4

Magnetic Circuits

4.1 INTRODUCTION

Two circuits are said to be coupled circuits when energy transfer takes place from one circuit to the other without having any electrical connection between them. Such coupled circuits are frequently used in network analysis and synthesis. Common examples of coupled circuits are transformer, gyrator, etc. In this chapter, we will discuss self and mutual inductance, magnetically coupled circuits, dot conventions and tuned circuits.

4.2 SELF-INDUCTANCE

Consider a coil of N turns carrying a current i as shown in Fig. 4.1.

When current flows through the coil, a flux ϕ is produced in the coil. The flux produced by the coil links with the coil itself. If the current flowing through the coil changes, the flux linking the coil also changes. Hence, an emf is induced in the coil. This is known as self-induced emf. The direction of this emf is given by *Lenz's law*.

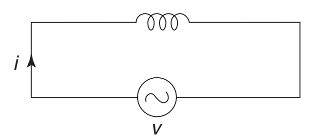


Fig. 4.1 *Coil carrying current*

We know that

$$\phi \propto i$$

$$\frac{\phi}{i} = k, \text{ a constant}$$

$$\phi = k\, i$$

Hence, rate of change of flux = $k \times$ rate of change of current

$$\frac{d\phi}{dt} = k\frac{di}{dt}$$

According to Faraday's laws of electromagnetic induction, a self-induced emf can be expressed as

$$v = -N\frac{d\phi}{dt} = -Nk\frac{di}{dt} = -N\frac{\phi}{i}\frac{di}{dt} = -L\frac{di}{dt}$$

where $L = \dfrac{N\phi}{i}$ and is called coefficient of self-inductance.

The property of a coil that opposes any change in the current flowing through it is called self-inductance or inductance of the coil. If the current in the coil is increasing, the self-induced emf is set up in such a direction so as to oppose the rise in current, i.e., the direction of self-induced emf is opposite to that of the applied voltage. Similarly, if the current in the coil is decreasing, the self-induced emf will be in the same direction as the applied voltage. Self-inductance does not prevent the current from changing, it serves only to delay the change.

4.3 MUTUAL INDUCTANCE

If the flux produced by one coil links with the other coil, placed closed to the first coil, an emf is induced in the second coil due to change in the flux produced by the first coil. This is known as mutually induced emf.

Consider two coils 1 and 2 placed adjacent to each other as shown in Fig. 4.2. Let Coil 1 has N_1 turns while Coil 2 has N_2 turns.

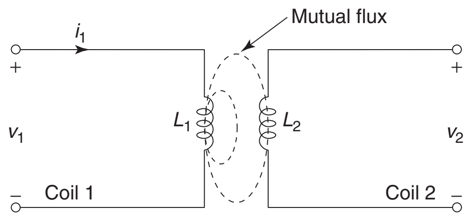


Fig. 4.2 *Two adjacent coils*

If a current i_1 flows in Coil 1, flux is produced and a part of this flux links Coil 2. The emf induced in Coil 2 is called mutually induced emf.

We know that

$$\phi_2 \propto i_1$$

$$\frac{\phi_2}{i_1} = k, \text{ a constant}$$

$$\phi_2 = k\, i_1$$

Hence, rate of change of flux = $k \times$ rate of change of current i_1

$$\frac{d\phi_2}{dt} = k\frac{di_1}{dt}$$

According to Faraday's law of electromagnetic induction, the induced emf is expressed as

$$v_2 = -N_2\frac{d\phi_2}{dt} = -N_2\, k\frac{di_1}{dt} = -N_2\frac{\phi_2}{i_1}\frac{di_1}{dt} = -M\frac{di_1}{dt}$$

where $M = \dfrac{N_2\,\phi_2}{i_1}$ and is called *coefficient of mutual inductance*.

4.4 COEFFICIENT OF COUPLING (k)

The coefficient of coupling (k) between coils is defined as fraction of magnetic flux produced by the current in one coil that links the other.

Consider two coils having number of turns N_1 and N_2 respectively. When a current i_1 is flowing in Coil 1 and is changing, an emf is induced in Coil 2.

$$M = \frac{N_2\,\phi_2}{i_1}$$

Let
$$k_1 = \frac{\phi_2}{\phi_1}$$
$$\phi_2 = k_1 \phi_1$$
$$M = \frac{N_2 k_1 \phi_1}{i_1} \qquad \ldots(4.1)$$

If the current i_2 is flowing in Coil 2 and is changing, an emf is induced in Coil 1,
$$M = \frac{N_1 \phi_1}{i_2}$$

Let
$$k_2 = \frac{\phi_1}{\phi_2}$$
$$\phi_1 = k_2 \phi_2$$
$$M = \frac{N_1 k_2 \phi_2}{i_2} \qquad \ldots(4.2)$$

Multiplying Eqs (4.1) and (4.2),
$$M^2 = k_1 k_2 \times \frac{N_1\phi_1}{i_1} \times \frac{N_2\phi_2}{i_2} = k^2 L_1 L_2$$
$$M = k\sqrt{L_1 L_2}$$
where
$$k = \sqrt{k_1 k_2}$$

4.5 INDUCTANCES IN SERIES

1. Cumulative Coupling Figure 4.3 shows two coils 1 and 2 connected in series, so that currents through the two coils are in the same direction in order to produce flux in the same direction. Such a connection of two coils is known as *cumulative coupling*.

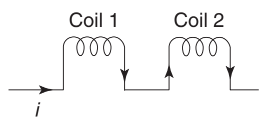


Fig. 4.3 *Cumulative coupling*

Let L_1 = coefficient of self-inductance of Coil 1
L_2 = coefficient of self-inductance of Coil 2
M = coefficient of mutual inductance

If the current in the coil increases by *di* amperes in *dt* seconds then

Self-induced emf in Coil 1 $= -L_1 \frac{di}{dt}$

Self-induced emf in Coil 2 $= -L_2 \frac{di}{dt}$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= -M \frac{di}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= -M \frac{di}{dt}$

Total induced emf
$$v = -(L_1 + L_2 + 2M)\frac{di}{dt} \qquad \ldots(4.3)$$

If L is the equivalent inductance then total induced emf in that single coil would have been

$$v = -L\frac{di}{dt} \qquad \ldots(4.4)$$

Equating Eqs (4.3) and (4.4),

$$L = L_1 + L_2 + 2M$$

2. *Differential Coupling* Figure 4.4 shows the coils connected in series but the direction of current in Coil 2 is now opposite to that in 1. Such a connection of two coils is known as *differential coupling*.

Hence, total induced emf in coils 1 and 2.

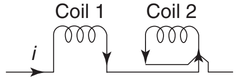


Fig. 4.4 *Differential coupling*

$$v = -L_1\frac{di}{dt} - L_2\frac{di}{dt} + 2M\frac{di}{dt} = -(L_1 + L_2 - 2M)\frac{di}{dt}$$

Coils 1 and 2 connected in series can be considered as a single coil with equivalent inductance L. The induced emf in the equivalent single coil with same rate of change of current is given by,

$$v = -L\frac{di}{dt}$$

$$-L\frac{di}{dt} = -(L_1 + L_2 - 2M)\frac{di}{dt}$$

$$L = L_1 + L_2 - 2M$$

4.6 INDUCTANCES IN PARALLEL

1. *Cumulative Coupling* Figure 4.5 shows two coils 1 and 2 connected in parallel such that fluxes produced by the coils act in the same direction. Such a connection of two coils is known as cumulative coupling.

Let L_1 = coefficient of self-inductance of Coil 1

L_2 = coefficient of self-inductance of Coil 2

M = coefficient of mutual inductance

If the current in the coils changes by di amperes in dt seconds then

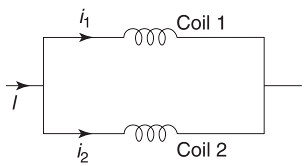


Fig. 4.5 *Cumulative coupling*

Self-induced emf in Coil 1 $= -L_1\dfrac{di_1}{dt}$

Self-induced emf in Coil 2 $= -L_2\dfrac{di_2}{dt}$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= -M\dfrac{di_2}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= -M\dfrac{di_1}{dt}$

Total induced emf in Coil 1 $= -L_1\dfrac{di_1}{dt} - M\dfrac{di_2}{dt}$

Total induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt} - M \dfrac{di_1}{dt}$

As both the coils are connected in parallel, the emf induced in both the coils must be equal.

$$-L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

$$L_1 \frac{di_1}{dt} - M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_2}{dt}$$

$$(L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} \qquad \text{...(4.5)}$$

Now,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 - M}{L_1 - M} + 1\right)\frac{di_2}{dt}$$

$$= \left(\frac{L_1 + L_2 - 2M}{L_1 - M}\right)\frac{di_2}{dt} \qquad \text{...(4.6)}$$

If L is the equivalent inductance of the parallel combination then the induced emf is given by

$$v = -L\frac{di}{dt}$$

Since induced emf in parallel combination is same as induced emf in any one coil,

$$L\frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}\left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right)$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} + M\frac{di_2}{dt}\right]$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt} \qquad \text{...(4.7)}$$

Substituting Eq. (4.6) in Eq. (4.7),

$$\left(\frac{L_1 + L_2 - 2M}{L_1 - M}\right)\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt}$$

$$L = \frac{L_1\left(\dfrac{L_2 - M}{L_1 - M}\right) + M}{\dfrac{L_1 + L_2 - 2M}{L_1 - M}}$$

$$= \frac{L_1 L_2 - L_1 M + L_1 M - M^2}{L_1 + L_2 - 2M}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

2. *Differential Coupling* Figure 4.6 shows two coils 1 and 2 connected in parallel such that fluxes produced by the coils act in the opposite direction. Such a connection of two coils is known as differential coupling.

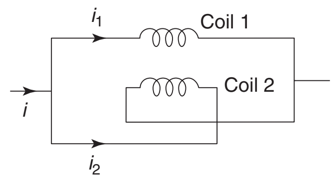


Fig. 4.6 *Differential coupling*

Self-induced emf in Coil 1 $= -L_1 \dfrac{di_1}{dt}$

Self-induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt}$

Mutually induced emf in Coil 1 due to change of current in Coil 2 $= M \dfrac{di_2}{dt}$

Mutually induced emf in Coil 2 due to change of current in Coil 1 $= M \dfrac{di_1}{dt}$

Total induced emf in Coil 1 $= -L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}$

Total induced emf in Coil 2 $= -L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt}$

As both the coils are connected in parallel, the emf induced in the coils must be equal.

$$-L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$L_1 \frac{di_1}{dt} + M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_2}{dt}$$

$$(L_1 + M)\frac{di_1}{dt} = (L_2 + M)\frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 + M}{L_1 + M}\right)\frac{di_2}{dt} \qquad \text{...(4.8)}$$

Now,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 + M}{L_1 + M}\right)\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$= \left(\frac{L_2 + M}{L_1 + M} + 1\right)\frac{di_2}{dt}$$

$$= \left(\frac{L_1 + L_2 + 2M}{L_1 + M}\right)\frac{di_2}{dt} \qquad \text{...(4.9)}$$

If L is the equivalent inductance of the parallel combination then the induced emf is given by

$$v = -L\frac{di}{dt}$$

Since induced emf in parallel combination is same as induced emf in any one coil,

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} - M\frac{di_2}{dt}\right)$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right)\frac{di_2}{dt} - M\frac{di_2}{dt}\right]$$

$$= \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right) - M\right]\frac{di_2}{dt} \qquad \text{...(4.10)}$$

Substituting Eq. (4.9) in Eq. (4.10),

$$\left(\frac{L_1+L_2+2M}{L_1+M}\right)\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2+M}{L_1+M}\right) - M\right]\frac{di_2}{dt}$$

$$L = \frac{L_1\left(\frac{L_2+M}{L_1+M}\right) - M}{\frac{L_1+L_2+2M}{L_1+M}}$$

$$= \frac{L_1L_2 + L_1M - L_1M - M^2}{L_1+L_2+2M}$$

$$= \frac{L_1L_2 - M^2}{L_1+L_2+2M}$$

Example 4.1 *The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on relative directions of currents in the two coils. If one of the coils has a self-inductance of 0.2 H, find (a) mutual inductance, and (b) coefficient of coupling.*

Solution

$$L_1 = 0.2\,\text{H}, \quad L_{\text{diff}} = 0.1\,\text{H}, \quad L_{\text{cum}} = 0.6\text{ H}$$

(a) Mutual inductance

$$L_{\text{cum}} = L_1 + L_2 + 2M = 0.6 \qquad \text{...(i)}$$

$$L_{\text{diff}} = L_1 + L_2 - 2M = 0.1 \qquad \text{...(ii)}$$

Adding Eqs (i) and (ii),

$$2(L_1 + L_2) = 0.7$$

$$L_1 + L_2 = 0.35$$

$$L_2 = 0.35 - 0.2 = 0.15\text{ H}$$

Subtracting Eqs (ii) from Eqs (i),

$$4M = 0.5$$
$$M = 0.125 \text{ H}$$

(b) Coefficient of coupling

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.125}{\sqrt{0.2 \times 0.15}} = 0.72$$

Example 4.2 *Two coils with a coefficient of coupling of 0.6 between them are connected in series so as to magnetise in (a) same direction, and (b) opposite direction. The total inductance in the same direction is 1.5 H and in the opposite direction is 0.5 H. Find the self-inductance of the coils.*

