Graph Theory

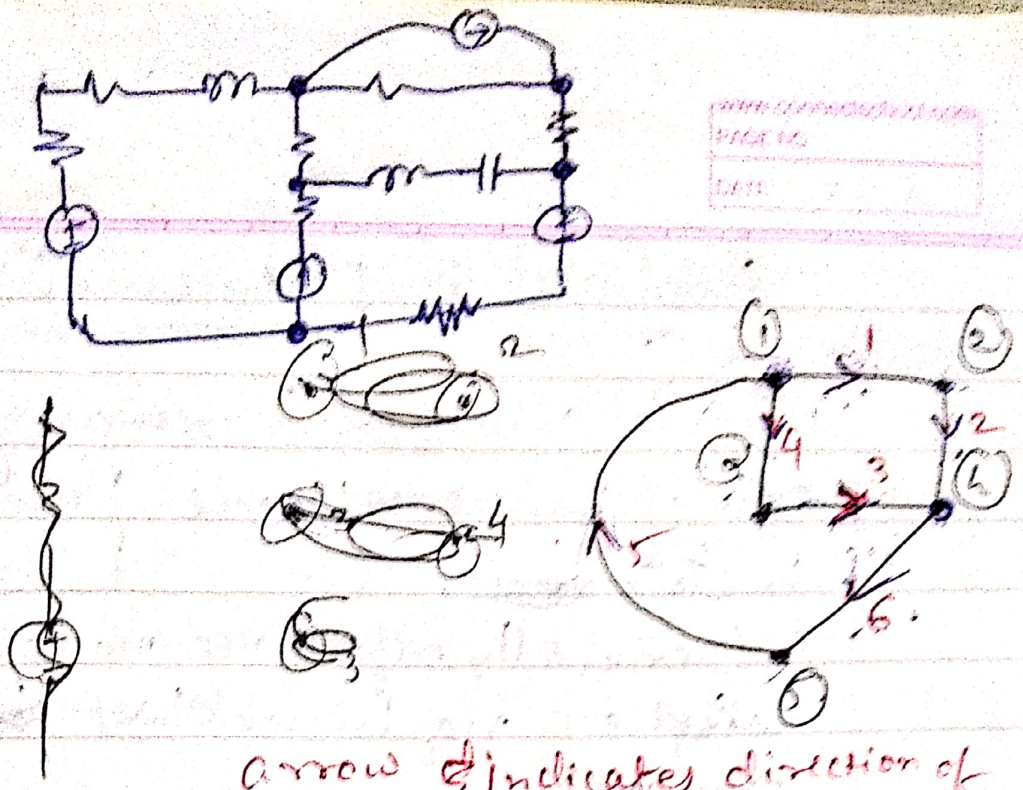
1) Mark min. no. of principle nodes ie. the junctions from where current can split, divide or combine.

2) Give highest no. to reference node.

3) Draw lines passing through R, L, C & vlg source.

4) • Open ckt cfn source.
• prefer straight lines. If not possible then only curve.
• Crossing of lines or jumpers are not allowed.
• Do not make any extra corners in the graph other wise they will be considered as principle nodes in further analysis.

5) Give random no.s & direc^{ns} to all the branches. Graph obtained is known as oriented graph.



arrow indicates direction of assumed branch current.

Complete incidence matrix.

1 : If branch direc. goes away from the node.

-1 : If branch direc goes towards the nodes.

0 : If branch is not connected to the node.

$$A_a = \begin{array}{c} \text{nodes} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \text{Branches} \\ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}_{5\times 6} \end{array}$$

Addition of elements of every single individual column = 0.

Every column has single element 1 & single element -1.; all other elements are zero.

Reduced incidence matrix

It is obtained by removing row of reference node from above matrix ~~node~~.

- Generally reference node has highest no. hence last ~~row~~ is removed.

