

Tutorial 1

Signals & Systems

Vaibhav Singh
58, DGA

Page No.

Date 15/02/22

Q. Q.

(1) $x(t) = \cos\left(\frac{\pi}{3}t\right) + \sin\left(\frac{\pi t}{4}\right)$

Soln. $x_1(t) = \cos\left(\frac{\pi}{3}t\right), 2\pi f_0 = \frac{\pi}{3}, f_0 = \frac{1}{6}, T_1 = 6$

$x_2(t) = \sin\left(\frac{\pi t}{4}\right), 2\pi f_0 = \frac{\pi}{4}, f_0 = \frac{1}{8}, T_2 = 8$

$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4}$ rational number.

as $x_1(t)$ and $x_2(t)$ are periodic
 $\Rightarrow x(t)$ is periodic

$T = \text{LCM}(6, 8) = 24$ T = 24 samples

(2) $x[n] = \cos^2\left[\frac{\pi n}{8}\right]$

Method.1 $x[n] = \frac{1}{2} + \cos\left[\frac{\pi n}{4}\right]/2 \quad \leftarrow (\because \cos^2\theta = \frac{1+\cos 2\theta}{2})$

Standard form $x[n] = B + A \cos 2\pi f_0 n$

$2\pi f_0 = \frac{\pi}{4} \quad f_0 = \frac{1}{8}$

T = 8 samples

As $x[n]$ is a sinusoidal signal
 $\therefore x[n]$ is periodic.

Method 2

$$x[n] = \cos^2\left(\frac{\pi n}{8}\right)$$

$$x[n] = \frac{1 + \cos\left(\frac{\pi n}{4}\right)}{2} \quad \leftarrow \left(\because \cos^2\theta = \frac{1 + \cos 2\theta}{2}\right)$$

let M and N be integers,

$$\underline{x[n+N] = \frac{1 + \cos\left(\frac{\pi}{4}(n+N)\right)}{2}}$$

$$\underline{= \frac{1 + \cos\left(\frac{n\pi}{4} + \frac{N\pi}{4}\right)}{2}}$$

$$\text{Since, } \cos(\theta + 2\pi M) = \cos\theta$$

\therefore for periodicity $\frac{N\pi}{4} = 2\pi M$, integral multiple of 2π .

$$\therefore N = 8M$$

$$\text{Let } M = 1, N = 8$$

$$\underline{x[n+N] = \frac{1 + \cos\left[\frac{n\pi}{4} + 2\pi M\right]}{2}}$$

$$\underline{= \frac{1 + \cos\left[\frac{n\pi}{4}\right]}{2}}$$

$$\therefore x[n+N] = \cos^2\left(\frac{n\pi}{8}\right)$$

As $x[n] = x[n+N] \rightarrow x[n]$ is periodic

N = T = 8 samples \rightarrow Fundamental Period.

$$3. \quad x[n] = \sin\left[\left(\frac{6\pi}{7}\right)n + 1\right]$$

Method.1

$$\because \sin(A+B) = \sin A \cdot \cos B + \cos A \sin B$$

$$x[n] = \sin\left[\left(\frac{6\pi}{7}\right)n\right] \cdot \cos(1) + \cos\left[\left(\frac{6\pi}{7}\right)n\right] \cdot \sin(1)$$

$$x[n] = \sin\left[\left(\frac{6\pi}{7}\right)n\right]$$

$x[n]$ is sinusoidal

\therefore it is periodic.

Comparing with standard form,

$$x[n] = A \sin[2\pi f_0 n]$$

$$2\pi f_0 = \frac{6\pi}{7}, \quad f_0 = \frac{3\pi}{7} \text{ cycles per samples}$$

$$f_0 = \frac{M}{N}, \quad M \text{ is an integer.}$$

N = fundamental period $\therefore \boxed{N = 7 \text{ samples}}$

Method 2

$$x[n+N] = \sin\left[\frac{6\pi}{7}(n+N) + 1\right]$$

$$= \sin\left[\frac{6\pi n}{7} + 1 + \frac{6\pi N}{7}\right]$$

$$\therefore \sin(\theta + 2\pi M) = \sin \theta$$

\therefore for periodicity $\frac{6\pi N}{7}$ = integral multiple of 2π

$$\therefore \frac{6\pi N}{7} = 2\pi M, \quad M, N \text{ are integers.}$$

Here $N \rightarrow$ integer, if $M = 3, 6, 9, 12 \dots$

Let $M = 3$, $\therefore N = 7$.

where $N = 7$ samples,

$$\begin{aligned} x[n+N] &= \sin\left[\frac{6\pi n}{7} + 1 + \frac{6\pi}{7} \times 7\right] \\ &= \sin\left[\frac{6\pi n}{7} + 1 + 3(2\pi)\right] \end{aligned}$$

$$x[n+N] = \sin\left[\frac{6\pi n}{7} + 1\right] = x[n] \quad \because \sin(\theta + 2\pi M) = \sin\theta$$

$$\text{As } x[n+N] = x[n]$$

It is periodic with $N = 7$ samples as fundamental period.

4. $x(t) = \sin\left(\frac{2\pi t}{6}\right) - \cos(\pi t)$

$$\text{Let } x_1(t) = \sin\left(\frac{2\pi t}{6}\right), \quad x_2 = \cos(\pi t)$$

Standard form: $A \sin(2\pi f_0 t)$

Comparing with $x_1(t)$, we get

$$2\pi f_0 = \pi/3$$

$$f_0 = \frac{1}{6}, \quad T = \frac{1}{f_0} = 6.$$

Standard form: $A \cos(2\pi f_0 t)$, comparing with $x_2(t)$

$$2\pi f_0 = \pi, \quad f_0 = \frac{1}{2}, \quad T = 2$$

$\therefore T_1/T_2 = 6/2 = 3$. Since $x_1(t)$ and $x_2(t)$ are both periodic and ratio is rational

$\Rightarrow x(t)$ is periodic

$$\text{LCM}(6, 2) = \boxed{6 \text{ samples} = T} \rightarrow \text{fundamental period.}$$

Q.Q1.

$$x(t) = 0.9e^{-3t} \cdot u(t)$$

it is an exponential aperiodic signal
 \therefore Energy signal.

$$x(t) = 0.9e^{-3t}, u(t); \text{ for all } t$$

$$x(t) = 0.9e^{-3t}; \text{ for } t \geq 0$$

$$\begin{aligned} \therefore \int_{-T}^T |x(t)|^2 dt &= \int_0^T |0.9e^{-3t}|^2 dt \\ &= (0.9)^2 \int_0^T e^{-6t} dt \\ &= 0.81 \left[\frac{e^{-6t}}{-6} \right]_0^T \\ &= \frac{0.81}{6} [e^{-6T} - 1] \end{aligned}$$

$$\begin{aligned} E = \text{Energy} &= \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{0.81}{6} [e^{-6T} - 1] \\ &= \frac{0.81}{6} [e^{-\infty} - 1] = \frac{0.81}{6} (-1) \end{aligned}$$

$$E = 0.135 \text{ J} \rightarrow \text{Energy is constant}$$

\therefore power = 0

$$\begin{aligned} P = \text{Power} &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \times \frac{0.81}{6} [e^{-6T} - 1] \\ &= \frac{1}{2 \cdot \infty} \times \frac{0.81}{6} \times [e^{-\infty} - 1] = [0 = P] \end{aligned}$$

$\therefore x(t)$ is an energy signal
 As $x(t)$ is aperiodic signal with constant energy of 0.135J.

Q.

$$x[n] = u[n]$$

$x[n]$ is a discrete signal.

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

∴ range = 0 to ∞

$$\text{Energy} = E = \sum_{n=-\infty}^{n=\infty} |x[n]|^2 = \sum_{n=+\infty}^{n=\infty} u[n] = 1 + 1 + 1 + 1 + \dots \infty$$

$E = \infty \rightarrow \text{infinite energy}$

$$\begin{aligned} \text{Power} = P &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2 \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} u[n] \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \underbrace{[1+1+1+\dots+1]}_{N+1 \text{ terms}} \end{aligned}$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} (N+1)$$

$$= \lim_{N \rightarrow \infty} \frac{N \left(1 + \frac{1}{N}\right)}{N \left(2 + \frac{1}{N}\right)} = \frac{1 + \frac{1}{\infty}}{2 + \frac{1}{\infty}} = \frac{1+0}{2+0}$$

$P = \frac{1}{2} \text{ Watts} \rightarrow \text{finite power}$

∴ E is infinite and P is finite

$x[n]$ is a power signal.

Q.

$$x[n] = \left(\frac{1}{6}\right)^n u[n]$$

$x[n]$ is a discrete signal

$u[n]$ is a unit step signal

$$\begin{aligned} x[n] &= \left(\frac{1}{6}\right)^n u[n] = (0.166)^n u[n]; \text{ for all } n \\ &= (0.166)^n; \text{ for } n \geq 0 \end{aligned}$$

Range = 0 to ∞

$$\begin{aligned} \text{Energy} = E &= \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} (0.166)^{2n} \\ &= \sum_{n=0}^{\infty} (0.0275)^n \quad \left(\because \sum_{n=0}^{\infty} t^n = \frac{1}{1-t} \right) \\ &= \frac{1}{1-0.0275} \end{aligned}$$

$$E = 1.0282 \text{ J} \rightarrow \text{Finite Energy}$$

$$\begin{aligned} \text{Power} = P &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (0.166^{2n}) \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (0.0275)^n \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} (0.0275)^n \\ &= \frac{1}{2 \cdot 0 + 1} \end{aligned}$$

$$P = 0 \text{ Watts} \rightarrow \text{zero Power}$$

Since E is finite and P is zero,

$\therefore x[n]$ is an Energy signal.

$$\underline{4.} \quad x(t) = 3 \cos(5\omega_0 t)$$

$x(t)$ is continuous signal.

$$x(t) = 3 \cos(5\omega_0 t) \quad \omega_0 = 2\pi f_0$$

Also cosinusoidal signal

⇒ periodic and power signal

$$\text{Energy} = \infty$$

$$\therefore \int_{-T}^T |x(t)|^2 dt = \int_{-T}^T |3 \cos(5\omega_0 t)|^2 dt \\ = 9 \int_{-T}^T \cos^2(5\omega_0 t) dt$$

$$= 9 \int_{-T}^T \frac{1 + \cos(10\omega_0 t)}{2} dt$$

$$= 9 \left[\frac{1}{2} \int_{-T}^T dt + \frac{1}{2} \cdot \cancel{\int_{-T}^T \cos(10\omega_0 t) dt}^0 \right]$$

$$= 9 \left[\frac{t}{2} \right]_{-T}^T = \frac{9}{2} \times 2T = \boxed{9T}$$

$$\text{Energy} = E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{9T}{2} = \infty$$

$$\boxed{E = \infty}$$

$$\text{Power} = P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \times 9T = \lim_{T \rightarrow \infty} \frac{9}{2} = \boxed{4.5 \text{ Watts}}$$

∴ E is infinite and P is constant

$x(t)$ is periodic and power signal.

