2012 TAMS Tournament

Pre-Calculus (math with $\sin(x)$ and its friends)

11.6.2012

This is a 10 question, 50 minute test. Questions 1-3 are worth 3 points; 4-6 are worth 5 points; 7-10 are worth 10 points. There is no guessing penalty. Give all answers as a closed form expression.

1. Given $\sum_{i=1}^{\infty} \tan a_i = 2 \sin \sum_{i=1}^{\infty} a_i$ find $(\cos a_1)(\cos a_2)(\cos a_3) \cdots (\cos a_n)$ as n approaches infinity.
2. Find all $a \in [0, \pi]$ such that $(1 - 2\sin(a)\cos(a))(1 + 4\sin(\frac{a+b}{2})\cos(\frac{a+b}{2})\cos(a) - 2\sin(b)\cos(a))$.
3. Find $\sum_{n=3}^{\infty} \frac{1}{n^2 - 4}$
4. For which of the 36 complex roots of $x^{36} + x^{35} + \cdots + x + 1 = 0$ is $\mathrm{Re}(z) - \mathrm{Im}(z)$ the least?
5. Find the eighth terms of the sequence $2012, 220, 2250, \ldots$, whose terms are formed by multiplying the corresponding terms of two arithmetic sequences.
6. A triangle has sides 9, 10, and 11. Find the sine of the largest angle.
7. Find all continuous functions f where $f(x+1) = f(x)^2$ for all integers x.
8. Solve the equation $56\cos(2\theta) + 28\cos(4\theta) + 8\cos(6\theta) + \cos(8\theta) + 93 = 0$ for $-2\pi \leq \theta \leq 2\pi$.
9. Equilateral triangle ABC with side length 1 is suspended in space. Lines dropped from A, B, and C parallel to the z axis intersect xy plane at A', B', and C' respectively. Find the area of $\triangle A'B'C'$.
10. Calculate $\cos(\frac{\pi}{5})$.