

Frequency Response of amplifiers

* Extended hybrid- π equivalent circuit of BJT

Fig a) shows an npn bipolar transistor in a common emitter configuration, along with the small signal voltages & currents.

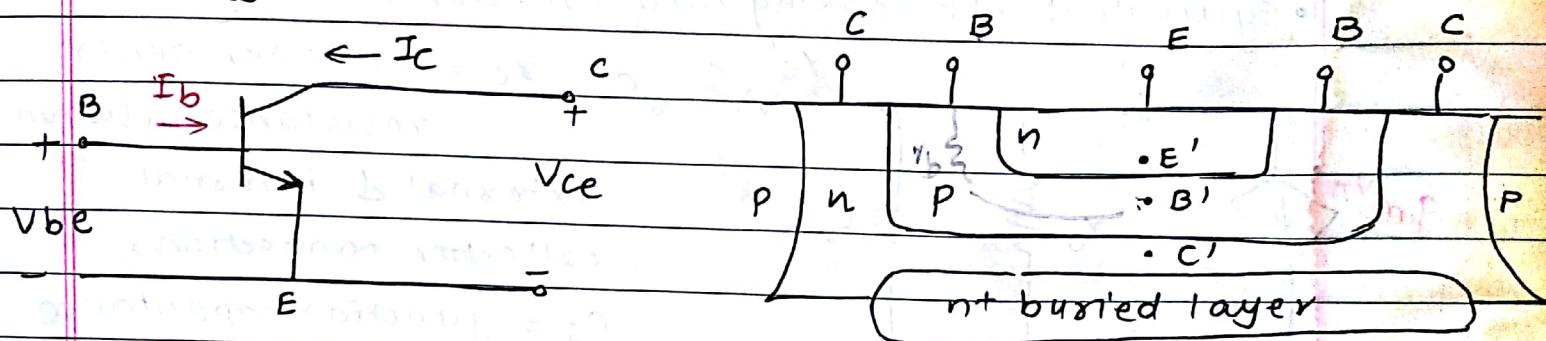


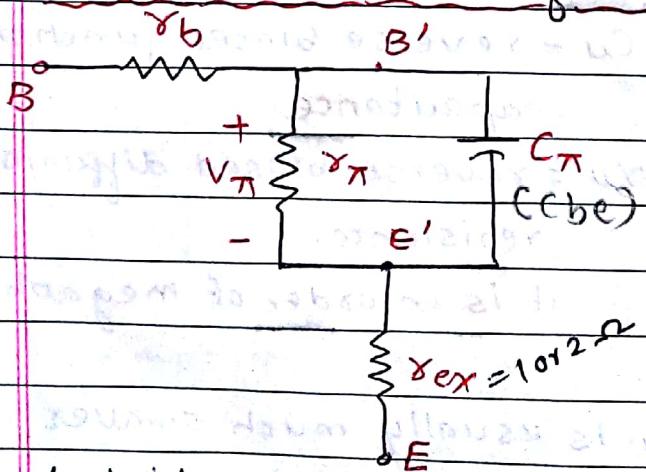
fig a) CE npn BJT

fig b) cross-section of npn BJT
for the hybrid π model.

fig b) is a cross-section of npn transistor in a (BJT amplifier ckt) classic integrated circuit configuration.

- The C, B & E terminals are the external connections to the transistor & C', B' & E' are the idealized internal collector, base and emitter regions.

Equivalent circuit of the transistor:-



- Resistance r_b is the base series resistance betw the external base terminal B & internal base region B'.

- the B'-E' junction is forward biased, therefore C_{π} is the forward biased junction capacitance &

hybrid π equivalent ckt
base to emitter

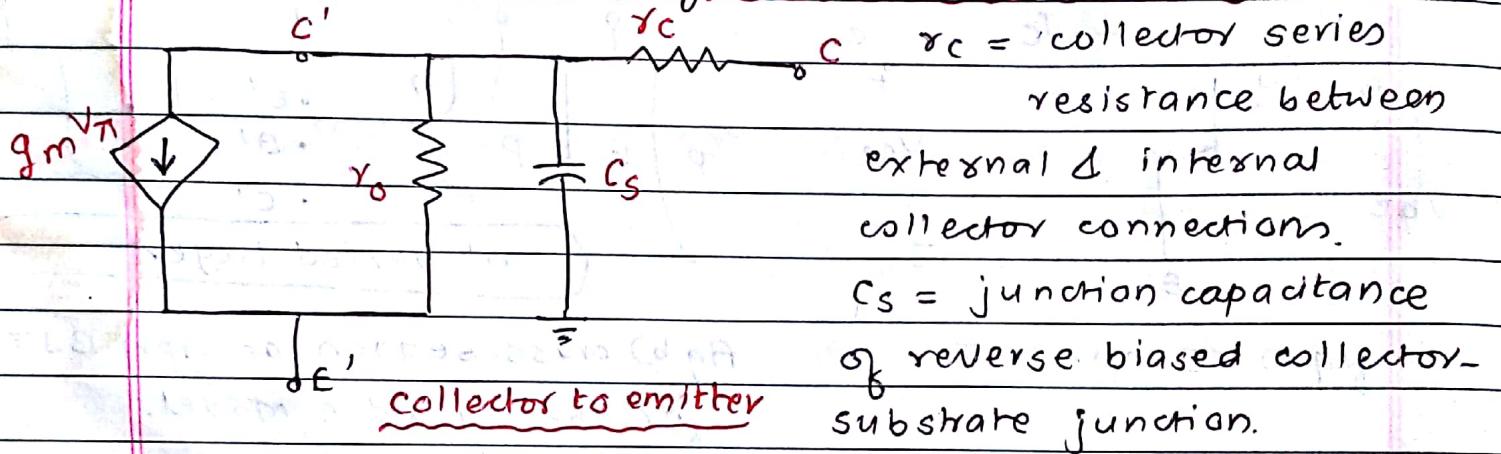
r_n is the forward biased junction diffusion resistance.

C_n & r_n are the function of the junction current.

r_{ex} is the emitter series resistance between the external emitter terminal & the internal emitter region.

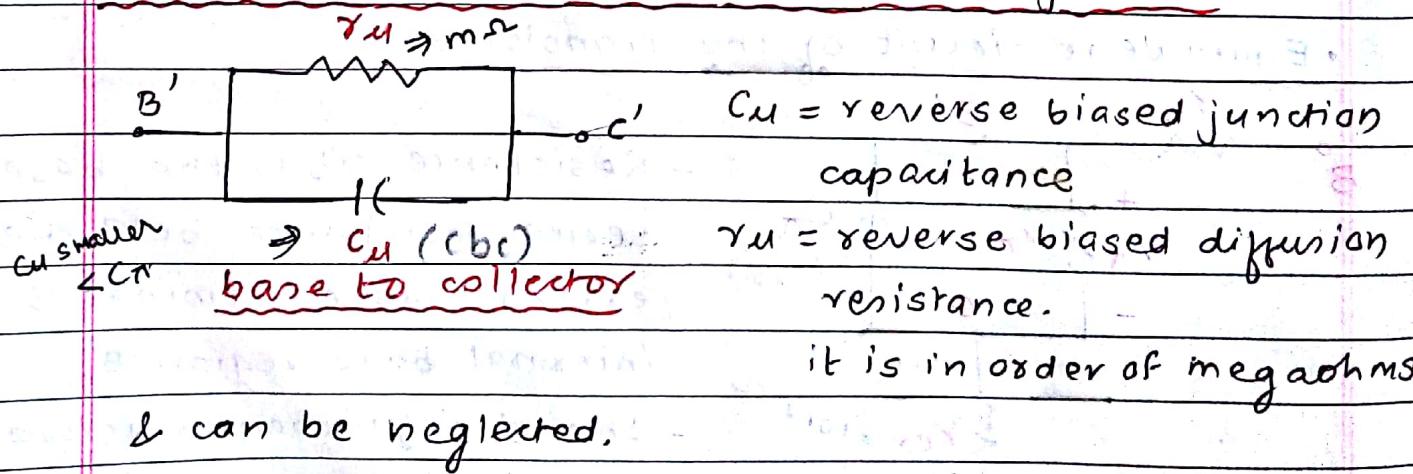
r_{ex} = usually very small in order of 1 to 2 Ω .

- Equivalent ckt looking into collector terminal :-



- The dependent current source $g_m V_n$ is the transistor collector current controlled by the internal base-emitter voltage.
- r_0 = inverse of op conductance g_0 & is due primarily to the Early effect.

- Equivalent ckt of reverse biased B'-c' junction -



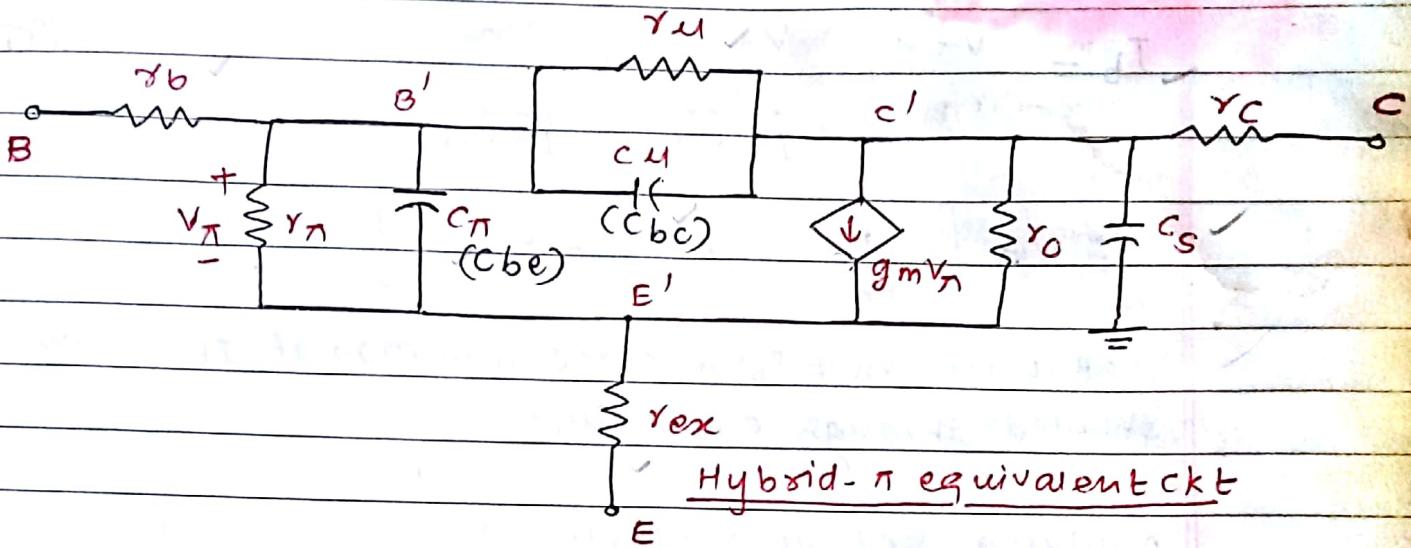
& can be neglected.

- **NOTE:-** The value of C_u is usually much smaller than C_n . However because of a phenomenon known as 'Miller effect', C_u usually cannot be neglected.

* Hybrid π provides more accurate model for HF effects.

FOR LF to MF $\Rightarrow \gamma_b \ll \text{small so S.C., } r_c \text{ & } r_{ex} \ll \text{small} \Rightarrow \text{S.C. 3}$
 $\gamma_u \gg \text{large} \Rightarrow j_3 \gamma_u = 0. \text{ C. so equivalent } c_{eb} = \text{same as } r_e \text{ model}$

- Complete hybrid- π equivalent ckt for BJT :-

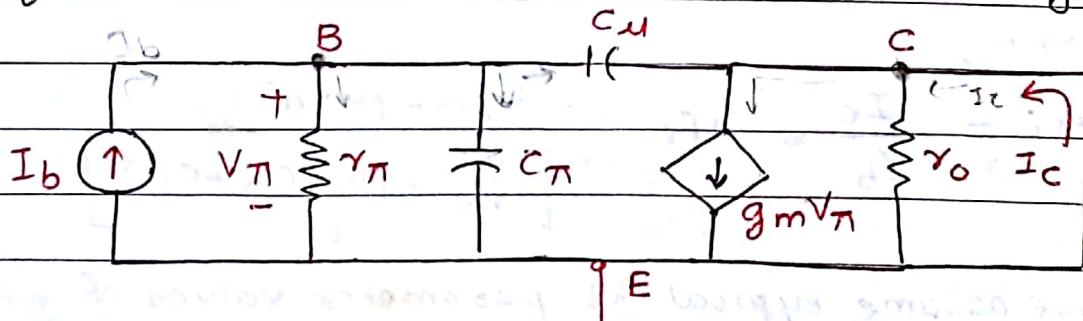


- The capacitances lead to frequency effects in the transistors.
- The gain is a function of the input signal frequency.

* Short-circuit current Gain:-

We can neglect the parasitic resistances γ_b , γ_c & r_{ex} , the base-collector B-C diffusion resistance γ_u , and the substrate capacitance C_s . Collector is connected to signal ground.

- To understand the frequency effects of BJT we first determine the short circuit current gain



simplified hybrid π equivalent ckt for determining s.c. current gain

- We will determine small signal current gain $A_i = \frac{I_c}{I_b}$

writing a KCL equation at i/p node,

$$I_b = \frac{V_\pi}{r_\pi} + \frac{V_\pi}{j\omega C_\pi} + \frac{V_\pi}{j\omega C_u} \quad \text{eqn ①}$$
$$= V_\pi \left[\frac{1}{r_\pi} + j\omega(C_\pi + C_u) \right]$$

But now $V_\pi \neq I_b r_\pi$ since a portion of I_b is now shunted through C_π & C_u .

Applying KCL at o/p node,

$$\frac{V_\pi}{j\omega C_u} + I_c = g_m V_\pi \quad I_c = V_\pi(g_m - j\omega C_u)$$

$$\text{OR} \quad I_c = V_\pi(g_m - j\omega C_u)$$

The i/p voltage V_π can be

$$V_\pi = \frac{I_c}{(g_m - j\omega C_u)} \quad \text{put this in eqn ①}$$

$$I_b = I_c \cdot \frac{\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_u) \right]}{g_m - j\omega C_u}$$

The small signal current gain usually designated as h_{fe} becomes,

$$A_i = \frac{I_c}{I_b} = h_{fe} = \frac{(g_m - j\omega C_u)}{\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_u) \right]}$$

If we assume typical ckt parameter values of

$$C_u = 0.2 \mu F \quad \& \quad f_{max} = 100 \text{ MHz}$$

$$g_m = 50 \text{ mA/V}$$

then we see that $\omega C_u \ll g_m$

Therefore to a good approximation, the small signal

current gain is

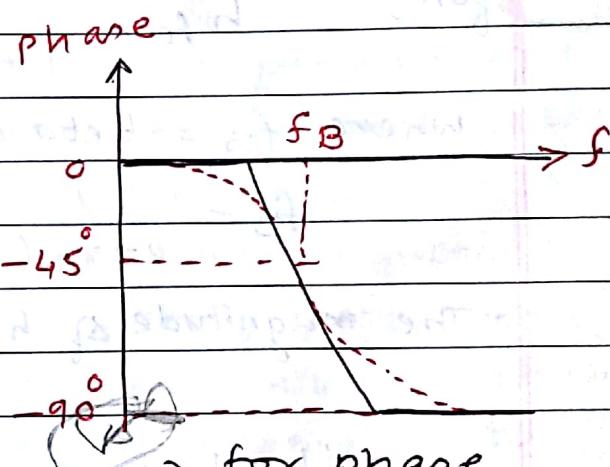
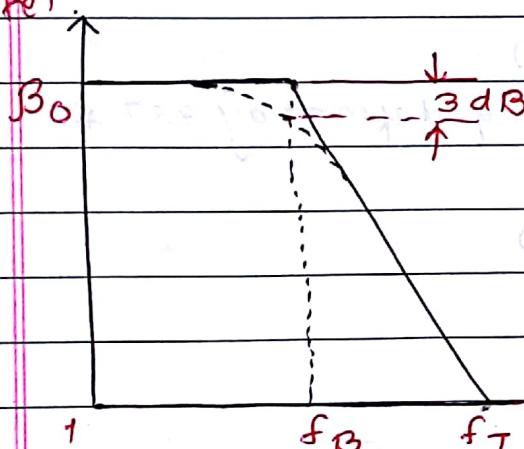
$$h_{fe} \approx \frac{g_m}{\left[\frac{1}{r_\pi} + jw(C_\pi + C_u) \right]} = \frac{g_m r_\pi}{1 + jw r_\pi (C_\pi + C_u)} \quad \text{Eqn ②}$$

since $g_m r_\pi = \beta$ then low freq current gain = is just β

Eqn ② shows that in a BJT, the magnitude & phase of current gain are both functions of the frequency.

Bode plot of short ckt current gain magnitude-

(In red)



- The corner frequency, which is also the beta cutoff frequency f_B in this case is given by

$$f_B = \frac{1}{2\pi r_\pi (C_\pi + C_u)} \quad \text{(A) '}$$

from fig b) above,

- As the frequency increases, the small signal collector current is no longer in phase with the small signal base current.

- At high frequencies, the collector current lags the i/p current by 90° .

* f_B is determined by a set of parameters employed in hybrid π Model.

PAGE NO.	
DATE	

* Cut-off frequency -

Bode plot in previous fig a) shows that the magnitude of small signal current gain decreases with increasing frequency.

- At frequency f_T , which is the cutoff frequency, this gain goes to 1.
- The cut off frequency is a figure of merit of transistors.

From Eqn ② $h_{fe} \approx \frac{g_m r_n}{1 + j\omega r_n (C_n + C_u)}$

We can write the small signal current gain in the form

$$h_{fe} = \frac{\beta_0}{1 + j(f/f_B)}$$

where f_B = beta cutoff freq defined by eqn ③ *

$$f_B = \frac{1}{2\pi r_n (C_n + C_u)}$$

- The magnitude of h_{fe} is

$$|h_{fe}| = \frac{\beta_0}{\sqrt{1 + \left(\frac{f}{f_B}\right)^2}} \quad ③$$

- At cut off freq f_T , $|h_{fe}| = 1$ & eqn ③ can be modified as

$$|h_{fe}| = 1 = \frac{\beta_0}{\sqrt{1 + \left(\frac{f_T}{f_B}\right)^2}}$$

Normally, $\beta_0 \gg 1$, which implies that $f_T \gg f_B$

$$1 \approx \frac{\beta_0}{\sqrt{\left(\frac{f_T}{f_B}\right)^2}} = \frac{\beta_0 f_B}{f_T} \quad \text{or} \quad f_T = \beta_0 f_B$$

- f_B is also called bandwidth of the transistor.

$$\boxed{f_T = \text{gain BW product}}$$

PAGE NO.

7

DATE

$\therefore f_T = \beta_0 f_B$, the cutoff freq f_T is the gain bandwidth product of transistor, or more commonly the unity gain bandwidth.

from eq (A),

The unit gain bandwidth is

$$f_T = \beta_0 \left[\frac{1}{2\pi r_n (C_{n\bar{n}} + C_{d\bar{d}})} \right] = \frac{g_m}{2\pi (C_{n\bar{n}} + C_{d\bar{d}})}$$

(if $\beta_0 = g_m R$)

- since the capacitances are a function of the size of the transistor, we see again that high frequency transistors imply small device sizes.

- The cutoff frequency f_T is also a function of the dc collector current I_C , and the general characteristic of f_T versus I_C is shown in fig c)



cut off frequency vs.
collector current

- The transconductance g_m is directly proportional to I_C , but only a portion of $C_{n\bar{n}}$ is related to I_C .

- The cut-off frequency is therefore lower at low collector current levels.

However, the cut off frequency also decreases at high current levels, in the same way that β decreases at large currents.

$$\boxed{f_B = \frac{f_T}{\beta_0}}$$

- The cut off frequency or unity gain bandwidth of a transistor is usually specified on the device data sheets.

* Miller Effect and Miller Capacitance:-

- The Miller effect, or feedback effect, is a multiplication effect of C_V in circuit applications.

- Next figure ① is a common-emitter circuit with a signal current source at the input. We will determine the small-signal current gain $A_i = i_o/i_s$ of the circuit.

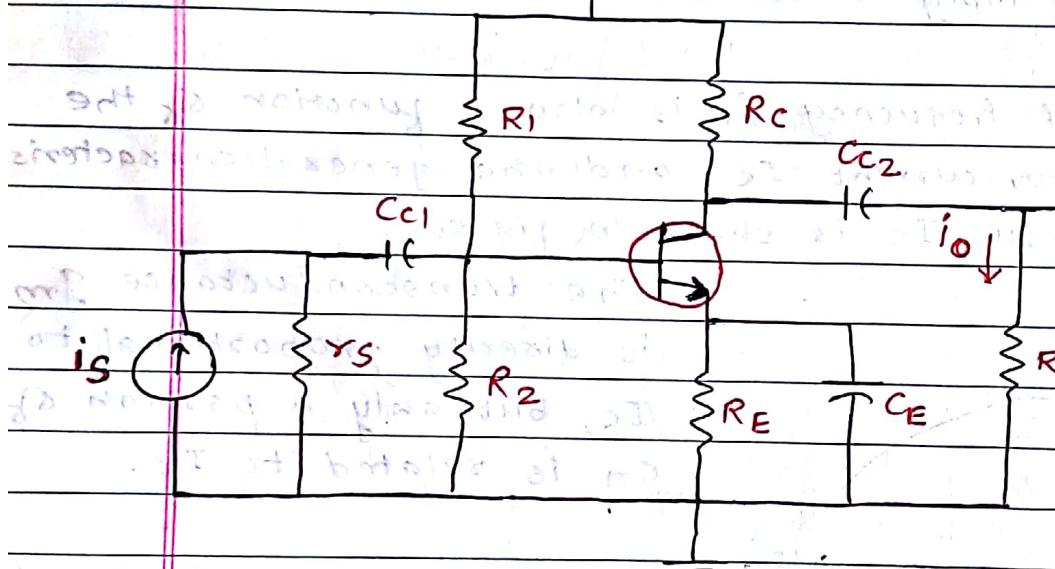


fig ① common-emitter ckt with common current source, i/p.

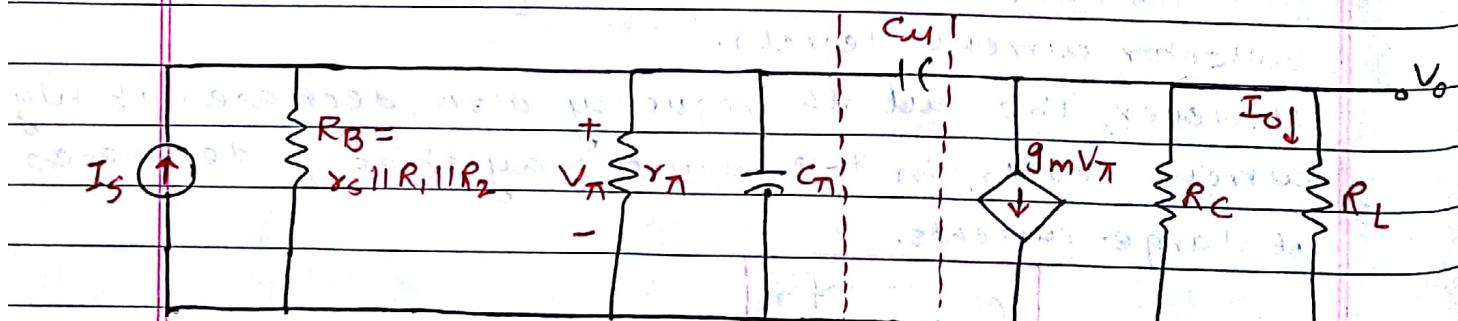


fig ② small signal equivalent ckt with simplified hybrid-pi Model

- fig ② is the small signal current gain equivalent ckt, assuming the frequency is sufficiently high for the coupling & bypass capacitors to act as short cks.

- The capacitor C_M is a feedback element that connects the output back to the input.
- The o/p voltage & current will therefore influence the input characteristics.
- We can determine the effect of C_M on the input characteristics by finding an equivalent impedance Z_A across the plane A-A in Fig ③, producing the equivalent ckt shown in Fig ④.
- The current I_1 from figure ③ can be written as

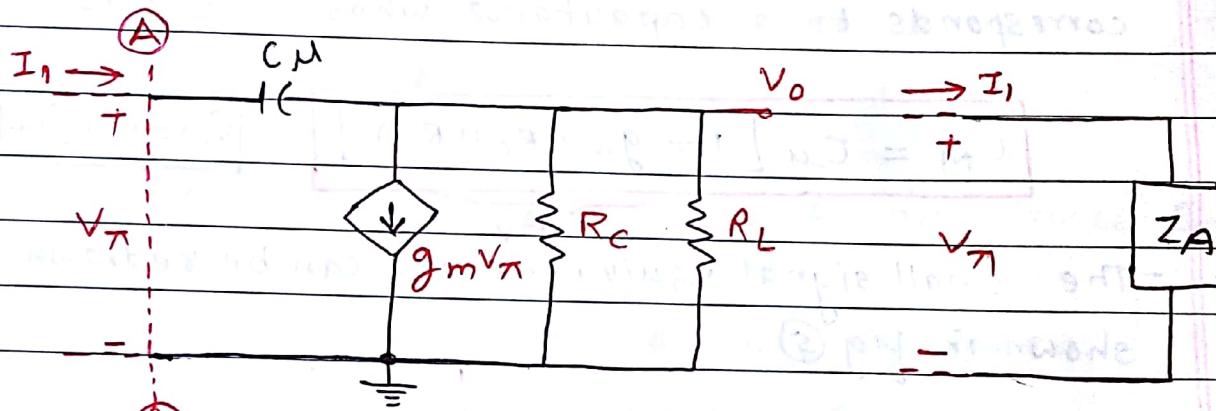


Fig ③ o/p position of small signal equivalent ckt

fig ④ equivalent

impedance of this position of the ckt

$$I_1 = \frac{V_\pi - V_o}{(1/jwC_M)} = (V_\pi - V_o) (jwC_M)$$

summing the currents at o/p gives

$$I_1 = (V_\pi - V_o) (jwC_M) = g_m V_\pi + \frac{V_o}{R_C \| R_L}$$

$V_\pi jwC_M - V_o jwC_M = g_m V_\pi + V_o / R_C \| R_L$
or combining terms, we have

$$V_\pi (jwC_M - g_m) = V_o \left(\frac{1}{R_C \| R_L} + jwC_M \right)$$

But $|jwC_M| \ll g_m$

$$\therefore -g_m V_\pi \approx V_o \left(\frac{1}{R_C \| R_L} \right)$$

$$V_o = -g_m (R_c \parallel R_L) V_\pi$$

substituting this expression for V_o into eqn ① yields

$$I_i = \{V_\pi - [-g_m (R_c \parallel R_L) V_\pi]\} j w C_M$$

or

$$I_i = V_\pi \cdot j w C_M [1 + g_m (R_c \parallel R_L)]$$

$$\star C_M = \frac{1}{j w C_M}$$

The equivalent impedance Z_A in fig ④ then corresponds to a capacitance whose value is

$$C_M = C_M [1 + g_m (R_c \parallel R_L)]$$

$$C_M = C_M [1 + |A_v|]$$

The small signal equivalent ckt can be redrawn as shown in fig ⑤.

