6 Time Domain Analysis of RLC Circuits

6.1 INTRODUCTION

Whenever a network containing energy storage elements such as inductor or capacitor is switched from one condition to another, either by change in applied source or change in network elements, the response current and voltage change from one state to the other state. The time taken to change from an initial steady state to the final steady state is known as the *transient period*. This response is known as *transient response* or *transients*. The response of the network after it attains a final steady value is independent of time and is called the steady-state response. The complete response of the network is determined with the help of a differential equation.

6.2 INITIAL CONDITIONS

In solving the differential equations in the network, we get some arbitary constant. Initial conditions are used to determine these arbitrary constants. It helps us to know the behaviour of elements at the instant of switching.

To differentiate between the time immediately before and immediately after the switching, the signs '–' and '+' are used. The conditions existing just before switching are denoted as $i\,(0^-)$, $v\,(0^-)$, etc. Conditions just after switching are denoted as $i\,(0^+)$, $v\,(0^+)$.

Sometimes conditions at $t = \infty$ are used in the evaluation of arbitrary constants. These are known as *final conditions*.

In solving the problems for initial conditions in the network, we divide the time period in the following ways:

1. Just before switching (from $t = -\infty$ *to* $t = 0^-$)
2. Just after switching (at $t = 0^+$)
3. After switching (for $t > 0$)

If the network remains in one condition for a long time without any switching action, it is said to be under steady-state condition.

1. Initial Conditions for the Resistor For a resistor, current and voltage are related by $v(t) = Ri(t)$. The current through a resistor will change instantaneously if the voltage changes instantaneously. Similarly, the voltage will change instantaneously if the current changes instantaneously.

2. Initial Conditions for the Inductor For an inductor, current and voltage are related by,

$$v(t) = L\frac{di}{dt}$$

Voltage across the inductor is proportional to the rate of change of current. It is impossible to change the current through an inductor by a finite amount in zero time. This requires an infinite voltage across the inductor. An inductor does not allow an abrupt change in the current through it.

The current through the inductor is given by,

$$i(t) = \frac{1}{L}\int_0^t v(t)dt + i(0)$$

where $i(0)$ is the initial current through the inductor.

If there is no current flowing through the inductor at $t = 0^-$, the inductor will act as an open circuit at $t = 0^+$. If a current of value I_0 flows through the inductor at $t = 0^-$, the inductor can be regarded as a current source of I_0 ampere at $t = 0^+$.

3. Initial Conditions for the Capacitor For the capacitor, current and voltage are related by,

$$i(t) = C\frac{dv(t)}{dt}$$

Current through a capacitor is proportional to the rate of change of voltage. It is impossible to change the voltage across a capacitor by a finite amount in zero time. This requires an infinite current through the capacitor. A capacitor does not allow an abrupt change in voltage across it.

The voltage across the capacitor is given by,

$$v(t) = \frac{1}{C}\int_0^t i(t)dt + v(0)$$

where $v(0)$ is the initial voltage across the capacitor.

If there is no voltage across the capacitor at $t = 0^-$, the capacitor will act as a short circuit at $t = 0^+$. If the capacitor is charged to a voltage V_0 at $t = 0^-$, it can be regarded as a voltage source of V_0 volt at $t = 0^+$. These conditions are summarized in Fig. 6.1.

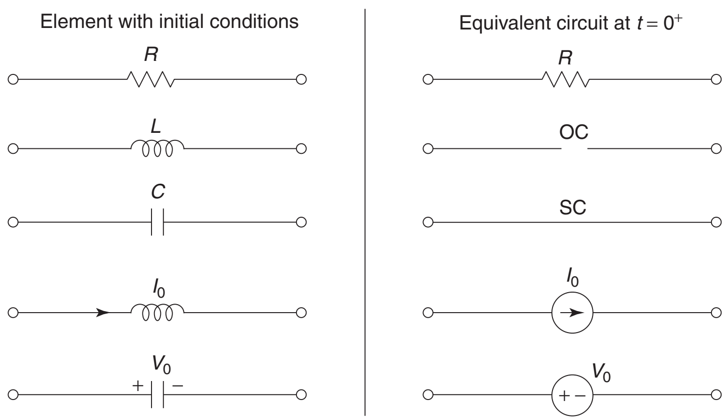


Fig. 6.1 *Initial conditions*

Similarly, we can draw the chart for final conditions as shown in Fig. 6.2

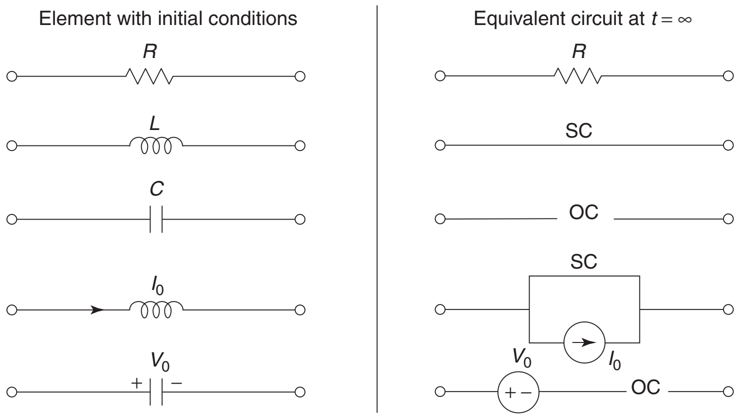


Fig. 6.2 *Final conditions*

4. Procedure for Evaluating Initial Conditions

(a) Draw the equivalent network at $t = 0^-$. Before switching action takes place, i.e., for $t = -\infty$ to $t = 0^-$, the network is under steady-state conditions. Hence, find the current flowing through the inductors $i_L(0^-)$ and voltage across the capacitor $v_C(0^-)$.

(b) Draw the equivalent network at $t = 0^+$, i.e., immediately after switching. Replace all the inductors with open circuits or with current sources $i_L(0^+)$ and replace all capacitors by short circuits or voltage sources $v_C(0^+)$. Resistors are kept as it is in the network.

(c) Initial voltages or currents in the network are determined from the equivalent network at $t = 0^+$.

(d) Initial conditions, i.e., $\frac{di}{dt}(0^+), \frac{dv}{dt}(0^+), \frac{d^2i}{dt^2}(0^+), \frac{d^2v}{dt^2}(0^+)$ are determined by writing integro-differential equations for the network for $t > 0$, i.e., after the switching action by making use of initial condition.

Example 6.1 *In the given network of Fig. 6.3, the switch is closed at t = 0. With zero current in the inductor, find i,* $\frac{di}{dt}$ *and* $\frac{d^2i}{dt^2}$ *at t =* 0^+.

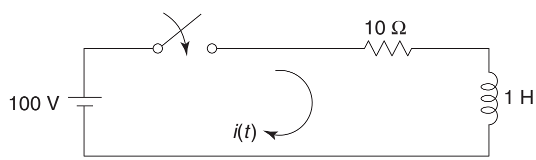


Fig. 6.3

Solution

At $t = 0^-$, no current flows through the inductor.

$$i(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.4.

At $t = 0^+$, the inductor acts as an open circuit.

$$i(0^+) = 0$$

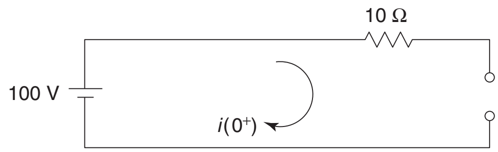


Fig. 6.4

For $t > 0$, the network is shown in Fig. 6.5.
Writing the KVL equation for $t > 0$,

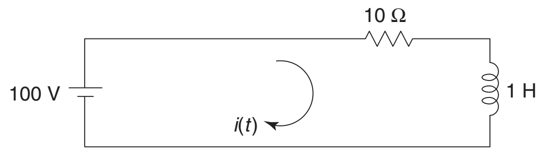


Fig. 6.5

$$100 - 10i - 1\frac{di}{dt} = 0 \qquad \ldots\text{(i)}$$

$$\frac{di}{dt} = 100 - 10i \qquad \ldots\text{(ii)}$$

At $t = 0^+$,

$$\frac{di}{dt}(0^+) = 100 - 10i(0^+) = 100 - 10(0) = 100 \text{ A/s}$$

Differentiating Eq. (ii),

$$\frac{d^2i}{dt^2} = -10\frac{di}{dt}$$

At $t = 0^+$,

$$\frac{d^2i}{dt^2}(0^+) = -10\frac{di}{dt}(0^+) = -10(100) = -1000 \text{ A/s}^2$$

Example 6.2 *In the network of Fig. 6.6, the switch is closed at t = 0. With the capacitor uncharged, find value for* i, $\frac{di}{dt}$ *and* $\frac{d^2i}{dt^2}$ *at* $t = 0^+$.

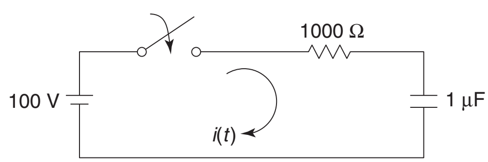


Fig. 6.6

Solution

At $t = 0^-$, the capacitor is uncharged.

$$v_C(0^-) = 0$$

$$i(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.7.
At $t = 0^+$, the capacitor acts as a short circuit.

$$v_C(0^+) = 0$$

$$i(0^+) = \frac{100}{1000} = 0.1 \text{ A}$$

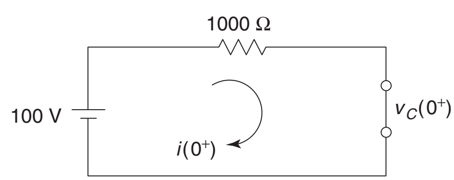


Fig. 6.7

For $t > 0$, the network is shown in Fig. 6.8.
Writing the KVL equation for $t > 0$,

$$100 - 1000i - \frac{1}{1 \times 10^{-6}} \int_0^t i\, dt = 0 \qquad \ldots(\text{i})$$

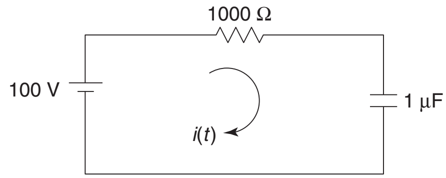


Fig. 6.8

Differentiating Eq. (i),

$$0 - 1000\frac{di}{dt} - 10^6 i = 0$$

$$\frac{di}{dt} = -\frac{10^6}{1000} i \qquad \ldots(\text{ii})$$

At $t = 0^+$,

$$\frac{di}{dt}(0^+) = -\frac{10^6}{1000} i(0^+) = -\frac{10^6}{1000}(0.1) = -100 \text{ A/s}$$

Differentiating Eq. (ii),

$$\frac{d^2i}{dt^2} = -\frac{10^6}{1000}\frac{di}{dt}$$

At $t = 0^+$,

$$\frac{d^2i}{dt^2}(0^+) = -\frac{10^6}{1000}\frac{di}{dt}(0^+) = -\frac{10^6}{1000}(-100) = 10^5 \text{ A/s}^2$$

Example 6.3 *In the network shown in Fig. 6.9, the switch is closed. Assuming all initial conditions as zero, find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.*

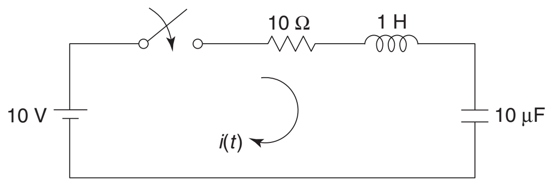


Fig. 6.9

Solution

At $t = 0^-$,

$$i(0^-) = 0$$

$$v_C(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.10.

At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit.

$$i(0^+) = 0$$

$$v_C(0^+) = 0$$

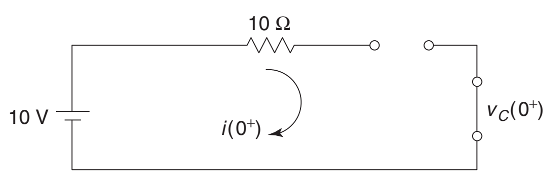


Fig. 6.10

For $t > 0$, the network is shown in Fig. 6.11.
Writing the KVL equation for $t > 0$,

$$10 - 10i - 1\frac{di}{dt} - \frac{1}{10\times 10^{-6}}\int_0^t i\,dt = 0 \qquad \ldots\text{(i)}$$

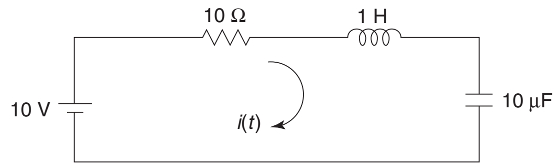


Fig. 6.11

At $t = 0^+$, $\quad 10 - 10i(0^+) - \frac{di}{dt}(0^+) - 0 = 0$

$$\frac{di}{dt}(0^+) = 10\text{A/s}$$

Differentiating Eq. (i),

$$0 - 10\frac{di}{dt} - \frac{d^2i}{dt^2} - \frac{1}{10\times 10^{-6}}i = 0$$

At $t = 0^+$, $$0 - 10\frac{di}{dt}(0^+) - \frac{d^2i}{dt^2}(0^+) - \frac{1}{10^{-5}}i(0^+) = 0$$

$$\frac{d^2i}{dt^2}(0^+) = -10\times 10 = -100\ \text{A/s}^2$$

Example 6.4 *In the network shown in Fig. 6.12, at $t = 0$, the switch is opened. Calculate v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t = 0^+$.*

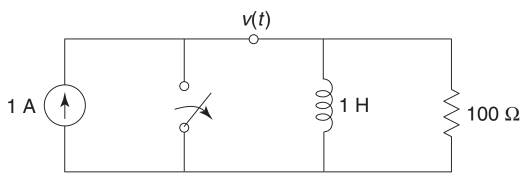


Fig. 6.12

Solution At $t = 0^-$, the switch is closed. Hence, no current flows through the inductor.

$$i_L(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.13.
At $t = 0^+$, the inductor acts as an open circuit.

$$i_L(0^+) = 0$$

$$v(0^+) = 100\times 1 = 100\ \text{V}$$

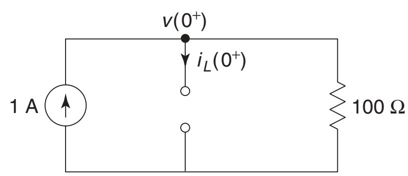


Fig. 6.13

For $t > 0$, the network is shown in Fig. 6.14.
Writing the KCL equation for $t > 0$,

$$\frac{v}{100} + \frac{1}{1}\int_0^t v\,dt = 1 \qquad \ldots\text{(i)}$$

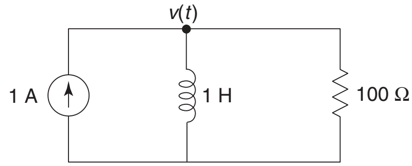


Fig. 6.14

Differentiating Eq. (i),

$$\frac{1}{100}\frac{dv}{dt} + v = 0 \qquad \ldots\text{(ii)}$$

At $t = 0^+$,
$$\frac{dv}{dt}(0^+) = -100v(0^+) = -100 \times 100 = -10000 \text{ V/s}$$

Differentiating Eq. (ii),

$$\frac{1}{100}\frac{d^2v}{dt^2} + \frac{dv}{dt} = 0$$

At $t = 0^+$,
$$\frac{d^2v}{dt^2}(0^+) = -100\frac{dv}{dt}(0^+) = -100 \times (-10^4) = 10^6 \text{ V/s}^2$$

Example 6.5 *In the given network of Fig. 6.15, the switch is opened at $t = 0$. Solve for v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t = 0^+$.*

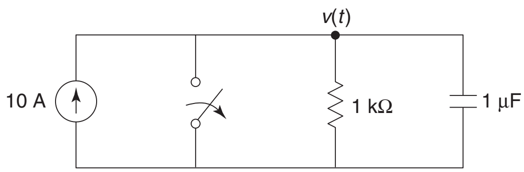


Fig. 6.15

Solution At $t = 0^-$, switch is closed. Hence, the voltage across the capacitor is zero.

$$v(0^-) = v_C(0^-) = 0$$

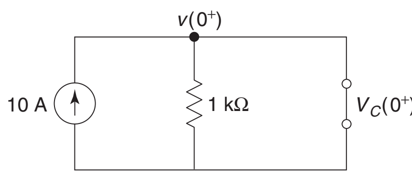


Fig. 6.16

At $t = 0^+$, the network is shown in Fig. 6.16.
At $t = 0^+$, the capacitor acts as a short circuit.

$$v(0^+) = v_C(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.17.
Writing the KCL equation for $t > 0$,

$$\frac{v}{1000} + 10^{-6}\frac{dv}{dt} = 10 \qquad \ldots\text{(i)}$$

At $t = 0^+$,
$$\frac{v(0^+)}{1000} + 10^{-6}\frac{dv}{dt}(0^+) = 10$$

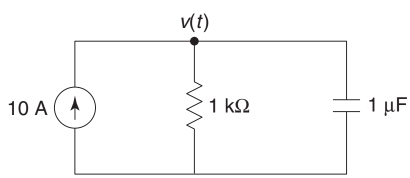


Fig. 6.17

$$\frac{dv}{dt}(0^+) = \frac{10}{10^{-6}} = 10 \times 10^6 \text{ V/s}$$

Differentiating Eq. (i),

$$\frac{1}{1000}\frac{dv}{dt} + 10^{-6}\frac{d^2v}{dt^2} = 0$$

At $t = 0^+$,
$$\frac{1}{1000}\frac{dv}{dt}(0^+) + 10^{-6}\frac{d^2v}{dt^2}(0^+) = 0$$

$$\frac{d^2v}{dt^2}(0^+) = -\frac{1}{1000 \times 10^{-6}} \times 10 \times 10^6 = -10 \times 10^9 \text{ V/s}^2$$

Example 6.6 *For the network shown in Fig. 6.18, the switch is closed at $t = 0$, determine v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t = 0^+$.*

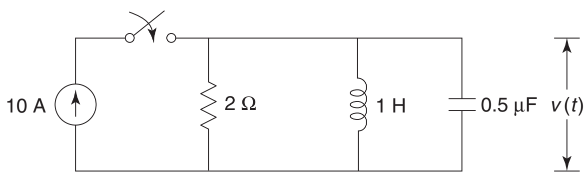


Fig. 6.18

Solution At $t = 0^-$, no current flows through the inductor and there is no voltage across the capacitor.

$$i_L(0^-) = 0$$
$$v(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.19.

At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit.

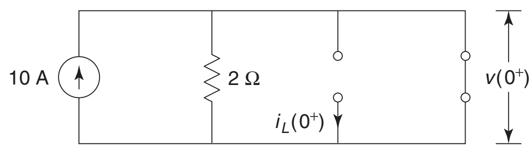


Fig. 6.19

$$i_L(0^+) = 0$$
$$v(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.20.

Writing the KCL equation for $t > 0$,

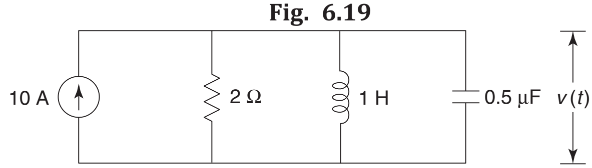


Fig. 6.20

$$\frac{v}{2} + \frac{1}{1}\int_1^t v\,dt + 0.5 \times 10^{-6}\frac{dv}{dt} = 10 \qquad \ldots\text{(i)}$$

At $t = 0^+$,
$$\frac{v(0^+)}{2} + 0 + 0.5 \times 10^{-6}\frac{dv}{dt}(0^+) = 10$$

$$\frac{dv}{dt}(0^+) = 20 \times 10^6 \text{ V/s}$$

Differentiating Eq. (i),

$$\frac{1}{2}\frac{dv}{dt}+v+0.5\times10^{-6}\frac{d^2v}{dt^2}=0$$

At $t = 0^+$,
$$\frac{1}{2}\frac{dv}{dt}(0^+)+v(0^+)+0.5\times10^{-6}\frac{d^2v}{dt^2}(0^+)=0$$

$$\frac{d^2v}{dt^2}(0^+)=-20\times10^{12}\ \text{V/s}^2$$

Example 6.7 *In the network shown in Fig. 6.21, the switch is changed from the position 1 to the position 2 at t = 0, steady condition having reached before switching. Find the values of* i, $\frac{di}{dt}$ *and* $\frac{d^2i}{dt^2}$ *at* $t = 0^+$.

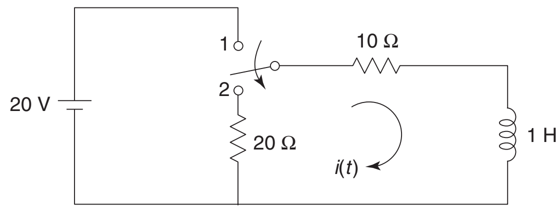


Fig. 6.21

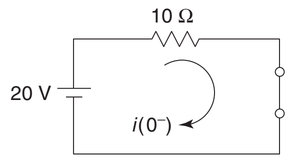


Fig. 6.22

Solution At $t = 0^-$, the network attains steady-state condition. Hence, the inductor acts as a short circuit.

$$i(0^-)=\frac{20}{10}=2\ \text{A}$$

At $t = 0^+$, the network is shown in Fig. 6.23.
At $t = 0^+$, the inductor acts as a current source of 2 A.

$$i(0^+)=2\ \text{A}$$

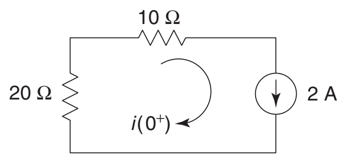


Fig. 6.23

For $t > 0$, the network is shown in Fig. 6.24.
Writing the KVL equation for $t > 0$,

$$-20i-10i-1\frac{di}{dt}=0 \qquad \ldots\text{(i)}$$

At $t = 0^+$,
$$-30i(0^+)-\frac{di}{dt}(0^+)=0$$

$$\frac{di}{dt}(0^+)=-30\times2=-60\ \text{A/s}$$

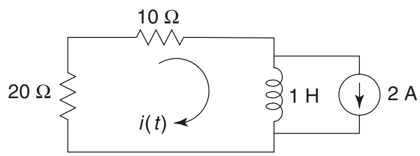


Fig. 6.24

Differentiating Eq. (i),

$$-30\frac{di}{dt}-\frac{d^2i}{dt^2}=0$$

At $t = 0^+$,
$$-30\frac{di}{dt}(0^+) - \frac{d^2 i}{dt^2}(0^+) = 0$$
$$\frac{d^2 i}{dt^2}(0^+) = 1800 \text{ A/s}^2$$

Example 6.8 *In the network shown in Fig.6.25, the switch is changed from the position 1 to the position 2 at t = 0, steady condition having reached before switching. Find the values of* i, $\frac{di}{dt}$ *and* $\frac{d^2 i}{dt^2}$ *at* $t = 0^+$.

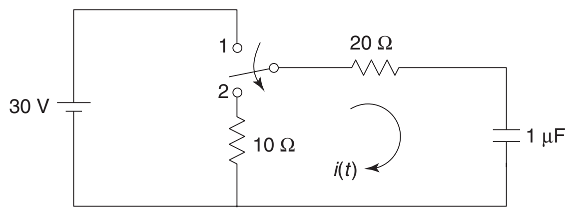


Fig. 6.25

Solution At $t = 0^-$, the network attains steady-state condition. Hence, the capacitor acts as an open circuit.

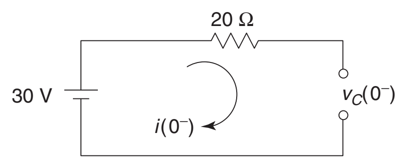


Fig. 6.26

$$v_C(0^-) = 30 \text{ V}$$
$$i(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.27.
At $t = 0^+$, the capacitor acts as a voltage source of 30 V.

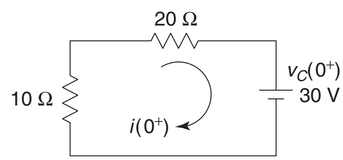


Fig. 6.27

$$v_C(0^+) = 30 \text{ V}$$
$$i(0^+) = -\frac{30}{30} = -1 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.28.
Writing the KVL equation for $t > 0$,

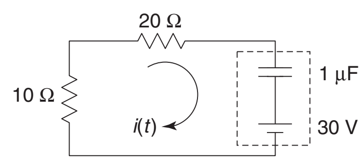


Fig. 6.28

$$-10i - 20i - \frac{1}{1\times10^{-6}}\int_0^t i\,dt - 30 = 0 \qquad \ldots\text{(i)}$$

Differentiating Eq. (i),
$$-30\frac{di}{dt} - 10^6 i = 0 \qquad \ldots\text{(ii)}$$

At $t = 0^+$,
$$-30\frac{di}{dt}(0^+) - 10^6 i(0^+) = 0$$
$$\frac{di}{dt}(0^+) = \frac{10^6(-1)}{30} = 0.33\times10^5 \text{ A/s}$$

Differentiating Eq. (ii),

$$-30\frac{d^2i}{dt^2}-10^6\frac{di}{dt}=0$$

At $t = 0^+$,
$$-30\frac{d^2i}{dt^2}(0^+)-10^6\frac{di}{dt}(0^+)=0$$

$$\frac{d^2i}{dt^3}(0^+)=-\frac{10^6\times 0.33\times 10^5}{30}=-1.1\times 10^9\ \text{A/s}^2$$

Example 6.9 *In the network shown in Fig. 6.29, the switch is changed from the position 1 to the position 2 at t = 0, steady condition having reached before switching. Find the values of* i, $\frac{di}{dt}$ *and* $\frac{d^2i}{dt^2}$ *at* $t = 0^+$.

