Module No.1 Introduction to signals and Systems

Mrs.Manisha Joshi

Definition of signal

Any physical phenomenon that conveys or carries some information can be called a signal. The music, speech, motion pictures, still photos, heart beat, etc., are examples of signals that we normally encounter in day to day life.

Usually, the information carried by a signal will be a function of an independent variable. The independent variable can be time, spatial coordinates, intensity of colours, pressure, temperature, etc.,. The most popular independent variable in signals is time and it is represented by the letter "t".

The value of a signal at any specified value of the independent variable is called its ***amplitude***. The sketch or plot of the amplitude of a signal as a function of independent variable is called its ***waveform***.

Mathematically, any signal can be represented as a function of one or more independent variables.Therefore, *a **signal** is defined as any physical quantity that varies with one or more independent variables.*

For example, the functions $x_1(t)$ and $x_2(t)$ as defined by the equations (1.1) and (1.2) represents two signals: one that varies linearly with time "t" and the other varies quadratically with time "t". The equation (1.3) represents a signal which is a function of two independent variables "p" and "q".

$$x_1(t) = 0.7t \qquad .....(1.1)$$

$$x_2(t) = 1.8t^2 \qquad .....(1.2)$$

$$x(p,q) = 0.6p + 0.5q + 1.1q^2 \qquad .....(1.3)$$

The signals can be classified in number of ways. Some way of classifying the signals are,

I. Depending on the number of sources for the signals.

1.One-channel signals

2. Multichannel signals

II. Depending on the number of dependent variables.

1. One-dimensional signals

2. Multidimensional signals

III. Depending on whether the dependent variable is continuous or discrete.

1. Analog or Continuous signals

2. Discrete signals

2. Multichannel signals

Signals that are generated by multiple sources or sensors are called multichannel signals.

The audio output of two stereo speakers is an example of two-channel signal. The record of ECG (Electro Cardio Graph) at eight different places in a human body is an example of eight-channel signal.

3. One-dimensional signals

A signal which is a function of single independent variable is called one-dimensional signal.

The signals represented by equation (1.1) and (1.2) are examples of one-dimensional signals.

The music, speech, heart beat, etc., are examples of one-dimensional signals where the single independent variable is time.

4. Multidimensional signals

A signal which is a function of two or more independent variables is called multidimensional signal.

The equation (1.3) represents a two dimensional signal.

A photograph is an example of a two-dimensional signal. The intensity or brightness at each point of a photograph is a function of two spatial coordinates "x" and "y", (and so the spatial coordinates are independent variables). Hence, the intensity or brightness of a photograph can be denoted by b(x, y).

4. Multidimensional signals

A signal which is a function of two or more independent variables is called multidimensional signal.

The equation (1.3) represents a two dimensional signal.

A photograph is an example of a two-dimensional signal. The intensity or brightness at each point of a photograph is a function of two spatial coordinates "x" and "y", (and so the spatial coordinates are independent variables). Hence, the intensity or brightness of a photograph can be denoted by b(x, y).

The motion picture of a black and white TV is an example of a three-dimensional signal. The intensity or brightness at each point of a black and white motion picture is a function of two spatial coordinates "x" and "y", and time "t". Hence, the intensity or brightness of a black and white motion picture can be denoted by b(x, y, t).

5. Analog or Continuous signals

When a signal is defined continuously for any value of independent variable, it is called analog or continuous signal. Most of the signals encountered in science and engineering are analog in nature. When the dependent variable of an analog signal is time, it is called continuous time signal.

6. Discrete signals

When a signal is defined for discrete intervals of independent variable, it is called discrete signal . When the dependent variable of a discrete signal is time, it is called discrete time signal. Most of the discrete signals are either sampled version of analog signals for processing by digital systems or output of digital systems.

1.1.1 Continuous Time Signal

In a signal with time as independent variable, if the signal is defined continuously for any value of the independent variable time "t", then the signal is called ***continuous time signal***. The continuous time signal is denoted as "x(t)".

The continuous time signal is defined for every instant of the independent variable time and so the magnitude (or the value) of continuous time signal is continuous in the specified range of time. Here both the magnitude of the signal and the independent variable are continuous.

1.1.2 Discrete Time Signal

In a signal with time as independent variable, if the signal is defined only for discrete instants of the independent variable time, then the signal is called ***discrete time signal***.

In discrete time signal the independent variable time "t" is uniformely divided into discrete intervals of time and each interval of time is denoted by an integer "n", where "n" stands for discrete interval of time and "n" can take any integer value in the range $-\infty$ to $+\infty$. Therefore, for a discrete time signal the independent variable is "n" and the magnitude of the discrete time signal is defined only for integer values of independent variable "n". The discrete time signal is denoted by "x(n)".

1.1.3 Digital Signal

The quantized and coded version of the discrete time signals are called ***digital signals***. In digital signals the value of the signal for every discrete time "n" is represented in binary codes. The process of conversion of a discrete time signal to digital signal involves quantization and coding.

Normally, for binary representation, a standard size of binary is chosen. In m-bit binary representation we can have 2^m binary codes. The possible range of values of the discrete time signals are usually divided into 2^m steps called ***quantization levels***, and a binary code is attached to each quantization level. The values of the discrete time signals are approximated by rounding or truncation in order to match the nearest quantization level.

2.2 Standard Continuous Time Signals

1. Impulse signal

The impulse signal is a signal with infinite magnitude and zero duration, but with an area of A. Mathematically, impulse signal is defined as,

$$\text{Impulse Signal, } \delta(t) = \infty\,;\; t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\,dt = A$$
$$= 0\,;\; t \neq 0$$

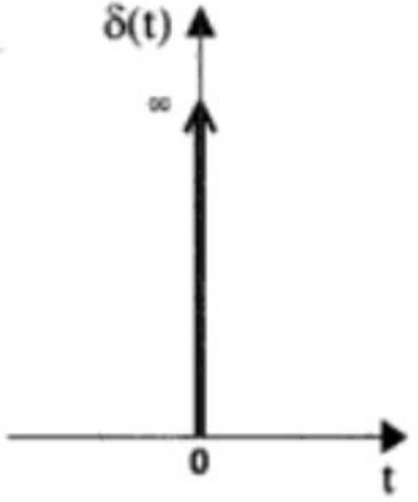


Fig 2.1 : Impulse signal (or Unit Impulse signal).

The unit impulse signal is a signal with infinite magnitude and zero duration, but with unit area. Mathematically, unit impulse signal is defined as,

The unit impulse signal is a signal with infinite magnitude and zero duration, but with unit area. Mathematically, unit impulse signal is defined as,

$$\text{Unit Impulse Signal, } \delta(t) = \infty\,;\; t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$
$$= 0\,;\; t \neq 0$$

2. *Step signal*

The step signal is defined as,

$$x(t) = A \; ; t \geq 0$$
$$= 0 \; ; t < 0$$

The unit step signal is defined as,

$$x(t) = u(t) = 1 \; ; t \geq 0$$
$$= 0 \; ; t < 0$$

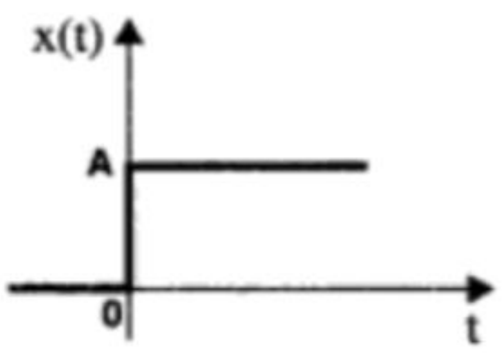


Fig 2.2 : Step signal.

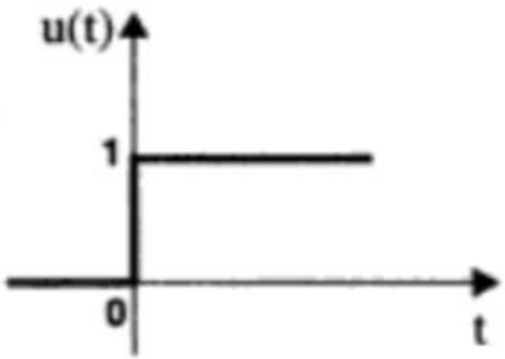


Fig 2.3 : Unit step signal.

3. Ramp signal

The ramp signal is defined as,

$$x(t) = At \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

The unit ramp signal is defined as,

$$x(t) = t \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

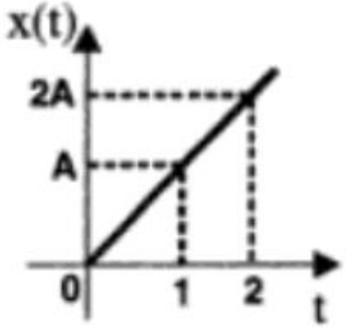


Fig 2.4 : Ramp signal.

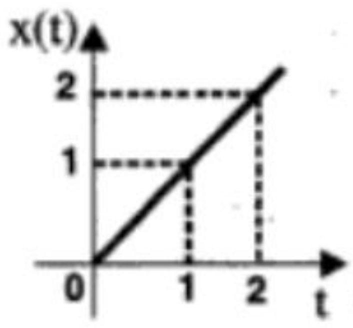


Fig 2.5 : Unit ramp signal.

4. Parabolic signal

The parabolic signal is defined as,

$$x(t) = \frac{At^2}{2} \quad ; \text{ for } t \geq 0$$

$$= 0 \quad ; \quad t < 0$$

The unit parabolic signal is defined as,

$$x(t) = \frac{t^2}{2} \quad ; \text{ for } t \geq 0$$

$$= 0 \quad ; \quad t < 0$$

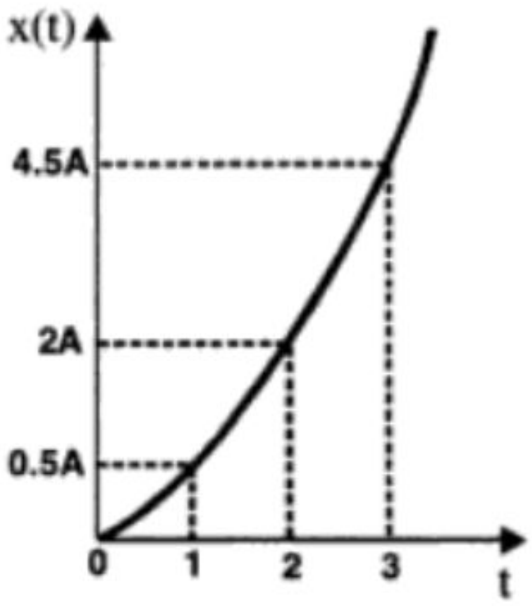


Fig 2.6 : Parabolic signal.