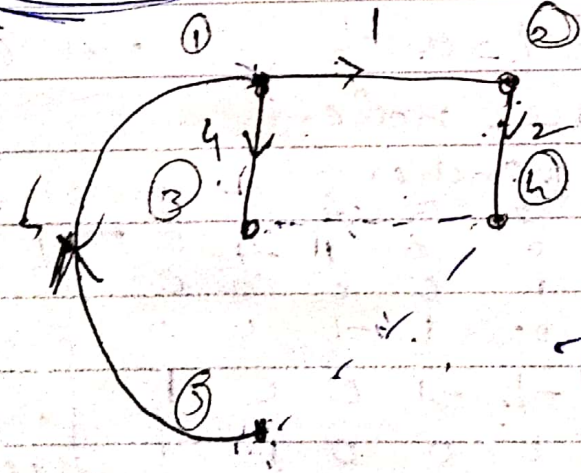
$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Tree

No. of nodes = 1

$= 5 - 1$

$= 4$



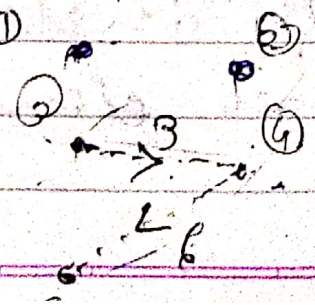
$A \cdot A^T$

Total No. of Tree

Twigs : 1, 2, 4, 5

Links / chords : 3, 6

Co-tree :



Tree	Co-tree
① It can not have close loop	① May have closed
② Its branches are known as twigs	② Links / chords.
③ No. of twigs = total nodes - 1	③ No. of links = total branches - No. of twigs.
④ There is a unique path to go from any one node to remaining nodes	④ There may be 0 or 1 or multiple path to go from any one node to remaining nodes.
⑤ It is directly obtained from graph.	⑤ It can be obtained only after making tree.
⑥ It is drawn using solid line	⑥ Dotted line
⑦ It can not have isolated or hanging node	⑦ May have
⑧ As an example show tree & co-tree for any 1 graph. Home work --	⑧ →

TL

A Fundamental Tieset matrix

TI is also known as loop matrix & ckt matrix.

Connect only one link at a time to tree, so that one closed loop is formed.

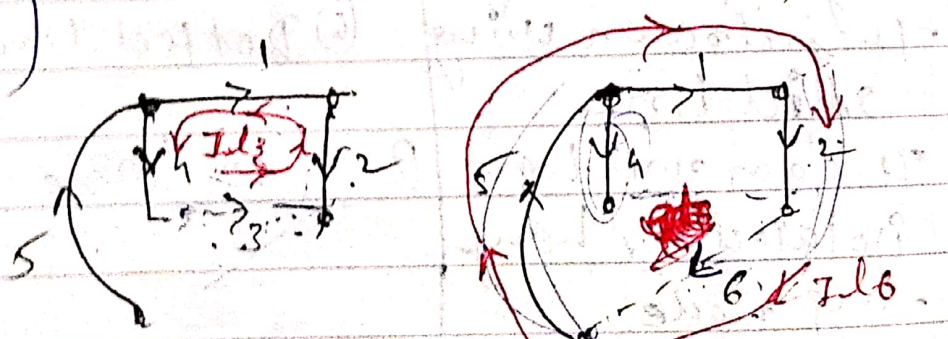
Direction of c/n is considered same as that of link, hence known as link c/n.

1 : match
-1 : opp.
0 : not concerned.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3×3

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



Branches

Links

$$B = \begin{matrix} 3 \\ 6 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ -1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Links are 3 & 6 ∴ column no. 3 & 6 forms unit matrix.

