Coupled Circuits

An electric circuit is said to be a coupled circuit, when there exists a mutual inductance between the coils (or inductors) present in that circuit. Coil is nothing but the series combination of resistor and inductor.

Magnetic Coupling

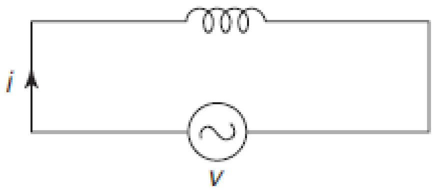
Magnetic coupling occurs, when there is no physical connection between two coils (or inductors). This coupling can be of either aiding type or opposing type.

Types of Inductance

Two types of inductance are there:

- Self Induction
- Mutual Induction

SELF-INDUCTANCE



When current flows through the coil, a flux ϕ is produced in the coil. The flux produced by the coil links with the coil itself. If the current flowing through the coil changes, the flux linking the coil also changes. Hence, an emf is induced in the coil. This is known as self-induced emf.

The direction of this emf is given by *Lenz's law.*

We know that

$$\phi \propto i$$

$$\frac{\phi}{i} = k, \text{ a constant}$$

$$\phi = k\,i$$

Hence, rate of change of flux = $k \times$ rate of change of current

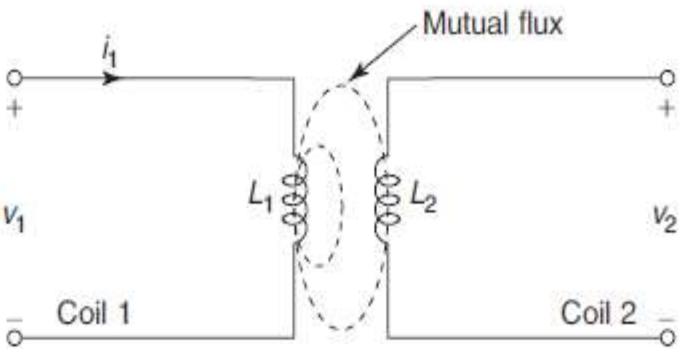
$$\frac{d\phi}{dt} = k\frac{di}{dt}$$

According to Faraday's laws of electromagnetic induction, a self-induced emf can be expressed as

$$v = -N\frac{d\phi}{dt} = -Nk\frac{di}{dt} = -N\frac{\phi}{i}\frac{di}{dt} = -L\frac{di}{dt}$$

where $L = \frac{N\phi}{i}$ and is called coefficient of self-inductance.

MUTUAL INDUCTANCE



If the flux produced by one coil links with the other coil, placed closed to the first coil, an emf is induced in the second coil due to change in the flux produced by the first coil. This is known as mutually induced emf.

If a current i_1 flows in Coil 1, flux is produced and a part of this flux links Coil 2. The emf induced in Coil 2 is called mutually induced emf.

We know that

$$\phi_2 \propto i_1$$

$$\frac{\phi_2}{i_1} = k, \text{ a constant}$$

$$\phi_2 = k\, i_1$$

Hence, rate of change of flux = $k \times$ rate of change of current i_1

$$\frac{d\phi_2}{dt} = k\frac{di_1}{dt}$$

According to Faraday's law of electromagnetic induction, the induced emf is expressed as

$$v_2 = -N_2\frac{d\phi_2}{dt} = -N_2\, k\frac{di_1}{dt} = -N_2\frac{\phi_2}{i_1}\frac{di_1}{dt} = -M\frac{di_1}{dt}$$

where $M = \frac{N_2\,\phi_2}{i_1}$ and is called *coefficient of mutual inductance.*

COEFFICIENT OF COUPLING (k)

The coefficient of coupling (k) between coils is defined as fraction of magnetic flux produced by the current in one coil that links the other.

Consider two coils having number of turns N_1 and N_2 respectively. When a current i_1 is flowing in Coil 1 and is changing, an emf is induced in Coil 2.

$$M = \frac{N_2 \phi_2}{i_1}$$

Let

$$k_1 = \frac{\phi_2}{\phi_1}$$

$$\phi_2 = k_1 \phi_1$$

$$M = \frac{N_2 k_1 \phi_1}{i_1} \qquad \ldots(4.1)$$

If the current i_2 is flowing in Coil 2 and is changing, an emf is induced in Coil 1,

$$M = \frac{N_1 \phi_1}{i_2}$$

Let

$$k_2 = \frac{\phi_1}{\phi_2}$$

$$\phi_1 = k_2 \phi_2$$

$$M = \frac{N_1 k_2 \phi_2}{i_2} \qquad \ldots(4.2)$$

Multiplying Eqs (4.1) and (4.2),

$$M^2 = k_1 k_2 \times \frac{N_1 \phi_1}{i_1} \times \frac{N_2 \phi_2}{i_2} = k^2 L_1 L_2$$

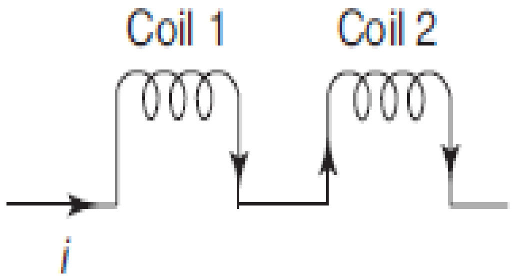
$$M = k\sqrt{L_1 L_2}$$

where

$$k = \sqrt{k_1 k_2}$$

INDUCTANCES IN SERIES

1. *Cumulative Coupling*



two coils 1 and 2 connected
in series, so that currents through the two coils are in the same direction in order to produce flux in the same direction. Such a connection of two coils is known as *cumulative coupling*.

Let L_1 = coefficient of self-inductance of Coil 1

L_2 = coefficient of self-inductance of Coil 2

M = coefficient of mutual inductance

If the current in the coil increases by di amperes in dt seconds then

Self-induced emf in Coil 1 $= -L_1 \dfrac{di}{dt}$

Self-induced emf in Coil 2 $= -L_2 \dfrac{di}{dt}$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= -M \dfrac{di}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= -M \dfrac{di}{dt}$

Total induced emf $$v = -(L_1 + L_2 + 2M)\frac{di}{dt} \qquad \ldots(4.3)$$

If L is the equivalent inductance then total induced emf in that single coil would have been

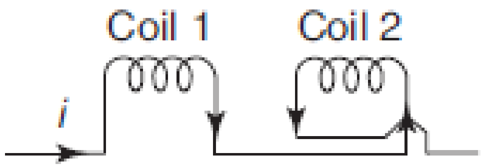
$$v = -L\frac{di}{dt} \qquad \ldots(4.4)$$

Equating Eqs (4.3) and (4.4),

$$L = L_1 + L_2 + 2M$$

2. *Differential Coupling*

the coils connected in series



but the direction of current in Coil 2 is now opposite to that in 1. Such a connection of two coils is known as *differential coupling*.

Hence, total induced emf in coils 1 and 2.

$$v = -L_1\frac{di}{dt} - L_2\frac{di}{dt} + 2M\frac{di}{dt} = -(L_1 + L_2 - 2M)\frac{di}{dt}$$

Coils 1 and 2 connected in series can be considered as a single coil with equivalent inductance L. The induced emf in the equivalent single coil with same rate of change of current is given by,

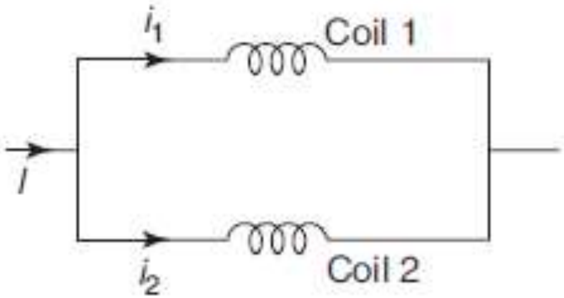
$$v = -L\frac{di}{dt}$$

$$-L\frac{di}{dt} = -(L_1 + L_2 - 2M)\frac{di}{dt}$$

$$L = L_1 + L_2 - 2M$$

INDUCTANCES IN PARALLEL

1. Cumulative Coupling



two coils 1 and 2 connected in parallel such that fluxes produced by the coils act in the same direction. Such a connection of two coils is known as cumulative coupling.

