3. Angle Modulation and Demodulation

3.1 Frequency and Phase modulation (FM and PM): Basic concepts, mathematical analysis, FM wave (time and frequency domain), sensitivity, phase and frequency deviation, modulation index, deviation ratio, bandwidth requirement of angle modulated waves, narrowband FM and wideband FM.

3.2 Varactor diode modulator, FET reactance modulator, stabilized, AFC, Direct FM transmitter, indirect FM Transmitter, noise triangle, pre-emphasis and de-emphasis

3.3 FM demodulation: Balanced slope detector, Foster-Seely discriminator, Ratio detector, FM demodulator using Phase lock loop, amplitude limiting and thresholding, Applications of FM and PM.

Advantages over AM:

- Freedom from interference: all natural and external noise consist of amplitude variations, thus receiver usually cannot distinguish between amplitude of noise or desired signal. AM is noisy than FM.
- Operate in very high frequency band (VHF): 88MHz-108MHz
- Can transmit musical programs with higher degree of fidelity.

Types of Modulation

Amplitude Modulation

$$V \cdot \sin(\omega * t + \Phi)$$

Phase Modulation

$$V \cdot \sin(\omega * t + \Phi)$$

Frequency Modulation

$$V \cdot \sin(\omega * t + \Phi)$$

With very few exceptions, phase modulation is used for digital information

FREQUENCY MODULATION PRINCIPLES

- In FM the **carrier amplitude remains constant** , the **carrier frequency varies with the amplitude of modulating signal** .

- The **amount of change (** the relative displacement of carrier frequency in hertz in respect to it un-modulated value**) in carrier frequency** produced by the modulating signal is known as ***frequency deviation.***

PHASE MODULATION (PM) PRINCIPLES

- The process by which changing the **phase of carrier signal in accordance with the instantaneous of message signal**. The **amplitude** and **frequency remains constant** after the modulation process.

- Mathematical analysis:

  Let message signal:

$$v_m(t) = V_m \cos \varpi_m t$$

  And carrier signal:

$$v_c(t) = V_c \sin[\varpi_c t + \theta]$$

What is Angle Modulation?

$$v_c(t) = \underset{\text{Amplitude}}{V} \cdot \sin(2 \cdot \pi \cdot \underset{\text{Frequency}}{f_c} \cdot t + \underset{\text{Phase}}{\text{phase}})$$

$$\text{Phase} = \Phi$$

$$\text{Frequency} = \frac{\Delta\Phi}{\Delta t}$$

Angle modulation is a variation of one of these two parameters.

Understanding Angle Modulation

Frequency Modulation

Phase Modulation

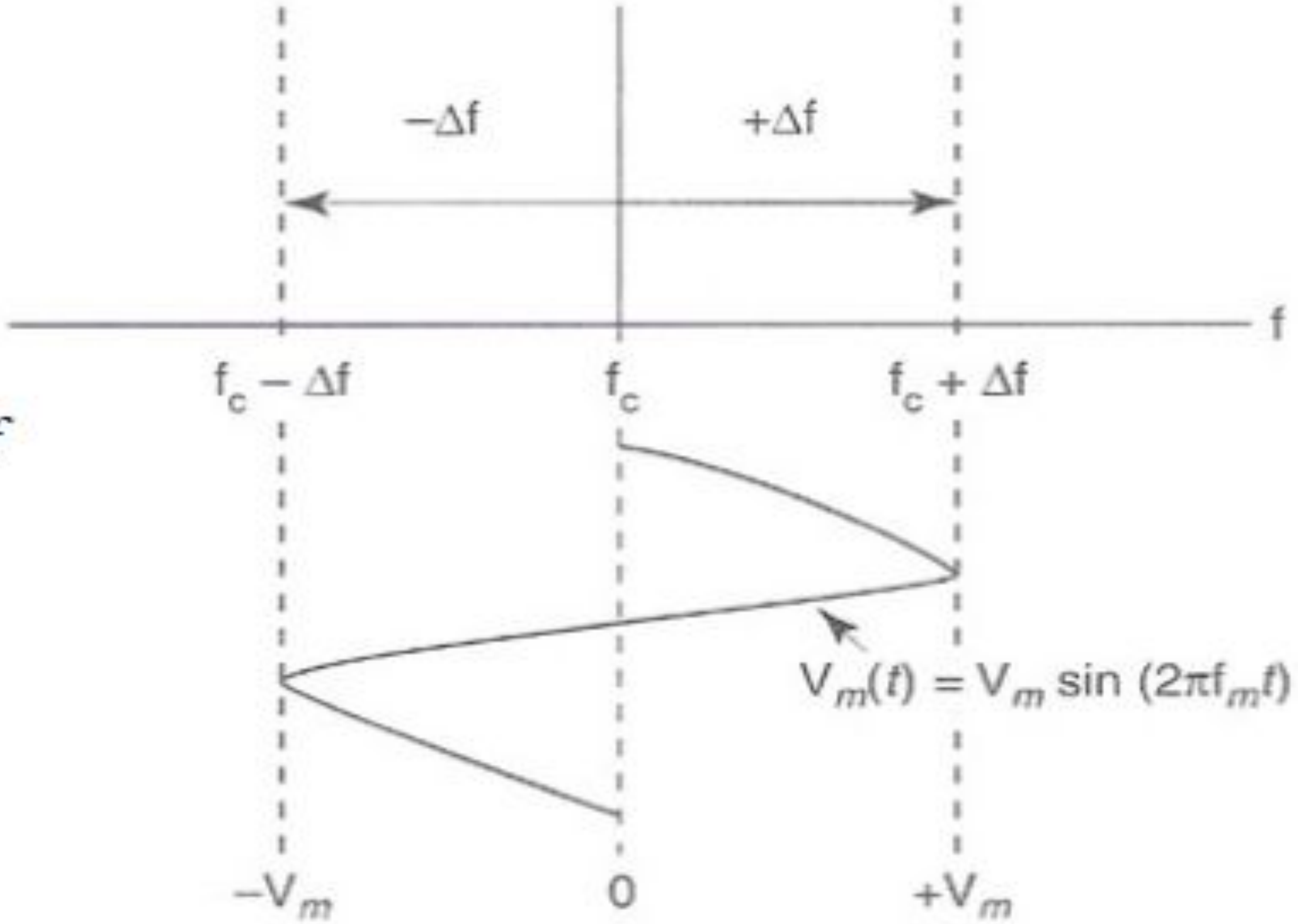
Angle modulation, either PM or FM, varies the frequency or phase of the carrier wave. Because of the practicalities of implementation, FM is predominant; analog PM is only used in rare cases.

$$V \cdot \sin(\omega * t + \Phi)$$

Vary one of these parameters

4.1.1 : Angle Modulation Representation in Frequency and Time Domain

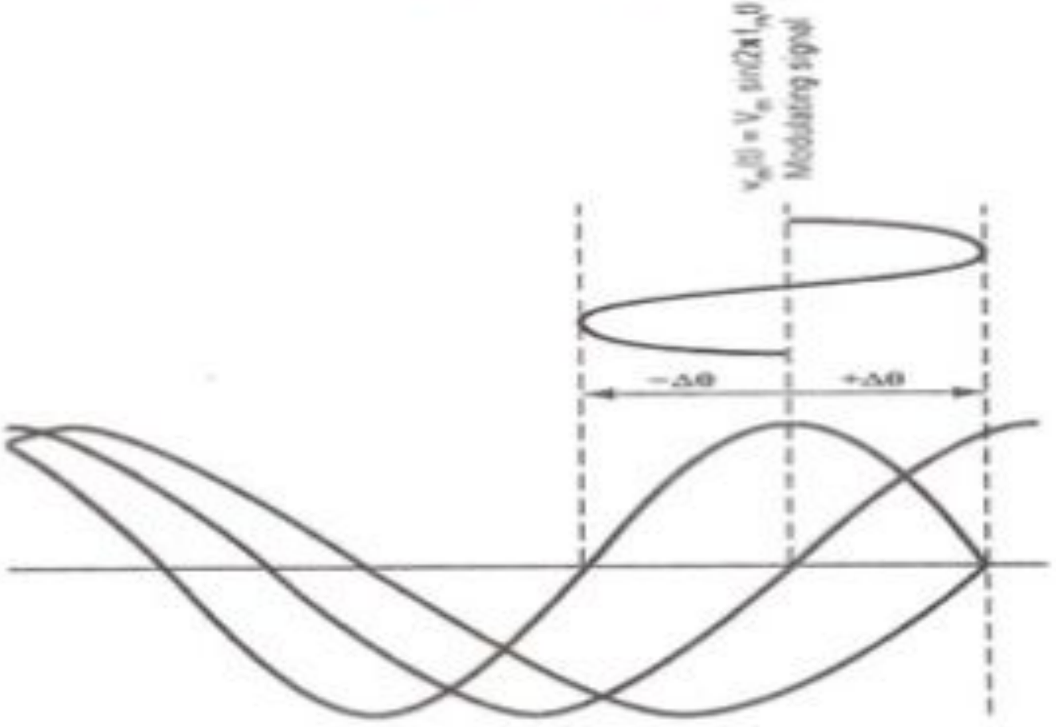
- An angle modulated signal in the frequency domain :
  - the carrier frequency, f_c is changed when acted on by the modulating signal.
  - the magnitude and direction of the frequency deviation, Δf is proportional to the amplitude and polarity of the modulating signal.