3x3

Fundamental Cutset matrix

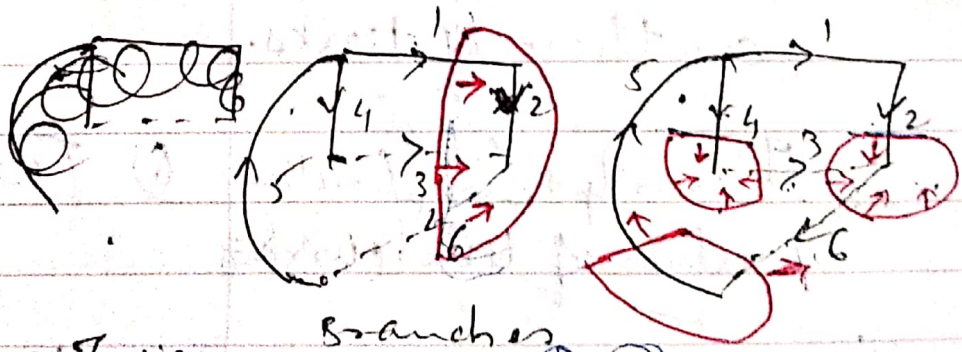
Cut passes through only one twig & at a time & remaining possible links.

Cut is a closed fig we should be able to close the cut from inside or outside the graph.

Cut cannot pass through same branch twice.

Direction of cut is taken same as that of twig.

1 : match
-1 : opp.
0 : X not concerned.



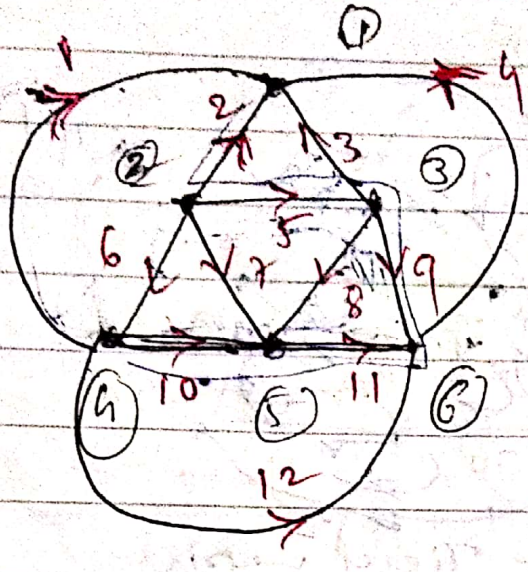
Branches

$$Q = \begin{array}{c} \text{Twigs} \\ 1 \\ 2 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

twigs are 1, 2, 4, 5 & their column no. 1, 2, 4 & 5 forms unit matrix.

Fundamental Tieset matrix	Fundamental cutset matrix
1) Its rows are links.	1) Twigs.
2) Only one link is considered at a time.	2) Twig
3) Direc^n is taken same as that of link	3) Twig
4) Matrix element value is 1 if direc^n of branch matches direc^n of link.	4) Twig
5) Unit matrix is formed by columns whose no. is same as those of links.	5) Twigs
6) It helps in finding values of link currents.	6) Twig V/gs.
7) It is required in KVL	7) KCL
8) As an example show Tieset & cutset matrices for any one tree for any graph as demonstration. H.W →	8)

(18)



Branches

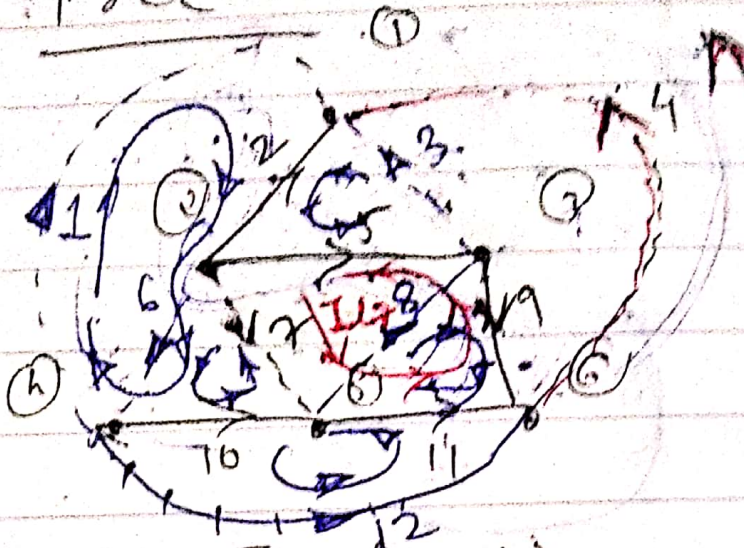
Nodes	1	2	3	4	5	6	7	8	9	10	11	12
A_a = 1	+1	-1	-1	-1	0	0	0	0	0	0	0	0
2	0	1	0	0	1	1	1	0	0	0	0	0
3	0	0	1	0	-1	0	0	1	1	0	0	0
4	1	0	0	0	0	-1	0	0	0	1	0	1
5	0	0	0	0	0	0	-1	-1	0	-1	1	0
6	0	0	0	1	0	0	0	0	-1	0	-1	-1

