Algebra

Directions: You will have 50 minutes to complete this 10 question short-answer algebra test. Not all questions will require only algebra, but the test primarily focuses on algebraic concepts. You will earn 1 point for each correct answer. There is no guessing penalty. No calculators will be allowed. Tiebreakers will be decided in order of time turned in. Good luck!

1.

2. a)
   b)

3.

4.

5.

6.

7.

8.

9.

10.

Algebra

1. The sum of the first 50 terms of an arithmetic sequence is $\frac{1}{4}$ the sum of the next 50 terms. Find the ratio of the common difference to the first term. (10 points)
2. Given $4x^2 + 9y^2 - 48x + 72y + 144 = 0$,
   a) Indicate the type of conic section formed by the graph. (2 points)
   b)Find the equation in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (8 points)
3. Find the smallest integer k for which $(log_6 11)$ $(log_{11} 16)$ $(log_{16} 21)(...)(log_{5k+1}(5k+6))$ is an integer. (10 points)
4. There are six fish in a pond, three of which are tagged. If a man catches three fish selected at random without replacement, what is the probability that exactly two of the fish caught are tagged? (10 points)
5. Given $f(x)$ such that $f(35 - x^3) + (35 - x^3)f(x^3)$=48, if $f(8)$ can be expressed as $\frac{m}{n}$, find $m + n$. (10 points)
6. Let $x = \sqrt{20} + \frac{13}{\sqrt{20}+\frac{13}{\sqrt{20}+\frac{13}{\cdots}}}$
   Find the sum of the cubes of all possible values of x. (10 points)
7. When polynomial $f(x)$ is divided by $(x - 2013)$, the remainder is 2103. When $f(x)$ is divided by $(x-2103)$, the remainder is 2013. If the remainder of $f(x)$ when divided by $(x - 2013)(x - 2103)$ is $r(x)$, what is $r(1337)$? (10 points)
8. Determine the value of the sum $\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+...+\frac{2013}{2011!+2012!+2013!}(10 points)$
9. For which integer n is $\frac{1}{n}$ closest to $\sqrt{8464} - \sqrt{8463}$? (10 points)
10. Consider the polynomial $3x^5 - x^4 + 2x^3 + x^2 - 5$.
    If r_1, r_2, r_3, r_4, and r_5 are its roots, find ${r_1}^4$+${r_3}^4$+${r_3}^4$+${r_4}^4$+${r_5}^4$.$(10 points)$