CHAPTER 6
INTRODUCTION TO DIFFERENTIAL AMPLIFIER

INTRODUCTION TO DIFFERENTIAL AMPLIFIER

- Differential amplifier is fundamental building block of Analog Circuits. It is input stage of virtually every opamp and is the basis of a high speed digital logic circuits.
- The MOSFET is by far the most widely used transistor in both digital and analog circuits, and it is the backbone of modern electronics.
- One of the most common uses of the MOSFET in analog circuits is the construction of differential amplifiers.
- The latter are used as input stages in op-amps, video amplifiers, high-speed comparators, and many other analog-based circuits.
- MOSFET differential amplifiers are used in integrated circuits, such as operational amplifiers, they provide a high input impedance for the input terminals.
- A properly designed differential amplifier with its current-mirror biasing stages is made from matched-pair devices to minimize imbalances from one side of the differential amplifier to the other.

BASICS OF DIFFERENTIAL AMPLIFIER

Figure 11.1 Difference amplifier block diagram

In this chapter, we introduce another basic transistor circuit configuration called the differential amplifier. This amplifier, also called a diff-amp, is the input stage to virtually all op-amps and is probably the most widely used amplifier building block in analog integrated circuits. Figure 11.1 is a block diagram of the diff-amp. There are two input terminals and one output terminal. Ideally, the output signal is proportional to only the difference between the two input signals.

BASICS OF DIFFERENTIAL AMPLIFIER

The ideal output voltage can be written as

$$v_o = A_{vol}(v_1 - v_2) \tag{11.1}$$

where A_{vol} is called the open-loop voltage gain. In the ideal case, if $v_1 = v_2$, the output voltage is zero. We only obtain a nonzero output voltage if v_1 and v_2 are not equal.

We define the differential-mode input voltage as

$$v_d = v_1 - v_2 \tag{11.2}$$

and the common-mode input voltage as

$$v_{cm} = \frac{v_1 + v_2}{2} \tag{11.3}$$

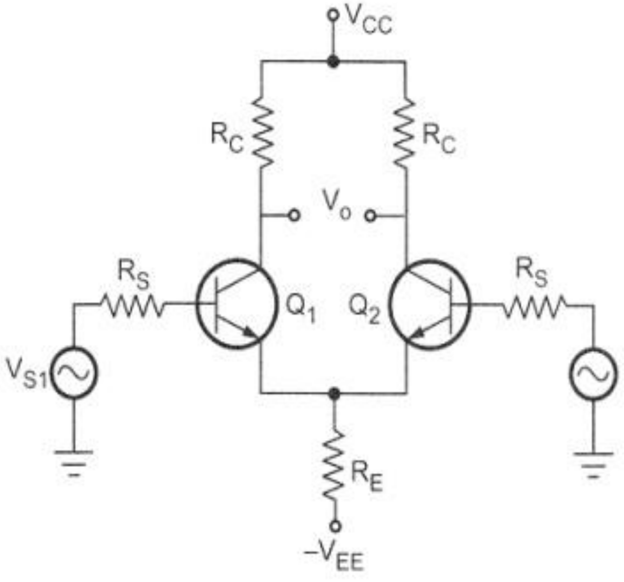
BASICS OF DIFFERENTIAL AMPLIFIER

These equations show that if $v_1 = v_2$, the differential-mode input signal is zero and the common-mode input signal is $v_{cm} = v_1 = v_2$.

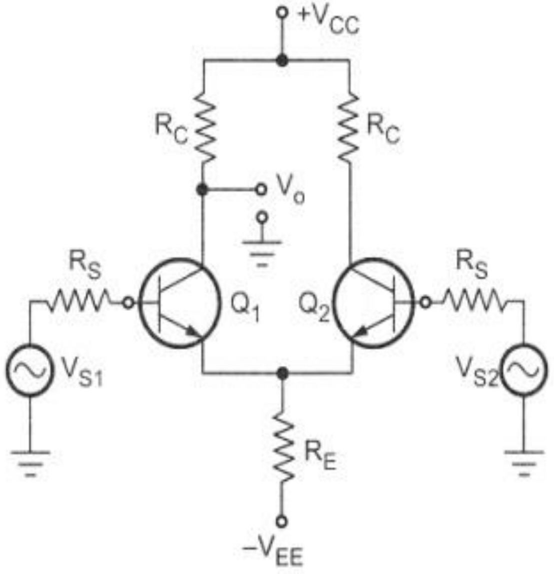
If, for example, $v_1 = +10\,\mu\text{V}$ and $v_2 = -10\,\mu\text{V}$, then the differential-mode voltage is $v_d = 20\,\mu\text{V}$ and the common-mode voltage is $v_{cm} = 0$. However, if $v_1 = 110\,\mu\text{V}$ and $v_2 = 90\,\mu\text{V}$, then the differential-mode input signal is still $v_d = 20\,\mu\text{V}$, but the common-mode input signal is $v_{cm} = 100\,\mu\text{V}$. If each pair of input voltages were applied to the ideal difference amplifier, the output voltage in each case would be exactly the same. However, amplifiers are not ideal, and the common-mode input signal does affect the output. One goal of the design of differential amplifiers is to minimize the effect of the common-mode input signal.