Let L_1 = coefficient of self-inductance of Coil 1
L_2 = coefficient of self-inductance of Coil 2
M = coefficient of mutual inductance

If the current in the coils changes by di amperes in dt seconds then

$$\text{Self-induced emf in Coil 1} = -L_1 \frac{di_1}{dt}$$

$$\text{Self-induced emf in Coil 2} = -L_2 \frac{di_2}{dt}$$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= -M \dfrac{di_2}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= -M \dfrac{di_1}{dt}$

Total induced emf in Coil 1 $= -L_1 \dfrac{di_1}{dt} - M \dfrac{di_2}{dt}$

Total induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt} - M \dfrac{di_1}{dt}$

As both the coils are connected in parallel, the emf induced in both the coils must be equal.

$$-L_1\frac{di_1}{dt}-M\frac{di_2}{dt}=-L_2\frac{di_2}{dt}-M\frac{di_1}{dt}$$

$$L_1\frac{di_1}{dt}-M\frac{di_1}{dt}=L_2\frac{di_2}{dt}-M\frac{di_2}{dt}$$

$$(L_1-M)\frac{di_1}{dt}=(L_2-M)\frac{di_2}{dt}$$

$$\frac{di_1}{dt}=\left(\frac{L_2-M}{L_1-M}\right)\frac{di_2}{dt} \qquad \text{...(4.5)}$$

Now,

$$i=i_1+i_2$$

$$\frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}$$

$$=\left(\frac{L_2-M}{L_1-M}\right)\frac{di_2}{dt}+\frac{di_2}{dt}$$

$$=\left(\frac{L_2-M}{L_1-M}+1\right)\frac{di_2}{dt}$$

$$=\left(\frac{L_1+L_2-2M}{L_1-M}\right)\frac{di_2}{dt} \qquad \text{...(4.6)}$$

If L is the equivalent inductance of the parallel combination then the induced emf is given by

$$v = -L\frac{di}{dt}$$

Since induced emf in parallel combination is same as induced emf in any one coil,

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right)$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} + M\frac{di_2}{dt}\right]$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt} \qquad ...(4.7)$$

Substituting Eq. (4.6) in Eq. (4.7),

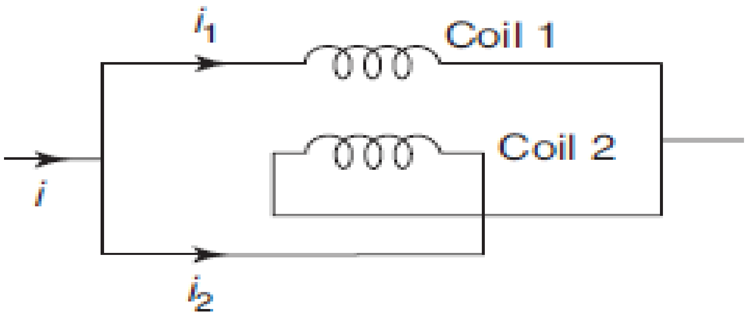
$$\left(\frac{L_1 + L_2 - 2M}{L_1 - M}\right)\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt}$$

$$L = \frac{L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M}{\frac{L_1 + L_2 - 2M}{L_1 - M}}$$

$$= \frac{L_1 L_2 - L_1 M + L_1 M - M^2}{L_1 + L_2 - 2M}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

2. *Differential Coupling*



two coils 1 and 2 connected in parallel such that fluxes produced by the coils act in the opposite direction. Such a connection of two coils is known as differential coupling.

$$\text{Self-induced emf in Coil 1} = -L_1 \frac{di_1}{dt}$$

$$\text{Self-induced emf in Coil 2} = -L_2 \frac{di_2}{dt}$$

$$\text{Mutually induced emf in Coil 1 due to change of current in Coil 2} = M \frac{di_2}{dt}$$

$$\text{Mutually induced emf in Coil 2 due to change of current in Coil 1} = M \frac{di_1}{dt}$$

Total induced emf in Coil 1 $= -L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}$

Total induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt}$

As both the coils are connected in parallel, the emf induced in the coils must be equal.

$$-L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$L_1 \frac{di_1}{dt} + M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_2}{dt}$$

$$(L_1 + M) \frac{di_1}{dt} = (L_2 + M) \frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 + M}{L_1 + M} \right) \frac{di_2}{dt} \qquad \text{...(4.8)}$$

Now,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 + M}{L_1 + M}\right)\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 + M}{L_1 + M} + 1\right)\frac{di_2}{dt}$$

$$= \left(\frac{L_1 + L_2 + 2M}{L_1 + M}\right)\frac{di_2}{dt} \qquad ...(4.9)$$

If L is the equivalent inductance of the parallel combination then the induced emf is given by

$$v = -L\frac{di}{dt}$$

Since induced emf in parallel combination is same as induced emf in any one coil,

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} - M\frac{di_2}{dt}\right)$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right)\frac{di_2}{dt} - M\frac{di_2}{dt}\right]$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right) - M\right]\frac{di_2}{dt} \qquad \text{...(4.10)}$$

Substituting Eq. (4.9) in Eq. (4.10),

$$\left(\frac{L_1 + L_2 + 2M}{L_1 + M}\right)\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 + M}{L_1 + M}\right) - M\right]\frac{di_2}{dt}$$

$$L = \frac{L_1\left(\dfrac{L_2 + M}{L_1 + M}\right) - M}{\dfrac{L_1 + L_2 + 2M}{L_1 + M}}$$

$$= \frac{L_1 L_2 + L_1 M - L_1 M - M^2}{L_1 + L_2 + 2M}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Problems- *Two coils having self-inductances of 4 mH and 7 mH respectively are connected in parallel. If the mutual inductance between them is 5 mH, find the equivalent inductance.*

Solution

$$L_1 = 4 \text{ mH}, \quad L_2 = 7 \text{ mH}, \quad M = 5 \text{ mH}$$

For cumulative coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4 \times 7 - (5)^2}{4 + 7 - 2(5)} = 3 \text{ mH}$$

For differential coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{4 \times 7 - (5)^2}{4 + 7 + 2(5)} = 0.143 \text{ mH}$$

Problems- *The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on relative directions of currents in the two coils. If one of the coils has a self-inductance of 0.2 H, find (a) mutual inductance, and (b) coefficient of coupling.*

Solution $L_1 = 0.2\,\text{H}, \quad L_{\text{diff}} = 0.1\,\text{H}, \quad L_{\text{cum}} = 0.6\,\text{H}$

(a) Mutual inductance

$$L_{\text{cum}} = L_1 + L_2 + 2M = 0.6 \qquad \text{...(i)}$$

$$L_{\text{diff}} = L_1 + L_2 - 2M = 0.1 \qquad \text{...(ii)}$$

Adding Eqs (i) and (ii),

$$2(L_1 + L_2) = 0.7$$

$$L_1 + L_2 = 0.35$$

$$L_2 = 0.35 - 0.2 = 0.15\,\text{H}$$

Subtracting Eqs (ii) from Eqs (i),

$$4M = 0.5$$

$$M = 0.125\ \mathrm{H}$$

(b) Coefficient of coupling

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.125}{\sqrt{0.2 \times 0.15}} = 0.72$$

Two inductors are connected in parallel. Their equivalent inductance when the mutual inductance aids the self-inductance is 6 mH and it is 2 mH when the mutual inductance opposes the self-inductance. If the ratio of the self- inductances is 1:3 and the mutual inductance between the coils is 4 mH, find the self-inductances.

Solution

$$L_{\text{cum}} = 6 \text{ mH}, \quad L_{\text{diff}} = 2 \text{ mH}, \quad \frac{L_1}{L_2} = 1.3, \quad M = 4 \text{ mH}$$

For cumulative coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$6 = \frac{L_1 L_2 - (4)^2}{L_1 + L_2 - 2(4)}$$

$$6 = \frac{L_1 L_2 - 16}{L_1 + L_2 - 8} \qquad \text{...(i)}$$

For differential coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$2 = \frac{L_1 L_2 - (4)^2}{L_1 + L_2 + 8}$$

$$2 = \frac{L_1 L_2 - 16}{L_1 + L_2 + 8} \qquad \text{...(i}$$

From Eqs (i) and (ii),

$$2(L_1 + L_2 + 8) = 6(L_1 + L_2 - 8)$$

$$L_1 + L_2 + 8 = 3L_1 + 3L_2 - 24$$

$$L_1 + L_2 = 16$$

But

$$\frac{L_1}{L_2} = 1.3$$

$$1.3\, L_2 + L_2 = 16$$

$$2.3\, L_2 = 16$$

$$L_2 = 6.95 \text{ mH}$$

$$L_1 = 1.3\, L_2 = 9.035 \text{ mH}$$

DOT CONVENTION

Consider two coils of inductances L_1 and L_2 respectively connected in series as shown in Fig. Each coil will contribute the same mutual flux (since it is in a series connection, the same current flows through L_1 and L_2) and hence, same mutual inductance (M).

