Digital Signal Processing Laboratory

EC3P002

Lab report

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Experiment 4 : Analytical and Recursive solution of difference equations

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Contents

1 Solve the following second order difference equation using mathematics.

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

When the input is $x(n) = 4^n u(n)$. Find the analytical solution and plot it.

1.1 Analytical Solution

Given equation

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Taking z transform on both sides

$Y(z) - 3z^{-1}Y(z) - 4z^{-2}Y(z) = X(z) + 2z^{-1}X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1}}{1 - 3z^{-1} - 4z^{-2}}$ = Transfer function

When $x(n) = 4^n u(n) \longleftrightarrow X(z) = \frac{1}{1-4z^{-1}}$

$Y(z) = H(z)X(z) = \frac{(1+2z^{-1})}{(1-3z^{-1}-4z^{-2})(1-4z^{-1})}$

$\frac{Y(z)}{z} = \frac{z(z+2)}{(z-4)^2(z+1)}$

Let $\frac{Y(z)}{z} = P(z)$

$P(z) = \frac{z(z+2)}{(z-4)^2(z+1)} = \frac{A}{(z-4)} + \frac{B}{(z-4)^2} + \frac{C}{(z+1)}$

$C = (z+1)P(z)\Big|_{z=1} = -1/25$

$B = (z-4)^2 P(z)\Big|_{z=4} = 24/5$

$A = \frac{d}{dz}\left((z-4)^2 P(z)\right)\Big|_{z=4} = 26/25$

$$\frac{Y(z)}{z} = \frac{36}{25}\frac{z^{-1}}{(1-4z^{-1})} + \frac{24}{5}\frac{z^{-2}}{(1-4z^{-1})^2} - \frac{1}{25}\frac{z^{-1}}{(1+z^{-1})}$$

$$Y(z) = \frac{36}{25}\frac{1}{(1-4z^{-1})} + \frac{24}{5}\frac{z^{-1}}{(1-4z^{-1})^2} - \frac{1}{25}\frac{1}{(1+z^{-1})}$$

if $x(n) \longleftrightarrow \frac{1}{1-4z^{-1}}$ $\quad x(n) = 4^n u(n)$

$$n x(n) \longleftrightarrow -z\frac{dX(z)}{dz} = \frac{-z(-1)(-4)(-1)z^{-2}}{(1-4z^{-1})^2}$$

$$n\,4^n u(n) = \frac{4z^{-1}}{(1-4z^{-1})^2}$$

$$Y(n) = \frac{36}{25}4^n u(n) + \frac{6}{5}n\,4^n u(n) - \frac{1}{25}(-1)^n u(n)$$

