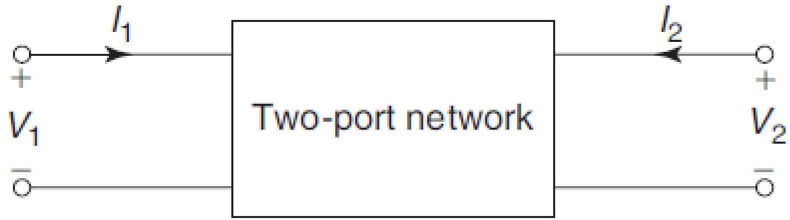
Two port Networks

Two-Port Networks

A two-port network has two pairs of terminals, one pair at the input known as *input port* and one pair at the output known as *output port* as shown in Fig. 9.1. There are four variables V_1, V_2, I_1 and I_2 associated with a two-port network. Two of these variables can be expressed in terms of the other two variables. Thus, there will be two dependent variables and two independent variables. The number of possible combinations generated by four variables taken two at a time is ${}^4C_2$, i.e., six. There are six possible sets of equations describing a two-port network.



Two–port parameters

Parameter	***Variables***		***Equation***
	Express	***In terms of***	
Open-Circuit Impedance	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
Short-Circuit Admittance	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
Transmission	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Inverse Transmission	V_2, I_2	V_1, I_1	$V_2 = A' V_1 - B' I_1$ $I_2 = C' V_1 - D' I_1$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
Inverse Hybrid	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

OPEN-CIRCUIT IMPEDANCE PARAMETERS (Z PARAMETERS)

The Z parameters of a two-port network may be defined by expressing two-port voltages V_1 and V_2 in terms of two-port currents I_1 and I_2.

$$(V_1, V_2) = f(I_1, I_2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero.

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

where Z_{11} is the driving-point impedance with the output port open-circuited. It is also called *open-circuit input impedance.*

Similarly,

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

where Z_{21} is the transfer impedance with the output port open-circuited. It is also called *open-circuit forward transfer impedance*.

Case 2 When input port is open-circuited, i.e., $I_1 = 0$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

where Z_{12} is the transfer impedance with the input port open-circuited. It is also called *open-circuit reverse transfer impedance*.

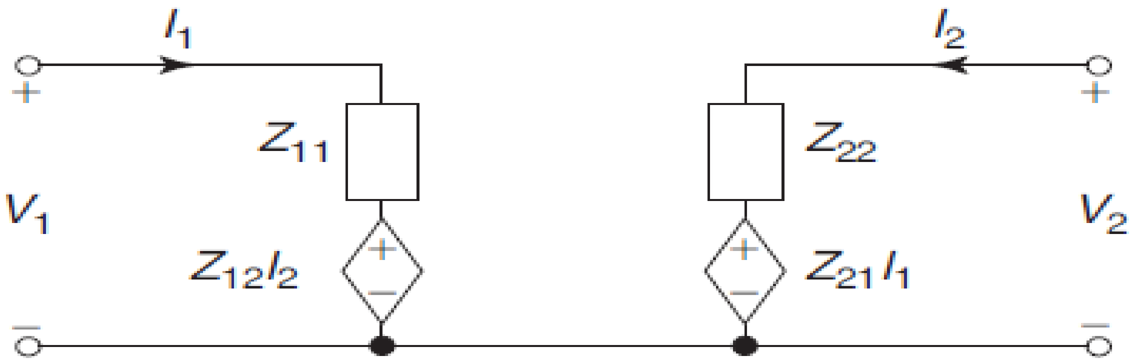
Similarly,

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

where Z_{22} is the open-circuit driving-point impedance with the input port open-circuited. It is also called *open circuit output impedance*.

As these impedance parameters are measured with either the input or output port open-circuited, these are called *open-circuit impedance parameters*.

The equivalent circuit of the two-port network in terms of Z parameters is shown in Fig.

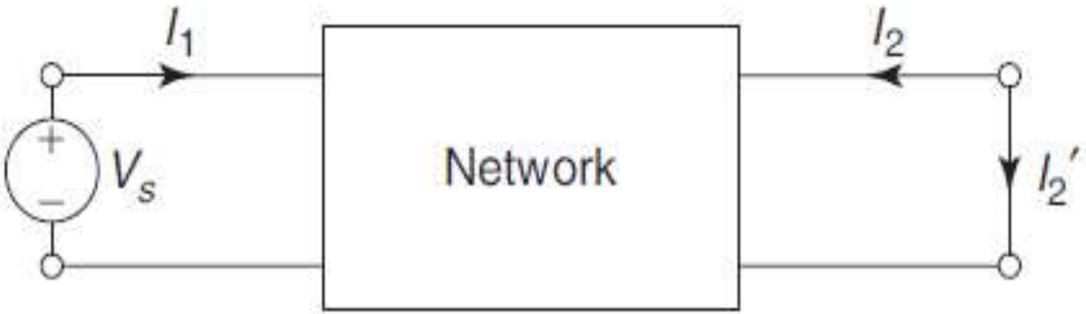


Equivalent circuit of the two-port network in terms of Z parameter

Condition for Reciprocity

A network is said to be reciprocal if the ratio of excitation at one port to response at the other port is same if excitation and response are interchanged.

(a) As shown in Fig. 9.3, voltage V_s is applied at the input port with the output port short-circuited.



i.e.,

$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2 = -I_2'$$

From the Z-parameter equations,

$$V_s = Z_{11} I_1 - Z_{12} I_2'$$
$$0 = Z_{21} I_1 - Z_{22} I_2'$$
$$I_1 = \frac{Z_{22}}{Z_{21}} I_2'$$
$$V_s = Z_{11} \frac{Z_{22}}{Z_{21}} I_2' - Z_{12} I_2'$$
$$\frac{V_s}{I_2'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

(b) As shown in Fig. 9.4, voltage V_s is applied at the output port with input port short-circuited.

i.e.,
$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1 = -I_1'$$

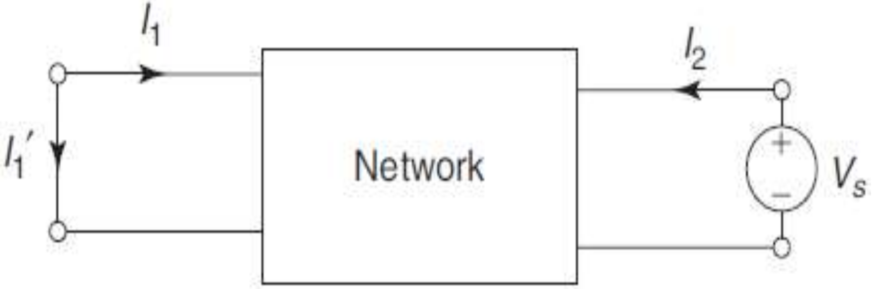


Fig. 9.4 *Network for deriving condition for reciprocity*

From the Z-parameter equations,
$$0 = -Z_{11}\, I_1' + Z_{12}\, I_2$$
$$V_s = -Z_{21}\, I_1' + Z_{22}\, I_2$$
$$I_2 = \frac{Z_{11}}{Z_{12}}\, I_1'$$
$$V_s = -Z_{21} I_1' + Z_{22}\,\frac{Z_{11}}{Z_{12}}\, I_1'$$
$$\frac{V_s}{I_1'} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{12}}$$

Hence, for the network to be reciprocal,

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

i.e.,

$$Z_{12} = Z_{21}$$

Condition for Symmetry

For a network to be symmetrical, the voltage-to-current ratio at one port should be the same as the voltage-to-current ratio at the other port with one of the ports open-circuited.