Flow

A = 1
2
3
4
5

6 → eliminate

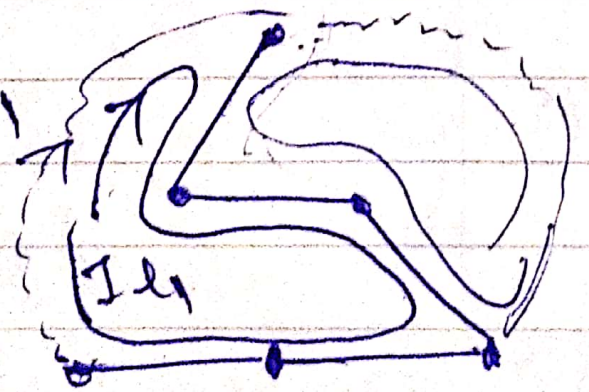
Tree



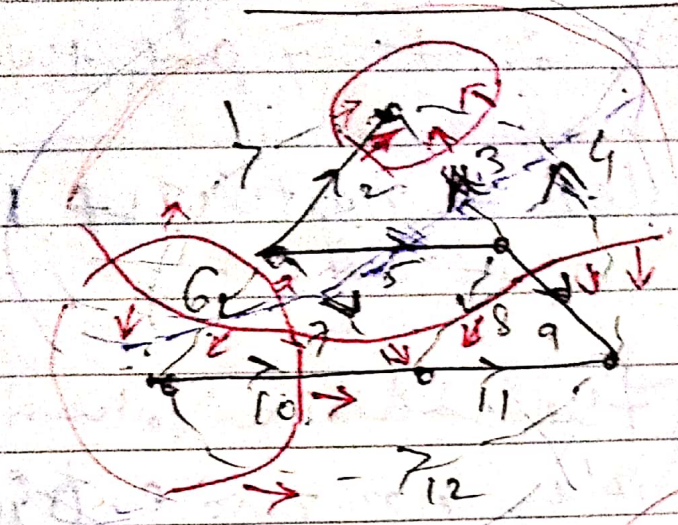
Fundamental Tieset matrix.

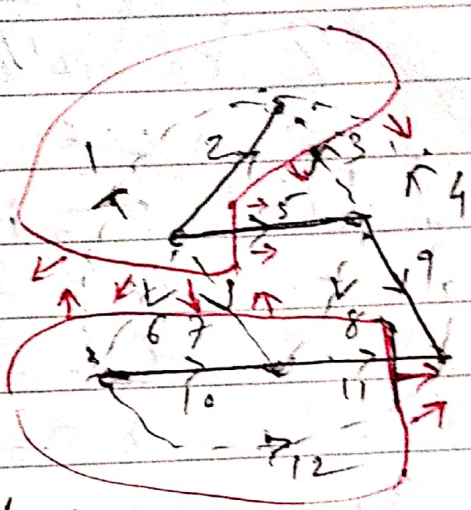
Links	1	2	3	4	5	6	7	8	9	10	11	12
B = 1	1	-1	0	0	0	0	0	0	-1	0	1	0
3	0	-1	1	0	1	0	0	0	0	0	0	0
4	0	-1	0	1	1	0	0	0	1	0	0	0
6	0	0	0	0	-1	1	0	0	-1	1	1	0
7	0	0	0	0	-1	0	1	0	-1	1	1	0
8	0	0	0	0	0	0	0	1	-1	0	1	0
12	0	1	0	0	0	0	0	0	-1	-1	1	1