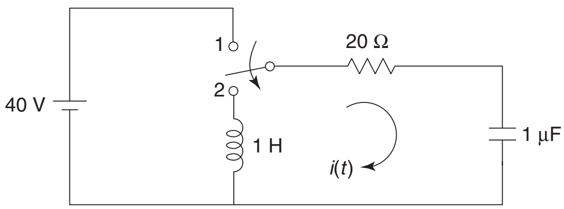


Fig. 6.29

Solution At $t = 0^-$, the network attains steady state. Hence, the capacitor acts as an open circuit.

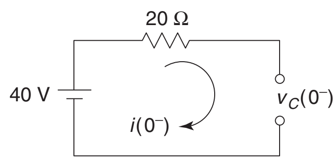


Fig. 6.30

$$v_C(0^-)=40\text{ V}$$

$$i(0^-)=0$$

At $t=0^+$, the network is shown in Fig. 6.31.

At $t = 0^+$, the capacitor acts as a voltage source of 40 V and the inductor acts as an open circuit.

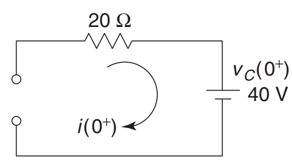


Fig. 6.31

$$v_C(0^+)=40\text{ V}$$

$$i(0^+)=0$$

For $t>0$, the network is shown in Fig. 6.32.

Writing the KVL equation fo $t > 0$,

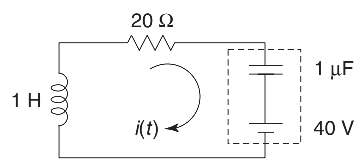


Fig. 6.32

$$-1\frac{di}{dt}-20i-\frac{1}{1\times 10^{-6}}\int_0^t i\,dt-40=0 \qquad \ldots\text{(i)}$$

At $t = 0^+$,
$$-\frac{di}{dt}(0^+)-20i(0^+)-0-40=0$$

$$\frac{di}{dt}(0^+)=-40\text{ A/s}$$

Differentiating Eq. (i),

$$-\frac{d^2i}{dt^2}-20\frac{di}{dt}-10^6 i-0=0$$

At $t=0^+$,

$$-\frac{d^2i}{dt^2}(0^+)-20\frac{di}{dt}(0^+)-10^6 i(0^+)=0$$

$$\frac{d^2i}{dt^2}(0^+)=800\ \text{A/s}^2$$

Example 6.10 *In the network of Fig. 6.33, the switch is changed from the position 'a' to 'b' at $t = 0$. Solve for i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.*

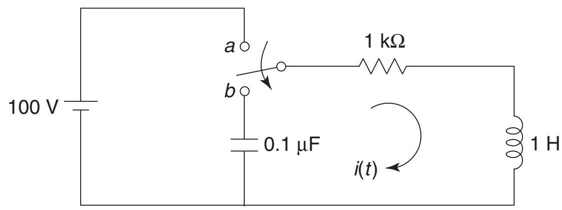


Fig. 6.33

Solution At $t = 0^-$, the network attains steady condition. Hence, the inductor acts as a short circuit.

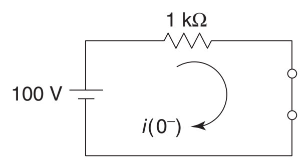


Fig. 6.34

$$i(0^-)=\frac{100}{1000}=0.1\ \text{A}$$

$$v_C(0^-)=0$$

At $t=0^+$, the network is shown in Fig. 6.35.

At $t = 0^+$, the inductor acts as a current source of 0.1 A and the capacitor acts as a short circuit.

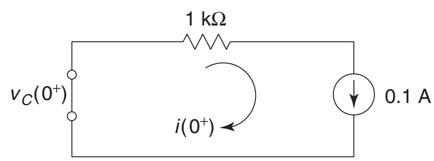


Fig. 6.35

$$i(0^+)=0.1\ \text{A}$$

$$v_C(0^+)=0$$

For $t>0$, the network is shown in Fig. 6.36.

Writing the KVL equation for $t > 0$,

$$-\frac{1}{0.1\times10^{-6}}\int_0^t i\,dt-1000i-1\frac{di}{dt}=0 \qquad \ldots\text{(i)}$$

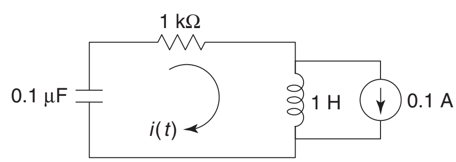


Fig. 6.36

At $t = 0^+$, $$-0 - 1000i(0^+) - \frac{di}{dt}(0^+) = 0$$

$$\frac{di}{dt}(0^+) = -1000\, i(0^+) = -1000 \times 0.1 = -100 \text{ A/s}$$

Differentiating Eq. (i),

$$-\frac{1}{10^{-7}} i - 1000\frac{di}{dt} - \frac{d^2 i}{dt^2} = 0$$

At $t = 0^+$, $$-10^7 i(0^+) - 1000\frac{di}{dt}(0^+) - \frac{d^2 i}{dt^2}(0^+) = 0$$

$$\frac{d^2 i}{dt^2}(0^+) = -10^7(0.1) - 1000(-100) = -9 \times 10^5 \text{ A/s}^2$$

Example 6.11 *The network of Fig. 6.37 attains steady-state with the switch closed. At $t = 0$, the switch is opened. Find the voltage across the switch v_K and $\frac{dv_K}{dt}$ at $t = 0^+$.*

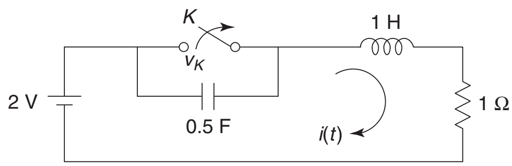


Fig. 6.37

Solution: At $t = 0^-$, the network is shown in Fig. 6.38. At $t = 0^-$, the network attains steady-state condition. The capacitor acts as an open circuit and the inductor acts as a short circuit.

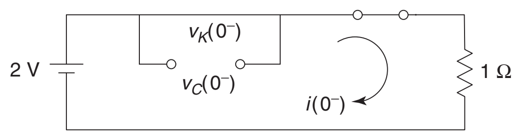


Fig. 6.38

$$i(0^-) = \frac{2}{1} = 2 \text{ A}$$

$$v_C(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.39.

At $t = 0^+$, the capacitor acts as a short circuit and the inductor acts as a current source of 2 A.

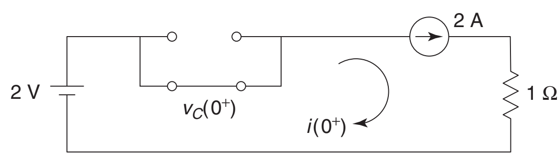


Fig. 6.39

$$i(0^+) = 2 \text{ A}$$

$$v_C(0^+) = 0$$

$$v_K(0^+) = 0$$

Also,
$$v_K = \frac{1}{C}\int i\,dt$$
$$\frac{dv_K}{dt} = \frac{i}{C}$$

At $t = 0^+$,
$$\frac{dv_K}{dt}(0^+) = \frac{i(0^+)}{C} = \frac{2}{0.5} = 4\text{ A/s}$$

Example 6.12 *In the network shown in Fig. 6.40, assuming all initial conditions as zero, find* $i_1(0^+), i_2(0^+), \frac{di_1}{dt}(0^+), \frac{di_2}{dt}(0^+), \frac{d^2 i_1}{dt^2}(0^+)$ *and* $\frac{d^2 i_2}{dt^2}(0^+)$.

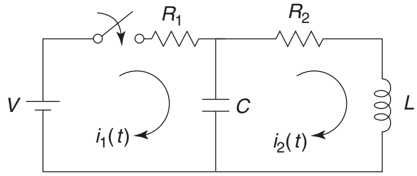


Fig. 6.40

Solution At $t = 0^-$, all initial conditions are zero.

$$v_C(0^-) = 0$$
$$i_1(0^-) = 0$$
$$i_2(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.41.

At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit.

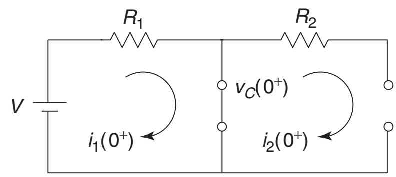


Fig. 6.41

$$i_1(0^+) = \frac{V}{R_1}$$
$$i_2(0^+) = 0$$
$$v_C(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.42.

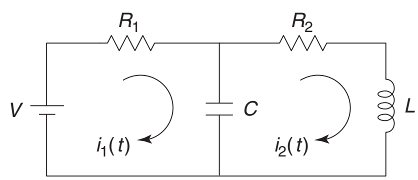


Fig. 6.42

Writing the KVL equations for two meshes for $t > 0$,

$$V - R_1 i_1 - \frac{1}{C}\int_0^t (i_1 - i_2)dt = 0 \qquad \text{...(i)}$$

and
$$-\frac{1}{C}\int (i_2 - i_1)dt - R_2 i_2 - L\frac{di_2}{dt} = 0 \qquad \text{...(ii)}$$

From Eq. (ii), at $t = 0^+$,

$$-\frac{1}{C}\int_0^{0^+}(i_2 - i_1)dt - R_2 i_2(0^+) - L\frac{di_2}{dt}(0^+) = 0$$

$$\frac{di_2}{dt}(0^+) = 0$$

Differentiating Eq. (i),

$$0 - R_1\frac{di_1}{dt} - \frac{1}{C}(i_1 - i_2) = 0 \qquad \ldots\text{(iii)}$$

At $t = 0^+$,

$$0 - R_1\frac{di_1}{dt}(0^+) - \frac{1}{C}i_1(0^+) + \frac{1}{C}i_2(0^+) = 0$$

$$R_1\frac{di_1}{dt}(0^+) + \frac{1}{C}\frac{V}{R_1} = 0$$

$$\frac{di_1}{dt}(0^+) = -\frac{V}{R_1^2 C}$$

Differentiating Eq. (iii),

$$-R_1\frac{d^2 i_1}{dt^2} - \frac{1}{C}\frac{di_1}{dt} + \frac{1}{C}\frac{di_2}{dt} = 0$$

At $t = 0^+$,

$$-R_1\frac{d^2 i_1}{dt^2}(0^+) - \frac{1}{C}\frac{di_1}{dt}(0^+) + \frac{1}{C}\frac{di_2}{dt}(0^+) = 0$$

$$\frac{d^2 i_1}{dt^2}(0^+) = \frac{V}{R_1^3 C^2}$$

Differentiating Eq. (ii),

$$-\frac{1}{C}(i_2 - i_1) - R_2\frac{di_2}{dt} - L\frac{d^2 i_2}{dt^2} = 0$$

At $t = 0^+$,

$$\frac{d^2 i_2}{dt^2}(0^+) = -\frac{R_2}{L}\frac{di_2}{dt}(0^+) - \frac{1}{LC}[i_2(0^+) - i_1(0^+)] = \frac{V}{R_1 LC}$$

Example 6.13 *In the network shown in Fig. 6.43, assuming all initial conditions as zero, find* $i_1, i_2, \frac{di_1}{dt}, \frac{di_2}{dt}, \frac{d^2 i_1}{dt^2}$ *and* $\frac{d^2 i_2}{dt^2}$ *at* $t = 0^+$.

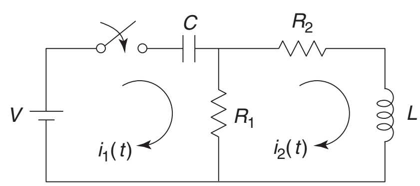


Fig. 6.43

Solution At $t = 0^-$, all initial conditions are zero.

$$v_C(0^-) = 0$$
$$i_1(0^-) = 0$$
$$i_2(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.44.

At $t = 0^+$, the capacitor acts as a short circuit and the inductor acts as an open circuit.

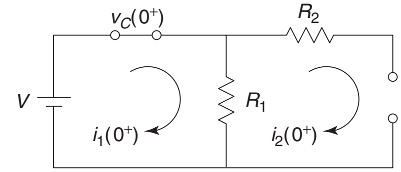


Fig. 6.44

$$i_1(0^+) = \frac{V}{R_1}$$
$$i_2(0^+) = 0$$
$$v_C(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.45.

Writing the KVL equation for $t > 0$,

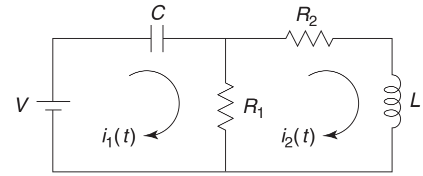


Fig. 6.45

$$V - \frac{1}{C}\int_0^t i_1\, dt - R_1(i_1 - i_2) = 0 \qquad \ldots\text{(i)}$$

and
$$-R_1(i_2 - i_1) - R_2 i_2 - L\frac{di_2}{dt} = 0 \qquad \ldots\text{(ii)}$$

From Eq. (ii),

$$\frac{di_2}{dt} = \frac{1}{L}[R_1 i_1 - (R_1 + R_2)i_2] \qquad \ldots\text{(iii)}$$

At $t = 0^+$,
$$\frac{di_2}{dt}(0^+) = \frac{1}{L}[R_1 i_1(0^+) - (R_1 + R_2)i_2(0^+)] = \frac{1}{L}\left[R_1\frac{V}{R_1} - (R_1 + R_2)0\right] = \frac{V}{L}$$

Differentiating Eq. (i),

$$0 - \frac{i_1}{C} - R_1\frac{di_1}{dt} + R_1\frac{di_2}{dt} = 0$$
$$\frac{di_1}{dt} = \frac{di_2}{dt} - \frac{i_1}{R_1 C} \qquad \ldots\text{(iv)}$$

At $t = 0^+$,
$$\frac{di_1}{dt}(0^+) = \frac{di_2}{dt}(0^+) - \frac{i_1(0^+)}{R_1 C} = \frac{V}{L} - \frac{V}{R_1^2 C}$$

Differentiating Eq. (iii),

$$\frac{d^2 i_2}{dt^2} = \frac{1}{L}\left[R_1\frac{di_1}{dt} - (R_1 + R_2)\frac{di_2}{dt}\right]$$

At $t = 0^+$,
$$\frac{d^2 i_2}{dt^2}(0^+) = -V\left(\frac{1}{R_1 LC} + \frac{R_2}{L^2}\right)$$

Differentiating Eq. (iv),

$$\frac{d^2 i_1}{dt^2} = \frac{d^2 i_2}{dt^2} - \frac{1}{R_1 C}\frac{di_1}{dt}$$

At $t = 0^+$, $\dfrac{d^2 i_1}{dt^2}(0^+) = \dfrac{d^2 i_2}{dt^2}(0^+) - \dfrac{1}{R_1 C}\dfrac{di_1}{dt}(0^+) = -\dfrac{V}{R_1 LC} - \dfrac{VR_2}{L^2} - \dfrac{1}{R_1 C}\left(\dfrac{V}{L} - \dfrac{V}{R_1^2 C}\right) = \dfrac{V}{R_1^3 C^2} - \dfrac{2V}{R_1 LC} - \dfrac{VR_2}{L^2}$

Example 6.14 *In the network shown in Fig. 6.46, a steady state is reached with the switch open. At t = 0, the switch is closed. For the element values given, determine the value of $v_a(0^-)$, $v_b(0^-)$, $v_a(0^+)$ and $v_b(0^+)$.*

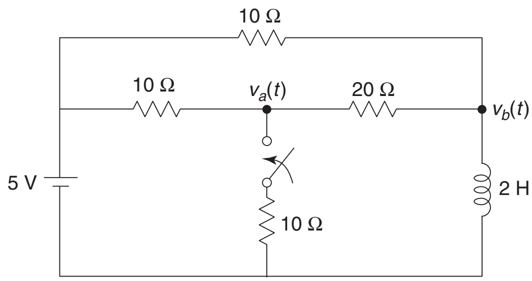


Fig. 6.46

Solution At $t = 0^-$, the network is shown in Fig. 6.47.

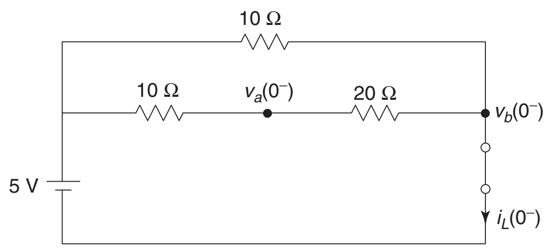


Fig. 6.47

At $t = 0^-$, the network attains steady-state condition. Hence, the inductor acts as a short circuit.

$$i_L(0^-) = \frac{5}{(10 \parallel 30)} = \frac{5}{7.5} = \frac{2}{3}\text{ A}$$

$$v_b(0^-) = 0$$

$$v_a(0^-) = 5 \times \frac{20}{30} = 3.33\text{ V}$$

At $t = 0^+$, the network is shown in Fig. 6.48.

At $t = 0^+$, the inductor acts as a current source of $\dfrac{2}{3}$ A.

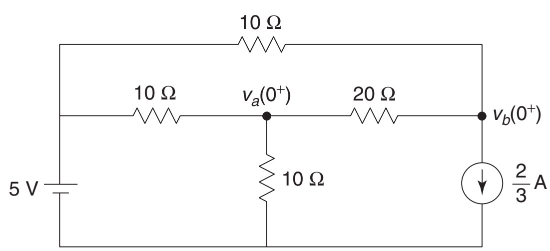


Fig. 6.48

$$i_L(0^+) = \frac{2}{3}\text{ A}$$

Writing the KCL equations at $t = 0^+$,

$$\frac{v_a(0^+) - 5}{10} + \frac{v_a(0^+)}{10} + \frac{v_a(0^+) - v_b(0^+)}{20} = 0$$

and
$$\frac{v_b(0^+)-v_a(0^+)}{20}+\frac{v_b(0^+)-5}{10}+\frac{2}{3}=0$$

Solving these two equations,

$$v_a(0^+)=1.9 \text{ V}$$
$$v_b(0^+)=-0.477 \text{ V}$$

Example 6.15 *In the accompanying Fig. 6.49 is shown a network in which a steady state is reached with switch open. At $t = 0$, switch is closed. Determine $v_a(0^-)$, $v_a(0^+)$, $v_b(0^-)$ and $v_b(0^+)$.*

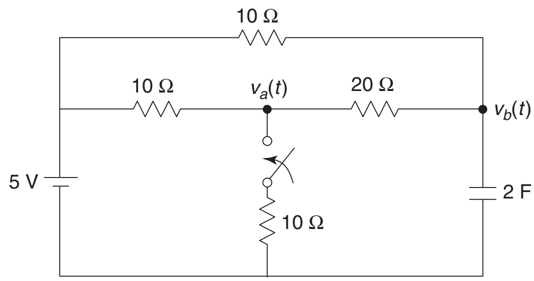


Fig. 6.49

Solution At $t = 0^-$, the network is shown in Fig. 6.50.

At $t = 0^-$, the network attains steady-state condition. Hence, the capacitor acts as an open circuit.

$$v_a(0^-)=5 \text{ V}$$
$$v_b(0^-)=5 \text{ V}$$

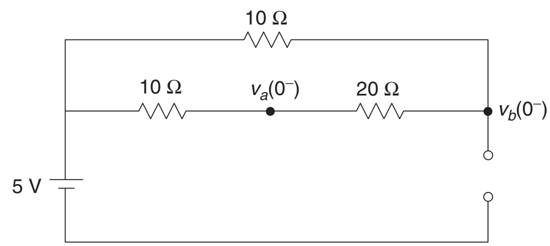


Fig. 6.50

At $t = 0^+$, the network is shown in Fig. 6.51.

At $t = 0^+$, the capacitor acts as a voltage source of 5 V.

$$v_b(0^+)=5 \text{ V}$$

Writing the KCL equation at $t = 0^+$,

$$\frac{v_a(0^+)-5}{10}+\frac{v_a(0^+)}{10}+\frac{v_a(0^+)-5}{20}=0$$
$$0.25v_a(0^+)=0.75$$
$$v_a(0^+)=3 \text{ V}$$

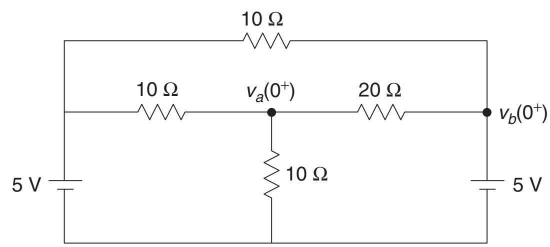


Fig. 6.51

Example 6.16 *The network shown in Fig. 6.52 has two independent node pairs. If the switch is opened at $t = 0$. Find $v_1, v_2, \dfrac{dv_1}{dt}$ and $\dfrac{dv_2}{dt}$ at $t = 0^+$.*

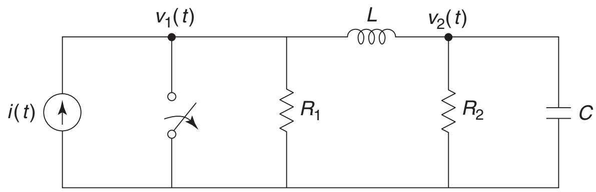


Fig. 6.52

Solution At $t = 0^-$, no current flows through the inductor and there is no voltage across the capacitor.

$$i_L(0^-) = 0$$

$$v_C(0^-) = v_2(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.53.
At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit.

$$i_L(0^+) = 0$$

$$v_1(0^+) = R_1\, i(0^+)$$

$$v_2(0^+) = 0$$

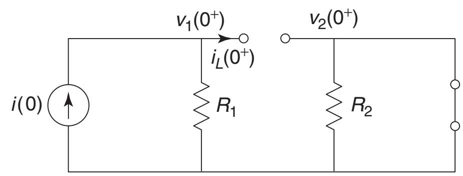


Fig. 6.53

For $t > 0$, the network is shown in Fig. 6.54.
Writing the KCL equation at Node 1 for $t > 0$,

$$\frac{v_1}{R_1} + \frac{1}{L}\int_0^t (v_1 - v_2)dt = i(t) \qquad \ldots\text{(i)}$$

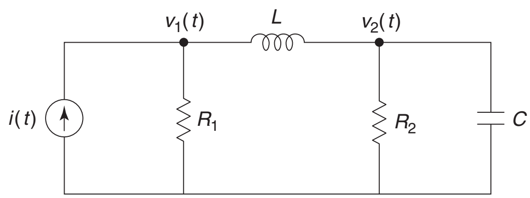


Fig. 6.54

Differentiating Eq. (i),

$$\frac{1}{R_1}\frac{dv_1}{dt} + \frac{1}{L}(v_1 - v_2) = \frac{di}{dt}$$

At $t = 0^+$,

$$\frac{dv_1}{dt}(0^+) = R_1\left[\frac{di}{dt}(0^+) - \frac{1}{L}R_1 i(0^+)\right]$$

Writing the KCL equation at Node 2 for $t > 0$,

$$\frac{1}{L}\int_0^t (v_2 - v_1)dt + \frac{v_2}{R_2} + C\frac{dv_2}{dt} = 0 \qquad \ldots\text{(ii)}$$

At $t = 0^+$,

$$0 + \frac{v_2(0^+)}{R_2} + C\frac{dv_2}{dt}(0^+) = 0$$

$$\frac{dv_2}{dt}(0^+) = 0$$

Example 6.17 *In the network shown in Fig. 6.55, the switch is closed at t = 0, with zero capacitor voltage and zero inductor current. Solve for* $v_1, v_2, \dfrac{dv_1}{dt}, \dfrac{dv_2}{dt}$ *and* $\dfrac{d^2v_2}{dt^2}$ *at* $t = 0^+$.

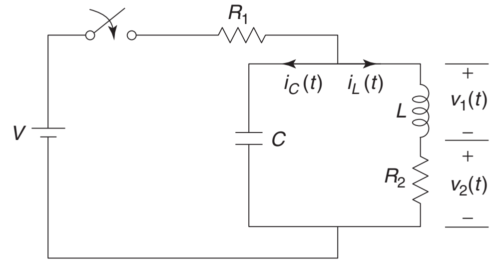


Fig. 6.55

Solution At $t = 0^-$, no current flows through the inductor and there is no voltage across the capacitor.

$$v_C(0^-) = 0$$

$$v_1(0^-) = 0$$

$$v_2(0^-) = 0$$

$$i_L(0^-) = 0$$

$$i_C(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.56.

At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit.

$$v_C(0^+) = 0$$

$$v_1(0^+) = 0$$

$$v_2(0^+) = 0$$

$$i_L(0^+) = 0$$

$$i_C(0^+) = \frac{V}{R_1}$$

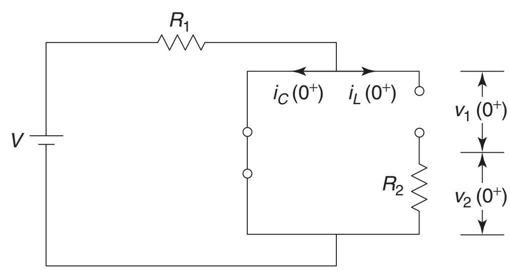


Fig. 6.56

For $t > 0$, the network is shown in Fig. 6.57.