Q. ③.

$$(1). \quad X(t) = t^3 + 3t$$

$$X(-t) = -t^3 - 3t$$

$$\text{Even part: } X_e(t) = \frac{1}{2} [X(t) + X(-t)] = \frac{1}{2} [t^3 + 3t - t^3 - 3t]$$

$$X_e(t) = 0$$

$$\text{Odd part: } X_o(t) = \frac{1}{2} [X(t) - X(-t)] = \frac{1}{2} [t^3 + 3t + t^3 - 3t]$$

$$X_o(t) = t^3 + 3t$$

$\therefore X(t)$ is an odd signal.

$$(2). \quad X[n] = \cos(n) + \sin(n) + \cos(n) \sin(n)$$

$$X[-n] = \cos(n) - \sin(n) - \sin(n) \cdot \cos(n)$$

↪ Folded Signal

$$\text{Even part: } X_e[n] = \frac{1}{2} (X[n] + X[-n])$$

$$X_e[n] = \frac{1}{2} [2\cos(n)] = \cos(n) = X_e[n]$$

$$\text{Odd part: } X_o[n] = \frac{1}{2} (X[n] - X[-n])$$

$$X_o[n] = \sin(n) + \cos(n) \cdot \sin(n)$$

$$3. \quad x(t) = \cos^2\left(\frac{\pi t}{2}\right)$$

$$x(t) = \frac{1}{2} + \frac{\cos(\pi t)}{2}, \quad \therefore \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\begin{aligned} x(-t) &= \frac{1}{2} + \frac{\cos(-\pi t)}{2} \\ &= \frac{1}{2} + \frac{\cos(\pi t)}{2} \end{aligned} \quad \therefore \cos(-\theta) = \cos \theta$$

Even part: $\frac{1}{2} [x(t) + x(-t)]$

$$x_e(t) = \frac{1}{2} [1 + \cos(\pi t)]$$

Odd part: $\frac{1}{2} [x(t) - x(-t)] = \frac{1}{2} \left[\frac{1 + \cos(\pi t)}{2} - \frac{1 + \cos(-\pi t)}{2} \right]$

$$x_o(t) = 0$$

$$4. \quad x[n] = u[n] - u[n-4]$$

$$u[n] = \begin{cases} 1 & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

$$u[n-4] = \begin{cases} 1 & ; n \geq 4 \\ 0 & ; n < 4 \end{cases}$$

$$x[n] = u[n] - u[n-4]$$

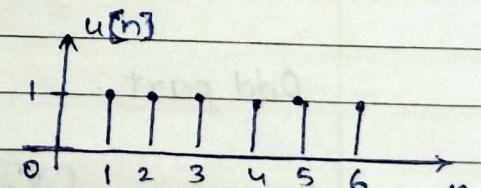
$$\text{when } n=0, \quad x[0] = 1 - 0 = 1$$

$$n=1, \quad x[1] = 1 - 0 = 1$$

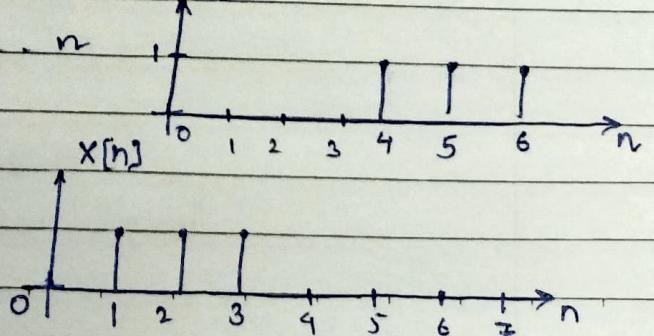
$$n=2, \quad x[2] = 1 - 0 = 1$$

$$n=3, \quad x[3] = 1 - 0 = 1$$

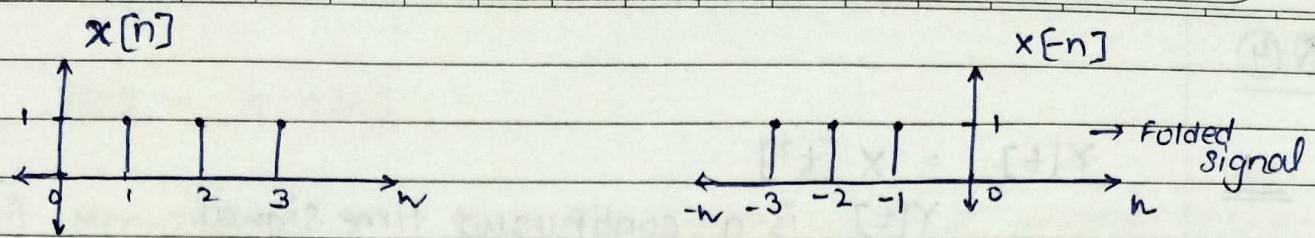
$$n=4, \quad x[4] = 1 - 1 = 0$$



$$u[n-4]$$

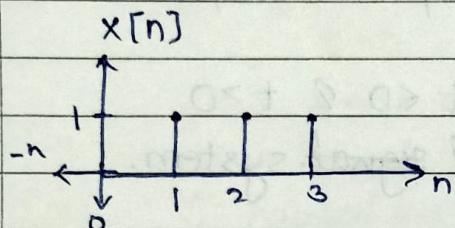


$$x[n]$$



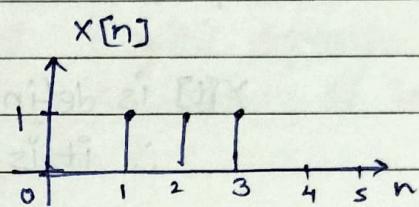
Even part $X_e[n]$

$$= \frac{1}{2} [x[n] + x[-n]]$$

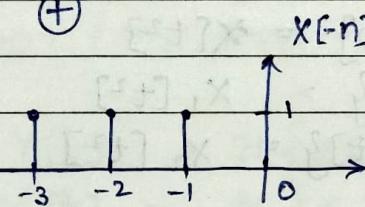


Odd part $X_o[n]$

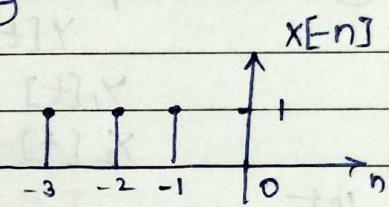
$$= \frac{1}{2} [x[n] - x[-n]]$$



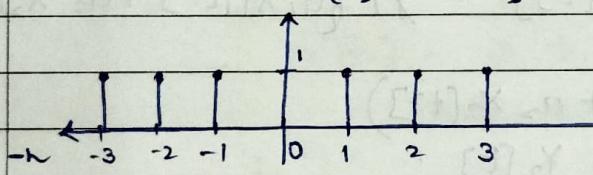
\oplus



\ominus

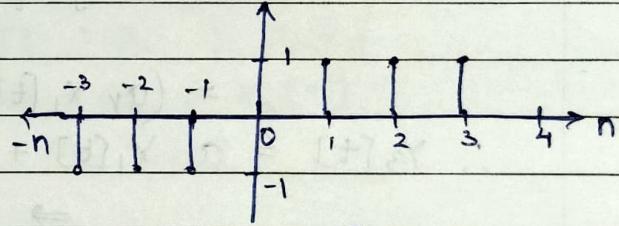


$$= x[n] + x[-n]$$

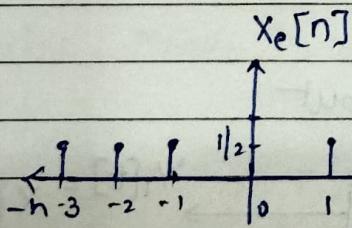


$\div 2$

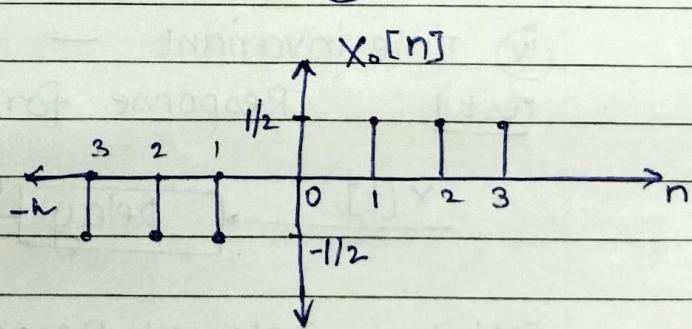
$$= x[n] - x[-n]$$



$\div 2$



Even Part.



Odd Part.

Q(4)

$$\underline{1.} \quad Y[t] = X[t^2]$$

$Y[t]$ is a continuous time signal.

(i) Memory - $\because Y(t)$ depends on future values of input less
it is not memory less. system.

(ii). causal / non-causal - $Y[t]$ depends on future values of input.

$Y[t]$ is defined for both $t < 0$ & $t > 0$

\therefore it is a non-causal signal. system.

(iii). linear / Non-linear -

$$Y[t] = H\{x(t)\} = X[t^2]$$

$$Y_1[t] = H\{x_1[t]\} = X_1[t^2]$$

$$Y_2[t] = H\{x_2[t]\} = X_2[t^2]$$

Let

$$X_3[t] = a_1 X_1[t] + a_2 X_2[t]$$

$$\therefore Y_3[t] = H\{x_3[t]\} = H\{a_1 X_1[t^2] + a_2 X_2[t^2]\}$$

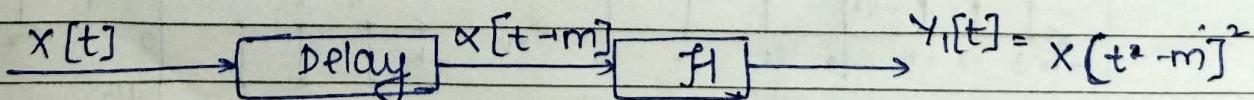
$$= (a_1 X_1[t^2] + a_2 X_2[t^2])$$

$$\therefore Y_3[t] = a_1 Y_1[t] + a_2 Y_2[t]$$

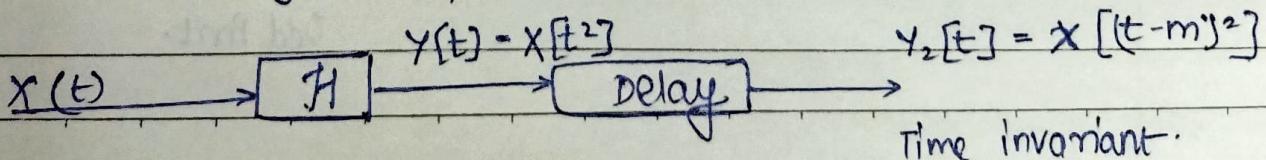
$\Rightarrow Y[t]$ is a linear system.

(iv) Time invariant -

Test 1 Response for delayed input



Test 2 Delayed Response



Time invariant.

Q.

$$Y[t] = t X[t]$$

(i) Memoryless — Since the system output depends only on the present values of input, it is a memoryless system.

(ii) causal / Non-causal —

$$\begin{aligned} t=0, \quad Y[0] &= 0 \times [0] \quad \left\{ \begin{array}{l} Y[t] \text{ depends on present} \\ \text{values of input} \end{array} \right. \\ t=1, \quad Y[1] &= 1 \times [1] \end{aligned}$$

∴ causal system.