1.2 Result

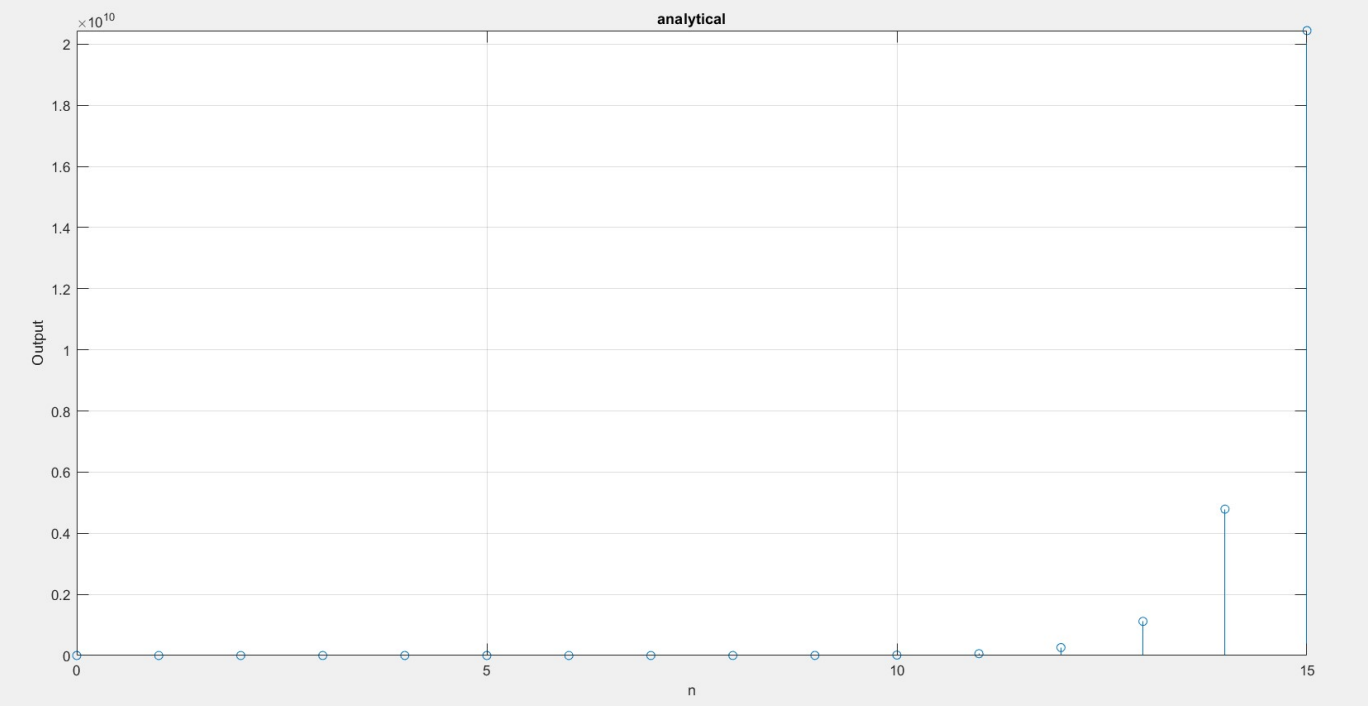


Figure 1: Output through analytical Function