Writing the KVL equation for $t > 0$,

$$v_C(t) = v_1(t) + v_2(t) \qquad \ldots\text{(i)}$$

Differentiating Eq. (i),

$$\frac{dv_C}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt} \qquad \ldots\text{(ii)}$$

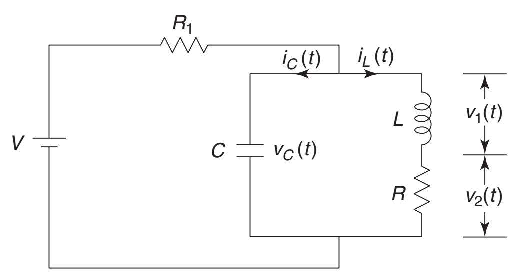


Fig. 6.57

Now, $$v_C = \frac{1}{C}\int_0^t i_C \, dt \qquad \ldots\text{(iii)}$$

$$\frac{dv_C}{dt} = \frac{i_C}{C}$$

At $t = 0^+$, $$\frac{dv_C}{dt}(0^+) = \frac{i_C(0^+)}{C} = \frac{V}{R_1 C} \text{ V/s}$$

Also $$v_1 = L\frac{di_L}{dt} \qquad \ldots\text{(iv)}$$

$$\frac{di_L}{dt} = \frac{v_1}{L} \qquad \ldots\text{(v)}$$

At $t = 0^+$, $$\frac{di_L}{dt}(0^+) = \frac{v_1(0^+)}{L} = 0$$

Also, $$v_2 = R_2 i_L \qquad \ldots\text{(vi)}$$

$$\frac{dv_2}{dt} = R_2\frac{di_L}{dt} \qquad \ldots\text{(vii)}$$

At $t = 0^+$, $$\frac{dv_2}{dt}(0^+) = R_2\frac{di_L}{dt}(0^+) = 0$$

$$\frac{dv_C}{dt}(0^+) = \frac{dv_1}{dt}(0^+) + \frac{dv_2}{dt}(0^+)$$

$$\frac{dv_1}{dt}(0^+) = \frac{V}{R_1 C} \text{ V/s}$$

Differentiating Eq. (vii),

$$\frac{d^2 v_2}{dt^2} = R_2\frac{d^2 i_L}{dt^2}$$

At $t = 0^+$, $$\frac{d^2 v_2}{dt^2}(0^+) = R_2\frac{d^2 i_L}{dt}(0^+)$$

Differentiating Eq. (v),

$$\frac{d^2 i_L}{dt^2} = \frac{1}{L}\frac{dv_1}{dt}$$

At $t = 0^+$, $$\frac{d^2 i_L}{dt^2}(0^+) = \frac{1}{L}\frac{dv_1}{dt}(0^+) = \frac{1}{L}\frac{V}{R_1 C}$$

$$\frac{d^2 v_2}{dt^2}(0^+) = \frac{R_2 V}{R_1 LC} \text{ V/s}^2$$

Example 6.18 *In the network shown in Fig. 6.58, a steady state is reached with switch open. At $t =$ 0, switch is closed. Find the three loop currents at $t = 0^+$.*

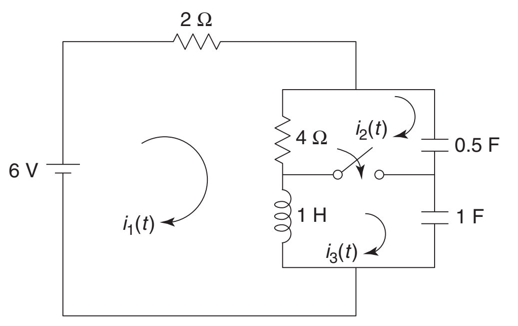


Fig. 6.58

Solution At $t = 0^-$, the network is shown in Fig. 6.59.

At $t = 0^-$, the network attains steady-state condition. Hence, the inductor act as a short circuit and the capacitors act as open circuits.

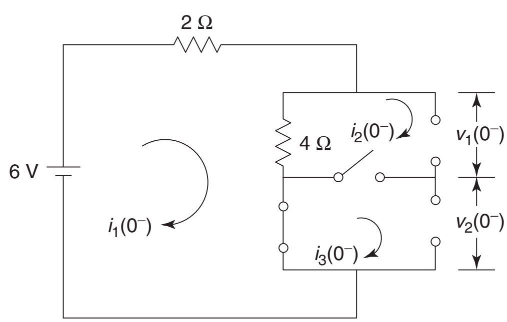


Fig. 6.59

$$i_{4\Omega}(0^-) = i_1(0^-) = \frac{6}{6} = 1\text{ A}$$

$$i_2(0^-) = 0$$

$$i_3(0^-) = 0$$

$$v_1(0^-) + v_2(0^-) = 6 \times \frac{4}{6} = 4\text{ V}$$

Since the charges on capacitors are equal when connected in series,

$$Q_1 = Q_2$$

$$C_1 v_1 = C_2 v_2$$

$$\frac{v_1(0^-)}{v_2(0^-)} = \frac{C_2}{C_1} = \frac{1}{0.5} = 2$$

$$v_1(0^-) = \frac{8}{3}\text{ V}$$

and $$v_2(0^-) = \frac{4}{3}\text{ V}$$

At $t = 0^+$, the network is shown in Fig. 6.60.

At $t = 0^+$, the inductor is replaced by a current source of 1 A and the capacitors are replaced by a voltage source of $\frac{8}{3}$ V and $\frac{4}{3}$ V respectively.

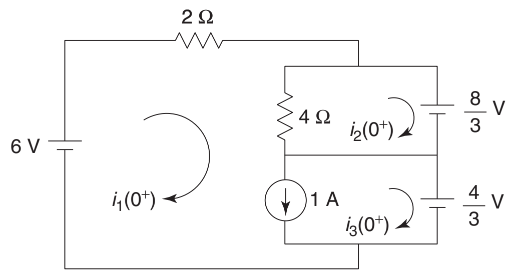


Fig. 6.60

$$v_1(0^+) = \frac{8}{3}\text{ V}$$

$$v_2(0^+) = \frac{4}{3}\text{ V}$$

At $t = 0^+$,

$$6 - 2i_1(0^+) - \frac{8}{3} - \frac{4}{3} = 0$$

$$i_1(0^+) = 1\text{ A}$$

Now,

$$i_1(0^+) - i_3(0^+) = 1$$

$$i_3(0^+) = 0$$

Writing the KVL equation for Mesh 2,

$$-4[i_2(0^+) - i_1(0^+)] - \frac{8}{3} = 0$$

$$-4i_2(0^+) + 4 - \frac{8}{3} = 0$$

$$i_2(0^+) = \frac{1}{3}\text{ A}$$

Example 6.19 *In the network shown in Fig. 6.61, the switch K is closed at $t = 0$ connecting a voltage $V_0 \sin \omega t$ to the parallel RL-RC circuit. Find (a) $i_1(0^+)$ and $i_2(0^+)$ (b) $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$.*

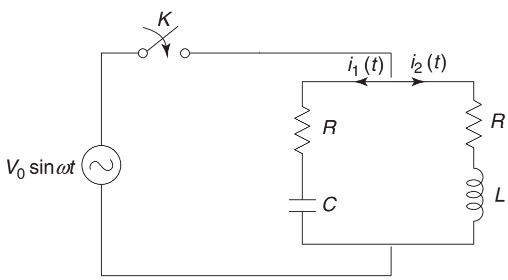


Fig. 6.61

Solution At $t = 0^-$, no current flows in the inductor and there is no voltage across the capacitor.

$$v_C(0^-) = 0$$

$$i_1(0^-) = 0$$

$$i_2(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.62.

At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit. The voltage source $V_0 \sin \omega t$ acts as a short circuit.

$$i_1(0^+) = 0$$

$$i_2(0^+) = 0$$

$$v_C(0^+) = 0$$

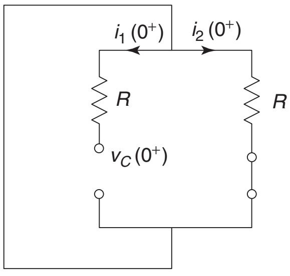


Fig. 6.62

For $t > 0$, the network is shown in Fig. 6.63.

Writing the KVL equation for $t > 0$,

$$V_0 \sin \omega t - R\, i_1 - \frac{1}{C}\int i_1 \, dt = 0 \qquad \ldots\text{(i)}$$

and

$$V_0 \sin \omega t - R\, i_2 - L\frac{di_2}{dt} = 0 \qquad \ldots\text{(ii)}$$

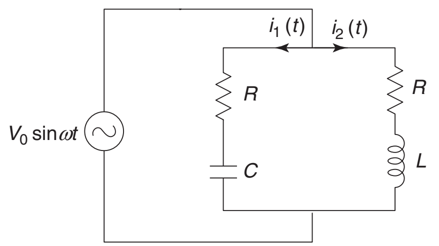


Fig. 6.63

Differentiating Eq. (i),

$$V_0 \,\omega \cos \omega t - R\frac{di_1}{dt} - \frac{i_1}{C} = 0$$

$$\frac{di_1}{dt} = \frac{V_0\omega}{R}\cos \omega t - \frac{i_1}{RC} \qquad \ldots\text{(iii)}$$

At $t = 0^+$,

$$\frac{di_1}{dt}(0^+) = \frac{V_0\omega}{R}\cos \omega t\bigg|_{t=0^+} - \frac{i_1(0^+)}{RC} = \frac{V_0\omega}{R}$$

From Eq. (ii),

$$\frac{di_2}{dt} = \frac{V_0}{L}\sin \omega t - \frac{R}{L}i_2$$

At $t = 0^+$,

$$\frac{di_2}{dt}(0^+) = \frac{V_0}{L}\sin \omega t\bigg|_{t=0^+} - \frac{R}{L}i_2(0^+) = 0$$

Example 6.20 *In the network of Fig. 6.64, the switch K is changed from 'a' to 'b' at $t = 0$ (a steady state having been established at the position a). Find i_1, i_2 and i_3 at $t = 0^+$.*

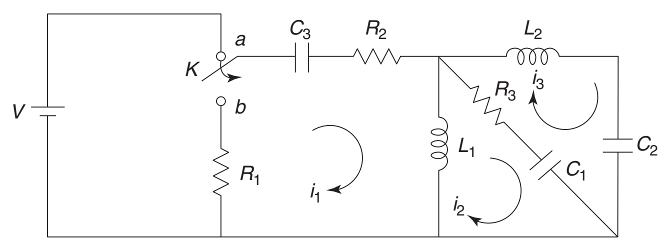


Fig. 6.64

Solution At $t = 0^-$, the network is shown in Fig. 6.65.

At $t = 0^-$, the network attains steady-state condition. Hence, the capacitors act as open circuits and inductors act as short circuits.

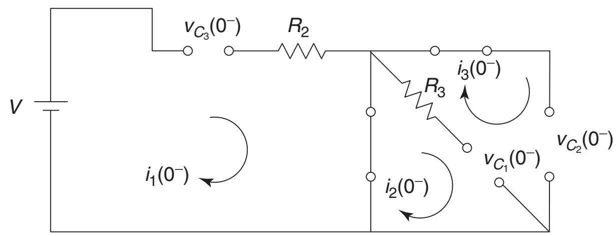


Fig. 6.65

$$i_1(0^-) = 0$$
$$i_2(0^-) = 0$$
$$i_3(0^-) = 0$$
$$v_{C_3}(0^-) = V$$
$$v_{C_2}(0^-) = 0$$
$$v_{C_1}(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.66.

At $t = 0^+$, the capacitor C_3 acts as a voltage source of V volts and capacitors C_1 and C_2 act as short circuts. The inductors act as open circuits.

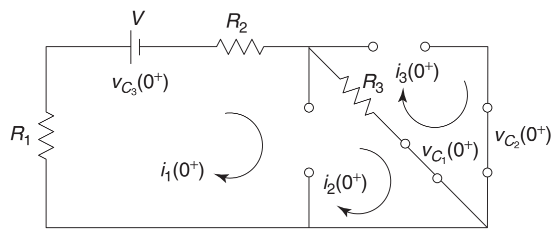


Fig. 6.66

$$i_1(0^+) = i_2(0^+) = -\frac{V}{R_1 + R_2 + R_3}$$

$$i_3(0^+) = 0$$

Example 6.21 *In the network of Fig. 6. 67, the switch K_1 has been closed for a long time prior to $t = 0$. At $t = 0$, the switch K_2 is closed. Find $v_C(0^+)$ and $i_C(0^+)$.*

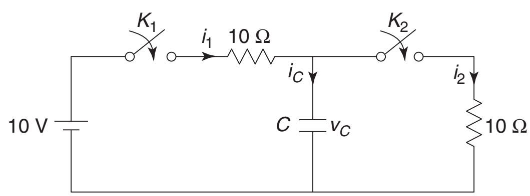


Fig. 6.67

Solution At $t = 0^-$, the network is shown in Fig. 6.68. At $t = 0^-$, the network attains steady-state condition. Hence, the capacitor acts as an open circuit.

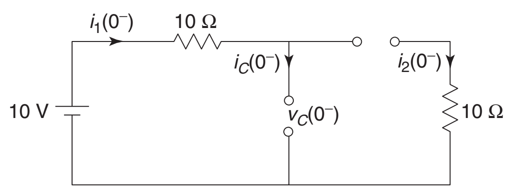


Fig. 6.68

$$i_1(0^-) = 0$$
$$i_2(0^-) = 0$$
$$i_C(0^-) = 0$$
$$v_C(0^-) = 10 \text{ V}$$

At $t = 0^+$, the network is shown in Fig. 6.69.

At $t = 0^+$, the capacitor acts as a voltage source of voltage V.

$$v_C(0^+) = 10 \text{ V}$$

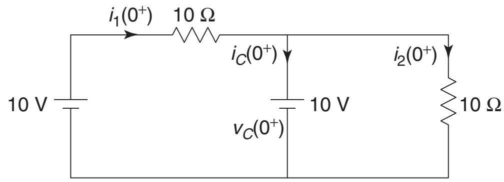


Fig. 6.69

Writing the KVL equation at $t = 0^+$,

$$10 - 10\, i_1(0^+) - 10 = 0$$

and $$10 - 10\, i_2(0^+) = 0$$

$$i_1(0^+) = 0$$

$$i_2(0^+) = -1 \text{ A}$$

$$i_1(0^+) = i_C(0^+) + i_2(0^+)$$

$$i_C(0^+) = 1 \text{ A}$$

Example 6.22 *In the network shown in Fig. 6.70, a steady state is reached with the switch open. At $t = 0$, the switch is closed. Determine $v_C(0^-)$, $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$.*

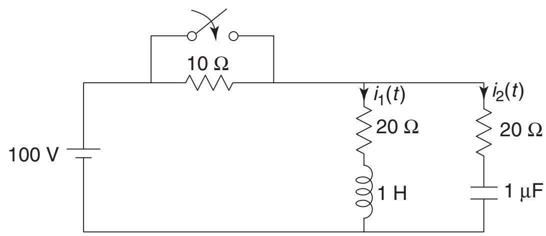


Fig. 6.70

Solution At $t = 0^-$, the network is shown in Fig. 6.71.

At $t = 0^-$, the network is in steady-state. Hence, the inductor acts as a short circuit and the capacitor acts as an open circuit.

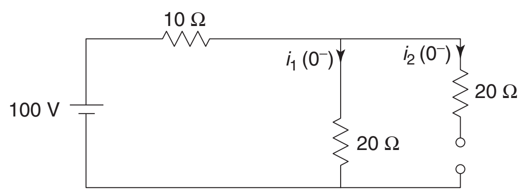


Fig. 6.71

$$v_C(0^-) = 100 \times \frac{20}{20 + 10} = 66.67 \text{ V}$$

$$i_1(0^-) = \frac{66.67}{20} = 3.33 \text{ A}$$

$$i_2(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.72.

At $t = 0^+$, the inductor acts as a current source of 3.33 A and the capacitor acts as a voltage source of 66.67 V.

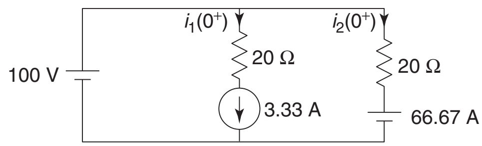


Fig. 6.72

$$v_C(0^+) = 66.67 \text{ V}$$
$$i_1(0^+) = 3.33 \text{ A}$$
$$i_2(0^+) = \frac{100 - 66.67}{20} = 1.67 \text{ A}$$

For $t > 0^-$, the network is shown in Fig. 6.73.

Writing the KVL equations for $t > 0$,

$$100 - 20i_1 - 1\frac{di_1}{dt} = 0 \quad \text{...(i)}$$

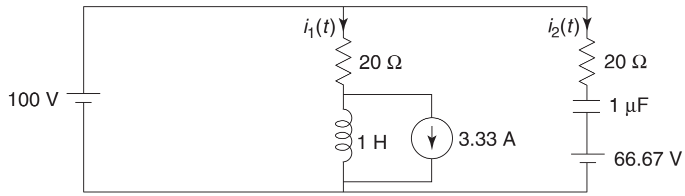


Fig. 6.73

and
$$100 - 20\, i_2 - \frac{1}{10^{-6}} \int i_2 \, dt - 66.67 = 0 \quad \text{...(ii)}$$

At $t = 0^+$,
$$\frac{di_1}{dt}(0^+) = 100 - 20i_1(0^+) = 100 - 20\,(3.33) = 33.3 \text{ A/s}$$

Differentiating Eq. (ii),

$$0 - 20\frac{di_2}{dt} - 10^6\, i_2 = 0$$

At $t = 0^+$,
$$20\frac{di_2}{dt}(0^+) = -10^6\; i_2(0^+)$$
$$\frac{di_2}{dt}(0^+) = -\frac{10^6}{20} \times 1.67 = -83500 \text{ A/s}^2$$

6.3 RESISTOR-INDUCTOR CIRCUIT

Consider a series *RL* circuit as shown in Fig. 6.74. The switch is closed at time $t = 0$. The inductor in the circuit is initially un-energised.

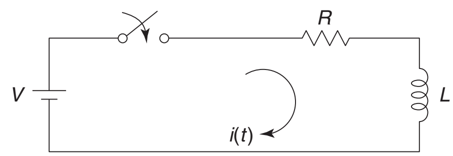


Fig. 6.74 *Series RL circuit*

Applying KVL to the circuit for $t > 0$,

$$V - Ri - L\frac{di}{dt} = 0$$

This is a linear differential equation of first order. It can be solved if the variables can be separated.

$$(V - Ri)\, dt = L\, di$$
$$\frac{L\, di}{V - Ri} = dt$$

Integrating both the sides,

$$-\frac{L}{R} l_n (V - Ri) = t + k$$

where l_n denotes that the logarithm is of base e and k is an arbitrary constant. k can be evaluated from the initial condition. In the circuit, the switch is closed at $t = 0$, i.e., just before closing the switch, the current in the inductor is zero. Since the inductor does not allow sudden change in current, at $t = 0^+$, just after the switch is closed, the current remains zero.

Setting $i = 0$ at $t = 0$,

$$-\frac{L}{R} l_n V = k$$

$$-\frac{L}{R} l_n (V - Ri) = t - \frac{L}{R} l_n V$$

$$-\frac{L}{R}\left[l_n (V - Ri) - l_n V\right] = t$$

$$\frac{V - Ri}{V} = e^{-\frac{R}{L}t}$$

$$V - Ri = Ve^{-\frac{R}{L}t}$$

$$Ri = V - Ve^{-\frac{R}{t}t}$$

$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t} \qquad \text{for } t > 0$$

The complete response is composed of two parts, the steady-state response or forced response or zero state response $\frac{V}{R}$ and transient response or natural response or zero input response $\frac{V}{R} e^{-\frac{R}{L}t}$.

The natural response is a characteristic of the circuit. Its form may be found by considering the source-free circuit. The forced response has the characteristics of forcing function, i.e., applied voltage. Thus, when the switch is closed, response reaches the steady-state value after some time interval as shown in Fig. 6.75.

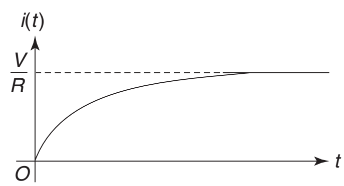


Fig. 6.75 *Current response of series RL circuit*

Here, the transient period is defined as the time taken for the current to reach its final or steady state value from its initial value.

The term $\frac{L}{R}$ is called time constant and is denoted by T.

$$T = \frac{L}{R}$$

At one time constant, the current reaches 63.2 per cent of its final value $\frac{V}{R}$.

$$i(T) = \frac{V}{R} - \frac{V}{R} e^{-\frac{1}{T}t} = \frac{V}{R} - \frac{V}{R} e^{-1} = \frac{V}{R} - 0.368\frac{V}{R} = 0.632\frac{V}{R}$$

Similarly,

$$i(2T) = \frac{V}{R} - \frac{V}{R}e^{-2} = \frac{V}{R} - 0.135\frac{V}{R} = 0.865\frac{V}{R}$$

$$i(3T) = \frac{V}{R} - \frac{V}{R}e^{-3} = \frac{V}{R} - 0.0498\frac{V}{R} = 0.950\frac{V}{R}$$

$$i(5T) = \frac{V}{R} - \frac{V}{R}e^{-5} = \frac{V}{R} - 0.0067\frac{V}{R} = 0.993\frac{V}{R}$$

After 5 time constants, the current reaches 99.33 per cent of its final value. The voltage across the resistor is

$$v_R = Ri = R \times \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$= V\left(1 - e^{-\frac{R}{L}t}\right) \qquad \text{for } t > 0$$

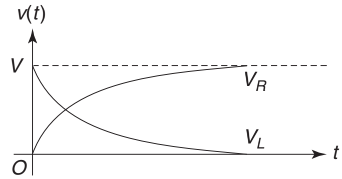


Fig. 6.76 *Voltage response of series RL circuit*

Similarly, the voltage across the inductor is

$$v_L = \frac{di}{dt} = L\frac{V}{R}\frac{d}{dt}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$= Ve^{-\frac{R}{L}t} \qquad \text{for } t > 0$$

Note:

1. Consider a homogeneous equation,

$$\frac{di}{dt} + Pi = 0 \qquad \text{where } P \text{ is a constant.}$$

The solution of this equation is given by,

$$i\,(t) = ke^{-Pt}$$

The value of k is obtained by putting $t = 0$ in the equation for $i\,(t)$.

2. Consider a non-homogeneous equation,

$$\frac{di}{dt} + Pi = Q$$

where P is a constant and Q may be a function of the independent variable t or a constant. The solution of this equation is given by,

$$i(t) = e^{-Pt}\int Q\, e^{Pt}\, dt + ke^{-Pt}$$

The value of k is obtained by putting $t = 0$ in the equation of $i\,(t)$.

Example 6.23 *In the network of Fig. 6.77, the switch is initially at the position 1. On the steady state having reached, the switch is changed to the position 2. Find current i(t).*

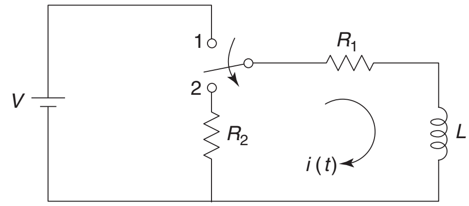


Fig. 6.77

Solution At $t = 0^-$, the network is shown in Fig. 6.78.

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

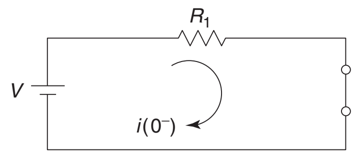


Fig. 6.78

$$i(0^-) = \frac{V}{R_1}$$

Since the inductor does not allow sudden change in current,

$$i(0^+) = \frac{V}{R_1}$$

For $t > 0$, the network is shown in Fig. 6.79.

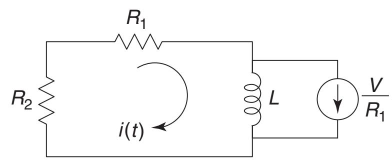


Fig. 6.79

Writing the KVL equation for $t > 0$,

$$-R_2 i - R_1 i - L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{(R_1 + R_2)}{L} i = 0$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = 0$,

$$P = \frac{R_1 + R_2}{L}$$

The solution of this differential equation is given by,

$$i(t) = k\, e^{-Pt}$$

$$i(t) = k\, e^{-\left(\frac{R_1 + R_2}{L}\right)t}$$

At $t = 0$, $i(0) = \dfrac{V}{R_1}$

$$\frac{V}{R_1} = k\, e^0 = k$$

$$i(t) = \frac{V}{R_1} e^{-\left(\frac{R_1 + R_2}{L}\right)t} \quad \text{for } t > 0$$

Example 6.24 *In the network shown in Fig. 6.80, the switch is closed at t = 0, a steady state having previously been attained. Find the current i (t).*

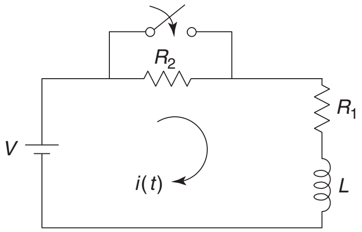


Fig. 6.80

Solution At $t = 0^-$, the network is shown in Fig. 6.81.

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

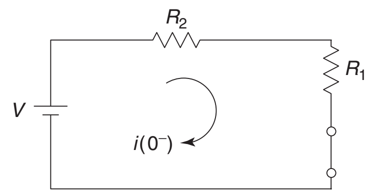


Fig. 6.81

$$i(0^-) = \frac{V}{R_1 + R_2}$$

Since the current through the inductor cannot change instantaneously,

$$i(0^+) = \frac{V}{R_1 + R_2}$$

For $t > 0$, the network is shown in Fig. 6.82.

