Block Diagram Reduction

- control systems consists of mathematical models.
- Transfer function is a mathematical representation of the individual physical system.
- To show the function performed by each component, we generally use a block diagram.
- To analyze complex control systems, it is desirable to reduce the block diagram in simple terms(by means of block diagram reduction techniques).
- block diagram is pictorial representation of entire system.
- In this manner it represent relationship between input and the output of entire system.

Block Diagram Reduction

- Output: It is defined as product of input and gain i.e   Output = gain x input

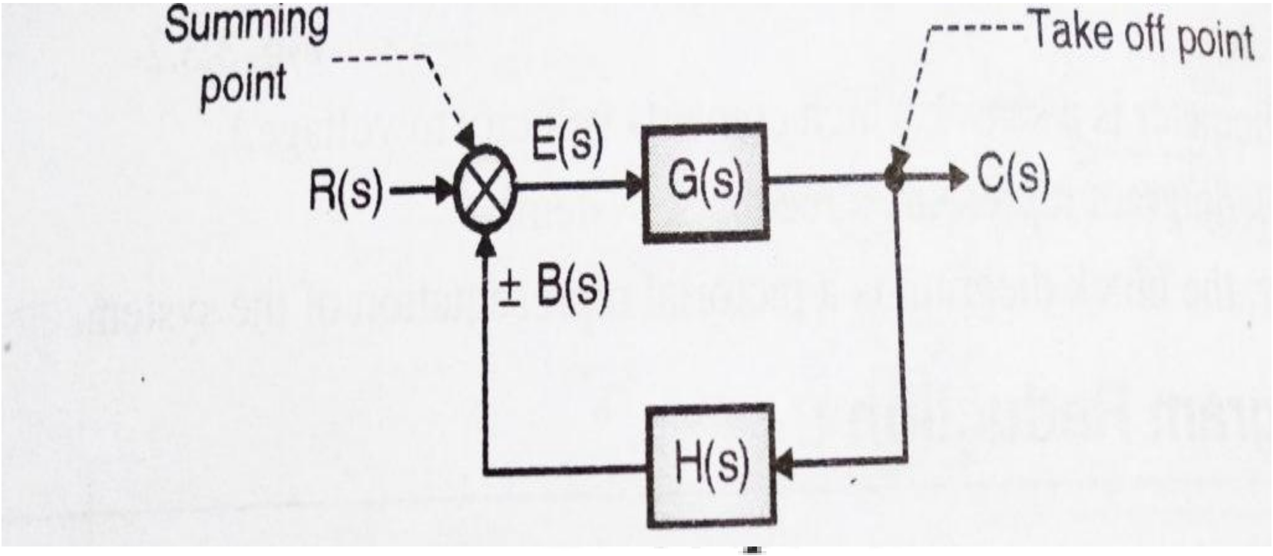
- Summing point: More than one signal can be added or subtracted at summing point.

- Take off point: From this point output can be again fed back to input, thus this point will be used for feedback purpose.

- Forward path: This path represent direction of the signal flow in the system from input side to output side.

- Feedback path: This path represent direction of the signal flow in the system from output side to input side.

simple closed loop system



$$C(s) = E(s) \cdot G(s)$$
$$E(s) = R(s) \pm B(s)$$

R(s) = Laplace of input signal

C(s) = Laplace of output signal

E(s) = Laplace of error signal

B(s) = Laplace of feedback signal

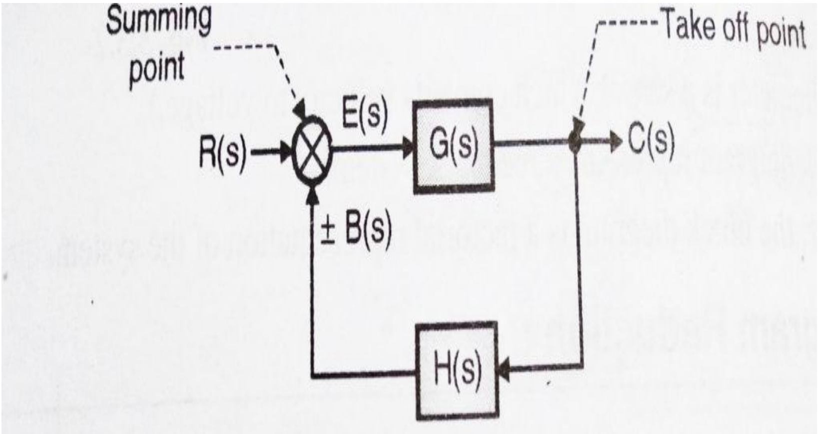
G(s) = forward transfer function

H(s) = feedback transfer function

Closed Loop (Feedback)System

$$C(s) = E(s) \cdot G(s)$$
$$E(s) = R(s) \pm B(s)$$

$$C(s) = [R(s) \pm B(s)]\,G(s)$$
$$\therefore\ C(s) = R(s)\,G(s) \pm B(s)\,G(s)$$
$$B(s) = C(s) \cdot H(s)$$



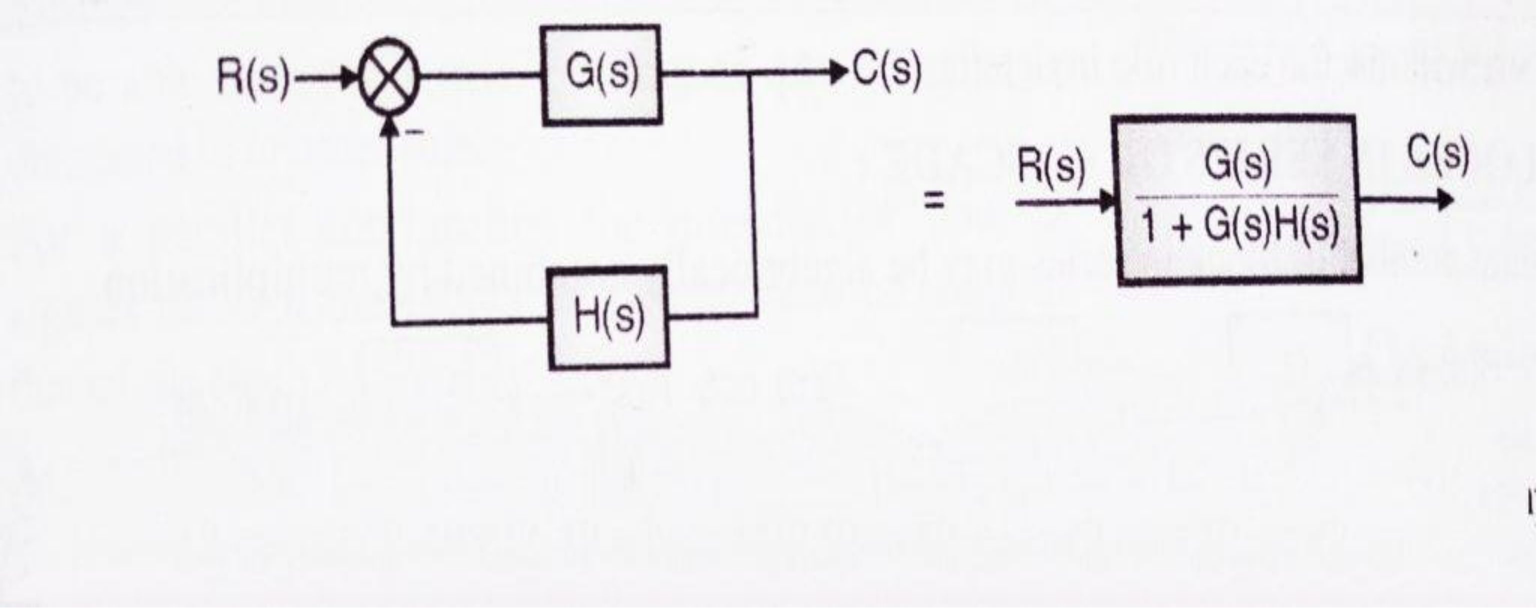
$$\therefore\ C(s) = R(s)\,G(s) \pm C(s)\,H(s)\,G(s)$$
$$\therefore C(s) \mp C(s)\,H(s)\,G(s) = R(s)\,G(s)$$
$$\therefore\ C(s)\,[1 \mp G(s)\,H(s)] = R(s)\,G(s)$$
$$\therefore\ \frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)\,H(s)}$$

If the system is a negative feedback

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

and if the system is a positive feedback

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\,H(s)}$$



If negative feedback is present in the system.
T.F=C(S)/R(S)=G(S)/(1+G(S)H(S))

If positive feedback is present in the system
T.F=C(S)/R(S)=G(S)/(1-G(S)H(S))

- For open loop transfer function, as no feedback present, transfer function can be given by:

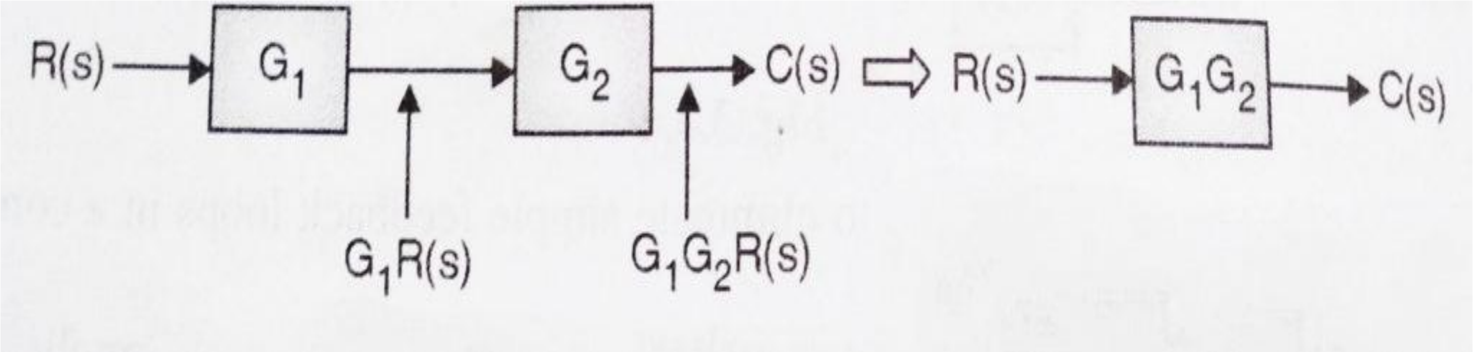
Open loop transfer function = G(s).H(s)

Block Diagram Reduction Rules

- Rule 1: Blocks in cascade/series
- Rule 2: Blocks in parallel
- Rule 3: Feedback loop elimination
- Rule 4: Associative law for summing
- Rule 5: Shifting a summing point before a block
- Rule 6: Shifting a summing point after a block
- Rule 7: Shifting take-off point before a block
- Rule 8: Shifting take-off point after a block
- Rule 9 : Shifting a Take-off point after a Summing Point
- Rule 9 : Shifting a Take-off point before a Summing Point

Rule 1 -Blocks in cascade/series

finite number of blocks in series can be combined together by multiplication

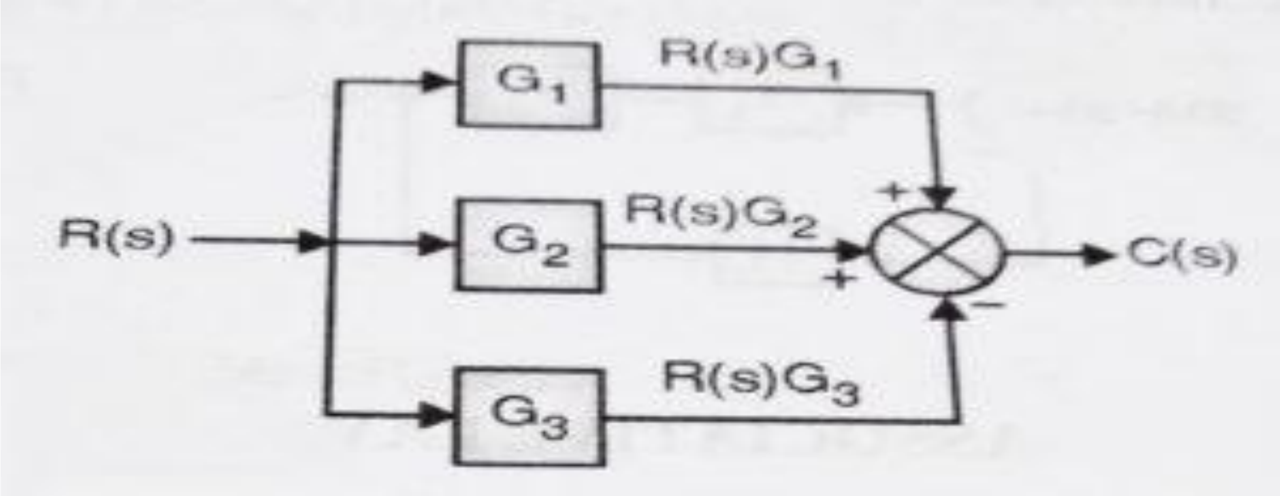


Output C(s) = G1 x G2 x R(s)

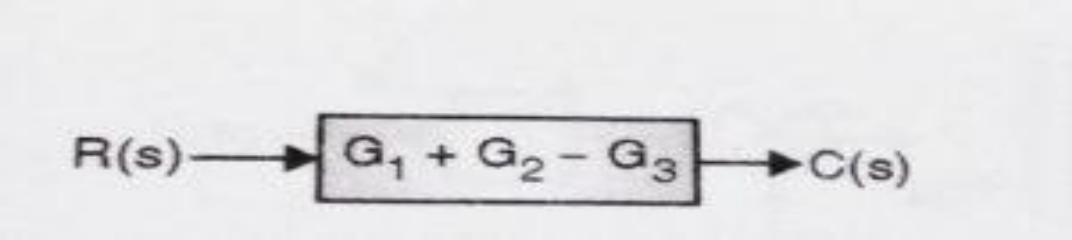
If there is a take-off point or summing point between the blocks, the blocks cannot be said to be in cascade/series. (The take-off / summing point has to be shifted before or after the block using another rule)

Rule 2 - Blocks in parallel

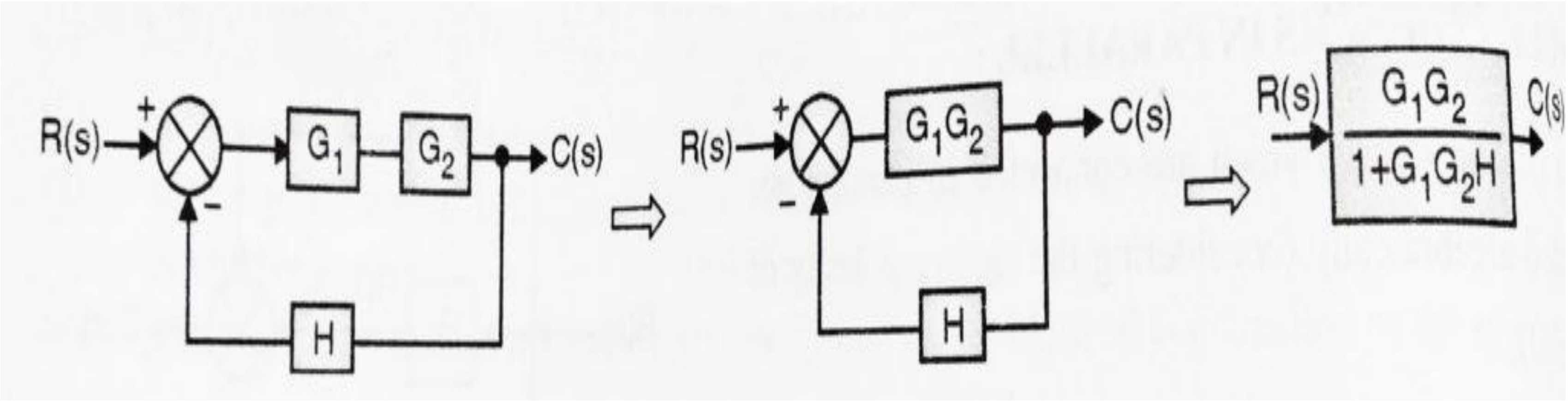
When the blocks are connected in parallel combination, they get added algebraically (considering the sign of the signal)



C(s) = R(s)G1 + R(s)G2 – R(s)G3

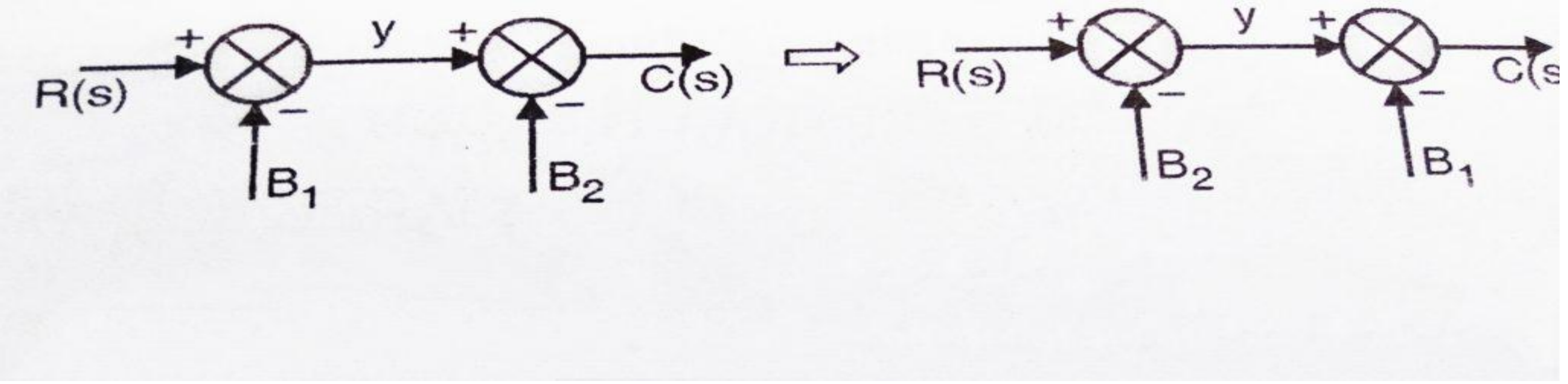


Rule 3: Elimination of feedback Loop



$$T.F = C(S)/R(S) = \frac{G(S)}{1+G(S)H(S)}$$

Rule 4: Associative Law For Summing Point



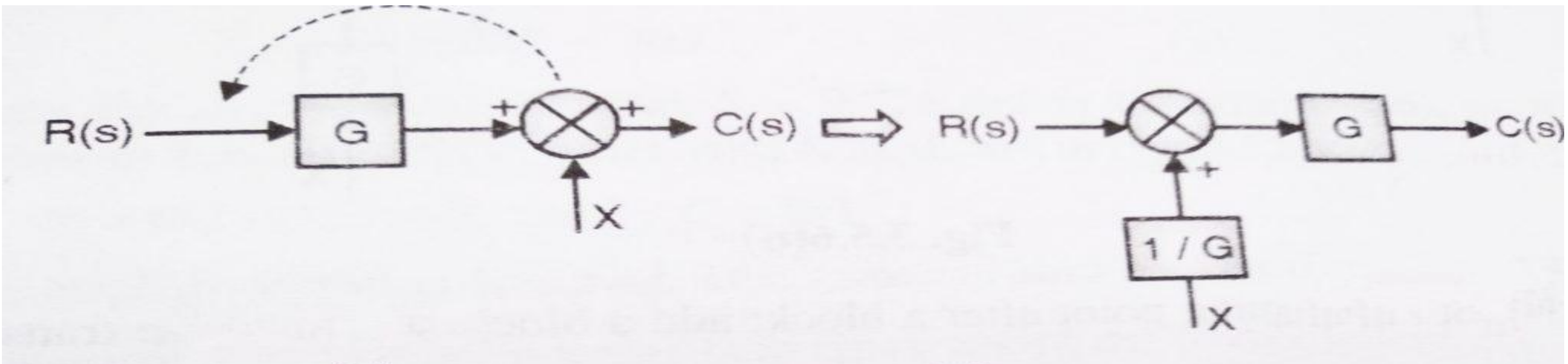
Y= R(s) – B1

C(s) = y – B2 = R(s) – B1 – B2

This law is applicable only to summing points which are connected directly to each other.

