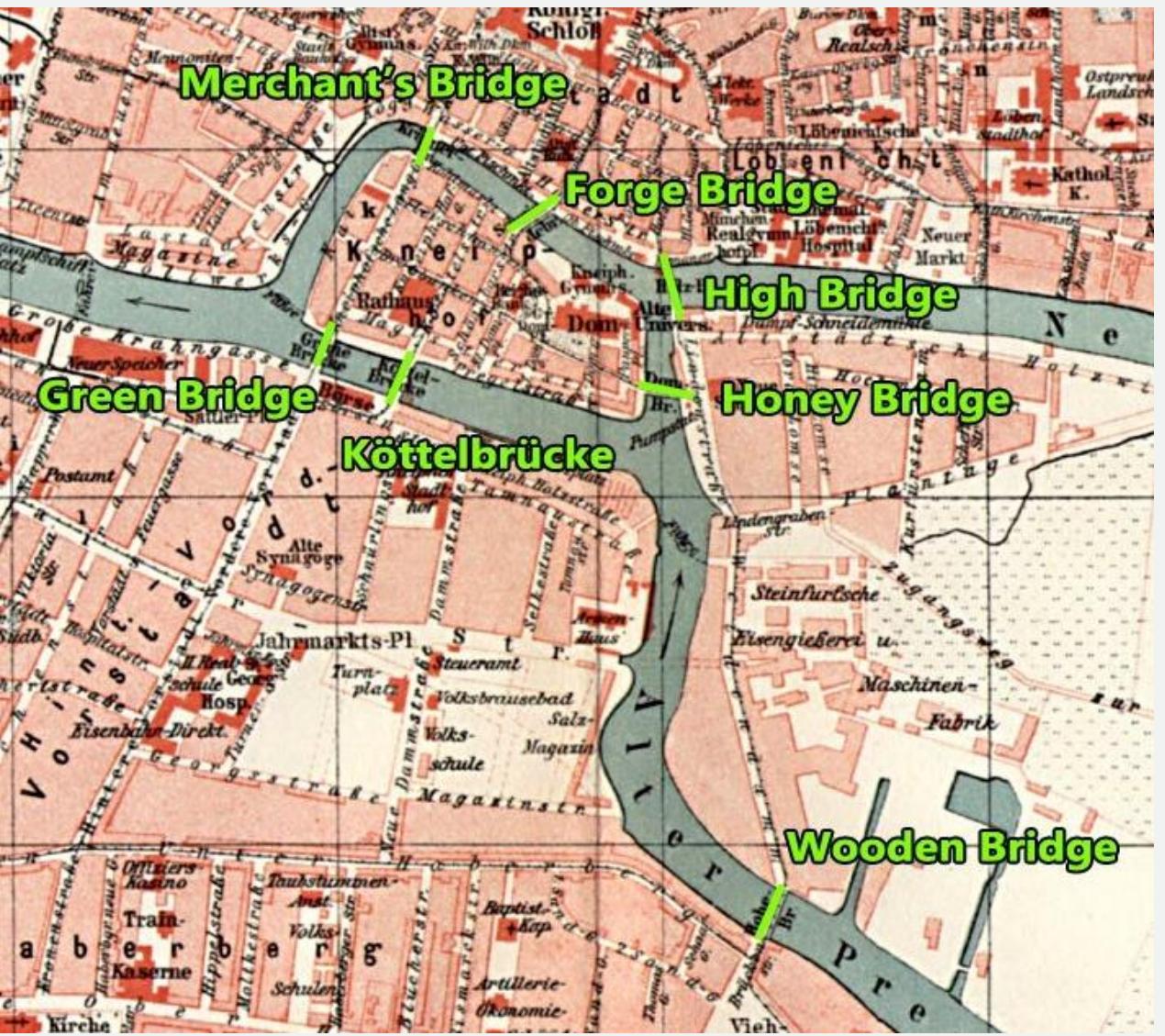
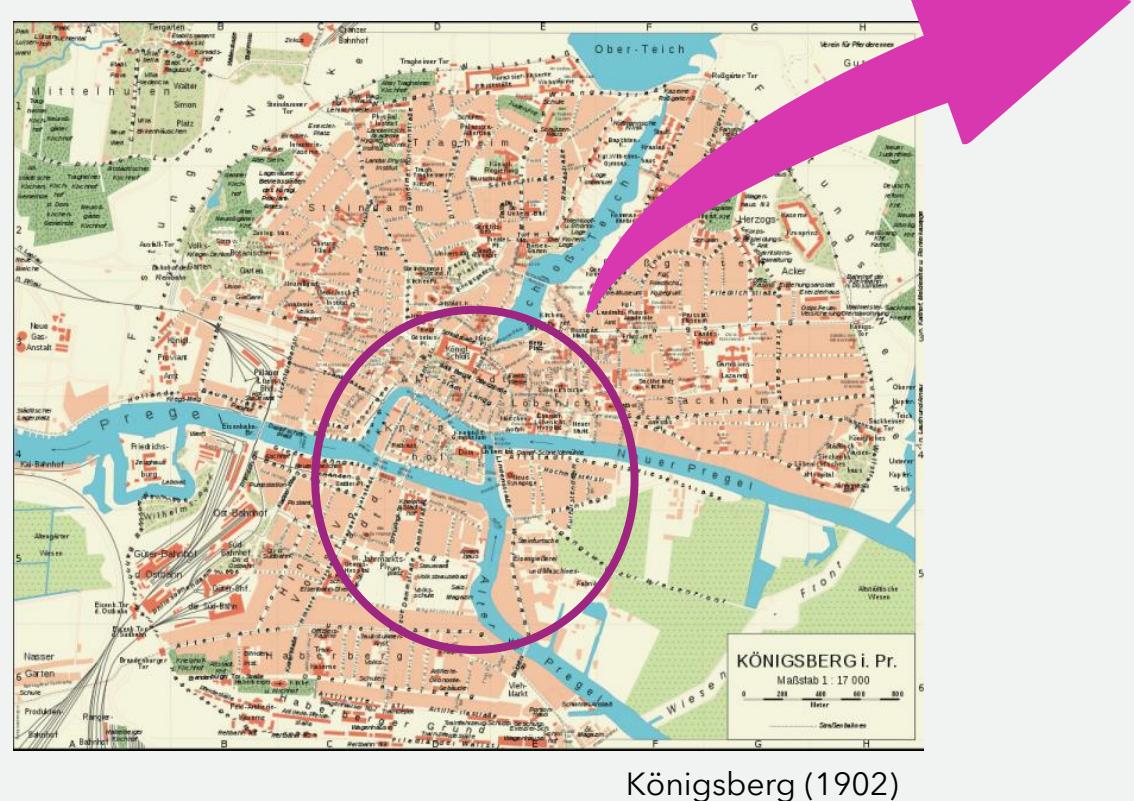
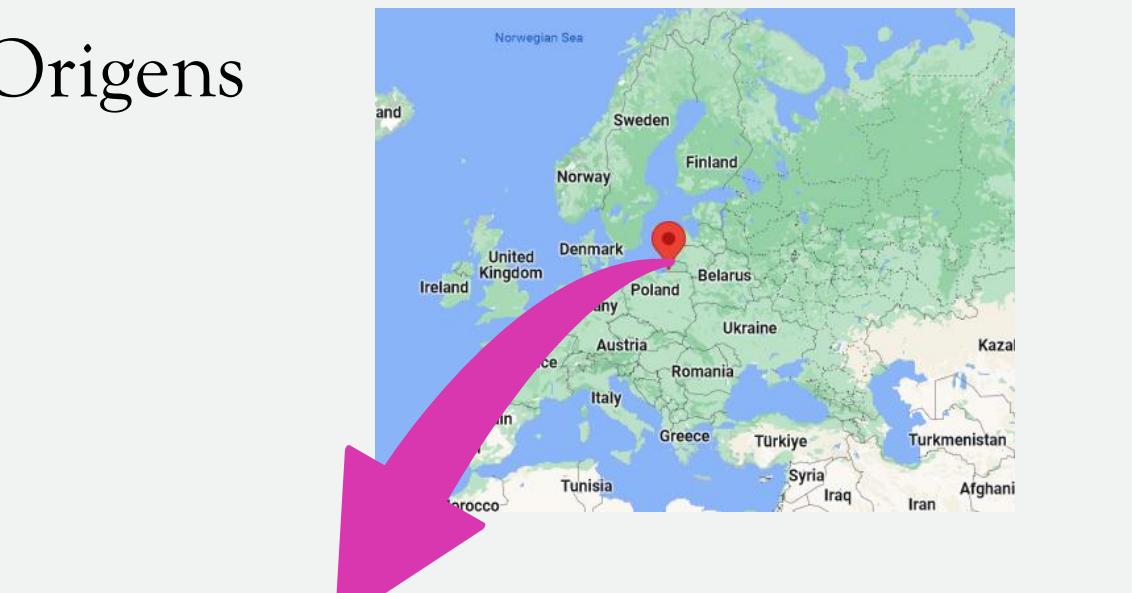




# Grafos aplicados



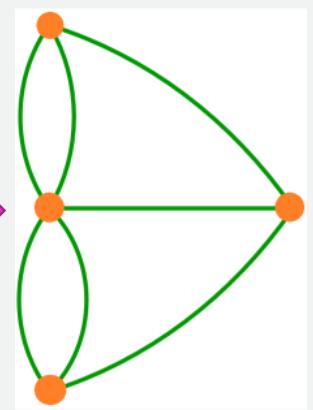
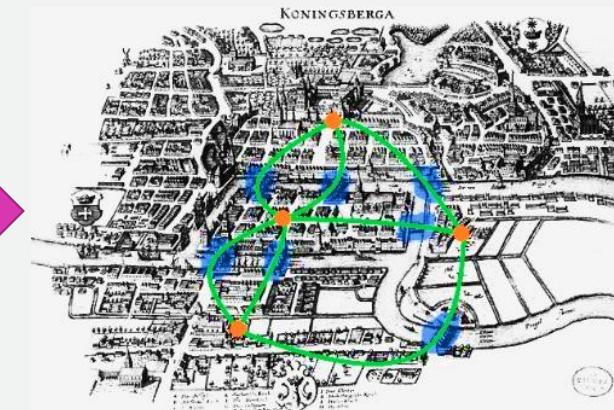
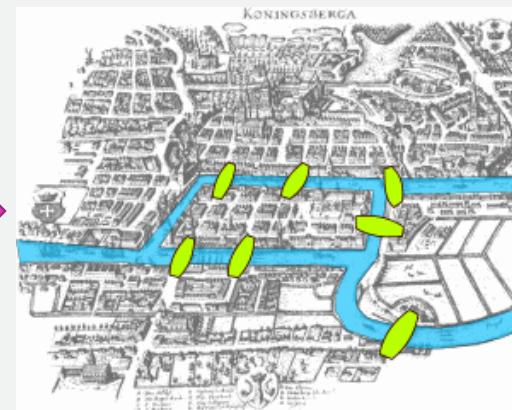
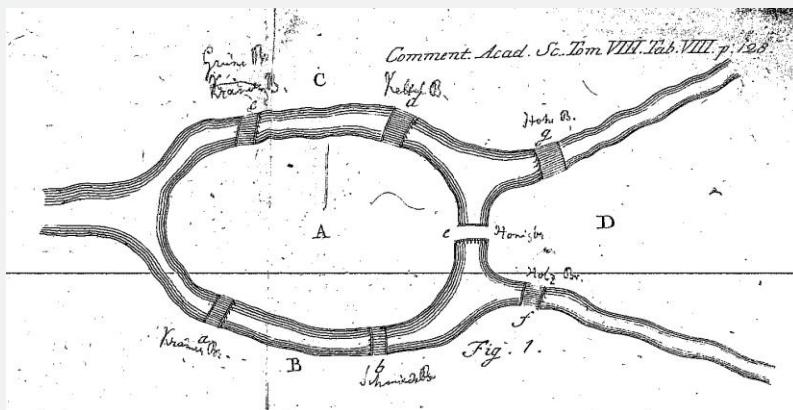
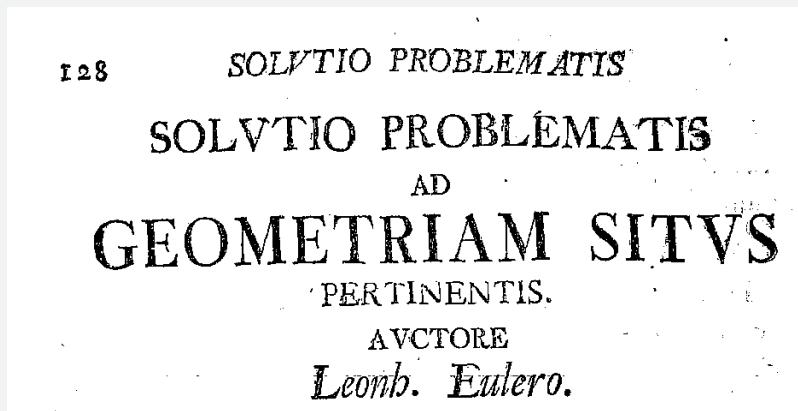
# Origens



## Problema:

É possível traçar um caminho através de Königsberg de modo a atravessar cada uma das sete pontes apenas uma vez e regressar à origem?

- As sete pontes de Königsberg (Kalinin grado)
  - Leonhard Euler (1736)



- O traçado é possível?



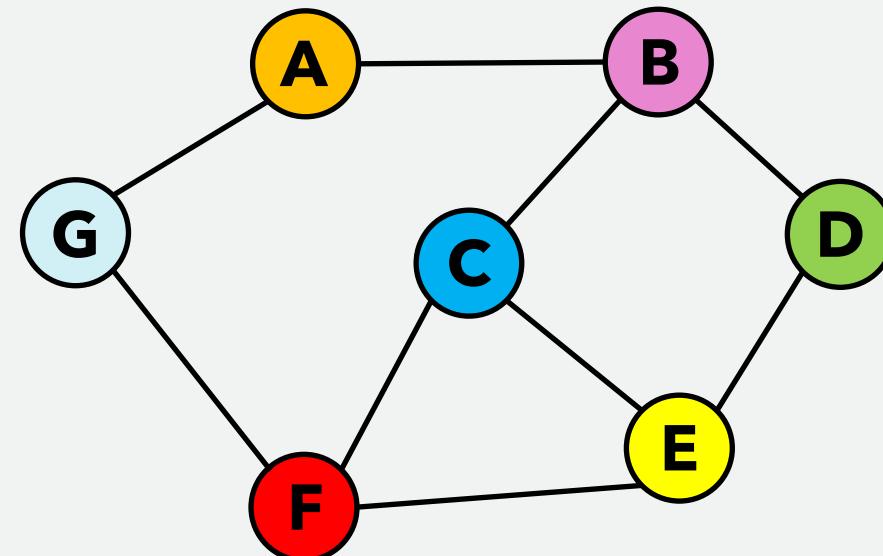
# O que é um grafo?

