Time Response Analysis

- Time Response
- Input Supplied System
- Steady State Response and Error
- Time Response specification
- Limitations

Time Response of Control Systems

- **Time Response of a system is defined as the output of a system when subjected to an input which is a function of time**

  Control systems generate an output or response for a given input

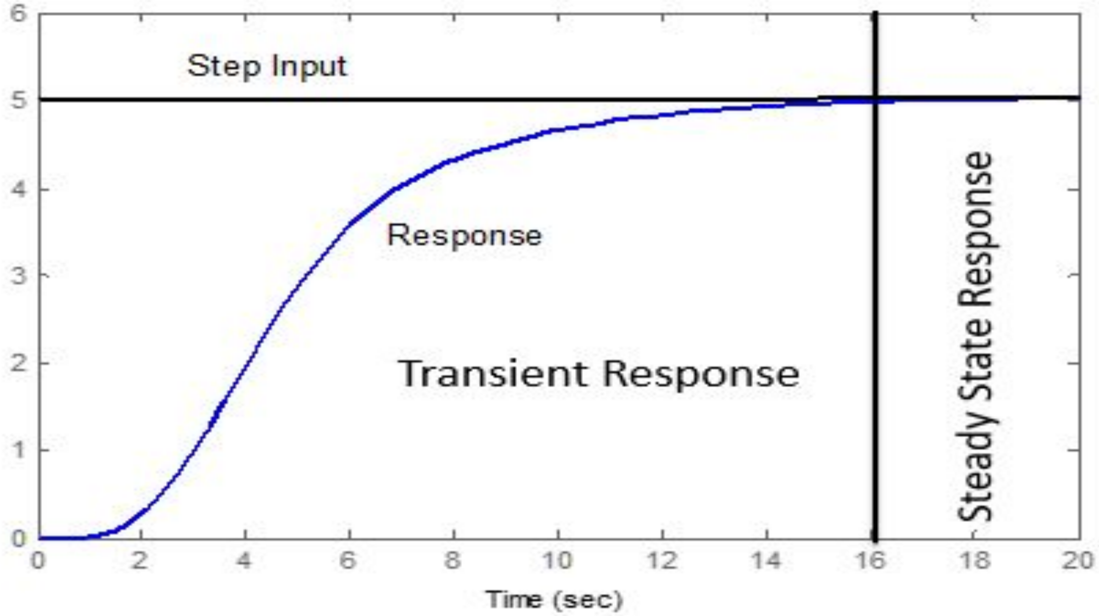
- The time response of any system has two components
  - Transient response
  - Steady-state response.

Time Response of Control Systems

- When the response of the system is changed from equilibrium it takes some time to settle down.

- This is called transient response.

- The response of the system after the transient response is called steady state response.

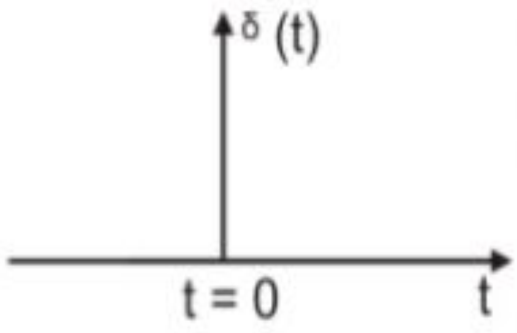


IMPULSE INPUT

- It is sudden change input. An impulse is infinite at t=0 and everywhere else.
- $r(t)= \delta(t)= 1 \qquad t=0$

  $= 0 \qquad t \neq 0$

  In laplase domain we have,
- $L[r(t)]= 1$

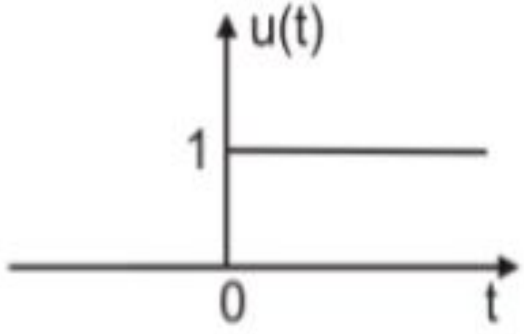


STEP INPUT

- It represents a constant command such as position. Like elevator is a step input.
- $r(t)= u(t)= A \qquad t \geq 0$

  $= 0 \qquad$ otherwise

  $L[r(t)]= A/s$



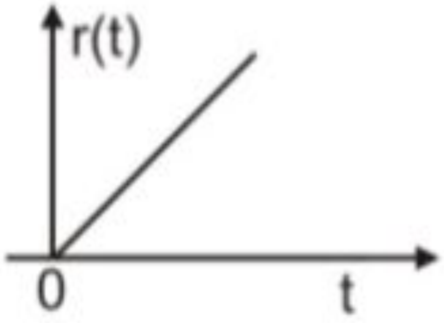
RAMP INPUT

- this represents a linearly increasing input command.
- $r(t) \quad = At \qquad t \geq 0$,Aslope