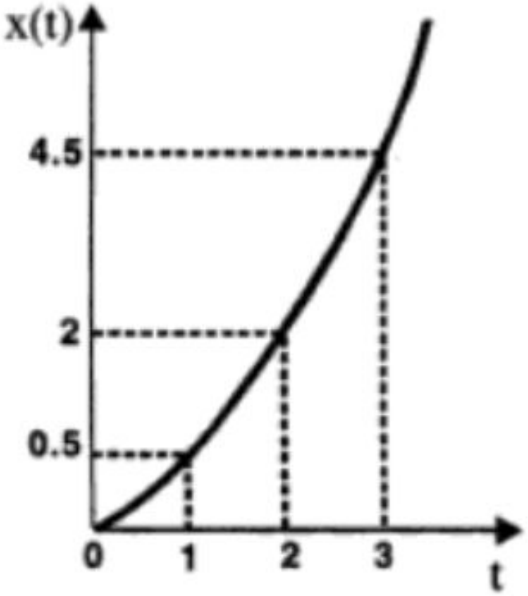


Fig 2.7 :Unit parabolic signal.

5. Unit pulse signal

The unit pulse signal is defined as,

$$x(t) = \Pi(t) = u\left(t+\frac{1}{2}\right) - u\left(t-\frac{1}{2}\right)$$

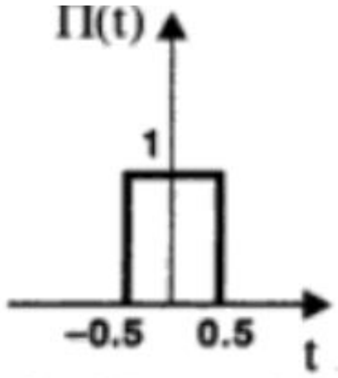


Fig 2.8 : Unit pulse signal.

6. *Sinusoidal signal*

Case i : ***Cosinusoidal signal***

The cosinusoidal signal is defined as,

$$x(t) = A\cos(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

When $\phi = 0$, $\quad x(t) = A\cos\Omega_0 t$

When ϕ = Positive, $\quad x(t) = A\cos(\Omega_0 t + \phi)$

When ϕ = Negative, $\quad x(t) = A\cos(\Omega_0 t - \phi)$

Case ii : ***Sinusoidal signal***

The sinusoidal signal is defined as,

$$x(t) = A\sin(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \frac{2\pi}{T}$ = Angular frequency in rad/sec

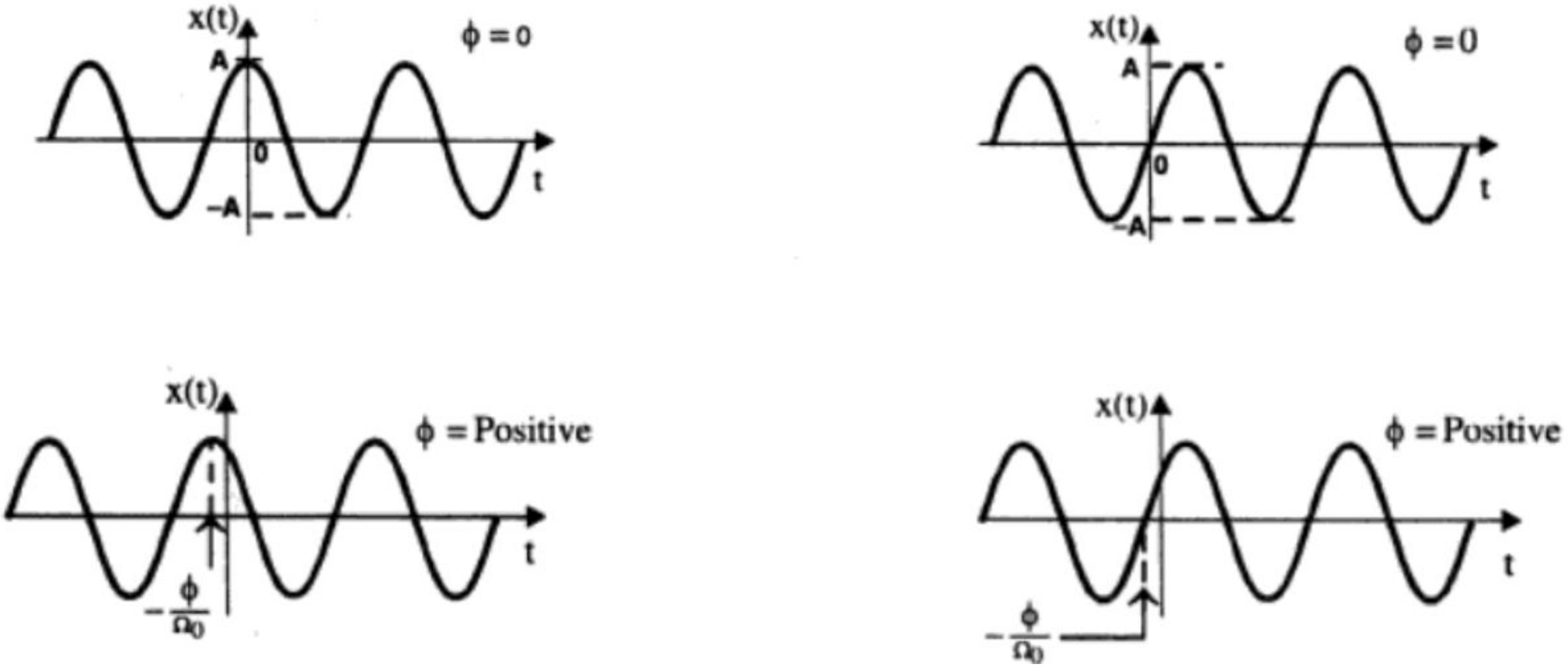
F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

When $\phi = 0$, $x(t) = A\sin\Omega_0 t$

When ϕ = Positive, $x(t) = A\sin(\Omega_0 t + \phi)$

When ϕ = Negative, $x(t) = A\sin(\Omega_0 t - \phi)$



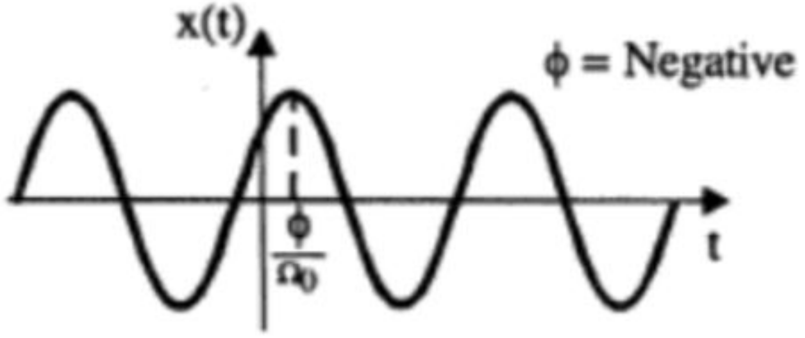


Fig 2.9 : Cosinusoidal signal.

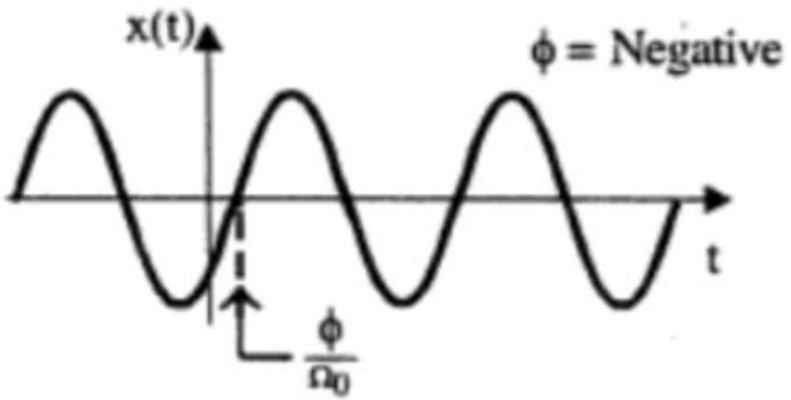


Fig 2.10 : Sinusoidal signal.

7. Exponential signal

Case i : Real exponential signal

The real exponential signal is defined as,

$$x(t) = A\, e^{bt}$$

where, A and b are real

Here, when b is positive, the signal x(t) will be an exponentially rising signal; and when b is negative the signal x(t) will be an exponentially decaying signal.

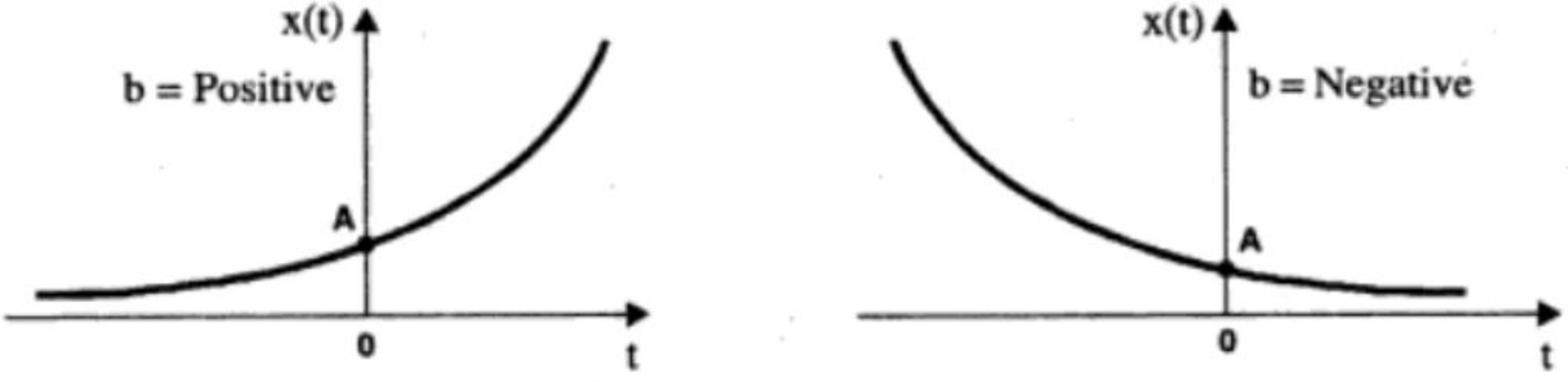


Fig 2.11 : Real exponential signal.

Case ii : ***Complex exponential signal***

The complex exponential signal is defined as,

$$x(t) = A\,e^{j\Omega_0 t}$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

The complex exponential signal can be represented in a complex plane by a rotating vector, which rotates with a constant angular velocity of Ω_0 rad/sec.