(iii). Linear / Non-linear —

$$Y[t] = H\{X[t]\} = t X[t]$$

$$Y_1[t] = H\{X_1[t]\} = t X_1[t]$$

$$Y_2[t] = H\{X_2[t]\} = t X_3[t]$$

$$\text{Let } X_3[t] = a_1 X_1[t] + a_2 X_2[t]$$

$$Y_3[t] = a_1 t X_1[t] + a_2 t X_2[t]$$

$$= a_1 \cdot t X_1[t] + a_2 \cdot t X_2[t]$$

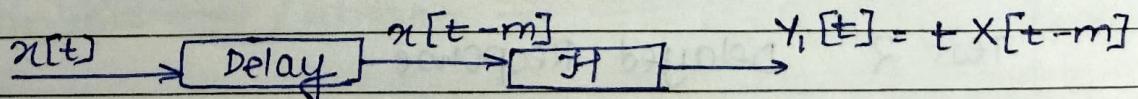
$$Y_3[t] = a_1 Y_1[t] + a_2 Y_2[t]$$

∴ linear system.

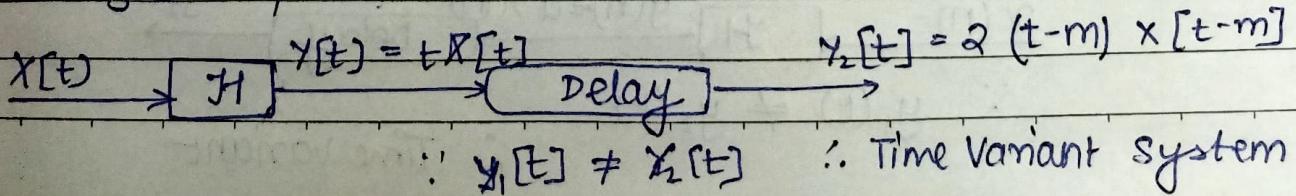
(iv). Time-invariant / Variant —

$$Y[t] = t X[t]$$

Test 1 Response for delayed input



Test 2 Delayed Response



$$\underline{3.} \quad y(n) = a^n x(n)$$

i) Memoryless - $n=0, y(0) = a^0 x(0)$ } $y(t)$ depends of present
 $n=1, y(1) = a^1 x(1)$ } values of input.
 \therefore it is memoryless.

ii) causal / Non-causal -

$n=0, y(0) = a^0 x(0)$ } Depends on present values of
 $n=1, y(1) = a^1 x(1)$ } input
 \therefore causal system.

iii) Linear / Non-linear -

$$y(n) = H\{x(n)\} = a^n x(n)$$

$$y_1(n) = H\{x_1(n)\} = a^n x_1(n),$$

$$y_2(n) = H\{x_2(n)\} = a^n x_2(n)$$

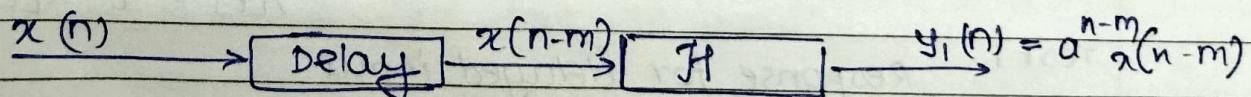
$$\text{let } x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$\begin{aligned} y_3(n) &= H\{a_1 x_1(n) + a_2 x_2(n)\} \\ &= a_1 a^n x_1(n) + a_2 a^n x_2(n) \end{aligned}$$

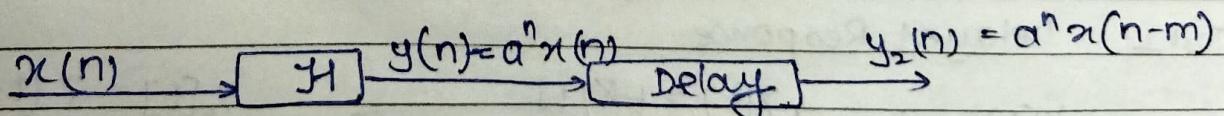
$$y_3(n) = a_1 y_1(n) + a_2 y_2(n) \quad \therefore \text{Linear System.}$$

iv) Time-invariant / Variant -

Test 1 Response for delayed input



Test 2 Delayed Response



$$\therefore y_1(t) \neq y_2(t)$$

\therefore Time variant

A. $y(n) = x(-n)$

i) Memoryless -

$n=1, y(1) = x(-1)$ } depends on past values of
 $n=2, y(2) = x(-2)$ } input
 \therefore Not a memoryless system.

ii) Causal / Non-causal - exists for $n \leq 0$ & $n > 0$

$n=-2, y(-2) = x(2)$

$n=-1, y(-1) = x(1)$ \rightarrow future input

$n=0, y(0) = x(0)$ \rightarrow present input

$n=1, y(1) = x(-1)$ \rightarrow past input

for $n < 0$ response depends on future input
 \therefore Non-causal.

iii) Linear / Non-linear.

$$y(n) = H \{ x(n) \} = x(-n)$$

$$y_1(n) = H \{ x_1(n) \} = x_1(-n)$$

$$y_2(n) = H \{ x_2(n) \} = x_2(-n)$$

Let,

$$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

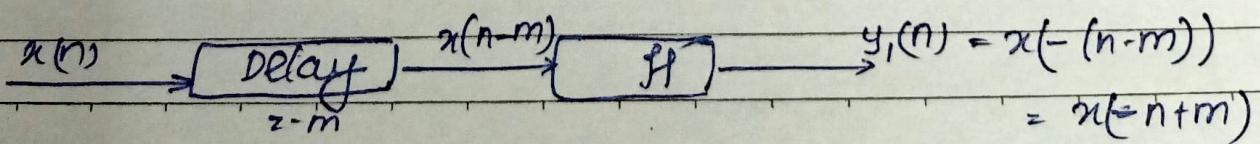
$$y_3(n) = H \{ x_3(n) \} = H \{ a_1 x_1(n) + a_2 x_2(n) \} \\ = a_1 y_1(n) + a_2 y_2(n)$$

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n) \quad \therefore \text{Linear system.}$$