(a) When the output port is open-circuited, i.e., $I_2 = 0$

From the *Z*-parameter equation,

$$V_s = Z_{11} I_1$$

$$\frac{V_s}{I_1} = Z_{11}$$

(b) When the input port is open-circuited, i.e., $I_1 = 0$

From the *Z*-parameter equation,

$$V_s = Z_{22} I_2$$

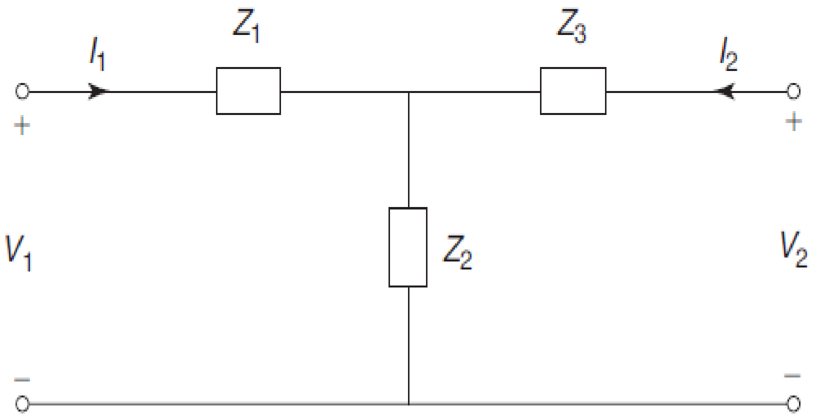
$$\frac{V_s}{I_2} = Z_{22}$$

Hence, for the network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$Z_{11} = Z_{22}$$

Find the Z parameters for the network shown in Fig.



Solution

First Method

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$.

Applying KVL to Mesh 1,

$$V_1 = (Z_1 + Z_2) I_1$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1 + Z_2$$

Also

$$V_2 = Z_2 I_1$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_2$$

Case 2 When the input port is open-circuited, i.e., $I_1 = 0$.
Applying KVL to Mesh 2,

$$V_2 = (Z_2 + Z_3)\, I_2$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3$$

Also

$$V_1 = Z_2\, I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_2$$

Hence, the Z-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

Second Method

The network is redrawn as shown in Fig. 9.6.

Applying KVL to Mesh 1,

$$V_1 = Z_1 I_1 + Z_2(I_1 + I_2)$$

$$= (Z_1 + Z_2)I_1 + Z_2 I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = Z_3 I_2 + Z_2(I_1 + I_2)$$

$$= Z_2 I_1 + (Z_2 + Z_3)I_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with *Z*-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

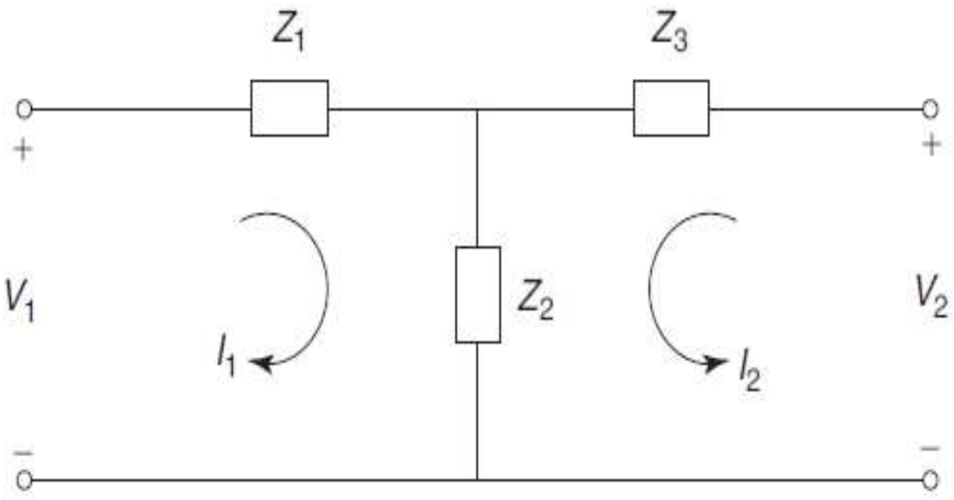
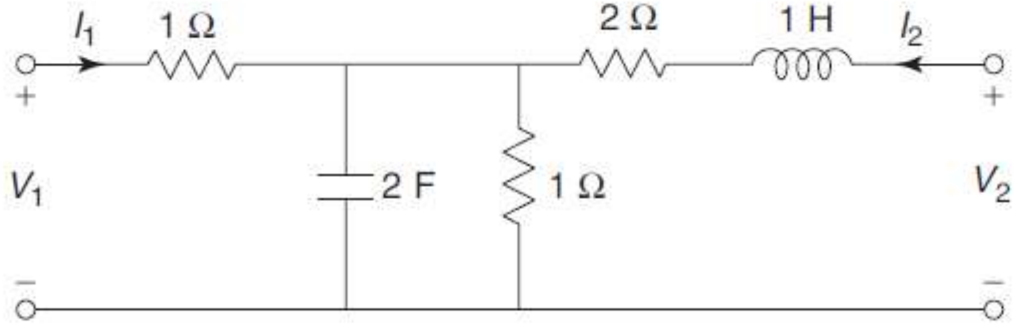
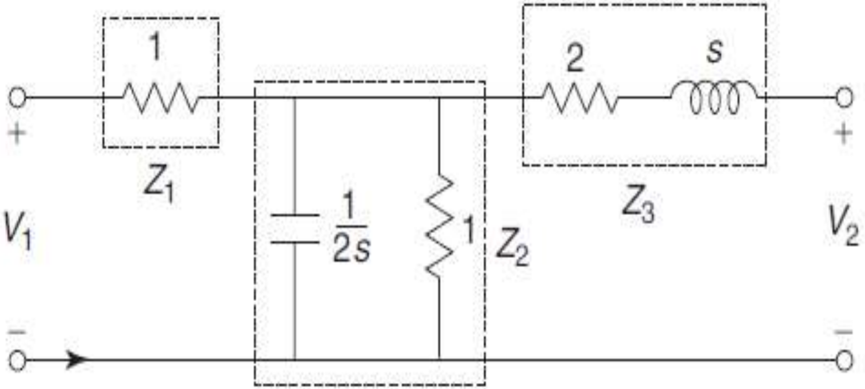


Fig. 9.6

Find Z-parameter for the network shown in Fig.



Solution The transformed network is shown in Fig. 9.8.



$$Z_1 = 1$$

$$Z_2 = \frac{\left(\frac{1}{2s}\right)(1)}{\frac{1}{2s}+1} = \frac{1}{2s+1}$$

$$Z_3 = s+2$$

From definition of Z-parameters,

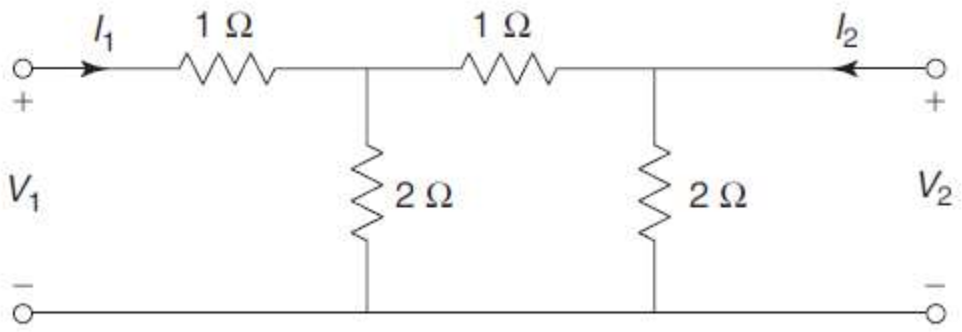
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1 + Z_2 = 1 + \frac{1}{2s+1} = \frac{2s+2}{2s+1},$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_2 = \frac{1}{2s+1},$$

