**Problem 5-1.**

Note that each legal path has exactly n+1 vertical and m+1 horizontal edges. In partical, it has n+m+2 edges in total. To choose a path means to choose n+1 positions from a set of n+m+2 positions:

\_|\_\_| ===> (2, 5), \_\_\_|| ===> (4, 5), ||\_\_\_ ===> (1,2).

So, there are C(n+m+2,n+1) different legal paths from source (0,0) to sink (n,m).

**Problem 5-2.**

1. A player who starts from a winning position will win, i. e. the player who can force his opponent to start from a losing position. The winning strategy for position S(n,m) is to determine whether S(n-2,m-1) or S(n-1,m-2) is a losing one and to take (2,1) or (1,2) rocks. In trivial situations, the first player always wins.

2. Position S(3,3) is losing because positions S(1,2) and S(2,1) are both not losing. Position S(6,4) is winning, because S(4,3) is losing.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 |  | W | W | W | W | W |
| 2 | W | W | W | W | W | W |
| 3 | W | W | L | L | L | L |
| 4 | W | W | L | L | W | W |
| 5 | W | W | L | W | W | W |
| 6 | W | W | L | W | W | L |

**Problem 5-3.**

Edit graph:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | **B** | **O** | **A** | **S** | **T** |
|  | 0 | -1 | -2 | -3 | -4 | -5 |
| **M** | -1 | ↘, -1 | ↘, -2 | ↘, -3 | ↘, -4 | →, -5 |
| **O** | -2 | ↘, -2 | ↘, 0 | →, -1 | →, -2 | →, -3 |
| **A** | -3 | ↘, -3 | ↓, -1 | ↘, 1 | →, 0 | →, -1 |
| **T** | -4 | ↓, -4 | ↓, -2 | ↓, 0 | ↘, 0 | ↘, 2 |

Optimal global alignment:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| M | O | A | - | T |
| B | O | A | S | T |

There is no other optimal global alignment:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | **B** | **O** | **A** | **S** | **T** |
|  | 0 | -1 | -2 | -3 | -4 | -5 |
| **M** | -1 | ↘, -1 | -2 | -3 | -4 | →, -5 |
| **O** | -2 | -2 | ↘, 0 | →, -1 | →, -2 | -3 |
| **A** | -3 | -3 | ↓, -1 | ↘, 1 | →, 0 | →, -1 |
| **T** | -4 | ↓, -4 | ↓, -2 | ↓, 0 | ↘, 0 | ↘, 2 |

Score:

2

**Problem 5-4.**

1.

Edit graph:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **G** | **G** | **A** | **C** | **G** | **T** | **A** | **C** | **G** |
|  | 0 | -1 | -2 | -3 | -4 | -5 | -6 | -7 | -8 | -9 |
| **T** | -1 | -1,↖ | -2,← | -3,← | -4,← | -5,← | -4,↖ | -5,← | -6,← | -7,← |
| **A** | -2 | -2,↑ | -2,↖ | -1,↖ | -2,← | -3,← | -4,← | -3,↖ | -4,← | -5,← |
| **C** | -3 | -3,↑ | -3,↑ | -2,↑ | 0,↖ | -1,← | -2,← | -3,← | -2,↖ | -3,← |
| **G** | -4 | -2,↖ | -2,↖ | -3,↑ | -1,↑ | 1,↖ | 0,← | -1,← | -2,← | -1,↖ |
| **G** | -5 | -3,↑ | -1,↖ | -2,← | -2,↑ | 0,↑ | 0,↖ | -1,← | -2,← | -1,↖ |
| **G** | -6 | -4,↑ | -2,↑ | -2,↖ | -3,↑ | -1,↑ | -1,↑ | -1,↖ | -2,← | -1,↖ |
| **T** | -7 | -5,↑ | -3,↑ | -3,↑ | -3,↖ | -2,↑ | 0,↖ | -1,← | -2,← | -2,↑ |
| **A** | -8 | -6,↑ | -4,↑ | -2,↖ | -3,← | -3,↑ | -1,↑ | 1,↖ | 0,← | -1,← |
| **T** | -9 | -7,↑ | -5,↑ | -3,↑ | -3,↖ | -4,↑ | -2,↑ | 0,↑ | 0,↖ | -1,← |

Global alignment:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| T | - | A | C | G | G | G | T | A | T |
| G | G | A | C | G | - | - | T | A | C |

Score:

-1

2.

Edit graph:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **G** | **G** | **A** | **C** | **G** | **T** | **A** | **C** | **G** |
|  | 0, | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **T** | 0 | 0,+ | 0,+ | 0,+ | 0,+ | 0,+ | 1,↖ | 0,+ | 0,+ | 0,+ |
| **A** | 0 | 0,+ | 0,+ | 1,↖ | 0,+ | 0,+ | 0,+ | 2,↖ | 1,← | 0,+ |
| **C** | 0 | 0,+ | 0,+ | 0,+ | 2,↖ | 1,← | 0,+ | 1,↑ | 3,↖ | 2,← |
| **G** | 0 | 1,↖ | 1,↖ | 0,+ | 1,↑ | 3,↖ | 2,← | 1,← | 2,↑ | 4,↖ |
| **G** | 0 | 1,↖ | 2,↖ | 1,← | 0,+ | 2,↑ | 2,↖ | 1,← | 1,↑ | 3,↑ |
| **G** | 0 | 1,↖ | 2,↖ | 1,← | 0,+ | 1,↑ | 1,↑ | 1,↖ | 0,+ | 2,↑ |
| **T** | 0 | 0,+ | 1,↑ | 1,↖ | 0,+ | 0,+ | 2,↖ | 1,← | 0,+ | 1,↑ |
| **A** | 0 | 0,+ | 0,+ | 2,↖ | 1,← | 0,+ | 1,↑ | 3,↖ | 2,← | 1,← |
| **T** | 0 | 0,+ | 0,+ | 1,↑ | 1,↖ | 0,+ | 1,↖ | 2,↑ | 2,↖ | 1,← |

Local alignment:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| # | # | # | # | # | T | A | C | G | G | G | T | A | T |
|  |  |  |  |  | | | | | | | | |  |  |  |  |  |
| G | G | A | C | G | T | A | C | G | # | # | # | # | # |

Score:

4

3.

Edit graph:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **G** | **G** | **A** | **C** | **G** | **T** | **A** | **C** | **G** |
|  | 0 |  |  |  |  |  |  |  |  |  |
| **T** |  | -1 |  |  |  |  |  |  |  |  |
| **A** |  |  | -2 |  |  |  |  |  |  |  |
| **C** |  |  |  | -3 |  |  |  |  |  |  |
| **G** |  |  |  |  | -4 |  |  |  |  |  |
| **G** |  |  |  |  |  | -3 |  |  |  |  |
| **G** |  |  |  |  |  |  | -4 |  |  |  |
| **T** |  |  |  |  |  |  |  | -5 |  |  |
| **A** |  |  |  |  |  |  |  |  | -6 |  |
| **T** |  |  |  |  |  |  |  |  |  | -7 |

Any other alignment scores < -20

Score:

-7

**Problem 5-5.**

1.