Solution $k = 0.6, \quad L_{\text{diff}} = 0.5 \text{ H}, \quad L_{\text{cum}} = 1.5 \text{ H}$

$$L_{\text{diff}} = L_1 + L_2 - 2M = 0.5 \qquad \text{...(i)}$$
$$L_{\text{cum}} = L_1 + L_2 + 2M = 1.5 \qquad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii),

$$4M = 1$$
$$M = 0.25 \text{ H}$$

Adding Eq. (i) and (ii),

$$2(L_1 + L_2) = 2$$
$$L_1 + L_2 = 1 \qquad \text{...(iii)}$$
$$k = \frac{M}{\sqrt{L_1 L_2}}$$
$$0.6 = \frac{0.25}{\sqrt{L_1 L_2}}$$
$$L_1 L_2 = 0.1736 \qquad \text{...(iv)}$$

Solving Eqs (iii) and (iv),

$$L_1 = 0.22 \text{ H}$$
$$L_2 = 0.78 \text{ H}$$

Example 4.3 *Two coils having self-inductances of 4 mH and 7 mH respectively are connected in parallel. If the mutual inductance between them is 5 mH, find the equivalent inductance.*

Solution $L_1 = 4 \text{ mH}, \quad L_2 = 7 \text{ mH}, \quad M = 5 \text{ mH}$

For cumulative coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4 \times 7 - (5)^2}{4 + 7 - 2(5)} = 3 \text{ mH}$$

For differential coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{4 \times 7 - (5)^2}{4 + 7 + 2(5)} = 0.143 \text{ mH}$$

Example 4.4 *Two inductors are connected in parallel. Their equivalent inductance when the mutual inductance aids the self-inductance is 6 mH and it is 2 mH when the mutual inductance opposes the self-inductance. If the ratio of the self- inductances is 1:3 and the mutual inductance between the coils is 4 mH, find the self-inductances.*

Solution

$$L_{\text{cum}} = 6 \text{ mH}, \quad L_{\text{diff}} = 2 \text{ mH}, \quad \frac{L_1}{L_2} = 1.3, \quad M = 4 \text{ mH}$$

For cumulative coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$6 = \frac{L_1 L_2 - (4)^2}{L_1 + L_2 - 2(4)}$$

$$6 = \frac{L_1 L_2 - 16}{L_1 + L_2 - 8} \qquad \text{...(i)}$$

For differential coupling,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$2 = \frac{L_1 L_2 - (4)^2}{L_1 + L_2 + 8}$$

$$2 = \frac{L_1 L_2 - 16}{L_1 + L_2 + 8} \qquad \text{...(ii)}$$

From Eqs (i) and (ii),

$$2(L_1 + L_2 + 8) = 6(L_1 + L_2 - 8)$$

$$L_1 + L_2 + 8 = 3L_1 + 3L_2 - 24$$

$$L_1 + L_2 = 16$$

But

$$\frac{L_1}{L_2} = 1.3$$

$$1.3\, L_2 + L_2 = 16$$

$$2.3\, L_2 = 16$$

$$L_2 = 6.95 \text{ mH}$$

$$L_1 = 1.3\, L_2 = 9.035 \text{ mH}$$

4.7 DOT CONVENTION

Consider two coils of inductances L_1 and L_2 respectively connected in series as shown in Fig. 4.7. Each coil will contribute the same mutual flux (since it is in a series connection, the same current flows through L_1 and L_2) and hence, same mutual inductance (M). If the mutual fluxes of the two coils aid each other as shown in Fig 4.7 (a), the inductances of each coil will be increased by M, i.e., the inductance of coils will become $(L_1 + M)$ and $(L_2 + M)$. If the mutual fluxes oppose each other as shown in Fig. 4.7 (b), inductance of the coils will become $(L_1 - M)$ and $(L_2 - M)$. Whether the two mutual fluxes aid to each other or oppose will depend upon the manner in which coils are wound. The method described above is very inconvenient because we have to include the pictures of the coils in the circuit. There is another simple method of defining the directions of currents in the coils. This is known as dot convention.

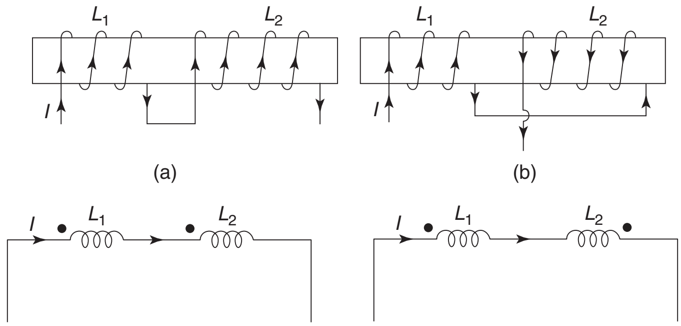


Fig. 4.7 *Dot convention*

Figure 4.7 shows the schematic connection of the two coils. It is not possible to state from Fig. 4.7(a) and Fig. 4.7(b) whether the mutual fluxes are additive or in opposition. However dot convention removes this confusion.

If the current enters from both the dotted ends of Coil 1 and Coil 2, the mutual fluxes of the two coils aid each other as shown in Fig. 4.7(c). If the current enters from the dotted end of Coil 1 and leaves from the dotted end of Coil 2, the mutual fluxes of the two coils oppose each other as shown in Fig. 4.7(d).

When two mutual fluxes aid each other, the mutual inductance is positive and polarity of the mutually induced emf is same as that of the self-induced emf. When two mutual fluxes oppose each other, the mutual inductance is negative and polarity of the mutually induced emf is opposite to that of the self-induced emf.

Example 4.5 *Obtain the dotted equivalent circuit for Fig. 4.8 shown below.*

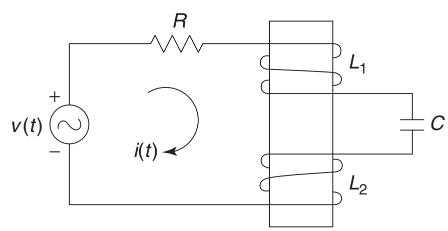


Fig. 4.8

Solution The current in the two coils is shown in Fig. 4.9. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

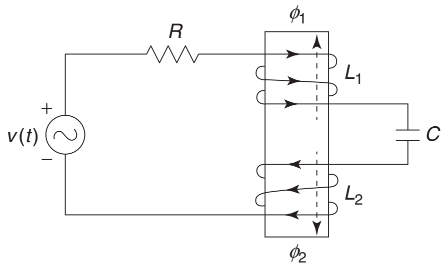


Fig. 4.9

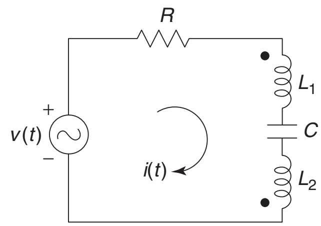


Fig. 4.10

From Fig. 4.9, it is seen that, the flux ϕ_1 is in upward direction in Coil 1, and flux ϕ_2 is in downward direction in Coil 2. Hence, fluxes are opposing each other. The mutual inductances are negative and mutually induced emfs have opposite polarities as that of self-induced emf. The dots are placed in two coils to illustrate these conditions. Hence, current $i(t)$ enters from the dotted end in Coil 1 and leaves from the dotted end in Coil 2.

The dotted equivalent circuit is shown in Fig. 4.10.

Example 4.6 *Obtain the dotted equivalent circuit for the circuit of Fig. 4.11.*

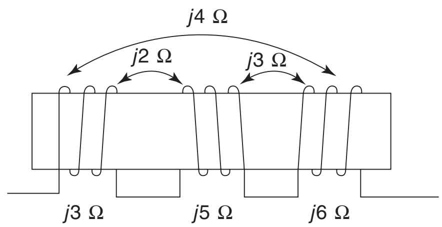


Fig. 4.11

Solution The current in the three coils is shown in Fig. 4.12. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

From Fig. 4.12, it is seen that the flux is towards the left in Coil 1, towards the right in Coil 2 and towards the left in Coil 3. Hence, fluxes ϕ_1 and ϕ_2 oppose each other in coils 1 and 2, fluxes ϕ_2 and ϕ_3 oppose each other in coils 2 and 3, and fluxes ϕ_1 and ϕ_3 aid each other in coils 1 and 3. The dots are placed in three coils to illustrate these conditions. Hence, current enters from the dotted end in Coil 1, leaves from the dotted end in Coil 2 and enters from the dotted end in Coil 3.

The dotted equivalent circuit is shown in Fig. 4.13.

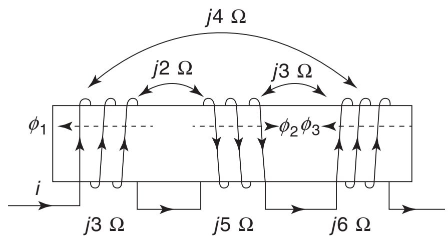


Fig. 4.12

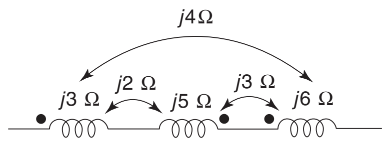


Fig. 4.13

Example 4.7 *Obtain the dotted equivalent circuit for the circuit shown in Fig. 4.14.*

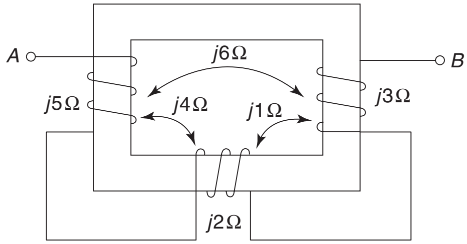


Fig. 4.14

Solution The current in the three coils is shown in Fig. 4.15. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

From Fig. 4.15, it is seen that all the three fluxes ϕ_1, ϕ_2, ϕ_3 aid each other. Hence, all the mutual reactances are positive and mutually induced emfs have same polarities as that of self-induced emfs. The dots are placed in three coils to illustrate these conditions. Hence, currents enter from the dotted end in each of the three coils. The dotted equivalent circuit is shown in Fig. 4.16.

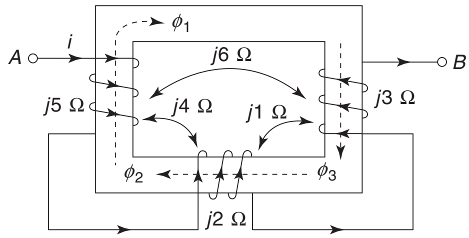


Fig. 4.15

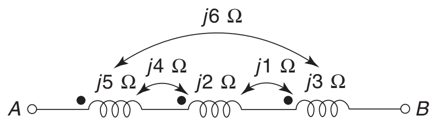


Fig. 4.16

Example 4.8 *Obtain the dotted equivalent circuit for the coupled circuit of Fig. 4.17.*

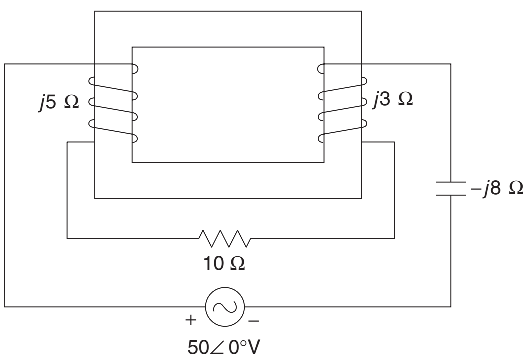


Fig. 4.17

Solution The current in the two coils is shown in Fig. 4.18 . The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

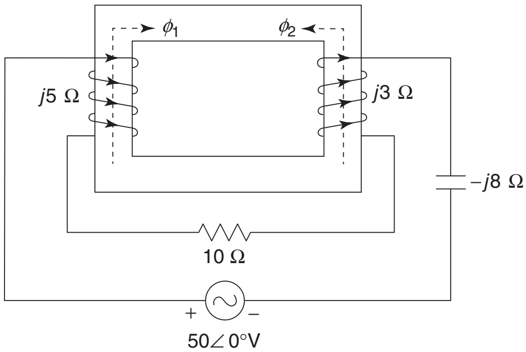


Fig. 4.18

From Fig. 4.18, it is seen that the flux ϕ_1 is in clockwise direction in Coil 1 and in anti-clockwise direction in Coil 2. Hence, fluxes are opposing each other. The dots are placed in two coils to illustrate these conditions. Hence, current enters from the dotted end in Coil 1 and leaves from the dotted end in Coil 2. The dotted equivalent circuit is shown in Fig. 4.19.

