

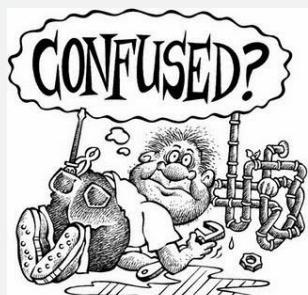
A complex network graph with numerous nodes represented by small white circles and edges represented by thin blue lines, set against a dark blue background.

Grafos aplicados

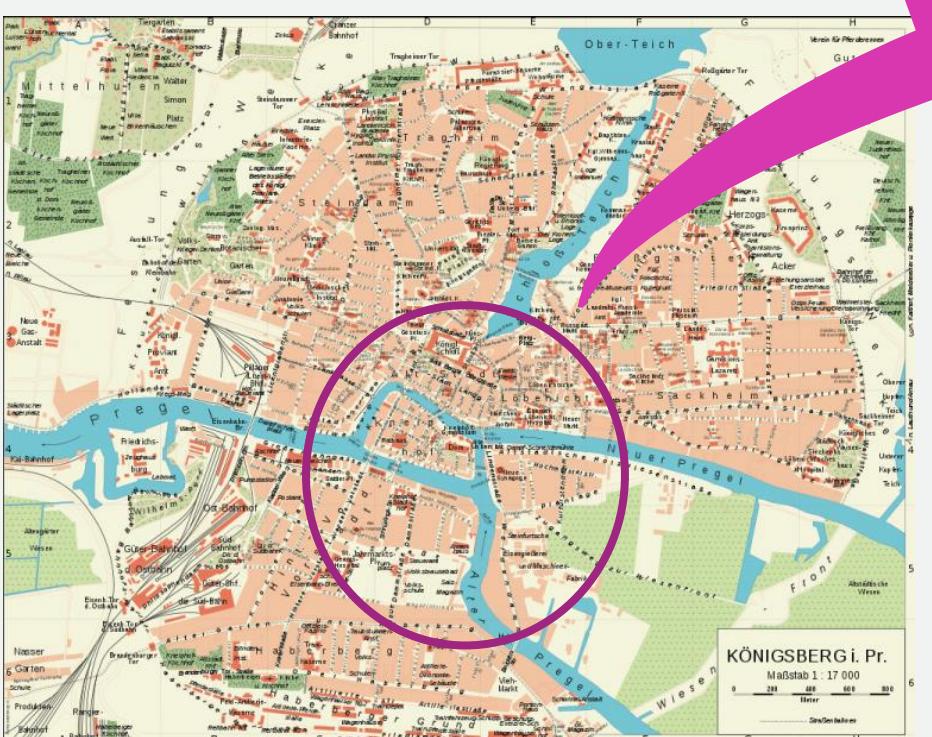


O que é um grafo?

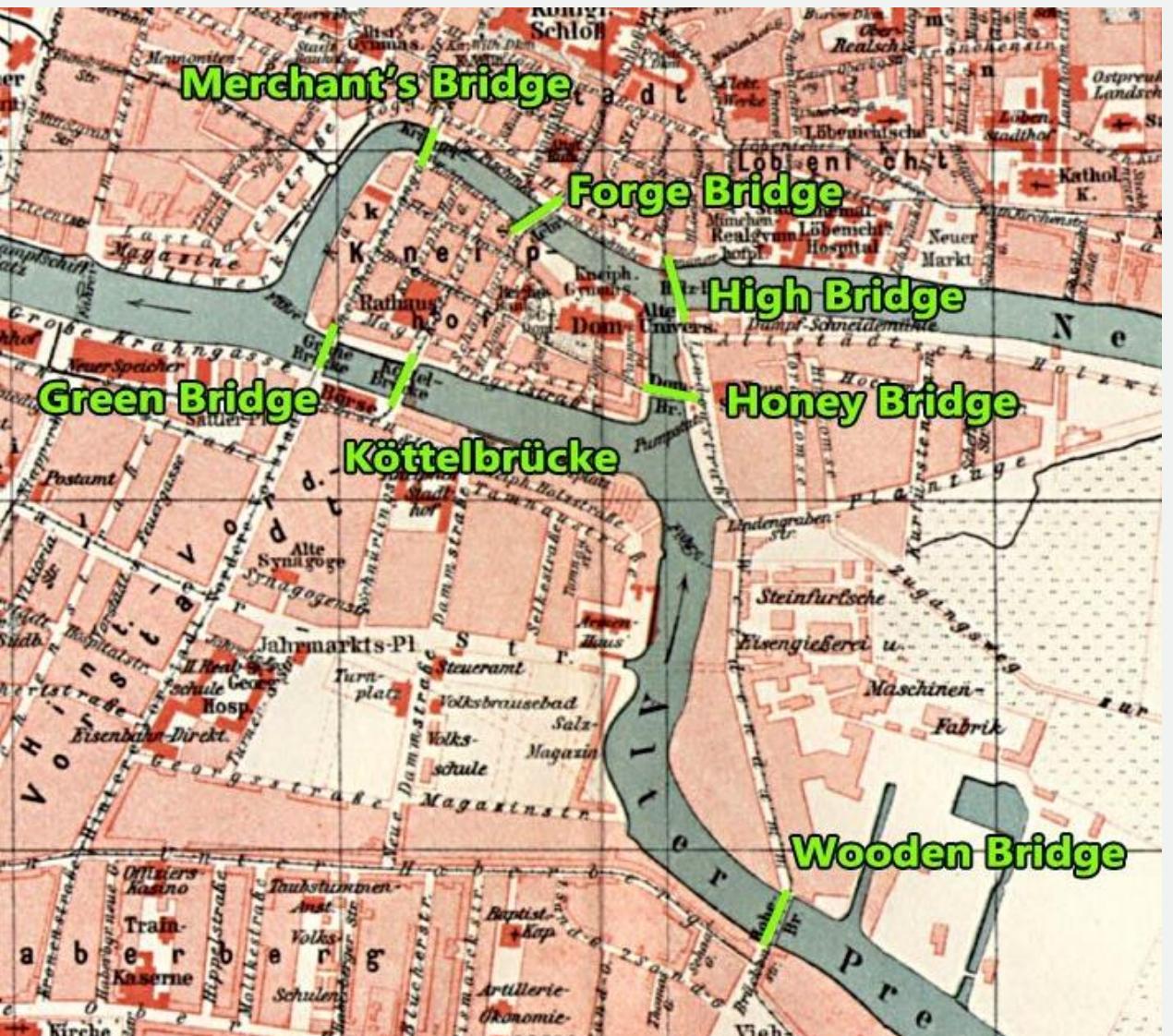
- Um grafo é uma abstração matemática com objetivo de representar um arranjo estrutural, no qual objetos podem estar relacionados sob certo aspecto.
- A teoria dos grafos é uma área da matemática discreta
- É representado como um par $G=(V, A)$ onde
 - V é um conjunto de vértices
 - A é um conjunto de arestas
- Geralmente é representada por um pictograma que expõe esta estrutura



Origens



Königsberg (1902)

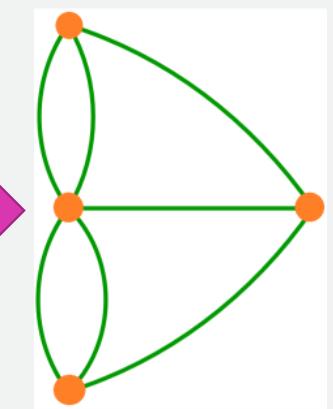
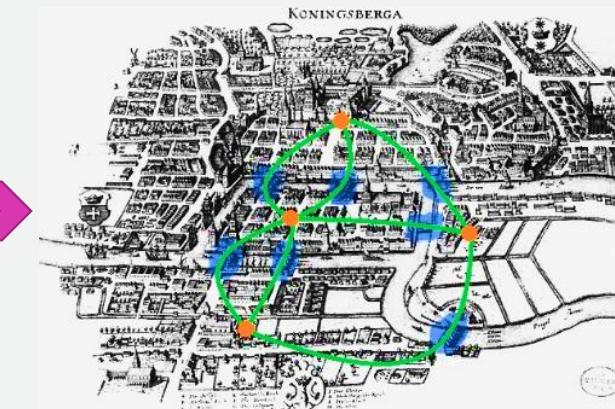
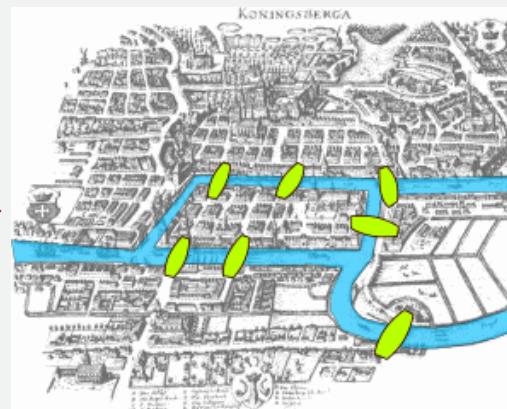
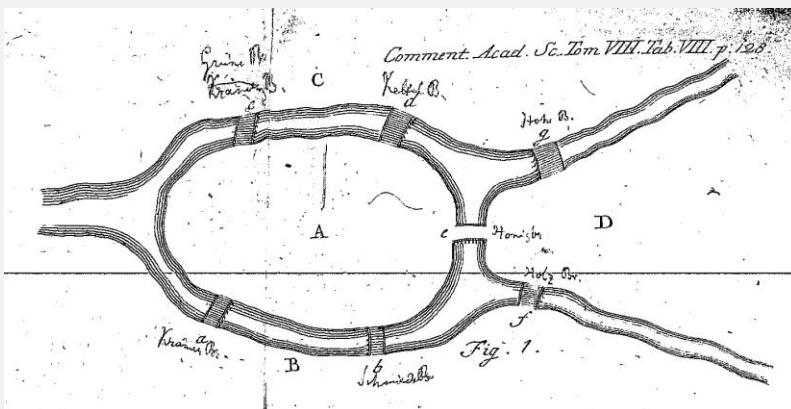


Problema:

É possível traçar um caminho através de Königsberg de modo a atravessar cada uma das sete pontes apenas uma vez e regressar à origem?

Origens

- As sete pontes de Königsberg (Kalininrado)
 - Leonhard Euler (1736)

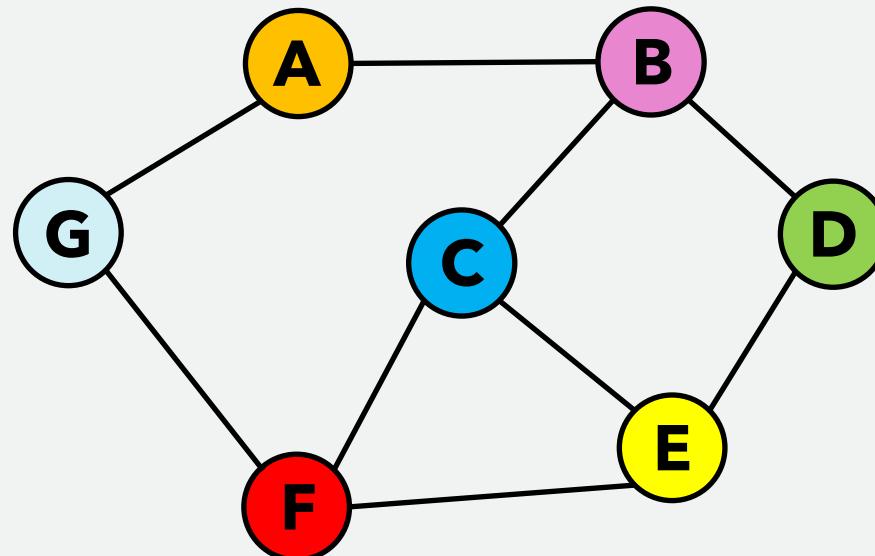


- O traçado é possível?

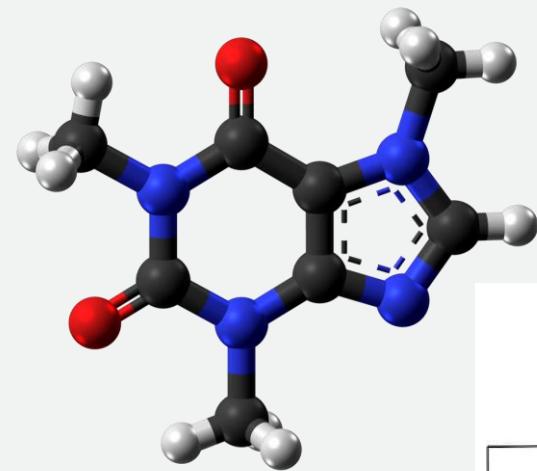


Representação gráfica (pictograma)

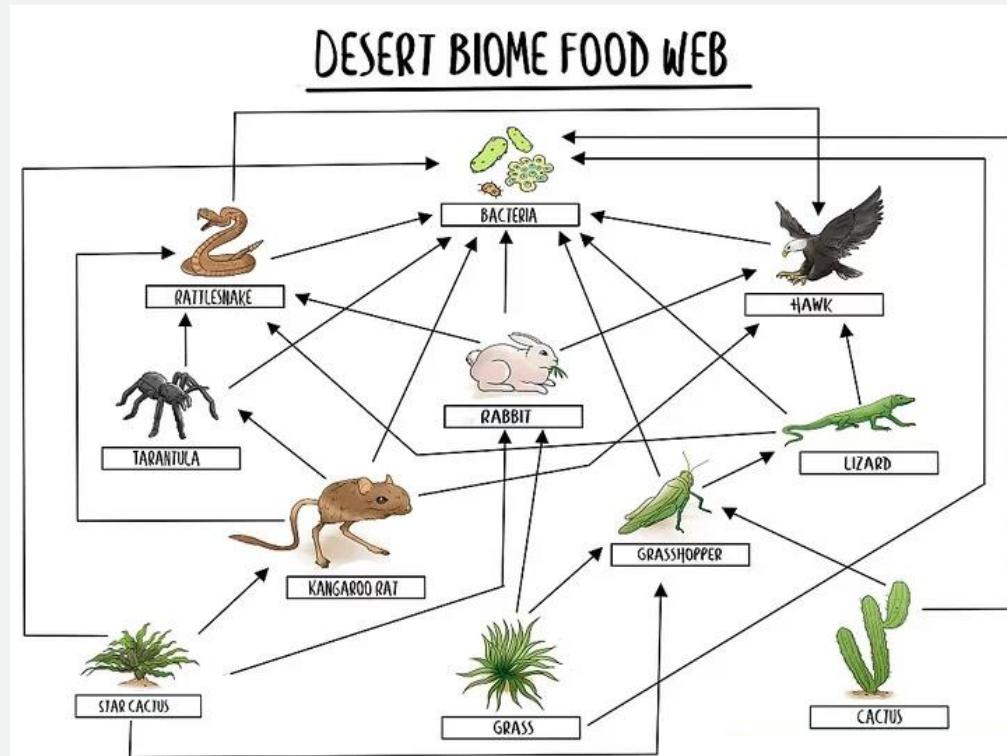
- Grafo $G = \{V, A\}$
- Vértices $\rightarrow V=\{A, B, C, D, E, F, G\}$
- Areias $\rightarrow A=\{(A,B),(B,C),(B,D),(D,E),(C,E),(C,F),(E,F),(F,G),(G,A)\}$



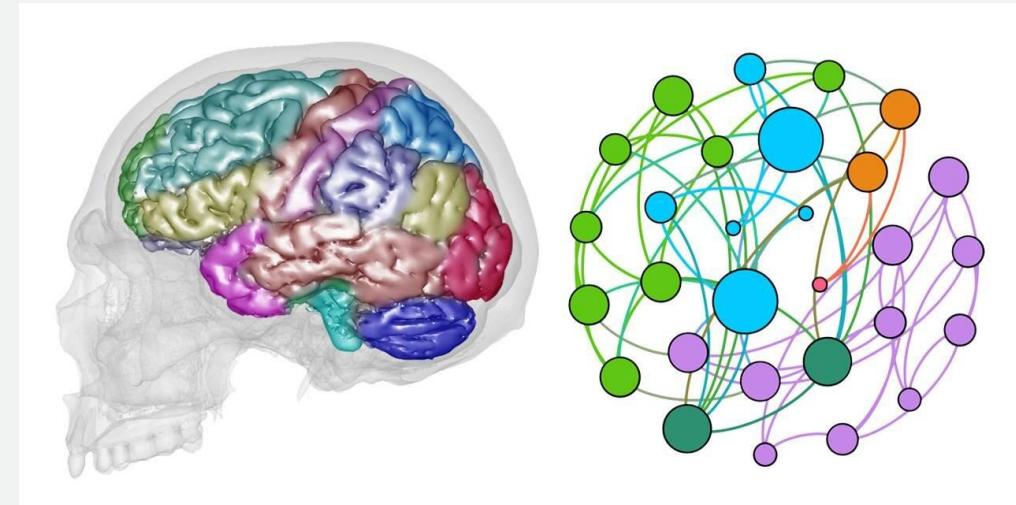
O quê os grafos podem representar?



