

with(DEtools);

[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

with(plots);

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

*ecdifl := diff(y(x), x) = 2 * x * (1 + y(x)^2);*

$$ecdifl := \frac{d}{dx} y(x) = 2x (1 + y(x)^2)$$

soll := dsolve(ecdifl, y(x));

$$soll := y(x) = \tan(x^2 + 2_CI)$$

y1 := unapply(rhs(soll), x, _CI);

$$y1 := (x, _CI) \mapsto \tan(x^2 + 2_CI)$$

plot([y1(x, 1), y1(x, 2), y1(x, 3)], x = -10 .. 10, y = -10 .. 10);

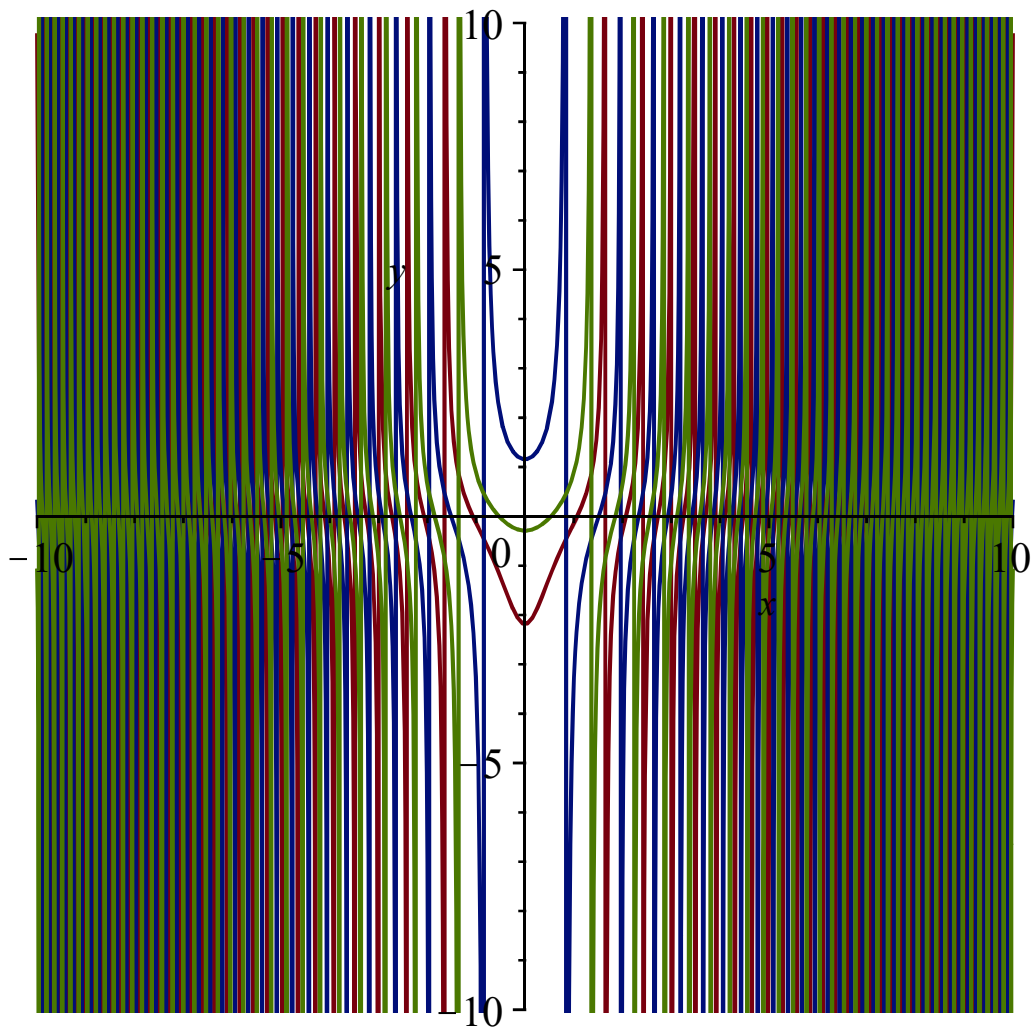
(1)

(2)

(3)

(4)

(5)



$$ecdif2 := (x^2 - 1) \cdot \text{diff}(y(x), x) + 2 \cdot x \cdot y(x)^2 = 0;$$

$$ecdif2 := (x^2 - 1) \left(\frac{d}{dx} y(x) \right) + 2 y(x)^2 x = 0 \quad (6)$$

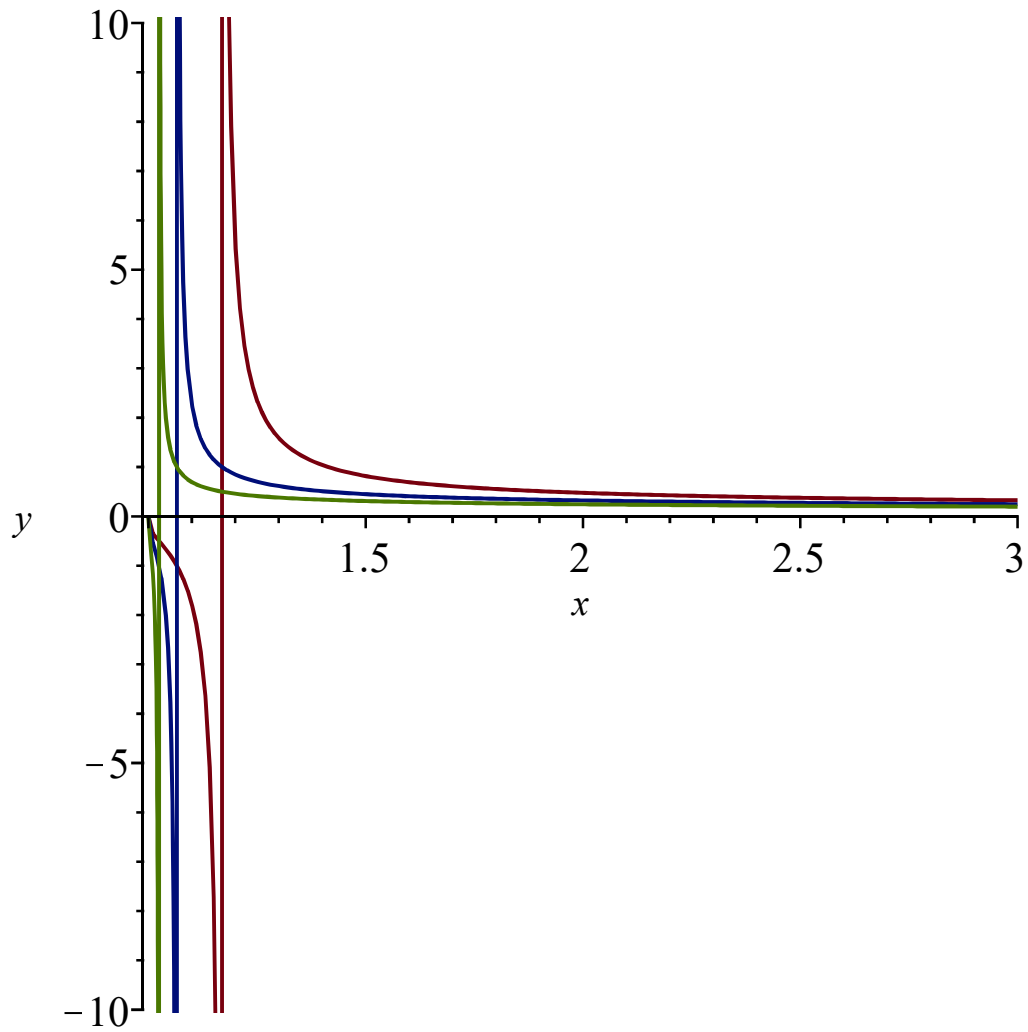
$$sol2 := \text{dsolve}(ecdif2, y(x));$$

$$sol2 := y(x) = \frac{1}{\ln(x-1) + \ln(x+1) + _CI} \quad (7)$$

$$y2 := \text{unapply}(\text{rhs}(sol2), x, _CI);$$

$$y2 := (x, _CI) \mapsto \frac{1}{\ln(x-1) + \ln(x+1) + _CI} \quad (8)$$

$$\text{plot}([y2(x, 1), y2(x, 2), y2(x, 3)], x = 1 .. 3, y = -10 .. 10);$$



$$ecdif3 := \text{diff}(y(x), x) = -\frac{x + y(x)}{y(x)};$$

$$ecdif3 := \frac{d}{dx} y(x) = -\frac{x + y(x)}{y(x)} \quad (9)$$

$$sol3 := \text{dsolve}(ecdif3, y(x));$$

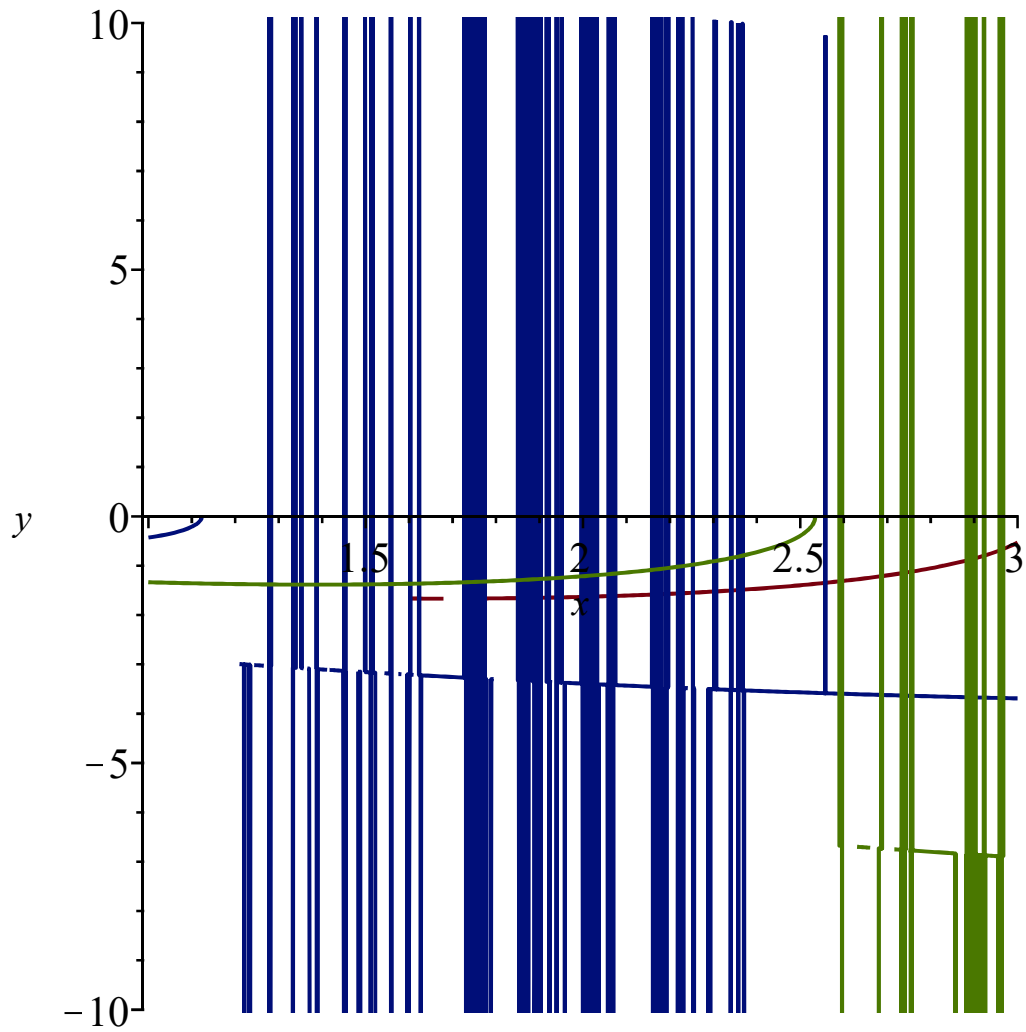
$$sol3 := y(x) = \frac{\sqrt{3} x \tan\left(\text{RootOf}\left(\sqrt{3} \ln\left(\frac{3x^2}{4} + \frac{3x^2 \tan(Z)^2}{4}\right) + 2\sqrt{3} _CI - 2_Z\right)\right)}{2} - \frac{x}{2} \quad (10)$$