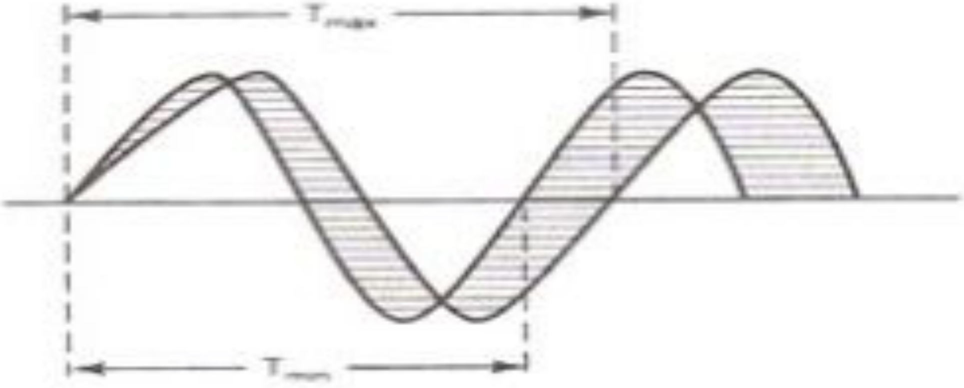
4.1.1 : Angle Modulation Representation in Frequency and Time Domain

- An angle modulated signal in the time domain :
  - the phase of the carrier is changing proportional to the amplitude of the modulating signal.
  - the phase shift is called phase deviation $\Delta\theta$. This shift is also produces a corresponding change in the frequency, known as frequency deviation Δf.
  - peak-to-peak frequency deviation is determine by (as shown in figure (b)),

$$\Delta f_{p-p} = \frac{1}{T_{\min}} - \frac{1}{T_{\max}} \qquad (3)$$



(a)



(b)

Angle Modulation – Basic Concepts

- Phase modulation (PM) & Freq modulation (FM) are special cases of angle modulated signaling.
- Complex envelope is $g(t)=A_c\, e^{j\theta(t)}$
  - Real envelope $R(t) = |g(t)| = A_c = constant$
  - Phase $\theta(t)$ is linear function of modulating signal $m(t)$
    - $g(t)$ being a nonlinear function of modulation
- Angle modulated signal $s(t) = A_c \cos[\underbrace{\omega_c t + \theta(t)}_{\Phi_i(t)}]$

Definitions:

- $\theta(t)$ is the instantaneous phase deviation (excess phase) - radian
- $\theta'(t)$ is the instantaneous frequency deviation – radian/sec
- $\Phi_i(t)=\omega_c t + \theta(t)$ is the instantaneous phase (exact) - radian
- $f_i(t)=(1/2p)d\Phi_i(t)/dt = d(\omega_c t + \theta(t))/dt$
  - This is the instantaneous frequency (exact) – radian/sec

$$f_i(t) = \frac{\omega_i(t)}{2\pi} = \frac{1}{2\pi}\left[\frac{d\Psi(t)}{dt}\right] = f_c + \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right] \qquad \phi_i(t) = 2\pi \int_{-\infty}^{t} f_i(\alpha)d\alpha$$

4.2 : Mathematical Analysis

- to differentiate between FM and PM, the following terms need to be defined :
  - *1. Instantaneous Phase Deviation*
    - the instantaneous change in the phase of the carrier at a given instant of time.

      Instantaneous phase deviation = $\theta(t)$ rad **(4)**
  - *2. Instantaneous phase*
    - the precise phase of the carrier at a given instant of time.

      Instantaneous phase = $\omega_c t + \theta(t)$ rad **(5)**
  - *3. Instantaneous frequency deviation*
    - the instantaneous change in the frequency of the carrier and is defined as the first time derivative of the instantaneous phase deviation.

      Instantaneous frequency deviation = $\theta'(t)$ rad/s **(6)**
  - *4. Instantaneous frequency*
    - the precise frequency of the carrier at a given instant of time and is defined as the first time derivative of the instantaneous phase.

      Instantaneous frequency = $\omega_i = \omega_c + \theta'(t)$ rad/s **(7)**

4.2 : Mathematical Analysis

- from the previous 4 terms, (3) ~ (7), PM and FM can be defined as :
  - PM : an angle modulation in which θ(t) is proportional to the amplitude of the modulating signal.
  - FM : an angle modulation in which θ'(t) is proportional to the amplitude of the modulating signal.
- For a modulating signal $v_m(t)$,

$$\theta(t) = K v_m(t) \ rad \qquad (8)$$

$$\theta'(t) = K_1 v_m(t) \ rad/s \qquad (9)$$

where K and K1 are constants and are the *deviation sensitivities* of the phase and frequency modulators, respectively.

4.2 : Mathematical Analysis

- substituting a modulating signal $v_m(t) = V_m cos(\omega mt)$, equation (8) and (9) into equation (1) yields

PM :

$$m(t) = V_c \cos[\omega_c t + \theta(t)]$$

$$= V_c \cos[\omega_c t + KV_m \cos(\omega_m t)] \qquad (10)$$

FM : as $\theta(t) = \int \theta'(t)$

$$m(t) = V_c \cos\left[\omega_c t + \int \theta'(t)\right]$$

$$= V_c \cos\left[\omega_c t + K_1 \int V_m \cos(\omega_m t) dt\right]$$

$$= V_c \cos\left[\omega_c t + \frac{K_1 Vm}{\omega_m} \sin(\omega_m t)\right] \qquad (11)$$

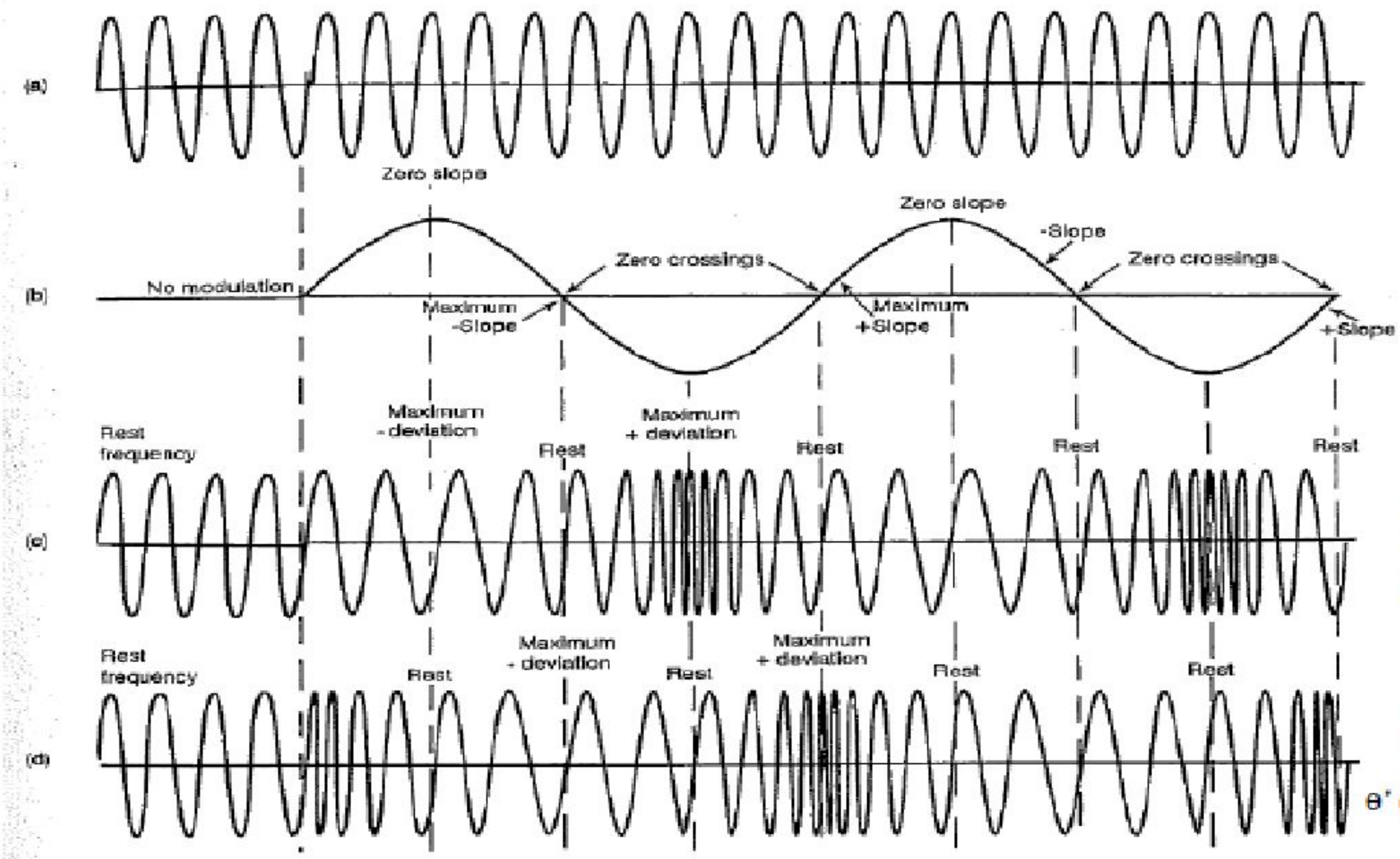
4.2 : Mathematical Analysis

- Summarized table :