Moléculas

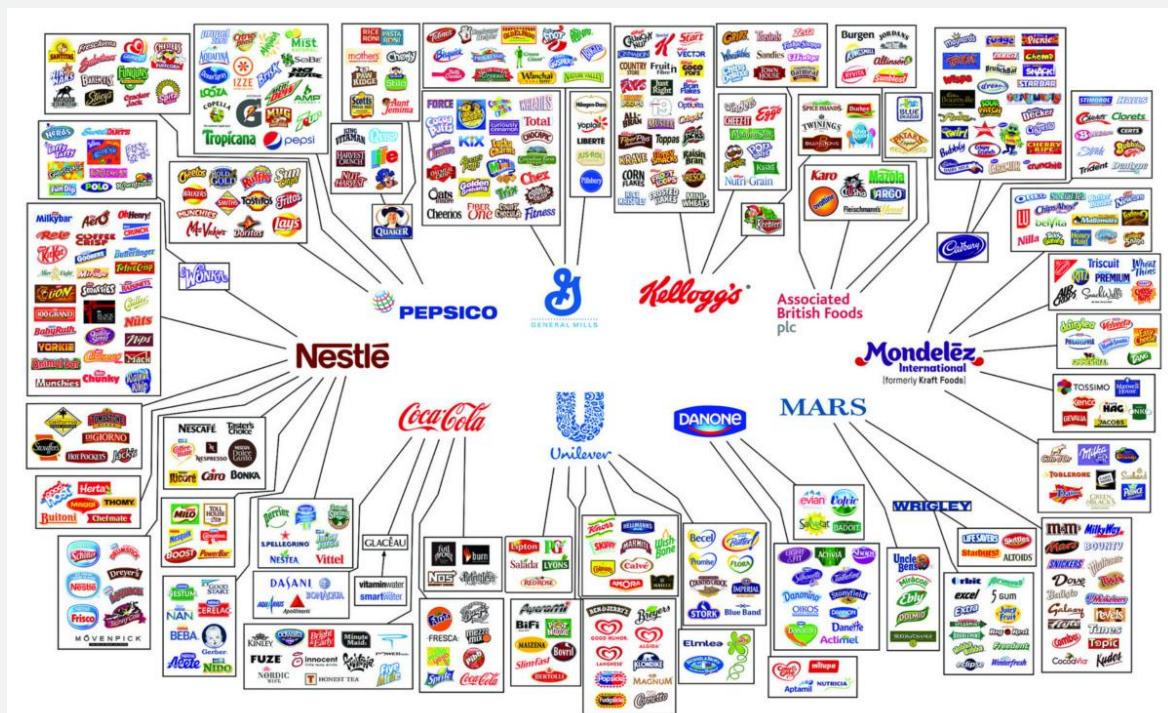
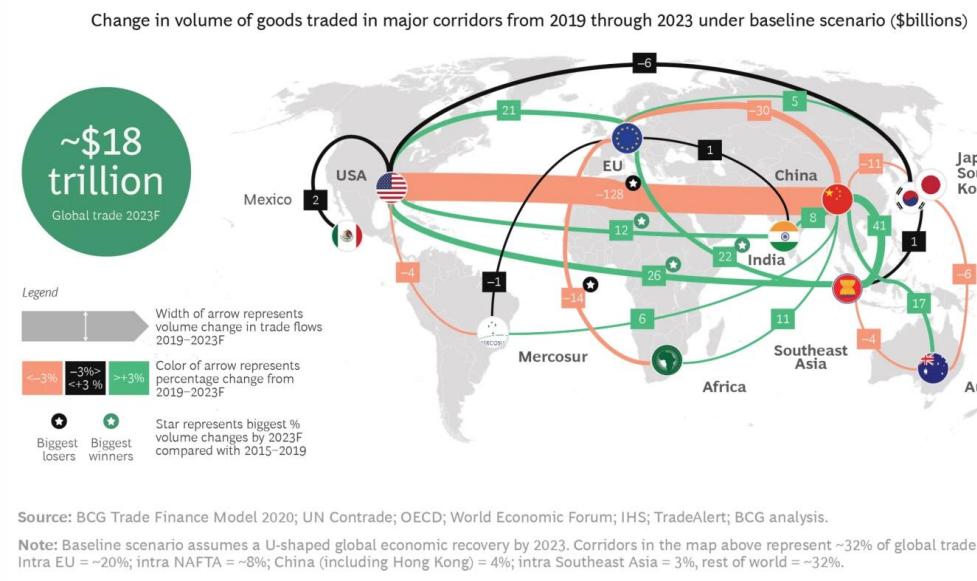


Cadeia alimentar

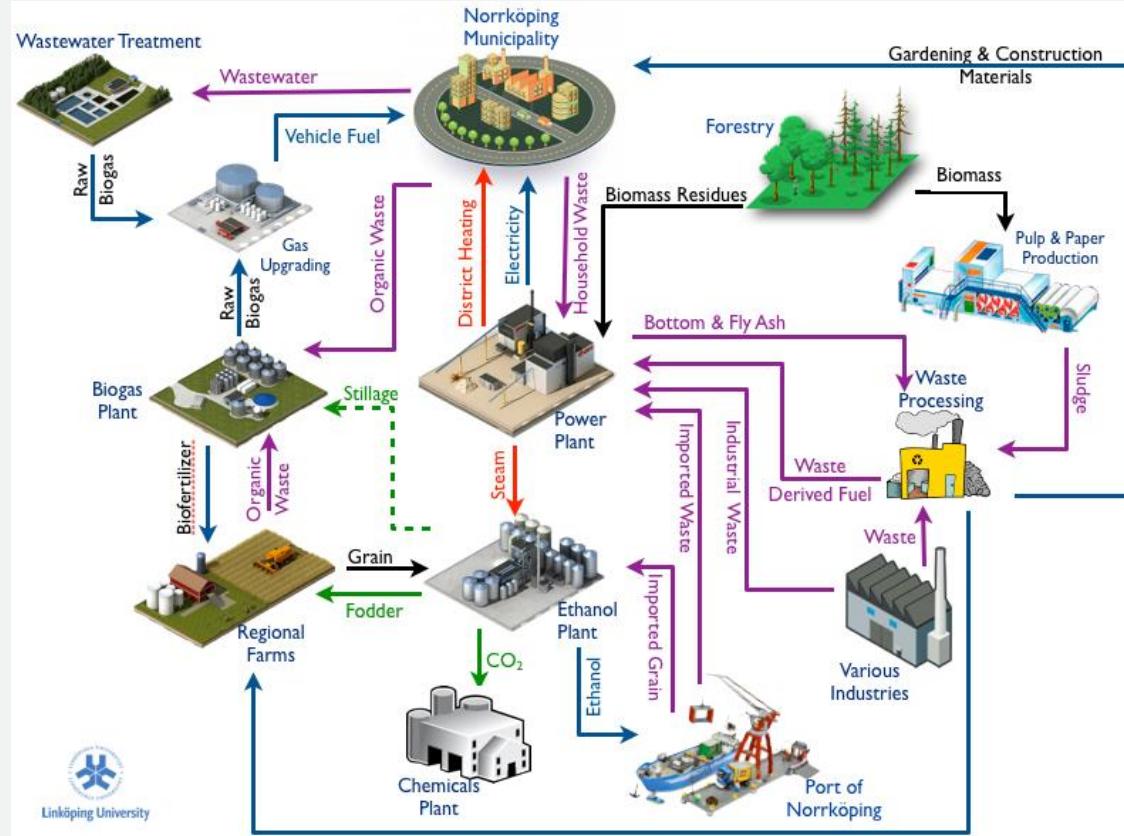


Estruturas biológicas

Exhibit 2 - Even If Trade Recovers by 2023, Expect Flows Between Blocs to Shift Dramatically

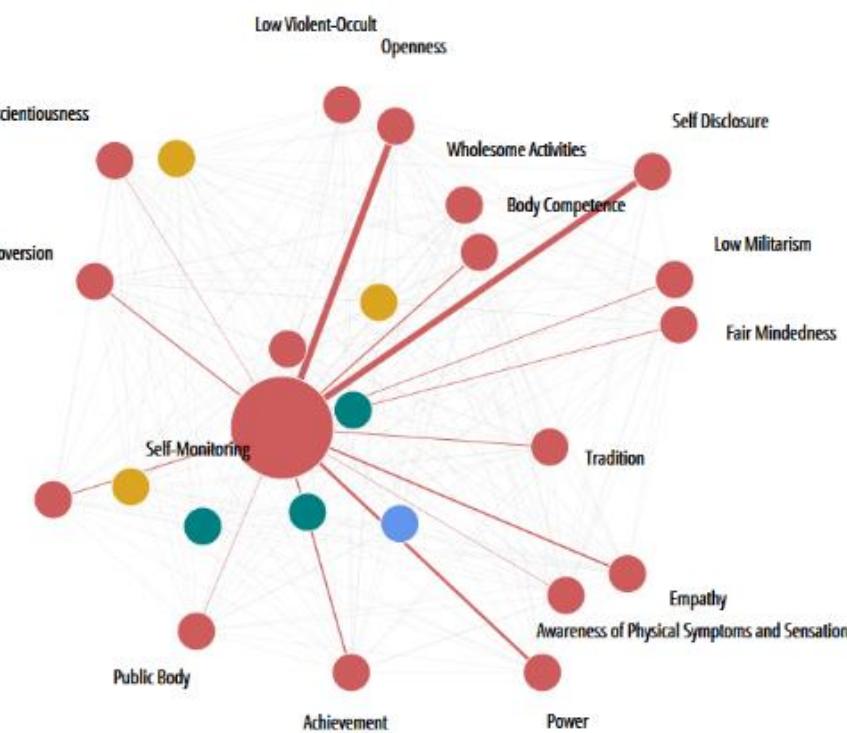
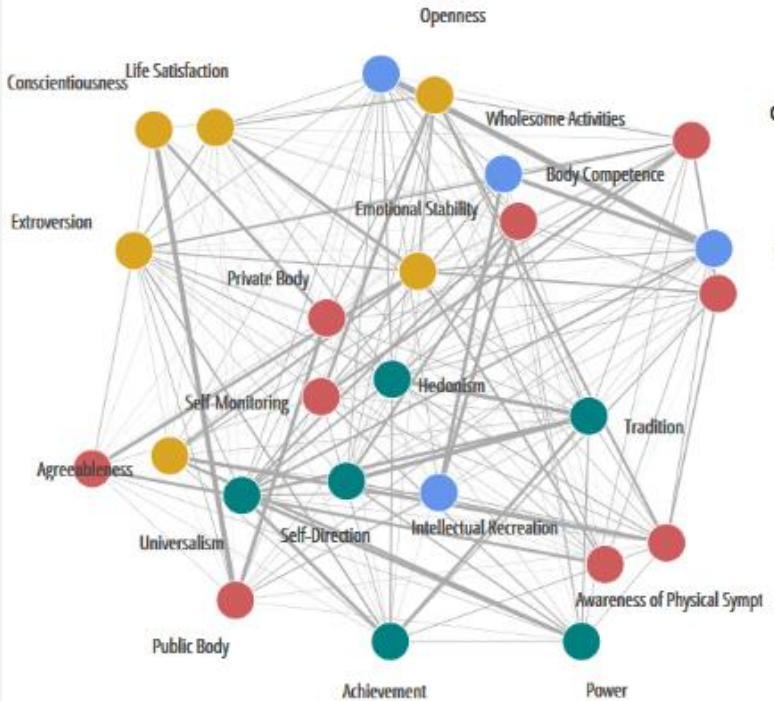


Comércio internacional



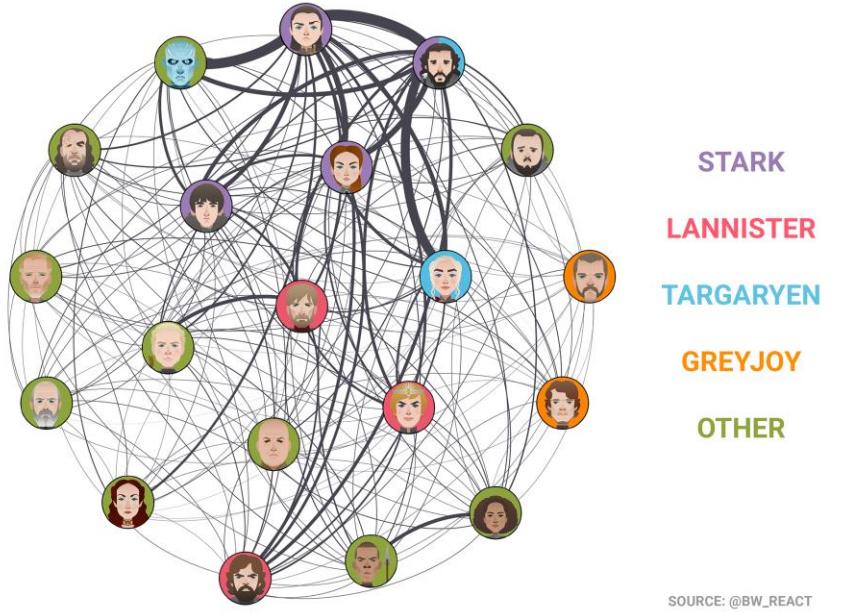
Cadeias produtivas e integrações

Relacionamentos entre empresas



About this trait : A high score on this trait describes a person who monitors her self-presentations, expressive behavior, and nonverbal affective displays.