Tieset



Fundamental cutset matrix





Branches

Twigs

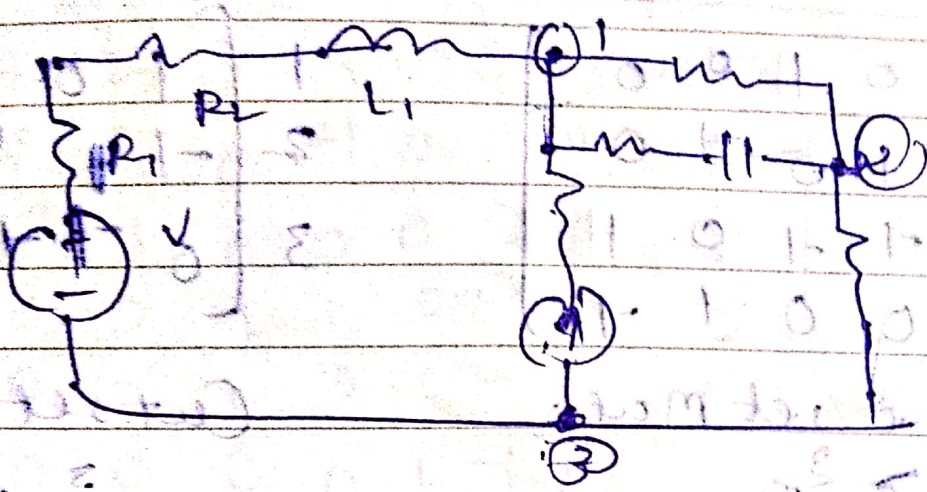
$$Q = \begin{array}{c} \\ 2 \\ 5 \\ 9 \\ 10 \\ 11 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & \\ 1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

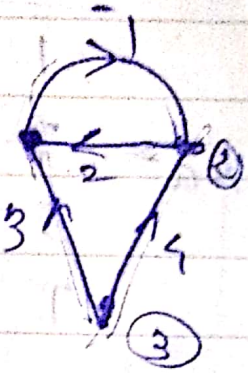
Ex: Obtain the graph for following F. cutset matrix,

Twigs

$$Q = \begin{array}{c} \\ 1 \\ 3 \\ 5 \\ 7 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(10) tie set & cut set





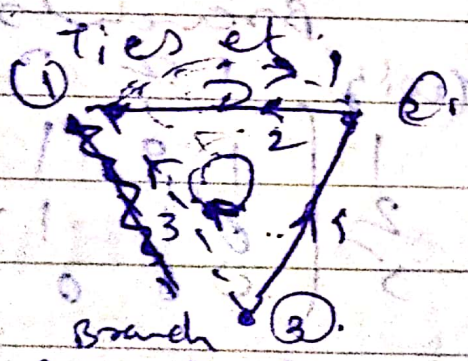
Complete Incidence Matrix

Branches

Nodes	1	2	3	4
1	1	-1	-1	0
2	-1	1	0	-1
3	0	0	1	1

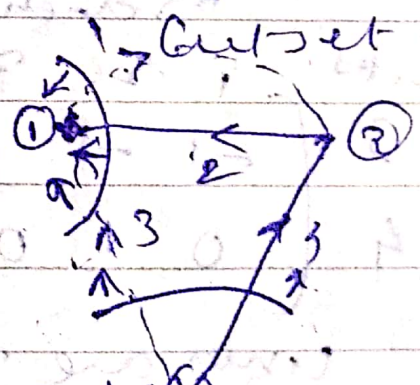
Reduced incidence mat

$$A = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & -1 & -1 & 0 \\ -1 & 1 & 0 & -1 \end{pmatrix} \end{array}$$