Note: if there is a block present between two summing points then this rule cannot be applied.

Rule 5: Shifting of a Summing Point before a block



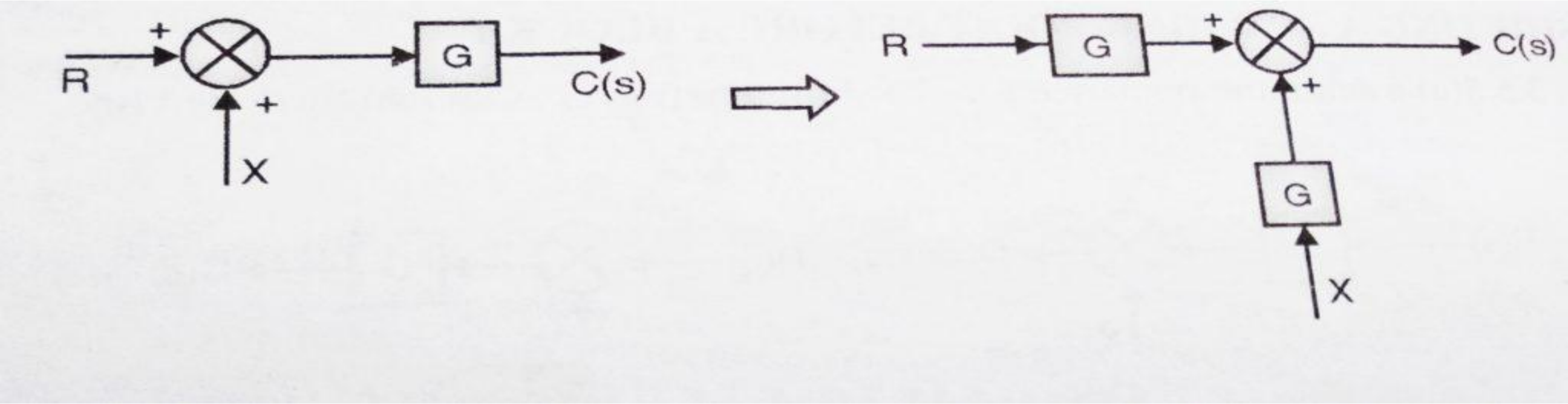
C(s) = GR(s) + X

After shifting the summing point, we will get

C(s) = [R +(X/G)] G = GR + X which is same as output in the first case.

Hence to shift a summing point before a block, we need o add another block of transfer function ‘1/G’ before the summing point .

Rule 6: Shifting of a Summing Point after a block



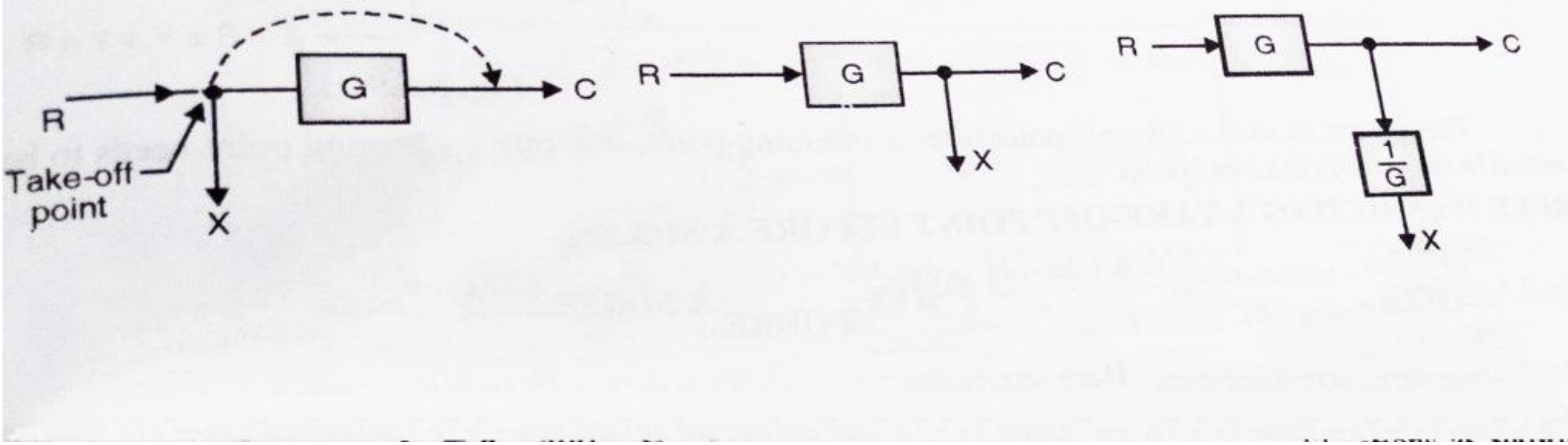
C(s) = (R + X)G

After shifting the summing point,

C(s) = (R +X) G = GR + XG which is same as output in the first case.

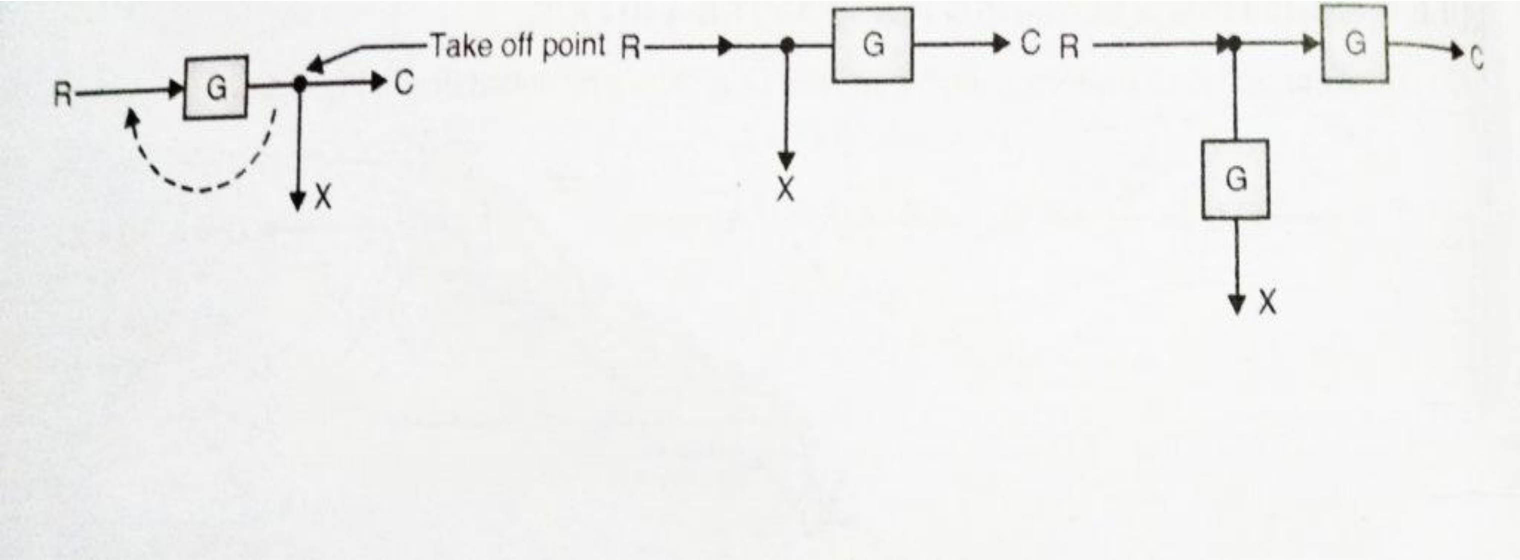
Hence to shift a summing point before a block, add another block having same transfer function at the summing point.

Rule 7: Shifting of Take-off point after a block



X = R and C = RG (initially)…….(I)
X = RG and C = RG
But as per equation (I) ,X=R
In order to achieve this, we need to add a block of transfer function ‘1/G’ in series with X.
X=(RG)*(1/G)=R and C=RG

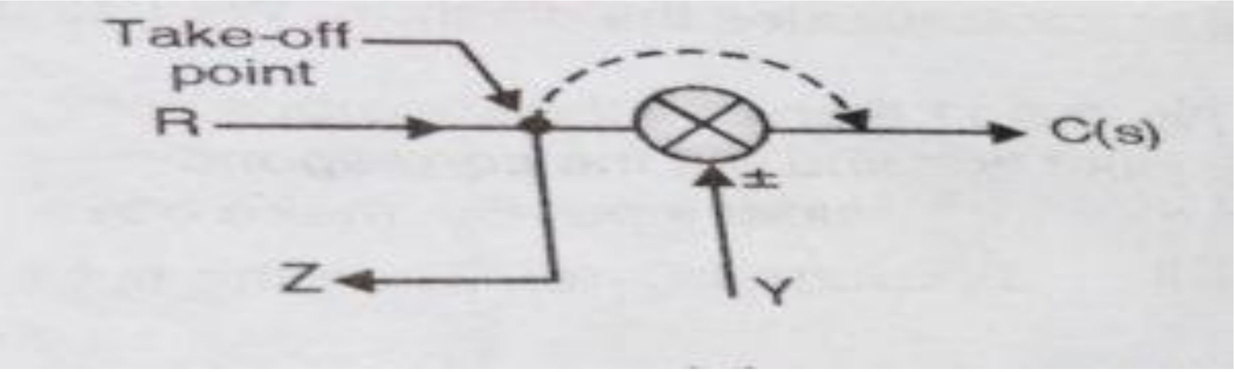
Rule 8 : Shifting of Take-off point before a block



Here we have X = RG and C = RG (initially)….(I)
X=R ,C=RG
But as per equation (I) ,X=RG
In order to achieve this we need to add a block of transfer function 'G' in series with X signal taking off from that point.
X=RG ,C=RG

CRITICAL RULES

Rule 9 : Shifting a Take-off point after a Summing Point



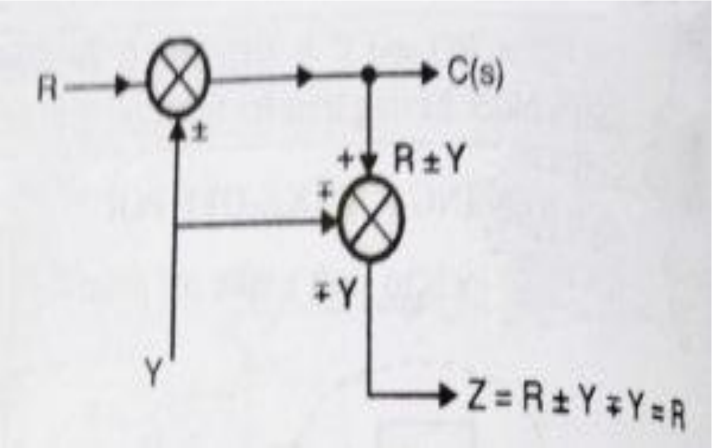
$Z = R$ and $C=R\pm Y$ (initially)…….(I)

If we directly shift take off point after summing point,then

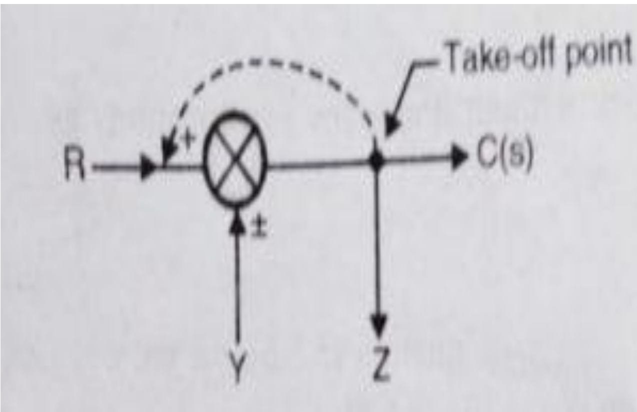
$Z = R\pm Y$ and $C=R\pm Y$

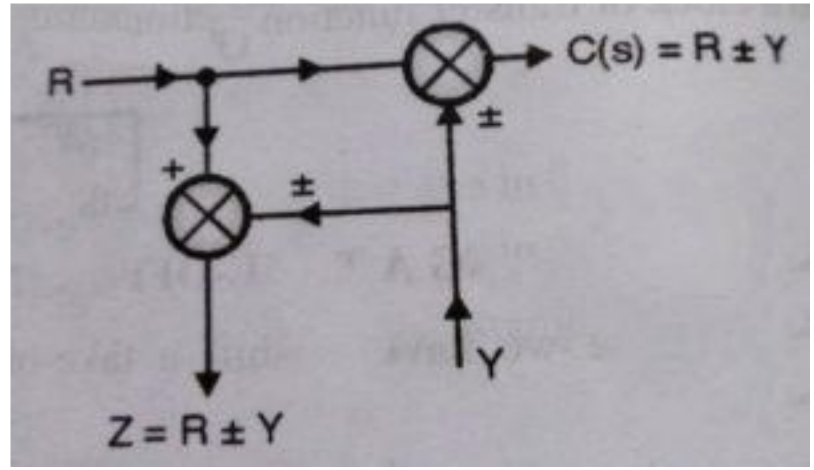
But as per equation (I) ,Z=R

In order to achieve this, we need to add one more summing point in series with take off point



Rule 10: Shifting a take-off point before a summing point





$C(s) = R \pm Y$

and $Z = R \pm Y$ (initially)….(I)

In order to satisfy equation (I), we need to add a summing point in series with the take-off point.

Procedure to solve Block Diagram Reduction Techniques

Step 1 : Reduce the blocks connected in series.

Step 2 : Reduce the blocks connected in parallel.

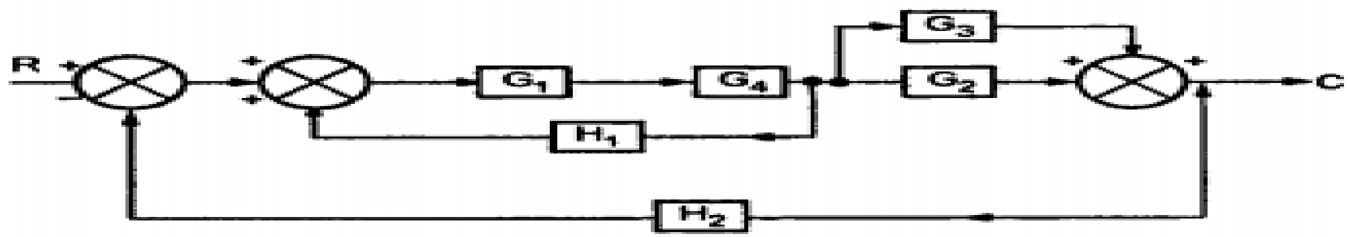
Step 3 : Reduce the minor internal feedback loops.

Step 4 : As far as possible try to shift takeoff points towards right and summing points to the left. Unless and until it is the requirement of problem do not use rule 10 and 11.

Step 5 : Repeat steps 1 to 4 till simple form is obtained.

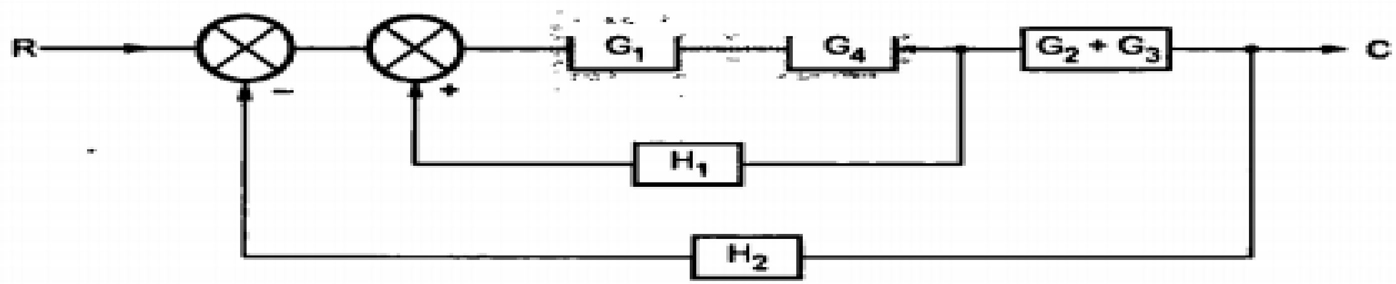
Step 6 : Using standard T.F. of simple closed loop system, obtain the closed loop T.F. $\frac{C(s)}{R(s)}$ of the overall system.