  $= 0 \qquad t < 0$

  $L[r(t)]= A/s^2$

  A= 1 then unit ramp

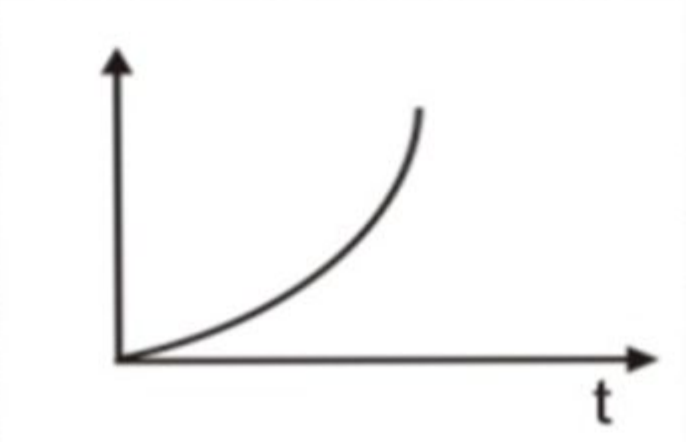


PARABOLIC INPUT

- Rate of change of velocity is acceleration. Acceleration is a parabolic function.
- r(t) = $At^2/2$ $t \geq 0$

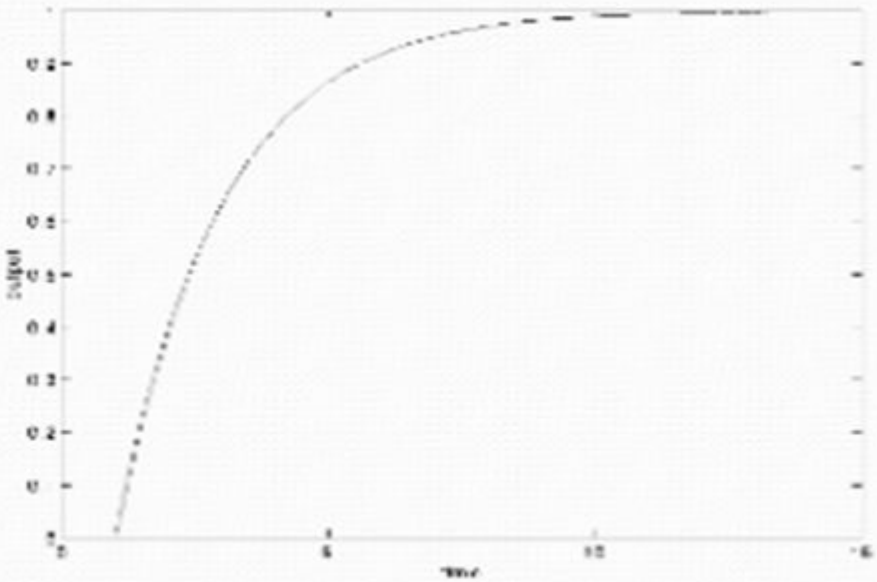
  = 0 $t < 0$

  $L[r(t)] = A/s^3$



SINUSOIDAL INPUT

- r(t) = $A \sin(wt)$ $t \geq 0$



Classification of Control Systems

- Control systems may be classified according to their ability to follow step inputs, ramp inputs, parabolic inputs, and so on.

Classification of Control Systems

- Consider the unity-feedback control system with the following open-loop transfer function

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1) \cdots (T_m s + 1)}{s^N (T_1 s + 1)(T_2 s + 1) \cdots (T_p s + 1)}$$

- It involves the term s^N in the denominator, representing N poles at the origin.

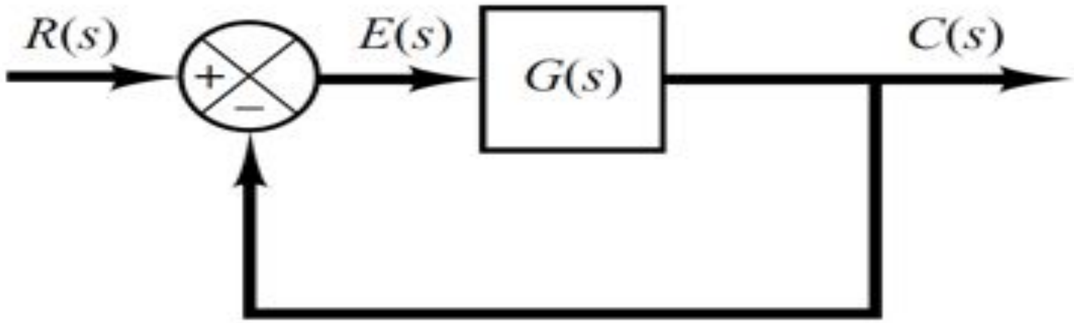
- A system is called type 0, type 1, type 2, ... , if N=0, N=1, N=2, ... , respectively.

