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Signals and Systems

Tutorial 3

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Vaibhav singh
58, D9A

Q. ①

① $x(t) = \delta(t-t_0)$

using time shifting property,

$$\mathcal{L}\{x(t \pm a)\} = e^{\pm as} X(s)$$

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-t_0 s} \cdot \mathcal{L}\{\delta(t)\}$$

$\delta(t)$ = impulse signal

$$\therefore \mathcal{L}\{\delta(t)\} = 1$$

$$\therefore \mathcal{L}\{\delta(t-t_0)\} = e^{-t_0 s}$$

ROC: entire s-plane

② $x(t) = u(t-t_0)$

using time shifting property

$$\mathcal{L}\{x(t \pm a)\} = e^{\pm as} X(s)$$

$$\mathcal{L}\{u(t-t_0)\} = e^{-t_0 s} \cdot \mathcal{L}\{u(t)\}$$

$u(t)$ = unit step signal

$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$\therefore \mathcal{L}\{u(t-t_0)\} = e^{-t_0 s} \cdot \frac{1}{s}$$

ROC: Right half of the s-plane

(3) $x(t) = e^{-2t}[u(t) - u(t-5)]$

$$x(t) = e^{-2t}.u(t) - e^{-2t}u(t-5)$$

$$= x_1(t) - x_2(t)$$

$$X(S) = X_1(S) - X_2(S) \quad \text{———— (A)}$$

$$x_1(t) = e^{-2t}.u(t)$$

By frequency shifting property,

$$L\{e^{\pm at}x(t)\} = X(s \mp a)$$

$$X_1(S) = \frac{1}{s+2} \quad \text{———— (1)}$$

$$x_2(t) = e^{-2t}u(t-5)$$

using Frequency shifting & Time shifting property

i.e. $L\{e^{\pm at}x(t)\} = X(s \mp a)$ and

$L\{x(t \pm a)\} = e^{\pm as}X(S)$ resp.

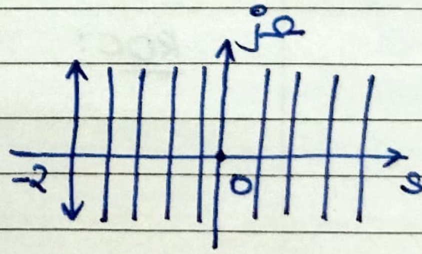
$$X_2(S) = \frac{1}{s+2} \cdot e^{-5(s+2)} \quad \text{———— (2)}$$

using (1) and (2) in (A)

$$X(S) = X_1(S) - X_2(S)$$

$$= \frac{1}{s+2} - e^{-5(s+2)}\frac{1}{s+2}$$

$$= \frac{1}{s+2}\left(1 - e^{-5(s+2)}\right)$$



ROC

Pole: at $s = -2$

causal and stable system.

ROC $\sigma > -2$

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Q ②

Given input: $x(t) = u(t)$

impulse response: $h(t) = e^{-2t} u(t)$

To find

output of CTS.

Soln. $y(t) = x(t).\, h(t)$, where $y(t)$ = output

Taking Laplace on both sides,

$Y(s) = X(s).\, H(s)$

$x(t) = u(t)$ and $h(t) = e^{-2t} u(t)$

$X(s) = \frac{1}{s}$ $= \frac{1}{s+2}$

$\therefore Y(s) = \frac{1}{s} . \frac{1}{s+2}$

Using Partial Fraction method,

$$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

we get $A = \frac{1}{2}$, $B = \frac{-1}{2}$

$$\therefore Y(s) = \frac{1}{2s} - \frac{1}{2(s+2)}$$

Taking inverse Laplace Transform,

$$y(t) = \frac{1}{2} u(t) - \frac{1}{2} e^{-2t} u(t)$$

$$y(t) = \frac{1}{2} u(t) \left[1 - e^{-2t}\right].$$

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(3) Given Transfer Function: $H(S) = \dfrac{S+3}{S^2+6S+8}$

