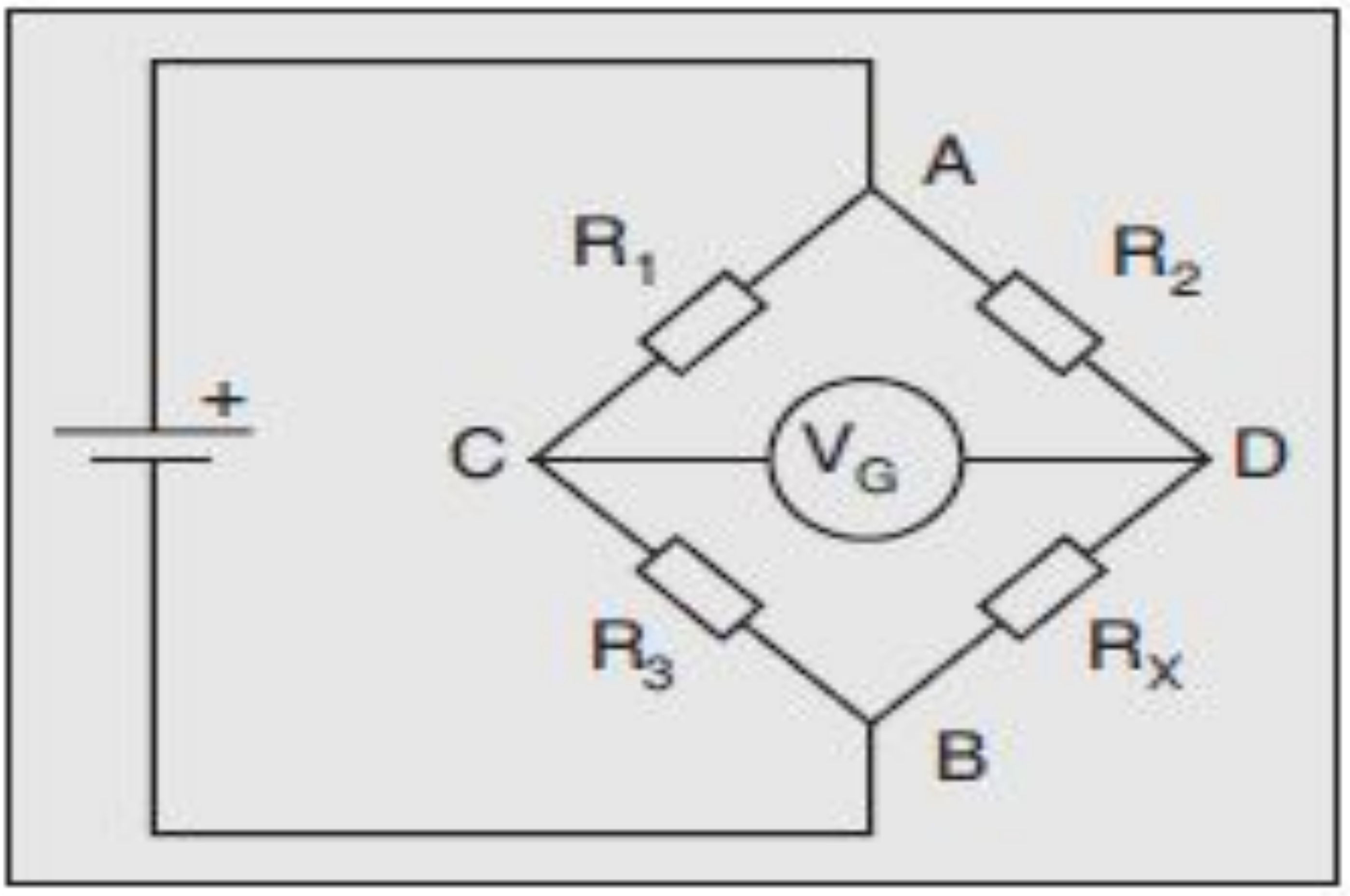
Module 2

Measurement of Resistance, Inductance and Capacitance

Concept of Measurement with bridge

Bridge Circuit



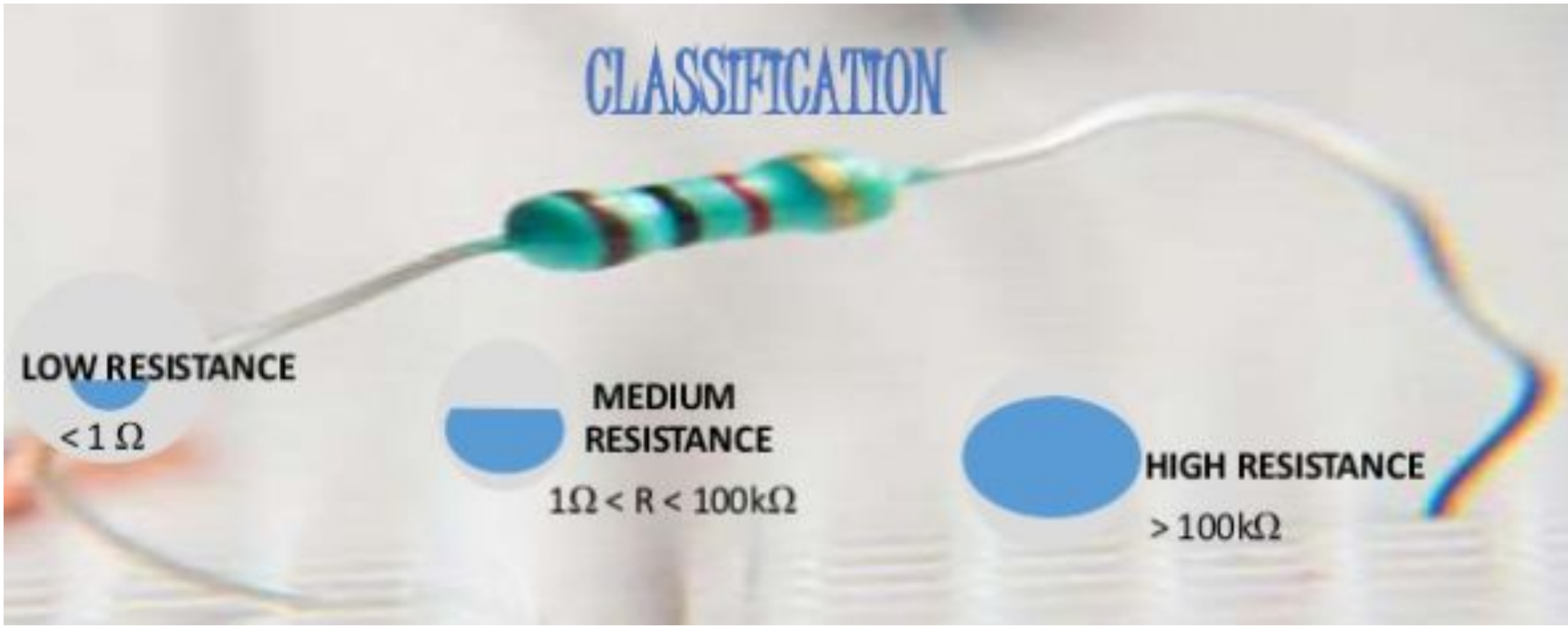
The bridge circuit construct from single or combination of passive circuit elements(resistors ,capacitors ,inductors).

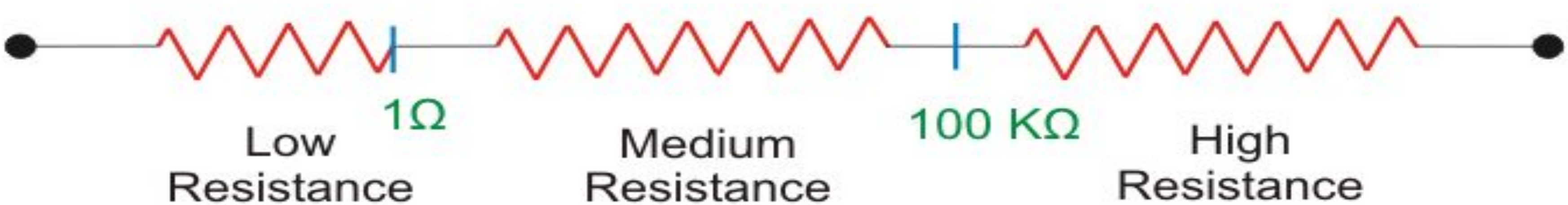
The bridge circuit construct from the resistors element,R1,R2,R3,where R3 is variable resistor and Rx is unknown resistor

- **Advantages of bridge circuits**

1. Accuracy in high measurements.
2. Accuracy is independent of null detector characteristics.
3. It can be used in control circuits.

Classification of Resistances





Types of Bridge circuits

- DC Bridge
- AC Bridge
- In DC bridge circuit , a DC source battery, galvanometer are used.
- In the AC bridge circuit, an AC source and a detector sensitive to AC voltage are used.