Example 1
Determine transfer function C(S)/R(S)

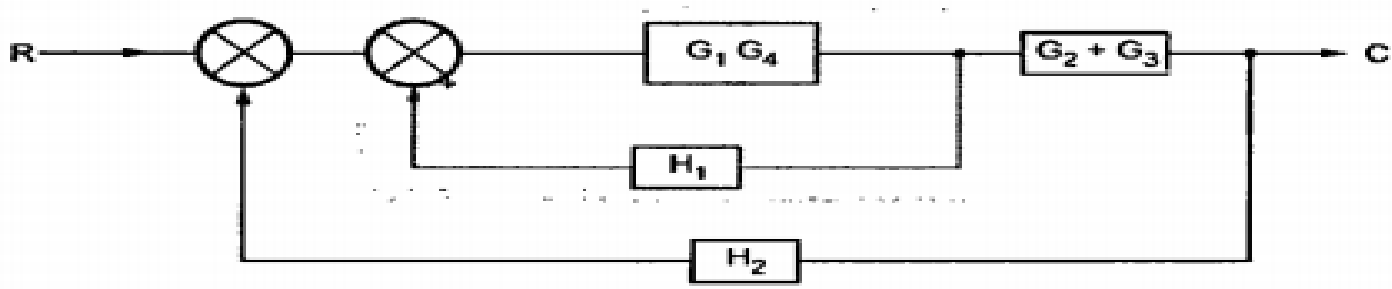


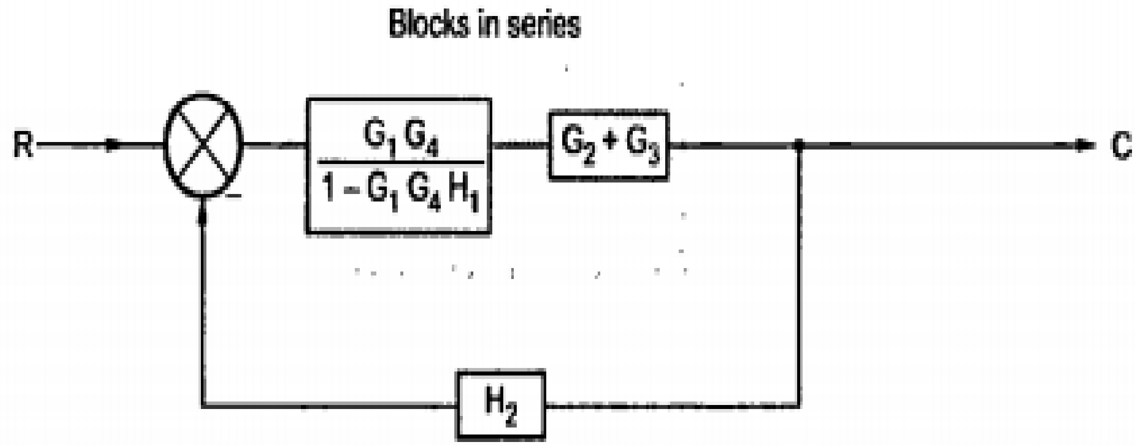
The blocks G_2 and G_3 are in parallel so combining them as (G_2+G_3)

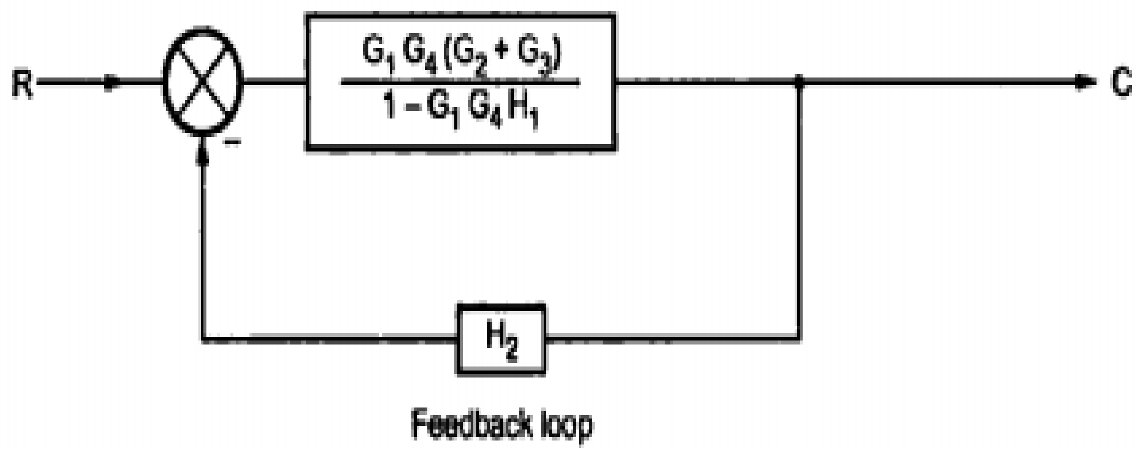
Blocks in series



Minor feedback loop



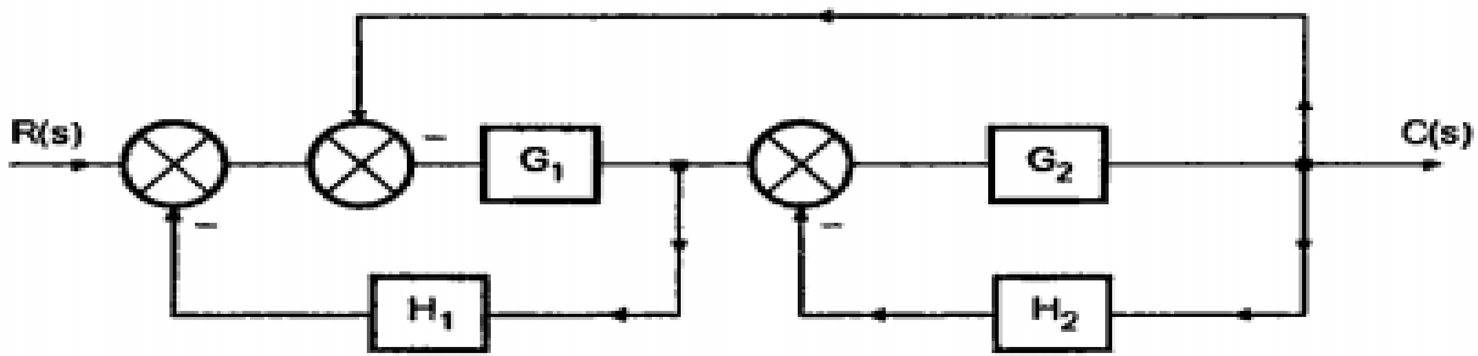




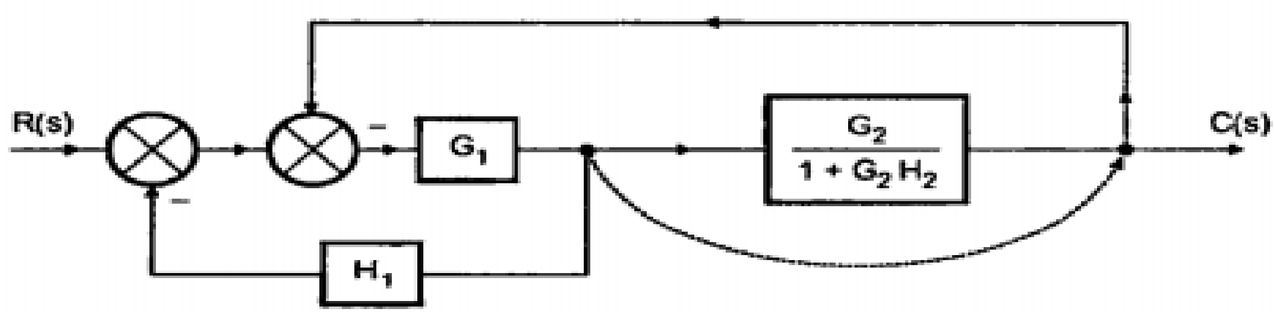
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_4 (G_2+G_3)}{1-G_1 G_4 H_1}}{1+\dfrac{G_1 G_4 (G_2+G_3) H_2}{1-G_1 G_4 H_1}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2+G_3)}{1-G_1 G_4 H_1+G_1 G_4 (G_2+G_3) H_2}$$

Example 2

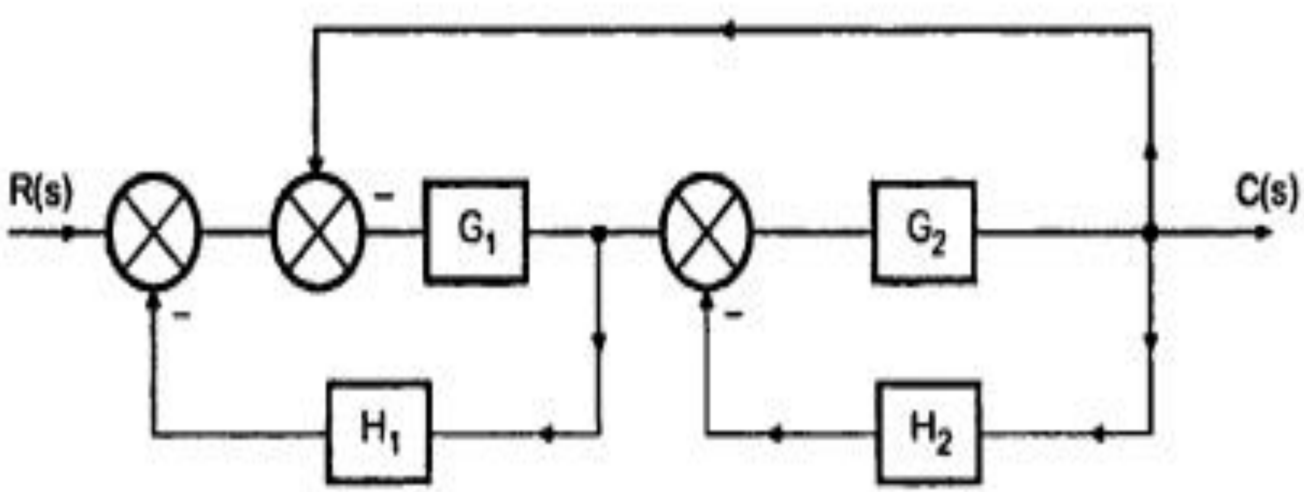


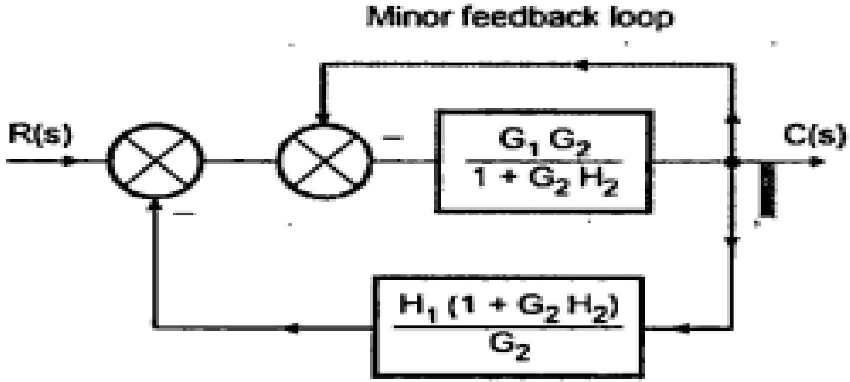
No blocks are connected in series or parallel. Blocks having transfer functions G_2 and H_2 form minor feedback loop so eliminating that loop

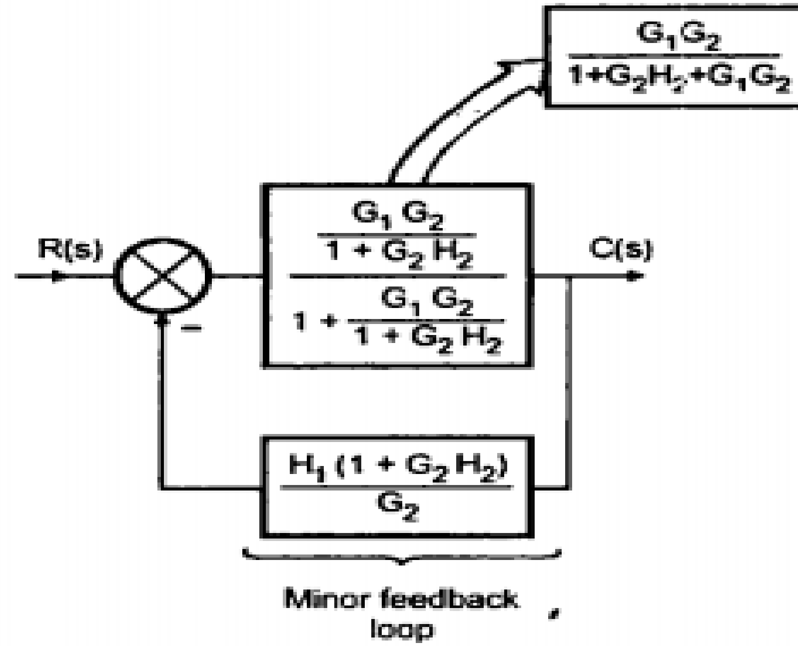


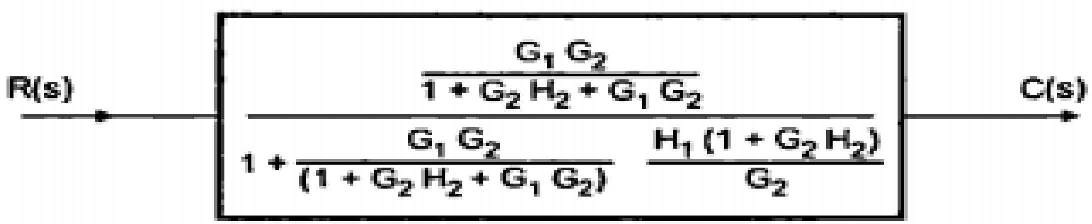
Key Point: *Always try to shift takeoff point towards right i.e. output side and summing point towards left i.e. input side.*

So shift takeoff point after G_1 to the right. While doing so, it is necessary to add a block having T.F. equal to reciprocal of the T.F. of the block after which takeoff point is to be shifted, in series with signal at that takeoff point. So in series with H_1 we get a block of $1/\left(\frac{G_2}{1+G_2H_2}\right)$ i.e $\frac{1+G_2H_2}{G_2}$ after shifting takeoff point.





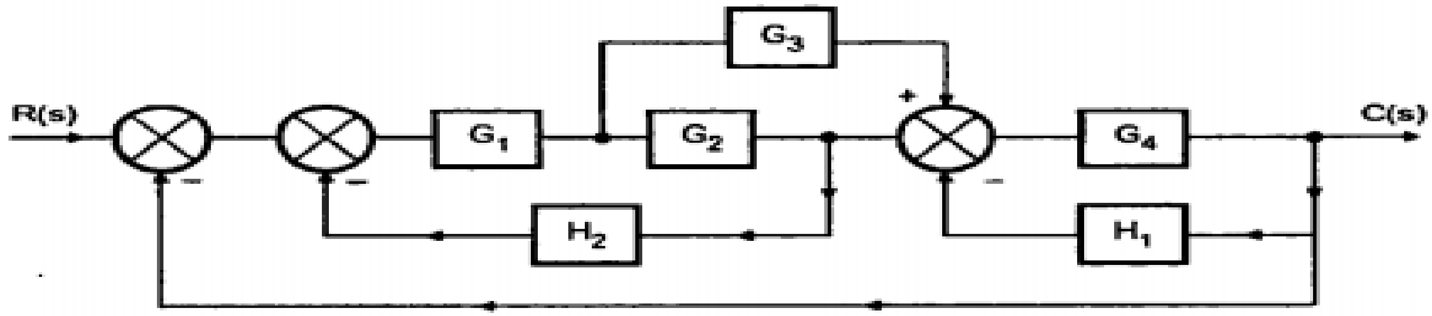




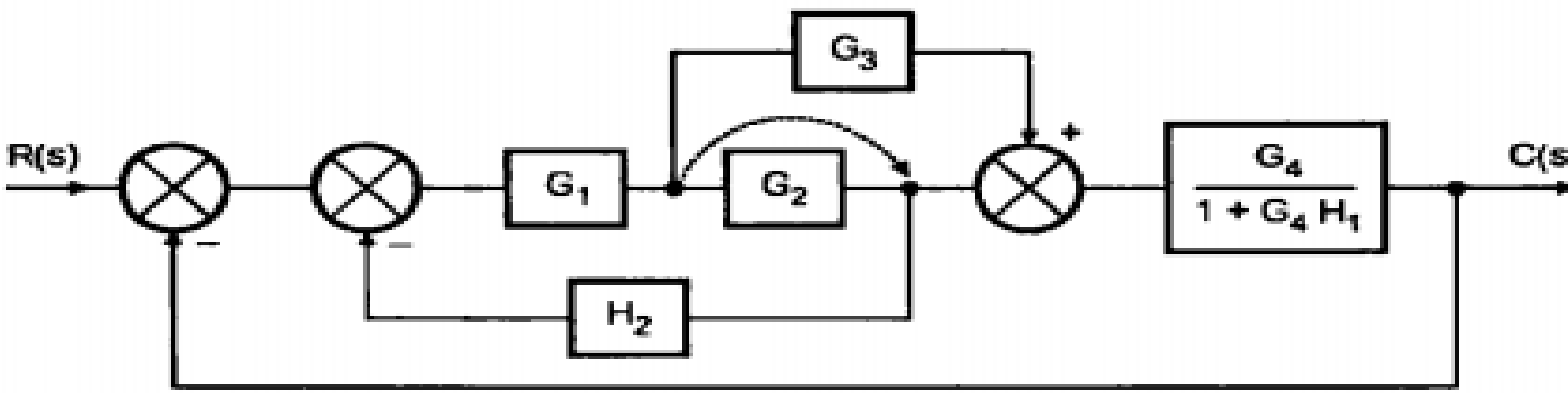
Simplifying,

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 + G_2 H_2 + G_1 H_1 + G_1 G_2 H_1 H_2}}$$