$$y3 := \text{unapply}(\text{rhs}(sol3), x, _CI);$$

$$y3 := (x, _CI) \quad (11)$$

$$\mapsto \frac{\sqrt{3} x \tan\left(\text{RootOf}\left(\sqrt{3} \ln\left(\frac{3x^2}{4} + \frac{3x^2 \tan(Z)^2}{4}\right) + 2\sqrt{3} _CI - 2_Z\right)\right)}{2} - \frac{x}{2}$$

$$\text{plot}([y3(x, 1), y3(x, 2), y3(x, 3)], x = 1..3, y = -10..10);$$



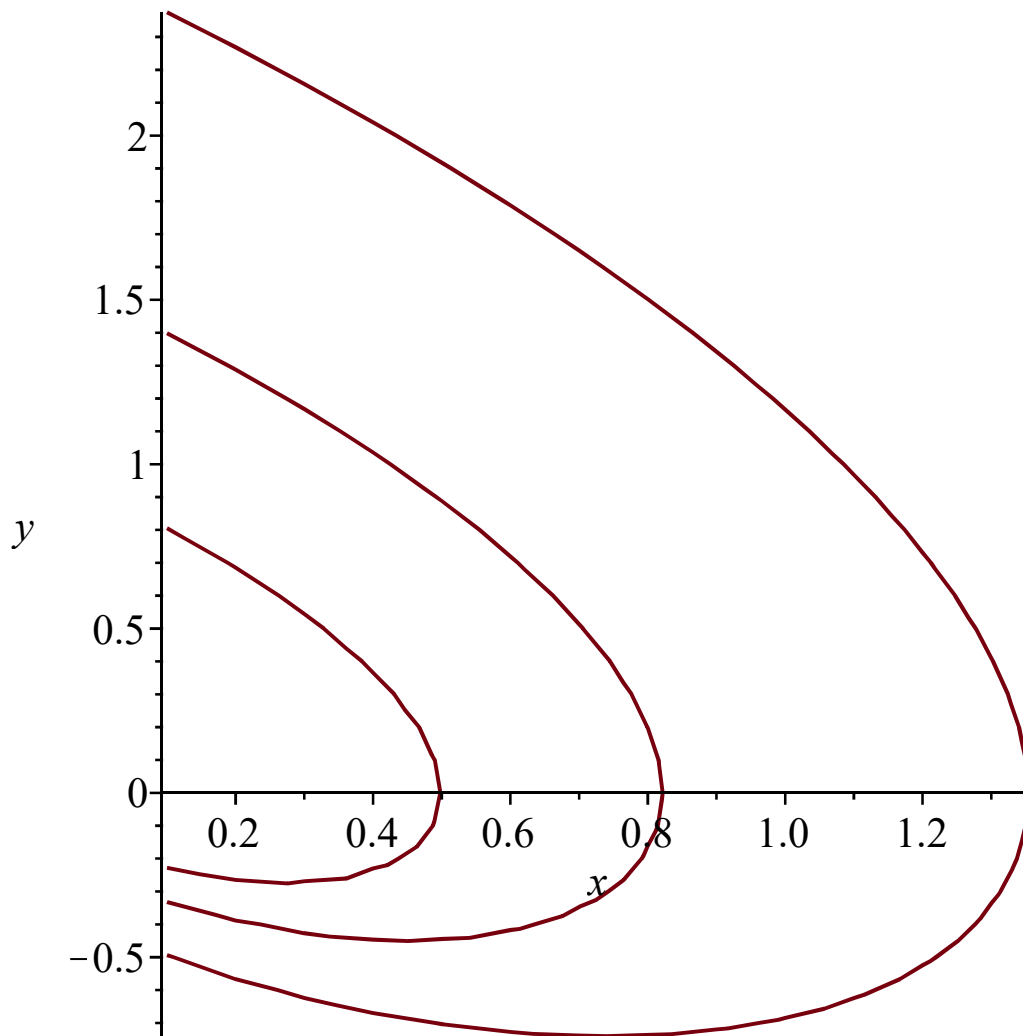
```
sol3 := dsolve(ecdif3, y(x), implicit);
```

$$sol3 := -\frac{\ln\left(\frac{x^2 + x y(x) + y(x)^2}{x^2}\right)}{2} + \frac{\sqrt{3} \arctan\left(\frac{(2 y(x) + x) \sqrt{3}}{3 x}\right)}{3} - \ln(x) - _CI = 0 \quad (12)$$

```
y3 := unapply(subs(y(x) = y, lhs(sol3)), x, y, \_CI);
```

$$y3 := (x, y, _CI) \mapsto -\frac{\ln\left(\frac{x^2 + y x + y^2}{x^2}\right)}{2} + \frac{\sqrt{3} \arctan\left(\frac{(2 y + x) \sqrt{3}}{3 x}\right)}{3} - \ln(x) - _CI \quad (13)$$

```
implicitplot([y3(x, y, 0) = 0, y3(x, y, 0.5) = 0, y3(x, y, 1) = 0], x = -5 .. 5, y = -5 .. 5, numpoints = 10000);
```



$$ecdif4 := \text{diff}(y(x), x) = -\frac{2 \cdot x \cdot y(x)}{1 + x^2 + 3 \cdot y(x)^2};$$

$$ecdif4 := \frac{d}{dx} y(x) = -\frac{2 x y(x)}{1 + x^2 + 3 y(x)^2} \quad (14)$$

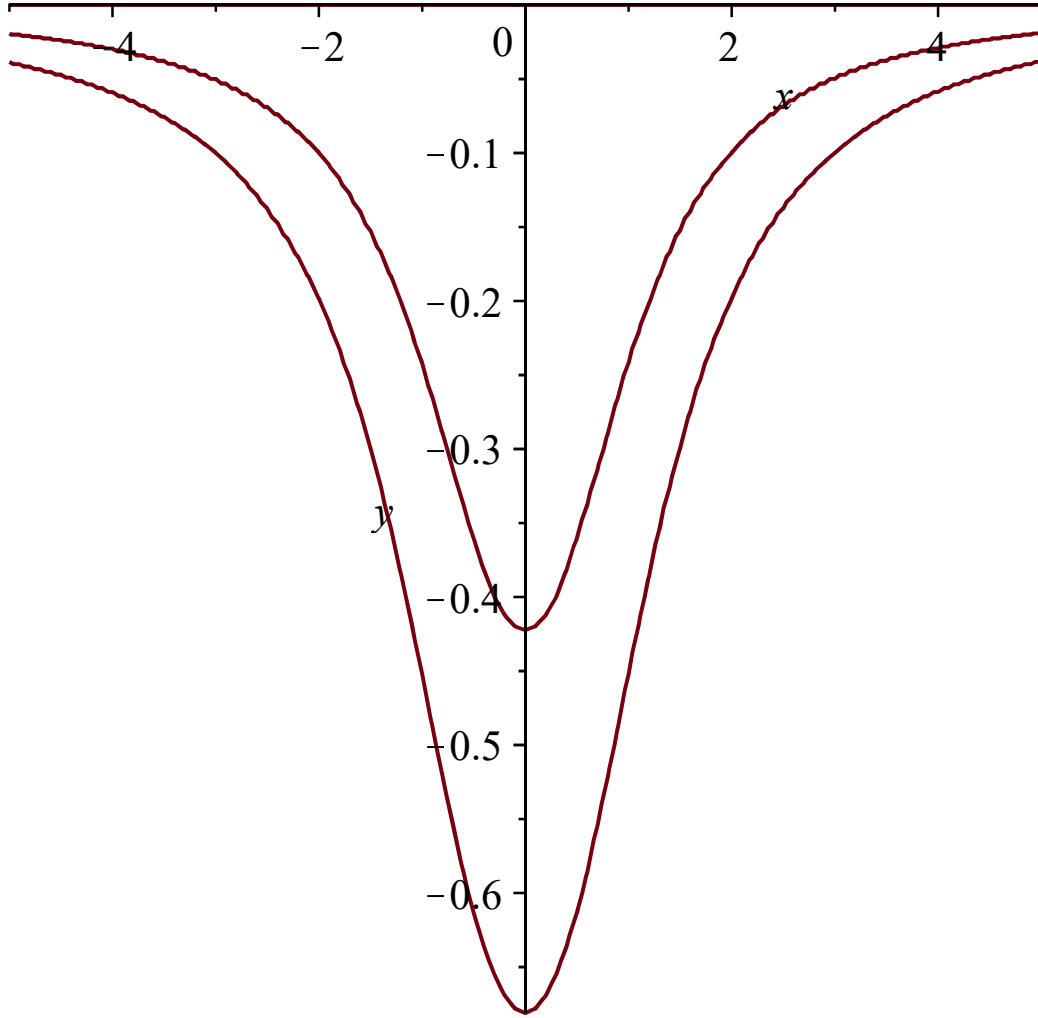
$$sol4 := \text{dsolve}(ecdif4, y(x), \text{implicit});$$

$$sol4 := x^2 y(x) + y(x)^3 + _CI + y(x) = 0 \quad (15)$$

$$y4 := \text{unapply}(\text{subs}(y(x) = y, \text{lhs}(sol4)), x, y, _CI);$$

$$y4 := (x, y, _CI) \mapsto x^2 y + y^3 + _CI + y \quad (16)$$

$$\text{implicitplot}([y4(x, y, 0) = 0, y4(x, y, 0.5) = 0, y4(x, y, 1) = 0], x = -5..5, y = -5..5, \text{numpoints} = 10000);$$