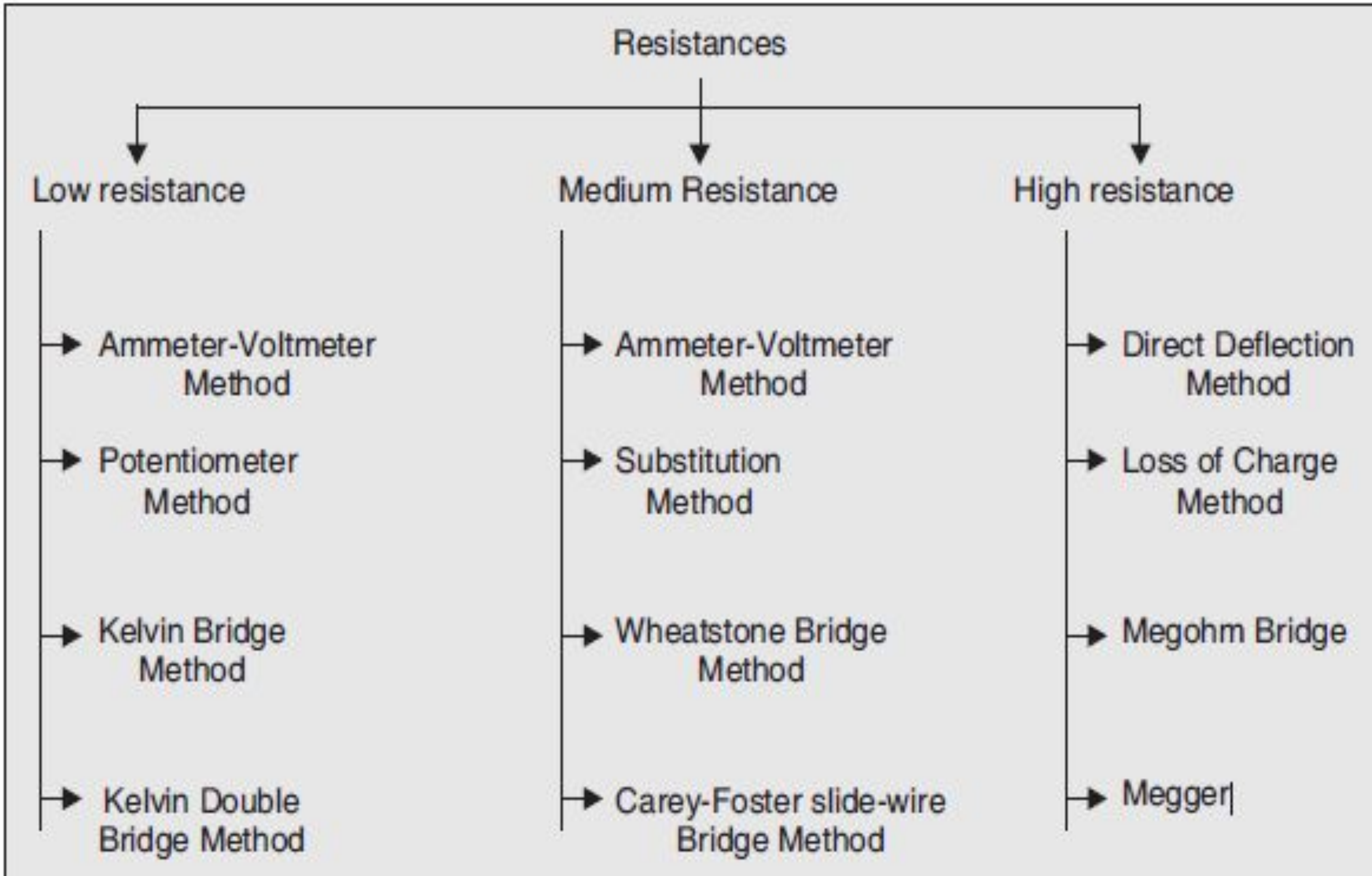
DC Bridge Circuits

- *Wheatstone Bridge Circuit*
- *Kelvin Bridge Circuit*
- *Double Kelvin Bridge Circuit*

AC Bridge Circuits

- *Maxwell Bridge Circuit*
- *Hay's Bridge Circuit*
- *Schering Bridge Circuit*
-

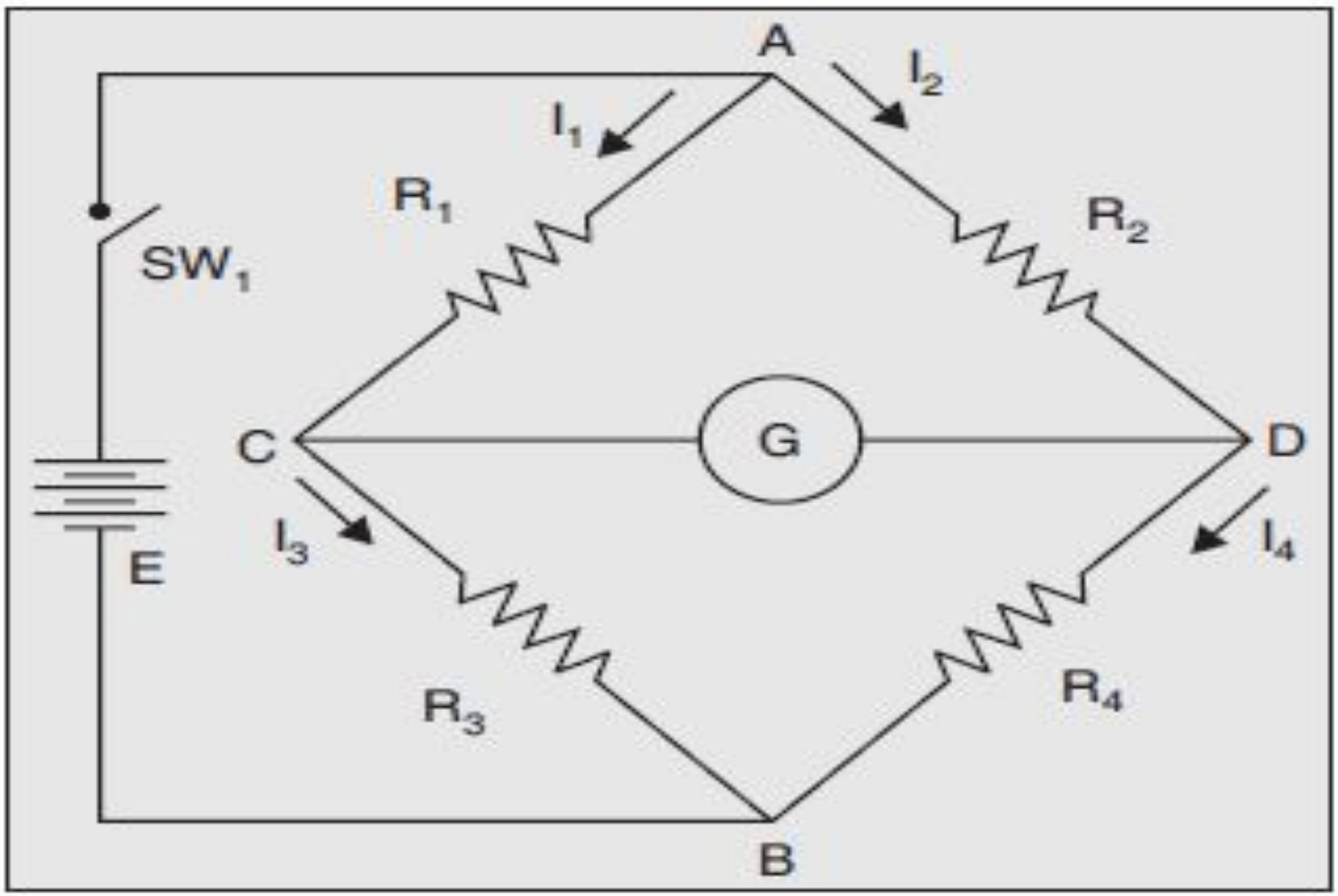
Measurement of Resistances



Measurement of Medium Resistance values

- 1. Ammeter-voltmeter Method
- 2. Substitution Method
- 3. Wheatstone Bridge
- 4. Carey-Foster slide-wire Bridge Method

Wheatstone Bridge Circuit



- The Wheatstone bridge circuit is used to compare an unknown resistance with a known resistance.
- The bridge is commonly used in control circuits.
- Wheatstone bridge is the most accurate method.
- The bridge has four resistive arms, together with a source of e.m.f and a null detector.
- *When* there is no current through the galvanometer, the pointer shows zero.
- The current in one direction cause the pointer to deflect on one side and current in the opposite direction cause the pointer to deflect to the other side.
- The bridge is said to be balanced when the potential difference across the galvanometer is '0V' so that there is no current through the galvanometer. This condition occurs when the voltage from point '*C' to point 'A' is equal to the voltage from point 'D' to point 'A'.*
- *By referring the* other battery terminal, the bridge is balanced when the voltage from point '*C' to point 'B' equals* the voltage from point '*D' to point 'B'.*

when the bridge is balanced,

- $I1\ R1 = I2\ R2$ …(i)

Applying Kirchhoff's Voltage Law in loop ABC when the galvanometer current is zero,

- $I1\ R1 + I3\ R3 – E = 0$

But since current $I1 = I3$,

- $I1 = I3 = E/(R1 + R3)$ …(ii)

Similarly, in loop ADB

- $I2 = I4 = E/(R2 + R4)$ …(iii)

Using equation (i) (ii) and (iii),

- $R1/(R1 + R3)=R2/(R2 + R4)$
- $R1\ R4 = R2\ R3$ …(iv)

- In balanced condition if three of the resistances have known values, then the value of fourth resistance is calculated from the equation (iv).

- If R4 is unknown resistor Rx, then the value of Rx,

- Rx = R3 R2/R1

- Resistor R3 is called the standard arm of the bridge, and resistors R2 and R1 are called the ratio arms.