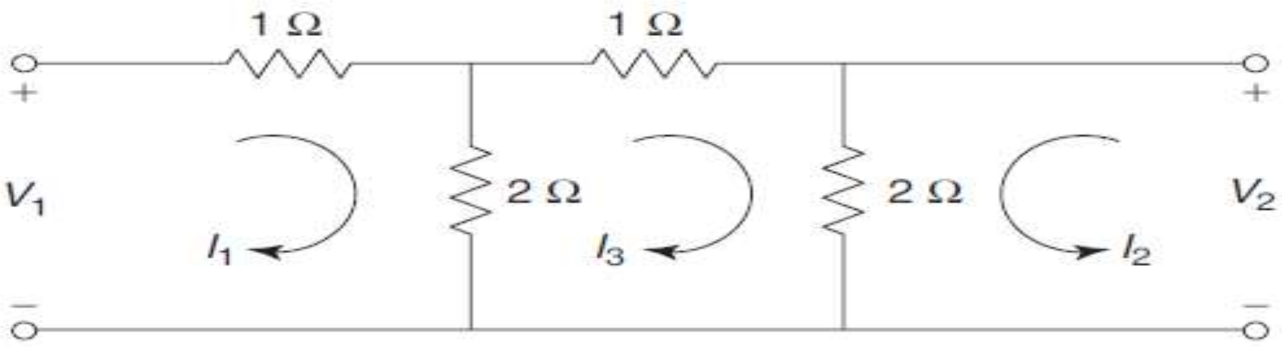
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_2 = \frac{1}{2s+1}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3 = \frac{1}{2s+1} + s + 2 = \frac{2s^2+5s+3}{2s+1}$$

Find Z-parameters for the network shown in Fig.



Solution The network is redrawn as shown in Fig.



Solution The network is redrawn as shown in

Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \qquad \dots(i)$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2I_3 \qquad \dots(ii)$$

Applying KVL to Mesh 3,

$$-2I_1 + 2I_2 + 5I_3 = 0$$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \qquad \dots(iii)$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2$$

$$= \frac{11}{5}I_1 + \frac{4}{5}I_2$$

Substituting Eq. (iii) in Eq. (ii),

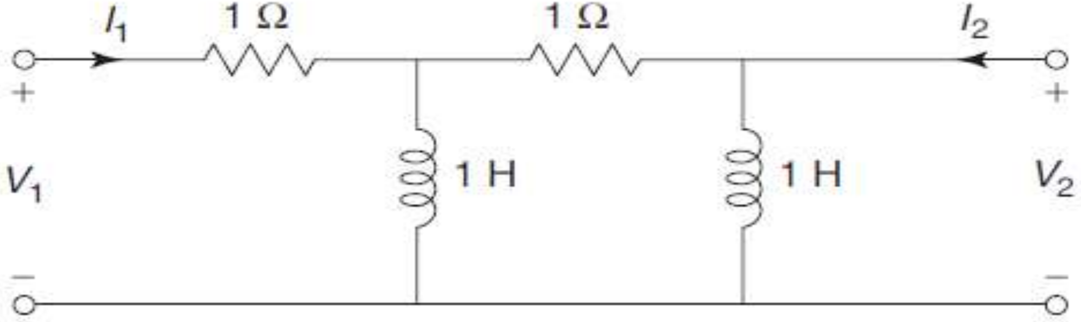
$$V_2 = 2I_2 + \frac{4}{5}I_1 - \frac{4}{5}I_2$$

$$= \frac{4}{5}I_1 + \frac{6}{5}I_2$$

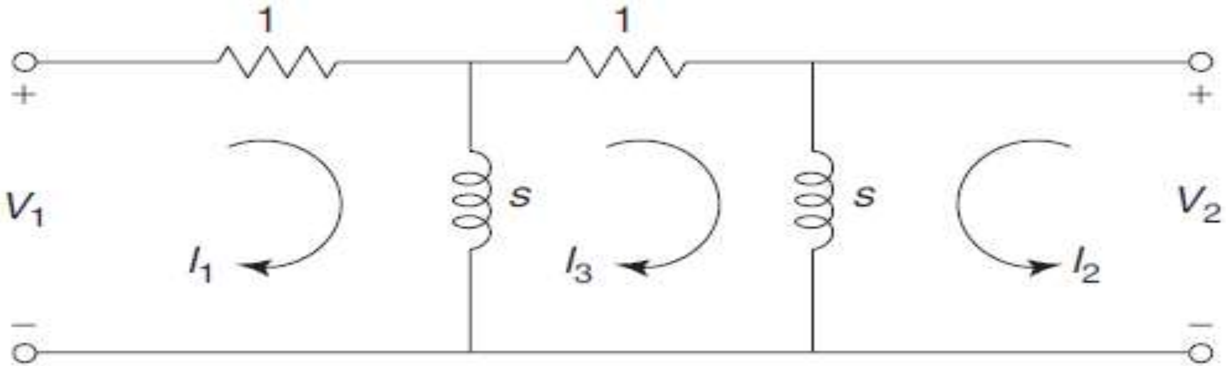
Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{6}{5} \end{bmatrix}$$

Find the Z-parameters for the network shown in Fig. 9.11.



Solution The transformed network is shown in Fig.



Applying KVL to Mesh 1,

$$V_1 = (s+1)I_1 - sI_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = sI_2 + sI_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-sI_1 + sI_2 + (2s+1)I_3 = 0$$

$$I_3 = \frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2 \qquad \cdots \text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = (s+1)I_1 - s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$= \left(\frac{s^2+3s+1}{2s+1}\right)I_1 + \left(\frac{s^2}{2s+1}\right)I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = sI_2 + s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$= \left(\frac{s^2}{2s+1}\right)I_1 + \left(\frac{s^2+s}{2s+1}\right)I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{2s+1} & \dfrac{s^2}{2s+1} \\ \dfrac{s^2}{2s+1} & \dfrac{s^2+s}{2s+1} \end{bmatrix}$$

SHORT-CIRCUIT ADMITTANCE PARAMETERS (Y PARAMETERS)

The Y parameters of a two-port network may be defined by expressing the two-port currents I_1 and I_2 in terms of the two-port voltages V_1 and V_2.

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

In matrix form, we can write

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual Y parameters for a given network can be defined by setting each of the port voltages equal to zero.

Case 1 When the output port is short-circuited, i.e., $V_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

where Y_{11} is the driving-point admittance with the output port short-circuited. It is also called *short-circuit input admittance.*

Similarly,

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

where Y_{21} is the transfer admittance with the output port short-circuited. It is also called *short-circuit forward transfer admittance.*

Case 2 When the input port is short-circuited, i.e., $V_1 = 0$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

where Y_{12} is the transfer admittance with the input port short-circuited. It is also called *short-circuit reverse transfer admittance.*

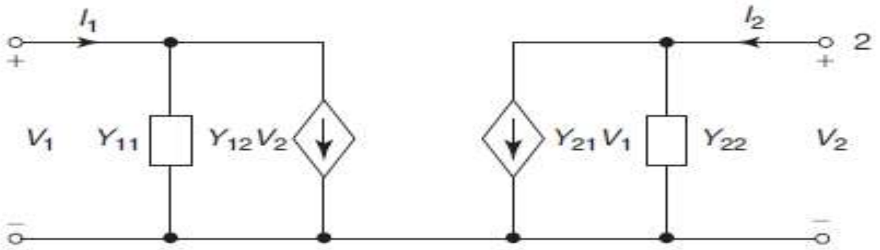
Similarly,

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

where Y_{22} is the short-circuit driving-point admittance with the input port short-circuited. It is also called the *short circuit output admittance.*

As these admittance parameters are measured with either input or output port short-circuited, these are called *short-circuit admittance parameters.*

The equivalent circuit of the two-port network in terms of Y parameters is shown in Fig. 9.15.



