Ex (3) Determine whether the set of vectors of the form (a,b,c) where $b = a+c$ forms a subspace of R^3 under usual addition and scaler multiplication.

Solution: Given: $V = \{(a,b,c) \in R^3 \,/\, b = a+c\}$

let $u = (a_1, b_1, c_1)$ and $v = (a_2, b_2, c_2)$ be any element of V and k be any scalar.

$\Rightarrow \quad b_1 = a_1 + c_1$ and $b_2 = a_2 + c_2$

$\therefore$ consider, $u + v = (a_1, b_1, c_1) + (a_2, b_2, c_2)$

$= (a_1 + a_2, \; b_1 + b_2, \; c_1 + c_2)$

$\therefore \; (b_1 + b_2) = [(a_1 + c_1) + (a_2 + c_2)]$

$= (a_1 + a_2) + (c_1 + c_2)$

Hence, $u + v = (a_1 + a_2, \; b_1 + b_2, \; c_1 + c_2) \in V$

and $ku = k(a_1, b_1, c_1) = (ka_1, kb_1, kc_1)$

$\therefore \; kb_1 = k(a_1 + c_1) = ka_1 + kc_1$

Hence, $ku = (ka_1, kb_1, kc_1) \in V$

that is $u+v \in V$ and $ku \in V$, for any $u, v \in V$ and $k \in F$

$\therefore$ By Necessary and sufficient condition V is a subspace of R^3

Ex. 4 If W is the set of all symmetric matrices of order $n \times n$ then W is a subspace of all $n \times n$ matrices V

Solution: let $W = \{ A \in V \mid A \text{ is symmetric} \}$

where, V is a vector space of all $n \times n$ matrices.

let A, B be any element of W and k be scaler

$\Rightarrow$ A, B are symmetric matrices

$\Rightarrow$ $A^T = A$ and $B^T = B$

Now, $(A+B)^T = A^T + B^T$

$= A + B$

Hence, $A+B$ is symmetric matrix.

$\therefore$ $A+B \in W$

and $(kA)^T = k(A^T) = kA$

Hence, kA is symmetric matrix

$\therefore$ $kA \in W$

i.e. $A+B \in W$ and $kA \in W$, for any $A, B \in W$ and for any scaler k

$\therefore$ By Necessary and sufficient condition,

W is the Subspace of V

Homework Q1. Determine whether following are subspace of vectors space of all $n \times n$ matrices.

i) W = set of all Lower traingular $n \times n$ matrices

ii) W = set of all diagonal $n \times n$ matrices.

Que 2 Determine whether the following are subspace of R^3

i) $W = \{(x,y,z) \mid x=1, z=1\}$ 2) $W = \{(x,y,z) \mid x+y+z=3\}$

3) $W = \{(x,y,z) \mid x^2 - y^2 = 0\}$ 4) $W = \{(x,y,z) \mid y \geq 0\}$

* vectors in n-dimensional vector space;

Note that!

* vectors in 1-dimensional vector space ($\mathbb{R}$) is of the form $\bar{u} = a$, $a \in \mathbb{R}$

* vectors in 2-dimensional vector space ($\mathbb{R}^2$) is of the form $\bar{u} = (a_1, a_2)$, $a_1, a_2 \in \mathbb{R}$

* vectors in 3-dimensional vector space ($\mathbb{R}^3$) is of the form $\bar{u} = (a_1, a_2, a_3)$, $a_1, a_2, a_3 \in \mathbb{R}$

⋮

* vectors in n-dimensional vector space ($\mathbb{R}^n$) is of the form $\bar{u} = (a_1, a_2, \cdots, a_n)$, All $a_i \in \mathbb{R}$, $i = 1, 2, 3, \cdots, n$

for examples:

i) $\bar{u} = (1, -2)$ is 2-dimensional vector

ii) $\bar{u} = (-5, 3, 0, 1)$ is 4-dimensional vector

Note that! if $\bar{u} = (u_1, u_2, u_3, \cdots u_n)$ and $\bar{v} = (v_1, v_2, \cdots v_n)$

then
1. $\bar{u} + \bar{v} = (u_1 + v_1, u_2 + v_2, \cdots, u_n + v_n)$
2. $\bar{u} - \bar{v} = (u_1 - v_1, u_2 - v_2, \cdots, u_n - v_n)$
3. $k\bar{u} = (ku_1, ku_2, \cdots, ku_n)$