CONFIGURATIONS OF DIFFERENTIAL AMPLIFIER

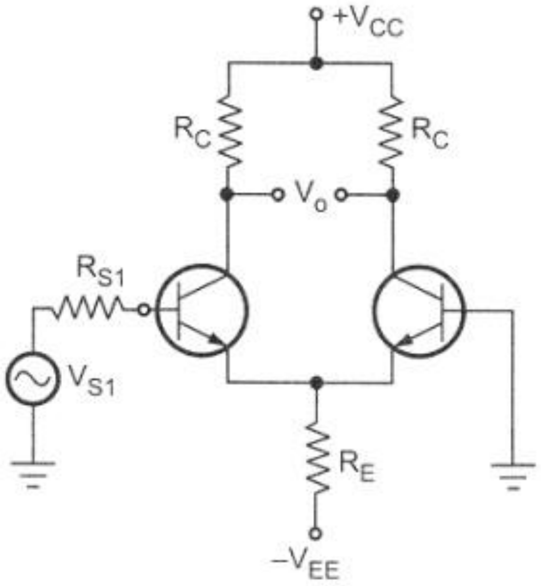
1. Dual Input Balanced Output
2. Dual Input Unbalanced Output
3. Single Input Balanced Output
4. Single Input Unbalanced Output