Condition for Reciprocity

(a) As shown in Fig. 9.16, voltage V_s is applied at input port with the output port short-circuited.

i.e,

$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2 = -I_2'$$

From the Y-parameter equation,

$$-I_2' = Y_{21} V_s$$
$$\frac{I_2}{V_s} = -Y_{21}$$

Fig. 9.16 *Network for deriving condition for reciprocity*

(b) As shown in Fig. 9.17, voltage V_s is applied at output port with the input port short-circuited.

i.e,

$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1 = -I_1'$$

From the Y-parameter equation,

$$-I_1' = Y_{12} V_s$$
$$\frac{I_1'}{V_s} = -Y_{12}$$

Fig. 9.17 *Network for deriving condition for reciprocity*

Hence, for the network to be reciprocal,

$$\frac{I_2'}{V_s} = \frac{I_1'}{V_s}$$

i.e,

$$Y_{12} = Y_{21}$$

Condition for Symmetry

(a) When the output port is short-circuited, i.e., $V_2 = 0$.
From the Y-parameter equation,

$$I_1 = Y_{11} V_s$$

$$\frac{V_s}{I_1} = \frac{1}{Y_{11}}$$

(b) When the input port is short-circuited, i.e., $V_1 = 0$.
From the Y-parameter equation,

$$I_2 = Y_{22} V_s$$

$$\frac{V_s}{I_2} = \frac{1}{Y_{22}}$$

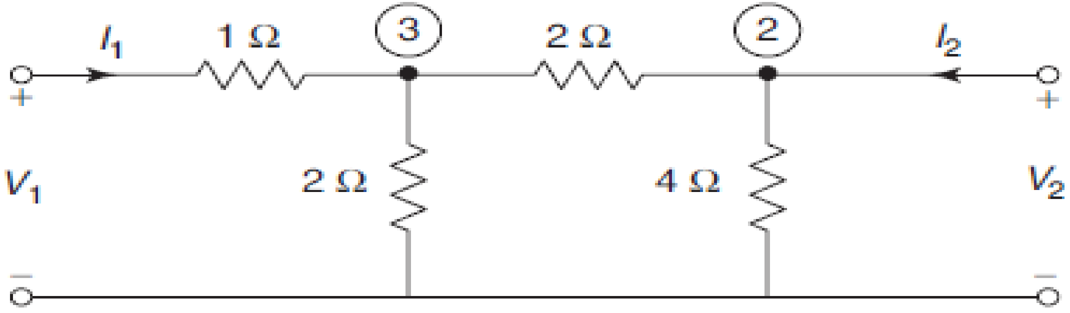
Hence, for the network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

i.e.,

$$Y_{11} = Y_{22}$$

Determine Y-parameters for the network shown in Fig. Determine whether the network is symmetrical and reciprocal.



Solution From Fig. 9.22,

$$I_1 = \frac{V_1 - V_3}{1}$$

$$= V_1 - V_3 \qquad \text{...(i)}$$

Applying KCL at Node 3,

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_2}{2}$$

$$= V_3 - \frac{V_2}{2} \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{4} + \frac{V_2 - V_3}{2}$$

$$= \frac{3}{4}V_2 - \frac{V_3}{2} \qquad \text{...(iii)}$$

Substituting Eq. (i) in Eq. (ii),

$$V_1 - V_3 = V_3 - \frac{V_2}{2}$$

$$V_3 = \frac{V_1}{2} + \frac{V_2}{4} \qquad \text{...(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1 = \frac{V_1}{2} + \frac{V_2}{4} - \frac{V_2}{2}$$

$$= \frac{V_1}{2} - \frac{V_2}{4} \qquad \text{...(v)}$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2 = \frac{3}{4}V_2 - \frac{1}{2}\left(\frac{V_1}{2} + \frac{V_2}{4}\right)$$

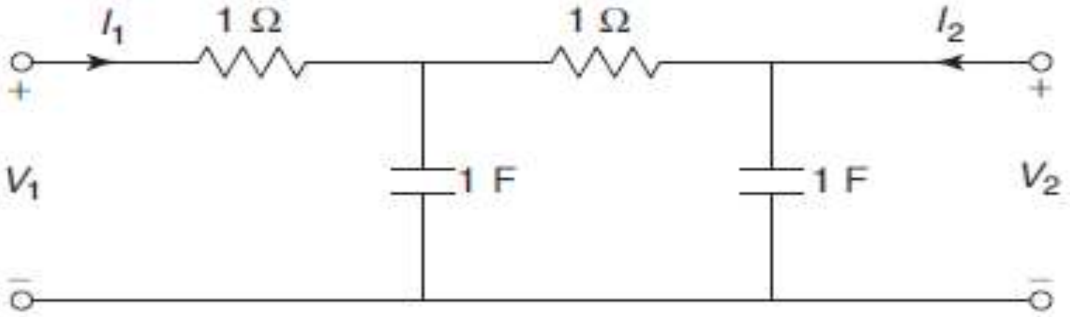
$$= -\frac{V_1}{4} + \frac{5V_2}{8} \qquad \text{...(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

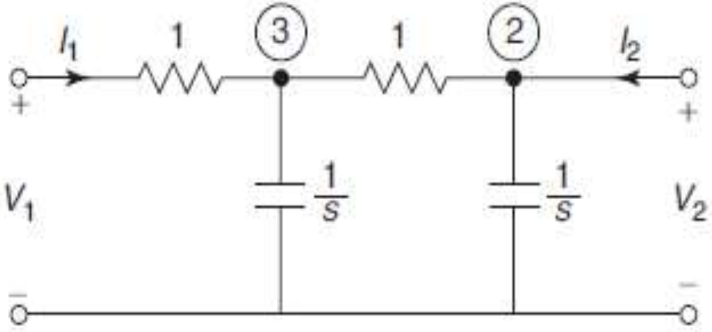
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{8} \end{bmatrix}$$

Since $Y_{11} \neq Y_{22}$, the network is not symmetrical.
Since $Y_{12} = Y_{21}$, the network is reciprocal.

Determine the short-circuit admittance parameters for the network shown in Fig.



Solution The transformed network is shown in Fig. 9.24.



From Fig. 9.24,

$$I_1 = \frac{V_1 - V_3}{1}$$

$$= V_1 - V_3 \qquad \ldots\text{(i)}$$

Applying KCL at Node 3,

$$I_1 = \frac{V_3}{\frac{1}{s}} + \frac{(V_3 - V_2)}{1}$$

$$= (s+1)\,V_3 - V_2 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{\frac{1}{s}} + \frac{(V_2 - V_3)}{1}$$

$$= (s+1)\,V_2 - V_3 \qquad \text{...(iii)}$$

Substituting Eq. (i) in Eq. (ii),

$$V_1 - V_3 = (s+1)\,V_3 - V_2$$

$$(s+2)\,V_3 = V_1 + V_2$$

$$V_3 = \frac{1}{s+2}V_1 + \frac{1}{s+2}V_2 \qquad \text{...(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1 = (s+1)\left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right) - V_2$$

$$= \frac{s+1}{s+2}V_1 - \frac{1}{s+2}V_2 \qquad \ldots(\text{v})$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2 = (s+1)\,V_2 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= -\frac{1}{s+2}V_1 + \frac{s^2+3s+1}{s+2}V_2 \qquad \ldots(\text{vi})$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s+1}{s+2} & -\dfrac{1}{s+2} \\ -\dfrac{1}{s+2} & \dfrac{s^2+3s+1}{s+2} \end{bmatrix}$$

TRANSMISSION PARAMETERS (ABCD PARAMETERS)

The transmission parameters or chain parameters or *ABCD* parameters serve to relate the voltage and current at the input port to voltage and current at the output port. In equation form,

$$(V_1, I_1) = f(V_2, -I_2)$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Here, the negative sign is used with I_2 and not for parameters B and D. The reason the current I_2 carries a negative sign is that in transmission field, the output current is assumed to be coming out of the output port instead of going into the port.