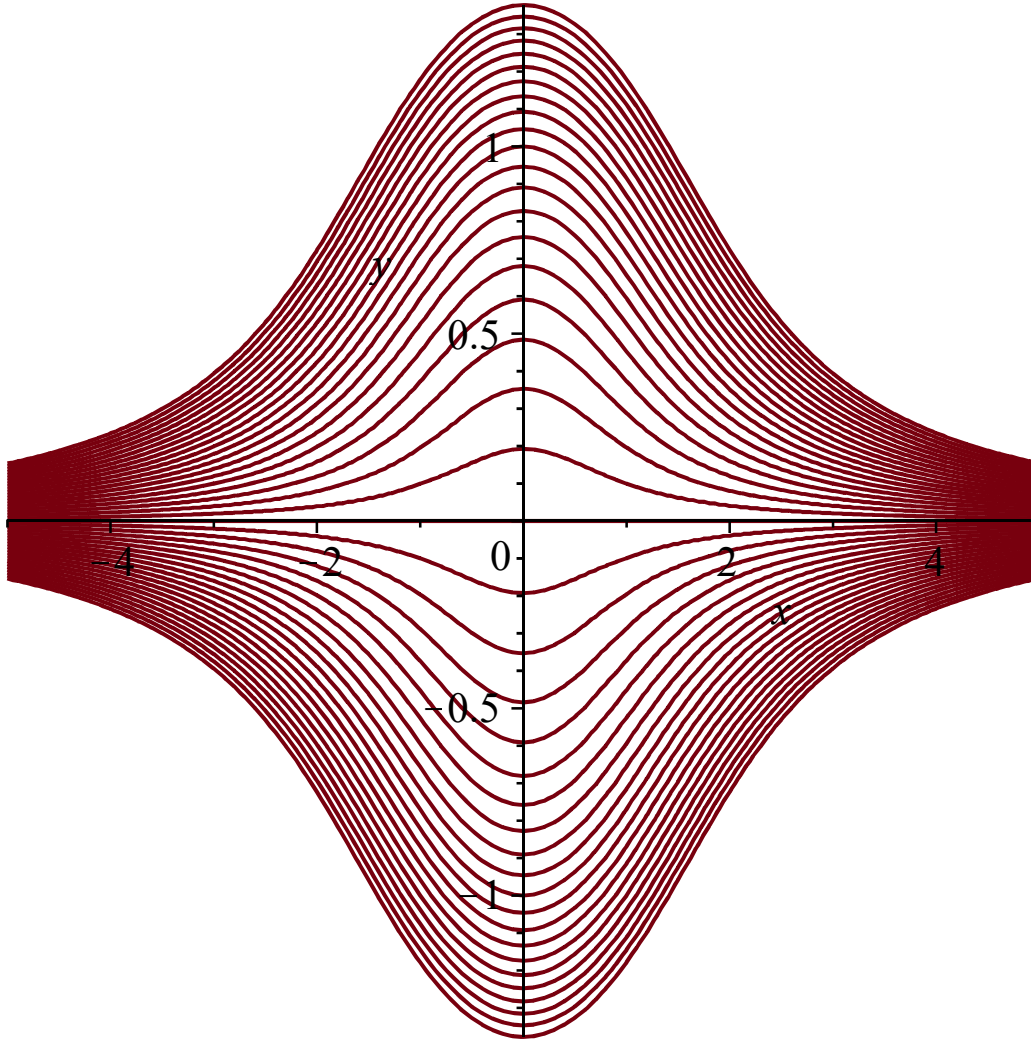


$$sir_sol := seq\left(y4\left(x, y, \frac{i}{5}\right) = 0, i = -20..20\right);$$

$$\begin{aligned}
 sir_sol := & x^2 y + y^3 + y - 4 = 0, x^2 y + y^3 - \frac{19}{5} + y = 0, x^2 y + y^3 - \frac{18}{5} + y = 0, x^2 y + y^3 \\
 & - \frac{17}{5} + y = 0, x^2 y + y^3 - \frac{16}{5} + y = 0, x^2 y + y^3 + y - 3 = 0, x^2 y + y^3 - \frac{14}{5} + y = 0, x^2 y \\
 & + y^3 - \frac{13}{5} + y = 0, x^2 y + y^3 - \frac{12}{5} + y = 0, x^2 y + y^3 - \frac{11}{5} + y = 0, x^2 y + y^3 + y - 2 = 0, \\
 & x^2 y + y^3 - \frac{9}{5} + y = 0, x^2 y + y^3 - \frac{8}{5} + y = 0, x^2 y + y^3 - \frac{7}{5} + y = 0, x^2 y + y^3 - \frac{6}{5} + y \\
 & = 0, x^2 y + y^3 + y - 1 = 0, x^2 y + y^3 - \frac{4}{5} + y = 0, x^2 y + y^3 - \frac{3}{5} + y = 0, x^2 y + y^3 - \frac{2}{5} + y \\
 & = 0, x^2 y + y^3 - \frac{1}{5} + y = 0, x^2 y + y^3 + y = 0, x^2 y + y^3 + \frac{1}{5} + y = 0, x^2 y + y^3 + \frac{2}{5} + y \\
 & = 0, x^2 y + y^3 + \frac{3}{5} + y = 0, x^2 y + y^3 + \frac{4}{5} + y = 0, x^2 y + y^3 + y + 1 = 0, x^2 y + y^3 + \frac{6}{5} + y \\
 & = 0, x^2 y + y^3 + \frac{7}{5} + y = 0, x^2 y + y^3 + \frac{8}{5} + y = 0, x^2 y + y^3 + \frac{9}{5} + y = 0, x^2 y + y^3 + y + 2
 \end{aligned} \tag{17}$$

$$\begin{aligned}
&=0, x^2 y + y^3 + \frac{11}{5} + y = 0, x^2 y + y^3 + \frac{12}{5} + y = 0, x^2 y + y^3 + \frac{13}{5} + y = 0, x^2 y + y^3 \\
&+ \frac{14}{5} + y = 0, x^2 y + y^3 + y + 3 = 0, x^2 y + y^3 + \frac{16}{5} + y = 0, x^2 y + y^3 + \frac{17}{5} + y = 0, x^2 y \\
&+ y^3 + \frac{18}{5} + y = 0, x^2 y + y^3 + \frac{19}{5} + y = 0, x^2 y + y^3 + y + 4 = 0
\end{aligned}$$

`implicitplot([sir_sol], x=-5..5, y=-5..5, numpoints=10000);`



`ecdif5 := diff(y(x), x, x) + y(x) = sin(x) + cos(x);`

$$ecdif5 := \frac{d^2}{dx^2} y(x) + y(x) = \sin(x) + \cos(x) \quad (18)$$

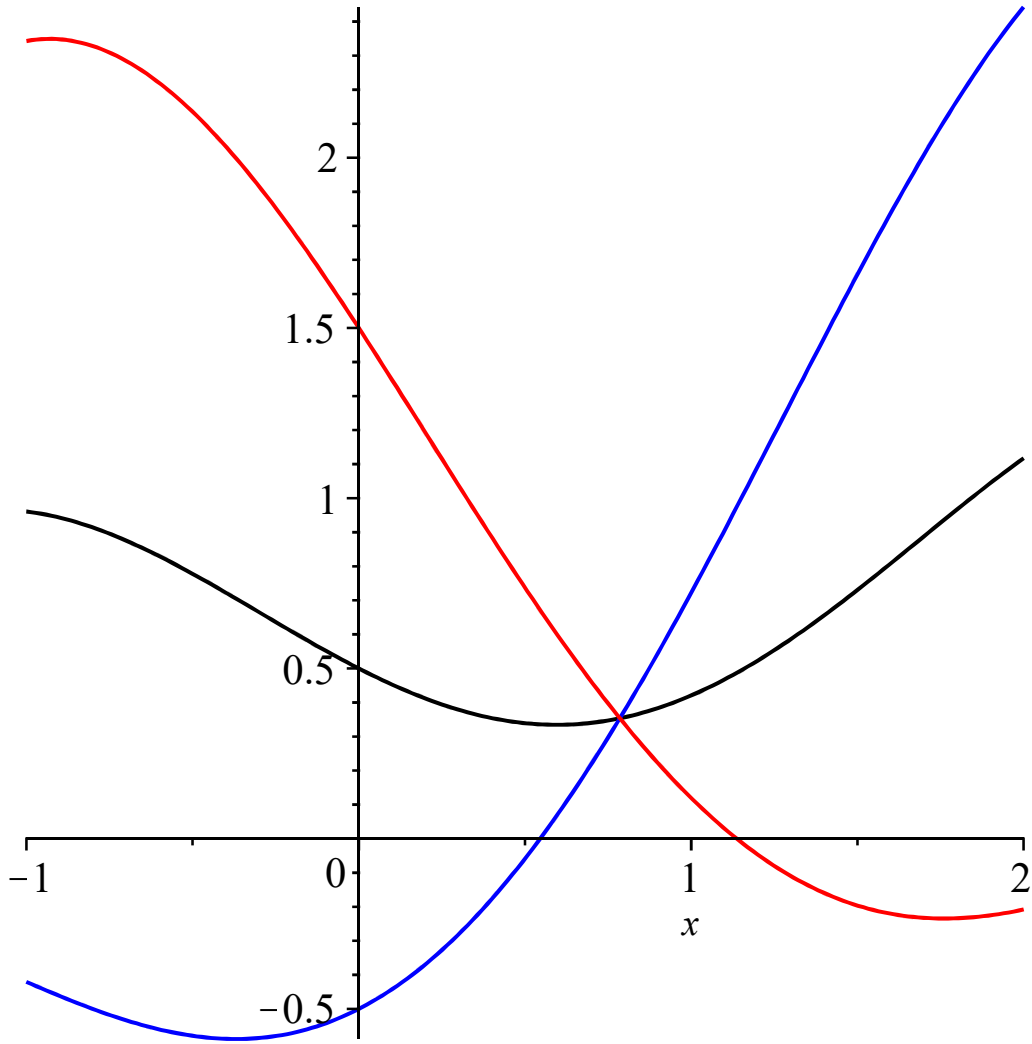
`sol5 := dsolve(ecdif5, y(x));`

$$sol5 := y(x) = \sin(x) _C2 + \cos(x) _C1 + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2} \quad (19)$$

`y5 := unapply(rhs(sol5), x, _C1, _C2);`

$$y5 := (x, _C1, _C2) \mapsto \sin(x) _C2 + \cos(x) _C1 + \frac{(1-x) \cos(x)}{2} + \frac{\sin(x) x}{2} \quad (20)$$

`plot([y5(x, 0, 0), y5(x, -1, 1), y5(x, 1, -1)], x=-1..2, color=[black, blue, red]);`