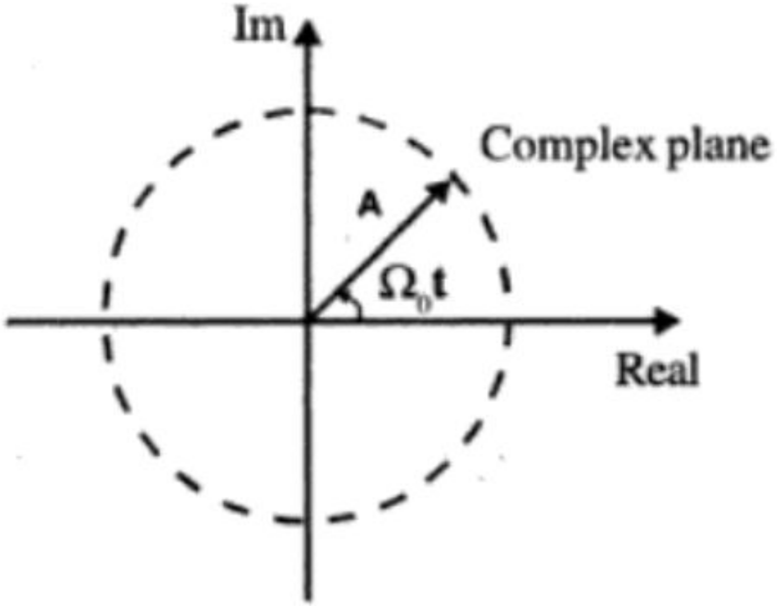


Fig 2.12 : Complex exponential signal.

The complex exponential signal can be resolved into real and imaginary parts as shown below,

$$x(t) = A\,e^{j\Omega_0 t} = A\,(\cos\Omega_0 t + j\sin\Omega_0 t)$$

$$= A\cos\Omega_0 t + jA\sin\Omega_0 t$$

$$\therefore \quad A\cos\Omega_0 t = \text{Real part of } x(t)$$

$$A\sin\Omega_0 t = \text{Imaginary part of } x(t)$$

8. Exponentially rising/decaying sinusoidal signal

The exponential rising/decaying sinusoidal signal is defined as,

$$x(t) = A\, e^{bt} \sin\Omega_0 t$$

$$\text{where, } \Omega_0 = 2\pi F_0 = \frac{2\pi}{T} = \text{Angular frequency in rad/sec}$$

$$F_0 = \text{Frequency in cycles/sec or Hz}$$

$$T = \text{Time period in sec}$$

Here, A and b are real constants. When b is positive, the signal x(t) will be an exponentially rising sinusoidal signal; and when b is negative, the signal x(t) will be an exponentially decaying sinusoidal signal.

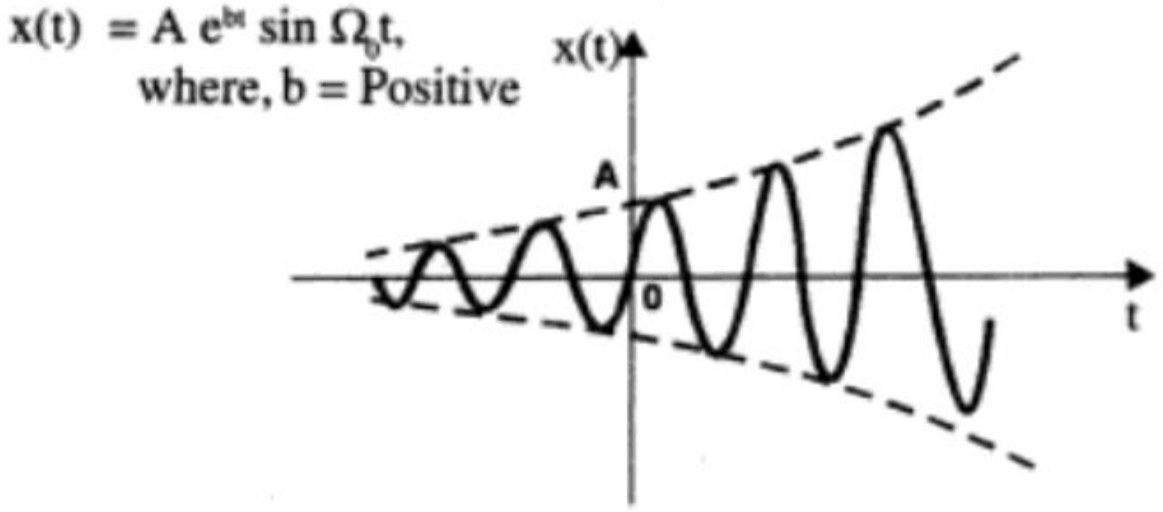


Fig 2.13 : Exponentially rising sinusoid.

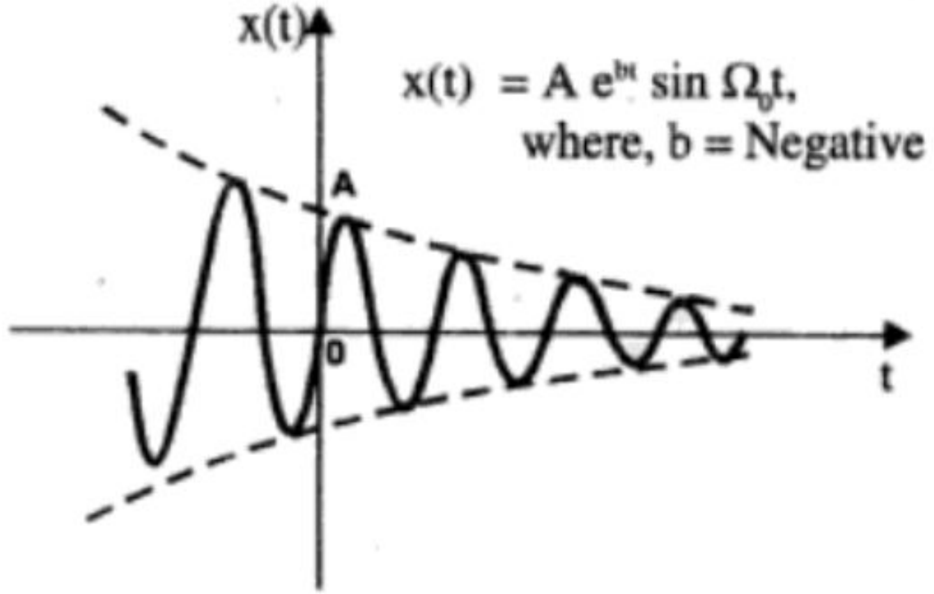


Fig 2.14 : Exponentially decaying sinusoid.

9. Triangular pulse signal

The Triangular pulse signal is defined as

$$x(t) = \Delta_a(t) = 1 - \frac{|t|}{a} \quad ; \quad |t| \le a$$
$$= 0 \quad ; \quad |t| > a$$

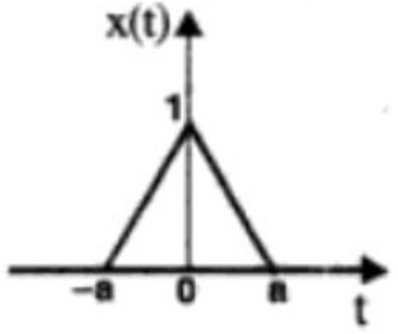


Fig 2.15 : Triangular pulse signal.

10. Signum signal

The Signum signal is defined as the sign of the independent variable t. Therefore, the Signum signal is expressed as,

$$x(t) = \text{sgn}(t) = 1 \quad ; \quad t > 0$$
$$= 0 \quad ; \quad t = 0$$
$$= -1 \quad ; \quad t < 0$$

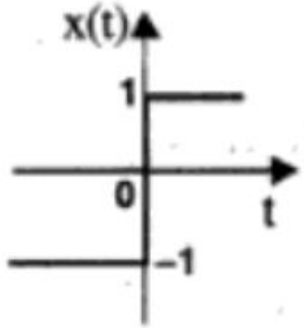


Fig 2.16 : Signum signal.

11. Sinc signal

The Sinc signal is defined as,

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t} \quad ; \quad -\infty < t < \infty$$

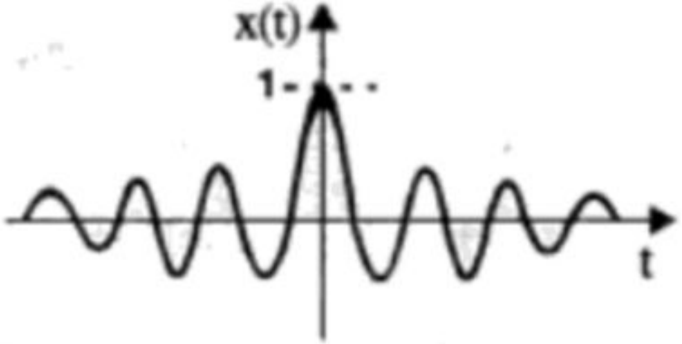


Fig 2.17 : Sinc signal.

12. Gaussian signal

The Gaussian signal is defined as,

$$x(t) = g_a(t) = e^{-a^2t^2} \quad ; \quad -\infty < t < \infty$$

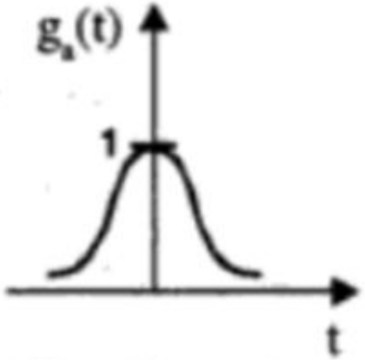


Fig 2.18 : Gaussian signal.

2.4 Mathematical Operations on Continuous Time Signals

2.4.1 Scaling of Continuous Time Signals

The two types of scaling continuous time signals are,

1. Amplitude Scaling
2. Time Scaling

1. Amplitude Scaling

The ***amplitude scaling*** is performed by multiplying the amplitude of the signal by a constant.

Let $x(t)$ be a continuous time signal. Now $Ax(t)$ is the amplitude scaled version of $x(t)$, where A is a constant.

When $|A| > 1$, then $Ax(t)$ is the amplitude magnified version of $x(t)$ and when $|A| < 1$, then $Ax(t)$ is the amplitude attenuated version of $x(t)$.