- Um grafo é uma abstração matemática com objetivo de representar um arranjo estrutural, no qual objetos podem estar relacionados sob certo aspecto.
- A teoria dos grafos é uma área da matemática discreta
- É representado como um par  $G=(V, A)$  onde
  - $V$  é um conjunto de vértices
  - $A$  é um conjunto de arestas
- Geralmente é apresentado por um pictograma que expõe esta estrutura

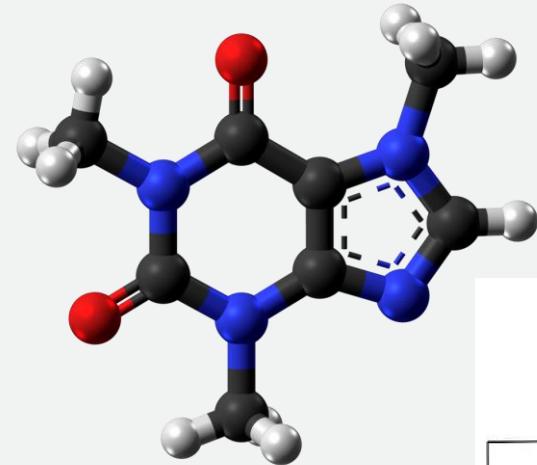


# Representação gráfica (pictograma)

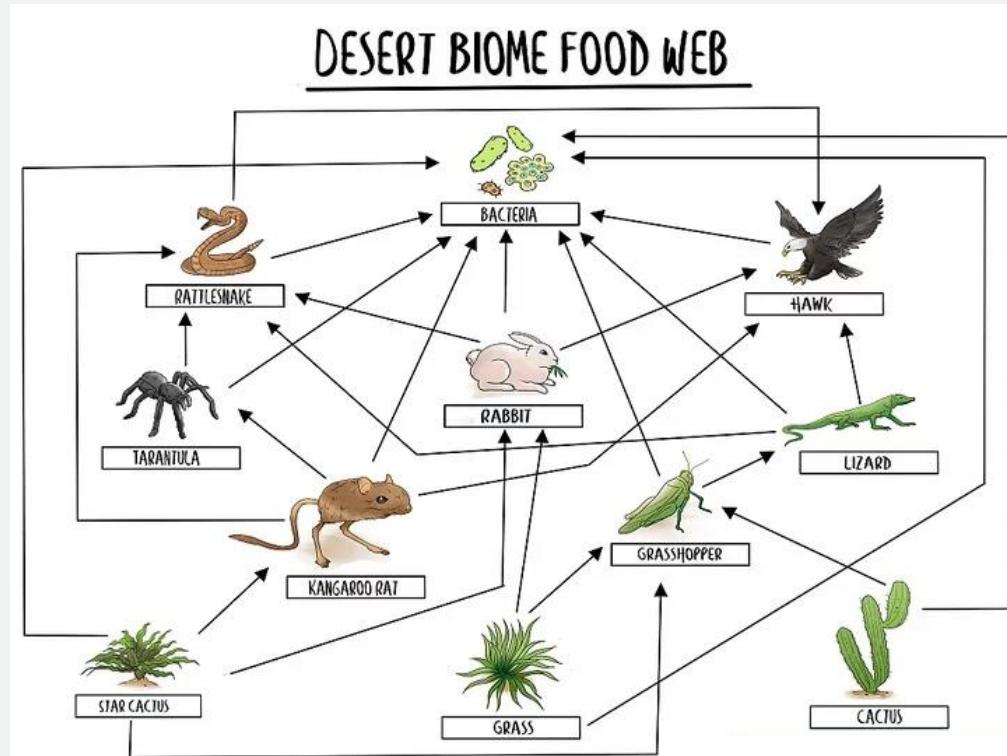
- Grafo  $G = \{V, A\}$
- Vértices  $\rightarrow V=\{A, B, C, D, E, F, G\}$
- Areias  $\rightarrow A=\{(A,B),(B,C),(B,D),(D,E),(C,E),(C,F),(E,F),(F,G),(G,A)\}$



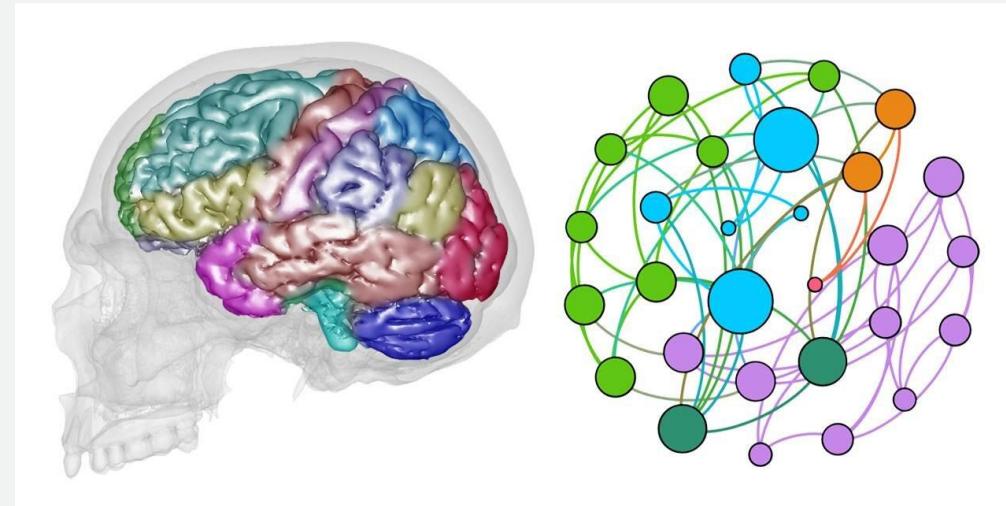
# O quê os grafos podem representar?



Moléculas

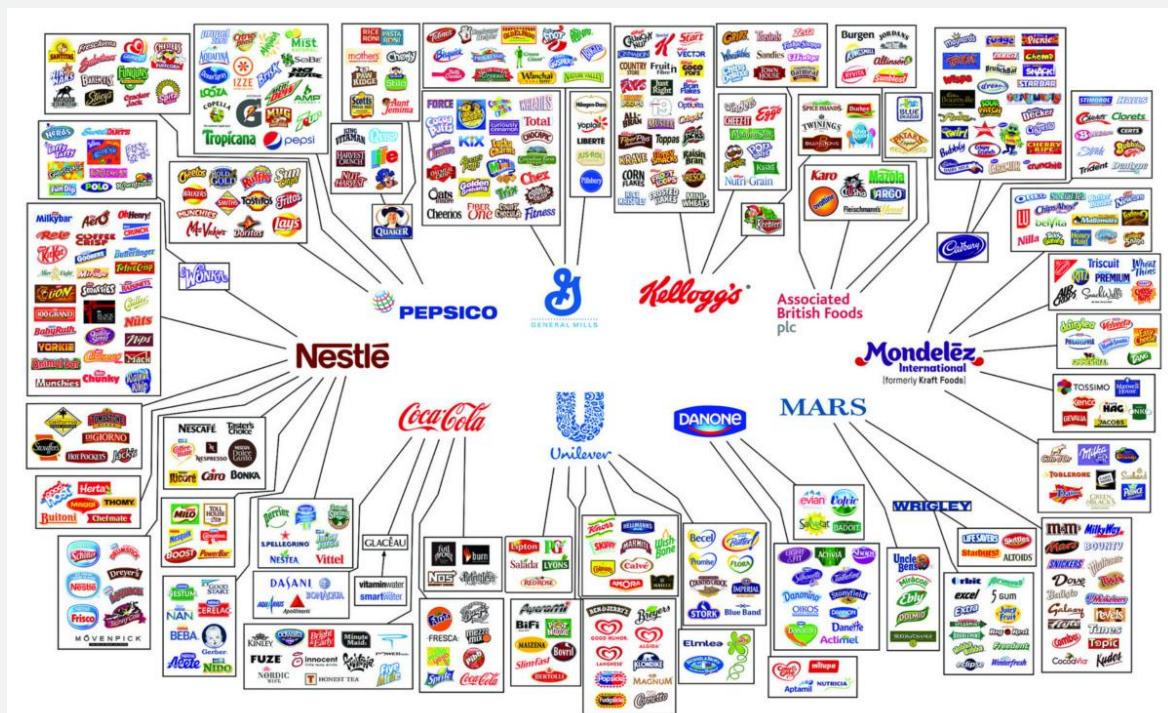
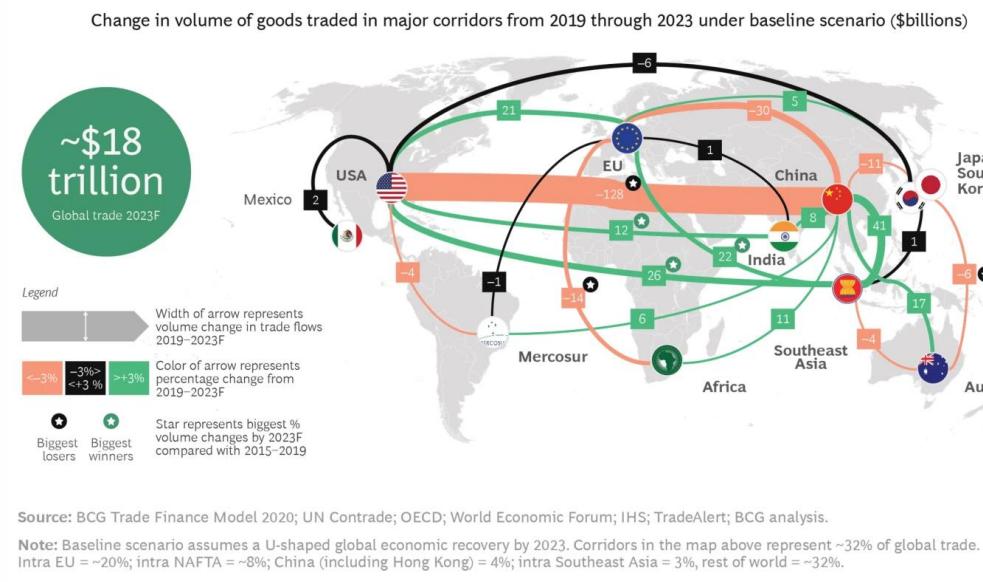


Cadeia alimentar

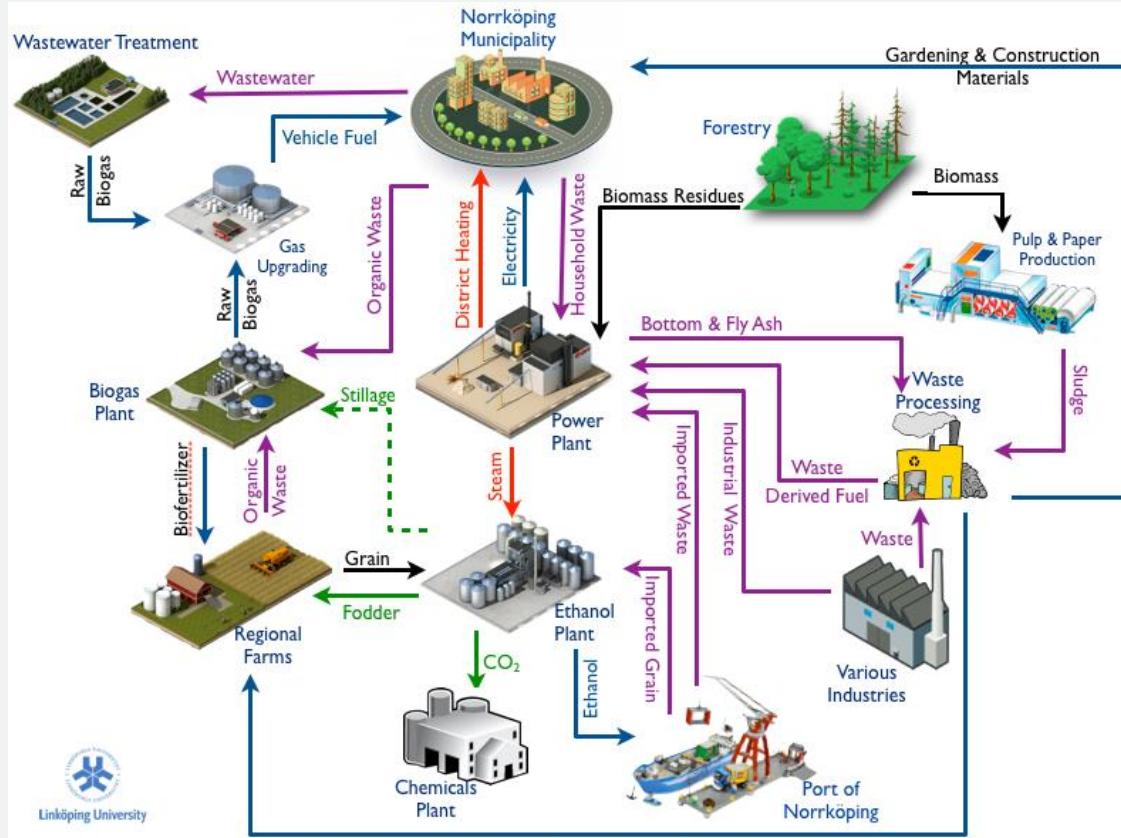


Estruturas biológicas

## Exhibit 2 - Even If Trade Recovers by 2023, Expect Flows Between Blocs to Shift Dramatically

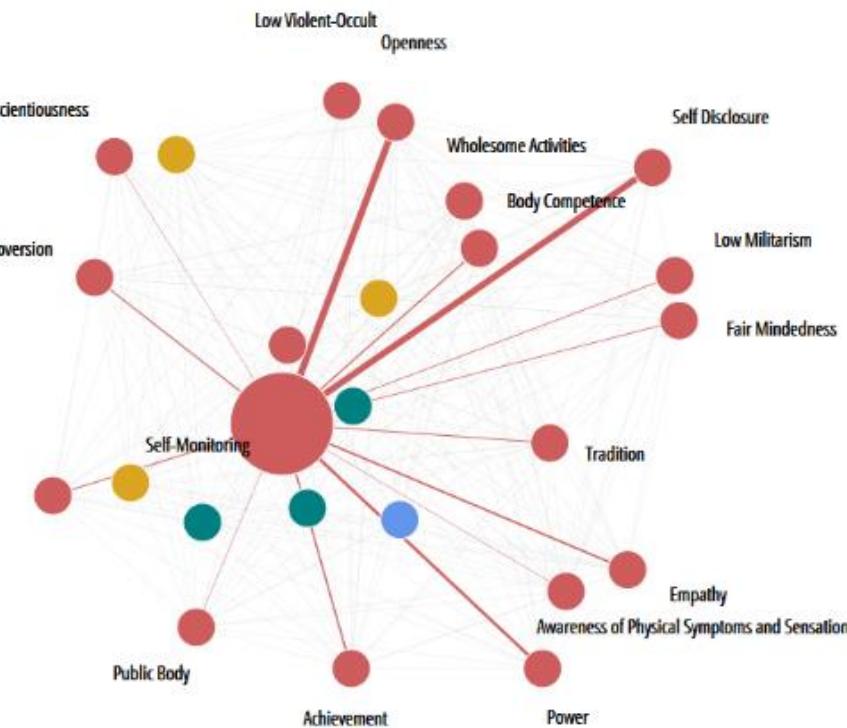
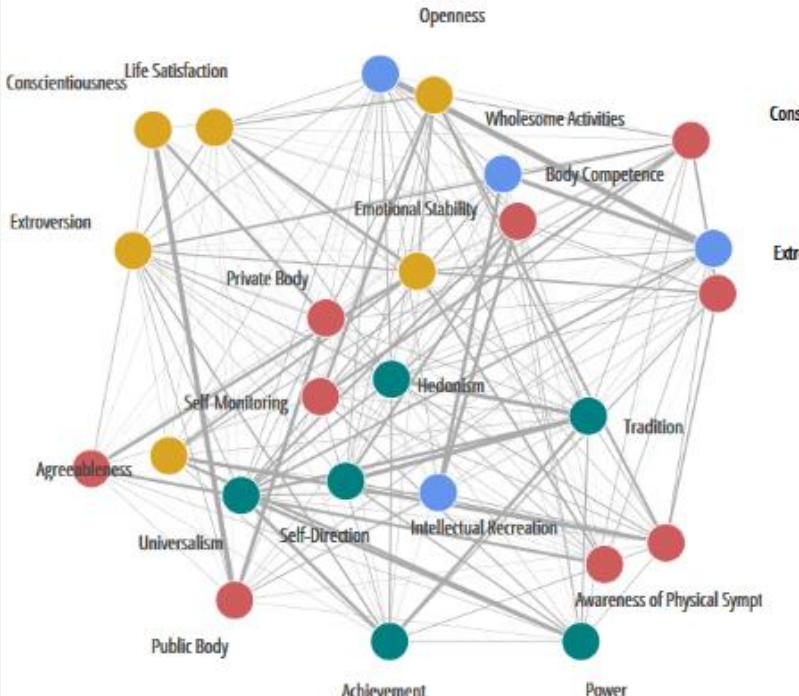


## Comércio internacional



Cadeias produtivas e integrações

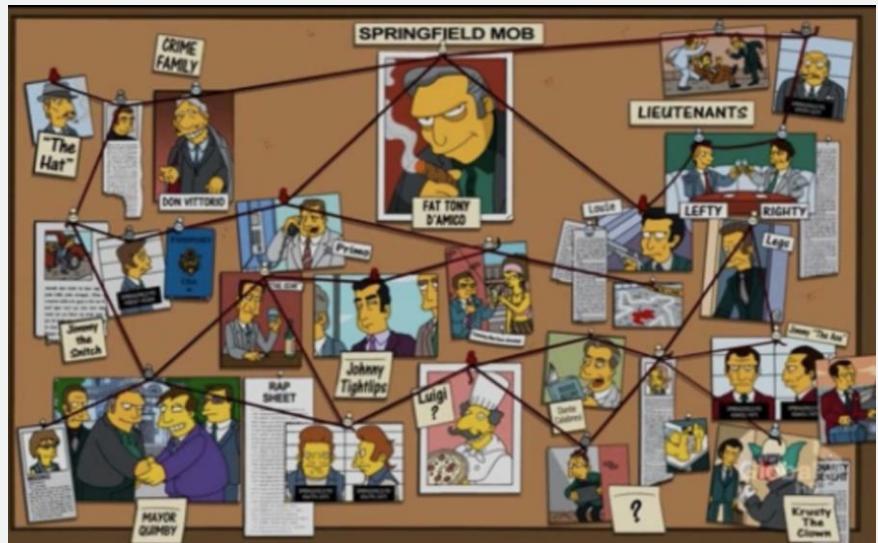
Relacionamentos entre empresas



About this trait : A high score on this trait describes a person who monitors her self-presentations, expressive behavior, and nonverbal affective displays.