The laboratory version of Wheatstone bridge instrument

Sensitivity of a Wheatstone Bridge

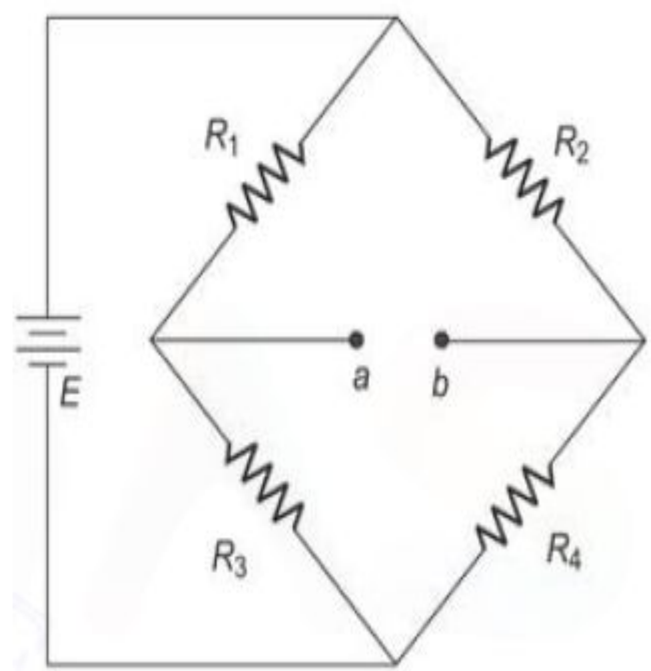
- When the bridge is in an unbalanced condition, current flows through the galvanometer, causing a deflection of its pointer.
- The amount of deflection is a function of the sensitivity of the galvanometer.
- Sensitivity is the deflection per unit current.
- A more sensitive galvanometer deflects by a greater amount for the same current.
- Deflection may be expressed in linear or angular units of measure, and sensitivity can be expressed in units of
- S = *mm/uA or* degree/uA or radians/uA.
- Therefore it follows that the total deflection *D is*
- $D = S \times I$

Unbalanced Wheatstone Bridge

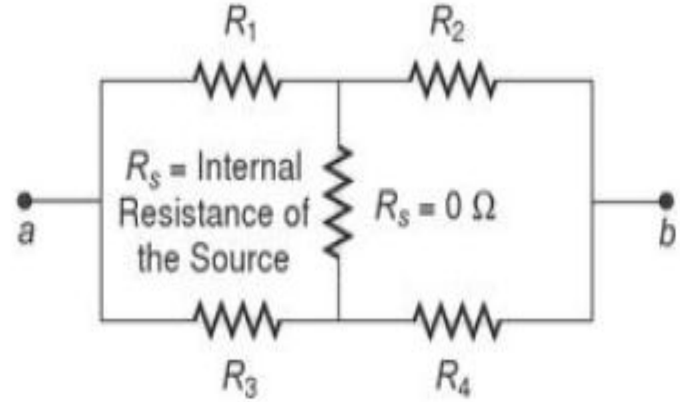
$$E_a = \frac{E \times R_3}{R_1 + R_3} \quad \text{and at point } b, \quad E_b = \frac{E \times R_4}{R_2 + R_4}$$

$$E_{th} = E_{ab} = E_a - E_b = \frac{E \times R_3}{R_1 + R_3} - \frac{E \times R_4}{R_2 + R_4}$$

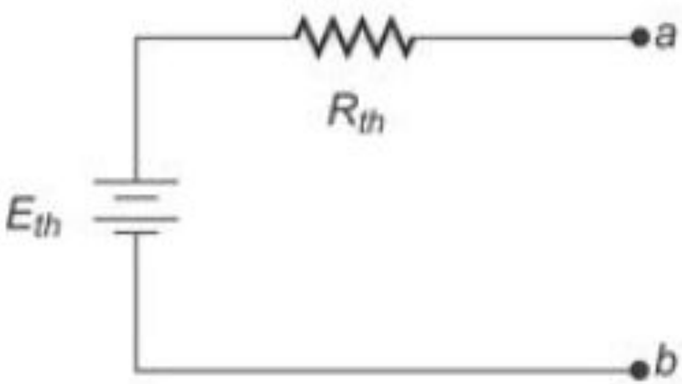
$$E_{ab} = E\left(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right)$$



The equivalent resistance of the circuit is $R_1//R_3$ in series with $R_2//R_4$ i.e. $R_1//R_3 + R_2//R_4$.

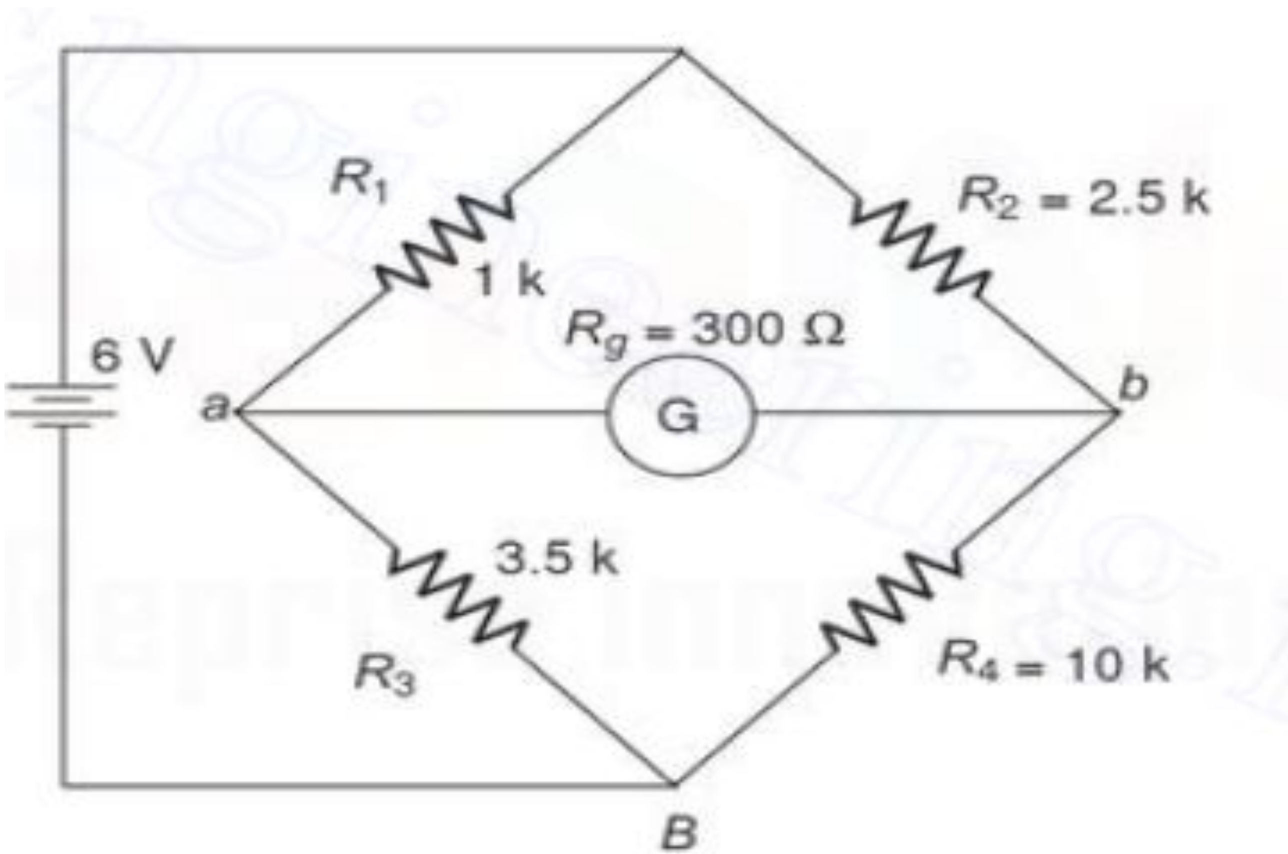


$$\therefore \qquad R_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$



$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

Example



Each of the ratio arms of a laboratory type Wheatstone bridge has guaranteed accuracy of ± 0.05 %, while the standard arm has a guaranteed accuracy of ± 0.1 %. The ratio arms are both set at 1000 Ω and bridge is balanced with standard arm adjusted to 3154 Ω.

Determine the upper and lower limits of the unknown resistance, based upon the guaranteed accuracies of the unknown bridge arms.

$$R_x = R_3 \frac{R_2}{R_1} = \frac{3154 \times 1000}{1000} = 3154\ \Omega$$

The percentage error in determination of R_X

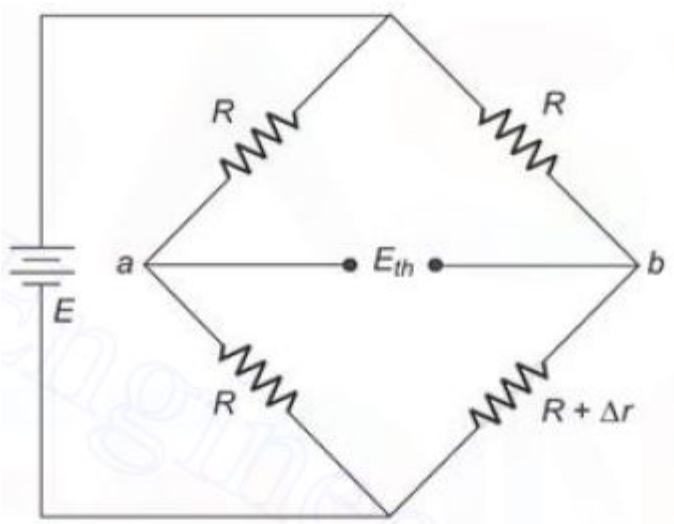
$$\frac{\delta R_X}{R_X} = \pm \frac{\delta R_1}{R_1} \pm \frac{\delta R_2}{R_2} \pm \frac{\delta R_3}{R_3}$$

$$= \pm 0.05 \pm 0.05 \pm 0.1 = \pm 0.2\ \%$$

Therefore, the limiting value of R_X is,

$$R_X = 3154 \pm 0.2\ \%$$

Slightly unbalanced Wheatstone Bridge



The voltage at point a is

$$E_a = \frac{E \times R}{R + R} = \frac{E \times R}{2R} = \frac{E}{2}$$

The voltage at point b is

$$E_b = \frac{R + \Delta r \times E}{R + R + \Delta r} = \frac{E(R + \Delta r)}{2R + \Delta r}$$

$$E_{th} = E_a - E_b = E\left(\frac{(R + \Delta r)}{2R + \Delta r} - \frac{1}{2}\right)$$

$$= E\left(\frac{2(R + \Delta r) - (2R + \Delta r)}{2(2R + \Delta r)}\right)$$

$$= E\left(\frac{2R + 2\Delta r - 2R - \Delta r}{4R + 2\Delta r}\right)$$

$$= E\left(\frac{\Delta r}{4R + 2\Delta r}\right)$$

If Δr is 5% of R or less, Δr in the denominator can be neglected without introducing appreciable error. Therefore, Thévenin's voltage is