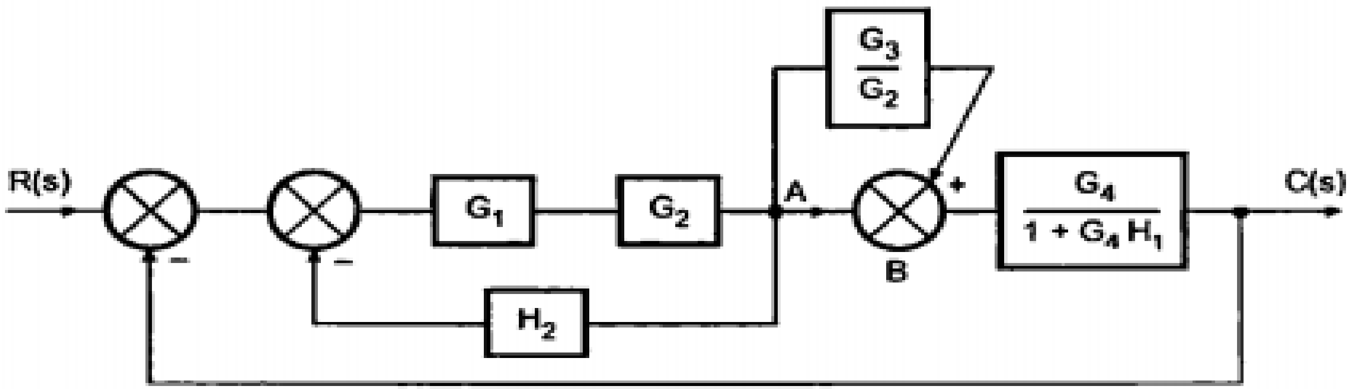
Example 3



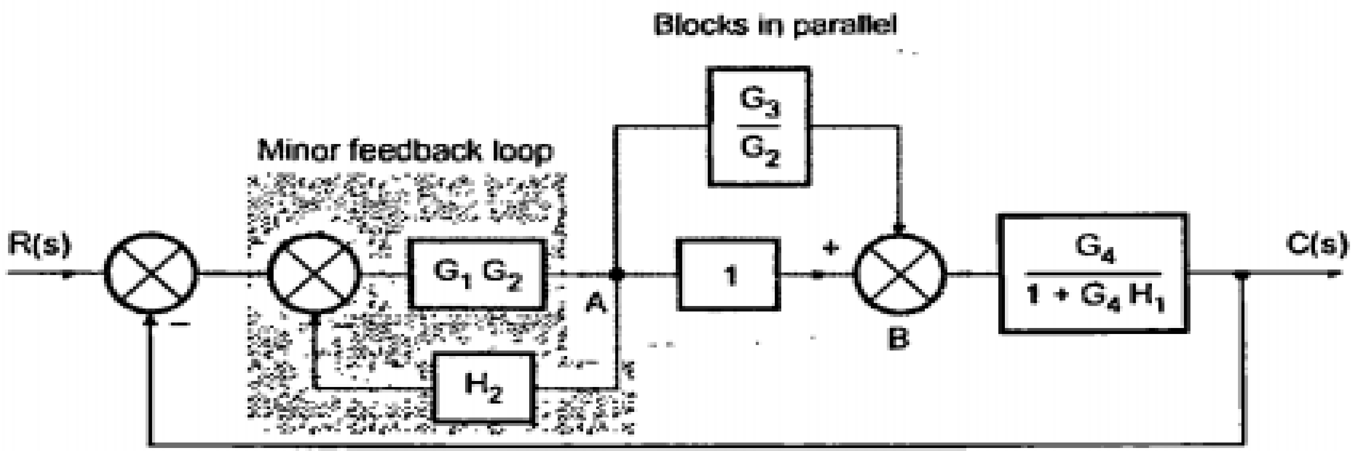
The blocks G_4 and H_1 form minor feedback loop,

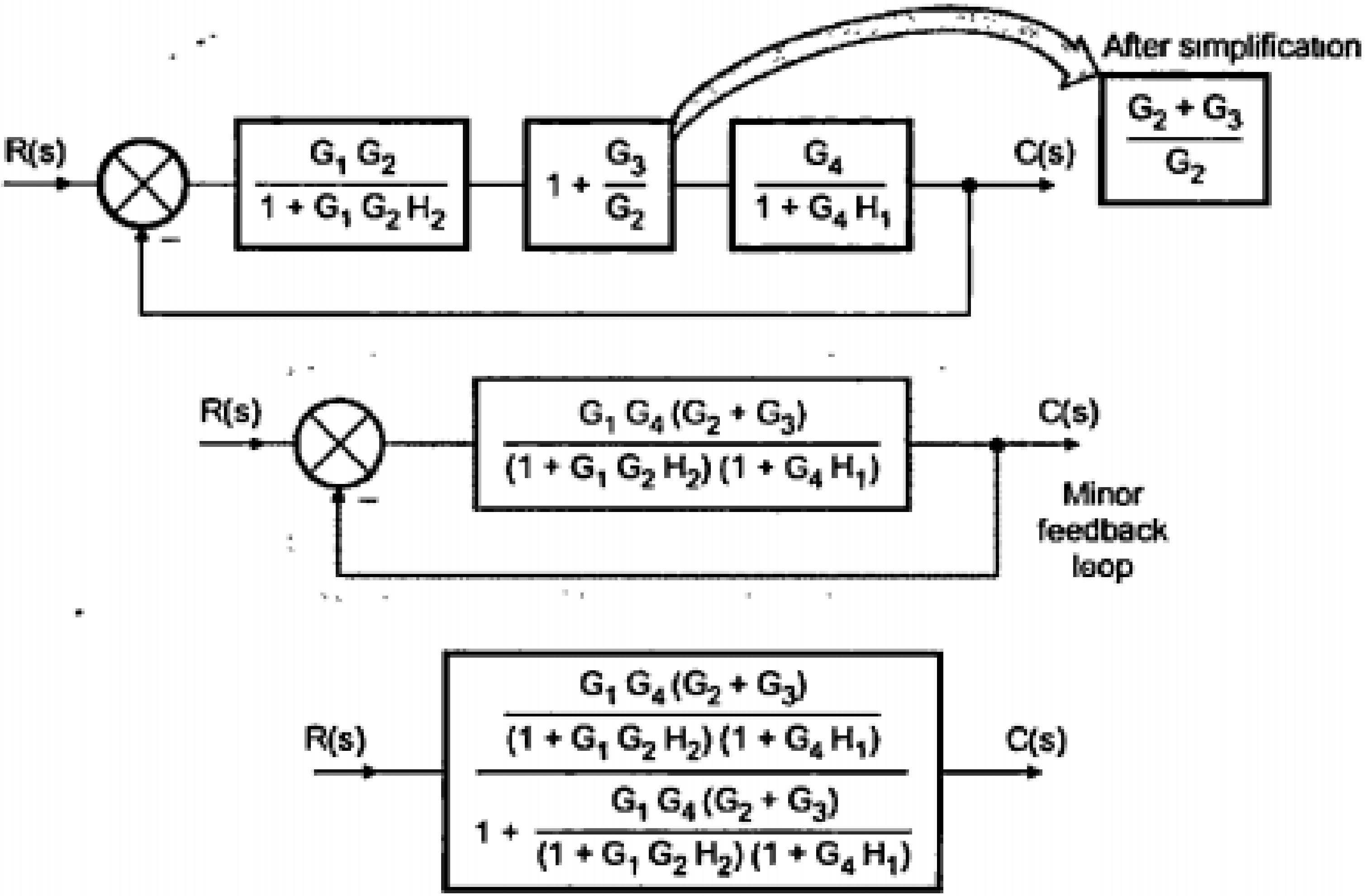


Shifting takeoff point



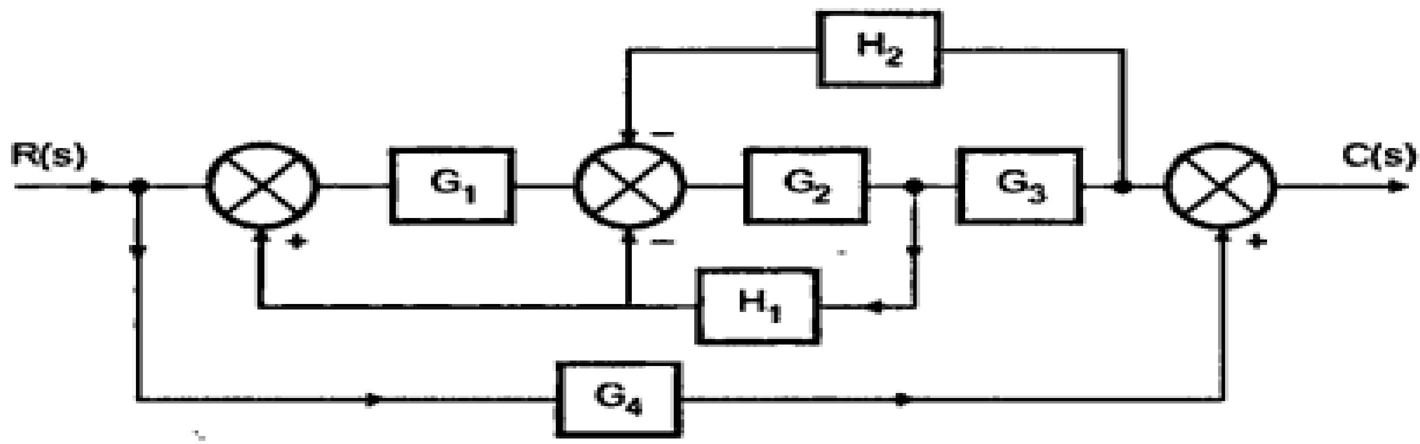
The signal from takeoff point A reaching to summing point B is without any block i.e. gain of that branch joining A to B is one. So introduce block of T.F. '1' in between A and B to avoid further confusion as shown below.

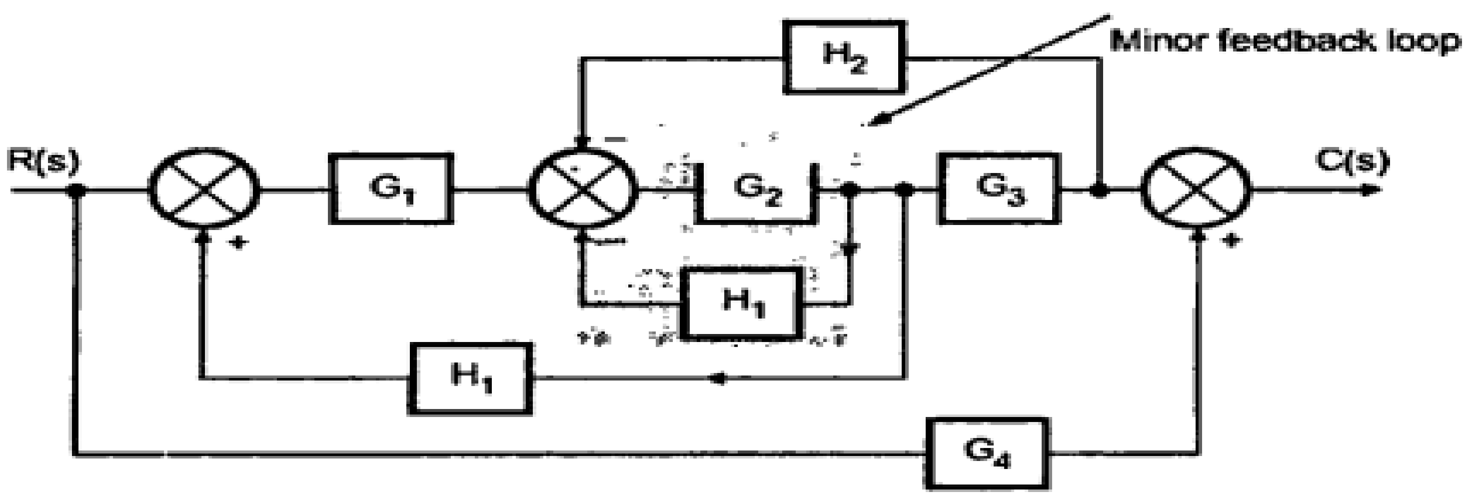


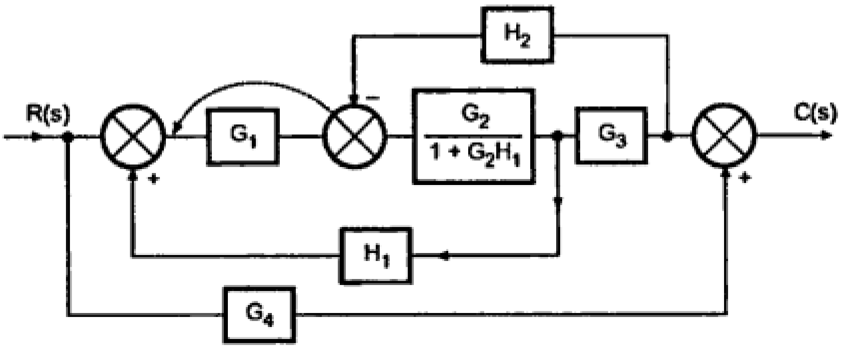


$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2 + G_3)}{1 + G_1 G_2 H_2 + G_4 H_1 + G_1 G_2 G_4 H_1 H_2 + G_1 G_4 (G_2 + G_3)}}$$

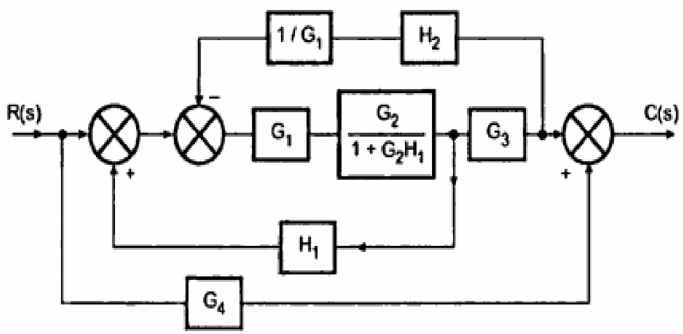
Obtain the closed loop transfer function C(s)/R(s)



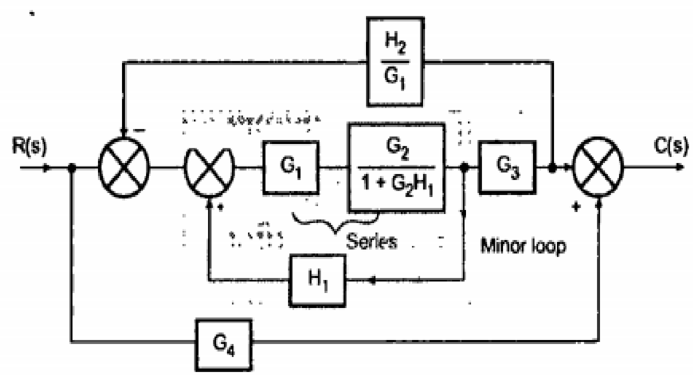


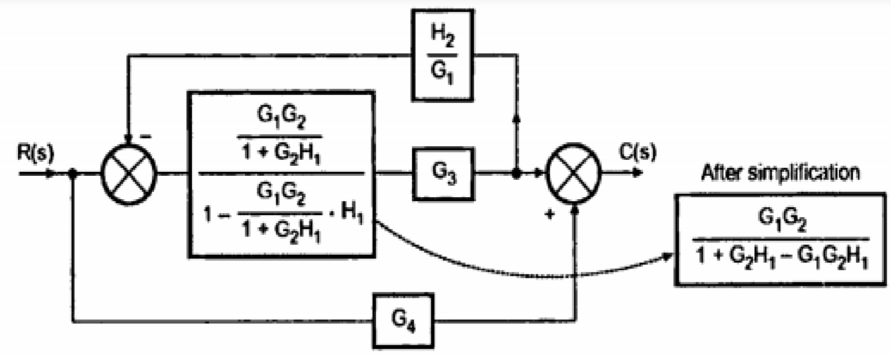


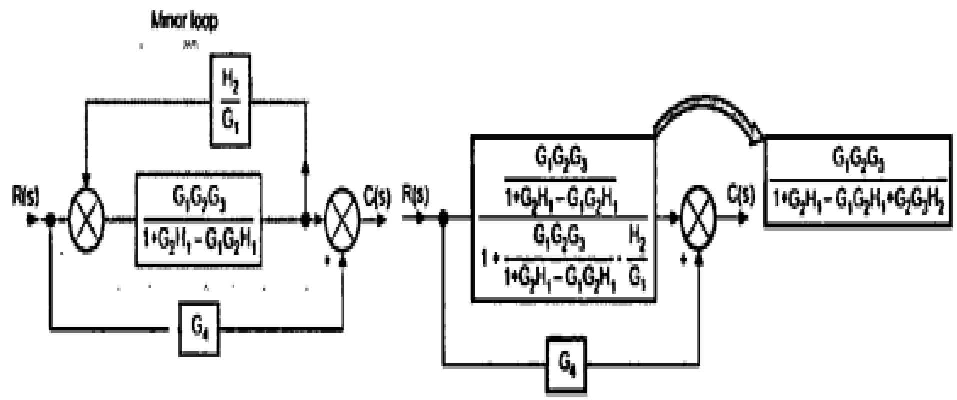
Shifting summing point behind the block having transfer function 'G_1'