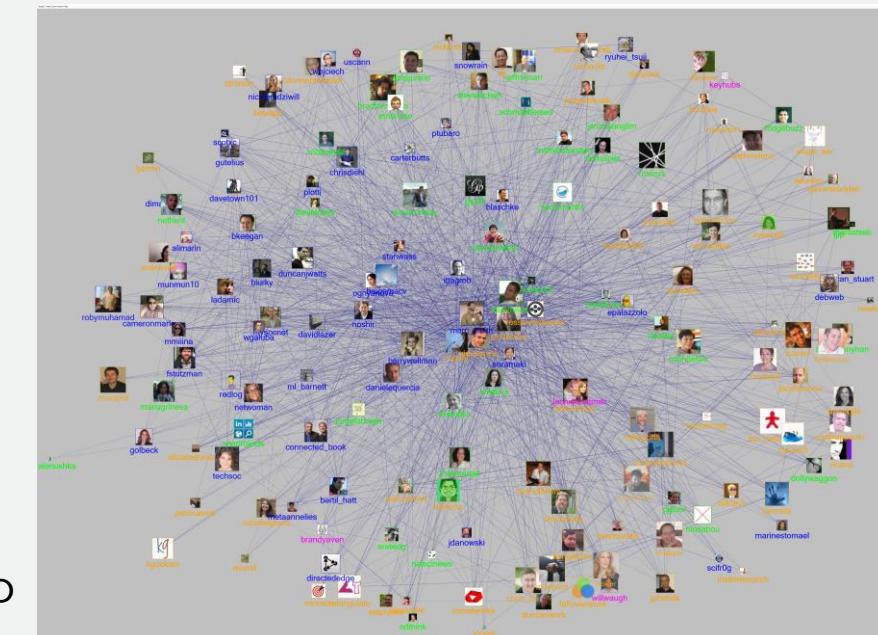
Self-monitoring has the strongest positive correlation (0.118) with Power  
Self-monitoring has the strongest negative correlation (-0.267) with Openness  
The measure used is Snyder's Self-Monitoring Scale

## Comportamento individual



## Aspectos de grupo

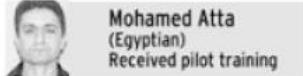
## Alcance de divulgação



## THE HIJACKERS ...

### American Airlines 11

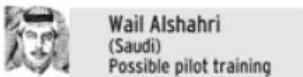
Crashed into WTC (north)



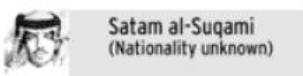
Mohamed Atta  
(Egyptian)  
Received pilot training



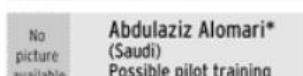
Waleed M. Alshehri  
(Saudi)  
Commercial pilot



Wail Alshahri  
(Saudi)  
Possible pilot training



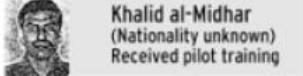
Satam al-Suqami  
(Nationality unknown)



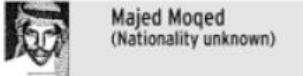
Abdulaziz Alomari\*  
(Saudi)  
Possible pilot training

### American Airlines 77

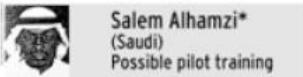
Crashed into Pentagon



Khalid al-Midhar  
(Nationality unknown)  
Received pilot training



Majed Moqed  
(Nationality unknown)



Salem Alhamzi\*  
(Saudi)  
Possible pilot training



Nawaf Alhamzi\*  
(Saudi)



Hani Hanjour  
(Saudi)

### UnitedAirlines 175

Crashed into WTC (south)



Marwan al-Shehhi  
(United Arab Emirates)  
Received pilot training



No picture available  
Fayed Ahmed  
(Believed to be Saudi)



Hamza Alghamdi  
(Believed to be Saudi)  
Possible pilot training



Mohald Alshehri  
(Nationality unknown)  
Possible pilot training

## AND HOW THEY WERE CONNECTED

### Attended same technical college

Hamburg, Germany  
Mohamed Atta  
Marwan al-Shehhi  
Ziad Jarrah

### Took flight classes together

Pilot schools  
in Florida  
Mohamed Atta  
Marwan al-Shehhi

Pilot schools  
in San Diago  
Khalid al-Midhar  
Nawaf Alhamzi

### Bought flight tickets using same address

- Mohamed Atta\*
- Marwan al-Shehhi
- Abdulaziz Alomari\*

\* Also used same credit card

- Waleed M. Alshehri
- Wail Alshahri

- Fayed Ahmed
- Mohald Alshehri

- Ahmed Alghamdi
- Hamza Alghamdi

\*Disputed identity

### Known to be together in week before attacks

Stayed together  
in a Florida  
motel  
Mohamed Atta  
Marwan al-Shehhi  
Hani Hanjour

Attended a gym  
in Maryland  
(Sept 2-6),  
also seen dining  
together  
Khalid al-Midhar  
Majed Moqed  
Salem Alhamzi  
Nawaf Alhamzi  
Hani Hanjour

### Last known address

Hollywood, Florida

Marwan al-Shehhi  
Waleed M. Alshehri  
Wail Alshahri  
Ziad Jarrah  
Hani Hanjour

Mohamed Atta  
Fayed Ahmed  
Ahmed Alghamdi  
Mohald Alshehri  
Khalid al-Midhar  
Ahmed Alhamzawi  
Ahmed Alnami  
Saeed Alghamdi

### Outside Florida

Satam al-Suqami

Hamza Alghamdi  
Abdulaziz Alomari  
Majed Moqed  
Salem Alhamzi  
Nawaf Alhamzi

Zakariya Essabar  
Said Bahaji  
Mounir El Motassadeq  
Zacarias Moussaoui  
Ramzi Bin al-Shibh  
Lotfi Raissi  
Agus Budiman  
Ahmed Khalil Ibrahim Samir Al-Ani  
Rayed Mohammed Abdullah  
Bandar Alhazmi  
Faisal Al Salmi

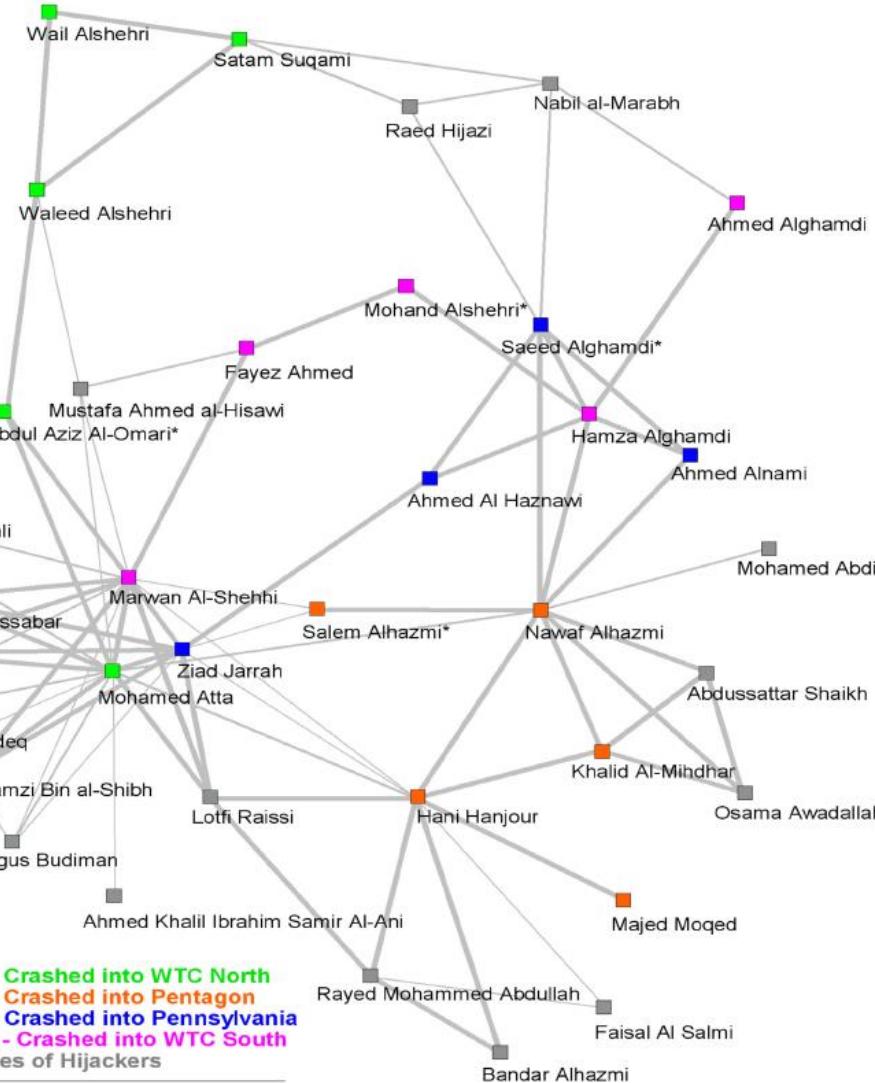
Mohamed Atta  
Ziad Jarrah  
Mohald Alshehri  
Khalid Al-Midhar  
Osama Awadallah  
Majed Moqed

Mamoun Darkazanli  
Mustafa Ahmed al-Hisawi  
Abdul Aziz Al-Omari\*  
Ahmed Alhaznawi  
Ahmed Alnami  
Saeed Alghamdi

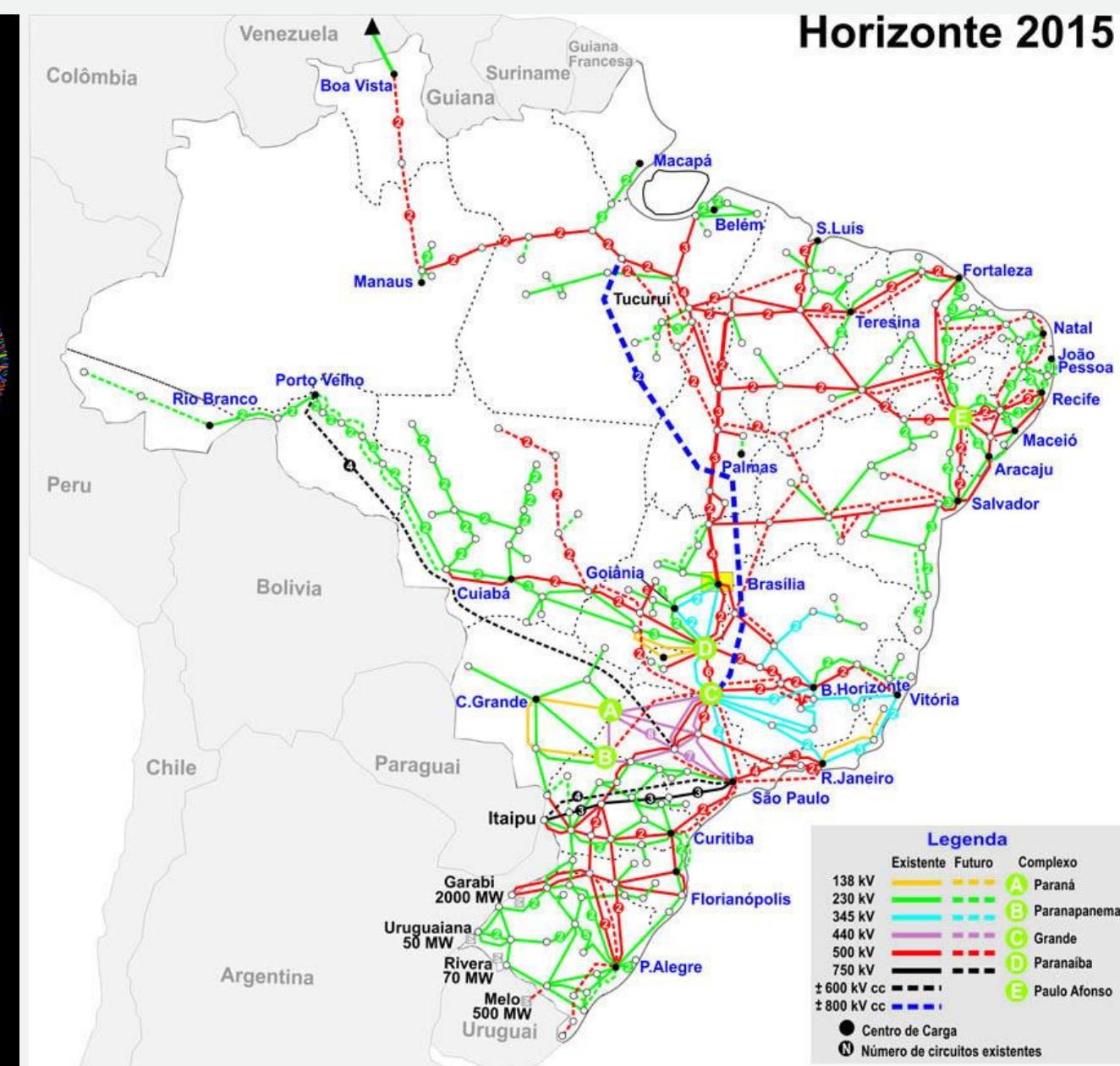
Marwan Al-Shehhi  
Salem Alhazmi\*  
Nawaf Alhazmi  
Abdussattar Shaikh