Writing the KVL equation for $t > 0$,

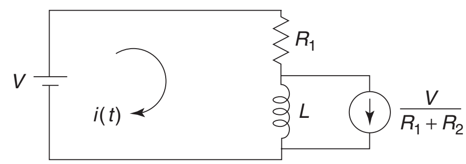


Fig. 6.82

$$V - R_1 i - L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R_1}{L} i = \frac{V}{L}$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = Q$,

$$P = \frac{R_1}{L}, \quad Q = \frac{V}{L}$$

The solution of this differential equation is given by,

$$i(t) = e^{-Pt}\int Qe^{Pt}\, dt + k\,e^{-Pt}$$

$$= e^{-\frac{R_1}{L}t}\int \frac{V}{L} e^{\frac{R_1}{L}t}\, dt + k\,e^{-\frac{R_1}{L}t}$$

$$= \frac{V}{R_1} + k\,e^{-\frac{R_1}{L}t}$$

At $t = 0$, $i(0) = \dfrac{V}{R_1 + R_2}$

$$\frac{V}{R_1 + R_2} = \frac{V}{R_1} + k$$

$$k = -\frac{VR_2}{R_1(R_1 + R_2)}$$

$$i(t) = \frac{V}{R_1} - \frac{VR_2}{R_1(R_1 + R_2)} e^{-\frac{R_1}{L}t}$$

$$= \frac{V}{R_1}\left(1 - \frac{R_2}{R_1 + R_2} e^{-\frac{R_1}{L}t}\right) \quad \text{for } t > 0$$

Example 6.25 *In the network of Fig. 6.83, a steady state is reached with the switch K open. At $t = 0$, the switch K is closed. Find the current $i(t)$ for $t > 0$.*

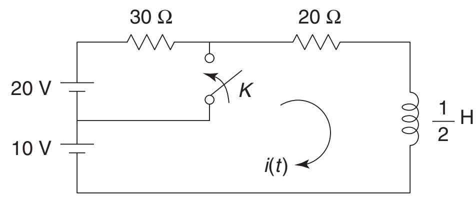


Fig. 6.83

Solution At $t = 0^-$, the network is shown in Fig. 6.84.

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

$$i(0^-) = \frac{20 + 10}{30 + 20} = 0.6 \text{ A}$$

Since the current through the inductor cannot change instantaneously,

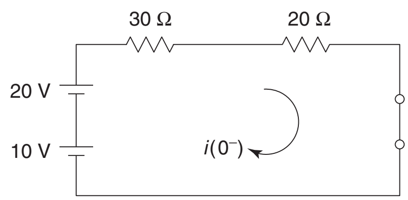


Fig. 6.84

$$i(0^+) = 0.6 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.85.

Writing the KVL equation for $t > 0$.

$$10 - 20i - \frac{1}{2}\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 40i = 20$$

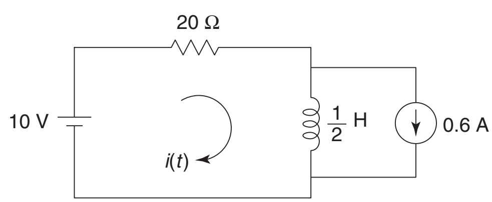


Fig. 6.85

Comparing with the differential equation $\frac{di}{dt} + Pi = Q$,

$$P = 40, \quad Q = 20$$

The solution of this differential equation is given by,

$$\begin{aligned} i(t) &= e^{-Pt} \int Q\, e^{Pt}\, dt + k\, e^{-Pt} \\ &= e^{-40t} \int 20\, e^{40t} dt + k\, e^{-40t} \\ &= \frac{20}{40} + k\, e^{-40t} \\ &= 0.5 + k\, e^{-40t} \end{aligned}$$

At $t = 0, i(0) = 0.6$ A

$$0.6 = 0.5 + k$$

$$k = 0.1$$

$$i(t) = 0.5 + 0.1\, e^{-40t} \qquad \text{for } t > 0$$

Example 6.26 *The network of Fig. 6.86 is under steady state with switch at the position 1. At t = 0, switch is moved to position 2. Find i (t).*

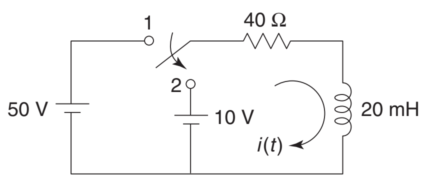


Fig. 6.86

Solution At $t = 0^-$, the network is shown in Fig. 6.87.

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

$$i(0^-) = \frac{50}{40} = 1.25 \text{ A}$$

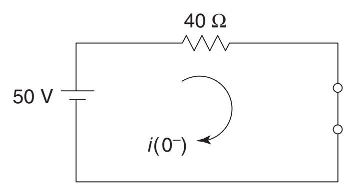


Fig. 6.87

Since current through the inductor cannot change instantaneously,

$$i(0^+) = 1.25 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.88.

Writing the KVL equation for $t > 0$,

$$10 - 40i - 20 \times 10^{-3} \frac{di}{dt} = 0$$

$$\frac{di}{dt} + 2000i = 500$$

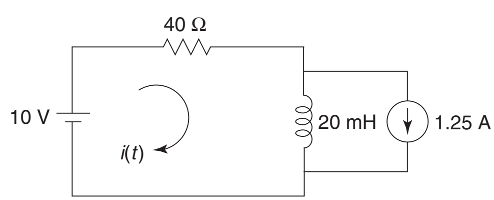


Fig. 6.88

Comparing with the differential equation $\frac{di}{dt} + Pi = Q,$

$$P = 2000, \ Q = 500$$

The solution of this differential equation is given by,

$$\begin{aligned} i(t) &= e^{-Pt} \int Q\, e^{Pt}\, dt + k\, e^{-Pt} \\ &= e^{-2000t} \int 500\, e^{2000t} + k\, e^{-2000t} \\ &= \frac{500}{2000} + k\, e^{-2000t} \\ &= 0.25 + k\, e^{-2000t} \end{aligned}$$

At $t = 0, \ i(0) = 1.25$ A

$$\begin{aligned} 1.25 &= 0.25 + k \\ k &= 1 \\ i(t) &= 0.25 + e^{-2000t} \qquad \text{for } t > 0 \end{aligned}$$

Example 6.27 *In the network of Fig. 6.89, the switch is moved from 1 to 2 at $t = 0$. Determine $i(t)$.*

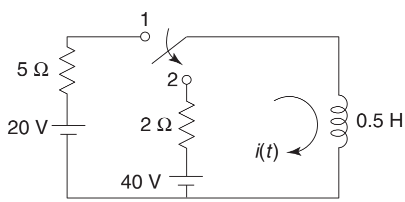


Fig. 6.89

Solution At $t = 0^-$, the network is shown in Fig. 6.90.

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

$$i(0^-) = \frac{20}{5} = 4 \text{ A}$$

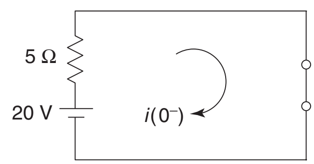


Fig. 6.90

Since the current through the inductor cannot change instantaneously,

$$i(0^+) = 4 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.91.

Writing the KVL equation for $t > 0$,

$$40 - 2i - 0.5\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 4i = 80$$

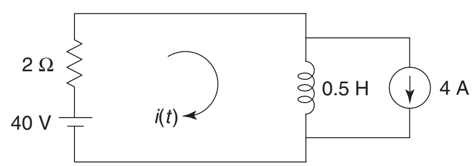


Fig. 6.91

Comparing with the differential equation $\frac{di}{dt} + Pi = Q,$

$$P = 4, \quad Q = 80$$

The solution of this differential equation is given by,

$$\begin{aligned} i(t) &= e^{-Pt}\int Q\, e^{Pt}\, dt + k\,e^{-Pt} \\ &= e^{-4t}\int 80\, e^{4t}\, dt + k\,e^{-4t} \\ &= \frac{80}{4} + k\,e^{-4t} \\ &= 20 + k\,e^{-4t} \end{aligned}$$

At $t = 0, i(0) = 4$ A

$$4 = 20 + k$$
$$k = -16$$
$$i(t) = 20 - 16e^{-4t} \qquad \text{for } t > 0$$

Example 6.28 *For the network shown in Fig. 6.92, steady state is reached with the switch closed. The switch is opened at t = 0. Obtain expressions for $i_L(t)$ and $v_L(t)$.*

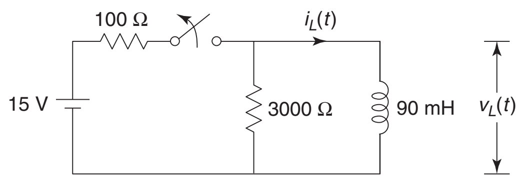


Fig. 6.92

Solution At $t = 0^-$, the network is shown in Fig. 6.93.

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

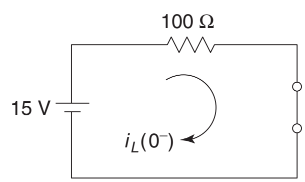


Fig. 6.93

$$i_L(0^-) = \frac{15}{100} = 0.15 \text{ A}$$

Since current through the inductor cannot change instantaneously,

$$i_L(0^+) = 0.15 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.94.

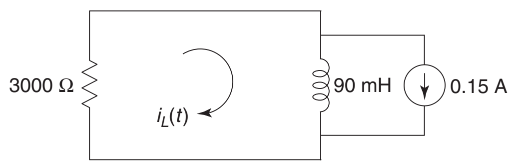


Fig. 6.94

Writing the KVL equation for $t > 0$,

$$-3000 i_L - 90\times 10^{-3}\frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + 33.33\times 10^3\, i_L = 0$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = 0,$

$$P = 33.33 \times 10^3$$

The solution of this differential equation is given by,

$$i_L(t) = k\,e^{-Pt}$$

$$i_L(t) = k\,e^{-33.33\times10^3\,t}$$

At $t = 0,\ i_L(0) = 0.15$ A

$$0.15 = k$$

$$i_L(t) = 0.15\,e^{-33.33\times10^3\,t} \qquad \text{for } t > 0$$

Also,

$$v_L(t) = L\frac{di_L}{dt}$$

$$= 90\times10^{-3}\,\frac{d}{dt}(0.15\,e^{-33.33\times10^3\,t})$$

$$= -90\times10^{-3}\times0.15\times33.33\times10^3\times e^{-33.33\times10^3\,t}$$

$$= -450\,e^{-33.33\times10^3\,t} \qquad \text{for } t > 0$$

Example 6.29 *In the network of Fig. 6.95, the switch is open for a long time and it closes at t = 0. Find i (t).*

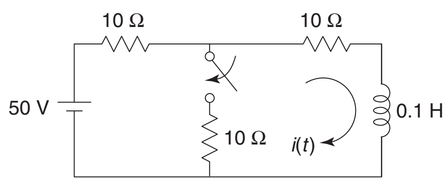


Fig. 6.95

Solution At $t = 0^-$, the network is shown in Fig. 6.96.

At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

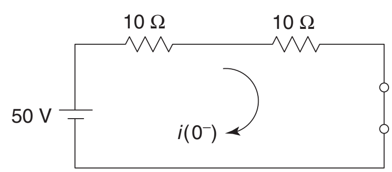


Fig. 6.96

$$i(0^-) = \frac{50}{10+10} = 2.5 \text{ A}$$

Since current through the inductor cannot change instantaneously,

$$i\,(0^+) = 2.5 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.97.

For $t > 0$, representing the network to the left of the inductor by Thevenin's equivalent network,

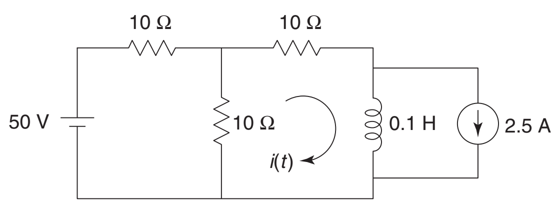


Fig. 6.97

$$V_{eq} = 50\times\frac{10}{10+10} = 25 \text{ V}$$

$$R_{eq} = (10\,||\,10) + 10 = 15\ \Omega$$

For $t > 0$, Thevenin's equivalent network is shown in Fig. 6.98.
Writing the KVL equation for $t > 0$,

$$25 - 15i - 0.1\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 150i = 250$$

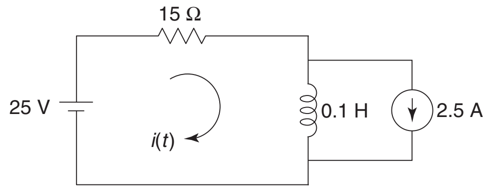


Fig. 6.98

Comparing with the differential equation $\frac{di}{dt} + Pi = Q$,

$$P = 150, \quad Q = 250$$

The solution of this differential equation is given by,

$$\begin{aligned} i(t) &= e^{-Pt}\int Q\, e^{Pt}\, dt + k\, e^{-Pt} \\ &= e^{-150t}\int 250\, e^{150t}\, dt + k\, e^{-Pt} \\ &= \frac{250}{150} + k\, e^{-150t} \\ &= 1.667 + k\, e^{-150t} \end{aligned}$$

At $t = 0$, $i(0) = 2.5$ A

$$2.5 = 1.667 + k$$

$$k = 0.833$$

$$i(t) = 1.667 + 0.833\, e^{-150t} \qquad \text{for } t > 0$$

Example 6.30 *In Fig. 6.99, the switch is closed at t = 0. Find i (t) for t > 0.*

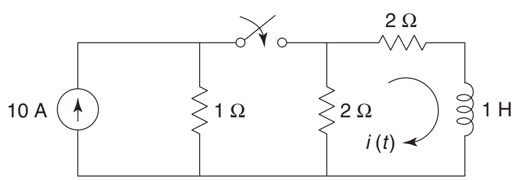


Fig. 6.99

Solution At $t = 0^-$, $$i(0^-) = 0$$

Since current through inductor cannot change instantaneously,

$$i(0^+) = 0$$

For $t > 0$, simplifying the network by source-transformation technique as shown in Fig. 6.100.

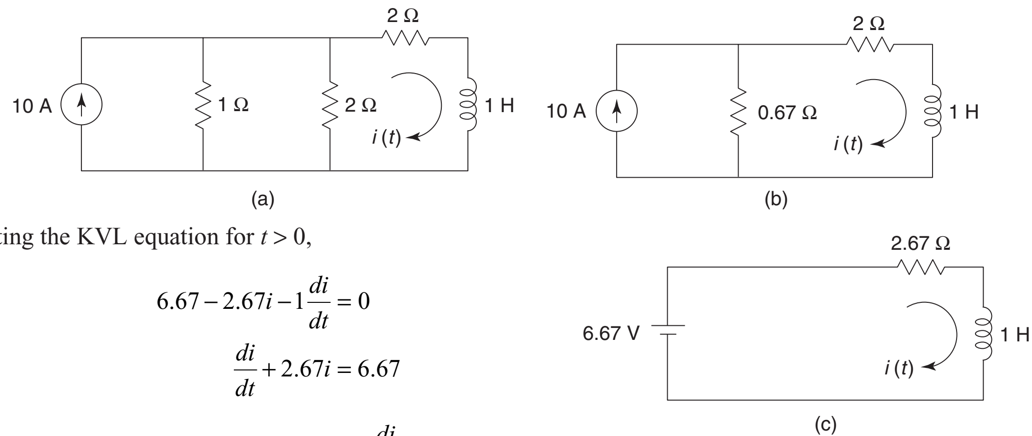


Fig. 6.100

Writing the KVL equation for $t > 0$,

$$6.67 - 2.67i - 1\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 2.67i = 6.67$$

Comparing with the differential equation $\frac{di}{dt} + Pi = Q$,

$$P = 2.67,\ Q = 6.67$$

The solution of this differential equation is given by,

$$\begin{aligned} i(t) &= e^{-Pt}\int Q\,e^{Pt}\,dt + k\,e^{-Pt} \\ &= e^{-2.67t}\int 6.67\,e^{2.67t}\,dt + k\,e^{-2.67t} \\ &= \frac{6.67}{2.67} + k\,e^{-2.67t} \\ &= 2.5 + k\,e^{-2.67t} \end{aligned}$$

At $t = 0$, $i\,(0) = 0$

$$0 = 2.5 + k$$

$$k = -2.5$$

$$\begin{aligned} i(t) &= 2.5 - 2.5\,e^{-2.67t} \\ &= 2.5(1 - e^{-2.67t}) \qquad \text{for } t > 0 \end{aligned}$$

Example 6.31 *Find the current i (t) for t > 0.*

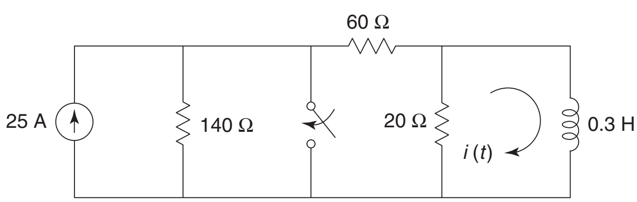


Fig. 6.101

Solution At $t = 0^-$, the inductor acts as a short circuit. Simplifying the network as shown in Fig. 6.102.

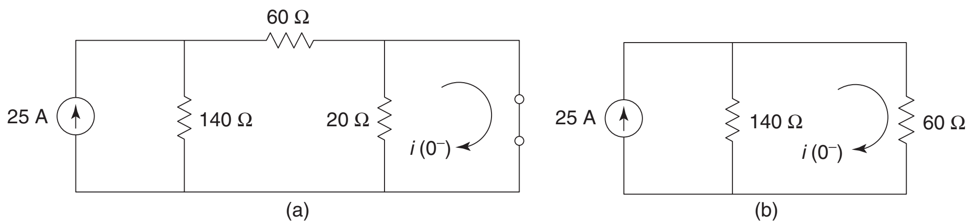


Fig. 6.102

$$i(0^-) = 25 \times \frac{140}{140+60} = 17.5 \text{ A}$$

Since current through the inductor cannot change instantaneously,

$$i(0^+) = 17.5 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.103.

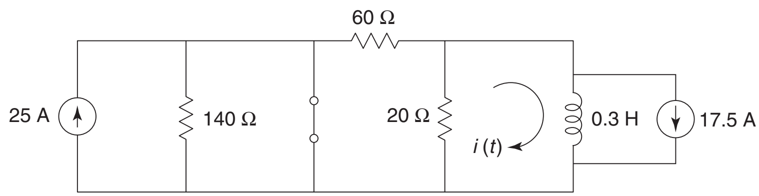


Fig. 6.103

Simplifying the network by source transformation as shown in Fig. 6.104,

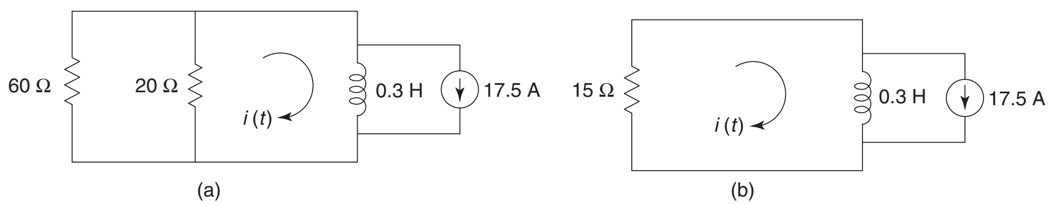


Fig. 6.104

Writing the KVL equation for $t > 0$,

$$-15i - 0.3\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 50i = 0$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = 0$,

$$P = 50$$

The solution of this differential equation is given by,

$$i(t) = k e^{-Pt} = k e^{-50t}$$

At $t = 0$, $i(0) = 17.5$ A

$$k = 17.5$$

$$i(t) = 17.5\, e^{-50t} \qquad \text{for } t > 0$$

Example 6.32 *In the network of Fig. 6.105, the switch is in position 'a' for a long time. At t = 0, the switch is moved from a to b. Find $v_2(t)$. Assume that the initial current in the 2 H inductor is zero.*

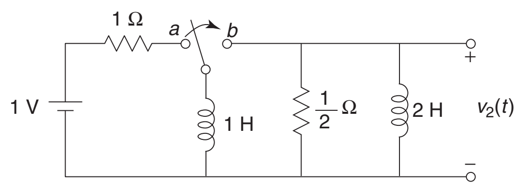


Fig. 6.105

Solution At $t = 0^-$, the switch is in the position *a*. The network has attained steady-state condition. Hence, the inductor acts as a short circuit.
Current through the 1 H inductor is given by

$$i(0^-) = \frac{1}{1} = 1 \text{ A}$$

$$v_2(0^-) = 0$$

Since current through the inductor cannot change instantaneously,

$$i(0^+) = 1 \text{ A}$$

$$v_2(0^+) = -1 \times \frac{1}{2} = -0.5 \text{ V}$$

For $t > 0$, the network is shown in Fig. 6.106.
Writing the KCL equation for $t > 0$,

$$\frac{1}{1}\int_0^t v_2 dt + 1 + \frac{v_2}{\frac{1}{2}} + \frac{1}{2}\int_0^t v_2 dt = 0 \qquad \ldots\text{(i)}$$

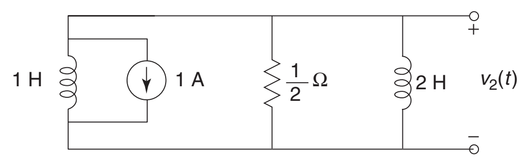


Fig. 6.106

Differentiating Eq. (i),

$$v_2 + 2\frac{dv_2}{dt} + \frac{1}{2}v_2 = 0$$

$$\frac{dv_2}{dt} + \frac{3}{4}v_2 = 0$$

Comparing with the differential equation $\dfrac{dv}{dt} + Pv = 0$,

$$P = \frac{3}{4}$$

The solution of this differential equation is given by,

$$v_2(t) = K\,e^{-Pt} = k\,e^{-\frac{3}{4}t}$$

At $t = 0$, $v_2(0) = -0.5$ V

$$-0.5 = k\,e^{\circ}$$
$$k = -0.5$$
$$v_2(t) = -0.5\,e^{-\frac{3}{4}t} \qquad \text{for } t > 0$$

Example 6.33 *In the network shown in Fig. 6.107, a steady-state condition is achieved with switch open. At t = 0 switch is closed. Find $v_a(t)$.*

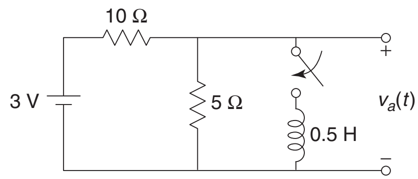


Fig. 6.107

Solution At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit.

$$i_L(0^-) = 0$$
$$v_a(0^-) = 3 \times \frac{5}{10+5} = 1\text{ V}$$

Since current through inductor cannot change instantaneously,

$$i_L(0^+) = 0$$
$$v_a(0^+) = 1\text{ V}$$

For $t > 0$, the network is shown in Fig. 6.108.

Writing the KCL equation for $t > 0$,

$$\frac{1}{0.5}\int_0^t v_a\,dt + \frac{v_a}{5} + \frac{v_a - 3}{10} = 0$$

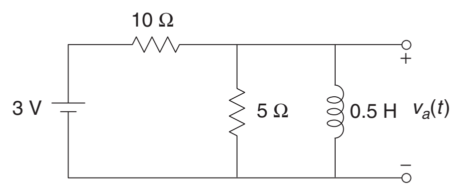


Fig. 6.108

Differentiating Eq. (i),

$$2v_a + 0.2\frac{dv_a}{dt} + 0.1\frac{dv_a}{dt} = 0$$
$$\frac{dv_a}{dt} + \frac{20}{3}v_a = 0$$

Comparing with the differential equation $\frac{dv}{dt} + Pv = 0,$

$$P = \frac{20}{3}$$

The solution of this differential equation is given by,

$$v_a(t) = k\,e^{-Pt} = k\,e^{-\frac{20}{3}t}$$

At $t = 0$, $v_a(0) = 1$ V

$$1 = k$$

$$v_a(t) = e^{-\frac{20}{3}t} \qquad \text{for } t > 0$$

Example 6.34 *In the network of Fig. 6.109, determine currents $i_1(t)$ and $i_2(t)$ when the switch is closed at $t = 0$.*

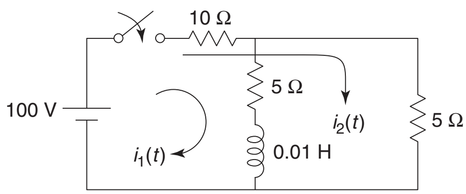


Fig. 6.109

Solution At $t = 0^-$,

$$i_1(0^-) = i_2(0^-) = 0$$

At $t = 0^+$,

$$i_1(0^+) = 0$$

$$i_2(0^+) = \frac{100}{15} = 6.67 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.110.