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission matrix.

For a given network, these parameters are determined as follows:

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

where A is the reverse voltage gain with the output port open-circuited.

Similarly,

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

where C is the transfer admittance with the output port open-circuited.

Case 2 When output port is short-circuited, i.e., $V_2 = 0$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

where B is the transfer impedance with the output port short-circuited.

Similarly,

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

where D is the reverse current gain with the output port short-circuited.

Condition for Reciprocity

(a) As shown in Fig. 9.28, voltage V_s is applied at the input port with the output port short-circuited.

i.e.,

$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2' = -I_2$$

From the transmission parameter equations,

$$V_s = B\, I_2'$$

$$\frac{V_s}{I_2'} = B$$

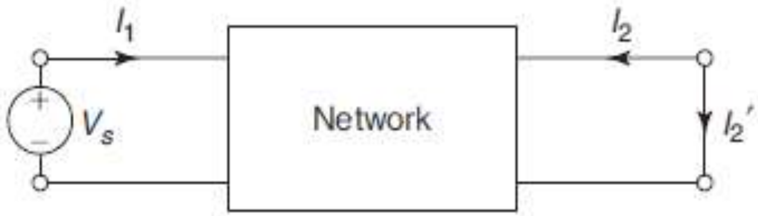


Fig. 9.28 *Network for deriving condition for reciprocity*

(b) As shown in Fig. 9.29, voltage V_s is applied at the output port with the input port short-circuited.

i.e.,
$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1' = -I_1$$

Fig. 9.29 *Network for deriving condition for reciprocity*

From the transmission parameter equations,

$$0 = AV_s - BI_2$$
$$-I_1' = CV_s - DI_2$$
$$I_2 = \frac{A}{B}V_s$$
$$-I_1' = CV_s - \frac{AD}{B}V_s$$
$$\frac{V_s}{I_1'} = \frac{B}{AD - BC}$$

Hence, for the network to be reciprocal,

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

i.e.,
$$B = \frac{B}{AD - BC}$$

i.e.,
$$AD - BC = 1$$

Condition for Symmetry

(a) When the output port is open-circuited, i.e., $I_2 = 0$.
From the transmission-parameter equations,

$$V_s = AV_2$$
$$I_1 = CV_2$$
$$\frac{V_s}{I_1} = \frac{A}{C}$$

(b) When the input port is open-circuited, i.e., $I_1 = 0$.
From the transmission parameter equation,

$$CV_s = DI_2$$
$$\frac{V_s}{I_2} = \frac{D}{C}$$

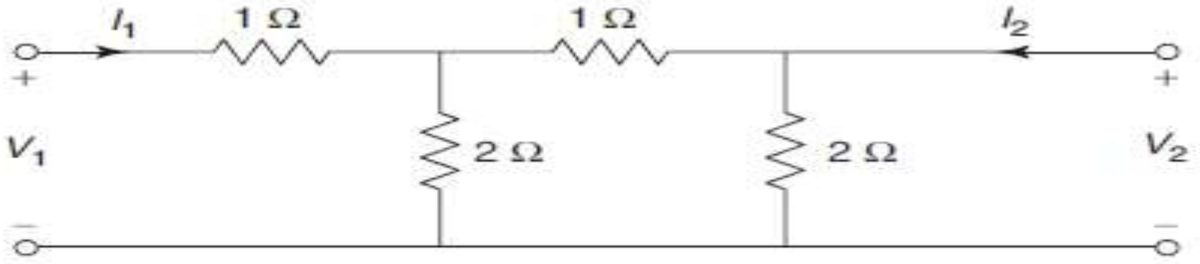
Hence, for network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

i.e.,

$$A = D$$

Obtain ABCD parameters for the network shown in Fig.



Solution The network is redrawn as shown in Fig. 9.33.

Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2I_3 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - I_3 - 2(I_3 + I_2) = 0$$

$$5I_3 = 2I_1 - 2I_2$$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \quad \ldots\text{(iii)}$$

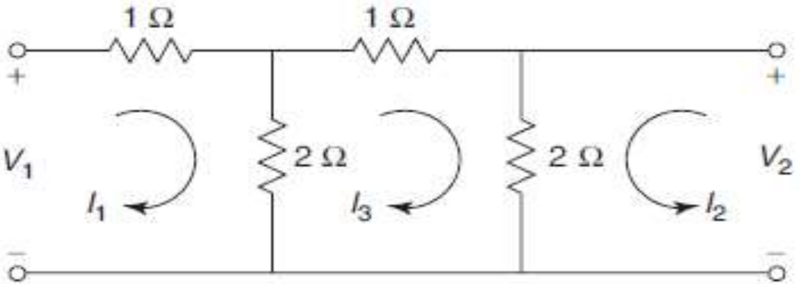


Fig. 9.33

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= \frac{11}{5}I_1 + \frac{4}{5}I_2 \quad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 2I_2 + 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= \frac{4}{5}I_1 + \frac{6}{5}I_2$$

$$\frac{4}{5}I_1 = V_2 - \frac{6}{5}I_2$$

$$I_1 = \frac{5}{4}V_2 - \frac{3}{2}I_2 \qquad \ldots\text{(v)}$$

Substituting Eq. (v) in Eq. (iv),

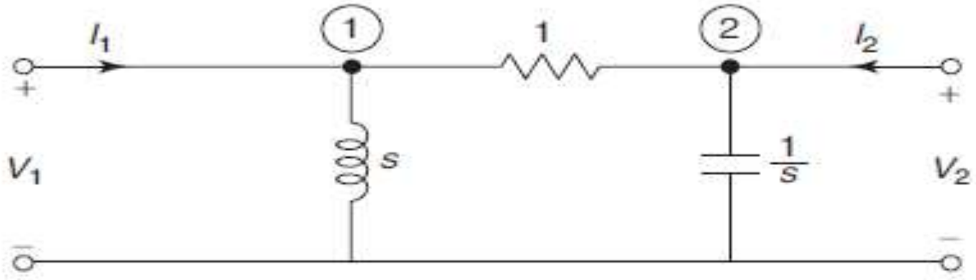
$$V_1 = \frac{11}{5}\left(\frac{5}{4}V_2 - \frac{3}{2}I_2\right) + \frac{4}{5}I_2$$

$$= \frac{11}{4}V_2 - \frac{5}{2}I_2 \qquad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with *ABCD* parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{11}{4} & \frac{5}{2} \\ \frac{5}{4} & \frac{3}{2} \end{bmatrix}$$

Determine the transmission parameters for the network shown in Fig.



Solution

Applying KCL at Node 1,

$$I_1 = \frac{V_1}{s} + (V_1 - V_2)$$

$$= \frac{s+1}{s} V_1 - V_2 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{\frac{1}{s}} + (V_2 - V_1)$$

$$= (s+1)\, V_2 - V_1$$

$$V_1 = (s+1)\, V_2 - I_2 \qquad \ldots\text{(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$I_1 = \frac{s+1}{s}[(s+1)V_2 - I_2] - V_2$$

$$= \left[\frac{(s+1)^2}{s} - 1\right]V_2 - \frac{s+1}{s}I_2$$

$$= \left(\frac{s^2+s+1}{s}\right)V_2 - \left(\frac{s+1}{s}\right)I_2 \qquad \ldots\text{(iii)}$$

Comparing Eqs (ii) and (iii) with *ABCD* parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} s+1 & -1 \\ \dfrac{s^2+s+1}{s} & \dfrac{s+1}{s} \end{bmatrix}$$

HYBRID PARAMETERS (*h* PARAMETERS)

The hybrid parameters of a two-port network may be defined by expressing the voltage of input port V_1 and current of output port I_2 in terms of current of input port I_1 and voltage of output port V_2.