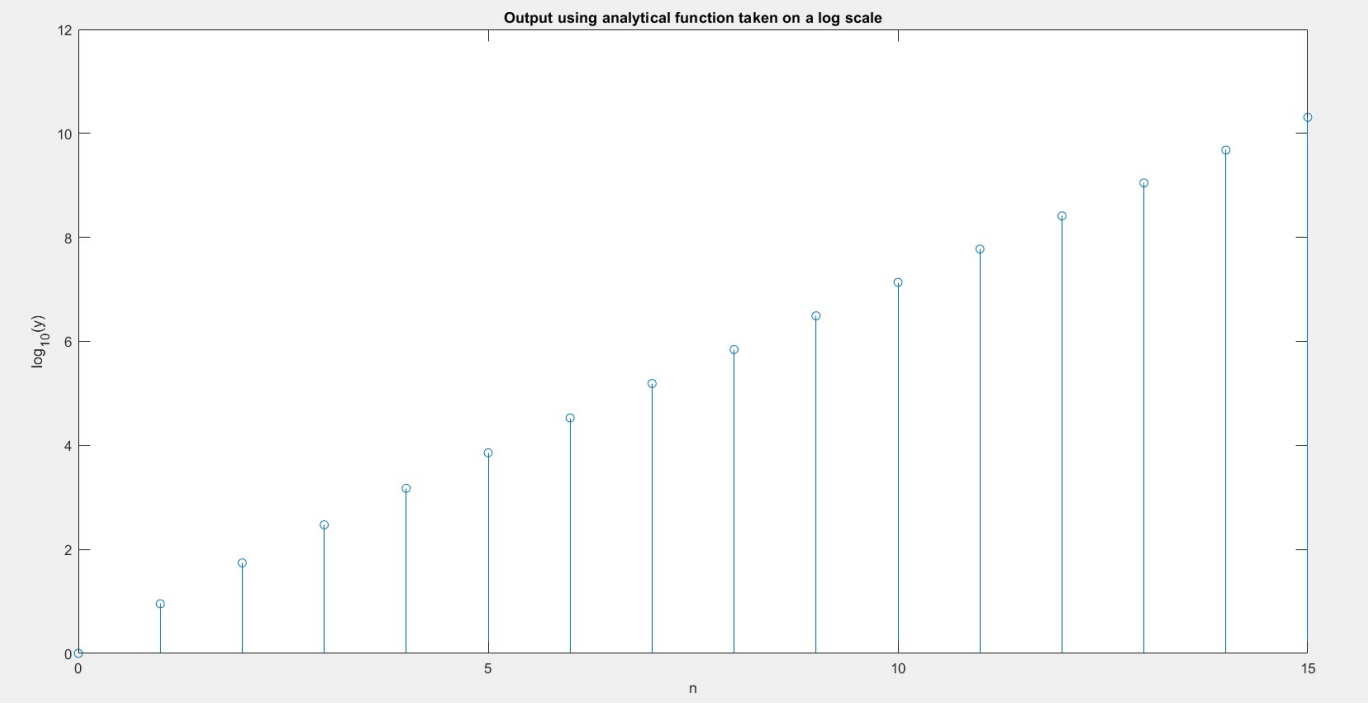


Figure 2: Output through analytical Function (on a log scale)