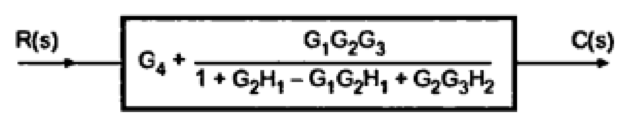


Use Associative law for the two summing points and interchange their positions



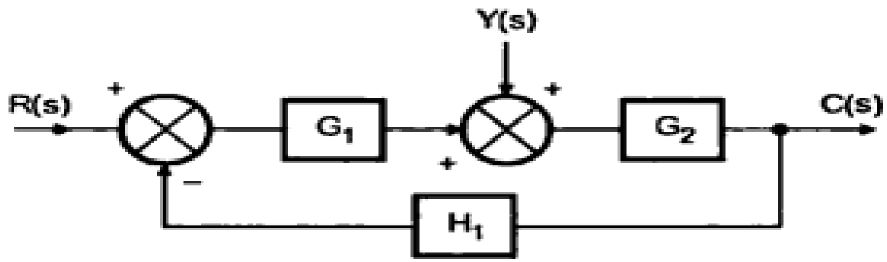




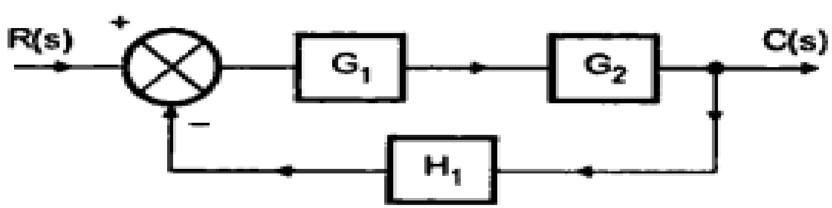


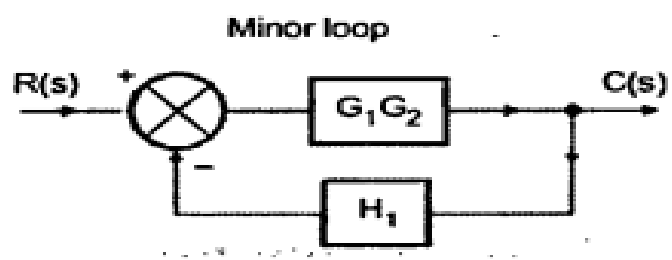
$$\frac{C(s)}{R(s)} = \frac{G_4 + G_4 G_2 H_1 - G_4 G_1 G_2 H_1 + G_2 G_3 G_4 H_2 + G_1 G_2 G_3}{1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2}$$

Obtain the resultant output C(s) interms of the inputs R(s) and Y(s).



As there are two inputs, consider each input separately. Consider R(s), assuming Y(s) = 0.

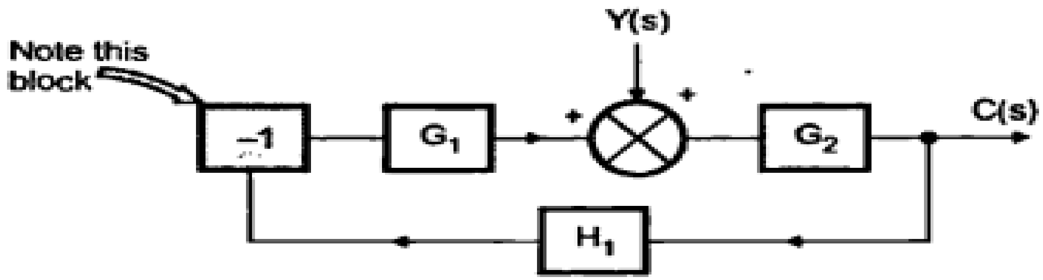




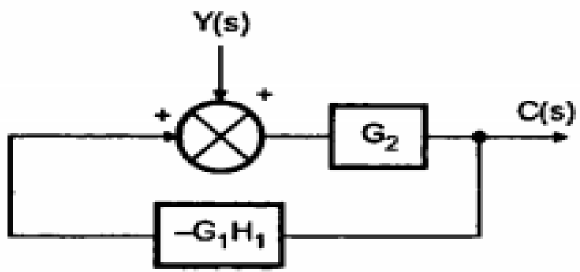
$$\frac{C(s)}{R(s)} = \frac{G_1\,G_2}{1+G_1\,G_2\,H_1} \quad \text{i.e.}\ C(s) = R(s)\left[\frac{G_1\,G_2}{1+G_1G_2\,H_1}\right]$$

Now consider Y(s) acting with R(s) = 0.

Now sign of signal obtained from H_1 is negative which must be carried forward, though summing point at R(s) is removed, as R(s) = 0



Combining the blocks G_1 H_1 and –1 as in series,



$$G_{eq} = G_2$$

$$H_{eq} = -G_1 H_1$$

Feedback sign positive at input summing point.

$$\therefore \frac{C(s)}{Y(s)} = \frac{G_{eq}}{1 - G_{eq}H_{eq}}$$

$$= \frac{G_2}{1 - G_2(-G_1H_1)} = \frac{G_2}{1 + G_1G_2H_1}$$

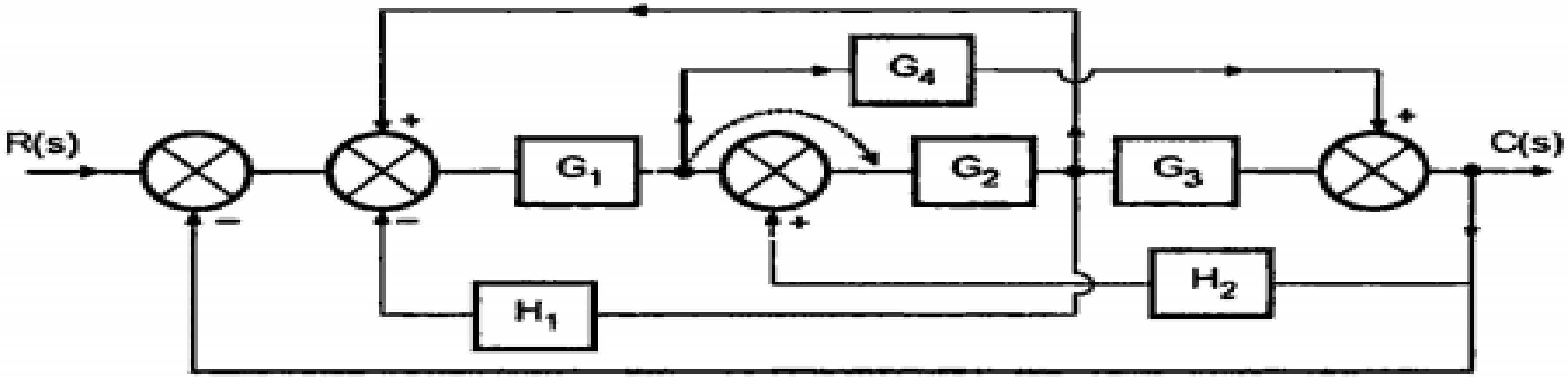
So part of C(s) due to Y(s) alone is,

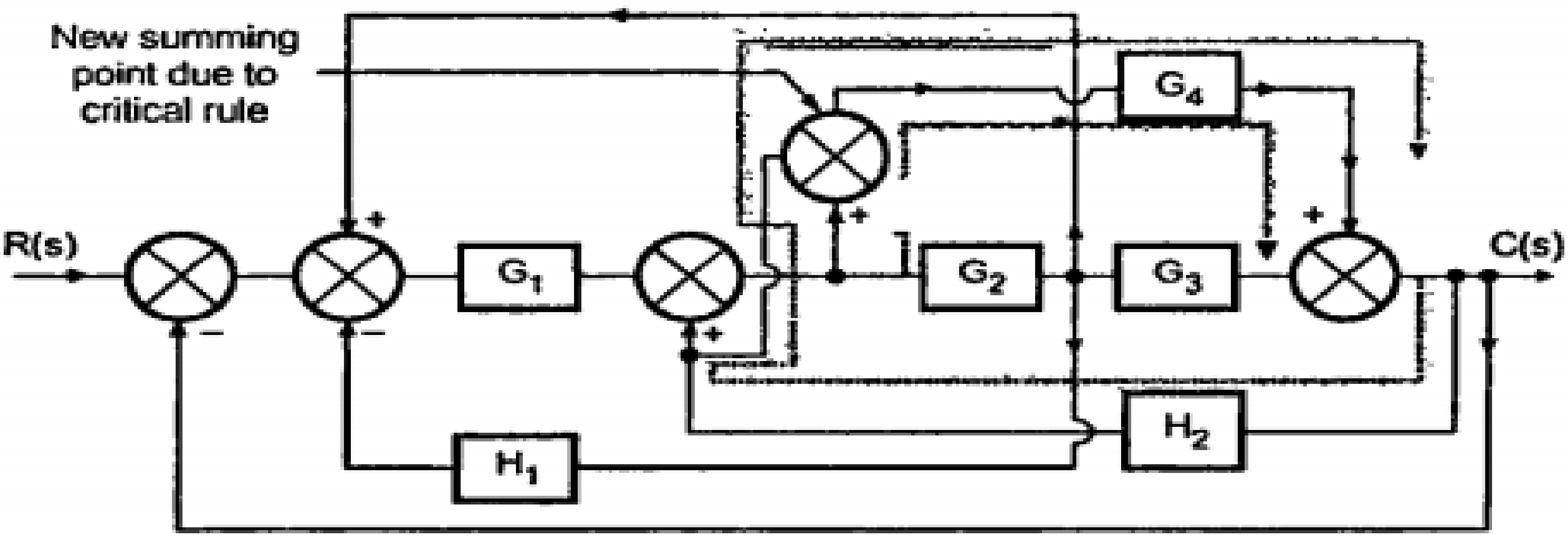
$$C(s) = Y(s)\left[\frac{G_2}{1 + G_1G_2H_1}\right]$$

Hence the net output C(s) is given by algebraically adding its two components,

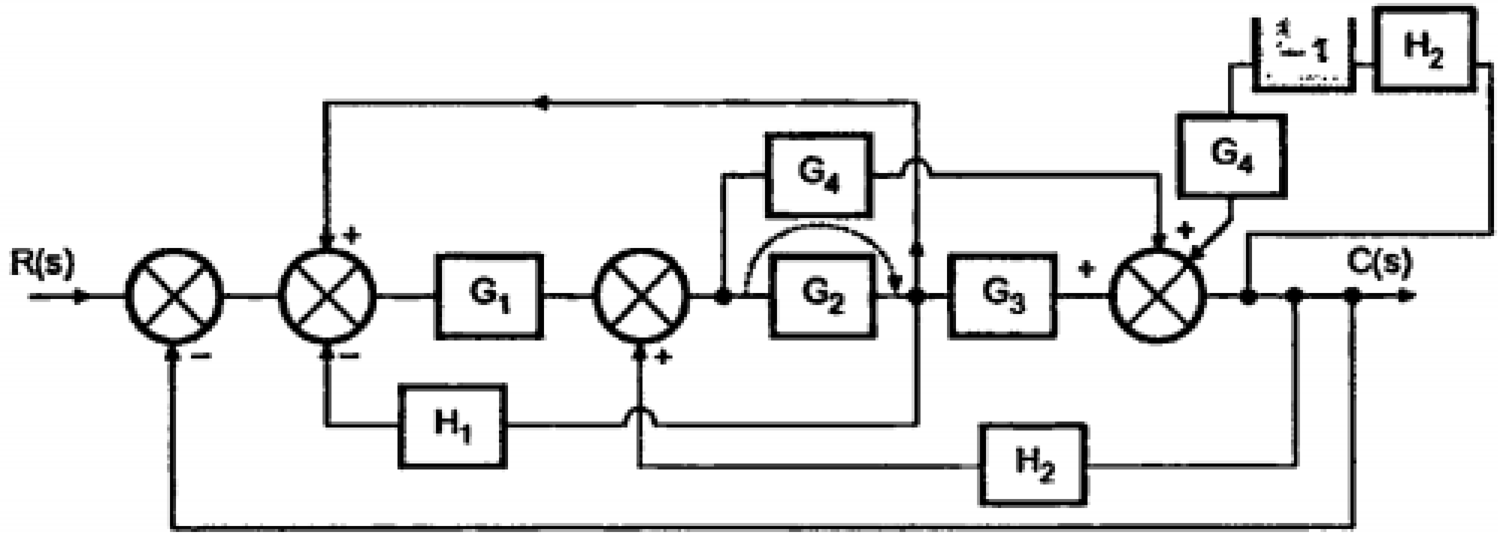
$$\boxed{C(s) = \frac{G_1G_2R(s) + G_2Y(s)}{1 + G_1G_2H_1}}$$

For the block diagram shown, obtain C(s)/R(s) by using reduction rules.



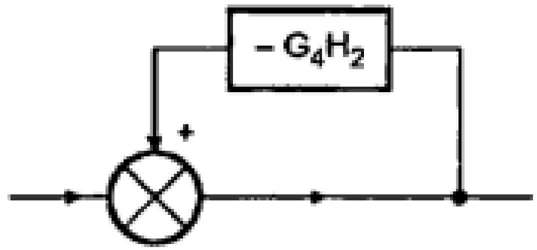


New summing point gets added due to use of critical rule . This summing point can be eliminated by separating the two paths which are linked by that summing point. The paths are shown dotted. So block diagram reduces as,



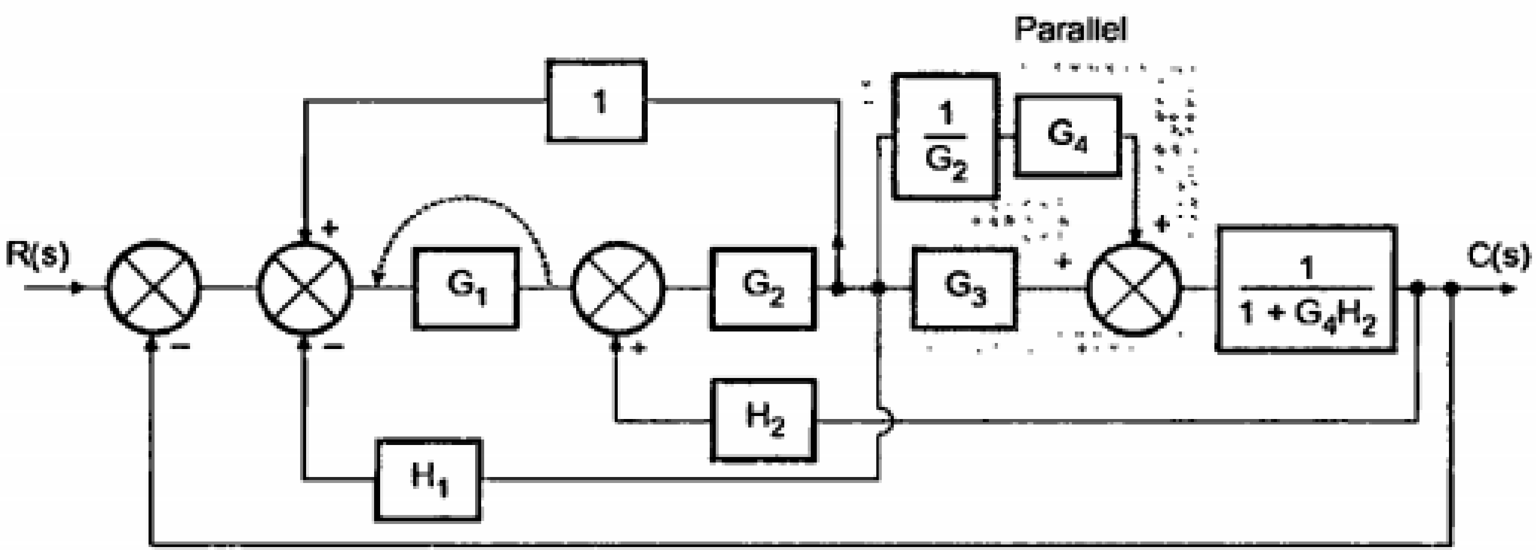
While removing summing point, as sign of one of the signal is negative, the block of transfer function '−1' is connected series with that signal.