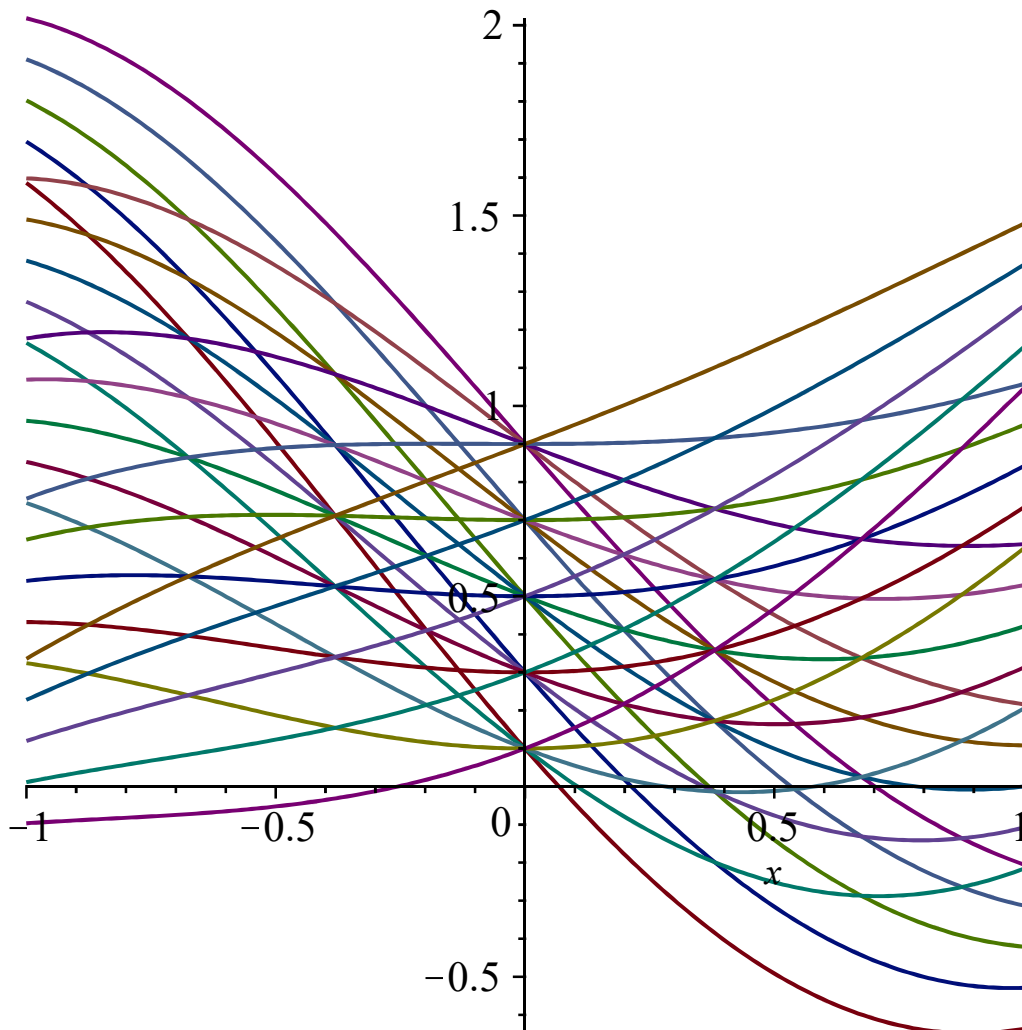


$sir_sol5 := seq(seq(y5(x, i/5, j/2), i=-2..2), j=-2..2);$

$$\begin{aligned}
 sir_sol5 := & -\sin(x) - \frac{2 \cos(x)}{5} + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\sin(x) - \frac{\cos(x)}{5} \\
 & + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\sin(x) + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\sin(x) \\
 & + \frac{\cos(x)}{5} + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\sin(x) + \frac{2 \cos(x)}{5} + \frac{(-x+1) \cos(x)}{2} \\
 & + \frac{\sin(x) x}{2}, -\frac{\sin(x)}{2} - \frac{2 \cos(x)}{5} + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\frac{\sin(x)}{2} \\
 & - \frac{\cos(x)}{5} + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\frac{\sin(x)}{2} + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, \\
 & -\frac{\sin(x)}{2} + \frac{\cos(x)}{5} + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\frac{\sin(x)}{2} + \frac{2 \cos(x)}{5} \\
 & + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\frac{2 \cos(x)}{5} + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, -\frac{\cos(x)}{5} \\
 & + \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, \frac{(-x+1) \cos(x)}{2} + \frac{\sin(x) x}{2}, \frac{\cos(x)}{5}
 \end{aligned} \tag{21}$$

$$\begin{aligned}
& + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \frac{2\cos(x)}{5} + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \frac{\sin(x)}{2} \\
& - \frac{2\cos(x)}{5} + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \frac{\sin(x)}{2} - \frac{\cos(x)}{5} + \frac{(-x+1)\cos(x)}{2} \\
& + \frac{\sin(x)x}{2}, \frac{\sin(x)}{2} + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \frac{\sin(x)}{2} + \frac{\cos(x)}{5} \\
& + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \frac{\sin(x)}{2} + \frac{2\cos(x)}{5} + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \\
& \sin(x) - \frac{2\cos(x)}{5} + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \sin(x) - \frac{\cos(x)}{5} \\
& + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \sin(x) + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \sin(x) \\
& + \frac{\cos(x)}{5} + \frac{(-x+1)\cos(x)}{2} + \frac{\sin(x)x}{2}, \sin(x) + \frac{2\cos(x)}{5} + \frac{(-x+1)\cos(x)}{2} \\
& + \frac{\sin(x)x}{2}
\end{aligned}$$

`plot([sir_sol5], x=-1..1);`



`ecdif6 := diff(y(x), x, x) - y(x) = exp(2 · x);`

$$ecdif6 := \frac{d^2}{dx^2} y(x) - y(x) = e^{2x} \quad (22)$$

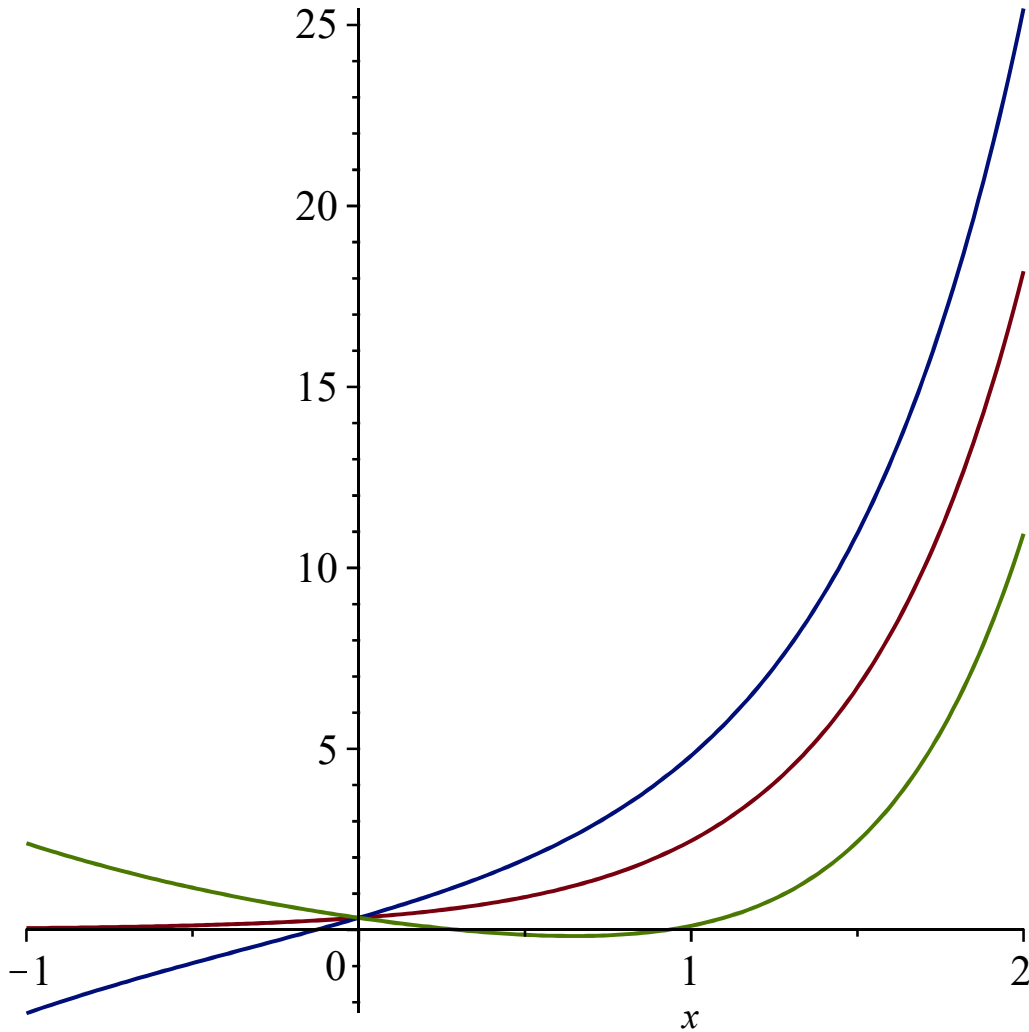
$sol6 := dsolve(ecdif6, y(x));$

$$sol6 := y(x) = e^x_C2 + e^{-x}_C1 + \frac{e^{2x}}{3} \quad (23)$$

$y6 := unapply(rhs(sol6), x, _C1, _C2);$

$$y6 := (x, _C1, _C2) \mapsto e^x_C2 + e^{-x}_C1 + \frac{e^{2x}}{3} \quad (24)$$

$plot([y6(x, 0, 0), y6(x, -1, 1), y6(x, 1, -1)], x=-1..2);$



$$ecdif7 := diff(y(x), x, x) + 4 \cdot y(x) = \frac{1}{\cos 2x};$$

$$ecdif7 := \frac{d^2}{dx^2} y(x) + 4 y(x) = \frac{1}{\cos 2x} \quad (25)$$

$$ecdif7 := diff(y(x), x, x) + 4 \cdot y(x) = \frac{1}{\cos(2 \cdot x)};$$

$$ecdif7 := \frac{d^2}{dx^2} y(x) + 4 y(x) = \frac{1}{\cos(2 x)} \quad (26)$$

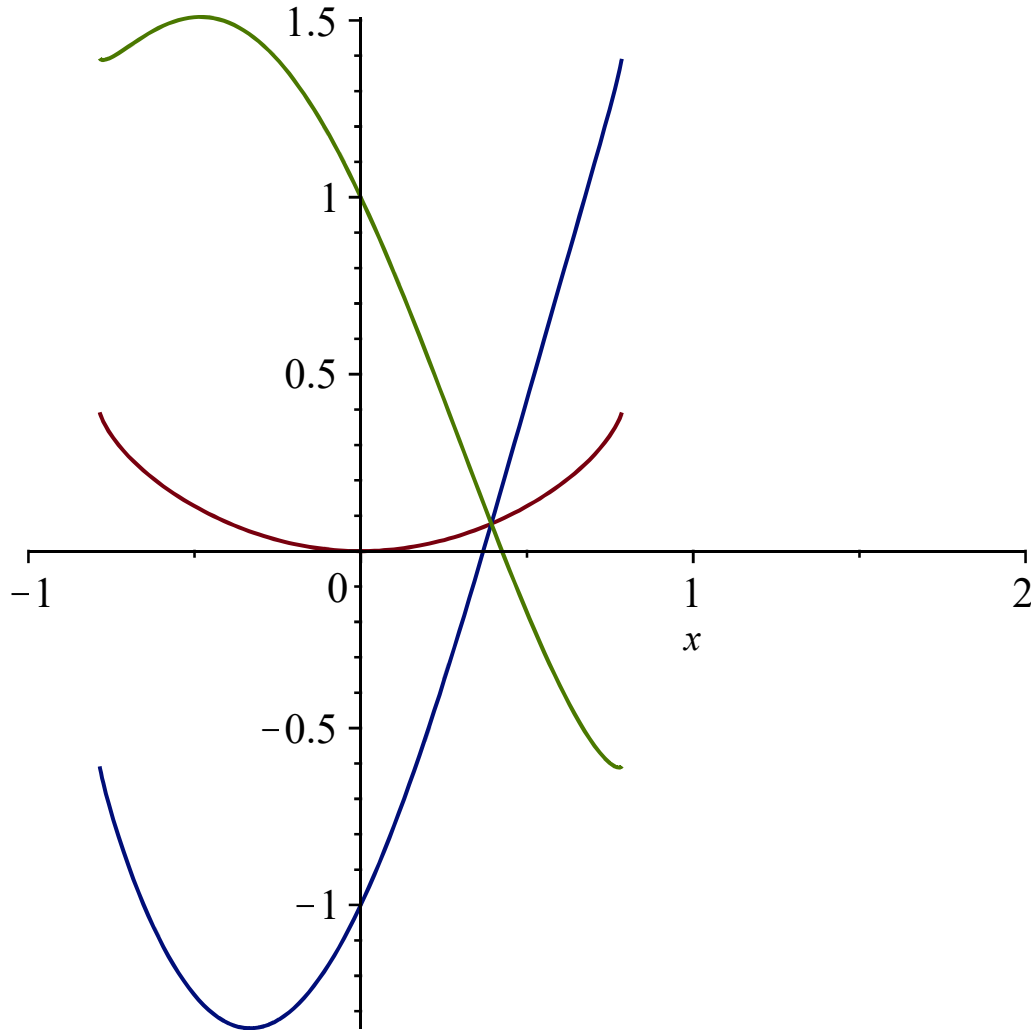
```
sol7 := dsolve(ecdif7, y(x));
```

$$sol7 := y(x) = \sin(2x) _C2 + \cos(2x) _C1 + \frac{x \sin(2x)}{2} + \frac{\ln(\cos(2x)) \cos(2x)}{4} \quad (27)$$

```
y7 := unapply(rhs(sol7), x, _C1, _C2);
```