Copyright ©, Valdis Krebs

Other Associates of Hijackers

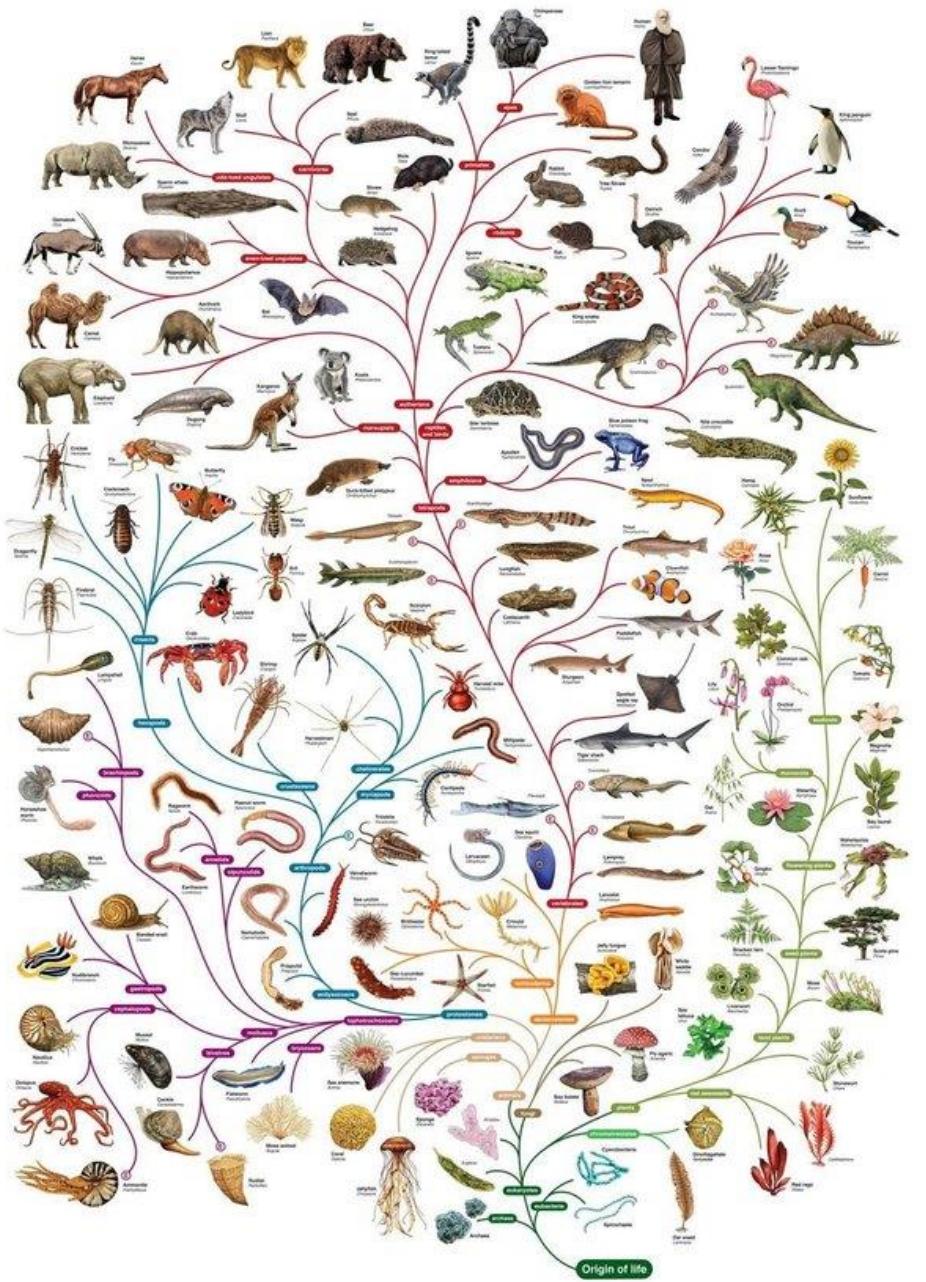


Associações criminosas

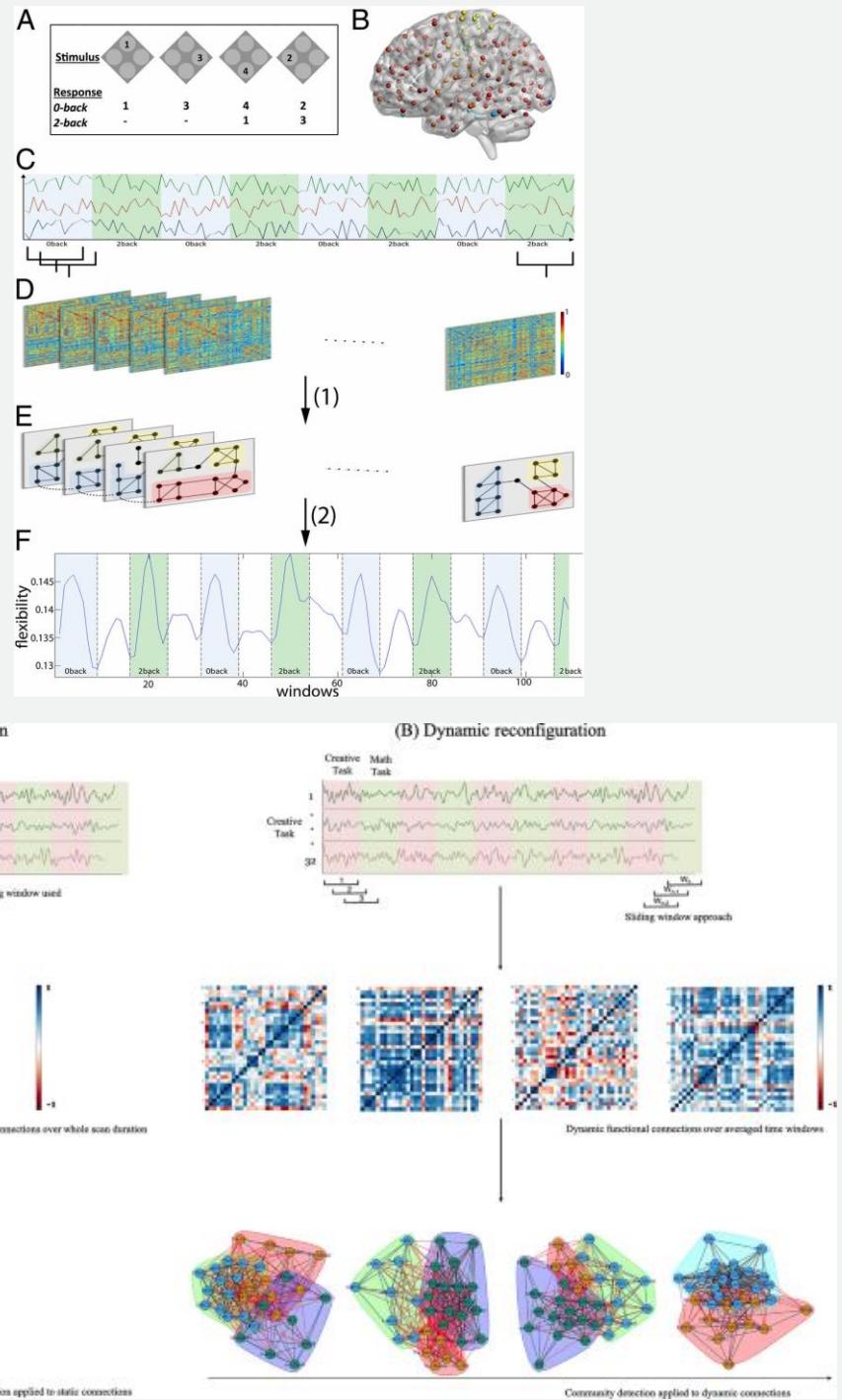


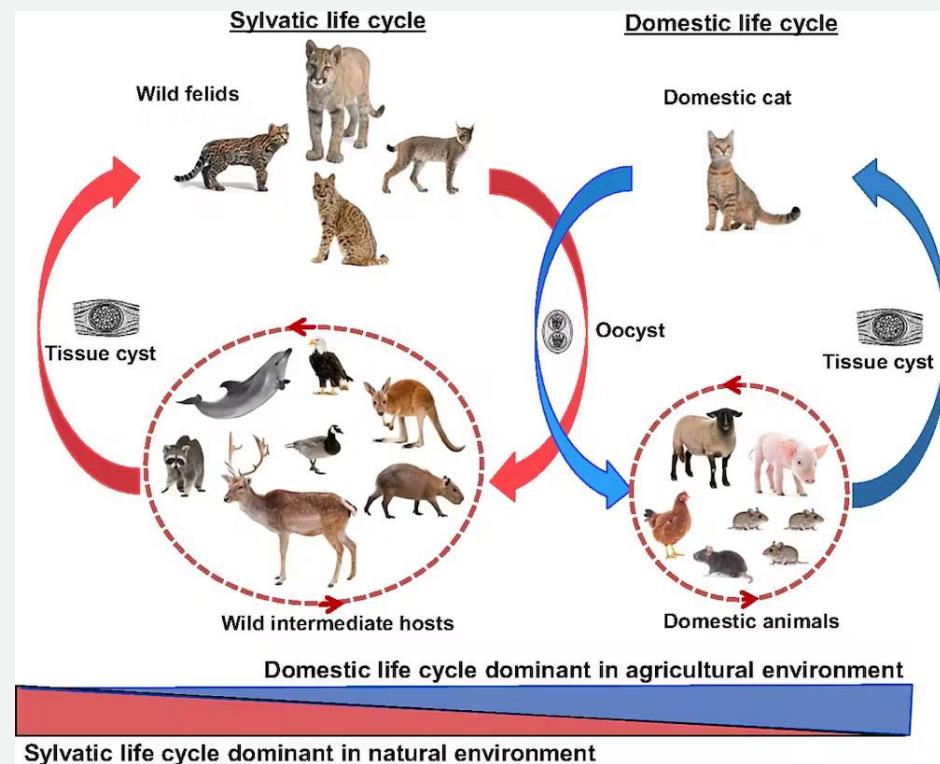
Recursos

# Filogenia

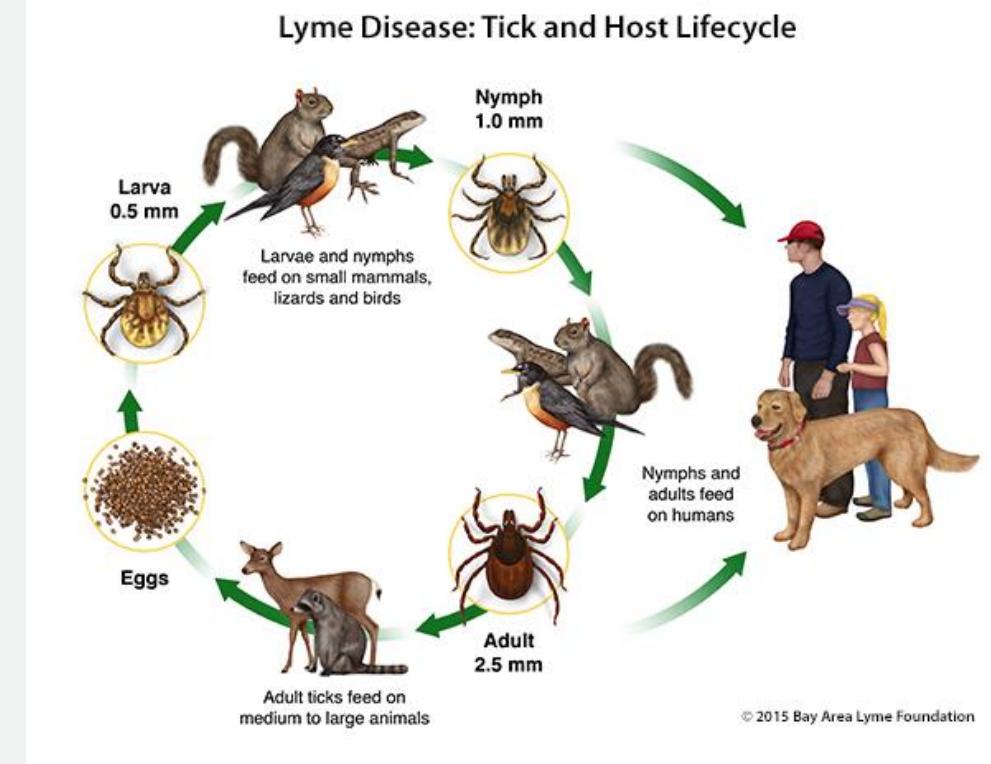


Processos  
dinâmicos





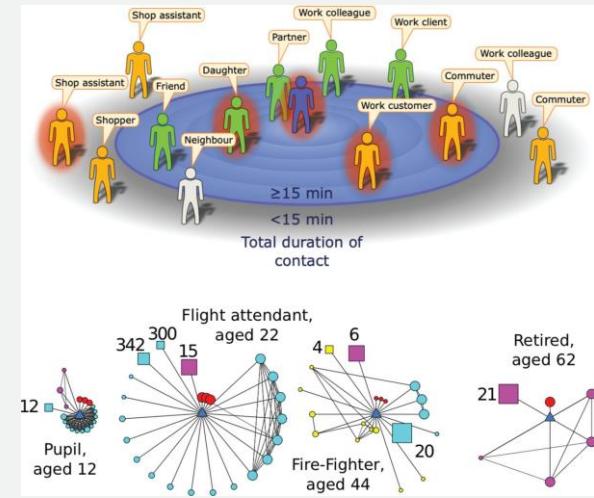
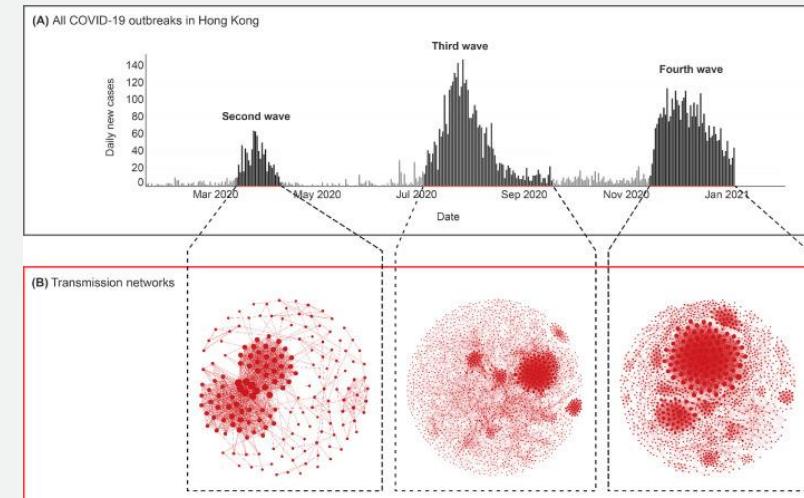
## Toxoplasmosis

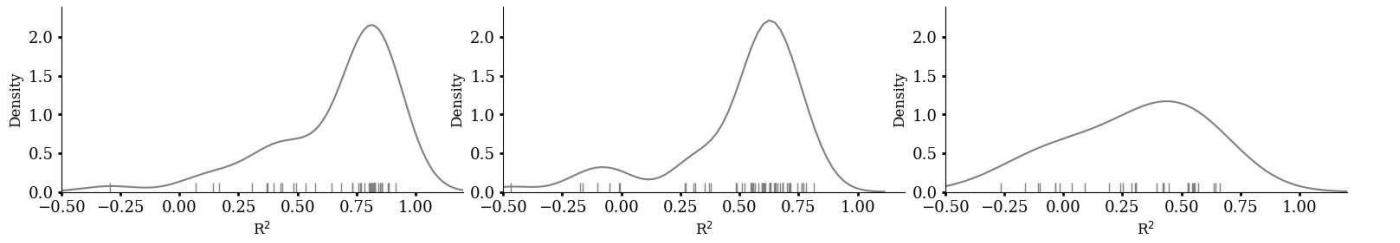
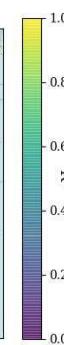
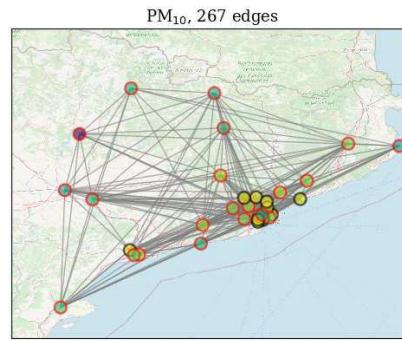
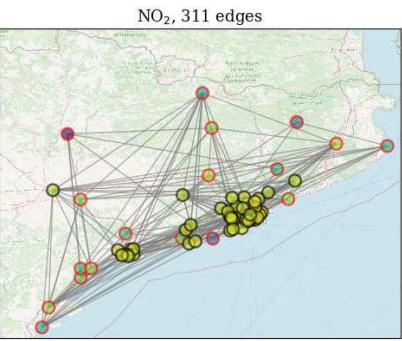
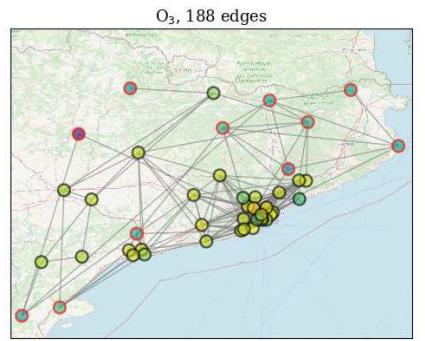