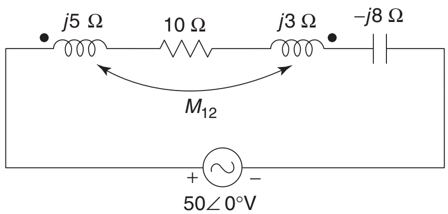


Fig. 4.19

Example 4.9 *Find the equivalent inductance of the network shown in Fig. 4.20.*

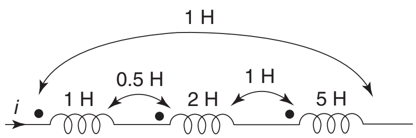


Fig. 4.20

Solution

$$
\begin{aligned}
L &= (L_1 + M_{12} + M_{13}) + (L_2 + M_{23} + M_{21}) + (L_3 + M_{31} + M_{32}) \\
&= (1 + 0.5 + 1) + (2 + 1 + 0.5) + (5 + 1 + 1) \\
&= 13 \text{ H}
\end{aligned}
$$

Example 4.10 *Find the equivalent inductance of the network shown in Fig. 4.21.*

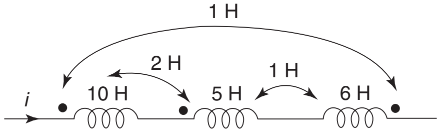


Fig. 4.21

Solution

$$
\begin{aligned}
L &= (L_1 + M_{12} - M_{13}) + (L_2 - M_{23} + M_{21}) + (L_3 - M_{31} - M_{23}) \\
&= (10 + 2 - 1) + (5 - 1 + 2) + (6 - 1 - 1) = 21 \text{ H}
\end{aligned}
$$

Example 4.11 *Find the equivalent inductance of the network shown in Fig. 4.22.*

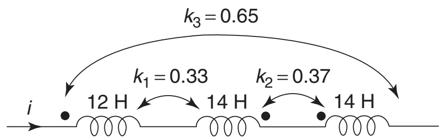


Fig. 4.22

Solution

$$M_{12} = M_{21} = k_1\sqrt{L_1 L_2} = 0.33\sqrt{(12)(14)} = 4.28 \text{ H}$$

$$M_{23} = M_{32} = k_2\sqrt{L_2 L_3} = 0.37\sqrt{(14)(14)} = 5.18 \text{ H}$$

$$M_{31} = M_{13} = k_3\sqrt{L_3 L_1} = 0.65\sqrt{(12)(14)} = 8.42 \text{ H}$$

$$\begin{aligned} L &= (L_1 - M_{12} + M_{13}) + (L_2 - M_{23} - M_{21}) + (L_3 + M_{31} - M_{32}) \\ &= (12 - 4.28 + 8.42) + (14 - 5.18 - 4.28) + (14 + 8.42 - 5.18) \\ &= 37.92 \text{ H} \end{aligned}$$

Example 4.12 *Find the equivalent inductance of the network shown in Fig. 4.23.*

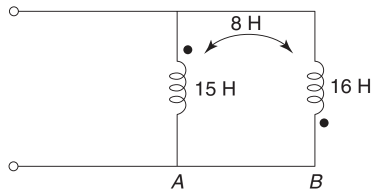


Fig. 4.23

Solution For Coil A,

$$L_A = L_1 - M_{12} = 15 - 8 = 7 \text{ H}$$

For Coil B,

$$L_B = L_2 - M_{12} = 16 - 8 = 8 \text{ H}$$

$$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B} = \frac{1}{7} + \frac{1}{8} = \frac{15}{56}$$

$$L = \frac{56}{15} = 3.73 \text{ H}$$

Example 4.13 *Find the equivalent inductance of the network shown in Fig. 4.24.*

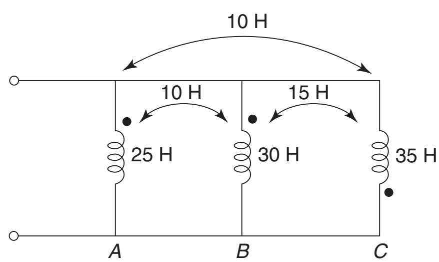


Fig. 4.24

Solution For Coil A,

$$L_A = L_1 + M_{12} - M_{13} = 25 + 10 - 10 = 25 \text{ H}$$

For Coil B,

$$L_B = L_2 - M_{23} + M_{21} = 35 - 15 + 10 = 25 \text{ H}$$

For Coil C,

$$L_C = L_3 - M_{32} - M_{31} = 35 - 15 - 10 = 10 \text{ H}$$

$$\frac{1}{L} = \frac{1}{L_A} + \frac{1}{L_B} + \frac{1}{L_C} = \frac{1}{25} + \frac{1}{25} + \frac{1}{10} = \frac{9}{50}$$

$$L = \frac{50}{9} = 5.55 \text{ H}$$

Example 4.14 *Find the equivalent impedance across the terminals A and B in Fig. 4.25.*

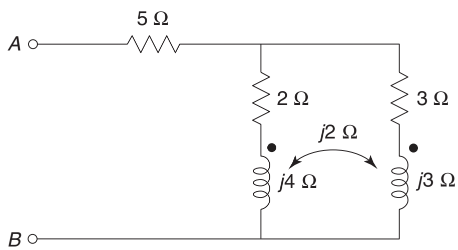


Fig. 4.25

Solution

$$\mathbf{Z}_1 = 5\,\Omega, \quad \mathbf{Z}_2 = (2 + j4)\,\Omega, \quad \mathbf{Z}_3 = (3 + j3)\,\Omega, \quad \mathbf{Z}_M = j2\,\Omega$$

$$\mathbf{Z} = \mathbf{Z}_1 + \frac{\mathbf{Z}_2\mathbf{Z}_3 - \mathbf{Z}_M^2}{\mathbf{Z}_2 + \mathbf{Z}_3 - 2\mathbf{Z}_M} = 5 + \frac{(2 + j4)\,(3 + j3) - (j2)^2}{2 + j4 + 3 + j3 - 2(j2)} = 6.9 \angle 24.16^\circ\ \Omega$$

4.8 COUPLED CIRCUITS

Consider two coils located physically close to one another as shown in Fig. 4.26.

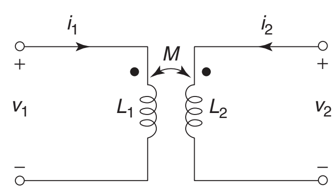


Fig. 4.26 *Coupled circuit*

When current i_1 flows in the first coil and $i_2 = 0$ in the second coil, flux ϕ_1 is produced in the coil. A fraction of this flux also links the second coil and induces a voltage in this coil. The voltage v_1 induced in the first coil is

$$v_1 = L_1 \left.\frac{di_1}{dt}\right|_{i_2 = 0}$$

The voltage v_2 induced in the second coil is

$$v_2 = M \left.\frac{di_1}{dt}\right|_{i_2 = 0}$$

The polarity of the voltage induced in the second coil depends on the way the coils are wound and it is usually indicated by dots. The dots signify that the induced voltages in the two coils (due to single current) have the same polarities at the dotted ends of the coils. Thus, due to i_1, the induced voltage v_1 must be positive at the dotted end of Coil 1. The voltage v_2 is also positive at the dotted end in Coil 2.

The same reasoning applies if a current i_2 flows in Coil 2 and $i_1 = 0$ in Coil 1. The induced voltages v_2 and v_1 are

$$v_2 = L_2 \left.\frac{di_2}{dt}\right|_{i_1=0}$$

and

$$v_1 = M \left.\frac{di_2}{dt}\right|_{i_1=0}$$

The polarities of v_1 and v_2 follow the dot convention. The voltage polarity is positive at the doted end of inductor L_2 when the current direction for i_2 is as shown in Fig. 4.26. Therefore, the voltage induced in Coil 1 must be positive at the dotted end also.

Now if both currents i_1 and i_2 are present, by using superposition principle, we can write

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

This can be represented in terms of dependent sources, as shown in Fig. 4.27.

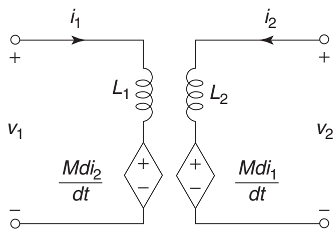


Fig. 4.27 *Equivalent circuit*

Now consider the case when the dots are placed at the opposite ends in the two coils, as shown in Fig. 4.28.

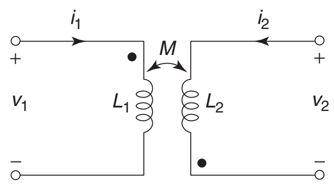


Fig. 4.28 *Coupled circuit*

Due to i_1, with $i_2 = 0$, the dotted end in Coil 1 is positive, so the induced voltage in Coil 2 is positive at the dot, which is the reverse of the designated polarity for v_2. Similarly, due to i_2, with $i_1 = 0$, the dotted ends have negative polarities for the induced voltages. The mutually induced voltages in both cases have polarities that are the reverse of terminal voltages and the equations are

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

This can be repressed in terms of dependent sources as shown in Fig. 4.29.

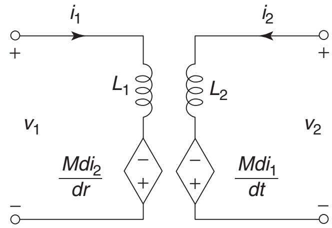


Fig. 4.29 *Equivalent circuit*

The various cases are summarised in the table shown in Fig. 4.30.

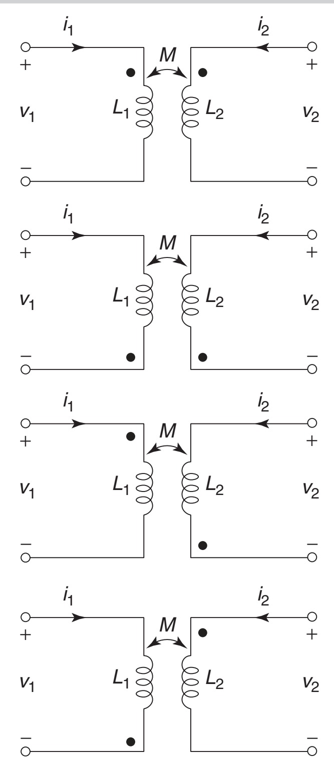
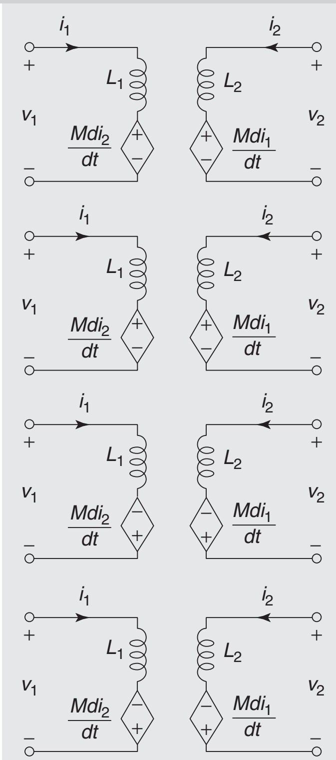
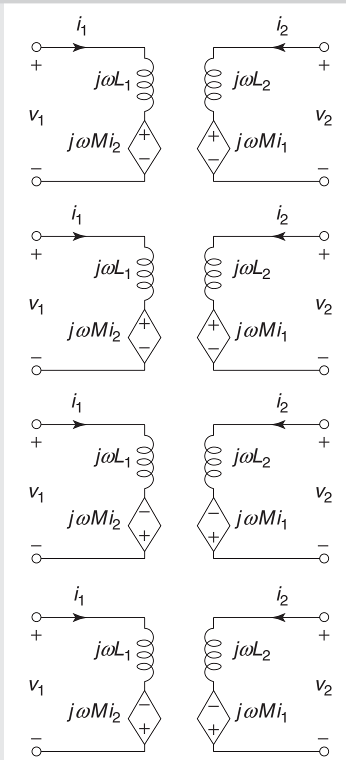
Coupled circuit	Time-domain equivalent circuit	Frequency-domain equivalent circuit
		

Fig. 4.30 *Coupled circuits for various cases*

Example 4.15 *Write mesh equations for the network shown in Fig. 4.31.*

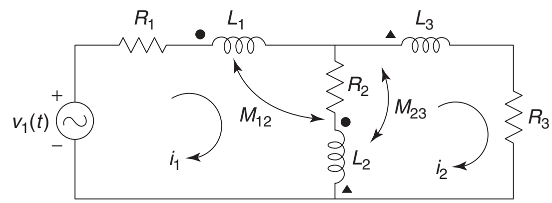


Fig. 4.31

Solution Coil 1 is magnetically coupled to Coil 2. Similarly, Coil 2 is magnetically coupled with Coil 1 and Coil 3. By applying dot convention, the equivalent circuit is drawn with the dependent sources.

The equivalent circuit in terms of dependent sources is shown in Fig. 4.32.

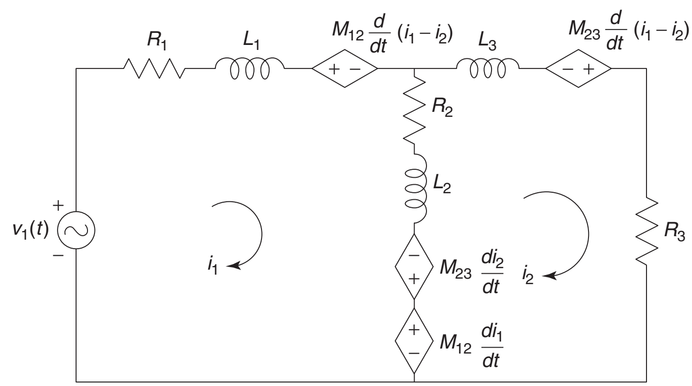


Fig. 4.32

(a) In Coil 1, there is a mutually induced emf due to current $(i_1 - i_2)$ in Coil 2. The polarity of the mutually induced emf is same as that of self-induced emf because currents i_1 and $(i_1 - i_2)$ enter in respective coils from the dotted ends.

(b) In Coil 2, there are two mutually induced emfs, one due to current i_1 in Coil 1 and the other due to current i_2 in Coil 3. The polarity of the mutually induced emf in Coil 2 due to the current i_1 is same as that of the self-induced emf because currents i_1 and $(i_1 - i_2)$ enter in respective coils from dotted ends. The polarity of the mutually induced emf in Coil 2 due to the current i_2 is opposite to that of the self-induced emf because current $(i_1 - i_2)$ leaves from the dotted end in Coil 2 and the current i_2 enters from the dotted end in Coil 3.

(c) In Coil 3, there is a mutually induced emf due to the current $(i_1 - i_2)$ in Coil 2. The polarity of the mutually induced emf is opposite to that of self-induced emf because the current $(i_1 - i_2)$ leaves from the dotted end in Coil 2 and the current i_2 enters from the dotted end in Coil 3.