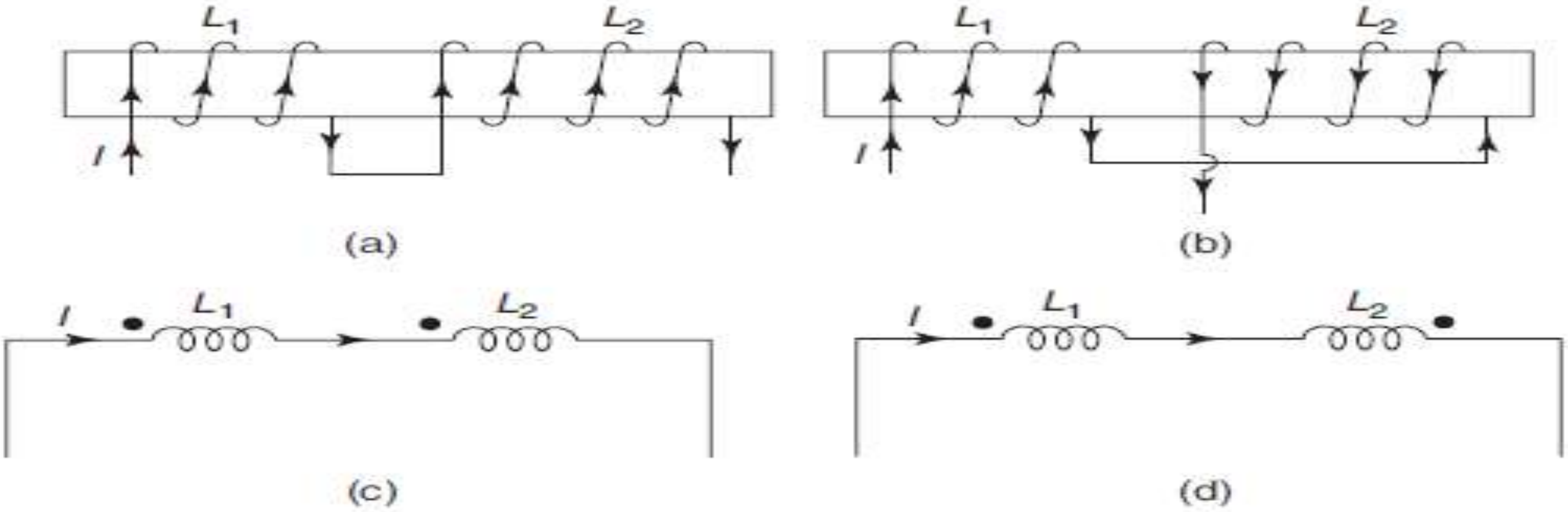
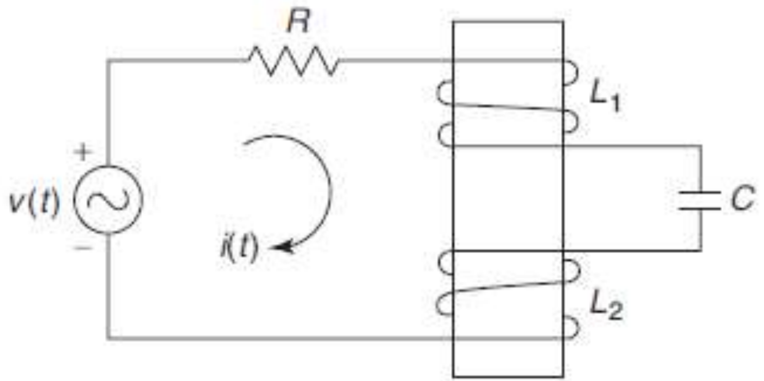
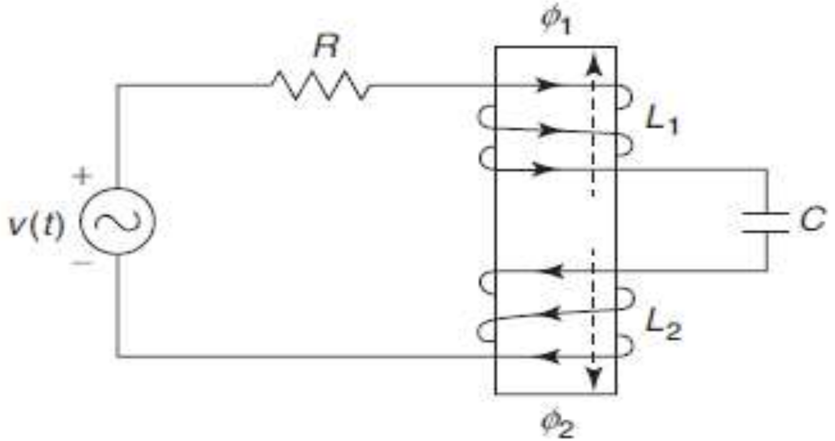
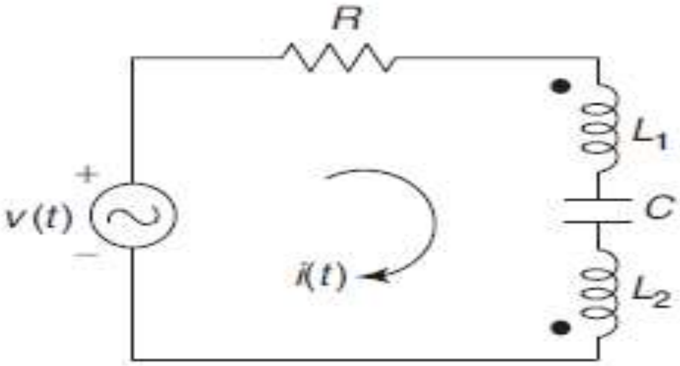


Fig. 4.7 *Dot convention*

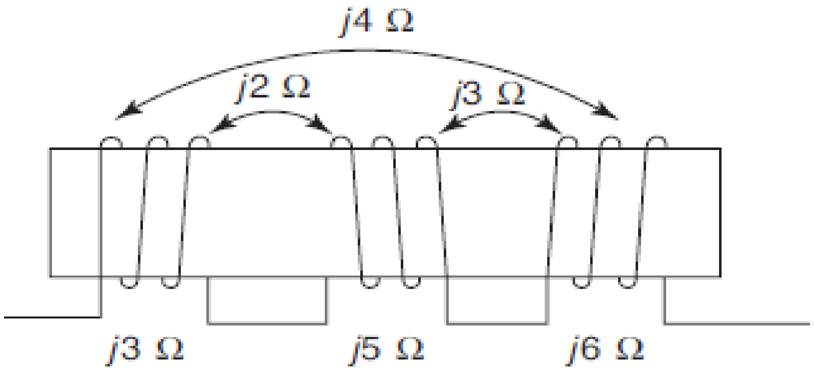
Obtain the dotted equivalent circuit for Fig.

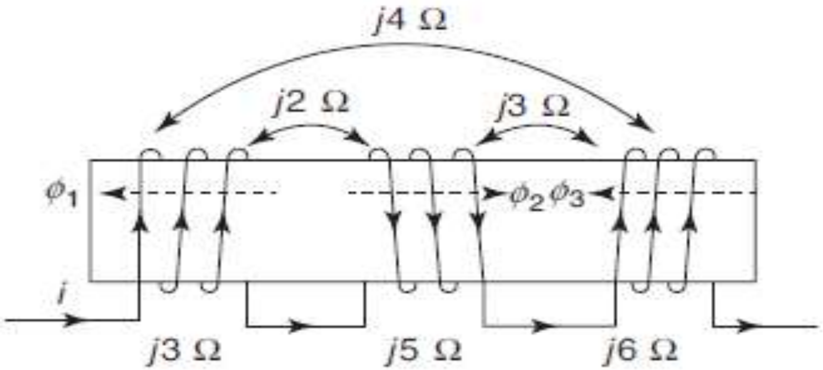


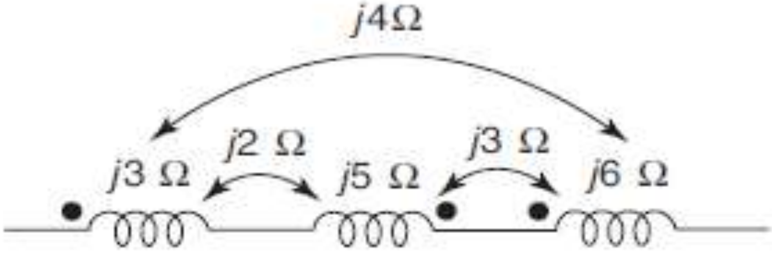




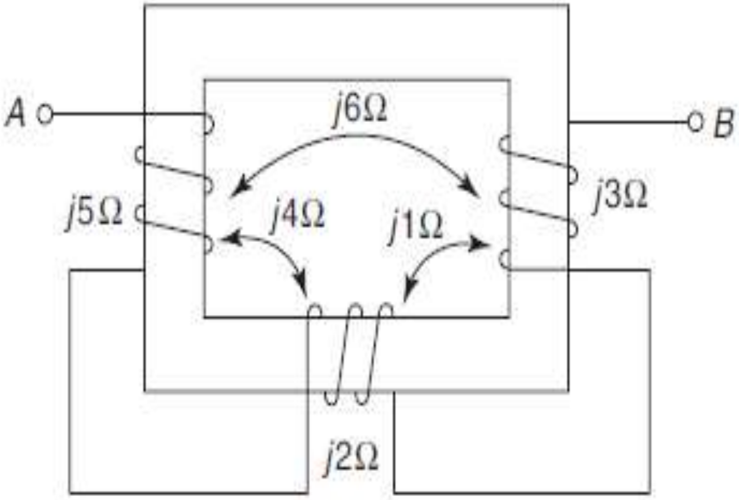
Obtain the dotted equivalent circuit for the circuit of Fig.