## Doença de Lyme (borrelia)

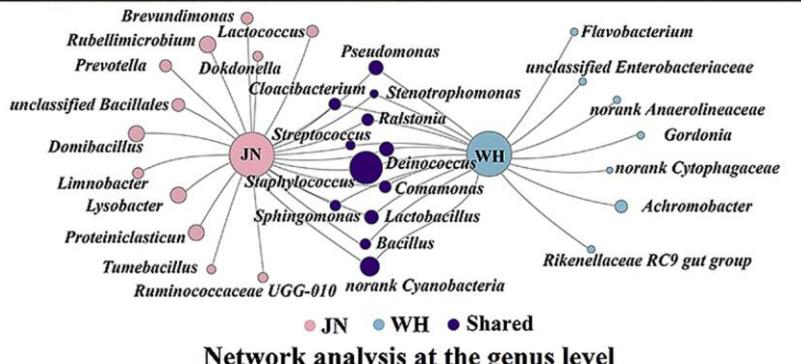
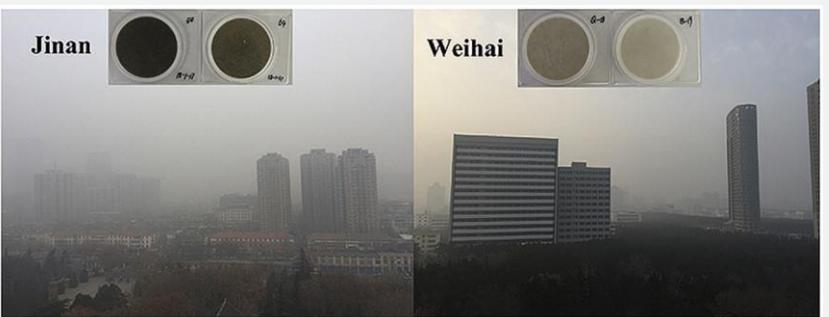


## Covid 19

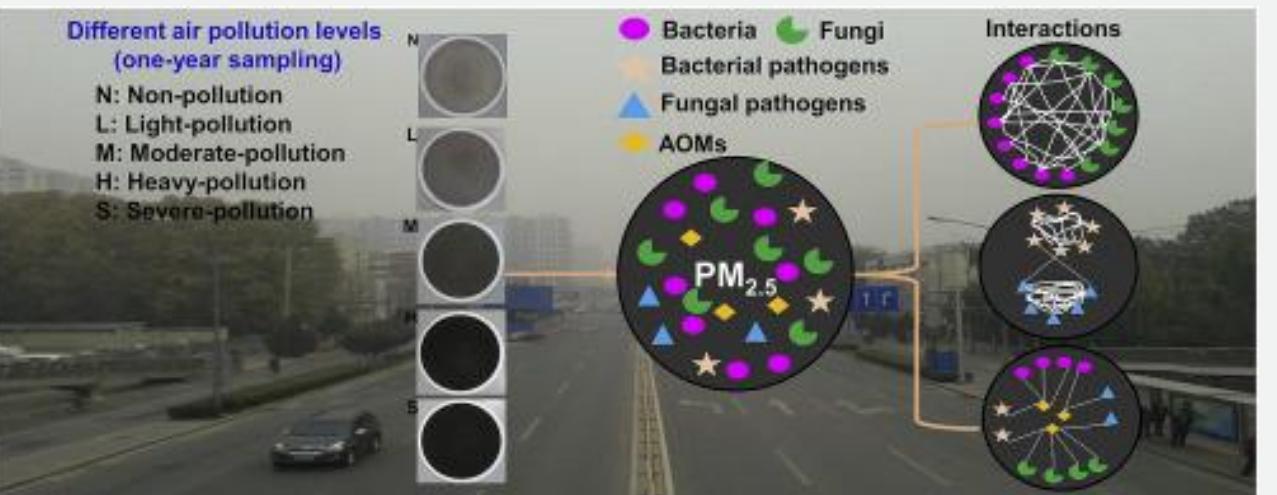




## Dinâmica de poluentes



## Qualidade do ar

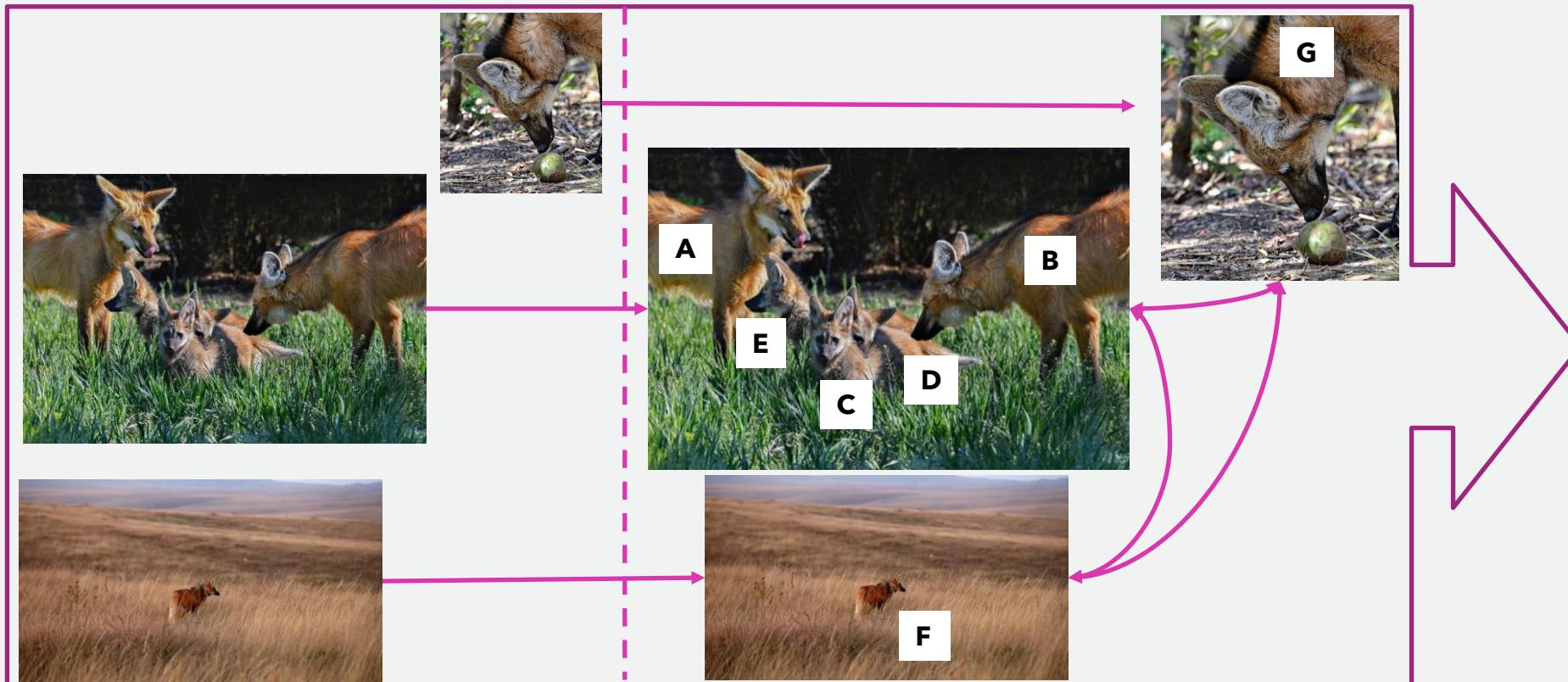


## Efeitos da poluição

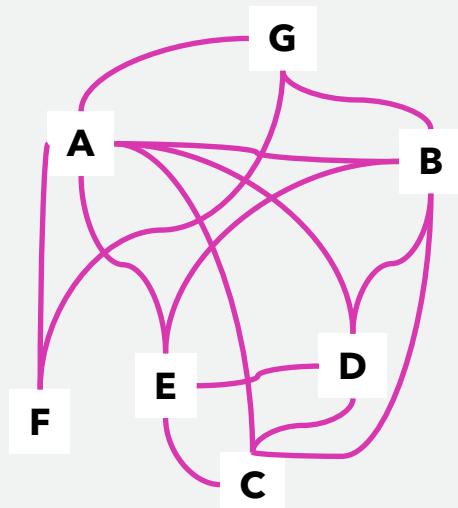
# Quais elementos são comuns a todos os casos?



Realidade	Exemplo	Abstração
Objetos	coisas, ideias, medidas, sentimentos, posições	Vértice (nó)
Vínculos	enlaces, conexões, influências, caminhos, associações	Aresta (linha)
Fenômenos	estrutura, disposição, arranjo, organização, configuração	Grafo (rede)



$$G = \{V, A\}$$



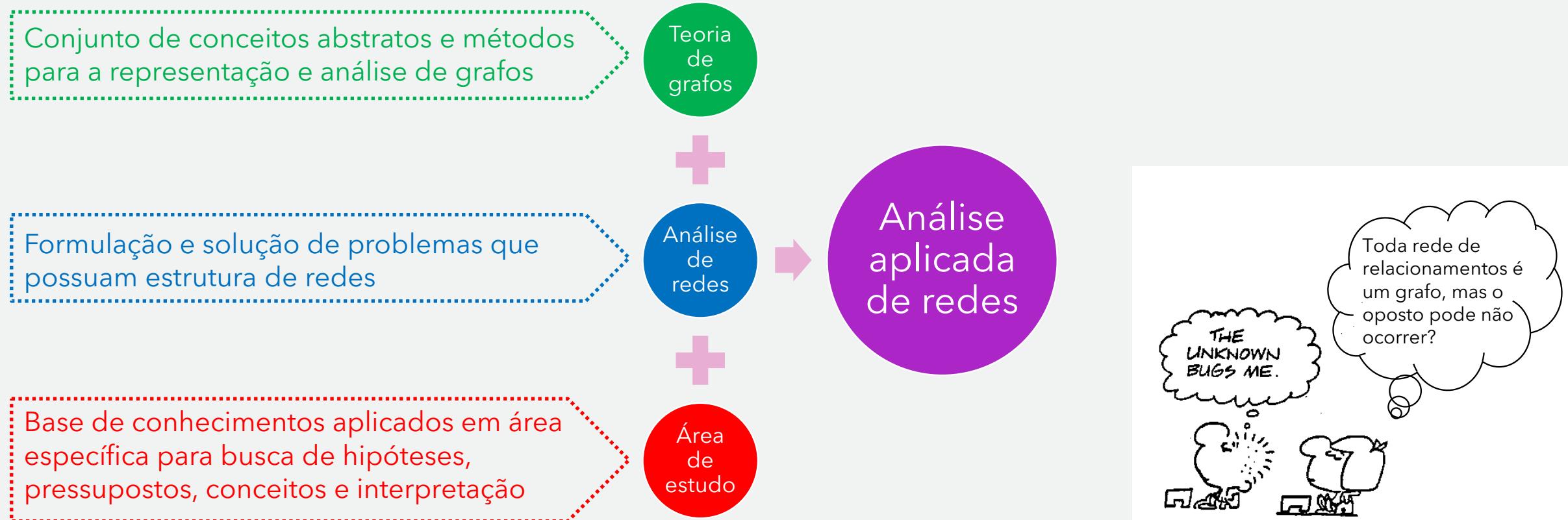
Qual é a diferença entre um "grafo" e uma "rede"?

Não há qualquer diferença. São apenas palavras diferentes para a mesma coisa, embora "grafo" tenda a ser mais comum em matemática e outras áreas formais e "rede" mais comum em áreas mais aplicadas.

# Como interpretar?

- A interpretação depende da área

Área	Pontos, círculos	Linhas
Matemática	Vértices	Arestas, arcos
Ciências da computação	Nós	Conexões, ligações
Física	Sites, locais	Vínculos
Sociologia	Atores, agentes	Laços, relações

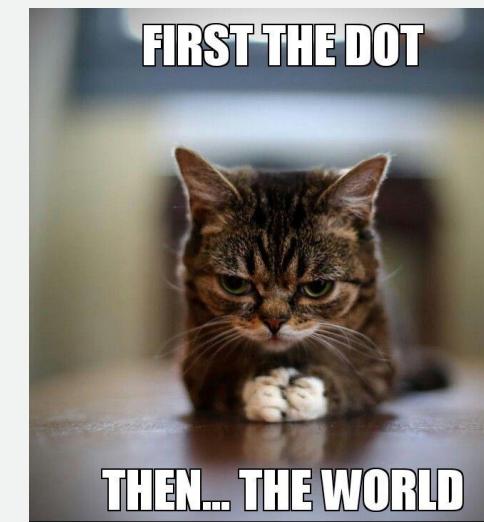


# Quais propriedades a avaliar em um grafo aplicado?

- Conjunto completo
- Componentes (vértices, arestas)
- Simplificações
- Representação (*layout*)
- Comportamento relacional
- Comportamento dinâmico
- ...



Resultados, discussões e conclusões  
conforme área de aplicação

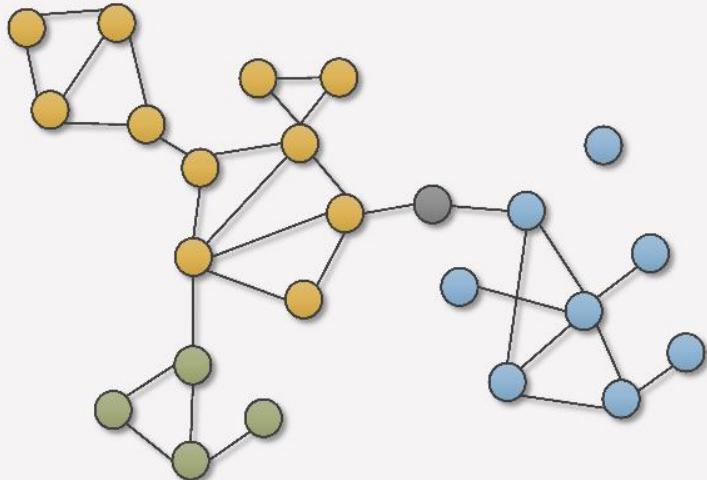


## **Topologia**

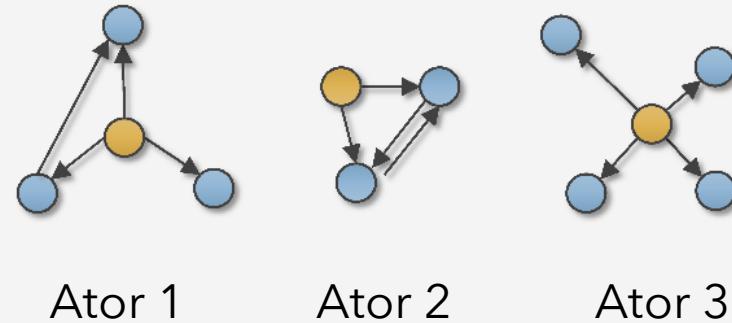
- É a forma como os nós e as arestas estão dispostos numa rede.
- As propriedades topológicas podem aplicar-se à rede como um todo ou a nós e arestas individuais.