Applying KVL to Mesh 1,

$$v_1(t) - R_1 i_1 - L_1\frac{di_1}{dt} - M_{12}\frac{d}{dt}(i_1 - i_2) - R_2(i_1 - i_2) - L_2\frac{d}{dt}(i_1 - i_2) + M_{23}\frac{di_2}{dt} - M_{12}\frac{di_1}{dt} = 0$$

$$(R_1+R_2)\,i_1+(L_1+L_2+2M_{12})\frac{di_1}{dt}-R_2 i_2-(L_2+M_{12}+M_{23})\frac{di_2}{dt}=v_1(t) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$M_{12}\frac{di_1}{dt}-M_{23}\frac{di_2}{dt}-L_2\frac{d}{dt}(i_2-i_1)-R_2(i_2-i_1)-L_3\frac{di_2}{dt}+M_{23}\frac{d}{dt}(i_1-i_2)-R_3 i_2=0$$

$$-R_2\,i_1-(L_2+M_{12}+M_{23})\frac{di_1}{dt}+(R_2+R_3)\,i_2+(L_2+L_3+2M_{23})\frac{di_2}{dt}=0 \qquad \ldots\text{(ii)}$$

Example 4.16 *Write KVL equations for the circuit shown in Fig. 4.33.*

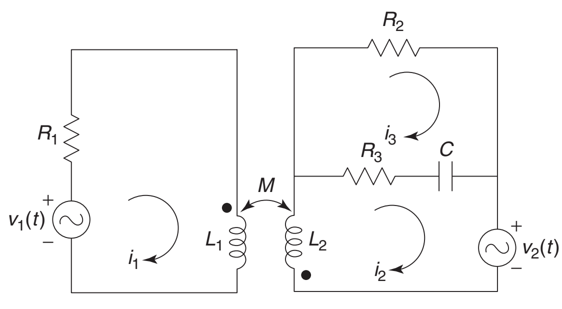


Fig. 4.33

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.34.

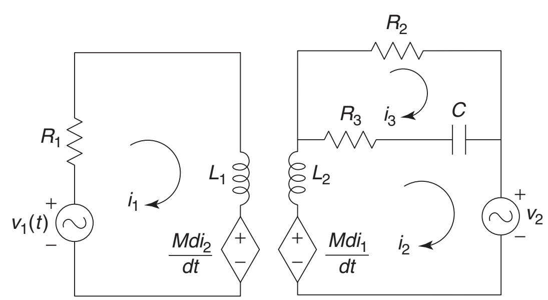


Fig. 4.34

Applying KVL to Mesh 1,

$$v_1(t)-R_1\,i_1-L_1\frac{di_1}{dt}-M\frac{di_2}{dt}=0$$

$$R_1\,i_1+L_1\frac{di_1}{dt}+M\frac{di_2}{dt}=v_1(t) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$M\frac{di_1}{dt}-L_2\frac{di_2}{dt}-R_3(i_2-i_3)-\frac{1}{C}\int_0^t(i_2-i_3)\,dt-v_2(t)=0$$

$$M\frac{di_1}{dt} - L_2\frac{di_2}{dt} - R_3(i_2 - i_3) - \frac{1}{C}\int_0^t (i_2 - i_3)\,dt = v_2(t) \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-R_2 i_3 - \frac{1}{C}\int_0^t (i_3 - i_2)dt - R_3(i_3 - i_2) = 0 \qquad \text{...(iii)}$$

Example 4.17 *Write down the mesh equations for the network shown in Fig. 4.35.*

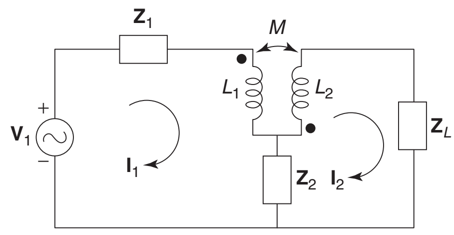


Fig. 4.35

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.36.

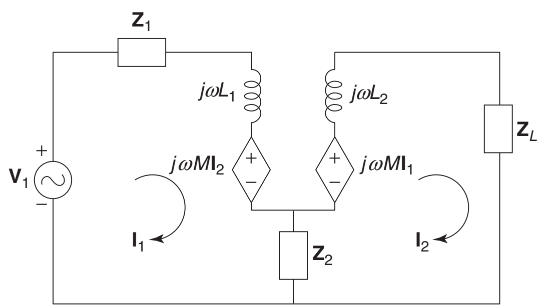


Fig. 4.36

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - \mathbf{Z}_1\mathbf{I}_1 - j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(\mathbf{Z}_1 + j\omega L_1 + \mathbf{Z}_2)\,\mathbf{I}_1 - (\mathbf{Z}_2 - j\omega M)\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) + j\omega M\mathbf{I}_1 - j\omega L_2\mathbf{I}_2 - \mathbf{Z}_L\mathbf{I}_2 = 0$$

$$-(\mathbf{Z}_2 - j\omega M)\,\mathbf{I}_1 + (\mathbf{Z}_2 + j\omega L_2 + \mathbf{Z}_L)\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Example 4.18 *Write mesh equations for the network shown in Fig. 4.37.*

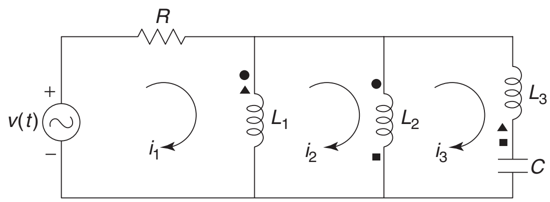


Fig. 4.37

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.38.

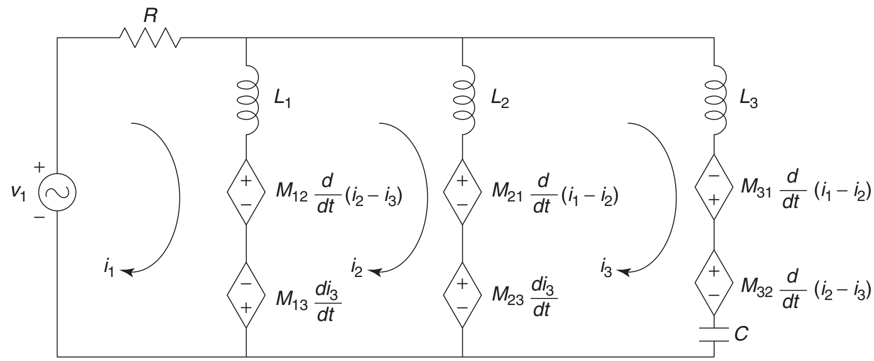


Fig. 4.38

Applying KVL to Mesh 1,

$$v(t) - Ri_1 - L_1\frac{d}{dt}(i_1 - i_2) - M_{12}\frac{d}{dt}(i_2 - i_3) + M_{13}\frac{di_3}{dt} = 0$$

$$Ri_1 + L_1\frac{d}{dt}(i_1 - i_2) + M_{12}\frac{d}{dt}(i_2 - i_3) - M_{13}\frac{di_3}{dt} = v(t) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-M_{13}\frac{di_3}{dt} + M_{12}\frac{d}{dt}(i_2 - i_3) - L_1\frac{d}{dt}(i_2 - i_1) - L_2\frac{d}{dt}(i_2 - i_3) - M_{21}\frac{d}{dt}(i_1 - i_2) - M_{23}\frac{di_3}{dt} = 0$$

$$M_{13}\frac{di_3}{dt} - M_{12}\frac{d}{dt}(i_2 - i_3) + L_1\frac{d}{dt}(i_2 - i_1) + L_2\frac{d}{dt}(i_2 - i_3) + M_{21}\frac{d}{dt}(i_1 - i_2) + M_{23}\frac{di_3}{dt} = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$M_{23}\frac{di_3}{dt} + M_{21}\frac{d}{dt}(i_1 - i_2) - L_2\frac{d}{dt}(i_3 - i_2) - L_3\frac{di_3}{dt} + M_{31}\frac{d}{dt}(i_1 - i_2) - M_{32}\frac{d}{dt}(i_2 - i_3) - \frac{1}{C}\int i_3\, dt = 0$$

$$-M_{23}\frac{di_3}{dt} - M_{21}\frac{d}{dt}(i_1 - i_2) + L_2\frac{d}{dt}(i_3 - i_2) + L_3\frac{di_3}{dt} - M_{31}\frac{d}{dt}(i_1 - i_2) + M_{32}\frac{d}{dt}(i_2 - i_3) + \frac{1}{C}\int i_3\, dt = 0 \qquad \ldots\text{(iii)}$$

Example 4.19 *Write KVL equations for the network shown in Fig. 4.39.*

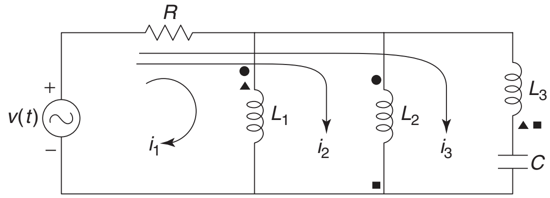


Fig. 4.39

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.40.

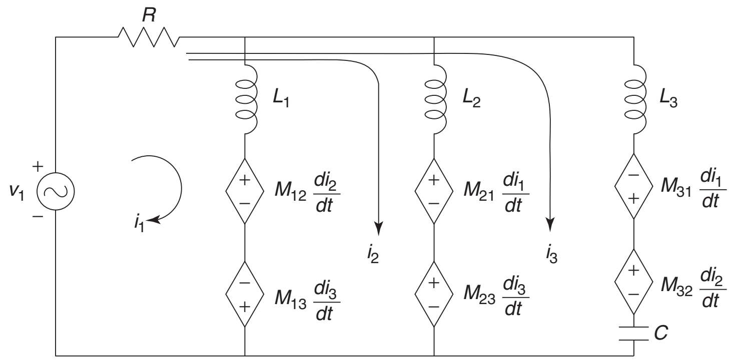


Fig. 4.40

Applying KVL to Loop 1,

$$v(t) - R(i_1 + i_2 + i_3) - L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt} = 0$$

$$R(i_1 + i_2 + i_3) + L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} - M_{13} \frac{di_3}{dt} = v(t) \qquad \ldots\text{(i)}$$

Applying KVL to Loop 2,

$$v(t) - R(i_1 + i_2 + i_3) - L_2 \frac{di_2}{dt} - M_{21} \frac{di_1}{dt} - M_{23} \frac{di_3}{dt} = 0$$

$$R(i_1 + i_2 + i_3) + L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt} + M_{23} \frac{di_3}{dt} = v(t) \qquad \ldots\text{(ii)}$$

Applying KVL to Loop 3,

$$v(t) - R(i_1 + i_2 + i_3) - L_3 \frac{di_3}{dt} + M_{31} \frac{di_1}{dt} - M_{32} \frac{di_2}{dt} - \frac{1}{C}\int i_3 \, dt = 0$$

$$R(i_1 + i_2 + i_3) + L_3 \frac{di_3}{dt} - M_{31} \frac{di_1}{dt} + M_{32} \frac{di_2}{dt} + \frac{1}{C}\int i_3 \, dt = v(t) \qquad \ldots\text{(iii)}$$

Example 4.20 *In the network shown in Fig. 4.41, find the voltages V_1 and V_2.*

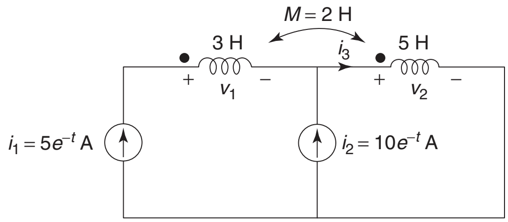


Fig. 4.41

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.42.

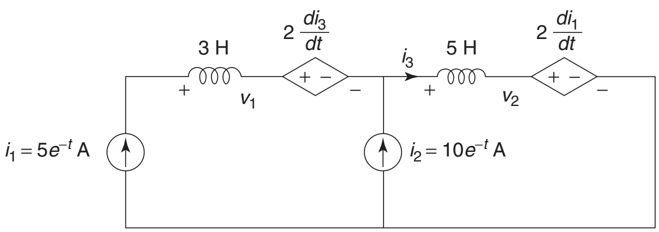


Fig. 4.42

From Fig. 4.42,

$$i_3 = i_1 + i_2 = 5\,e^{-t} + 10\,e^{-t} = 15\,e^{-t}\text{ A}$$

$$v_1 = 3\frac{di_1}{dt} + 2\frac{di_3}{dt} = 3\frac{d}{dt}(5\,e^{-t}) + 2\frac{d}{dt}(15\,e^{-t}) = -15\,e^{-t} - 30\,e^{-t} = -45\,e^{-t}\text{V}$$

$$v_2 = 5\frac{di_3}{dt} + 2\frac{di_1}{dt} = 5\frac{d}{dt}(15\,e^{-t}) + 2\frac{d}{dt}(5\,e^{-t}) = -75\,e^{-t} - 10\,e^{-t} = -85\,e^{-t}\text{V}$$

Example 4.21 *In the network shown in Fig. 4.43, find the voltages V_1 and V_2.*

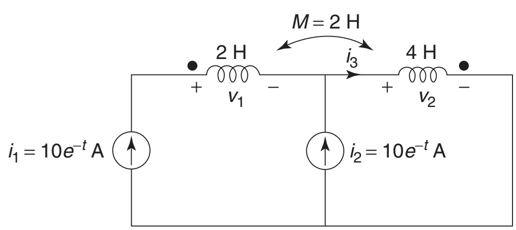


Fig. 4.43

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.44.