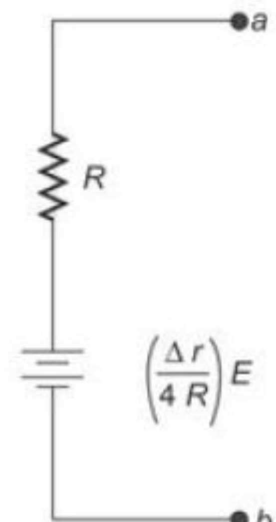
$$E_{th} = \frac{E \times \Delta r}{4R} = E\left(\frac{\Delta r}{4R}\right)$$

$$R_{th} = \frac{R \times R}{R + R} + \frac{R(R + \Delta r)}{R + R + \Delta r}$$

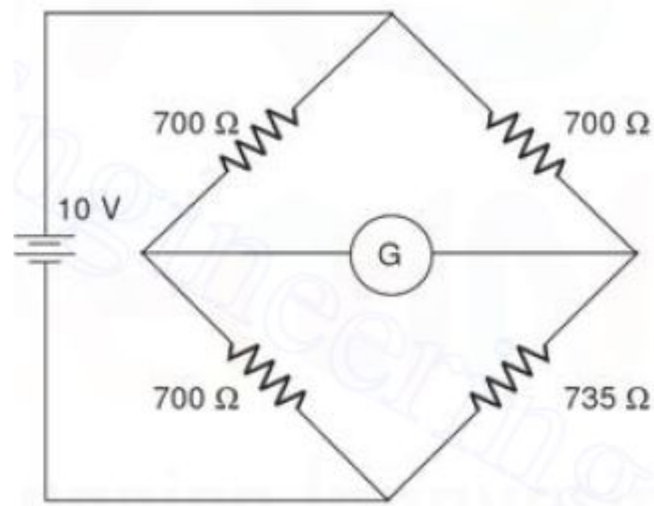
$$= \frac{R}{2} + \frac{R(R + \Delta r)}{2R + \Delta r}$$

Again, if Δr is small compared to R, Δr can be neglected. Therefore,

$$R_{th} = \frac{R}{2} + \frac{R}{2} = R$$



Given a centre zero 200 – 0 – 200 μA movement having an internal resistance of 125 Ω. Calculate the current through the galvanometer



$$E_{th} = \frac{E(\Delta r)}{4R}$$

$$= \frac{10 \times 35}{4 \times 700} = 0.125 \text{ V}$$

Thévenin's equivalent resistance is

$$R_{th} = R = 700 \ \Omega$$

The current through the galvanometer is

$$I_g = \frac{E_{th}}{R_{th} + R_g} = \frac{0.125 \text{ V}}{700 + 125} = \frac{0.125}{825} = 151.5 \ \mu\text{A}$$

Applications

- A Wheatstone bridge may be used to measure the dc resistance of various types of wire, either for the purpose of quality control of the wire itself.

- For example, the resistance of motor windings, transformers, and relay coils can be measured.

- Wheatstone's bridge is also used extensively by telephone companies and others to locate cable faults.

- The fault may be two lines shorted together, or a single line shorted to ground.

Limitations of Wheatstone Bridge

- For low resistance measurement, the resistance of the leads and contacts becomes significant and introduces an error. This can be eliminated by Kelvin's Double bridge.
- For high resistance measurements, the resistance presented by the bridge becomes so large that the galvanometer is insensitive to imbalance.
- Another difficulty in Wheatstone's bridge is the change in resistance of the bridge arms due to the heating effect of current through the resistance.
- The rise in temperature causes a change in the value of the resistance, and excessive current may cause a permanent change in value.

Measurement Errors in Wheatstone Bridge

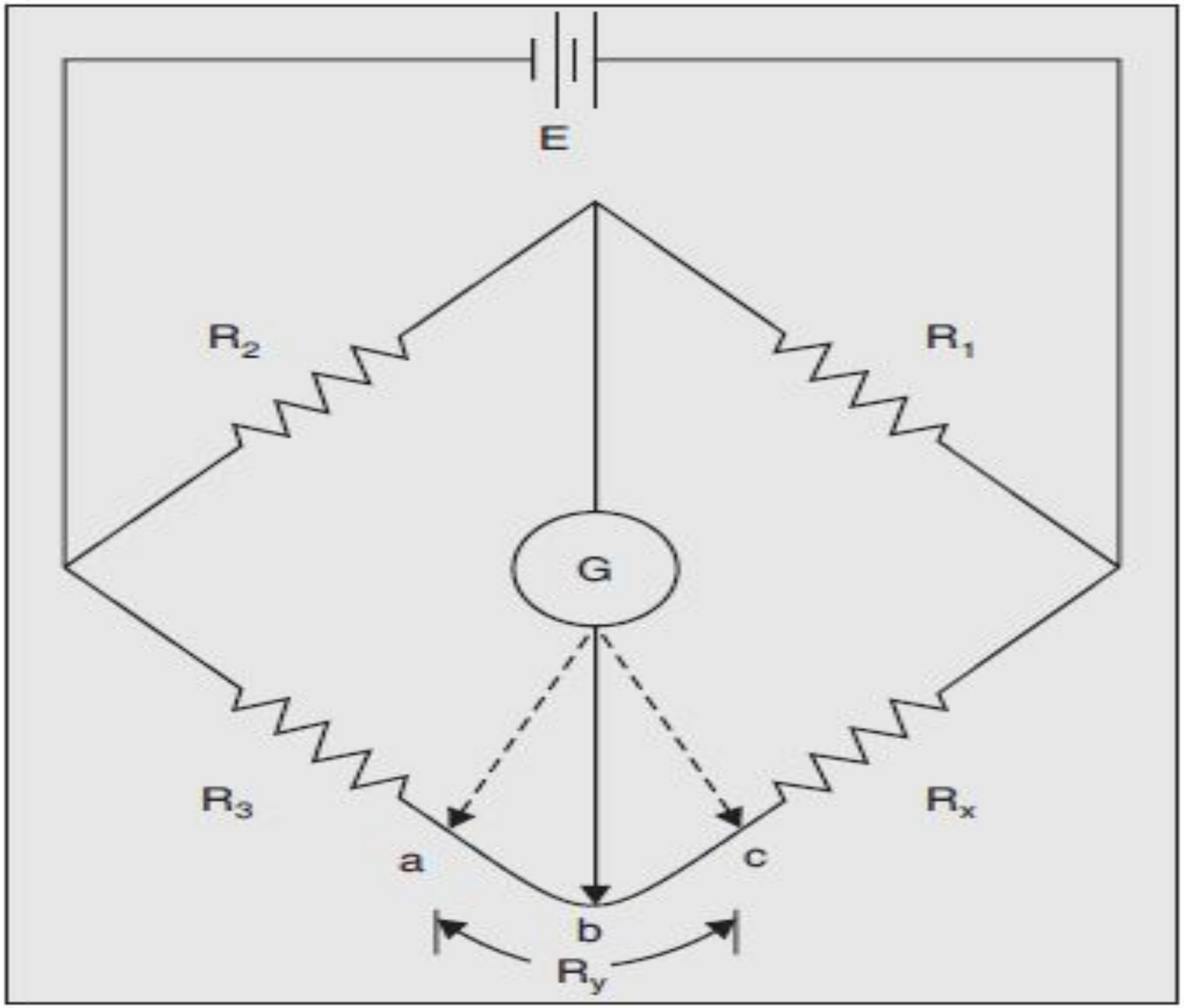
- The main source of measurement error is found in the limiting errors of three known resistors
- Insufficient sensitivity of the null detector
- Changes in resistance of the bridge arms due to the heating effect of the current through the resistors
- Thermal e.m.fs in the bridge circuit or the galvanometer circuit can also cause problems when low-value resistors are being measured.

Kelvin's Bridge

- This circuit provides great accuracy in the measurement of low value resistance generally below 1 ohm.

  It is used for measuring resistance values ranging from micro ohms to 1 ohm.

Kelvin's Bridge



- The resistance Ry represents the resistance of the conducting lead from R3 to Rx.
- The resistance Rx is the unknown resistance to be measured.
- The galvanometer can be connected either to point ‘c’ or to point ‘a’. When it is connected to point ‘a’, the resistance Ry of the connecting lead is added to the unknown resistance Rx.
- The measurement value of the resistance is too high than the actual value.
- When the galvanometer is connected to the point ‘c’, the resistance Ry of the connecting lead is added to the known resistance R3.
- The actual value of R3 is higher than the normal value by the resistance Ry and the resulting measurement of Rx is lower than the actual value.
- If the galvanometer is connected to point ‘b’, in between points ‘c’ and ‘a’, in such a way that the ratio of the resistance from ‘c’ to ‘b’ and that from ‘a’ to ‘b’ equals the ratio of resistance R1 and R2

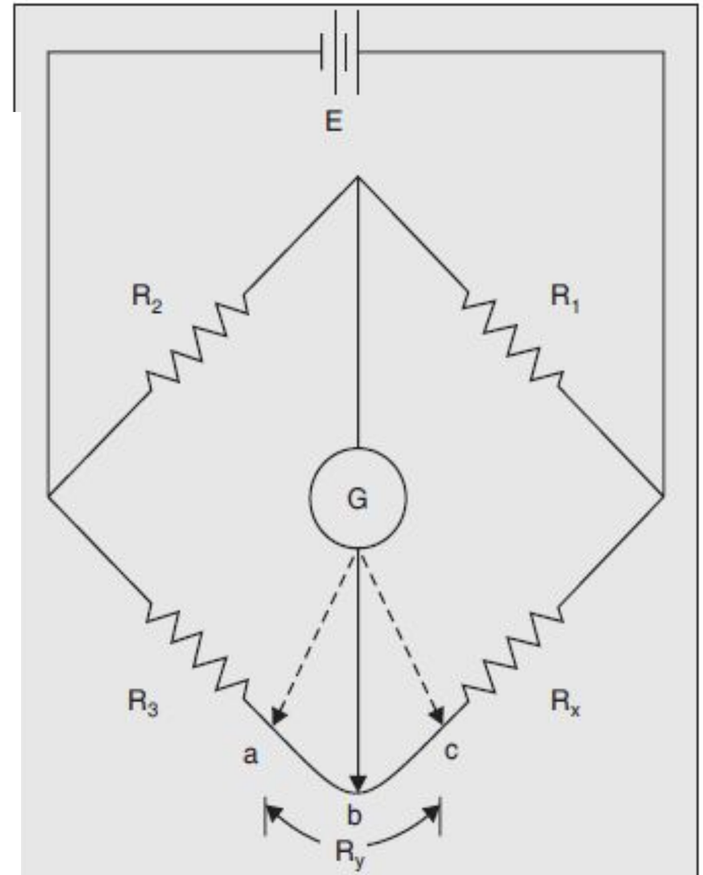
$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