Foco

Rede total  
(completa)



Atores específicos  
(egocêntrica)

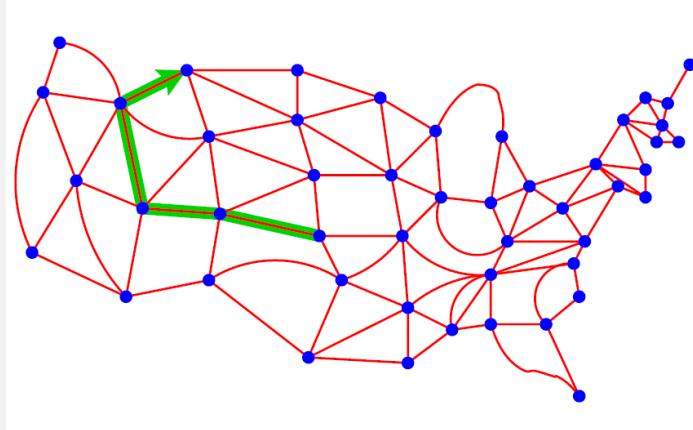


Não dirigido	Dirigido (dígrafo)	Bipartido	Valorado (ponderado)	Completo
Tipos				
Conectado	Desconectado	Cíclico	Acíclico	Pseudo
Simples	Multigrafo	Euleriano	Hamiltoniano	Nulo
Espenso	Denso	Trivial	Finito	Infinito

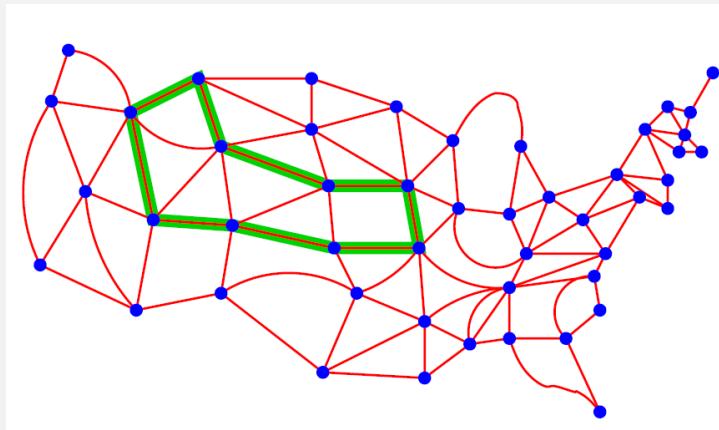
# Viagem ou rota

- Uma viagem (*trip*) não pode usar a mesma aresta mais do que uma vez, mas pode passar pelo mesmo vértice mais do que uma vez.

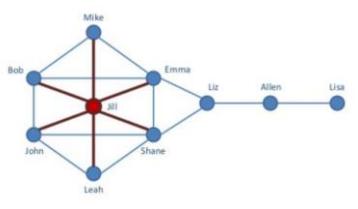
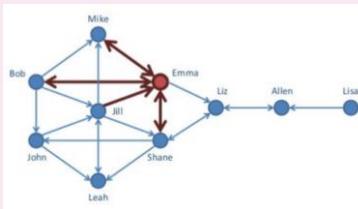
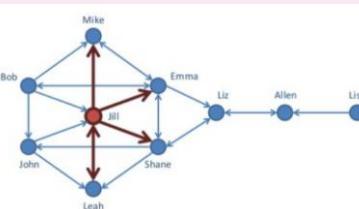
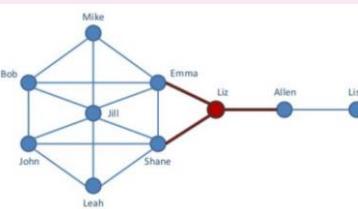
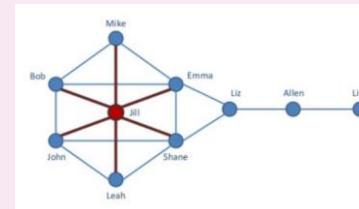
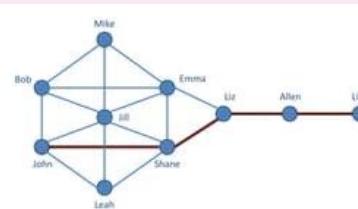
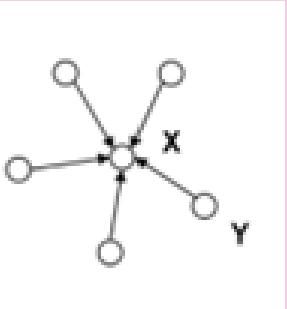
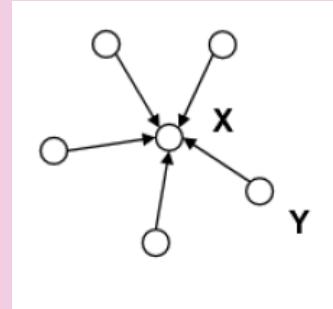
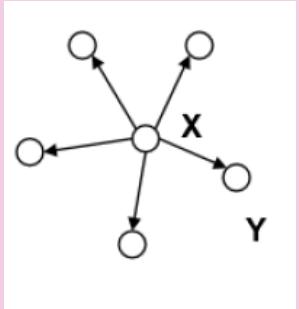
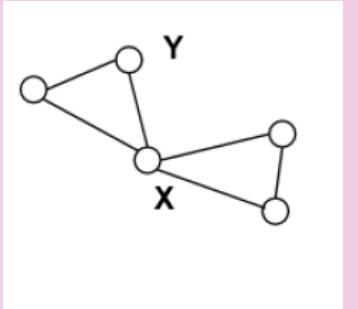
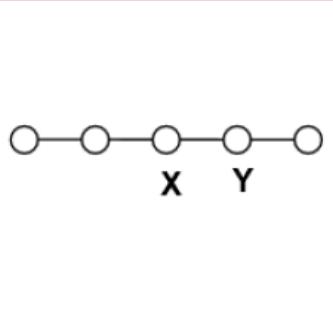
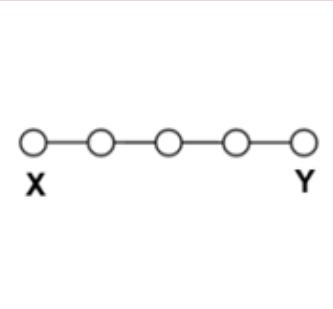
- Caminho (*path*)



- Circuito (*circuit*)



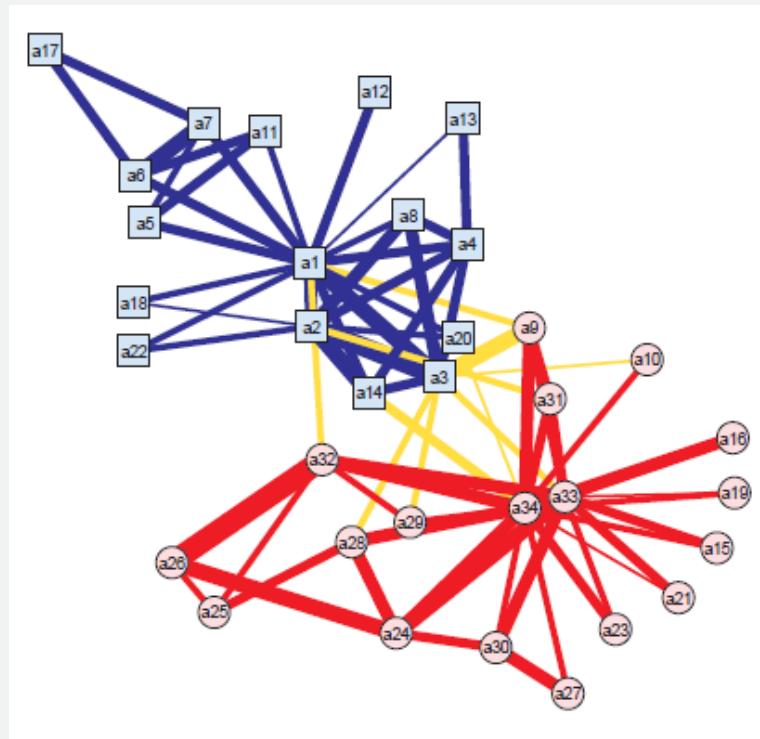
# Métricas da rede

	<b>Grau (degree)</b>		<b>Intermediação (betweenness)</b>	<b>Proximidade (closeness)</b>	<b>Diâmetro</b>
<b>Grau (degree)</b>	Grau de entrada (indegree)	Grau de saída (outdegree)			
Quantidade absoluta de vínculos que um nó estabelece (rede não direcionada)	Conexões direcionadas ao nó	Conexões originadas no nó	Considera a posição de um nó entre o caminho que conecta todos os outros nós	Observa a menor distância entre todos os nós	Caminho mais longo
					
					

<b>Prestígio ou notoriedade ou autoridade (prestige)</b>				<b>Centralidade por autovetor (eigenvector centrality)</b>
Grau	Proximidade	Rank ou ordem		
		PageRank	HITS (hyperlink induced topic search)	
Conexões direcionadas ao nó (somente em redes direcionadas). Um nó de elevado prestígio é aquele destinatário generalizado de ligações internas.	O prestígio de proximidade generaliza o prestígio ao considerar tanto os nós ligados diretamente e indiretamente a um determinado nó, dentro de sua área de influência, avaliando as menores distâncias de conexão.	O prestígio do nó depende do prestígio dos nós a que está ligado. Pondera conexões indo para o nó e originadas nele para outros nós. A importância de um nó não é determinada apenas pelo número de conexões do nó, mas também a soma de todas as pontuações do PageRank das páginas que apontam para ele.	Tal como o PageRank, explora a estrutura de conexões para classificar os nós, mas produz um conjunto maior de busca composto também pela classificação de hub. A intenção é que um bom hub aponta para boas autoridades e uma boa autoridade é apontada por um bom hub.	A pontuação da centralidade é calculada pelo autovetor da matriz de adjacências, tendo como base o número de conexões do nó com seus vizinhos.

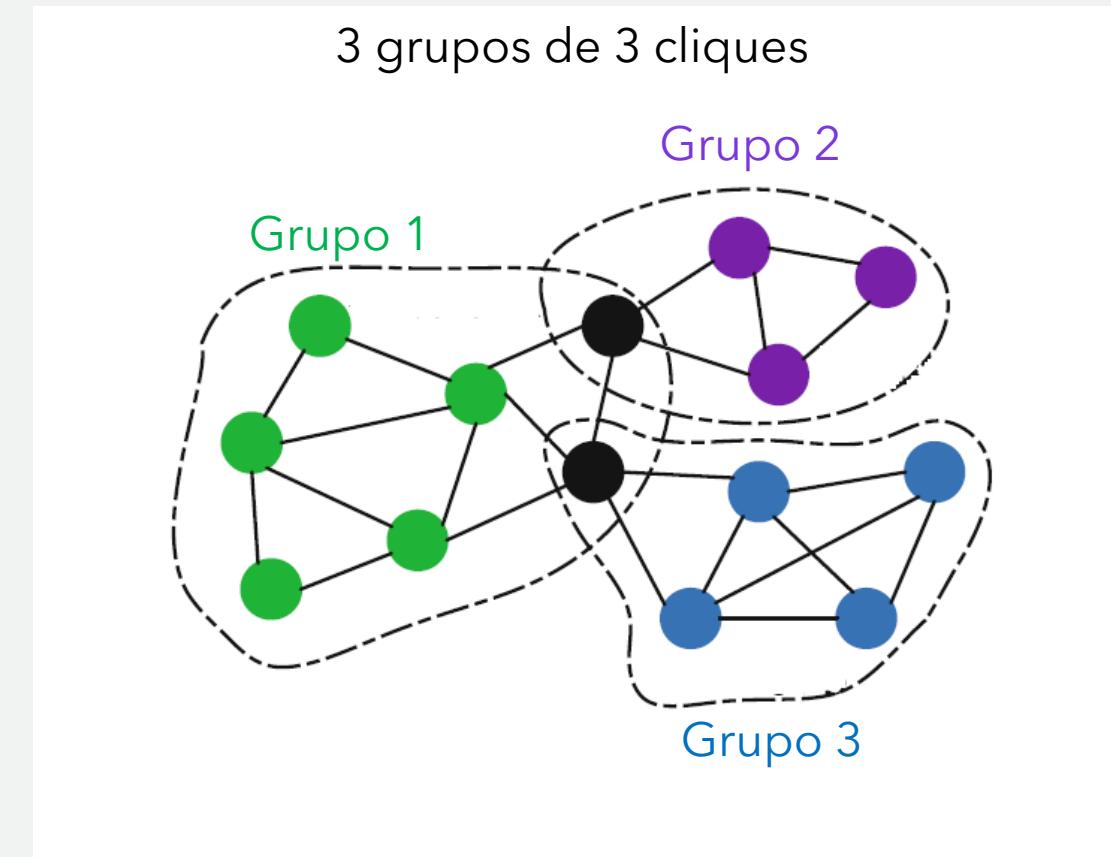
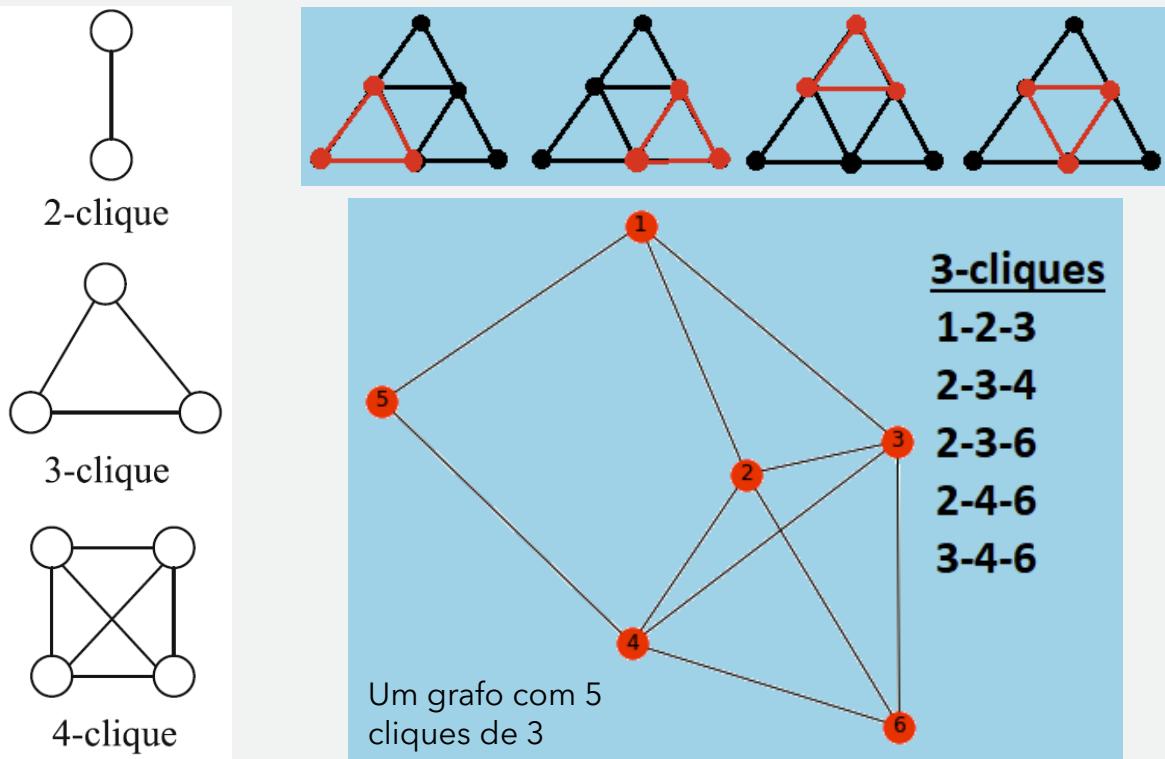
# Agrupamento (clustering)

- Mede a fração de triplos (grupos de três vértices) ligados que se aproximam para formar triângulos, também denominado de coeficiente de agrupamento ou transitividade.