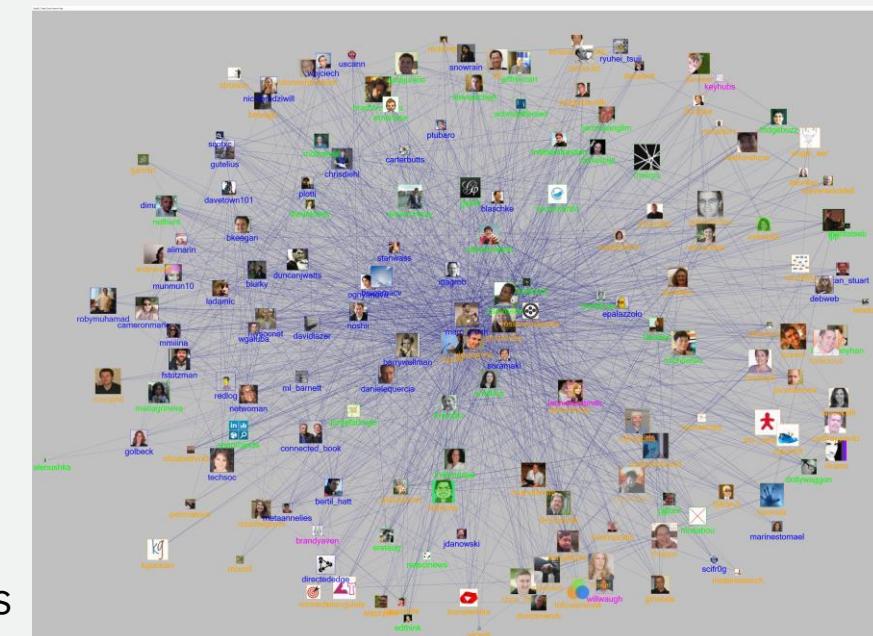
Self-monitoring has the strongest positive correlation (0.118) with Power
Self-monitoring has the strongest negative correlation (-0.267) with Openness
The measure used is Snyder's Self-Monitoring Scale



Comportamento individual

Associações e grupos

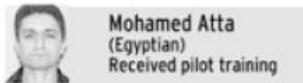
Redes sociais



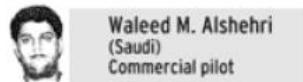
THE HIJACKERS ...

American Airlines 11

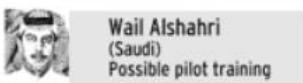
Crashed into WTC (north)



Mohamed Atta
(Egyptian)
Received pilot training



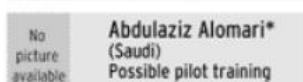
Waleed M. Alshehri
(Saudi)
Commercial pilot



Wail Alshahri
(Saudi)
Possible pilot training



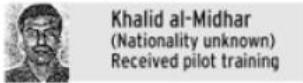
Satam al-Suqami
(Nationality unknown)



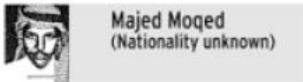
Abdulaziz Alomari*
(Saudi)
Possible pilot training

American Airlines 77

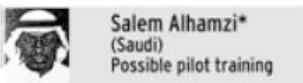
Crashed into Pentagon



Khalid al-Midhar
(Nationality unknown)
Received pilot training



Majed Moqed
(Nationality unknown)



Salem Alhamzi*
(Saudi)
Possible pilot training



Nawaf Alhamzi*
(Saudi)



Hani Hanjour
(Saudi)

UnitedAirlines 175

Crashed into WTC (south)



Marwan al-Shehhi
(United Arab Emirates)
Received pilot training



Fayed Ahmed
(Believed to be Saudi)



Ahmed Alghamdi
(Possibly Saudi)



Hamza Alghamdi
(Believed to be Saudi)
Possible pilot training



Mohald Alshehri
(Nationality unknown)
Possible pilot training

AND HOW THEY WERE CONNECTED

Attended same technical college

Hamburg, Germany
Mohamed Atta
Marwan al-Shehhi
Ziad Jarrah

Took flight classes together

Pilot schools
in Florida
Mohamed Atta
Marwan al-Shehhi

Pilot schools
in San Diago
Khalid al-Midhar
Nawaf Alhamzi

Known to be together in week before attacks

Stayed together
in a Florida
motel
Mohamed Atta
Marwan al-Shehhi
Hani Hanjour

Attended a gym in Maryland (Sept 2-6), also seen dining together

Other cities
in Florida
Mohamed Atta
Fayed Ahmed
Ahmed Alghamdi
Mohald Alshehri
Khalid al-Midhar
Majed Moqed
Salem Alhamzi
Nawaf Alhamzi
Hani Hanjour

Pilot schools in San Diago Khalid al-Midhar Nawaf Alhamzi

Last known address

Hollywood, Florida
Marwan al-Shehhi
Waleed M. Alshehri
Wail Alshahri
Ziad Jarrah

Attended a gym in Maryland (Sept 2-6), also seen dining together

Other cities
in Florida
Mohamed Atta
Fayed Ahmed
Ahmed Alghamdi
Mohald Alshehri
Khalid al-Midhar
Majed Moqed
Salem Alhamzi
Ahmed Alhaznawi
Ahmed Alnami
Saeed Alghamdi

Outside Florida

Satam al-Suqami
Hamza Alghamdi
Abdulaziz Alomari
Majed Moqed
Salem Alhamzi
Nawaf Alhamzi

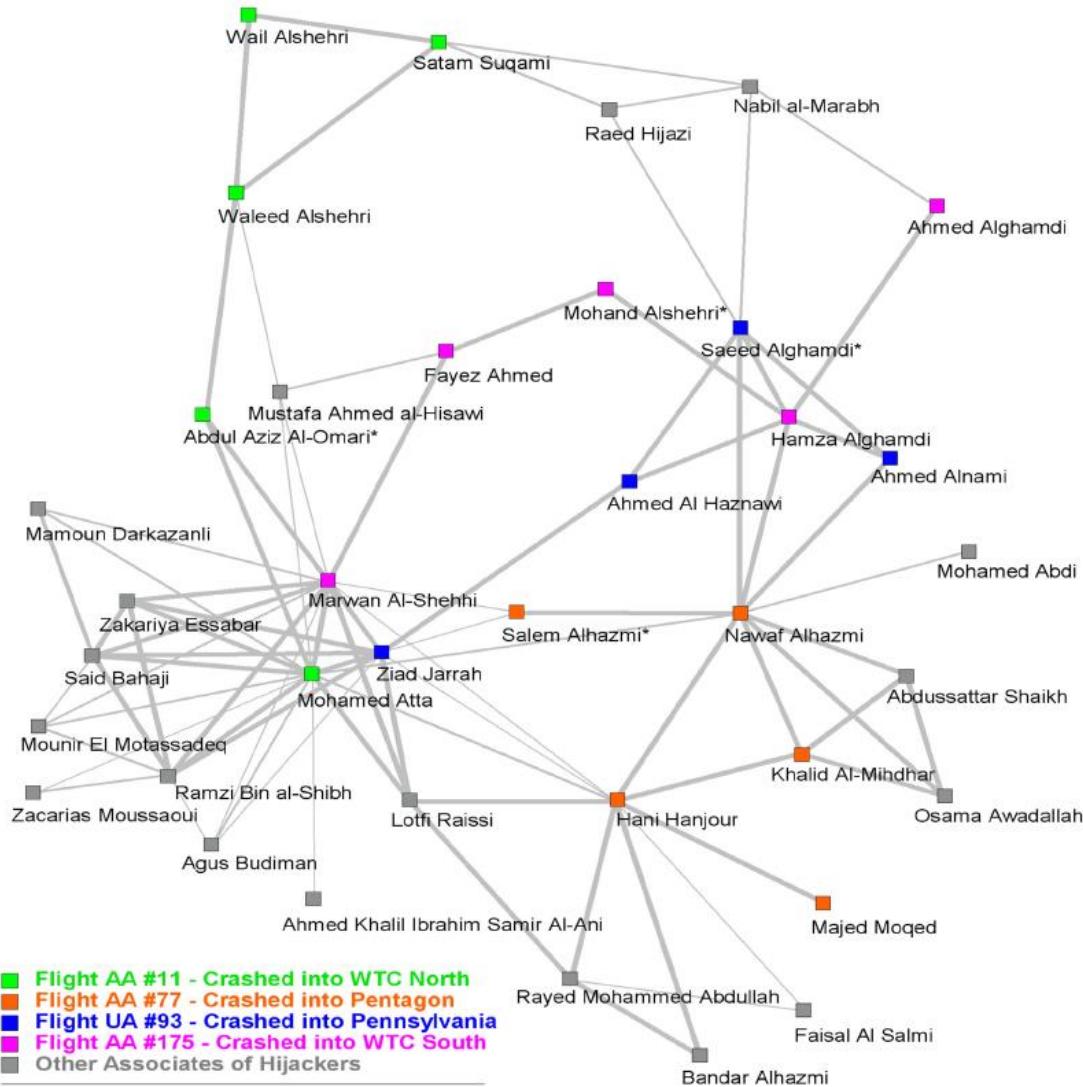
Picked up tickets bought earlier in Baltimore

Mohamed Atta
Ziad Jarrah
Ahmed Alhaznawi
Khalid al-Midhar
Majed Moqed

Bought from the same travel agent in Florida

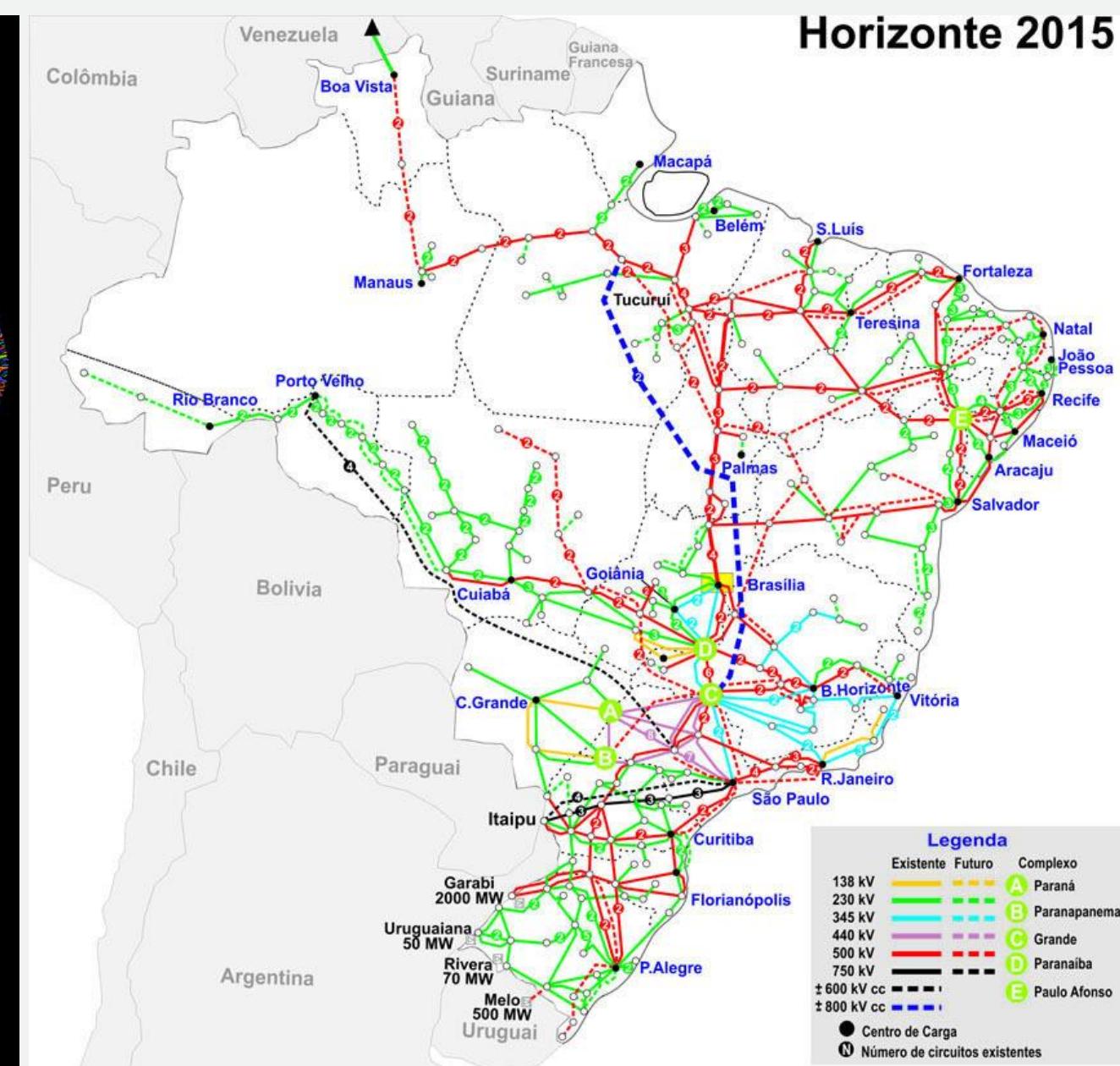
Ziad Jarrah
Mohamed Atta
Fayed Ahmed
Mohald Alshehri
Ahmed Alghamdi
Hamza Alghamdi

Other Associates of Hijackers



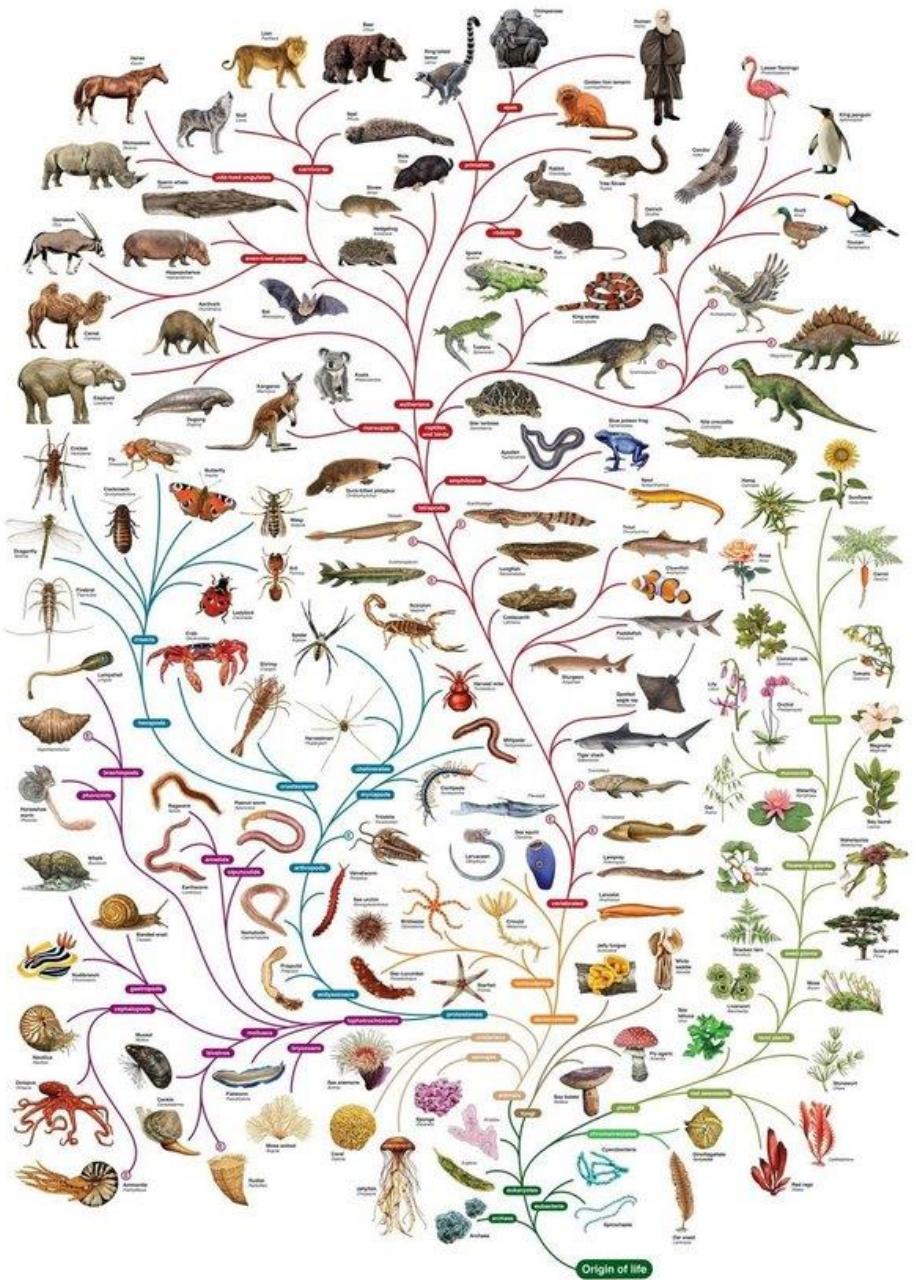
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Associações criminosas