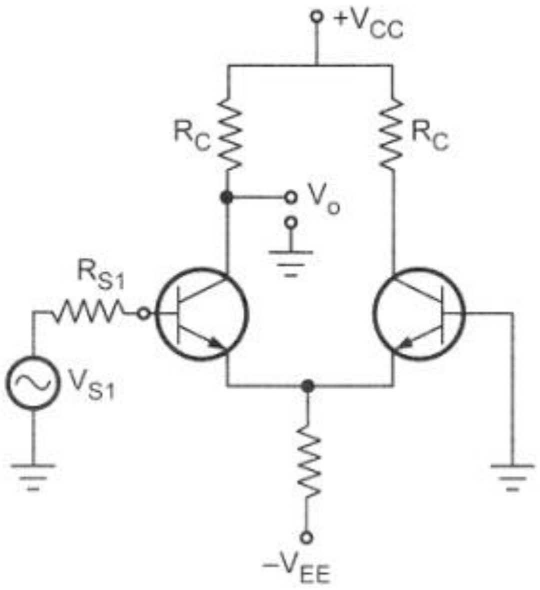
(a) Dual input balanced output



(b) Dual input unbalanced output

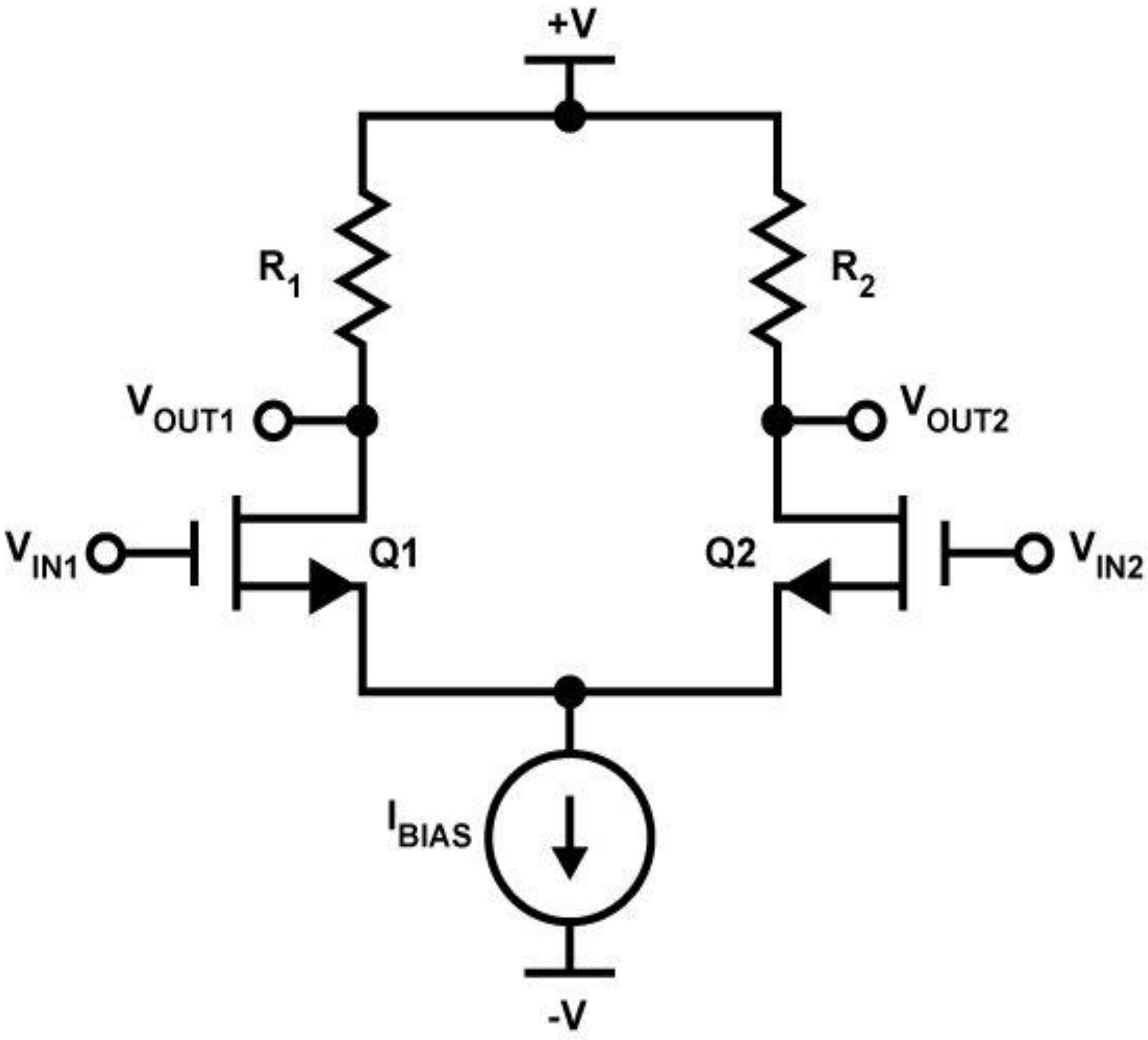


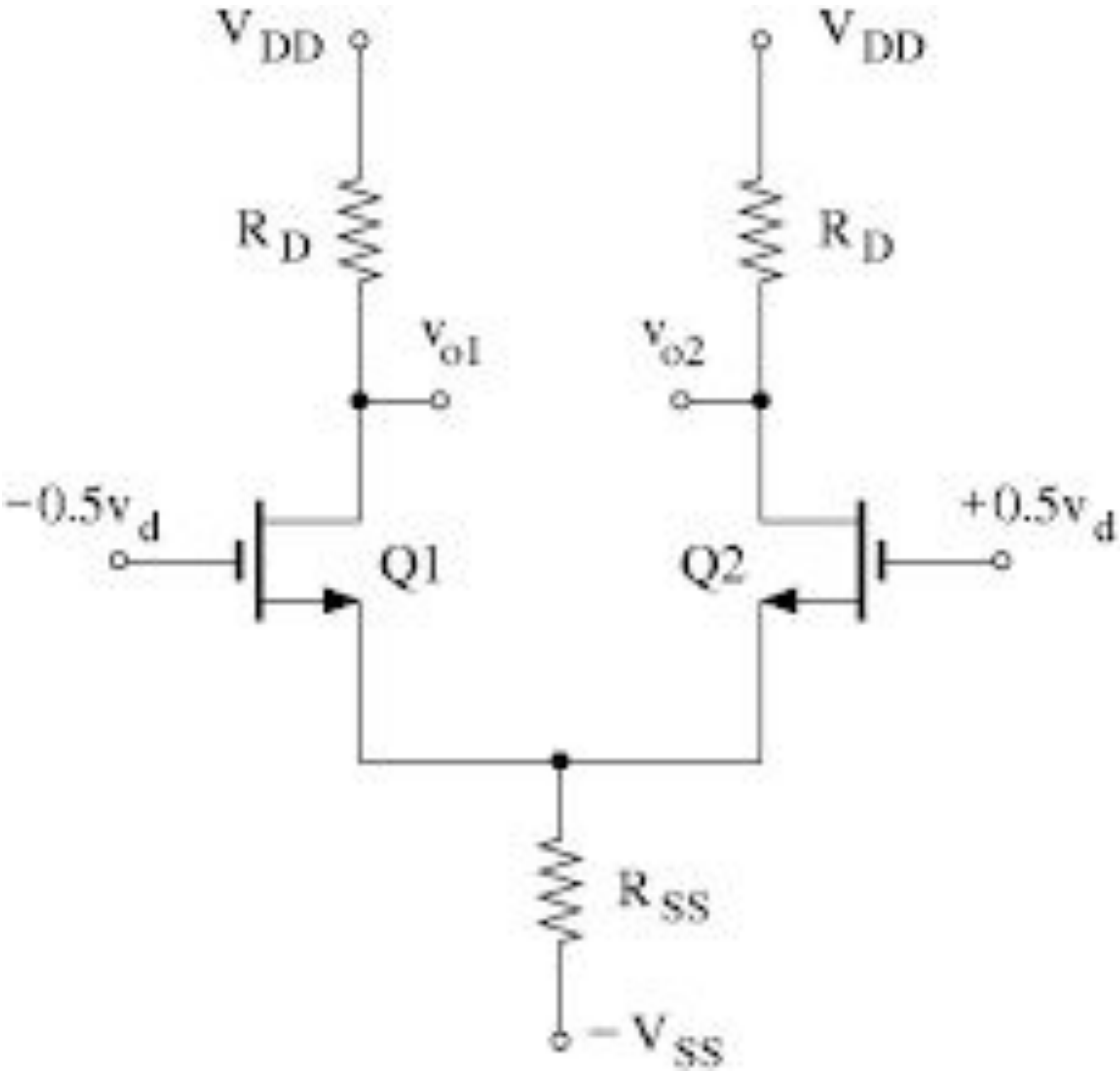
(c) Single input balanced output



(d) Single input unbalanced output

DIFFERENTIAL AMPLIFIER





DC TRANSFER CHARACTERISTICS

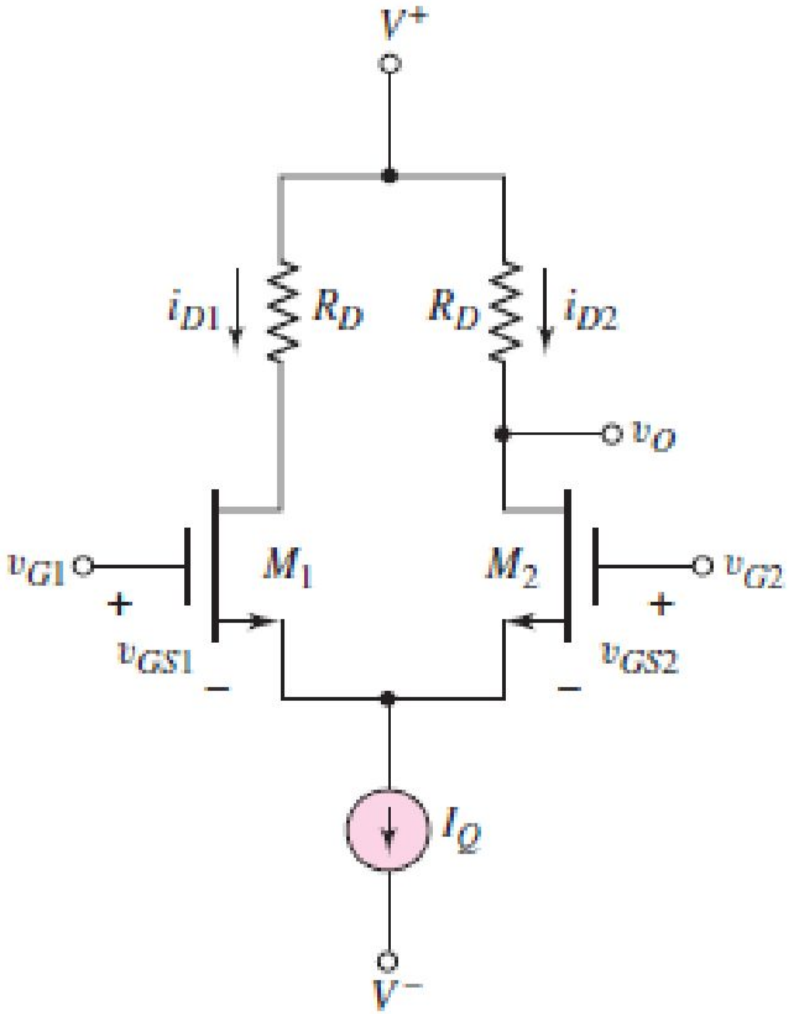


Figure 11.19 shows the basic MOSFET differential pair, with matched transistors M_1 and M_2 biased with a constant current I_Q. We assume that M_1 and M_2 are always biased in the saturation region.

Like the basic bipolar configuration, the basic MOSFET diff-amp uses both positive and negative bias voltages, thereby eliminating the need for coupling capacitors and voltage divider biasing resistors at the gate terminals. Even with $v_{G1} = v_{G2} = 0$, the transistors M_1 and M_2 can be biased in the saturation region by the current source I_Q. This circuit, then, is also a **dc-coupled** diff-amp.

Figure 11.19 Basic MOSFET differential pair configuration

DC TRANSFER CHARACTERISTICS

The dc transfer characteristics of the MOSFET differential pair can be determined from the circuit in Figure 11.19. Neglecting the output resistances of M_1 and M_2, and assuming the two transistors are matched, we can write

$$i_{D1} = K_n(v_{GS1} - V_{TN})^2 \tag{11.60(a)}$$

and