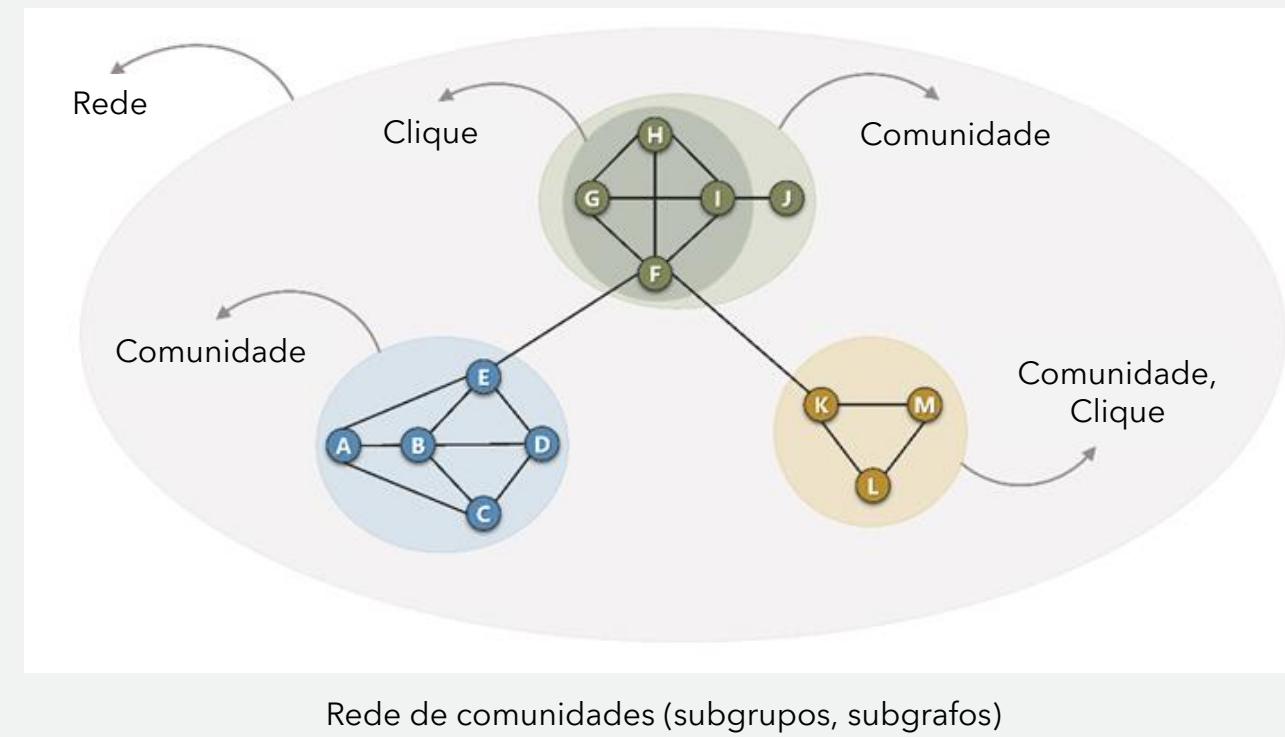
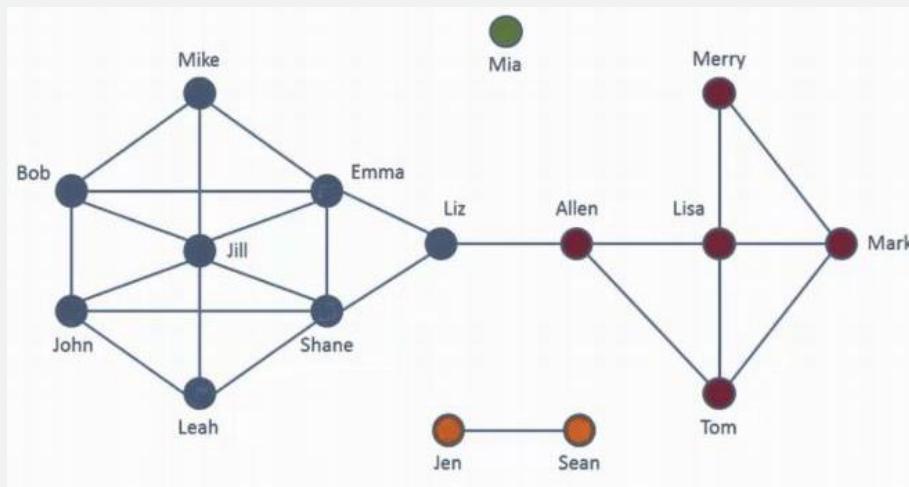
# Clique

- É um subconjunto de vértices de um grafo não direcionado, de tal forma que todos os dois vértices distintos da clique são adjacentes, ou seja, o seu subgrafo induzido é completo.



# Modularidade

- É uma medida da densidade da estrutura de vizinhança da rede
  - Avalia a relação dos nós com seus vizinhos, considerando quanto um nó tende a aparecer em um determinado grupo (quanto está conectado ao grupo)
- Calcula a divisão da rede em módulos (grupos, *clusters*, comunidades) através da mensuração da divisão da rede
- Valores de modularidade
  - Altos: conexões densas
  - Baixos: conexões esparsas



Rede de comunidades (subgrupos, subgrafos)

# NETWORK OF THRONES

ANDREW BEVERIDGE AND JIE SHAN

Math Horizons, Vol. 23, No. 4 (April 2016), pp. 18-22

<https://doi.org/10.4169/mathhorizons.23.4.18>

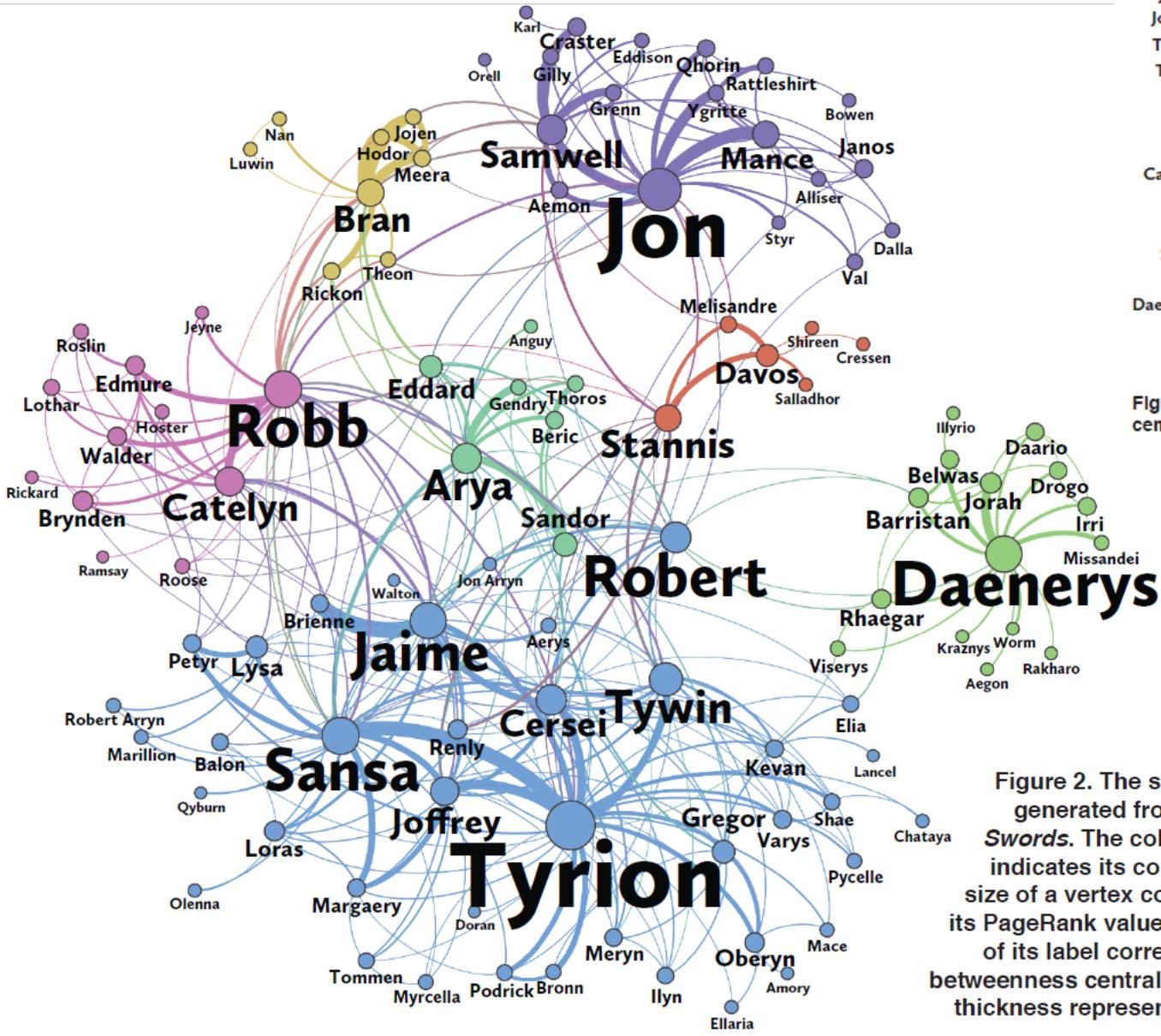


Figure 2. The social network generated from *A Storm of Swords*. The color of a vertex indicates its community. The size of a vertex corresponds to its PageRank value, and the size of its label corresponds to its betweenness centrality. An edge's thickness represents its weight.

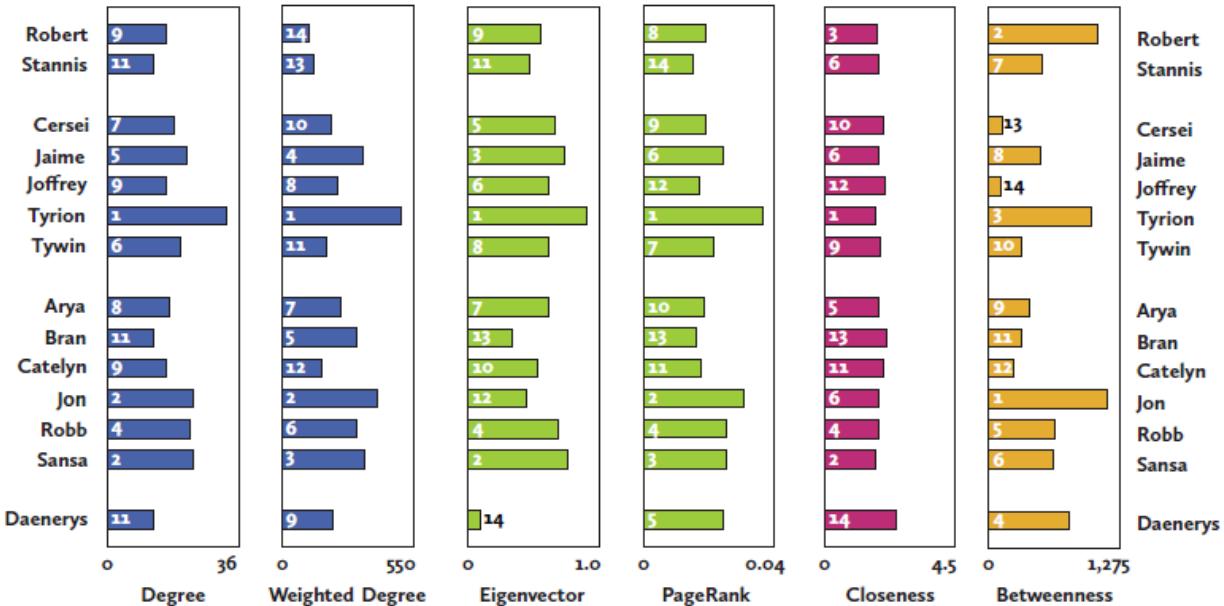
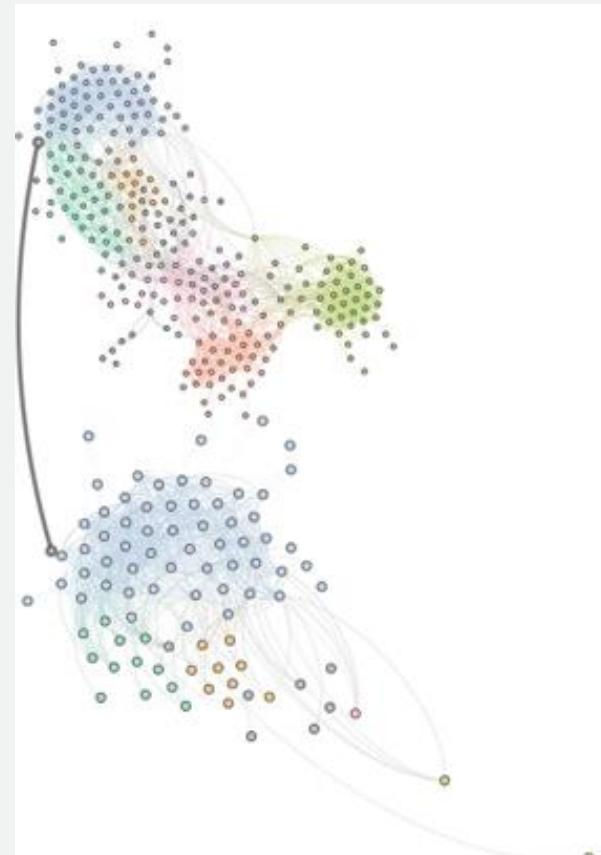
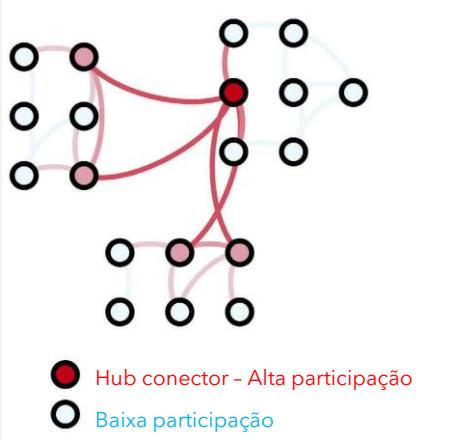
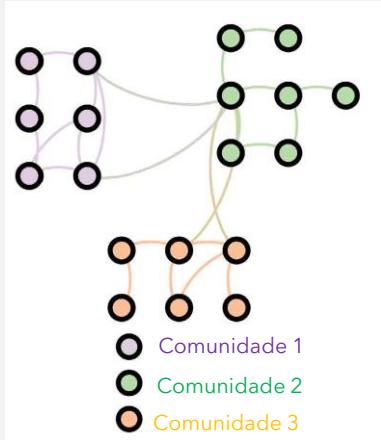


Figure 3. Centrality measures for the network. Larger values correspond to greater importance, except for closeness centrality, where smaller values are better. Numbers in the bars give the rankings of these characters.

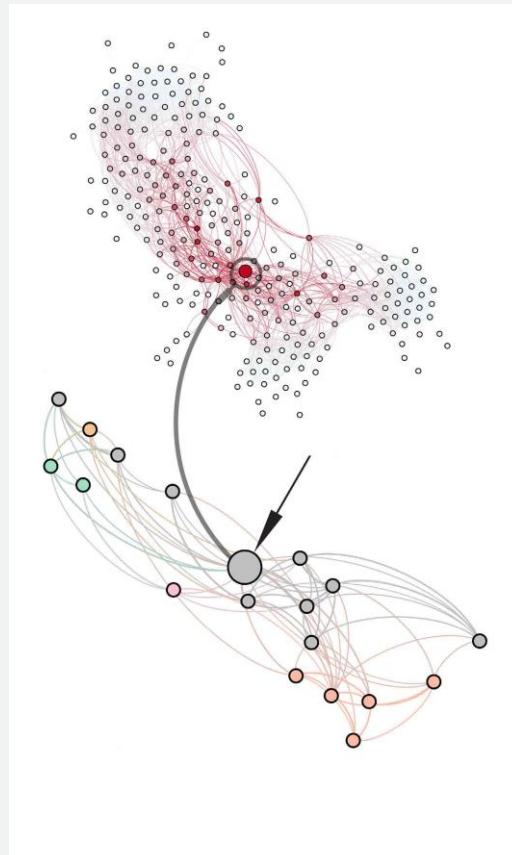
Todos juntos!!!



