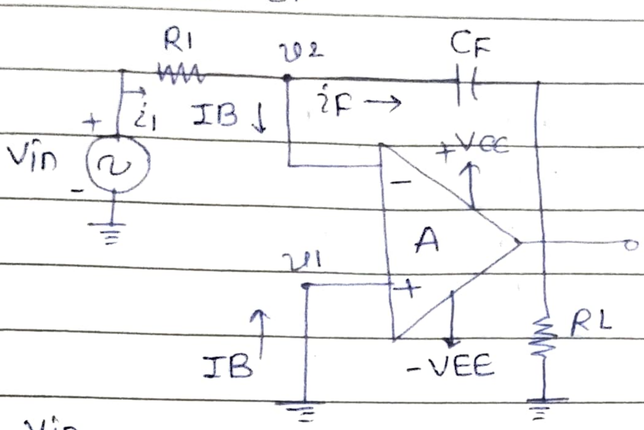
→ Integrator (Integrating ampʳ) :→

- In this ckt the O/P waveform is the integral of the I/P vtg. w/f.
- This ckt is obtained by using a basic inverting ampʳ config. if the f/b resi RF is replaced by a capacitor CF.

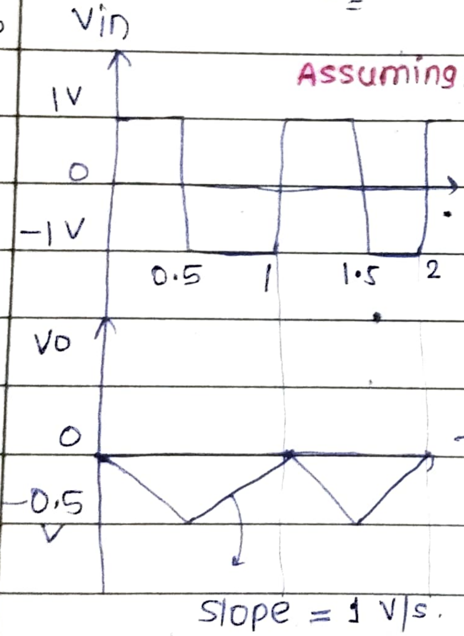
when Vin = 0
CF → open ckt
($X_{CF} = \infty$).
⇓
∴ Vio & part of IB
↓
error vtg at O/P
⇓ To overcome
RF connected



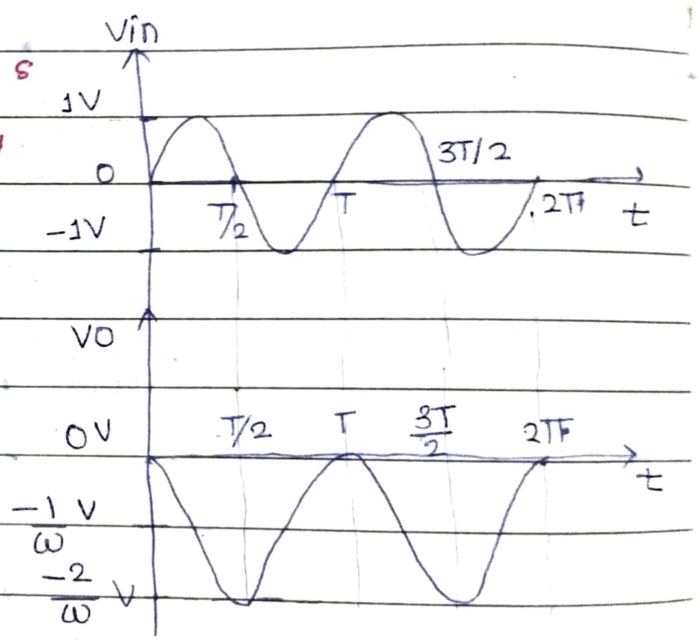
- If Vin = 0, CF will act as short ckt. Hence CF will charge due to IB & error vtg is produced

$$v_0 = -\frac{1}{R_1 \cdot C_F} \int_0^t v_{in}\, dt + C$$

- To remove this use RF.



Assuming, $R_1 C_F = 1\,s$
& $V_{OOT} = 0V$
i.e. $C = 0$.



- Using KCL at node v2,

$$i_1 = I_B + i_F$$

∵ IB is negligibly small (due to Ri >>)

$$i_1 \simeq i_F$$

We know the relation betⁿ current flowing thr' C & vtg. a/c C

$$i_c = C \cdot \frac{dv_c}{dt}$$

$$\therefore \frac{v_{in} - v_2}{R_1} = C_F \cdot \left(\frac{d}{dt}\right) \cdot (v_2 - v_0)$$

However, $v_1 = v_2 \cong 0$ because A is very large.

$$\therefore \frac{v_{in}}{R_1} = C_F \cdot \frac{d}{dt}(-v_0).$$

The O/P vtg. can be obtained by integrating both sides w.r.t time

$$\int_0^t \frac{v_{in}}{R_1} \cdot dt = \int_0^t C_F \cdot \frac{d}{dt}(-v_0) \cdot dt$$

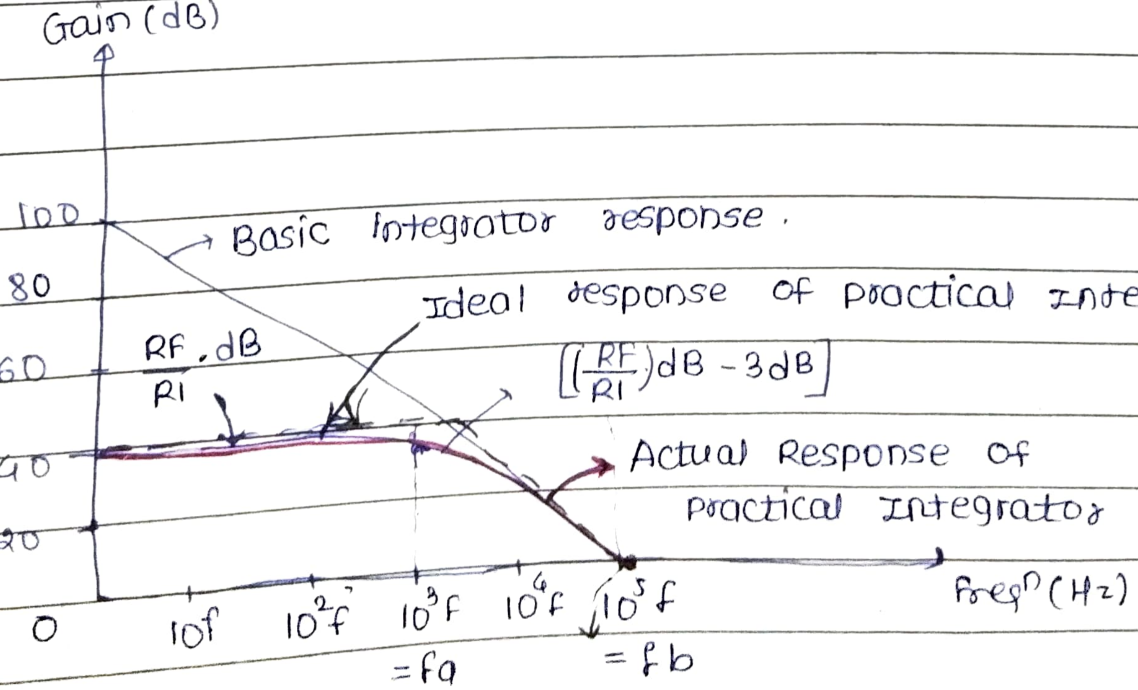
$$= C_F(-v_0) + C|_{t=0}$$

$$\therefore v_0 = -\frac{1}{R_1 \cdot C_F} \int_0^t v_{in} \cdot dt + C \longrightarrow (a)$$

where C is the integration constant & is proportional to the value of the output voltage v_0 at time $t = 0$ sec

eq^n (a) indicates that O/P vtg is directly $\propto$ to the -ve integral of the I/P vtg. & inversely $\propto$ to the time constant $R_1 C_F$.

→ **freq^n Response of Integrator ckt:**

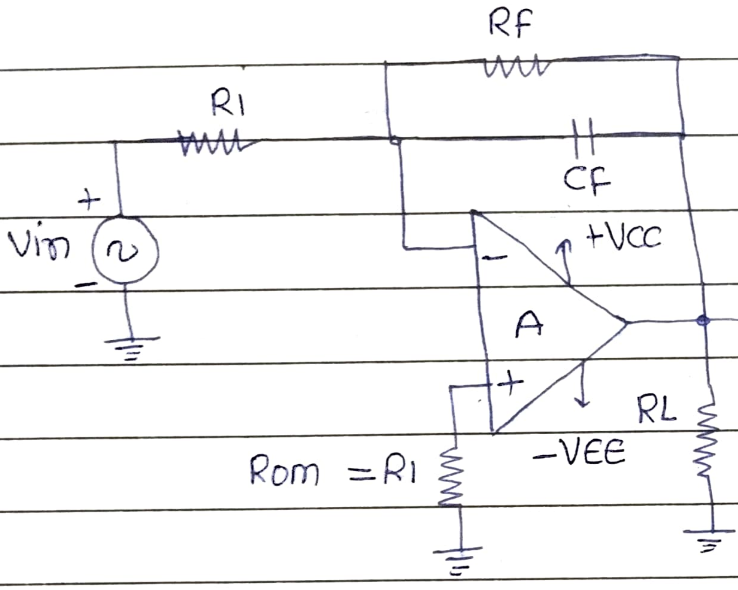


Here, f_a = 3 dB below max gain gives this freq^n i.e called as break freq^n $= \frac{1}{2\pi R_F \cdot C_F} = f_a$.

& f_b = freq^n at which gain is 0.

$$f_b = \frac{1}{2\pi R_1 C_F}$$

⇒ Practical Integrator :-



RF → minimises variations
- in O/P vtg.
- limits low freq^n gain.

$$v_o = -\frac{1}{R_1 \cdot C_F} \int_0^t v_{in} \cdot dt$$

with VOUT = 0V

- for proper operation of the ckt.
- value of f_a & in turn R_1C_F & R_FC_F should be selected such that $f_a < f_b$.

e.g if $f_a = f_b / 10$ then $R_F = 10 R_1$

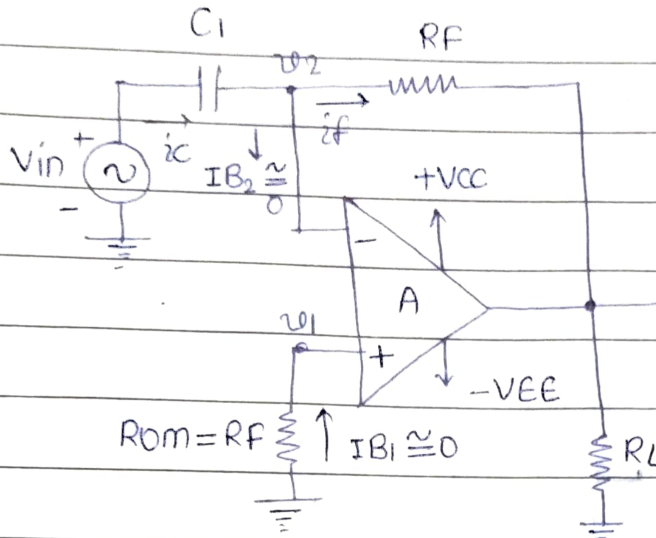
- i.e $T \geq R_FC_F$. $(R_1C_F \geq R_FC_F)$.

where, $R_FC_F = \frac{1}{2\pi f_a}$.

- Application:
- Analog computers
- ADC
- signal waveshaping ckts.