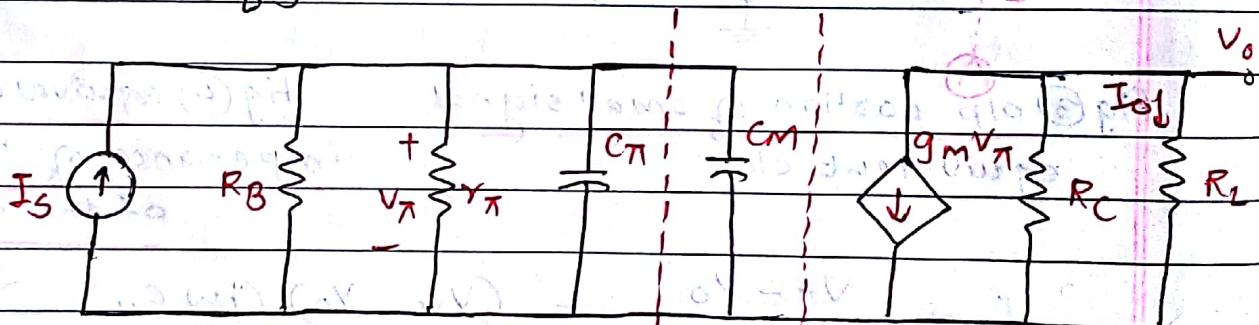


fig ⑤ small signal equivalent ckt including the equivalent Miller capacitance

Capacitance C_M is called the Miller capacitance & the multiplication effect of C_M is called the Miller effect.

from fig ⑤, the input capacitance is now $C_{pi} + C_M$ rather than just C_{pi} if C_M had been ignored.

[Ex1] Determine the 3dB frequency of the short circuit current gain of a bipolar transistor. consider a bipolar transistor with parameters $\gamma\pi = 2.6 \text{ k}\Omega$, $C\pi = 2 \text{ pF}$ & $Cu = 0.1 \text{ pF}$

soln. We know

$$f_B = \frac{1}{2\pi \gamma\pi (C\pi + Cu)}$$

$$\boxed{f_B = 29.1 \text{ MHz}}$$

[Ex2] calculate the bandwidth f_B & capacitance $C\pi$ of a bipolar transistor.

Consider a transistor that has parameters $f_T = 500 \text{ MHz}$ at $I_c = 1 \text{ mA}$, $\beta_0 = 100$, $Cu = 0.3 \text{ pF}$

soln Bandwidth is $f_B = \frac{f_T}{\beta_0} = \frac{500}{100} = 5 \text{ MHz}$

The transconductance is

$$g_m = \frac{I_c}{V_T} = \frac{1 \text{ mA}}{0.026} = 38.5 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C\pi + Cu)}$$

$$500 \times 10^6 = \frac{38.5 \times 10^{-3}}{2\pi(C\pi + 0.3 \times 10^{-12})}$$

$$\boxed{C\pi = 12.0 \text{ pF}}$$

Ex3) Determine 3dB frequency of the current gain for the circuit shown in fig 5 (in previous page) both with & without effect of C_M .

The circuit parameters are

$$R_C = R_L = 4k\Omega, \quad r_\pi = 2.6k\Omega, \quad R_B = 200k\Omega$$

$$C_N = 4\text{pF}, \quad C_M = 0.2\text{pF}, \quad g_m = 38.5\text{mA/V}$$

without effect
of C_M

$$f_{3dB} = \frac{1}{2\pi(R_B || r_\pi)(C_N + C_M)}$$

neglecting effect of C_M ($C_M = 0$)

$$f_{3dB} = \frac{1}{2\pi [(200 \times 10^3) || (2.6 \times 10^3)] (4 \times 10^{-12})}$$

$$= 15.5 \text{ MHz}$$

$$C_M = C_M [1 + g_m(R_C || R_L)]$$

$$= 0.2 [1 + (38.5)(4 || 4)]$$

$$C_M = 15.6 \text{ pF}$$

with effect
of C_M

$$f_{3dB} = \frac{1}{2\pi(R_B || r_\pi)(C_N + C_M)}$$

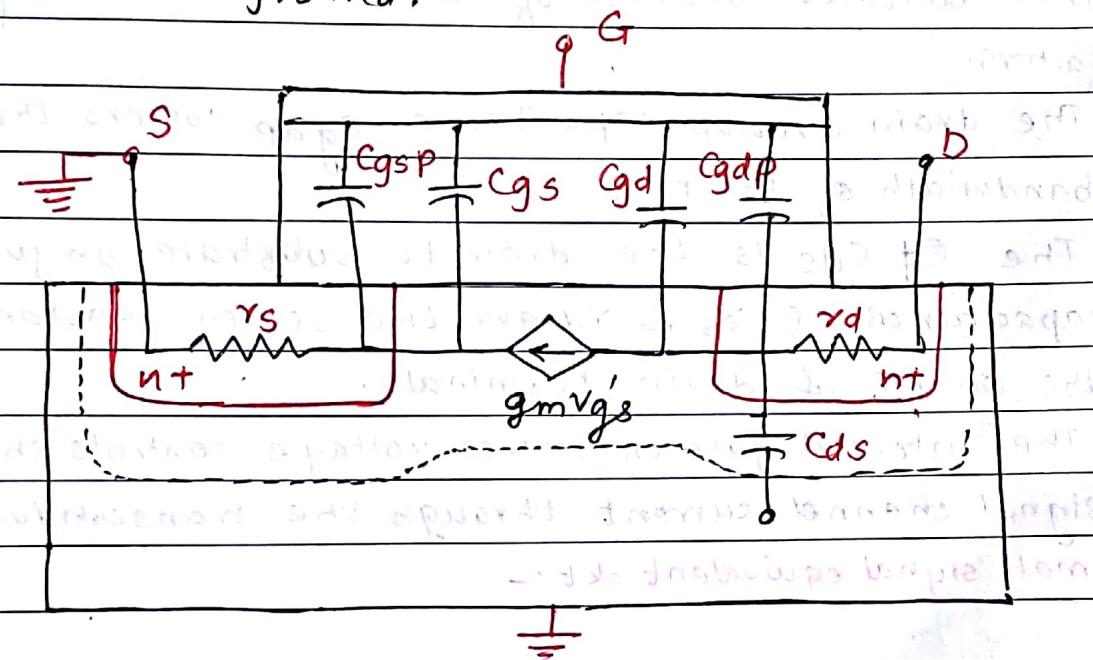
$$= \frac{1}{2\pi [(200 \times 10^3) || (2.6 \times 10^3)] [4 + 15.6] (10^{-12})}$$

$$[f_{3dB} = 3.16 \text{ MHz}]$$

NOTE: If the gain is reduced then the Miller capacitance will be reduced & the bandwidth will be increased.

* High frequency Equivalent circuit of FET

Next figure shows a model based on the inherent capacitances & resistances in the n-channel MOSFET, as well as the elements representing the basic device equation. Assumption :- The source & substrate are both tied to ground.



(a) Inherent resistance & capacitances in n-channel MOSFET

- Two capacitances C_{gs} & C_{gd} represent the interaction between the gate and the channel inversion charge near the source & drain elements resp.
- If the device is biased in the nonsaturation region & V_{DS} is small, the channel inversion charge is approximately uniform, which means that

$$C_{gs} \approx C_{gd} \approx \frac{1}{2} W L C_{ox}$$

Where $C_{ox} (\text{F/cm}^2) = E_{ox}/t_{ox}$

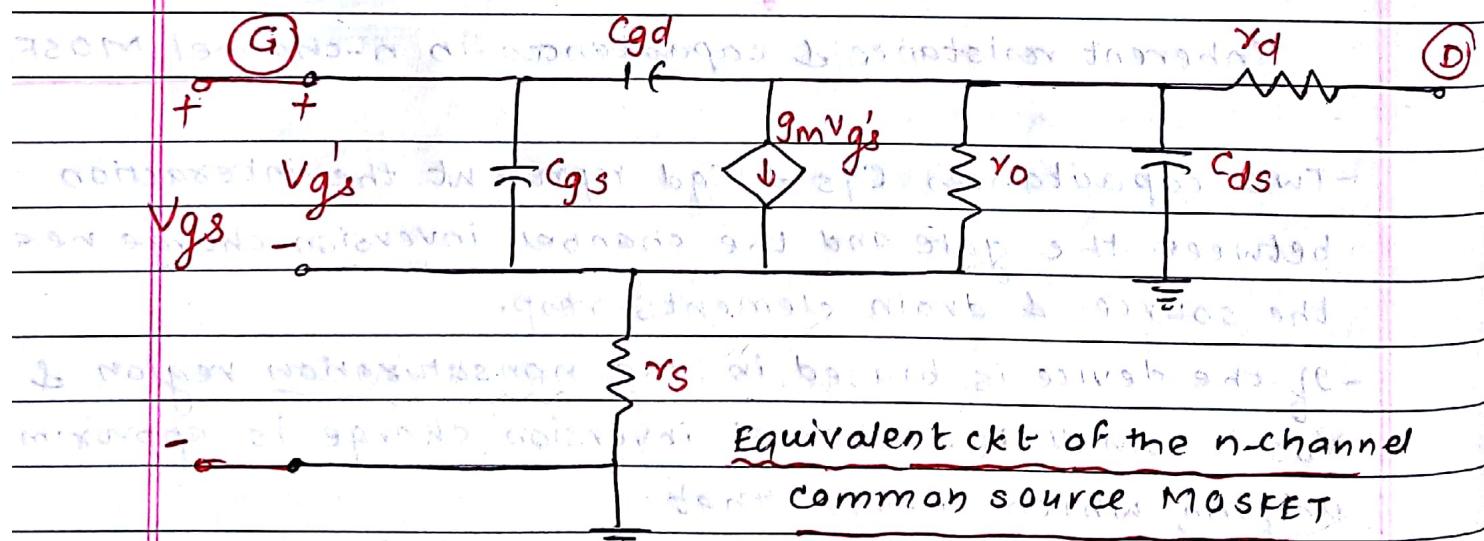
NOTE :- When the transistor is biased in the saturation region, the channel is pinched off at the drain & the inversion charge is no longer uniform. The value of C_{gd}

essentially goes to zero & C_{GS} approximately equals to $(2/3)WLCox$.

The value of C_{GS} changes as device size changes, but typical values are in the tenth of picofarad range.

- The remaining two capacitances C_{GSP} & C_{GDP} are parasitic or overlap capacitances so called because, in actual devices, the gate oxide overlaps the source & drain contacts, because of tolerances or other fabrication factors.
- The drain overlap capacitance C_{GDP} lowers the bandwidth of the FET.
- The ~~C_{DS}~~ C_{DS} is the drain to substrate pn junction capacitance & r_s & r_d are the series resistances of the source & drain terminals.
- The internal gate to source voltage controls the small signal channel current through the transconductance.

Small signal equivalent ckt:-



- V_{GS}' = internal gate to source voltage that controls the channel current.

- C_{GS} & C_{GD} contain the parasitic overlap capacitances.

- r_s is associated with the slope of I_D Versus V_{DS} .

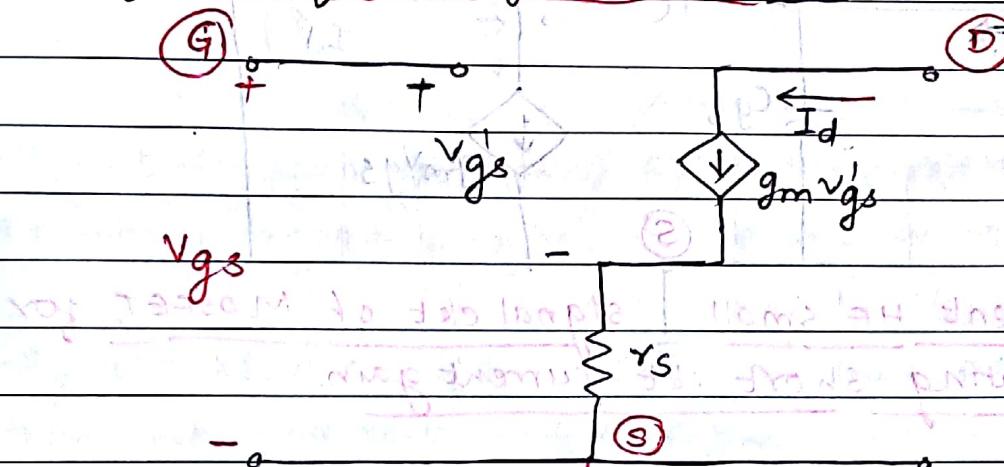
In ideal MOSFET biased in the saturation region, I_D is

independent of V_{DS} which means r_0 is infinite.

- However r_0 is finite in short channel length devices, because of channel length modulation & is therefore included in the equivalent ckt.

- Source resistance r_s 's can have a significant effect on the transistor characteristics. Very simple of sketch

simplified low frequency equivalent ckt -



simplified LF equivalent ckt of n-channel CS MOSFET including r_s but not resistance r_0

- The drain current is $I_d = g_m V_g s$
- & The relationship betn $V_g s$ & V_g 's is

$$\begin{aligned} V_g s &= V_g + (g_m V_g) r_s \\ &= (1 + g_m r_s) V_g \end{aligned}$$

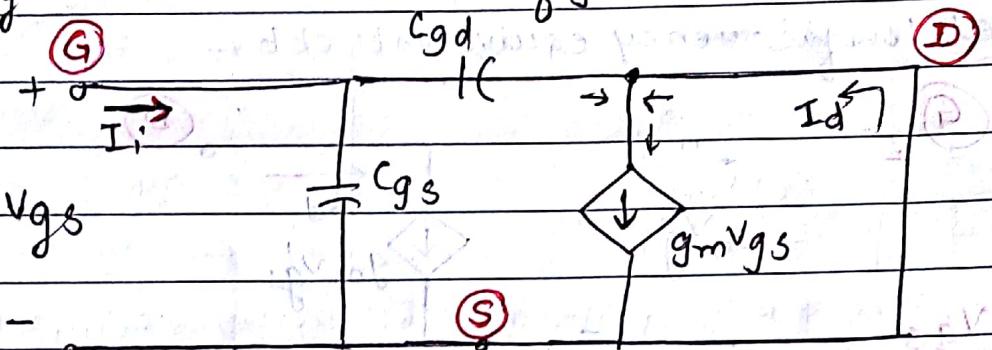
- The drain current can be written as

$$I_d = \left(\frac{g_m}{1 + g_m r_s} \right) V_g s = g_m' V_g$$

This eqn shows that source resistance reduces the effective transconductance or transistor gain.

* Unity Gain Bandwidth :-

- For a BJT, UGB or BW is a figure of merit for FETs.
- If we neglect r_s , r_d , r_o & C_{ds} and connect the drain to signal ground, the resulting equivalent small signal ckt is shown in figure.



Equivalent HF small signal ckt of MOSFET, for calculating short ckt current gain.

- At HF, i/p gate impedance is no longer infinite, we can define short ckt current gain.
- From that we can define & calculate UGB.

writing KCL eqn at i/p node,

$$I_i = \frac{V_{gs}}{\frac{1}{j\omega C_{gs}}} + \frac{V_{gs}}{\frac{1}{j\omega C_{gd}}} - \textcircled{1}$$

$$= V_{gs} [j\omega(C_{gs} + C_{gd})]$$

Apply KCL at output o/p node

$$\frac{V_{gs}}{\frac{1}{j\omega C_{gd}}} + I_d = g_m V_{gs}$$

OR

$$I_d = V_{gs} (g_m - j\omega C_{gd})$$

solving this eqn for v_{gs}

$$v_{gs} = \frac{Id}{g_m - j\omega C_{gd}} \quad \text{put this in eqn ①}$$

$$I_i = Id \frac{j\omega(C_{gs} + C_{gd})}{g_m - j\omega C_{gd}}$$

\therefore small signal current gain

$$A_i = \frac{Id}{I_i} = \frac{g_m - j\omega C_{gd}}{j\omega(C_{gs} + C_{gd})}$$

- The unity gain frequency f_T is defined as frequency at which magnitude of the s.c. current

- If we assume $C_{gd} = 0.05pF$ & $g_m = 1mA/V$ & $f = 100MHz$ then we find that $\omega C_{gd} \ll g_m$

- The small signal current gain to a good approximation is then,

$$A_i = \frac{Id}{I_i} \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})}$$

- The unity gain frequency f_T is defined as the frequency at which the magnitude of the short ckt current gain goes to 1.

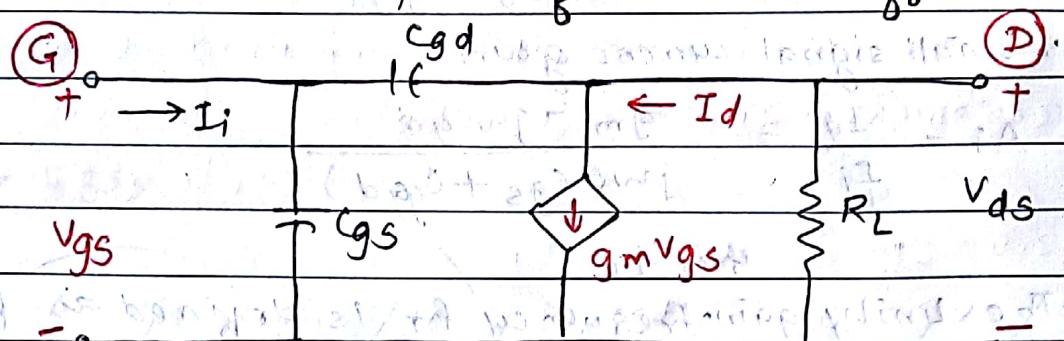
$$\therefore f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

- The unity gain frequency or bandwidth is a parameter of the transistor & is independent of the ckt.

* Miller effect and Miller capacitance:-

- fig a) is a simplified high frequency transistor model, with a load resistor R_L connected to the o/p.

- We will determine the current gain in order to demonstrate the impact of the Miller effect.



fig@ Equivalent High freq small signal ckt of a MOSFET with a load resistance R_L .

- writing a KCL eqn at the i/p gate node, we have

$$I_i = j\omega C_{gs} V_{gs} + j\omega C_{gd} (V_{gs} - V_{ds}) \quad \text{--- (1)}$$

where I_i is the i/p current.

- summing currents at the o/p drain node we have,

$$V_{ds} + g_m V_{gs} + j\omega C_{gd} (V_{ds} - V_{gs}) = 0 \quad \text{--- (2)}$$

we can combine eqn (1) & (2) to eliminate V_{ds} .

The i/p current is

$$I_i = j\omega \left\{ C_{gs} + C_{gd} \left[\frac{1 + g_m R_L}{1 + j\omega R_L C_{gd}} \right] \right\} V_{gs} \quad \text{--- (3)}$$

$(\omega R_L C_{gd})$ is much less than 1, therefore we can neglect $(j\omega R_L C_{gd})$ & eqn (3) becomes,

$$I_i = j\omega [C_{gs} + C_{gd}(1 + g_m R_L)] V_{gs} \quad \text{--- (4)}$$

Next fig ⑤ shows the equivalent ckt described by eqn ④.

The parameter C_M is the Miller capacitance & is given by

$$C_M = (g_d (1 + g_m R_L)) \quad \text{--- (5)}$$

Eq. ⑤ clearly shows the effect of the parasitic drain overlap capacitance.

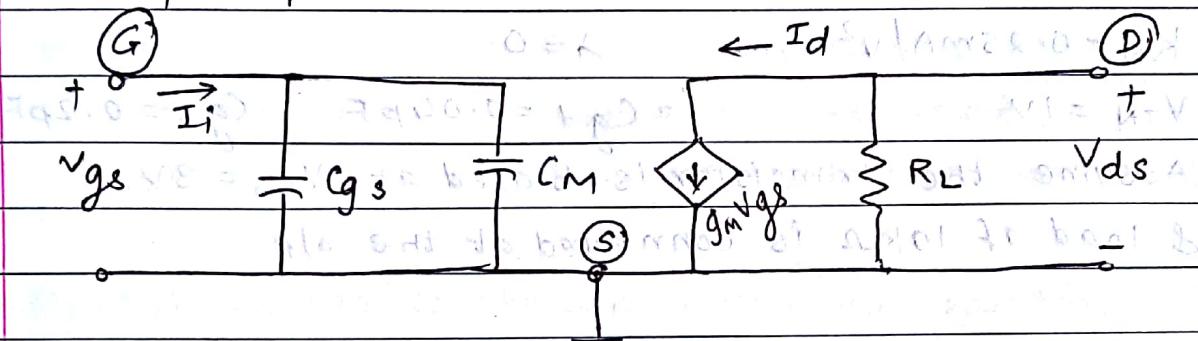


fig ⑤ MOSFET high frequency ckt, including equivalent Miller capacitance.

NOTE: C_{gd} is the overlap capacitance (or total gate to drain capacitance). This (overlap capacitance) is multiplied because of the Miller effect & may become a significant factor in the bandwidth of an amplifier.

- The cut off frequency f_T of a MOSFET is defined as the frequency at which the current gain = 1, or magnitude of iip current I_i is equal to the ideal load current I_d .

- From fig ⑤, we see that

$$I_i = j\omega (C_{gs} + C_M) V_{gs} \quad \& \quad \text{ideal load current is } I_d = g_m V_{gs}$$

\therefore magnitude of current gain is

$$|A_i| = \left| \frac{I_d}{I_i} \right| = \frac{g_m V_{gs}}{j\omega (C_{gs} + C_M) V_{gs}}$$

$$A_i = \frac{g_m}{2\pi f (C_{gs} + C_M)}$$

setting this eqⁿ(=1) equal to 1, we find cut off freq

$$f_T = \frac{g_m}{2\pi(C_{GS} + C_M)} = \frac{g_m}{2\pi C_M}$$

where $C_M = i/p$ gate capacitance.

Ex 1) Determine Miller capacitance & cut off freq of MOSFET

& determine unity gain bandwidth of an MOSFET.

consider an n-channel MOSFET with parameters

$$K_n = 0.25 \text{ mA/V}^2$$

$$\lambda = 0$$

$$V_{TN} = 1 \text{ V}$$

$$C_{GD} = 0.04 \text{ pF}$$

$$C_{GS} = 0.2 \text{ pF}$$

Assume the transistor is biased at $V_{GS} = 3 \text{ V}$,

& load of $10 \text{ k}\Omega$ is connected at the op.

Sol:-

The Transconductance is

$$g_m = 2K_n(V_{GS} - V_{TN})$$

$$= 2(0.25)(3-1)$$

$$g_m = 1 \text{ mA/V}$$

Unity-Gain-Bandwidth f_T (or freq) is given by

$$f_T = \frac{g_m}{2\pi(C_{GS} + C_{GD})} = \frac{1}{2\pi(0.2 + 0.04) \times 10^{-12}}$$

$$f_T = 6.63 \times 10^8 \text{ Hz} \quad \text{or} \quad f_T = 663 \text{ MHz}$$

Miller capacitance is given by the formula

$$C_M = C_{GD}(1 + g_m R_L) = 0.04 [1 + (1)(10)]$$

$$C_M = 0.44 \text{ pF}$$

With cut off frequency

$$f_T = \frac{g_m}{2\pi(C_{GS} + C_M)} = \frac{10^8}{2\pi(0.2 + 0.44) \times 10^{-12}} = 2.49 \times 10^8 \text{ Hz}$$

$$f_T = 249 \text{ MHz}$$

* NOTE: Miller capacitance reduce the cutoff frequency of MOSFET as well as BJT

* Frequency Response : Transistor Amplifiers with circuit capacitors :-

- Three types of capacitors are considered.

- ① coupling capacitor ② Load capacitors ③ Bypass capacitors.