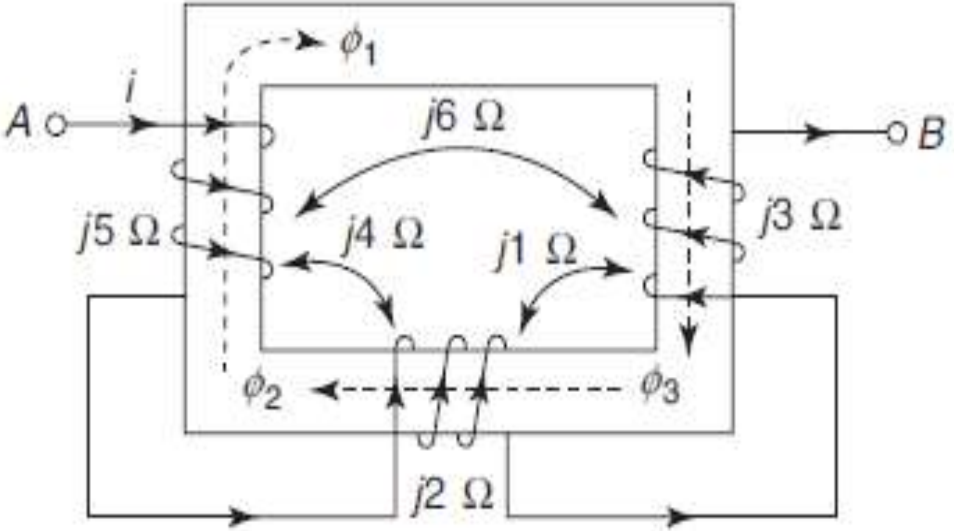


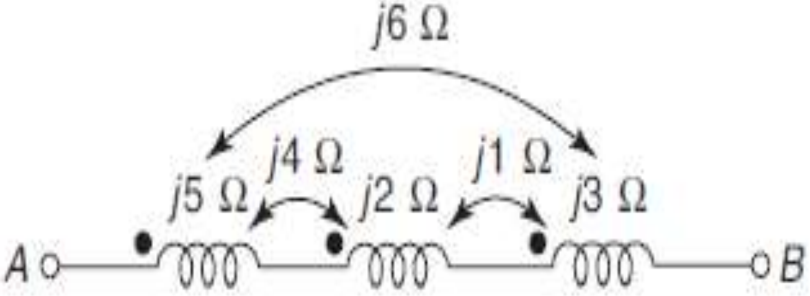




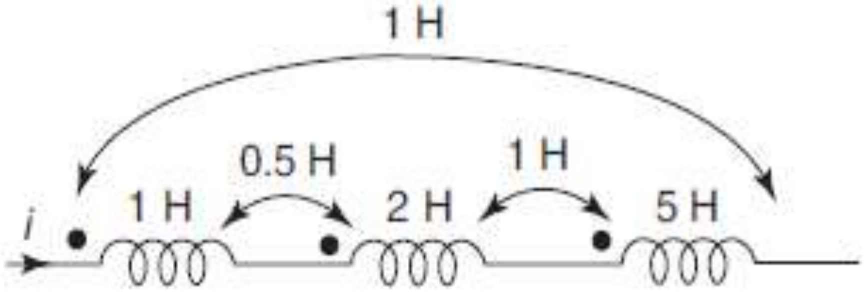
Obtain the dotted equivalent circuit for the circuit shown in Fig.







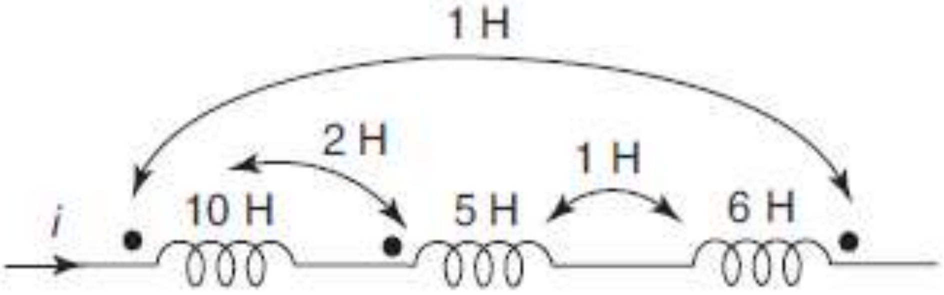
Find the equivalent inductance of the network shown in Fig.



Solution

$$L = (L_1 + M_{12} + M_{13}) + (L_2 + M_{23} + M_{21}) + (L_3 + M_{31} + M_{32})$$
$$= (1 + 0.5 + 1) + (2 + 1 + 0.5) + (5 + 1 + 1)$$
$$= 13\ \mathrm{H}$$

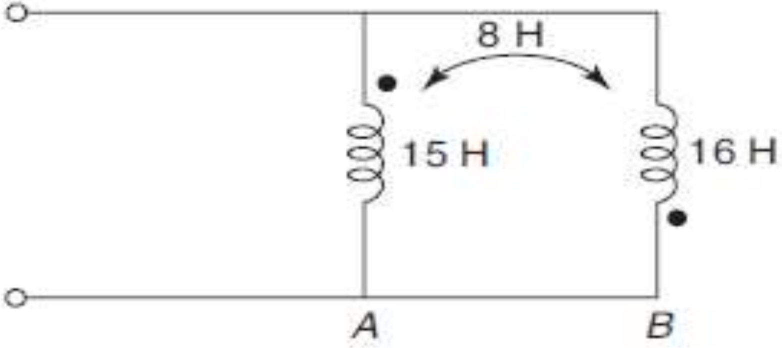
Find the equivalent inductance of the network shown in Fig.



Solution

$$L = (L_1 + M_{12} - M_{13}) + (L_2 - M_{23} + M_{21}) + (L_3 - M_{31} - M_{23})$$
$$= (10 + 2 - 1) + (5 - 1 + 2) + (6 - 1 - 1) = 21\ \text{H}$$

Find the equivalent inductance of the network shown in Fig.



Solution For Coil A,

$$L_A = L_1 - M_{12} = 15 - 8 = 7 \text{ H}$$

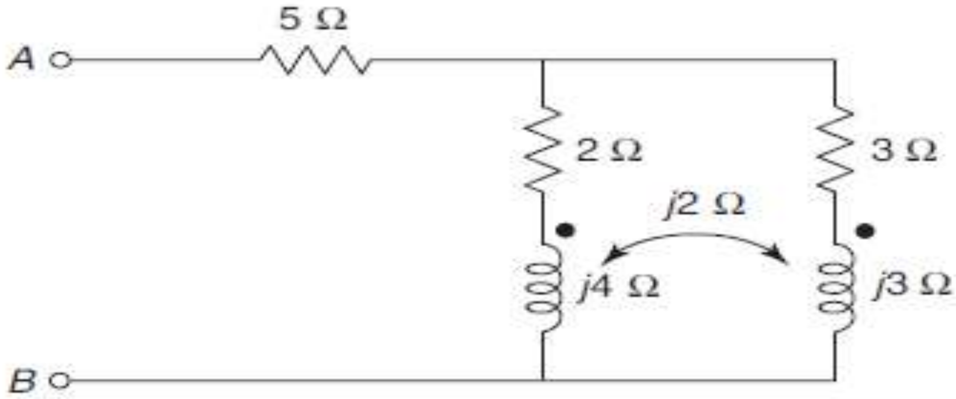
For Coil B,

$$L_B = L_2 - M_{12} = 16 - 8 = 8 \text{ H}$$

$$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B} = \frac{1}{7} + \frac{1}{8} = \frac{15}{56}$$

$$L = \frac{56}{15} = 3.73 \text{ H}$$

Find the equivalent impedance across the terminals A and B in Fig.



Solution

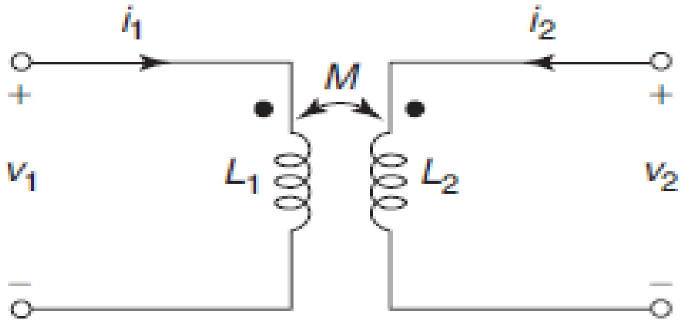
$$\mathbf{Z}_1 = 5\Omega, \quad \mathbf{Z}_2 = (2+j4)\ \Omega, \quad \mathbf{Z}_3 = (3+j3)\ \Omega, \quad \mathbf{Z}_M = j2\ \Omega$$

$$\mathbf{Z} = \mathbf{Z}_1 + \frac{\mathbf{Z}_2\mathbf{Z}_3 - \mathbf{Z}_M^2}{\mathbf{Z}_2 + \mathbf{Z}_3 - 2\mathbf{Z}_M} = 5 + \frac{(2+j4)(3+j3)-(j2)^2}{2+j4+3+j3-2(j2)} = 6.9\angle 24.16^\circ\ \Omega$$