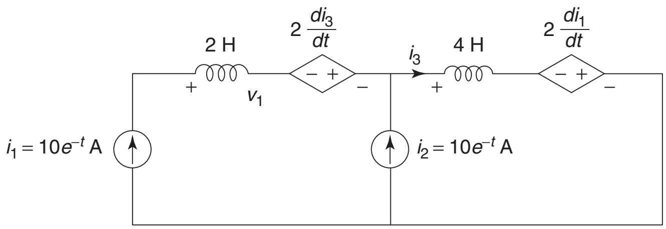


Fig. 4.44

From Fig. 4.44,

$$i_3 = i_1 + i_2 = 10\,e^{-t} + 10\,e^{-t} = 20\,e^{-t}\text{A}$$

$$v_1 = 2\frac{di_1}{dt} - 2\frac{di_3}{dt} = 2\frac{d}{dt}(10\,e^{-t}) - 2\frac{d}{dt}(20\,e^{-t}) = -20\,e^{-t} + 40\,e^{-t} = 20\,e^{-t}\text{A}$$

$$v_2 = 4\frac{di_3}{dt} - 2\frac{di_1}{dt} = 4\frac{d}{dt}(20\,e^{-t}) - 2\frac{d}{dt}(10\,e^{-t}) = -80\,e^{-t} + 20\,e^{-t} = -60\,e^{-t}\text{A}$$

Example 4.22 *Calculate the current $i_2(t)$ in the coupled circuit of Fig. 4.45.*

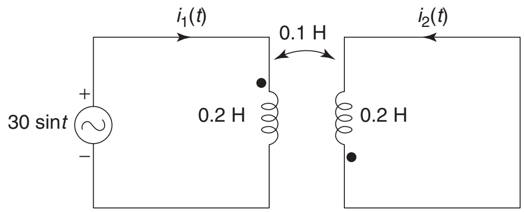


Fig. 4.45

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.46.

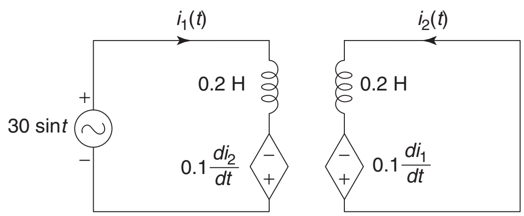


Fig. 4.46

Applying KVL to Mesh 1,

$$30 \sin t - 0.2\frac{di_1}{dt} + 0.1\frac{di_2}{dt} = 0 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-0.2\frac{di_2}{dt}+0.1\frac{di_1}{dt}=0$$

$$\frac{di_1}{dt}=2\frac{di_2}{dt} \qquad \ldots\text{(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$30\sin t-0.2\left(2\frac{di_2}{dt}\right)+0.1\frac{di_2}{dt}=0$$

$$0.3\frac{di_2}{dt}=30\sin t$$

$$\frac{di_2}{dt}=100\sin t$$

$$di_2=100\sin t\,dt$$

Integrating both the sides,

$$i_2\,(t)=100\int_0^t \sin t\,dt$$

$$=100\left[-\cos t\right]_0^t$$

$$=100\,(1-\cos t)$$

Example 4.23 *Find the voltage V_2 in the circuit shown in Fig. 4.47 such that the current in the left-hand loop (Loop 1) is zero.*

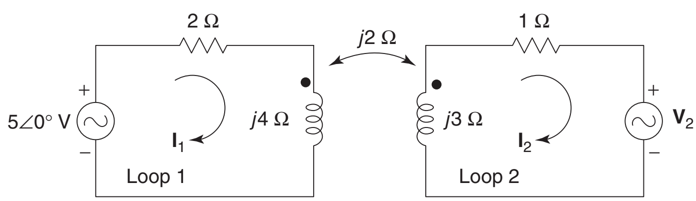


Fig. 4.47

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.48.

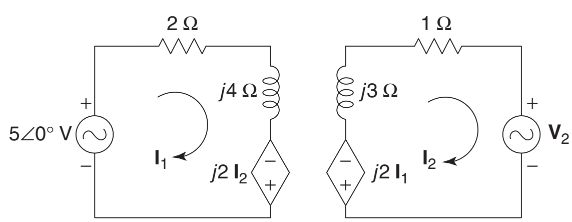


Fig. 4.48

Applying KVL to Loop 1,

$$5\angle 0° - 2\,\mathbf{I}_1 - j4\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$(2 + j4)\,\mathbf{I}_1 - j2\mathbf{I}_2 = 5\angle 0° \qquad \text{...(i)}$$

Applying KVL to Loop 2,

$$-j2\mathbf{I}_1 - j3\mathbf{I}_2 - 1\mathbf{I}_2 - \mathbf{V}_2 = 0$$

$$-j2\mathbf{I}_1 - (1 + j3)\,\mathbf{I}_2 = \mathbf{V}_2 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 2+j4 & -j2 \\ -j2 & -(1+j3) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5\angle 0° \\ \mathbf{V}_2 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 5\angle 0° & -j2 \\ \mathbf{V}_2 & -(1+j3) \end{vmatrix}}{\begin{vmatrix} 2+j4 & -j2 \\ -j2 & -(1+j3) \end{vmatrix}}$$

But $\mathbf{I}_1 = 0$.

$$\therefore -(5\angle 0°)(1 + j3) + j2\,\mathbf{V}_2 = 0$$

$$\mathbf{V}_2 = \frac{(5\angle 0°)(1 + j3)}{j2} = 7.91\angle -18.43°\text{ V}$$

Example 4.24 *Determine the ratio* $\dfrac{V_2}{V_1}$ *in the circuit of Fig. 4.49, if* $\boldsymbol{I}_1 = 0$.

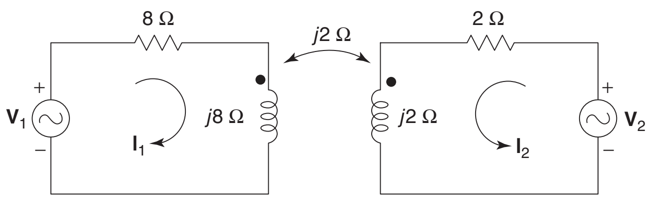


Fig. 4.49

Solution The equivalent circuit in terms of dependent sources is as shown in Fig. 4.50.

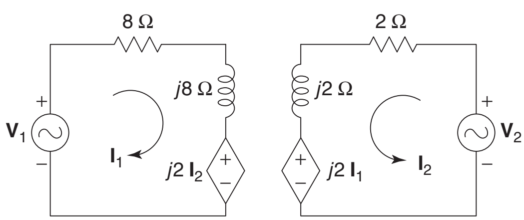


Fig. 4.50

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - 8\mathbf{I}_1 - j8\mathbf{I}_1 - j2\mathbf{I}_2 = 0$$

$$(8 + j8)\,\mathbf{I}_1 + j2\mathbf{I}_2 = \mathbf{V}_1 \qquad \text{...(i)}$$

Putting $\mathbf{I}_1 = 0$ in Eq (i),

$$j2\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$\mathbf{V}_2 - 2\mathbf{I}_2 - j2\mathbf{I}_2 - j2\mathbf{I}_1 = 0$$

$$j2\mathbf{I}_1 + (2 + j2)\,\mathbf{I}_2 = \mathbf{V}_2 \qquad \ldots\text{(iii)}$$

Putting $\mathbf{I}_1 = 0$ in Eq (iii),

$$(2 + j2)\,\mathbf{I}_2 = \mathbf{V}_2 \qquad \ldots\text{(iv)}$$

From Eqs (ii) and (iv),

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{(2 + j2)\mathbf{I}_2}{j2\mathbf{I}_2} = \frac{2 + j2}{j2} = 1.41\angle -45°\ \text{V}$$

Example 4.25 *For the coupled circuit shown in Fig. 4.51, find input impedance at terminals A and B.*

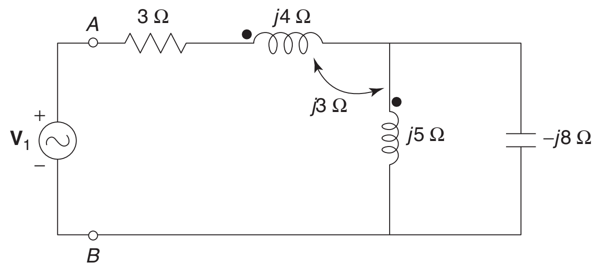


Fig. 4.51

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.52.

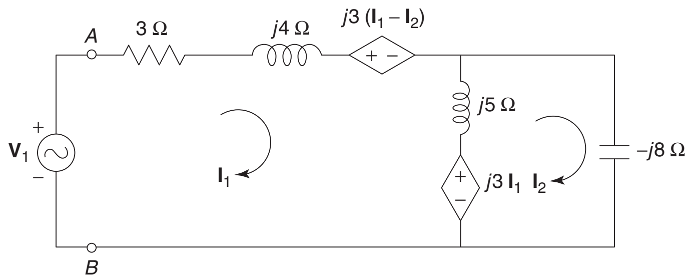


Fig. 4.52

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - 3\mathbf{I}_1 - j4\mathbf{I}_1 - j3\,(\mathbf{I}_1 - \mathbf{I}_2) - j5\,(\mathbf{I}_1 - \mathbf{I}_2) - j3\mathbf{I}_1 = 0$$

$$(3 + j15)\,\mathbf{I}_1 - j8\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$j3\,\mathbf{I}_1 - j5\,(\mathbf{I}_2 - \mathbf{I}_1) + j8\,\mathbf{I}_2 = 0$$

$$j8\,\mathbf{I}_1 + j3\,\mathbf{I}_2 = 0$$

$$\mathbf{I}_2 = -\frac{j8}{j3}\mathbf{I}_1 = -2.67\,\mathbf{I}_1 \qquad \ldots\text{(ii)}$$

Substituting Eq (ii) in Eq (i),

$$(3 + j15)\mathbf{I}_1 - j8\,(-2.67\,\mathbf{I}_1) = \mathbf{V}_1$$

$$(3 + j\,36.36)\,\mathbf{I}_1 = \mathbf{V}_1$$

$$\mathbf{Z}_i = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (3 + j\,36.36)\,\Omega = 36.48\,\angle\,85.28°\,\Omega$$

Example 4.26 *Find equivalent impedance of the network shown in Fig. 4.53.*

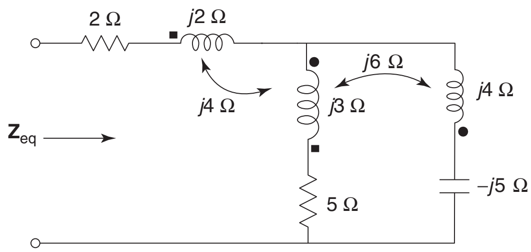


Fig. 4.53

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.54.

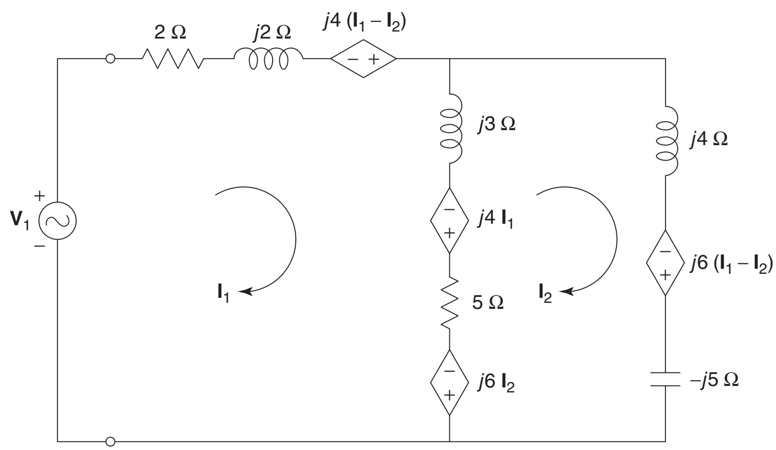


Fig. 4.54

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - 2\mathbf{I}_1 - j2\mathbf{I}_1 + j4\,(\mathbf{I}_1 - \mathbf{I}_2) - j3\,(\mathbf{I}_1 - \mathbf{I}_2) + j4\,\mathbf{I}_1 - 5(\mathbf{I}_1 - \mathbf{I}_2) + j6\mathbf{I}_2 = 0$$

$$(7 - j3)\,\mathbf{I}_1 - (5 + j5)\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-j6\mathbf{I}_2 - 5(\mathbf{I}_2 - \mathbf{I}_1) - j4\mathbf{I}_1 - j3(\mathbf{I}_2 - \mathbf{I}_1) - j4\mathbf{I}_2 + j6(\mathbf{I}_1 - \mathbf{I}_2) + j5\mathbf{I}_2 = 0$$

$$(5 + j5)\mathbf{I}_1 = (5 + j4)\,\mathbf{I}_2$$

$$\mathbf{I}_2 = \left(\frac{5 + j5}{5 + j4}\right)\mathbf{I}_1 \qquad \text{...(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$(7 - j3)\mathbf{I}_1 - (5 + j5)\left(\frac{5 + j5}{5 + j4}\right)\mathbf{I}_1 = \mathbf{V}_1$$

$$\mathbf{Z}_i = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 7 - j3 - \frac{(5 + j5)\,(5 + j5)}{5 + j4} = 5.63\,\angle -47.15°\ \Omega$$

Example 4.27 *Find the voltage across the 5 Ω resistor in Fig. 4.55 using mesh analysis.*

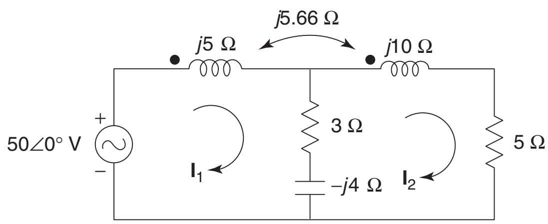


Fig. 4.55

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.56.

