Generating variates from the *normal* distribution requires numerical approximation techniques because the cdf cannot be inverted in closed form. Two of the best known approaches are the following.

Sum of Uniform Variates: Let U_i be a *uniform* (0,1) random variable. Then

$$Z = \frac{\sum_{i=1}^{n} U_i - n/2}{\sqrt{n/12}}$$

is approximately *normal* (0,1).

Box-Muller Method: Let U_1 and U_2 be independent *uniform* (0,1) random variables. Then

$$\begin{aligned} Z_1 &= \sqrt{-2\log U_1}\cos(2\pi U_2) \\ Z_2 &= \sqrt{-2\log U_1}\sin(2\pi U_2) \end{aligned}$$

are pairwise independent *normal* (0,1) random variables.

Assignment:

1. Write an R function that will generate a single *normal* (0,1) random variate using the sum-of-uniforms approach with $n = 12$. Then generate many *standard normal* (i.e., *normal* (0,1)) variates using your function, plot a histogram, and superimpose the theoretical *standard normal* pdf.

   Does the fit looks reasonable? What are the drawbacks of using this method? What is the possible range of values generated?

2. Write a separate R function that will generate a single *standard normal* random variate using the Box-Muller method. (Note that on the first call, your function should generate u_1 and u_2 but return only z_1; on the subsequent call, it should return z_2 without generating new values for u_1 and u_2. Subsequent pairs of calls will repeat this pairwise process.) Then generate many *standard normal* variates using your function, plot a histogram, and superimpose the theoretical *standard normal* pdf.

   Does the fit looks reasonable? In terms of efficiency, how does this approach compare to the sum-of-uniforms approach? What is the possible range of values generated?

Submitting: Include the following in your submission:

- the source code for both R functions;
- PNG/JPG/PDF of a histogram (with theoretical superimposed) created using the first approach;
- PNG/JPG/PDF of a histogram (with theoretical superimposed) created using the second approach;
- README discussing the range, drawbacks, & efficiency questions posed above.