$$y7 := (x, _C1, _C2) \mapsto \sin(2x) _C2 + \cos(2x) _C1 + \frac{x \sin(2x)}{2} + \frac{\ln(\cos(2x)) \cos(2x)}{4} \quad (28)$$

```
plot([y7(x, 0, 0), y7(x, -1, 1), y7(x, 1, -1)], x=-1..2);
```



```
restart;
```

```
with(DEtools);
```

```
[AreSimilar, Closure, DENormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, (29)
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DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols,

exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, infactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

ecdif1 := *diff*(*y*(*x*), *x*) = 1 + *y*(*x*)²;

$$ecdif1 := \frac{d}{dx} y(x) = 1 + y(x)^2 \quad (30)$$

cond_in1 := *y*(0) = 1;

$$cond_in1 := y(0) = 1 \quad (31)$$

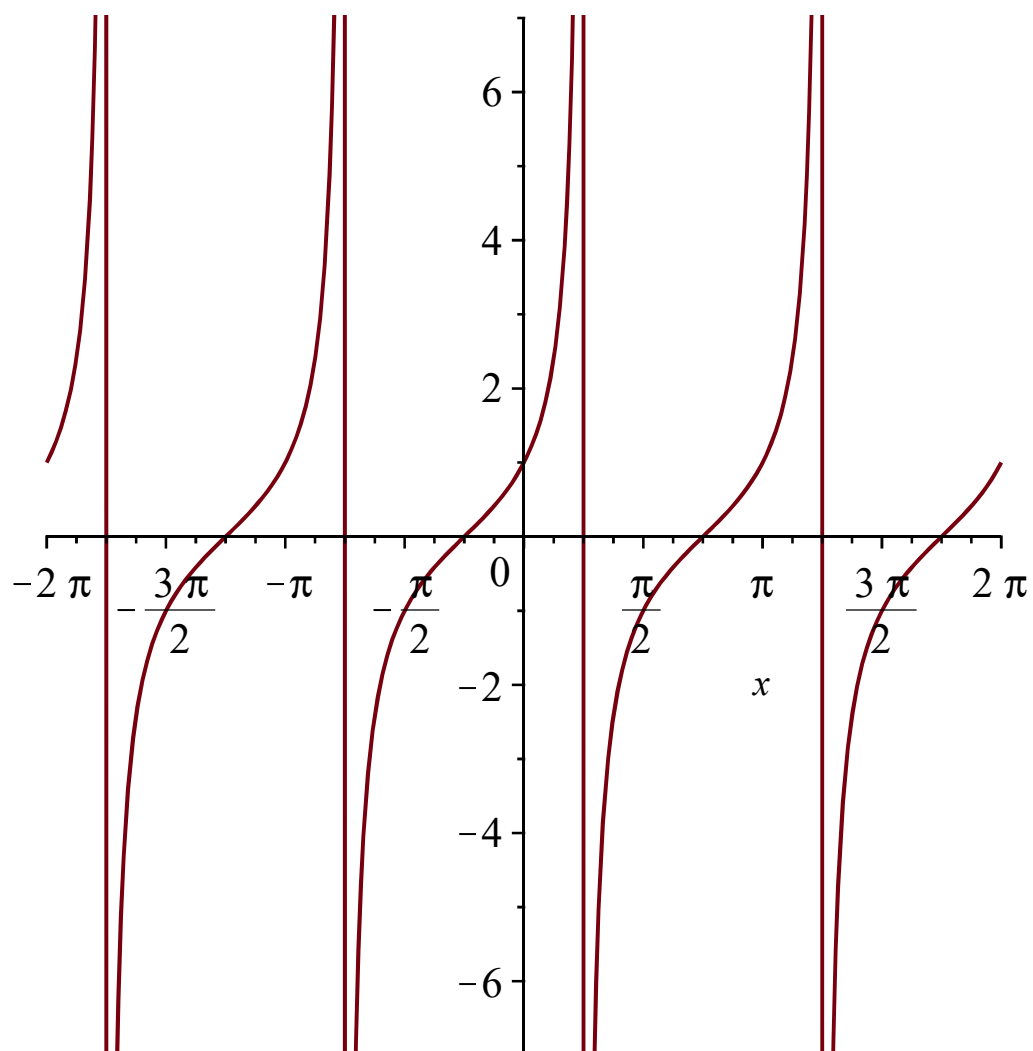
soll := *dsolve*({*ecdif1*, *cond_in1*}, *y*(*x*));

$$soll := y(x) = \tan\left(x + \frac{\pi}{4}\right) \quad (32)$$

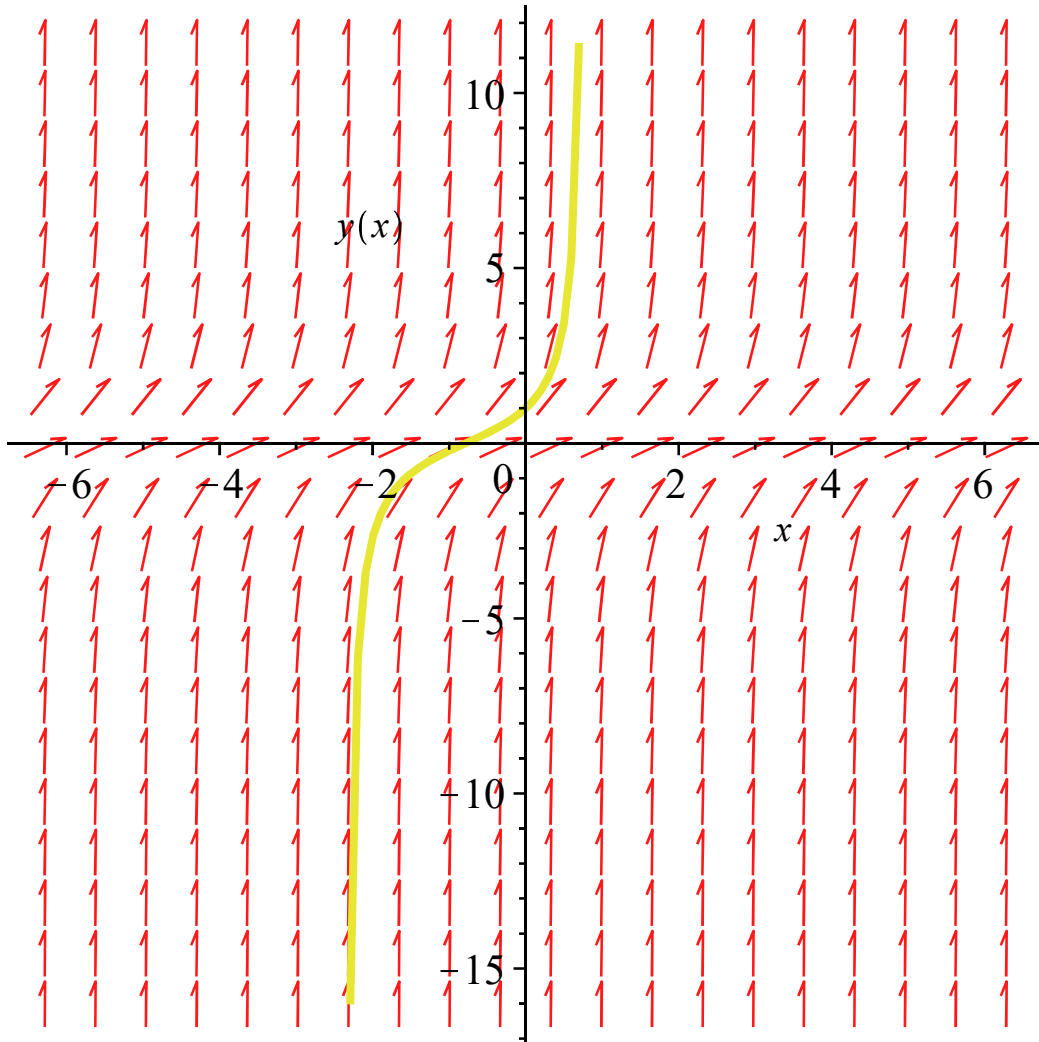
y1 := *unapply*(*rhs*(*soll*), *x*);

$$y1 := x \mapsto \tan\left(x + \frac{\pi}{4}\right) \quad (33)$$

plot(*y1*(*x*), *x* = -2·Pi .. 2·Pi);