So now there exists a minor feedback loop

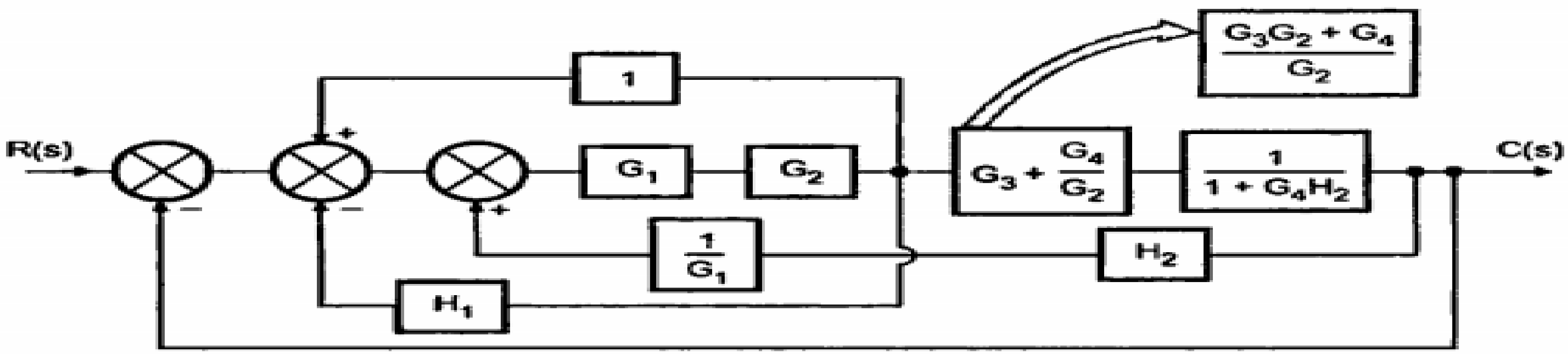


$$\approx \frac{1}{1-(-G_4H_2)} = \frac{1}{1+G_4H_2}$$

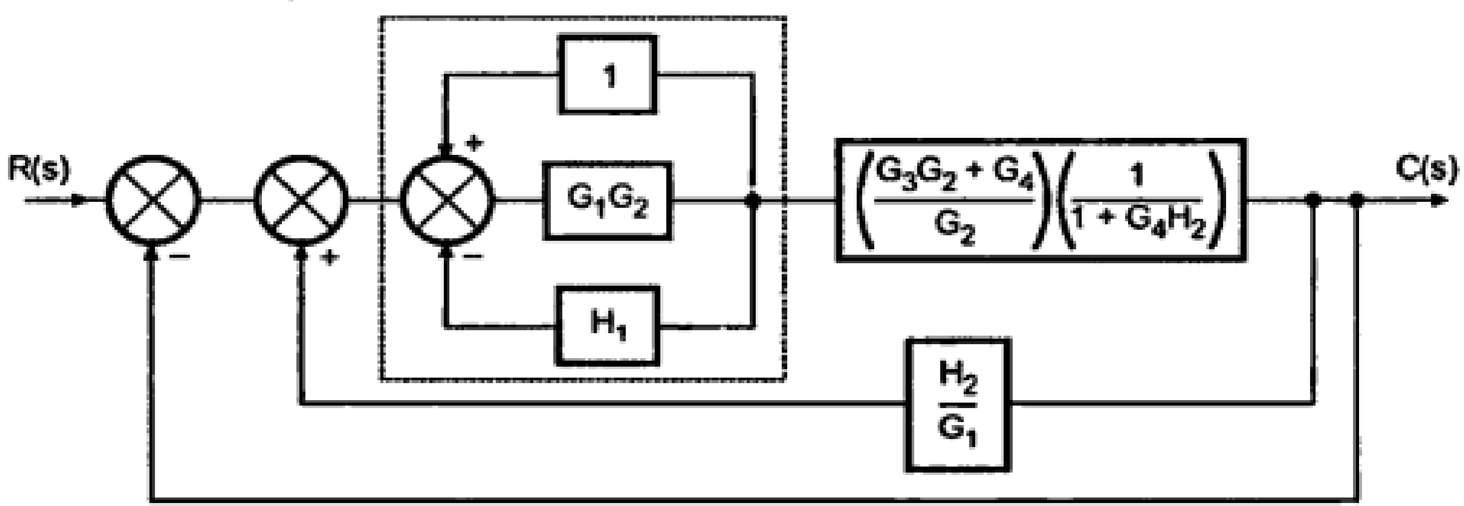
Shifting take off to the right of G_2



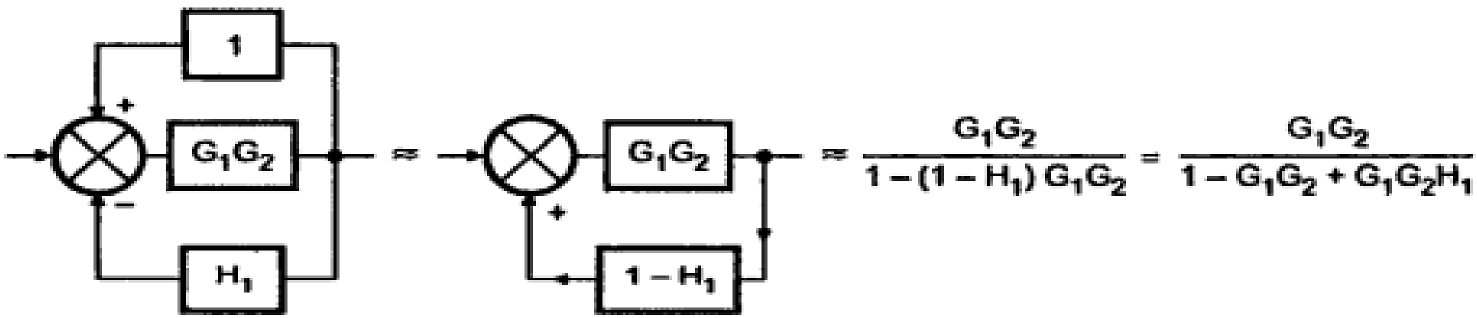
Combining blocks in parallel and shifting summing point to the left of 'G₁'.

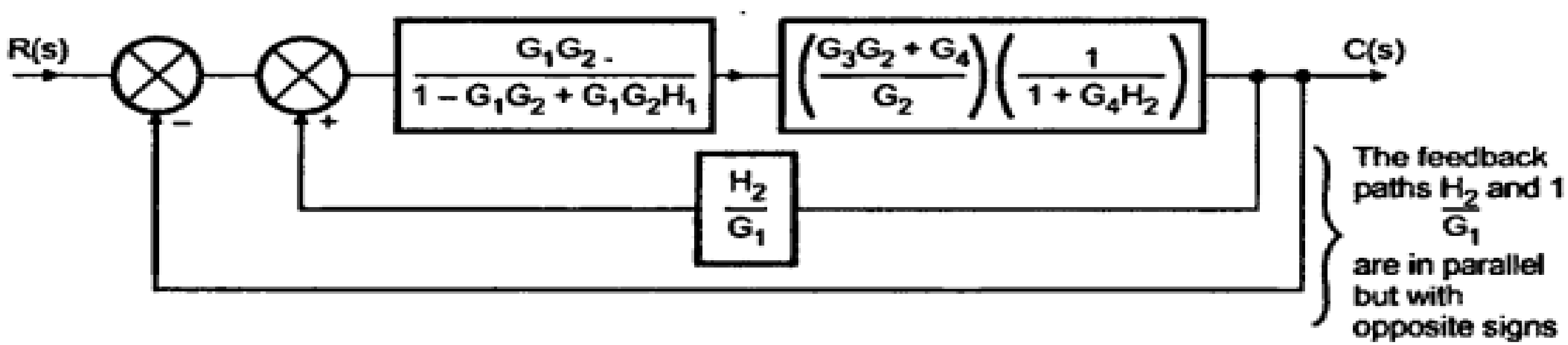


Interchanging the summing points using associative law.

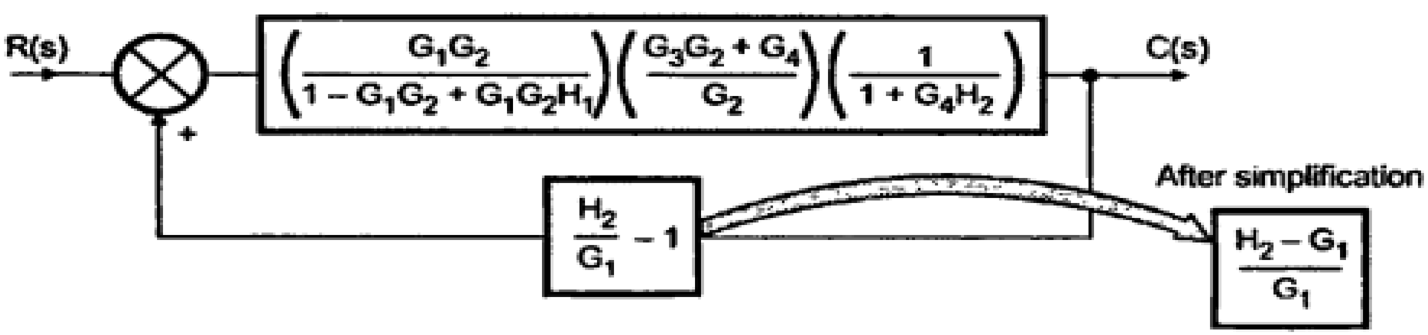


Solving minor feedback loop. The block of '1' and 'H_1' are in parallel





Combining the feedback blocks which are in parallel.



$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\left(\frac{G_1 G_2}{1 - G_1 G_2 + G_1 G_2 H_1}\right)\left(\frac{G_3 G_2 + G_4}{G_2}\right)\left(\frac{1}{1 + G_4 H_2}\right)}{1 - \left(\frac{G_1 G_2}{1 - G_1 G_2 + G_1 G_2 H_1}\right)\left(\frac{G_3 G_2 + G_4}{G_2}\right)\left(\frac{1}{1 + G_4 H_2}\right)\left(\frac{H_2 - G_1}{G_1}\right)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 - G_1 G_2 + G_1 G_2 H_1 - G_1 G_2 G_4 H_2 + G_1 G_2 G_4 H_1 H_2 + G_1 G_2 G_3 + G_1 G_4 - G_2 G_3 H_2}}$$

SIGNAL FLOW GRAPH

A signal-flow graph consists of a network in which nodes are connected by directed branches.

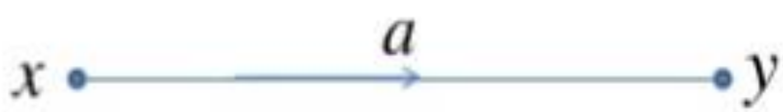
It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

- The signal flow graph of the equation is shown below;



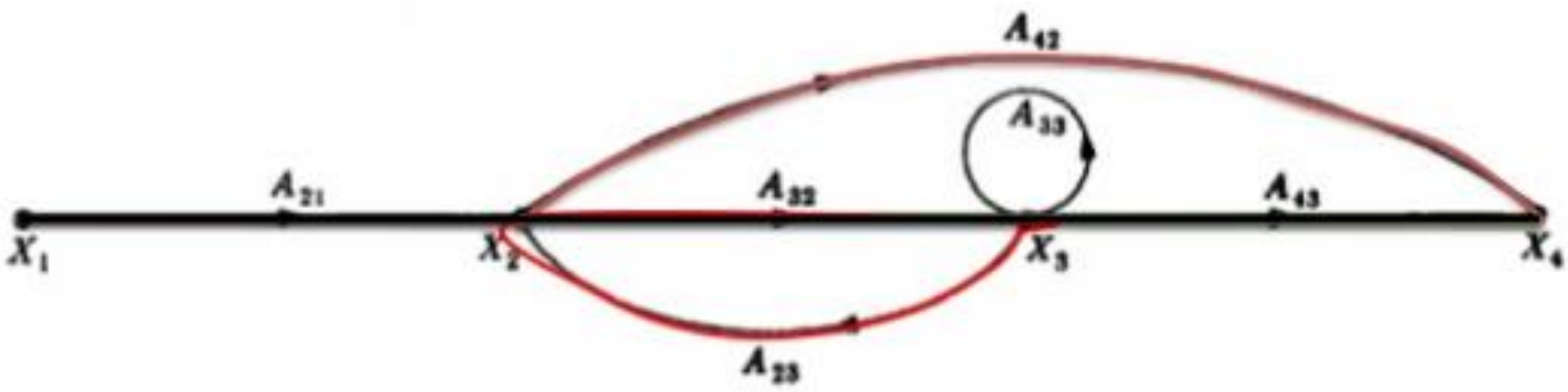
- Every variable in a signal flow graph is designed by a **Node**.
- Every transmission function in a signal flow graph is designed by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal flow.

Terminologies

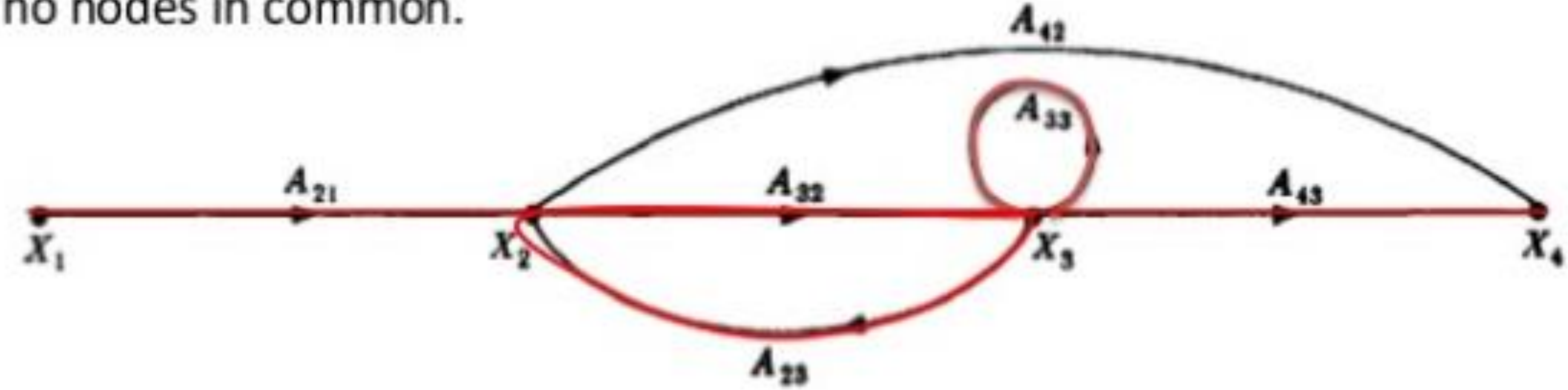
- An **input node** or source contain only the outgoing branches. i.e., X_1
- An **output node** or sink contain only the incoming branches. i.e., X_4
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e.,

  X_1 *to* X_2 *to* X_3 *to* X_4 X_1 *to* X_2 *to* X_4 X_2 *to* X_3 *to* X_4
- A **forward path** is a path from the input node to the output node. i.e.,

  X_1 *to* X_2 *to* X_3 *to* X_4, and X_1 *to* X_2 *to* X_4, are forward paths.
- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; X_2 *to* X_3 *and back to* X_2 is a feedback path.



- A self-loop is a feedback loop consisting of a single branch. i.e.; A_{33} is a self loop.
- The gain of a branch is the transmission function of that branch.
- The path gain is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X_1 to X_2 to X_3 to X_4 is $A_{21}A_{32}A_{43}$
- The loop gain is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be non-touching if they have no nodes in common.



Mason’s Gain Formula

Masons gain formula is used to obtain the over all gain (transfer function) of signal flow graphs.

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- On the other hand, Mason’s rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

Mason's Rule:

- The transfer function **T**, of a system represented by a signal-flow graph is;

$$T = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

Where,

n = number of forward paths.
P_i = the i^{th} forward-path gain.
Δ = Determinant of the system
Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function. Since $\Delta=0$ is the system characteristic equation.

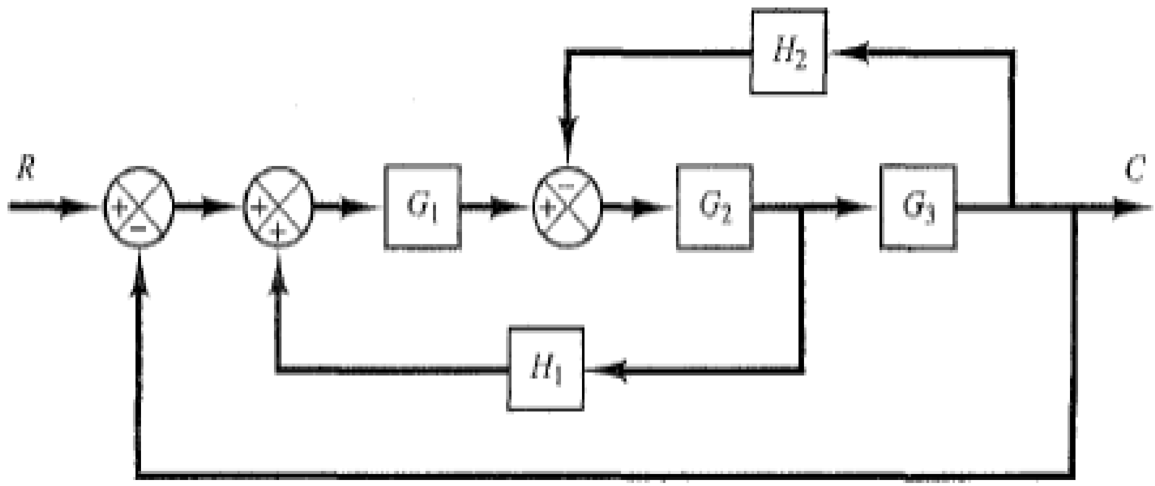
Δ=1-(sum of all individual loop gains)+(sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + ···

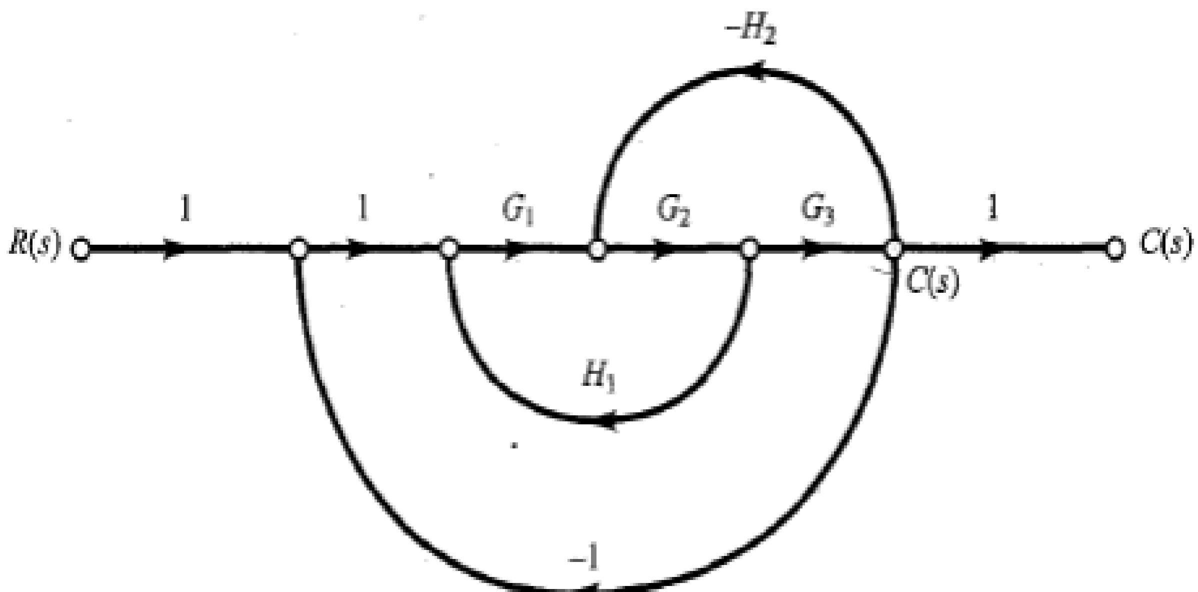
Δ_k is cofactor of k^{th} forward path determinant of graph with loops touching k^{th} forward path. It is obtained from Δ by removing the loops touching the path P_k.