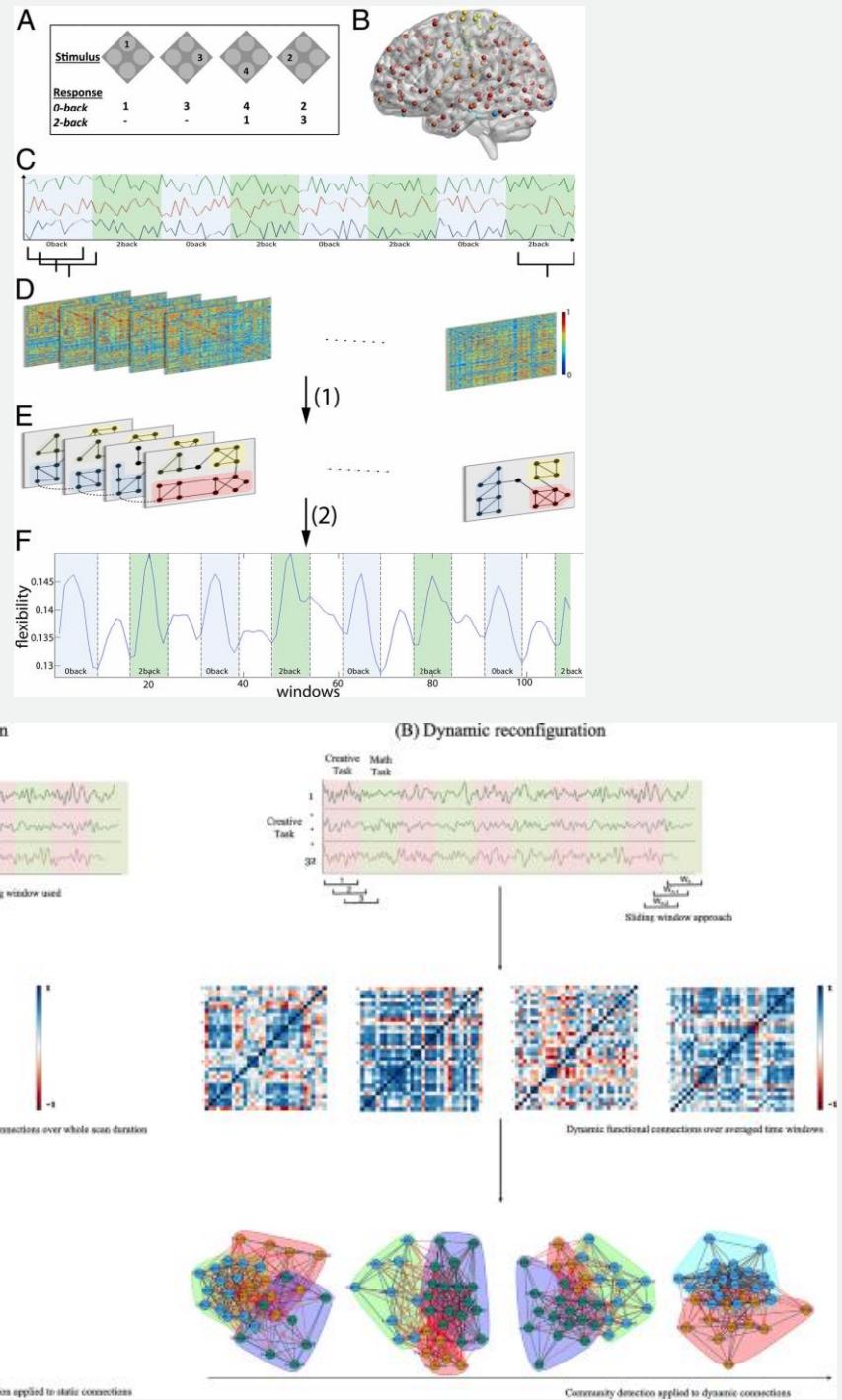


Recursos

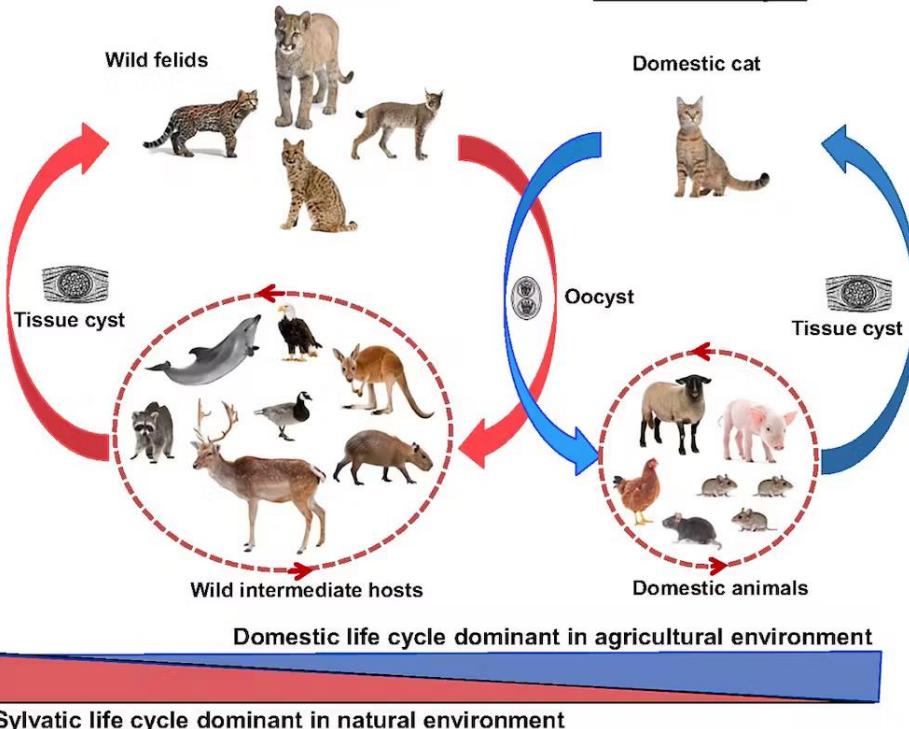
Filogenia



Processos
dinâmicos

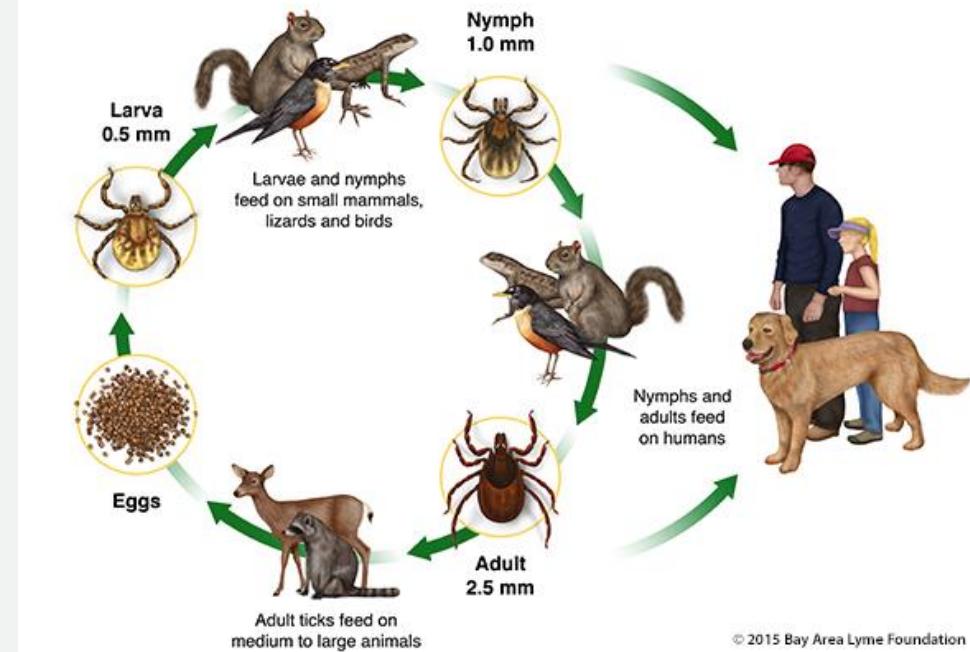


Sylvatic life cycle



Toxoplasmosis

Lyme Disease: Tick and Host Lifecycle

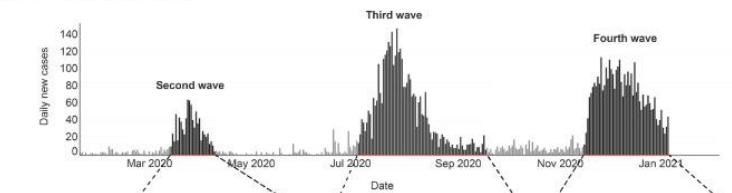


Doença de Lyme (borrelia)

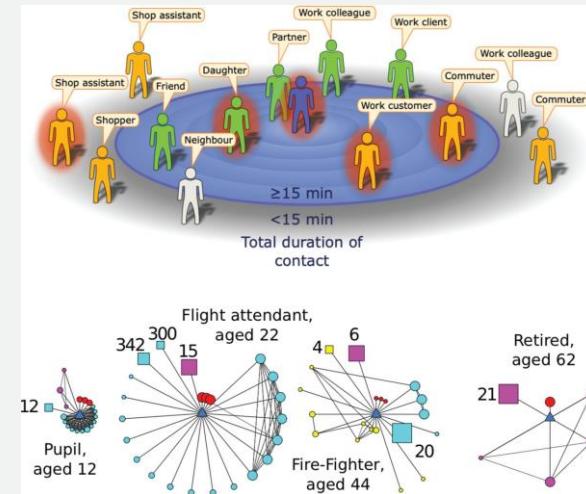
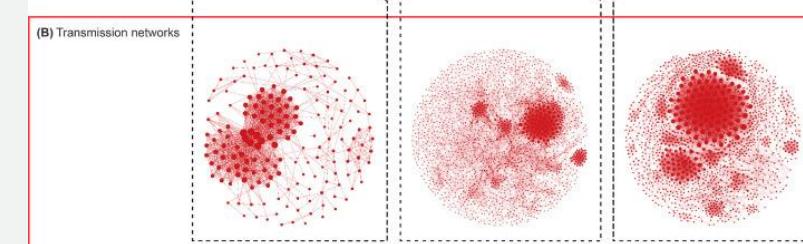
Covid 19

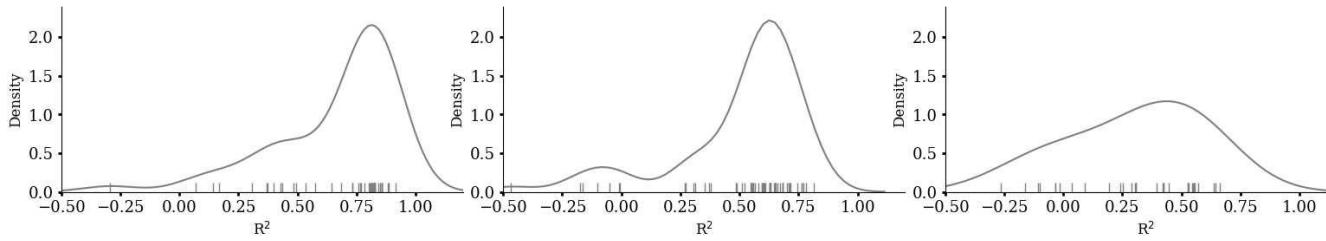
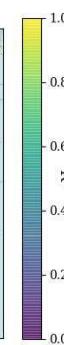
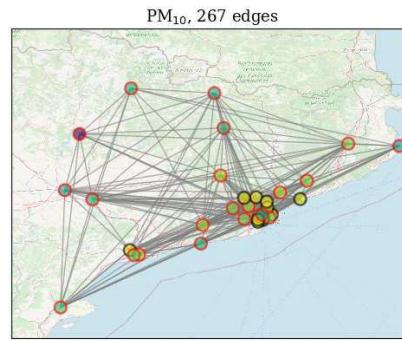
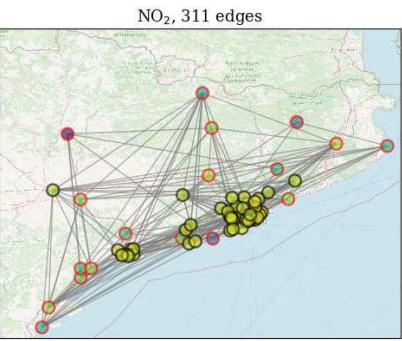
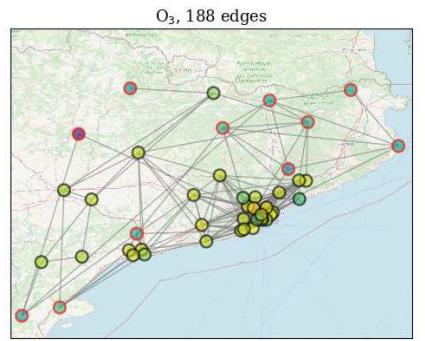


(A) All COVID-19 outbreaks in Hong Kong

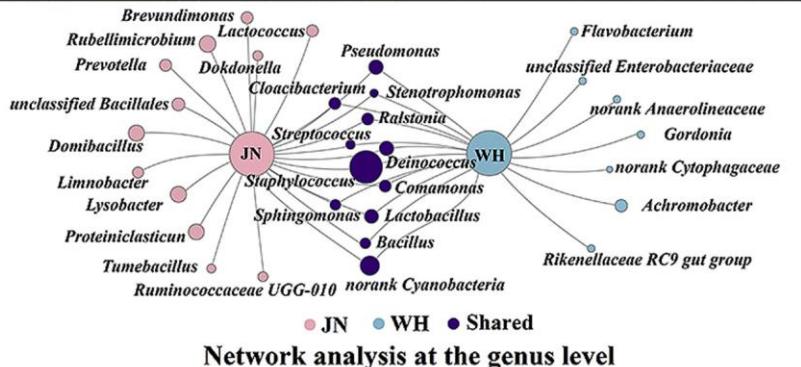
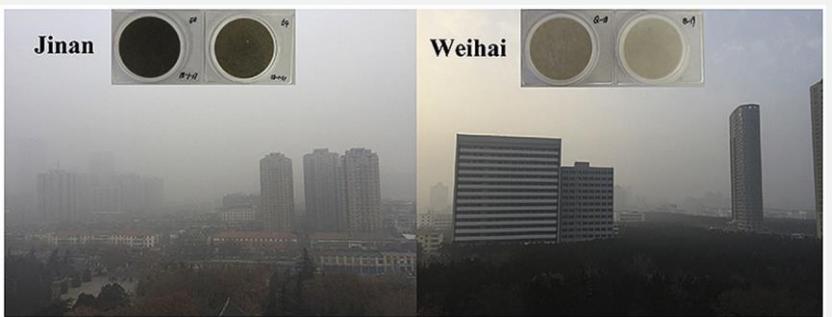