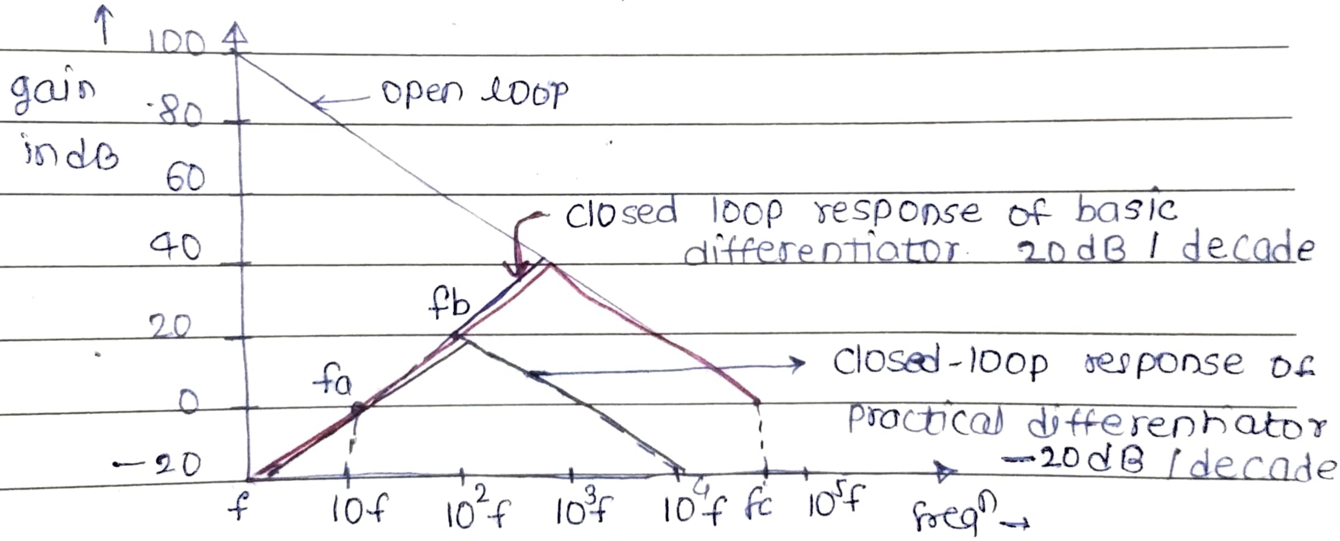
⇒ **Differentiator :→ (differentiation ampr)**

- The O/P w/f is the derivative of the I/P w/f.
- It can be constructed from a basic inv. ampr. if an I/P resistor R_1 is replaced by a capacitor C_1.



$$v_0 = -R_F \cdot C_1 \cdot \frac{dv_{in}}{dt}$$

: Basic differentiator :



: frequency Response :

Apply KCL at node v_2,

$$i_C = I_B + i_f$$

$\because$ $I_B \simeq 0$

$$i_C = i_f$$

$$C_1 \cdot \frac{d}{dt}(v_{in} - v_2) = \frac{v_2 - v_0}{R_F}$$

But $v_1 = v_2 \simeq 0V$, because A is very large.

$$\therefore \quad C_1 \frac{dv_{in}}{dt} = -\frac{v_0}{R_F}$$

or $v_0 = -R_F C_1 \cdot \frac{dv_{in}}{dt}$

- In freq^n response,

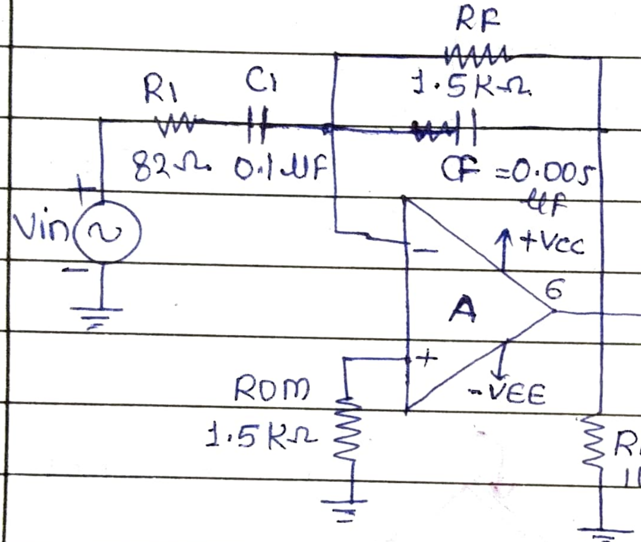
f_a = freq^n at which gain is 0.

$$= \frac{1}{2\pi R_F C_1}$$

~~fb = unity gain B/W.~~

f = relative operating freq^n.

⇒ Practical differentiator :-



$v_0 = -R_F \cdot C_1 \cdot \frac{dv_{in}}{dt}$

if $R_F C_1 > R_1 C_1$ or $R_F \cdot C_F$

R_1 & C_F :- To eliminate high freq^n noise problem.
- To increase stability.

f_b = gain limiting freq^n $= \frac{1}{2\pi R_1 C_1}$

- Generally value of f_b i.e R_1C_1 & R_FC_F should be selected such that,

$$f_a < f_b < f_c$$

where, $f_a = \frac{1}{2\pi R_F C_1}$

$$f_b = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi R_F C_F}.$$

f_c = unity gain freq^n in open loop

- for proper differentiation,

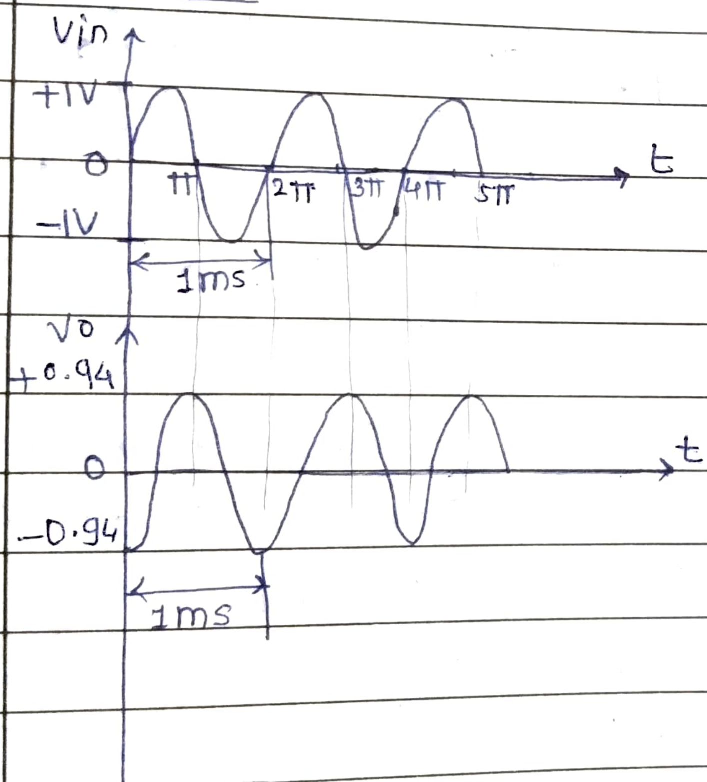
$$T \geq R_F \cdot C_1$$

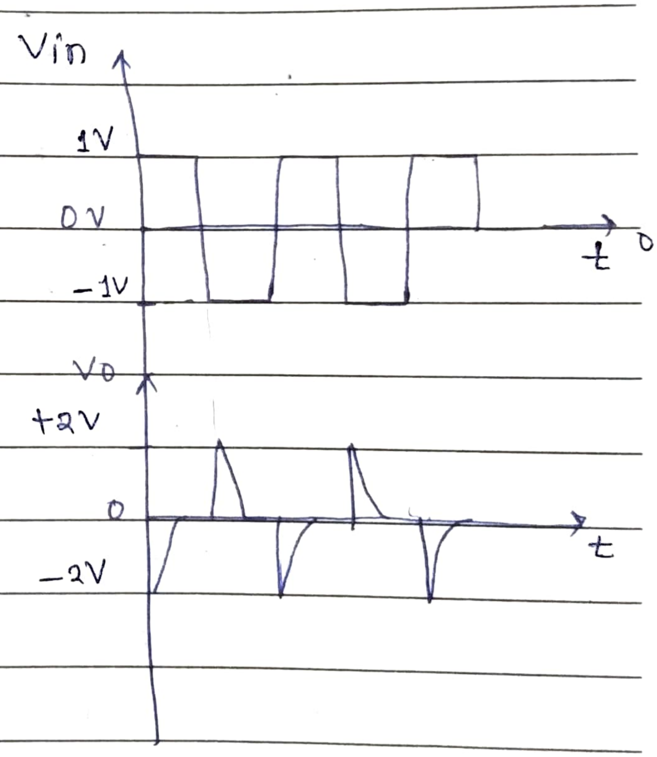
- For workable differentiator, Design steps are:
  - select f_a = highest I/P freq^n to be differentiated
  - Assume $C_1 < 1\mu F$ & calculate value of R_F.
  - choose $f_b = 20 f_a$ & calculate value of R_1 & C_F so that $R_1 C_1 = R_F C_F$.

→ Applications :-

- waveshaping ckts to detect high freq^n components in I/P signal
- As a rate of change detector in FM modulators.

→ waveforms:

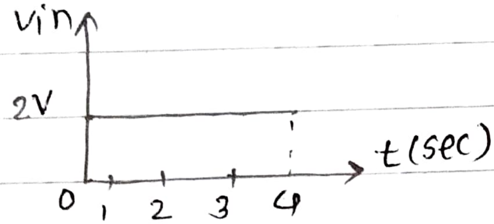


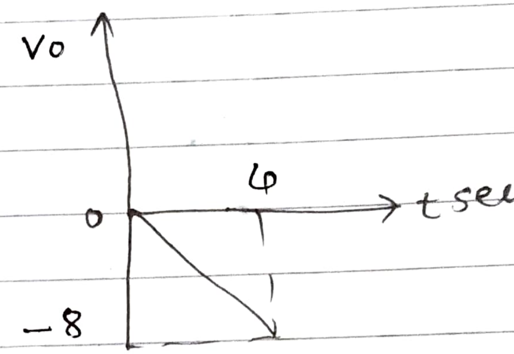


- gain of ckt ↑ with ↑ in I/P freq^n at rate 20dB/decade.
  & X_{C1} (I/P impedance) ↓ when I/P freq^n ↑.
- ∴ ckt becomes very susceptible to noise.
  when amplified this noise overrides the differentiated O/P.

Examples based on Integrator & differentiator:

1). In the basic integrator ckt, if $R_1 C_F$ = 1 sec & I/P vin is a step I/P (DC vtg) as shown below. Determine the O/P vtg. vo & sketch it. Assume that op-amp is initially nulled. ($V_{OOT} = 0$).





$$V_O = -\int_0^{t=4} 2\,dt$$

$$= -\left[\int_0^1 2\,dt + \int_1^2 2\,dt + \int_2^3 2\,dt + \int_3^4 2\,dt\right] = -8\,V.$$

2) Design a differentiator to differentiate an I/P signal that varies in freqn from 10 Hz to about 1 KHz.

→ 1. f_a = 1 KHz - max I/P freqn.

$$= \frac{1}{2\pi R_F C_1}$$

Let C_1 = 0.1 μF then

R_F = 1.59 KΩ ≙ 1.5 KΩ -- std. value.

2) f_b = 20 KHz (∵ $f_b = 20 f_a$). R_1 = 79.5Ω

($R_1 C_1 = R_1 C_F$). Let R_1 = 82Ω

∴ C_F = 0.0055 μF.

$R_{OM} = R_F$

Introduction

An electronic filter ~~is a~~ in general, is frequency selective circuit that allows a specified range of freq.ies to pass through & blocks or attenuates signals of frequencies outside this band.

A filter circuit has at least one pass band so that the signals can easily pass to the output. Similarly, it must have at least one attenuation band so that the signals can be blocked or attenuated to the output.

⇒ **Necessity of filter circuits**

Many times in electronic systems, the desired signal contains some unwanted signals called noise which must kept away in order to get correct information. Also, sometimes it is necessary that the signals of certain frequencies should pass at the output & remaining signals should be attenuated. ∴ there is necessity to filter circuits to pass certain range of frequencies & to attenuate other unwanted frequencies.

⇒ **Classification of filters**

→ **Analog or Digital filter**

The analog filters are designed to process the analog i/p signals ~~whereas the~~ also the o/p of filter is analog signal.

[freq.ies → frequencies]

The digital filters work entirely in digital domain. They use the digital data as the i/p signal.

→ **Passive & active filter**

Passive filter are using only passive components such as resistors, capacitors & inductors. They donot use active device such as FET or transistors or OPAMP. Also this filter have low efficiency & their frequency response characteristics are not sharp.

Active filters use active devices such as an OPAMP or transistors alongwith the passive components. This filter have more sharp frequency response characteristics & have many advantages.

→ **Audio frequency or Radio frequency filters.**

R-C filters can operate only in audio freq. range where as L-C filters or crystal filters are used in radio frequency range.

Filters

- Low pass filter
- High pass filter
- Bandpass filter
- Band stop filter
- All pass filter

Chintan Jotwa

→ Merits & Demerits of active filters over passive filters

— Merits

i) Flexibility in gain & frequency adjustment.

In passive filter i/p signal is attenuated while passing passing through filter but in active filter as an OPAMP can provide a gain, the i/p signal is not attenuated. In addition, the active filter is easy to tune, ∴ easy freq. adjustment is possible.

ii) No loading problem

The OPAMP has high i/p resistance & low o/p resistance ∴ active filter using OPAMP donot load the i/p source or load.

iii) Low Cost

Due to easy availability of variety of cheaper OP-AMPs & due to absence of inductors in the active filters, the active filters are more economical than the passive filters.

iv) No Insertion loss

Active filters do not exhibit any insertion loss.

v) Passband gain

Active filter can be designed to provide some passband gain. In passive filter not possible.

vi) Interstage isolation & control of inductor

Active filter allows for interstage isolation for control of i/p & o/p resistance.

vii) Small component size

Component required for active filters are of smaller size as compared to those used for passive filter.

viii) Use of inductors can be avoided

It is possible to avoid the use of inductor in the complex active filters, this reduces the size of filters to great extent.

→ **Demerits**

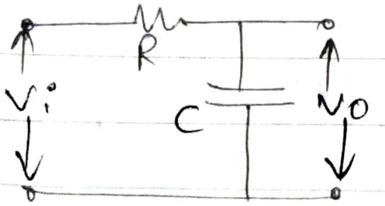
i) Active filters require dc power supply for their operation.

ii) Active filters are limited in their frequency range because an OPAMP has a finite gain bandwidth product.

iii) Active filter can not handle large amounts of power, while passive filter can handle hundreds of watts.