Writing the KVL equations for $t > 0$,

$$100 - 10(i_1 + i_2) - 5i_1 - 0.01\frac{di_1}{dt} = 0 \qquad \ldots\text{(i)}$$

and

$$100 - 10(i_1 + i_2) - 5i_2 = 0 \qquad \ldots\text{(ii)}$$

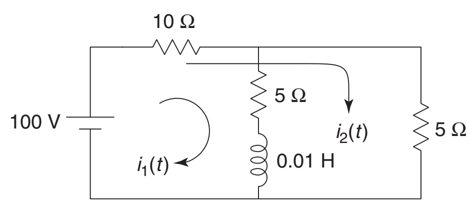


Fig. 6.110

From Eq. (ii),

$$i_2 = \frac{100 - 10i_1}{15}$$

Substituting in Eq. (i),

$$\frac{di_1}{dt} + 833i_1 = 3333$$

Comparing with the differential equation $\frac{di}{dt} + Pi = Q,$

$$P = 833,\ Q = 3333$$

The solution of this differential equation is given by,

$$i_1(t) = e^{-Pt}\int Q\,e^{Pt}\,dt + k\,e^{-Pt}$$
$$= e^{-833t}\int 3333\,e^{833t}\,dt + k\,e^{-833t}$$
$$= \frac{3333}{833} + k\,e^{-833t}$$
$$= 4 + k\,e^{-833t}$$

At $t = 0$, $i_1(0) = 0$

$$0 = 4 + k$$
$$k = -4$$
$$i_1(t) = 4 - 4\,e^{-833t}$$
$$= 4(1 - e^{-833t}) \qquad \text{for } t > 0$$
$$i_2(t) = \frac{100 - 10i_1}{15}$$
$$= \frac{100 - 10(4 - 4\,e^{-833t})}{15}$$
$$= 4 + 2.67\,e^{-833t} \qquad \text{for } t > 0$$

Example 6.35 *The switch in the network shown in Fig. 6.111 is closed at $t = 0$. Find $v_2(t)$ for all $t > 0$. Assume zero initial current in the inductor.*

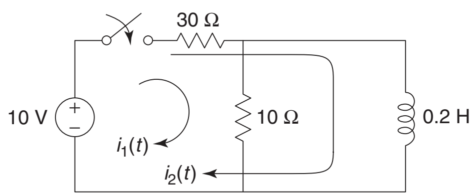


Fig. 6.111

Solution At $t = 0^-$,

$$i_1(0^-) = 0$$
$$i_2(0^-) = 0$$

Since current through the inductor cannot change instantaneously,

$$i_2(0^+) = 0$$
$$i_1(0^+) = \frac{10}{30 + 10} = 0.25\ \text{A}$$

For $t > 0$, the network is shown in Fig. 6.112.

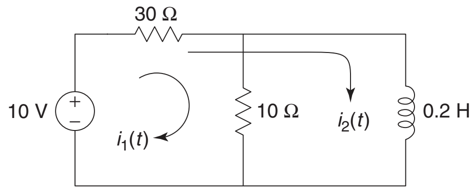


Fig. 6.112

Writing the KVL equations for $t > 0$,

$$10 - 30(i_1 + i_2) - 10\, i_1 = 0 \qquad \ldots\text{(i)}$$

and
$$10 - 30(i_1 + i_2) - 0.2\frac{di_2}{dt} = 0 \qquad \ldots\text{(ii)}$$

From Eq. (i),

$$i_1 = \frac{10 - 30\, i_2}{40} = 0.25 - 0.75\, i_2 \qquad \ldots\text{(iii)}$$

Substituting Eq. (iii) into Eq. (ii),

$$\frac{di_2}{dt} + 37.5\, i_2 = 2.5$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = Q$,

$$P = 37.5,\ Q = 2.5$$

The solution of this differential equation is given by,

$$\begin{aligned} i_2(t) &= e^{-Pt}\int Q\, e^{Pt}\, dt + k\, e^{-Pt} \\ &= e^{-37.5t}\int 2.5\, e^{37.5t}\, dt + k\, e^{-37.5t} \\ &= \frac{2.5}{37.5} + k\, e^{-37.5t} \\ &= 0.067 + k\, e^{-37.5t} \end{aligned}$$

At $t = 0,\ i_2(0) = 0$

$$\begin{aligned} 0 &= 0.067 + k \\ k &= -0.067 \\ i_2(t) &= 0.067 - 0.067\, e^{-37.5t} \\ v_2(t) &= 0.2\frac{di_2}{dt} \\ &= 0.2\frac{d}{dt}(0.067 - 0.067\, e^{-37.5t}) \\ &= 0.5\, e^{-37.5t} \qquad \text{for } t > 0 \end{aligned}$$

Example 6.36 *For the network shown in Fig. 6.113, find the current $i(t)$ when the switch is changed from the position 1 to 2 at $t = 0$.*

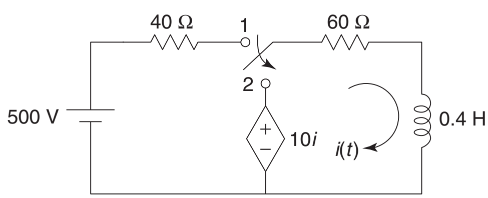


Fig. 6.113

Solution At $t = 0^-$, the network is shown in Fig. 6.114. At $t = 0^-$, the network attains steady-state condition. Hence, the inductor acts as a short circuit.

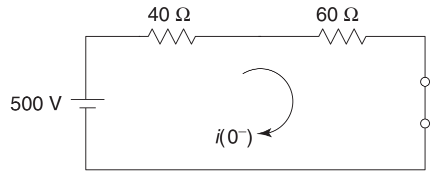


Fig. 6.114

$$i(0^-) = \frac{500}{40+60} = 5 \text{ A}$$

Since current through the inductor cannot change instantaneously,

$$i(0^+) = 5 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.115.

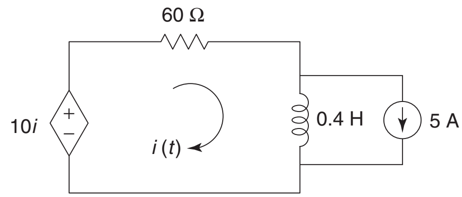


Fig. 6.115

Writing the KVL equation for $t > 0$,

$$10i - 60i - 0.4\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 125i = 0$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = 0$,

$$P = 125$$

The solution of this differential equation is given by,

$$i(t) = k\,e^{-Pt} = k\,e^{-125t}$$

At $t = 0, i(0) = 5$ A

$$5 = k$$

$$i(t) = 5e^{-125t} \qquad \text{for } t > 0$$

Example 6.37 *For the network shown in Fig. 6.116, find the current in the 20 Ω resistor when the switch is opened at $t = 0$.*

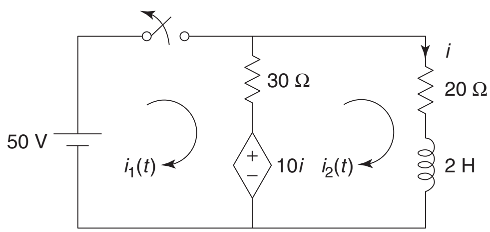


Fig. 6.116

Solution At $t = 0^-$, the network is shown in Fig. 6.117.

At $t = 0^-$, the network attains steady-state condition. Hence, the inductor acts as a short circuit.

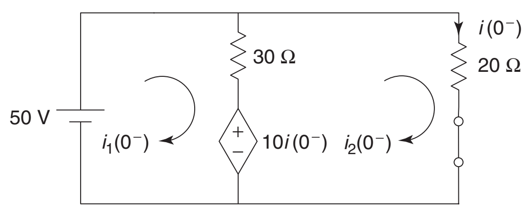


Fig. 6.117

$$i(0^-) = i_2(0^-)$$

Writing the KVL equations at $t = 0^-$,

$$50 - 30(i_1 - i_2) - 10i_2 = 0$$
$$10i_2 - 30(i_2 - i_1) - 20i_2 = 0$$

Solving these equations,

$$i_1(0^-) = 3.33 \text{ A}$$
$$i_2(0^-) = 2.5 \text{ A}$$

Since the current through the inductor cannot change instantaneously,

$$i_2(0^+) = 2.5 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.118.

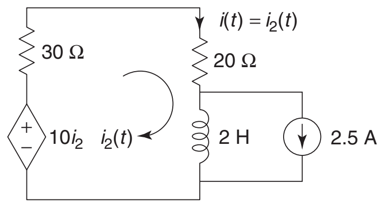


Fig. 6.118

Writing the KVL equation for $t > 0$,

$$10i_2 - 30i_2 - 20i_2 - 2\frac{di_2}{dt} = 0$$
$$\frac{di_2}{dt} + 20i_2 = 0$$

Comparing with the differential equation $\frac{di}{dt} + Pi = 0$,

$$P = 20$$

The solution of this differential equation is given by,

$$i_2(t) = k\,e^{-Pt} = k\,e^{-20t}$$

At $t = 0, i_2(0) = 2.5$ A

$$2.5 = k$$
$$i_2(t) = 2.5\,e^{-20t} \qquad \text{for } t > 0$$

Example 6.38 *In the network of Fig. 6.119, an exponential voltage $v(t) = 4\,e^{-3t}$ is applied at $t = 0$. Find the expression for current $i(t)$. Assume zero current through inductor at $t = 0$.*

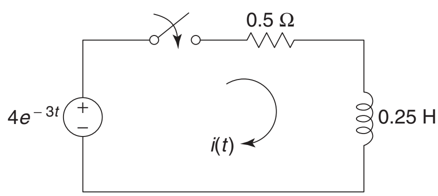


Fig. 6.119

Solution At $t = 0^-$, $\quad i(0^-) = 0$

Since current through the inductor cannot change instantaneously,

$$i(0^+) = 0$$

Writing the KVL equation for $t > 0$,

$$4e^{-3t} - 0.5i - 0.25\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 2i = 16e^{-3t}$$

Comparing with the differential equation $\frac{di}{dt} + Pi = Q$,

$$P = 2, \quad Q = 16\,e^{-3t}$$

The solution of this differential equation is given by,

$$\begin{aligned} i(t) &= e^{-Pt}\int Q\,e^{Pt}\,dt + k\,e^{-Pt} \\ &= e^{-2t}\int 16\,e^{-3t}\,e^{2t}\,dt + +k\,e^{-2t} \\ &= 16\,e^{-2t}\int e^{-t}\,dt + k\,e^{-2t} \\ &= -16\,e^{-3t} + k\,e^{-2t} \end{aligned}$$

At $t = 0$, $i(0) = 0$

$$0 = -16 + k$$

$$k = 16$$

$$i(t) = -16\,e^{-3t} + 16\,e^{-2t} \qquad \text{for } t > 0$$

Example 6.39 *For the network shown in Fig. 6.120, a sinusoidal voltage source $v = 150\sin(500t + \theta)$ volts is applied at a time when $\theta = 0$. Find the expression for the current $i(t)$.*

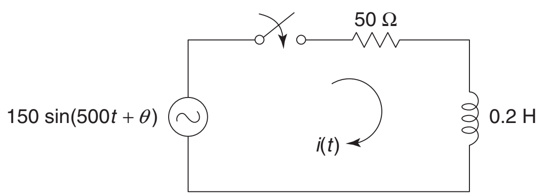


Fig. 6.120

Solution

Writing the KVL equation for $t > 0$,

$$150\sin(500t + \theta) - 50i - 0.2\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 250i = 750\sin(500t + \theta)$$

Comparing with the differential equation $\frac{di}{dt} + Pi = Q$,

$$P = 250, \quad Q = 750\sin(500t + \theta)$$

The solution of this differential equation is given by,

$$i(t) = e^{-Pt} \int Q e^{Pt} dt + k e^{-Pt}$$
$$= e^{-250t} \int 750 \sin(500t+\theta) e^{250t} + k e^{-250t}$$
$$= 750\, e^{-250t} \left[\frac{e^{250t}}{(250)^2 + (500)^2} \{250 \sin(500t+\theta) - 500 \cos(500t+\theta)\} \right] + k e^{-250t}$$
$$= 0.6 \sin(500t+\theta) - 1.2 \cos(500t+\theta) + k e^{-250t}$$

Let $$A \cos\phi = 0.6$$

and $$A \sin\phi = 1.2$$

$$A^2 \cos^2\phi + A^2 \sin^2\phi = (0.6)^2 + (1.2)^2 = 1.8$$
$$A = 1.342$$

and $$\phi = \tan^{-1}\left(\frac{1.2}{0.6}\right) = 63.43°$$

$$i(t) = 1.342 \cos(63.43°) \sin(500t+\theta) - 1.342 \sin(63.43°) \cos(500t+\theta) + k e^{-250t}$$
$$i(t) = 1.342 \sin(500t+\theta - 63.43°) + k e^{-250t}$$

At $t = 0, \theta = 0, i(0) = 0$

$$0 = 1.342 \sin(-63.43°) + k$$
$$k = 1.2$$
$$i(t) = 1.342 \sin(500t + \theta - 63.43°) + 1.2\, e^{-250t} \qquad \text{for } t > 0$$

Example 6.40 *For the network shown in Fig. 6.121, find the transient current when the switch is moved from the position 1 to 2 at $t = 0$. The network is in steady state with the switch in the position 1. The voltage applied to the network is $v = 150 \cos(200t + 30°)\, V$.*

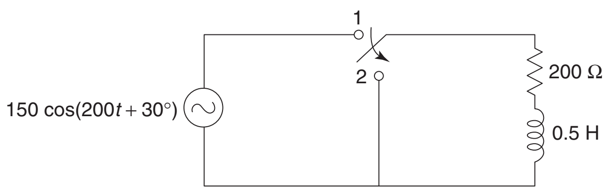


Fig. 6.121

Solution At $t = 0^-$, the network is shown in Fig 6.122.

At $t = 0^-$ the network attains steady-state condition.

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{150\angle 30°}{200 + j \times 200 \times 0.5} = 0.67\angle 3.43° \text{ A}$$

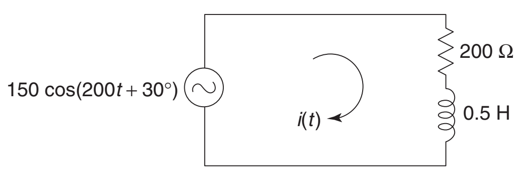


Fig. 6.122

The steady-state current passing through the network when the switch is in the position 1 is

$$i = 0.67\cos(200t + 3.43°) \qquad \text{...(i)}$$

For $t > 0$, the network is shown in Fig. 6.123.
Writing the KVL equation for $t > 0$,

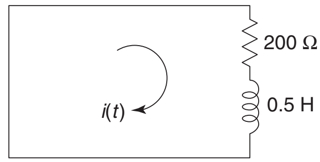


Fig. 6.123

$$-200i - 0.5\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 400i = 0$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = 0$,

$$P = 400$$

The solution of this differential equation is given by,

$$i(t) = k\,e^{-pt} = k\,e^{-400t} \qquad \text{...(ii)}$$

From Eqs (i) and (ii),

$$0.67\cos(200t + 3.43°) = k\,e^{-400t}$$

At $t = 0$,

$$0.67\cos(3.43°) = k$$

$$k = 6.67$$

$$i(t) = 0.67\,e^{-400t} \qquad \text{for } t > 0$$

6.4 RESISTOR–CAPACITOR CIRCUIT

Consider a series *RC* circuit as shown in Fig. 6.124. The switch is closed at time $t = 0$. The capacitor is initially uncharged. Applying KVL to the circuit for $t > 0$,

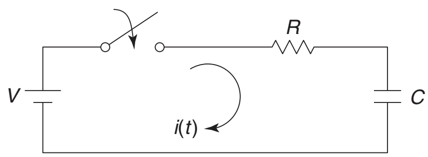


Fig. 6.124 *Series RC circuit*

$$V - Ri - \frac{1}{C}\int_0^1 i\,dt = 0$$

Differentiating the above equation,

$$0 - R\frac{di}{dt} - \frac{i}{C} = 0$$

$$\frac{di}{dt} + \frac{1}{RC}i = 0$$

This is a linear differential equation of first order. The variables may be separated to solve the equation.

$$\frac{di}{i} = -\frac{dt}{RC}$$

Integrating both the sides,

$$l_n i = -\frac{1}{RC}t + k$$

The constant k can be evaluated from initial condition. In the circuit shown, the switch is closed at $t = 0$. Since the capacitor never allows sudden change in voltage, it will act as short circuit at $t = 0^+$. Hence, current in the circuit at $t = 0^+$ is $\frac{V}{R}$.

Setting $i = \frac{V}{R}$ at $t = 0$,

$$l_n \frac{V}{R} = k$$

$$l_n i = -\frac{1}{RC}t + l_n \frac{V}{R}$$

$$l_n i - l_n \frac{V}{R} = -\frac{1}{RC}t$$

$$l_n \frac{i}{\left(\frac{V}{R}\right)} = -\frac{1}{RC}t$$

$$\frac{i}{\frac{V}{R}} = e^{-\frac{1}{RC}t}$$

$$i = \frac{V}{R}e^{-\frac{1}{RC}t} \quad \text{for } t > 0$$

When the switch is closed, the response decays with time as shown in Fig. 6.125(a).

The term RC is called time constant and is denoted by T.

$$T = RC$$

After 5 time constants, the current drops to 99 per cent of its initial value.

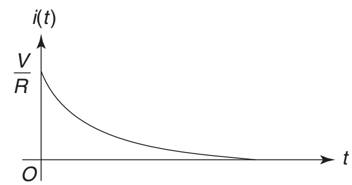


Fig. 6.125(a) *Current response of series RC circuit*

The voltage across the resistor is

$$v_R = Ri = R\frac{V}{R}e^{-\frac{1}{RC}t}$$

$$= Ve^{-\frac{1}{RC}t} \quad \text{for } t > 0$$

Similarly, the voltage across the capacitor is

$$v_C = \frac{1}{C}\int_0^t i\, dt$$

$$= \frac{1}{C}\int_0^t \frac{V}{R}e^{-\frac{1}{RC}t}$$

$$= -Ve^{-\frac{1}{RC}t} + k$$

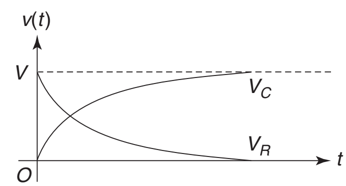


Fig. 6.125(b) *Voltage response of series RC circuit*

At $t = 0$, $v_C(0) = 0$

$$k = V$$

Hence, $$v_C = V\left(1 - e^{-\frac{1}{RC}t}\right) \quad \text{for } t > 0$$

Example 6.41 *The switch in the circuit of Fig. 6.126 is moved from the position 1 to 2 at $t = 0$. Find $v_C(t)$.*

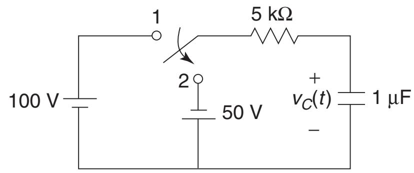


Fig. 6.126

Solution At $t = 0^-$, the network is shown in Fig. 6.127.

At $t = 0^-$, the network has attained steady-state condition. Hence, the capacitor acts as an open circuit.

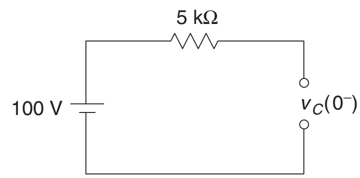


Fig. 6.127

$$v_C(0^-) = 100 \text{ V}$$

Since the voltage across the capacitor cannot change instantaneously,

$$v_C(0^+) = 100 \text{ V}$$

For $t > 0$, the network is shown in Fig. 6.128.

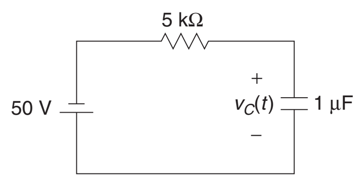


Fig. 6.128

Writing the KCL equation for $t > 0$,

$$1\times10^{-6}\frac{dv_C}{dt} + \frac{v_C + 50}{5000} = 0$$

$$\frac{dv_C}{dt} + 200v_C = 10^4$$

Comparing with the differential equation $\frac{dv}{dt} + Pv = Q$,

$$P = 200, \quad Q = 10^4$$

Solution of this differential equation is given by,

$$\begin{aligned}
v_C(t) &= e^{-Pt}\int Q\, e^{Pt}\, dt + k\, e^{-Pt}\\
&= e^{-200t}\int 10^4\, e^{200t}\, dt + k\, e^{-200t}\\
&= \frac{10^4}{200} + k\, e^{-200t}\\
&= -50 + k\, e^{-200t}
\end{aligned}$$

At $t = 0$, $v_C(0) = 100$ V

$$100 = -50 + k$$
$$k = 150$$
$$v_C(t) = -50 + 150\, e^{-200t} \qquad \text{for } t > 0$$

Example 6.42 *In the network shown in Fig. 6.129, the switch closes at t = 0. The capacitor is initially uncharged. Find $v_C(t)$ and $i_C(t)$.*

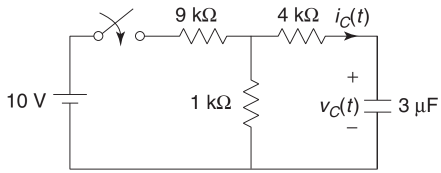


Fig. 6.129

Solution At $t = 0^-$, the capacitor is uncharged. Hence, it acts as a short circuit.

$$v_C(0^-) = 0$$
$$i_C(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.130.

Since voltage across the capacitor cannot change instantaneously,

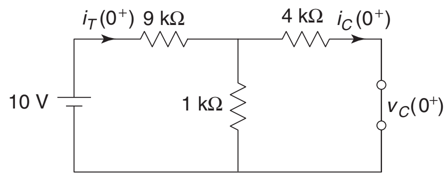


Fig. 6.130

$$v_C(0^+) = 0$$

At $t = 0^+$, $$i_T(0^+) = \left[\frac{10}{9\text{ k} + (4\text{ k} \parallel 1\text{ k})}\right] = \frac{10}{9.8\text{ k}} = 1.02\text{ mA}$$

$$i_C(0^+) = 1.02\text{ m} \times \frac{1\text{ k}}{1\text{ k} + 4\text{ k}} = 0.204\text{ mA}$$

For $t > 0$, the network is shown in Fig. 6.131.

For $t > 0$, representing the network to the left of the capacitor by Thevenin's equivalent network,

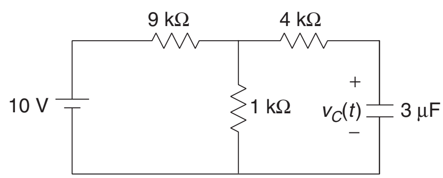


Fig. 6.131

$$V_{\text{eq}} = 10 \times \frac{1\text{ k}}{9\text{ k} + 1\text{ k}} = 1\text{ V}$$
$$R_{eq} = (9\text{ k} \parallel 1\text{ k}) + 4\text{ k} = 4.9\text{ k}\Omega$$

For $t > 0$, Thevenin's equivalent network is shown in Fig. 6.132.

Writing the KCL equation for $t > 0$,

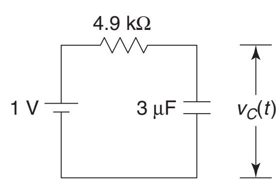


Fig. 6.132

$$3 \times 10^{-6} \frac{dv_C}{dt} + \frac{v_C - 1}{4.9 \times 10^3} = 0$$
$$\frac{dv_C}{dt} + 68.02\, v_C = 68.02$$

Comparing with the differential equation $\dfrac{dv}{dt} + Pv = Q$,

$$P = 68.02, \quad Q = 68.02$$

The solution of this differential equation is given by,

$$\begin{aligned} v_C(t) &= e^{-Pt}\int Q\, e^{Pt}\ dt + k\,e^{-Pt} \\ &= e^{-68.02t}\int 68.02\, e^{68.02t}\ dt + k\,e^{-68.02t} \\ &= 1 + k\,e^{-68.02t} \end{aligned}$$

At $t = 0, v_C(0) = 0$

$$\begin{aligned} 0 &= 1 + k \\ k &= -1 \\ v_C(t) &= 1 - e^{-68.02t} \qquad \text{for } t > 0 \\ i_C(t) &= C\frac{dv_C}{dt} \\ &= 3\times10^{-6}\frac{d}{dt}(1 - e^{-68.02t}) \\ &= 3\times10^{-6}\times 68.02e^{-68.02t} \\ &= 204.06\times10^{-6}\,e^{-68.02t} \qquad \text{for } t > 0 \end{aligned}$$

Example 6.43 *For the network shown in Fig. 6.133, the switch is open for a long time and closes at t = 0. Determine $v_C(t)$.*

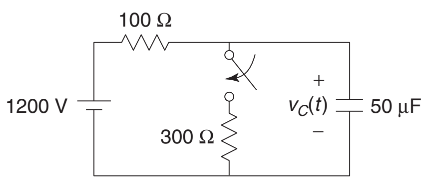


Fig. 6.133

Solution At $t = 0^-$, the network is shown in Fig. 6.134.

At $t = 0^-$, the network has attained steady-state condition. Hence, the capacitor acts as an open circuit.

$$v_C(0^-) = 1200 \text{ V}$$

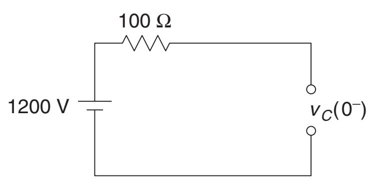


Fig. 6.134

Since the voltage across the capacitor cannot change instantaneously,

$$v_C(0^+) = 1200 \text{ V}$$

For $t > 0$, the network is shown in Fig. 6.135.

Writing the KCL equation for $t > 0$,

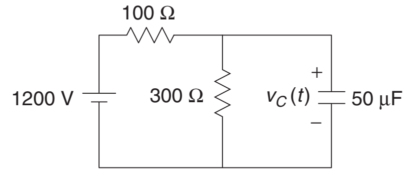


Fig. 6.135

$$50\times10^{-6}\frac{dv_C}{dt}+\frac{v_C}{300}+\frac{v_C-1200}{100}=0$$

$$\frac{dv_C}{dt}+266.67\,v_C=0.24\times10^6$$

Comparing with the differential equation $\frac{dv}{dt}+Pv=Q$,

$$P=266.67,\quad Q=0.24\times10^6$$

The solution of this differential equation is given by,

$$\begin{aligned}v_C(t)&=e^{-Pt}\int Q\,e^{Pt}\,dt+k\,e^{-Pt}\\&=e^{-266.67t}\int 0.24\times10^6\,e^{266.67t}\,dt+k\,e^{-266.67t}\\&=\frac{0.24\times10^6}{266.67}+k\,e^{-266.67t}\\&=900+k\,e^{-266.67t}\end{aligned}$$

At $t=0, v_C(0)=1200$ V

$$1200=900+k$$

$$k=300$$

$$v_C(t)=900+300\,e^{-266.67t}\qquad \text{for } t>0$$

Example 6.44 *In Fig. 6.136, the switch is closed at $t = 0$ Find $v_C(t)$ for $t > 0$.*

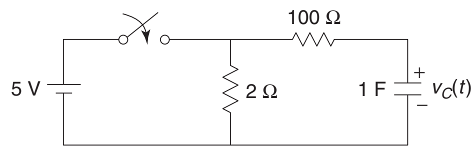


Fig. 6.136

Solution At $t = 0^-$, $v_C(0^-) = 0$

Since the voltage across the capacitor cannot change instantaneously,

$$v_C(0^+)=0$$

Since the resistor of 2 Ω is connected in parallel with the voltage source of 5 V, it becomes redundant.