Example : 1

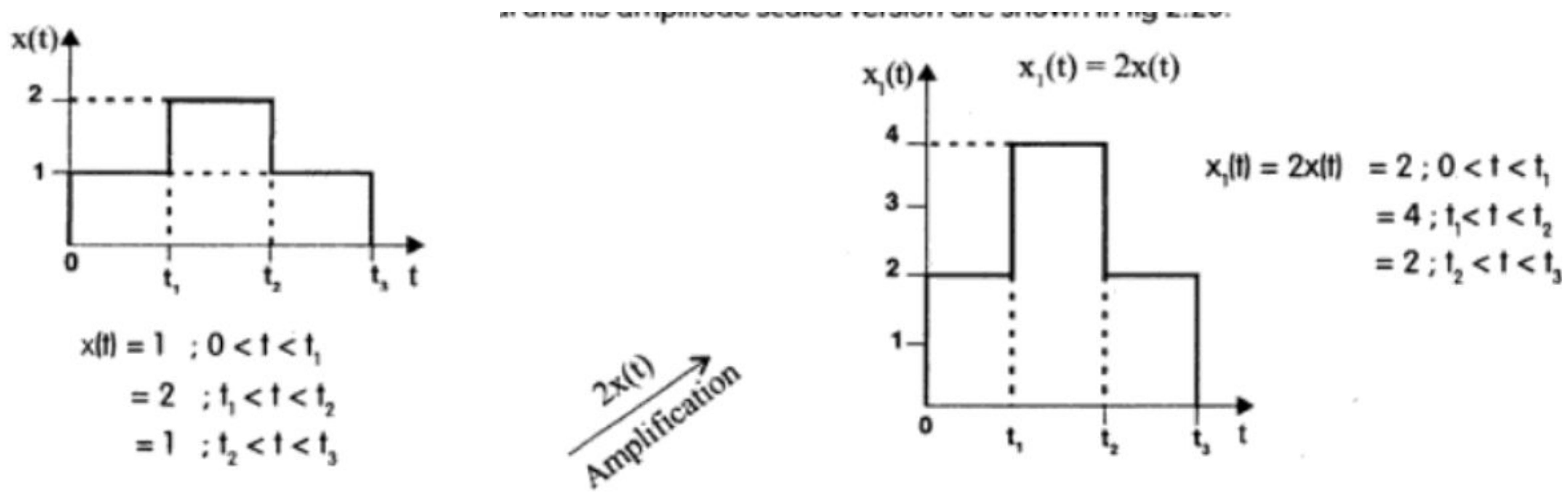
Let, $x(t) = at + be^{-ct}$

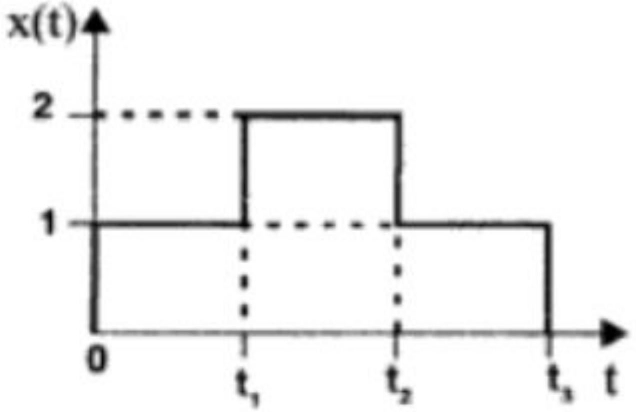
Let $x_1(t)$ and $x_2(t)$ be the amplitude scaled versions of $x(t)$, scaled by constants 4 and $\frac{1}{4}\left(\frac{1}{4} = 0.25\right)$ respectively.

Now, $x_1(t) = 4x(t) = 4(at + be^{-ct}) = 4at + 4be^{-ct}$

$x_2(t) = 0.25x(t) = 0.25(at + be^{-ct}) = 0.25at + 0.25be^{-ct}$

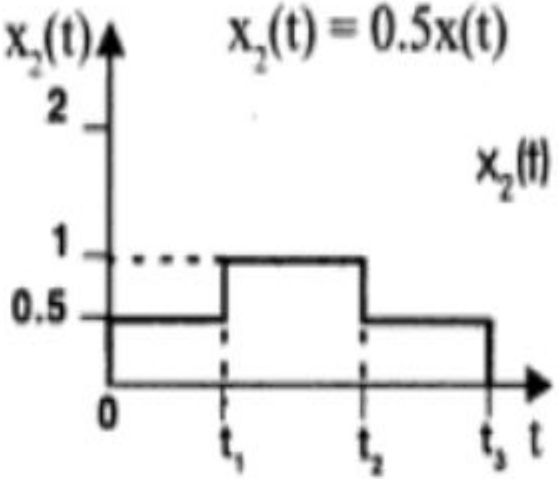
A continuous time signal and its amplitude scaled version are shown in fig 2.20.





$$
\begin{aligned}
x(t) &= 1 \quad ; 0 < t < t_1 \\
&= 2 \quad ; t_1 < t < t_2 \\
&= 1 \quad ; t_2 < t < t_3
\end{aligned}
$$

$x_2(t) = 0.5x(t)$



$$
\begin{aligned}
x_2(t) = 0.5x(t) &= 0.5 \; ; 0 < t < t_1 \\
&= 1 \quad ; t_1 < t < t_2 \\
&= 0.5 \; ; t_2 < t < t_3
\end{aligned}
$$

2. *Time Scaling*

The ***time scaling*** is performed by multiplying the variable time by a constant.

If x(t) is a continuous time signal, then x(At) is the time scaled version of x(t), where A is a constant.

When $|A| > 1$, then x(At) is the time compressed version of x(t) and when $|A| < 1$, then x(At) is the time expanded version of x(t).

Example : 1

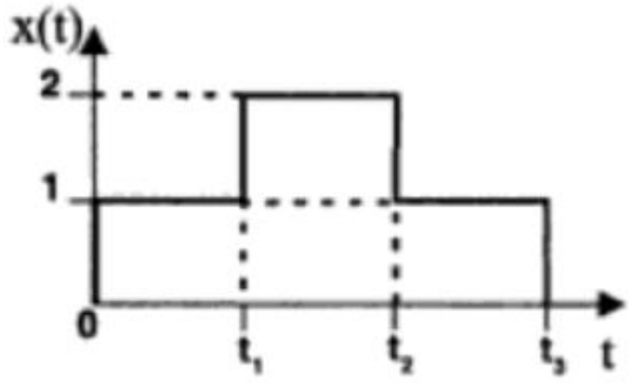
Let, $x(t) = at + be^{-ct}$

Let $x_1(t)$ and $x_2(t)$ be the time scaled versions of x(t), scaled by constants 4 and 1/4 (0.25)respectively.

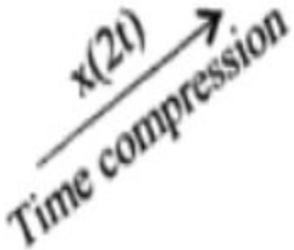
Now, $x_1(t) = x(4t) = a \times 4t + be^{-c \times 4t} = 4at + be^{-4ct}$

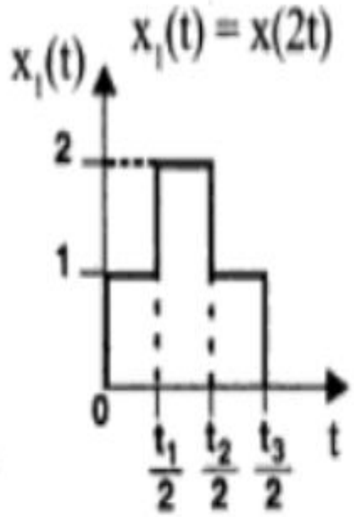
$x_2(t) = x(0.25t) = a \times 0.25t + be^{-c \times 0.25t} = 0.25at + be^{-0.25ct}$

A continuous time signal and its time scaled version are shown in fig 2.21.



$$
\begin{aligned}
x(t) &= 1 \; ; \; 0 < t < t_1 \\
&= 2 \; ; \; t_1 < t < t_2 \\
&= 1 \; ; \; t_2 < t < t_3
\end{aligned}
$$

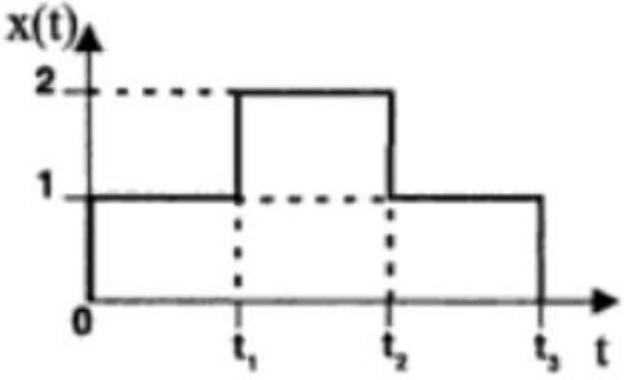




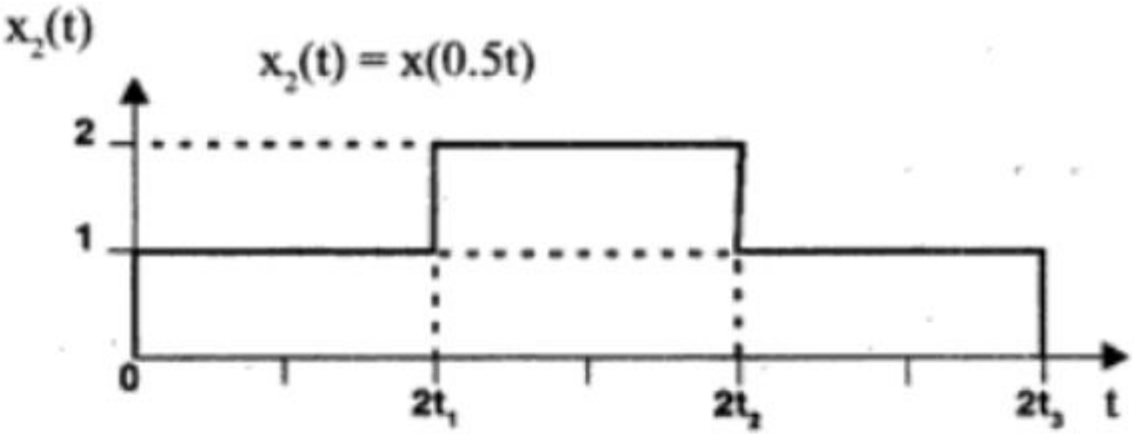
When $t = \frac{t_1}{2}$; $x_1\left(\frac{t_1}{2}\right) = x\left(2\frac{t_1}{2}\right) = x(t_1)$

When $t = \frac{t_2}{2}$; $x_1\left(\frac{t_2}{2}\right) = x\left(2\frac{t_2}{2}\right) = x(t_2)$

When $t = \frac{t_3}{2}$; $x_1\left(\frac{t_3}{2}\right) = x\left(2\frac{t_3}{2}\right) = x(t_3)$



$$x(t) = 1\ ;\ 0 < t < t_1$$
$$= 2\ ;\ t_1 < t < t_2$$
$$= 1\ ;\ t_2 < t < t_3$$



When $t = 2t_1$; $x_1(2t_1) = x(0.5 \times 2t_1) = x(t_1)$
When $t = 2t_2$; $x_1(2t_2) = x(0.5 \times 2t_2) = x(t_2)$
When $t = 2t_3$; $x_1(2t_3) = x(0.5 \times 2t_3) = x(t_3)$

2.4.2 Folding (Reflection or Transpose) of Continuous Time Signals

The *folding* of a continuous time signal x(t) is performed by changing the sign of time base t in the signal x(t).