① Coupling Capacitor Effects :-

Input coupling capacitor : common emitter circuit

* Note:- We will use a current-voltage analysis to determine the frequency response of the circuit

Then, we will use the equivalent time constant technique.

fig a) shows BJT CE ckt with a coupling capacitor, fig b) shows corresponding small signal equivalent ckt.

r_o (small signal o/p resistance) is assumed to be infinite ($\therefore r_o \gg R_C$ & $r_o \gg R_E$)

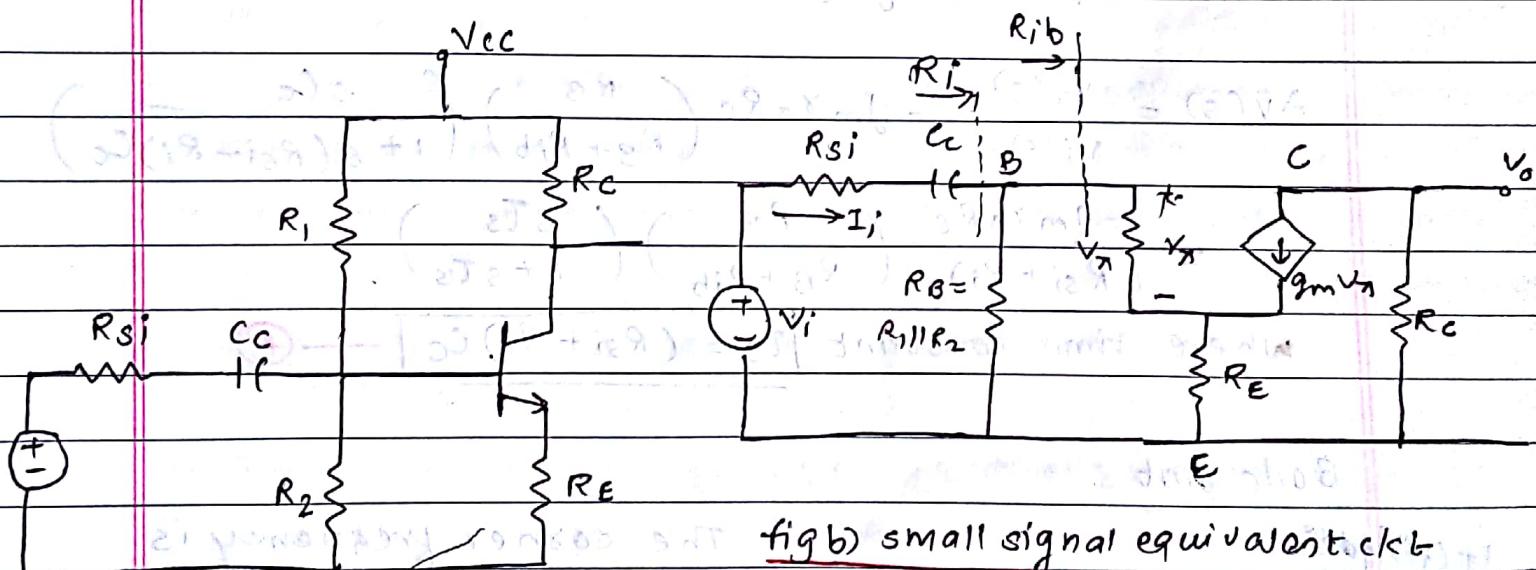


fig b) small signal equivalent ckt

fig 1.2 / fig a) CE ckt with coupling capacitor

(A) Current-Voltage Analysis :-

The input current can be written as,

$$I_i = \frac{V_i}{R_{si} + \frac{1}{sC_C} + R_1}$$

$$\text{where } R_i = R_B \parallel [r_A + (1+\beta)R_E] = R_B \parallel R_{ib} = R_B \parallel \frac{R_E}{1+\beta}$$

$$= R_B \parallel R_{ib}$$

$$I_i = \frac{V_i}{R_{si} + \frac{1}{sC} + R_i}$$

where R_i is the input resistance $= R_B || [r_n + (1+\beta)R_E]$
 $= R_B || R_{ib}$

Using current divider,

$$I_b = \left(\frac{R_B}{R_B + R_{ib}} \right) I_i \quad \text{and then } V_A = I_b r_n \quad \textcircled{3}$$

Op-amp is given by

$$V_o = -g_m V_A R_C \quad \text{, putting eqn } \textcircled{2} \text{ & } \textcircled{3}$$

$$= -g_m R_C (I_b r_n) \\ = -g_m r_n R_C \left(\frac{R_B}{R_B + R_{ib}} \right) I_i$$

$$V_o = -g_m r_n R_C \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{V_i}{R_{si} + \frac{1}{sC} + R_i} \right)$$

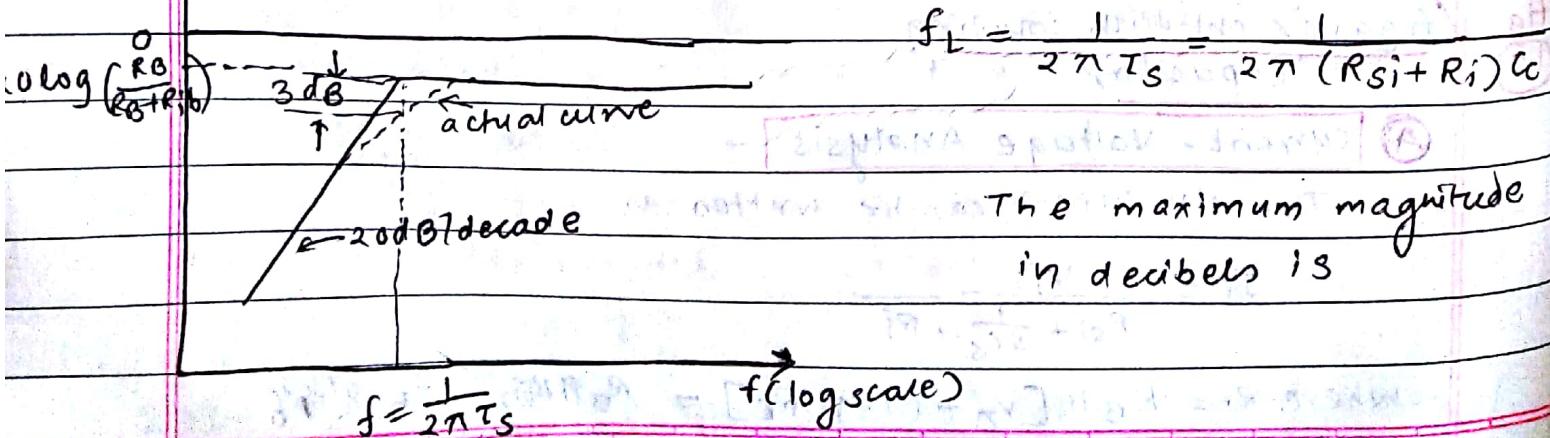
∴ small signal voltage gain is

$$A_V(s) = \frac{V_o(s)}{V_i(s)} = -g_m r_n R_C \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{sC}{1 + s(R_{si} + R_i)C} \right) \\ = \frac{-g_m r_n R_C}{(R_{si} + R_i)} \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{sT_s}{1 + sT_s} \right)$$

where time constant $T_s = (R_{si} + R_i)C$ — *

Bode plot —

$|T(jf)| \text{ in dB}$ The corner frequency is



The maximum magnitude in decibels is

$$|AV(\text{max})| \text{dB} = 20 \log_{10} \left(\frac{g_m \gamma_n R_C}{R_{S1} + R_i} \right) \left(\frac{R_B}{R_B + R_{iB}} \right)$$

(B) Time constant Technique -

- In general, we do not need to derive the complete circuit transfer function including capacitance effects in order to complete the bode plot & determine the frequency response.

- By looking at a circuit, we can determine if the amplifier is low-pass or high pass circuit. We can specify the Bode plot if we know the time constant & the maximum midband gain.

- The time constant determines the corner frequency.

- The time constant technique yields good results when all poles are real.

(NOTE: This technique does not determine corner frequencies due to system zeros.)

- The major benefit of using the time constant approach is that it gives information about which circuit elements affect the -3dB frequency of the ckt.

- The time constant for the ckt is a function of the equivalent resistance seen by the capacitor.

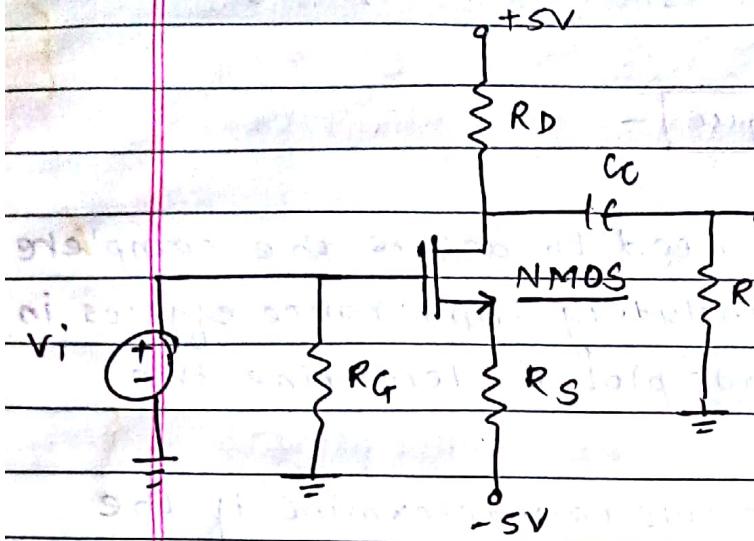
- from fig (b) (previous page), the equivalent resistance seen by the coupling capacitor C_C is $(R_{S1} + R_i)$.

\therefore The time constant is

$$T_S = (R_{S1} + R_i) C_C \quad (*) \quad \text{where } R_i = R_B || [\gamma_n + (1 + \beta) R_C]$$

so this is same as that determined by using current voltage analysis

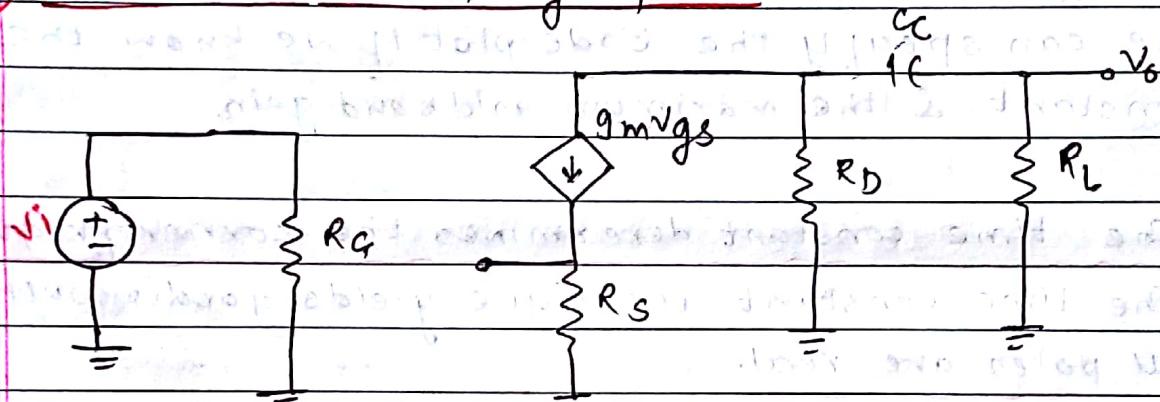
* Output coupling capacitor - Common Source circuit.



- o/p signal is connected to the load through a coupling capacitor.

Note - the resistance of the signal generator is much less than R_G . (Therefore R_G can be neglected.)

Fig 1. CS ckt with o/p coupling capacitor.



small-signal equivalent ckt

- The Time constant is a function of the effective resistance seen by capacitor C_C , which is determined by setting all independent sources equal to zero.

$$\text{If } V_I = 0 \text{ then } V_{GS} = 0 \text{ & } g_m V_{GS} = 0. \text{ So } R_{eq} = R_D + R_L$$

∴ Effective resistance seen by C_C is $(R_D + R_L)$

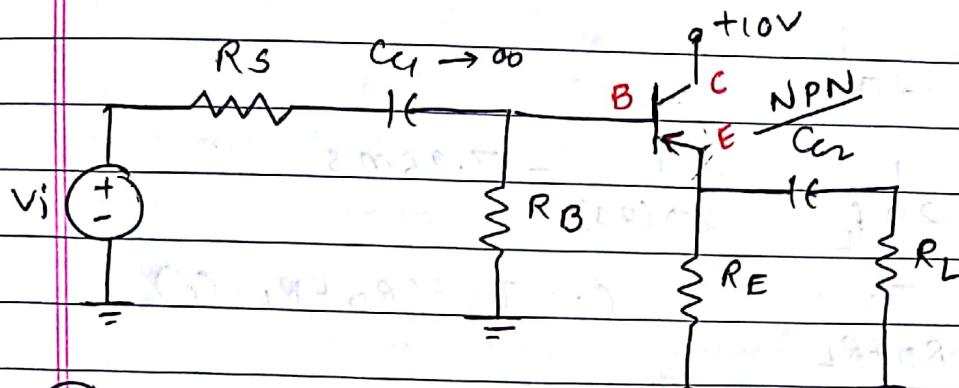
∴ The time constant is $\tau_s = (R_D + R_L) C_C$

$$\tau_s = (R_D + R_L) C_C$$

& corner frequency is $f_L = \frac{1}{2\pi\tau_s}$

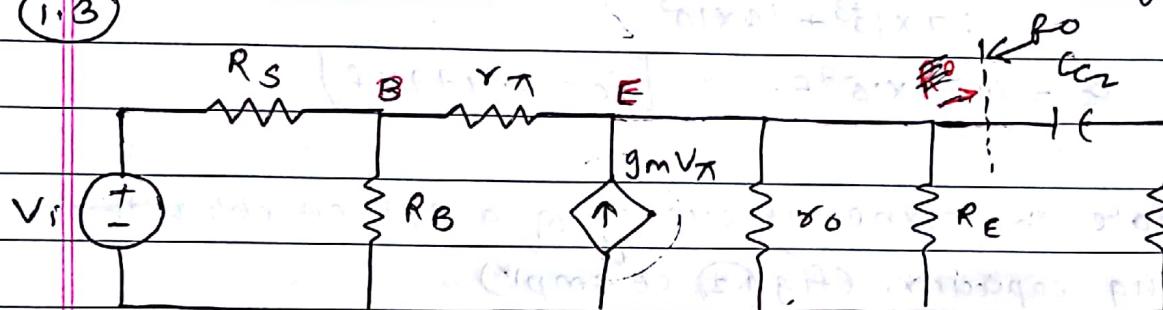
(W) * Output coupling capacitors: Emitter follower circuit

- Coupling capacitor C_{C1} is very large & it acts as a short circuit to the input signal.



fig(13) Emitter follower ckt - 10V with o/p coupling capacitor

(1.3)



small-signal equivalent ckt

The equivalent resistance seen by the coupling capacitor C_{C2} is $(R_o + R_L)$ & the time constant is

$$T_s = [R_o + R_L] C_{C2}$$

where $R_o = \text{o/p resistance}$

$$R_o = R_E \parallel \gamma_o \parallel \left\{ \frac{[\gamma_A + (R_S \parallel R_B)]}{1 + \beta} \right\}$$

NOTE - To find effect of one capacitor, assume all other capacitor as short circuited.

To find

B to E

Effective Resistance from Left terminal to Right \Rightarrow divide by (1.3)

Cap from Right (P) to Left term (B) \Rightarrow multiply by (1.3)

in fig 1.1 previous page (CS amp) (r)

Ex1) The ckt given is to be used as a simple audio amplifier.
Design a circuit such that the lower corner frequency

$$f_L = 20 \text{ Hz} \quad R_D = 5.7 \text{ k}\Omega \quad R_S = 5 \text{ k}\Omega \quad V_+ = 5 \text{ V} \\ R_G = 50 \text{ k}\Omega \quad R_L = 10 \text{ k}\Omega \quad V_- = -5 \text{ V}$$

Sol1) The corner freq in terms of time constant

$$f_L = \frac{1}{2\pi T_S}$$

$$T_S = (R_D + R_L) C_C \quad T_S = \frac{1}{2\pi f_L} = \frac{1}{2\pi (20)} = 7.96 \text{ ms}$$

$$C_C = \frac{T_S}{R_D + R_L} \quad (\because T_S = (R_D + R_L) C_C)$$

$$= 7.96 \times 10^{-3}$$

$$6.7 \times 10^3 + 10 \times 10^3$$

$$C_C = 4.77 \times 10^{-7} \text{ F}$$

$$\boxed{C_C = 0.477 \mu\text{F}}$$

Ex2) Calculate the corner frequency of a BJT CE ckt with a coupling capacitor. (fig 1.2 CE amp) (r)

The ckt parameters are: $R_1 = 51.2 \text{ k}\Omega$, $R_2 = 9.6 \text{ k}\Omega$

$R_C = 2 \text{ k}\Omega$, $R_E = 0.4 \text{ k}\Omega$, $R_{Si} = 0.1 \text{ k}\Omega$, $C_C = 1 \mu\text{F}$

$V_{CC} = 10 \text{ V}$, $V_{BE(on)} = 0.7 \text{ V}$, $\beta = 100$ & $V_A = \infty$ & $I_{CQ} = 1.81 \text{ mA}$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.81}{0.026} = 69.6 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 0.026}{1.81} = 1.44 \text{ k}\Omega$$

The f_{lp} Resistance

$$R_{lp} = R_1 || R_2 || [r_\pi + (1 + \beta) R_E]$$

$$\{ \text{ratio } R_1 : R_2 = 51.2 : 9.6 \Rightarrow [1.44 + (100)(0.4)] \text{ bny OT - STOU} \\ = 6.77 \text{ k}\Omega \quad \text{but this is not true as ratio is 20 ratios equal}$$

$$T_S = (R_{Si} + R_1) C_C$$

$$= (0.1 \times 10^3 + 6.77 \times 10^3) (1 \times 10^{-6})$$

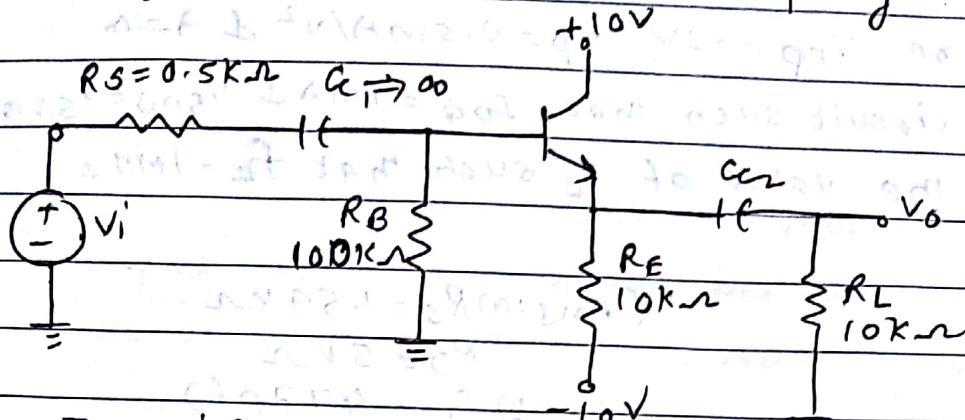
$$= 6.87 \times 10^{-3} \text{ s}$$

$$\boxed{T_S = 6.87 \text{ ms}}$$

$$\& f_L = \frac{1}{2\pi T_S} = 23.2 \text{ Hz}$$

$$\boxed{f_L = 23.2 \text{ Hz}}$$

(x3) Determine the 3dB frequency of an emitter follower amplifier circuit with an o/p coupling capacitor. fig(1.3)



Transistor parameters $\beta = 100$, $V_{BE(\text{CON})} = 0.7 \text{ V}$, $V_A = 120 \text{ V}$

The o/p coupling capacitance is $C_{c2} = 1 \mu\text{F}$, $I_{CQ} = 0.838 \text{ mA}$
 $r_o = 143 \text{ k}\Omega$

$$r_T = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 0.026}{0.838 \times 10^3} = 3.10 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.838 \times 10^{-3}}{0.026} = 32.2 \text{ mA/V}$$

$$R_o = R_E \parallel r_{o11} \left\{ \frac{[r_T + (R_S \parallel R_B)]}{1 + \beta} \right\}$$

$$= 10 \parallel 143 \parallel \left\{ \frac{[3.10 + (0.5 \parallel 100)]}{101} \right\}$$

$$\boxed{R_o \approx 35.5 \Omega}$$

$$T_s = [R_o + R_L]C_{c2}$$

$$= [35.5 + 10^4] (10^{-6})$$

$$\underline{T_s = 1 \times 10^{-2} \text{s}}$$

$$f_L = \frac{1}{2\pi T_s} = \frac{1}{2\pi (10^{-2})} = \underline{15.9 \text{ Hz}}$$

$$\boxed{f_L = 15.9 \text{ Hz}}$$

- E74) The PMOS ckt shown in fig ① (next page) has a load resistance $R_L = 10\text{ k}\Omega$. The transistor parameters are $V_{TP} = -2\text{ V}$, $k_p = 0.5\text{ mA/V}^2$ & $\gamma = 0$.
- H.W.
- ① Design the circuit such that $I_{DQ} = 1\text{ mA}$ & $V_{SDQ} = V_{SSQ}$
 - ② Determine the value of C_L such that $f_Z = 1\text{ MHz}$

$$(\text{Ans: a}) R_S = 1.59\text{ k}\Omega$$

$$R_D = 5\text{ k}\Omega$$

$$\text{b) } C_L = 47.7\text{ pF}$$

$$\left[\frac{E_{DD}(1 + \alpha)}{R_S + R_D} \right] \left(n_1, n_2, n_3, n_4 \right) = 1.9$$

$$\left[\frac{E_{DD}(1 + \alpha)}{R_S + R_D} \right] \left(n_1, n_2, n_3, n_4 \right) = 1.9$$

The corner frequency

$$f_L = \frac{1}{2\pi R_S} = \frac{1}{2\pi(6.87 \times 10^3)} = 23.2 \text{ Hz}$$

* Load capacitor Effects :-

The model of the load circuit input impedance is parallel with a resistance.

In addition there is a parasitic capacitance between ground and the line that connects the amplifier o/p to the load circuit.

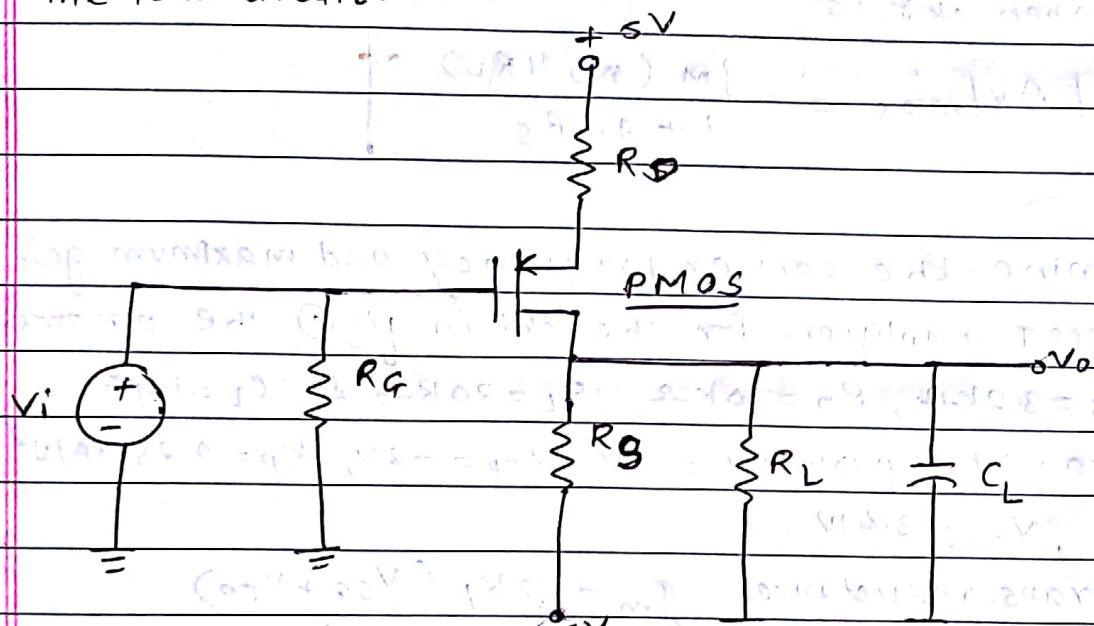


Fig ① MOSFET CS ckt with a load capacitor

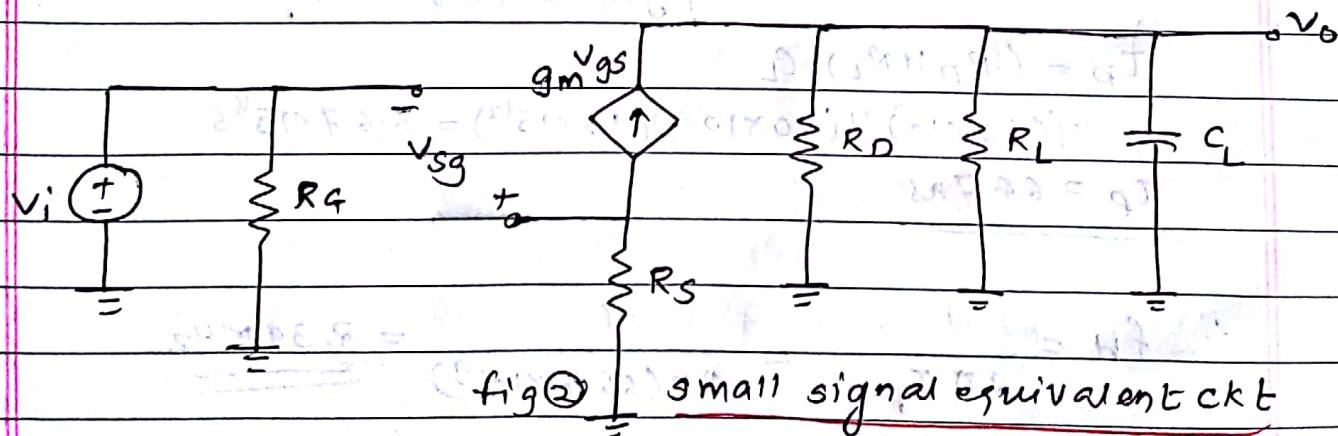


Fig ② small signal equivalent ckt

- fig ① shows a MOSFET CS amp with a load resistance R_L & load capacitance C_L connected to the o/p.

fig ② shows the small signal equivalent ckt.

o/p resist r_o is assumed to be infinite

- The equivalent resistance seen by the load capacitor C_L is $(R_D \parallel R_L)$

Since we set $V_i = 0$ then $g_m v_{gs} = 0$ which means that the dependent current source does not affect the equivalent resistance.

∴ The time constant for this ckt is

$$T_p = (R_D \parallel R_L) C_L$$

- The maximum gain, which is found by assuming C_L is an open ckt is

$$|AV|_{\max} = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_S}$$

Ex12 Determine the corner frequency and maximum gain of a MOSFET amplifier. For the ckt in fig①, the parameters are $R_S = 3.2\text{k}\Omega$, $R_D = 10\text{k}\Omega$, $R_L = 20\text{k}\Omega$ & $C_L = 10\text{pF}$.