(B) Transmission networks

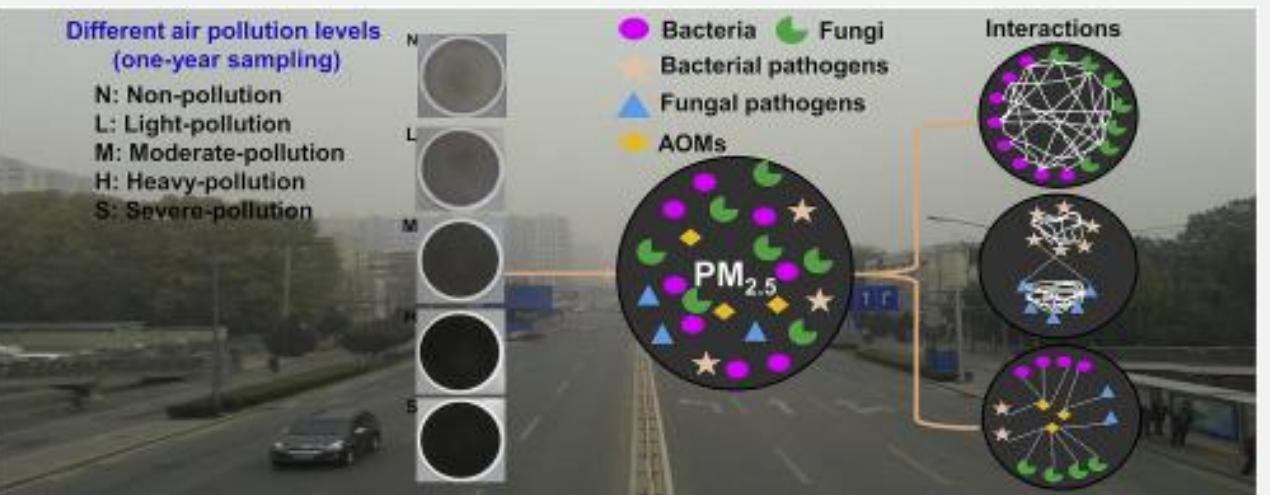




Dinâmica de poluentes



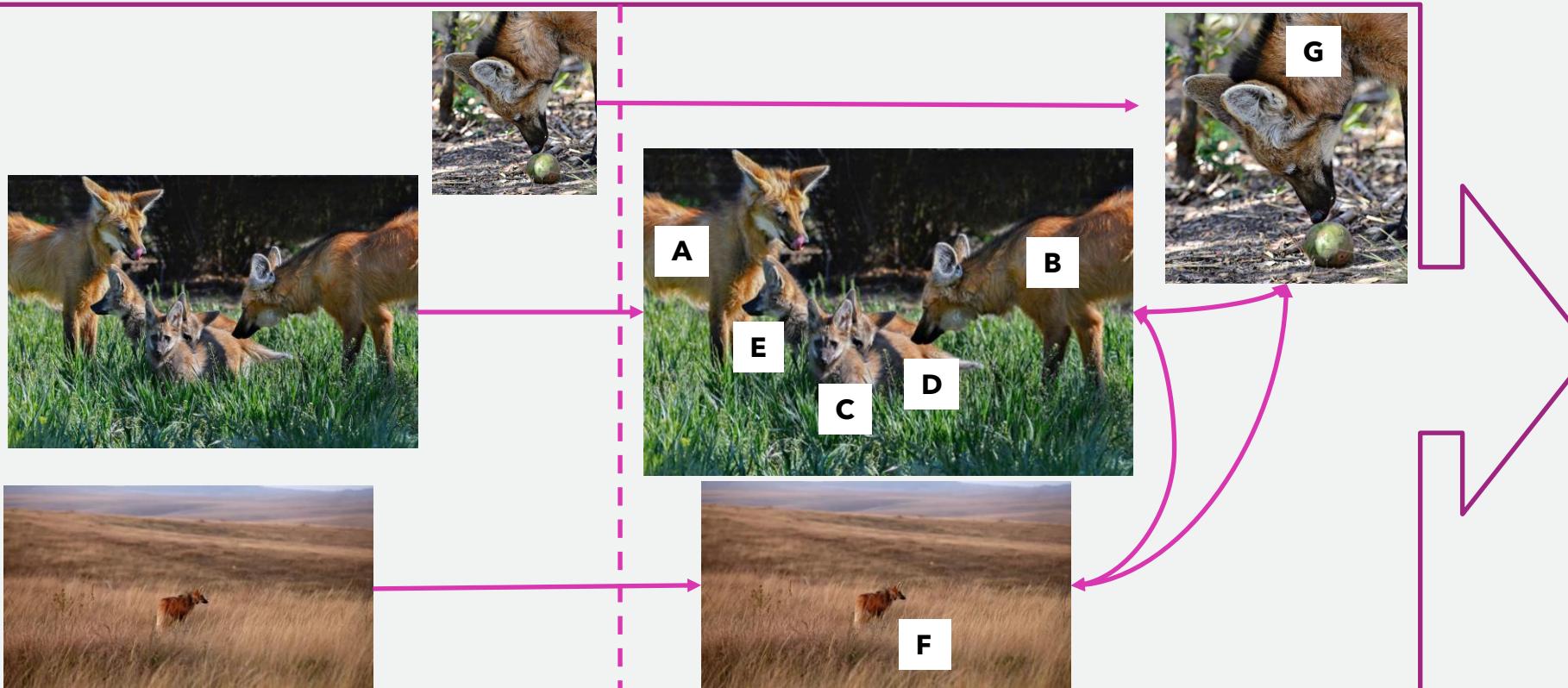
Qualidade do ar



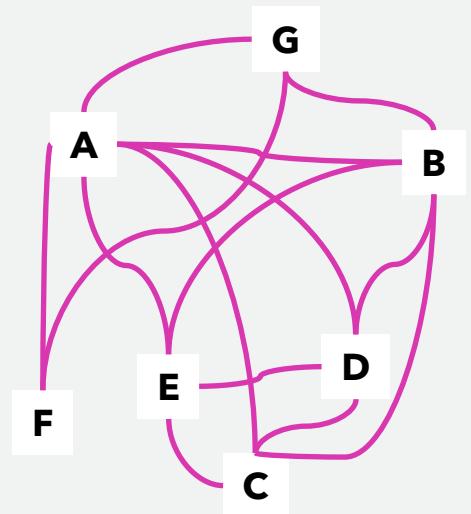
Efeitos da poluição

Quais elementos são comuns a todos os casos?

Realidade	Exemplo	Abstração
Objetos	coisas, ideias, medidas, sentimentos, posições	Vértice (nó)
Vínculos	enlaces, conexões, influências, caminhos, associações	Aresta (linha)
Fenômenos	estrutura, disposição, arranjo, organização, configuração	Grafo (rede)



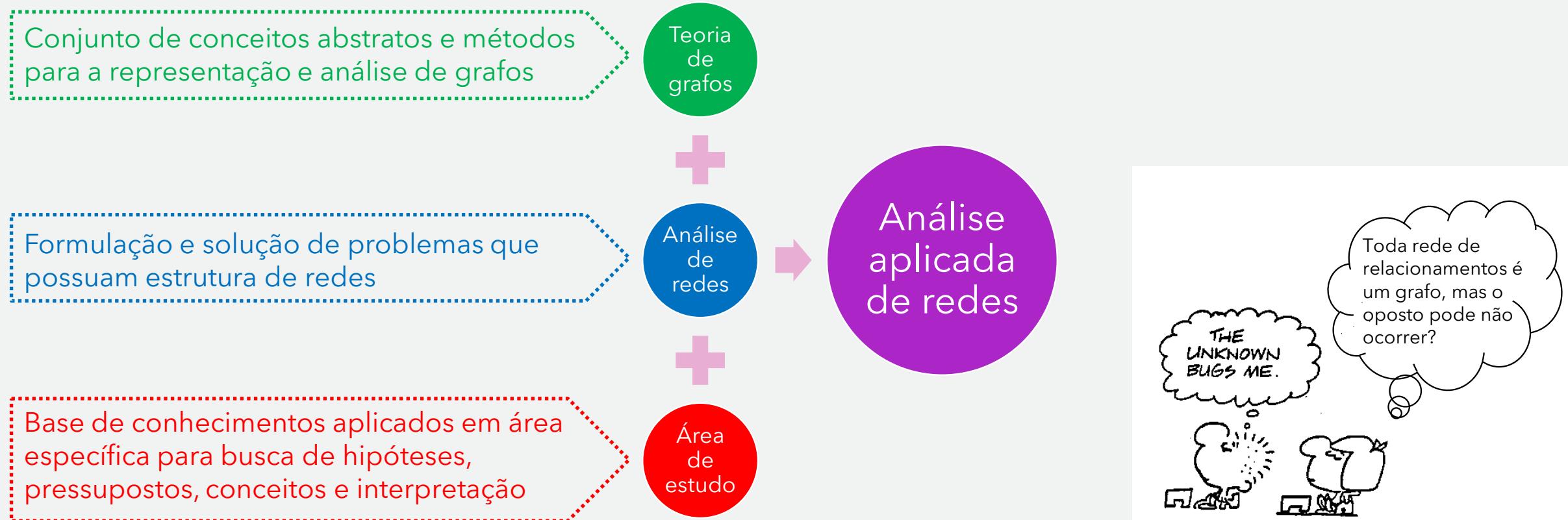
$$G = \{V, A\}$$



Como interpretar?

- A interpretação depende da área

Área	Pontos, círculos	Linhas
Matemática	Vértices	Arestas, arcos
Ciências da computação	Nós	Conexões, ligações
Física	Sites, locais	Vínculos
Sociologia	Atores, agentes	Laços, relações

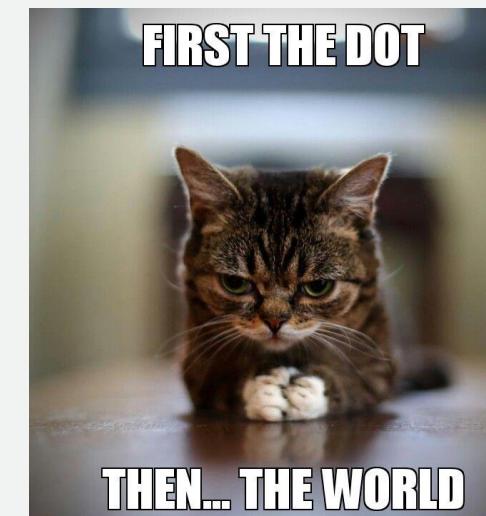


Quais propriedades a avaliar em um grafo aplicado?

- Conjunto completo
- Componentes (vértices, arestas)
- Simplificações
- Representação (*layout*)
- Comportamento relacional
- Comportamento dinâmico
- ...



Resultados, discussões e conclusões
conforme área de aplicação

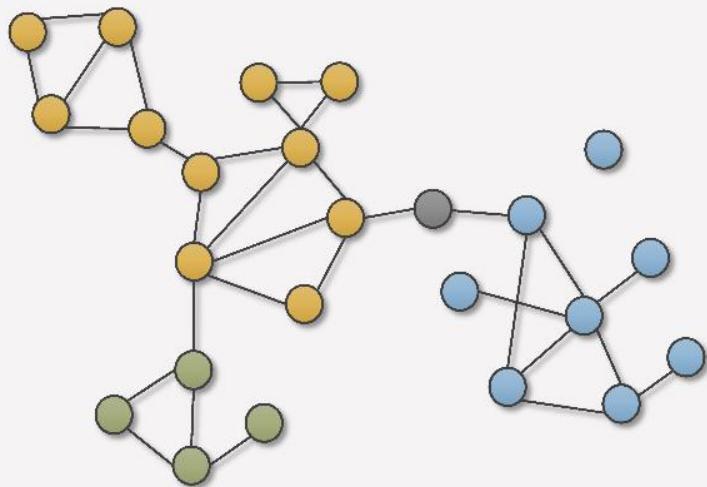


Topologia

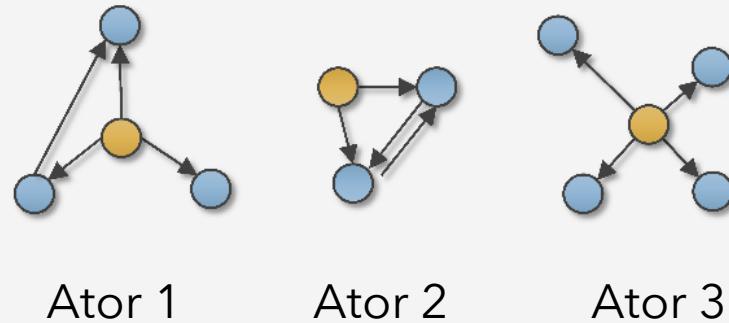
- É a forma como os nós e as arestas estão dispostos numa rede.
- As propriedades topológicas podem aplicar-se à rede como um todo ou a nós e arestas individuais.

Foco

Rede total
(completa)



Atores específicos
(egocêntrica)

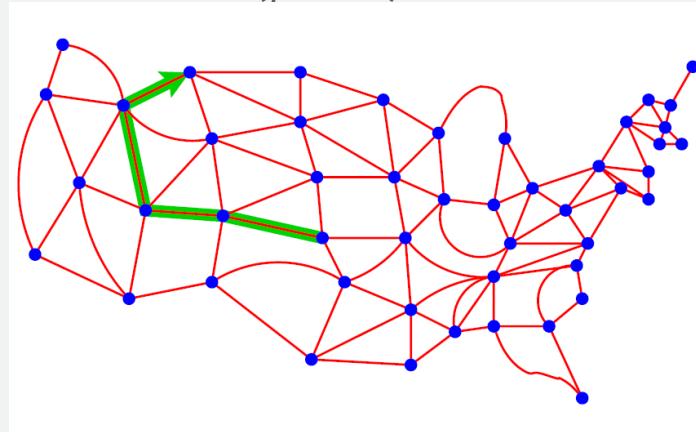


Não dirigido	Dirigido (dígrafo)	Bipartido	Valorado (ponderado)	Completo
Tipos				
Conectado	Desconectado	Cíclico	Acíclico	Pseudo
Simples	Multigrafo	Euleriano	Hamiltoniano	Nulo
Espenso	Denso	Trivial	Finito	Infinito

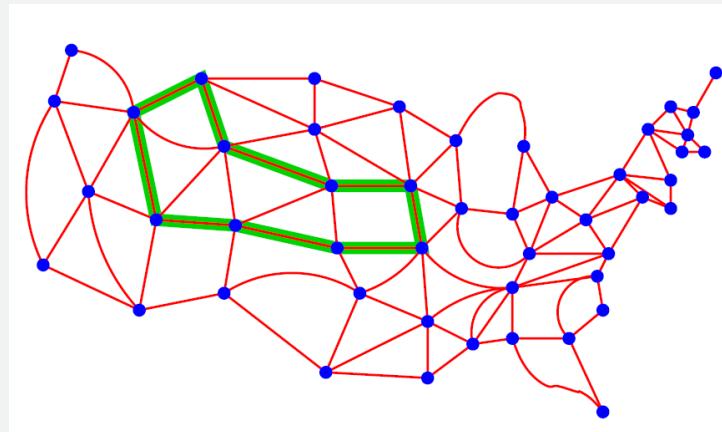
Viagem ou rota

- Uma viagem (*trip*) não pode usar a mesma aresta mais do que uma vez, mas pode passar pelo mesmo vértice mais do que uma vez.