$$i_{D2} = K_n(v_{GS2} - V_{TN})^2 \tag{11.60(b)}$$

Taking the square roots of Equations (11.60(a)) and (11.60(b)), and subtracting the two equations, we obtain

$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{K_n}(v_{GS1} - v_{GS2}) = \sqrt{K_n} \cdot v_d \tag{11.61}$$

where $v_d = v_{G1} - v_{G2} = v_{GS1} - v_{GS2}$ is the differential-mode input voltage. If $v_d > 0$, then $v_{G1} > v_{G2}$ and $v_{GS1} > v_{GS2}$, which implies that $i_{D1} > i_{D2}$. Since

$$i_{D1} + i_{D2} = I_Q \tag{11.62}$$

DC TRANSFER CHARACTERISTICS

then Equation (11.61) becomes

$$(\sqrt{i_{D1}} - \sqrt{I_Q - i_{D1}})^2 = (\sqrt{K_n} \cdot v_d)^2 = K_n v_d^2 \tag{11.63}$$

when both sides of the equation are squared. After the terms are rearranged, Equation (11.63) becomes

$$\sqrt{i_{D1}(I_Q - i_{D1})} = \frac{1}{2}(I_Q - K_n v_d^2) \tag{11.64}$$

If we square both sides of this equation, we develop the quadratic equation

$$i_{D1}^2 - I_Q i_{D1} + \frac{1}{4}(I_Q - K_n v_d^2)^2 = 0 \tag{11.65}$$

Applying the quadratic formula, rearranging terms, and noting that $i_{D1} > I_Q/2$ and $v_d > 0$, we obtain

$$i_{D1} = \frac{I_Q}{2} + \sqrt{\frac{K_n I_Q}{2}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2} \tag{11.66}$$

Using Equation (11.62), we find that

$$i_{D2} = \frac{I_Q}{2} - \sqrt{\frac{K_n I_Q}{2}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2} \tag{11.67}$$