Balance equation for the bridge is given by relation,

$$\frac{R_x + R_{cb}}{R_3 + R_{ab}} = \frac{R_1}{R_2}$$

$$(R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \qquad \text{...(i)}$$

Rab + Rbc = Ry



$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

balance equations for the bridge give the relationship

$$(R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab})$$

$$R_{ab} + R_{cb} = R_y \text{ and } \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$R_{ab} = \frac{R_2\, R_y}{R_1 + R_2} \quad \text{and as} \quad R_{ab} + R_{cb} = R_y$$

$$R_{cb} = R_y - R_{ab} = R_y - \frac{R_2\, R_y}{R_1 + R_2}$$

$$R_{cb} = \frac{R_1\, R_y + R_2\, R_y - R_2\, R_y}{R_1 + R_2} = \frac{R_1\, R_y}{R_1 + R_2}$$

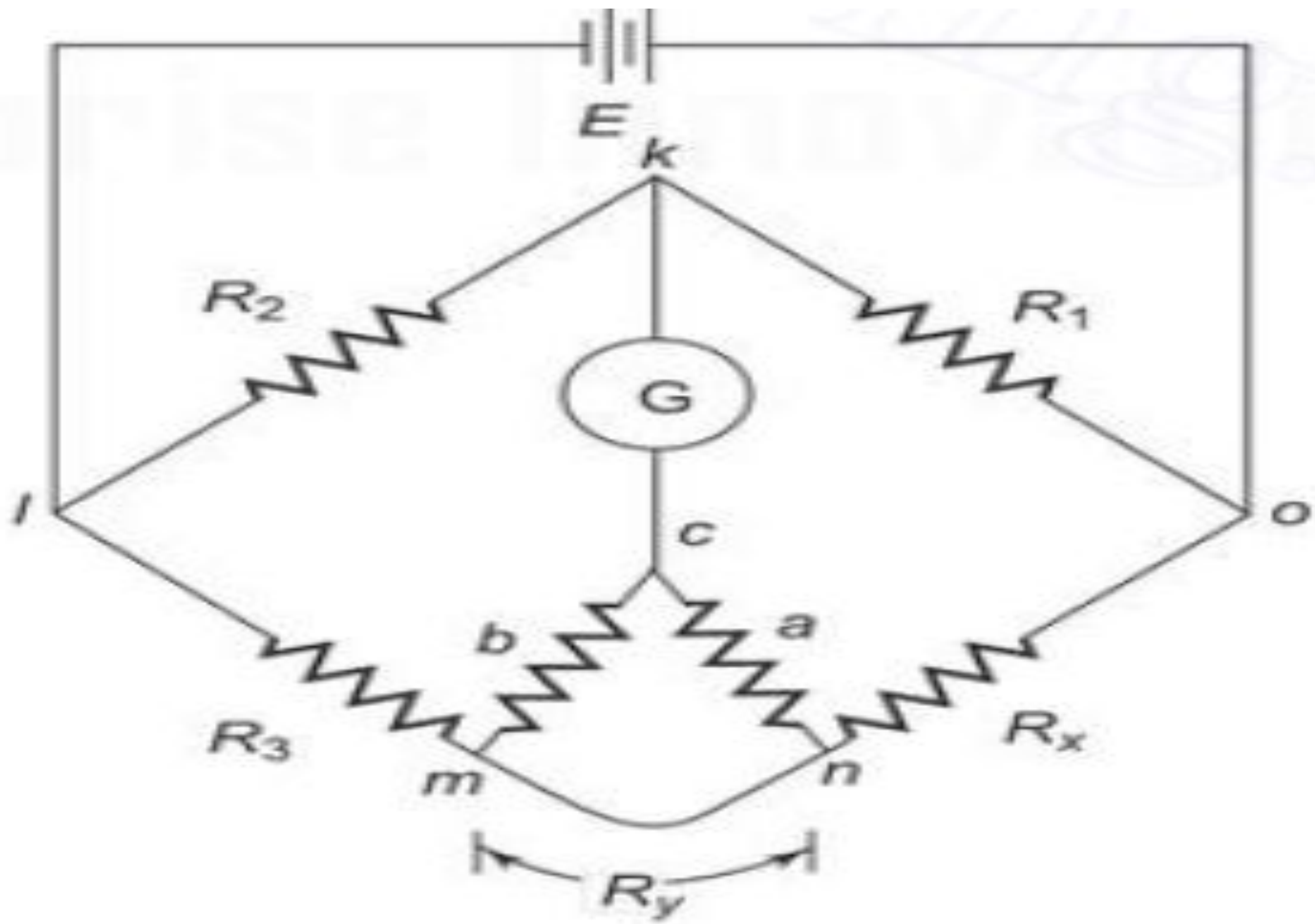
$$R_x + \frac{R_1\, R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2\, R_y}{R_1 + R_2} \right)$$

$$R_x + \frac{R_1\, R_y}{R_1 + R_2} = \frac{R_1\, R_3}{R_2} + \frac{R_1\, R_2\, R_y}{R_2\,(R_1 + R_2)}$$

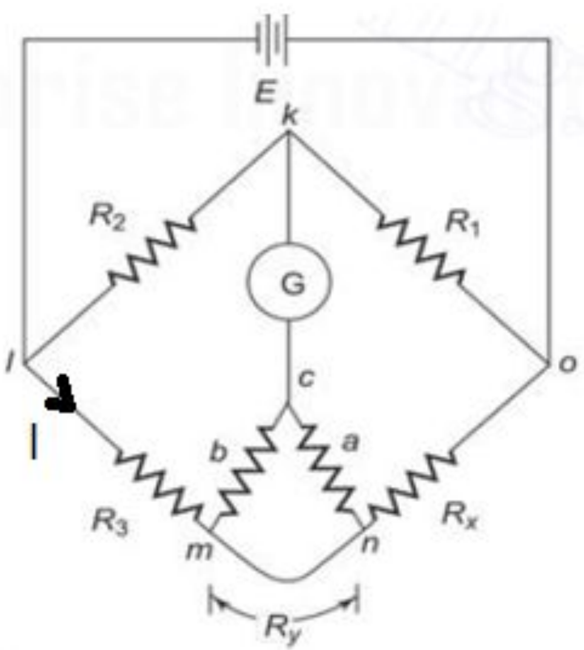
$$R_x = \frac{R_1\, R_3}{R_2}$$

This is the standard equation of the bridge balance. The equation does not depend on the resistance of connecting lead from *R3 to Rx. The effect of lead and contact resistances is completely* eliminated by connecting the galvanometer to the intermediate position ‘*b’.*

Kelvin’s Double Bridge



The second set of arms, a and b, connects the galvanometer to a point c at the appropriate potential between m and n connection, i.e. R_y. The ratio of the resistances of arms a and b is the same as the ratio of R_1 and R_2. The galvanometer indication is zero when the potentials at k and c are equal.



$$E_{lk} = E_{lmc}$$

$$E_{lk} = \frac{R_2}{R_1 + R_2} \times E \qquad \text{...(i)}$$

$$E = I\left(R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right)$$

$$E_{lk} = \frac{R_2}{R_1 + R_2} \times I\left(R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right) \qquad \text{...(ii)}$$

Similarly, $E_{lmc} = I \cdot \left(R_3 + \frac{b R_y}{a + b + R_y} \right)$

But $\quad E_{lk} = E_{lmc}$

i.e. $\frac{I R_2}{R_1 + R_2} \left(R_3 + R_x + \frac{(a+b)R_y}{a + b + R_y} \right) = I \cdot \left(R_3 + \frac{b R_y}{a + b + R_y} \right)$

$$\therefore \quad R_3 + R_x + \frac{(a+b)R_y}{a + b + R_y} = \frac{R_1 + R_2}{R_2} \left(R_3 + \frac{b R_y}{a + b + R_y} \right)$$

$$\therefore \quad R_3 + R_x + \frac{(a+b)R_y}{a + b + R_y} = \left(\frac{R_1}{R_2} + 1 \right) \left(R_3 + \frac{b R_y}{a + b + R_y} \right)$$

$$R_x + \frac{(a+b)R_y}{a + b + R_y} + R_3 = \frac{R_1 R_3}{R_2} + R_3 + \frac{b R_1 R_y}{R_2 (a + b + R_y)} + \frac{b R_y}{a + b + R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (a + b + R_y)} + \frac{b R_y}{a + b + R_y} - \frac{(a+b)R_y}{a + b + R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (a + b + R_y)} + \frac{b R_y - a R_y - b R_y}{a + b + R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (a + b + R_y)} - \frac{a R_y}{a + b + R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{(a + b + R_y)} \left(\frac{R_1}{R_2} - \frac{a}{b} \right)$$

$$\frac{R_1}{R_2} = \frac{a}{b}$$

$$R_x = \frac{R_1 R_3}{R_2}$$

This is the usual equation for Kelvin's bridge. It indicates that the resistance of the connecting lead R_y, has no effect on the measurement, provided that the ratios of the resistances of the two sets of ratio arms are equal. In a typical Kelvin's bridge the range of a resistance covered is 1 – 0.00001 Ω (10 µohm) with an accuracy of ± 0.05% to ± 0.2%.