Concept of Passive & Active filter.

→ Passive RC low pass filter.



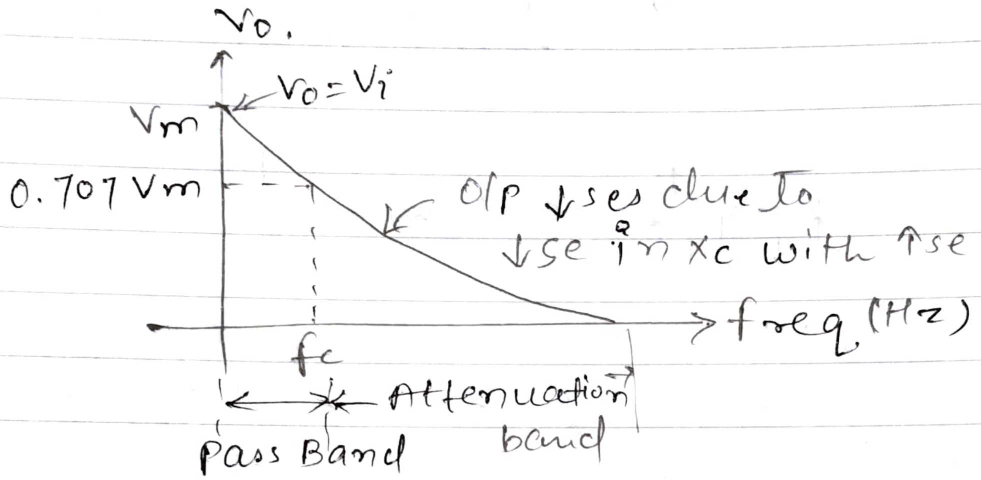
It is basically an RC integrator circuit. Here we have used only passive components.

o/p is taken across capacitor.

$$V_o = \frac{X_c}{\sqrt{R^2 + X_c^2}} V_i$$

→ At $f = 0$ the reactance $X_C = \frac{1}{2\pi f C} = \infty$. ∴ $V_o = V_i$

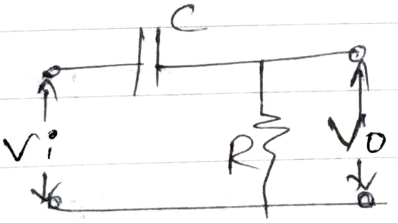
→ But if freq. ↑ses, the X_C i.e. capacitor reactance X_C will decrease., Hence the o/p voltage will ↓se because as o/p is, $\frac{X_C}{\sqrt{R^2+X_C^2}} \times V_i$



→ This circuit is called as low pass filter since it passes the low freq. signal.

→ The cutoff freq. fc corresponds to an o/p v/g of ~~0.707~~ 0.707 Vm or 3 dB below maximum. This freq. will separates the pass band & stop band of the filter.

→ RC passive high pass filter



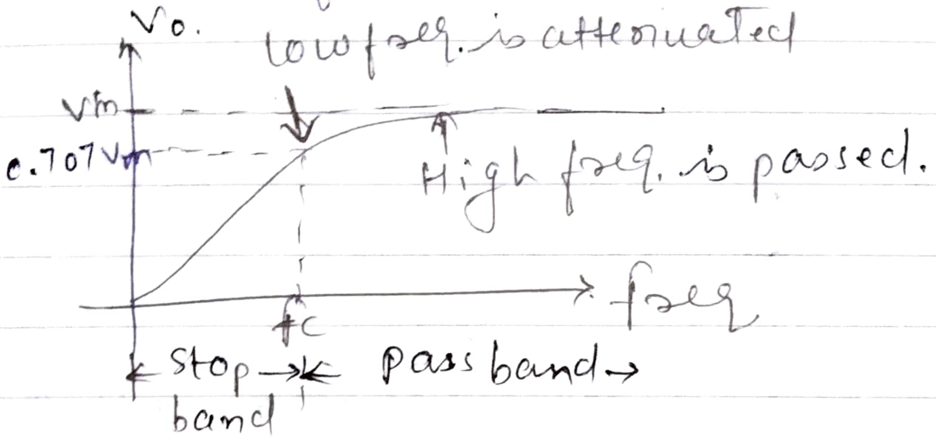
This circuit diagram shows that this is an RC passive high pass filter.

Here o/p is now being taken across the R rather than capacitor.

o/p voltage is given by $V_o = \frac{R}{R - jX_C} V_i$

$$\therefore V_o = \frac{R}{\sqrt{R^2+X_C^2}} V_i$$

→ At $f=0$, $X_C=\infty$, hence $V_0=0$. But freq. of i/p. signal ↑ses, X_C will ↓se & o/p voltage will ↑se as shown in fig.



→ This circuit attenuates low freq. & passes High freq. signal. ∴ called as high pass filter.

→ But at very high freqies the reactance of C is almost '0' & o/p voltage is equal to i/p v/g.

But Disadvantages of Passive filter.

→ These circuit can not provide gain.

→ There is no isolation between i/p & o/p.

→ If inductors are used then circuit becomes bulky.

2. Advantages of Passive filter

→ They donot need additional dc power supplies for operation

→ No limitation on the freq. range.

→ They can handle Large voltages, currents & powers.

Concept of Active filter

It uses passive as well as active device such as OPAMP.
If we are using OPAMP then the value of R, & Rf decides value of passband gain.

→ Advantages of Active filter
→ Gain is provided.
→ Frequency response is more sharp.

→ Comparison between active & passive filter.

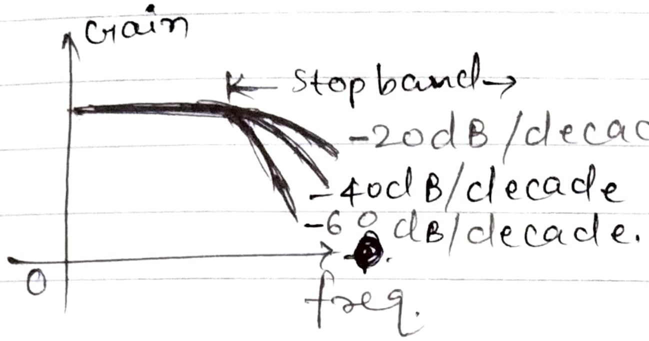
Parameter	Passive filter	Active filter
Component used	R & C or L & C	RC or LC & OPAMP
Active component	Not used	OPAMP
Gain	Less than 1	Greater than 1
Isolation betn i/p & o/p	Absent	Present.
Freq. Response.	Not sharp	sharp.

→ Cut off frequency → It is the frequency which separates pass band & stopband.

→ Pass band → It is the band or range of frequency which are allowed to pass through to the o/p by filter without any attenuation.

→ Stopband → It is the band or range of freq.ies which are not allowed to pass through to the o/p by the filter.

→ Roll off Rate

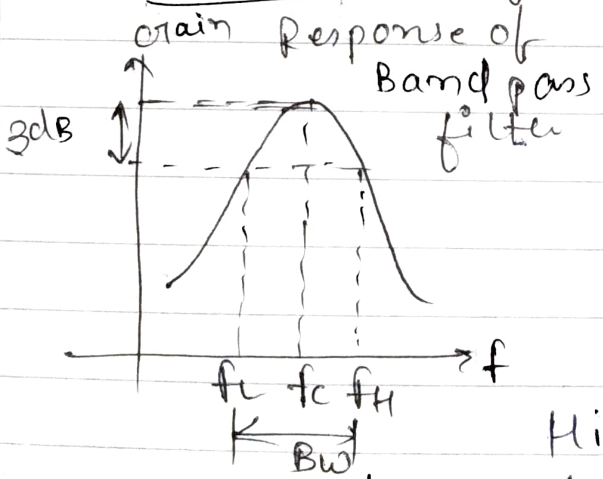


The rate at which it falls off is called as the roll off rate.

→ The roll of rate is decided by the order of the filter. If we ↑se the filter order by 1, then roll off rate is ↑sed by 20dB/decade.

→ If we ↑se the order of the filter then circuit become too much complex but response becomes more & more sharp.

→ Q (quality factor) & Bandwidth of filter



Relation between center freq., Q & bandwidth is as follows. $Q = \frac{f_c}{BW}$

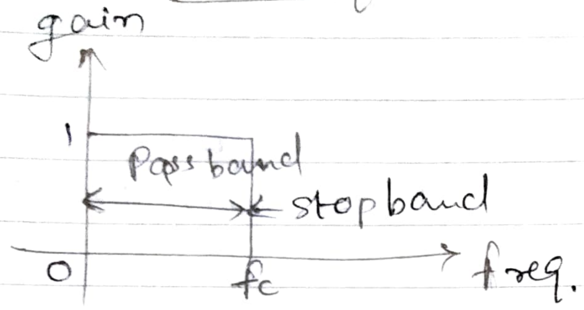
Higher the value of 'Q' sharper the freq. response.

→ Bandwidth is defined as the difference between two cut off freq.ies f_H & f_L

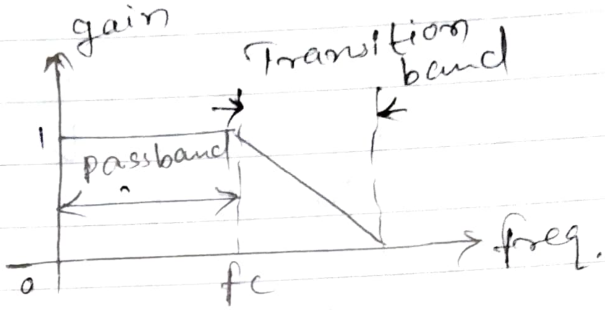
$\therefore BW = f_H - f_L$

Ideal & Actual characteristics of filter.

→ **Low pass filter**



Ideal

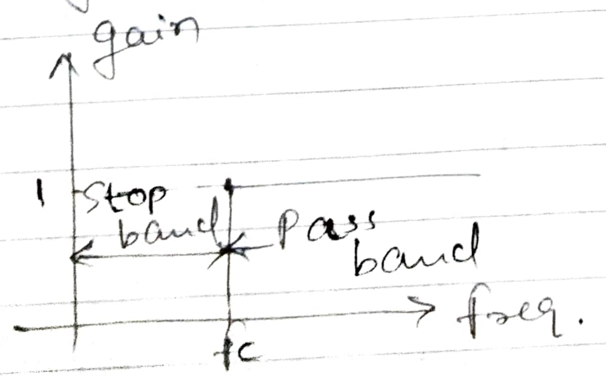


Practical

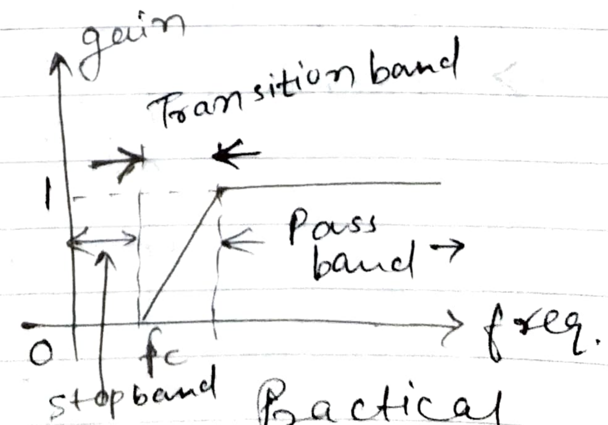
→ In ideal chara. gain is const. from 0 Hz to High cut off freq. & these frequencies are known as passband frequencies & beyond this freq. are known as stopband freq.ies. At f = fc gain makes sudden transition to zero. ∴ freq.ies beyond fc are completely attenuated.

→ In Practical chara. filter gain doesnot change suddenly at f = fc. Instead as f ↑ses, gain reduces gradually. But by using special design techniques, precision component values & high speed OPAMPs it is possible to design practical filter having chara. very close to ideal chara.

→ **High Pass filter**



Ideal

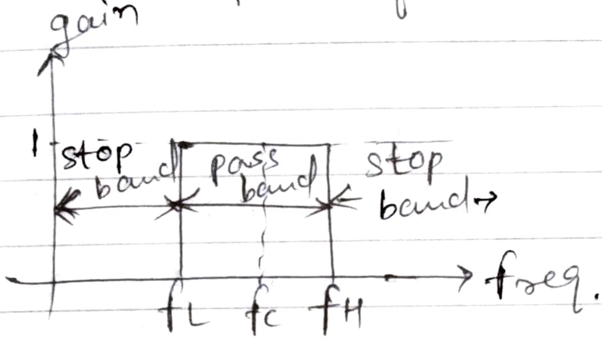


Practical

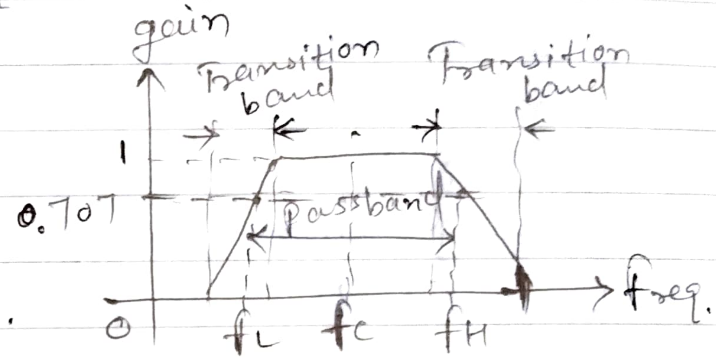
→ In ideal chara, stop band extends from $f=0$ to $f=f_c$, where f_c is the cutoff freq., The 'pass band' will be for all freq.ies above 'f_c'. The gain makes sudden transition from 0 to 1 at $f=f_c$.