Classification of Control Systems

- As the type number is increased, accuracy is improved.
- However, increasing the type number aggravates the stability problem.
- A compromise between steady-state accuracy and relative stability is always necessary.

Steady State Error of Unity Feedback Systems

- Consider the system shown in following figure.



- The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} \qquad G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N (T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

Steady State Error of Unity Feedback Systems

- Steady state error is defined as the error between the input signal and the output signal when $t \to \infty$.

- The transfer function between the error signal E(s) and the input signal R(s) is $$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$
- The final-value theorem provides a convenient way to find the steady-state performance of a stable system.
- Since E(s) is $$E(s) = \frac{1}{1+G(s)}R(s)$$
- The steady state error is

Static Position Error Constant (K_p)

- The steady-state error of the system for a unit-step input is

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s}}{1 + G(s)} \frac{1}{\cancel{s}}$$

$$= \frac{1}{1 + G(0)}$$

- The static position error constant K_p is defined by

$$K_p = \lim_{s \to 0} G(s) = G(0)$$

- Thus, the steady-state error in terms of the static position error constant K_p is given by

$$e_{ss} = \frac{1}{1 + K_p}$$

Static Position Error Constant (K_p)

- For a Type 0 system

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For Type 1 or higher order systems

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 1$$

- For a unit step input the steady state error e_{ss} is

$$e_{\mathrm{ss}} = \frac{1}{1 + K}, \qquad \text{for type 0 systems}$$

$$e_{\mathrm{ss}} = 0, \qquad \text{for type 1 or higher systems}$$

Static Velocity Error Constant (K_v)

- The steady-state error of the system for a unit-ramp input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s^2}$$
$$= \lim_{s \to 0} \frac{1}{sG(s)}$$

- The static velocity error constant K_v is defined by

$$K_v = \lim_{s \to 0} sG(s)$$

- Thus, the steady-state error in terms of the static velocity error constant K_v is given by

$$e_{ss} = \frac{1}{K_v}$$

Static Velocity Error Constant (K_v)

- For a Type 0 system

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- For Type 1 systems

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For type 2 or higher order systems

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 2$$

Static Velocity Error Constant (K_v)

- For a ramp input the steady state error e_{ss} is

$$e_{\mathrm{ss}} = \frac{1}{K_v} = \infty, \qquad \text{for type 0 systems}$$

$$e_{\mathrm{ss}} = \frac{1}{K_v} = \frac{1}{K}, \qquad \text{for type 1 systems}$$

$$e_{\mathrm{ss}} = \frac{1}{K_v} = 0, \qquad \text{for type 2 or higher systems}$$

Static Acceleration Error Constant (K_a)

- The steady-state error of the system for parabolic input is

$$e_{ss} = \lim_{s\to 0} \frac{s}{1 + G(s)} \frac{1}{s^3}$$

$$= \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

- The static acceleration error constant K_a is defined by

$$K_a = \lim_{s\to 0} s^2 G(s)$$

- Thus, the steady-state error in terms of the static acceleration error constant K_a is given by

$$e_{ss} = \frac{1}{K_a}$$

Static Acceleration Error Constant (K_a)

- For a Type 0 system

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1) \cdots}{(T_1 s + 1)(T_2 s + 1) \cdots} = 0$$

- For Type 1 systems

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1) \cdots}{s(T_1 s + 1)(T_2 s + 1) \cdots} = 0$$

- For type 2 systems

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1) \cdots}{s^2(T_1 s + 1)(T_2 s + 1) \cdots} = K$$

- For type 3 or higher order systems

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1) \cdots}{s^N(T_1 s + 1)(T_2 s + 1) \cdots} = \infty, \qquad \text{for } N \geq 3$$

Static Acceleration Error Constant (K_a)

- For a parabolic input the steady state error e_{ss} is

$$e_{ss} = \infty, \quad \text{for type 0 and type 1 systems}$$

$$e_{ss} = \frac{1}{K}, \quad \text{for type 2 systems}$$

$$e_{ss} = 0, \quad \text{for type 3 or higher systems}$$

Summary

	Step Input $r(t) = 1$	Ramp Input $r(t) = t$	Acceleration Input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1+K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$