AC Bridges

- AC bridge are the best and most usual methods for the precise measurement of self and mutual inductance and capacitance.
- These bridges are used to determine the value of inductance, capacitance and frequency.

- ***Measurement of Inductance***
- **1. Maxwell Bridge**
- **2. Hay Bridge**
-
- ***Measurement of Capacitance***
- **1. Schering Bridge**

- Detectors used for A.C bridges

Head Phones: Head phones are widely used as detectors at frequencies of 250 Hz upto 3 or 4 kHz.

2. Vibration Galvanometers: Vibration galvanometers are extremely used for power and low audio frequency ranges.

- These work at frequencies ranging from 5 Hz to 1000 Hz.
- The vibration galvanometers are used for power frequency range and low range of audio frequency as these instruments are very sensitive and selective for this frequency range.

3. Tunable Amplifier Detectors: The transistor amplifier can be tuned electrically to any desired frequency and then it can be made to respond to a narrow bandwidth at a bridge frequency.

- These detectors can be used over a frequency range of 10 Hz to 100 kHz.

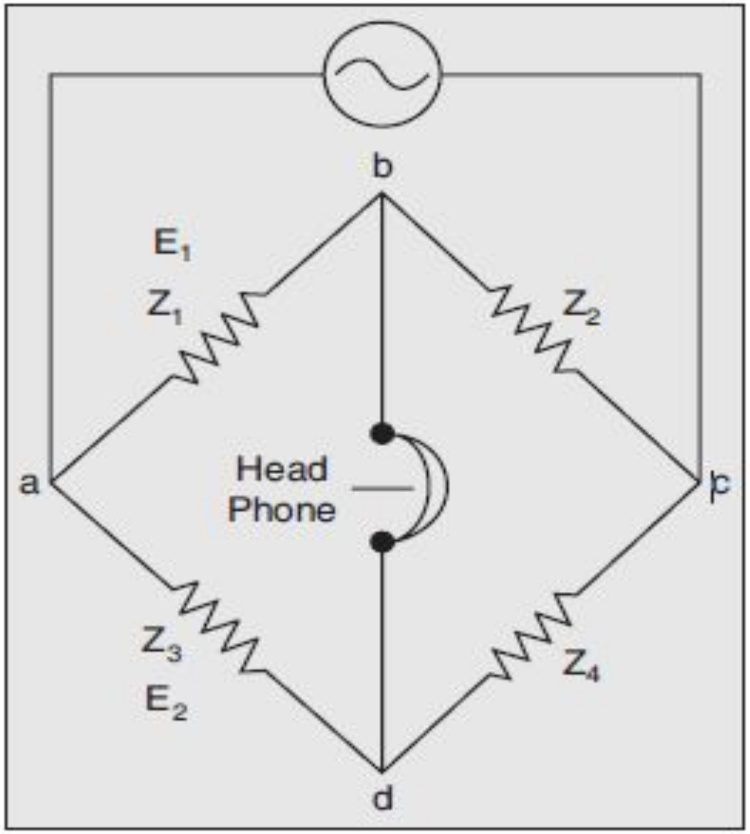
AC Bridge Circuit

The four bridge arms *Z1, Z2, Z3, and Z4 are* impedances and the detector is represented by headphones.

In balanced condition ,there is no current through the detector.

The potential difference between point ‘*b’ and ‘d’ should* be zero.

The voltage drop from ‘*a’ to ‘b’ equals to the voltage drop from point ‘a’ to ‘d’ in magnitude*
and phase.



- Z1Z4 = Z2Z3

If the impedance is written in form $Z = Z\angle\theta$ where Z represents the magnitude and θ represent the phase angle of the complex impedance

$$(Z_1\angle\theta_1)\,(Z_4\angle\theta_4) = (Z_2\angle\theta_2)\,(Z_3\angle\theta_3)$$

$$Z_1Z_4\,(\angle\theta_1 + \angle\theta_4) = Z_2Z_3\,(\angle\theta_2 + \angle\theta_3)$$

Condition 1: The products of the magnitudes of impedances of the opposite arms must be equal, *i.e.,* Z1Z4 = Z2Z3

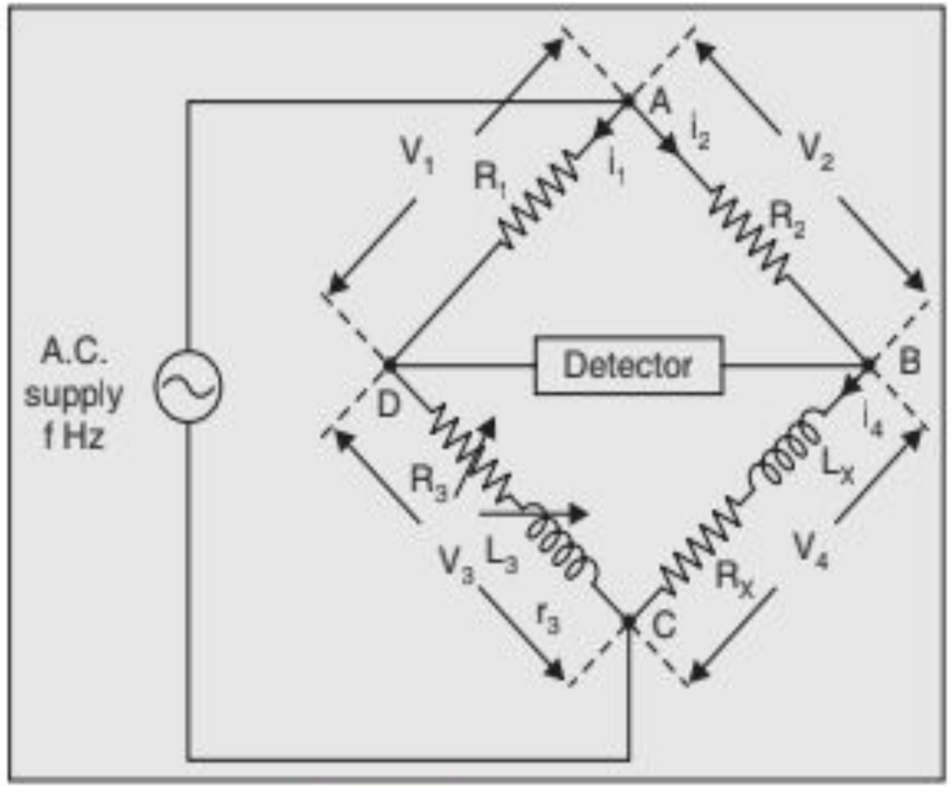
Condition 2: The sum of phase angles impedances in the opposite arms must be equal, i.e., $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

The following data relates to the basic AC bridge $\overline{Z_1} = 50\ \angle 80°$; $\overline{Z_2} = 125\ \Omega$ *and* $\overline{Z_3} = 200\ \angle 30°$. *Find the value of* $\overline{Z_4}$.

Measurement of Inductance using maxwell Bridge and Hay's Bridge

Maxwell Inductance Bridge

- This method is suitable f
accurate measurement c
medium inductance.
- This circuit measures the
inductance by comparisc
with a variable standard
self-inductance.
- LX is an unknown
self-inductance of resistc
RX, L3 is a known variabl
inductance of fixed
resistor r3 and variable
resistance R3, R2 and R1
are pure resistances and
is a detector.
- The bridge is balanced b
varying L3 and one of the
resistance R2 and R1.



Maxwell Inductance Bridge

The general equation of the bridge,

$$Z_1 Z_x = Z_2 Z_3 \qquad \ldots(i)$$

$$Z_x = \frac{Z_2 Z_3}{Z_1}$$

The values of Y_1, Z_2, Z_3 and Z_X are

$$Z_1 = R_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3 + j\omega L_3 + r_3$$

$$Z_X = R_X + j\omega L_X$$

Substituting all the above values in equation (*i*) we get,

$$R_1\,(R_X + j\omega L_X) = R_2\,(R_3 + j\omega L_3 + r_3)$$

$$R_X R_1 + j\omega L_X\,R_1 = R_3 R_2 + j\omega L_3\,R_2 + r_3 R_2 \qquad \ldots(ii)$$

Comparing the real terms on both sides of the above equation, we get,

$$R_X = \frac{R_2}{R_1}\,(R_3 + r_3) \qquad \ldots(iii)$$

Comparing the imaginary term on both sides of the equation (*ii*), we get,

$$\omega L_X R_1 = \omega L_3 R_2$$

$$L_X = \frac{L_3 R_2}{R_1} \qquad \ldots(iv)$$

The value of unknown resistance R_X and inductance L_X is given by equation (*iii*) and (*iv*) respectively. The quality factor (Q-factor) of coil is given by,

$$Q = \frac{\omega L_X}{R_X}$$

Maxwell Inductance-Capacitance Bridge

Maxwell's bridge, shown in Fig. 11.22, measures an unknown inductance in terms of a known capacitor. The use of standard arm offers the advantage of compactness and easy shielding. The capacitor is almost a loss-less component. One arm has a resistance R_1 in parallel with C_1, and hence it is easier to write the balance equation using the admittance of arm 1 instead of the impedance.