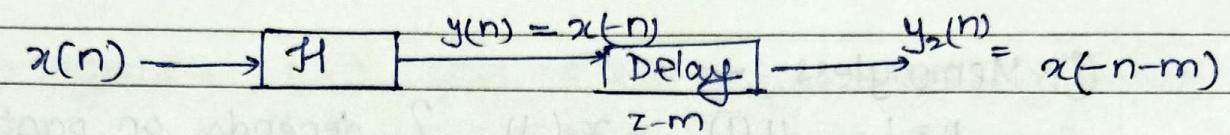
iv) Time invariant / variant -

Test 1

Response for delayed input



Test 2 Delayed Response



$$y_1(t) \neq y_2(t)$$

Time variant

Q.5

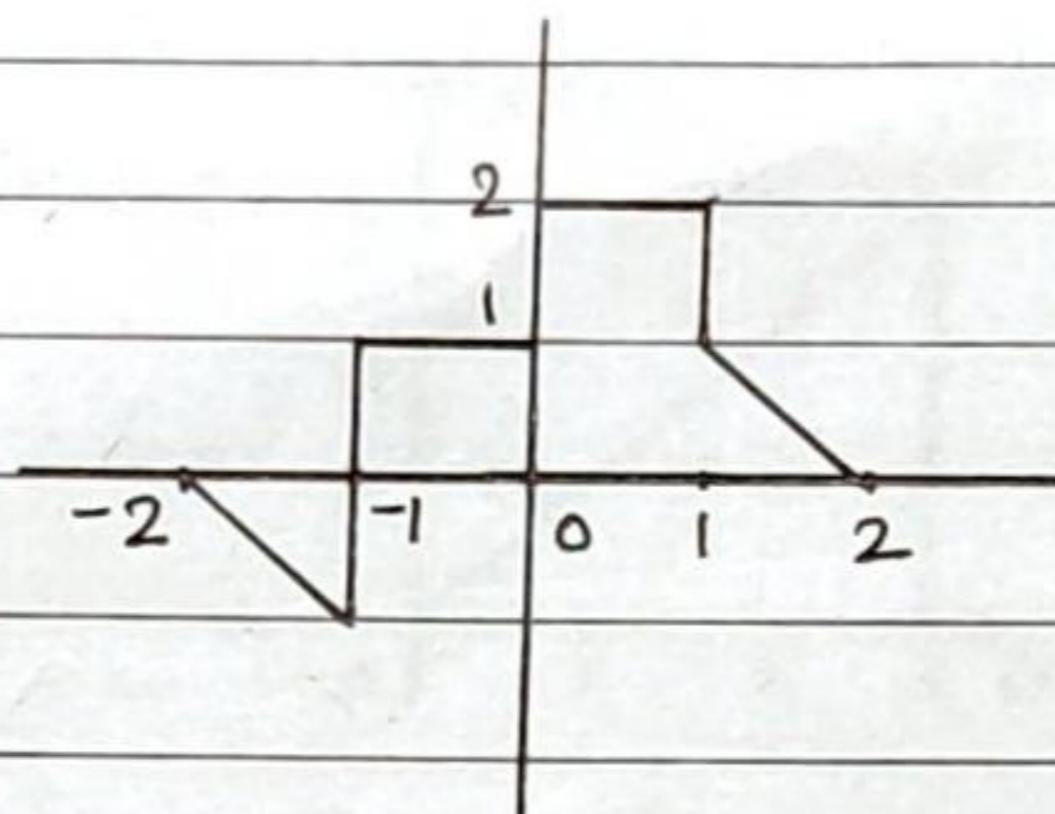
For signal $x(t)$ sketch

i) $x(-t)$

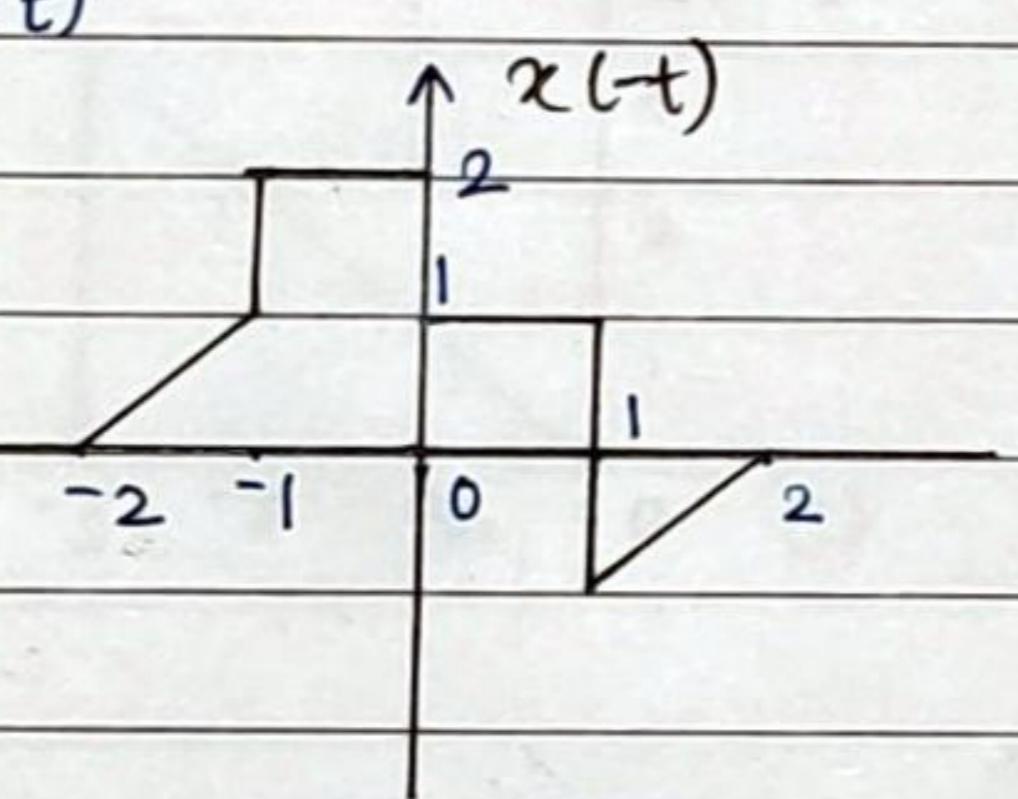
ii) $x(t+6)$

iii) $x(3t)$

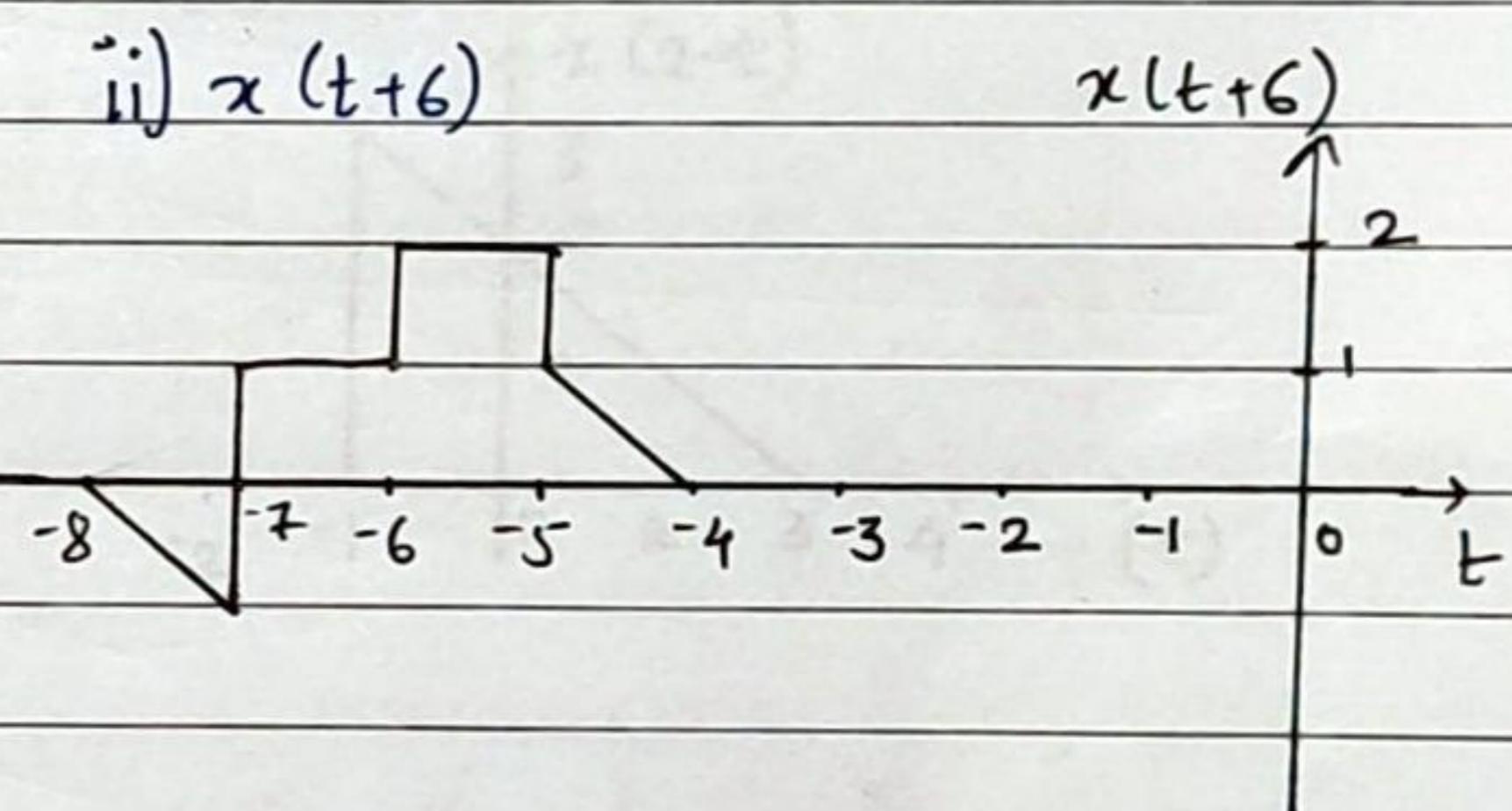
iv) $x\left(\frac{t}{2}\right)$