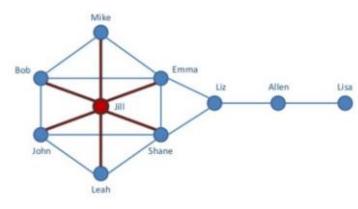
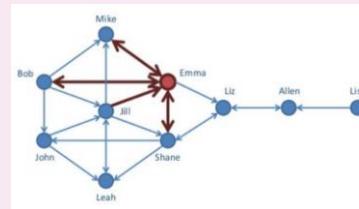
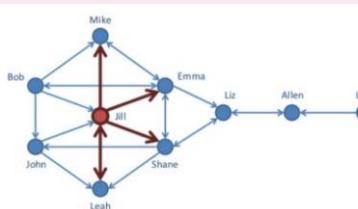
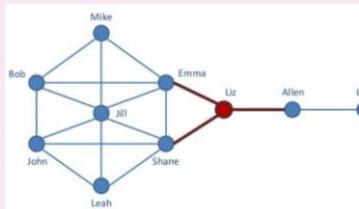
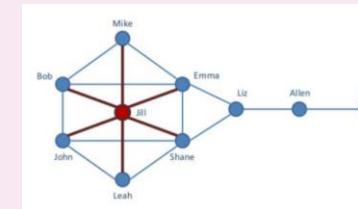
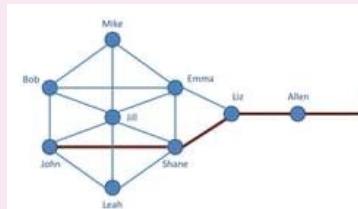
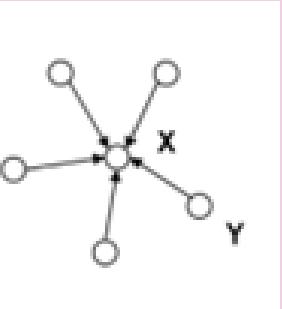
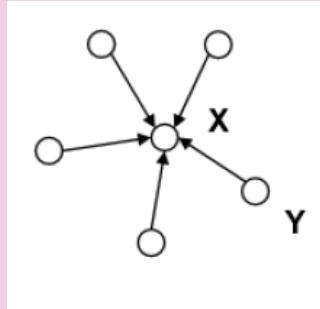
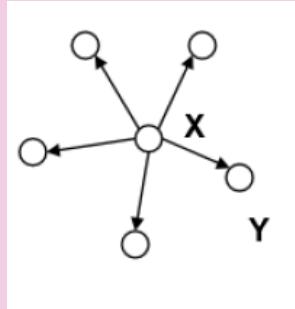
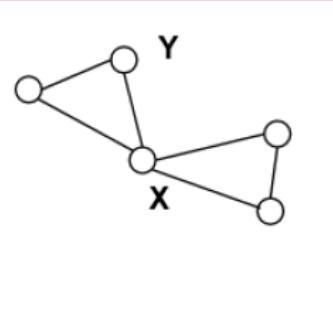
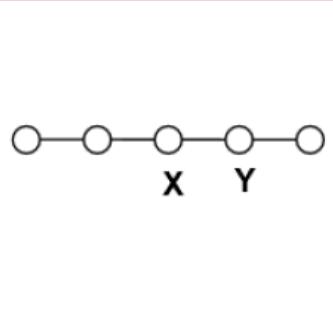
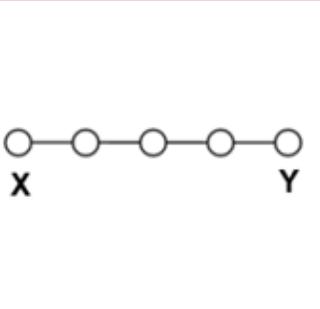
- Caminho (*path*)



- Circuito (*circuit*)



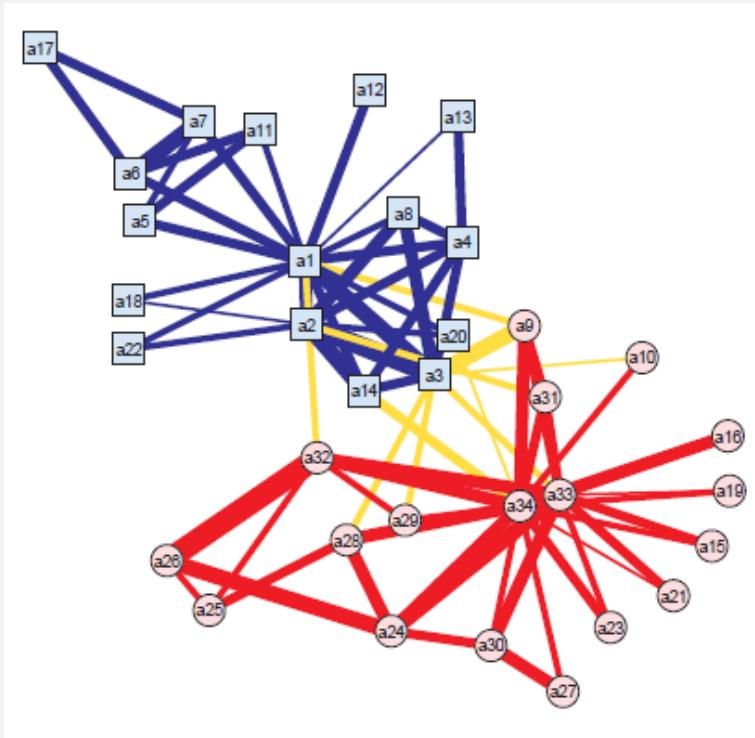
Métricas da rede

	Grau (degree)		Intermediação (betweenness)	Proximidade (closeness)	Diâmetro
Grau (degree)	Grau de entrada (indegree)	Grau de saída (outdegree)			
Quantidade absoluta de vínculos que um nó estabelece (rede não direcionada)	Conexões direcionadas ao nó	Conexões originadas no nó	Considera a posição de um nó entre o caminho que conecta todos os outros nós	Observa a menor distância entre todos os nós	Caminho mais longo
					
					

Prestígio ou notoriedade ou autoridade (prestige)				Centralidade por autovetor (eigenvector centrality)
Grau	Proximidade	Rank ou ordem		
		PageRank	HITS (hyperlink induced topic search)	
Conexões direcionadas ao nó (somente em redes direcionadas). Um nó de elevado prestígio é aquele destinatário generalizado de ligações internas.	O prestígio de proximidade generaliza o prestígio ao considerar tanto os nós ligados diretamente e indiretamente a um determinado nó, dentro de sua área de influência, avaliando as menores distâncias de conexão.	O prestígio do nó depende do prestígio dos nós a que está ligado. Pondera conexões indo para o nó e originadas nele para outros nós. A importância de um nó é determinada apenas pelo número de conexões do nó, mas também a soma de todas as pontuações do PageRank das páginas que apontam para ele.	Tal como o PageRank, explora a estrutura de conexões para classificar os nós, mas produz um conjunto maior de busca composto também pela classificação de hub. A intenção é que um bom hub aponta para boas autoridades e uma boa autoridade é apontada por um bom hub.	A pontuação da centralidade é calculada pelo autovetor da matriz de adjacências, tendo como base o número de conexões do nó com seus vizinhos.

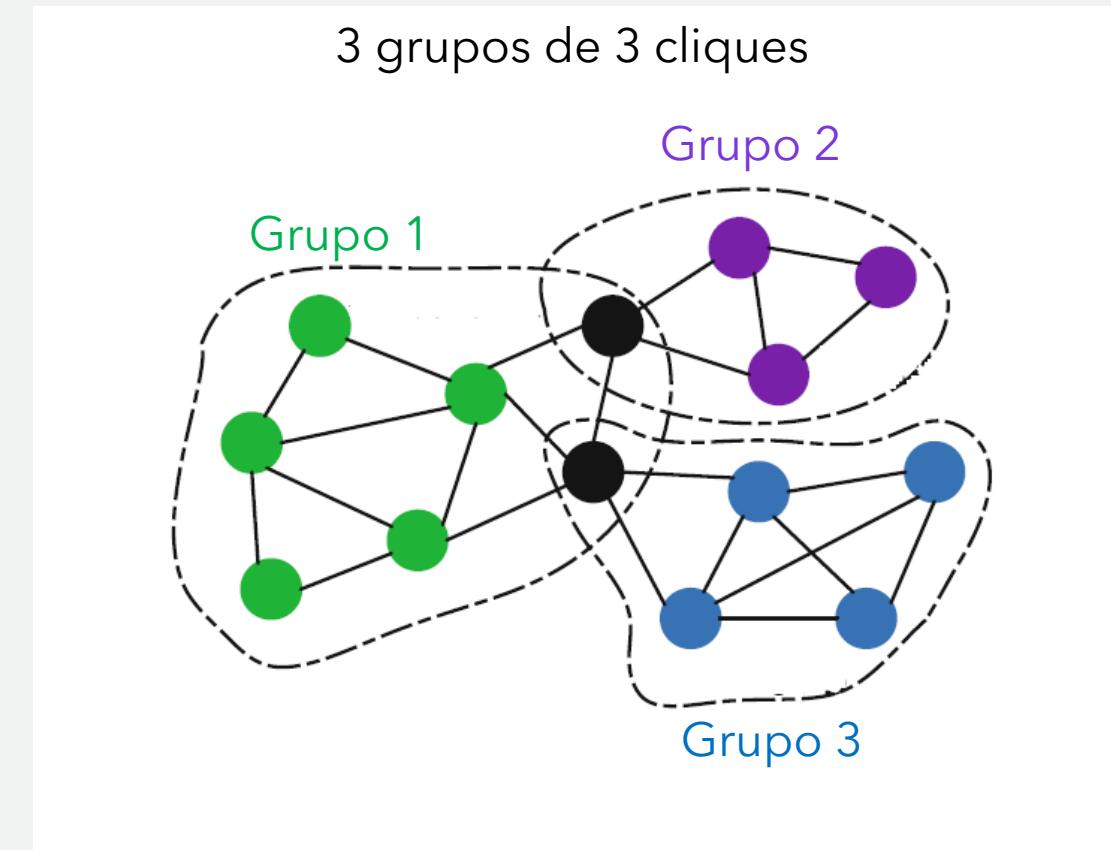
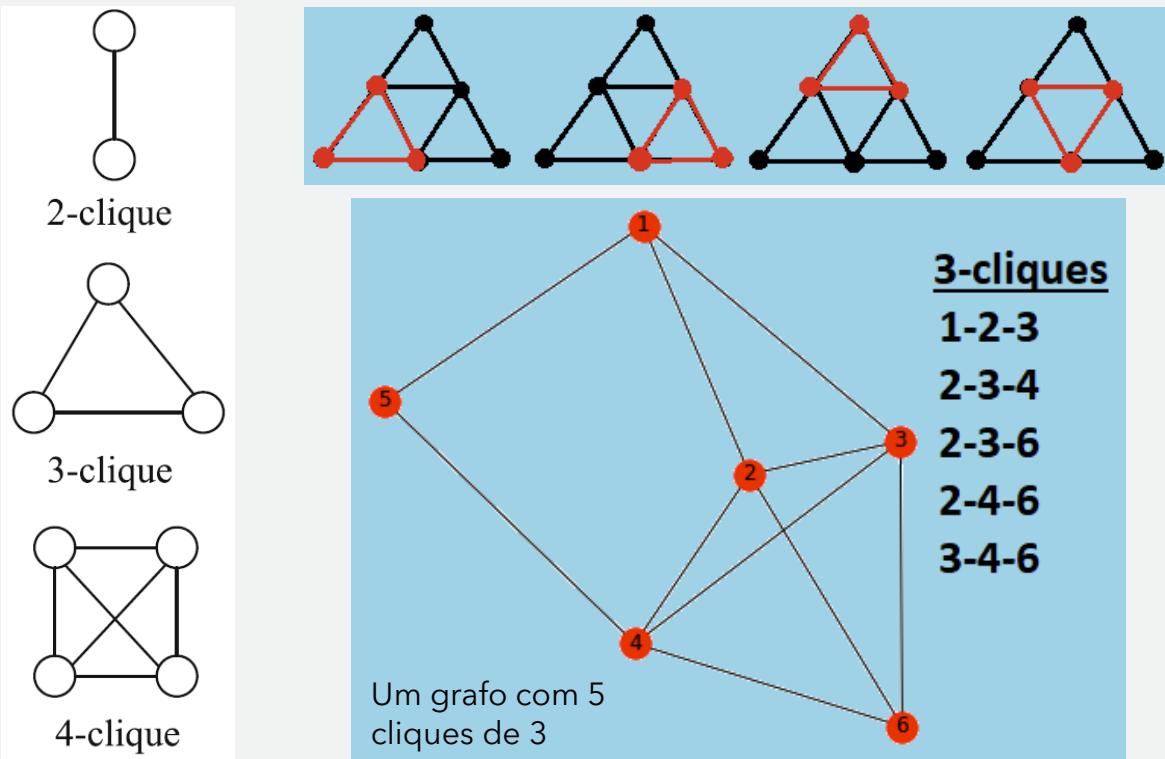
Agrupamento (clustering)

- Mede a fração de triplos (grupos de três vértices) ligados que se aproximam para formar triângulos, também denominado de coeficiente de agrupamento ou transitividade.