2 Recursive Solution through MATLAB.

Note:Refer appendix Section at the end of the document for the complete code

2.1 Results

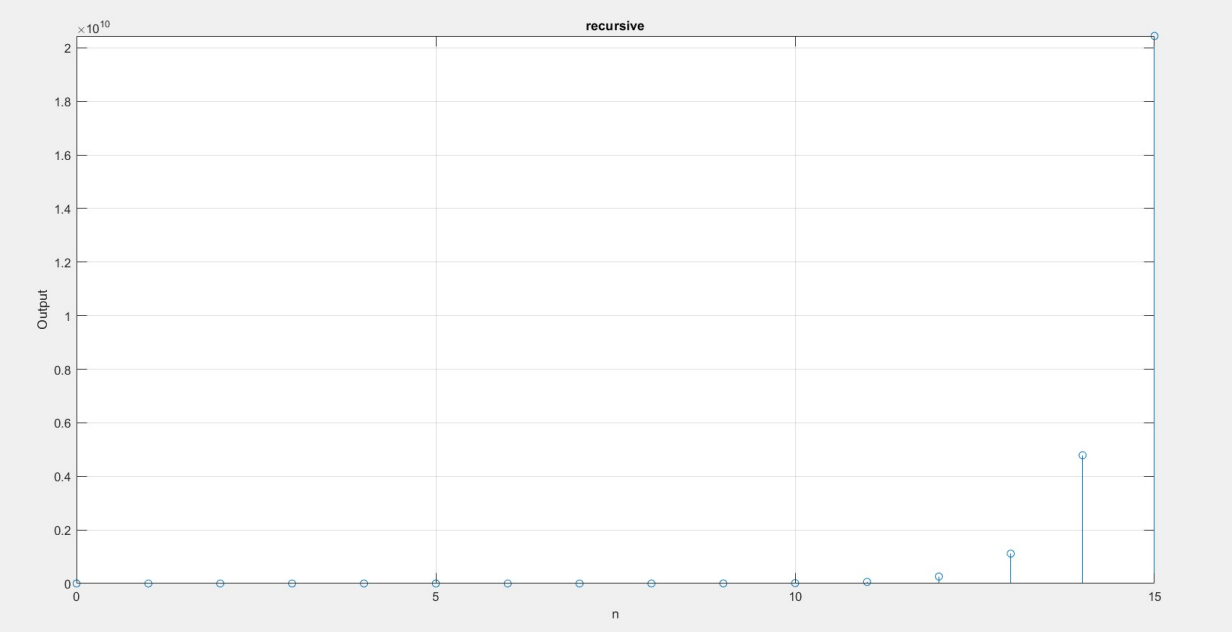


Figure 3: Output through recursive Function

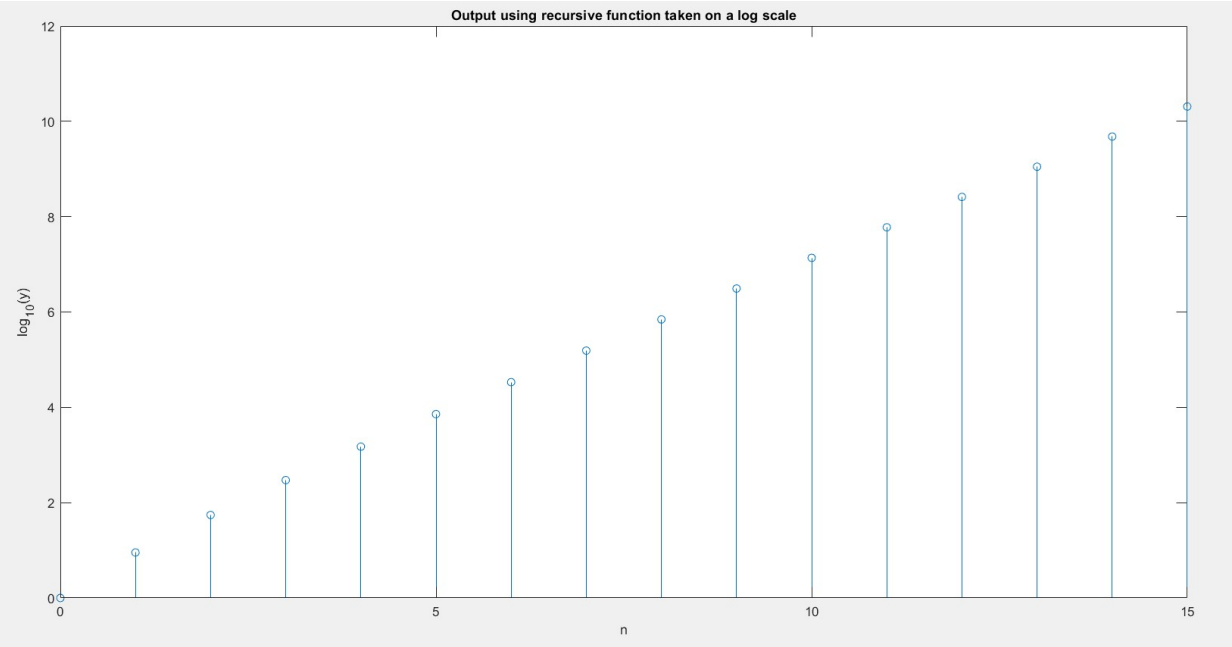


Figure 4: Output through Recursive Function (on a log scale)

3 Observations

3.1 Command window output

For Analytical Solution

```
y =

  Columns 1 through 5

    1.000000000000000e+00    9.000000000000000e+00    5.500000000000000e+01    2.970000000000000e+02    1.495000000000000e+03

  Columns 6 through 10

    7.209000000000000e+03    3.375099999999999e+04    1.546650000000000e+05    6.973030000000000e+05    3.103785000000000e+06

  Columns 11 through 15

    1.367343100000000e+07    5.972688999999999e+07    2.590402150000000e+08    1.116691497000000e+09    4.788888535000000e+09

  Column 16

    2.044404432900000e+10
```

