Portfolio Evaluation: Consider two stocks, a and b, and let $S_a(t)$ and $S_b(t)$ be the prices of the two stocks at time t. At time $t = 0$, you buy n_a shares of stock A and n_b shares of stock B. Then your initial wealth is

$$W_0 = n_a S_a(0) + n_b S_b(0).$$

Suppose your investment horizon is $T \in \mathbb{R}$ years, after which your terminal wealth W_T is given by

$$W_T = n_a S_a(T) + n_b S_b(T).$$

(This presumes you do not trade any of your stock in the time interval $[0, T]$.)

Assume that $S_a \sim GBM(\mu_a, \sigma_a)$ and $S_b \sim GBM(\mu_b, \sigma_b)$, where $GBM(\mu_b, \sigma_b)$ corresponds to a *Geometric Brownian Motion* distribution given by the following equations:

$$S_a(T) = S_a(0) \exp\left((\mu_a - \sigma_a^2/2)T + \sigma_a B_a(T)\right)$$
$$S_b(T) = S_b(0) \exp\left((\mu_b - \sigma_b^2/2)T + \sigma_b B_b(T)\right)$$

and where $B_a(T)$ and $B_b(T)$ are given by a *Standard Brownian Motion* distribution, i.e., a *Normal* $(0, \sqrt{T})$ distribution. Assume that $B_a(T)$ and $B_b(T)$ are independent.

You would like to estimate

$$\Pr\left(\frac{W_T}{W_0} \leq 0.9\right),$$

i.e., the probability that the value of your portfolio drops by more than 10%.

Let L be the (loss) event that $W_T/W_0 \leq 0.9$. You can estimate the probability of L (that your portfolio drops by more than 10%) using the following characteristic function, where $\mathbf{X} = (S_a(T), S_b(T))$:

$$I_L(\mathbf{X}) = \begin{cases} 1, & \text{if } \dfrac{n_a S_a(T) + n_b S_b(T)}{n_a S_a(0) + n_b S_b(0)} \leq 0.9 \\ 0, & \text{otherwise} \end{cases}$$

computing N realizations of this characteristic function, and dividing the sum by N:

$$\hat{\theta}_N = \frac{I_L(\mathbf{X}_1) + I_L(\mathbf{X}_2) + \cdots + I_L(\mathbf{X}_N)}{N}$$

Use Monte Carlo simulation to estimate this probability. Use the following parameter values:

- $T = 0.5$ years
- $\mu_a = 0.15, \sigma_a = 0.20$
- $\mu_b = 0.12, \sigma_b = 0.18$
- $S_a(0) = \$100, S_b(0) = \$75$
- $n_a = n_b = 100$ shares

(Note that these parameter values give $W_0 = \$17,500$.)

Implement a Monte Carlo simulation model for this portfolio using R. Experiment with various values for N. Experiment with various values for T, the terminal time in years. Generate some meaningful histograms, superimposing the sample mean as well as a sample variance statistic of your choice (e.g., 5th and 95th percentiles, or 25th and 75th percentiles — use `quantile`). What kind of decisions can you make based on your results?

Submitting: Submit your work (R source, appropriately labeled histograms as PNGs or PDFs, a README containing appropriate discussion) to Lyceum.