For $t > 0$, the network is as shown in Fig. 6.137.

Writing KCL equation for $t > 0$,

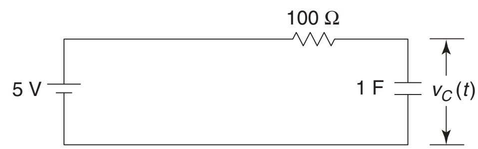


Fig. 6.137

$$\frac{v_C-5}{100}+1\frac{dv_C}{dt}=0$$

$$100\frac{dv_C}{dt}+v_C=5$$

$$\frac{dv_C}{dt} + 0.01 v_C = 0.05$$

Comparing with the differential equation $\dfrac{dv}{dt} + Pv = Q$,

$$P = 0.01, \quad Q = 0.05$$

The solution of this differential equation is given by,

$$\begin{aligned} v_C(t) &= e^{-Pt} \int Q e^{Pt}\, dt + k e^{-Pt} \\ &= e^{-0.01} \int 0.05\, e^{0.01t}\, dt + k e^{-0.01t} \\ &= \frac{0.05}{0.01} + k e^{-0.01t} \\ &= 5 + k e^{-0.01t} \end{aligned}$$

At $t = 0, v_C(0) = 0$

$$\begin{aligned} 0 &= 5 + k \\ k &= -5 \\ v_C(t) &= 5 - 5e^{-0.01t} \\ &= 5(1 - e^{-0.01t}) \qquad \text{for } t > 0 \end{aligned}$$

Example 6.45 *In the network shown, the switch is shifted to position b at t = 0. Find v (t) for t > 0.*

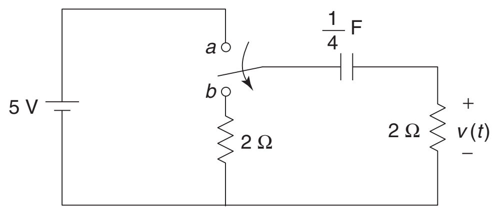


Fig. 6.138

Solution At $t = 0^-$, the network is shown in Fig. 6.139.

At $t = 0^-$, the network has attained steady-state condition. Hence, the capacitor acts as an open circuit.

$$v_C(0^-) = 5 \text{ V}$$

$$v(0^-) = 0$$

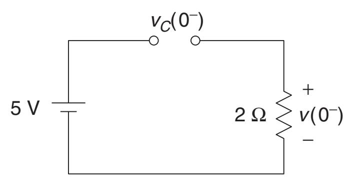


Fig. 6.139

At $t = 0^+$, the network is shown in Fig. 6.140.

At $t = 0^+$, the capacitor acts as a voltage source of 5 V.

$$i(0^+) = -\frac{5}{4} = -1.25 \text{ A}$$

$$v(0^+) = -1.25 \times 2 = -2.5 \text{ V}$$

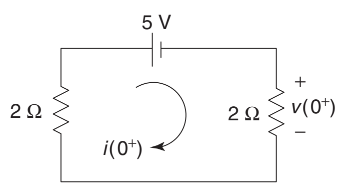


Fig. 6.140

For $t > 0$, the network is shown in Fig. 6.141.
Writing the KVL equation for $t > 0$,

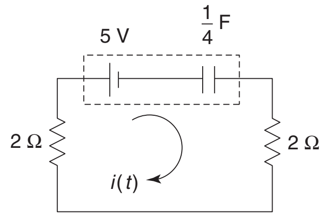


Fig. 6.141

$$-2i - 5 - \frac{1}{\frac{1}{4}}\int_0^t i\, dt - 2i = 0 \qquad \ldots\text{(i)}$$

Differentiating Eq. (i),

$$-4\frac{di}{dt} - 4i = 0$$

$$\frac{di}{dt} + i = 0$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = 0$,

$$P = 1$$

The solution of this differential equation is given by,

$$i(t) = k\,e^{-Pt} = k\,e^{-t}$$

At $t = 0$, $i(0) = -1.25$ A

$$k = -1.25$$

$$i(t) = -1.25\, e^{-t} \quad \text{for } t > 0$$

$$v(t) = 2i(t)$$

$$= -2.5e^{-t} \quad \text{for } t > 0$$

Example 6.46 *In the network of Fig. 6.142, the switch is open for a long time and at t = 0, it is closed. Determine $v_2(t)$.*

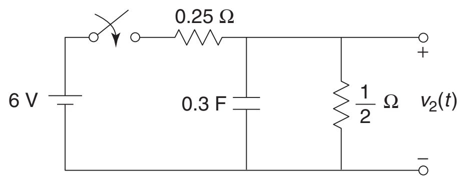


Fig. 6.142

Solution At $t = 0^-$, the switch is open.

$$v_2(0^-) = 0$$

Since voltage across capacitor cannot change instantaneously,

$$v_2(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.143.

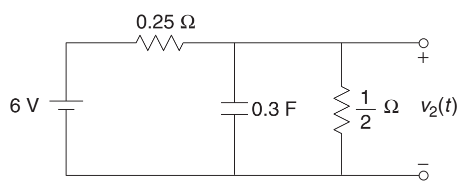


Fig 6.143

Writing KCL equation for $t > 0$,

$$\frac{v_2}{\frac{1}{2}} + 0.3\frac{dv_2}{dt} + \frac{v_2 - 6}{0.25} = 0$$

$$\frac{dv_2}{dt} + 20v_2 = 80$$

Comparing with the differential equation $\dfrac{dv}{dt} + Pv = Q$,

$$P = 20, \quad Q = 80$$

The solution of this differential equation is given by,

$$\begin{aligned} v(t) &= e^{-Pt}\int Q\, e^{Pt}\, dt + k\, e^{-Pt} \\ &= e^{-20t}\int 80\, e^{20t}\, dt + k\, e^{-20t} \\ &= \frac{80}{20} + k\, e^{-20t} \\ v_2 t &= 4 + k\, e^{-20t} \end{aligned}$$

At $t = 0, v_2(0) = 0$

$$\begin{aligned} 0 &= 4 + k \\ k &= -4 \\ v_2(t) &= 4 - 4e^{-20t} \\ &= 4(1 - e^{-20t}) \qquad \text{for } t > 0. \end{aligned}$$

Example 6.47 *The switch is moved from the position a to b at t = 0, having been in the position a for a long time before t = 0. The capacitor C_2 is uncharged at t = 0. Find i (t) and v_2 (t) for t > 0.*

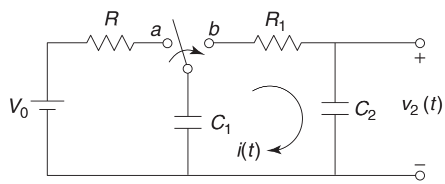


Fig. 6.144

Solution At $t = 0^-$, the network has attained steady-state condition. Hence, the capacitor C_1 acts as an open circuit and it will charge to V_0 volt.

$$v_{C_1}(0^-) = V_0$$
$$v_{C_2}(0^-) = 0$$

Since the voltage across the capacitor cannot change instantaneously,

$$v_{C_1}(0^+) = V_0$$
$$v_{C_2}(0^+) = 0$$
$$i(0^+) = \frac{V_0}{R_1}$$

For $t > 0$, the network is shown in Fig. 6.145.
Writing the KVL equation for $t > 0$,

$$V_0 - \frac{1}{C_1}\int_0^t i\,dt - R_1 i - \frac{1}{C_2}\int_0^t i\,dt = 0 \qquad \text{...(i)}$$

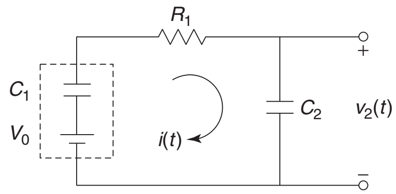


Fig. 6.145

Differentiating Eq. (i),

$$-\frac{i}{C_1} - R_1\frac{di}{dt} - \frac{i}{C_2} = 0$$

$$\frac{di}{dt} + \frac{1}{R_1}\left(\frac{C_1+C_2}{C_1 C_2}\right)i = 0$$

Comparing with the differential equation $\frac{di}{dt} + Pi = 0$,

$$P = \frac{1}{R_1}\left(\frac{C_1+C_2}{C_1 C_2}\right)$$

The solution of this differential equation is given by,

$$i(t) = k\,e^{-Pt} = k\,e^{-\frac{1}{R_1}\left(\frac{C_1+C_2}{C_1 C_2}\right)t}$$

At $t = 0$, $i(0) = \dfrac{V_0}{R_1}$

$$k = \frac{V_0}{R_1}$$

$$i(t) = \frac{V_0}{R_1}e^{-\frac{1}{R_1}\left(\frac{C_1+C_2}{C_1 C_2}\right)t}$$

$$= \frac{V_0}{R_1}e^{-\frac{1}{R_1 C}t} \qquad \text{where, } C = \frac{C_1 C_2}{C_1+C_2}$$

$$v_2(t) = \frac{1}{C_2}\int_0^t i\,dt$$

$$= \frac{1}{C_2}\int_0^t \frac{V_0}{R_1}e^{-\frac{t}{R_1 C}}dt$$

$$= \frac{V_0}{R_1 C_2}R_1 C\left(1 - e^{-\frac{1}{R_1 C}t}\right)$$

$$= \frac{V_0 C_1}{C_1+C_2}\left(1 - e^{-\frac{t}{R_1 C}t}\right) \qquad \text{for } t > 0$$

Example 6.48 *For the network shown in Fig.6.146, the switch is opened at $t = 0$. Find $v\ (t)$ for* $t > 0$.

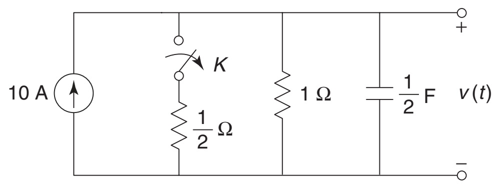


Fig. 6.146

Solution At $t = 0^-$, the network is shown in Fig. 6.147.

At $t = 0^-$, the network attains steady-state condition. Hence, the capacitor acts as an open circuit.

$$v_C(0^-) = 0$$

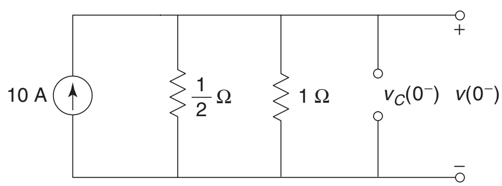


Fig. 6.147

Writing the KCL equation at $t = 0^-$,

$$\frac{v(0^-)}{1} + \frac{v(0^-)}{\frac{1}{2}} = 10$$

$$3v(0^-) = 10$$

$$v(0^-) = 3.33 \text{ V}$$

Since the voltage across the capacitor cannot change instantaneously,

$$v_C(0^+) = v(0^+) = 3.33 \text{ V}$$

For $t > 0$, the network is shown in Fig. 6.148.

Writing the KCL equation for $t > 0$,

$$\frac{1}{2}\frac{dv}{dt} + \frac{v}{1} = 10$$

$$\frac{dv}{dt} + 2v = 20$$

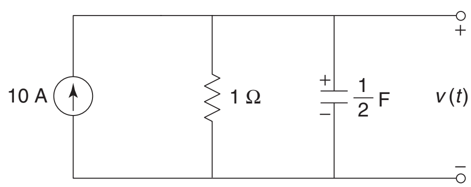


Fig. 6.148

Comparing with the differential equation $\dfrac{dv}{dt} + Pv = Q$,

$$P = 2, \quad Q = 20$$

The solution of this differential equation is given by,

$$\begin{aligned} v(t) &= e^{-Pt}\int Q\, e^{Pt}\ dt + k\,e^{-Pt} \\ &= e^{-2t}\int 20\, e^{-2t}\ dt + k\,e^{-2t} \\ &= \frac{20}{2} + k\,e^{-2t} \\ &= 10 + k\,e^{-2t} \end{aligned}$$

At $t = 0$, $v(0) = 3.33$ V

$$3.33 = 10 + k$$

$$k = 6.67$$

$$v(t) = 10 + 6.67\, e^{-2t}$$

Example 6.49 *For the network shown in Fig. 6.149, find the current i(t) when the switch is opened at $t = 0$.*

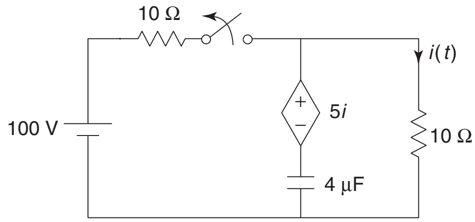


Fig. 6.149

Solution At $t = 0^-$, the network is shown in Fig. 6.150.

At $t = 0^-$, the network attains steady-state condition. Hence, the capacitor acts as open circuit.

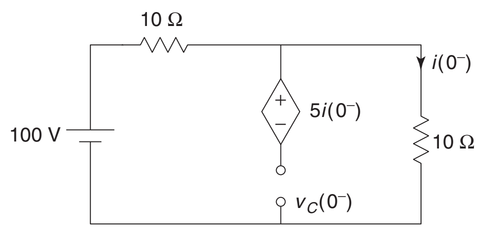


Fig. 6.150

$$i(0^-) = \frac{100}{10+10} = 5 \text{ A}$$

$$v_C(0^-) = 100 - 10i(0^-) - 5i(0^-) = 100 - 10(5) - 5(5) = 25 \text{ V}$$

At $t = 0^+$, the network is shown in Fig. 6.151.

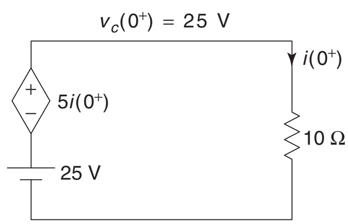


Fig. 6.151

$$25 + 5i(0^-) - 10i(0^+) = 0$$

$$i(0^+) = 5 \text{ A}$$

For $t > 0$, the network is shown in Fig. 6.152.

$$25 - \frac{1}{4 \times 10^{-6}} \int_0^t i \, dt + 5i - 10i = 0 \qquad \ldots\text{(i)}$$

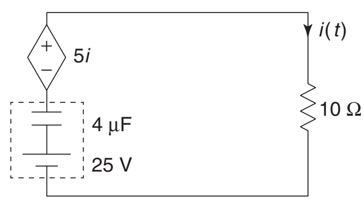


Fig. 6.152

Differentiating Eq. (i),

$$0 - 0.25 \times 10^6\, i + 5\frac{di}{dt} - 10\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 50000\, i = 0$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = 0$,

$$P = 50000$$

The solution of this differential equation is given by,

$$i(t) = k\,e^{-Pt} = k\,e^{-50000t}$$

At $t = 0,\quad i(0) = 5$ A

$$5 = k$$

$$i(t) = 5e^{-50000t} \qquad \text{for } t > 0$$

Example 6.50 *For the network shown in Fig. 6.153, find the current i(t) when the switch is opened at t = 0.*

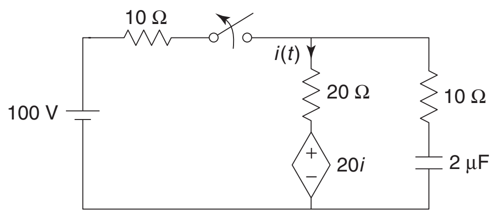


Fig. 6.153

Solution At $t = 0^-$, the network is shown in Fig. 6.154. At $t = 0^-$, the network attains steady-state condition. Hence, the capacitor acts as an open circuit.

Writing the KVL equation at $t = 0^-$,

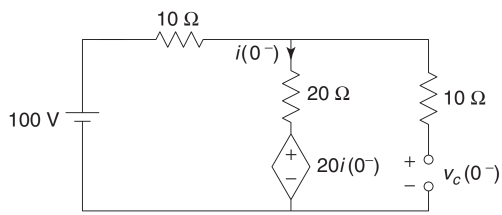


Fig. 6.154

$$100 - 10\,i(0^-) - 20\,i(0^-) - 20\,i(0^-) = 0$$

$$i(0^-) = 2 \text{ A}$$

Also, $$20\,i(0^-) + 20\,i(0^-) - 0 - v_C(0^-) = 0$$

$$v_C(0^-) = 40\,i(0^-) = 40(2) = 80 \text{ V}$$

At $t = 0^+$, the network is shown in Fig. 6.155.

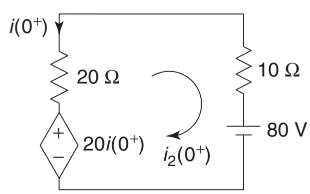


Fig. 6.155

From Fig. 6.155, $i(0^+) = -i_2(0^+)$

$$20\,i(0^+) - 20\,i_2(0^+) - 10\,i_2(0^+) - 80 = 0$$

$$20\,i(0^+) + 20\,i(0^+) + 10\,i(0^+) - 80 = 0$$

$$i(0^+) = 1.6 \text{ A}$$

$$v_C(0^+) = 80 \text{ V}$$

For $t > 0$, the network is shown in Fig. 6.156.

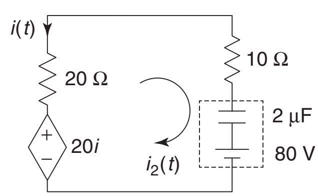


Fig. 6.156

From Fig. 6.156, $i(t) = -i_2(t)$

Writing the KVL equation for $t > 0$,

$$20i - 20i_2 - 10i_2 - \frac{1}{2\times10^{-6}}\int_0^t i_2\,dt - 80 = 0$$

$$20i + 20i_2 + 10i_2 + \frac{1}{2\times10^{-6}}\int_0^t i\,dt - 80 = 0$$

$$50i + \frac{1}{2\times10^{-6}}\int i\,dt - 80 = 0 \qquad \text{...(i)}$$

Differentiating Eq. (i),

$$50\frac{di}{dt} + 5\times10^5 i = 0$$

$$\frac{di}{dt} + 1\times10^4 i = 0$$

Comparing with the differential equation $\frac{di}{dt} + Pi = 0$,

$$P = 1\times10^4$$

The solution of this differential equation is given by,

$$i(t) = k\,e^{-Pt} = k\,e^{-1\times10^4 t}$$

At $t = 0$, $i(0) = 1.6$ A

$$1.6 = k$$

$$i(t) = 1.6\,e^{-1\times10^4 t} \qquad \text{for } t > 0$$

Example 6.51 *In the network of Fig. 6.157, an exponential voltage $4e^{-5t}$ is applied at time $t = 0$. Find the expression for current $i(t)$. Assume zero voltage across the capacitor at $t = 0$.*

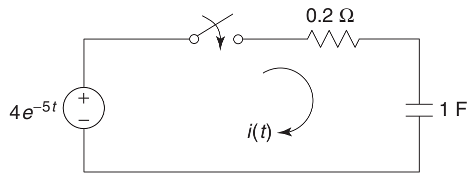


Fig. 6.157

Solution At $t = 0^-$,

$$v_C(0^-) = 0$$

$$i(0^-) = 0$$

At $t = 0^+$, the network is shown in Fig. 6.158.

Since voltage across the capacitor cannot change instantaneously,

$$v_C(0^+) = 0$$

$$i(0^+) = \frac{4}{0.2} = 20 \text{ A}$$

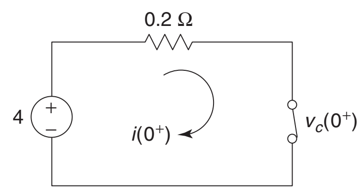


Fig. 6.158

Writing the KVL equation for $t > 0$,

$$4e^{-5t} - 0.2i - \frac{1}{1}\int_0^t i\,dt = 0 \qquad \text{...(i)}$$

Differentiating Eq. (i),

$$-20e^{-5t} - 0.2\frac{di}{dt} - i = 0$$

$$\frac{di}{dt} + 5i = -100e^{-5t}$$

Comparing with the differential equation $\frac{di}{dt} + Pi = Q$,

$$P = 5, \quad Q = -100e^{-5t}$$

The solution of this differential equation is given by,

$$\begin{aligned} i(t) &= e^{-Pt}\int Q\,e^{Pt}\,dt + k\,e^{-Pt} \\ &= e^{-5t}\int -100e^{-5t}e^{5t}\,dt + k\,e^{-5t} \\ &= -100t\,e^{-5t} + k\,e^{-5t} \end{aligned}$$

At $t = 0$, $i(0) = 20$ A

$$20 = k$$

$$i(t) = -100\,t\,e^{-5t} + 20e^{-5t} \qquad \text{for } t > 0$$

Example 6.52 *In the network shown in Fig. 6.159, the switch is closed at $t = 0$ connecting a source e^{-t} to the network. At $t = 0$, $v_C(0) = 0.5$ V. Determine $v(t)$.*

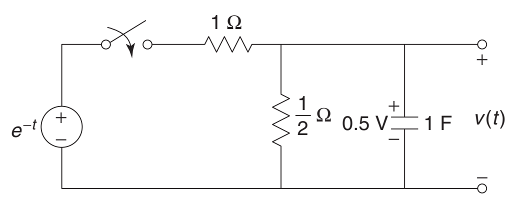


Fig. 6.159

Solution At $t = 0^-$, $v(0) = v_C(0^-) = 0.5$ V

Since voltage across the capacitor cannot change instantaneously,

$$v(0^+) = v_C(0^+) = 0.5 \text{ V}$$

Writing the KCL equation for $t > 0$,

$$\frac{v - e^{-t}}{1} + \frac{v}{\frac{1}{2}} + 1\frac{dv}{dt} = 0$$

$$\frac{dv}{dt} + 3v = e^{-t}$$

Comparing with the differential equation $\frac{dv}{dt} + Pv = Q$,

$$P = 3, \quad Q = e^{-t}$$

The solution of this differential equation is given by,

$$\begin{aligned}v(t) &= e^{-Pt}\int Q\, e^{Pt}\, dt + k\, e^{-Pt}\\ &= e^{-3t}\int e^{-t}\, e^{3t}\, dt + k\, e^{-3t}\\ &= e^{-3t}\int e^{2t}\, dt + k\, e^{-3t}\\ &= \frac{1}{2}e^{-t} + k\, e^{-3t}\end{aligned}$$

At $t = 0$, $v(0) = 0.5$ V

$$\begin{aligned}0.5 &= \frac{1}{2} + k\\ k &= 0\\ v(t) &= 0.5\, e^{-t}\end{aligned}$$

Example 6.53 *In the network shown in Fig. 6.160, a sinusoidal voltage $v = 100\sin(500t + \theta)$ volts is applied to the circuit at a time corresponding to $\theta = 45°$. Obtain the expression for the current $i(t)$.*

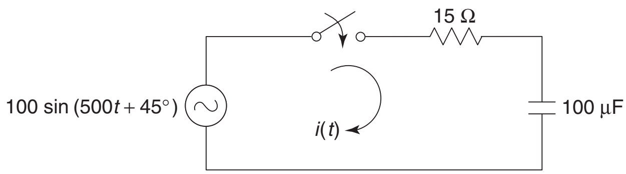


Fig. 6.160

Solution

Writing the KVL equation for $t > 0$,

$$100\sin(500t + 45°) - 15i - \frac{1}{100 \times 10^{-6}}\int_0^t i\, dt = 0 \qquad \ldots\text{(i)}$$

Differentiating Eq. (i),

$$(100)(500)\cos(500t + 45°) - 15\frac{di}{dt} - 10^4 i = 0$$

$$\frac{di}{dt} + 666.67i = 3333.33\cos(500t + 45°)$$

Comparing with the differential equation $\dfrac{di}{dt} + Pi = Q$,

$$P = 666.67,\ Q = 3333.33\cos(500t + 45°)$$

The solution of this differential equation is given by,

$$\begin{aligned}i(t) &= e^{-Pt}\int Q\, e^{Pt}\, dt + k\, e^{-Pt}\\ &= e^{-666.67t}\int 3333.33\cos(500t + 45°)\, e^{666.67t} + k\, e^{-666.67t}\end{aligned}$$

$$= 3333.33\, e^{-666.67t}\left[\frac{e^{666.67t}}{(666.67)^2+(500)^2}\{666.67\cos(500t+45°)+500\sin(500t+45°)\}\right]+k\,e^{-666.67t}$$

$$= 3.2\cos(500t+45°)+2.4\sin(500t+45°)+k\,e^{-666.67t}$$

Let $\quad A\sin\phi = 3.2$

and $\quad A\cos\phi = 2.4$

$$A^2\sin^2\phi + A^2\cos^2\phi = (3.2)^2+(2.4)^2 = 16$$

$$A = 4$$

and $\quad \phi = \tan^{-1}\left(\frac{3.2}{2.4}\right) = 53.13°$

$$i(t) = 4\sin(53.13°)\cos(500t+45°)+4\cos(53.13°)\sin(500t+45°)+k\,e^{-666.67t}$$

$$= 4\sin(500t+98.13°)+k\,e^{-666.67t}$$

Putting $t=0,$ in Eq. (i),

$$100\sin(45°)-15\,i(0)-0=0$$

$$i(0) = 4.71$$

$$4.71 = 4\sin(98.13°)+k$$

$$k = 0.75$$

$$i(t) = 4\sin(500t+98.13°)+0.75\,e^{-666.67t} \qquad \text{for } t>0$$

Example 6.54 *In the network of Fig. 6.161, the switch is moved from the position 1 to 2 at $t = 0$. The switch is in the position 1 for a long time. Initial charge on the capacitor is 7×10^{-4} coulombs. Determine the current expression $i(t)$, when $\omega = 1000$ rad/s.*

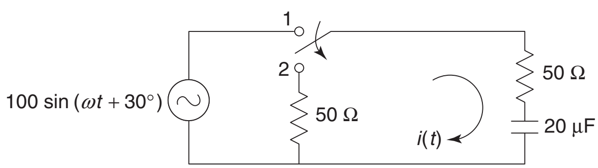


Fig. 6.161

Solution At $t = 0^-$, the network is shown in Fig. 6.162. At $t = 0^-$, the network attains steady-state condition.