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual h parameters can be defined by setting $I_1 = 0$ and $V_2 = 0$.

Case 1 When the output port is short-circuited i.e., $V_2 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$$

where h_{11} is the short-circuit input impedance.

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$

where h_{21} is the short-circuit forward current gain.

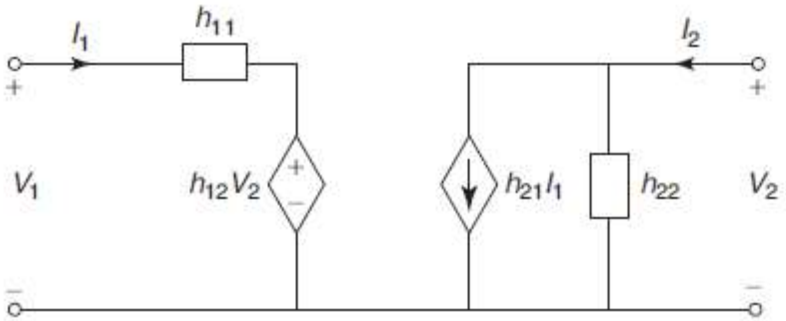
Case 2 When the input port is open-circuited, i.e., $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

where h_{12} is the open-circuit reverse voltage gain.

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

where h_{22} is the open-circuit output admittance.



6 *Equivalent circuit of the two-port network in terms of h-parameters*

Condition for Reciprocity

(a) As shown in Fig. 9.37, voltage V_s is applied at the input port and the output port is short-circuited.

i.e.,

$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2' = -I_2$$

Fig. 9.37 *Network for deriving condition for reciprocity*

From the h-parameter equations,

$$V_s = h_{11}\, I_1$$
$$-I_2' = h_{21}\, I_1$$
$$\frac{V_s}{I_2'} = -\frac{h_{11}}{h_{21}}$$

(b) As shown in Fig. 9.38, voltage V_s is applied at the output port with the input port short-circuited.

i.e.,

$$V_1 = 0$$
$$V_2 = V_s$$
$$I_1 = -I_1'$$

Fig. 9.38 *Network for deriving condition for reciprocity*

From the h-parameter equations,

$$0 = h_{11} I_1 + h_{12}\, V_s$$
$$h_{12}\, V_s = -h_{11}\, I_1 = h_{11}\, I_1'$$
$$\frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}}$$

Hence, for the network to be reciprocal,

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

i.e.,

$$h_{21} = -h_{12}$$

Condition for Symmetry

The condition for symmetry is obtained from the Z-parameters.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{h_{11}I_1 + h_{12}V_2}{I_1}\right|_{I_2=0} = h_{11} + h_{12}\frac{V_2}{I_1}$$

But with $I_2 = 0$,

$$0 = h_{21}\, I_1 + h_{22}\, V_2$$

$$\frac{V_2}{I_1} = -\frac{h_{21}}{h_{22}}$$

$$Z_{11} = h_{11} - \frac{h_{12}h_{21}}{h_{22}} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

where $\qquad \Delta h = h_{11}h_{22} - h_{12}h_{21}$

Similarly,

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

With $I_1 = 0$,

$$I_2 = h_{22} V_2$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$$

For a symmetrical network, $$Z_{11} = Z_{22}$$

i.e., $$\frac{\Delta h}{h_{22}} = \frac{1}{h_{22}}$$

i.e., $$\Delta h = 1$$

i.e., $$h_{11} h_{22} - h_{12} h_{21} = 1$$

Conditions for reciprocity and symmetry

Parameter	Condition for Reciprocity	Condition for Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
T	$AD - BC = 1$	$A = D$
h	$h_{12} = -h_{21}$	$h_{11}\,h_{22} - h_{12}\,h_{21} = 1$

INTER-RELATIONSHIPS BETWEEN THE PARAMETERS
Z-parameters in Terms of Other Parameters

1. *Z-parameters in Terms of Y-parameters* We know that

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

By Cramer's rule,

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22} I_1 - Y_{12} I_2}{Y_{11} Y_{22} - Y_{12} Y_{21}} = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2$$

where $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

Comparing with $V_1 = Z_{11} I_1 + Z_{12}, I_2,$

$$Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

Also,

$$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta Y} = \frac{Y_{11}}{\Delta Y} I_2 - \frac{Y_{21}}{\Delta Y} I_1$$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2,$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y}$$

2. *Z-parameter in Terms of ABCD Parameters* We know that

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Rewriting the second equation,

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2$,

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

Also, $$V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - BI_2 = \frac{A}{C} I_1 + \left[\frac{AD}{C} - B\right] I_2 = \frac{A}{C} I_1 + \left[\frac{AD - BC}{C}\right] I_2$$

Comparing with $V_1 = Z_{11} I_1 + Z_{12} I_2$,

$$Z_{11} = \frac{A}{C}$$

$$Z_{12} = \frac{AD - BC}{C}$$

3. *Z-parameters in Terms of Hybrid Parameters* We know that

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the second equation,

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

Comparing with

$$V_2 = Z_{21} I_1 + Z_{22} I_2,$$

$$Z_{21} = -\frac{h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Also, $V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right] = h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12} h_{21}}{h_{22}} I_1 = \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$

Comparing with

$$V_1 = Z_{11} I_1 + Z_{12} I_2,$$

$$Z_{11} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

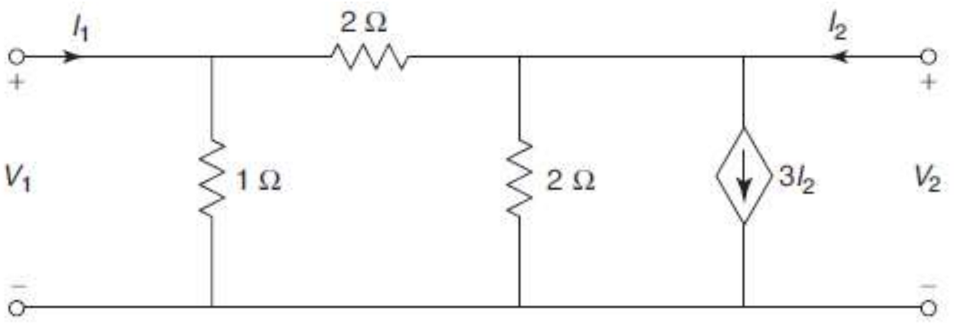
$$Z_{12} = \frac{h_{12}}{h_{22}}$$

Inter-relationship between parameters

$$\Delta X = X_{11} X_{22} - X_{12} X_{21}$$

	In terms of			
	[Z]	**[Y]**	**[T]**	**[h]**
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$
[T]	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$

For the network shown in Fig. 9.44, find Z and Y-parameters.



Solution The network is redrawn by source transformation technique as shown in Fig. 9.45.