Table 7-1 Equations for Phase- and Frequency-Modulated Carriers

Type of Modulation	Modulating Signal	Angle-Modulated Wave, $m(t)$
(a) Phase	$v_m(t)$	$V_c \cos[\omega_c t + K v_m(t)]$
(b) Frequency	$v_m(t)$	$V_c \cos[\omega_c t + K_1 \int v_m(t)\, dt]$
(c) Phase	$V_m \cos(\omega_m t)$	$V_c \cos[\omega_c t + K V_m \cos(\omega_m t)]$
(d) Frequency	$V_m \cos(\omega_m t)$	$V_c \cos\left[\omega_c t + \frac{K_1 V_m}{\omega_m} \sin(\omega_m t)\right]$

Frequency VS Phase Modulation



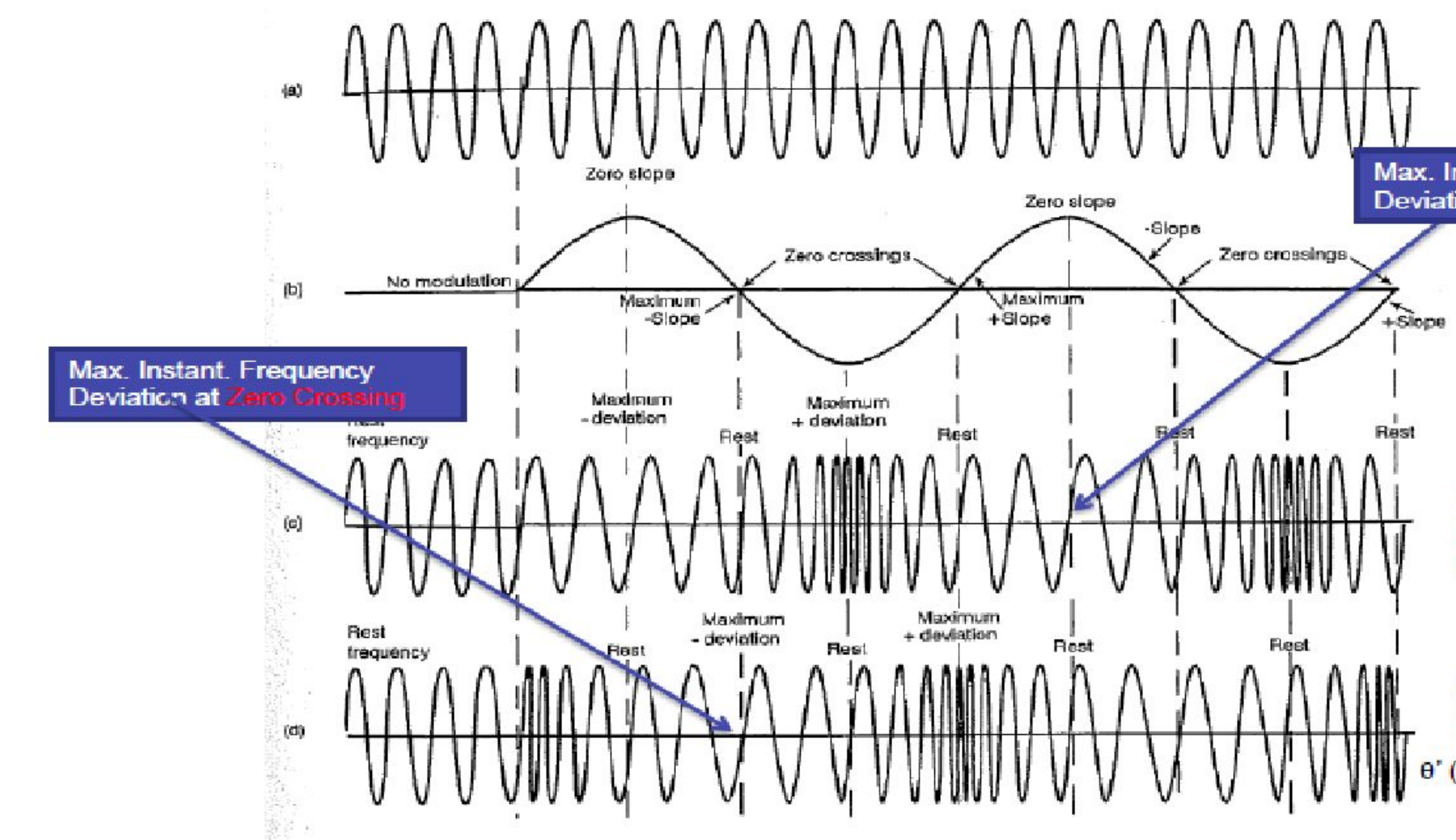
Frequency Modulation

$\theta'(t) = Df \cdot m(t)$

Phase Modulation

$\theta'(t) = Dp \cdot d[m(t)] / dt$

Frequency VS Phase Modulation



Frequency Modulation
$\theta'(t) = Df \cdot m(t)$

Phase Modulation
$\theta'(t) = Dp \cdot d[m(t)] / dt$

4.3 : FM and PM Waveforms

- FM and PM waveforms are identical except for their time relationship.
- for FM, the maximum frequency deviation occurs during the maximum positive and negative peaks of the modulating signal.
- for PM, the maximum frequency deviation occurs during the zero crossings of the modulating signal (i.e. the frequency deviation is proportional to the slope of first derivative of the modulating signal).

4.4 : Modulation Index and Percent Modulation

- comparing equation (10) and (11), equation (1) can be rewritten in general form as

$$m(t) = V_c \cos\left[\omega_c t + m\cos(\omega_m t)\right] \tag{12}$$

where m is called the modulation index.

4.4.1 : Modulation Index and Percent Modulation for PM

- for PM, the modulation index is also known as *peak phase deviation $\Delta\theta$*, and is proportional to the amplitude of the modulating signal and is expressed as

$$m = \Delta\theta = KV_m(radians) \tag{13}$$

where m = modulation index

K = deviation sensitivity (radians/volt)

V_m = peak modulating signal amplitude (volt)

4.4.1 : Modulation Index and Percent Modulation for PM

- therefore, for PM :

$$m(t) = V_c \cos\left[\omega_c t + KV_m \cos(\omega_m t)\right]$$
$$= V_c \cos\left[\omega_c t + \Delta\theta \cos(\omega_m t)\right]$$
$$= V_c \cos\left[\omega_c t + m \cos(\omega_m t)\right] \qquad (14)$$

4.4.2 : Modulation Index and Percent Modulation for FM

- for FM, the modulation index is directly proportional to the amplitude of the modulating signal and inversely proportional to the frequency of the modulating signal.

$$m = \frac{K_1 V_m}{\omega_m} = \frac{K_1 V_m}{f_m} (unitless) \qquad (15)$$

where $\mathbf{K_1}$ = deviation sensitivities (radians/second per volt or cycles/second per vol

Vm = peak modulating signal amplitude (volt)

$\boldsymbol{\omega_m}$ = radian frequency (radians/second)

$\mathbf{f_m}$ = cyclic frequency (cycles/second or hertz)

4.4.2 : Modulation Index and Percent Modulation for FM

- also for FM, the peak frequency deviation Δf is simply the product of the deviation sensitivity and the peak modulating signal voltage. I.e.