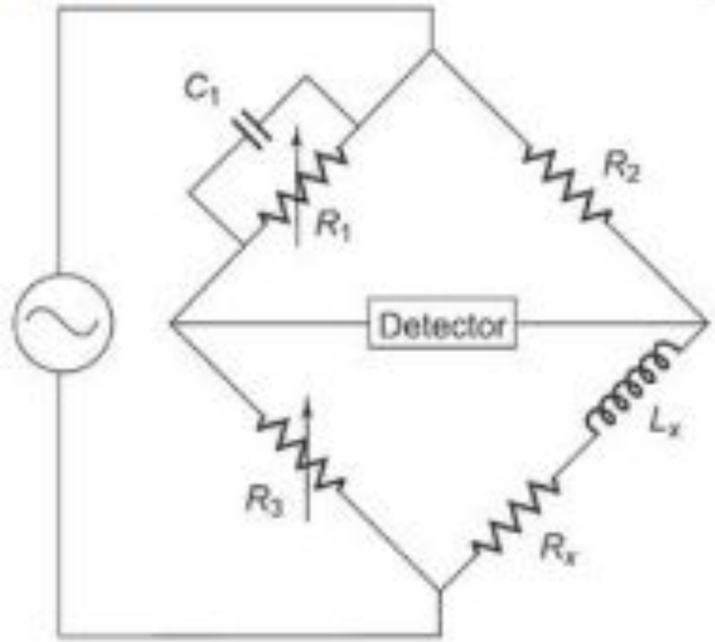


Fig. 11.22 Maxwell's bridge

The general equation for bridge balance is

$$Z_1 Z_x = Z_2 Z_3$$

i.e.
$$Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1 \qquad (11.14)$$

Where
$$Z_1 = R_1 \text{ in parallel with } C_1 \text{ i.e. } Y_1 = \frac{1}{Z_1}$$

$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

Z_1

$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_x = R_x \text{ in series with } L_x = R_x + j\omega L_x$$

From Eq. (11.14) we have

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Equating real terms and imaginary terms we have

$$R_x = \frac{R_2 R_3}{R_1} \text{ and } L_x = C_1 R_2 R_3 \tag{11.15}$$

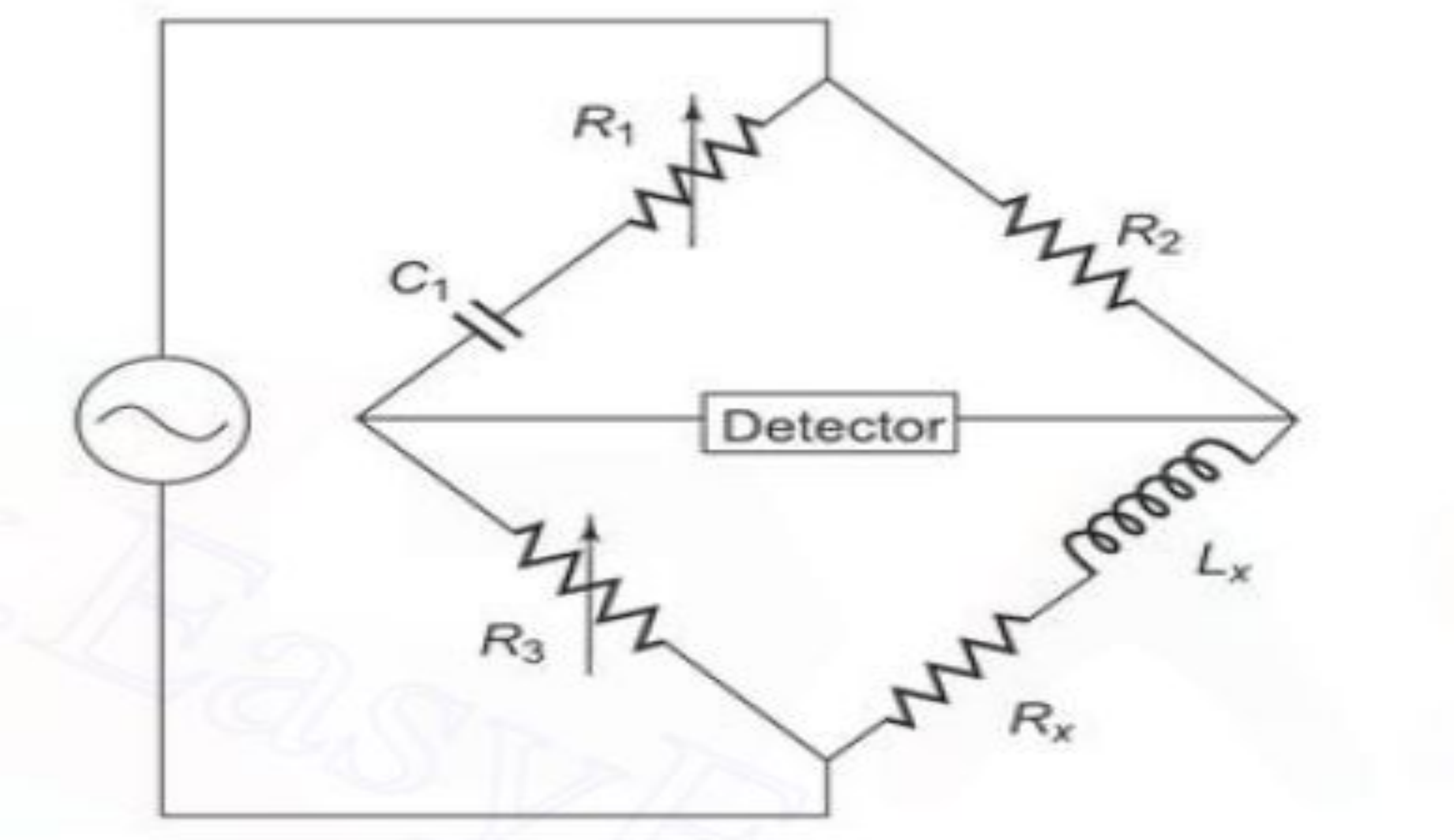
Also $$Q = \frac{\omega L_x}{R_x} = \frac{\omega C_1 R_2 R_3 \times R_1}{R_2 R_3} = \omega C_1 R_1$$

Maxwell's bridge is limited to the measurement of low Q values (1 – 10). The measurement is independent of the excitation frequency. The scale of the resistance can be calibrated to read inductance directly.

The Maxwell bridge using a fixed capacitor has the disadvantage that there is an interaction between the resistance and reactance balances. This can be avoided by varying the capacitances, instead of R_2 and R_3, to obtain a reactance balance. However, the bridge can be made to read directly in Q.

The bridge is particularly suited for inductances measurements, since comparison with a capacitor is more ideal than with another inductance. Commercial bridges measure from 1 – 1000 H, with ± 2% error. (If the Q is very large, R_1 becomes excessively large and it is impractical to obtain a satisfactory variable standard resistance in the range of values required).

HAY'S BRIDGE



At balance $$Z_1 Z_x = Z_2 Z_3, \text{ where}$$

$$Z_1 = R_1 - j/\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_x = R_x + j\omega L_x$$

Substituting these values in the balance equation we get

$$\left(R_1 - \frac{j}{\omega c_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} - \frac{j R_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$$

Equating the real and imaginary terms we have

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3$$

and $$\frac{R_x}{\omega C_1} = \omega L_x R_1$$

$$R_1 (\omega^2 R_1 C_1 L_x) + \frac{L_x}{C_1} = R_2 R_3$$

$$\omega^2 R_1^2 C_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

Multiplying both sides by C_1

$$\omega^2 R_1^2 C_1^2 L_x + L_x = R_2 R_3 C_1$$

Therefore, $$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Substituting for L_x

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 R_1^2 C_1^2}$$

The term ω appears in the expression for both L_x and R_x. This indicates that the bridge is frequency sensitive.

The inductance balance equation depends on the losses of the inductor (or Q) and also on the operating frequency.

$$L_x = \frac{R_2 R_3 C_1}{1 + (1/Q)^2}$$

For a value of Q greater than 10, the term $1/Q^2$ will be smaller than 1/100 and can be therefore neglected.

Therefore $L_x = R_2 R_3 C_1$, which is the same as Maxwell's equation. But for inductors with a Q less than 10, the $1/Q^2$ term cannot be neglected. Hence this bridge is not suited for measurements of coils having Q less than 10.

A commercial bridge measure from 1 μ H – 100 H with ± 2% error.

Measurement of Capacitance using Schering Bridge

Measurement of Capacitance
Schering Bridge

A very important bridge used for the precision measurement of capacitors and their insulating properties is the Schering bridge. Its basic circuit arrangement is given in Fig. 11.26. The standard capacitor C_3 is a high quality mica capacitor (low-loss) for general measurements, or an air capacitor (having a very stable value and a very small electric field) for insulation measurement.

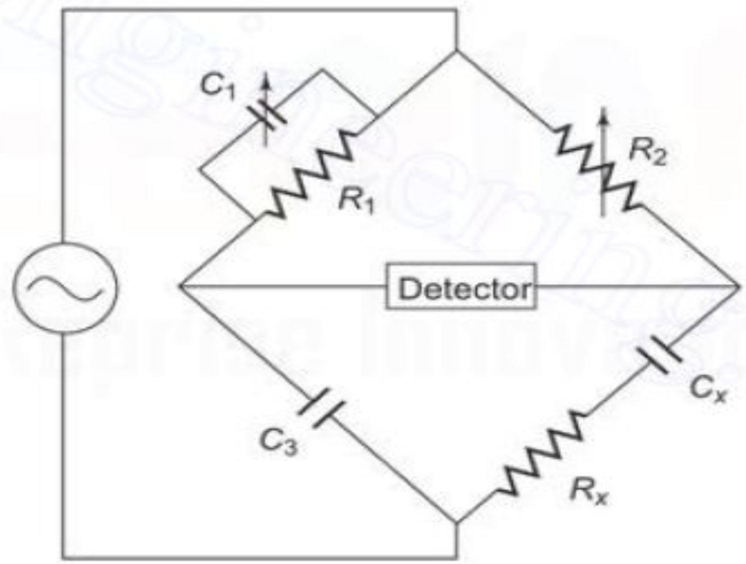


Fig. 11.26 Schering's bridge

For balance, the general equation is

$$Z_1 Z_x = Z_2 Z_3$$

$$\therefore \qquad Z_x = \frac{Z_2 Z_3}{Z_1}, Z_x = Z_2 Z_3 Y_1$$

where $\quad Z_x = R_x - j/\omega C_x$

$Z_2 = R_2$

$Z_3 = -j/\omega C_3$

Measurement of Capacitance
Schering Bridge

$$Z_1 Z_x = Z_2 Z_3$$

$\therefore$ $$Z_x = \frac{Z_2 Z_3}{Z_1}, \; Z_x = Z_2 Z_3 Y_1$$

where $$Z_x = R_x - j/\omega C_x$$
$$Z_2 = R_2$$
$$Z_3 = -j/\omega C_3$$
$$Y_1 = 1/R_1 + j\,\omega C_1$$

as $$Z_x = Z_2 Z_3 Y_1$$

$\therefore$ $$\left(R_x - \frac{j}{\omega C_x}\right) = R_2 \left(\frac{-j}{\omega C_3}\right) \times \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$\left(R_x - \frac{j}{\omega C_x}\right) = \frac{R_2 (-j)}{R_1 (\omega C_3)} + \frac{R_2 C_1}{C_3}$$

Equating the real and imaginary terms, we get

$$R_x = \frac{R_2 C_1}{C_3} \qquad [11.20(a)]$$

and $$C_x = \frac{R_1}{R_2} C_3 \qquad [11.20(b)]$$

Numerical based on Maxwell's Bridge

Example 11.8 (a) *A Maxwell bridge is used to measure an inductive impedance. The bridge constants at balance are*

$C_1 = 0.01\ \mu F$, $R_1 = 470\ k\Omega$, $R_2 = 5.1\ k\Omega$, and $R_3 = 100\ k\Omega$.