Links 1 2 3 4

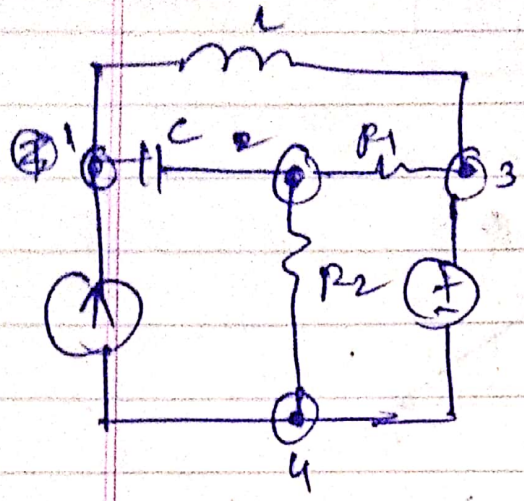
$$B = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{pmatrix} \end{array}$$

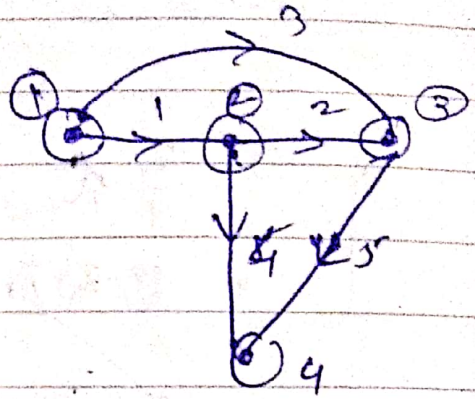


Trigs. Branches 1 2 3 4

$$Q = \begin{array}{c} \\ 2 \\ 4 \end{array} \begin{array}{c} \\ \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

11) · Tieset & cutset.





Incidence matrix

Branches

Nodes 1 2 3 4 5

$$A_a = \begin{matrix}1\\2\\3\\4\end{matrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

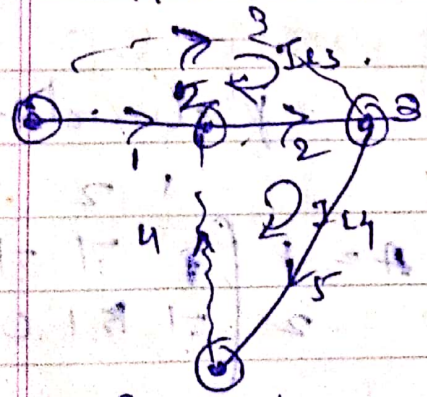
reduced matrix

Branches

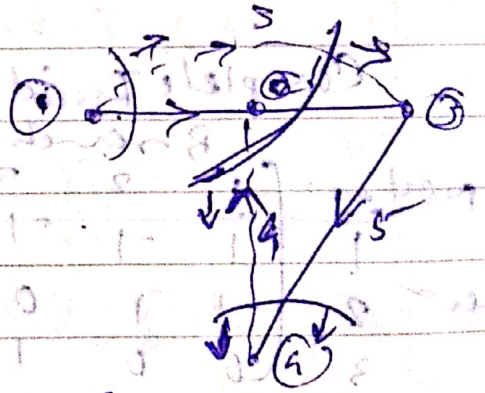
Nodes 1 2 3 4 5

$$\begin{matrix}1\\2\\3\end{matrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{pmatrix}$$

Tie set Matrix



Cut set matrix



Branches

Links 1 2 3 4 5

$$\begin{matrix}3\\4\end{matrix}\begin{pmatrix} -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Links are 3 & 4