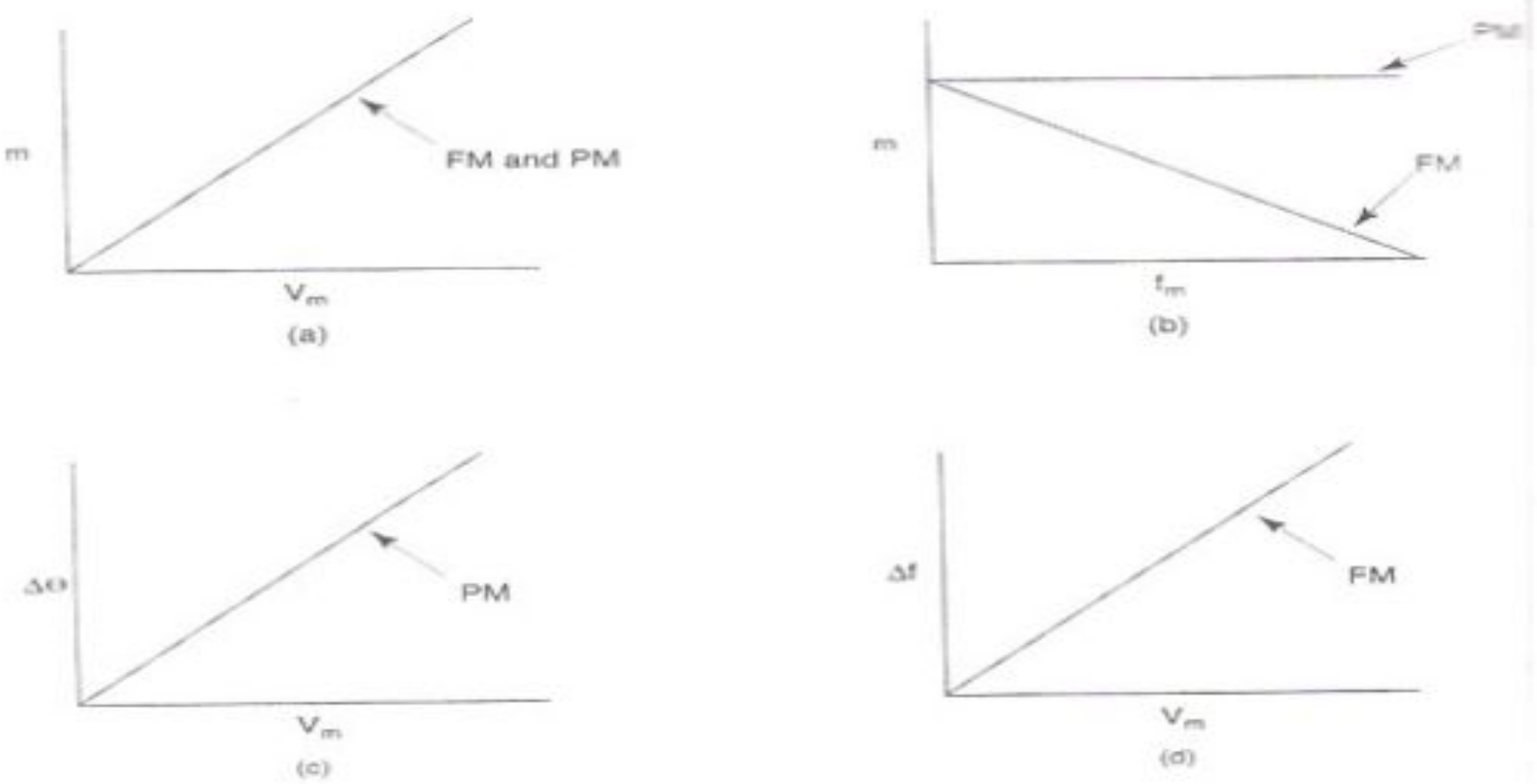
$$\Delta f = K_1 V_m \Rightarrow m = \frac{\Delta f}{f_m} (unitless) \qquad (16)$$

- therefore, for FM, equation (11) can be rewritten as

$$\begin{aligned} m(t) &= V_c \cos\left[\omega_c t + \frac{K_1 V_m}{f_m} \sin(w_m t)\right] \\ &= V_c \cos\left[\omega_c t + \frac{\Delta f}{f_m} \sin(w_m t)\right] \\ &= V_c \cos\left[\omega_c t + m \sin(w_m t)\right] \end{aligned} \qquad (17)$$

- figure 7.4 & table 7.2

4.4.2 : Modulation Index and Percent Modulation for FM



- Relationship between modulation index, frequency deviation and phase deviation in respect to the modulation signal amplitude and frequency : (a) modulation index vs amplitude (b) frequency deviation vs modulating frequency (c) phase deviation vs amplitude (d) frequency deviation vs amplitude

Modulation Index

Definition:

Modulation Index is defined as the ratio of frequency deviation (δ) to the modulating frequency (f_m).

$$\text{M.I.} = \frac{\text{Frequency Deviation}}{\text{Modulating Frequency}}$$

$$mf = \frac{\delta}{fm}$$

In FM M.I.>1

Modulation Index of FM decides –

(i) Bandwidth of the FM wave.

(ii) Number of sidebands in FM wave.

Deviation Ratio

The modulation index corresponding to maximum deviation and maximum modulating frequency is called deviation ratio.

$$\text{Deviation Ratio} = \frac{\text{Maximum Deviation}}{\text{Maximum modulating Frequency}}$$

$$= \frac{\delta max}{fmax}$$

In FM broadcasting the maximum value of deviation is limited to **75 kHz.** The maximum modulating frequency is also limited to **15 kHz.**

Percentage M.I. of FM

4.4.3 : Percent Modulation

- percent modulation for angle modulation is determined in different manner than for amplitude modulation.
- with angle modulation, percent modulation is the ratio of frequency deviation actually produced to the maximum frequency deviation allowed, stated in percent form

$$Percent\ modulation = \frac{\Delta f_{(actual)}}{\Delta f_{(max)}} \times 100\% \qquad (18)$$

Summary

	FM	PM
Modulated wave	$m(t) = V_c \cos\left[\omega_c t + \frac{K_1 V_m}{f_m} \sin(\omega_m t)\right]$	$m(t) = V_c \cos[\omega_c t + K V_m \cos(\omega_m t)]$
or	$m(t) = V_c \cos[\omega_c t + m \sin(\omega_m t)]$	$m(t) = V_c \cos[\omega_c t + m \cos(\omega_m t)]$
or	$m(t) = V_c \cos\left[\omega_c t + \frac{\Delta f}{f_m} \sin(\omega_m t)\right]$	$m(t) = V_c \cos[\omega_c t + \Delta\theta \cos(\omega_m t)]$
Deviation sensitivity	K_1 (Hz/V) =Df	K (rad/V) =Dp
Deviation	$\Delta f = K_1 V_m$ (Hz)	$\Delta\theta = K V_m$ (rad)
Modulation index	$m = \frac{K_1 V_m}{f_m}$ (unitless)	$m = K V_m$ (rad)
or	$m = \frac{\Delta f}{f_m}$ (unitless)	$m = \Delta\theta$ (rad)
Modulating signal	$v_m(t) = V_m \sin(\omega_m t)$	$v_m(t) = V_m \cos(\omega_m t)$
Modulating frequency	$\omega_m = 2\pi f_m$ rad/s	$\omega_m = 2\pi f_m$ rad/s
or	$\omega_m/2\pi = f_m$ (Hz)	$\omega_m/2\pi = f_m$ (Hz)
Carrier signal	$V_c \cos(\omega_c t)$	$V_c \cos(\omega_c t)$
Carrier frequency	$\omega_c = 2\pi f_c$ (rad/s)	$\omega_c = 2\pi f_c$ (rad/s)
or	$\omega_c/2\pi = f_c$ (Hz)	$\omega_c/2\pi = f_c$ (Hz)

Note K = D = Sensitivity; Vm = max [m(t)]=max [Vm(t)] = Modulating Signal
m modulation index; $\Delta F = \Delta f$;

4.5 : Frequency and Bandwidth Analysis of Angle-Modulated Waves

- frequency analysis of the angle-modulated wave is much more complex compared to the amplitude modulation analysis.
- in phase/frequency modulator, a modulating signal produces an infinite number of side frequencies pairs (i.e. it has infinite bandwidth), where each side frequency is displaced from the carrier by an integral multiple of the modulating frequency.

4.5.1 : Bessel Function

- from equation (12), the angle-modulated wave is expressed as

$$m(t) = V_c \cos\left[\omega_c t + m\cos(\omega_m t)\right]$$

- based on the above equation, the individual frequency components of the angle-modulated wave is not obvious.

4.5.1 : Bessel Function

- Bessel function identities can be used to determine the side frequencies components

$$\cos(\alpha + m\cos\beta) = \sum_{n=-\infty}^{\infty} J_n(m)\cos(\alpha + n\beta + \frac{n\pi}{2}) \qquad (19)$$

where $J_n(m)$ is the Bessel function of the first kind.