The folding operation produces a signal x(–t) which is a mirror image of the original signal x(t) with respect to the time origin t = 0.

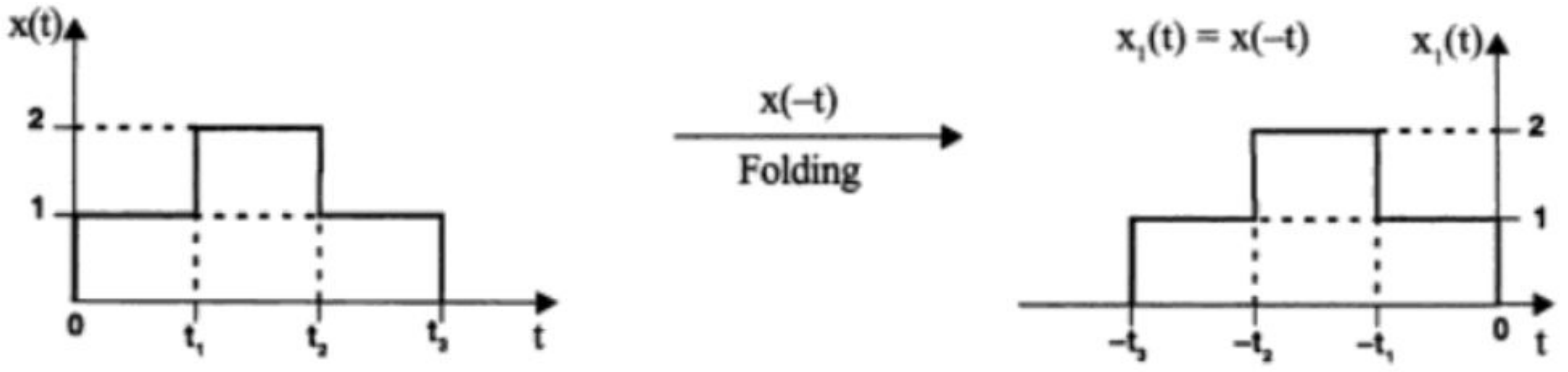
Example : 1

Let, $x(t) = at + be^{-ct}$

Let $x_1(t)$ be folded version of x(t).

Now, $x_1(t) = x(-t) = a(-t) + be^{-c(-t)} = -at + be^{ct}$

A continuous time signal and its folded version is shown in fig :



$$x(t) = 1 \;;\quad 0 < t < t_1$$
$$= 2 \;;\quad t_1 < t < t_2$$
$$= 1 \;;\quad t_2 < t < t_3$$

When $t = -t_1$; $x_1(-t_1) = x(-(-t_1)) = x(t_1)$

When $t = -t_2$; $x_1(-t_2) = x(-(-t_2)) = x(t_2)$

When $t = -t_3$; $x_1(-t_3) = x(-(-t_3)) = x(t_3)$

Fig 2.22 : A continuous time signal and its folded version.

2.4.3 Time Shifting of Continuous Time Signals

The ***time shifting*** of a continuous time signal x(t) is performed by replacing the independent variable t by t – m, to get the time shifted signal x(t – m), where m represents the time shift in seconds.

In x(t – m), if m is positive, then the time shift results in a delay by m seconds. The ***delay*** results in shifting the original signal x(t) to right, to generate the time shifted signal x(t – m).

In x(t – m), if m is negative, then the time shift results in an advance of the signal by |m| seconds. The ***advance*** results in shifting the original signal x(t) to left, to generate the time shifted signal x(t – m).

Example : 1

Let, $x(t) = at + be^{-ct}$

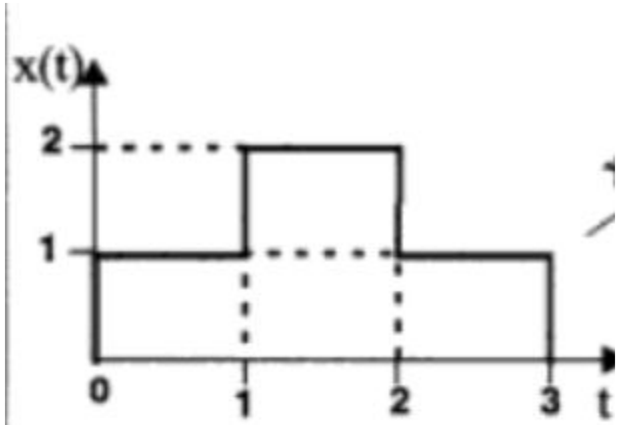
Let $x_1(t)$ and $x_2(t)$ be time shifted version of $x(t)$, shifted by m units of time.

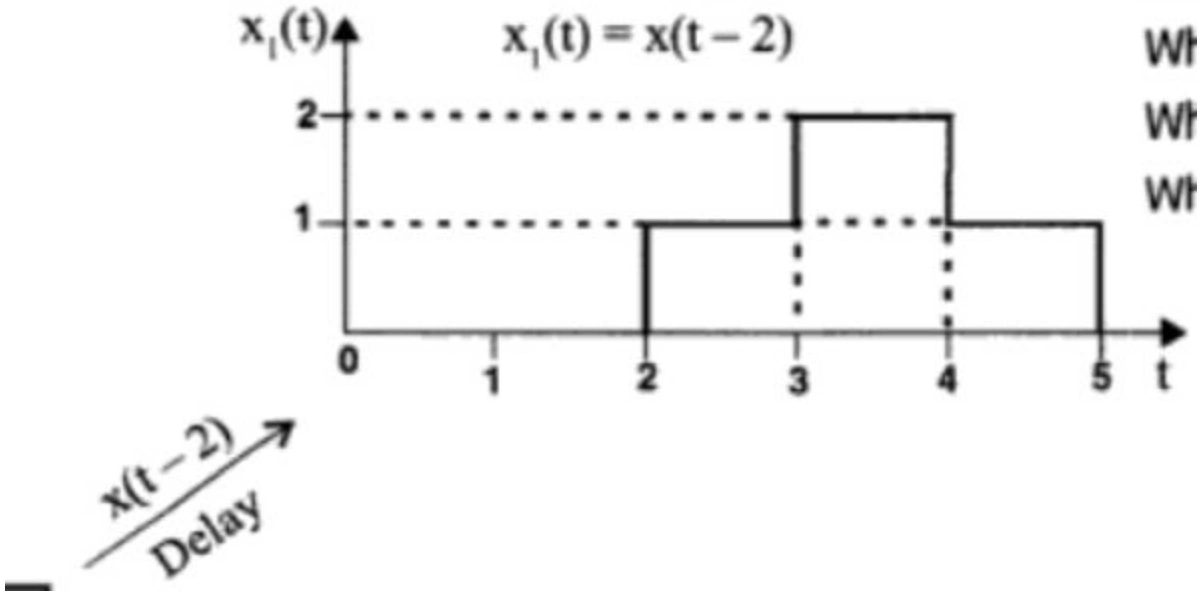
Let $x_1(t)$ be delayed version of $x(t)$ and $x_2(t)$ be advanced version of $x(t)$.

Now, $x_1(t) = a(t - m) + be^{-c(t - m)}$

$x_2(t) = a(t + m) + be^{-c(t + m)}$

A signal and its shifted version are shown in fig 2.23.





$x(t) = 1 \quad ; 0 < t < 1$

$\quad\;\; = 2 \quad ; 1 < t < 2$

$\quad\;\; = 1 \quad ; 2 < t < 3$

When $t = 2$; $x_1(2) = x(2-2) = x(0) = 1$

When $t = 3$; $x_1(3) = x(3-2) = x(1) = 1$

When $t = 4$; $x_1(4) = x(4-2) = x(2) = 2$

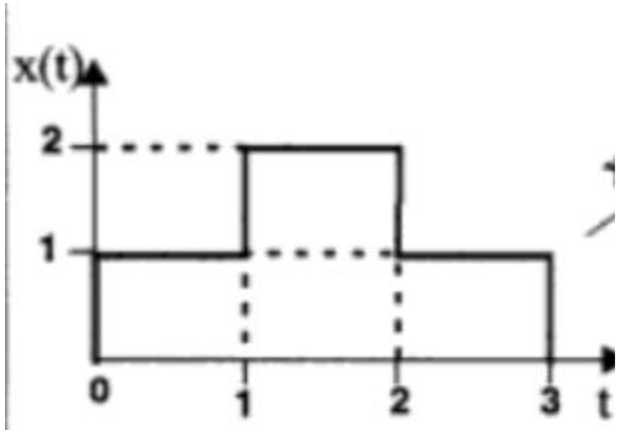
When $t = 5$; $x_1(5) = x(5-2) = x(3) = 1$

When $t = -2$; $x_1(-2) = x(-2 + 2) = x(0) = 1$

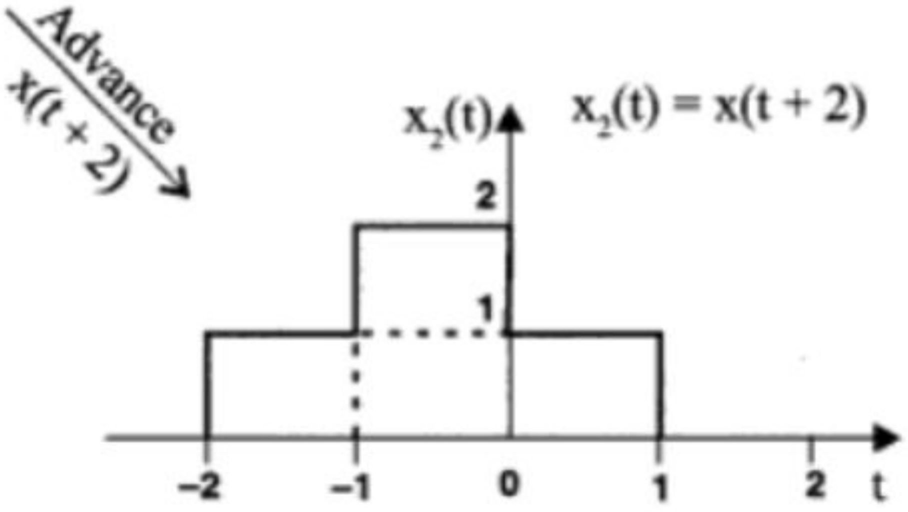
When $t = -1$; $x_1(-1) = x(-1 + 2) = x(1) = 1$

When $t = 0$; $x_1(0) = x(0 + 2) = x(2) = 2$

When $t = 1$; $x_1(1) = x(1 + 2) = x(3) = 1$



$$x(t) = \begin{cases} 1 & ; 0 < t < 1 \\ 2 & ; 1 < t < 2 \\ 1 & ; 2 < t < 3 \end{cases}$$



Delayed Unit Impulse Signal

The unit impulse signal is defined as,

$$\delta(t) = \infty \,;\, t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0 \,;\, t \neq 0$$

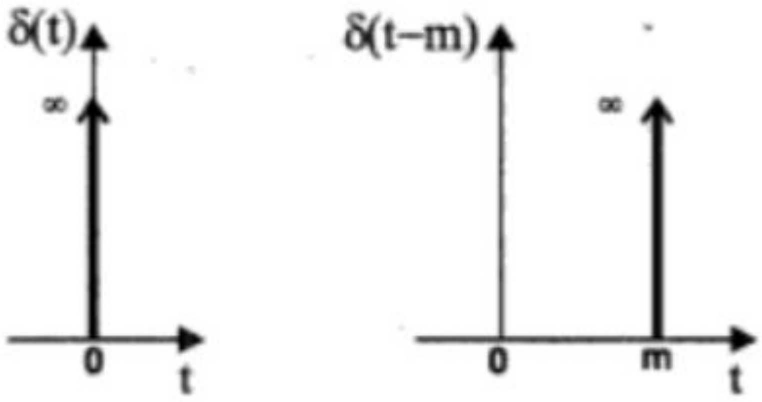


Fig 2.24a : Impulse. *Fig 2.24b : Delayed impulse.*

Fig 2.24 : Impulse and delayed impulse signal.