```
DEplot(ecdif1, y(x), x=-2·Pi..2·Pi, [[cond_in1]], stepsize=0.1);
Warning, plot may be incomplete, the following error(s) were
issued:
    cannot evaluate the solution further right of .78539807,
probably a singularity
    cannot evaluate the solution further left of -2.3561943,
probably a singularity
```



$$ecdif2 := diff(y(x), x) = \frac{1}{1-x^2} \cdot y(x) + 1 + x;$$

$$ecdif2 := \frac{d}{dx} y(x) = \frac{y(x)}{-x^2 + 1} + 1 + x \quad (34)$$

$$cond_in2 := y(0) = 0;$$

$$cond_in2 := y(0) = 0 \quad (35)$$

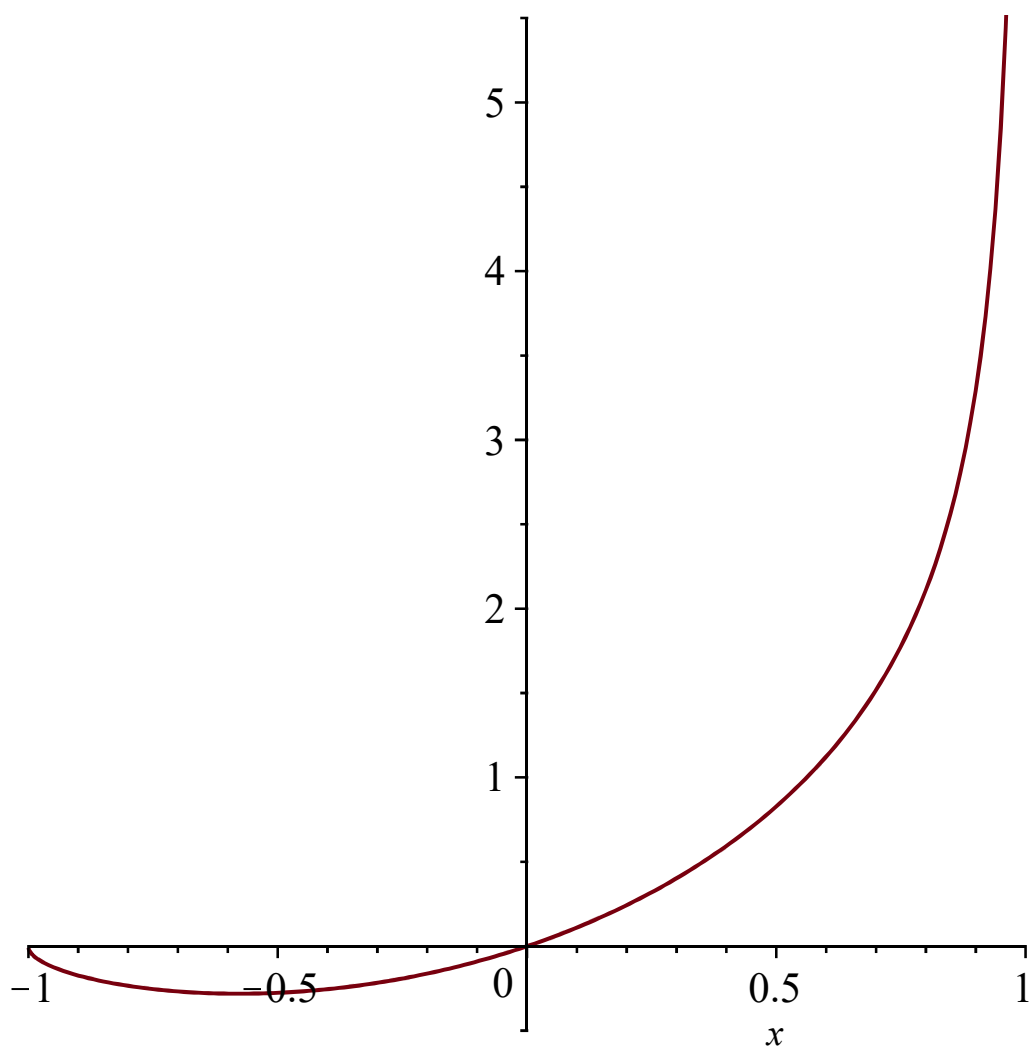
$$sol2 := dsolve(\{ecdif2, cond_in2\}, y(x));$$

$$sol2 := y(x) = \frac{\left(\frac{x\sqrt{-x^2+1}}{2} + \frac{\arcsin(x)}{2} \right) (x+1)}{\sqrt{-x^2+1}} \quad (36)$$

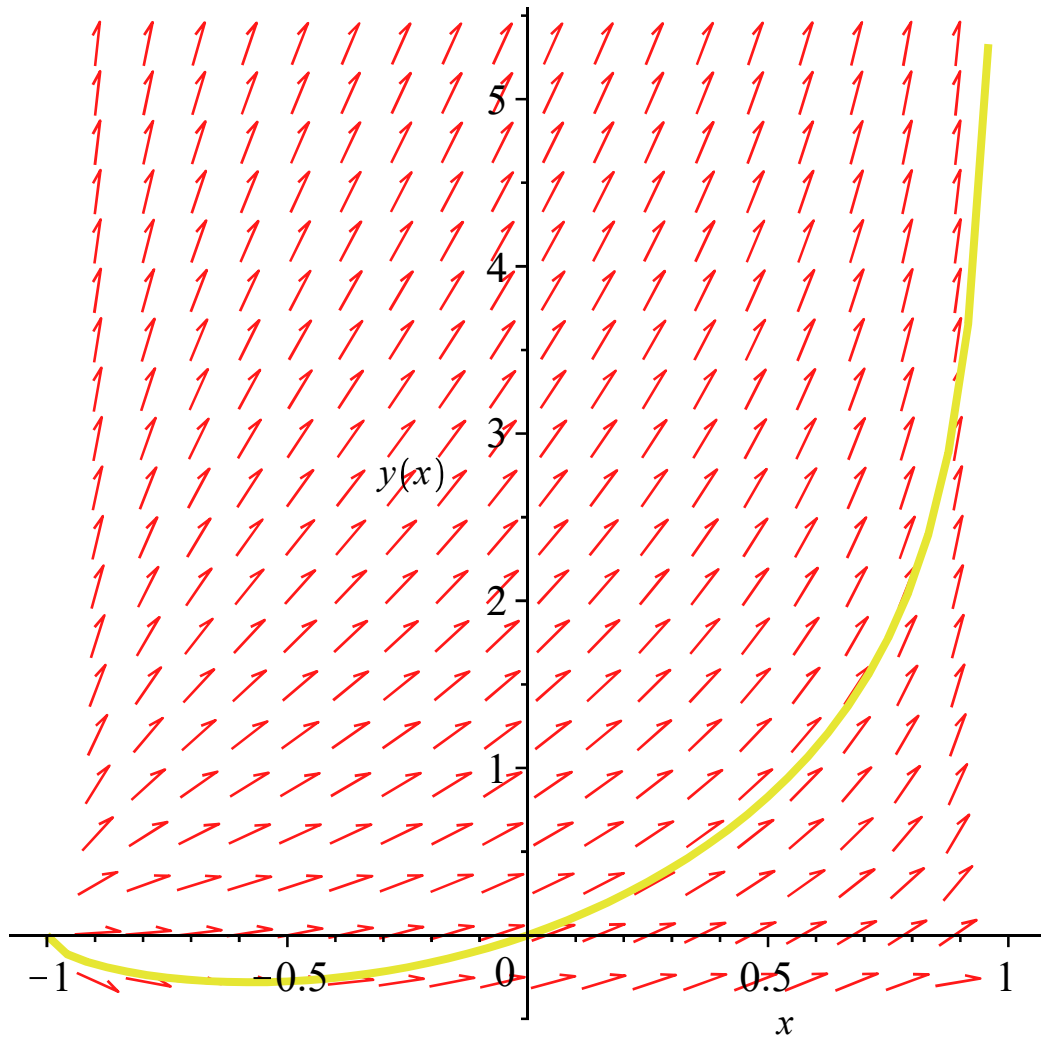
$$y2 := unapply(rhs(sol2), x);$$

$$y2 := x \mapsto \frac{\left(\frac{x\sqrt{-x^2+1}}{2} + \frac{\arcsin(x)}{2} \right) (x+1)}{\sqrt{-x^2+1}} \quad (37)$$

$$plot(y2(x), x=-1..1);$$