The transistor parameters are $V_{TP} = -2\text{V}$, $K_p = 0.25\text{mA/V}^2$ & $\lambda = 0$, $V_{GSQ} = 3.41\text{V}$

2015

The transconductance

$$\begin{aligned} g_m &= 2K_p (V_{SG} + V_{TP}) \\ &= 2(0.25)(3.41 - 2) \\ &= 0.705\text{mA/V} \end{aligned}$$

$$T_p = (R_D \parallel R_L) \cdot C_L$$

$$= ((10 \times 10^3) \parallel (20 \times 10^3)) (10 \times 10^{-12}) = 6.67 \times 10^{-8}\text{s}$$

$$T_p = 66.7\text{ns}$$

$$\therefore f_H = \frac{1}{2\pi T_p} = \frac{1}{2\pi(66.7 \times 10^{-9})} = 2.39\text{MHz}$$

$$|AV|_{\max} = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_S} = \frac{0.705(10 \parallel 20)}{1 + (0.705 \times 3.2)} = 1.44$$

$$|AV|_{\max} = 1.44$$

Next Example $\oplus \otimes$ (previous page)

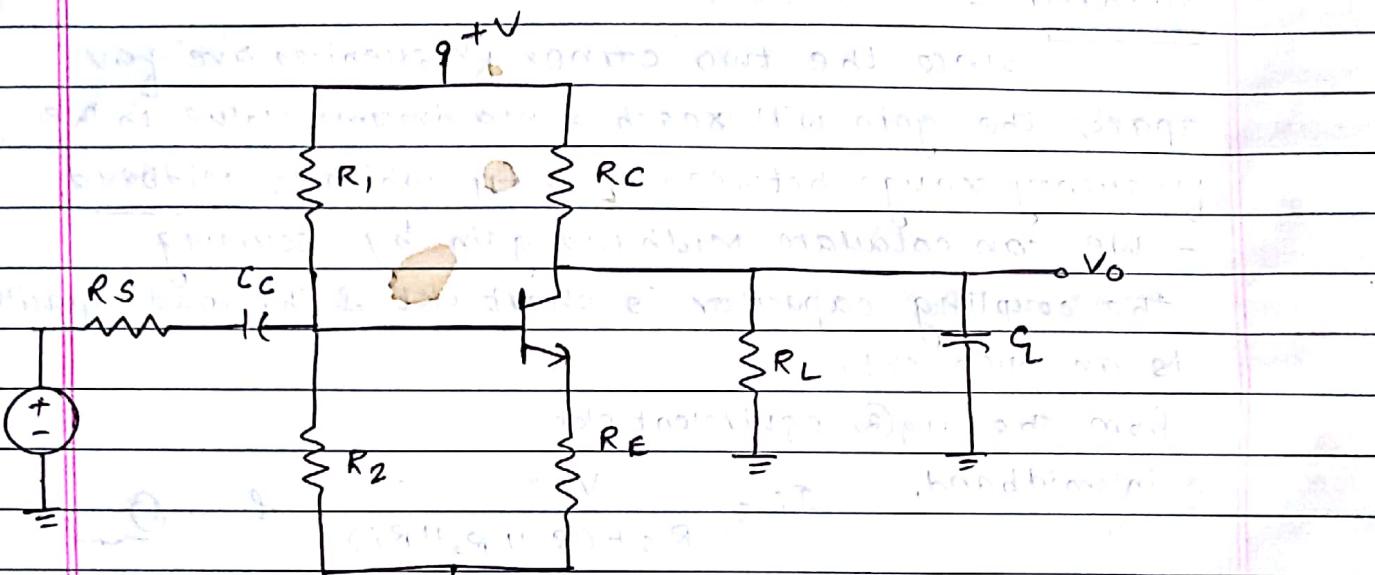
At LF, we treat load capacitor as O.C. $\Rightarrow T_S \Rightarrow$ T.C. associated with C_L
 At HF, we treat coupling capacitor as S.C. $\Rightarrow T_P = T.C.$ associated with C_P

PAGE NO. _____
 DATE _____

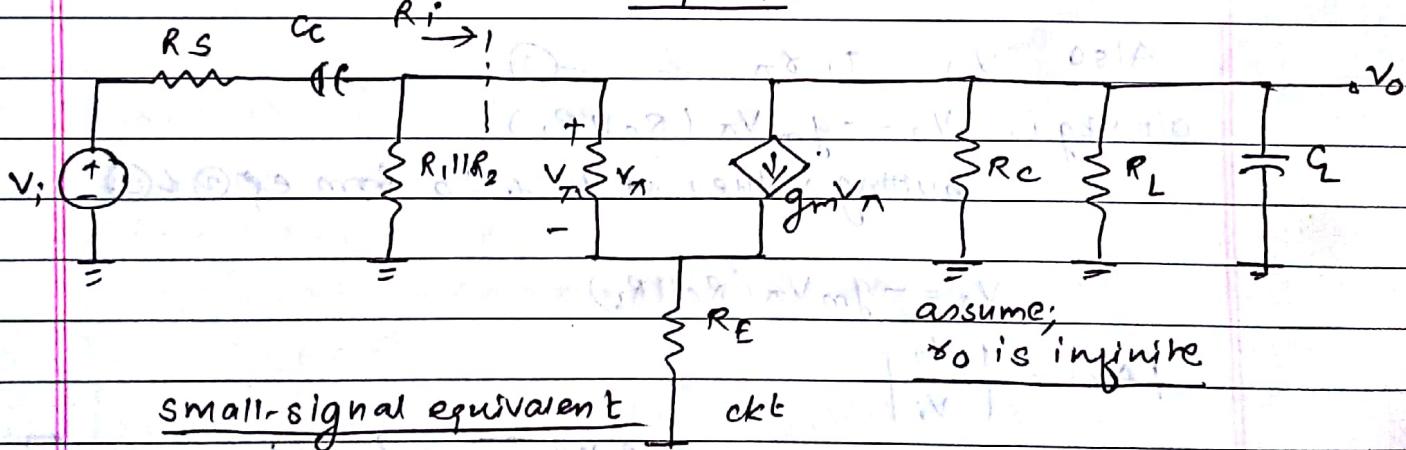
$T_S = (R_s + R_p) C_L$
 $T_P = (C_R || R_p) C_P$ (31)

* Coupling and Load capacitors :-

- since the values of the coupling capacitance & load capacitance differ by orders of magnitude, the corner frequencies are far apart & can be treated separately.



ckt with both a coupling & a load capacitor.



small-signal equivalent

ckt

- The lower corner frequency f_L is given by $f_L = \frac{1}{2\pi T_S}$

Where $T_S = \text{time constant associated with } C_C$

$$T_S = [R_s + (R_1 || R_2 || R_f)] C_C$$

$$\text{Where } R_f = r_o + (1 + \beta) R_E$$

(T_S is equivalent resistance associated with the coupling capacitor by setting signal source to zero.)

- The upper corner frequency f_H is given by

$$f_H = \frac{1}{2\pi R_p C_L}$$

$$T_p = (R_C || R_L) C_L = \text{Time constant related to } Q$$

Midband :-

since the two corner frequencies are far apart, the gain will reach a maximum value in the frequency range between f_L & f_H which is Midband.

- We can calculate midband gain by assuming the coupling capacitor is short ckt & the load capacitor is an open ckt.

from the fig②, equivalent ckt

$$\text{in midband, } I_i = \frac{V_i}{R_s + (R_1 || R_2 || R_i)} \quad \text{--- (1)}$$

$$I_b = \left(\frac{R_1 || R_2}{(R_1 || R_2) + R_i} \right) I_i \quad \text{--- (2)}$$

$$\text{Also } V_A = I_b r_n \quad \text{--- (3)}$$

$$\text{O/p vbg is } V_o = -g_m V_A (R_C || R_L)$$

putting values of V_A & I_b from eqn② & ③

$$V_o = -g_m V_A (R_C || R_L)$$

$$|AV| = \left| \frac{V_o}{V_i} \right|$$

$$= g_m r_n (R_C || R_L) \left(\frac{R_1 || R_2}{(R_1 || R_2) + R_i} \right) \left(\frac{1}{R_s + (R_1 || R_2 || R_i)} \right) \quad \text{--- (4)}$$

* Determine the midband gain, corner frequencies and bandwidth of a circuit containing both a coupling capacitor & a load capacitor.

Consider the ckt in fig 9) with transistor parameters

$$V_{BE(ON)} = 0.7V, \beta = 100, \& V_A = \infty, I_{CQ} = 0.99mA$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.99}{0.026} = 38.1 \text{ mA/V}$$

$$\gamma_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.99} = 2.63 \text{ k}\Omega$$

Input resistance R_i :

$$R_i = \gamma_\pi + (1+\beta) R_E = 2.63 + (101)(0.5) = 53.1 \text{ k}\Omega$$

from Eq "④",

$$\begin{aligned} |AV|_{\max} &= \left| \frac{V_o}{V_i} \right|_{\max} \\ &= g_m \gamma_\pi (R_C || R_L) \left(\frac{R_1 || R_2}{(R_1 || R_2) + R_i} \right) \left(\frac{1}{[R_s + (R_1 || R_2) || R_i]} \right) \\ &= (38.1)(2.63)(5)(10) \left(\frac{40115.7}{(40115.7) + 53.1} \right) \left(\frac{1}{[0.1 + (40115.7 || 53.1)]} \right) \end{aligned}$$

$$|AV|_{\max} = 6.16$$

$$T_s = (R_s + R_1 || R_2 || R_i) C_C$$

$$= (0.1 \times 10^3 + (5.7 \times 10^3)) || (40 \times 10^3 || 53.1 \times 10^3) (10 \times 10^{-6})$$

$$= 4.46 \times 10^{-2} \text{ s}$$

$$T_s = 4.46 \text{ ms}$$

$$T_p = (R_C || R_L) C_L$$

$$= ((5 \times 10^3) || (10 \times 10^3)) (15 \times 10^{-12})$$

$$= 5 \times 10^{-8} \text{ s}$$

$$T_p = 5 \text{ ns}$$

$$f_L = \frac{1}{2\pi T_s} = \frac{1}{2\pi (4.46 \times 10^{-3})} = 3.42 \text{ Hz}$$

$$f_H = \frac{1}{2\pi T_p} = \frac{1}{2\pi (5 \times 10^{-9})} = 3.18 \text{ MHz}$$

$$\begin{aligned} \text{BW} &= f_H - f_L \\ \text{BW} &\approx 3.18 \text{ MHz} \end{aligned}$$

* Frequency Response of BJT Amplifier:-



Fig ① - f_H & f_C (on log scale)

by making f_L

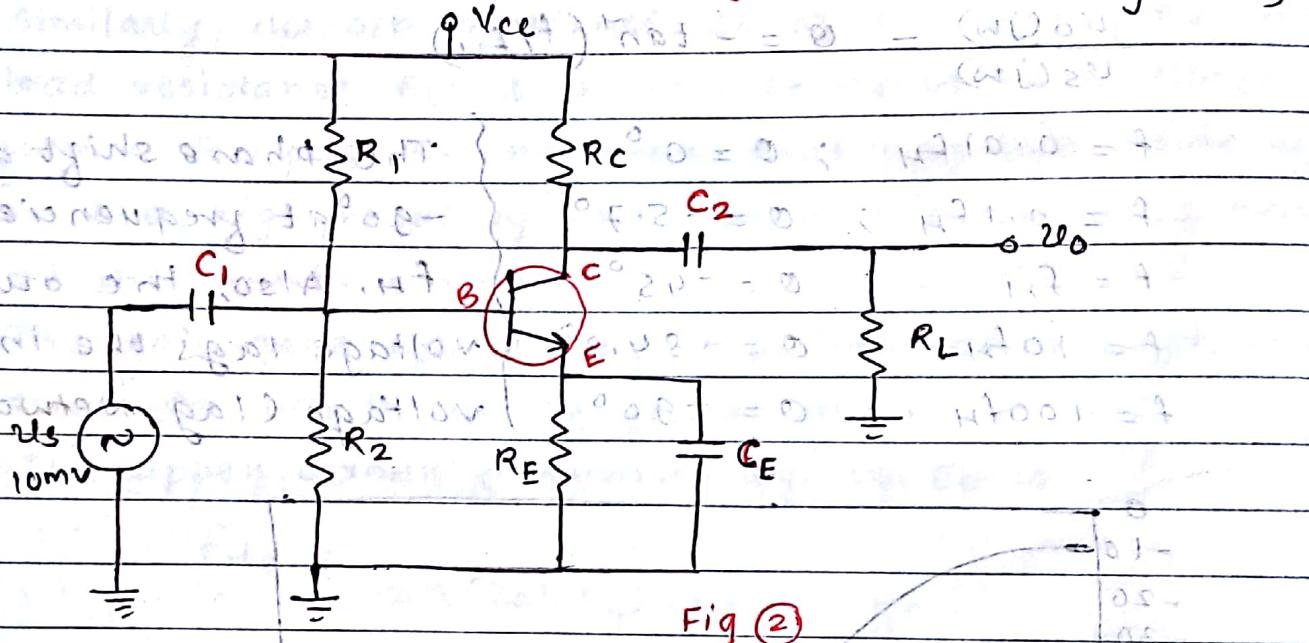


Fig ②

* In HF & MF region, short circuit the capacitors.

$$X_{C_1} = \frac{1}{2\pi f C_1} \quad 2\pi f \text{ & } C_1 \text{ are constant}$$

$$\downarrow X_{C_1} \propto \frac{1}{f}$$

- ~~short circuit the capacitors at high frequency~~

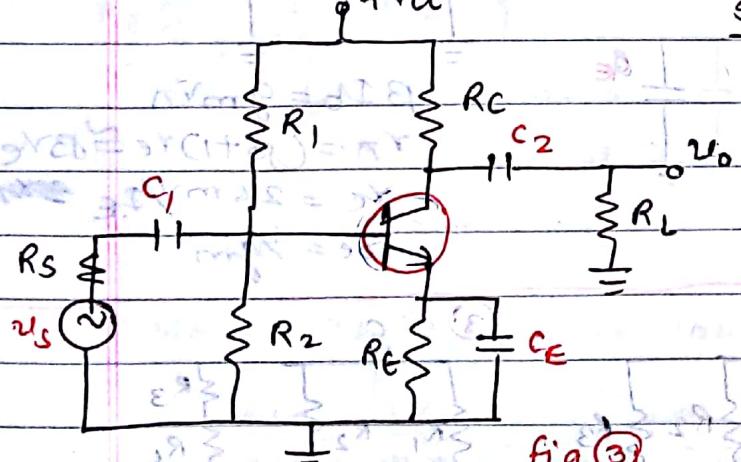
* In HF region, coupling & bypass capacitor are short circuited. & Av decreases due to junction capacitance, exist in the f_H .

- In LF region, gain increases due to coupling & bypass capacitor, as we know $X_C \propto 1/f$
ie AS $f \uparrow$, $X_C \downarrow$ & then more & more current flow

through the i/p terminal and o/p current increased &
 \therefore o/p v_{tg} increases, ie A_v also increase.

- At Freq. f_L (lower cut off freq), coupling & bypass capacitors will become short.
- The frequency region between lower cut off freq (f_L) & upper cut off freq (f_H) is called mid frequency region.
- At **MF**, these capacitors will remain short circuited.
 so a constant i/p current will flow through the amplifier
 so o/p current constant & o/p v_{tg} remains constant.
 ie v_{tg} gain remains constant in MF region.
- At a freq f_H (Upper cut off freq), the junction and parasitic capacitance will exist.
- At beyond f_H , as freq increases, in HF region,
 gain decreases due to this parasitic capacitance.

* Low Frequency Analysis of BJT :-



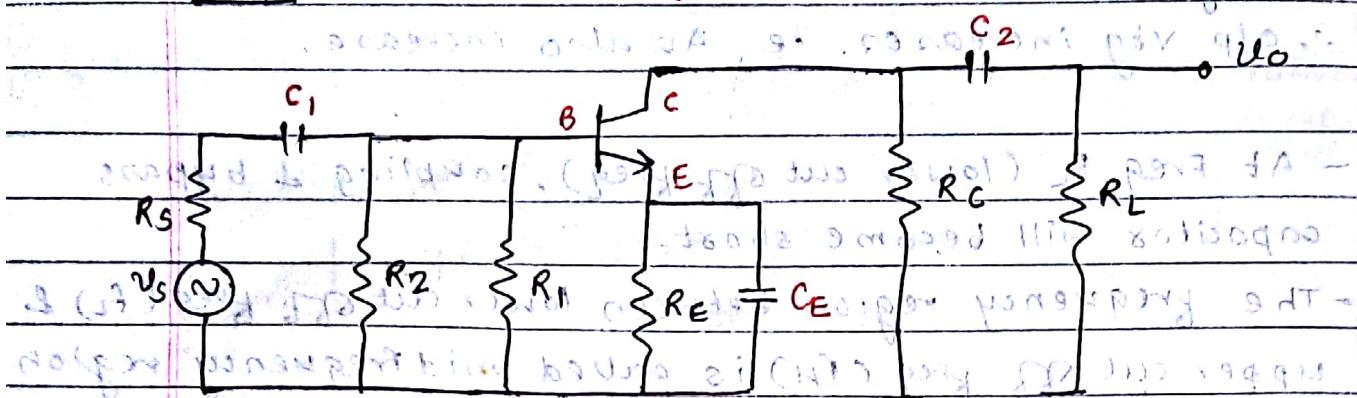
- 1: connect +Vcc to GND ckt.
- 2: draw the small signal model for BJT (with coupling & bypass capacitors.)
- 3: connect appropriate components to appropriate terminal of BJT.
- 4: To find effect of one capacitor assume all other capacitor as shorted.

Fig ③

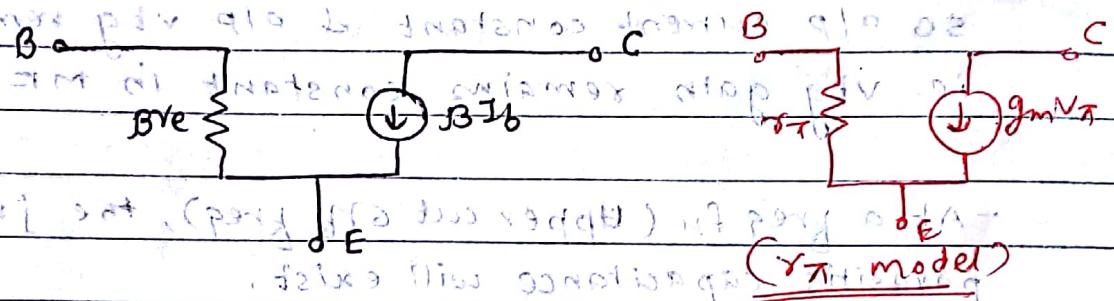
* Hybrid- π Model can be quite accurate for L-F ckt & can easily be adopted for higher freq ckt with addition of appropriate inter-electrode capacitance & other parasitic elements.

PAGE No.

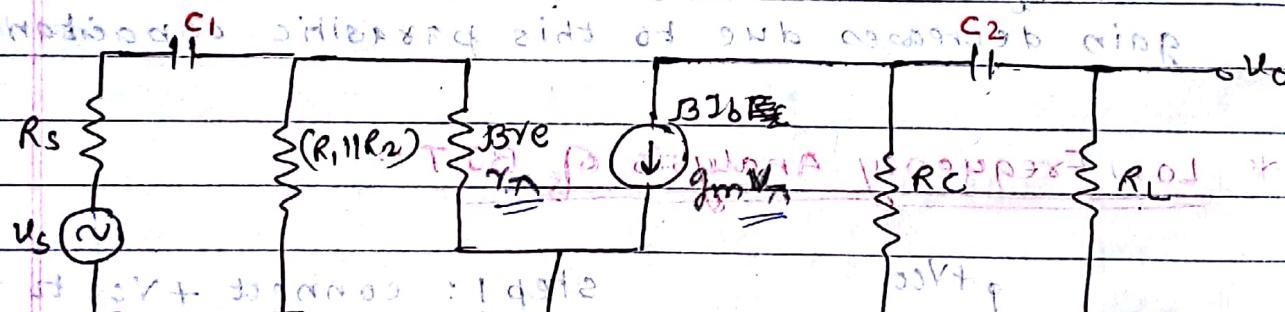
Step①: diagram ③ / fig ③



Step②: small signal model (using π) for CE amplifier

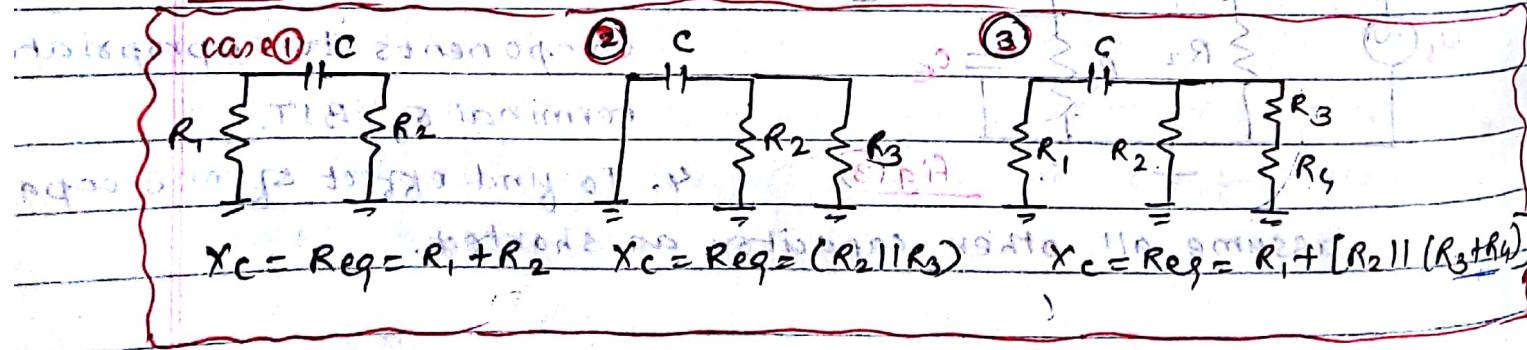


Step③: connect appropriate components to appropriate terminals.



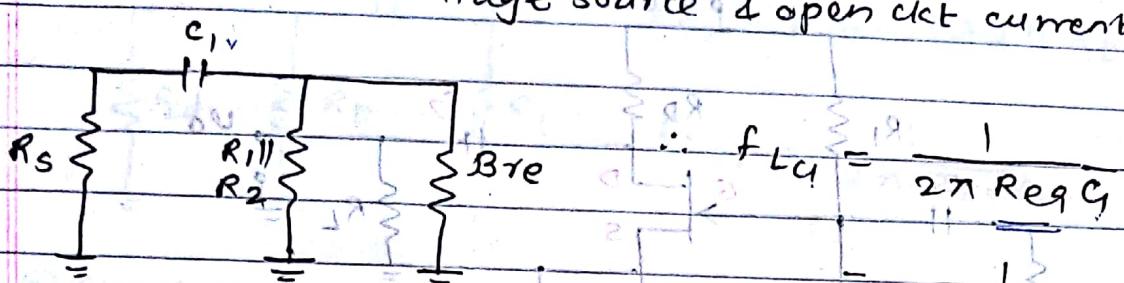
$$\text{Compensation gain } \beta = \frac{\beta_{1e}}{\gamma_e} \quad \beta_{1e} = g_m V_A \quad \gamma_e = 26 mV/2E \quad \beta_{1e} = 1/g_m$$

Note: β_{1e} is determined by E



Step ④: Effect of C_1

- short ckt voltage source & open ckt current source.

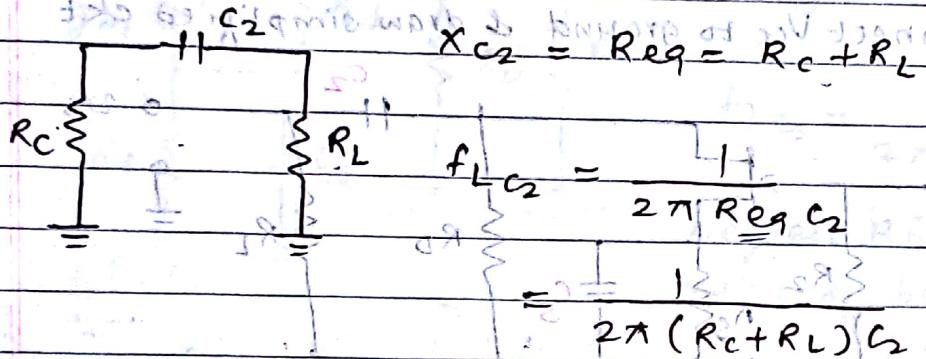


$$f_{LC1} = \frac{1}{2\pi R_{eq} C_1}$$

$$X_{C1} = R_{eq} = R_S + [R_1 || R_2 || BYe]$$

$$R_{eq} = R_S + [R_B || R_{ib}] \text{ where } R_{ib} = r_A + (1+\beta)R_E$$

⑤ Effect of C_2

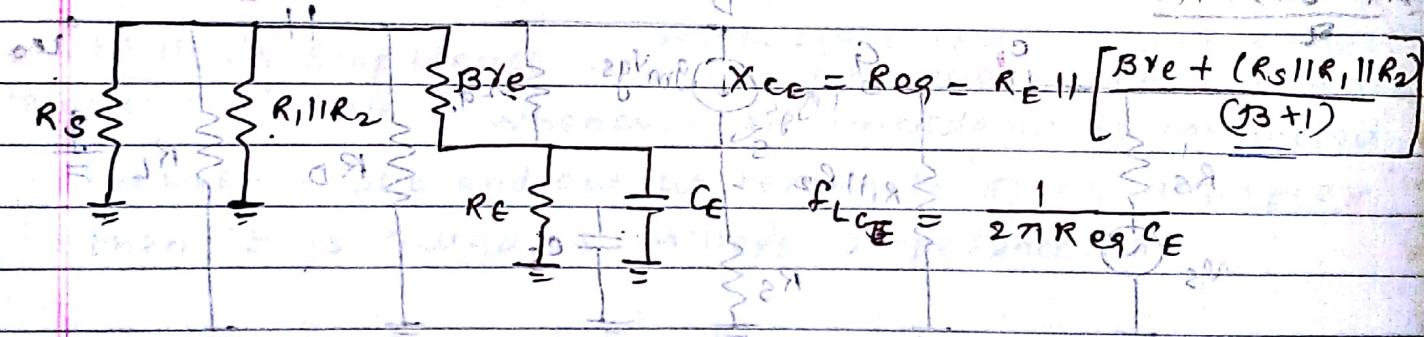


$$X_{C2} = R_{eq} = R_C + R_L$$

$$f_{LC2} = \frac{1}{2\pi R_{eq} C_2}$$

$$= \frac{1}{2\pi (R_C + R_L) C_2}$$

⑥ Effect of C_E



$$X_{CE} = R_{eq} = R_E || \left[BYe + (R_S || R_1 || R_2) \right] \frac{1}{(j\beta + 1)}$$

$$f_{LC_E} = \frac{1}{2\pi R_{eq} C_E}$$

Note: The effective lower cut off frequency among f_{LC1} , f_{LC2} & f_{LC_E} is the highest frequency among those three.