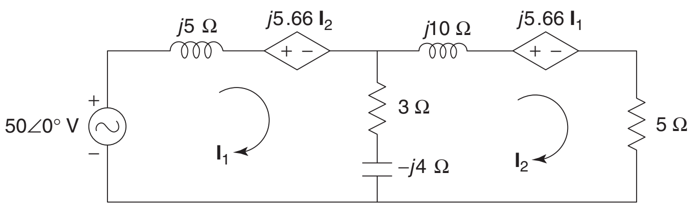


Fig. 4.56

Applying KVL to Mesh 1,

$$50\,\angle\,0° - j5\,\mathbf{I}_1 - j\,5.66\,\mathbf{I}_2 - (3 - j4)\,(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(3 + j\,1)\,\mathbf{I}_1 - (3 - j\,9.66)\,\mathbf{I}_2 = 50\,\angle\,0° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(3 - j4)\,(\mathbf{I}_2 - \mathbf{I}_1) - j\,10\mathbf{I}_2 - j\,5.66\,\mathbf{I}_1 - 5\mathbf{I}_2 = 0$$

$$-(3 - j\,9.66)\,\mathbf{I}_1 + (8 + j6)\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 3+j1 & -(3-j9.66) \\ -(3-j9.66) & 8+j6 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 3+j1 & 50\angle 0^\circ \\ -(3-j9.66) & 0 \end{vmatrix}}{\begin{vmatrix} 3+j1 & -(3-j9.66) \\ -(3-j9.66) & 8+j6 \end{vmatrix}} = 3.82\angle -112.14^\circ \text{ A}$$

$$\mathbf{V}_{5\,\Omega} = 5\,\mathbf{I}_2 = 5\,(3.82\angle -112.14^\circ) = 19.1\angle -112.14^\circ \text{ V}$$

Example 4.28 *Find the voltage across the 5 Ω resistor in Fig. 4.57 using mesh analysis.*

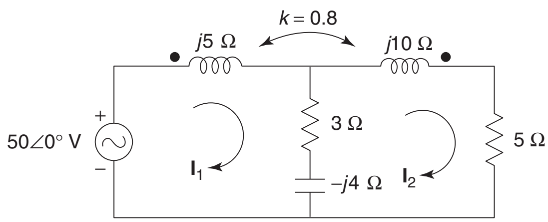


Fig. 4.57

Solution For a magnetically coupled circuit,

$$\begin{aligned} X_M &= k\sqrt{X_{L_1} X_{L_2}} \\ &= 0.8\sqrt{(5)\,(10)} \\ &= 5.66\ \Omega \end{aligned}$$

The equivalent circuit in terms of dependent sources is shown in Fig. 4.58.

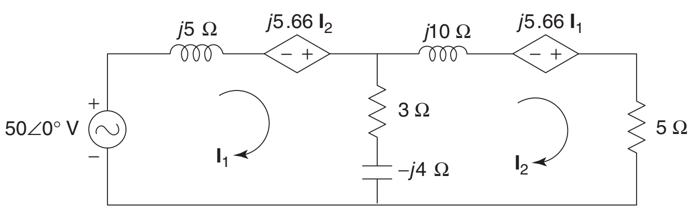


Fig. 4.58

Applying KVL to Mesh 1,

$$50\angle 0^\circ - j5\,\mathbf{I}_1 + j\,5.66\,\mathbf{I}_2 - (3-j4)\,(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$(3+j1)\,\mathbf{I}_1 - (3+j1.66)\,\mathbf{I}_2 = 50\angle 0^\circ \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(3-j4)(\mathbf{I}_2-\mathbf{I}_1)-j\,10\mathbf{I}_2+j\,5.66\,\mathbf{I}_1-5\mathbf{I}_2=0$$

$$-(3+j\,1.66)\,\mathbf{I}_1+(8+j6)\,\mathbf{I}_2=0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 3+j\,1 & -(3+j\,1.66) \\ -(3+j\,1.66) & 8+j\,6 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}=\begin{bmatrix} 50\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2=\frac{\begin{vmatrix} 3+j\,1 & 50\angle 0^\circ \\ -(3+j\,1.66) & 0 \end{vmatrix}}{\begin{vmatrix} 3+j\,1 & -(3+j\,1.66) \\ -(3+j\,1.66) & 8+j6 \end{vmatrix}}=8.62\angle -24.79^\circ \text{ A}$$

$$\mathbf{V}_{5\,\Omega}=5\,\mathbf{I}_2=5\,(8.62\angle -24.79^\circ)=43.1\angle -24.79^\circ \text{ A}$$

Example 4.29 *Find the current through the capacitor in Fig. 4.59 using mesh analysis.*

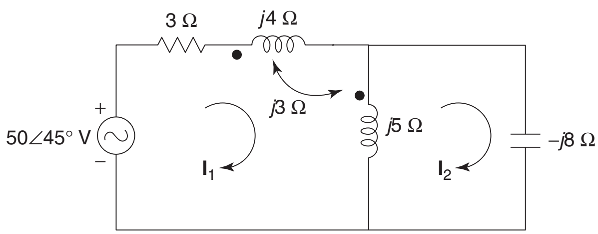


Fig. 4.59

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.60.

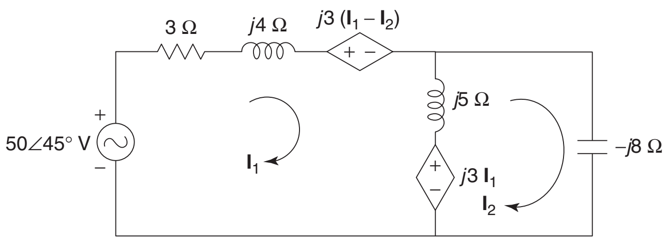


Fig. 4.60

Applying KVL to Mesh 1,

$$50\angle 45^\circ-(3+j4)\,\mathbf{I}_1-j3(\mathbf{I}_1-\mathbf{I}_2)-j5\,(\mathbf{I}_1-\mathbf{I}_2)-j3\,\mathbf{I}_1=0$$

$$(3+j\,15)\,\mathbf{I}_1-j\,8\,\mathbf{I}_2=50\angle 45^\circ \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$j3\,\mathbf{I}_1 - j5\,(\mathbf{I}_2 - \mathbf{I}_1) + j8\,\mathbf{I}_2 = 0$$

$$-j8\,\mathbf{I}_1 - j3\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 3+j\,15 & -j8 \\ -j8 & -j3 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle 45^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 3+j\,15 & 50\angle 45^\circ \\ -j8 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j\,15 & -j8 \\ -j8 & -j3 \end{vmatrix}} = 3.66\angle -310.33^\circ \text{ A}$$

$$\mathbf{I}_C = \mathbf{I}_2 = 3.66\angle -310.33^\circ\text{A}$$

Example 4.30 *Find the voltage across the 15 Ω resistor in Fig. 4.61 using mesh analysis.*

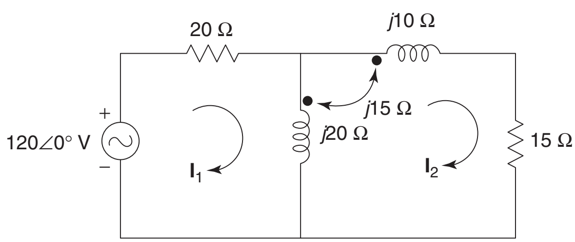


Fig. 4.61

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.62.

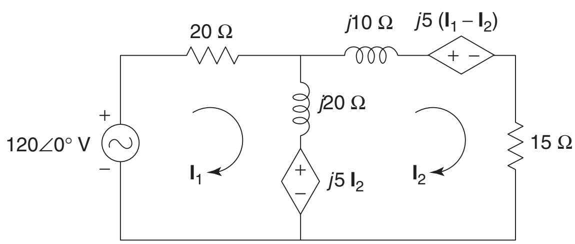


Fig. 4.62

Applying KVL to Mesh 1,

$$120\angle 0^\circ - 20\,\mathbf{I}_1 - j20(\mathbf{I}_1 - \mathbf{I}_2) - j5\,\mathbf{I}_2 = 0$$

$$(20 + j\,20)\,\mathbf{I}_1 - j\,15\,\mathbf{I}_2 = 120\angle 0^\circ \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$j5\,\mathbf{I}_2 - j\,20\,(\mathbf{I}_2 - \mathbf{I}_1) - j10\,\mathbf{I}_2 - j5\,(\mathbf{I}_1 - \mathbf{I}_2) - 15\,\mathbf{I}_2 = 0$$

$$-j15\,\mathbf{I}_1 + (15 + j20)\,\mathbf{I}_2 = 0 \qquad \text{...(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 20 + j\,20 & -j15 \\ -j15 & 15 + j\,20 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120 \angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 20 + j\,20 & 120 \angle 0° \\ -j15 & 0 \end{vmatrix}}{\begin{vmatrix} 20 + j\,20 & -j15 \\ -j15 & 15 + j\,20 \end{vmatrix}} = 2.53 \angle 10.12°\text{A}$$

$$\mathbf{V}_{15\,\Omega} = 15\,\mathbf{I}_2 = 15\,(2.53 \angle 10.12°) = 37.95 \angle 10.12°\text{V}$$

Example 4.31 *Find the current through the 6 Ω resistor in Fig. 4.63 using mesh analysis.*

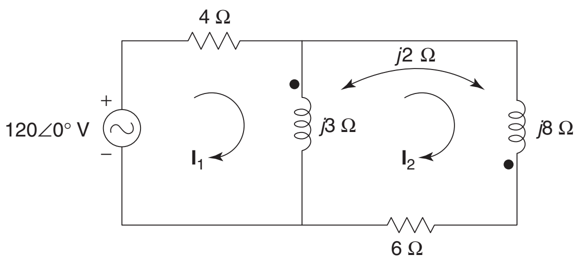


Fig. 4.63

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.64.

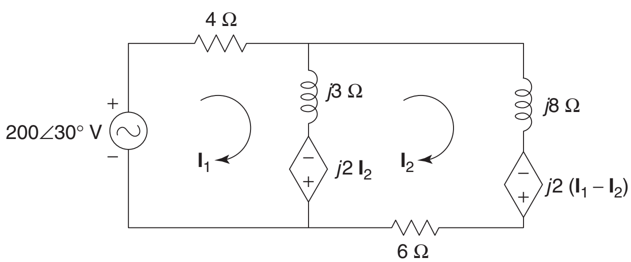


Fig. 4.64

Applying KVL to Mesh 1,

$$120 \angle 0° - 4\,\mathbf{I}_1 - j3(\mathbf{I}_1 - \mathbf{I}_2) + j2\,\mathbf{I}_2 = 0$$

$$(4 + j\,3)\,\mathbf{I}_1 - j\,5\,\mathbf{I}_2 = 120 \angle 0° \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-j\,2\,\mathbf{I}_2 - j\,3\,(\mathbf{I}_2 - \mathbf{I}_1) - j8\,\mathbf{I}_2 + j2\,(\mathbf{I}_1 - \mathbf{I}_2) - 6\,\mathbf{I}_2 = 0$$

$$-j5\,\mathbf{I}_1 + (6 + j15)\,\mathbf{I}_2 = 0 \qquad \ldots\text{(ii)}$$

Writing Eqs (i) and (ii) in matrix form,

$$\begin{bmatrix} 4 + j\,3 & -j5 \\ -j5 & 6 + j\,15 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120\angle 0^\circ \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 4 + j\,3 & 120\angle 0^\circ \\ -j5 & 0 \end{vmatrix}}{\begin{vmatrix} 4 + j\,3 & -j5 \\ -j5 & 6 + j\,15 \end{vmatrix}} = 7.68\angle 2.94^\circ\text{A}$$

Example 4.32 *Determine the mesh current I_3 in the network of Fig. 4.65.*

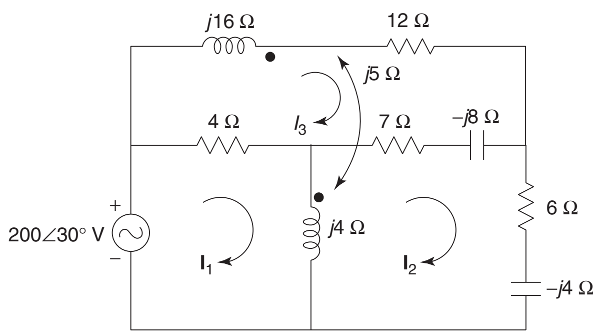


Fig. 4.65

Solution The equivalent circuit in terms of dependent sources is shown in Fig. 4.66.