```
DEplot(ecdif2, y(x), x=-1..1, [[cond_in2]], stepsize=0.1);  
Warning, plot may be incomplete, the following error(s) were  
issued:  
cannot evaluate the solution further right of .99999999,  
probably a singularity
```



$$ecdif3 := \text{diff}(y(x), x) - 2 \cdot y(x) = -x^2;$$

$$ecdif3 := \frac{d}{dx} y(x) - 2 y(x) = -x^2 \quad (38)$$

$$cond_in3 := y(0) = \frac{1}{4};$$

$$cond_in3 := y(0) = \frac{1}{4} \quad (39)$$

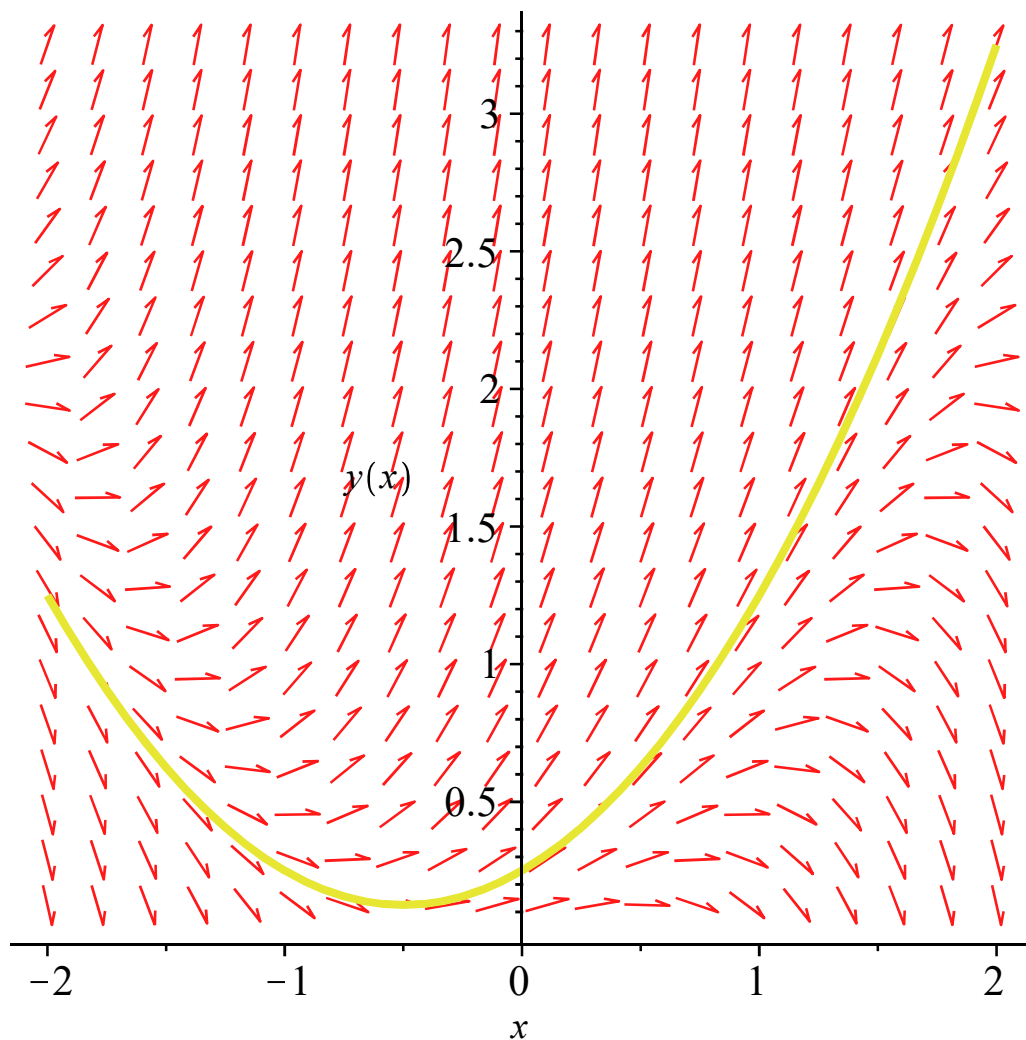
$$sol3 := \text{dsolve}(\{ecdif3, cond_in3\}, y(x));$$

$$sol3 := y(x) = \frac{1}{2} x^2 + \frac{1}{2} x + \frac{1}{4} \quad (40)$$

$$y3 := \text{unapply}(\text{rhs}(sol3), x);$$

$$y3 := x \mapsto \frac{1}{2} x^2 + \frac{1}{2} x + \frac{1}{4} \quad (41)$$

$$DEplot(ecdif3, y(x), x = -2 .. 2, [[cond_in3]], \text{stepsize} = 0.1);$$



$$ecdif4 := diff(y(x), x, x) - 5 \cdot diff(y(x), x) + 4 \cdot y(x) = 0;$$

$$ecdif4 := \frac{d^2}{dx^2} y(x) - 5 \frac{d}{dx} y(x) + 4 y(x) = 0 \quad (42)$$

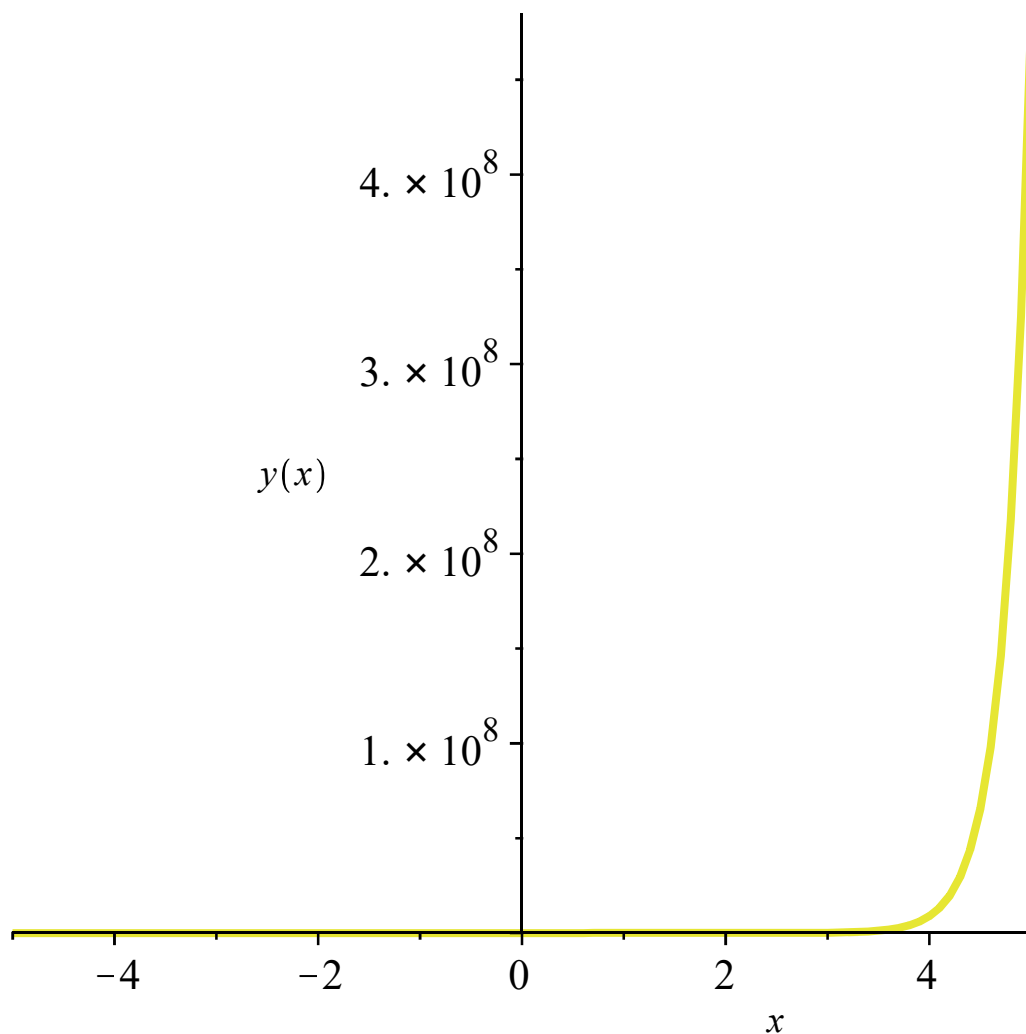
$$cond_in4 := y(0) = 5, D(y)(0) = 8;$$

$$cond_in4 := y(0) = 5, D(y)(0) = 8 \quad (43)$$

$$sol4 := dsolve(\{ecdif4, cond_in4\}, y(x));$$

$$sol4 := y(x) = e^{4x} + 4 e^x \quad (44)$$

$$DEplot(ecdif4, y(x), x = -5 .. 5, [[cond_in4]], stepsize = 0.1);$$



$$ecdif5 := \text{diff}(y(x), x\$2) - 4 \cdot \text{diff}(y(x), x) + 5 \cdot y(x) = 2 \cdot x^2 \cdot \exp(x);$$

$$ecdif5 := \frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 5 y(x) = 2 x^2 e^x \quad (45)$$

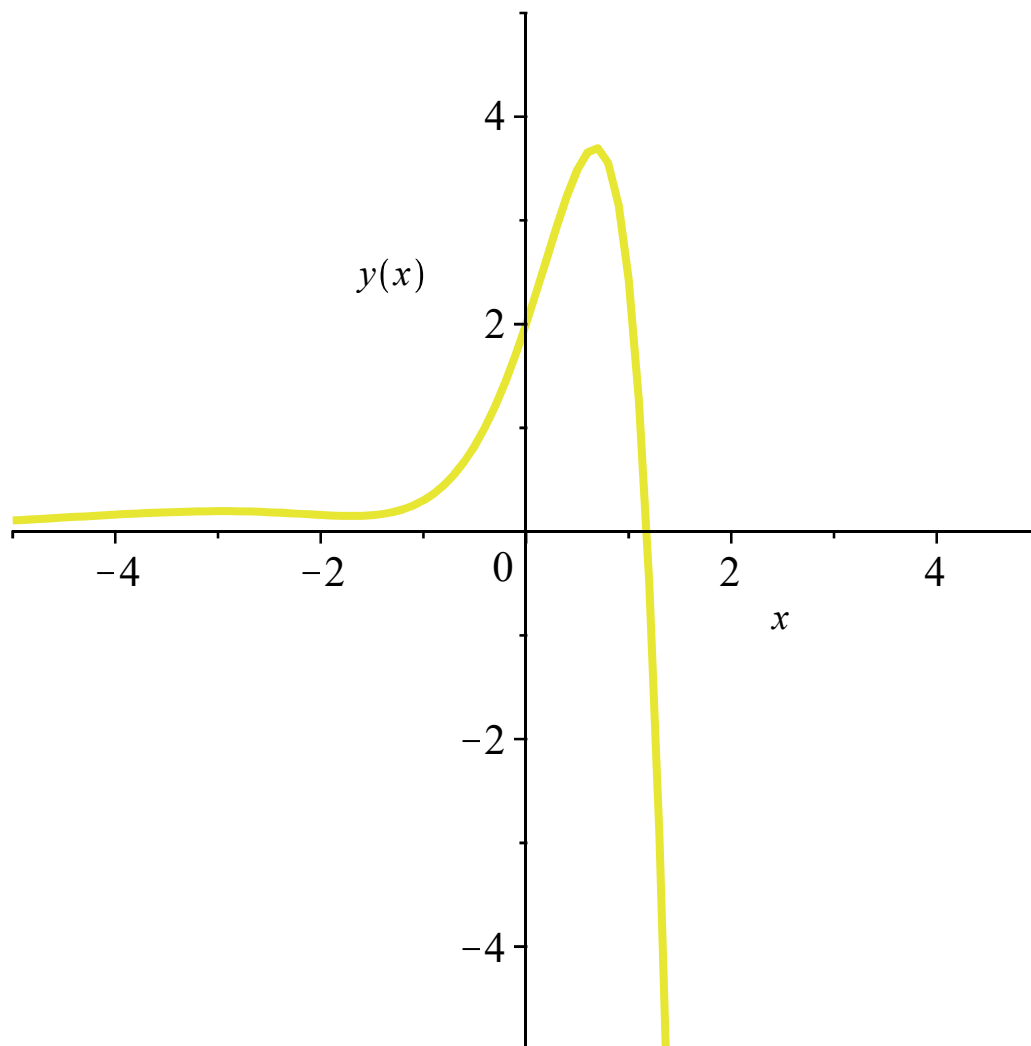
$$cond_in5 := y(0) = 2, D(y)(0) = 3;$$

$$cond_in5 := y(0) = 2, D(y)(0) = 3 \quad (46)$$

$$sol5 := \text{dsolve}(\{ecdif5, cond_in5\}, y(x));$$

$$sol5 := y(x) = -2 e^{2x} \sin(x) + e^{2x} \cos(x) + (x + 1)^2 e^x \quad (47)$$

$$DEplot(ecdif5, y(x), x = -5 .. 5, y = -5 .. 5, [[cond_in5]], stepsize = 0.1);$$



$$ecdif6 := diff(y(x), x\$2) + 4 \cdot y(x) = 4 \cdot (\sin(2 \cdot x) + \cos(2 \cdot x));$$

$$ecdif6 := \frac{d^2}{dx^2} y(x) + 4 y(x) = 4 \sin(2 x) + 4 \cos(2 x) \quad (48)$$

$$cond_in6 := y(\text{Pi}) = 2 \cdot \text{Pi}, D(y)(\text{Pi}) = 2 \cdot \text{Pi};$$

$$cond_in6 := y(\pi) = 2 \pi, D(y)(\pi) = 2 \pi \quad (49)$$

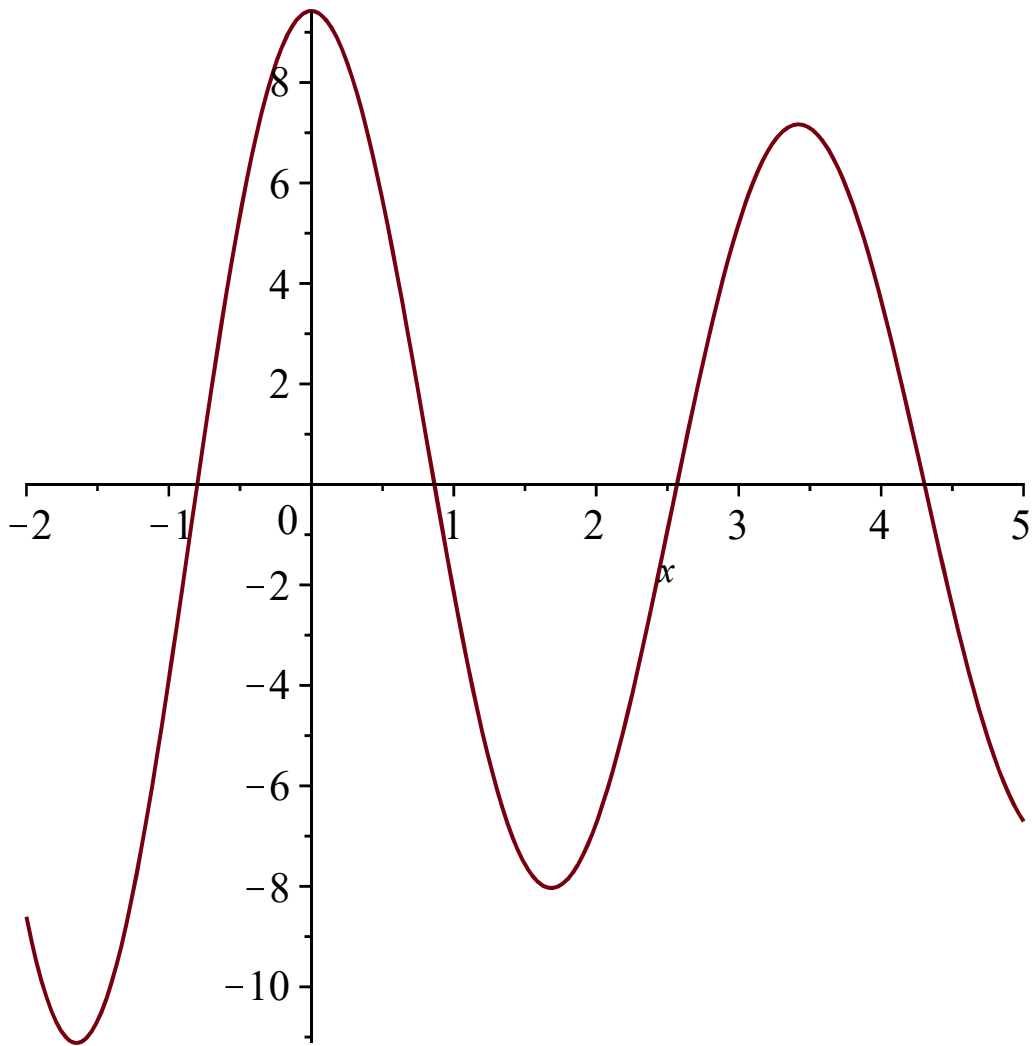
$$sol6 := dsolve(\{ecdif6, cond_in6\}, y(x));$$

$$sol6 := y(x) = \frac{\sin(2 x)}{2} + \cos(2 x) \left(3 \pi - \frac{1}{2} \right) + \frac{\cos(2 x)}{2} + \sin(2 x) x - \cos(2 x) x \quad (50)$$

$$y6 := unapply(rhs(sol6), x);$$

$$y6 := x \mapsto \frac{\sin(2 x)}{2} + \cos(2 x) \left(3 \pi - \frac{1}{2} \right) + \frac{\cos(2 x)}{2} + \sin(2 x) x - \cos(2 x) x \quad (51)$$

$$plot(y6(x), x = -2 .. 5);$$



restart;

$$ecdif := \text{diff}(y(x), x) - \frac{1}{2} \cdot y(x) = \cos(x);$$

$$ecdif := \frac{d}{dx} y(x) - \frac{y(x)}{2} = \cos(x) \quad (52)$$

sol := dsolve(ecdif, y(x));

$$sol := y(x) = -\frac{2 \cos(x)}{5} + \frac{4 \sin(x)}{5} + e^{\frac{x}{2}} _CI \quad (53)$$

y := unapply(rhs(sol), x, _CI);

$$y := (x, _CI) \mapsto -\frac{2 \cos(x)}{5} + \frac{4 \sin(x)}{5} + e^{\frac{x}{2}} _CI \quad (54)$$

restart;

$$ecdif := \text{diff}(y(x), x) - \frac{1}{2} \cdot y(x) = \cos(x);$$