Branches 3, 4 form

Unit matrix

Branches

Twigs 1 2 3 4 5

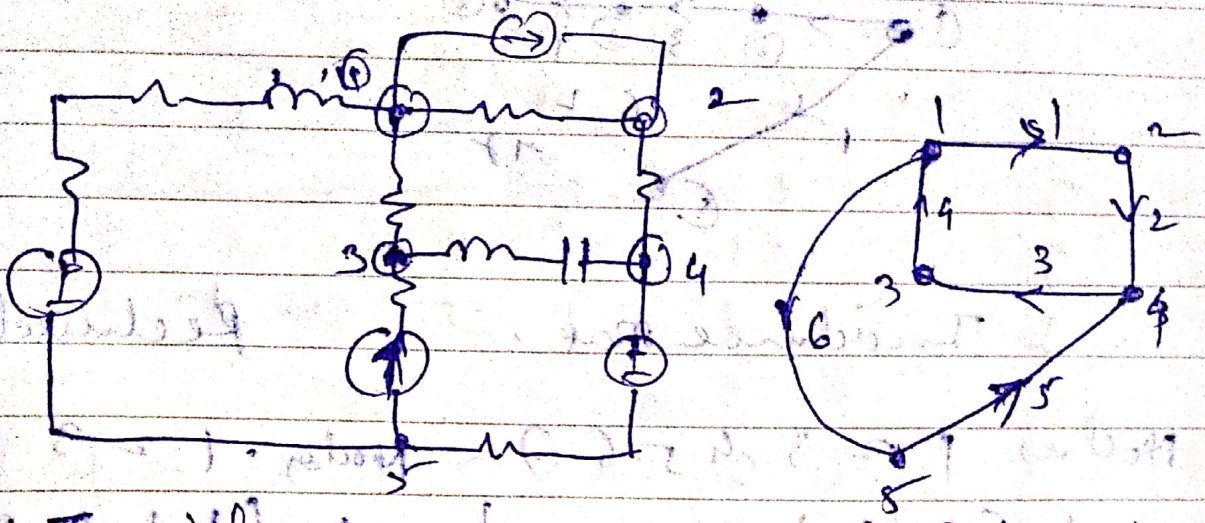
$$\begin{matrix}1\\2\\5\end{matrix}\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

Twigs are 1, 2, 5

Branches 1, 2, 5

form unit matrix

(12) Tieset & cutset incl.



Incidence matrix

Branches

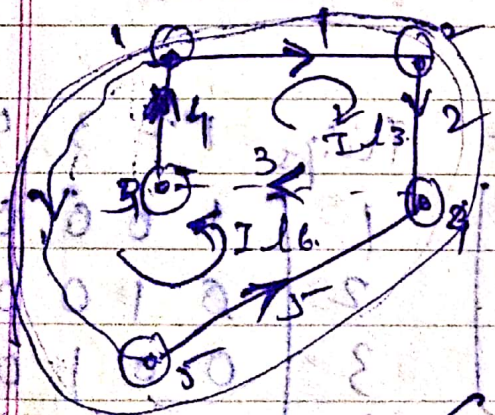
Nodes	1	2	3	4	5	6
1	1	0	0	-1	0	1
2	-1	1	0	0	0	0
3	0	0	-1	1	0	0
4	0	-1	1	0	-1	0
5	0	0	0	0	1	-1

Reduced matrix

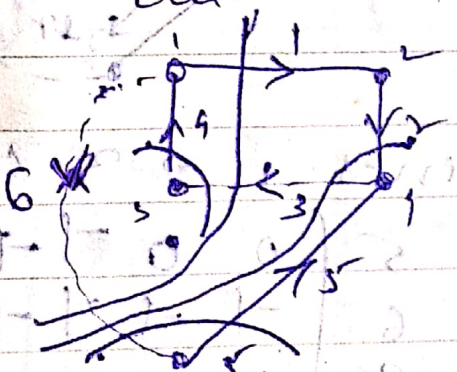
Branches

Nodes	1	2	3	4	5	6
1	1	0	0	-1	0	1
2	-1	1	0	0	0	0
3	0	0	-1	1	0	0
4	0	-1	1	0	-1	0

Tieset.



Cutset.



Links 1 2 3 4 5 6.

	1	2	3	4	5	6
3	1	1	1	1	0	0
6	0	0	1	0	1	1

wrong check it.