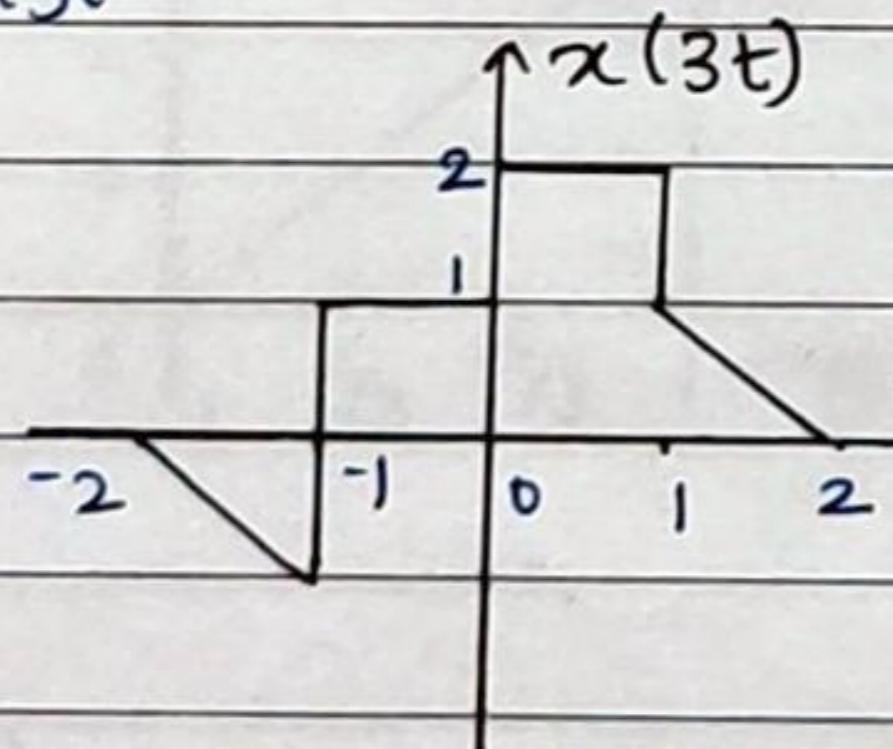
i) $x(-t)$



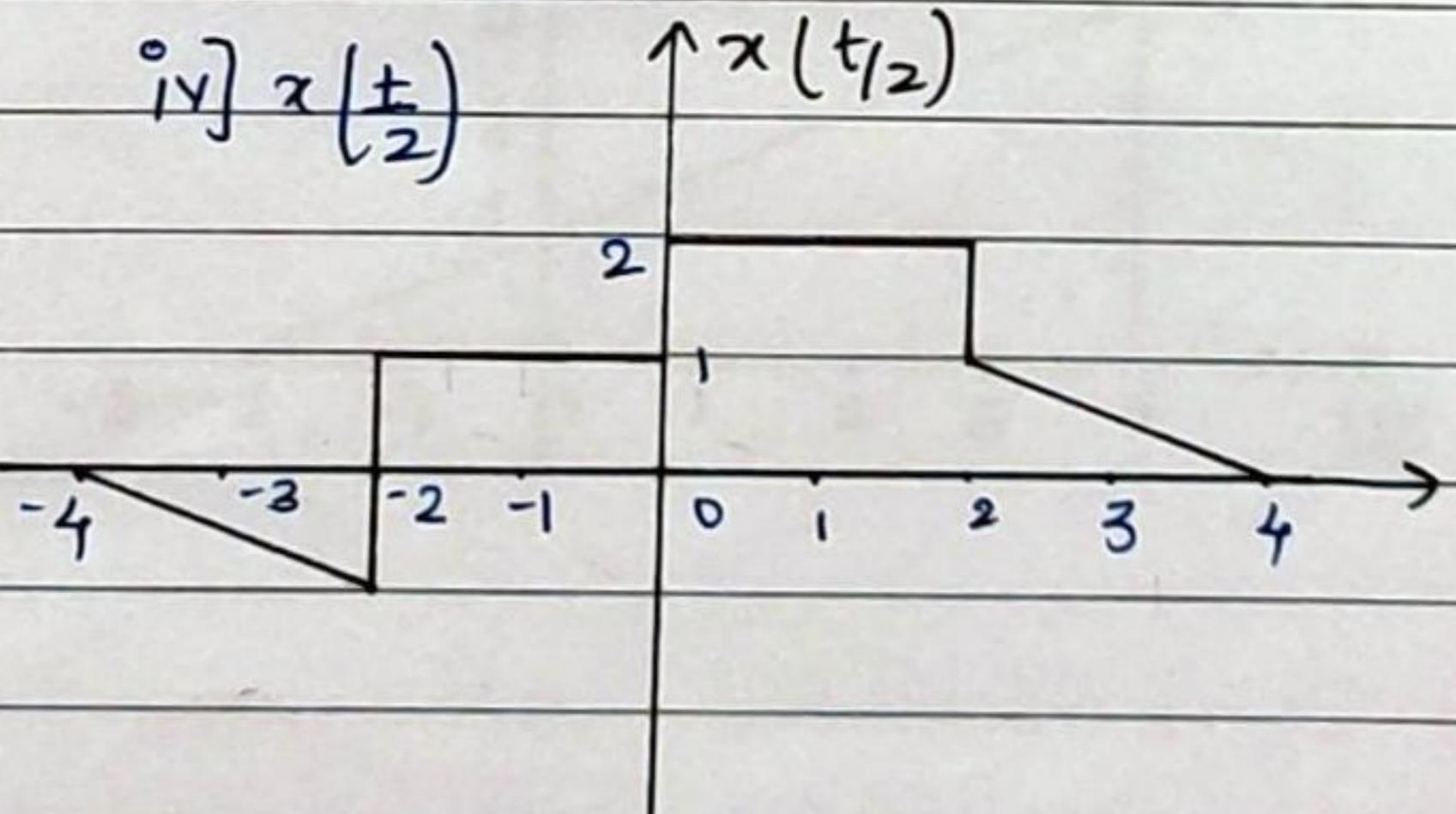
ii) $x(t+6)$



iii) $x(3t)$



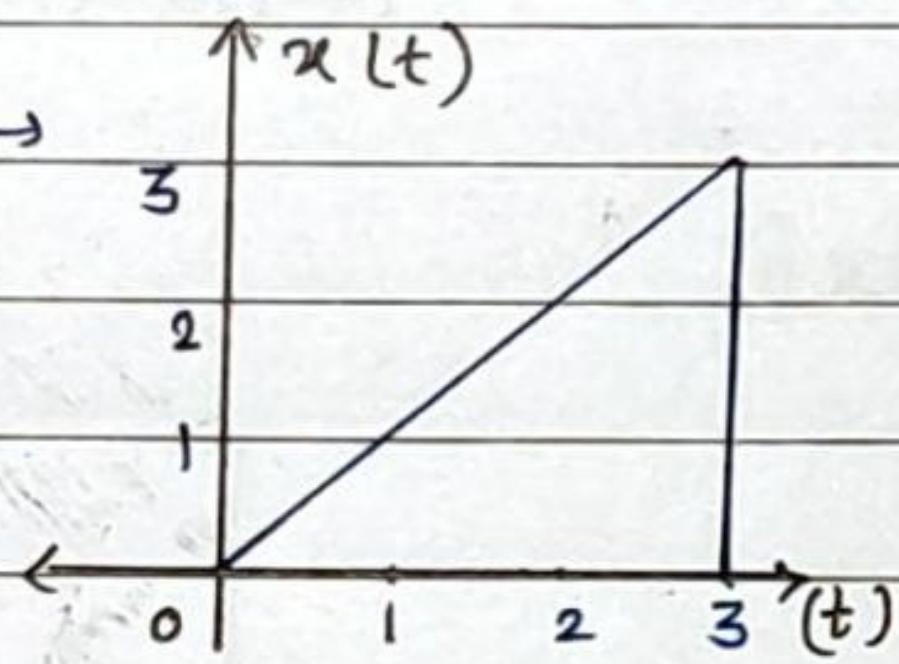
iv) $x\left(\frac{t}{2}\right)$



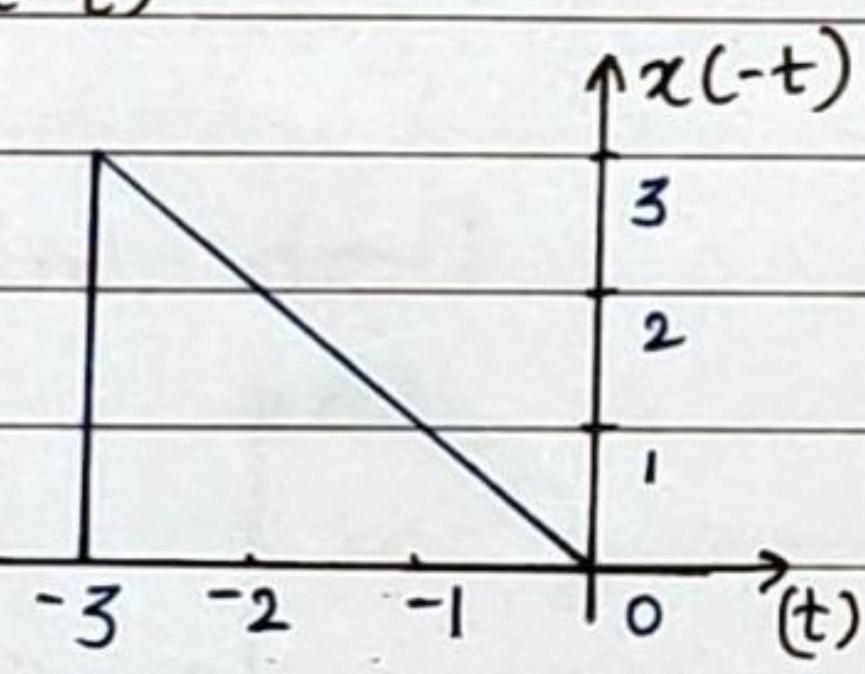
Q. 6

A continuous signal $x(t) = t$; ~~$0 \leq t \leq 3$~~ Sketch, i) $x(-t)$ ii) $x(2-t)$
 $x(t) = 0$; $t > 0$ iii) $x(3t)$ iv) $x(0.5t+1)$