Applying KVL to Mesh 1,

$$V_1 = I_1 - I_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2(I_2 + I_3) - 6I_2$$
$$= -4I_2 + 2I_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-(I_3 - I_1) - 2I_3 - 2(I_2 + I_3) + 6I_2 = 0$$
$$5I_3 = I_1 + 4I_2$$
$$I_3 = \frac{1}{5}I_1 + \frac{4}{5}I_2 \qquad \ldots\text{(iii)}$$

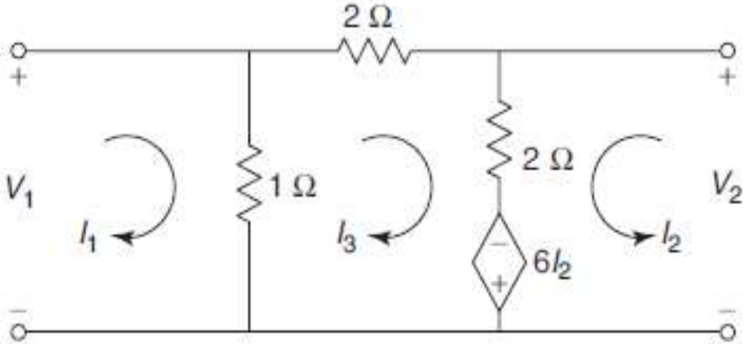


Fig. 9.45

Substituting Eq. (iii) in Eq. (i),

$$V_1 = I_1 - \frac{1}{5}I_1 - \frac{4}{5}I_2$$

$$= \frac{4}{5}I_1 - \frac{4}{5}I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = -4I_2 + 2\left(\frac{1}{5}I_1 + \frac{4}{5}I_2\right)$$

$$= \frac{2}{5}I_1 - \frac{12}{5}I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{4}{5} \\ \frac{2}{5} & -\frac{12}{5} \end{bmatrix}$$

***Y*-parameters**

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21} = \left(\frac{4}{5}\right)\left(-\frac{12}{5}\right) - \left(-\frac{4}{5}\right)\left(\frac{2}{5}\right) = -\frac{40}{25} = -\frac{8}{5}$$

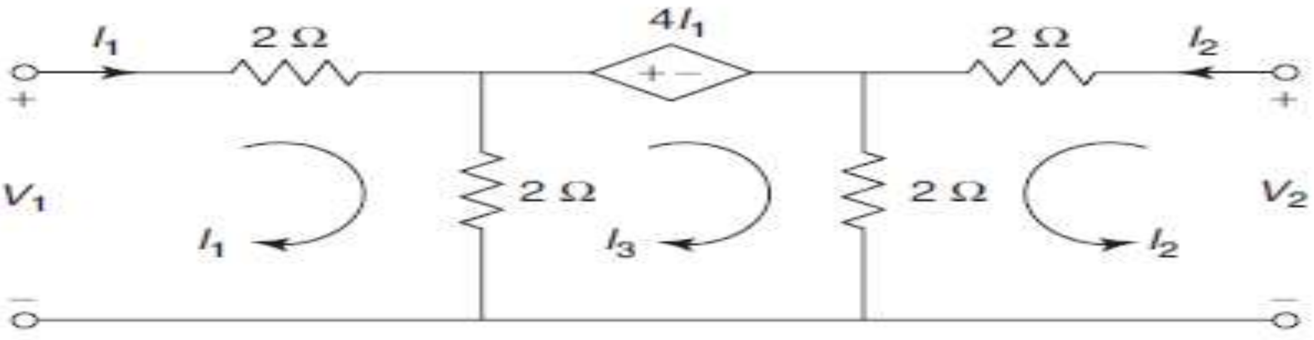
$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{-\frac{12}{5}}{-\frac{8}{5}} = \frac{3}{2}\,\mho, \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = \frac{-\frac{4}{5}}{-\frac{8}{5}} = -\frac{1}{2}\,\mho$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = \frac{-\frac{2}{5}}{-\frac{8}{5}} = \frac{1}{4}\,\mho, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{\frac{4}{5}}{-\frac{8}{5}} = -\frac{1}{2}\,\mho$$

Hence, the *Y*-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

Find Z and h-parameters for the network shown in Fig. 9



Solution Applying KVL to Mesh 1,

$$V_1 = 2I_1 + 2(I_1 - I_3)$$

$$= 4I_1 - 2I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2(I_2 + I_3)$$

$$= 4I_2 + 2I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 4I_1 - 2(I_3 + I_2) = 0$$

$$I_1 + I_2 = -2I_3 \qquad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 4I_1 + I_1 + I_2$$

$$= 5I_1 + I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 4I_2 - I_1 - I_2$$

$$= -I_1 + 3I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

h-parameters

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (5)(3) - (1)(-1) = 15 + 1 = 16$$

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{16}{3}\,\Omega, \qquad h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{1}{3}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = \frac{1}{3}, \qquad h_{22} = \frac{1}{Z_{22}} = \frac{1}{3}\,\mho$$

Hence, the h-parameters are

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{22} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{16}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

INTERCONNECTION OF TWO-PORT NETWORKS

We shall now discuss the various types of interconnections of two-port networks, namely, cascade, parallel, series, series-parallel and parallel-series. We shall derive the relation between the input and output quantities of the combined two-port networks.

9.7.1 Cascade Connection

Transmission Parameter Representation Figure 9.56 shows two-port networks connected in cascade. In the cascade connection, the output port of the first network becomes the input port of the second network. Since it is assumed that input and output currents are positive when they enter the network, we have

$$I_1' = -I_2$$

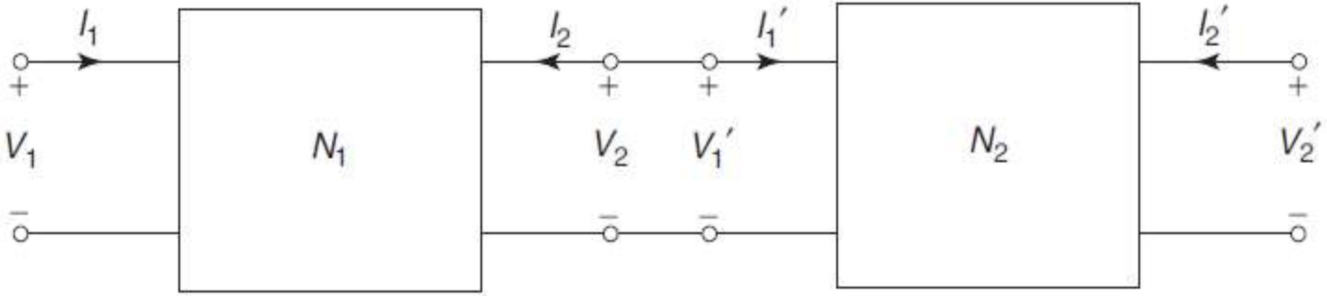


Fig. 9.56 *Cascade Connection*

Let A_1, B_1, C_1, D_1 be the transmission parameters of the network N_1 and A_2, B_2, C_2, D_2 be the transmission parameters of the network N_2.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Parallel Connection

Figure 9.62 shows two-port networks connected in parallel. In the parallel connection, the two networks have the same input voltages and the same output voltages.

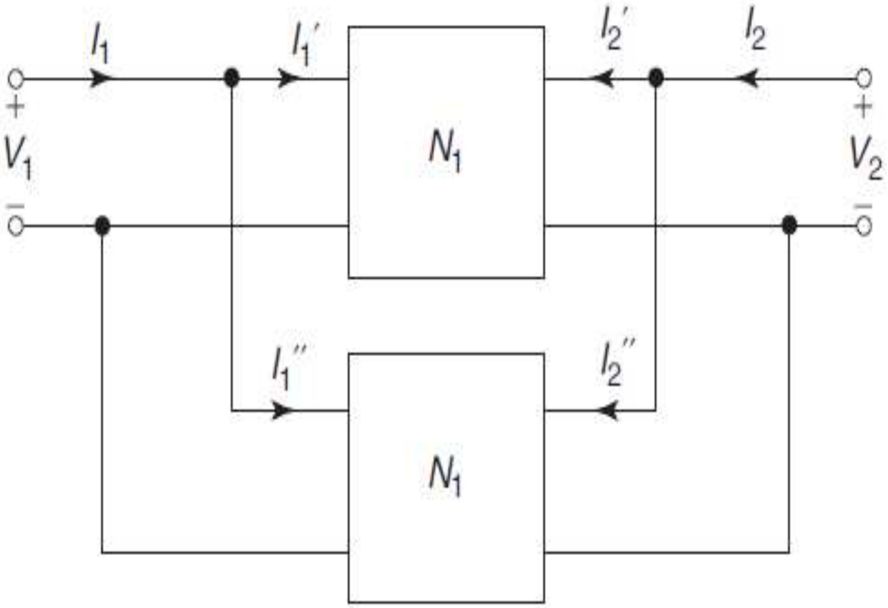


Fig. 9.62 *Parallel connection*

Let $Y_{11}', Y_{12}', Y_{21}', Y_{22}'$ be the Y-parameters of the network N_1 and $Y_1'', Y_{12}'', Y_{21}'', Y_{22}''$ be the Y-parameters of the network N_2.