- applying equation (19) to equation (12) yields,

$$m(t) = Vc\sum_{n=-\infty}^{\infty} J_n(m)\cos(\omega_c t + n\omega_m t + \frac{n\pi}{2}) \qquad (20)$$

4.5.1 : Bessel Function

- expanding (20),

$$m(t) = V_c \left\{ J_0(m)\cos(\omega_c t) + J_1(m)\cos\left[(\omega_c + \omega_m)t + \frac{\pi}{2}\right] \right.$$

$$- J_1(m)\cos\left[(\omega_c - \omega_m)t - \frac{\pi}{2}\right] + J_2(m)\cos\left[(\omega_c + 2\omega_m)t\right]$$

$$\left. + \ldots Jn(m)\ldots \right\}$$

where $m(t)$ = angle modulated wave

m = modulation index

V_c = peak carrier ampitude

$J_0(m)$ = carrier component

$J_1(m)$ = first set of side frequencies displaced from carrier by ω_m

$J_2(m)$ = second set of side frequencies displaced from carrier by $2\omega_m$

$J_n(m)$ = *nth* set of side frequencies displaced from carrier by $n\,\omega_m$

4.5.1 : Bessel Function

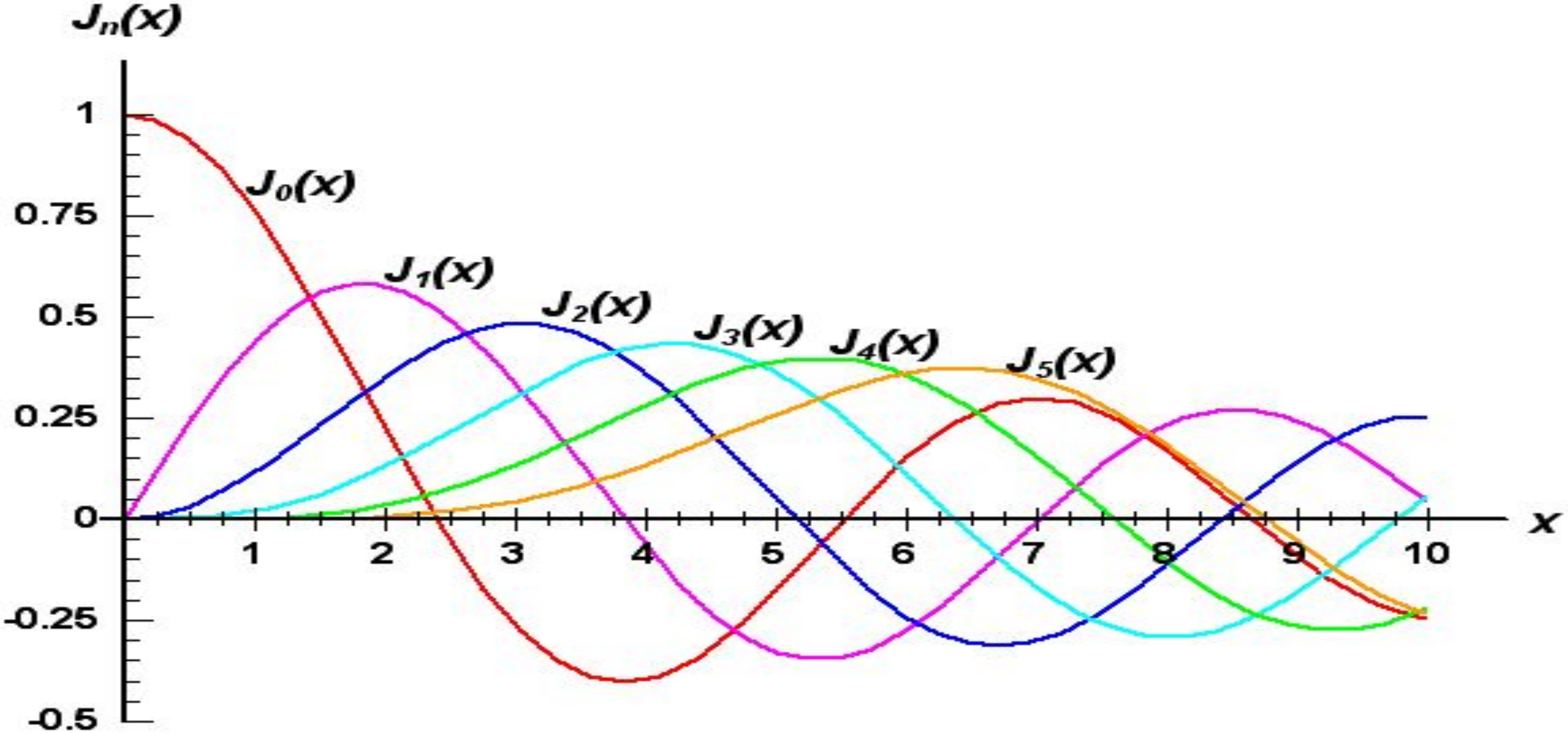
- in other words, angle modulation produces infinite number of sidebands, called as first-order sidebands, second-order sidebands, and so on. Also their magnitude are determined by the coefficients $J_1(m)$, $J_2(m)$,...$J_n(m)$.
- Bessel function of the first kind for several values of modulation index.

Table of Bessel functions of the first kind

Modulation index	Carrier	Side frequency pairs								
β_f	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
0.00	1.00	---	---	---	---	---	---	---	---	---
0.25	0.98	0.12	---	---	---	---	---	---	---	---
0.5	0.94	0.24	0.03	---	---	---	---	---	---	---
1.0	0.77	0.44	0.11	0.02	---	---	---	---	---	---
1.5	0.51	0.56	0.23	0.06	0.01	---	---	---	---	---
2.0	0.22	0.58	0.35	0.13	0.03	---	---	---	---	---
2.4	0	0.52	0.43	0.20	0.06	0.02	---	---	---	---
2.5	-0.05	0.50	0.45	0.22	0.07	0.02	0.01	---	---	---
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	---	---	---
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	---	---
5.0	-0.18	-0.13	0.05	0.36	0.39	0.26	0.13	0.05	0.02	---
5.45	0	-0.34	-0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02

4.5.1 : Bessel Function

- Curves for the relative amplitudes of the carrier and several sets of side frequencies for values of *m* up to 10.



4.5.1 : Bessel Function

- Conclusion from the table & graph :
  - modulation index m of 0 produces zero side frequencies.
  - the larger the m, the more sets of side frequencies are produced.
  - values shown for J_n are relative to the amplitude of the unmodulated carrier.
  - as the m decreases below unity, the amplitude of the higher-order side frequencies rapidly becomes insignificant.
  - as the m increases from 0, the magnitude of the carrier $J_0(m)$ decreases.
  - the negative values for J_n simply indicate the relative phase of that side frequency set
  - a side frequency is not considered significant unless its amplitude is equal or greater that 1% of the unmodulated carrier amplitude ($J_n \geq 0.01$).
  - as m increases, the number of significant side frequencies increases. I.e. the bandwidth of an angle-modulated wave is a function of the modulation index.

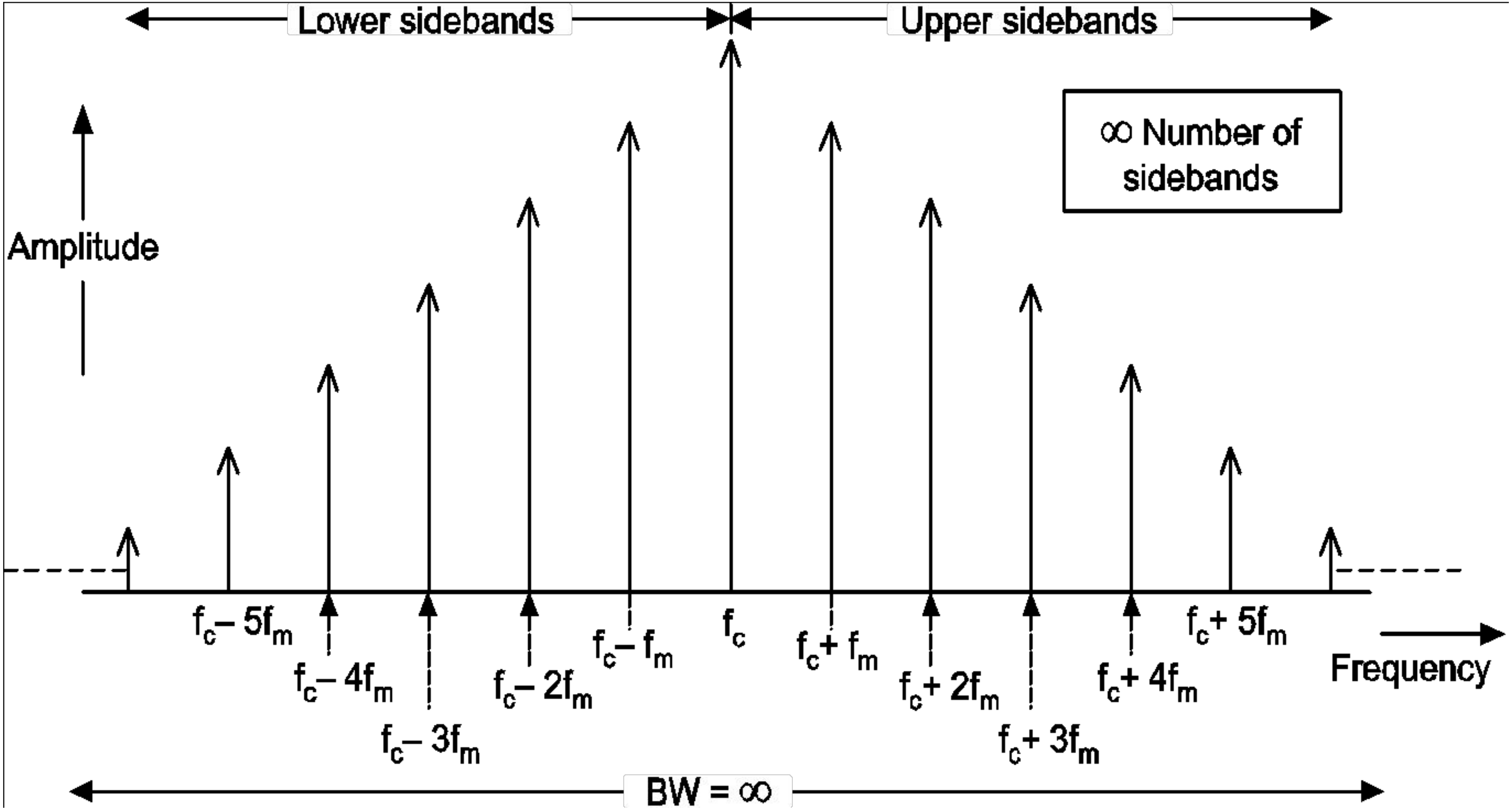


Fig. : Ideal Frequency Spectrum of FM

Example (A)