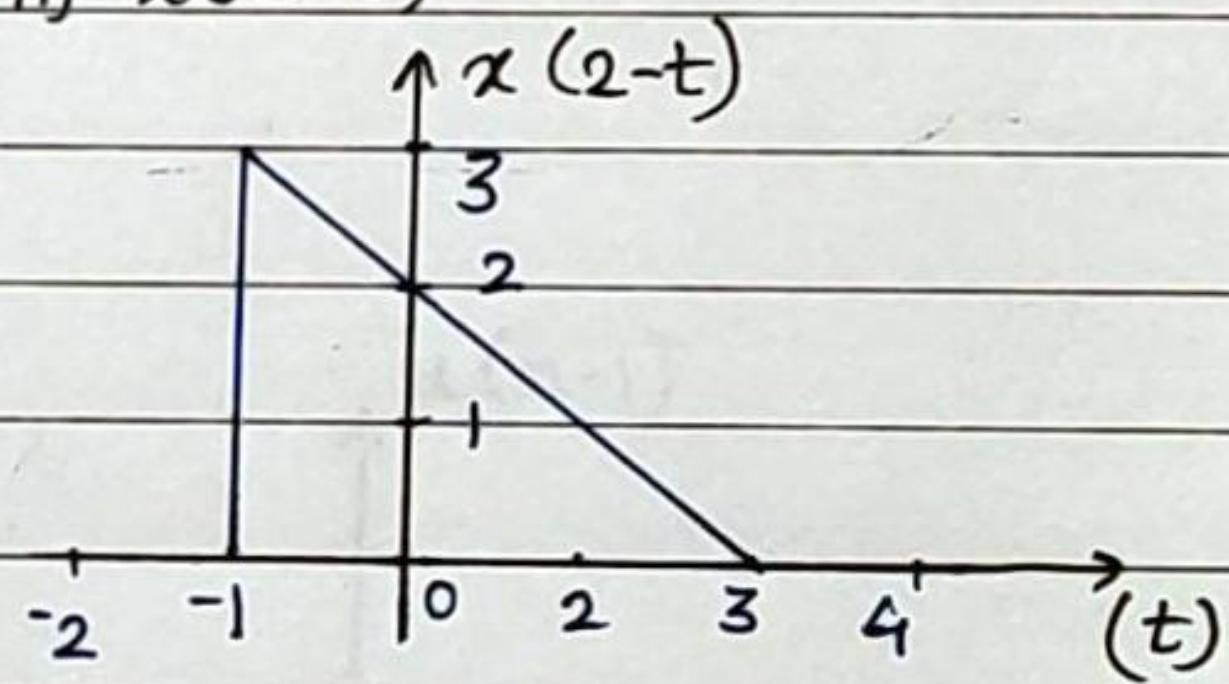
∴ The $x(t)$ signal is →



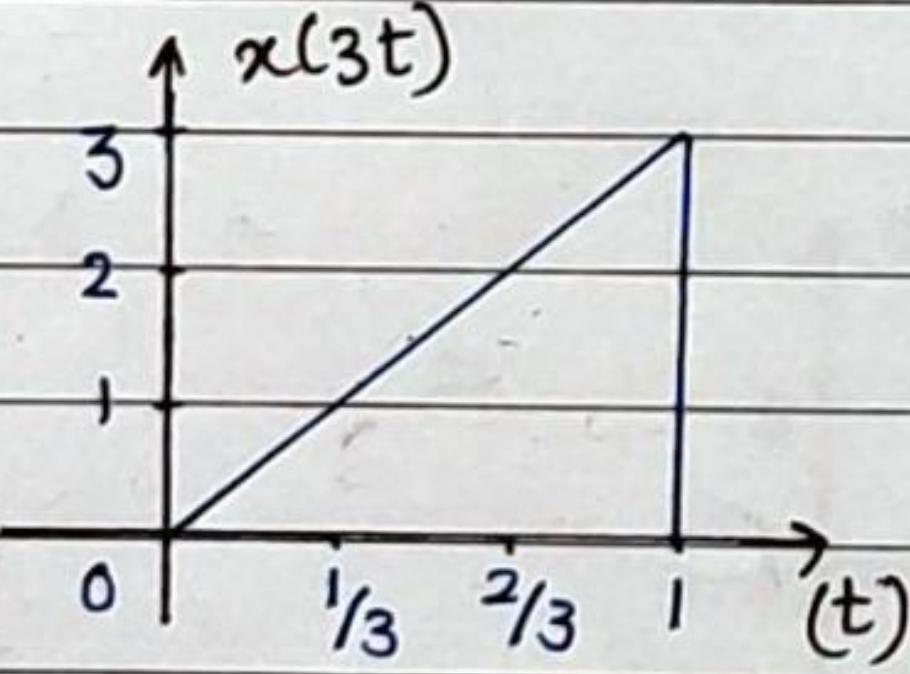
i) $x(-t)$



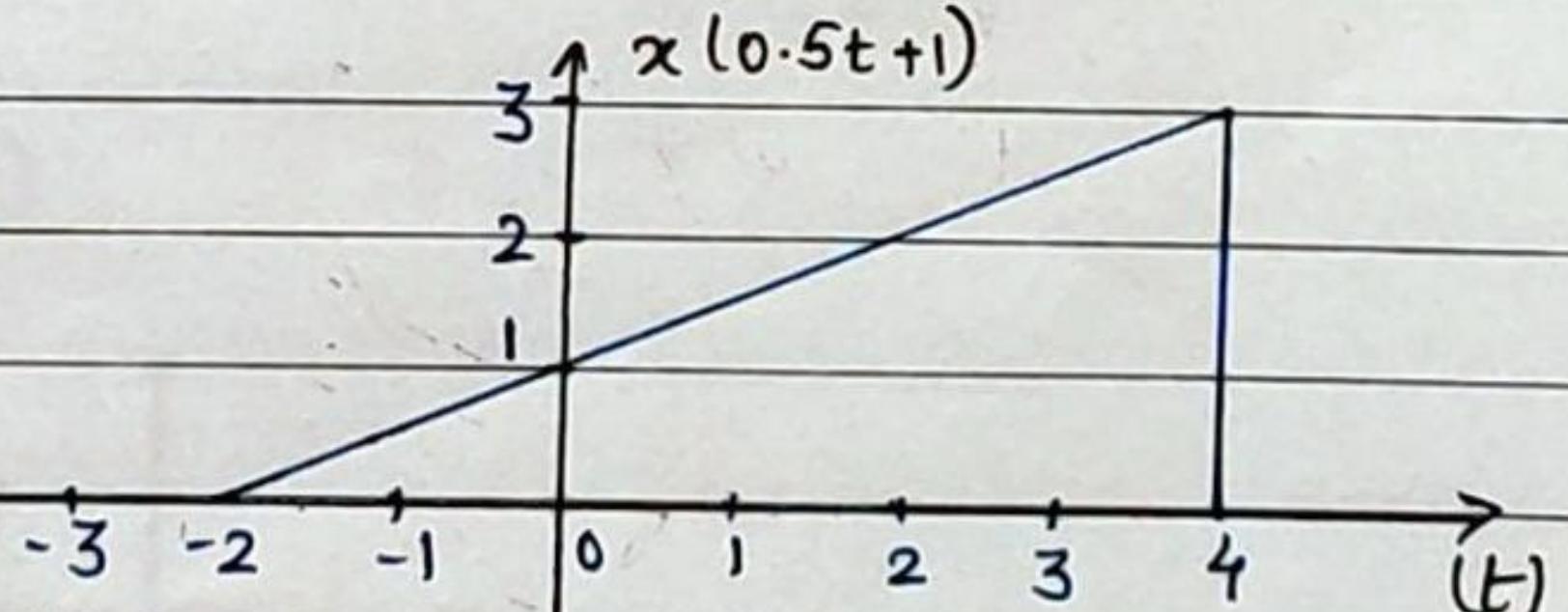
ii) $x(2-t)$



iii) $x(3t)$



iv) $x(0.5t+1)$

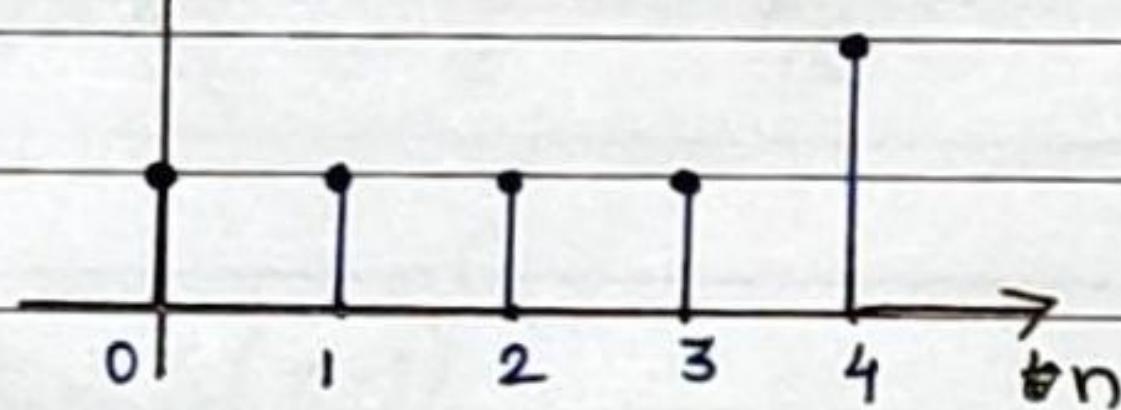


Q.7

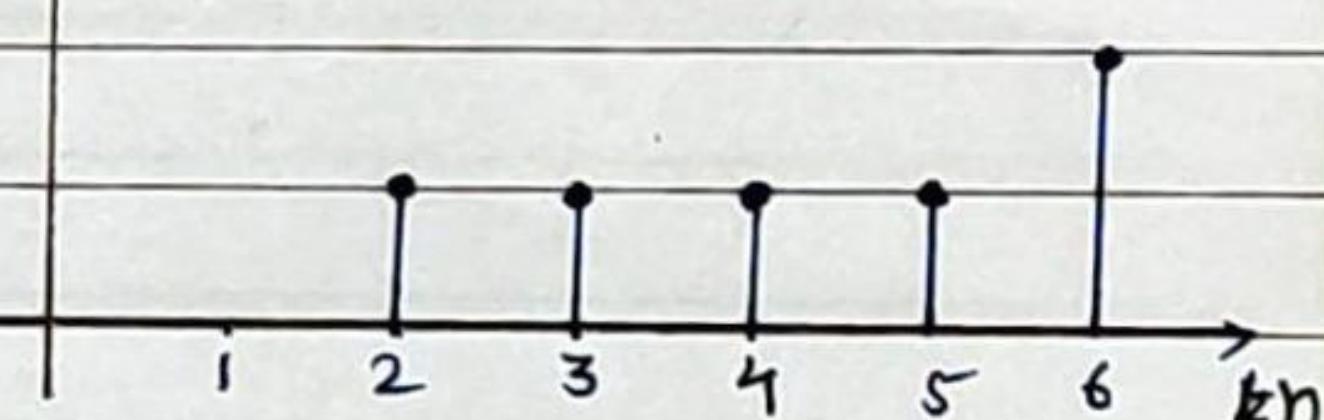
A discrete time signal is given by $x(n) = \{1, 1, 1, 1, 2\}$. Sketch

- i) $x(n)$
- ii) $x(n-2)$
- iii) $x(n) \cdot u(n-1)$
- iv) $x(3-n)$
- v) $x(n-1) \cdot \delta(n-1)$

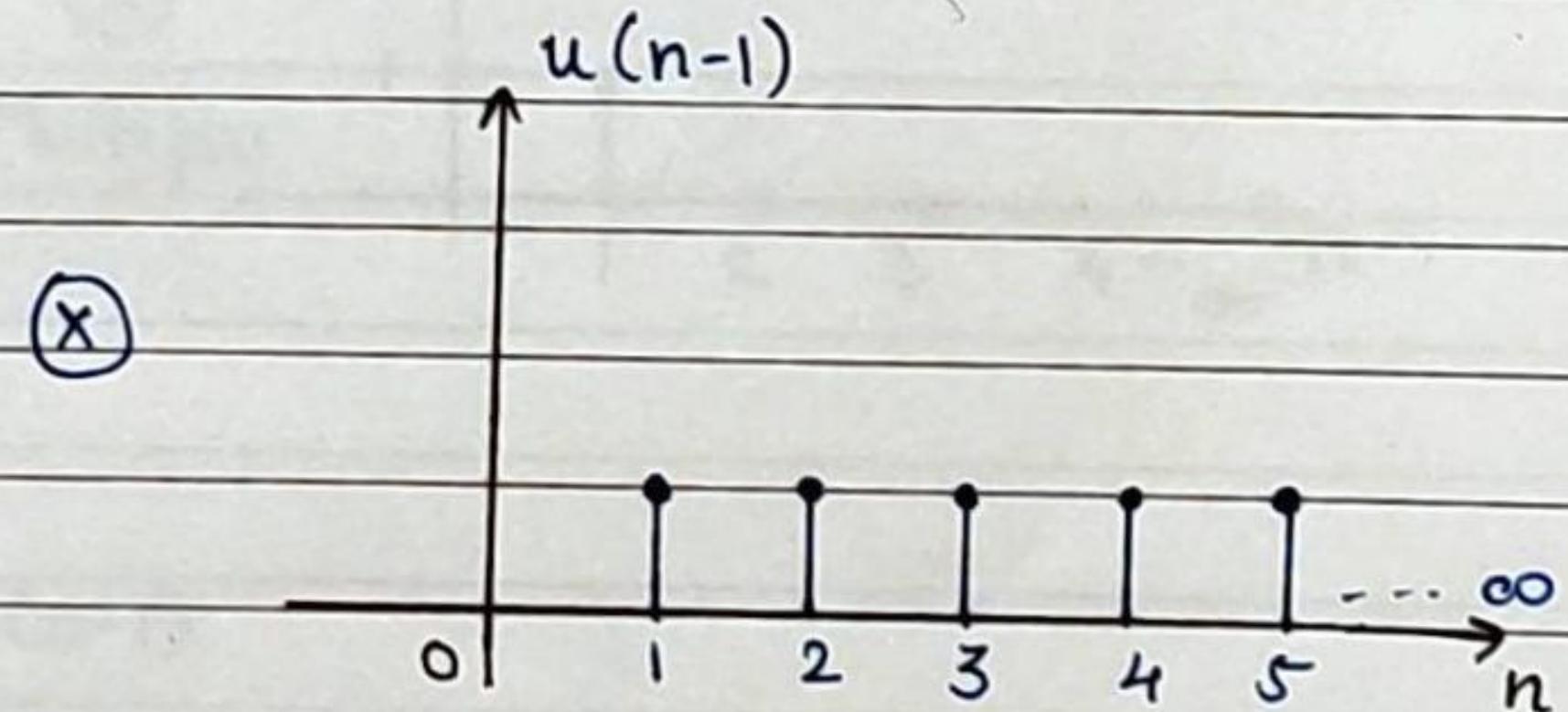
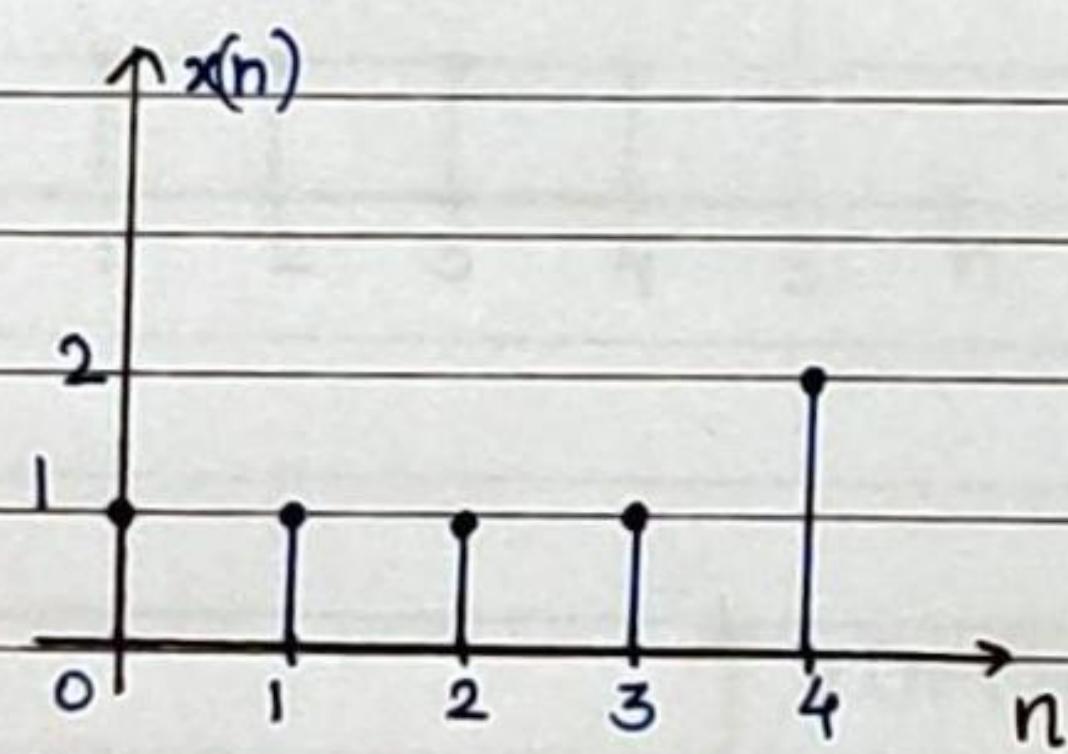
i) $x(n) =$