Procedure

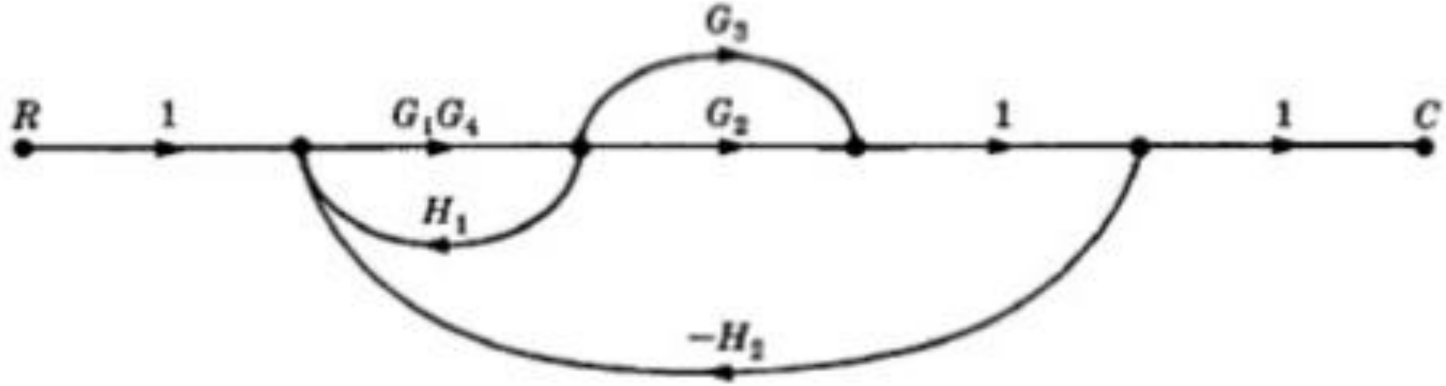
1. Calculate forward path gain P_i for each forward path *i*.
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path *i*

Using Mason's Formula, Find the T.F. $C(s)/R(s)$





Example=1



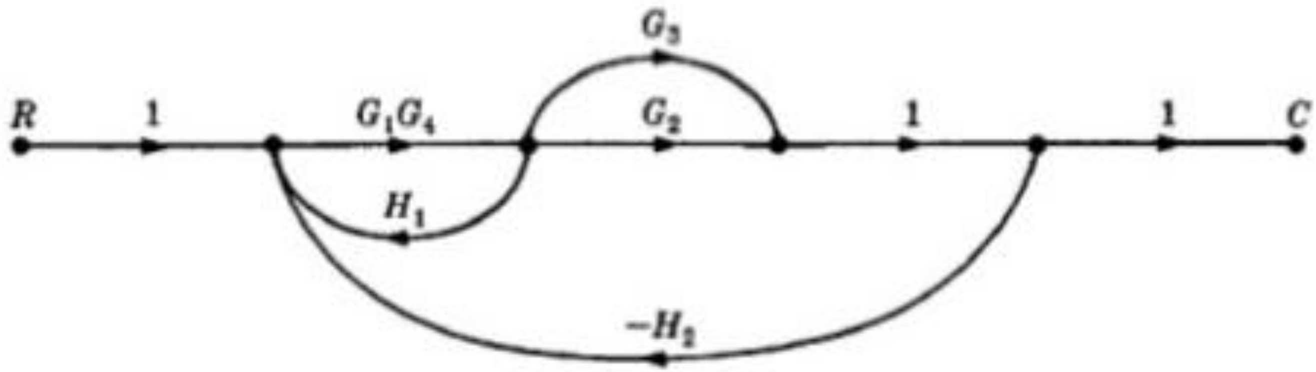
There are no non-touching loops, therefore

Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - (G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2)$$

Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



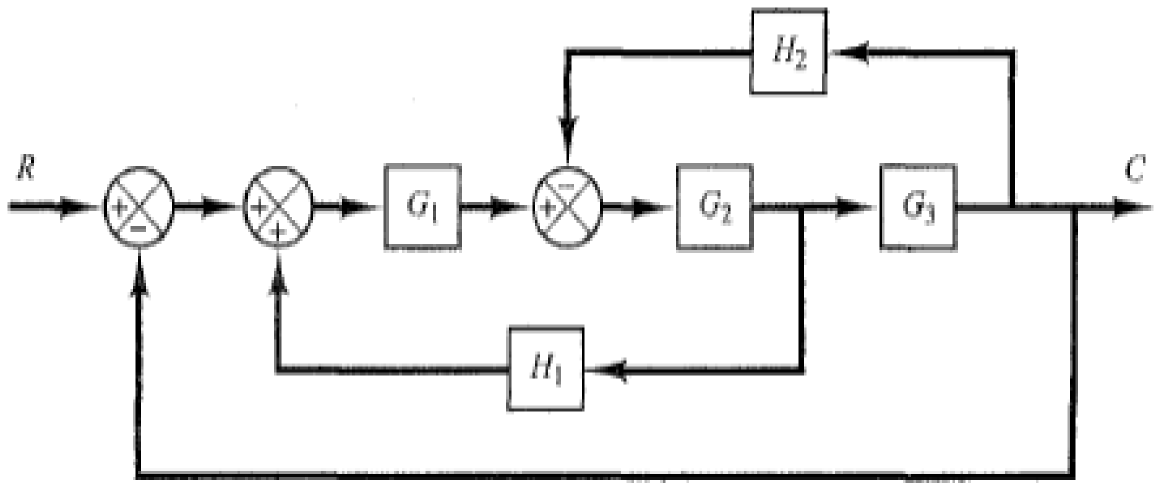
Eliminate forward path-1

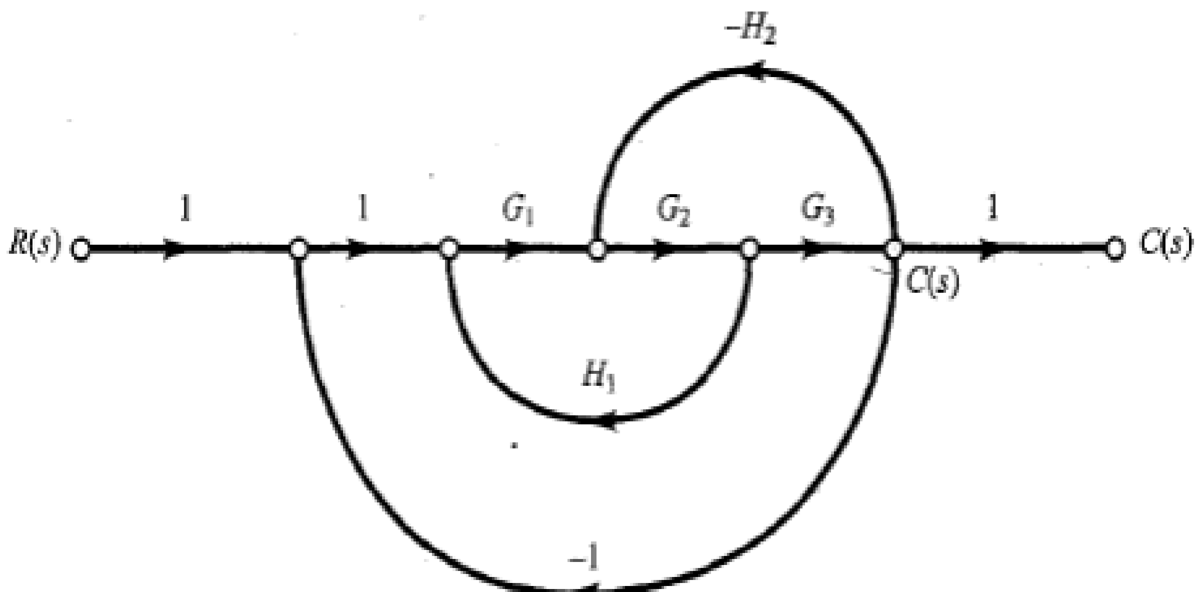
$$\Delta_1 = 1 - \text{(sum of all individual loop gains)} + \ldots$$
$$\Delta_1 = 1$$

Eliminate forward path-2

$$\Delta_2 = 1 - \text{(sum of all individual loop gains)} + \ldots$$
$$\Delta_2 = 1$$

Using Mason's Formula, Find the T.F. $C(s)/R(s)$





In this system there is only one forward path between the input $R(s)$ and the output $C(s)$. The forward path gain is

$$P_1 = G_1G_2G_3$$

there are three individual loops. The gains of these loops are

$$L_1 = G_1G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_1G_2G_3$$

since all three loops have a common branch, there are no non-touching loops. Hence, the determinant Δ is given by

$$\begin{aligned}\Delta &= 1 - (L_1 + L_2 + L_3) \\ &= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3\end{aligned}$$

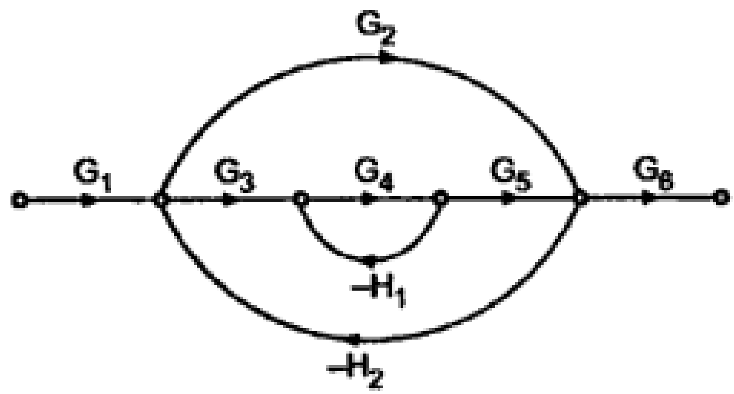
The cofactor Δ_1 of the determinant along the forward path connecting the input node and output node is obtained from Δ by removing the loops that touch this path. Since path P_1 touches all three loops, we obtain

$$\Delta_1 = 1$$

Therefore, the overall gain between the input $R(s)$ and the output $C(s)$, or the closed-loop transfer function, is given by

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

Find the overall T.F. by using Mason's gain formula for the signal flow graph



Solution : Two forward paths, K = 2,

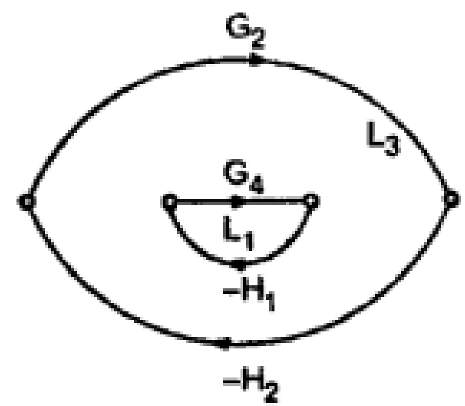
$$T_1 = G_1 G_3 G_4 G_5 G_6$$

$$T_2 = G_1 G_2 G_6$$

Loops are, $L_1 = -G_4 H_1$

$$L_2 = -G_3 G_4 G_5 H_2$$

$$L_3 = -G_2 H_2$$



Non touching loops

Out of these, L_1 and L_3 is combination of 2 non touching loops

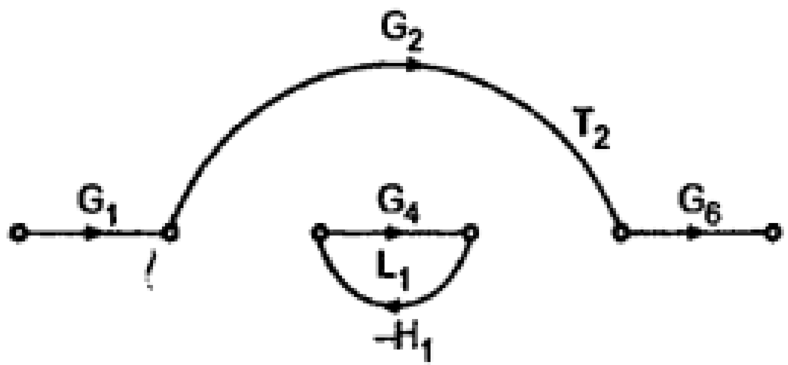
$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$$

Δ_1 = Eliminate L_1, L_2, L_3 as all are touching to T_1 from Δ

$\therefore \quad \Delta_1 = 1$

Δ_2 = Eliminate L_2 and L_3, as they are touching to T_2, from Δ. But L_1 is non touching hence keep it as it is in Δ.

$\therefore \quad \Delta_2 = 1 - [L_1]$



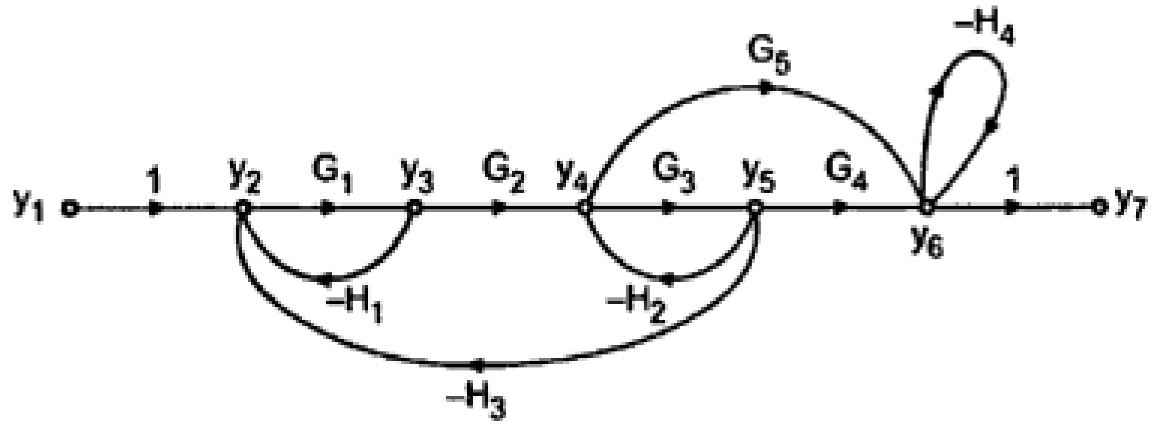
L_1 Non touching to T_2

Substitute in Mason's gain formula,

$$\text{T.F.} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$\boxed{\text{T.F.} = \frac{G_1 G_3 G_4 G_5 G_6 [1] + G_1 G_2 G_6 [1 + G_4 H_1]}{1 + G_4 H_1 + G_3 G_4 G_5 H_2 + G_2 H_2 + G_2 G_4 H_1 H_2}}$$

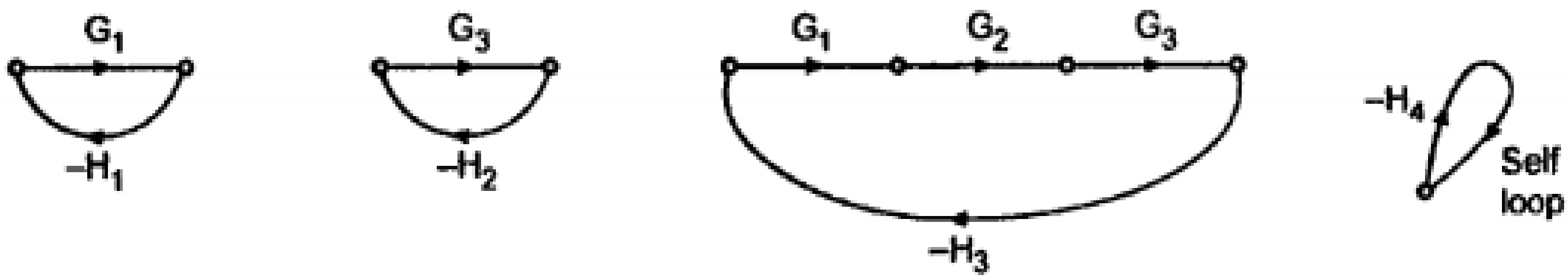
Calculate $\frac{y_7}{y_2}$ of the system, whose signal flow graph is given below.



Solution : Forward paths for y_1 to y_7 are two

$$T_1 = G_1 G_2 G_3 G_4 , \quad T_2 = G_1 G_2 G_5$$

Individual feedback loops are



$L_1 = -G_1 H_1$ $\quad$ $L_2 = -G_3 H_2$ $\quad$ $L_3 = -G_1 G_2 G_3 H_3$ $\quad$ $L_4 = -H_4$

Combinations of two non touching loops

$$L_1 L_2 = +G_1 G_3 H_1 H_2, \qquad L_1 L_4 = +G_1 H_1 H_4$$

$$L_2 L_4 = +G_3 H_2 H_4, \qquad L_3 L_4 = +G_1 G_2 G_3 H_3 H_4$$

One combination of three non touching

$$L_1 L_2 L_4 = -G_1 G_3 H_1 H_2 H_4$$

$$\therefore \quad \Delta = 1-[L_1+L_2+L_3+L_4]+[L_1 L_2+L_1 L_4+L_2 L_4+L_3 L_4]-[L_1 L_2 L_4]$$

$$\Delta_1 = 1 \text{ all loops are touching}$$

$$\Delta_2 = 1 \text{ all loops are touching}$$

$$\frac{y_7}{y_1} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_5}{\Delta}$$

Now to find the ratio $\frac{y_2}{y_1}$

Forward paths for y_1 to y_2 is one. $T_1 = 1$ Now Δ is same as above.