- Assume FM modulation with modulation index of 1
- $m(t) = V_m \sin(2.pi.1000t)$ and $Vc(t) = =10\sin(2.pi.500.10^3 t)$
- Find the following:
  - Number of sets of significant side frequencies (G(f))
  - Amplitude of freq. components
  - Draw the frequency component

n \ β:	0.5	1
0	0.9385	0.7652
1	0.2423	0.4401
2	0.03060	0.1149
3	0.002564	0.01956
4		0.002477
5		

Solution a. From Table 7-3, a modulation index of 1 yields a reduced carrier component and three sets of significant side frequencies.

b. The relative amplitudes of the carrier and side frequencies are

$$J_0 = 0.77(10) = 7.7\ \text{V}$$

$$J_1 = 0.44(10) = 4.4\ \text{V}$$

$$J_2 = 0.11(10) = 1.1\ \text{V}$$

$$J_3 = 0.02(10) = 0.2\ \text{V}$$

c. The frequency spectrum is shown in Figure 7-6.

If the FM modulator used in Example 7-3 were replaced with a PM modulator and the same carrier and modulating signal frequencies were used, a peak phase deviation of 1 rad would produce exactly the same frequency spectrum.

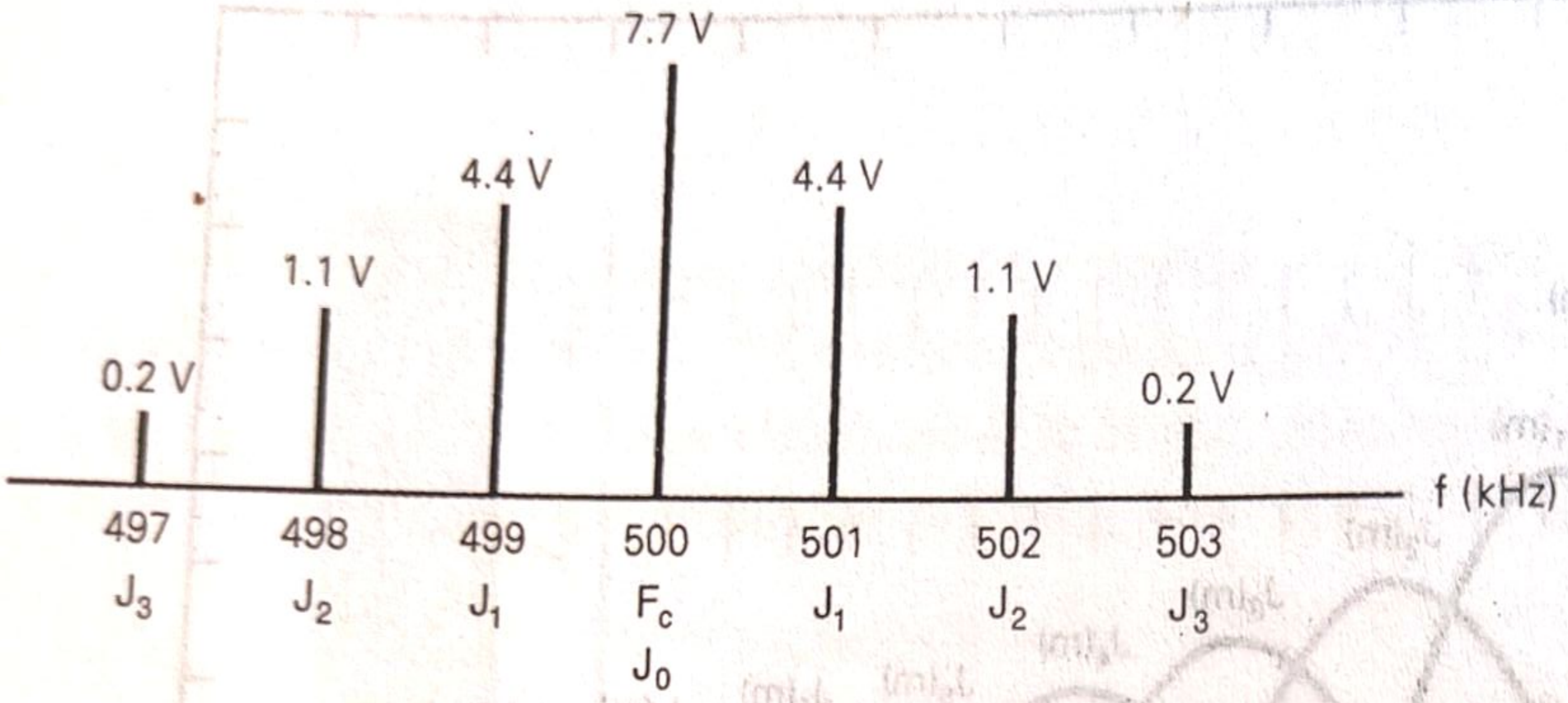


FIGURE 7-6 Frequency spectrum for Example 6-2

4.5.2 : Bandwidth Requirement

- angle-modulated wave consumes larger bandwidth than an amplitude-modulated wave.
- bandwidth of an angle-modulated wave is a function of the modulating signal and the modulation index.
- the actual bandwidth required to pass all the significant sidebands for an angle-modulated wave is equal to 2 times the product of the highest modulating signal frequency and the number of significant sidebands determined from the table of Bessel function.
  - I.e. the minimum bandwidth for angle-modulated wave using the Bessel table,

$$B = 2(n \times f_m)\,Hz \qquad (21)$$

- Carson's Rule
  - it is a general rule to estimate the bandwidth for all angle-modulated systems regardless of the modulation index.
  - the Carson's Rule states that the bandwidth necessary to transmit an angle-modulated wave as twice the sum of the peak frequency deviation and the highest modulating signal frequency.

4.5.2 : Bandwidth Requirement

- Carson's Rule

$$B = 2(\Delta f + f_m)\,Hz \tag{22}$$

  - for a low modulation index (f_m is much larger than Δf),

$$B = 2 f_m (Hz) \tag{23}$$

  - for a high modulation index (Δf is much larger than f_m)

$$B = 2\Delta f (Hz) \tag{24}$$

  - Carson's Rule approximate and gives a narrower bandwidth than the bandwidth determined using Bessel function. Therefore, a system designed using Carson's Rule would have a narrower bandwidth but a poorer performance than system designed using the Bessel table.
  - for modulation index above 5, Carson's Rule is a close approximation to the actual bandwidth required.

Example 7-4

For an FM modulator with a peak frequency deviation $\Delta f = 10$ kHz, a modulating-signal frequency $f_m = 10$ kHz, $V_c = 10$ V, and a 500-kHz carrier, determine

a. Actual minimum bandwidth from the Bessel function table.

b. Approximate minimum bandwidth using Carson's rule.

Then

c. Plot the output frequency spectrum for the Bessel approximation.