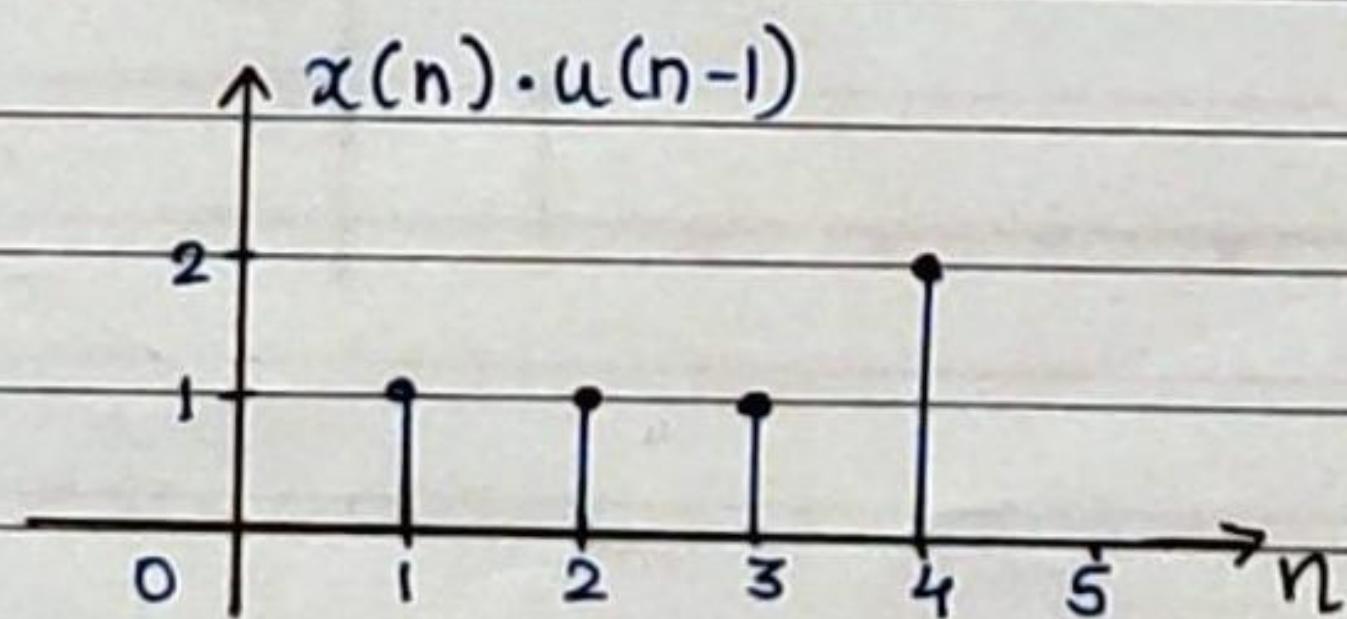
ii) $x(n-2)$



iii) $x(n) \cdot u(n-1)$

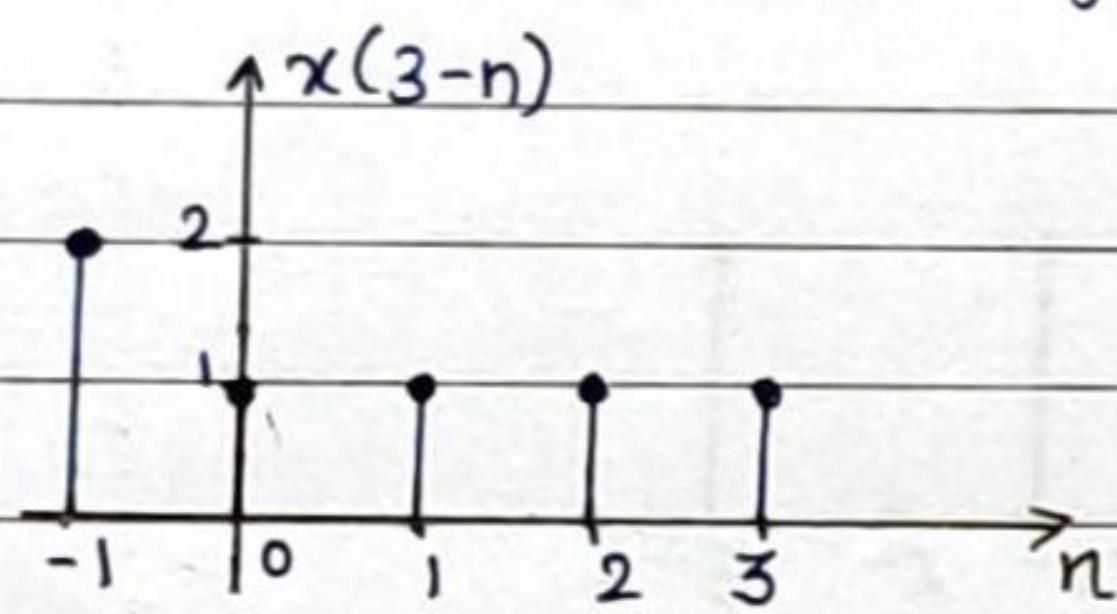


=

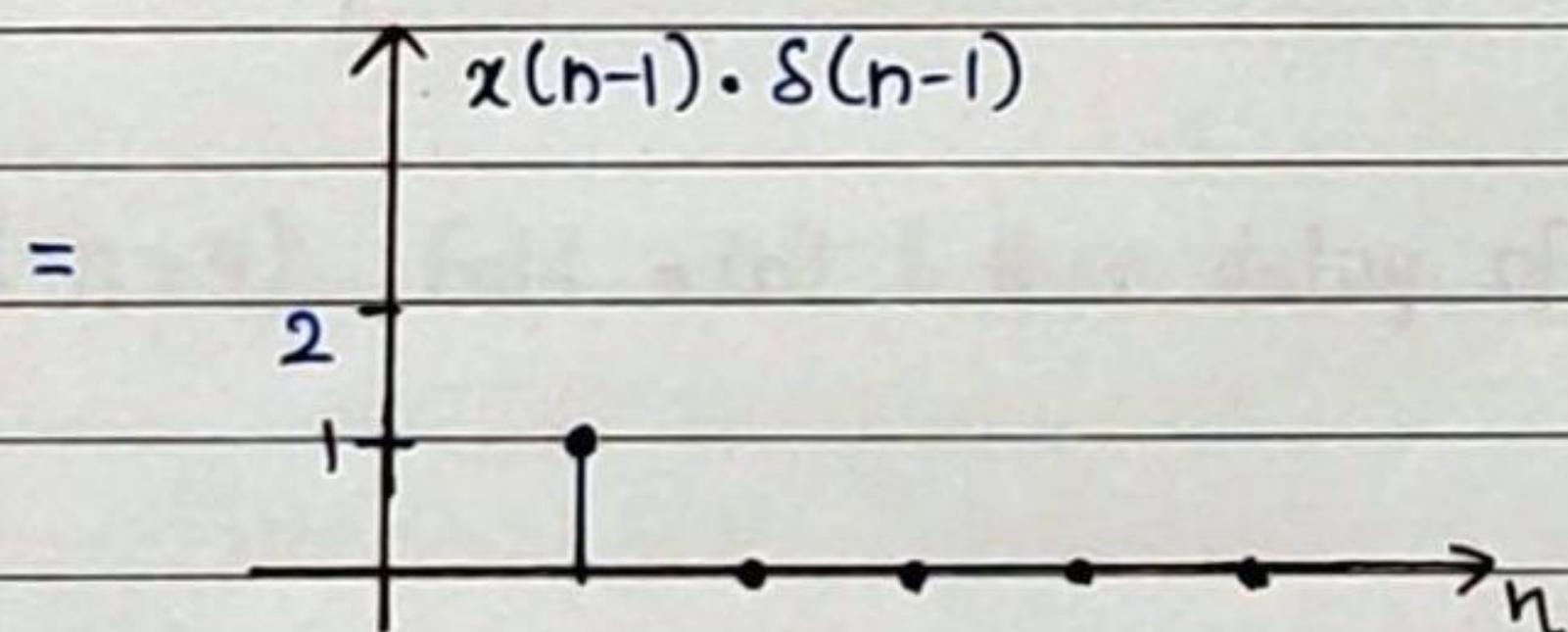
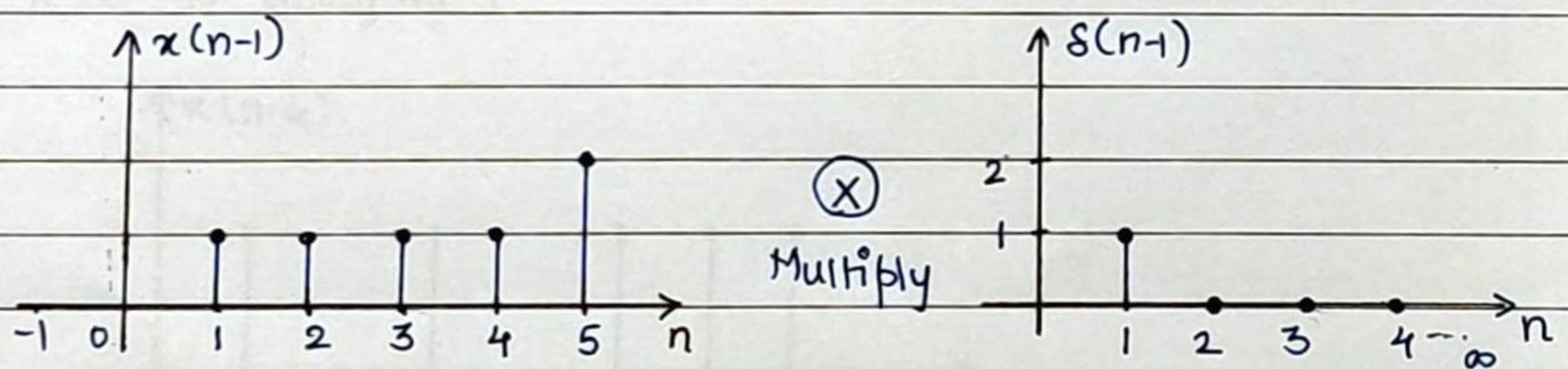