COUPLED CIRCUITS

Case-1

Consider two coils located physically close to one another as shown



When current i_1 flows in the first coil and $i_2 = 0$ in the second coil, flux ϕ_1 is produced in the coil. A fraction of this flux also links the second coil and induces a voltage in this coil. The voltage v_1 induced in the first coil is

$$v_1 = L_1 \left.\frac{di_1}{dt}\right|_{i_2=0}$$

The voltage v_2 induced in the second coil is

$$v_2 = M \left.\frac{di_1}{dt}\right|_{i_2=0}$$

The same reasoning applies if a current i_2 flows in Coil 2 and $i_1 = 0$ in Coil 1. The induced voltages v_2 and v_1 are

$$v_2 = L_2 \frac{di_2}{dt}\bigg|_{i_1=0}$$

and

$$v_1 = M \frac{di_2}{dt}\bigg|_{i_1=0}$$

Now if both currents i_1 and i_2 are present, by using superposition principle, we can write

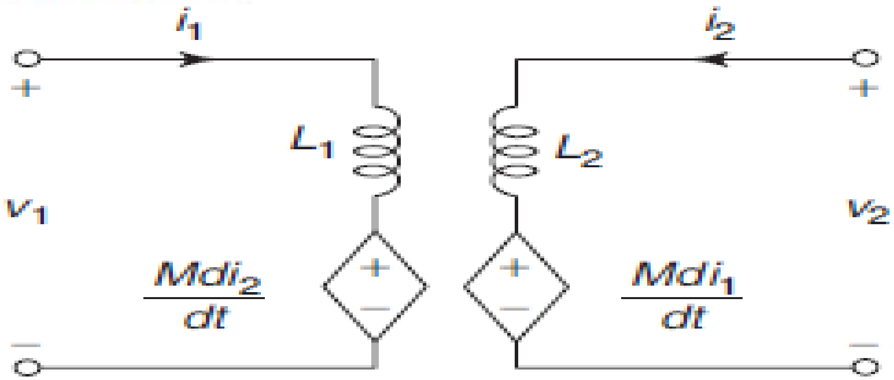
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

This can be represented in terms of dependent sources,

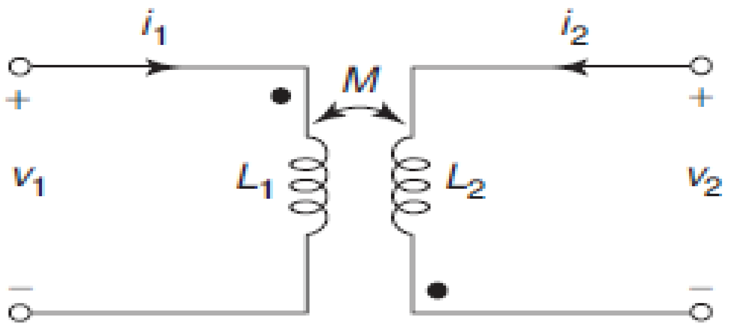
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



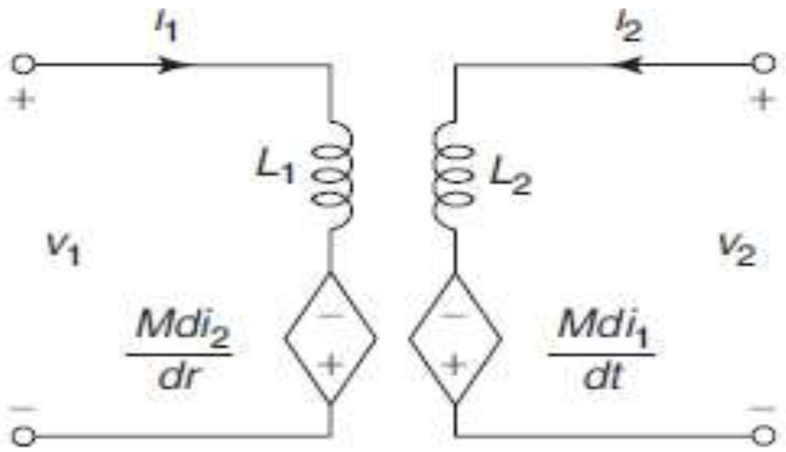
Equivalent circuit

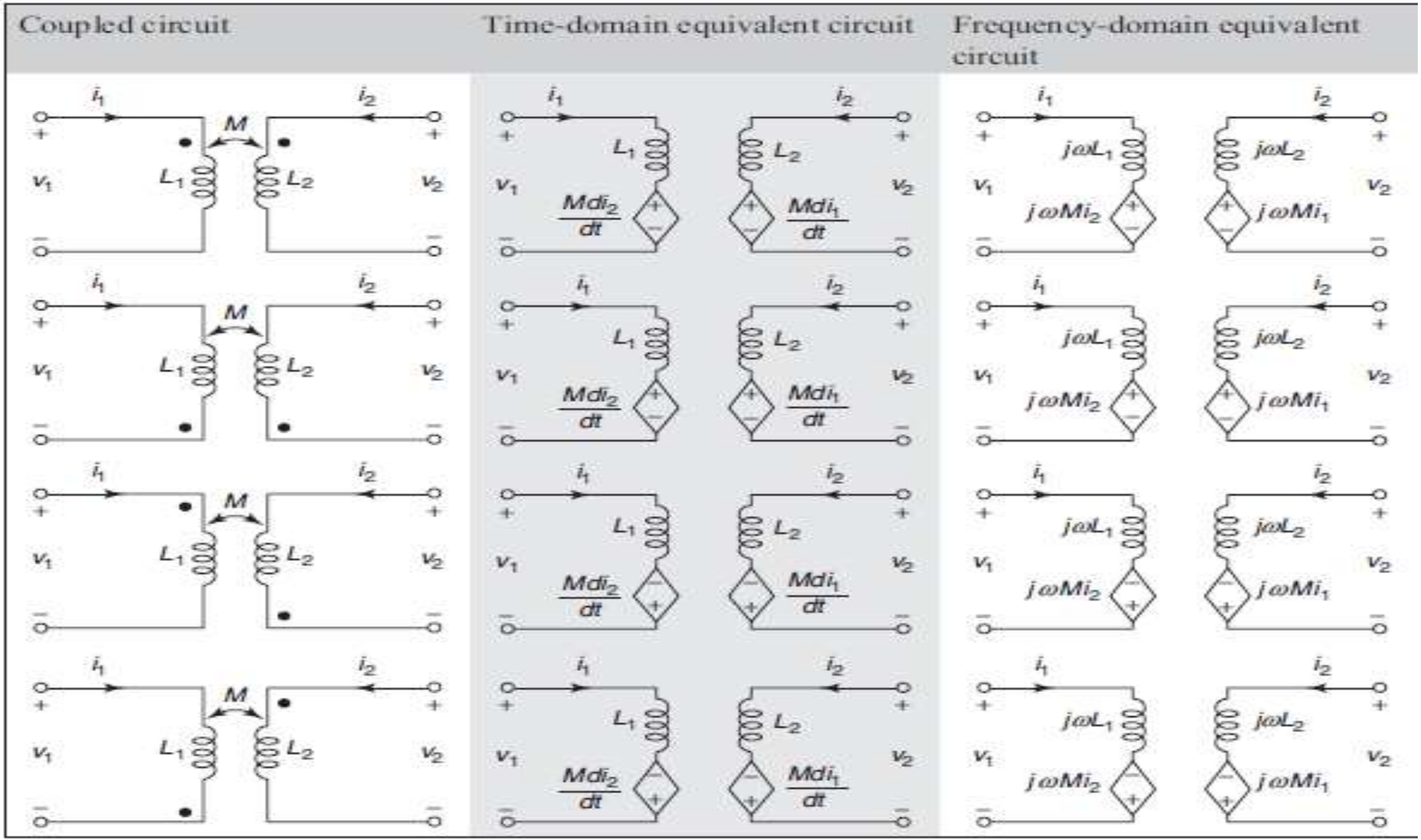
Case-2



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

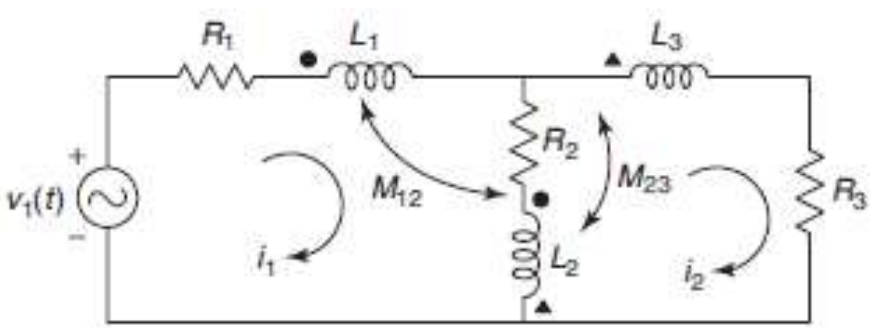
$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