DC TRANSFER CHARACTERISTICS

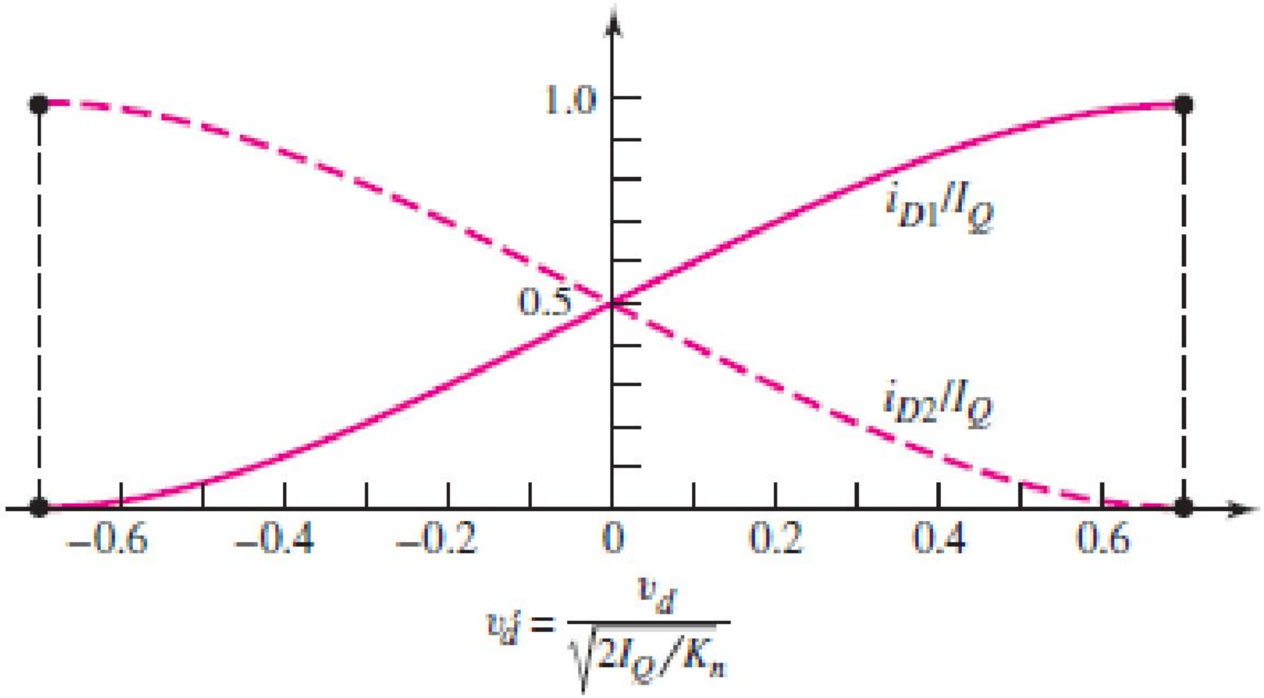


Figure 11.21 Normalized dc transfer characteristics, MOSFET differential amplifier

Using Equation (11.62), we find that

$$i_{D2} = \frac{I_Q}{2} - \sqrt{\frac{K_n I_Q}{2}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2}$$

The normalized drain currents are

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2}$$

and

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \sqrt{\frac{K_n}{2I_Q}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2}$$

These equations describe the dc transfer characteristics for this circuit. They are plotted in Figure 11.21 as a function of a normalized differential input voltage $v_d / \sqrt{(2I_Q / K_n)}$.

DC TRANSFER CHARACTERISTICS

We can see from Equations (11.68) and (11.69) that, at a specific differential input voltage, bias current I_Q is switched entirely to one transistor or the other. This occurs when

$$|v_d|_{\max} = \sqrt{\frac{I_Q}{K_n}} \tag{11.70}$$

The forward transconductance is defined as the slope of the i_{D1} versus v_d transfer characteristic evaluated at $v_d = 0$, or

$$g_f(\max) = \left.\frac{di_{D1}}{dv_d}\right|_{v_d=0} \tag{11.71}$$

Using Equation (11.66), we find that

$$g_f(\max) = \sqrt{\frac{K_n I_Q}{2}} = \frac{g_m}{2} \tag{11.72}$$

where g_m is the transconductance of each transistor. The slope of the i_{D2} characteristic curve at $v_d = 0$ is the same, except it is negative.

