side length 1. Sides AB and CB are extended past B to meet the circle at D and E, respectively. What is the area of the shaded region bounded by BD, BE, and the minor arc connecting D and E?

5. Pentagon $ABCDE$ has $\overline{AB} = 2, \overline{BC} = 3, \overline{CD} = 4, \overline{DE} = 5, \overline{BE} = 6$, and $m\angle B = m\angle D = 90°$. Find $(\overline{AE})^2$.

6. Jack and Jill are 10 and 40 meters from the base of a 9 meter tall billboard. They have equivalent views, meaning that the angle between the top of the billboard, Jack, and the bottom of the billboard is the same as the angle between the top of the billboard, Jill, and the bottom of the billboard. How far, in meters, is the bottom of the billboard from the ground?

7. Regular dodecahedron (12-gon) $A_1 A_2 A_3 \cdots A_{12}$ has side length 1. Lines $A_1 A_6, A_2, A_7, \ldots A_{12} A_5$ are drawn to form a star within the the dodecahedron. What is the area between the star and the dodecahedron?

8. A fox stands at each vertex of this dodecahedron (from number 7). At the same time, each fox begins chasing the fox to its left so that the fox at A_1 chases the one at A_2, the one at A_2 chases the one at A_3, and so on. If each fox runs at 1 unit per second, how long will it take for all of them to meet at the center?

9. Suppose, in triangle ABC, that the medians from A and B are perpendicular. If the lengths of AC is 6 and the length of BC is 7, what is the length of AB?

10. $ABCDEF$ is a convex hexagon. P, Q, R, S, T, U are the midpoints of sides AB, BC, CD, DE, EF, FA respectively. The areas of tringles $ABR, BCS, CDT, DEU, EPF, FAQ$ are $12, 34, 56, 12, 34, 56$ respectively. Find the area of the hexagon.