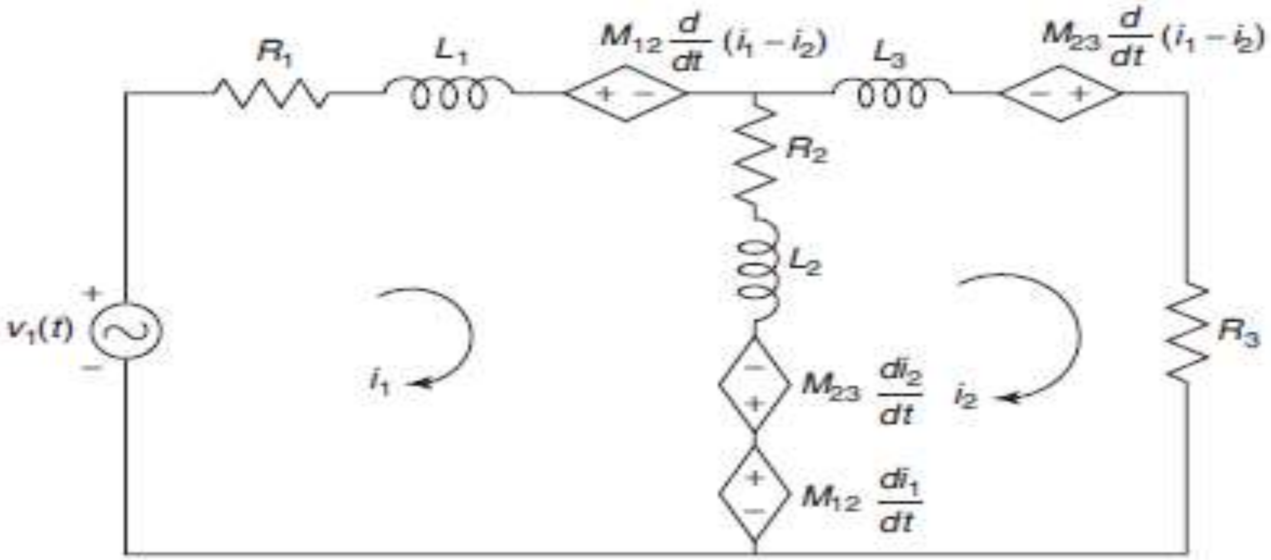




Coupled circuit
Time-domain equivalent circuit
Frequency-domain equivalent circuit
i1
i2
M
L1
L2
v1
v2
Mdi2/dt
Mdi1/dt
jωL1
jωL2
jωMi2
jωMi1

Write mesh equations for the network shown in Fig.





Applying KVL to Mesh 1,

$$v_1(t) - R_1 i_1 - L_1 \frac{di_1}{dt} - M_{12} \frac{d}{dt}(i_1 - i_2) - R_2 (i_1 - i_2) - L_2 \frac{d}{dt}(i_1 - i_2) + M_{23} \frac{di_2}{dt} - M_{12} \frac{di_1}{dt} = 0$$

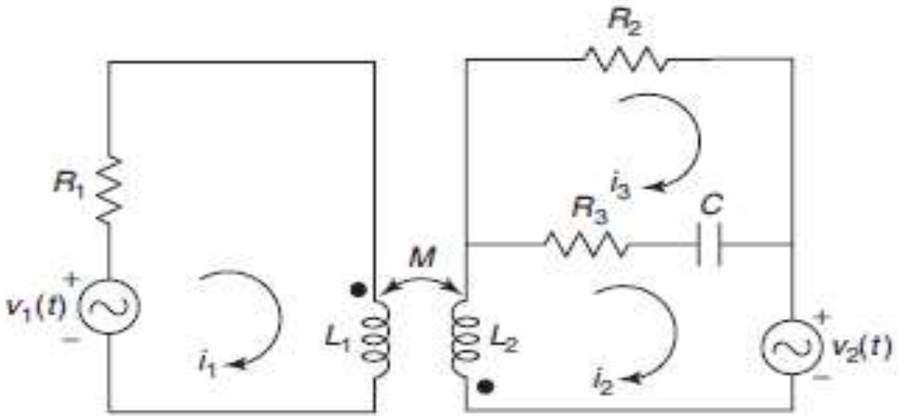
$$(R_1 + R_2) i_1 + (L_1 + L_2 + 2M_{12}) \frac{di_1}{dt} - R_2 i_2 - (L_2 + M_{12} + M_{23}) \frac{di_2}{dt} = v_1(t) \qquad \ldots(i)$$

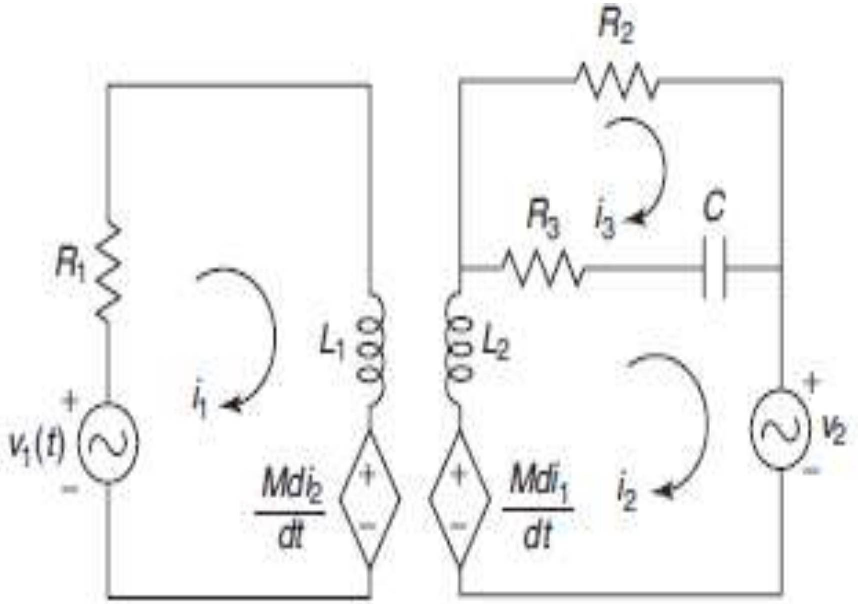
Applying KVL to Mesh 2,

$$M_{12} \frac{di_1}{dt} - M_{23} \frac{di_2}{dt} - L_2 \frac{d}{dt}(i_2 - i_1) - R_2 (i_2 - i_1) - L_3 \frac{di_2}{dt} + M_{23} \frac{d}{dt}(i_1 - i_2) - R_3 i_2 = 0$$

$$-R_2 i_1 - (L_2 + M_{12} + M_{23}) \frac{di_1}{dt} + (R_2 + R_3) i_2 + (L_2 + L_3 + 2M_{23}) \frac{di_2}{dt} = 0 \qquad \ldots(ii)$$

Write KVL equations for the circuit shown in Fig.





Applying KVL to Mesh 1,

$$v_1(t) - R_1 i_1 - L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0$$

$$R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = v_1(t) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

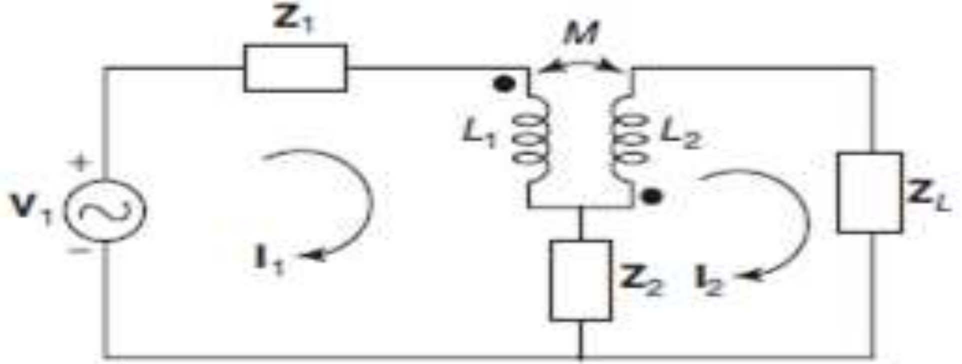
$$M \frac{di_1}{dt} - L_2 \frac{di_2}{dt} - R_3(i_2 - i_3) - \frac{1}{C}\int_0^t (i_2 - i_3)\, dt - v_2(t) = 0$$

$$M \frac{di_1}{dt} - L_2 \frac{di_2}{dt} - R_3(i_2 - i_3) - \frac{1}{C}\int_0^t (i_2 - i_3)\, dt = v_2(t) \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

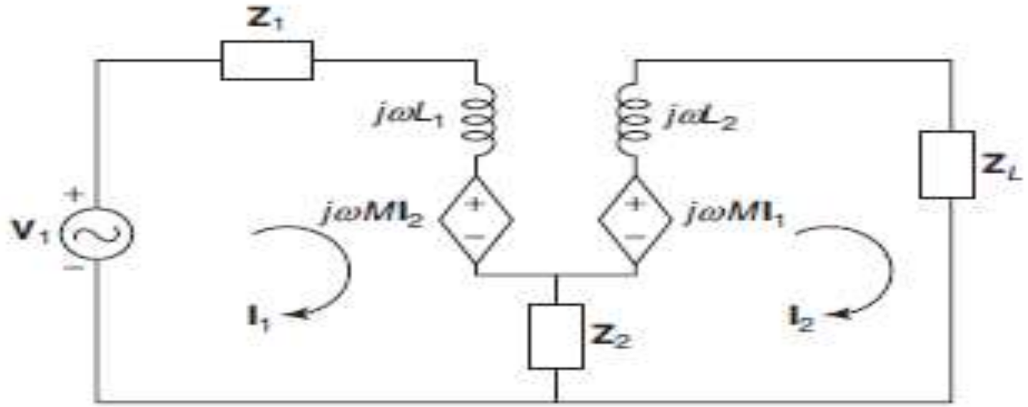
$$-R_2 i_3 - \frac{1}{C}\int_0^t (i_3 - i_2) dt - R_3(i_3 - i_2) = 0 \qquad \ldots\text{(iii)}$$

Write down the mesh equations for the network shown in Fig.