The unit impulse signal delayed by m units of time is denoted as $\delta(t-m)$, and it is defined as,

$$\delta(t-m) = \infty ;\, t = m \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t-m)\, dt = 1$$
$$= 0 \,;\, t \neq m$$

Delayed Unit Step Signal

The unit step signal is defined as,

$$u(t) = 1 \; ; \text{ for } \; t \geq 0$$
$$= 0 \; ; \text{ for } \; t < 0$$

The unit step signal delayed by m units of time is denoted as u(t – m), and it is defined as,

$$u(t-m) = 1 \; ; \; t \geq m$$
$$= 0 \; ; \; t < m$$

Fig 2.25a : Unit step signal.

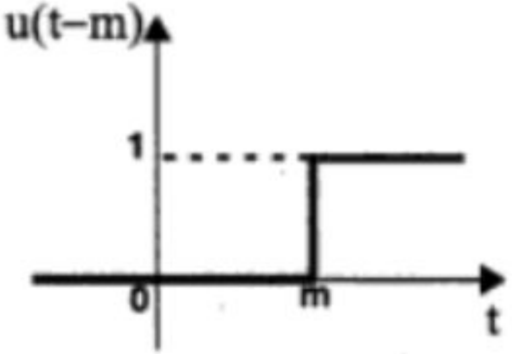


Fig 2.25b : Delayed unit step signal.

Fig 2.25 : Unit step and delayed unit step signal.

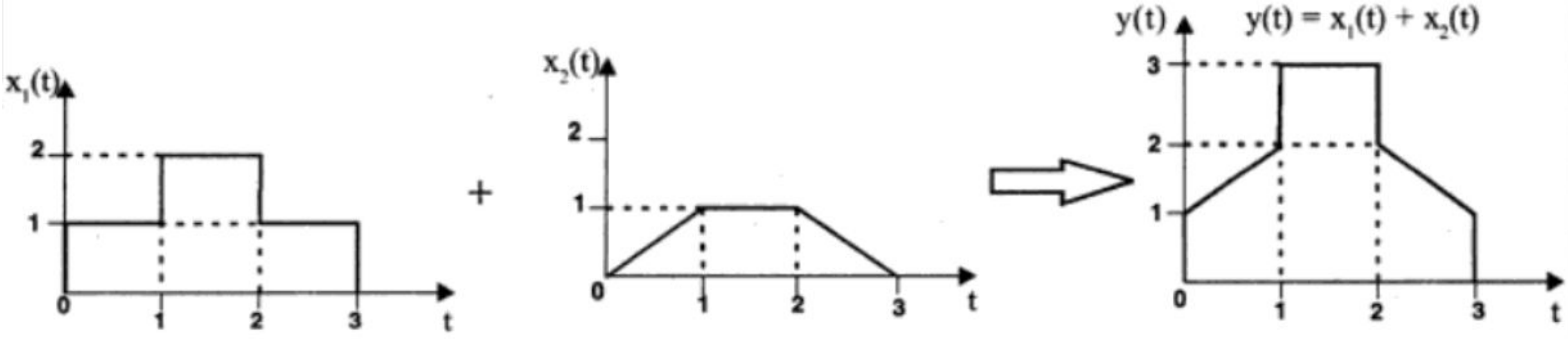
2.4.4 Addition of Continuous Time Signals

The ***addition*** of two continuous time signals is performed by adding the value of the two signals corresponding to the same instant of time.

The sum of two signals $x_1(t)$ and $x_2(t)$ is a signal $y(t)$, whose value at any instant is equal to the sum of the value of these two signals at that instant.

i.e., $y(t) = x_1(t) + x_2(t)$

Graphical addition of two continuous time signals is shown in fig 2.26.



$$x_1(t) = 1 \quad ; 0 < t < 1$$
$$= 2 \quad ; 1 < t < 2$$
$$= 1 \quad ; 2 < t < 3$$

$$x_2(t) = t \quad ; 0 < t < 1$$
$$= 1 \quad ; 1 < t < 2$$
$$= 3 - t \; ; 2 < t < 3$$

For $t = 0$ to 1 ; $y(t) = x_1(t) + x_2(t) = 1 + t$

For $t = 1$ to 2 ; $y(t) = x_1(t) + x_2(t) = 2 + 1 = 3$

For $t = 2$ to 3 ; $y(t) = x_1(t) + x_2(t) = 1 + 3 - t = 4 - t$

Fig 2.26 : Addition of two continuous time signals.

2.4.5 Multiplication of Continuous Time Signals

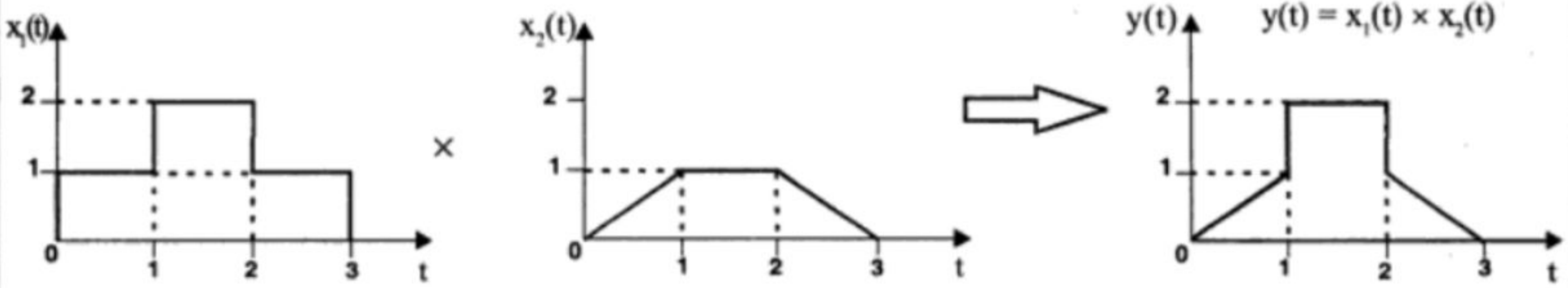
The ***multiplication*** of two continuous time signals is performed by multiplying the value of the two signals corresponding to the same instant of time.

The product of two signals $x_1(t)$ and $x_2(t)$ is a signal $y(t)$, whose value at any instant is equal to the product of the values of these two signals at that instant.

i.e., $y(t) = x_1(t) \times x_2(t)$

Example :

Graphical multiplication of two continuous time signals is shown in fig 2.27.



$x_1(t) = 1 \,;\, 0 < t < 1$
$\quad = 2 \,;\, 1 < t < 2$
$\quad = 1 \,;\, 2 < t < 3$

$x_2(t) = t \,;\, 0 < t < 1$
$\quad = 1 \,;\, 1 < t < 2$
$\quad = 3 - t \,;\, 2 < t < 3$

For $t = 0$ to 1; $y(t) = x_1(t) \times x_2(t) = 1$ ˝ $t = t$
For $t = 1$ to 2; $y(t) = x_1(t) \times x_2(t) = 2$ ˝ $1 = 2$
For $t = 2$ to 3; $y(t) = x_1(t) \times x_2(t) = 1$ ˝ $(3 - t) = 3 - t$

Fig 2.27 : Multiplication of two continuous time signals.

Example 2.5

A continuous time signal is defined as,

$$x(t) = t \quad ; \; 0 \le t \le 3$$
$$\quad = 0 \quad ; \; t > 3$$

Sketch the waveform of $x(-t)$ and $x(2-t)$.

Solution

The given signal is shown in fig 1.

The signal $x(-t)$ is the folded version of $x(t)$. The signal $x(-t)$ is shown in fig 2.

The signal $x(2-t) = x(-t+2)$ is the advanced version of the folded signal. The signal $x(-t+2)$ is shown in fig 3.

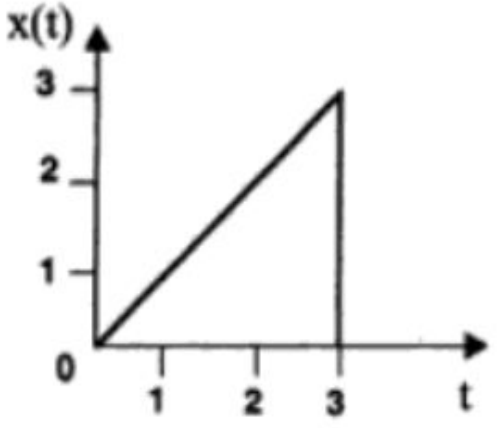


Fig 1 : x(t).

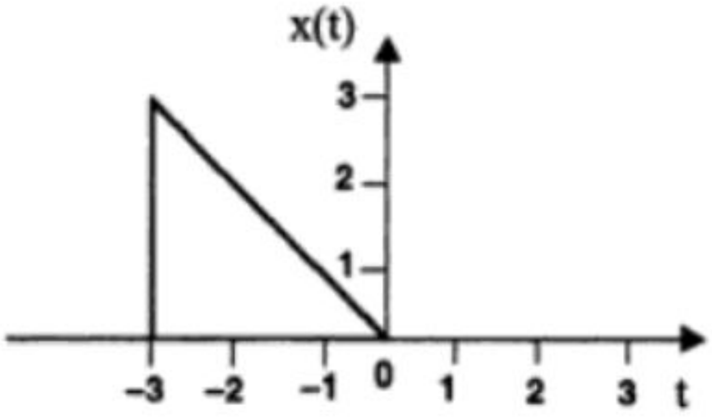


Fig 2 : x(–t).