Solution **a.** Substituting into Equation 7-22 yields

$$m = \frac{10\text{ kHz}}{10\text{ kHz}} = 1$$

From Table 7-3, a modulation index of 1 yields three sets of significant sidebands. Substituting into Equation 7-33, the bandwidth is

$$B = 2(3 \times 10\text{ kHz}) = 60\text{ kHz}$$

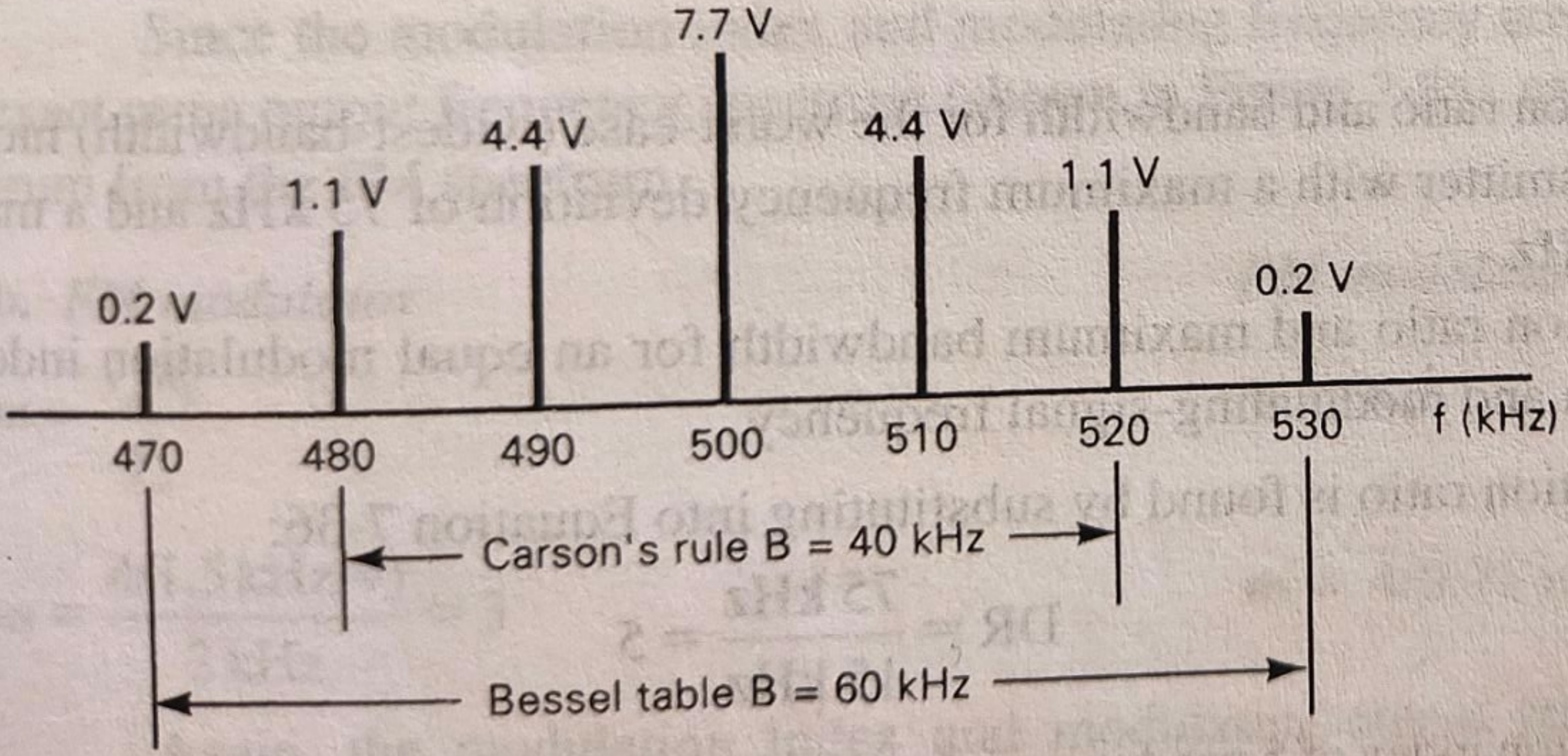


FIGURE 7-7 Frequency spectrum for Example 7-4

b. Substituting into Equation 7-34, the minimum bandwidth is

$$B = 2(10\text{ kHz} + 10\text{ kHz}) = 40\text{ kHz}$$

c. The relative amplitudes of the carrier and sideband frequencies are

$$J_0 = 0.77(10) = 7.7\text{ V}$$
$$J_1 = 0.44(10) = 4.4\text{ V}$$
$$J_2 = 0.11(10) = 1.1\text{ V}$$
$$J_3 = 0.02(10) = 0.2\text{ V}$$

The output frequency spectrum for the Bessel approximation is shown in Figure 7-7.

From Example 7-4, it can be seen that there is a significant difference in the minimum bandwidth determined from Carson's rule and the minimum bandwidth determined from the Bessel table. The bandwidth from Carson's rule is less than the actual minimum bandwidth required to pass all the significant sideband sets as defined by the Bessel table. Therefore, a system that was designed using Carson's rule would have a narrower bandwidth and, thus, poorer performance than a system designed using the Bessel table. For modulation indexes above 5, Carson's rule is a close approximation to the actual bandwidth required.

Example (

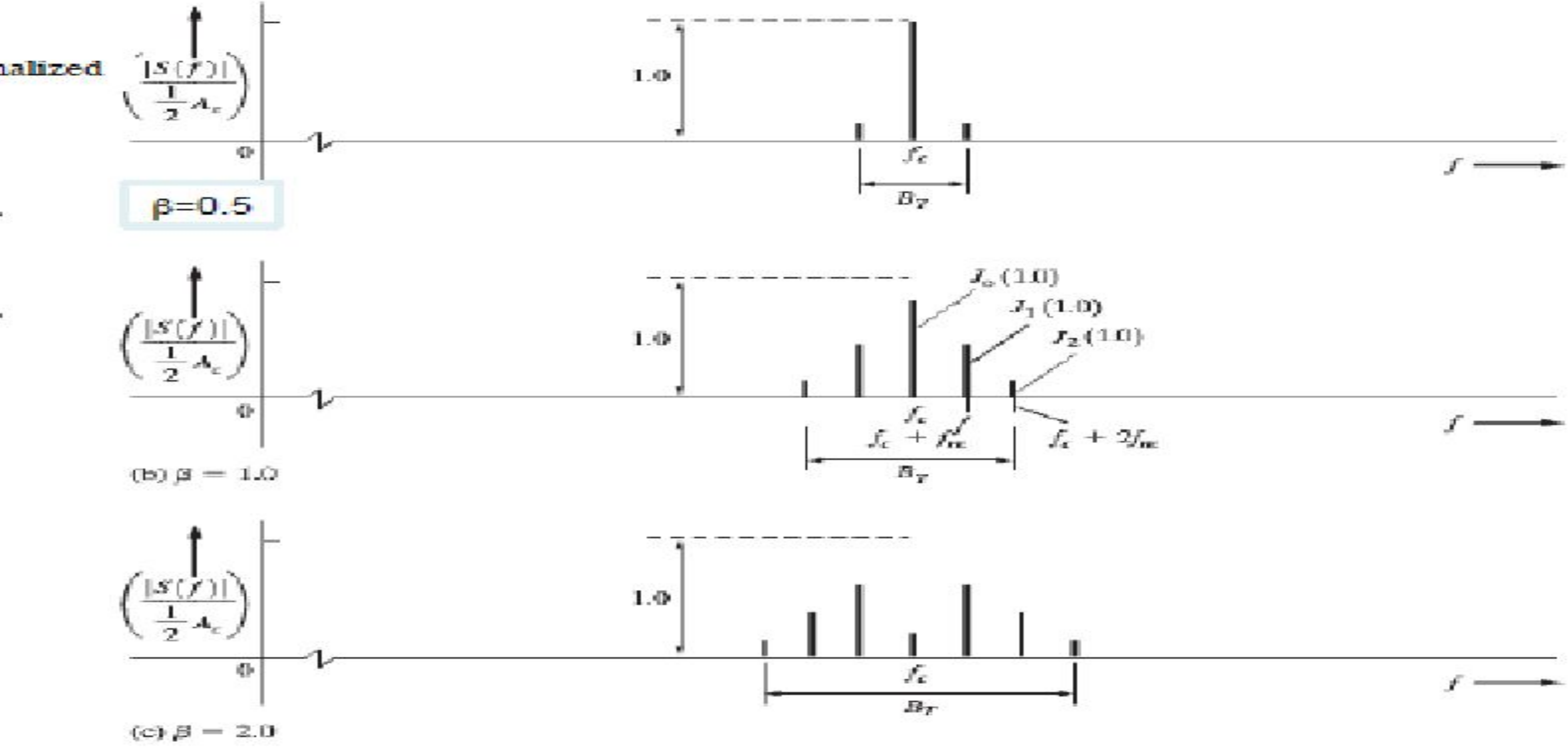
- Plot the spectrum from the modulated FM signal for β=0.5, 1, 2

n \ β:	0.5	1	2
0	0.9385	0.7652	0.2239
1	0.2423	0.4401	0.5767
2	0.03060	0.1149	0.3528
3	0.002564	0.01956	0.1289
4		0.002477	0.03400
5			0.007040
6			0.001202
7			