Solution

The equivalent circuit in terms of dependent sources is shown in Fig.



Applying KVL to Mesh 1,

$$\mathbf{V}_1 - \mathbf{Z}_1\mathbf{I}_1 - j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

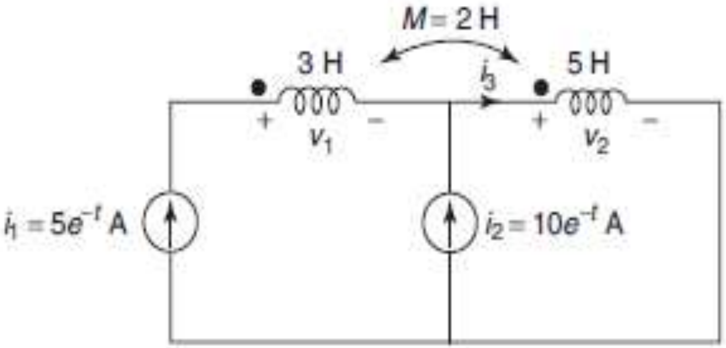
$$(\mathbf{Z}_1 + j\omega L_1 + \mathbf{Z}_2)\,\mathbf{I}_1 - (\mathbf{Z}_2 - j\omega M)\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

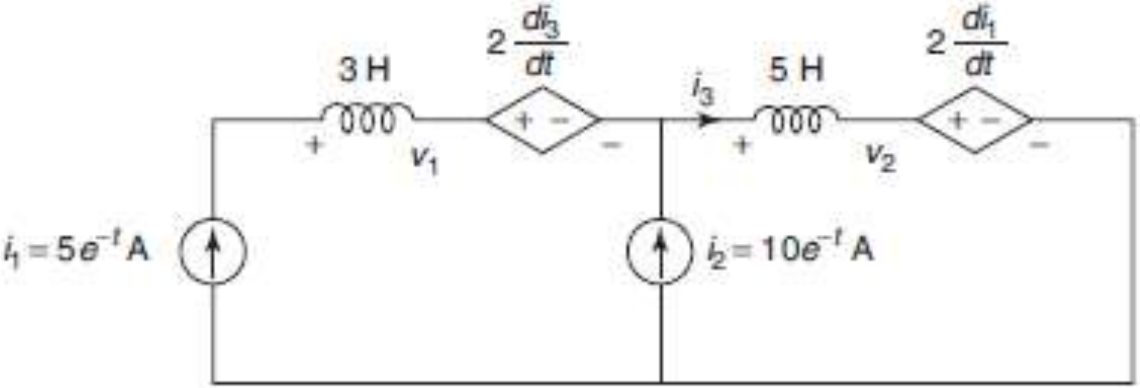
$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) + j\omega M\mathbf{I}_1 - j\omega L_2\mathbf{I}_2 - \mathbf{Z}_L\mathbf{I}_2 = 0$$

$$-(\mathbf{Z}_2 - j\omega M)\,\mathbf{I}_1 + (\mathbf{Z}_2 + j\omega L_2 + \mathbf{Z}_L)\,\mathbf{I}_2 = 0 \qquad \ldots\text{(ii)}$$

In the network shown in Fig. 4.41, find the voltages V_1 and V_2.



Solution The equivalent circuit in terms of dependent sources is shown in Fig.

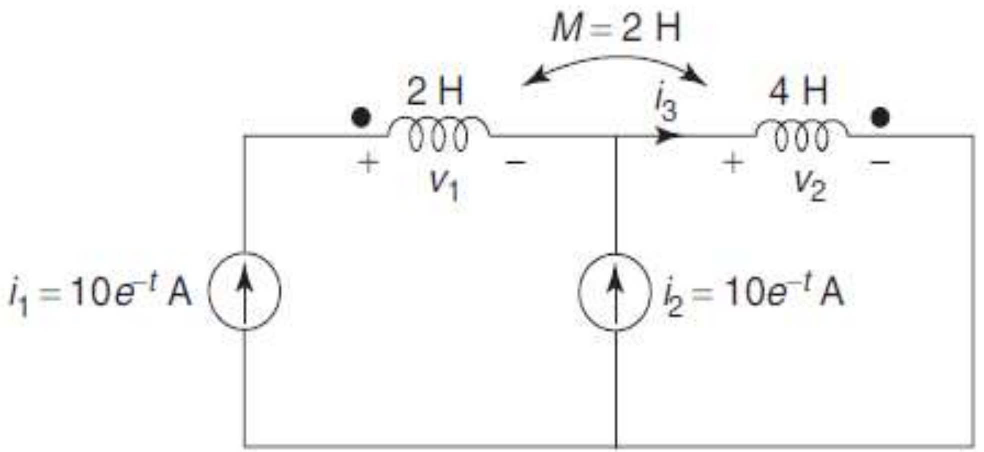


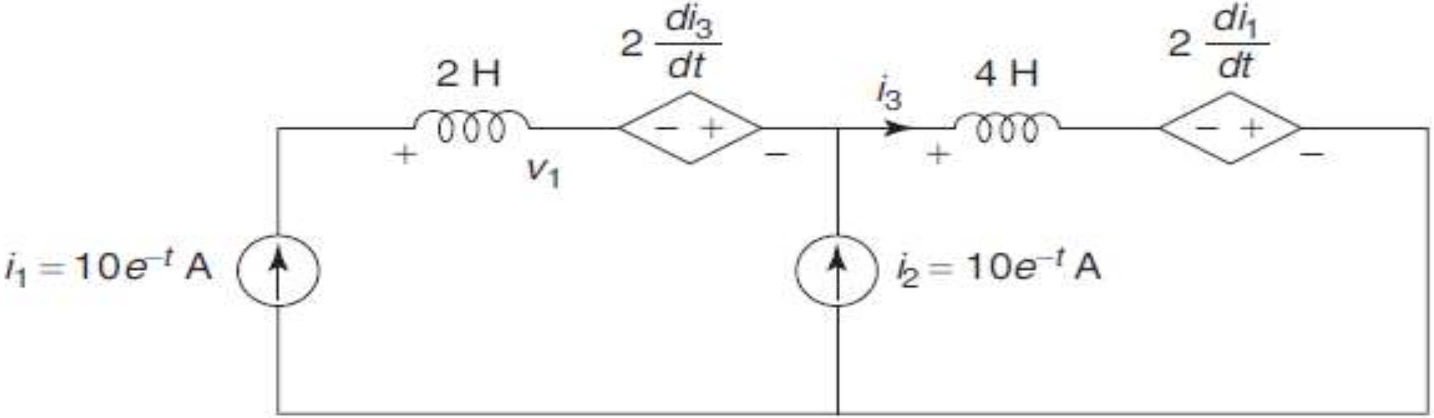
$$i_3 = i_1 + i_2 = 5\,e^{-t} + 10\,e^{-t} = 15\,e^{-t}\ \text{A}$$

$$v_1 = 3\frac{di_1}{dt} + 2\frac{di_3}{dt} = 3\frac{d}{dt}(5\,e^{-t}) + 2\frac{d}{dt}(15\,e^{-t}) = -15\,e^{-t} - 30\,e^{-t} = -45\,e^{-t}\text{V}$$

$$v_2 = 5\frac{di_3}{dt} + 2\frac{di_1}{dt} = 5\frac{d}{dt}(15\,e^{-t}) + 2\frac{d}{dt}(5\,e^{-t}) = -75\,e^{-t} - 10\,e^{-t} = -85\,e^{-t}\text{V}$$

In the network shown in Fig. find the voltages V_1 and V_2.