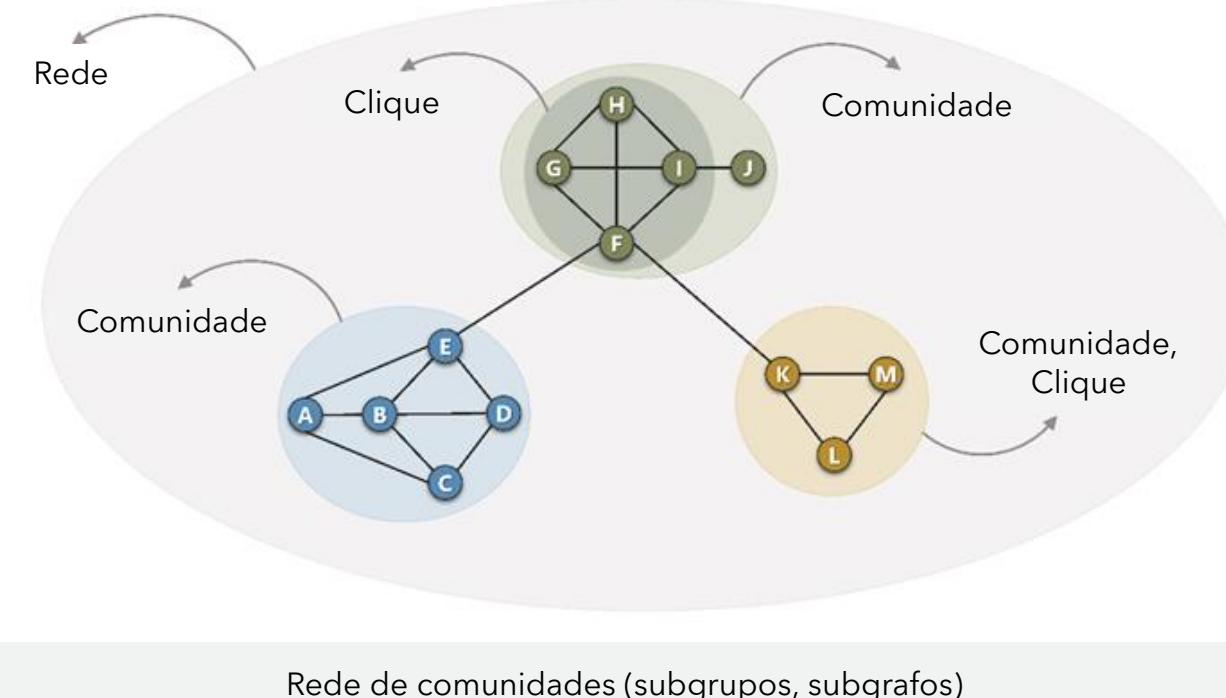
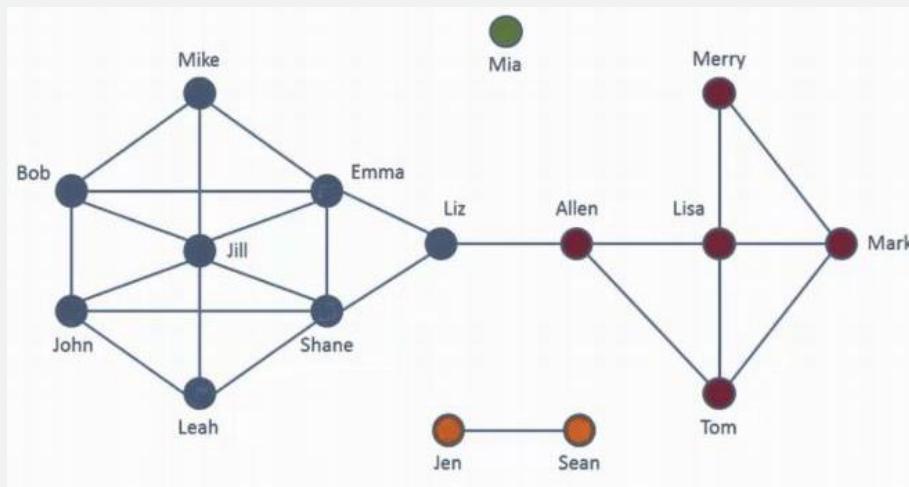
Clique

- É um subconjunto de vértices de um grafo não direcionado, de tal forma que todos os dois vértices distintos da clique são adjacentes, ou seja, o seu subgrafo induzido é completo.



Modularidade

- É uma medida da densidade da estrutura de vizinhança da rede
 - Avalia a relação dos nós com seus vizinhos, considerando quanto um nó tende a aparecer em um determinado grupo (quanto está conectado ao grupo)
- Calcula a divisão da rede em módulos (grupos, *clusters*, comunidades) através da mensuração da divisão da rede
- Valores de modularidade
 - Altos: conexões densas
 - Baixos: conexões esparsas



Rede de comunidades (subgrupos, subgrafos)

NETWORK OF THRONES

ANDREW BEVERIDGE AND JIE SHAN

Math Horizons, Vol. 23, No. 4 (April 2016), pp. 18-22

<https://doi.org/10.4169/mathhorizons.23.4.18>

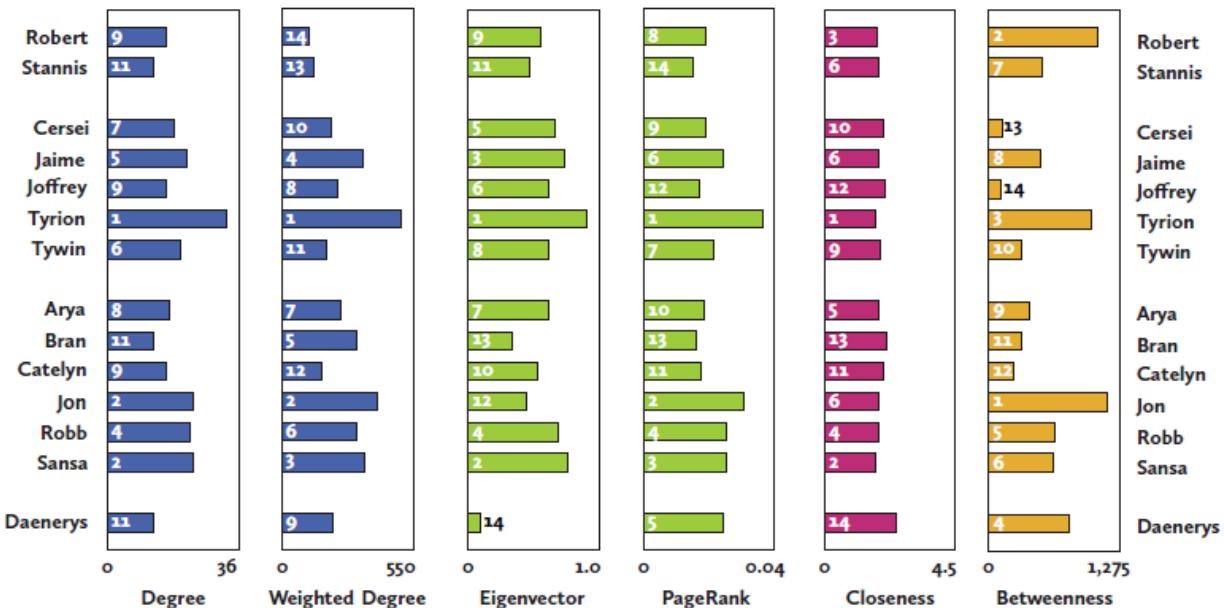
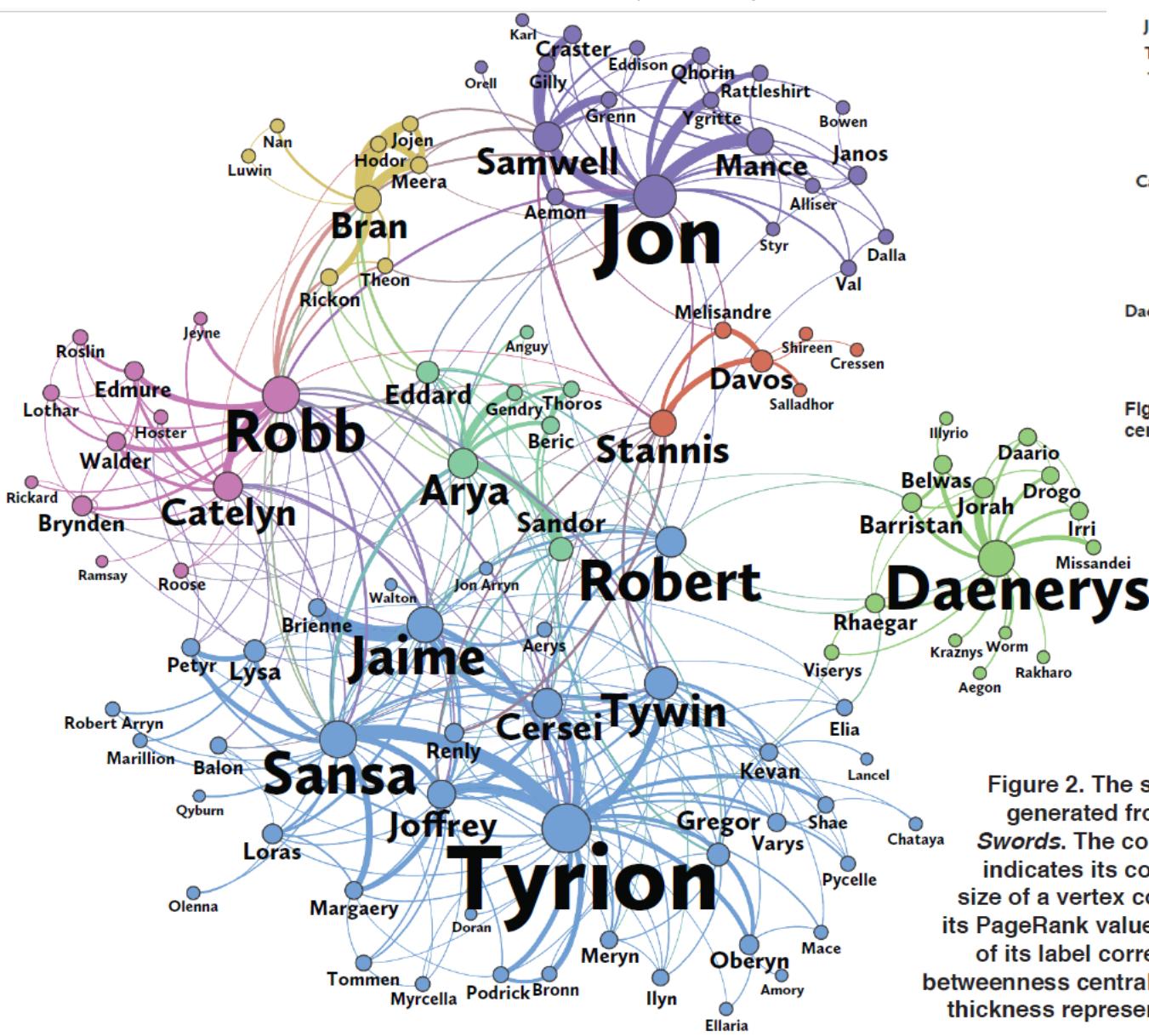


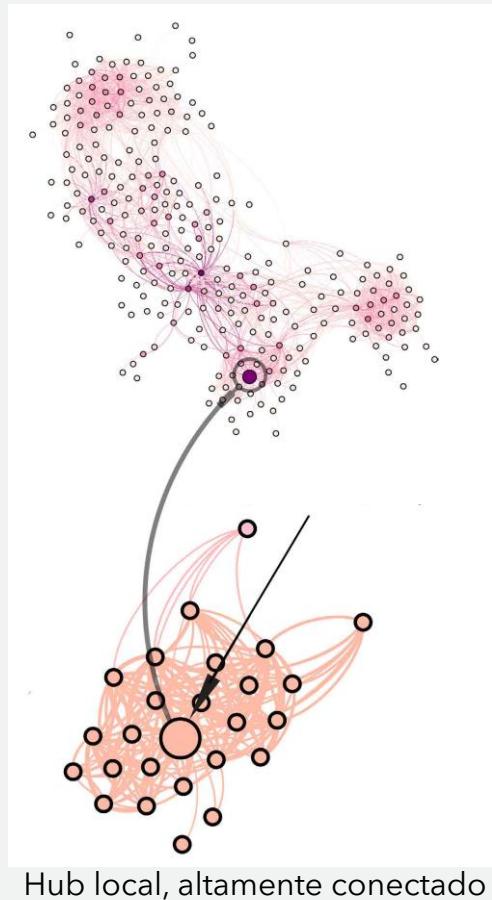
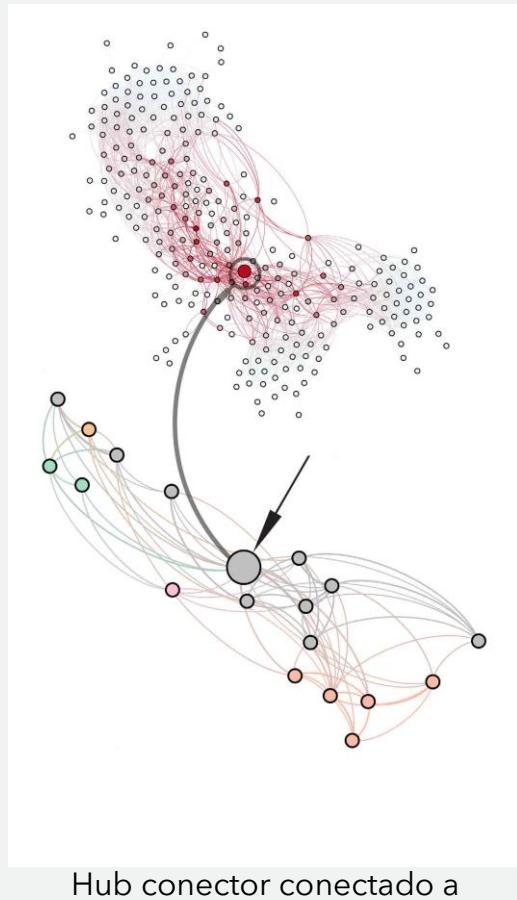
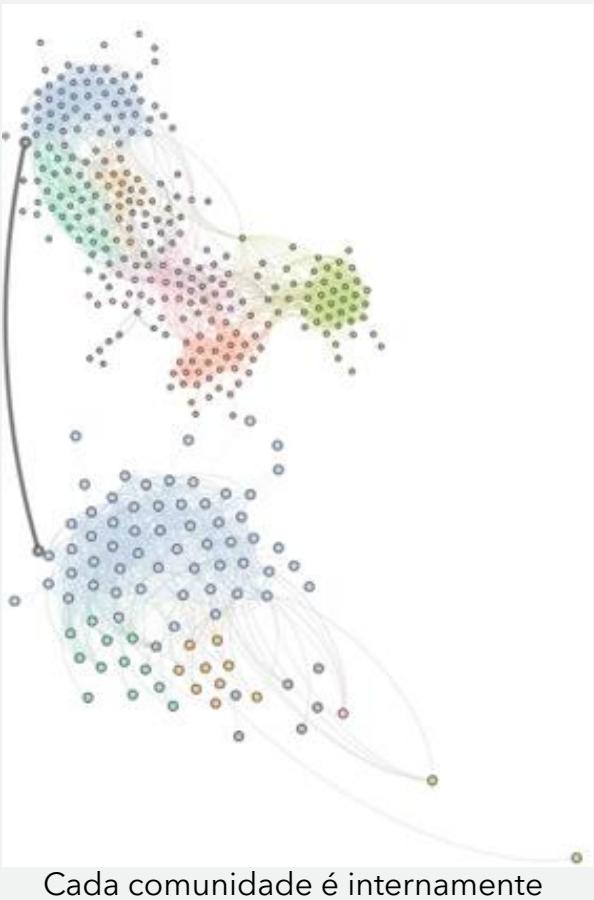
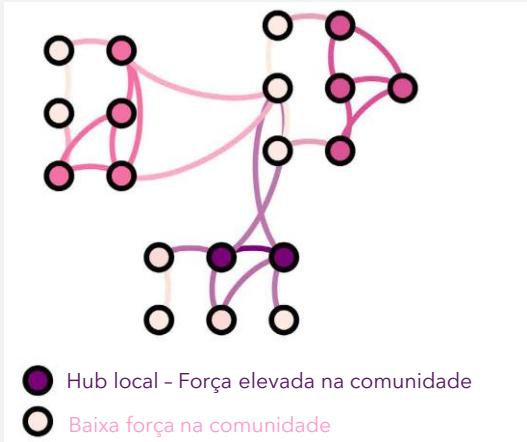
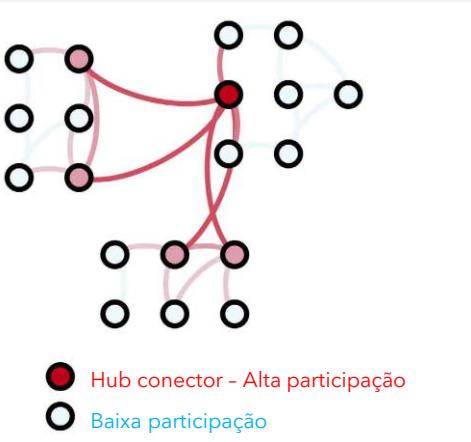
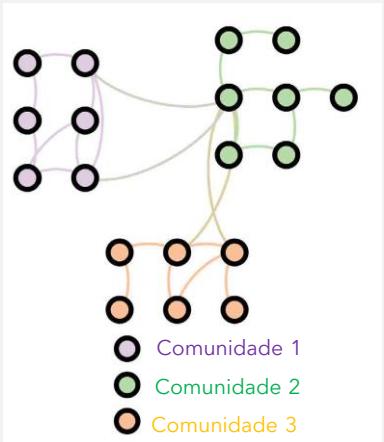
Figure 3. Centrality measures for the network. Larger values correspond to greater importance, except for closeness centrality, where smaller values are better. Numbers in the bars give the rankings of these characters.