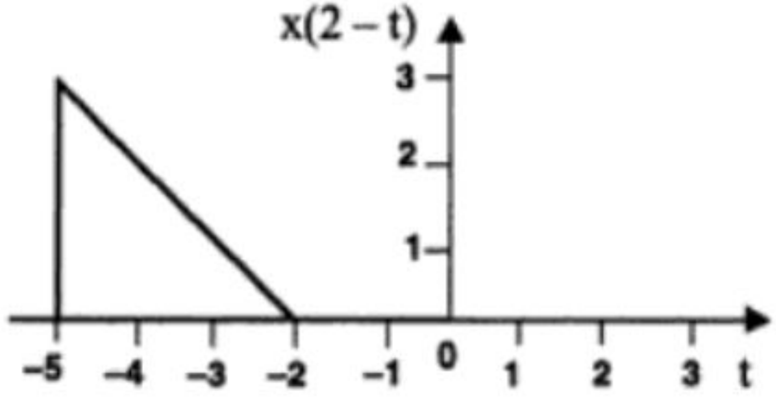
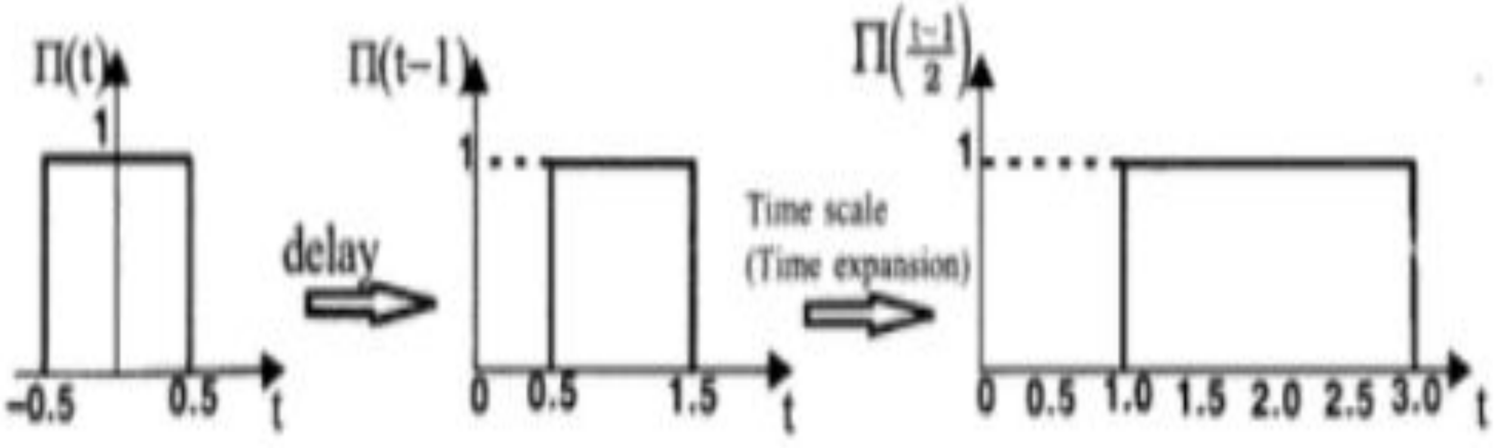
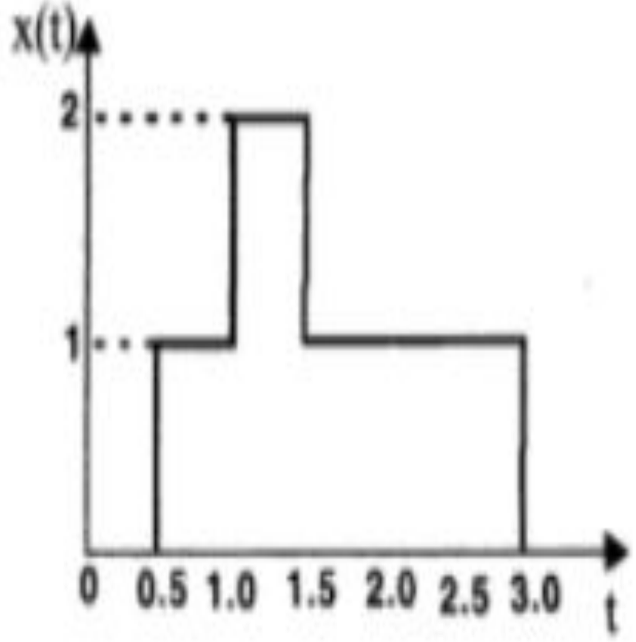


Fig 3 : x(2 – t).

Q2.11 ***Sketch the signal,*** $x(t) = \Pi\left(\frac{t-1}{2}\right) + \Pi(t-1)$

Solution





Q2.12 ***A continuous time signal is shown in fig Q 2.12. Sketch the following versions of the signal.***

a) $x(t-3)$ b) $-2\,x(t)$ c) $x(t-3)-2\,x(t)$ d) $\dfrac{d\,x(t)}{dt}$

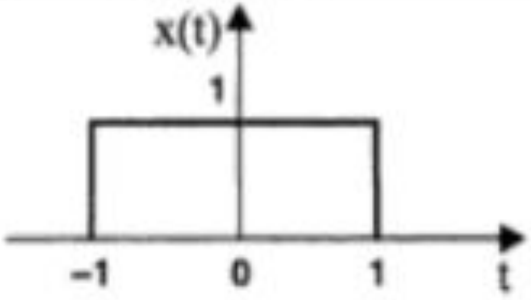
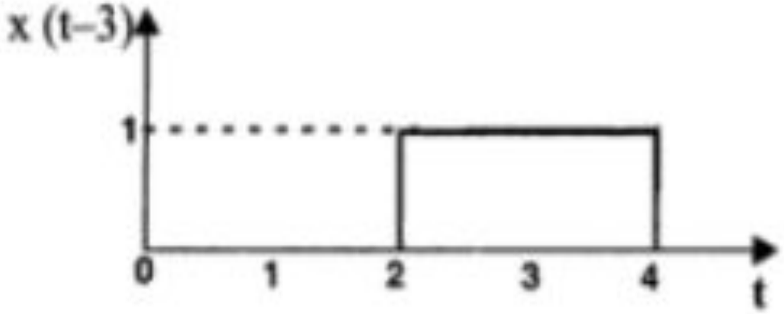


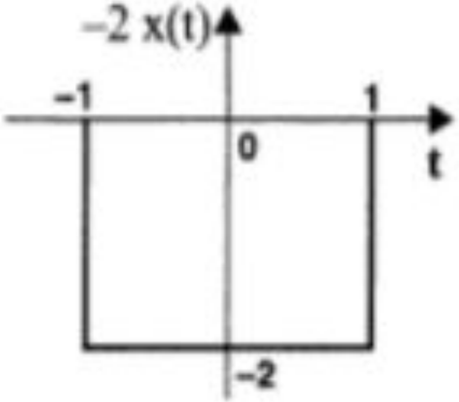
Fig Q : 2.12.

Solution

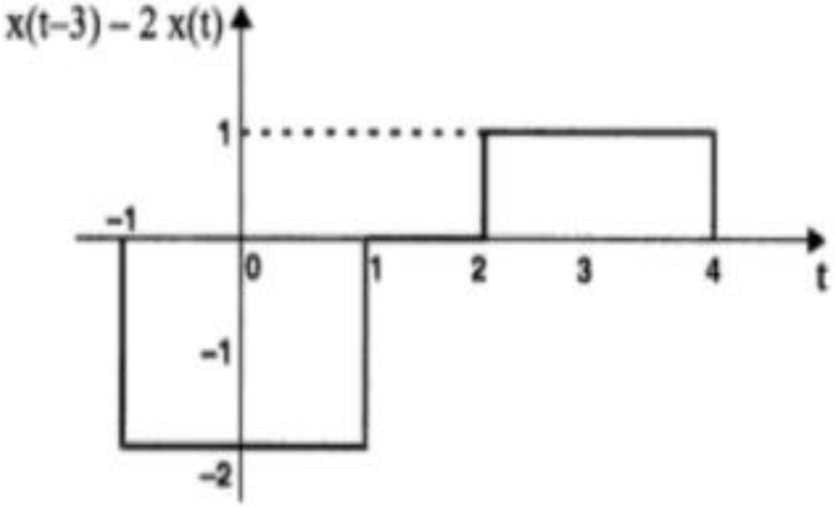
a)



b)



c)

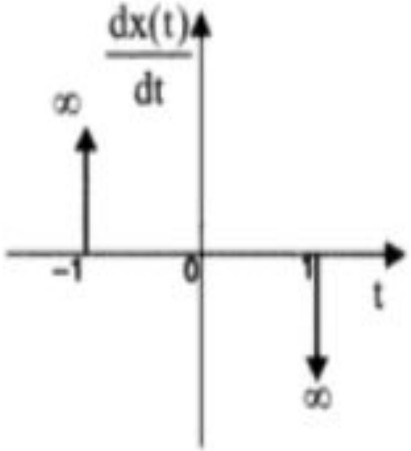


d) $x(t) = u(t+1) - u(t-1)$

$$\therefore \frac{d}{dt}x(t) = \frac{d}{dt}u(t+1) - \frac{d}{dt}u(t-1)$$

$$= \delta(t+1) - \delta(t-1)$$

$\frac{d}{dt}u(t) = \delta(t)$
Using time invariant property $\frac{d}{dt}u(t \pm k) = \delta(t \pm k)$



Q2.13 *A continuous time signal is shown in fig Q 2.13. Find the following versions of the signal.*

a) x(– t) *b) – x(t)*

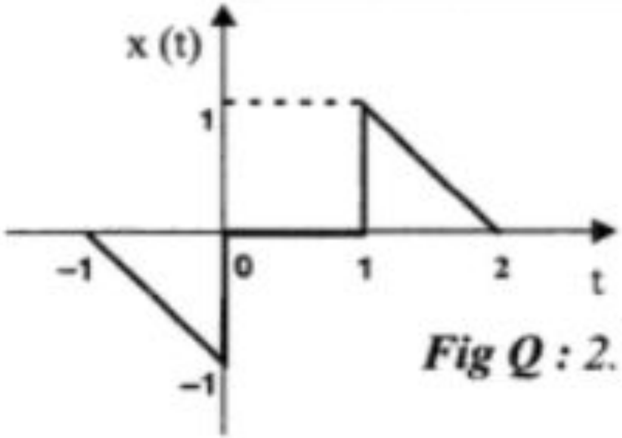
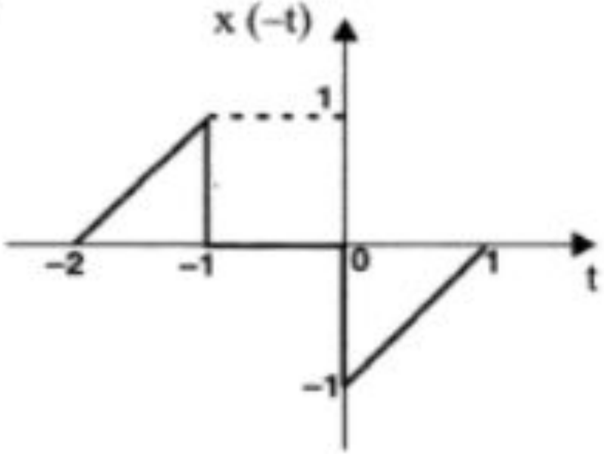