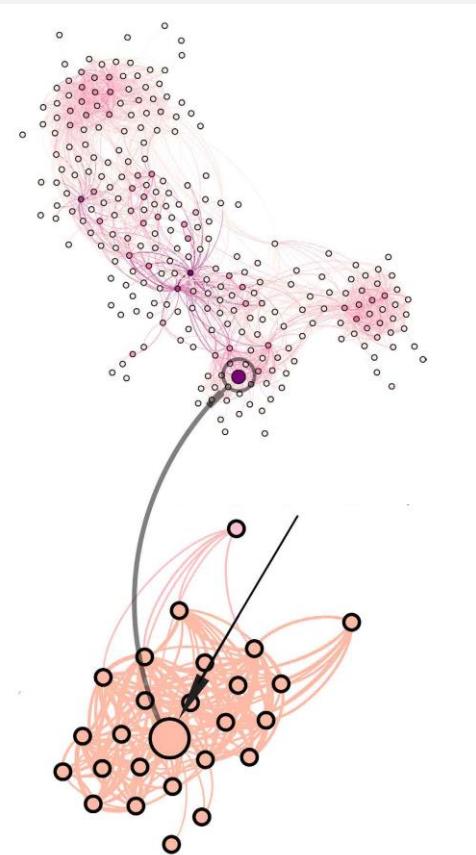
# Hubs e conectores



Cada comunidade é internamente densa e externamente esparsa



Hub conector conectado a diferentes comunidades

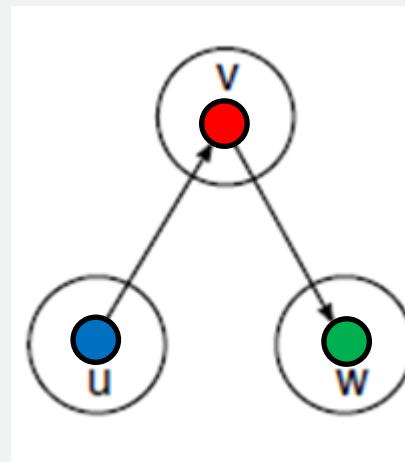
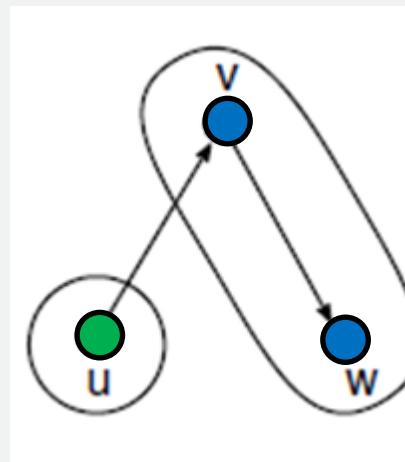
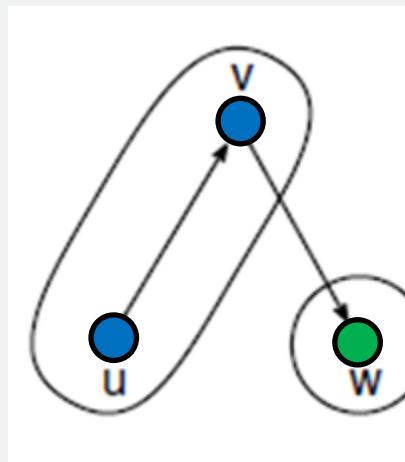
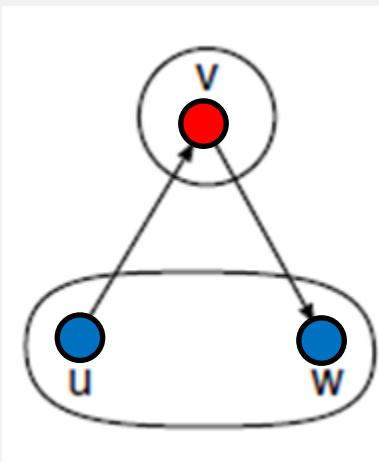
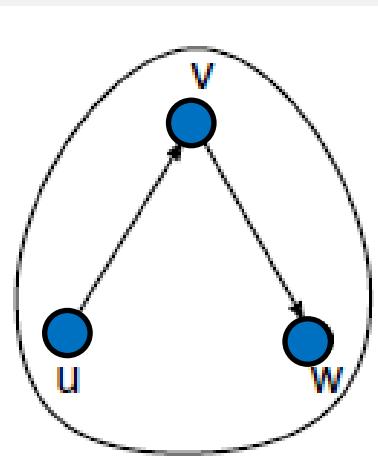


Hub local, altamente conectado a sua comunidades local

# Corretores (*brokers*) e pontes (*bridges*)

\*Comportamento de mediação do nó v em relação às tríades

Entre membros de um mesmo grupo		Entre membros de grupos diferentes		
Coordenador	Mediador externo ou corretor itinerante	Representante ou delegado	Guardião ou porteiro	Articulador
Participa do grupo	Não participa do grupo	Regula o fluxo dentro de seu grupo para outros	Regula o fluxo externo para o seu grupo	Mediador entre membros de grupos diferentes, não participa de nenhum deles

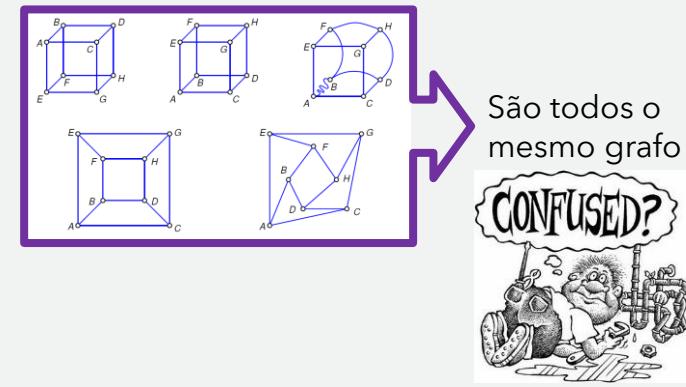


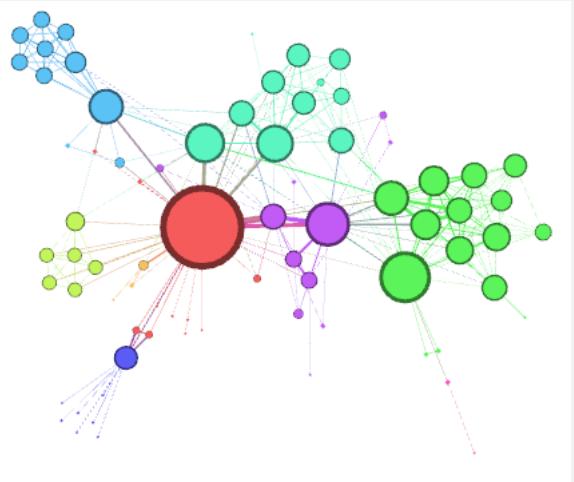
# Layout (disposição, representação)

- O objetivo é ressaltar diferentes aspectos da topologia da rede



Layout	Tipo	Número de nós do grafo	Usa peso das arestas
Force Atlas	Força	$\leq 10.000$	Sim
Force Atlas 2	Força	$\leq 1.000.000$	Sim
Fruchterman-Reingold	Força	$\leq 10.000$	Não
Yifan Hu (multinível)	Força e multinível	100 a 100.000	Não
OpenOrd	Força e anelamento simulado	100 a 1.000.000	Sim
Circular	Circular	$\leq 1.000.000$	
Radial	Circular	$\leq 1.000.000$	
Geo	Geográfico	$\leq 1.000.000$	
Kamada-Kawai	Força	$\leq 1.000.000$	Sim

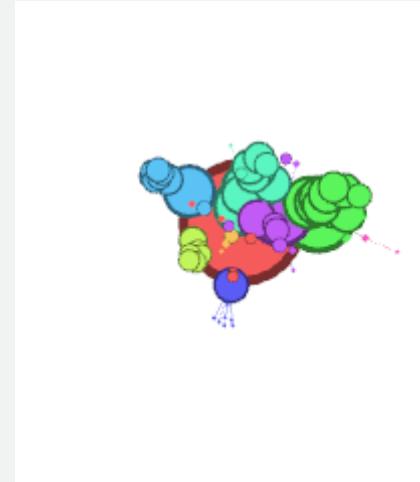




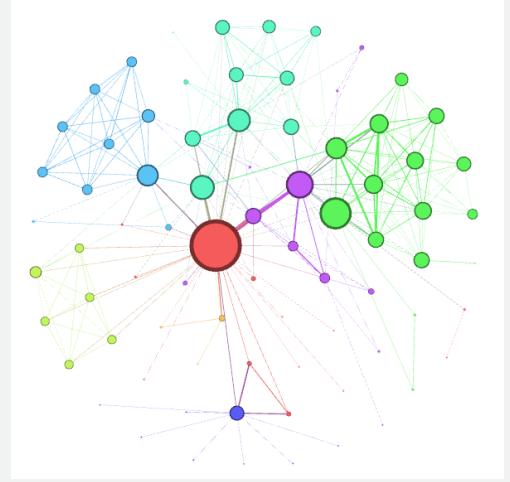
Original  
(Les Miserables - Gephi)



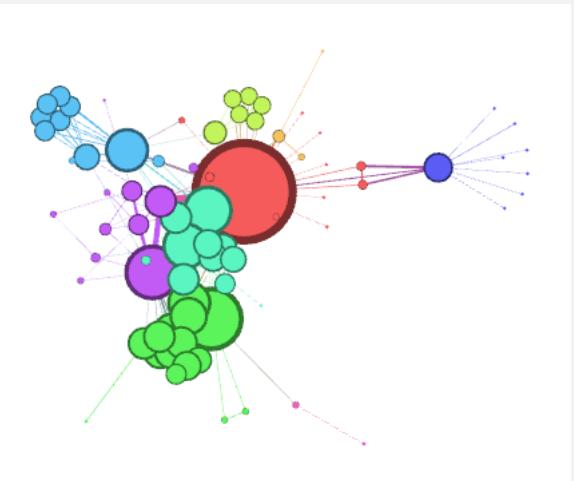
Force Atlas



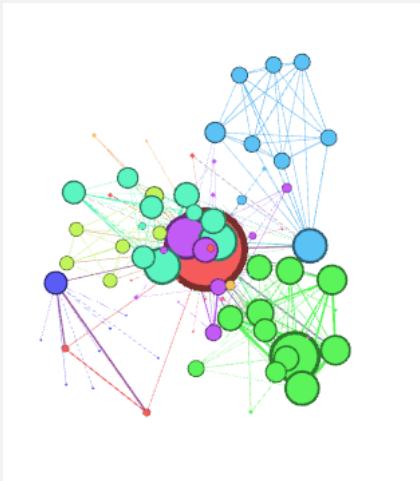
Force Atlas 2



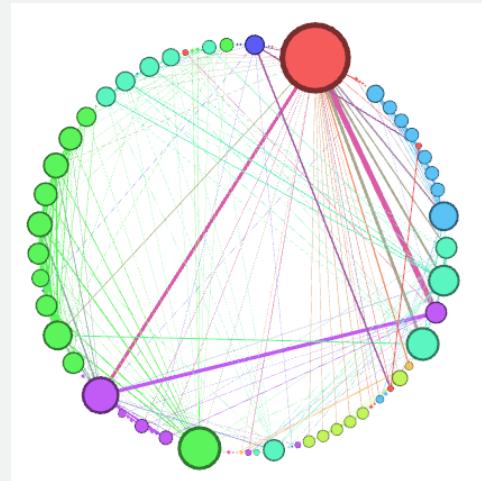
Fruchterman-Reingold



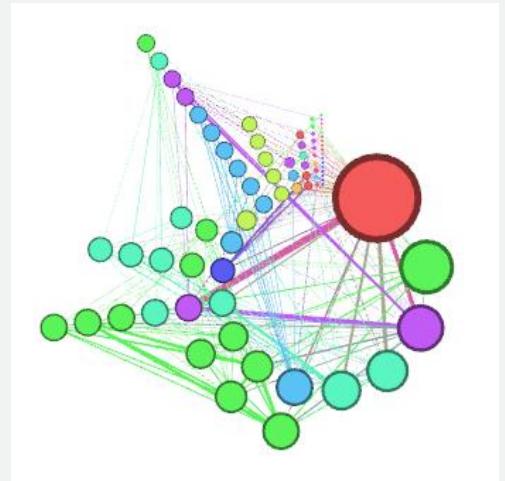
Yifan Hu



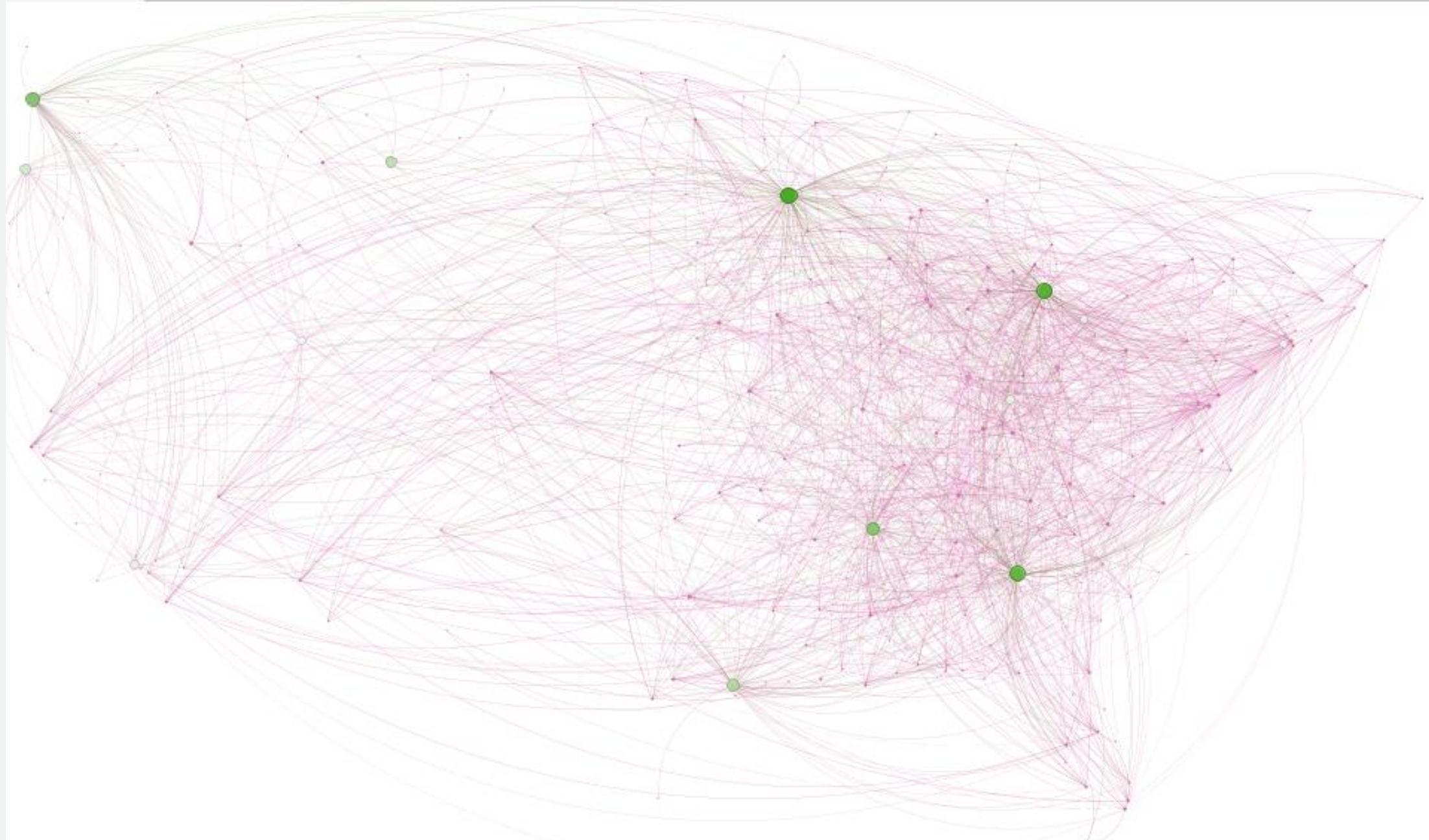
OpenOrd



Circular



Radial



Geo  
(US Airports - Gephi)

Próximos passos...



Next...



Next, it's your turn.

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