AC Equivalent Circuit of MOSFET Differential Amplifier

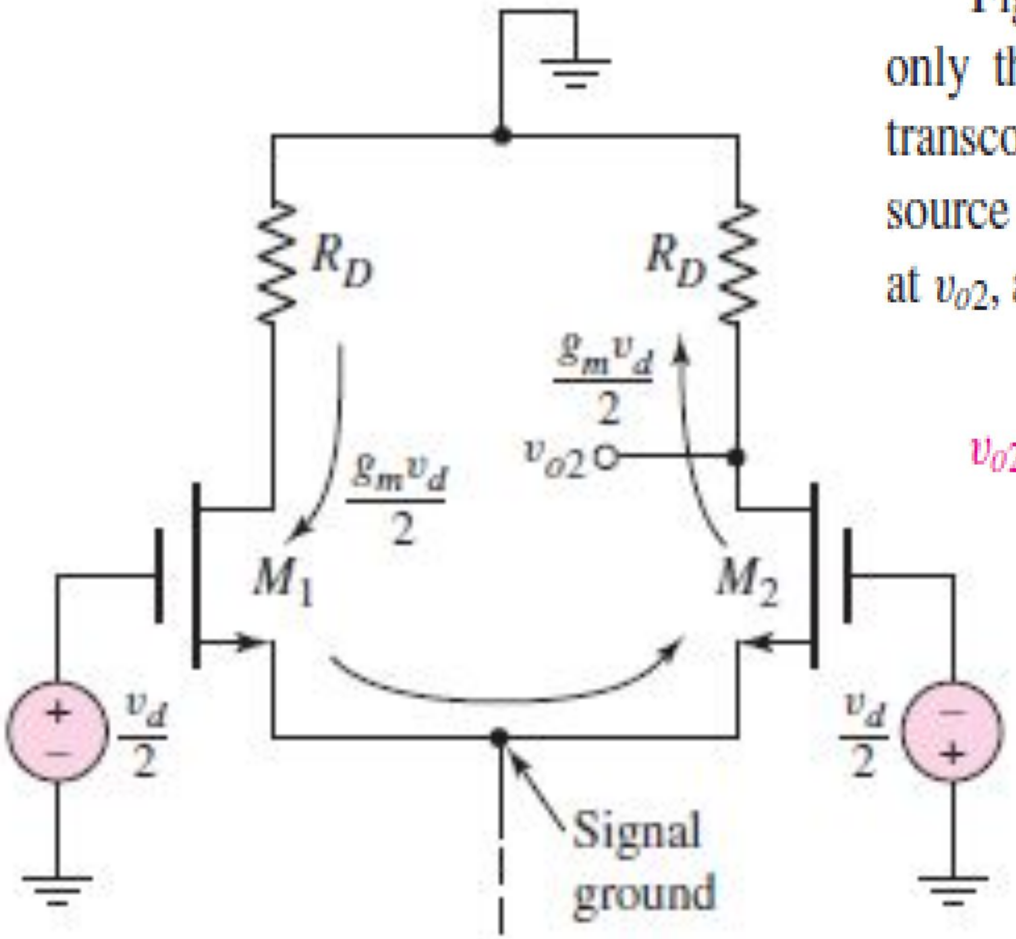


Figure 11.22 AC equivalent circuit, MOSFET differential amplifier

Figure 11.22 is the ac equivalent circuit of the diff-amp configuration, showing only the differential voltage and signal currents as a function of the transistor transconductance g_m. We assume that the output resistance looking into the current source is infinite. Using this equivalent circuit, we find the one-sided output voltage at v_{o2}, as follows:

$$v_{o2} \equiv v_o = +\left(\frac{g_m v_d}{2}\right) R_D \tag{11.73}$$

The differential voltage gain is then

$$A_d = \frac{v_o}{v_d} = \frac{g_m R_D}{2} = \sqrt{\frac{K_n I_Q}{2}} \cdot R_D$$

DIFFERENTIAL AND COMMON MODE INPUT IMPEDANCES

At low frequencies, the input impedance of a MOSFET is essentially infinite, which means that both the differential- and common-mode input resistances of a MOSFET diff-amp are infinite. Also, we know that the differential input resistance of a bipolar pair can be in the low kilohm range. A design trade-off, then, would be to use a MOSFET diff-amp with infinite input resistance, and sacrifice the differential-mode voltage gain.

SMALL SIGNAL EQUIVALENT CIRCUIT ANALYSIS

We can determine the basic relationships for the differential-mode gain, common-mode gain, and common-mode rejection ratio from an analysis of the small-signal equivalent circuit.

Figure 11.23 shows the small-signal equivalent circuit of the MOSFET differential pair configuration. We assume the transistors are matched, with $\lambda = 0$ for each transistor, and that the constant-current source is represented by a finite output resistance R_o. All voltages are represented by their phasor components. The two transistors are biased at the same quiescent current, and $g_{m1} = g_{m2} \equiv g_m$.