→ In practical high pass filter gain changes gradually, ∴ it has finite transition band.

→ Band pass filter



Ideal

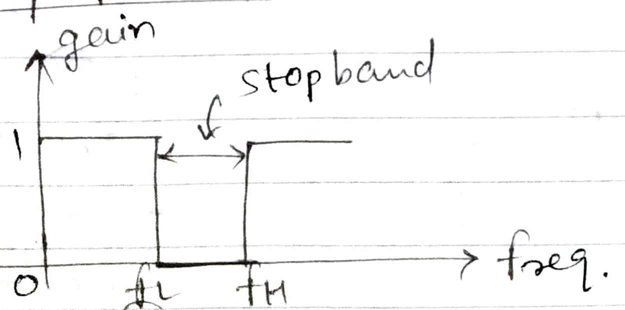


Practical

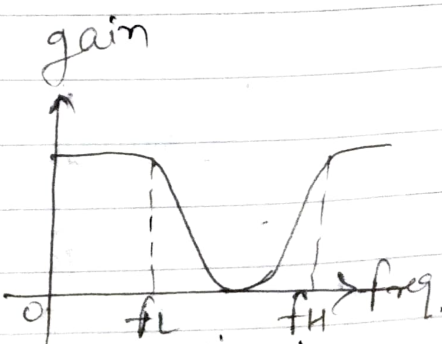
→ In ideal, passband extends between the two cut off frequencies f_L & f_H with $f_H > f_L$. The freq.ies outside this passband lie in the stopband or attenuation band. The gain of an ideal filter will make sudden transition from 0 to 1 at $f=f_L$ & from 1 to 0 at $f=f_H$.

→ In practical, gain does not change suddenly at $f=f_L$ & $f=f_H$ instead it changes gradually.

→ Bandstop filter



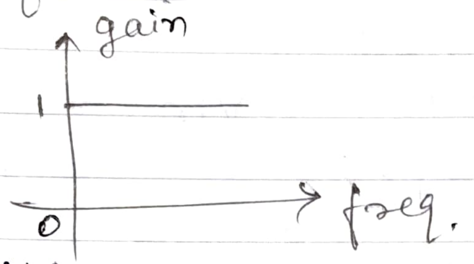
Ideal.



practical.

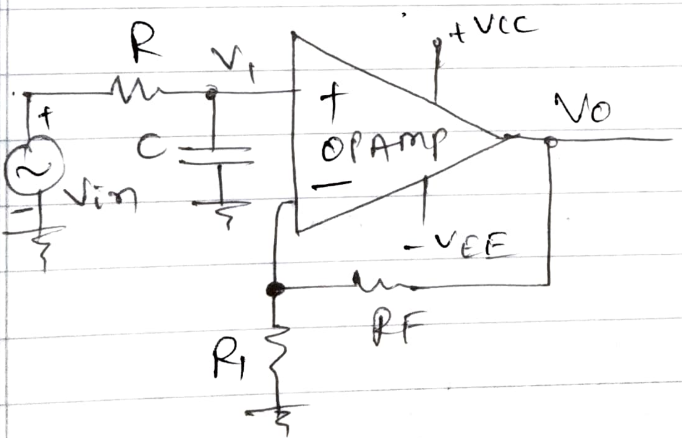
→ It is opposite to Bandpass filter ie. complementary to the bandpass filter.
It blocks out the freq. within stop band.
→ In ideal case it will suddenly take transition but practically it will not.

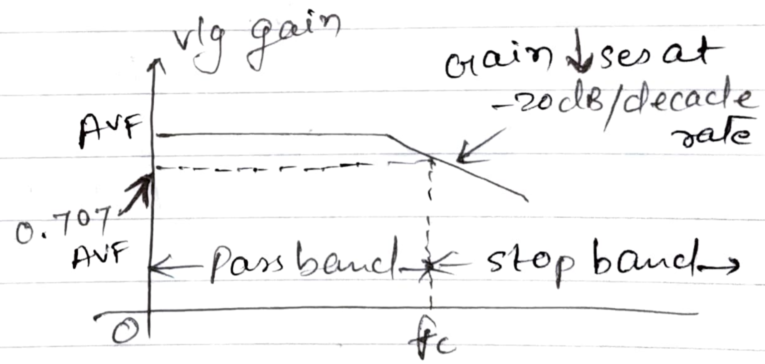
→ All pass filter



This filter passes all the freq. ∴ it is known as all pass filter.

First order Low pass filter





→ First order low-pass filter which is also known as one pole lowpass filter.
→ This circuit uses R-C n/w for filtering & the OPAMP is used in non-inverting configuration.
→ R & C values decides the cut off frequency of filter whereas R_1 & RF will decide its gain.

As the opamp is used in non-inverting config.

$$A_{VF} = 1 + R_F/R_1$$

→ We want to obtain the expression for V_1. It is nothing but voltage across 'C'. The R & 'C' forms a voltage divider across i/p v/g. Vin.

$$\therefore V_1 = \frac{-jX_C}{R - jX_C} \cdot V_{in}$$

But $X_C = 1/2\pi fC$

$$\therefore V_1 = \frac{-j(1/2\pi fC)}{R - j(1/2\pi fC)} V_{in} = \frac{-jV_{in}}{2\pi fRC - j}$$

Dividing numerator & denominator by $-j$,

$$\therefore V_1 = \frac{V_{in}}{1 - \frac{2\pi fRC}{j}}$$

But $1/j = -j$

$$V_1 = \frac{V_{in}}{1 + j2\pi fRC}$$

Now, $V_0 = A_{VF} \cdot V_1$

$$\therefore V_0 = \left(1 + \frac{R_F}{R_1}\right) \frac{V_{in}}{1 + j2\pi fRC}$$

$$\therefore \frac{V_0}{V_{in}} = \frac{A_{VF}}{1 + j2\pi fRC}$$

Now put $2\pi RC = 1/f_c$

& f_c = Cut off freq. = $1/2\pi RC$

$$\therefore \frac{V_0}{V_{in}} = \frac{A_{VF}}{1 + j(f/f_c)}$$

Now taking mod on both sides,

$$\left|\frac{V_0}{V_{in}}\right| \cancel{A_{VF}} = \frac{A_{VF}}{\sqrt{1 + (f/f_c)^2}}$$

Now there are three cases,

I> At very low frequencies $f < f_c$,

$$\left|\frac{V_o}{V_{in}}\right| \approx A_{VF} \quad \text{ie. maximum gain.}$$

II) At $f = f_c$

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{[1+1]^{1/2}} = \frac{A_{VF}}{\sqrt{2}} = 0.707\, A_{VF}$$

At $f = f_c$ the gain of the filter reduces to 3dB since dB value of 0.707 is -3dB.

III) At very High frequency ie. $f > f_c$

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{(f/f_c)^2}} = \frac{A_{VF}}{f/f_c}$$

& As the value of f/f_c is greater than '1'

$$\therefore \left|\frac{V_o}{V_{in}}\right| < A_{VF}$$

ie. the filter gain will keep decreasing with ↑se in frequency. This ↓se takes place at the rate of -20 dB/decade.

It means if frequency is ↑sed 10 times then gain is ↓sed 20dB which is shown by '−ve' sign.

→ Designing Steps

→ Choose the ~~ct~~ cut off freq. f_c,

→ Select a value of 'C' usually between 0.001 to 0.1 μf.

→ Calculate R using $R = 1/2\pi f_c C$

→ Select the value of R_1 & R_F depending on desired value of A_{VF} ie. $A_{VF} = 1 + R_F/R_1$

Eg Design first order low pass filter for pass band gain = 2
cut off freq $f_c = 10\,kHz$

→ gain is, $A_{VF} = 1 + \frac{R_F}{R_1}$

But $A_{VF} = 2$

$\therefore \frac{R_F}{R_1} = 1$

Let us choose $R_F = 10\,k\Omega$

$\therefore R_F = R_1 = 10\,k\Omega$

Now choose $C = 0.001\,\mu F$

But $f_c = \frac{1}{2\pi RC}$

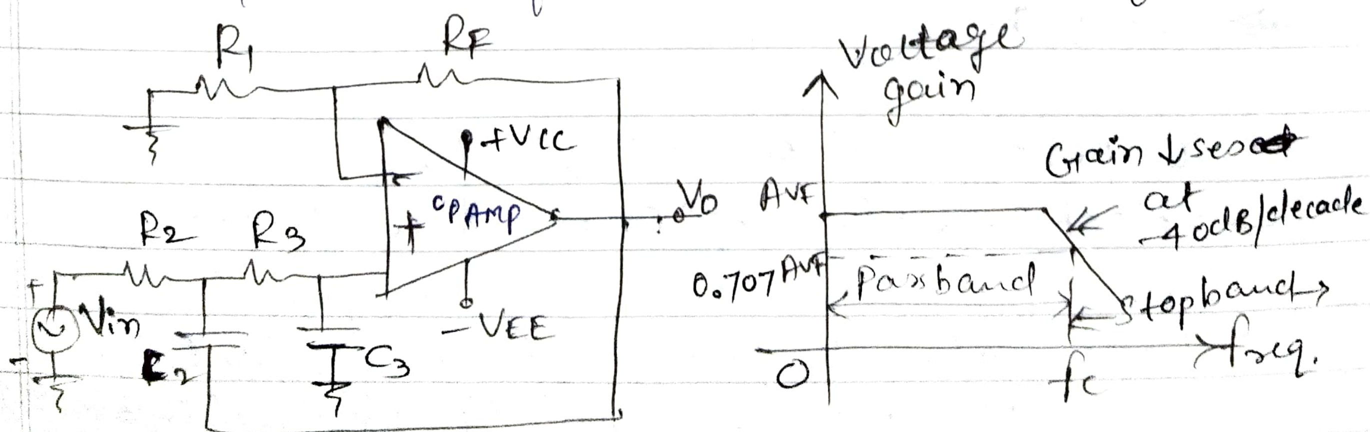
$\therefore R = \frac{1}{2\pi f_c C}$

$\therefore R = \frac{1}{2\pi \times 10 \times 10^3 \times 0.001 \times 10^{-6}}$

$\therefore R = 15.9\,k\Omega$

✱ Second order Low pass filter

The second order low pass filter can be obtained simply by inserting an additional RC n/w into first order low pass filter.



The response shows that in stopband rolloff rate is $-40dB/decade$.

→ The resistors R_F & R_1 will decide the gain of the filter.

→ Cut off freq. 'f_c' is determined by R_2, C_2, R_3, C_3 as follows,

$$f_c = \frac{1}{2\pi \cdot \sqrt{R_2 R_3 C_2 C_3}}$$

The voltage gain magnitude is given by,

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{1+(f/f_c)^4}}$$

where $A_{VF} = 1 + \frac{R_F}{R_1}$

f = Freq. of i/p signal.

f_c = Cut off frequency.

→ **Design Procedure.**

→ Choose the cutoff freq. f_c.

→ Assume $R_2 = R_3 = R$ & $C_2 = C_3 = C$ & then choose 'C' in the range of $0.001\,\mu f$ to $0.1\,\mu f$.