Hence,
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' + I_1'' \\ I_2' + I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus, the resultant Y-parameter matrix for parallel connected networks is the sum of Y matrices of each individual two-port networks.

Series Connection

Figure 9.72 shows two-port networks connected in series. In a series connection, both the networks carry the same input current. Their output currents are also equal.

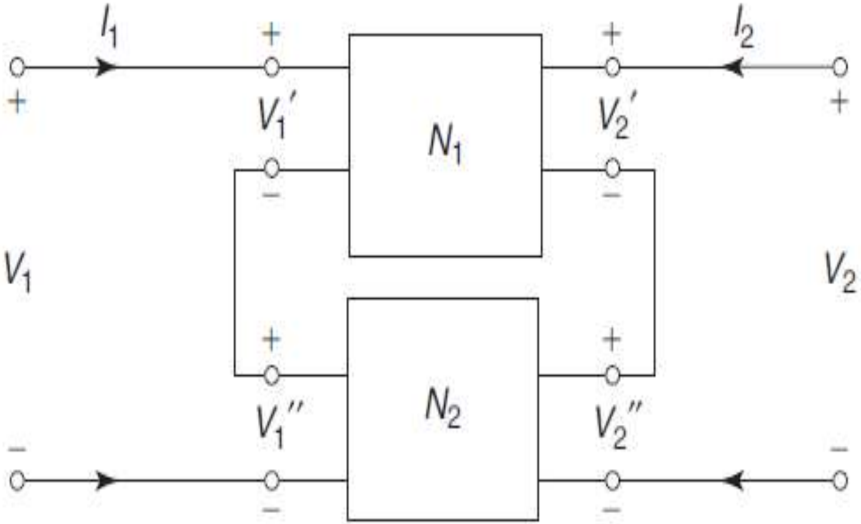


Fig. 9.72 *Series connection*

Let $Z_{11}', Z_{12}', Z_{21}', Z_{22}'$ be the Z-parameters of the network N_1 and $Z_{11}'', Z_{12}'', Z_{21}'', Z_{22}''$ be the Z-parameters of the network N_2.

For the combined network $\quad V_1 = V_1' + V_1''$ and $V_2 = V_2' + V_2''$.

Hence,
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' + V_1'' \\ V_2' + V_2'' \end{bmatrix} = \begin{bmatrix} Z_{11}' + Z_{11}'' & Z_{12}' + Z_{12}'' \\ Z_{21}' + Z_{21}'' & Z_{22}' + Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus, the resultant Z-parameter matrix for the series-connected networks is the sum of Z matrices of each individual two-port network.

T-NETWORK

Any two-port network can be represented by an equivalent *T* network as shown in Fig. 9.80. The elements of the equivalent *T* network may be expressed in terms of *Z*-parameters.

Applying KVL to Mesh 1,

$$V_1 = Z_A I_1 + Z_C(I_1 + I_2)$$
$$= (Z_A + Z_C)I_1 + Z_C I_2 \qquad \text{...(9.9)}$$

Applying KVL to Mesh 2,

$$V_2 = Z_B I_2 + Z_C(I_2 + I_1)$$
$$= Z_C I_1 + (Z_B + Z_C)I_2 \qquad \text{...(9.10)}$$

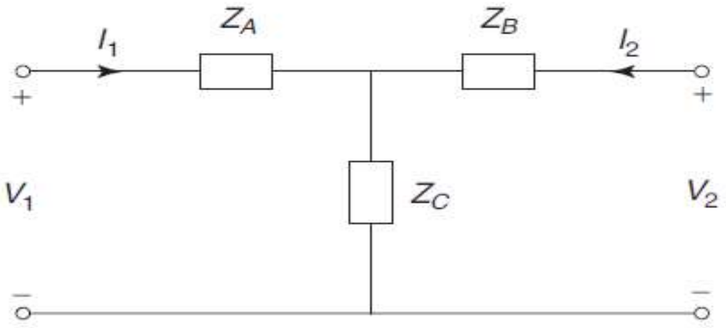


Fig. 9.80 *T-Network*

Comparing Eqs (9.9) and (9.10) with *Z*-parameter equations,

$$Z_{11} = Z_A + Z_C$$
$$Z_{12} = Z_C$$
$$Z_{21} = Z_C$$
$$Z_{22} = Z_B + Z_C$$

Solving the above equations,

$$Z_A = Z_{11} - Z_{12} = Z_{11} - Z_{21}$$
$$Z_B = Z_{22} - ZZ_{21} = Z_{22} - Z_{12}$$
$$Z_C = Z_{12} = Z_{21}$$

PI (π)-NETWORK

Any two-port network can be represented by an equivalent pi (π) network as shown in Fig. 9.81.
Applying KCL at Node 1,

$$I_1 = Y_A V_1 + Y_B (V_1 - V_2)$$
$$= (Y_A + Y_B) V_1 - Y_B V_2 \qquad \text{...(9.11)}$$

Applying KCL at Node 2,

$$I_2 = Y_C V_2 + Y_B (V_2 - V_1)$$
$$= -Y_B V_1 + (Y_B + Y_C) V_2 \qquad \text{...(9.12)}$$

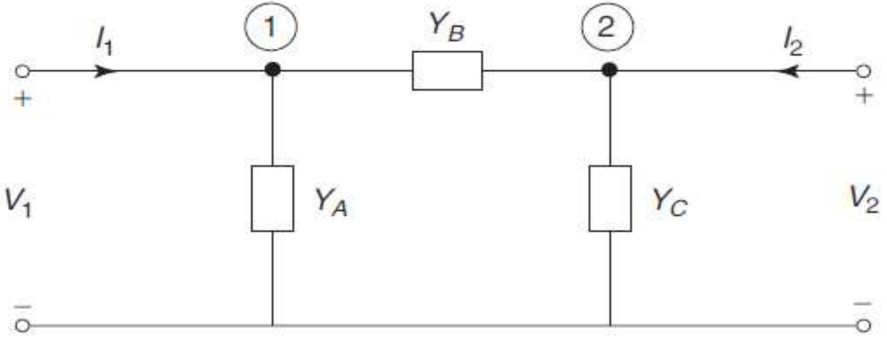


Fig. 9.81 *π-network*

Comparing Eqs (9.11) and (9.12) with *Y*-parameter equations,

$$Y_{11} = Y_A + Y_B$$
$$Y_{12} = -Y_B$$
$$Y_{21} = -Y_B$$
$$Y_{22} = Y_B + Y_C$$

Solving the above equations,

$$Y_A = Y_{11} + Y_{12} = Y_{11} + Y_{21}$$
$$Y_B = -Y_{12} = -Y_{21}$$
$$Y_C = Y_{22} + Y_{12} = Y_{22} + Y_{21}$$