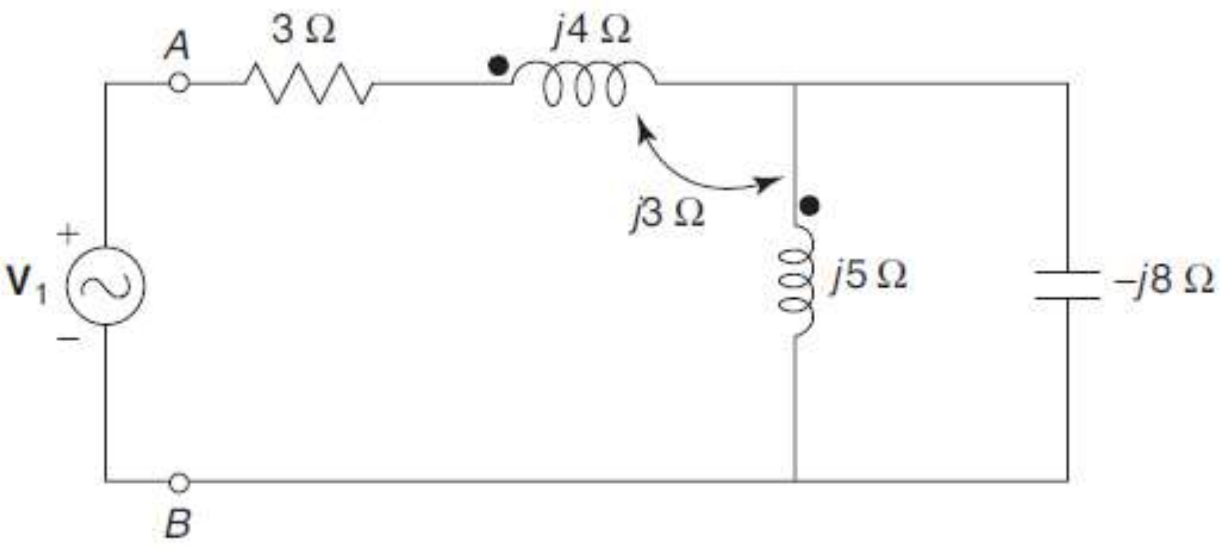


$$i_3 = i_1 + i_2 = 10\,e^{-t} + 10\,e^{-t} = 20\,e^{-t}\,\mathrm{A}$$

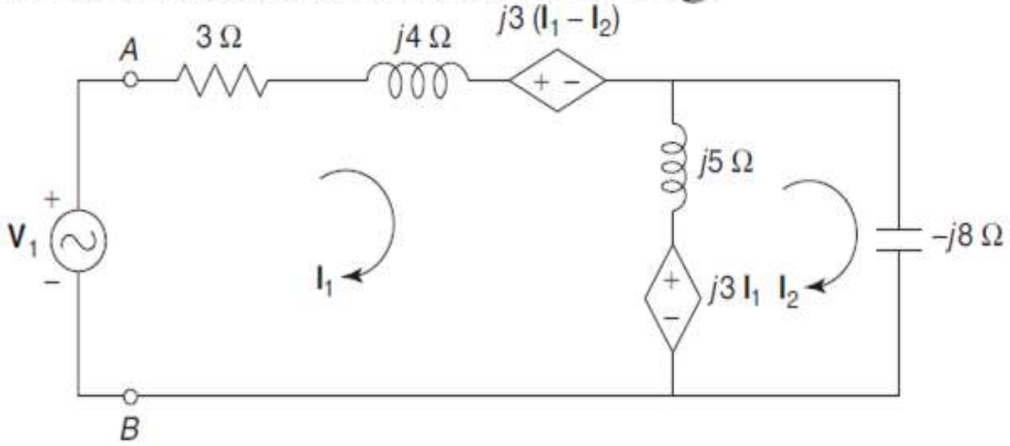
$$v_1 = 2\frac{di_1}{dt} - 2\frac{di_3}{dt} = 2\frac{d}{dt}(10\,e^{-t}) - 2\frac{d}{dt}(20\,e^{-t}) = -20\,e^{-t} + 40\,e^{-t} = 20\,e^{-t}\,\mathrm{A}$$

$$v_2 = 4\frac{di_3}{dt} - 2\frac{di_1}{dt} = 4\frac{d}{dt}(20\,e^{-t}) - 2\frac{d}{dt}(10\,e^{-t}) = -80\,e^{-t} + 20\,e^{-t} = -60\,e^{-t}\,\mathrm{A}$$

For the coupled circuit shown in Fig. find input impedance at terminals A and B.



The equivalent circuit in terms of dependent sources is shown in Fig.



Applying KVL to Mesh 1,

$$V_1 - 3I_1 - j4I_1 - j3(I_1 - I_2) - j5(I_1 - I_2) - j3I_1 = 0$$

$$(3 + j15)\,\mathbf{I}_1 - j8\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \ldots(i)$$

Applying KVL to Mesh 2,

$$j3\,I_1 - j5(I_2 - I_1) + j8\,I_2 = 0$$

$$j8\,I_1 + j3\,I_2 = 0$$

$$I_2 = -\frac{j8}{j3} I_1 = -2.67\,I_1 \qquad \ldots(ii)$$

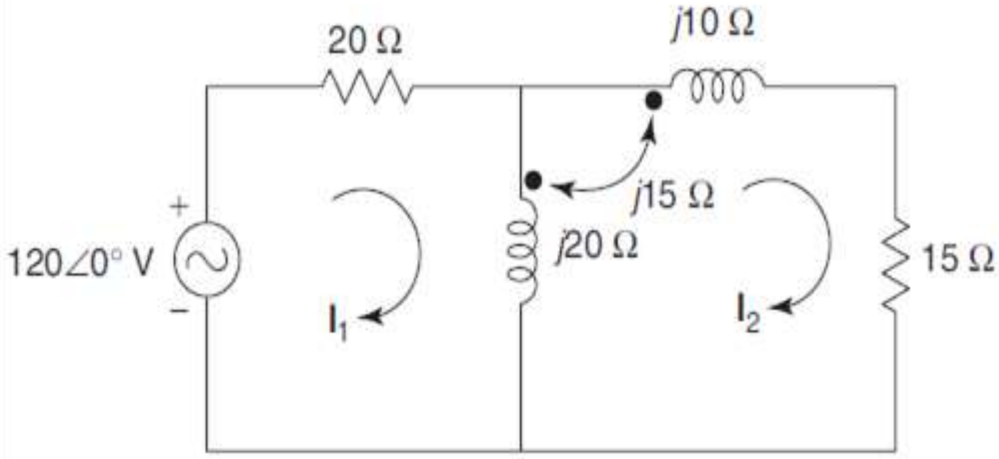
Substituting Eq (ii) in Eq (i),

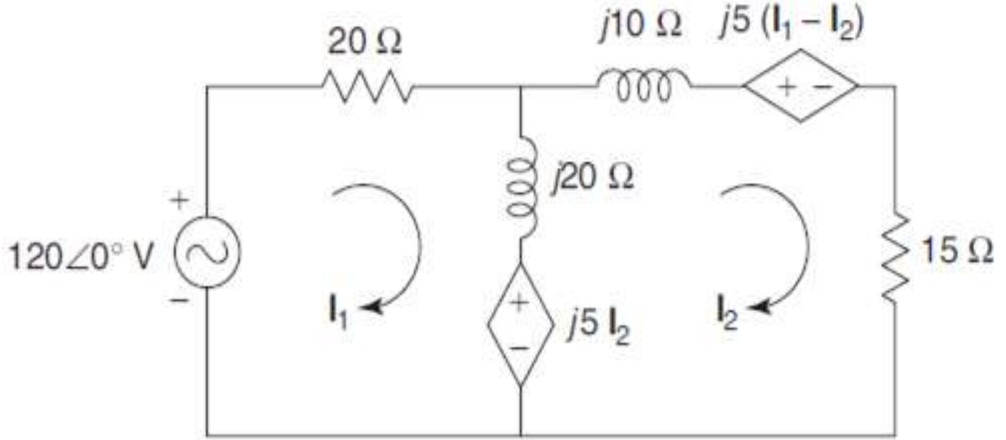
$$(3 + j15)\mathbf{I}_1 - j8\,(-2.67\,\mathbf{I}_1) = \mathbf{V}_1$$

$$(3 + j\,36.36)\,\mathbf{I}_1 = \mathbf{V}_1$$

$$\mathbf{Z}_i = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (3 + j\,36.36)\,\Omega = 36.48\angle 85.28^\circ\,\Omega$$

Find the voltage across the 15 Ω resistor in Fig.using mesh analysis.





Applying KVL to Mesh 1,

$$120\angle 0^\circ - 20\,\mathbf{I}_1 - j20(\mathbf{I}_1 - \mathbf{I}_2) - j5\,\mathbf{I}_2 = 0$$

$$(20 + j\,20)\,\mathbf{I}_1 - j\,15\,\mathbf{I}_2 = 120\angle 0^\circ \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$j5\,\mathbf{I}_2 - j\,20\,(\mathbf{I}_2 - \mathbf{I}_1) - j10\,\mathbf{I}_2 - j5\,(\mathbf{I}_1 - \mathbf{I}_2) - 15\,\mathbf{I}_2 = 0$$

$$-j15\,\mathbf{I}_1 + (15 + j20)\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 20 + j\,20 & -j15 \\ -j15 & 15 + j\,20 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 20 + j\,20 & 120\angle 0^\circ \\ -j15 & 0 \end{vmatrix}}{\begin{vmatrix} 20 + j\,20 & -j15 \\ -j15 & 15 + j\,20 \end{vmatrix}} = 2.53\angle 10.12^\circ\,\text{A}$$

$$\mathbf{V}_{15\,\Omega} = 15\,\mathbf{I}_2 = 15\,(2.53\angle 10.12^\circ) = 37.95\angle 10.12^\circ\,\text{V}$$