→ Calculate the value of R as,

$$R = \frac{1}{2\pi f_c C}$$

→ Now depending upon gain choose the value of R_1 & R_F. Select gain $= 1.586$ $\therefore R_F = 0.586 R_1$

→ Case I At $f < f_c$, $\left|\frac{V_o}{V_{in}}\right| \approx A_{VF}$ ie max gain

Case II At $f = f_c$, $\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{2}} = 0.707\, A_{VF}$

Case III At $f > f_c$

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{(f/f_c)^4}} \quad \therefore \left|\frac{V_o}{V_{in}}\right| < A_{VF}$$

Filter:

Derive expression for second-order LPF (Butterworth filter). FH

$$FH = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$$

f: For analysis, we will use Laplace transform.
All components & ckt parameters are expressed in s domain where $s = j\omega$

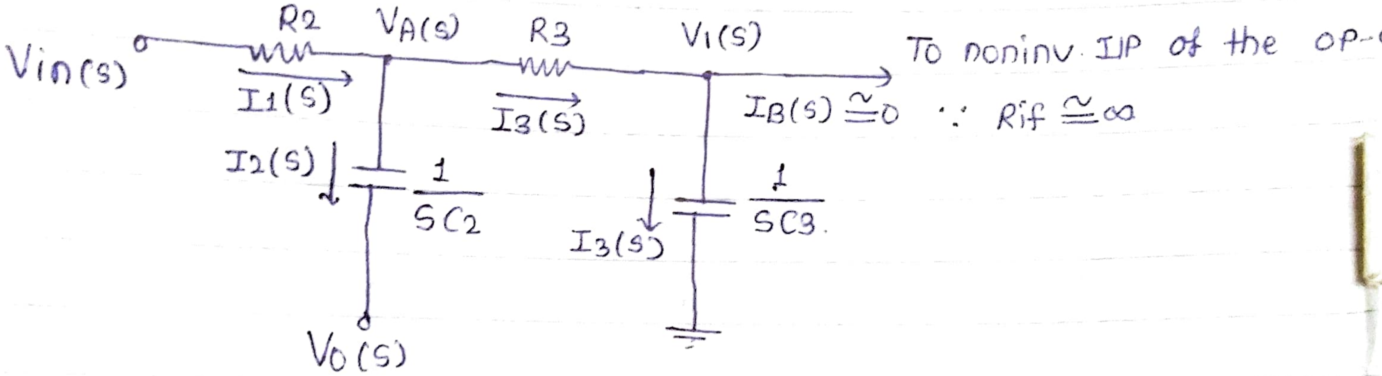


Fig: transformed into S domain:

By Kirchhoff's current law at node VA(s),

$$I_1 = I_2 + I_3.$$

or
$$\frac{V_{in} - V_A}{R_2} = \frac{V_A - V_o}{1/sC_2} + \frac{V_A - V_1}{R_3} \quad \text{--} (1)$$

But
$$V_1 = \frac{1/sC_3}{R_3 + (1/sC_3)} \cdot V_A \qquad \because R_{if} \cong \infty,\ I_B \cong 0A.$$

$$= \frac{V_A}{R_3 C_3 s + 1}.$$

or $V_A = (R_3 C_3 s + 1) \cdot V_1$.

substitute value of VA in eqⁿ (1).

$$\therefore \cancel{\frac{V_{in}}{R_2} - \left[\frac{1/sC_3}{R_3 + (1/sC_3)} \cdot \frac{V_A}{R_2}\right]} =$$

$$\frac{V_{in} - (R_3 C_3 s + 1) \cdot V_1}{R_2} = \frac{(R_3 C_3 s + 1) \cdot V_1 - V_o}{1/sC_2} + \frac{(R_3 C_3 s + 1) \cdot V_1 - V_1}{R_3}$$

$$\therefore V_1 = \frac{R_3 \cdot V_{in} + (R_3 R_2 C_2 s) \cdot V_o}{(R_3 C_3 s + 1)(R_2 + R_3 + R_3 R_2 C_2 s) - R_2}$$

But $V_o = A_F \cdot V_i$

where $A_F = 1 + \frac{R_F}{R_1}$ ... NON-INV config.

$$\therefore \quad V_o = \frac{(A_F) \cdot [(R_3)(V_{in}) + (R_3 R_2 C_2 S)(V_o)]}{(R_3 C_3 S + 1)(R_2 + R_3 + R_3 R_2 C_2 S) - R_2}.$$

Re-arrange to get $\frac{V_o}{V_{in}}$

$$\frac{V_o}{V_{in}} = \frac{A_F}{S^2 + \frac{(R_3 C_3 + R_2 C_3 + R_2 C_2 - A_F R_2 C_2) S}{R_2 R_3 C_2 C_3} + \frac{1}{R_2 R_3 C_2 C_3}}$$

For freq^{ns} above f_H, gain of 2nd order LPF rolls off at the rate of -40 dB/dec.

$\therefore$ The denominator quadratic in the gain (V_o/V_{in}) eq^n must have two real & equal roots.

This means that,

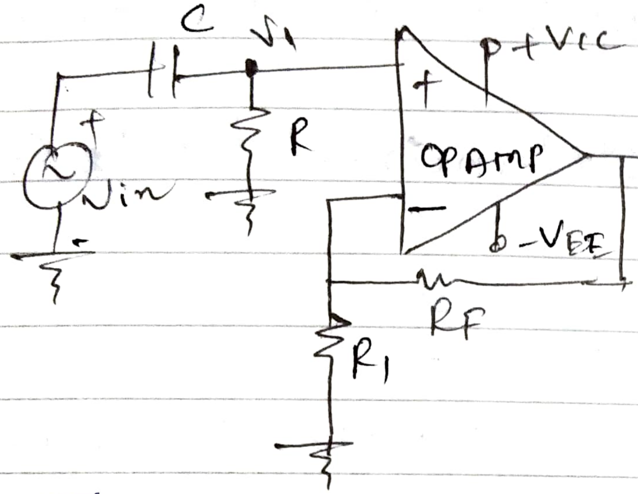
$$\omega^2_H = \frac{1}{R_2 R_3 C_2 C_3}$$

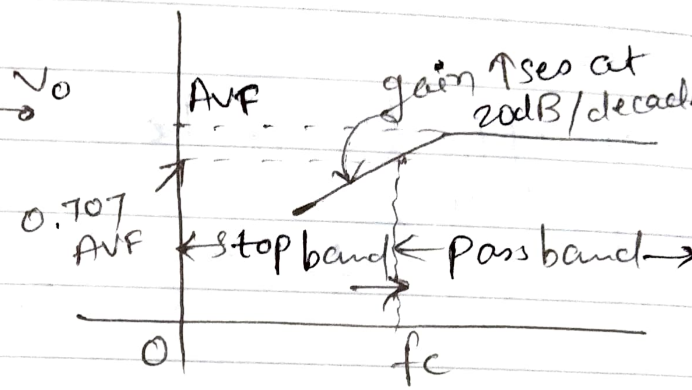
Or

$$\omega_H = \frac{1}{\sqrt{R_2 R_3 C_2 C_3}}$$

$$\boxed{\therefore \; f_H = \frac{1}{2\pi \sqrt{R_2 R_3 C_2 C_3}}}$$

First Order High pass filter





The RC component decide the cut off freq. of high pass filter & RF and R_1 decides the gain.

$$A_{VF} = \left[1 + \frac{R_F}{R_1}\right]$$

→ Determining the V_1 using the concept of voltage divider. V_1 is nothing but voltage across R.

$$\therefore V_1 = \frac{R}{R - jX_C} V_{in}$$

But $X_C = 1/2\pi f C$

$$\therefore V_1 = \frac{R}{R - j/2\pi f C} V_{in} = \frac{1}{1 - j/2\pi f C R} V_{in}$$

substitute $2\pi RC = 1/f_c$

where f_c = cut off freq.

$$\therefore f_c = \frac{1}{2\pi RC}$$

$$\therefore V_1 = \frac{V_{in}}{1 - j(f_c/f)}$$

Now, $V_o = A_{VF} \cdot V_1$

$$\therefore V_o = \frac{A_{VF}}{1 - j(f_c/f)} \cdot V_{in}$$

$$\therefore \frac{V_o}{V_{in}} = \frac{A_{VF}}{1 - j(f_c/f)}$$

$$\therefore \left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{1 + (f_c/f)^2}}$$

Case I At low freq. ie. $f < f_c$

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{(f_c/f)^2}} = \frac{A_{VF}}{f_c/f}$$

Since $f_c/f > 1$

$$\left|\frac{V_o}{V_{in}}\right| < A_{VF}$$

Case II At $f = f_c$ ie. cut off freq = i/p freq.

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{2}} = 0.707\, A_{VF}$$

Thus filter gain is down by 3dB as 0.707 corresponds to −3dB.

Case III At $f > f_c$

$$\left| \frac{V_o}{V_{in}} \right| \approx A_{VF} \quad \text{i.e. constant}$$

Thus the gain will remain constant in the pass band.

→ **Design Procedure**

→ choose the value of cut off freq f_c.

→ select the value of 'C' betn 0.001µf & 0.1µf

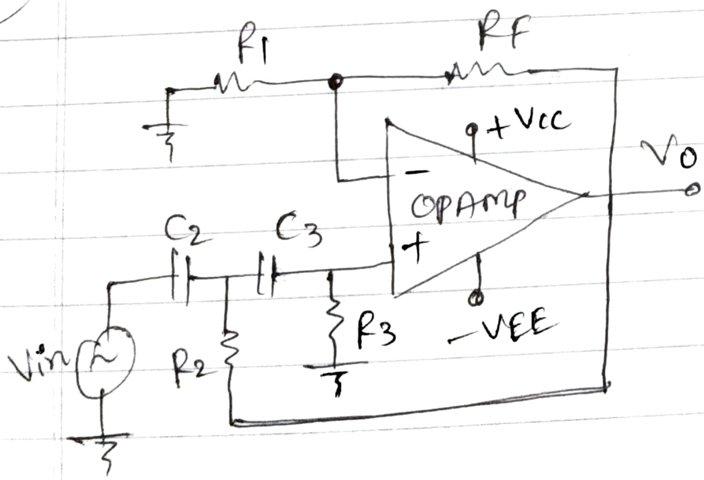
→ Calculate R using,

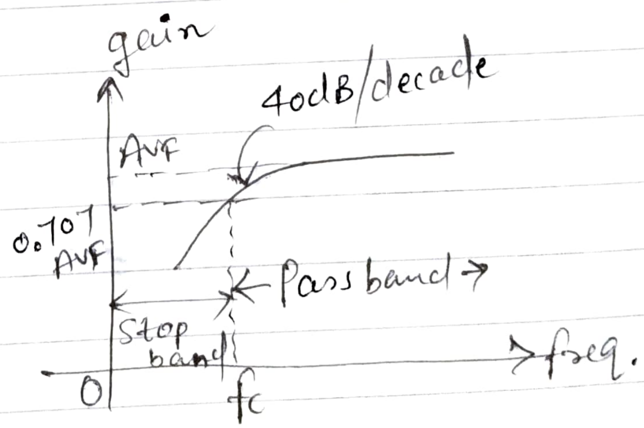
$$R = \frac{1}{2\pi f_c C}$$

→ Select the values of R_1 & R_f depending on the desired value of pass band gain.

$$A_{VF} = 1 + \frac{R_F}{R_1}$$

Second order High Pass filter





It is obtained by simply inserting CR n/w into first order high pass filter.
The R_1 & R_F will decide the gain of the high pass filter.

Cut off freq. 'f_c' is determined by R_2, R_3, C_2 & C_3.

$$f_c = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$$

gain,

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{1+(f_c/f)^4}}$$

where $A_{VF} = 1 + \frac{R_f}{R_1}$ = gain of filter.

f = i/p signal frequency

f_c = Low cut off freq.

From the response it is clear that gain ↑ses at a rate of 40dB/decade. which is twice the rate of first order & because of this the frequency response becomes sharper.

Case I when $f < f_c$

$$\therefore \left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{(f_c/f)^4}}$$

$$\therefore \left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{(f_c/f)^2}$$

Since $f_c > 1$

$$\therefore \left|\frac{V_o}{V_{in}}\right| < A_{VF}$$

Case II when $f = f_c$,

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{1+(f_c/f)^4}}$$

$$\therefore \left|\frac{V_o}{V_{in}}\right| = \frac{A_{VF}}{\sqrt{2}} = 0.707 A_{VF}$$

Case III At $f > f_c$

$$\left|\frac{V_o}{V_{in}}\right| \simeq A_{VF}$$

Designing steps

- Choose cutoff freq. f_c.
- Assume $R_2 = R_3 = R$ & $C_2 = C_3 = C$.
  Then choose value of C betⁿ 0.001μf & 0.1μf.

$$R = \frac{1}{2\pi f_c C}$$

- Calculate the value of R.
- Calculate the value of R_1 & R_F depending upon the value of gain.
  ie. $A_{VF} = 1 + \frac{R_F}{R_1}$, Select gain = 1.586
  $\therefore R_F = 0.586 R_1$

Band Pass filter

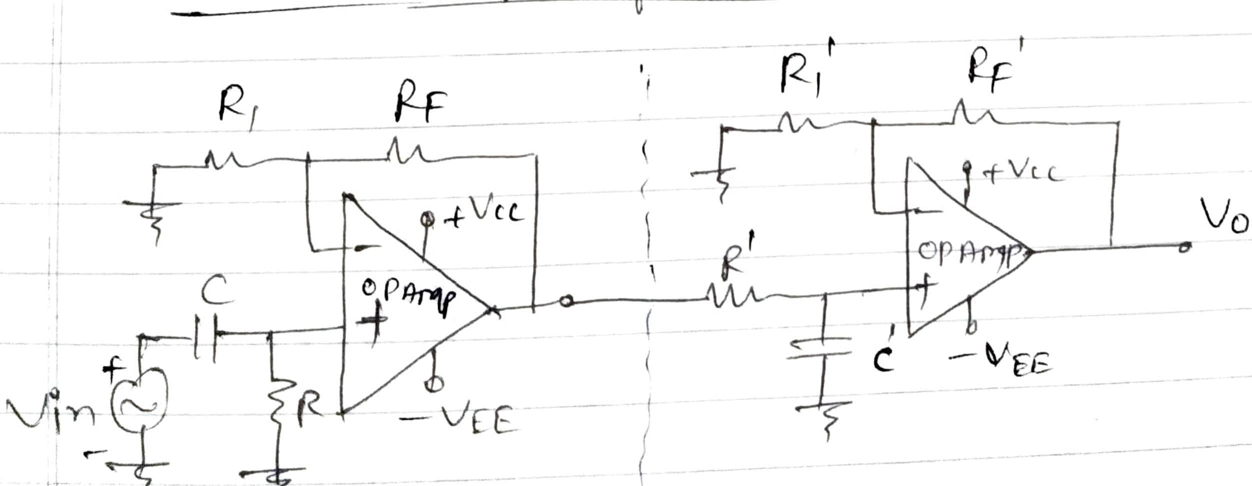
Bandpass filter has passband between two cutoff freq.ies f_H & f_L, such that $f_H > f_L$. Any i/p freq. outside this passband will be attenuated.