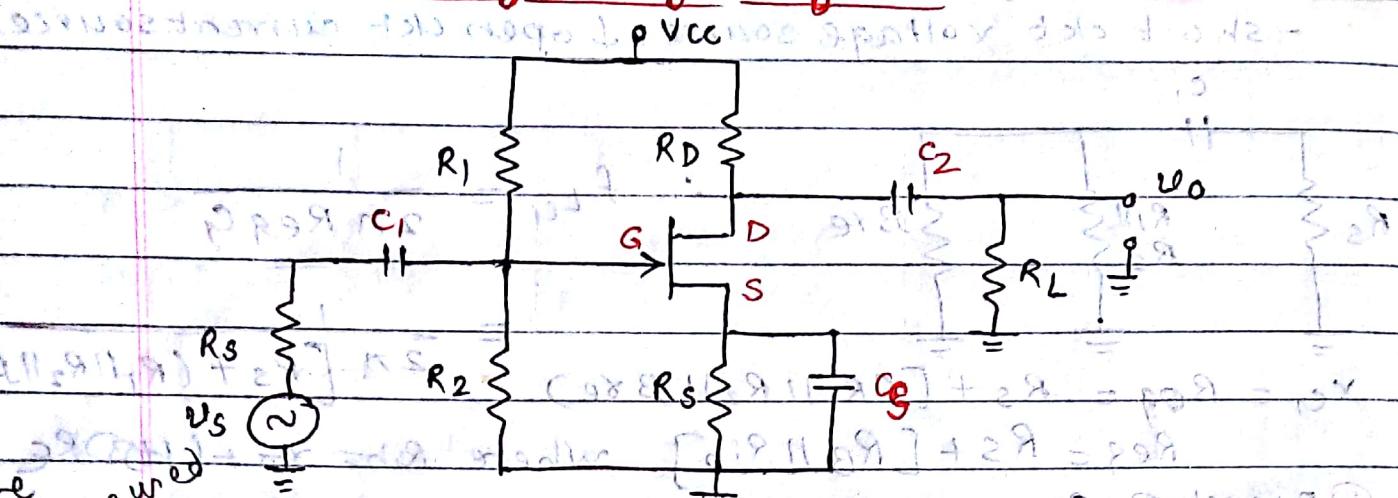
$$f_L = \text{Highest}(f_{LC1}, f_{LC2}, f_{LC_E})$$

$$\Delta V_{cat}(at f=f_L) = \frac{A_{Vm id}}{\sqrt{2}}$$

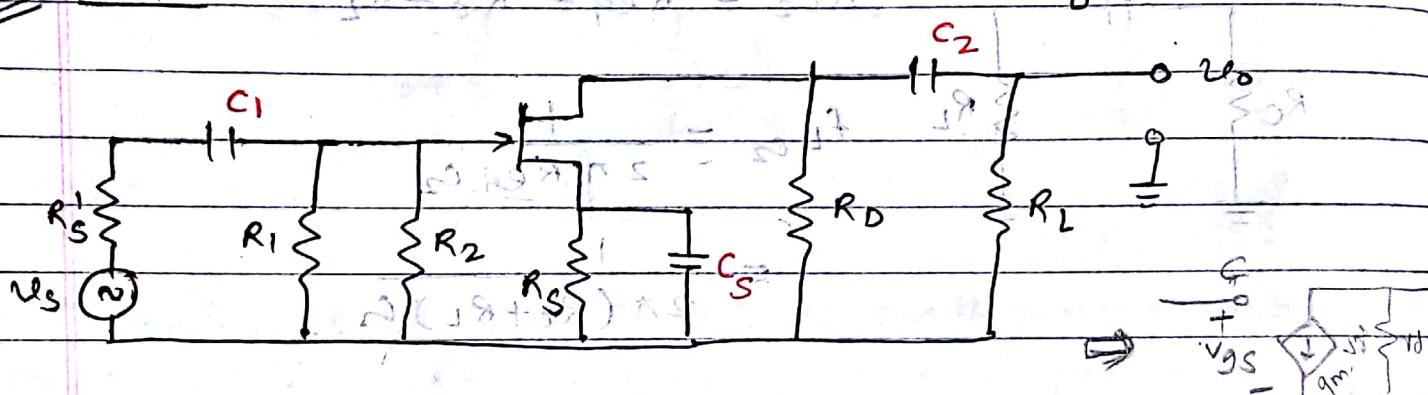
$$\Delta V_{cat(Any LF)} = \frac{A_{Vm id}}{\sqrt{1 + (f_L/f)^2}}$$

$$A_{Vm id} = -\frac{(R_C || R_L)}{r_e} \quad A_{Vm id} = -g_m (R_C || R_L)$$

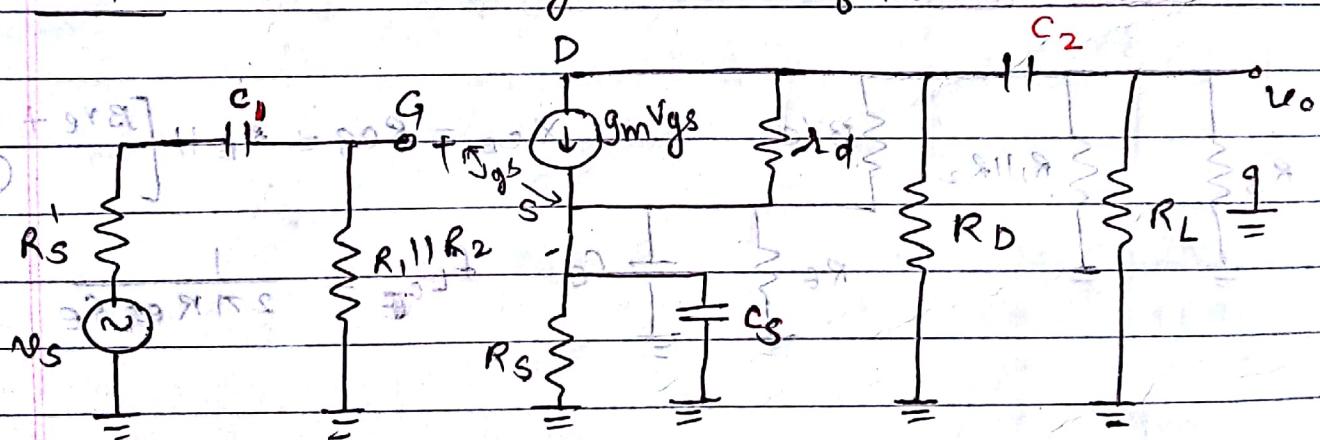
* Low Frequency Analysis of FET:-



Step①: connect V_{CC} to ground & draw simplified ckt



Step②: draw small signal model of FET



Step③: @ effect of C_D - small signal effect : state

$$R_{eq} = R_s + (R_1 || R_2)$$

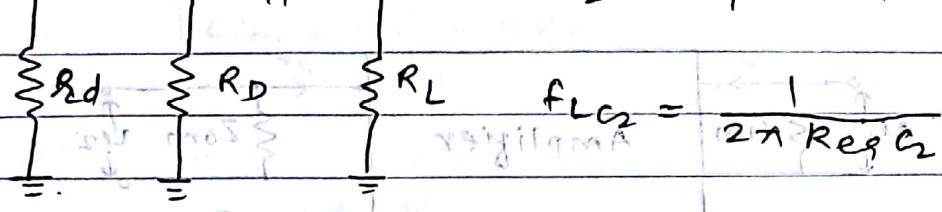
$$f_{LC} = \frac{1}{2\pi R_{eq} C_D}$$

* In HF region, capacitance of importance are interelectrode capacitor & wiring capacitor

15

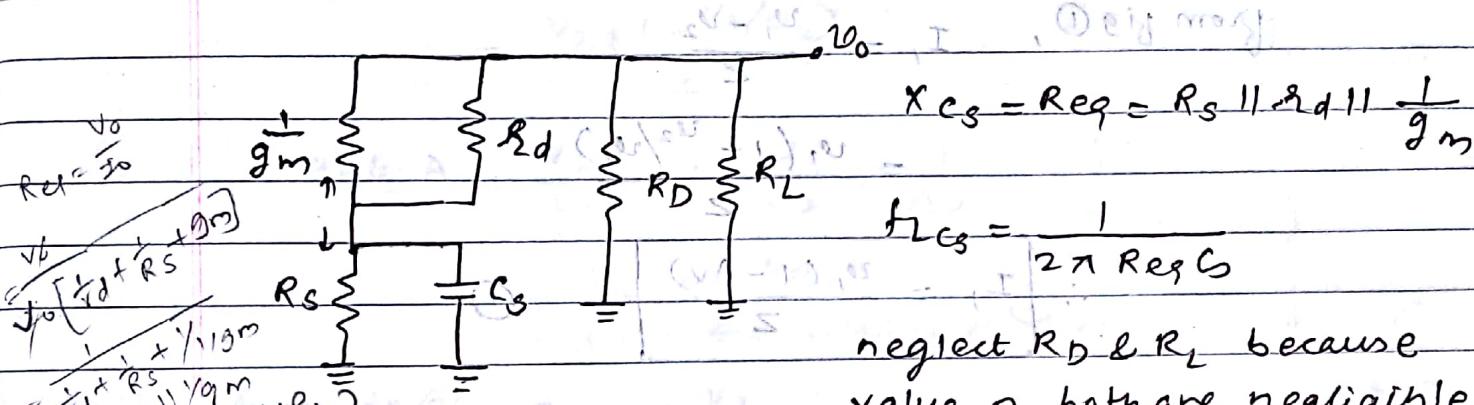
④ Effect of C_2 :-

$$X_{C_2} = R_{eq} = (R_d || R_D) + R_L$$



$$f_{LC_2} = \frac{1}{2\pi R_{eq} C_2}$$

⑤ Effect of C_3 :-



neglect R_D & R_L because

value of both are negligible.

effective lower cut off freq is the highest freq component among f_{Ld} , f_{LC} and f_{LC_3} .

* Miller's Impedance :- Miller effect idea is used to convert an impedance across an amplv into two separate ones at IIP & OIP. whenever an impedance is connected between input and output terminals of an amplifier then it is called as Miller's Impedance.

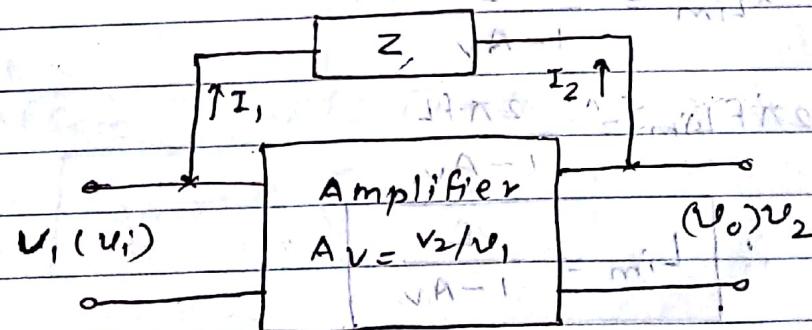


Fig ①

u_i, I_1 is IIP vbg & IIP current resp.

u_o, I_2 is OIP vbg & OIP current resp.

(VA - 1)

Z_{im} : Miller i/p impedance
 Z_{om} : Miller o/p impedance

PAGE NO.	
DATE	

The miller(effect) impedance affect on i/p side (Z_{im}) & o/p side (Z_{om}) as shown in fig below.

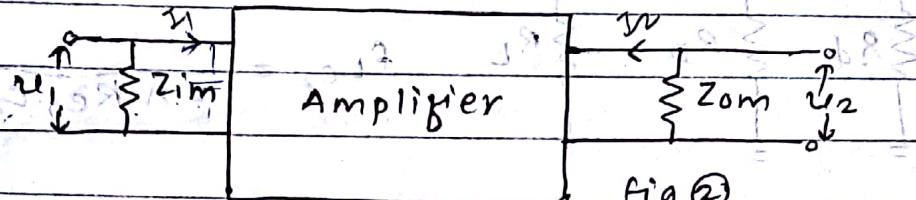


Fig ②

To find Z_{im} :-

From fig ①, $I_i = \frac{u_1 - u_2}{Z}$

$$= \frac{u_1 (1 - \frac{u_2}{u_1})}{Z}$$

$$\therefore I_i = \frac{u_1 (1 - A_v)}{Z}$$

from fig ②, $Z_{im} = \frac{u_1}{I_i}$ put value of I_i from ①

$$Z_{im} = \frac{u_1}{\frac{u_1 (1 - A_v)}{Z}}$$

$$Z_{im} = \frac{Z}{1 - A_v}$$

For Inductive Reactance instead of Z :-

$$x_{lim} = \frac{x_L}{1 - A_v}$$

$$2\pi f L_{lim} = \frac{2\pi f L}{1 - A_v}$$

$$\therefore L_{lim} = \frac{x_L}{1 - A_v}$$

For Capacitive Reactance :-

$$x_{cim} = \frac{x_C}{1 - A_v}$$

$$2\pi f C_{im} = \frac{V_2 - V_1}{Z}$$

$$\therefore C_{im} = C \left(1 - \frac{V_1}{V_2} \right)$$

To find Z_{om} (from fig ①)

$$I_2 = \frac{V_2 - V_1}{Z}$$

$$= \frac{V_2 \left(1 - \frac{V_1}{V_2} \right)}{Z}$$

$$\text{But } A_V = \frac{V_2}{V_1} \quad \therefore \frac{V_1}{V_2} = \frac{1}{A_V}$$

$$\therefore I_2 = \frac{V_2 \left(1 - \frac{1}{A_V} \right)}{Z} \quad \text{②}$$

from fig ②, $Z_{om} = \frac{V_2}{I_2}$ put value of I_2 from eqn ②,

$$Z_{om} = \frac{V_2}{\frac{V_2 \left(1 - \frac{1}{A_V} \right)}{Z}}$$

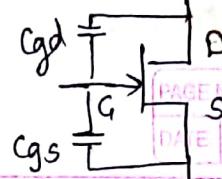
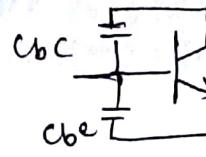
$$Z_{om} = \frac{Z}{\left(1 - \frac{1}{A_V} \right)}$$

For Capacitive Reactance -

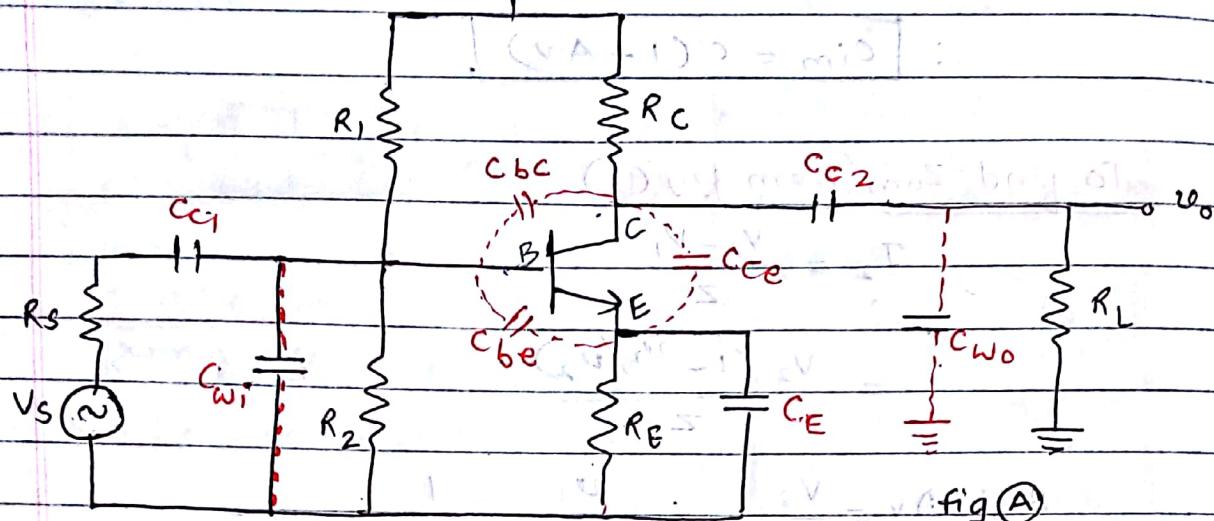
$$X_{com} = \frac{X_C}{\left(1 - \frac{1}{A_V} \right)}$$

$$2\pi f C_{com} = \frac{1}{2\pi f C \left(1 - \frac{1}{A_V} \right)}$$

$$\therefore C_{com} = C \left(1 - \frac{1}{A_V} \right)$$



* High Frequency Analysis of BJT :-



C_{wi} = i/p wiring capacitor

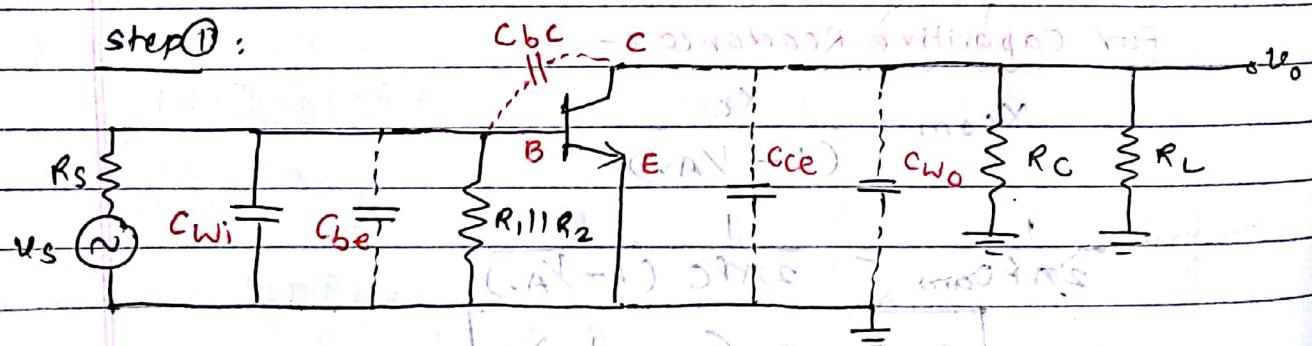
C_{wo} = o/p wiring capacitor

Step① : short det coupling & bypass capacitors & connect +Vcc to ground, and redraw simplified det.

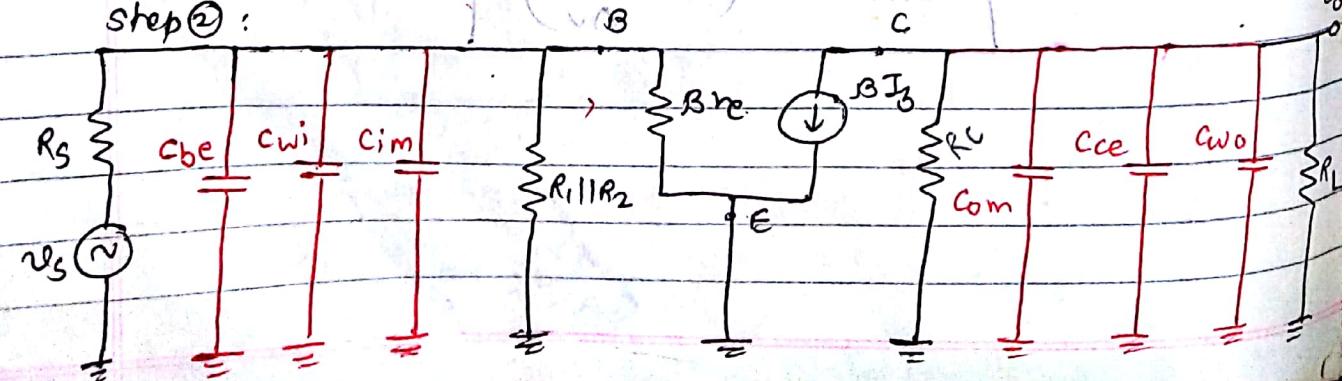
Step② : draw the small signal model for BJT & connect appropriate components to appropriate terminals.

Step③ : calculate F_{HI} & F_{HO} for i/p & o/p capacitors resp.

Step① :



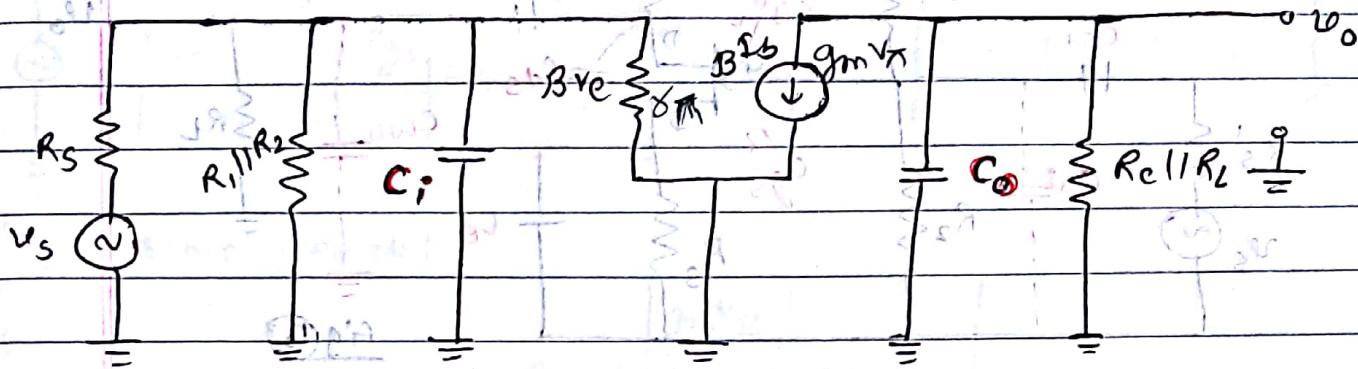
Step② :



where $C_{im} = C_{bc} (1 - \frac{1}{A_v})$

$$C_{im} = C_{bc} (1 - \frac{1}{A_v})$$

simplified ckt :-



where $C_i = C_{im} + C_{be} + C_{wi}$ (parallel)

$$C_o = C_{ce} + C_{wo} + C_{om}$$

step ③ :-

a) effect of C_i :-

$$f_{H1} = \frac{1}{2\pi \cdot R_{eq} \cdot C_i}$$

$$\text{where } C_i = C_{im} + C_{be} + C_{wi} \quad \& \quad C_{im} = (1 - A_v) C_{bc}$$

To find R_{eq} , short ckt vs.

$$\therefore R_{eq} = R_s \parallel (R_1 \parallel R_2) \parallel B \cdot r_e$$

b) effect of C_o :-

$$f_{H2} = \frac{1}{2\pi R_L \cdot C_o}$$

$$\text{where } C_o = C_{ce} + C_{wo} + C_{om} \quad \& \quad C_{om} = C_{bc} (1 - \frac{1}{A_v})$$

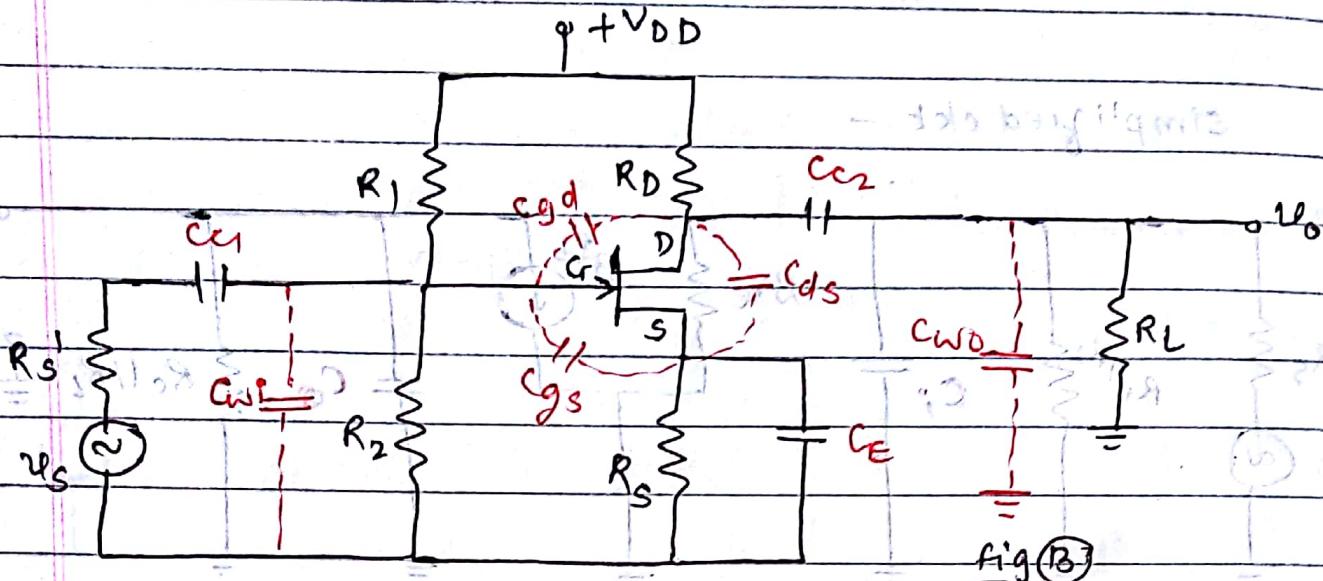
To find R_{eq} , open ckt current source.

$$\therefore R_{eq} = R_L \parallel R_L$$

Note: Effective higher cut off freq betw f_{H1} & f_{H2} is the lowest freq component betw f_{H1} & f_{H2} .

$$A_v (\text{at } f=f_{H1}) = \frac{A_{vmid}}{\sqrt{2}} \quad A_v (\text{at any high freq}) = \frac{A_{vmid}}{\sqrt{1 + (f/f_{H1})^2}}$$

* High frequency Analysis of FET (BJT) amplifier

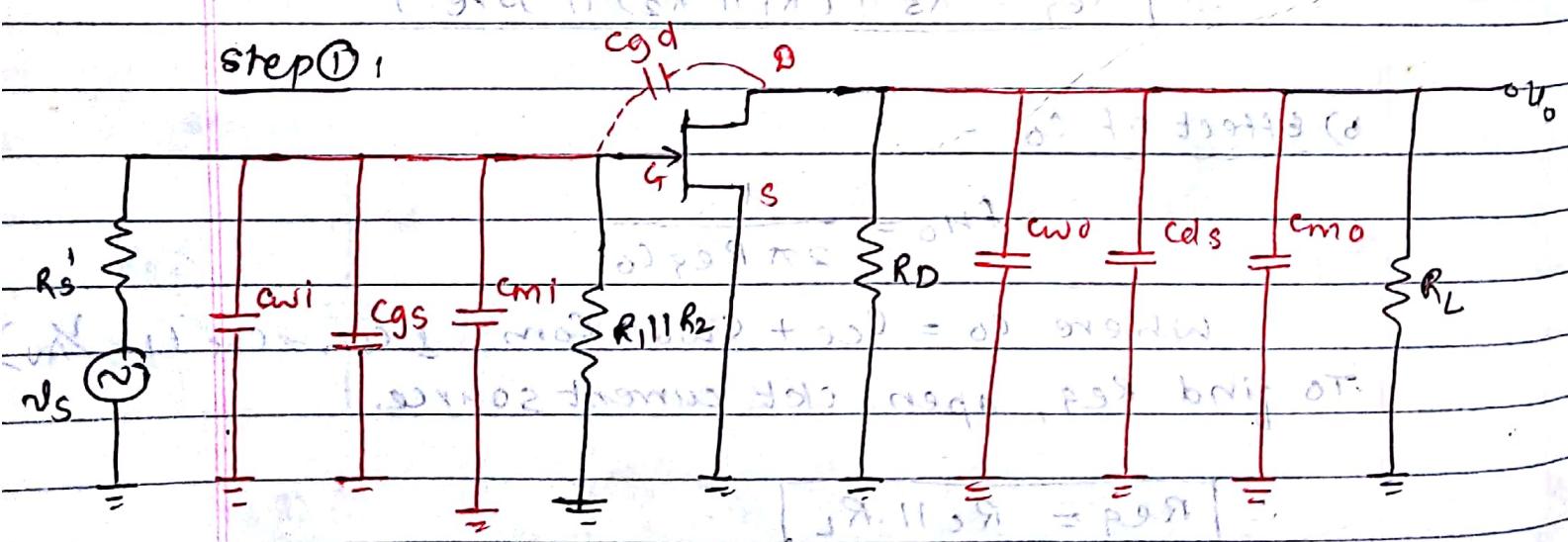


fig(B)

Step ①: short ckt coupling & bypass capacitors.
connect +VDD to ground & draw simplified ckt.