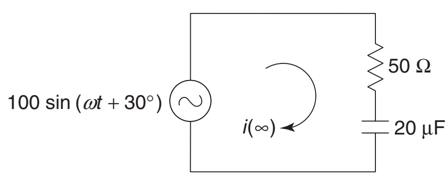


Fig. 6.162

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{100\angle 30°}{50 - j\dfrac{1}{1000\times 20\times 10^{-6}}} = 1.41\angle 75°\text{ A}$$

The steady-state current passing through the network when the switch is in the position 1 is

$$i = 1.41\sin(1000t+75°) \qquad \ldots\text{(i)}$$

For $t > 0$, the network is as shown in Fig. 6.163.
Writing the KVL equation for $t > 0$,

$$-50i - 50i - \frac{1}{20 \times 10^{-6}} \int_0^t i\,dt - v_C(0) = 0 \qquad \ldots\text{(ii)}$$

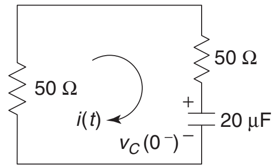


Fig. 6.163

Differentiating Eq. (ii),

$$-50\frac{di}{dt} - 50\frac{di}{dt} - \frac{1}{20 \times 10^{-6}} i = 0$$

$$\frac{di}{dt} + 500i = 0$$

Comparing with differential equation $\frac{di}{dt} + Pi = 0$,

$$P = 500$$

The solution of this differential equation is given by,

$$i(t) = k\,e^{-Pt} = k\,e^{-500t} \qquad \ldots\text{(iii)}$$

From Eqs (i) and (ii),

$$1.41\sin(1000t + 75°) = k\,e^{-500t}$$

At $t = 0$,

$$1.41\sin(75°) = k$$

$$k = 1.36$$

$$i(t) = 1.36\,e^{-500t} \quad \text{for } t > 0$$

6.5 RESISTOR–INDUCTOR–CAPACITOR CIRCUIT

Consider a series *RLC* circuit as shown in Fig. 6.164. The switch is closed at time $t = 0$. The capacitor and inductor are initially uncharged.

Applying KVL to the circuit for $t > 0$,

$$V - Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,dt = 0$$

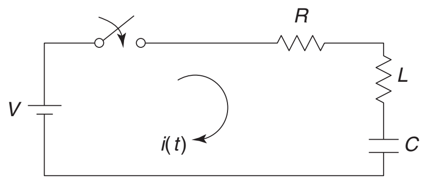


Fig. 6.164 *Series RLC circuit*

Differentiating the above equation,

$$0 - R\frac{di}{dt} - L\frac{d^2i}{dt^2} - \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

This is a second-order differential equation. The auxiliary equation or characteristic equation will be given by,

$$s^2 + \left(\frac{R}{L}\right)s + \left(\frac{1}{LC}\right) = 0$$

Let s_1 and s_2 be the roots of the equation.

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \beta$$

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - \beta$$

where $$\alpha = \frac{R}{2L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

and $$\beta = \sqrt{\alpha^2 - \omega_0^2}$$

The solution of the above second-order differential equation will be given by,

$$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

where k_1 and k_2 are constants to be determined and s_1 and s_2 are the roots of the equation.
Now depending upon the values of α and ω_0, we have 3 cases of the response.

Case I *When* $\alpha > \omega_0$

i.e., $$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$$

The roots are real and unequal and it gives an overdamped response.
In this case, the solution is given by,

$$i = e^{-\alpha t}(k_1 e^{\beta t} + k_2 e^{-\beta t})$$

or $$i = k_1 e^{s_1 t} + k_2 e^{s_2 t} \quad \text{for } t > 0$$

The current curve for an overdamped case is shown in Fig. 6.165.

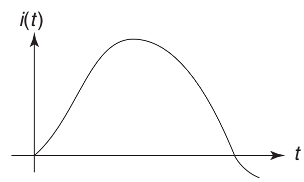


Fig. 6.165 *Overdamped response*

Case II *When* $\alpha = \omega_0$

i.e., $$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$$

The roots are real and equal and it gives a critically damped response.
In this case the solution is given by,

$$i = e^{-\alpha t}(k_1 + k_2 t) \quad \text{for } t > 0$$

The current curve for critically damped case is shown in Fig. 6.166.

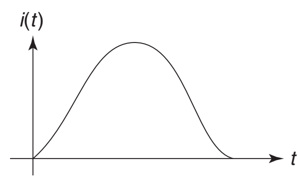


Fig. 6.166 *Critically damped response*

Case III *When* $\alpha < \omega_0$

i.e., $$\frac{R}{2L} < \frac{1}{\sqrt{LC}}$$

The roots are complex conjugate and it gives an underdamped response.

In this case, the solution is given by,

$$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

where $$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 + \omega_0^2}$$

Let $$\sqrt{\alpha^2 - \omega_0^2} = \sqrt{-1}\sqrt{\omega_0^2 - \alpha^2} = j\omega_d$$

where $$j = \sqrt{-1}$$

and $$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Hence, $$i(t) = e^{-\alpha t}(k_1 e^{j\omega_d t} + k_2 e^{-j\omega_d t})$$

$$= e^{-\alpha t}\left\{(k_1 + k_2)\left[\frac{e^{j\omega_d t} + e^{-j\omega_d t}}{2}\right] + j(k_1 - k_2)\left[\frac{e^{j\omega_d t} - e^{-j\omega_d t}}{j2}\right]\right\}$$

$$= e^{-\alpha t}\left[(k_1 + k_2)\cos\omega_d t + j(k_1 - k_2)\sin\omega_d t\right] \quad \text{for } t > 0$$

The current curve for an underdamped case is shown in Fig. 6.167.

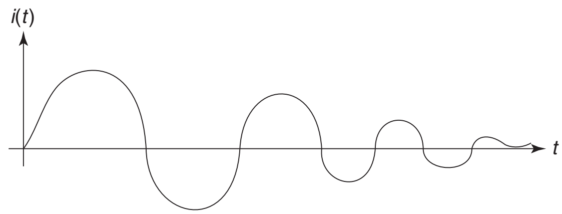


Fig. 6.167 *Underdamped response*

Example 6.55 *In the network of Fig. 6.168, the switch is closed at $t = 0$. Obtain the expression for current $i(t)$ for $t > 0$.*

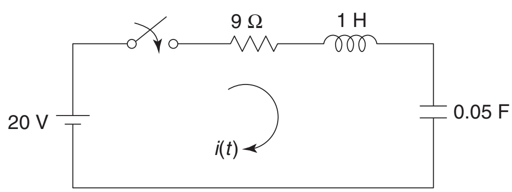


Fig. 6.168

Solution At $t = 0^-$, the switch is open.

$$i(0^-) = 0$$

$$v_C(0^-) = 0$$

Since current through the inductor and voltage across the capacitor cannot change instantaneously,

$$i(0^+) = 0$$

$$v_C(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.169.

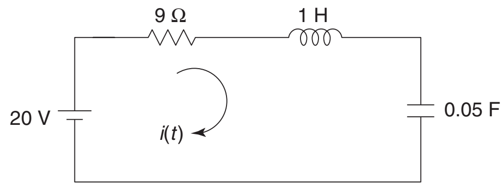


Fig. 6.169

Writing the KVL equation for $t > 0$,

$$20 - 9i - 1\frac{di}{dt} - \frac{1}{0.05}\int_0^t i\,dt = 0 \qquad \text{...(i)}$$

Differentiating Eq. (i),

$$0 - 9\frac{di}{dt} - \frac{d^2i}{dt^2} - 20i = 0$$

$$\frac{d^2i}{dt^2} + 9\frac{di}{dt} + 20i = 0$$

$$(D^2 + 9D + 20)i = 0$$

$$D_1 = -4,\ D_2 = -5$$

The solution of this differential equation is given by,

$$i(t) = k_1 e^{-4t} + k_2 e^{-5t} \qquad \text{...(ii)}$$

Differentiating Eq. (ii),

$$\frac{di}{dt} = -4k_1 e^{-4t} - 5k_2 e^{-5t} \qquad \text{...(iii)}$$

At $t = 0,\ i(0) = 0$

$$0 = k_1 + k_2 \qquad \text{...(iv)}$$

$$\frac{di}{dt}(0) = -4k_1 - 5k_2 \qquad \text{...(v)}$$

Putting $t = 0$ in Eq. (i),

$$20 - 9i(0^+) - \frac{di}{dt}(0^+) - 0 = 0$$

$$\frac{di}{dt}(0^+) = 20 - 9i(0^+) = 20 \text{ A/s}$$

From Eq. (v),

$$20 = -4k_1 - 5k_2 \qquad \text{...(vi)}$$

Solving Eqs (iv) and (vi),

$$k_1 = 20$$

$$k_2 = 20$$

$$i(t) = 20e^{-4t} - 20e^{-5t} \qquad \text{for } t > 0$$

Example 6.56 *In the network shown in Fig. 6.170, the switch is moved from the position 1 to 2 at $t = 0$. The switch is in the position 1 for a long time. Determine the expression for the current $i(t)$.*

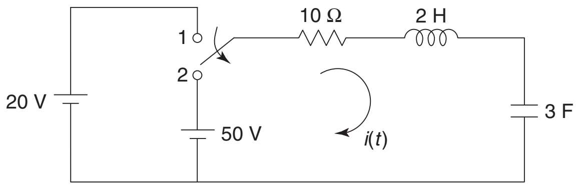


Fig. 6.170

Solution At $t = 0^-$, the network is shown in Fig. 6.171.

At $t = 0^-$, the network attains steady-state condition. Hence, the inductor acts as a short circuit and the capacitor acts as an open circuit.

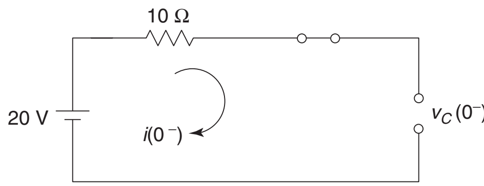


Fig. 6.171

$$v_C(0^-) = 20 \text{ V}$$
$$i(0^-) = 0$$

Since the current through the inductor and the voltage across the capacitor cannot change instaneously,

$$v_C(0^+) = 20 \text{ V}$$
$$i(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.172.

Writing the KVL equation for $t > 0$,

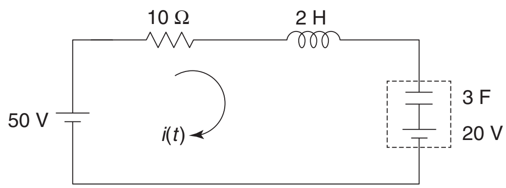


Fig. 6.172

$$50 - 10i - 2\frac{di}{dt} - \frac{1}{3}\int_0^t i\,dt - 20 = 0 \qquad \ldots\text{(i)}$$

Differentiating Eq. (i),

$$0 - 10\frac{di}{dt} - 2\frac{d^2i}{dt^2} - \frac{1}{3}i - 0 = 0$$
$$\frac{d^2i}{dt^2} + 5\frac{di}{dt} + \frac{1}{6}i = 0$$
$$\left(D^2 + 5D + \frac{1}{6}\right)i = 0$$
$$D_1 = -0.03,\ D_2 = -4.97$$

The solution of this differential equation is given by,

$$i(t) = k_1\, e^{-0.03t} + k_2\, e^{-4.97t} \qquad \ldots\text{(ii)}$$

Differentiating Eq. (ii),

$$\frac{di}{dt} = -0.03k_1\, e^{-0.03t} - 4.97k_2\, e^{-4.97t} \qquad \ldots\text{(iii)}$$

At $t = 0, i(0) = 0$

$$0 = k_1 + k_2 \qquad \ldots\text{(iv)}$$

$$\frac{di}{dt}(0) = -0.03\, k_1 - 4.97\, k_2 \qquad \ldots\text{(v)}$$

Putting $t = 0$ in Eq. (i),

$$20 - 10\, i(0^+) - 2\frac{di}{dt}(0^+) - 0 = 0$$

$$\frac{di}{dt}(0^+) = \frac{20 - 10\, i(0^+)}{2} = 10 \text{ A/s}$$

From Eq. (v),

$$10 = -0.03\, k_1 - 4.97\, k_2 \qquad \text{...(vi)}$$

Solving Eqs (iv) and (vi),

$$k_1 = 2.02$$

$$k_2 = 2.02$$

$$i(t) = 2.02\, e^{-0.03t} - 2.02\, e^{-4.97t} \quad \text{for } t > 0$$

Example 6.57 *In the network of Fig. 6.173, the switch is closed and a steady state is reached in the network. At $t = 0$, the switch is opened. Find the expression for the current $i_2(t)$ in the inductor.*

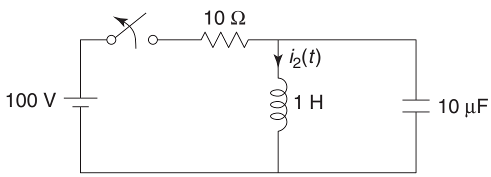


Fig. 6.173

Solution At $t = 0^-$, the network is shown in Fig. 6.174.

At $t = 0^-$, the network attains steady-state condition. Hence, the inductor acts as a short circuit and the capacitor acts as an open circuit.

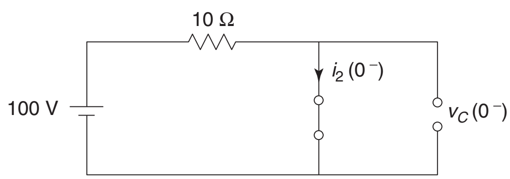


Fig. 6.174

$$i_2(0^-) = \frac{100}{10} = 10 \text{ A}$$

$$v_C(0^-) = 0$$

Since current through the inductor and voltage across capacitor cannot change instantaneously,

$$i_2(0^+) = 10 \text{ A}$$

$$v_C(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.175.

Writing the KVL equation for $t > 0$,

$$-1\frac{di_2}{dt} - \frac{1}{10 \times 10^{-6}}\int_0^t i\, dt = 0 \qquad \text{...(i)}$$

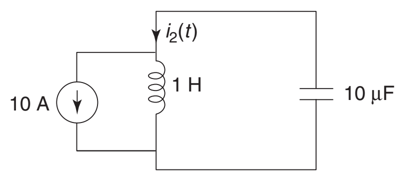


Fig. 6.175

Differentiating Eq. (i),

$$-\frac{d^2 i_2}{dt^2} - 10^5 i = 0$$

$$\frac{d^2 i_2}{dt^2} + 10^5 i = 0$$

$$(D^2 + 10^5) i = 0$$

$$D_1 = j\,316,\ D_2 = -j\,316$$

The solution of this differential equation is given by,

$$i_2(t) = k_1 \cos 316t + k_2 \sin 316t \qquad \text{...(ii)}$$

Differentiating Eq. (ii),

$$\frac{di_2}{dt} = -316\,k_1 \sin 316t + 316\,k_2 \cos 316t \qquad \text{...(iii)}$$

At $t = 0,\ i_2(0) = 10$ A

$$10 = k_1 \qquad \text{...(iv)}$$

$$\frac{di_2}{dt}(0) = 316\,k_2 \qquad \text{...(v)}$$

Putting $t = 0$ in Eq. (i),

$$-\frac{di}{dt}(0^+) - 0 = 0$$

$$\frac{di}{dt}(0^+) = 0$$

From Eq. (v),

$$0 = 316\,k_2$$

$$k_2 = 0$$

$$i_2(t) = 10 \cos 316t \quad \text{for } t > 0$$

Example 6.58 *In the network of Fig. 6.176, capacitor C has an initial voltage $v_c(0^-)$ of 10 V and at the same instant, current in the inductor L is zero. The switch is closed at time $t = 0$. Obtain the expression for the voltage $v(t)$ across the inductor L.*

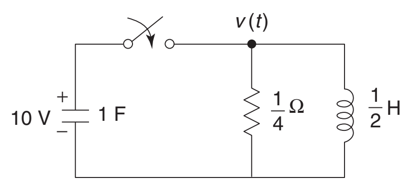


Fig. 6.176

Solution At $t = 0^-$,

$$i_L(0^-) = 0$$

$$v(0^-) = v_C(0^-) = 10 \text{ V}$$

Since current through the inductor and voltage across capacitor cannot change instantaneously,

$$i_L(0^+) = 0$$

$$v(0^+) = v_C(0^+) = 10 \text{ V}$$

For $t > 0$, the network is shown in Fig. 6.177.

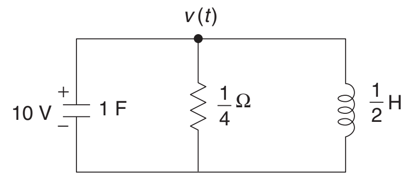


Fig. 6.177

Writing the KCL equation for $t > 0$,

$$1\frac{dv}{dt} + \frac{v}{\frac{1}{4}} + \frac{1}{\frac{1}{2}}\int_0^t v\, dt = 0 \qquad \ldots\text{(i)}$$

Differentiating Eq. (i),

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 2v = 0$$

$$(D^2 + 4D + 2)v = 0$$

$$D_1 = -1, D_2 = -3$$

The solution of this differential equation is given by,

$$v(t) = k_1 e^{-t} + k_2 e^{-3t} \qquad \ldots\text{(ii)}$$

Differentiating Eq. (iii),

$$\frac{dv}{dt} = -k_1 e^{-t} - 3k_2 e^{-3t} \qquad \ldots\text{(iii)}$$

At $t = 0$, $v(0) = 10$ V

$$10 = k_1 + k_2 \qquad \ldots\text{(iv)}$$

$$\frac{dv}{dt}(0) = -k_1 - 3\,k_2 \qquad \ldots\text{(v)}$$

Putting $t = 0$ in Eq. (i),

$$\frac{dv}{dt}(0^+) + 4v(0) + 0 = 0$$

$$\frac{dv}{dt}(0^+) = -40 \text{ V/s}$$

From Eq. (v),

$$-40 = -k_1 - 3\,k_2 \qquad \ldots\text{(vi)}$$

Solving Eqs (iv) and (vi),

$$k_1 = -5$$

$$k_2 = 15$$

$$v(t) = -5\,e^{-t} + 15\,e^{-3t} \qquad \text{for } t > 0$$

Example 6.59 *In the network of Fig. 6.178, the switch is opened at $t = 0$ obtain the expression for $v(t)$. Assume zero initial conditions.*

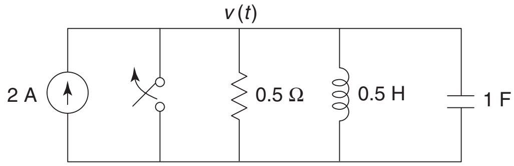


Fig. 6.178

Solution At $t = 0^-$,

$$i_L(0^-) = 0$$
$$v(0^-) = v_C(0^-) = 0$$

Since current through the inductor and voltage across the capacitor can not change instantaneously,

$$i_L(0^+) = 0$$
$$v(0^+) = v_C(0^+) = 0$$

For $t > 0$, the network is shown in Fig. 6.179.

Writing the KCL equation for $t > 0$,

$$\frac{v}{0.5} + \frac{1}{0.5}\int_0^t v\,dt + 1\frac{dv}{dt} = 2 \qquad \ldots\text{(i)}$$

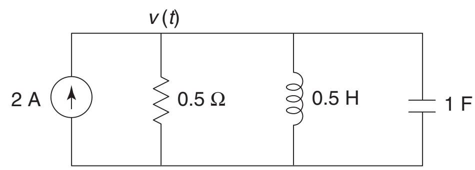


Fig. 6.179

Differentiating Eq. (i),

$$2\frac{dv}{dt} + 2v + \frac{d^2v}{dt^2} = 0$$
$$\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + 2v = 0$$
$$(D^2 + 2D + 2)v = 0$$
$$D_1 = -1 + j1, \quad D_2 = -1 - j1$$

The solution of this differential equation is given by,

$$v(t) = e^{-t}(k_1 \cos t + k_2 \sin t) \qquad \ldots\text{(ii)}$$

Differentiating Eq. (ii),

$$\frac{dv}{dt} = -e^{-t}(k_1 \cos t + k_2 \sin t) + e^{-t}(-k_1 \sin t + k_2 \cos t)$$
$$= e^{-t}[-k_1(\cos t + \sin t) + k_2(\cos t - \sin t)] \qquad \ldots\text{(iii)}$$

At $t = 0$, $v(0) = 0$

$$0 = k_1 \qquad \ldots\text{(iv)}$$
$$\frac{dv}{dt}(0) = -k_1 + k_2 \qquad \ldots\text{(v)}$$

Putting $t = 0$ in Eq. (i),

$$2v(0) + 0 + \frac{dv}{dt}(0) = 2$$

$$\frac{dv}{dt}(0) = 2\ V/s$$

From Eq. (v),

$$2 = -k_1 + k_2 \qquad \text{...(vi)}$$

Solving Eq. (iv) and (vi),

$$k_1 = 0$$
$$k_2 = 2$$
$$v(t) = 2\,e^{-t}\sin t \qquad \text{for } t > 0$$

Example 6.60 *The network shown in Fig. 6.180, a sinusoidal voltage $v = 150\sin(200t+\phi)$ is applied at $\phi = 30°$. Determine the current $i(t)$.*

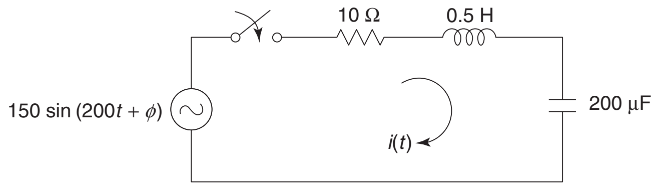


Fig. 6.180

Solution Writing the KVL equation for $t > 0$,

$$150\sin(200t+30°) - 10i - 0.5\frac{di}{dt} - \frac{1}{200\times10^{-6}}\int_0^t i\,dt = 0 \qquad \text{...(i)}$$

Differentiating Eq. (i),

$$30000\cos(200t+30°) - 10\frac{di}{dt} - 0.5\frac{d^2i}{dt^2} - 5000i = 0$$
$$\frac{d^2i}{dt^2} + 20\frac{di}{dt} + 10000i = 60000\cos(200t+30°)$$
$$(D^2 + 20D + 10000)i = 60000\cos(200t+30°) \qquad \text{...(ii)}$$

The roots of the characteristic equation are

$$D_1 = -10 + j\,99.5, \quad D_2 = -10 - j\,99.5$$

The complimentary function is

$$i_C = e^{-10t}(K_1\cos 99.5t + K_2\sin 99.5t)$$

Let the particular function be

$$i_P = A\cos(200t+30°) + B\sin(200t+30°)$$
$$i_P' = -200A\sin(200t+30°) + 200B\cos(200t+30°)$$
$$i_P'' = -40000A\cos(200t+30°) - 40000B\sin(200t+30°)$$

Substituting these values in Eq. (ii),

$$-40000\,A\cos(200t+30°)-40000\,B\sin(200t+30°)+20[-200\,A\sin(200t+30°)+200\,B\cos(200t+30°)]$$
$$+10000[A\cos(200t+30°)+B\sin(200t+30°)]=60000\cos(200t+30°)$$

$$(-40000\,B-4000\,A+10000\,B)\sin(200\text{t}+30°)+(-40000\,A+4000\,B+10000\,A)\cos(200t+30°)$$
$$=60000\cos(200t+30°)$$

Equating the coefficients,

$$-40000\,B-4000\,A+10000\,B=0$$
$$-40000\,A+4000\,B+10000\,A=60000$$

Solving these equations,

$$A=-1.97$$
$$B=0.26$$
$$i_P=-1.97\cos(200t+30°)+0.26\sin(200t+30°)$$

Let $\qquad A\sin\phi=-1.97$

and $\qquad A\cos\phi=0.26$

$$A^2\sin^2\phi+A^2\cos^2\phi=(-1.97)^2+(0.26)^2=3.95$$
$$A=1.987$$

and $\qquad \phi=\tan^{-1}\left(\dfrac{-1.97}{0.26}\right)=-82.48°$

$$\begin{aligned} i_P &= 1.987\sin(-82.48°)\cos(200t+30°)+1.987\cos(-82.48°)\sin(200t+30°)\\ &= 1.987\sin(200t+30°-82.48°)\\ &= 1.987\sin(200t-52.48°)\end{aligned}$$

The solution of the differential equation is given by,

$$i(t)=e^{-10t}(k_1\cos 99.5t+k_2\sin 99.5t)+1.987\sin(200t-52.48°) \qquad \ldots\text{(iii)}$$