→ there are two types of bandpass filter.

i) Wide band pass filter. $Q < 10$

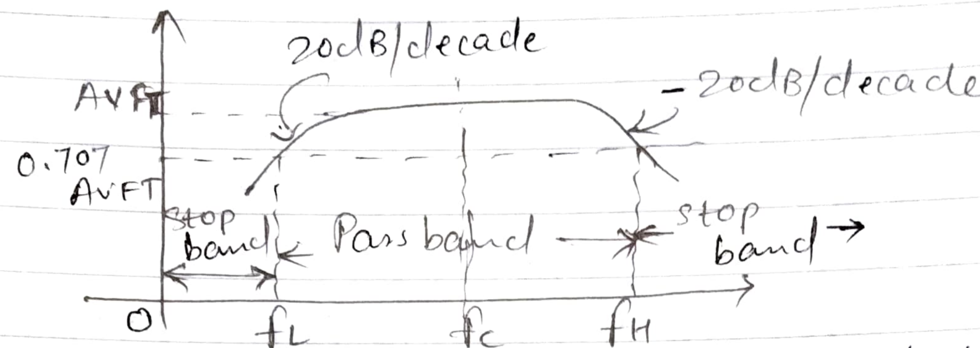
ii) Narrow band pass filter. $Q > 10$

Wide bandpass filter



← First order Highpass section → ← First order low-pass section →

It is formed by ~~s~~ cascading a 1st order high pass filter & first order low pass filter.



'fH' is cutoff freq. of low pass filter while 'fL' is cut off freq. for high pass filter.

Total pass band gain $A_{VFT} = A_{VF_1} \times A_{VF_2}$

A_{VF_1} = Gain of high pass filter

A_{VF_2} = Gain of low pass filter.

If we want sharper frequency response then ~~th~~ we can cascade second order high pass & low pass filter instead of first order.

$$f_L = \cancel{\frac{1}{2\pi\sqrt{RC}}} = \frac{1}{2\pi RC}$$

$$\& \quad f_H = \cancel{\frac{1}{2\pi\sqrt{R'C'}}} = \frac{1}{2\pi R'C'}$$

Center frequency of bandpass filter is,

$$f_c = \sqrt{f_H f_L}$$

$$Q = \frac{f_c}{f_H - f_L}$$

& the $f_H - f_L = BW$.

$$\therefore \quad Q = \frac{f_c}{BW}$$

⇒ Designing steps

→ from cut off freq of low pass filter (f_H), calculate values of R' & C'.

→ from cut off freq. of high pass filter (f_L) calculate values of R & C.

→ $A_{VFT} = A_{VF1} \times A_{VF2}$

Assume value of A_{VF1} & A_{VF2}, then calculate values of R_F, R_1, R_F' & R_1'.

Eg Design wide band pass filter for $f_L = 100Hz$, $f_H = 1KHz$ & passband gain $= 4$. Also calculate value of Q.

⇒ Assuming $C' = 0.01 \mu f$

$$\therefore R' = \frac{1}{2\pi f_H C} = \frac{1}{2\pi \times 10^3 \times 0.01 \times 10^{-6}} = 15.9 K\Omega$$

Gain of low pass filter $A_{VF2} = 2$

$$\therefore 2 = 1 + \frac{R_F'}{R_1'}$$

$$\therefore R_F' = R_1'$$

choose $R_1' = 10k\Omega$ $\therefore R_F' = 10k\Omega$

Components for low pass filter are,

$R_1' = 10k\Omega$, $R' = 15.9 K\Omega$

$R_F' = 10K\Omega$, $C = 0.01 \mu f$

Now assume $C = 0.05 \mu f$.

$$\therefore R = \frac{1}{2\pi f_L C} = \frac{1}{2\pi \times 100 \times 0.05 \times 10^{-6}}$$

$$\therefore R = 3.183 K\Omega$$

The gain of High pass filter $A_{VF1} = 2$

$$\therefore 2 = 1 + \frac{R_F}{R_1} \qquad \therefore R_F = R_1$$

Choose $R_1 = 10k\Omega$ $\therefore R_F = 10k\Omega$

Thus components for high pass filter are,

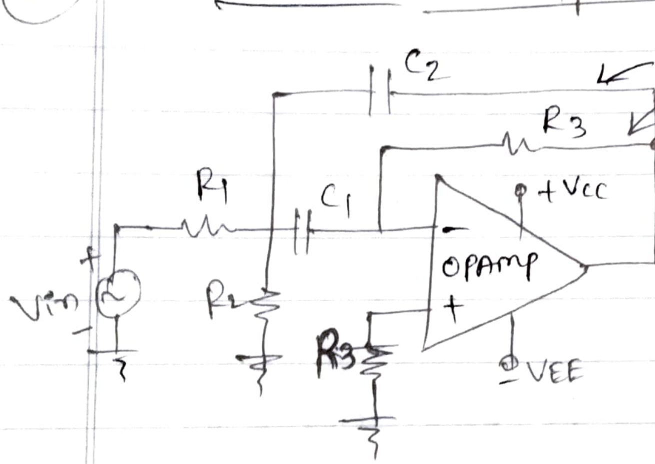
$R_1 = 10k\Omega$, $R = 31.83k\Omega$

$R_F = 10k\Omega$, $C = 0.05\mu f$

$$Q = \frac{f_c}{f_H - f_L} \quad \text{But } f_c = \sqrt{f_H \cdot f_L}$$

$$\therefore Q = \frac{\sqrt{f_H f_L}}{f_H - f_L} = \frac{\sqrt{1000 \times 100}}{(1000-100)} = 0.351$$

Narrow Bandpass filter



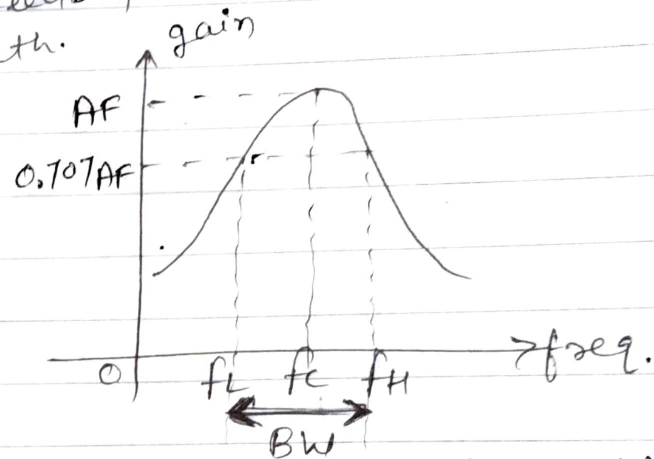


Fig. shows the circuit of narrow bandpass filter. The main advantage is it uses only one OPAMP.

→ Features

- → It uses one OPAMP.
- → It has multipath feedback
- → OPAMP is used as inverting amp.

→ Meaning of narrow bandpass.

- → It is the bandpass filter with small bandwidth & $BW = f_H - f_L$.
- → Freq. response of filter is sharper than wide BPF.
- → Q is higher than wide BPF.

→ Advantages of Narrow BPF

i) It uses only one OPAMP.
ii) We can change center freq. fc to a new value without changing gain or bandwidth.

→ Designing steps

→ select $C_1 = C_2 = C$

→ $R_1 = \dfrac{Q}{2\pi f_c C A_F}$

→ $R_2 = \dfrac{Q}{2\pi f_c C (2Q^2 - A_F)}$

→ $R_3 = \dfrac{Q}{\pi f_c C}$

→ A_F is gain at $f = f_c$ & $A_F = \dfrac{R_3}{2R_1}$

& gain A_F is $A_F < 2Q^2$

→ To change center freq 'f_c' to new value of f_c' by changing the resistance R_2 to R_2'

$$R_2' = R_2 \left(f_c / f_c' \right)^2$$

Band Reject filter (Band Stop filter)

This filter will attenuate (Stop) the frequencies in the stopband & pass all the frequencies outside the stopband.

→ Types of Band reject filter

i) Wide Band reject filters.

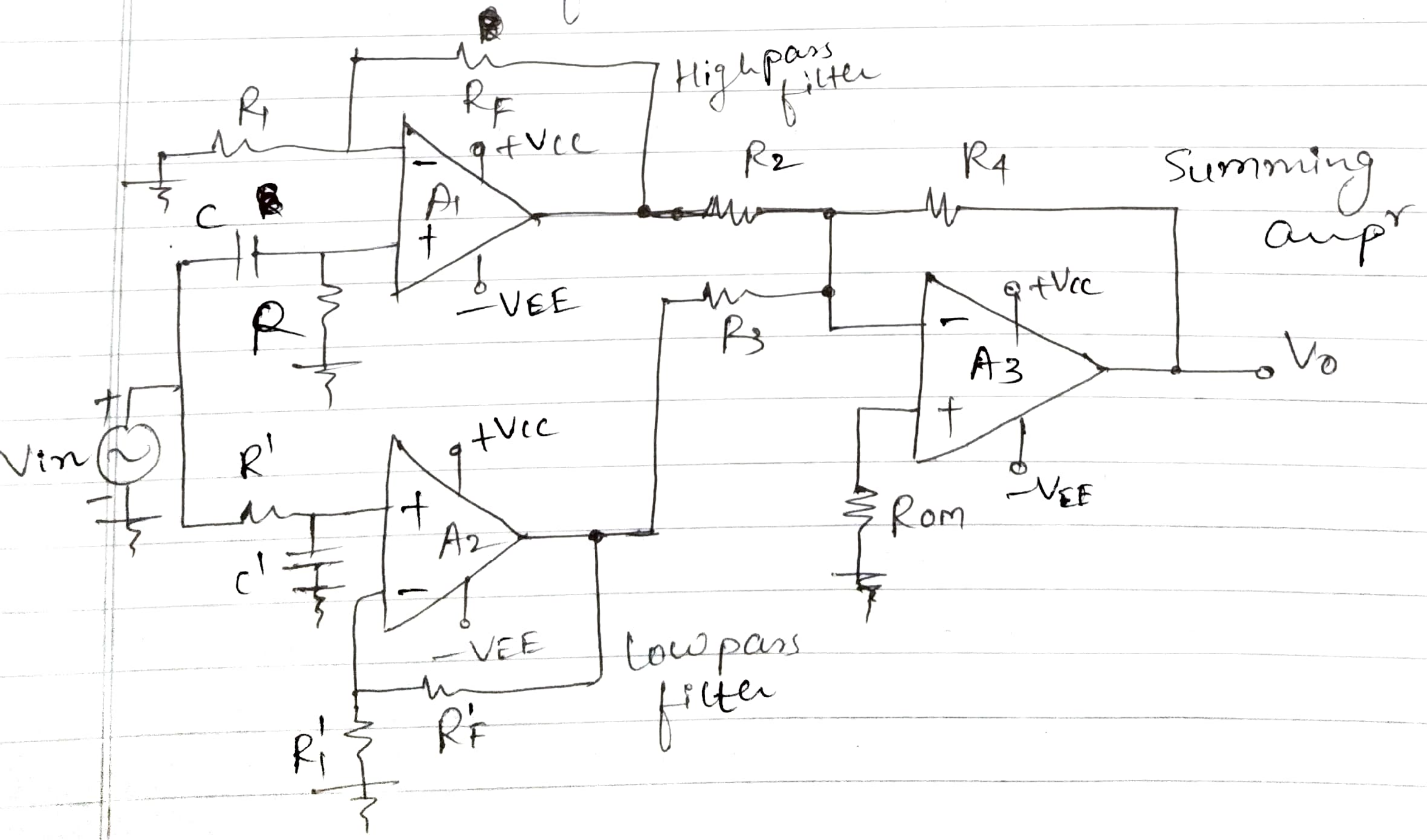
ii) Narrow Band reject filters.