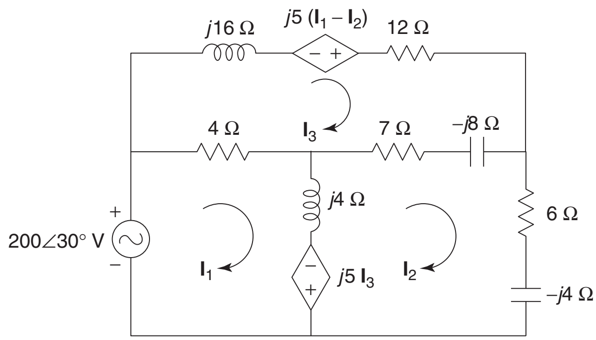


Fig. 4.66

Applying KVL to Mesh 1,

$$200\angle 30° - 4(\mathbf{I}_1 - \mathbf{I}_3) - j4(\mathbf{I}_1 - \mathbf{I}_2) + j5\,\mathbf{I}_3 = 0$$

$$(4 + j4)\,\mathbf{I}_1 - j4\,\mathbf{I}_2 - (4 + j5)\,\mathbf{I}_3 = 200\angle 30° \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j5\,\mathbf{I}_3 - j4(\mathbf{I}_2 - \mathbf{I}_1) - (7 - j8)(\mathbf{I}_2 - \mathbf{I}_3) - (6 - j4)\,\mathbf{I}_2 = 0$$

$$-j4\,\mathbf{I}_1 + (13 - j8)\,\mathbf{I}_2 - (7 - j13)\,\mathbf{I}_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-j16\,\mathbf{I}_3 + j5(\mathbf{I}_1 - \mathbf{I}_2) - 12\,\mathbf{I}_3 - (7 - j8)(\mathbf{I}_3 - \mathbf{I}_2) - 4(\mathbf{I}_3 - \mathbf{I}_1) = 0$$

$$-(4 + j5)\,\mathbf{I}_1 - (7 - j13)\,\mathbf{I}_2 + (23 + j8)\,\mathbf{I}_3 = 0 \qquad \ldots\text{(iii)}$$

Writing Eqs. (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 4 + j4 & -j4 & -(4 + j5) \\ -j4 & 13 - j8 & -(7 - j13) \\ -(4 + j5) & -(7 - j13) & 23 + j8 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 200\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 4 + j4 & -j4 & 200\angle 30° \\ -j4 & 13 - j8 & 0 \\ -(4 + j5) & -(7 - j13) & 0 \end{vmatrix}}{\begin{vmatrix} 4 + j4 & -j4 & -(4 + j5) \\ -j4 & 13 - j8 & -(7 - j13) \\ -(4 + j5) & -(7 - j13) & 23 + j8 \end{vmatrix}} = 16.28\angle 16.87°\,\text{A}$$

Example 4.33 *Obtain the dotted equivalent circuit for the coupled circuit shown in Fig. 4.67 and find mesh currents. Also find the voltage across the capacitor.*

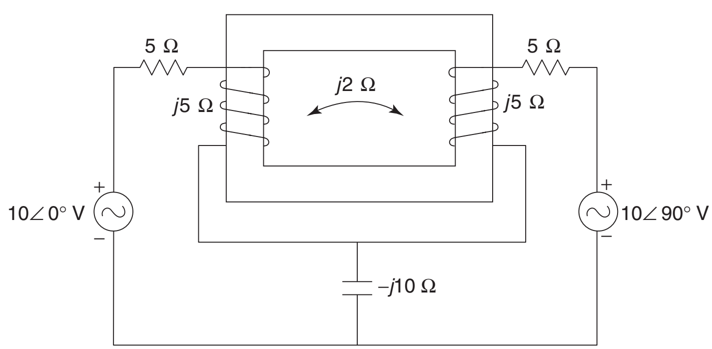


Fig. 4.67

Solution The currents in the coils are as shown in Fig. 4.68. The corresponding flux due to current in each coil is also drawn with the help of right-hand thumb rule.

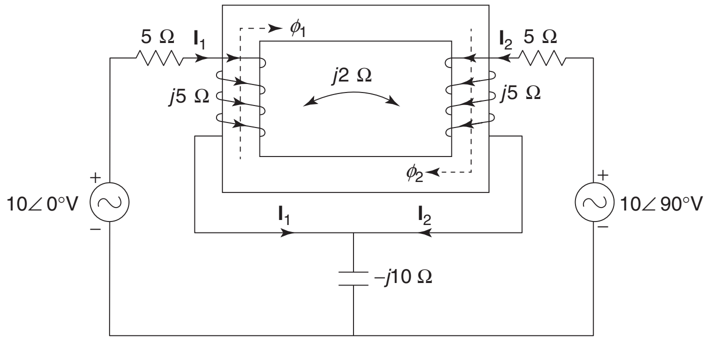


Fig. 4.68

From Fig. 4.68, it is seen that two fluxes ϕ_1 and ϕ_2 aid each other. Hence, dots are placed at the two coils as shown in Fig. 4.69.

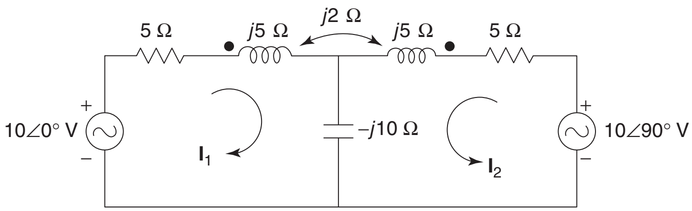


Fig. 4.69

The equivalent circuit in terms of dependent sources is shown in Fig. 4.70.

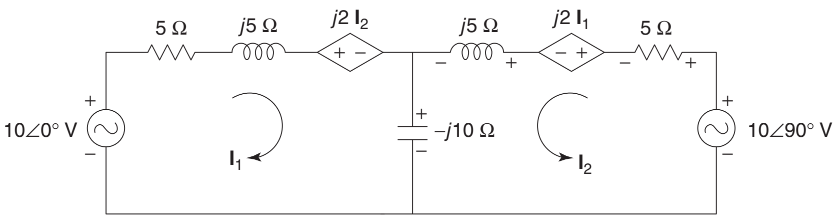


Fig. 4.70

Applying KVL to Mesh 1,

$$10\angle 0^\circ - (5 + j5)\,\mathbf{I}_1 - j2\,\mathbf{I}_2 + j10\,(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$(5 - j5)\,\mathbf{I}_1 - j8\,\mathbf{I}_2 = 10\angle 0^\circ \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-j10\,(\mathbf{I}_2 + \mathbf{I}_1) + j5\,\mathbf{I}_2 - j2\,\mathbf{I}_1 + 5\,\mathbf{I}_2 - 10\angle 90^\circ = 0$$

$$-j8\,\mathbf{I}_1 + (5 - j5)\,\mathbf{I}_2 = 10\angle 90^\circ \qquad \ldots\text{(ii)}$$

Writing Eqs. (i) and (ii) in matrix form,

$$\begin{bmatrix} 5 - j5 & -j8 \\ -j8 & 5 - j5 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 10\angle 90^\circ \end{bmatrix}$$

By Cramer's rule,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 10 \angle 0^\circ & -j8 \\ 10 \angle 90^\circ & 5 - j5 \end{vmatrix}}{\begin{vmatrix} 5 - j5 & -j8 \\ -j8 & 5 - j5 \end{vmatrix}} = 0.72 \angle -82.97^\circ \text{A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 5 - j5 & 10 \angle 0^\circ \\ -j8 & 10 \angle 90^\circ \end{vmatrix}}{\begin{vmatrix} 5 - j5 & -j8 \\ -j8 & 5 - j5 \end{vmatrix}} = 1.71 \angle 106.96^\circ \text{A}$$

$$\mathbf{V}_C = -j\,10\,(\mathbf{I}_1 + \mathbf{I}_2) = (-j\,10)\,(0.72 \angle -82.97^\circ + 1.71 \angle 106.96^\circ \text{ A})$$

$$= 10.08 \angle 24.03^\circ \text{V}$$

4.9 CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS

For simplifying circuit analysis, it is desirable to replace a magnetically coupled circuit with an equivalent circuit called conductively coupled circuit. In this circuit, no magnetic coupling is involved. The dot convention is also not needed in the conductively coupled circuit.

Consider a coupled circuit as shown in Fig. 4.71.

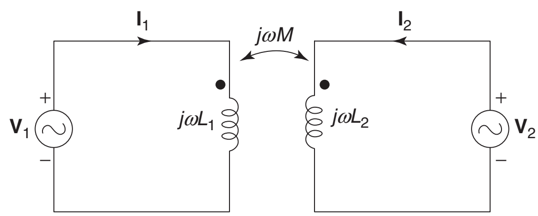


Fig. 4.71 *Coupled circuit*

The equivalent circuit in terms of dependent sources is shown in Fig. 4.72.

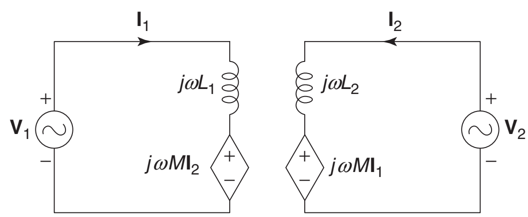


Fig. 4.72 *Equivalent circuit*

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - j\omega L_1\, \mathbf{I}_1 - j\omega M\, \mathbf{I}_2$$

$$j\omega L_1\, \mathbf{I}_1 + j\omega M\, \mathbf{I}_2 \qquad \ldots(4.11)$$

Applying KVL to Mesh 2,

$$\mathbf{V}_2 - j\omega L_2\, \mathbf{I}_2 - j\omega M\, \mathbf{I}_1 = 0$$

$$j\omega M\, \mathbf{I}_1 + j\omega L_2\, \mathbf{I}_2 = \mathbf{V}_2 \qquad \ldots(4.12)$$

Writing Eqs (4.11) and (4.12) in matrix form,

$$\begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad \ldots(4.13)$$

Consider a T-network as shown in Fig. 4.73.

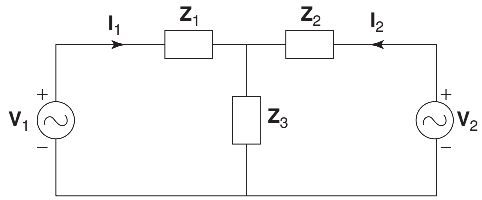


Fig. 4.73 *T-network*

Applying KVL to Mesh 1,

$$\mathbf{V}_1 - \mathbf{Z}_1\,\mathbf{I}_1 - \mathbf{Z}_3(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$(\mathbf{Z}_1 + \mathbf{Z}_3)\,\mathbf{I}_1 + \mathbf{Z}_3\,\mathbf{I}_2 = \mathbf{V}_1 \qquad \ldots(4.14)$$

Applying KVL to Mesh 2,

$$R_{\text{Th}} = [(2 \,||\, 12) + 1] \,||\, 3 = 1.43\,\Omega \qquad \ldots(4.15)$$

Writing Eqs (4.14) and (4.15) in matrix form,

$$\begin{bmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & \mathbf{Z}_3 \\ \mathbf{Z}_3 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Comparing matrix equations,

$$\mathbf{Z}_1 + \mathbf{Z}_3 = j\,\omega\,L_1$$

$$\mathbf{Z}_3 = j\,\omega\,M$$

$$\mathbf{Z}_2 + \mathbf{Z}_3 = j\,\omega\,L_2$$

Solving these equations,

$$\mathbf{Z}_1 = j\,\omega\,L_1 - j\,\omega\,M = j\,\omega(L_1 - M)$$

$$\mathbf{Z}_2 = j\,\omega\,L_2 - j\,\omega\,M = j\,\omega(L_2 - M)$$

$$\mathbf{Z}_3 = j\,\omega\,M$$

Hence, the conductively coupled circuit of a magnetically coupled circuit is shown in Fig. 4.74.

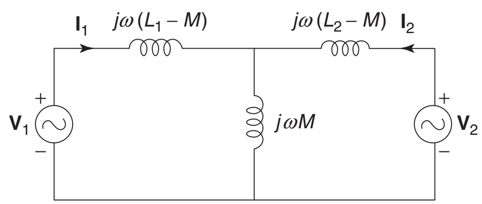


Fig. 4.74 *Conductively coupled equivalent circuit*

Example 4.34 *Find the conductively coupled equivalent circuit for the network shown in Fig. 4.75.*

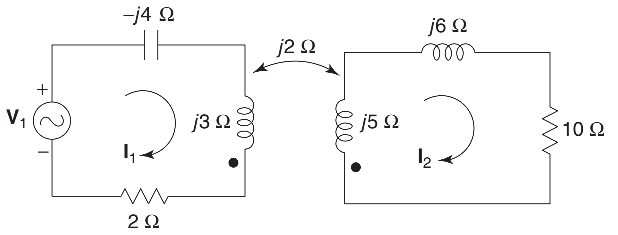


Fig. 4.75

Solution The current $\mathbf{I}_1$ leaves from the dotted end and $\mathbf{I}_2$ enters from the dotted end. Hence, mutual inductance M is negative.

In the conductively coupled equivalent circuit,

$$\mathbf{Z}_1 = j\omega(L_1 - M) = j\omega L_1 - j\omega M = j3 - j2 = j1\,\Omega$$

$$\mathbf{Z}_2 = j\omega(L_2 - M) = j\omega L_2 - j\omega M = j5 - j2 = j3\,\Omega$$

$$\mathbf{Z}_3 = j\omega M = j2\,\Omega$$

The conductively coupled equivalent circuit is shown in Fig. 4.76.

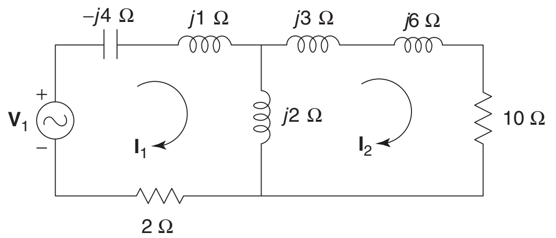


Fig. 4.76

Example 4.35 *Draw the conductively coupled equivalent circuit of Fig. 4.77.*

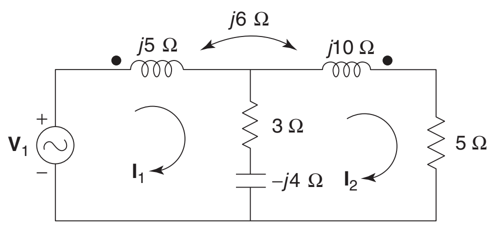


Fig. 4.77

Solution The current $\mathbf{I}_1$ enters from the dotted end and $\mathbf{I}_2$ leaves from the dotted end. Hence, the mutual inductance M is negative.