Differentiating Eq. (iii),

$$\begin{aligned}\frac{di}{dt} &= e^{-10t}(-99.5\,k_1\sin 99.5t+99.5\,k_2\cos 99.5t)\\ &\quad -10\,e^{-10t}(k_1\cos 99.5t+k_2\sin 99.5t)+(1.987)(200)\cos(200t-52.48°)\end{aligned}$$

At $t=0, \quad i(0)=0$

$$0=k_1+1.987\sin(-52.48°)$$
$$k_1=1.58 \qquad \ldots\text{(iv)}$$

$$\begin{aligned}\frac{di}{dt}(0) &= 99.5\,k_2-10\,k_1+(1.987)(200)\cos(-52.48°)\\ &= 99.5\,k_2-10(1.58)+242.03\\ &= 99.5\,k_2+226.23 \qquad \ldots\text{(v)}\end{aligned}$$

Putting $t = 0$ in Eq. (i),

$$150\sin(30°) - 10(0) - 0.5\frac{di}{dt}(0) - 0 = 0$$

$$\frac{di}{dt}(0) = 150 \text{ A/s}$$

From Eq. (v),

$$150 = 99.5\,k_2 + 226.23$$

$$k_2 = -0.77$$

$$i(t) = e^{-10t}(1.58\cos 99.5t - 0.77\sin 99.5t) + 1.987\sin(200t - 52.48°) \quad \text{for } t > 0$$

Example 6.61 *The switch in the network of Fig. 6.181 is opened at t = 0. Find i (t) for t > 0 if,*

(a) $L = \frac{1}{2}$ *H and C = 1 F* *(b) L = 1 H and C = 1 F* *(c) L = 5 H and C = 1 F*

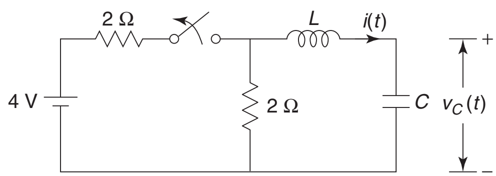


Fig. 6.181

Solution At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit and the capacitor acts as an pen circuit.

$$v_C(0^-) = 4 \times \frac{2}{2+2} = 2 \text{ V}$$

$$i(0^-) = 0$$

Since current through the inductor and voltage across the capacitor cannot change instantaneously,

$$v_C(0^+) = 2 \text{ V}$$

$$i(0^+) = 0$$

Case I *When* $R = 2\ \Omega$, $L = \frac{1}{2}$ *H*, $C = 1$ *F*

$$\alpha = \frac{R}{2L} = \frac{2}{2 \times \frac{1}{2}} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times 1}} = \frac{1}{\sqrt{0.5}} = 1.414$$

$$\alpha > \omega_0$$

This indicates an overdamped case.

$$i(t) = A_1 e^{s_1 t} - A_2 e^{s_2 t}$$

where, $$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -2 - \sqrt{4-2} = -2 - \sqrt{2} = -3.414$$

and $$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2 + \sqrt{2} = -0.586$$

$$i(t) = k_1 e^{-3.414t} + k_2 e^{-0.586t}$$

At $t = 0$, $i(0) = 0$

$$k_1 + k_2 = 0 \qquad \ldots\text{(i)}$$

Also $v_L(0^+) + v_C(0^+) + v_R(0^+) = 0$

$$v_L(0^+) = -v_R(0^+) - v_C(0^+) = -2i(0^+) - v_C(0^+) = -2\text{ V} \qquad \ldots\text{(ii)}$$

$$v_L(0^+) = L\frac{di}{dt}(0^+)$$

$$\frac{di}{dt}(0^+) = \frac{v_L(0^+)}{L} = -\frac{2}{0.5} = -4\text{ A/s}$$

Differentiating the equation of $i(t)$ and putting the condition at $t = 0$,

$$-3.414\,k_1 - 0.586\,k_2 = -4 \qquad \ldots\text{(iii)}$$

Solving Eqs (i) and (iii), we get

$$k_1 = 1.414 \quad \text{and} \quad k_2 = -1.414$$

$$i(t) = 1.414(e^{-3.414t} - e^{-0.586t}) \qquad \text{for } t > 0$$

Case II *When $R = 2\ \Omega$, $L = 1\ H$, $C = 1\ F$*

$$\alpha = \frac{R}{2L} = \frac{2}{2\times 1} = \frac{2}{2} = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1}} = 1$$

$$\alpha = \omega_0$$

This indicates a critically damped case.

$$i(t) = e^{-\alpha t}(k_1 + k_2 t) = e^{-t}(k_1 + k_2 t)$$

At $t = 0$, $i(0) = 0$

$$k_1 = 0$$

Also, $$v_L(0^+) = L\frac{di}{dt}(0^+)$$

$$\frac{di}{dt}(0^+) = \frac{v_L(0^+)}{L} = -\frac{2}{1} = -2\text{ A/s}$$

Differentiating the equation of $i(t)$ and putting the condition at $t = 0$,

$$\left.\frac{di}{dt}\right|_{t=0} = -k_1 + k_2 = -2$$

$$k_2 = -2$$

$$i(t) = -2t\,e^{-t} \qquad \text{for } t > 0$$

Case III *When $R = 2\ \Omega$, $L = 5\ H$, $C = 1\ F$*

$$\alpha = \frac{R}{2L} = \frac{2}{10} = 0.2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5}} = 0.447$$

$$\alpha < \omega_0$$

This indicates an underdamped case.

$$i\ (t) = e^{-\alpha t}\,(B_1 \cos \omega_d\, t + B_2 \sin \omega_d\, t)$$

where, $$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{(0.447)^2 - (0.2)^2} = 0.4$$

$$s_{1,2} = -\alpha \pm j\omega_d = -0.2 \pm j0.4$$

$$i(t) = e^{-0.2t}\,(B_1 \cos 0.4t + B_2 \sin 0.4t)$$

Applying the initial condition,

$$i(0^+) = 0$$

and $$\frac{di}{dt}(0^+) = -\frac{v_L(0^+)}{L} = -\frac{2}{5}$$

$$B_1 = i(0) = 0$$

$$B_2 = -1$$

$$i(t) = -e^{-0.2t} \sin 0.4t \qquad \text{for } t > 0$$

Exercises

6.1 The switch in Fig. 6.182 is moved from the position a to b at $t = 0$, the network having been in steady state in the position a. Determine $i_1(0^+), i_2(0^+), i_3(0^+), \frac{di_2}{dt}(0^+)$ and $\frac{di_3}{dt}(0^+)$.

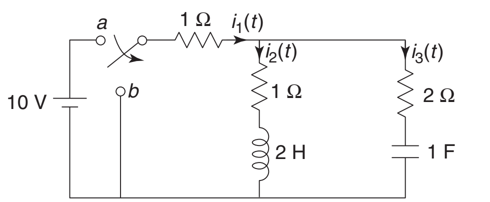


Fig. 6.182

[1.66 A, 5 A, –3.33 A, –3.33 A/s, 2.22 A/s]

6.2 The switch K is closed at $t = 0$ in the network shown in Fig. 6.183. Determine $i(0^+), \frac{di}{dt}(0^+)$ and $\frac{d^2i}{dt^2}(0^+)$.

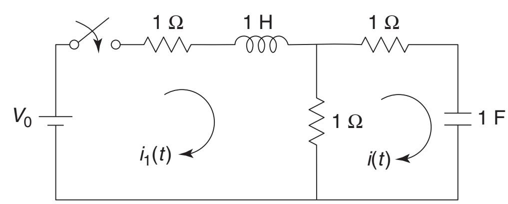


Fig. 6.183

$$\left[0, \frac{1}{2}V_0, -V_0\right]$$

6.3 In the network of Fig. 6.184, the switch K is closed at $t = 0$. At $t = 0^-$, all capacitor voltages and inductor currents are zero. Find $v_1, \dfrac{dv_1}{dt}, v_2, \dfrac{dv_2}{dt}, v_3$ and $\dfrac{dv_3}{dt}$ at $t = 0^+$.

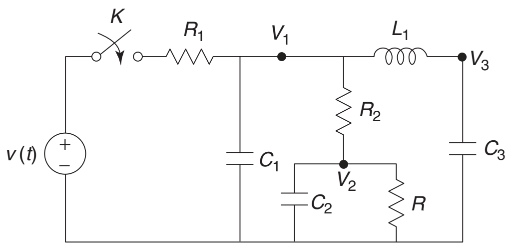


Fig. 6.184

$$\left[0, \frac{1}{C}\frac{v(0^+)}{R_1}, 0, 0, 0, 0\right]$$

6.4 In the network at Fig. 6.185, the capacitor C_1 is charged to voltage 1000 V and the switch K is closed at $t = 0$. Find $\dfrac{d^2 i_2}{dt^2}$ at $t = 0^+$.

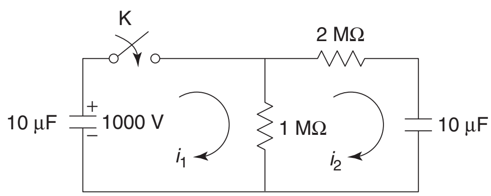


Fig. 6.185

$$\left[\frac{17}{400000}\text{A/s}^2\right]$$

6.5 In the network shown in Fig. 6.186, switch is closed at $t = 0$. Obtain the current $i_2(t)$.

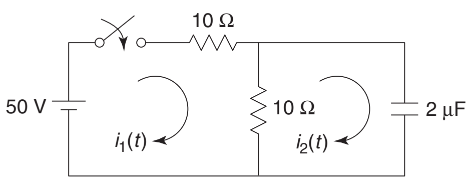


Fig. 6.186

$[i_2(t) = 5\, e^{-100000t}]$

6.6 The network shown in Fig. 6.187 is under steady-state when the switch is closed. At $t = 0$, it is opened. Obtain an expression for $i(t)$.

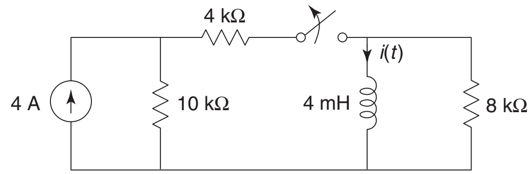


Fig. 6.187

$[i(t) = 2.857\, e^{-2\times10^6\, t}]$

6.7 The switch in Fig. 6.188 is open for a long time and closes at $t = 0$. Determine $i(t)$ for $t > 0$.

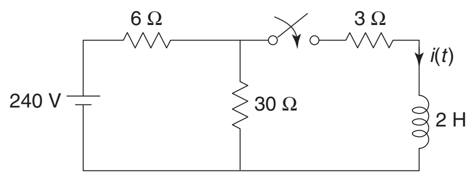


Fig. 6.188

$[i(t) = 25(1 - e^{-4t})]$

6.8 In the network shown in Fig. 6.189, the steady state is reached with the switch open. At $t = 0$, the switch is closed Find $v_C(t)$ for $t > 0$.

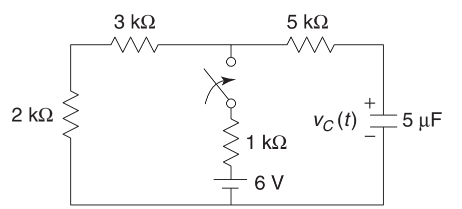


Fig. 6.189

$[v_C(t) = 5e^{-20t}]$

6.9 The circuit shown in Fig. 6.190 has acquired steady state before switching at $t = 0$.

(i) Obtain $v_C(0^+)$, $v_C(0^-)$, $i(0^+)$ and $i(0^-)$.
(ii) Obtain time constant for $t > 0$.
(iii) Find current $i(t)$ for $t > 0$.

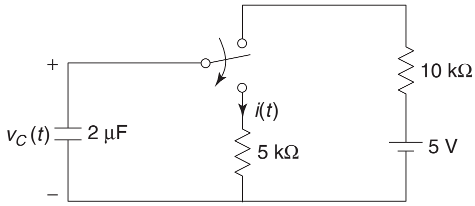


Fig. 6.190

[(i) 5 V, 5 V, 1 mA, 0, (ii) 0.01 s, (iii) $e^{-100\,t}$ mA]

6.10 In the network shown in Fig. 6.191, the switch is initially at the position 1 for a long time. At $t = 0$, the switch is changed to the position 2. Find current $i\ (t)$ for $t > 0$.

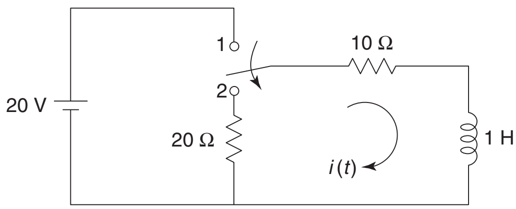


Fig. 6.191

$[i\ (t) = 2\,e^{-30t}]$

6.11 In the network shown in Fig. 6.192, the switch is closed at $t = 0$. Find $v\ (t)$ for $t > 0$.

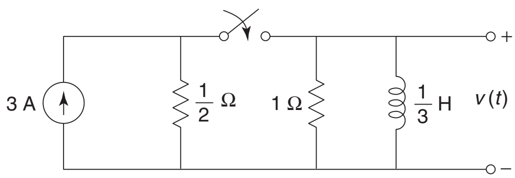


Fig. 6.192

$[v(t) = e^{-t}]$

6.12 In the network shown in Fig. 6.193, the switch is in the position 1 for a long time and at $t = 0$, the switch is moved to the position . Find $v\ (t)$ for $t > 0$.

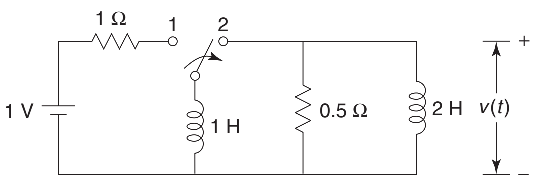


Fig. 6.193

$[v\ (t) = -0.5\ e^{-\frac{3}{4}t}]$

6.13 In Fig. 6.194, the switch is open until time $t = 100$ seconds and is closed for all times thereafter. Find $v\ (t)$ for all times greater than 100 if $v\ (100) = -3$ V.

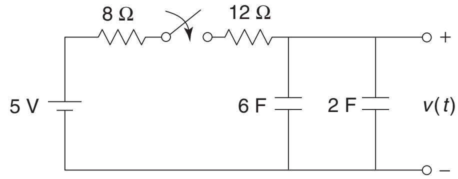


Fig. 6.194

$$\left[v(t) = 5 - 8e^{-\frac{(t-100)}{160}} \right]$$

6.14 A series *RL* circuit shown in Fig. 6.195 has a constant voltage V applied at $t = 0$. At what time does $v_R = v_L$.

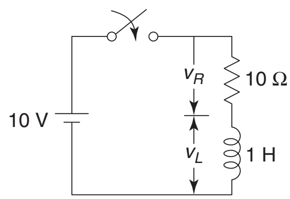


Fig. 6.195

[0.0693 s]

6.15 In the circuit shown in Fig. 6.196, at time $t = 0$, the voltage across the capacitor is zero and the switch is moved to the position y. The switch is kept at position y for 20 seconds and then moved to position z and kept in that position thereafter. Find the voltage across the capacitor at $t = 30$ seconds.

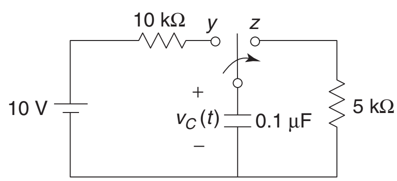


Fig. 6.196

[0]

6.16 Determine whether *RLC* series circuit shown in Fig 6.197 is underdamped, overdamped or critically damped. Also, find $v_L(0^+)$, $\dfrac{di}{dt}(0^+)$ and $i\,(\infty)$.

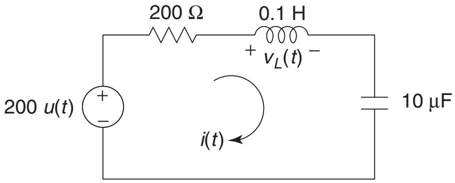


Fig. 6.197

[critically damped, 200 V, 2000 A/s, 0]

6.17 Determine whether *RLC* circuit of Fig 6.198 is underdamped, overdamped or critically damped. Also find $v_L(0^+)$, $\dfrac{di}{dt}(0^+)$, $\dfrac{d^2v}{dt^2}(0^+)$ if $v(t) = u(t)$.

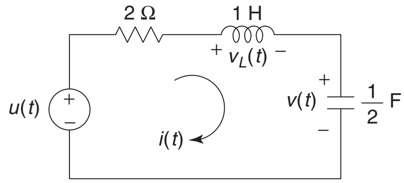


Fig. 6.198

[underdamped 1 V, 1 A/s, 2 V/s^2]

Objective-Type Questions

6.1 The voltages v_{C_1}, v_{C_2} and v_{C_3} across the capacitors in the circuit in Fig. 6.199 under steady state are respectively

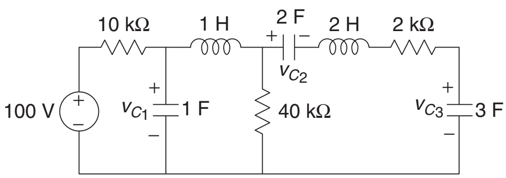


Fig. 6.199

(a) 80 V, 32 V, 48 V

(b) 80 V, 48 V, 32 V

(c) 20 V, 8 V, 12 V

(d) 20 V, 12 V, 8 V

6.2 In the circuit of Fig. 6.200, the voltage $v(t)$ is

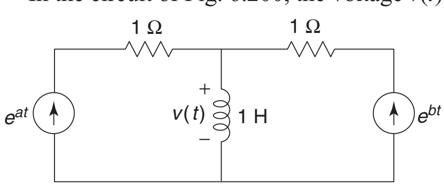


Fig. 6.200

(a) $e^{at} - e^{bt}$ (b) $e^{at} + e^{bt}$

(c) $ae^{dt} - be^{bt}$ (d) $ae^{at} + be^{bt}$

6.3 The differential equation for the current $i(t)$ in the circuit of Fig. 6.201 is

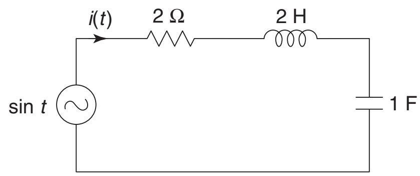


Fig. 6.201

(a) $2\dfrac{d^2i}{dt^2} + 2\dfrac{di}{dt} + i(t) = \sin t$

(b) $\dfrac{d^2i}{dt^2} + 2\dfrac{di}{dt} + 2i(t) = \cos t$

(c) $2\dfrac{d^2i}{dt^2} + 2\dfrac{di}{dt} + i(t) = \cos t$

(d) $\dfrac{d^2i}{dt^2} + 2\dfrac{di}{dt} + 2i(t) = \sin t$

6.4 At $t = 0^+$, the current i_1 in Fig. 6.202 is

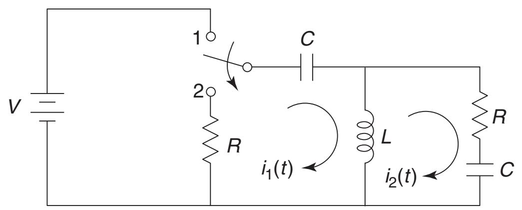


Fig. 6.202

(a) $-\dfrac{V}{2R}$ (b) $-\dfrac{V}{R}$

(c) $-\dfrac{V}{4R}$ (d) zero

6.5 For the circuit shown in Fig. 6.203, the time constant $RC = 1$ ms. The input voltage is $v_i(t) = \sqrt{2}\sin 10^3 t$. The output voltage $v_0(t)$ is equal to

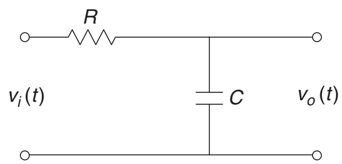


Fig. 6.203

(a) $\sin(10^3 - 45°)$ (b) $\sin(10^3 t + 45°)$

(c) $\sin(10^3 t - 53)$ (d) $\sin(10^3 t + 53°)$

6.6 For the *RL* circuit shown in Fig. 6.204, the input voltage $v_i(t) = u(t)$. The current $i(t)$ is

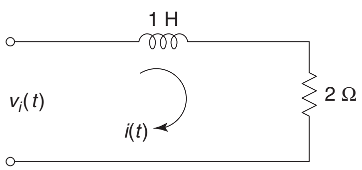
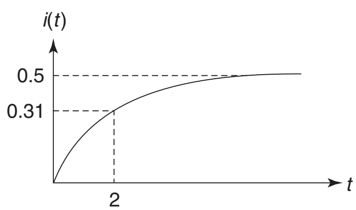
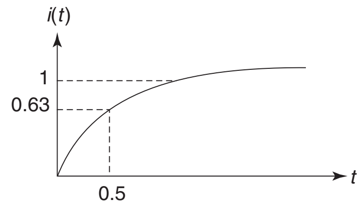


Fig. 6.204

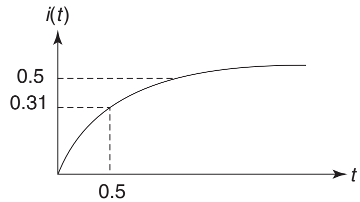
(a)



(b)



(c)



(d)

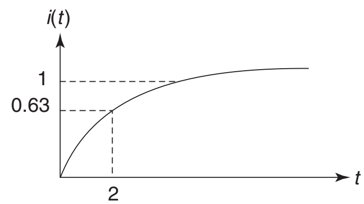


Fig. 6.205

6.7 The condition on R, L and C such that the step response $v(t)$ in Fig. 6.206 has no oscillations is

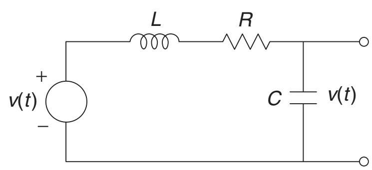


Fig. 6.206

(a) $R \geq \dfrac{1}{2}\sqrt{\dfrac{L}{C}}$ (b) $R \geq \sqrt{\dfrac{L}{C}}$

(c) $R \geq 2\sqrt{\dfrac{L}{C}}$ (d) $R = \dfrac{1}{\sqrt{LC}}$

6.8 The switch S in Fig. 6.207 closed at $t = 0$. If $v_2(0) = 10$ V and $v_g(0) = 0$ respectively, the voltages across capacitors in steady state will be

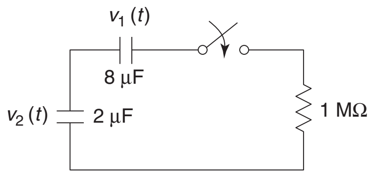


Fig. 6.207

(a) $v_2(\infty) = v_1(\infty) = 0$
(b) $v_2(\infty) = 2$ V, $v_1(\infty) = 8$ V
(c) $v_2(\infty) = v_1(\infty) = 8$ V
(d) $v_2(\infty) = 8$ V, $v_1(\infty) = 2$ V

6.9 The time constant of the network shown in Fig. 6.208 is

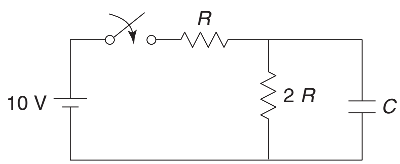


Fig. 6.208

(a) 2 RC (b) 3 RC
(c) $\frac{1}{2}RC$ (d) $\frac{2}{3}RC$

6.10 In the series RC circuit shown in Fig. 6.209, the voltage across C starts increasing when the dc source is switched on. The rate of increase of voltage across C at the instant just after the switch is closed i.e., at $t = 0^+$ will be

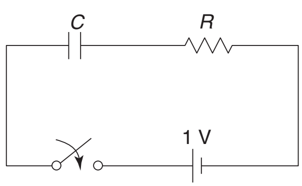


Fig. 6.209

(a) zero (b) infinity
(c) RC (d) $\frac{1}{RC}$

6.11 The $v - i$ characteristic as seen from the terminal pair ($A - B$) of the network of Fig. 6.210(a) is shown in Fig. 6.210(b). If an inductance of value 6 mH is connected across the terminal pair, the time constant of the system will be

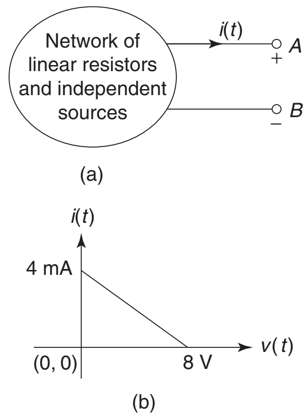


Fig. 6.210

(a) 3 μs
(b) 12 s
(c) 32 s
(d) unknown, unless actual network is specified

6.12 In the network shown in Fig. 6.211, the circuit was initially in the steady-state condition with the switch K closed. At the instant when the switch is opened, the rate of decay of current through inductance will be

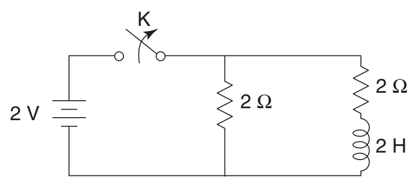


Fig. 6.211

(a) zero (b) 0.5 A/s
(c) 1 A/s (d) 2 A/s

6.13 A step function voltage is applied to an RLC series circuit having $R = 2\ \Omega$, $L = 1$ H and $C = 1$ F. The transient current response of the circuit would be

(a) over damped (b) critically damped
(c) under damped (d) none of these

Answers to Objective-Type Questions

6.1 (b)	6.2 (d)	6.3 (c)	6.4 (d)	6.5 (a)	6.6 (b)	6.7 (c)
6.8 (d)	6.9 (d)	6.10 (d)	6.11 (a)	6.12 (d)	6.13 (b)	