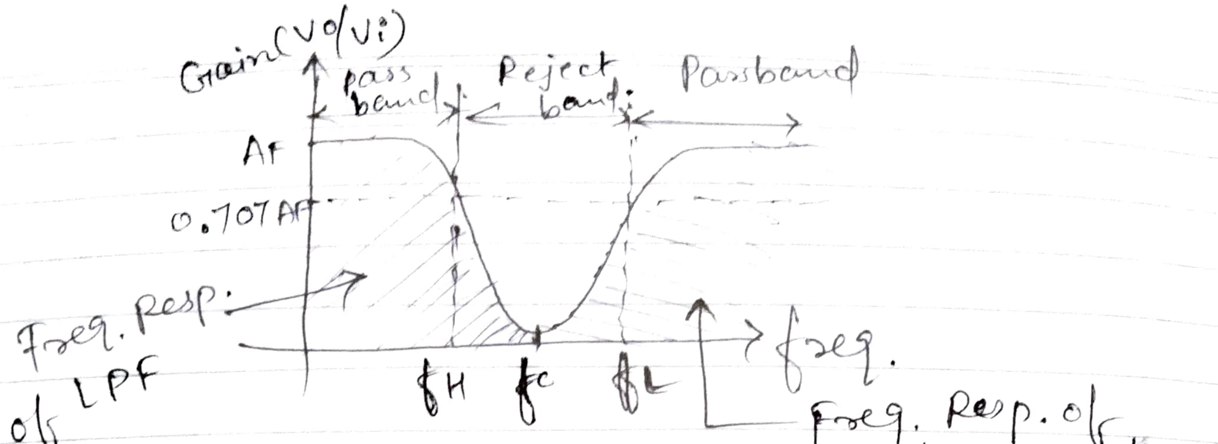
Wide band reject filter

It consist of low pass filter, high pass filter & summing ampr.

To practically obtain the freq. of wide band ~~reject~~ reject filter, we have to adjust the cut off freq. f_L of high pass filter to be higher than cut off freq. f_H of low pass filter.

ie. $f_L > f_H$.





f_H = cut off freq. of LPF
f_L = cut off freq. of HPF

Center freq. $\rightarrow$ $f_c = \sqrt{f_H \cdot f_L}$

$\rightarrow$ Design wide band reject filter having $f_H = 100Hz$ & $f_L = 2KHz$.

Sol^n

Design for High pass section

Let $C = 0.01\mu f$

$\therefore R = \dfrac{1}{2\pi f_L C} = \dfrac{1}{2\pi \times 2 \times 10^3 \times 0.01 \times 10^{-6}} = 7.96k\Omega$

Let gain = 2.

$\therefore A_{VF} = 1 + \dfrac{R_F}{R_1}$ $\quad\therefore 1 + \dfrac{R_F}{R_1} = 2$

$\therefore R_F = R_1$ & let us assume $R_F = R_1 = 10k\Omega$

Design for low pass section

Let $C' = 0.1\mu f$

$\therefore R' = \dfrac{1}{2\pi f_H C} = \dfrac{1}{2\pi \times 100 \times 0.1 \times 10^{-6}} = 15.9k\Omega$

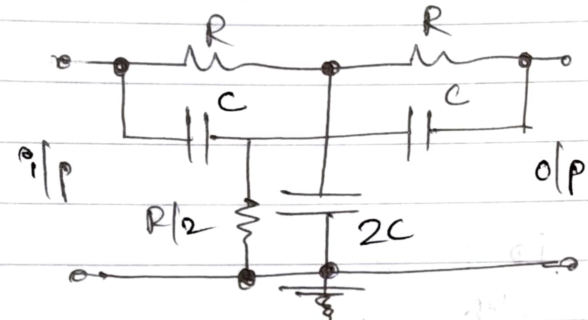
Assuming same gain $\therefore A_{VF} = 2$ $\quad R_F' = R_1' - 10k\Omega$
selecting

For summing amp. Assuming $R_2 = R_3 = R_4 = 10k\Omega$

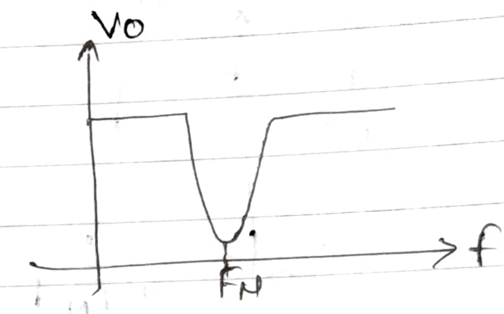
(*) Narrow Band Reject filter (Notch filter)

→ It is also called as notch filter. The 'Q' of this filter is higher than that of wide band reject filter (i.e. Q > 10)

→ Therefore it has very sharp frequency response. Notch filter commonly uses 'Twin-T' n/w as a frequency rejecting n/w.



Twin-T n/w



Freq. Response.

→ The first 'T' contains two resistors & one capacitor where as second one contains two capacitor & one resistor.

→ Maximum attenuation is obtained at a freq. called notch-out frequency 'F_N',

$$F_N = \frac{1}{2\pi RC}$$

→ The main drawback of 'T' n/w is that its 'Q' is low. Hence freq. response is not sharp.

→ This problem of low Q can be solved by using voltage follower after Twin-T n/w.

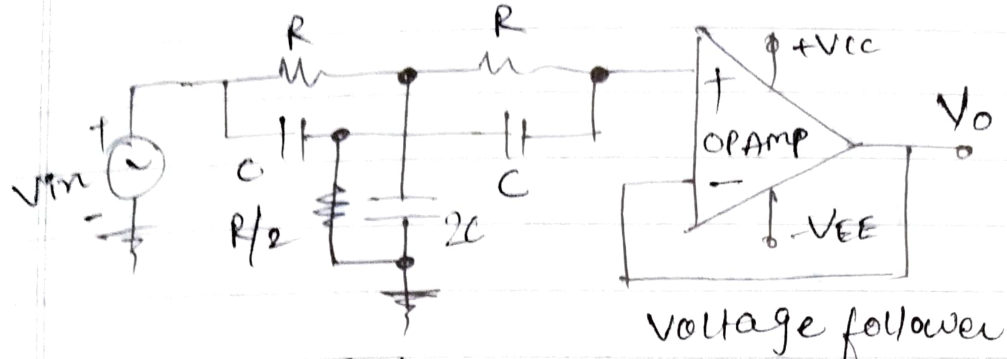
$$f_N = \frac{1}{2\pi RC}$$

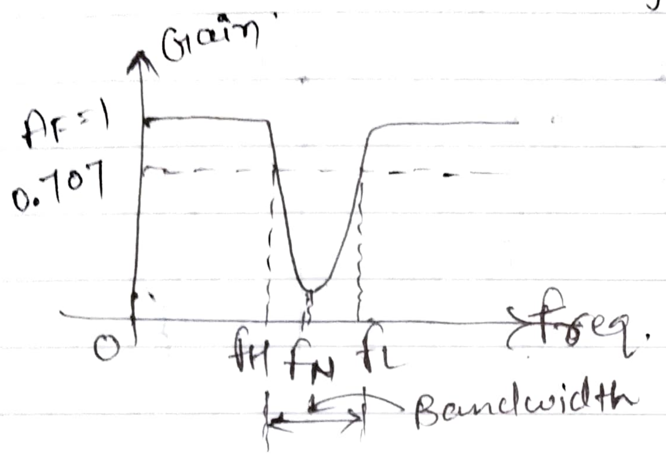
App^n

→ In Communication circuits

→ In biomedical in order to eliminate undesired freq.

→ Active notch filter





→ Design Procedure

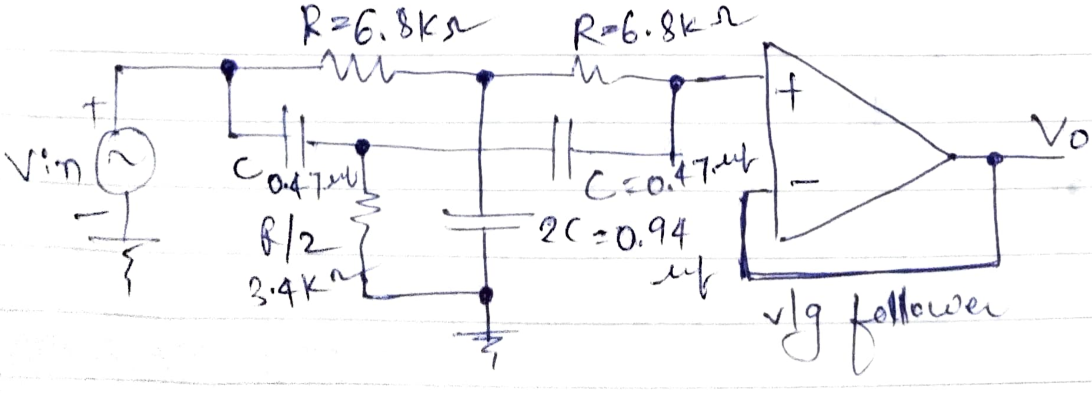
- → Select capacitor C of value less than 1 μf.
- → Calculate $R = \frac{1}{2\pi f_N C}$

→ Design active notch filter for rejecting mains frequency 50Hz.

Sol^n $f_N = 50Hz$

let us, $C = 0.47 \mu f$.

$$\therefore R = \frac{1}{2\pi f_N C} = \frac{1}{2\pi\, 50 \times 0.47 \times 10^{-6}} = 6.77 k\Omega$$

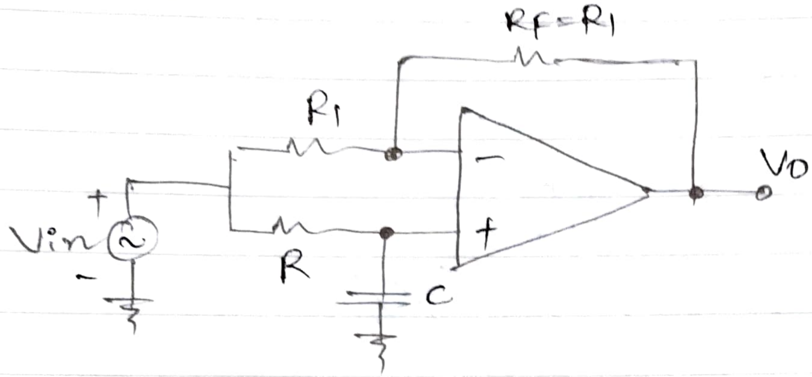


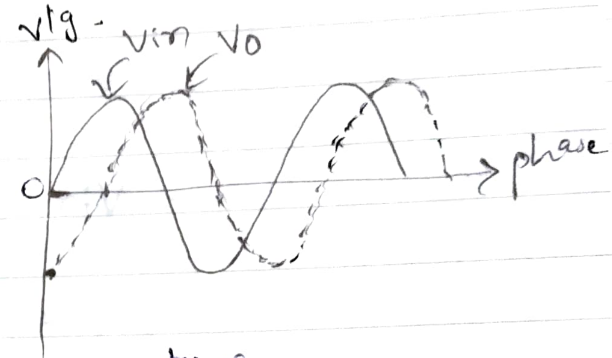
All Pass filter (Phase corrector)

→ It is the special type of filter which passes all freq. component of i/p s/g to o/p without any attenuation.

→ But it introduces phase shift for different frequency components of i/p s/g.

→ They also called as phase corrector.





Case I Considering Vin is applied at inverting i/p & non-inv. terminal connected to ground.

$$V_o' = -\frac{R_F}{R_1} V_{in}$$

But $R_F = R_1$

$$\therefore V_o' = -V_{in}$$

Case II Considering i/p is applied to non inv. terminal through RC & inv. terminal is grounded.

$$V_o'' = \text{Gain} \times \text{v/g across 'C'}$$

$$\therefore V_o'' = \left[1 + \frac{R_F}{R_1}\right] \times \left(\frac{-jX_C}{R - jX_C}\right) V_{in}$$

$$V_o'' = 2\,\frac{-jX_C}{R - jX_C}\,V_{in} \quad \because R_F = R_1$$

Now $V_o = V_o' + V_o''$.

$$\therefore V_o = -V_{in} - \frac{2jX_c V_{in}}{R - jX_c}$$

But $-j = 1/j$ & $X_c = \frac{1}{2\pi f C}$

$$\therefore V_o = V_{in}\left[-1 + \frac{2\left(1/2\pi f C\right)}{j\left(R + \frac{1}{j2\pi f C}\right)}\right]$$

$$= V_{in}\left[-1 + \frac{\left(1/\pi f C\right) \times j2\pi f C}{j\left(j2\pi f RC + 1\right)}\right]$$

$$= V_{in}\left[-1 + \frac{2}{1 + j2\pi f RC}\right]$$

$$= V_{in}\left[\frac{-1 - j2\pi f RC + 2}{1 + j2\pi f RC}\right]$$

$$\cancel{= V_{in}\left[\frac{-1 + j2(1 + \pi f RC)}{1 + j2\pi f RC}\right]}$$

$$V_o = \frac{V_{in}\left[1 - j2\pi f RC\right]}{\left(1 + j2\pi f RC\right)}$$

$$\therefore \frac{V_o}{V_{in}} = \frac{1 - j2\pi f RC}{1 + j2\pi f RC}$$

By taking mod on both side will get,

$$\frac{|V_o|}{|V_{in}|} = 1 \quad \text{ie.} \quad |V_o| = |V_{in}|$$

That means amplitude of $\frac{V_o}{V_{in}}$ will be 1.

App^n

→ In telephone n/w for compensation of phase change.