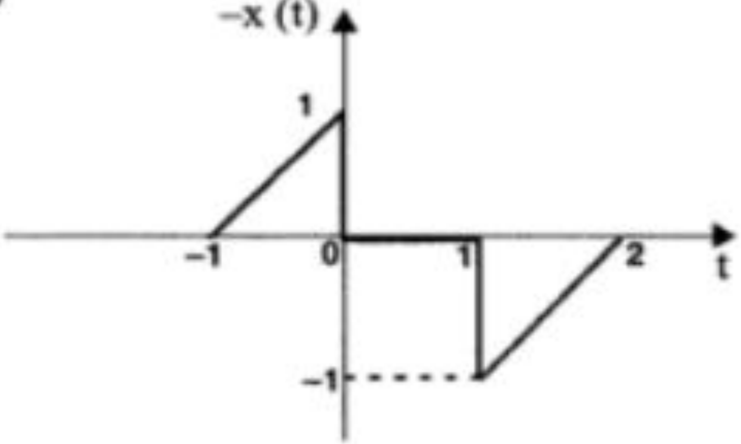
Fig Q : 2.13.

Solution

a)



b)



2.5 Impulse Signal

The impulse signal is a special signal which can be derived as follows.

Consider a pulse signal, $P_\Delta(t)$ with height A/Δ and width Δ as shown in fig 2.28. Now, the pulse signal, $P_\Delta(t)$ can be defined as,

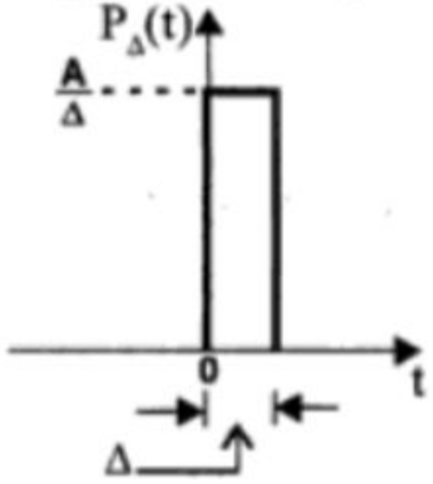


Fig 2.28 : Pulse signal.

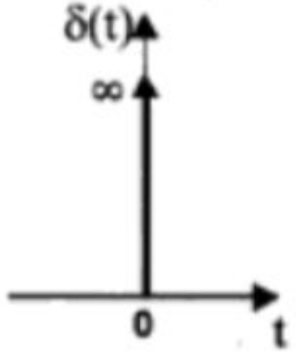
$$P_\Delta(t) = \frac{A}{\Delta} \quad ; \; 0 \le t \le \Delta$$

$$= 0 \quad ; \; t > \Delta$$

The area of the pulse signal for any value of t is given by,

$$\text{Area} = \text{Height} \times \text{Width} = \frac{A}{\Delta} \times \Delta = A$$

In the signal, $P_\Delta(t)$ if the width Δ is reduced, then the height A/Δ increases, but the area of the pulse remains same as A. When the width Δ tends to zero, the height A/Δ tends to infinity. This limiting value of the pulse signal is called ***impulse signal***, $\delta(t)$. Even when the width Δ tends to zero, the area of the pulse remains as A.



$$\therefore \text{ Impulse Signal, } \delta(t) = \underset{\Delta\to 0}{\text{Lt}}\ P_\Delta(t) = \underset{\Delta\to 0}{\text{Lt}}\ \frac{A}{\Delta} \quad ; \ t = 0$$

$$= 0 \quad ; \ t \neq 0$$

$$\therefore \text{ Impulse Signal, } \delta(t) = \infty \ ; \ t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\,dt = A$$

$$= 0 \ ; \ t \neq 0$$

Fig 2.29 : Impulse signal (or Unit impulse signal).

In the above impulse signal if A=1, then the impulse signal is called a ***unit impulse signal.*** The impulse signal or unit impulse signal can be represented graphically as shown in fig 2.29. An impulse with infinite magnitude and zero duration is a mathematical fiction and does not exist in reality. However a signal with large magnitude and short duration (when compared to time constant of a system) can be considered as an impulse signal. Practically, the magnitude of the impulse is measured by its area.

Definition of impulse signal : The impulse signal is a signal with infinite magnitude and zero duration, but with an area of A. Mathematically, an impulse signal is defined as,

$$\text{Impulse Signal, } \delta(t) = \infty \; ; \; t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\,dt = A$$

$$= 0 \; ; \; t \neq 0$$

Definition of unit impulse signal : The unit impulse signal is a signal with infinite magnitude and zero duration, but with unit area. Mathematically, a unit impulse signal is defined as,

$$\text{Unit Impulse Signal, } \delta(t) = \infty \; ; \; t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

$$= 0 \; ; \; t \neq 0$$

2.5.1 Properties of Impulse Signal

Property-1: $$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

Property-2: $$\int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt = x(0)$$

Property-3: $$\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

Property-4: $$\int_{-\infty}^{+\infty} x(\lambda)\,\delta(t-\lambda)\,d\lambda = x(t)$$

Property-5: $$\delta(at) = \frac{1}{|a|}\delta(t)$$

2.3 Classification of Continuous Time Signals

The continuous time signals are classified depending on their characteristics. Some ways of classifying continuous time signals are,

1. Deterministic and Nondeterministic signals
2. Periodic and Nonperiodic signals
3. Symmetric and Antisymmetric signals (Even and Odd signals)
4. Energy and Power signals
5. Causal and Noncausal signals

2.3.1 Deterministic and Nondeterministic Signals

The signal that can be completely specified by a mathematical equation is called a ***deterministic signal***. The step, ramp, exponential and sinusoidal signals are examples of deterministic signals.

Examples of deterministic signals: $x_1(t) = At$

$$x_2(t) = X_m \sin \Omega_0 t$$

The signal whose characteristics are random in nature is called a ***nondeterministic signal.*** The noise signals from various sources like electronic amplifiers, oscillators, radio receivers, etc., are best examples of nondeterministic signals.

2.3.2 Periodic and Nonperiodic Signals

A periodic signal will have a definite pattern that repeats again and again over a certain period of time. Therefore the signal which satisfies the condition,

$\boxed{x(t + T) = x(t)}$ is called a ***periodic signal.***

A signal which does not satisfy the condition, $x(t + T) = x(t)$ is called an ***aperiodic or nonperiodic signal***. In periodic signals, the term T is called the ***fundamental time period*** of the signal. Hence, inverse of T is called the ***fundamental frequency,*** F_0 in cycles/sec or Hz, and $2\pi F_0 = \Omega_0$ is called the ***fundamental angular frequency*** in rad/sec.

The sinusoidal signals and complex exponential signals are always periodic with a periodicity of T, where, $T = \dfrac{1}{F_0} = \dfrac{2\pi}{\Omega_0}$. The proof of this concept is given below.

Example 2.1

Verify whether the following continuous time signals are periodic. If periodic, find the fundamental period.

a) $x(t) = 2\cos\frac{t}{4}$ b) $x(t) = e^{\alpha t}$; $\alpha > 1$ c) $x(t) = e^{\frac{-j2\pi t}{7}}$ d) $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$ e) $x(t) = \cos^2\left(2t - \frac{\pi}{4}\right)$

Solution

a) Given that, $x(t) = 2\cos\frac{t}{4}$

The given signal is a cosinusoidal signal, which is always periodic.

On comparing x(t) with the standard form "$A\cos 2\pi F_0 t$" we get,

$$2\pi F_0 = \frac{1}{4} \quad \Rightarrow \quad F_0 = \frac{1}{8\pi}$$

Period, $T = \frac{1}{F_0} = 8\pi$

$\therefore$ x(t) is periodic with period, $T = 8\pi$.

b) Given that, $x(t) = e^{\alpha t}$; $\alpha > 1$

$$\therefore x(t + T) = e^{\alpha(t + T)}$$

$$= e^{\alpha t} e^{\alpha T}$$

For any value of α, $e^{\alpha T} \neq 1$ and so $x(t + T) \neq x(t)$

Since $x(t + T) \neq x(t)$, the signal $x(t)$ is non-periodic.

c) Given that, $x(t) = e^{\frac{-j2\pi t}{7}}$

The given signal is a complex exponential signal, which is always periodic.

On comparing x(t) with the standard form "$Ae^{-j2\pi F_0 t}$"

We get, $F_0 = \frac{1}{7}$

$$\therefore \text{ Peroid, } T = \frac{1}{F_0} = 7$$

$\therefore$ x(t) is periodic with period, T = 7.

d) Given that, $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$

The given signal is a cosinusoidal signal, which is always periodic.

$$\therefore\ x(t+T) = 3\cos\left(5(t+T) + \frac{\pi}{6}\right) = 3\cos\left(5t + 5T + \frac{\pi}{6}\right) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 5T\right)$$

Let $5T = 2\pi$, $\quad \therefore\ T = \frac{2\pi}{5}$

$$\therefore\ x(t+T) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 5 \times \frac{2\pi}{5}\right) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 2\pi\right)$$

$$= 3\cos\left(5t + \frac{\pi}{6}\right) = x(t)$$

For integer values of M,
$\cos(\theta + 2\pi M) = \cos\theta$

Since $x(t+T) = x(t)$, the signal $x(t)$ is periodic with period, $T = \frac{2\pi}{5}$

e) Given that, $x(t) = \cos^2\left(2t - \frac{\pi}{3}\right)$

$$x(t) = \cos^2\left(2t - \frac{\pi}{3}\right) = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2}$$

$$\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$\therefore\ x(t + T) = \frac{1 + \cos\left(4(t + T) - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t + 4T - \frac{2\pi}{3}\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4T\right)}{2}$$

Let $4T = 2\pi,\ \therefore\ T = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\therefore\ x(t + T) = \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4 \times \frac{\pi}{2}\right)}{2} = \frac{1 + \cos\left(\left(4t - \frac{2\pi}{3}\right) + 2\pi\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \cos^2\left(2t - \frac{\pi}{3}\right) = x(t)$$

Since $x(t + T) = x(t)$, the signal $x(t)$ is periodic with period, $T = \frac{\pi}{2}$

For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$