To find : (i). impulse response and step response

Soln. T.F $H(S) = \dfrac{S+3}{S^2+6S+8} = \dfrac{I(S)}{D(S)}$

(i) $\delta(t)$ = impulse signal , $h(t) = \dfrac{i(t)}{\delta(t)}$

$$\mathcal{L}\{\delta(t)\} = 1 = D(S)$$

$$\therefore H(S) = I(S)$$

$$\therefore I(S) = \frac{S+3}{S^2+6S+8} = \frac{S+3}{(S+2)(S+4)}$$

By Partial Fraction,

$$\frac{S+3}{(S+2)(S+4)} = \frac{A}{S+2} + \frac{B}{S+4}$$

$$S+3 = A(S+4) + B(S+2)$$

$S = -4$, $S = -2$

$B = \frac{+1}{2}$, $A = \frac{1}{2}$

$\therefore I(S) = \dfrac{1}{2(S+2)} + \dfrac{1}{2(S+4)}$ $\longrightarrow$ Laplace of impulse response

Taking inverse Laplace Transform,

$$i(t) = \frac{1}{2}e^{-2t}u(t) + \frac{1}{2}e^{-4t}u(t)$$

$i(t) = u(t). \dfrac{[e^{-2t} + e^{-4t}]}{2}$ $\Rightarrow$ Impulse response.

(ii) step response

$u(t)$ = unit step signal

$$U(S) = \frac{1}{S}$$

T.F $H(S) = \frac{S(S)}{U(S)}$, $h(t) = \frac{s(t)}{u(t)}$

$\therefore S(S) = H(S).U(S)$

$S(S) = \frac{s+3}{s(s+2)(s+4)}$ ← Laplace of step Response.

By Partial Fraction,

$$\frac{s+3}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$\frac{s+3}{s(s+2)(s+4)} = \frac{A(s+2)(s+4) + B(s)(s+4) + C(s)(s+2)}{s(s+2)(s+4)}$$

At $s=0$, , At $s=-2$, , At $s=-4$

$3 = 8A$ $\quad B = \frac{-1}{4}$ $\quad C = \frac{-1}{8}$

$A = \frac{3}{8}$

$$\therefore S(S) = \frac{3}{8s} - \frac{1}{4(s+2)} - \frac{1}{8(s+4)}$$

$$s(t) = \frac{3}{8}u(t) - \frac{e^{-2t}u(t)}{4} - \frac{e^{-4t}.u(t)}{8}$$

↳ step Response.

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(4) Given: $x(t) = u(t)$ and

$y(0) = 1$; $\frac{d\,y(0)}{dt} = 3$

system equation : $\frac{d^2y(t)}{dt^2} + 6\frac{d\,y(t)}{dt} + 8y(t) = \frac{d\,x(t)}{dt} + x(t)$

To find: Complete response

Soln.

(i) Zero input Response i.e. $x(t) = 0$, $\frac{d\,x(t)}{dt} = 0$

$\therefore$ Taking laplace Transform of the equation,

$$s^2 y(s) + 6s.\,y(s) + 8.y(s) = 0$$

$$\therefore \quad y(s)[s^2+6s+8] = 0$$

$$\therefore y(s) = 0 \quad , \quad \therefore \underline{y(t) = 0}$$

zero input response

(ii) Zero State Response:

input $x(t) = u(t)$, $X(s) = \frac{1}{s}$

using shifting property of unilateral Laplace Transform.

$$\left[s^2y(s) - s\,y(0) - \frac{d\,y(t)}{dt}\bigg|_{t=0}\right] + 6\left[s\,y(s) - y(0)\right] + 8\,y(s)$$

$$= s.X(s) + X(s)$$

Given,

$y(0) = 1$, $\frac{dy(0)}{dt} = 3$, $x(t) = u(t)$

$\therefore X(s) = \frac{1}{s}$

$\therefore Y(s)[s^2+6s+8] - s - 9 = \frac{s+1}{s}$

$\therefore Y(s) = \frac{s^2+10s+1}{s(s^2+6s+8)}$

using Partial fraction fraction, method,

$$Y(s) = \frac{s^2+10s+1}{s(s+2)(s+4)} = \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+4}$$