Step ②: Draw small signal model for FET & connect appropriate components to appropriate terminals.

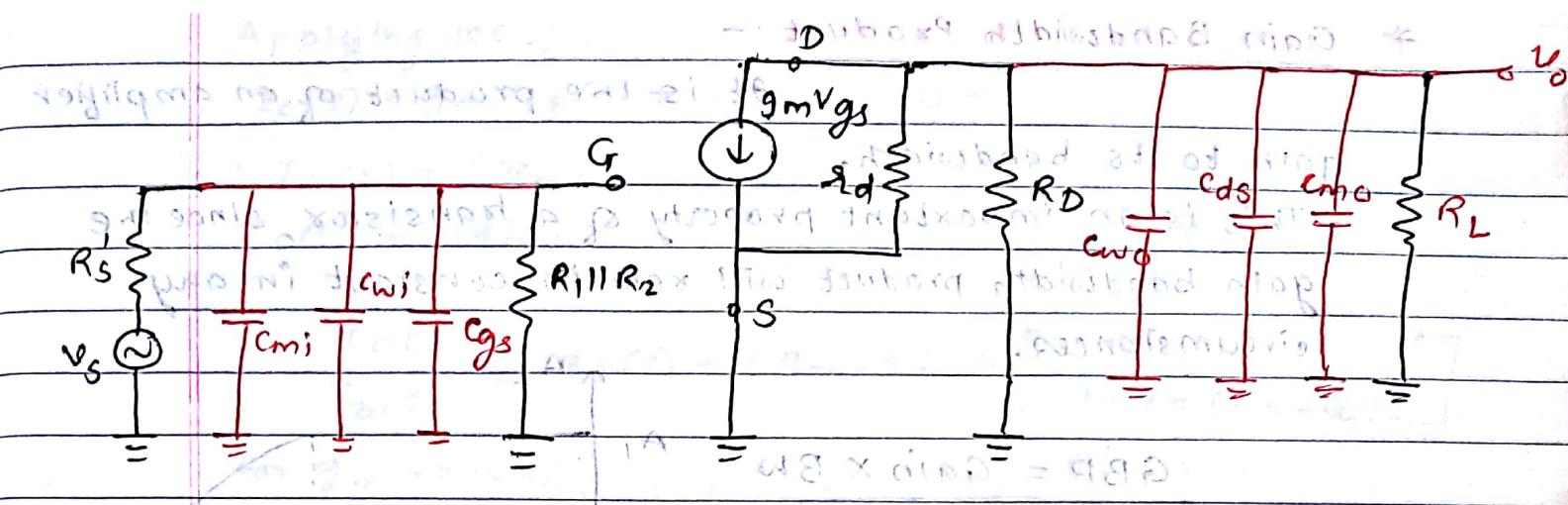
Step ③: find f_Hi & f_Ho corresponding to C_m & C_o resp.



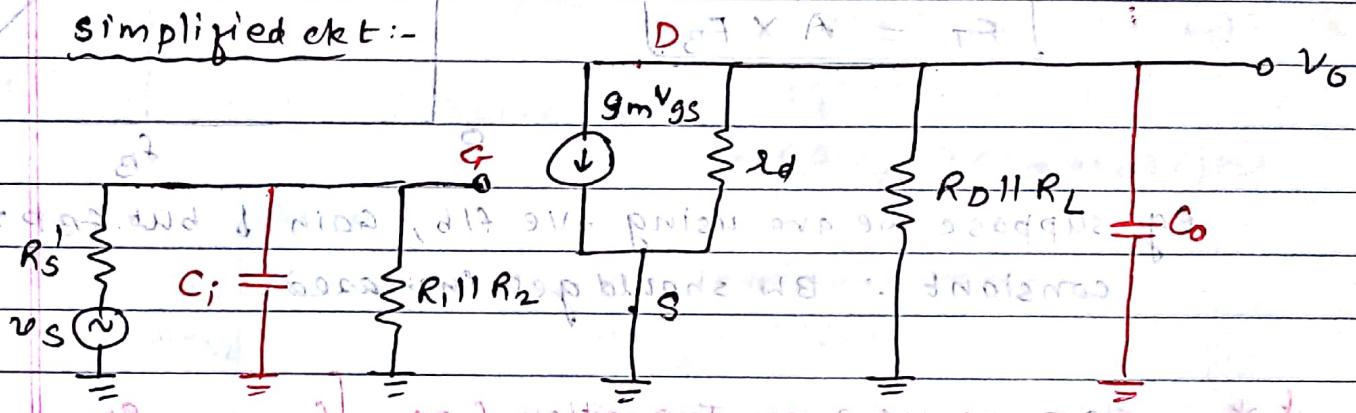
Step ④: small signal model - find equivalent circuit

$$C_{mi} = C_{gd} (1 - A_v)$$

$$C_{mo} = C_{gd} (1 - V_A v)$$



simplified ckt :-



$$\text{where } C_i = C_{mi} + C_{wi} + C_{gs}$$

→ and so $C_o = C_{mo} + C_{wo} + C_{ds}$ is parallel.

→ so f_{hi} priority may be resistances not parasitics

step II (a) Effect of C_i :-

$$f_{hi} = \frac{1}{2\pi R_{eq} C_i} \quad \text{where } C_i = C_{mi} + C_{wi} + C_{gs}$$

or To find R_{eq} , short ckt w.r.t. source (v_s).

$$R_{eq} = R_s' || R_1 || R_2$$

(a) Effect of C_o :-

$$f_{ho} = \frac{1}{2(\omega^2 + R_o^2) \times \pi \times 2\pi R_{eq} C_o} \quad \text{where } C_o = C_{mo} + C_{ds} + C_{wo}$$

To find R_{eq} , open ckt current source.

$$R_{eq} = r_d || R_D || R_L$$

$$f_{hi} \Rightarrow \text{lowest}(f_{hi} & f_{ho})$$

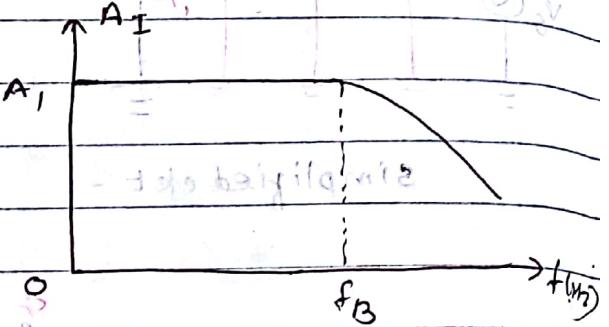
* Gain Bandwidth Product :-

It is the product of an amplifier gain to its bandwidth.

- This is an important property of a transistor, since the gain bandwidth product will remain constant in any circumstances.

$$GBP = \text{Gain} \times \text{BW}$$

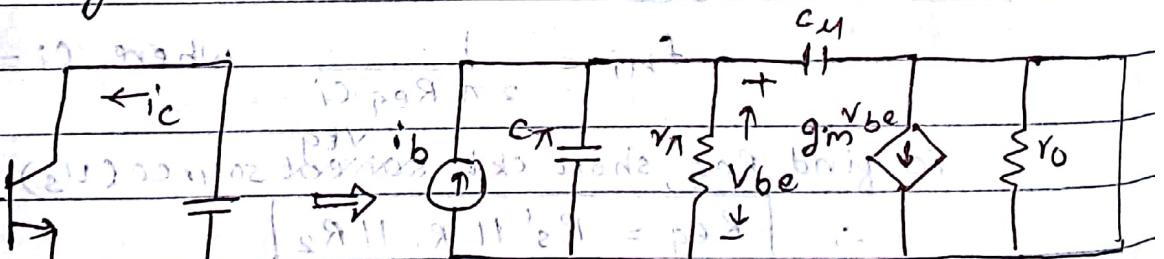
$$f_T = A \times f_B$$



e.g. suppose we are using f_B , gain ↓ but GBP remains constant \therefore BW should get increased.

Proof \Rightarrow $GBP \text{ or } UGB \text{ or Transition freq } = f_T = \frac{g_m}{2\pi(C_n + C_L)}$

Proof: applying a test current i_b to the base and short circuiting the collector for ac, for finding short circuit current gain B_f



$$V_{be}(s) = \left[r_n \frac{1}{(C_n + C_L)s} \right] I_b(s)$$

$$= \left[r_n \times \frac{1}{(C_n + C_L)s} \right] I_b(s)$$

$$= \left[r_n \times \frac{1}{(C_n + C_L)s} \right] I_b(s)$$

$$= \left[\frac{r_n}{1 + r_n(C_n + C_L)s} \right] I_b(s)$$

Applying KCL,

$$I_C(s) - g_m V_{BE}(s) + s C_U V_{BE}(s) = 0$$

$$\therefore I_C(s) = (g_m - s C_U) V_{BE}(s)$$

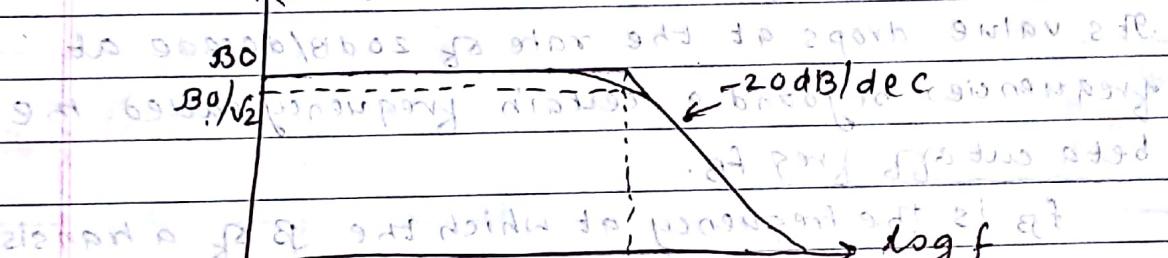
$$I_C(s) = (g_m - s C_U) \left[\frac{r_\pi}{1 + r_\pi (C_\pi + C_U)s} \right] I_B(s)$$

$$\therefore \frac{I_C(s)}{I_B(s)} = \beta_f(s) = (g_m - s C_U) \left[\frac{r_\pi}{1 + r_\pi (C_\pi + C_U)s} \right]$$

$$\text{for } g_m \gg w C_U, \beta_f(j\omega) = \frac{g_m r_\pi}{1 + r_\pi (C_\pi + C_U)(j\omega)}$$

$$\beta_f(j\omega) = \frac{\beta_0}{1 + (\beta_0/g_m)(C_\pi + C_U)(j\omega)}$$

β_0 is called current gain at zero frequency.



∴ current gain β_f is a function of frequency w as
when freq increases, current gain decreases at a rate of -20 dB per decade. (Low pass characteristics)

$$\text{At } 3 \text{ dB freq } \beta_f(j\omega_B) = \frac{\beta_0}{1 + j1} \quad \text{where } \beta_0 \approx \beta_f, \text{ low freq current gain.}$$

$$\therefore \beta_f(C_\pi + C_U) \omega_B = 1$$

$$\therefore \omega_B = 2\pi f_B = \frac{g_m}{\beta_0 (C_\pi + C_U)} \quad \text{ie } f_B = \frac{g_m}{2\pi \beta_0 (C_\pi + C_U)}$$

∴ current gain will be infinity when $| \beta_f(j\omega) | = 1$

$$\therefore UGB = w_T = \frac{g_m}{2(C_\pi + C_U)} \text{ rad/sec or } f_T = \frac{g_m}{2\pi(C_\pi + C_U)} \text{ Hz}$$

$$f_T = \frac{1}{2\pi r_e(C_\pi + C_U)} \quad \therefore \frac{1}{r_e} = g_m$$

for a MOSFET, $f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd} + C_{gb})}$

f_B = Beta cutoff freq

$f_T \approx$ is the figure of merit of a transistor for linear

application.

$\approx 100 \text{ MHz}$ to few GHz

$\approx 300 \text{ MHz}$ for BC147A, BC147B, BF911 etc.

(With $C_{gd} + C_{gb} \ll C_{gs}$)

* High frequency performance (response) of a transistor

* is affected by the interelectrode capacitances that causes β to be frequency dependent.

g_m 's value drops at the rate of 20 dB/decade at frequencies beyond a certain frequency called the beta cut off freq f_B .

f_B is the frequency at which the β of a transistor is 0.707 times its low freq value β_0 .

if frequency is increased beyond f_B , β continues to decrease until it eventually reaches a value of 1.

* The frequency at which $\beta = 1$, is called transition freq (f_T) or unity gain bandwidth.

For a BJT

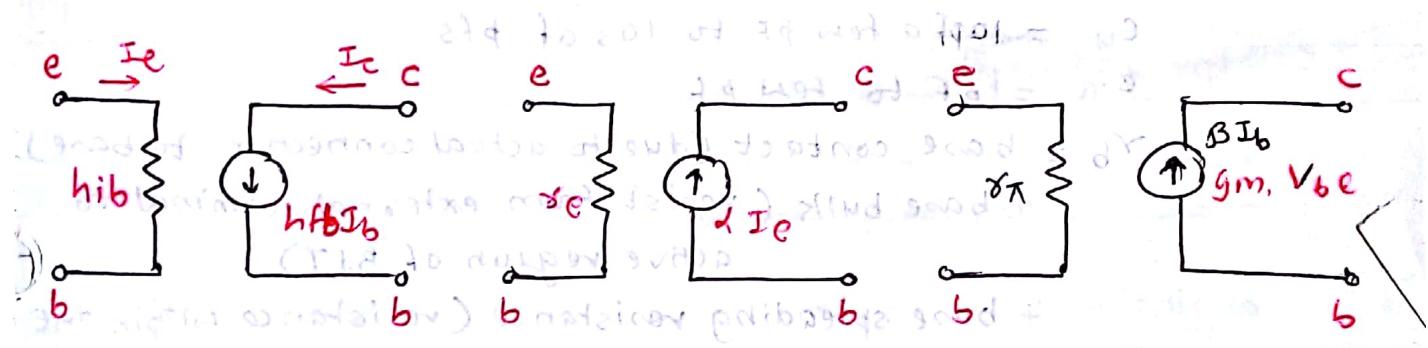
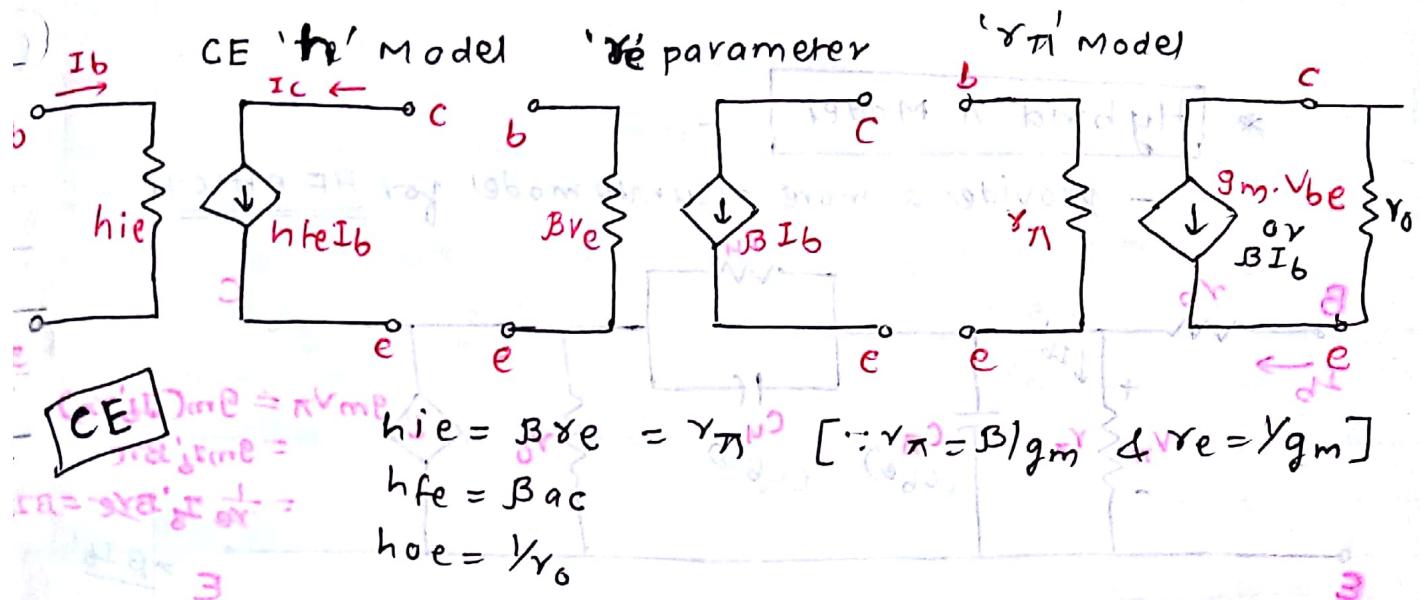
The Beta cut off freq $f_B = \frac{g_m}{2\pi\beta_0(C_{\pi} + C_{\mu})}$

The transition frequency (GBP), $f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})}$

For a FET

The gain-bandwidth product (GBP), $f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$

Models of BJT

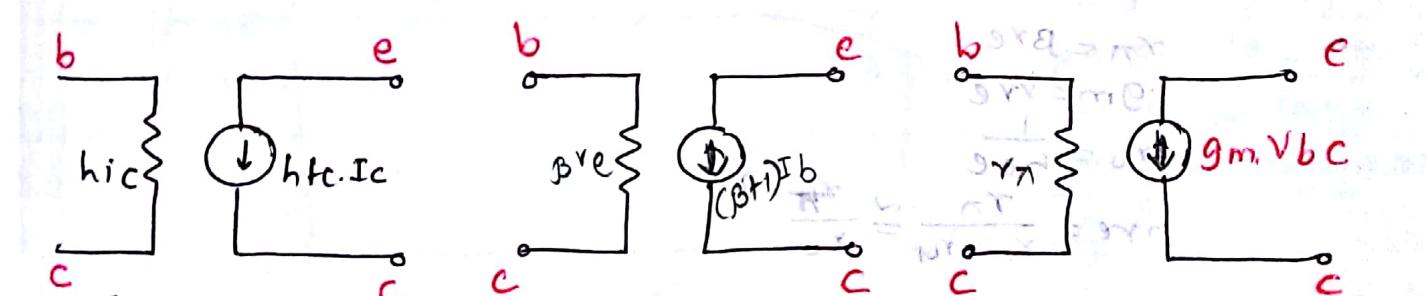


h parameter h_{ib} y_e model π model

$h_{ib} = y_e \left(\frac{h_{ie}}{1+h_{fe}} \right)$ $g_m = \frac{I_c}{V_T}$

$h_{fb} = -1 = -1 \left(\frac{-h_{fe}}{1+h_{fe}} \right)$ $y_e = \frac{V_T}{I_e} \approx \frac{1}{g_m}$

$I_c = B I_B$



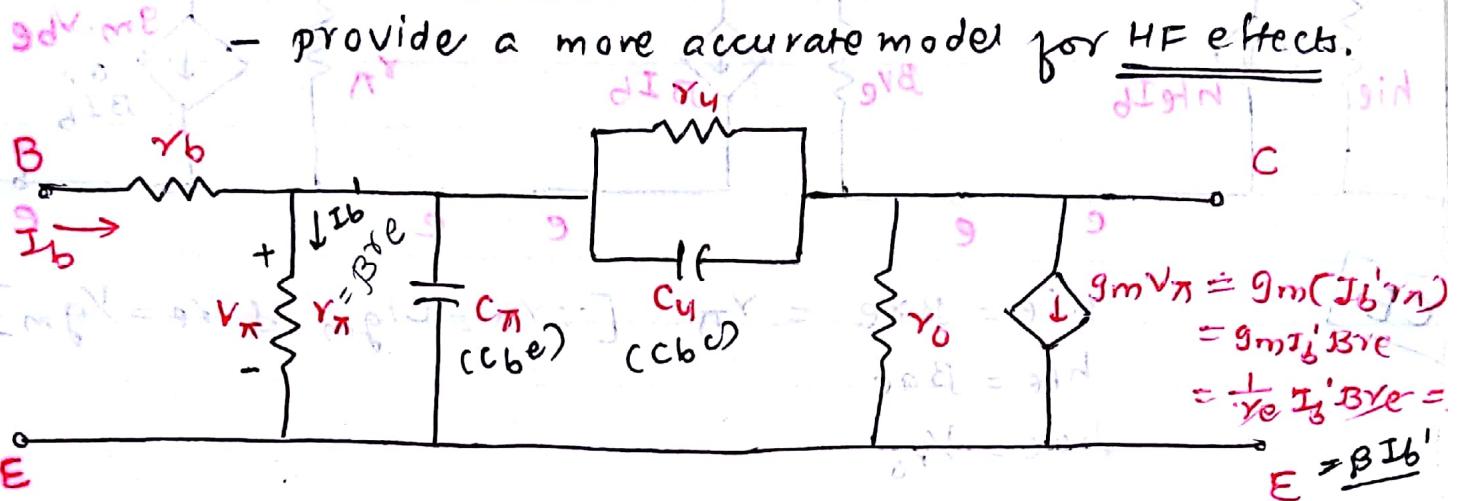
CC **h parameter** h_{ic} y_e model π model

$h_{ic} = h_{ie} = B \cdot y_e$

$h_{fc} = -(1+h_{fe})$

$\left(\frac{1}{h_{ic}} \right) = \left(\frac{1}{h_{oe}} \right)$

* Hybrid π Model :-



$C_U = 10\text{pf}$ a few μF to $10\text{s of }\mu\text{Fs}$

$C_{\pi} = 1\text{pf}$ to few μF

R_b = base contact (due to actual connection to base + base bulk (resist. from external terminal to active region of BJT))

+ base spreading resistance (resistance within the active-base-region)

$r_u = 'u'$ union it provides betw C & B terminal
= very large R provides H/L path from o/p to i/l in the equivalent model.