* **Norm of Vector :**

let $\mathbb{R}^n$ be the n-dimensional vector space and $\bar{u} = (u_1, u_2, u_3, \dots, u_n) \in \mathbb{R}^n$ then Norm of $\bar{u}$ is denoted by '$\|\bar{u}\|$' and is given by

$$\|u\| = \sqrt{u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2}$$

* **Dot product:**

If $\bar{u}$ and $\bar{v}$ are vectors in $\mathbb{R}^n$ then dot product of $\bar{u}$ and $\bar{v}$ is

$$\boxed{\bar{u} \cdot \bar{v} = \|\bar{u}\| \cdot \|\bar{v}\| \cdot \cos\theta}$$

where 'θ' is angle between $\bar{u}$ and $\bar{v}$

Note that : ① The angle between two vectors $\bar{u}$ and $\bar{v}$ is

$$\theta = \cos^{-1}\left(\frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \cdot \|\bar{v}\|}\right)$$

② If $\bar{u} = (u_1, u_2, \dots, u_n)$ and $\bar{v} = (v_1, v_2, \dots, v_n)$ then usual product of $\bar{u}$ and $\bar{v}$ is

$$\bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2 + \dots + u_n v_n$$

* Cauchy - schwartz inequality in R^n

Statement: If $u = (u_1, u_2, \cdots, u_n)$ and $v(v_1, v_2, \cdots, v_n)$ are any two vectors in R^n

then $|u \cdot v| \leq \|u\| \cdot \|v\|$

Proof: we prove this for vectors in R^2 and R^3

let u, v be any two vectors in R^2 or R^3

then $\cos\theta = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$

$\Rightarrow u \cdot v = \|u\| \cdot \|v\| \cos\theta$

By Applying modulus to both side we get

$|u \cdot v| = \|u\| \cdot \|v\| \cdot |\cos\theta|$

But we know that $|\cos\theta| \leq 1$

Hence, $|u \cdot v| \leq \|u\| \cdot \|v\|$

Hence the proof.

Example: ① varify cauchy - schwartz inequality for the vectors $u = (2,3,1)$ and $v = (3,0,4)$, Also find the angle between u and v

solution: Given: $u = (2,3,1)$ and $v = (3,0,4)$

i) $\|u\| = \sqrt{(2)^2 + (3)^2 + (1)^2} = \sqrt{14}$

and $\|v\| = \sqrt{(3)^2 + (0)^2 + (4)^2} = \sqrt{25} = 5$.

$\therefore \|u\| \cdot \|v\| = \sqrt{14} \cdot 5 = 5\sqrt{14}$

Now, $|u \cdot v| = |(2,3,1) \cdot (3,0,4)| = |(2\times3)+(3\times0)+(1\times4)|$

$\Rightarrow |u \cdot v| = |6+0+4| = |10| = 10$

$\Rightarrow |u \cdot v| = 10 < 5\sqrt{14} = \|u\| \cdot \|v\|$

that is $|u \cdot v| \leq \|u\| . \|v\|$

Hence, Cauchy - schwartz inequality is varified.

ii) Note that Angle between $\bar{u}$ and $\bar{v}$ is

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{\|u\| \cdot \|v\|}\right) = \cos^{-1}\left(\frac{10}{5\sqrt{14}}\right) = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$$

$$\therefore \boxed{\theta = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)}$$

Homework.

