Tutorial 5

Signals and systems

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Vaibhav Singh
58, D9A

Q①. z transform of

(i). $x(n) = \left(\frac{-1}{4}\right)^n \cdot n \cdot u(n) * \left(\frac{-1}{6}\right)^{-n} u(-n)$

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Taking z- transform on both sides,

$X(z) = A(z) \cdot B(z)$

using, $z\{n \cdot a^n \cdot u(n)\} = \frac{az}{(z-a)^2}$ and

$z\{u(-n)\} = \frac{1}{1-z}$

$A(z) = \frac{1/4\, z}{\left(z - \left(\frac{-1}{4}\right)\right)^2}$, $B(z) = z\left\{\left(\frac{-1}{6}\right)^n \cdot u(-n)\right\}$

using scaling property

$B(z) = \frac{6}{6+z}$

$\therefore X(z) = \frac{\frac{1}{4} z}{\left(z+\frac{1}{4}\right)^2} \cdot \frac{6}{(6+z)}$

$X(z) = \frac{2/3 \cdot z}{\left(z+\frac{1}{4}\right)^2 \cdot z+6}$ Ans.

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(ii). $x(n) = u(n-6) - u(n-10)$

$$z\{u(n)\} = \frac{z}{z-1}$$

By time shifting property,

$$z(x(n-a)) = z^{-a} . X(z)$$

$$\therefore z\{u(n-6)\} = z^{-6} . \frac{z}{z-1} = \frac{z^{-5}}{z-1}$$

$$z\{u(n-10)\} = z^{-10} . \frac{z}{z-1} = \frac{z^{-9}}{z-1}$$

$$\therefore X(z) = \frac{z^{-5}}{z-1} - \frac{z^{-9}}{z-1}$$

$$X(z) = \frac{z^{-5} - z^{-9}}{z-1}$$ Ans.

Q.2 Convolving $x(n) = \left(\frac{1}{3}\right)^n . u(n)$ and $h(n) = \left(\frac{1}{2}\right)^n . u(n)$ using z-transform.

Soln

$$x(n) = \left(\frac{1}{3}\right)^n \quad \text{for } n \geq 0$$

$$h(n) = \left(\frac{1}{2}\right)^n \quad \text{for } n \geq 0$$

$$y(n) = \sum_{m=0}^{n} \left(\frac{1}{3}\right)^m . \left(\frac{1}{2}\right)^{n-m}$$

$$= \left(\frac{1}{2}\right)^n . \sum_{m=0}^{n} \left(\frac{1}{3}\right)^m . \left(\frac{1}{2}\right)^{-m}$$

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$$= 3\left(\frac{1}{2}\right)^n \cdot \left[1 - \left(\frac{2}{3}\right)^{n+1}\right]$$

By z-transform,

$$y(z) = \frac{z}{z-1/3} \cdot \frac{z}{z-1/2}$$

$$\frac{y(z)}{z} = \frac{z}{(z-1/3)(z-1/2)}$$

By Partial Fraction,

$$\frac{Y(z)}{z} = \frac{k_1}{z-1/3} + \frac{k_2}{z-1/2}, \quad k_1 = -2, \quad k_2 = 3$$

$$\therefore Y(z) = \frac{-2z}{z-1/3} + \frac{3z}{z-1/2}$$

By inverse z-transform,

$$y(n) = -2\left(\frac{1}{3}\right)^n u(n) + 3\left(\frac{1}{2}\right)^n \cdot u(n)$$

$$y(n) = 3\left(\frac{1}{2}\right)^n \left[1 - \left(\frac{2}{3}\right)^{n+1}\right]$$

Q.③ LTI system function, $H(z) = \dfrac{z}{(z-\frac{1}{4})(z+\frac{1}{4})(z-\frac{1}{2})}$

Soln Poles: at

$z = \frac{1}{4}, \frac{-1}{4}, \frac{1}{2}$

As the ROC of the system is exterior of circle, LTI system is causal. As all the poles lie inside the unit circle it is stable system.

Impulse Response,

$$\frac{H(z)}{z} = \frac{1}{(z-\frac{1}{4})(z+\frac{1}{4})(z-\frac{1}{2})}$$

By Partial Fraction method,

when $z = 1/4$, $k_1 = -8$

$z = -1/4$, $k_2 = 8/3$

$z = 1/2$, $k_3 = 16/3$

where,

$$\frac{H(z)}{z} = \frac{k_1}{(z-1/4)} + \frac{k_2}{(z+1/4)} + \frac{k_3}{(z-1/2)}$$

$$\therefore H(z) = \frac{-8z}{(z-1/4)} + \frac{8/3\, z}{(z+1/4)} + \frac{16/3\, z}{(z-1/2)}$$

By z-inverse transform,

$$h(n) = -8\left(\frac{1}{4}\right)^n u(n) + \frac{8}{3}\left(\frac{-1}{4}\right)^n u(n) + \frac{16}{3}\left(\frac{1}{2}\right)^n u(n)$$

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Q. 4 causal LTI system: $y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n)$

(i) Determine impulse response of the system.

$\frac{Y(z)}{X(z)} = H(z) \rightarrow$ z-transform of impulse response

$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n)$

Taking z-transform on both sides,

$Y(z) = \frac{3}{4} \cdot z^{-1} Y(z) - \frac{1}{8} z^{-2} Y(z) + X(z)$

$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}} = \frac{z^2}{z^3 - \frac{3}{4} z + \frac{1}{8}}$

By Partial Fraction,

$\frac{H(z)}{z} = \frac{z}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)} = \frac{A}{(z - 1/2)} + \frac{B}{(z - 1/4)}$

$z = A(z - 1/4)$, $z = B(z - 1/2)$
when $z = 1/2$ | when $z = 1/4$
$A = 2$ | $B = -1$

$\therefore H(z) = \frac{2z}{(z - 1/2)} - \frac{z}{(z - 1/4)}$

Taking inverse-z transforms,

$h(n) = 2 \cdot \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$ Ans.