$r_\pi = BRe = \text{symbol } \pi \text{ for hybrid } \pi \text{ terminology.}$

$r_o = o/p \text{ resistance across an applied load.} = 1/h_o$

$$r_\pi = BRe$$

$$1/g_m = 1/R_e$$

$$r_o = h_{o/e}$$

$$h_{re} = \frac{r_\pi}{r_\pi + r_u} \approx \frac{r_\pi}{r_u}$$

For Low to MF $r_b \ll \text{small so } S.C.$
 $r_u \gg \text{large} \gg BRe \Rightarrow O.C.$

so equivalent o/p in LF & MF \Rightarrow same as ' r_e ' model

- * At HF, there are 2 factors that define \rightarrow -3dB cutoff pt.
- h_{fe} or β variation**:-
- i) N/W capacitance (parasitic 'C')
 - ii) freq depedance of h_{FE} (f_B)

We know that $A_V = \frac{1}{1+j(f/f_2)}$

similarly, $h_{FE} = \frac{h_{FEmid}}{1+j(f/f_B)}$

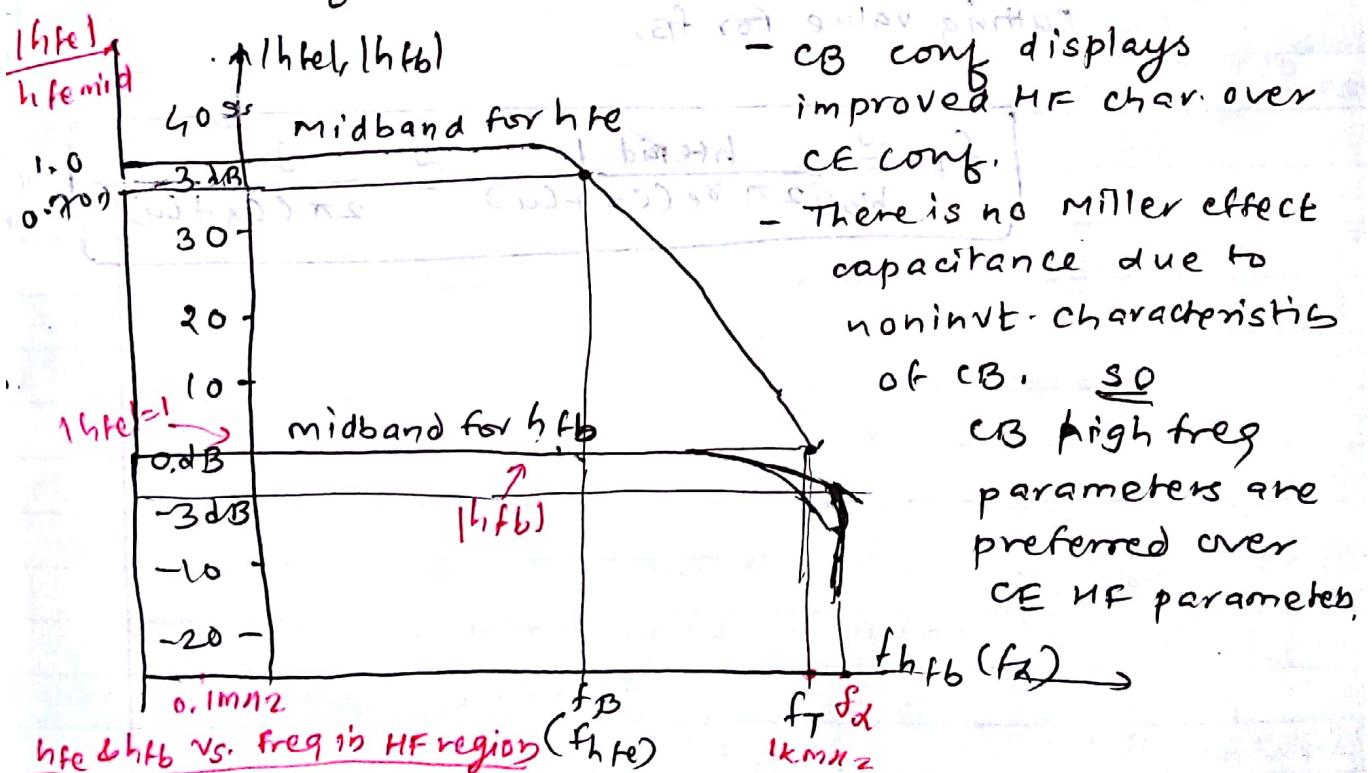
f_B is determined by a set of parameters employed in hybrid π or Giacoleto Model

$$f_B = f_{hFE} = \frac{1}{2\pi \gamma \pi (C_n + C_s)}$$

But $\gamma = \beta r_e = h_{FEmid} r_e$

$$\therefore f_B = \frac{1}{h_{FEmid} \cdot r_e \cdot 2\pi (C_n + C_s)} = \frac{1}{2\pi \beta_{mid} r_e (C_n + C_s)}$$

r_e = is a function of N/W design
 f_B = function of bias configuration.



Relation for direct conversion,

$$f_B = f_2 (1-\lambda)$$

A quantity GBP is defined for transistor by cond'n, $\left| \frac{h_{fe}(\text{mid})}{1+j(f/f_B)} \right| = 1$

$$\text{so that } h_{fe}(\text{dB}) = 20 \log_{10} \left| \frac{h_{fe}(\text{mid})}{1+j(f/f_B)} \right| = 20 \log_{10} 1 = 0 \text{ dB}$$

The frequency at which $h_{fe}(\text{dB}) = 0 \text{ dB}$ is f_T

h_{fe} at defined cond'n p.t. ($f_T \gg f_B$) is given by

$$\frac{h_{fe}(\text{mid})}{\sqrt{1 + (f_T/f_B)^2}} \approx \frac{h_{fe}(\text{mid})}{f_T/f_B} = 1$$

$$f_T \approx h_{fe(\text{mid})} \cdot f_B \quad \text{GBP} \quad (\because f_B \approx \text{BW})$$

$$f_T \approx \beta_{\text{mid}} f_B \quad \& \quad f_B = f_T / \beta_{\text{mid}}$$

putting value for f_B ,

$$f_T \approx \frac{h_{fe(\text{mid})}}{\frac{h_{fe(\text{mid})}}{2\pi \sigma_e (C_A + C_D)}} \approx \frac{g_m}{2\pi (C_A + C_D)}$$

Ex 1) Determine the 3dB freq & shortcircuit current gain of a bipolar transistor with the following parameters.

$$\gamma_{\pi} = 2.6K \quad C_{\pi} = 2 \text{ pF} \quad C_{\mu} = 0.1 \text{ pF}$$

$$\beta_0 = (\lambda f_B)^2$$

Sol:- The 3dB freq $f_B = \frac{g_m}{2\pi \beta_f (C_{\pi} + C_{\mu})}$

$$= \frac{1}{2\pi \times 2\pi (\beta_f/g_m)(C_{\pi} + C_{\mu})}$$

$$= \frac{1}{2\pi \times 2\pi \times 2.6K \times (2 \times 10^{-12} + 0.1 \times 10^{-12})}$$

$$f_B = 29.16 \text{ MHz}$$

2) calculate the bandwidth & capacitance C_{π} of a BJT if the unity GBW is 500MHz at $I_C = 1 \text{ mA}$, $\beta_f = 100$

$$\text{& } C_{\mu} = 0.3 \text{ pF}$$

Sol:- $UGB = f_T = f_B \times \beta_f$ Bandwidth $f_B = \frac{500 \times 10^6}{100} = 5 \text{ MHz}$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{26 \text{ mV}} = 38.5 \text{ mS}$$

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})} \Rightarrow 500 \times 10^6 = \frac{38.5 \times 10^3}{2\pi \times 500 \times 10^6}$$

$$\therefore C_{\pi} = 11.9 \text{ pF}$$

$$C_{\pi} + C_{\mu} = 12.2 \text{ pF}$$

3) calculate the UGB of a n-channel MOSFET with the parameters $K_n = 0.25 \text{ mA/V}^2$, $V_{TN} = 1 \text{ V}$, $C_{gd} = 0.09 \text{ pF}$

$$C_{gs} = 0.2 \text{ pF} \text{ & transistor is biased at } V_{GS} = 3 \text{ V}$$

Sol:- $I_D = K_n (V_{GS} - V_{TN})^2$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2 K_n (V_{GS} - V_{TN}) = 2(0.25)(3-1)$$

$$= 1 \text{ mS}$$

$$f_T = \frac{g_m}{2\pi(C_{gd} + C_{gs})}$$

$$= \frac{1 \times 10^{-3}}{2\pi(0.09 \times 10^{-12} + 0.2 \times 10^{-12})}$$

$$f_T = 663.5 \text{ MHz}$$

(Ex4) a) Determine the lower cut off frequency for the network shown

using following parameters:-

$$C_S = 10 \mu F \quad R_S = 1 k\Omega$$

$$C_E = 20 \mu F$$

$$R_1 = 40 k\Omega$$

$$C_C = 1 \mu F \quad R_2 = 10 k\Omega$$

$$\beta_B = 100 \quad R_E = 2 k\Omega$$

$$r_o = 20 k\Omega \quad R_D = 4 k\Omega$$

$$V_{CC} = 20 V$$

$$R_L = 2.2 k\Omega$$

fig @

b) sketch the frequency response using a Bode plot.

sol:- a) To determine for dc conditions,

$$\beta_B R_E > 10 R_2 \quad \beta_B R_E = (100)(2 k\Omega) = 200 k\Omega > 10 R_2 = 100 k\Omega$$

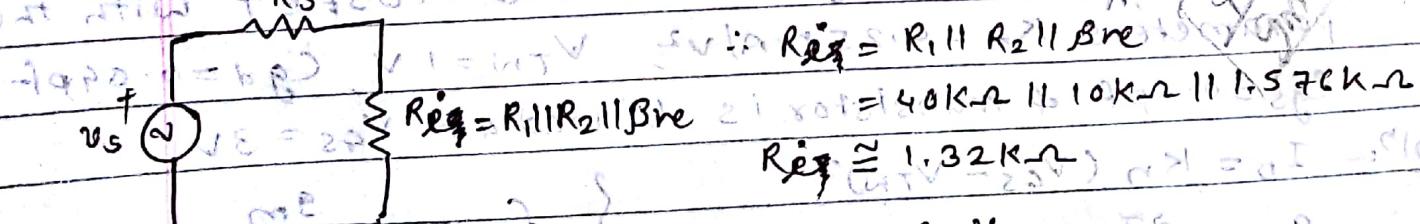
$$V_B = \frac{R_2 \cdot V_{CC}}{R_2 + R_1} = \frac{10 k\Omega \cdot (20 V)}{10 k\Omega + 40 k\Omega} = 4 V$$

$$I_E = \frac{V_E}{R_E} = \frac{4 V - 0.7}{2 k\Omega} = 1.65 \text{ mA}$$

$$r_e = \frac{26 mV}{I_E} = \frac{26 mV}{1.65 \text{ mA}} = 15.76 \Omega$$

$$g_m = \frac{I_E}{26 mV} \quad g_m = 100 (15.76) = 1576 \Omega = 1.576 k\Omega$$

$$A_v = \frac{v_o}{v_i} = \frac{-R_C \parallel R_L}{r_e} = \frac{-(4 k\Omega) \parallel (2.2 k\Omega)}{15.76} \approx -91$$



$$v_i = \frac{R_E \cdot v_s}{R_E + R_S} = \frac{R_E \cdot v_s}{R_E + 1 k\Omega}$$

$$v_o = \frac{R_C \cdot v_i}{R_C + R_S} = \frac{1.32 k\Omega \cdot \frac{R_E \cdot v_s}{R_E + 1 k\Omega}}{1.32 k\Omega + 1 k\Omega} = 0.569 v_s$$

$$f_{LC} = \frac{1}{2\pi(R_C + R_L)C_C}$$

$$= \frac{1}{6.28(4k\Omega + 2.4k\Omega)1\mu F} = 1.28(4k\Omega + 2.4k\Omega)1\mu F$$

$$f_{LC} \approx 25.68 \text{ Hz}$$

$$\text{effect of } C_E = f_{LE} = \frac{1}{2\pi R_E C_E}$$

$$R_E = R_{E1} \parallel \left[R_{E2} + (R_S \parallel R_1 \parallel R_2) \right] \quad 27$$

$$R_E = 24.35 \quad \therefore f_{LE} = 327 \text{ Hz}$$

$$A_{VS} = \frac{U_o}{U_s} = \frac{U_o}{U_i} \times \frac{U_i}{U_s} = (-90)(0.5869) = -51.21$$

$$f_{LC} = 1.76 \text{ Hz}$$

$$f_{LE} = 25.6 \text{ Hz}$$

$$f_{VS} = 327 \text{ Hz}$$

$$\therefore A_{VS} = -51.21$$

$$\text{effect of } C_S = f_{LS} = \frac{1}{2\pi(R_S + R_S)C_S} = \frac{1}{(6.28)(1k\Omega + 1.32k\Omega)(10\mu F)} = 6.86 \text{ Hz}$$

where $R_E = R_1 + R_2 + R_{RE} + R_S$

$$f_{LS} \approx 6.86 \text{ Hz}$$

$$\therefore \text{Lower cut-off freq} = 327 \text{ Hz} \quad (\text{highest among } f_{LS}, f_{LC} \text{ & } f_{LE})$$

Ex 5) a) Determine the lower cut-off frequency for the network

shown in fig @ using the following parameters.

LF

$$C_S = 10\mu F \quad R_S = 1k\Omega$$

$$C_E = 20\mu F \quad R_1 = 40k\Omega$$

EF2
u.w

$$C_C = 1\mu F \quad R_2 = 10k\Omega$$

$$B = 100 \quad R_E = 1.2k\Omega$$

$$(1.2k\Omega + 10k\Omega + 40k\Omega) = 51.2k\Omega \quad R_C = 4k\Omega$$

$$V_{CC} = 10V \quad R_L = 10k\Omega$$

b) sketch the frequency response using a Bode plot.

Sol:- a) To determine r_e for dc conditions,

$$\beta R_E = 100(1.2k\Omega) = 120k\Omega > 10k\Omega = 100k\Omega$$

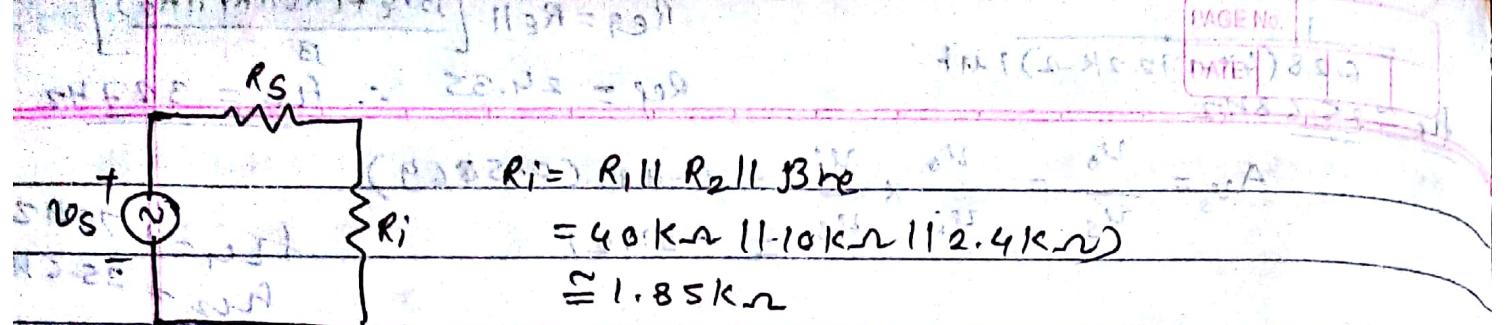
$$V_B = \frac{R_2 V_{CC}}{R_1 + R_2} = \frac{10k\Omega (10V)}{10k\Omega + 40k\Omega} = \frac{100V}{50} = 2V$$

$$I_E = \frac{V_E}{R_E} = \frac{2V - 0.7}{1.2k\Omega} = 1.083 \text{ mA}$$

$$\gamma_T = \frac{\beta V_T}{I_E} \quad r_e = \frac{2GMV}{I_E} = \frac{2GMV}{1.083 \text{ mA}} \approx 24.01 \text{ }\Omega$$

$$g_m = \frac{I_E}{V_T} \quad \beta R_E = 100(24.01 \text{ }\Omega) = 2400 \text{ }\Omega = 2.4k\Omega$$

$$\text{Midband gain: } A_{VS} = \frac{U_o}{U_i} = \frac{-R_C || R_L}{r_e} = \frac{-(4k\Omega) || (10k\Omega)}{24.01} = -119$$



$$R_i = R_1 \parallel R_2 \parallel g_{BE}$$

$$= 40k\Omega \parallel 10k\Omega \parallel 112.4k\Omega$$

$$\approx 1.85k\Omega$$

$$U_i = \frac{R_i \cdot U_s}{R_i + R_s}$$

$$\frac{U_o}{U_s} = \frac{R_L}{R_i + R_s} = \frac{1.85k\Omega}{(1.85+1)k\Omega} = 0.65$$

$$A_{vS} = \frac{U_o}{U_s} = \frac{U_o \cdot 20}{U_i \cdot 20} = -11.9 \times 0.65 = -77.35$$

Effect of C_B :-

$$f_{LS} = \frac{1}{2\pi(R_{ce} + R_L)C_B} = \frac{1}{2\pi(1.85k\Omega + 10k\Omega)C_B} = \frac{(6.28)(1.85k\Omega + 10k\Omega)}{2\pi \times 10^4} =$$

$$f_{LS} \approx 5.59 \text{ Hz}$$

Effect of C_C :-

$$f_{LC} = \frac{1}{2\pi(R_{ce} + R_L)C_C} = \frac{1}{2\pi(1.85k\Omega + 10k\Omega)C_C} = \frac{(6.28)(1.85k\Omega + 10k\Omega)}{2\pi \times 10^4} =$$

$$f_{LC} = 11.68 \text{ Hz}$$

Effect of C_E :-

$$f_{LE} = \frac{1}{2\pi R_{ce} C_E} = \frac{1}{2\pi [R_{ce} \left(\frac{g_{BE} + (R_s \parallel R_1 \parallel R_2)}{g_{BE}} \right)] C_E}$$

~~$$f_{LE} = \frac{248.6 \text{ Hz}}{2 \times 10^4 \times (1.85k\Omega + 10k\Omega)}$$~~

∴ Lower cutoff freq = highest among f_{LS} , f_{LC} & f_{LE} $\Rightarrow 248.6 \text{ Hz}$

$$f_{cutoff} = \frac{V_{rms}}{4\pi R_{ce} I_c} = \frac{V_{rms}}{4\pi \times 10^4 \times 1.85} = 35$$

$$V_{rms} = 50 \text{ mV} = (5 \times 10^{-5}) \text{ V} = 0.5 \text{ mV}$$

Next Example - $\left. \begin{array}{l} R_L = 2.2k\Omega \\ V_{CC} = 20V \end{array} \right\} f_{HI} = 738.25 \text{ kHz} \rightarrow R_{EF} = 0.531k\Omega, C_i = 40fF$

$f_{HO} = 8.6 \text{ MHz} \rightarrow R_{EF} = 1.418k\Omega, C_o = 13.03 \text{ pF}$

$C_{im} = [1 - (-90)] \mu \text{F}$

$C_{om} = (1 - 1/90) \mu \text{F}$

(31)

EX 7) Use the network of Fig (A), with the parameters,

HF $R_S = 1k\Omega$ $C_S = 10 \mu\text{F}$ & $C_{pi}(C_{be}) = 3.6 \text{ pF}$

BJT $R_1 = 40k\Omega$ $C_C = 1 \mu\text{F}$ $C_{pi}(C_{bc}) = 4 \text{ pF}$

$R_2 = 10k\Omega$ $C_E = 20 \mu\text{F}$ $C_{ce} = 1 \text{ pF}$

(Ex 3) $R_E = 1.2k\Omega$ $A_{m8} B = 100$ $C_{wi} = 2 \text{ pF}$

$R_C = 4k\Omega$ $V_{RE} = 0 \Omega$ $C_{wo} = 8 \text{ pF}$

$R_L = 10k\Omega$ $V_{CC} = 10V$

a) Determine f_{HI} & f_{HO}

$$x_e = \frac{26mV}{I_E} \quad I_E = \frac{V_E}{Z_E} = \frac{V_B - V_{BE}}{R_E}$$

$$V_B = R_2 V_{CC} / (R_1 + R_2)$$

$$x_e = 15.76 \text{ nA} \quad (\text{from Ex 6})$$

$$I_E = 15.76 \text{ nA}$$

$$g_m = 0.03 \text{ S}$$

Soln:- $R_i = R_1 || R_2 || R_S = 1.85k\Omega$

$g_m = \frac{I_E}{V_T} \quad A_{mid} = \frac{V_o}{V_i} = - (R_C || R_L) = -119$

$\gamma_T = \frac{BVT}{I_E g_m} \quad C_i = C_{wi} + C_{be} + C_{mi}$

$= C_{wi} + C_{be} + (1 - A_v) C_{be}$

$I_E = 1 \text{ mA} \quad = 6 \text{ pF} + 3.6 \text{ pF} + [1 - (-119) 4] \text{ pF}$

$V_T = 26 \text{ mV} \quad = 522 \text{ pF}$

$g_m = 0.03 \text{ S} \quad f_{HI} = \frac{1}{2\pi R_{eq} C_i}$

$R_{eq} = R_S || R_1 || R_2 || R_L = 2.86k\Omega$

$f_{HI} = \frac{1}{2\pi (2.86k\Omega) (522 \text{ pF})} = 470 \text{ kHz}$

$R_{eq} = R_C || R_L = 4k\Omega || 10k\Omega = 2.86k\Omega$

$C_o = C_{wo} + C_{ce} + C_{mo} = 8 \text{ pF} + 1 \text{ pF} + (1 - \frac{1}{119}) 4 \text{ pF}$

$C_o = 13.03 \text{ pF}$

$$f_{HO} = \frac{1}{2\pi R_{eq} C_o} = \frac{1}{2\pi (2.86k\Omega) (13.03 \text{ pF})} = 8.542 \text{ MHz}$$

$f_{HI} = 470 \text{ kHz}$

$f_{HO} = 8.542 \text{ MHz}$

Select Lowest value of f_H among f_{HI} & f_{HO}

$f_H = \text{Lowest} (f_{HI}, f_{HO})$

Ex 8) Determine the high-cut off frequencies for the network

in fig (B) using following parameters,

$$\begin{array}{lll}
 C_F = 0.01\text{nF} & R_S = 1\text{k}\Omega & C_{gd} = 2\text{pF} \\
 C_D = 0.5\text{nF} & R_L = 2.2\text{k}\Omega & C_{gs} = 4\text{pF} \\
 C_S = 2\text{nF} & I_{DSS} = 8\text{mA} & C_{ds} = 0.5\text{pF} \\
 R_S' = 10\text{k}\Omega & V_p = -4\text{V} & C_{wi} = 5\text{pF} \\
 R_Q = 1\text{M}\Omega & r_d = \infty \Omega & C_{wo} = 6\text{pF} \\
 R_D = 4.7\text{k}\Omega & V_{DD} = 20\text{V} & g_m = g_m \left(1 - \frac{V_{GS}}{V_p}\right) \\
 & & g_m = 2I_{DSS} \left(1 - \frac{V_{GS}}{V_p}\right)
 \end{array}$$

$$SOL:- R_{eq} = R_S' || R_Q = 10\text{k}\Omega || 1\text{M}\Omega = 9.9\text{k}\Omega \quad (g_m = 2\text{mV})$$

$$\boxed{\text{MOSFET}} \quad C_i = C_{wi} + C_{gs} + (1 - A_v) C_{gd} \quad A_{vmid} = g_m \left(R_D || R_L \right)$$

$$K_n = 0.25 \text{mV}^2 = 5\text{pF} + 4\text{pF} + (1+3)2\text{pF}$$

$$V_{TN} = 1\text{V}$$

$$C_i = 17\text{pF}$$

$$V_{GS} = 3\text{V} \quad f_{H1} = \frac{1}{2\pi R_{eq} C_i} = \frac{2\pi (9.9\text{k}\Omega) \times 17\text{pF}}{2\pi (9.9\text{k}\Omega) + 3\text{pF}} = 945.67\text{kHz}$$

$$R_{eq} = R_S' || R_Q$$

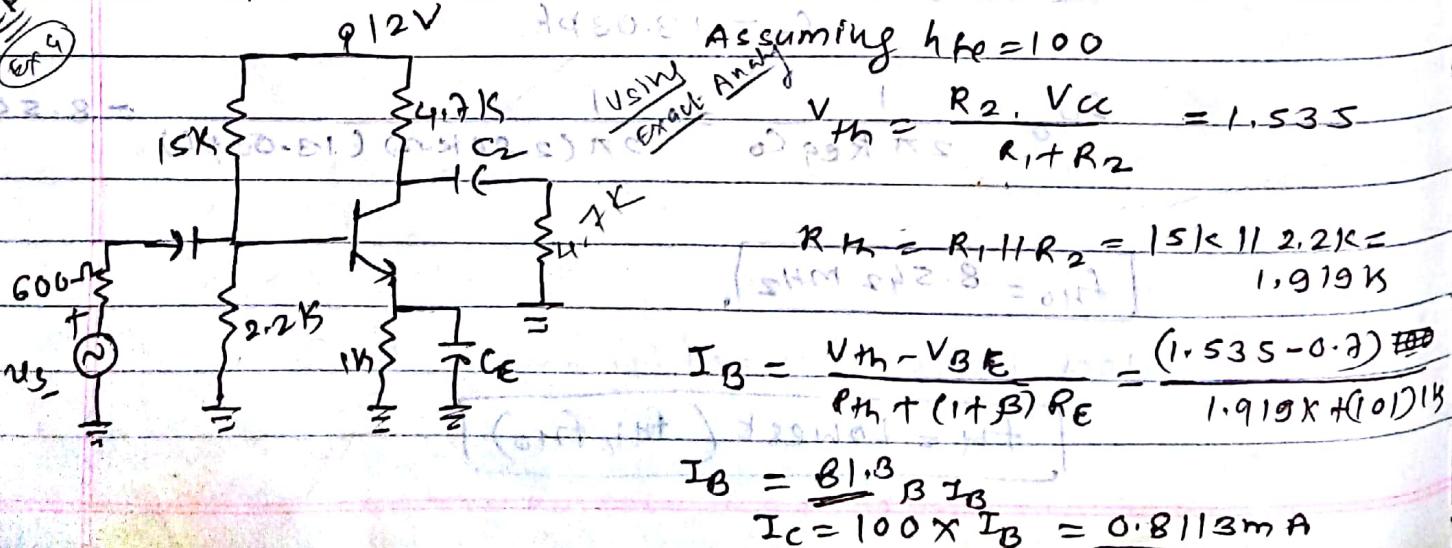
$$R_{eq} = R_D || R_L = 4.7\text{k}\Omega || 2.2\text{k}\Omega = 1.5\text{k}\Omega$$

$$C_o = C_{wo} + C_{ds} + C_{mo} = 6\text{pF} + 0.5\text{pF} + \left(1 - \frac{1}{3}\right)2\text{pF} = 9.17\text{pF}$$

$$f_{H2} = \frac{1}{2\pi R_{eq} C_o} = \frac{1}{2\pi (1.5\text{k}\Omega) (9.17\text{pF})} = 1.157\text{MHz}$$

Select lowest value $f_H = 945.67\text{kHz}$

~~design problem~~ Ex 9) For the CE amplifier shown, calculate the size of bypass & coupling capacitors to provide $f_L = 10\text{Hz}$