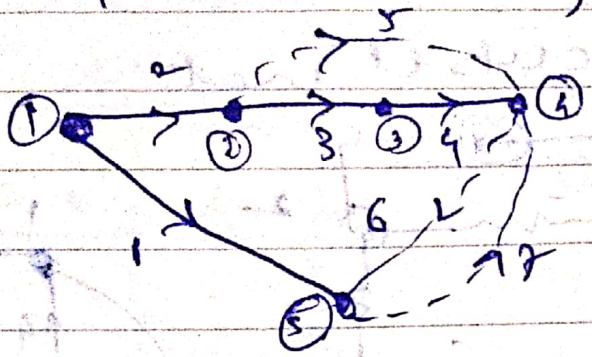
Same as

Twigs 1 2 3 4 5 6.

	1	2	3	4	5	6
1	1	0	-1	0	0	-1
2	0	1	-1	0	0	-1
4	0	0	-1	1	0	0
5	0	0	0	0	1	1

Same as

15). Find incidence, Tie-set & cut-set matrix



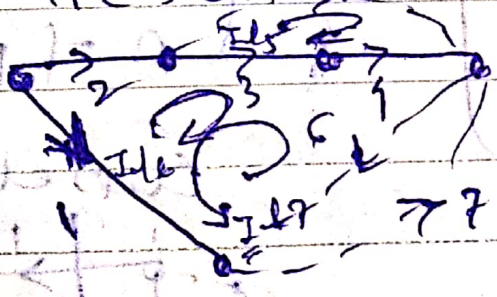
Incidence Mat.

Nodes	1	2	3	4	5	6	7
1	1	1	0	0	0	0	0
2	0	-1	1	0	1	0	0
3	0	0	-1	1	0	0	0
4	0	0	0	-1	-1	1	-1
5	-1	0	0	0	0	-1	1

Reduced matrix

Nodes	1	2	3	4	5	6	7
1	1	1	0	0	0	0	0
2	0	-1	1	0	1	0	0
3	0	0	-1	1	0	0	0
4	0	0	0	-1	-1	1	-1

Tie set :-



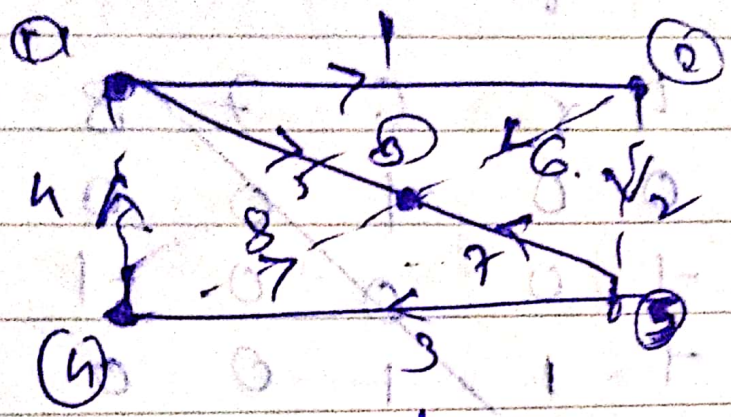
Cut set

Links	1	2	3	4	5	6	7
5	0	0	-1	-1	1	0	0
6	-1	-1	-1	-1	0	1	0
7	1	-1	0	0	0	0	1

Twigs

	1	2	3	4	5	6	7
1	1	0	0	0	0	1	-1
2	0	1	0	0	0	-1	1
3	0	0	1	0	1	-1	1
4	0	0	0	1	-1	1	-1

(18) Find incidence, tieset, cutset-



Incidence

Nodes: 1 2 3 4 5 6 7 8

1	-1	0	0	-1	1	0	0	0
2	-1	1	0	0	0	1	0	0
3	0	0	0	0	-1	-1	-1	-1
4	0	0	-1	1	0	0	0	1
5	0	-1	1	0	0	0	1	0

Tieset