SMALL SIGNAL EQUIVALENT CIRCUIT ANALYSIS

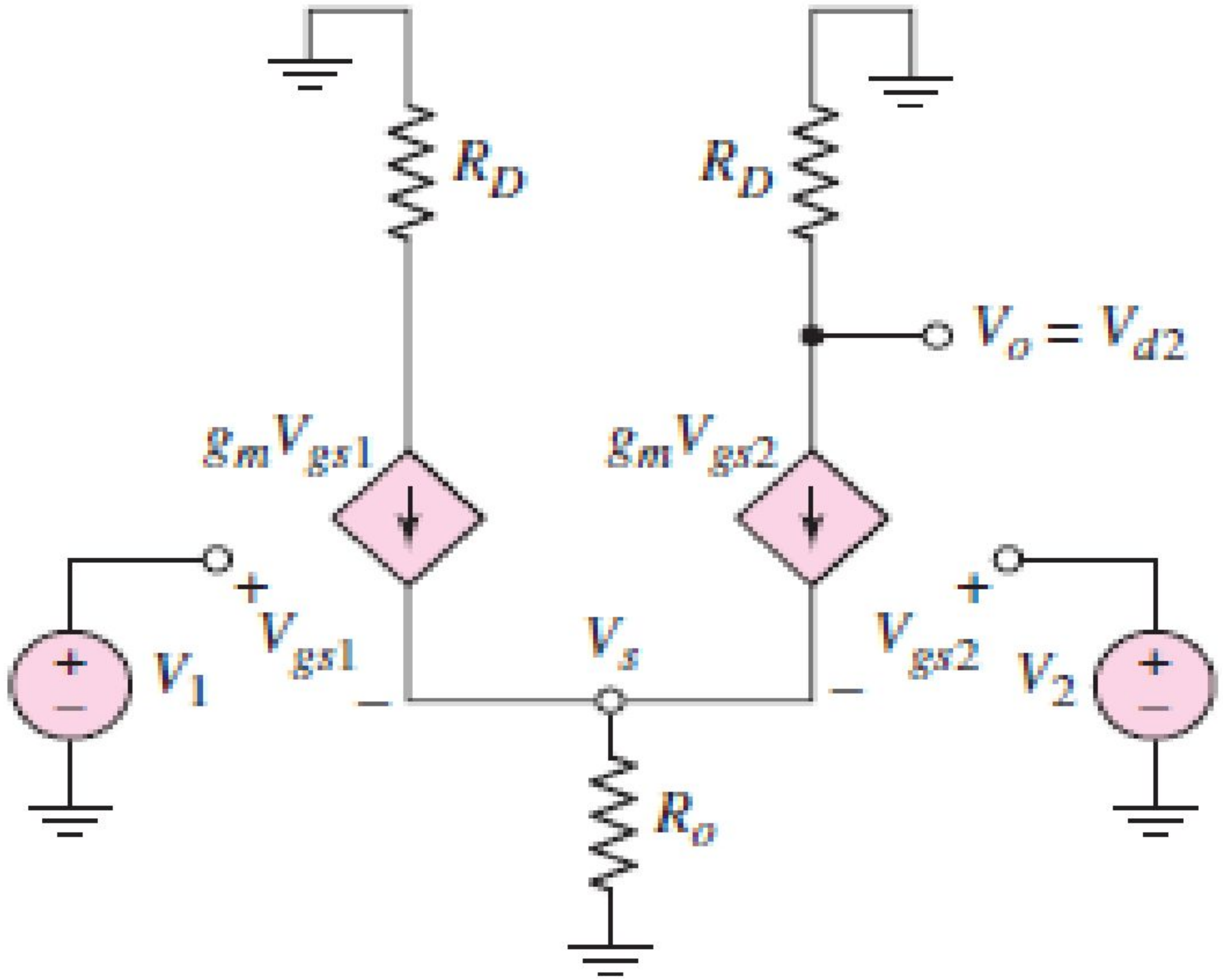


Figure 11.23 Small-signal equivalent circuit, MOSFET differential amplifier

SMALL SIGNAL EQUIVALENT CIRCUIT ANALYSIS

Writing a KCL equation at node V_s, we have

$$g_m V_{gs1} + g_m V_{gs2} = \frac{V_s}{R_o} \tag{11.75}$$

From the circuit, we see that $V_{gs1} = V_1 - V_s$ and $V_{gs2} = V_2 - V_s$. Equation (11.75) then becomes

$$g_m(V_1 + V_2 - 2V_s) = \frac{V_s}{R_o} \tag{11.76}$$

Solving for V_s we obtain

$$V_s = \frac{V_1 + V_2}{2 + \frac{1}{g_m R_o}} \tag{11.77}$$

For a one-sided output at the drain of M_2, we have

$$V_o = V_{d2} = -(g_m V_{gs2})R_D = -(g_m R_D)(V_2 - V_s) \tag{11.78}$$

Small Signal Equivalent Circuit Analysis

Substituting Equation (11.77) into (11.78) and rearranging terms yields

$$V_o = -g_m R_D \left[\frac{V_2\left(1 + \frac{1}{g_m R_o}\right) - V_1}{2 + \frac{1}{g_m R_o}} \right] \tag{11.79}$$

Based on the relationships between the input voltages V_1 and V_2 and the differential- and common-mode voltages, as given by Equation (11.29), Equation (11.79) can be written

$$V_o = \frac{g_m R_D}{2} V_d - \frac{g_m R_D}{1 + 2g_m R_o} V_{cm} \tag{11.80}$$

The output voltage, in general form, is

$$V_o = A_d V_d + A_{cm} V_{cm} \tag{11.81}$$

The transconductance g_m of the MOSFET is

$$g_m = 2\sqrt{K_n I_{DQ}} = \sqrt{2K_n I_Q}$$

SMALL SIGNAL EQUIVALENT CIRCUIT ANALYSIS

Comparing Equations (11.80) and (11.81), we develop the relationships for the differential-mode gain,

$$A_d = \frac{g_m R_D}{2} = \sqrt{2K_n I_Q}\left(\frac{R_D}{2}\right) = \sqrt{\frac{K_n I_Q}{2}} \cdot R_D \tag{11.82(a)}$$

and the common-mode gain

$$A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_o} = \frac{-\sqrt{2K_n I_Q} \cdot R_D}{1 + 2\sqrt{2K_n I_Q} \cdot R_o} \tag{11.82(b)}$$

We again see that for an ideal current source, the common-mode gain is zero since $R_o = \infty$.

From Equations (11.82(a)) and (11.82(b)), the common-mode rejection ratio, $\text{CMRR} = |A_d/A_{cm}|$, is found to be

$$\text{CMRR} = \frac{1}{2}\left[1 + 2\sqrt{2K_n I_Q} \cdot R_o\right] \tag{11.83}$$

This demonstrates that the CMRR for the MOSFET diff-amp is also a strong function of the output resistance of the constant-current source.

EXAMPLE

Objective: Determine the differential-mode voltage gain, common-mode voltage gain, and CMRR for a MOSFET diff-amp.

Consider a MOSFET diff-amp with the configuration in Figure 11.20. Assume the same transistor parameters as given in Example 11.8 except assume $\lambda = 0.01\ \mathrm{V}^{-1}$ for M_4.