In the conductively coupled equivalent circuit,

$$\mathbf{Z}_1 = j\omega(L_1 - M) = j\omega L_1 - j\omega M = j5 - j6 = -j1\,\Omega$$
$$\mathbf{Z}_2 = j\omega(L_2 - M) = j\omega L_2 - j\omega M = j10 - j6 = j4\,\Omega$$
$$\mathbf{Z}_3 = j\omega M = j6\,\Omega$$

The conductively coupled equivalent circuit is shown in Fig. 4.78.

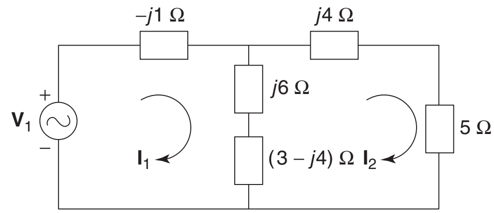


Fig. 4.78

Example 4.36 *Find the conductively coupled equivalent circuit of the network in Fig. 4.79.*

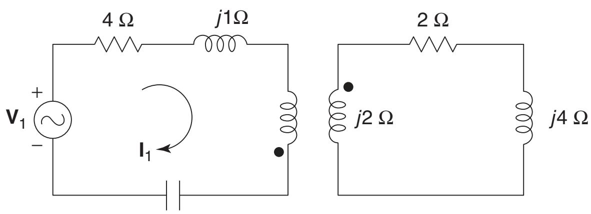


Fig. 4.79

Solution The currents $\mathbf{I}_1$ and $\mathbf{I}_2$ leave from the dotted terminals. Hence, mutual inductance is positive. In the conductively coupled equivalent circuit,

$$\mathbf{Z}_1 = j\omega(L_1 + M) = j\omega L_1 + j\omega M = j4 + j2 = j6\,\Omega$$
$$\mathbf{Z}_2 = j\omega(L_2 + M) = j\omega L_2 + j\omega M = j2 + j2 = j4\,\Omega$$
$$\mathbf{Z}_3 = -j\omega M = -j2\,\Omega$$

The conductively coupled equivalent circuit is shown in Fig. 4.80.

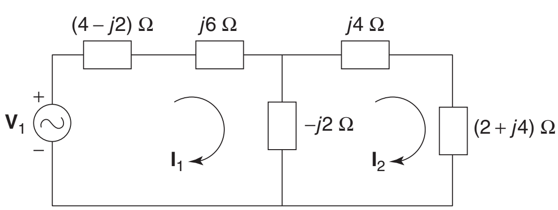


Fig. 4.80

Exercises

4.1 Two coupled coils have inductances of 0.8 H and 0.2 H. The coefficient of coupling is 0.90. Find the mutual inductance and the turns ratio $\dfrac{N_1}{N_2}$.

[0.36 H, 2]

4.2 Two coils with coefficient of coupling 0.5 are connected in such a way that they magnetise (i) in the same direction, and (ii) in opposite directions. The corresponding equivalent inductances are 1.9 H and 0.7 H. Find self-inductances of the two coils and the mutual inductance between them.

[0.4 H, 0.9 H, 0.3 H]

4.3 Two coils having 3000 and 2000 turns are wound on a magnetic ring. 60% of the flux produced in the first coil links with the second coil. A current of 3 A produce a flux of 0.5 mwb in the first coil and 0.3 mwb in the second coil. Determine the mutual inductance and coefficient of coupling.

[0.2 H, 0.63]

4.4 Find the equivalent inductance of the network shown in Fig. 4.81.

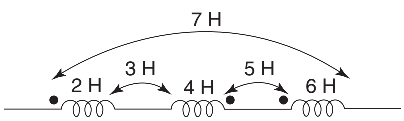


Fig. 4.81

[10 H]

4.5 Find the effective inductance of the network shown in Fig. 4.82.

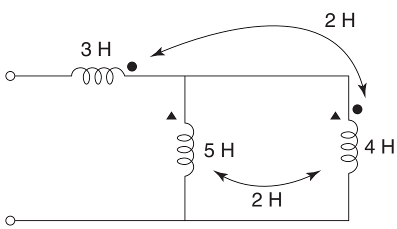


Fig. 4.82

[4.8 H]

4.6 Write mesh equations of the network shown in Fig. 4.83.

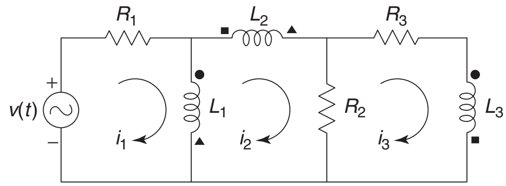


Fig. 4.83

$$\left[\begin{array}{l} v = i_1R_1 + L_1\dfrac{d}{dt}(i_1 - i_2) + M_{12}\dfrac{di_2}{dt} + M_{13}\dfrac{di_3}{dt} \\ R_2(i_3 - i_2) + R_3 i_3 + L_3\dfrac{di_3}{dt} + M_{13}\dfrac{d}{dt}(i_1 - i_2) \\ -M_{23}\dfrac{di_2}{dt} = 0 \end{array}\right]$$

4.7 Find the input impedance at terminals *AB* of the coupled circuits shown in Fig. 4.84 to 4.85.

(i)

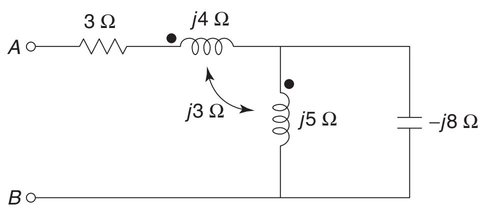


Fig. 4.84

(ii)

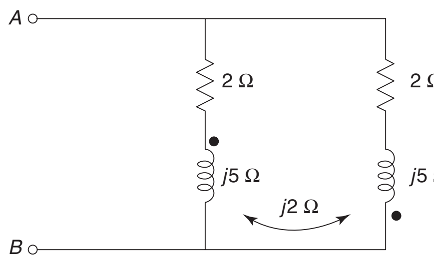


Fig. 4.85

(iii)

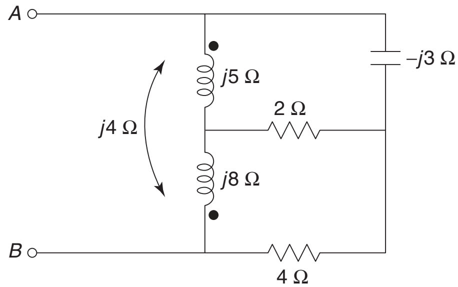


Fig. 4.86

$$\begin{bmatrix} \text{(a)}\,(3+j36.3)\,\Omega \quad \text{(b)}\,(1+j1.5)\,\Omega \\ \text{(c)}\,(6.22+j4.65)\,\Omega \end{bmatrix}$$

4.8 In the coupled circuit shown in Fig. 4.87, find $\mathbf{V}_2$ for which $\mathbf{I}_1 = 0$. What voltage appears at the 8 Ω inductive reactance under this condition?

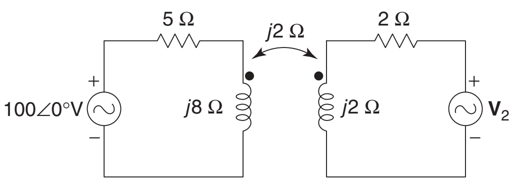


Fig. 4.87

[141.5∠ − 45°V, 100∠0°V]

4.9 For the coupled circuit shown in Fig. 4.88, find the components of the current $\mathbf{I}_2$ resulting from each source $\mathbf{V}_1$ and $\mathbf{V}_2$.

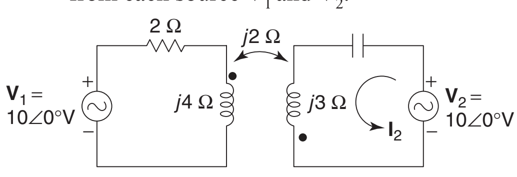


Fig. 4.88

[0.77∠112.6°A, 1.72∠86.05°A]

4.10 Find the voltage across the 5 Ω resistor in the network shown in Fig. 4.89.

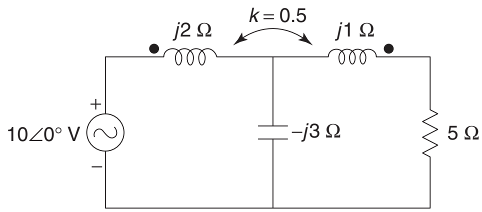


Fig. 4.89

[19.2∠ − 33.02°V]

4.11 Find the power dissipated in the 5 Ω resistor in the network of Fig. 4.90.

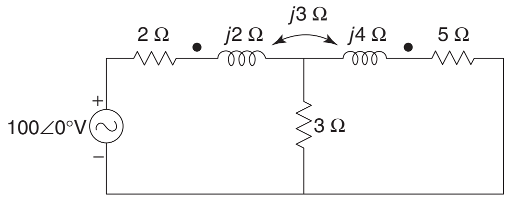


Fig. 4.90

[668.16 W]

4.12 Find the current **I** in the circuit of Fig. 4.91.

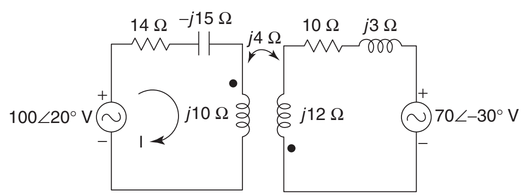


Fig. 4.91

[7.07∠45° V, 1.

4.13 Obtain a conductively coupled circuit for the circuit shown in Fig. 4.92.

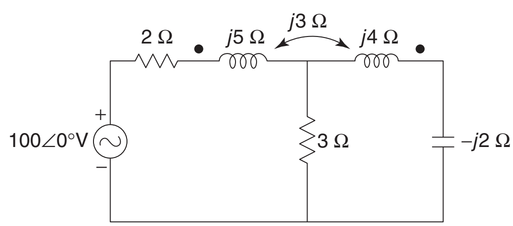


Fig. 4.92

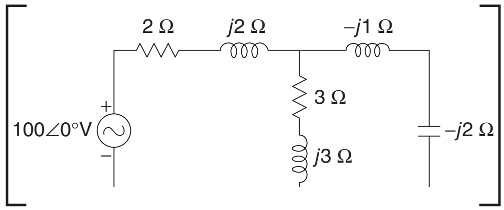


Fig. 4.93

Objective-Type Questions

4.1 Two coils are wound on a common magnetic core. The sign of mutual inductance M for finding out effective inductance of each coil is positive if the

(a) two coils are wound in the same sense.
(b) fluxes produced by the two coils are equal
(c) fluxes produced by the coils act in the same direction
(d) fluxes produced by the two coils act in opposition

4.2 When two coils having self-inductances of L_1 and L_2 are coupled through a mutual inductance M, the coefficient of coupling k is given by

(a) $k = \dfrac{M}{\sqrt{2L_1L_2}}$ (b) $k = \dfrac{M}{\sqrt{L_1L_2}}$

(c) $k = \dfrac{2M}{\sqrt{L_1L_2}}$ (d) $k = \dfrac{L_1L_2}{M}$

4.3 The overall inductance of two coils connected in series, with mutual inductance aiding self-inductance is L_1; with mutual inductance opposing self-inductance, the overall inductance is L_2. The mutual inductance M is given by

(a) $L_1 + L_2$ (b) $L_1 - L_2$

(c) $\dfrac{1}{4}(L_1 - L_2)$ (d) $\dfrac{1}{2}(L_1 + L_2)$

4.4 Consider the following statements:
The coefficient of coupling between two oils depends upon
1. Orientation of the coils
2. Core material
3. Number of turns on the two coils
4. Self-inductance of the two coils
of these statements,
(a) 1, 2 and 3 are correct
(b) 1 and 2 are correct
(c) 3 and 4 are correct
(d) 1, 2 and 4 are correct

4.5 Two coupled coils connected in series have an equivalent inductance of 16 mH or 8 mH depending on the inter connection.
Then the mutual inductance M between the coils is
(a) 12 mH (b) $8\sqrt{2}$ mH
(c) 4 mH (d) 2 mH

4.6 Two coupled coils with $L_1 = L_2 = 0.6$ H have a coupling coefficient of $k = 0.8$. The turns ratio $\dfrac{N_1}{N_2}$ is
(a) 4 (b) 2
(c) 1 (d) 0.5

4.7 The coupling between two magnetically coupled coils is said to be ideal if the coefficient of coupling is
(a) zero (b) 0.5
(c) 1 (d) 2

4.8 The mutual inductance between two coupled coils is 10 mH . If the turns in one coil are doubled and that in the other are halved then the mutual inductance will be
(a) 5 mH (b) 10 mH
(c) 14 mH (d) 20 mH

4.9 Two perfectly coupled coils each of 1 H self-inductance are connected in parallel so as to aid each other. The overall inductance in henrys is

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) Zero

4.10 The impedance Z as shown in Fig. 4.94 is

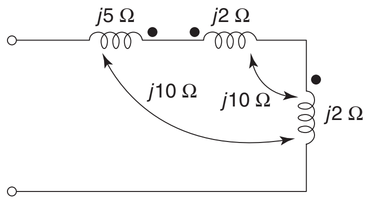


Fig. 4.94

(a) $j\,29\ \Omega$ (b) $j9\ \Omega$

(c) $j19\ \Omega$ (d) $j39\ \Omega$

Answers to Objective-Type Questions

4.1 (c)	4.2 (b)	4.3 (c)	4.4 (d)	4.5 (d)	4.6 (c)
4.7 (c)	4.8 (b)	4.9 (b)	4.10 (b)		