e modulated FM

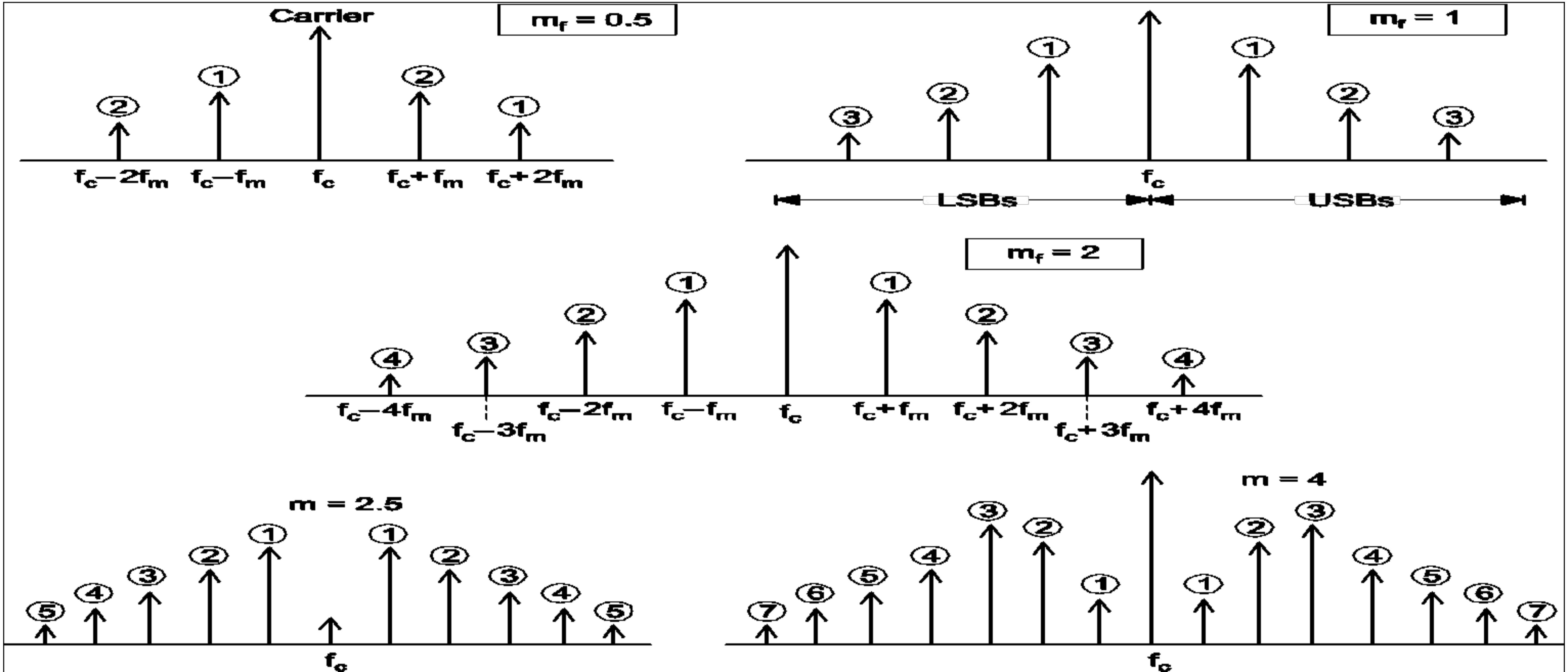
$$S(f) = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)],$$

$$G(f) = A_c \sum_{n=-\infty}^{n=\infty} J_n(\beta)\delta(f - nf_m)$$



Effect of Modulation Index on Sidebands

Modulation index	0.5	1	2	2.5	4
Number of significant sideband on either side of carrier	2	3	4	5	7



4.6 : Deviation Ratio

- ***Deviation ratio DR*** is the worst case modulation index and is equal to the maximum peak frequency deviation divided by the maximum modulating-signal frequency – producing the widest frequency spectrum.

$$DR = \frac{\Delta f_{(max)}}{f_{m(max)}}$$

where DR = deviation ratio (unitless)

$\Delta f_{(max)}$ = maximum peak frequency deviation (Hertz)

$f_{m(max)}$ = maximum modulating-signal frequency (Hertz)

4.6 : Deviation Ratio

- Ex : a. Determine the deviation ratio and bandwidth for the worst-case (widest bandwidth) modulation index for an FM broadcast-band transmitter with a maximum frequency deviation of 75 kHz and a maximum modulating-signal frequency of 15 kHz.

$$DR = \frac{75kHz}{15kHz} = 5$$

From Bessel Table, a modulation index of 5 produces 8 significant sidebands

Thus, the bandwidth is

$$B = 2(8 \times 15000) = 240 \text{ kHz}$$

- Ex : b. For a 37.5 kHz frequency deviation and a modulating-signal frequency fm = 7.5 kHz, the modulation index is

$$m = \frac{37.5kHz}{7.5kHz} = 5$$

and the bandwidth is B = 2(8 x 7500) = 120 kHz

4.6 : Deviation Ratio

- From Ex. a & b, although the same modulation index (5) was achieved with 2 different modulating-signal frequencies and amplitudes, 2 different bandwidths were produced.
- The widest bandwidth will only be produced from the maximum modulating-signal frequency and maximum frequency deviation.
- The same condition applies in the case of using the Carson's rule :

$$
\begin{aligned}
B &= 2\left[\Delta f_{(\max)} + f_{m(\max)}\right] \\
&= 2\left(75\,kHz + 15\,kHz\right) \\
&= 180\,kHz
\end{aligned}
$$

FM Power Distribution

- As seen in Bessel function table, it shows that as the sideband relative amplitude increases, the carrier amplitude,J_0 decreases.
- This is because, in FM, the total transmitted power is always constant and the total average power is equal to the unmodulated carrier power, that is the amplitude of the FM remains constant whether it is modulated or not.
- The total power in angle-modulated wave is equal to the power of the un-modulated wave.

FM Power Distribution (cont'd)

- In effect, in FM, the total power that is originally in the carrier is redistributed between all components of the spectrum, in an amount determined by the modulation index, m_f, and the corresponding Bessel functions.

- At certain value of modulation index, the carrier component goes to zero, where in this condition, the power is carried by the sidebands only.

Average Power

- The average power in **unmodulated** carrier

$$\boxed{P_c = \frac{V_c^2}{2R}}$$

- The total power in the **angle modulated** carrier.

$$P_t = \frac{m(t)^2}{R} = \frac{V_c^2}{R}\cos^2[\omega_c t + \theta(t)]$$

$$P_t = \frac{V_c^2}{R}\left\{\frac{1}{2} + \frac{1}{2}\cos[2\omega_c t + 2\theta(t)]\right\} = \frac{V_c^2}{2R}$$

- The modulated carrier power is the sum of the powers of the carrier and the side frequency components as follow;

$$\boxed{P_t = P_0 + P_1 + P_2 + .. + P_n = \frac{V_c^2}{2R} + \frac{2(V_1)^2}{2R} + \frac{2(V_2)^2}{2R} + .. + \frac{2(V_n)^2}{2R}}$$

V_c=peak unmodulated carrier voltage (volts), V_n= sidebands voltage (volts), P_c=Carrier power, R=Resisitive load (ohms)

Types of Frequency Modulation

FM (Frequency Modulation)

Narrowband FM
(NBFM)
[When modulation index is small]

Wideband FM
(WBFM)
[When modulation index is large]

Comparison between Narrowband and Wideband FM

Sr. No.	Parameter	NBFM	WBFM
1.	Modulation index	Less than or slightly greater than 1	Greater than 1
2.	Maximum deviation	5 kHz	75 kHz
3.	Range of modulating frequency	20 Hz to 3 kHz	20 Hz to 15 kHz
4.	Maximum modulation index	Slightly greater than 1	5 to 2500
5.	Bandwidth	Small approximately same as that of AM BW = $2f_m$	Large about 15 times greater than that of NBFM. BW = $2(\delta+fmmax)$
6.	Applications	FM mobile communication like police wireless, ambulance, short range ship to shore communication etc.	Entertainment broadcasting (can be used for high quality music transmission)

Example (A)

- Assume FM modulation with modulation index of 1
- $m(t) = V_m \sin(2.pi.1000t)$ and $Vc(t) = = 10\sin(2.pi.500.10^3 t)$
- Find the following:
  - Number of sets of significant side frequencies (G(f))
  - Amplitude of freq. components
  - Draw the frequency component

- B) For an FM modulator with peak frequency deviation of 10kHz, a modulating frequency of 10kHz, Vc=10 and 500kHz carrier determine a. actual minimum bandwidth from Bessel function table
- b. Approximate minimum bandwidth using carson’s rule
- c. Plot the output frequency spectrum from Bessel approximation.

- C) FM modulator: Deviation sensitivity= 1.5kHz/v, carrier frequency= 500kHz, modulating signal= 2 sin(2*pi*2k*t).
- PM modulator: Deviation sensitivity= 0.75rad/v, carrier frequency= 500kHz, modulating signal= 2 sin(2*pi*2k*t).

a. Determine the modulation indexs and sketch the output spectrums for both modulators.

b. Change the modulating signal amplitude for both modulators to 4Vp and repeat step a).

c. Change the modulating signal frequency for both modulators to 1kHz and repeat step a).

D) Determine the unmodulated carrier power if the Vc=10v and load resistance of 50ohm. Modulation index is 1.

Also determine the total power in the angle modulated wave.