$$ecdif := \frac{d}{dx} y(x) - \frac{y(x)}{2} = \cos(x) \quad (55)$$

sol := dsolve(ecdif, y(x));

$$sol := y(x) = -\frac{2 \cos(x)}{5} + \frac{4 \sin(x)}{5} + e^{\frac{x}{2}} _CI \quad (56)$$

with(DEtools);

[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, infactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

y := unapply(rha(sol), x, _CI);

$$y := (x, _CI) \mapsto rha\left(y(x) = -\frac{2 \cos(x)}{5} + \frac{4 \sin(x)}{5} + e^{\frac{x}{2}} _CI\right) \quad (58)$$

DEplot(ecdif1, y(x, 1), x = -2 * Pi .. 2 * Pi, y = -1 .. 1); =

DEplot(ecdif1, y(x, 1), x = -2 * Pi .. 2 * Pi, y = -1 .. 1)

restart;

with(DEtools);

[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, infactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon,

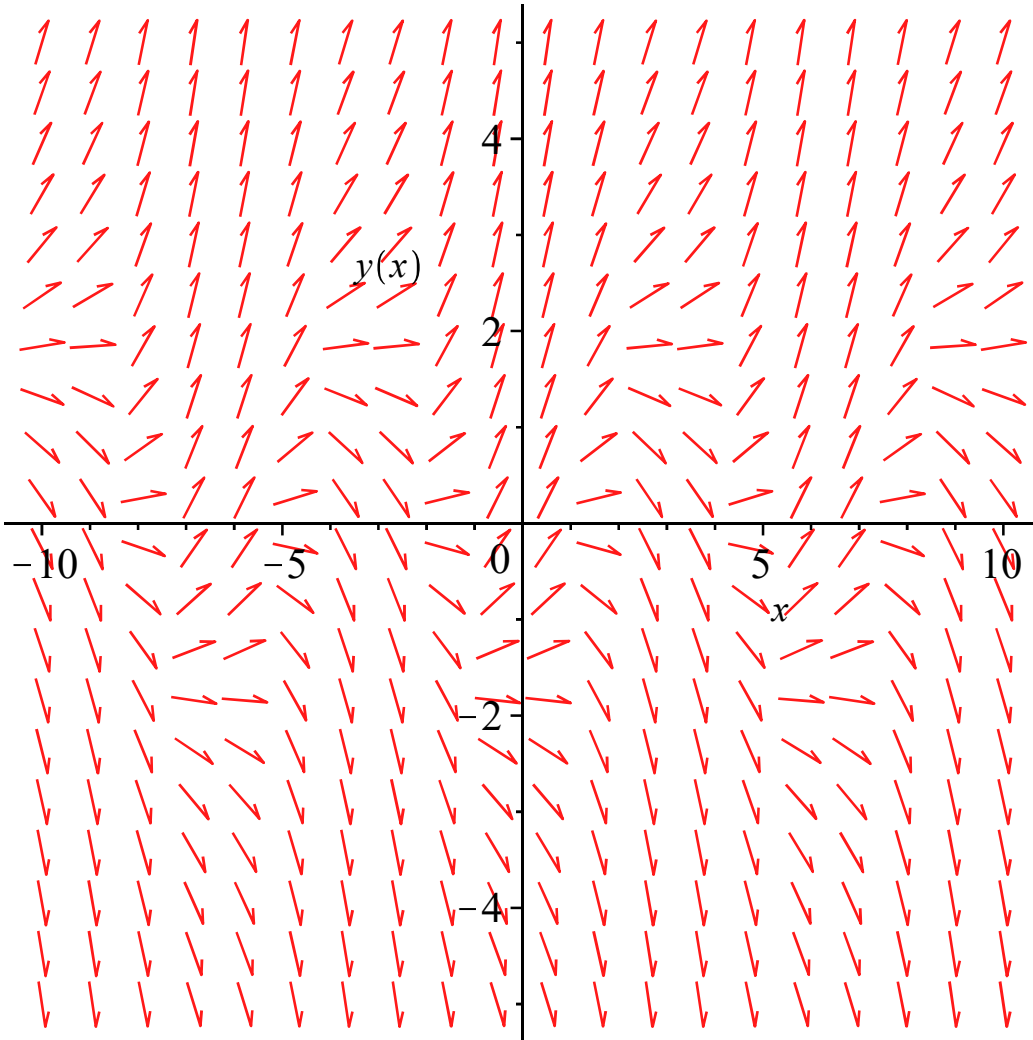
normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightrightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

$ecdif := \text{diff}(y(x), x) - \frac{1}{2} \cdot y(x) = \cos(x);$

$$ecdif := \frac{d}{dx} y(x) - \frac{y(x)}{2} = \cos(x)$$

(60)

$DEplot(ecdif, y(x), x=-10..10, y=-5..5);$



$cond1 := y(0) = \frac{1}{10};$

$$cond1 := y(0) = \frac{1}{10}$$

(61)

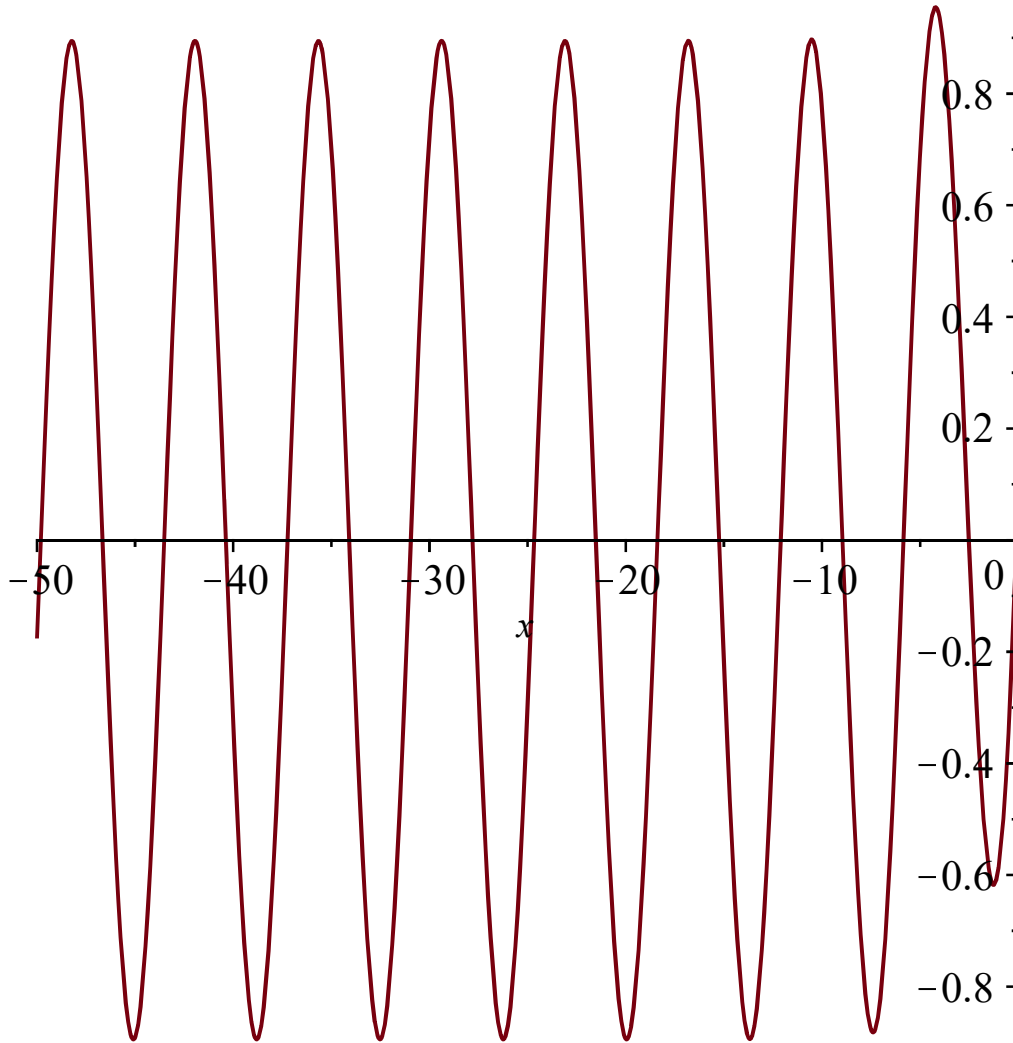
$soll := \text{dsolve}(\{ecdif, cond1\}, y(x));$

$$sol1 := y(x) = -\frac{2 \cos(x)}{5} + \frac{4 \sin(x)}{5} + \frac{e^{\frac{x}{2}}}{2} \quad (62)$$

$y1 := unapply(rhs(sol1), x);$

$$y1 := x \mapsto -\frac{2 \cos(x)}{5} + \frac{4 \sin(x)}{5} + \frac{e^{\frac{x}{2}}}{2} \quad (63)$$

$plot(y1(x), x=-50..0);$



$restart;$

$with(DEtools);$

[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, (64)

dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

with(plots);

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot] (65)

ecdif := diff(y(x), x) = a·y(x) + b;

$$ecdif := \frac{d}{dx} y(x) = a y(x) + b \quad (66)$$

sol := dsolve(ecdif, y(x));

$$sol := y(x) = -\frac{b}{a} + e^{ax} _C1 \quad (67)$$

cond := D(y)(x) = a·y(x) + b;

$$cond := D(y)(x) = a y(x) + b \quad (68)$$

sol := dsolve({cond, ecdif}, y(x));

$$sol := \left\{ y(x) = \frac{e^{ax} _C1 a - b}{a} \right\} \quad (69)$$

cond := D(y)(x) = 0;

$$cond := D(y)(x) = 0 \quad (70)$$

sol := dsolve({cond, ecdif}, y(x));

Error, (in dsolve) found the following equations not depending on the unknowns of the input system: {ecdif}

restart;

with(DEtools);

[AreSimilar, Closure, DENormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, (71)

Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

ecdifl := *diff*(*y*(*x*), *x*, *x*) - *diff*(*y*(*x*), *x*) - 2 * *y*(*x*) = 0;

$$ecdifl := \frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) - 2 y(x) = 0 \quad (72)$$

condl := *y*(0) = *a*, *D*(*y*)(0) = 2;

$$condl := y(0) = a, D(y)(0) = 2 \quad (73)$$

soll := *dsolve*({*condl*, *ecdifl* }, *y*(*x*));

$$soll := y(x) = \left(\frac{a}{3} + \frac{2}{3} \right) e^{2x} + \left(-\frac{2}{3} + \frac{2a}{3} \right) e^{-x} \quad (74)$$

y := *unapply*(*rhs*(*soll*), *x*, *a*);

$$y := (x, a) \mapsto \left(\frac{a}{3} + \frac{2}{3} \right) e^{2x} + \left(-\frac{2}{3} + \frac{2a}{3} \right) e^{-x} \quad (75)$$