Todos juntos!!!

Figure 2. The social network generated from *A Storm of Swords*. The color of a vertex indicates its community. The size of a vertex corresponds to its PageRank value, and the size of its label corresponds to its betweenness centrality. An edge's thickness represents its weight.



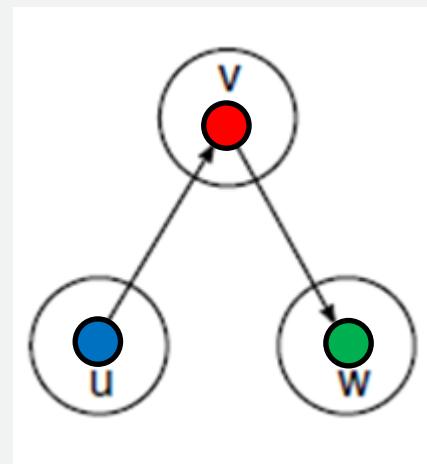
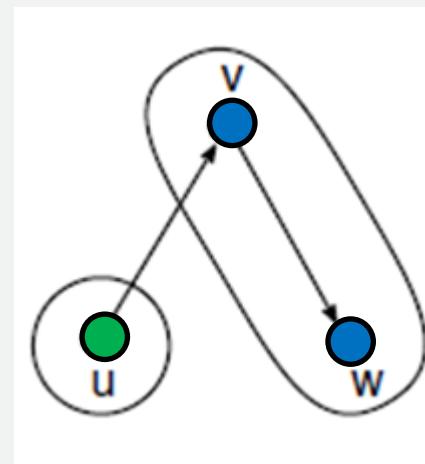
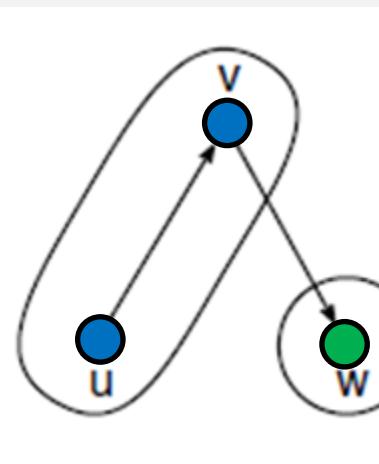
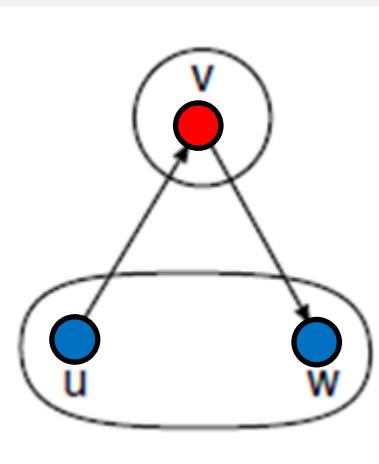
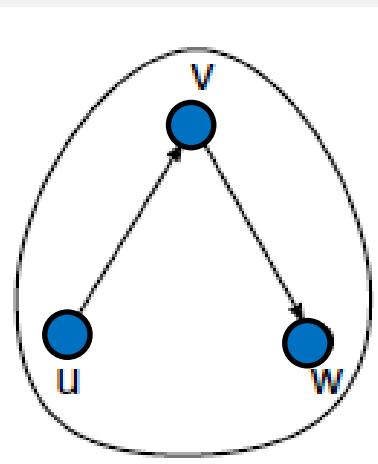
Hubs e conectores



Corretores (*brokers*) e pontes (*bridges*)

*Comportamento de mediação do nó v em relação às tríades

Entre membros de um mesmo grupo		Entre membros de grupos diferentes		
Coordenador	Mediador externo ou corretor itinerante	Representante ou delegado	Guardião ou porteiro	Articulador
Participa do grupo	Não participa do grupo	Regula o fluxo dentro de seu grupo para outros	Regula o fluxo externo para o seu grupo	Mediador entre membros de grupos diferentes, não participa de nenhum deles

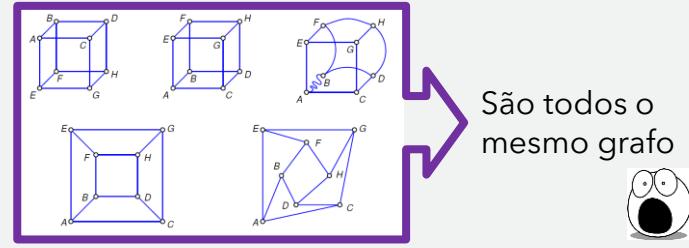


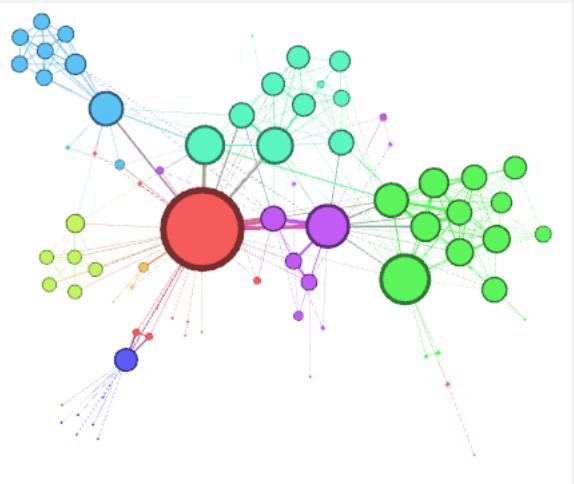
Layout (disposição, representação)

- O objetivo é ressaltar diferentes aspectos da topologia da rede



Layout	Tipo	Número de nós do grafo	Usa peso das arestas
Force Atlas	Força	≤ 10.000	Sim
Force Atlas 2	Força	$\leq 1.000.000$	Sim
Fruchterman-Reingold	Força	≤ 10.000	Não
Yifan Hu (multinível)	Força e multinível	100 a 100.000	Não
OpenOrd	Força e anelamento simulado	100 a 1.000.000	Sim
Circular	Circular	$\leq 1.000.000$	
Radial	Circular	$\leq 1.000.000$	
Geo	Geográfico	$\leq 1.000.000$	
Kamada-Kawai	Força	$\leq 1.000.000$	Sim

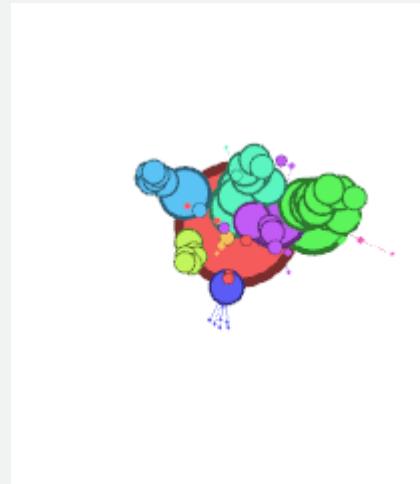




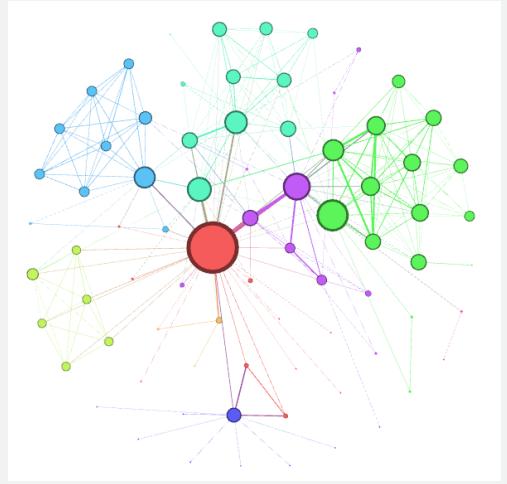
Original
(Les Miserables - Gephi)



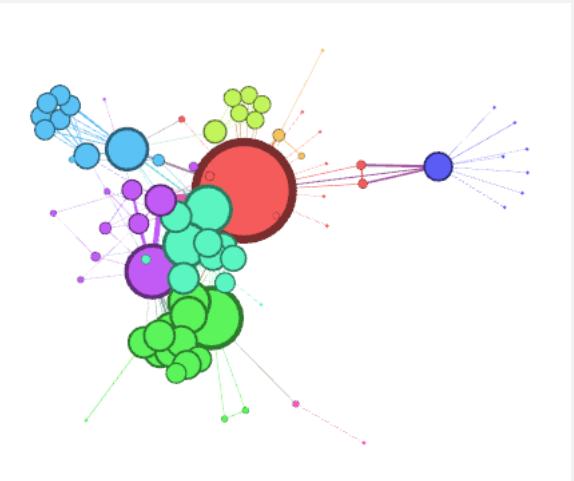
Force Atlas



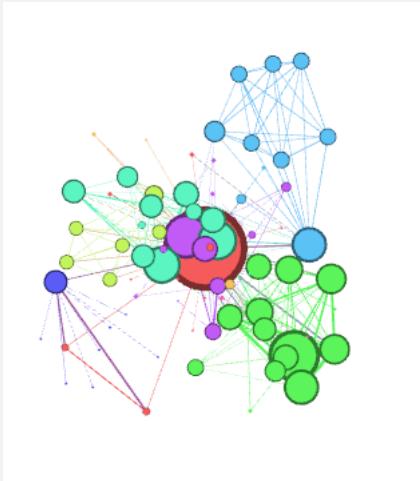
Force Atlas 2



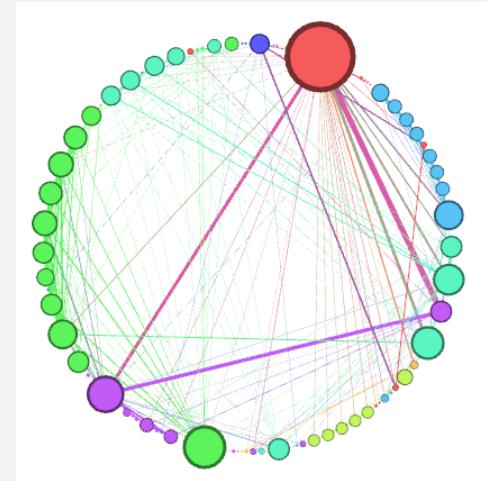
Fruchterman-Reingold



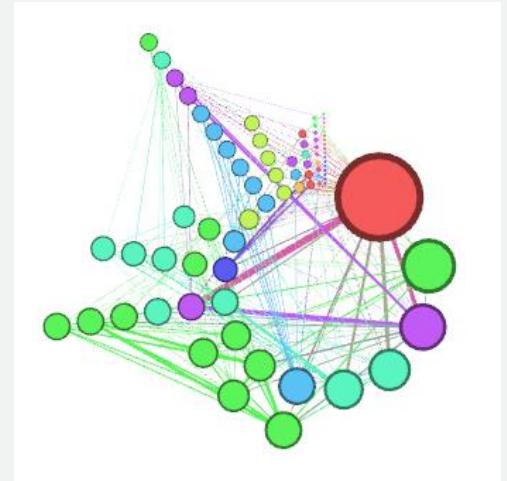
Yifan Hu



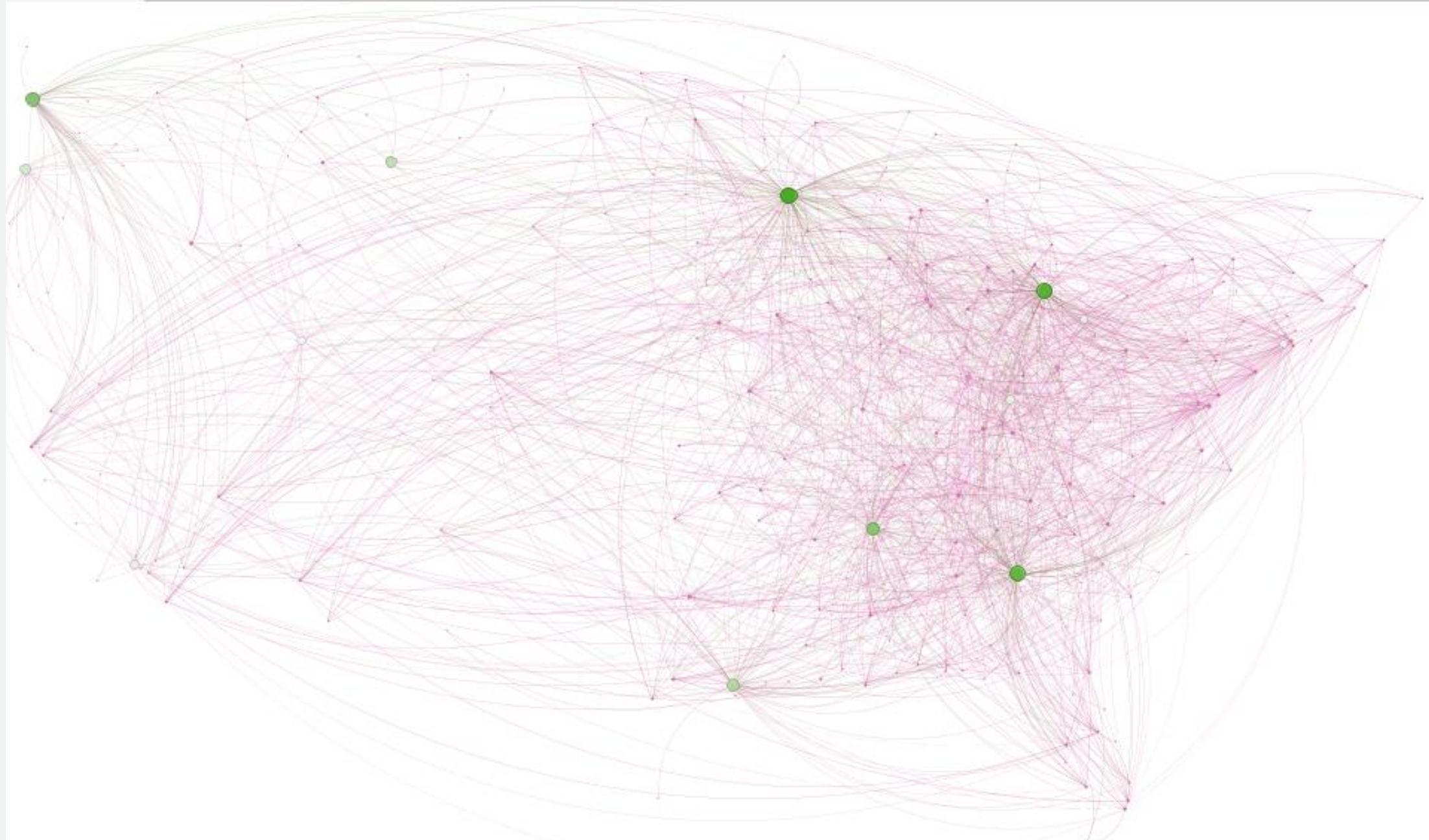
OpenOrd



Circular



Radial



Geo

(US Airports - Gephi)

Próximos passos...



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