Using
Approx

$$\text{Analysis} \quad V_B = \frac{R_2 \cdot V_{CC}}{R_2 + R_1} =$$

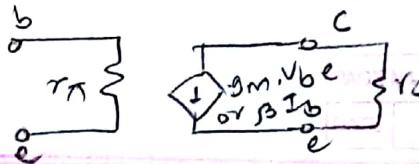
$$I_E = V_E / R_E =$$

$$r_e = \frac{2GMV}{I_E} = r_T = \beta V_E$$

$$r_T = \frac{\beta}{I_C Q} = \frac{\beta}{9.8113 \times 10^{-3}} = 0.8113 \times 10^3 = 811.3 \text{ k}\Omega$$

$$r_T = \frac{\beta V_T}{I_C Q} = \frac{15 \times 26 \text{ mV}}{1.12 \text{ k}\Omega} = 1.12 \text{ k}\Omega$$

$$g_m = \frac{I_C Q}{V_T} = \frac{1.12 \text{ k}\Omega}{26 \text{ mV}} = 4.27 \text{ S}$$



33

$$f_{LCE} = 10 \text{ Hz} = \frac{1}{2\pi R_{\text{Rep}} C_E} \quad R_{\text{Rep}} = R_E || \left[\frac{r_T + (R_1 || R_2 || R_S)}{1 + \beta} \right]$$

$$= 1 \text{ k}\Omega || \left[\frac{3.2 \text{ k} + (15 \times 112.2 \text{ k}) / 10.6}{10.6} \right] = 34.94 \text{ nF}$$

$$C_E = \frac{1}{2\pi R_{\text{Rep}} f_{LCE}} = 34.94 \text{ nF}$$

$$\therefore C_E = \frac{1}{2\pi (10) \times 34.94} = 4.70 \mu\text{F} \approx 4.70 \mu\text{F}$$

$$f_{LC_1} = \left(\frac{f_L}{10} \right) = 1 \text{ Hz} = \frac{1}{2\pi R_{\text{Rep}} C_1} \quad R_{\text{Rep}} = R_i + R_S = (R_1 || R_2 || r_T) + R_S = (1.12 + 600) = 611.2 \text{ k}\Omega$$

$$\therefore C_1 = \frac{1}{2\pi R_{\text{Rep}} f_{LC_1}} = \frac{1}{2\pi (1.12 \text{ k} + 600) \times 1} = 9.258 \mu\text{F} = 100 \mu\text{F}$$

$$f_{LC_2} = \left(\frac{f_L}{20} \right) = \frac{1}{20} = 0.5 = \frac{1}{2\pi R_{\text{Rep}} C_2} \quad R_{\text{Rep}} = R_i + R_L = 4.7 \text{ k} + 4.7 \text{ k} = 9.4 \text{ k}\Omega$$

$$\therefore C_2 = \frac{1}{2\pi R_{\text{Rep}} f_{LC_2}} = \frac{1}{2\pi (9.4 \text{ k} \times 0.5)} = 33.88 \mu\text{F} \approx 39 \mu\text{F}$$

$$\boxed{\begin{aligned} C_1 &= 100 \mu\text{F} \\ C_2 &= 39 \mu\text{F} \\ C_E &= 4.7 \mu\text{F} \end{aligned}}$$

Verification :- with the above capacitor values

$$f_L = f_{LC_1} + f_{LC_2} + f_{LC_3}$$

$$= \frac{1}{2\pi \times 34.94 \times 4.7 \mu\text{F}} + \frac{1}{2\pi (1.12 \text{ k} + 600) \times 100 \times 10^{-6}} + \frac{1}{2\pi (9.4 \text{ k}) \times 39 \times 10^{-6}}$$

$$= 9.696 + 0.925 + 0.434 \text{ Hz} \approx 10.05 \text{ Hz}$$

$$\boxed{f_L = 11 \text{ Hz}} \quad \text{& given } \boxed{f_L = 10 \text{ Hz}}$$

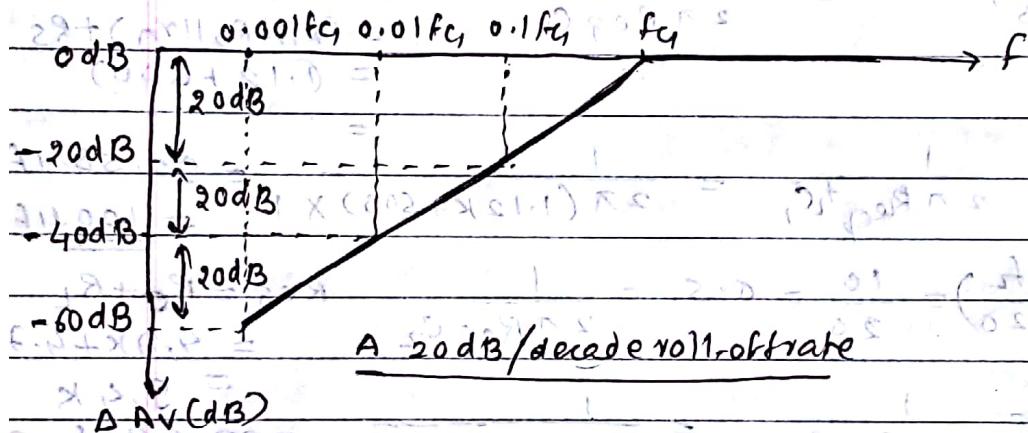
$\left\{ \begin{array}{l} \text{If we increase } C_E \approx 5 \text{ times} \\ f_L = 9.5 \text{ Hz} < 10 \text{ Hz} \end{array} \right.$

* Gain Roll-off :-

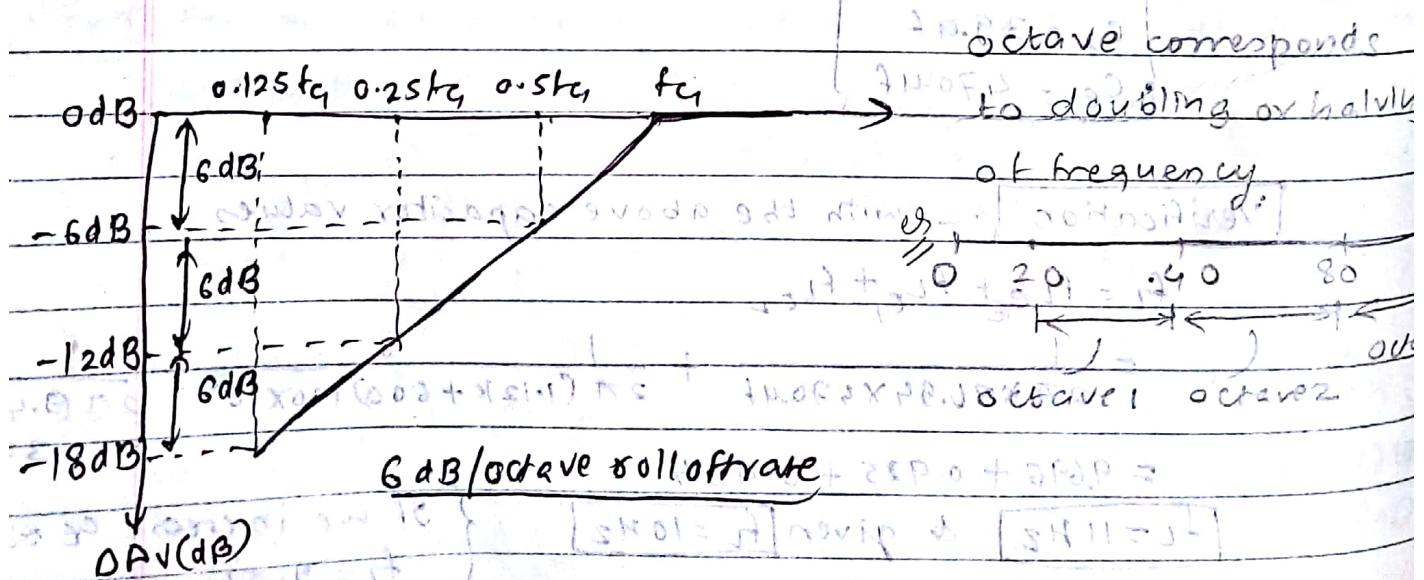
- The term roll-off rate is used to describe the rate at which an RC circuit causes the voltage gain of an amplifier to decrease once the f_1 or f_{c2} has been passed.

- Low-freq roll-off is calculated using, $\Delta A_V(dB) = 20 \log \frac{f}{f_1}$, where f = freq of operation.

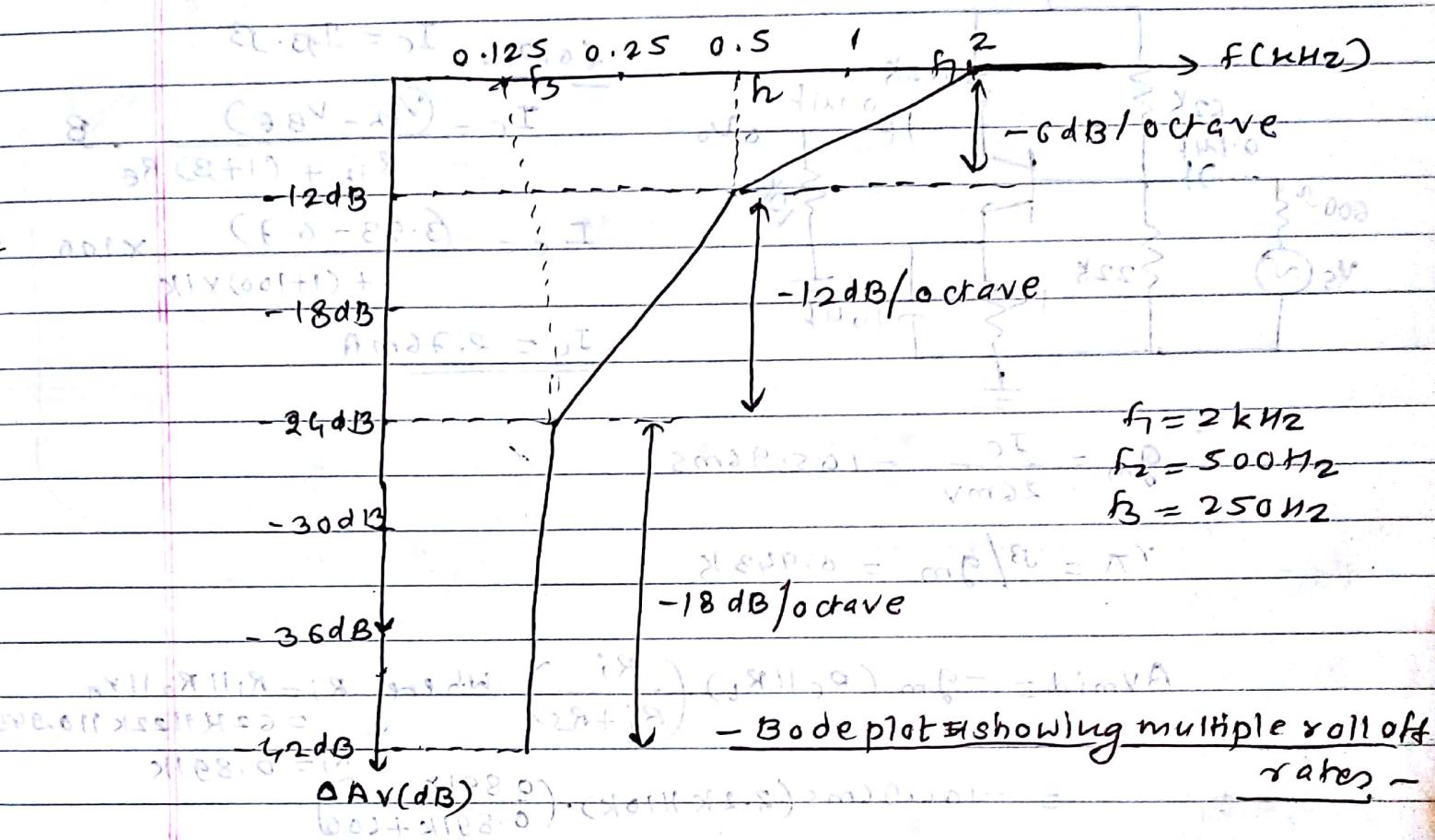
- The $\Delta A_V(dB)$ eqn can be used to show that every RC circuit has a roll-off rate of 20 dB per decade.



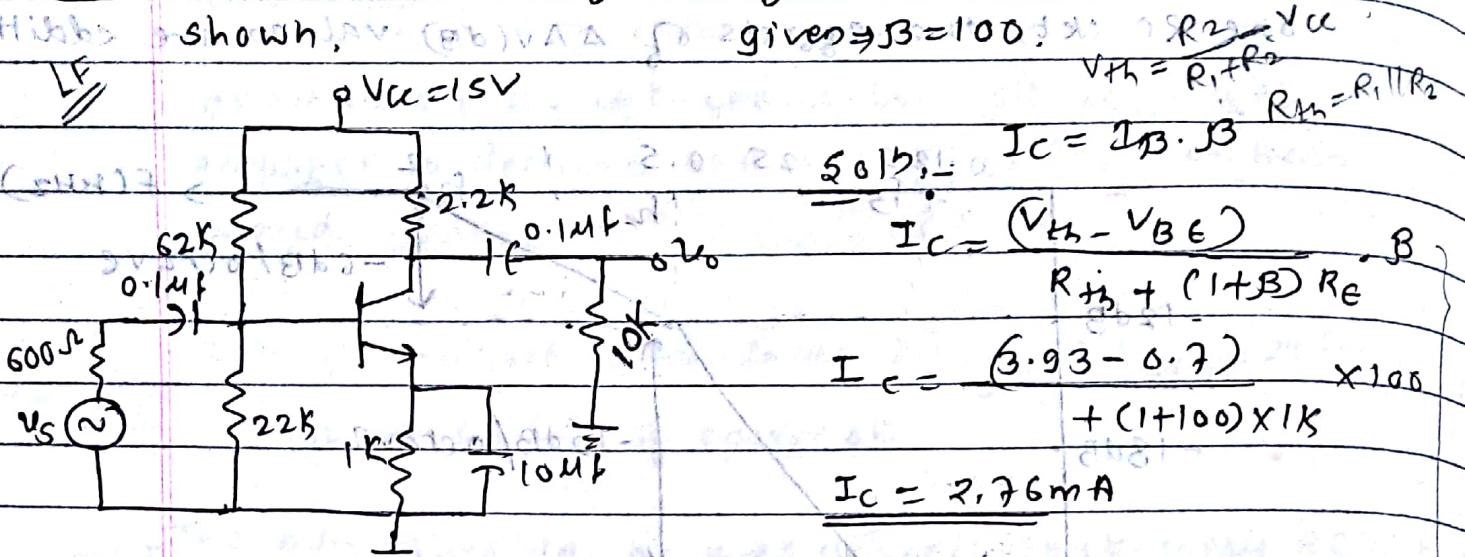
- The $\Delta A_V(dB)$ eqn can also be used to show that every RC circuit has a roll-off rate of 6 dB/octave.



When a circuit (such as BJT amplifier) contains more than one RC ckt, the effects of $\Delta A_V(dB)$ values are additive.



(Ex10). Determine the low frequency response of BJT amplifier



$$A_{v\text{mid}} = -g_m \left(R_c || R_L \right) \cdot \frac{R_i}{R_i + R_S} \quad \text{where } R_i = R_1 || R_2 || r_\pi \\ = -105.96 \text{ ms} \cdot (2.2k || 10k) \cdot \frac{0.891k}{0.891k + 100}$$

$$A_{v\text{mid}} = -115.26$$

$$A_{v\text{low}} = \frac{A_{v\text{mid}}}{\sqrt{2}} = \frac{-115.26}{\sqrt{2}} = -81.5 \quad (\text{at } f = f_2)$$

$$f_{LC_1} = \frac{1}{2\pi(R_i + R_S)C_1} = \frac{1}{2\pi(0.891k + 0.6k)(0.1 \times 10^{-6})} = 1067.98 \text{ Hz}$$

$$f_{LC_2} = \frac{1}{2\pi(R_C + R_L)C_2} = \frac{1}{2\pi(2.2k + 10k)(0.1 \times 10^{-6})} = 130.52 \text{ Hz}$$

$$f_{LCE} = \frac{1}{2\pi R_{CE} C_E} \quad R_{CE} = R_E || \left[\frac{r_\pi + (R_1 || R_2 || r_\pi)}{(B+1)} \right]$$

$$f_{LCE} = \frac{1}{2\pi \times 1k || [0.943k + (2.2k || 22k || 0.6k)] \times 10 \times 10^{-6}} \\ 10^6$$

$$f_{LCE} = 1072.87 \text{ Hz}$$

$$f_L = f_{Lc_1} + f_{Lc_2} + f_{LCE} = 1067.98 \text{ Hz} + 130.52 \text{ Hz} + 1072.87 \text{ Hz}$$

$$\boxed{f_L = 2.27 \text{ kHz}}$$

~~DC cond.~~

$$\text{or } V_B = \frac{R_2}{R_2 + R_1} \cdot V_{CC} = \frac{22K}{(22+62)K} \times 15V =$$

$$I_E = \frac{V_E}{R_E} = \frac{V_B - 0.7}{R_E} = \frac{-0.7}{1K\Omega} =$$

$$r_e = 26 \text{ mV}$$

$$26 \text{ mV} = 16.52 = 1.81 \text{ mA} = 9.59$$

$$g_m = \frac{1}{r_e} = \frac{1}{9.59} = 0.104 = 2.5 \times 10^{-3} \text{ A/V}$$

$$r_\pi = B/g_m = 100.$$

$$r_\pi = B r_e =$$

$$\text{midband gain} = A_{Vm} = -g_m (R_C \parallel R_L) = \frac{V_o}{V_i}$$

$$A_V = A_{Vm} \left(\frac{R_i}{R_i + R_S} \right) = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s}$$

$$\frac{V_i}{V_s} \text{ where } R_i = R_1 \parallel R_2 \parallel r_\pi$$

Ex 11) Design a CB amplifier to give a low 3dB freq of 150 Hz.

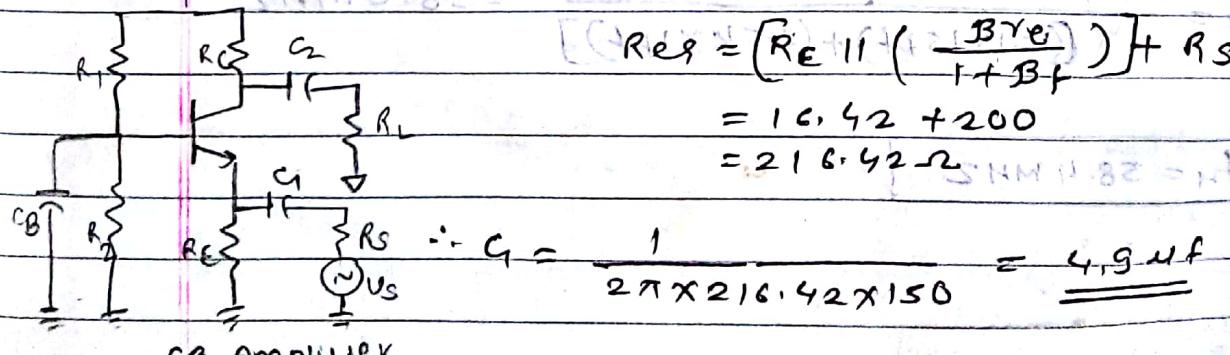
a) calculate values of C_1, C_2 & C_B , given $R_1 = 7K$, $R_2 = 4.3K$,

$$R_E = 330 \text{ }\mu\text{A}, R_C = 5K, R_L = 15K, R_S = 200 \text{ }\Omega, C_M = 15 \text{ pF}$$

$$C_M = 1 \text{ pF}, r_\pi = 1.4K, g_m = 57.15 \text{ mS}, JBF = 80$$

b) calculate A_{Vm} c) the high 3dB freq

$$\frac{2017}{2\pi R_E C_M} \text{ for CB, } f_L = f_Q = 150 = \frac{2\pi R_E C_M}{2\pi R_E C_M} = 1.42 \text{ Hz} \quad \boxed{r_\pi = B r_e = 1.4K}$$



$$R_{eq} = \left[R_E \parallel \left(\frac{B r_e}{1 + B_f} \right) \right] + R_S$$

$$= 16.42 + 200$$

$$= 216.42 \text{ }\Omega$$

$$\therefore C = \frac{1}{2\pi \times 216.42 \times 150} = 4.9 \text{ nF}$$

CB Amplifier

C_B = bypass capacitor

PAGE No.
DATE

$$f_{LCB} = \frac{f_L}{20} = 1.5 \text{ Hz} = \frac{1}{2\pi R_{eq} C_B}$$

$$R_{eq} = R_1 || R_2 || [\beta_{re} + (1+\beta) (R_e || R_s)] = 2.16 \text{ k}\Omega$$

$$\therefore C_B = \frac{1}{2\pi \times 2.16 \text{ k} \times 1.5} = 4.914 \mu\text{F}$$

$$\underline{C_B = 4.914 \mu\text{F}}$$

$$f_{LC2} = \frac{f_L}{20} = \frac{150}{20} = 7.5 \text{ Hz} = \frac{1}{2\pi R_{eq} C_2}$$

$$R_{eq} = R_c + R_L = 5\text{k} + 5\text{k} = \cancel{10\text{k}}, 2.5$$

$$C_2 = \frac{1}{2\pi \times 10\text{k} \times 7.5} = \cancel{0.0002\text{H}} \\ = 4.24 \mu\text{F}$$

$$\underline{C_2 = 4.24 \mu\text{F}}$$

$$A_{mid} = g_m (R_c || R_L) \left(\frac{z_1}{z_1 + R_s} \right) \text{ where } z_1 = R_e || \left(\frac{\beta \times e}{1 + \beta F} \right) = 16.42 \\ = (57.14 \times 10^3) (5\text{k} || 5\text{k}) \left(\frac{16.42}{16.42 + 200} \right)$$

$$\boxed{A_{mid} = 10.84}$$

$$f_H = \frac{1}{2\pi (R_{eq} C_{in} + R_{in} C_H)}$$

$$R_{eq} = R_s || R_e || \beta \times e || (Y_{gm})$$

$$= 200 || 300 || 1.4 \text{ k} || 12 \left(\frac{1}{57.14 \times 10^3} \right) \cdot 1.5 \text{ pF} + 1 \text{ pF} = 100$$

$$= 15.1 \text{ n} \Omega \text{ (approx)}$$

$$R_{in} = R_{eq} || R_L = 5\text{k} || 5\text{k} = 2.5 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi \left[(15.1 \times 15 \text{ pF}) + (2.5 \text{ k} \times 1 \text{ pF}) \right]} = 58.4 \text{ MHz}$$

$$\boxed{f_H = 58.4 \text{ MHz}}$$

Ex 12) Design a CC amplifier to give a lower 3dB freq

$f_L = 150\text{Hz}$ calculate values of C_B , G_1 , & C_2 given $R_i = 7\text{k}$,

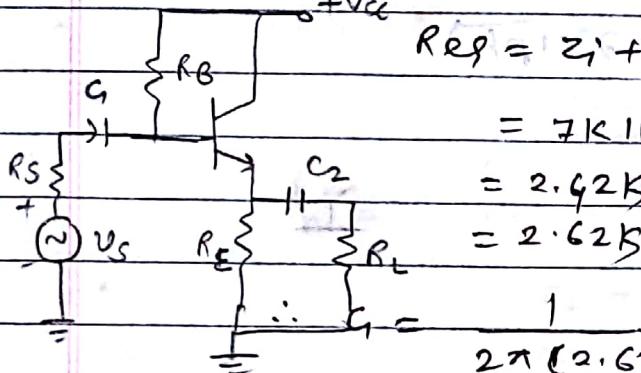
$R_2 = 4.3\text{k}$, $R_E = 330\text{-}\Omega$, $R_S = 200\text{-}\Omega$, $R_L = 5\text{k}$, $g_m = 57.16\text{ms}$

$B_f = 80$, $r_A = 1.4\text{k}$, $C_A = 1\text{spt}$, $C_U = 1\text{pt}$ calculate f_U

2017

$$r_A = \beta r_e$$

$$f_L = f_{LG} = 150\text{Hz} = \frac{1}{2\pi R_E C_1}$$



$$\begin{aligned} R_{eq} &= Z_i + R_S = [R_i || R_2 || (\beta r_e + (1 + \beta_f)(R_E || R_L))] + R_S \\ &= 7\text{k} || 4.3\text{k} || [1.4\text{k} + 81(330 || 5\text{k})] \\ &= 2.42\text{k} + 0.215 \\ &= 2.62\text{k} \\ \therefore C_1 &= \frac{1}{2\pi(2.62\text{k})(150)} = 0.405\text{nF} \end{aligned}$$

$$f_{LC_2} = \frac{f_L}{T_0} = 15\text{Hz} = \frac{1}{2\pi R_{eq} C_2}$$

$$\begin{aligned} R_{eq} &= Z_o + R_L \\ &= [R_E || (\frac{\beta r_e + (R_i || R_2 || R_S)}{1 + \beta_f})] + R_L \\ &= 330 || (\frac{1.4\text{k} + (7\text{k} || 4.3\text{k} || 200)}{81}) \\ &= 18.48\text{-}\Omega + 5\text{k} \\ &= 5.018\text{k} \end{aligned}$$

$$C_2 = \frac{1}{2\pi(18.48)(15)} \quad \frac{1}{2\pi(5.018\text{k})(15)}$$

$$\underline{C_2 = 2.11\text{nF}}$$

$$R_{C_2} = \beta r_e || \left[\frac{(R_i || R_2 || R_S) + (R_E || R_L)}{1 + g_m(R_E || R_L)} \right]$$

$$\begin{aligned} &= 1.4\text{k} || \left[\frac{(7\text{k} || 4.3\text{k} || 200) + (330 || 5\text{k})}{1 + 57.16 \times 10^{-3}(330 || 5\text{k})} \right] = 1.4\text{k} || 26.56 \\ &= 26.05\text{-}\Omega \end{aligned}$$

$$R_{Cu} = (R_1 || R_2 || R_S) || [B_{re} + (1 + \beta_f)(R_G || R_L)]$$

$$= 7k || 4.3k || 200 || [1.4k + 81(330 || 5k)]$$

$$\approx 184.7 \Omega$$

$$f_m = \frac{1}{2\pi (R_{Cu}(n + R_{Cu}C_u))}$$

$$f_m = 276.7 \text{ Hz}$$

$$= \frac{1}{(0.21)(4.5 \times 10^{-9})} = 222.2 \text{ Hz}$$