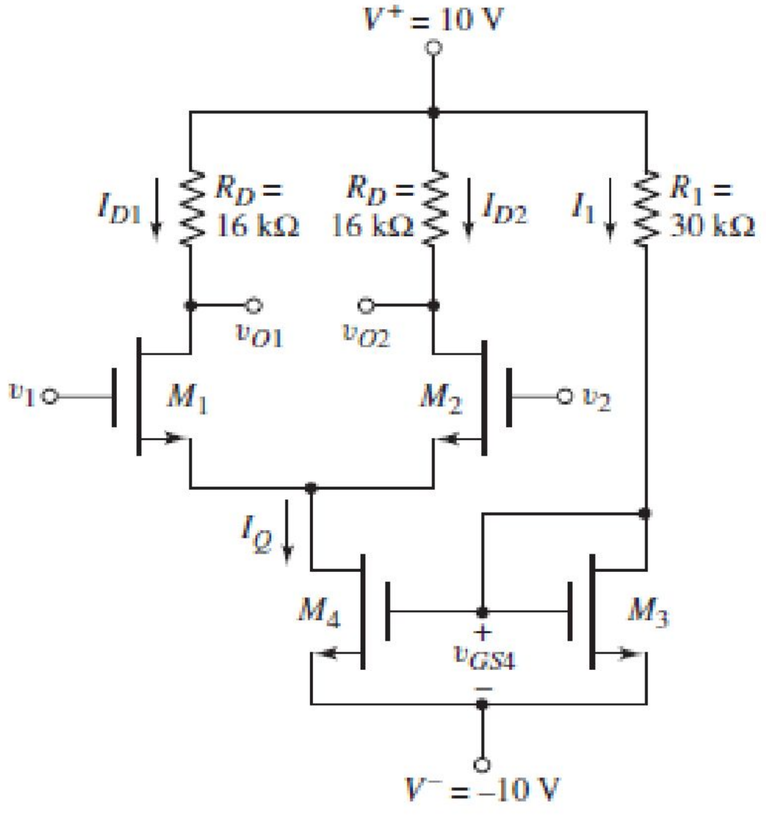


Figure 11.20 MOSFET differential amplifier for Example 11.8

SOLUTION

Solution: From Example 11.8, we found the bias current to be $I_Q = 0.587$ mA. The output resistance of the current source is then

$$R_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.587)} = 170\ \text{k}\Omega$$

The differential-mode voltage gain is

$$A_d = \sqrt{\frac{K_n I_Q}{2}} \cdot R_D = \sqrt{\frac{(1)(0.587)}{2}} \cdot (16) = 8.67$$

and the common-mode voltage gain is

$$A_{cm} = -\frac{\sqrt{2K_n I_Q} \cdot R_D}{1 + 2\sqrt{2K_n I_Q} \cdot R_o} = -\frac{\sqrt{2(1)(0.587)} \cdot (16)}{1 + 2\sqrt{2(1)(0.587)} \cdot (170)} = -0.0469$$

The common-mode rejection ratio is then

$$\text{CMRR}_{\text{dB}} = 20 \log_{10}\left(\frac{8.67}{0.0469}\right) = 45.3\ \text{dB}$$

Comment: As mentioned earlier, the differential-mode voltage gain of the MOSFET diff-amp is considerably less than that of the bipolar diff-amp, since the value of the MOSFET transconductance is, in general, much smaller than that of the BJT.

MOSPFET WITH TWO SIDED OUTPUT(BALANCED OUTPUT)

If we consider the two-sided output of an ideal MOSFET op-amp and define the output voltage as $V_o = V_{d2} - V_{d1}$, we can show that the differential-mode voltage gain is given by

$$A_d = g_m R_D \tag{11.84(a)}$$

and the common-mode voltage gain is given by

$$A_{cm} = 0 \tag{11.84(b)}$$

The result of $A_{cm} = 0$ for the two-sided output is a consequence of using matched devices and elements in the diff-amp circuit. We will reconsider a two-sided output and discuss the effects of mismatched elements in the next section.

MOSPFET WITH TWO SIDED OUTPUT(BALANCED OUTPUT)

EFFECT OF RD MISMATCH

We assume that R_{D1} and R_{D2} are the resistors in the drains of M_1 and M_2. If the two resistors are not matched, we assume that we can write $R_{D1} = R_D + \Delta R_D$ and $R_{D2} = R_D - \Delta R_D$. Using the small-signal equivalent circuit in Figure 11.23, we can find

$$A_d = g_m R_D$$

and

$$A_{cm} \cong \frac{\Delta R_D}{R_o}$$

The common-mode rejection ratio is then

$$\text{CMRR} = \left| \frac{A_d}{A_{cm}} \right| = \frac{g_m R_D}{(\Delta R_D / R_o)}$$

MOSPFET WITH TWO SIDED OUTPUT(BALANCED OUTPUT)

Effect of gm mismatch

We can consider the effect of transistor mismatch by considering the effect of a mismatch in the transconductance g_m. We assume g_{m1} and g_{m2} are the transconductance parameters of the two transistors in the diff-amp. We will assume that we can write $g_{m1} = g_m + \Delta g_m$ and $g_{m2} = g_m - \Delta g_m$. Again, using the small-signal equivalent circuit shown in Figure 11.23, we find the differential-mode voltage gain is

$$A_d = g_m R_D \tag{11.87(a)}$$

and the common-mode gain is

$$A_{cm} = \frac{R_D(2\Delta g_m)}{1 + 2R_o g_m} \tag{11.87(b)}$$

The common-mode rejection ratio now becomes

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \frac{1 + 2R_o g_m}{2(\Delta g_m / g_m)} \tag{11.88}$$

The CMRR of mismatched elements in the MOSFET diff-amp is identical with the results of mismatched elements in the BJT diff-amp.