2012 TAMS Tournament

Algebra (math with letters)

11.10.2012

This is a 10 question, 50 minute test. Questions 1-3 are worth 3 points; 4-6 are worth 5 points; 7-10 are worth 10 points. There is no guessing penalty. Give all answers as a closed form expression.

1. When polynomial $P(x)$ is divided by $x - 2012$, the remainder is 2102. If the remainder when divided by $(x - 2012)^2$ is $r(x)$, what is $r(2777)$?
2. A flat saltine craker measure 2 by 3 inches. How many times can it be halved until its total surface area is less than 1?
3. a and b are integers such that $x^2 + x + 2012$ is a factor of $ax^3 + bx^2 + 2012^2$. What is b?
4. Evaluate $\frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \cdots + \frac{2012}{2010!+2011!+2012!}$.
5. What number(s) has the property that moving the decimal point to the left cubes it?
6. $f(x) = 13x^3 + 91x^2 - 221x - 2795$. If a, d, c are the roots of f, what is $(a + 1)(b + 1)(c + 1)$?
7. A point (a, b) i chosen within two units of the origin. What is the probability that an ellipse tangent to the y axis and the line $y = b$ can fit within a unit square?
8. For how many distinct real polynomials $P(x)$ which are not multiples of each other is $(x^{2012} - 1)/P(x)$ is also a real polynomial?
9. Consider the following system of equations.

$$\begin{aligned} x + y + z &= 1 \\ xy^2 + yz^2 + zx^2 &= 2 \\ x^3(z - y) + y^3(x - z) + z^3(y - x) &= 3 \end{aligned}$$

   Find $x^2y + y^2z + z^2x$.
10. What is the sum of the roots of $(2 - x)^{2012} - x^{2012} = 0$?