iv] $x(3-n)$

$x(-n)$ then delayed by 3

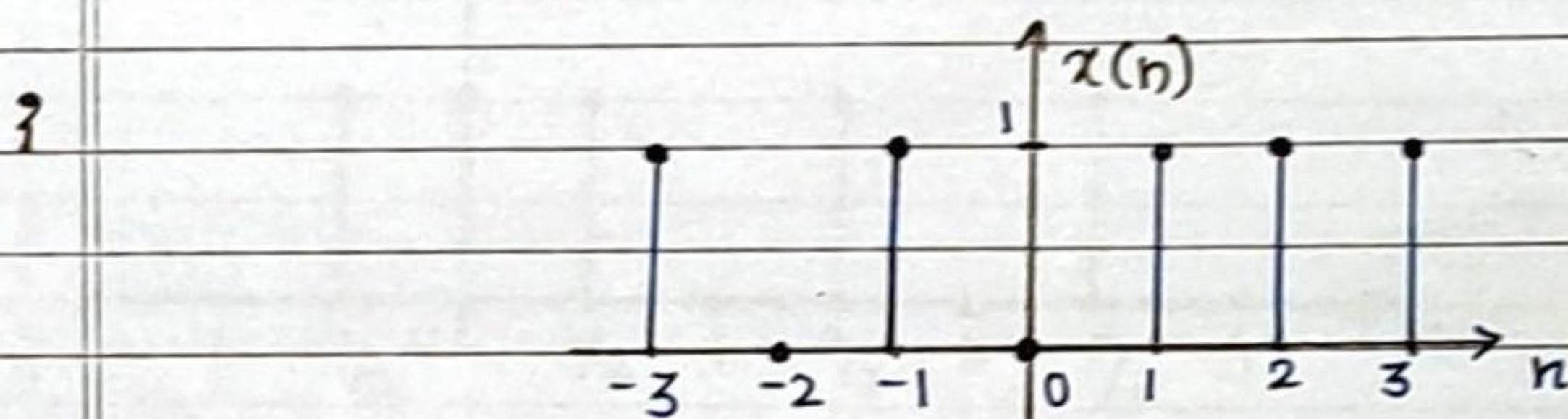


v] $x(n-1) \cdot \delta(n-1)$

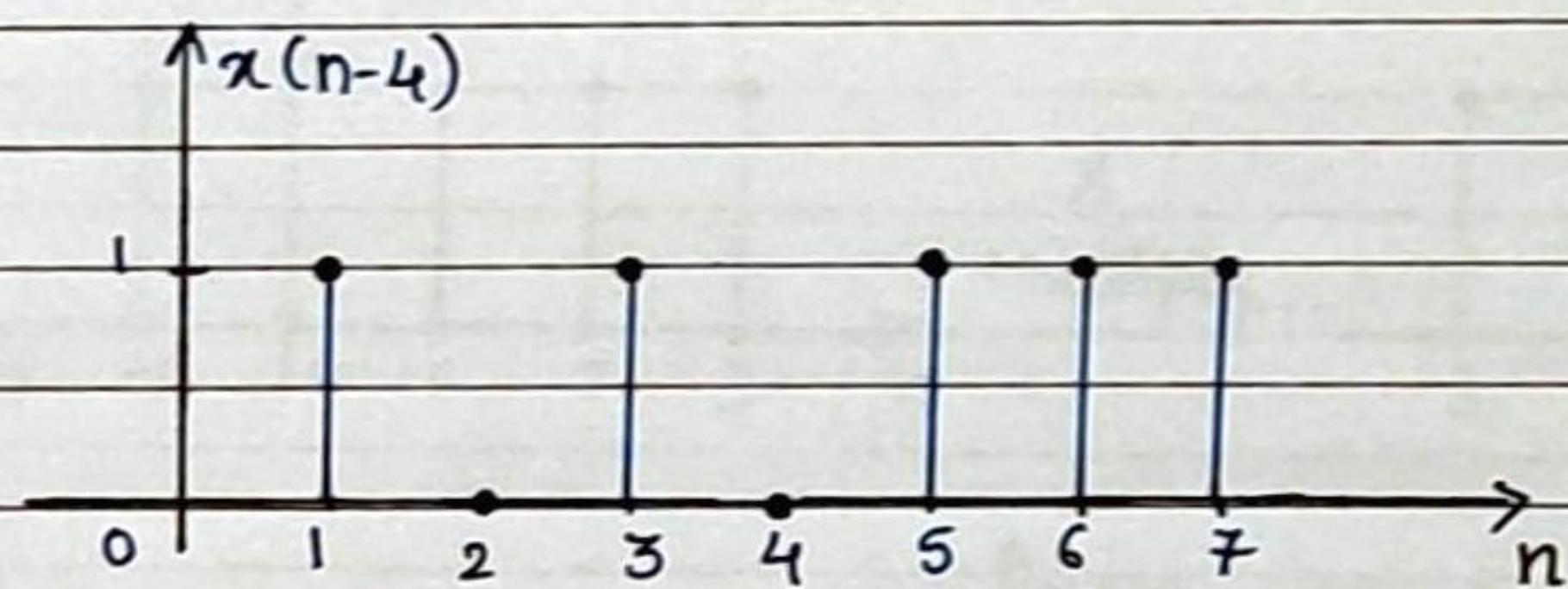


Q.8

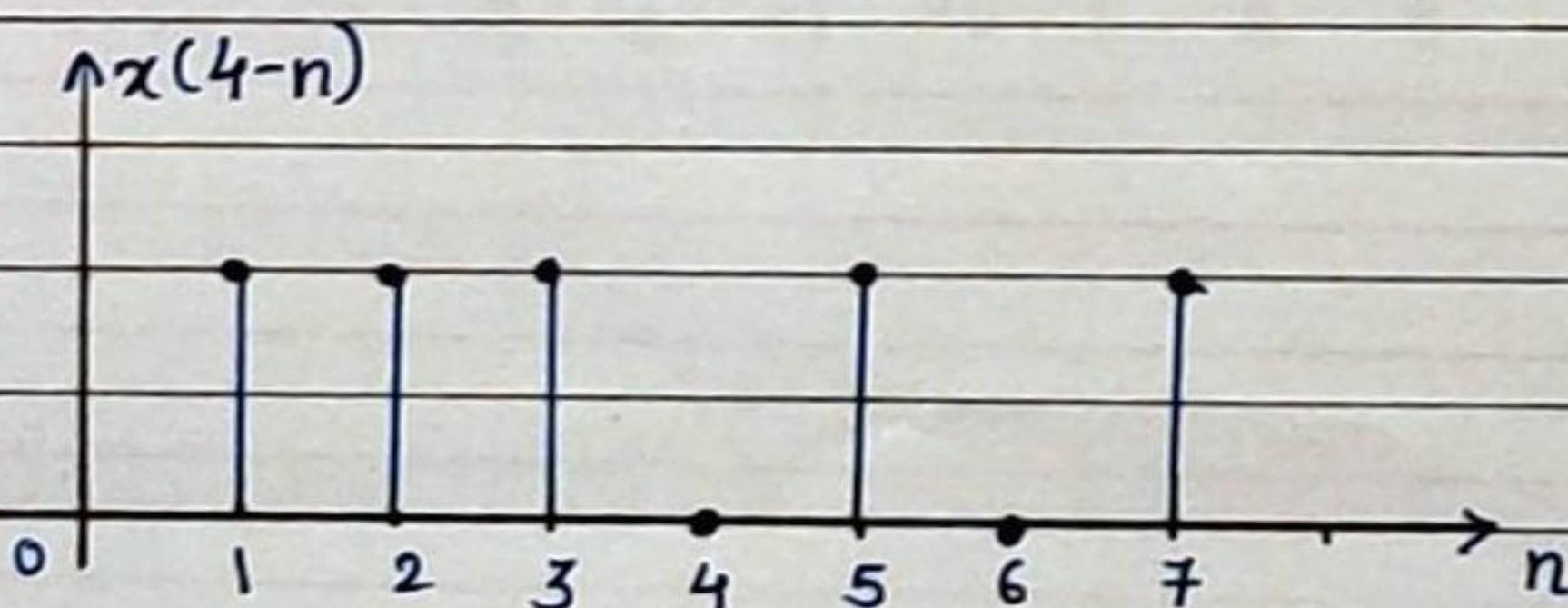
A DT signal has been shown. Sketch, i) $x(n-4)$ ii) $x(4-n)$
ii) $x(-2n+2)$ iv) $x(n) \cdot u(3-n)$.



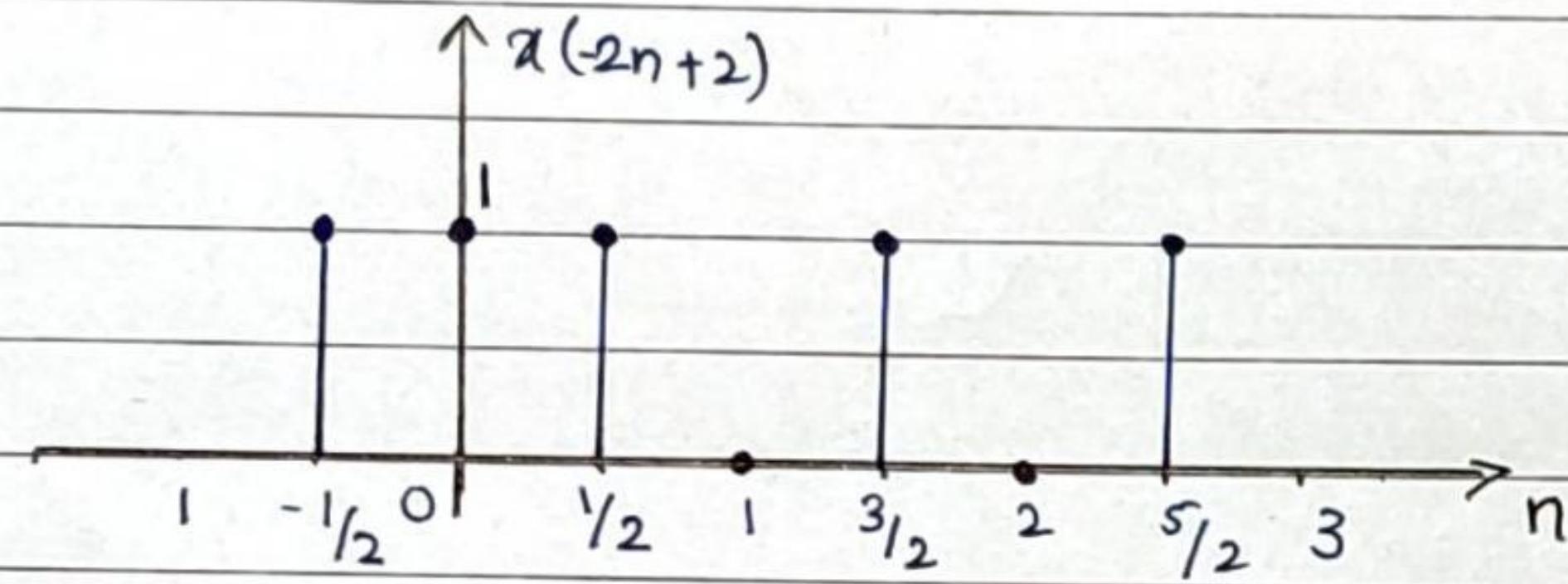
i) $x(n-4)$ delay by 4



ii) $x(4-n) = x(-n+4)$ Fold $x(n)$ & then delay of 4

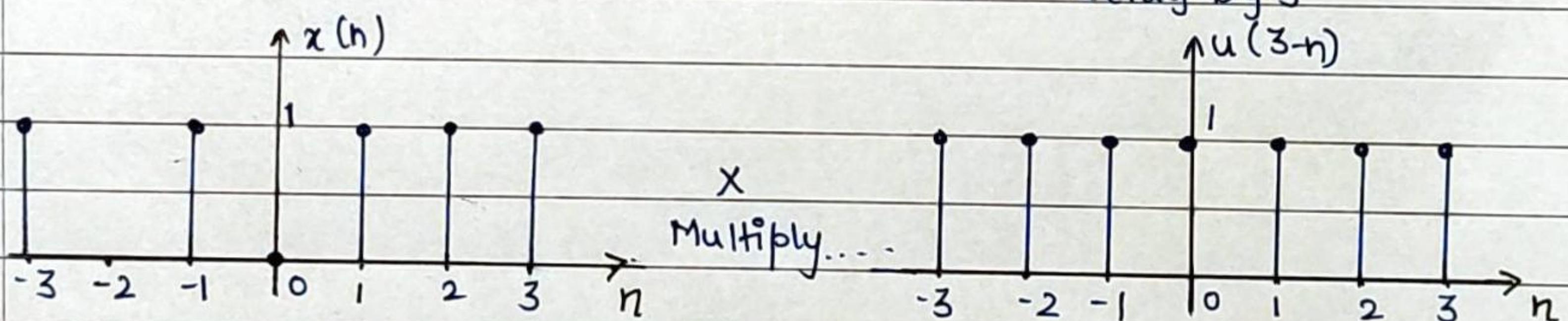


iii) $x(-2n+2)$ Fold $x(n)$ & delay by 2 and then compress by 2



iv) $x(n) \cdot u(3-n)$.

$u(-n) \rightarrow$ delay by 3



=