→ In T.V. to compensate phase change because phase contains the information.

→ Design a band pass filter for $Q = 3$, gain $= 5$ & $f_c = 1KHz$

Sol^n $Q < 10$ ∴ given circuit is wide band pass filter.

$Q = 3$ & $f_c = 1KHz$

$$Q = \frac{f_c}{BW} = \frac{\sqrt{f_H \cdot f_L}}{f_H - f_L}$$

$$3 = \frac{1KHz}{f_H - f_L} \quad \therefore f_H - f_L = \frac{1\times10^3}{3} = 333.33Hz$$

$$\therefore f_H = f_L + 333.33$$

Now, $f_c = \sqrt{f_H \cdot f_L}$

$$\therefore f_c^2 = (1\times10^3)^2 = f_H \cdot f_L$$

$$\therefore f_H \cdot f_L = 10^6$$

$$\therefore (f_L + 333.33) f_L = 10^6$$

$$\therefore f_L^2 + 333.33 f_L - 10^6 = 0.$$

This is quadratic equation.

$$\therefore f_L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-333.33 \pm \sqrt{(333.33)^2 - 4\times1(-10^6)}}{2\times1}$$

$$= \frac{-333.33 \pm \sqrt{4111108.889}}{2}$$

$$f_L = \frac{-333.33 \pm 2027.29}{2}$$

$$\therefore f_L = \frac{-333.33 + 2027.29}{2} \quad \text{or} \quad \frac{-333.33 - 2027.29}{2}$$

$$\therefore f_L = 847.13Hz \quad \because \text{freq} \neq -\text{ve}.$$

$$\therefore f_H = 847.13 + 333.33 = 1180.46 Hz$$

For HPF, $f_L = 847.13 Hz$

choosing $C = 0.1\mu f$.

$$\therefore f_L = \frac{1}{2\pi RC} \quad \therefore R = \frac{1}{2\pi f_L C} = 1.87K\Omega$$

For LPF,

$f_H = 1180.5 Hz$

choosing $C' = 0.01 \mu f$.

$f_H = \frac{1}{2\pi R'C'}$

$\therefore R' = \frac{1}{2\pi f_H C'} = 13.48 K\Omega$

Gain of filter = 5

$\therefore A_{FT} = A_{F1} \cdot A_{F2} = 5$

$A_{FT} = \left(1+\frac{R_f}{R_1}\right)\left(1+\frac{R_f'}{R_1'}\right) = 5$

Assuming $R_f = \cancel{R_f} R_f'$ & $R_1 = R_1'$

$\therefore \left(1+\frac{R_f}{R_1}\right)^2 = (\sqrt{5})^2$

$\therefore 1+\frac{R_f}{R_1} = 2.236$

$\therefore \frac{R_f}{R_1} = 1.236$

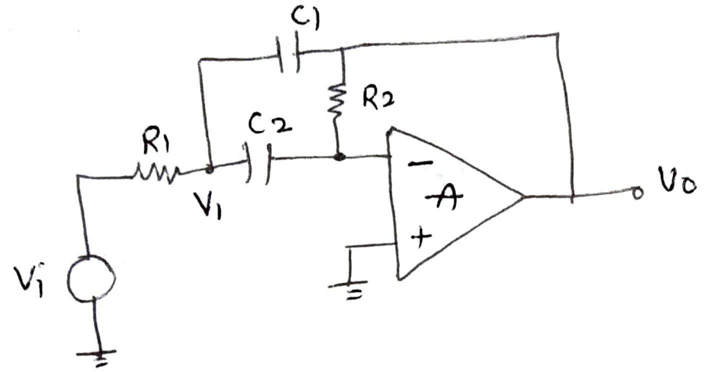
$\therefore R_f = 1.236 R_1$

Let $R_1 = 1K\Omega$ $\therefore R_f = 1.236 K\Omega$

Multiple f/b filters :-

- more than one f/b path.
- exploit full open-loop gain & also referred to as ∞ gain filters.
- with RRC filters, they are most popular single op-amp realization of the 2nd order responses.

Band pass filters :-



The ckt. acts as differentiator w.r.t. V_1.

$$\therefore \quad V_o = -sR_2C_2V_1 \quad \text{---(1)}$$

summing currents at node V_1,

$$\frac{V_i - V_1}{R_1} + \frac{V_o - V_1}{1/sC_1} + \frac{0 - V_1}{1/sC_2} = 0.$$

Eliminating V_1, Let $s \to j\omega$ & rearrange terms

$$\therefore \quad H(j\omega) = \frac{V_o}{V_i} = \frac{-j\omega R_2C_2}{1 - \omega^2 R_1R_2C_1C_2 + j\omega R_1(C_1+C_2)} \quad \text{---(2)}$$

To put eqn (2) in std. form,

$H(j\omega) = H_{0BP}H_{BP}(j\omega)$, we impose

$\omega^2 R_1R_2C_1C_2 = (\omega/\omega_0)^2$ to get

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} \quad \text{---(3)}$$

& $j\omega R_1(C_1+C_2) = (j\omega/\omega_0)/Q$ to get,

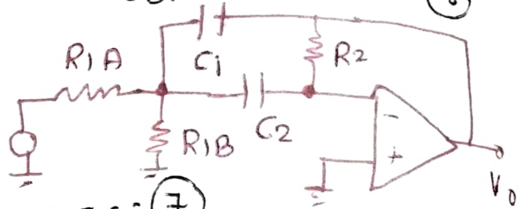
$$Q = \frac{\sqrt{R_2/R_1}}{\sqrt{C_1/C_2} + \sqrt{C_2/C_1}} \quad \text{---(4)}$$

Finally we impose, $-j\omega R_2 C_2 = H_{OBP} \times (j\omega/\omega_0)/Q$ to get

$$H_{OBP} = \frac{-R_2/R_1}{1 + C_1/C_2} \quad \text{---- (5)}$$

filter is of inverting type. Let $C_1 = C_2 = C$

$$\therefore \omega_0 = \frac{1}{\sqrt{R_1 \cdot R_2} \cdot C} \qquad Q = 0.5\sqrt{R_1/R_2} \quad \& \; H_{OBP} = -2Q^2 \text{ -- (6)}$$



The corresponding design eqns are.

$$R_1 = 1/2\omega_0 Q C \quad , \quad R_2 = 2Q/\omega_0 C. \quad \text{---- (7)}$$

Let resonance gain magnitude as $H_0 = |H_{OBP}|$

If $H_0 < 2Q^2$, replace R_1 with a voltage divider and,

$$\therefore R_{1A} = Q/H_0\omega_0 C \quad , \quad R_{1B} = R_{1A}/(2Q^2/H_0 - 1).$$

Example: Design a multiple f/b BPF with $f_0 = 1\,KHz$, $Q = 10$ & $H_0 = 20\,dB$.

soln :- Let $C_1 = C_2 = 10\,nF$.

$$\therefore R_2 = 2 \times 10/(2\pi \times 10^3 \times 10^{-8}) = 318.3\,K\Omega$$
$$= 316\,K\Omega \text{ --- std.}$$

$H_0 = 20\,dB \Rightarrow 10\,V/V$, which is less than $2Q^2 = 200$,

i.e $H_0 < 2Q^2$.

$\therefore$ we need I/P attenuator.

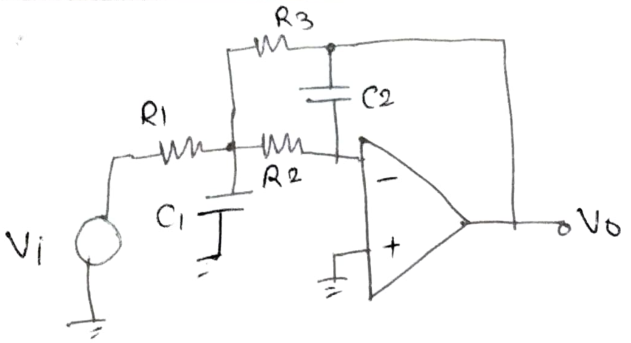
Thus $R_{1A} = Q/H_0\omega_0 C$

$$= 10/(10 \times 2\pi\, 10^3 \times 10^{-8}) = 15.92\,K\Omega$$
$$= 15.8\,K\Omega \text{ --- std.}$$

& $R_{1B} = R_{1A}/([2Q^2/H_0] - 1).$

$$= 15.92/[(200/10) - 1)] = 837.7\,\Omega.$$
$$= 845\,\Omega \text{ --- std.}$$

Low Pass filter :- (∞ gain).



- $R_1 - C_1$ (Low pass stage) followed by integrator stage (R_2, C_2).
- presence of + ve f/b via R_3 should allow for Q control.
- AC analysis gives,

$$\frac{V_o}{V_i} = H_{OLP} \cdot H_{LP} \text{ where}$$

$$H_{OLP} = \frac{-R_3}{R_1}, \quad \omega_0 = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}} \quad \text{---} (1)$$

$$Q = \frac{\sqrt{C_1/C_2}}{\sqrt{R_2 R_3 / R_1^2} + \sqrt{R_3/R_2} + \sqrt{R_2/R_3}} \quad \text{---} (2)$$

i.e $\updownarrow R_3$ to adjust ω_0, & $\updownarrow R_1$ to adjust Q.

Example: Design multiple f/b LPF with $H_0 = 2V/V$, $f_0 = 10KHz$ & $Q = 4$.

Sol^n:- Design procedure:

1. choose C_2 & calculate C_1

   $C_1 = nC_2$, where $n \rightarrow$ capacitance spread.

   $n \geq 4Q^2 (1 + H_0)$.

2. $H_0 \rightarrow$ desired dc gain magnitude.

$$\therefore R_3 = \frac{1 + \sqrt{1 - 4Q^2(1+H_0)/n}}{2 \omega_0 Q C_2}$$

$$R_1 = \frac{R_3}{H_0}$$

$$R_2 = \frac{1}{\omega_0^2 R_3 C_1 C_2}$$

$Q \uparrow, \uparrow H_O \Rightarrow n \uparrow$

sol^n:- $n \geq 192$.

Let $n = 200$.

Let $C_2 = 1\,nF$

$\therefore C_1 = 0.2\,\mu F$

$R_3 = 2.387\,k\Omega$

$= 2.37\,k\Omega$ ... std.

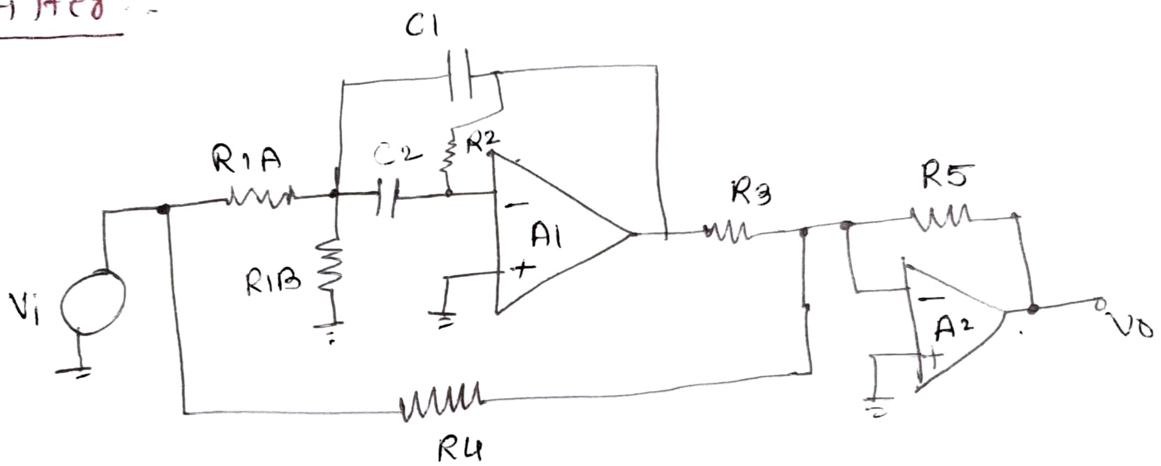
$R_1 = 1.194\,k\Omega$ ...

$= 1.18\,k\Omega$ ... std.

$R_3 = 530.5\,\Omega$

$= 536\,\Omega$ ..... std.

Notch filter:-



$$V_O = -\frac{R_5}{R_3}\left[-H_O \cdot H_{BP}\right] V_i - (R_5/R_4) \cdot V_i$$

$$= -\frac{R_5}{R_4}\left[1 - (H_O R_4/R_3) \cdot H_{BP}\right] \cdot V_i$$

If $H_O R_4/R_3 = 1$,

$\therefore V_o/V_i = H_{ON} H_N$, $H_{ON} = -R_5/R_4$.

Ex: Design a Notch filter with $f_o = 1\,KHz$, $Q = 10$ & $H_{ON} = 0\,dB$.

sol^n:- $f_o = 1\,KHz$, $Q = 10$ & $H_O = 1\,V/V$,

$C_1 = C_2 = 10\,nF$

$R_2 = 318.3\,K\Omega$

$R_{1A} = 159.2\,K$

$R_{1B} = 799.8\,\Omega$

select $R_3 = R_4 = R_5 = 10\,K\Omega$.