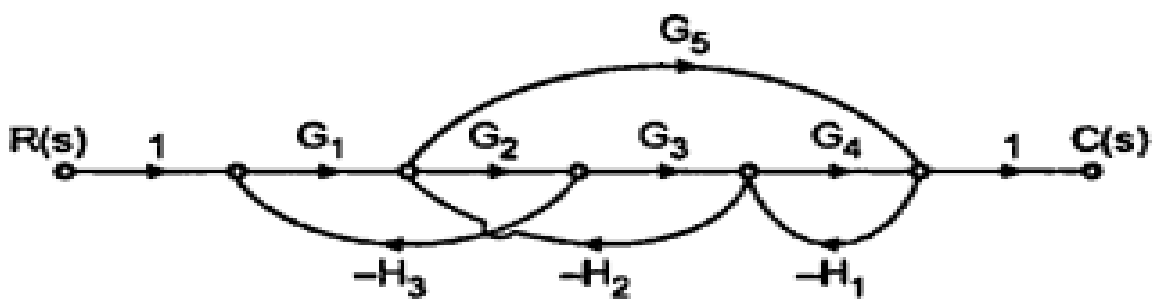
and $\quad \Delta_1 = 1 - L_2 - L_4 + L_2 L_4 \quad$ ... L_2 and L_4 nontouching to $T_1 = 1$

$$= 1 + G_3 H_2 + H_4 + G_3 H_2 H_4$$

$$\therefore \quad \frac{y_2}{y_1} = \frac{T_1 \Delta_1}{\Delta} = \frac{1+G_3 H_2 + H_4 + G_3 H_2 H_4}{\Delta}$$

$$\therefore \quad \boxed{\frac{y_7}{y_2} = \frac{\frac{y_7}{y_1}}{\frac{y_2}{y_1}} = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_5 (1+G_3 H_2)}{1+G_3 H_2 + H_4 + G_3 H_2 H_4}}$$

Find $\frac{C(s)}{R(s)}$ for S.F.G. shown in following figure.



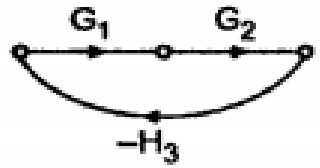
Number of forward paths = K = 2

$$\therefore \qquad T.F. = \frac{\sum_{K=1}^{2} T_K \Delta_K}{\Delta} \qquad \text{using Mason's gain Formula}$$

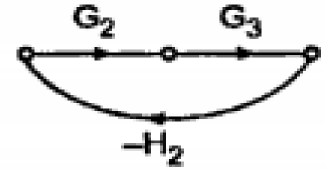
$$T_1 = G_1 G_2 G_3 G_4$$

$$T_2 = G_1 G_5$$

Individual feedback loops,

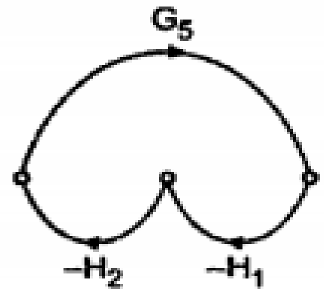


$L_1 = -G_1G_2H_3$



$L_2 = -G_2G_3H_2$

$L_3 = -G_4H_1$



$L_4 = +G_5H_1H_2$

Loops L_1 and L_3 are non touching loops.

$$\therefore \quad \Delta = \left[L_1 + L_2 + L_3 + L_4\right] + \left[L_1 L_3\right]$$

$$= 1 - [-G_1G_2H_3 - G_2G_3H_2 - G_4H_1 + G_5H_1H_2] + [G_1G_2G_4H_1H_3]$$

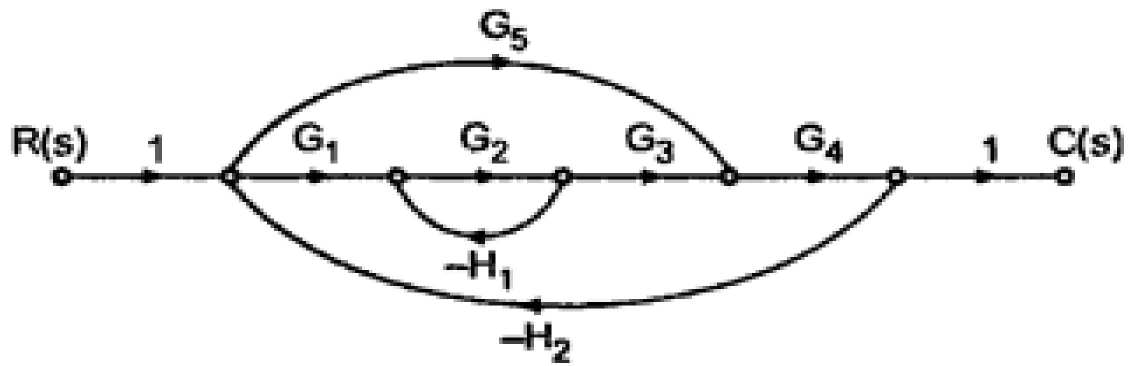
Consider T_1, all loops are touching $\therefore \quad \Delta_1 = 1$

Consider T_2, all loops are touching $\therefore \quad \Delta_2 = 1$

$$\therefore \text{ T.F.} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 G_4 \cdot 1 + G_1 G_5 \cdot 1}{1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_5 H_1 H_2 + G_1 G_2 G_4 H_1 H_3}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_5 H_1 H_2 + G_1 G_2 G_4 H_1 H_3}}$$

Find $\frac{C(s)}{R(s)}$ *by using Mason's gain formula.*



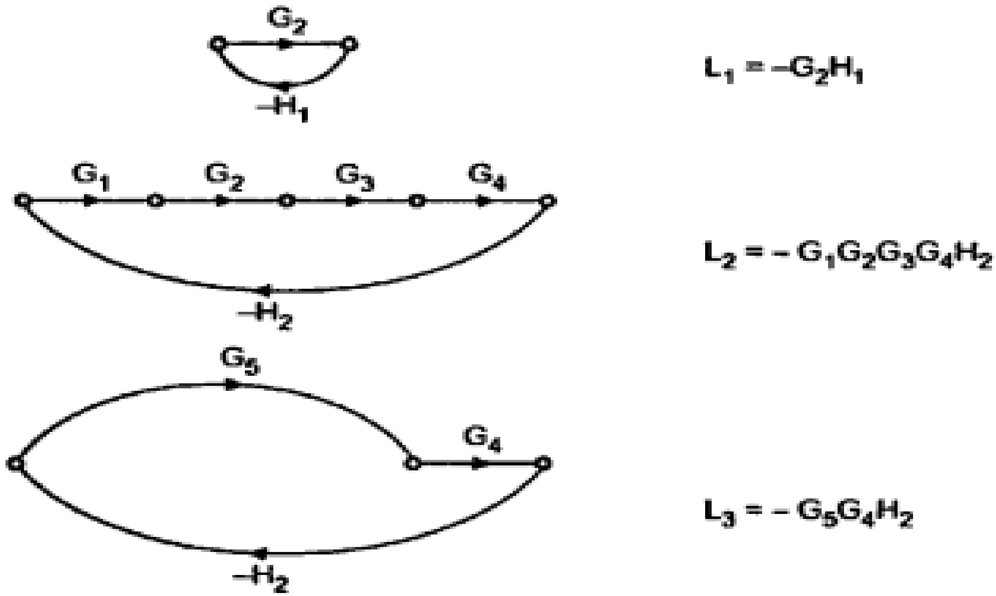
Number of forward paths K = 2

Mason's gain formula,

$$\text{T.F.} = \frac{\sum_{K=1}^{2} T_K \Delta_K}{\Delta} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$T_1 = G_1 G_2 G_3 G_4, \quad T_2 = G_5 G_4$$

Individual feedback loops are,



L_1 and L_3 are two non touching loops.

$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$$

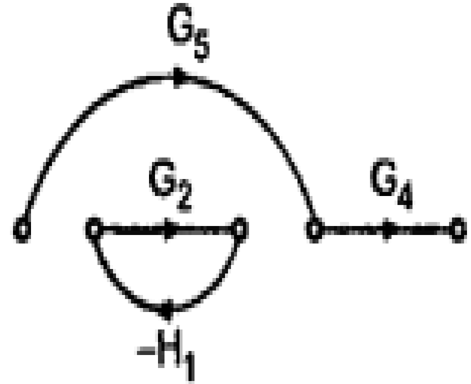
$$= 1 - [-G_2 H_1 - G_1 G_2 G_3 G_4 H_2 - G_5 G_4 H_2] + [G_2 H_1 G_5 G_4 H_2]$$

$$= 1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_5 G_4 H_2 + G_2 G_5 G_4 H_1 H_2$$

For T_1 all loops are touching

$\therefore$ $\Delta_1 = 1$ eliminating all loop gains and products from Δ

Consider T_2,



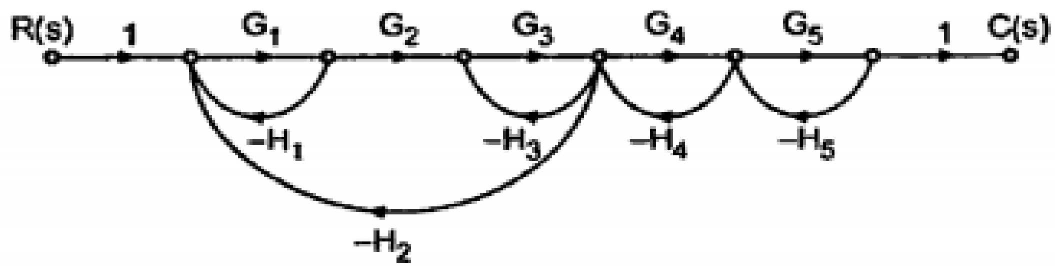
L_1 is non touching to T_2.

$\therefore \quad \Delta_2 = 1 - [L_1] = 1 - (-G_2H_1) = 1 + G_2H_1$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1\,\Delta_1 + T_2\,\Delta_2}{\Delta} = \frac{G_1\,G_2\,G_3\,G_4 \cdot 1 + G_5\,G_4\,(1 + G_2\,H_1)}{\Delta}$$

$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1\,G_2\,G_3\,G_4 + G_4\,G_5\,(1 + G_2\,H_1)}{1 + G_2\,H_1 + G_1\,G_2\,G_3\,G_4\,H_2 + G_5\,G_4\,H_2 + G_2\,G_5\,G_4\,H_1\,H_2}}$$

Find $\frac{C(s)}{R(s)}$

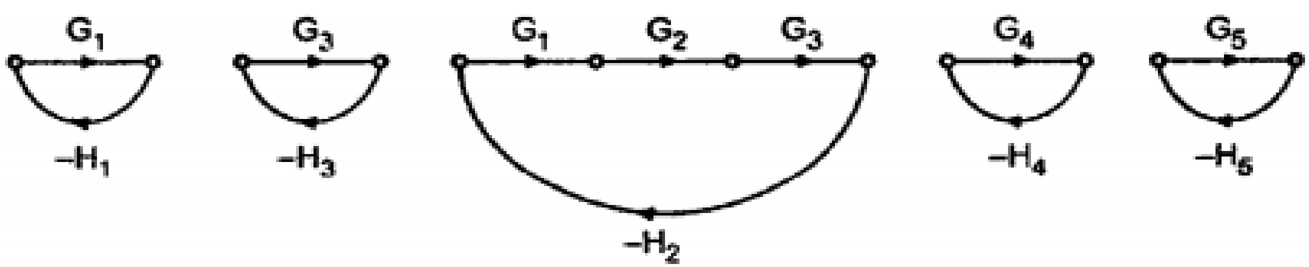


Number of forward paths = K = 1

$\therefore$ $$\text{T.F.} = \frac{T_1\,\Delta_1}{\Delta}$$ .... Mason's gain formula

$$T_1 = G_1\,G_2\,G_3\,G_4\,G_5$$

Individual feedback loops,



$L_1 = -G_1H_1$ $L_2 = -G_3H_3$ $L_3 = -G_1G_2G_3H_2$ $L_4 = -G_4H_4$ $L_5 = -G_5H_5$

Combinations of two non touching loops,

i) L_1 and L_2 ii) L_1 and L_5 iii) L_1 and L_4

iv) L_2 and L_5 v) L_3 and L_5

Combination of three non touching loops, is L_1, L_2 and L_5.

$$\therefore \quad \Delta = 1-[L_1+L_2+L_3+L_4+L_5]+[L_1L_2+L_1L_5+L_1L_4+L_2L_5+L_3L_5]-[L_1L_2L_5]$$

$$\therefore \quad \Delta = 1+G_1H_1+G_3H_3+G_1G_2G_3H_2+G_4H_4+G_5H_5$$
$$+G_1G_3H_1H_3+G_1G_4H_1H_4+G_1G_5H_1H_5+G_3G_5H_3H_5$$
$$+G_1G_2G_3G_5H_2H_5+G_1G_3G_5H_1H_3H_5$$

Now considering $T_1 = G_1G_2G_3G_4G_5$

All loops are touching to this forward path hence,

$$\Delta_1 = 1$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4G_5\cdot 1}{\Delta}$$

$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5}{\begin{array}{l}1+G_1H_1+G_3H_3+G_1G_2G_3H_2+G_4H_4+G_5H_5\\ +G_1G_3H_1H_3+G_1G_4H_1H_4+G_1G_5H_1H_5\\ +G_3G_5H_3H_5+G_1G_2G_3G_5H_2H_5\\ +G_1G_3G_5H_1H_3H_5\end{array}}}$$

Construct the signal flow graph for the following set of system equations.

$$Y_2 = G_1 Y_1 + G_3 Y_3 \quad \text{... (1)}$$

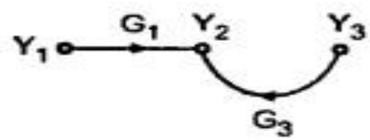
$$Y_3 = G_4 Y_1 + G_2 Y_2 + G_5 Y_3 \quad \text{... (2)}$$

$$Y_4 = G_6 Y_2 + G_7 Y_3 \quad \text{... (3)}$$

where Y_4 is output. Find transfer function $\frac{Y_4}{Y_1}$.

System node variables are Y_1 , Y_2 , Y_3 , Y_4

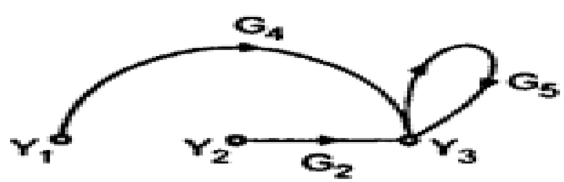
Consider equation 1 : This indicates Y_2 depends on Y_1 and Y_3



S.F.G. for equation (1)

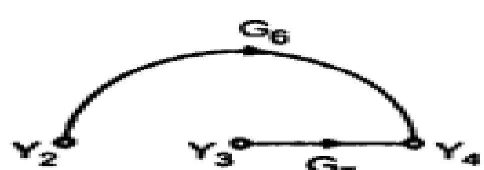
Consider equation 2 : This indicates Y_3 depends on Y_1 , Y_2 and Y_3

Consider equation 2 : This indicates Y_3 depends on Y_1 , Y_2 and Y_3



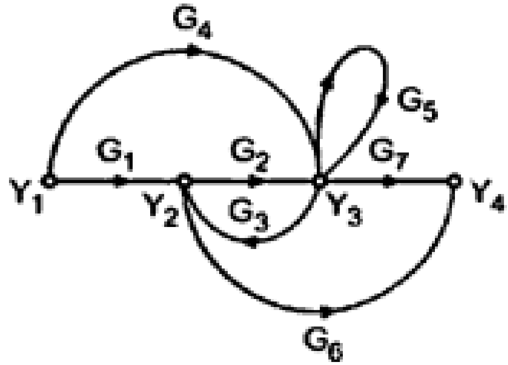
S.F.G. for equation (2)

Consider equation 3 : This indicates Y_4 depends on, Y_3 and Y_2



S.F.G. for equation (3)

Combining all three we get, complete S.F.G. as shown,



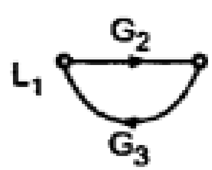
No. of forward paths = K = 4

$$\therefore \qquad \text{T.F.} = \sum_{K=1}^{4} \frac{T_K \, \Delta_K}{\Delta} = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3 + T_4 \Delta_4}{\Delta}$$

... Mason's gain formula

$T_1 = G_1 G_2 G_7$, $\quad T_2 = G_4 G_7$, $T_3 = G_1 G_6$, $\quad T_4 = G_4 G_3 G_6$

Individual loops are,



$L_1 = G_2 G_3 \qquad\qquad L_2 = G_5$

$$\therefore \qquad \Delta = 1 - [L_1 + L_2] = 1 - G_2 G_3 - G_5$$

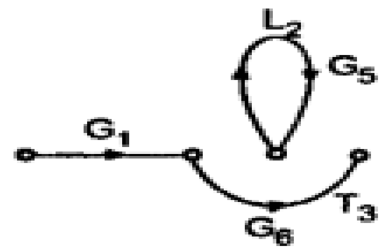
No non touching loop combinations.

Consider T_1, both loops are touching $\therefore \Delta_1 = 1$

T_2, both loops are touching $\therefore \Delta_2 = 1$

T_3, for this 'G5' self loop is non touching, $\therefore \Delta_3 = 1 - G_5$

T_1, both loops are touching $\therefore \Delta_4 = 1$



L_2 non touching to T_3

$$\frac{Y_4}{Y_1} = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3 + T_4 \Delta_4}{\Delta}$$

$$= \frac{G_1G_2G_7.1 + G_4G_7.1 + G_1G_6(1-G_5) + G_4G_3G_6.1}{\Delta}$$

$$\therefore \boxed{\frac{Y_4}{Y_1} = \frac{G_1 G_2 G_7 + G_4 G_7 + G_1 G_6 (1-G_5) + G_4G_3G_6}{1 - G_2 G_3 - G_5}}$$