For Recursive Solution

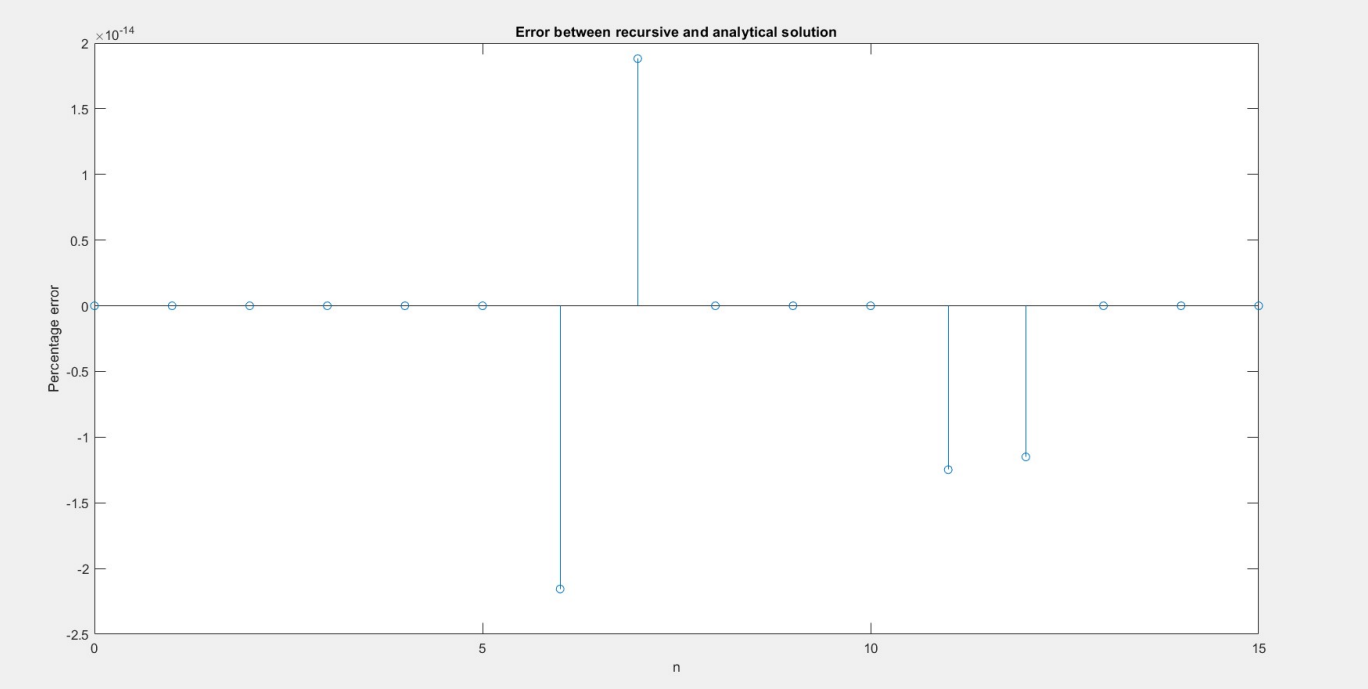
```
out =

  Columns 1 through 6

    1.000000000000000e+00    9.000000000000000e+00    5.500000000000000e+01    2.970000000000000e+02    1.495000000000000e+03    7.209000000000000e+03

  Columns 7 through 12

    3.375100000000000e+04    1.546650000000000e+05    6.973030000000000e+05    3.103785000000000e+06    1.367343100000000e+07    5.972688900000000e+07

  Columns 13 through 16

    2.590402150000000e+08    1.116691497000000e+09    4.788888535000000e+09    2.044404432900000e+10
```

For error

```
>> y-out

ans =

  Columns 1 through 6

                         0                        0                        0                        0                        0                        0

  Columns 7 through 12

   -7.275957614183426e-12    2.910383045673370e-11                        0                        0                        0   -7.450580596923828e-09

  Columns 13 through 16

   -2.980232238769531e-08                        0                        0                        0
```

3.2 Error

It can be observed that the error is of the range $10^{(-14)}$. Hence we can say that the output obtained is same both through analytical and recursive solution



4 Conclusion

Thus, The analytical solution (Using Z transform) as well as the recursive solution (Through MATLAB function) for the given problem statement was found. The outputs for both the cases were plotted and were found the same.
For better view of the output, it was also plotted on a log scale. Then, the error was found, which was of the range 10^{-14}, which can be neglected (and was maybe caused because our output was shooting up to a very large value, and some minute computational error could be there for those cases).
All the analysis was done for n=1:15.For large values of n, the code took a very long time to run as we are using a recursive function which has a time complexity of 2^n.

5 Appendix

```
clc
clear all
close all
n=0:1:15;
%code by saurabh 19EE01008 :DSP LAB EXP 4
u_n = [n>=0];
y = (26/25)*(4.^n).*(u_n) + (6/5)*n.*(4.^n).*u_n - (((-
1).^n).*u_n)/25;
figure()
stem(n,y)
title('analytical')
xlabel('n');
ylabel('Output');
grid on
axis tight
```

```
out =[];
for i=0:1:15
    out = [out y_output(i)];
end
figure()
stem(n,out)
title('recursive')
xlabel('n');
ylabel('Output');
grid on
axis tight

e = ((y-out)./out)* 100;
figure()
stem(n,e);
xlabel('n');
ylabel('Percentage error');
title('Error between recursive and analytical solution')

figure()
stem(n,log10(out));
xlabel('n')
ylabel('log_1_0(y)');
title('Output using recursive function taken on a log scale');

figure()
stem(n,log10(y));
xlabel('n')
ylabel('log_1_0(y)');
title('Output using analytical function taken on a log scale');
```

```
function p = x_input(n)
if(n<0)
    p = 0;
else
    p = 4^n;
end
end


function q = y_output(n)
if(n<0)
    q=0;
else
    q = 3*y_output(n-1) +4*y_output(n-2) + x_input(n) + 2*x_input(n-
1);
end
end
```