$A_1 = 1/8, \quad A_2 = \frac{15}{4}, \quad A_3 = \frac{-23}{8}$

$$\therefore Y(s) = \frac{1}{8}\cdot\frac{1}{s} + \frac{15}{4}\cdot\frac{1}{s+2} - \frac{23}{8}\cdot\frac{1}{s+4}$$

Taking inverse laplace Transform,

$$y(t) = \underbrace{\frac{1}{8}u(t) + \frac{15}{4}e^{-2t}.u(t) - \frac{23}{8}e^{-4t}u(t)}_{\text{zero state response}}$$

Therefore,

completeter reoponse is,

zero input reoponse + zero state reopouse

$$y(t) = 0 + \frac{1}{8}u(t) + \frac{15}{4}e^{-2t}u(t) - \frac{23}{8}.e^{-4t}.u(t)$$

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Q.5

1. $x(t) = \delta(t)$

$\delta(t)$ is the unit impulse signal.

$\delta(t) = 1, \quad t = 0$

$\quad\quad 0 \quad\quad t \neq 0$

$$L\{x(t)\} = L\{\delta(t)\} = \int_{-\infty}^{+\infty} \delta(t) \cdot e^{-st} \cdot dt$$

$\delta(t)$ is defined at $t=0$,

$$\therefore L\{\delta(t)\} = 1$$ Ans

2. $x(t) = \delta'(t) \longrightarrow$ Derivative of $\delta(t)$

By time differentiation,

$$L\left\{\frac{d}{dt} x(t)\right\} = s\,X(s) - X(0)$$

$$L\{x(t)\} = L\left\{\frac{d}{dt}\delta(t)\right\} = s \cdot X(s)$$

$$= s \times 1$$

as $L\{\delta(t) = 1\}$

$$\therefore L\{\delta'(t) \cancel{\;}\} = s.$$ Ans

3. $t \cdot u(t)$

By frequency differentiation,

$$L\{t^n \cdot x(t)\} = (-1)^n \frac{d^n X(s)}{ds^n} \quad , \quad L\{t\,x(t)\} = -\frac{dX(s)}{ds}$$

$$\therefore L\{t \cdot x(t)\} = -\frac{d}{ds} X_1(s) = -\frac{d}{ds}\left(\frac{1}{s}\right)$$

$$x_1(t) = u(t)$$

$$X_1(s) = L\{u(t)\} = \frac{1}{s}$$

$$\therefore L\{t\,u(t)\} = \frac{1}{s^2}$$ Ans

④ $e^{-at}.u(t)$

By frequency shifting,

$$L\{e^{\pm at}x(t)\} = X(s \mp a)$$

$$\therefore L\{e^{-at}.u(t)\} = \frac{1}{s+a}$$

$$\therefore L\{e^{-at}\ u(t)\} = \frac{1}{s+a}$$

⑤ $t.e^{-at}.u(t)$

By frequency shifting,

$$L\{e^{\pm at}.x(t)\} = X(s \mp a)$$

and then by frequency differentiation,

$$L\{t^n.x(t)\} = (-1)^n \frac{d^n}{ds^n} X(s)$$

$$\therefore L\{t.e^{-at}.u(t)\} = \frac{-d}{ds}\left(\frac{1}{s+a}\right)$$

$$= \frac{1}{(s+a)^2}$$

⑥ $x(t) = \cos(\omega_0 t).\ u(t)$

$= \cos(\omega_0 t)$ for $t \geq 0$

$$= \int_0^\infty \cos(\omega_0 t).\ e^{-st}.dt$$

$$= \int_0^\infty \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} . e^{-st}.dt$$

$$= \frac{1}{2}\left(\frac{2s}{s^2+\omega_0^2}\right)$$

$$\therefore L\{\cos(\omega_0 t).\ u(t)\} = \frac{s}{s^2+\omega_0^2}$$