Find the series equivalent of the unknown impedance.

Solution We need to find R_x and L_x.

$$R_x = \frac{R_2 R_3}{R_1} = \frac{100\text{ k} \times 5.1\text{ k}}{470\text{ k}} = 1.09\text{ k}\Omega$$

$$\begin{aligned} L_x &= R_2 R_3 C_1 \\ &= 5.1\text{ k} \times 100\text{ k} \times 0.01\ \mu\text{f} \\ &= 5.1\text{ H} \end{aligned}$$

The equivalent series circuit is shown in Fig. 11.23.

Numerical based on HAY'S Bridge

Example 11.9 (b) *Four arms of a Hay Bridge are arranged as follows: AD is coil of unknown impedance Z, DC is a non-inductive resistance of 1 kΩ, CB is a non-inductive resistance of 800 Ω in series with a standard capacitor of 2 μF, BA is a non-inductive resistance of 16500 Ω, if the supply frequency is 50 Hz. Calculate the value of L and R of coil When the bridge is balanced.*

Solution Given $R_2 = 1000\ \Omega$, $R_3 = 16500\ \Omega$, $R_4 = 800\ \Omega$, $C_4 = 2\ \mu F$, $f = 50$ Hz

Step 1:

$$\therefore \quad \omega = 2\pi f = 2 \times 3.14 \times 50 = 314 \text{ and } \omega^2 = (314)^2 = 98596$$

Step 2:

$$L_x = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2}, \ R_x = \frac{\omega^2 C_4^2 R_4 R_2 R_3}{1 + \omega^2 C_4^2 R_4^2}$$

$$\therefore \quad L_1 = \frac{(1000) \times 16500 \times 2 \times 10^{-6}}{1 + 98596 \times (2\mu F)^2 \times (800)^2} = 26.4 \text{ H}$$

Step 3: $$R_x = \frac{\omega^2 C_4^2 R_4 R_2 R_3}{1 + \omega^2 C_4^2 R_4^2}$$

$$R_1 = \frac{(314)^2 \times (2\mu F)^2 \times 16500 \times 800 \times 1000}{1 + (314)^2 \times (2\mu F)^2 \times (800)^2} = 4.18 \text{ k}\Omega$$

Numerical based on HAY'S Bridge

Example 11.9 (c) *Find the unknown resistance and inductance having the following bridge arms*

$C_4 = 1\ \mu F,\ R_2 = R_3 = R_4 = 1000\ \Omega,\ \omega = 314\ rad/s$

Solution To find R_1 and L_1

Step 1: Given

$$L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 C_4^2 R_4^2},\ R_1 = \frac{\omega^2 C_4^2 R_4 R_2 R_3}{1+\omega^2 C_4^2 R_4^2}$$

$$\therefore \qquad L_1 = \frac{1000 \times 1000 \times 1 \times 10^{-6}}{1+(314)^2 \times (1\ \mu\text{F})^2 \times (1000)^2} = 0.91\ \text{H}$$

Step 2:

$$R_x = \frac{\omega^2 C_4^2 R_4 R_2 R_3}{1+\omega^2 C_4^2 R_4^2}$$

$$R_1 = \frac{(314)^2 \times (1\mu\text{F})^2 \times 1000 \times 1000 \times 1000}{1+(314)^2 \times (1\ \mu\text{F})^2 \times (1000)^2} \approx 89.79\ \Omega$$

Numerical based on Schering Bridge

Example 11.10 (a) *An ac bridge has the following constants (refer Fig. 11.27).*

Arm AB — capacitor of 0.5 μF in parallel with 1 kΩ resistance

Arm AD — resistance of 2 kΩ

Arm BC — capacitor of 0.5 μF

Arm CD — unknown capacitor C_x and R_x in series

Frequency — 1 kHz

Determine the unknown capacitance and dissipation factor.

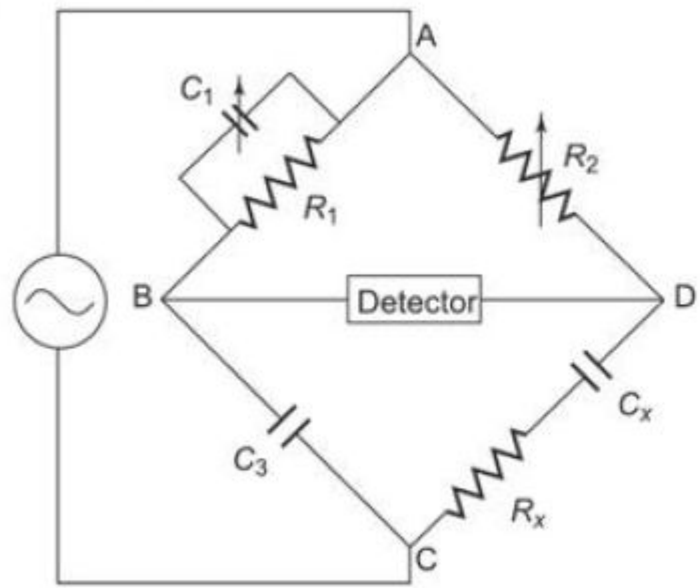


Fig. 11.27

Numerical based on Schering Bridge

Solution From Eqs 11.20(a) and 11.20(b), we have

$$R_x = \frac{C_1}{C_3} R_2 = \frac{0.5\ \mu\text{F}}{0.5\ \mu\text{F}} \times 2\ \text{k} = 2\ \text{k}\Omega$$

$$C_x = \frac{R_1}{R_2} \times C_3 = \frac{1\ \text{k}}{2\ \text{k}} \times 0.5\ \mu\text{F} = 0.25\ \mu\text{F}$$

The dissipation factor is given by

$$\begin{aligned} D &= \omega C_x R_x \\ &= 2 \times 3.142 \times 1000 \times 2\ \text{k} \times 0.25\ \mu\text{F} \\ &= 4 \times 3.142 \times 0.25 \\ &= 3.1416 \end{aligned}$$

Example *A Maxwell bridge is used to measure inductance and impedance. The bridge constants at balance are:*

$$R_1 = 235\ k\Omega$$
$$C_1 = 0.012\ \mu F$$
$$R_2 = 2.5\ k\Omega \text{ and}$$
$$R_3 = 50\ k\Omega$$

Find the series equivalent of the unknown impedance.

Example *Find the series equivalent inductance and resistance of the network that causes an opposite angle (Hay Bridge) to null the following bridge arms.*

$$\omega = 3000\ rad/s$$
$$R_2 = 9\ k\Omega$$
$$R_1 = 1.8\ k\Omega$$
$$C_1 = 0.9\ \mu F \text{ and}$$
$$R_3 = 0.9\ k\Omega$$

. Anderson bridge for measuring the inductance L and resistance R of the coil. Find R and L, if balance is obtain when,

$$R_4 = R_2 = 1\ k\Omega,\ R_3 = 500\ \Omega,\ r = 100\ \Omega \text{ and } C = 0.5\ \mu F.$$

Example *The Schering Bridge shown in Fig. 5.14 has the following constants* $R_1 = 1.5\ k\Omega$, $C_1 = 0.4\ \mu F$, $R_2 = 3\ k\Omega$ *and* $C_3 = 0.4\ \mu F$ *at frequency 1 kHz. Determine the unknown resistance and capacitance of the bridge circuit and dissipation factor.*

Example *Determine the equivalent parallel resistance and capacitance that causes a Wien bridge to null with the following component values:*

$$R_1 = 2.8\ k\Omega,\ C_1 = 4.8\ \mu F,\ R_2 = 20\ k\Omega,\ R_4 = 80\ k\Omega \text{ and } f = 2\ kHz$$

A Maxwell bridge is used to measure inductive impedance. The bridge constants at balance are:

$$R_1 = 470\ \text{k}\Omega,$$
$$C_1 = 0.01\ \mu\text{F},$$
$$R_2 = 5.1\ \text{k}\Omega \text{ and}$$
$$R_3 = 100\ \text{k}\Omega$$

Find the series equivalent of the unknown impedance.

(**Ans.** $R_X = 1.09\ \text{k}\Omega$; $L_X = 5.1$ H)

The four arms of a Maxwell capacitance bridge at balance are unknown inductance L_x having inherent resistance R_x. The resistance R_2 = 1 kΩ and R_3 = 1 kΩ. The capacitor of 0.5 µF in parallel with a resistance of 1000 Ω. Derive the equation for the bridge and determine the value of L_x and R_x.

(**Ans.** L_x = 0.5 H, R_X = 1000 Ω)

In the Schering Bridge has the constants R_1 = 1 kΩ, C_1 = 0.5 µF, R_2 = 3 kΩ and C_3 = 0.5 µF at frequency 1 kHz. Determine the unknown resistance and capacitance of the bridge circuit and dissipation factor.

(**Ans.** 2 Ω, 0.25 µF, 3.1416)