Ex ② varify cauchy schwartz inequality for

$u = (-4, 2, 1)$ and $v = (8, -4, -2)$

* **Unit vector:**

— A vector whose norm is equal to one is called unit vector

Note that: If $\bar{u}$ is not unit vector then we can make unit vector by using u as

$$\boxed{\hat{u} = \frac{1}{\|u\|} \cdot \bar{u}}$$

for ex. If $u = (2, 4, -5)$

then $\|u\| = \sqrt{(2)^2 + (4)^2 + (-5)^2} = 3\sqrt{5}$

$\therefore$ unit vector is

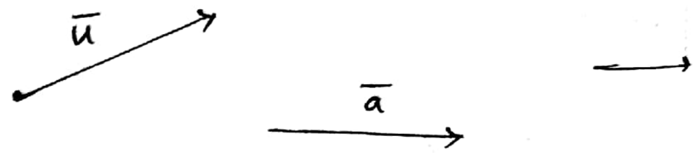
$$\hat{u} = \frac{1}{\|u\|} \cdot u = \frac{1}{3\sqrt{5}} (2, 4, -5)$$

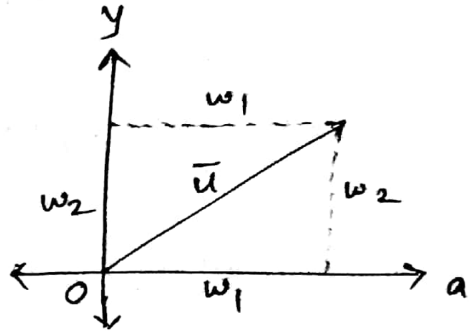
$$\hat{u} = \left(\frac{2}{3\sqrt{5}}, \frac{4}{3\sqrt{5}}, \frac{-5}{3\sqrt{5}} \right)$$

* orthogonal projection:

let $\bar{u}$ and $\bar{a}$ be the two vectors then the orthogonal projection of u on $\bar{a}$ is defined as

$$\boxed{\text{proj}_a \bar{u} = \frac{(\bar{u} \cdot \bar{a})}{\|\bar{a}\|} \cdot \bar{a}}$$





<u>Note that</u>: The vector component of u orthogonal to a is given by

$$\boxed{\bar{u} - \text{proj}_a \bar{u} = \bar{u} - \frac{\bar{u} \cdot \bar{a}}{\|\bar{a}\|} \cdot \bar{a}}$$

<u>Ex</u> ① find the projection of $\bar{u} = (1, -2, 3)$ along $v = (1, 2, 1)$ in R^3

<u>Solution</u>: $\bar{u} \cdot \bar{v} = (1, -2, 3) \cdot (1, 2, 1) = (1 \times 1) + (-2 \times 2) + (3 \times 1)$

$\Rightarrow \bar{u} \cdot \bar{v} = 0$

$$\therefore \text{proj}_{\bar{v}} \bar{u} = \frac{\bar{u} \cdot \bar{v}}{\|\bar{v}\|} \cdot \bar{v} = \frac{0}{\|\bar{v}\|} \cdot \bar{v} = 0$$

$$\therefore \boxed{\text{proj}_v \bar{u} = 0}$$

Scanned by CamScanner

Ex ② find the projection of $u = (3, 1, 3)$ along and perpendicular to $v = (4, -2, 2)$

Solution:

$$\bar{u} \cdot \bar{v} = (3)(4) + (1)(-2) + (3)(2) = 16$$

$$\|v\|^2 = \left(\sqrt{(4)^2 + (-2)^2 + (2)^2}\right)^2 = 24$$

$$\therefore \text{proj}_{\bar{v}} \bar{u} = \frac{\bar{u} \cdot \bar{v}}{\|v\|^2} \cdot \bar{v} = \frac{16}{24}(4, -2, 2) = \left(\frac{8}{3}, \frac{-4}{3}, \frac{4}{3}\right)$$

and the projection of $\bar{u}$ perpendicular to $\bar{v}$ is

$$u - \text{proj}_v \bar{u} = \bar{u} - \frac{\bar{u} \cdot \bar{v}}{\|v\|^2} \cdot \bar{v} = (3, 1, 3) - \left(\frac{8}{3}, \frac{-4}{3}, \frac{4}{3}\right)$$

$$= (3, 1, 3) + \left(\frac{-8}{3}, \frac{4}{3}, \frac{-4}{3}\right)$$

$$= \left(\frac{1}{3}, \frac{7}{3}, \frac{5}{3}\right)$$