

1.6.1 Addition of CT Signals

Consider the signals $x_1(t)$ and $x_2(t)$ which are shown in Figures 1.23(a) and (b). The amplitude of these two signals at each instant of time is added to get their sum. The following table is prepared.

From Table 1.1, $x(t) = x_1(t) + x_2(t)$ is plotted and is shown in Figure 1.23(c).

Table 1.1

t	-3	-2	-1	0	1	2
$x_1(t)$	0	1	2	2	0	0
$x_2(t)$	1	-2	-2	1	3	0
$x(t) = x_1(t) + x_2(t)$	1	-1	0	3	3	0

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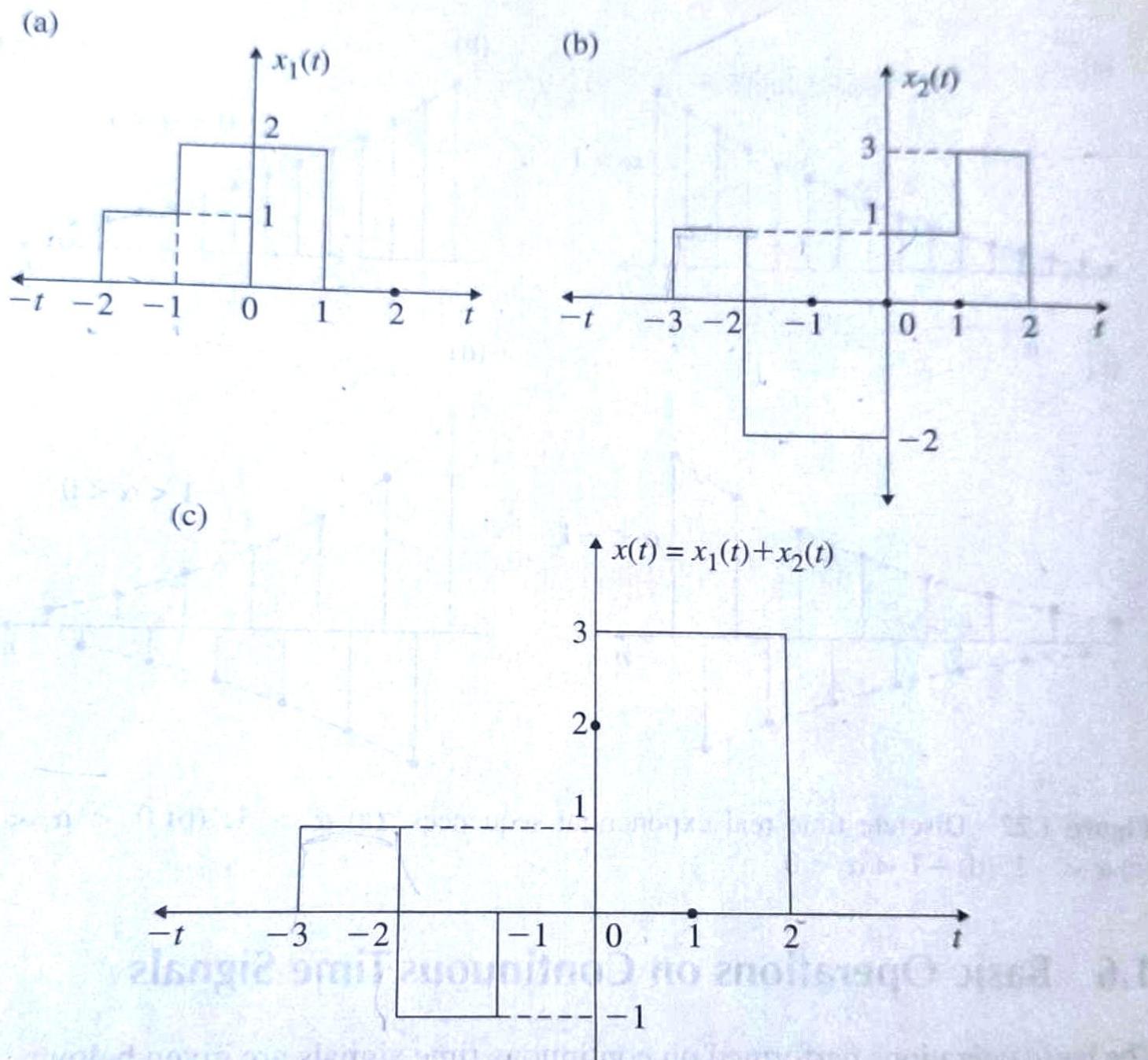


Figure 1.23 Additions of two CT signals.

1.6.2 Multiplications of CT Signals

Consider the two signals $x_1(t)$ and $x_2(t)$ shown in Figures 1.23(a) and (b) respectively. These signals $x_1(t)$ and $x_2(t)$ are multiplied to get $x(t)$

$$x(t) = x_1(t) \times x_2(t)$$

The functions $x_1(t)$ and $x_2(t)$ at different time intervals are determined from Figure and multiplied. Table 1.2 is prepared to get $x(t)$ at different time intervals. The Table 1.2 is transformed to plot $x(t) = x_1(t) \times x_2(t)$ which is shown in Figure 1.24.

Table 1.2

t	-3	-2	-1	0	1	2
$x_1(t)$	0	1	2	2	0	0
$x_2(t)$	1	-2	-2	1	3	0
$x(t) = x_1(t) \times x_2(t)$	0	-2	-4	2	0	0

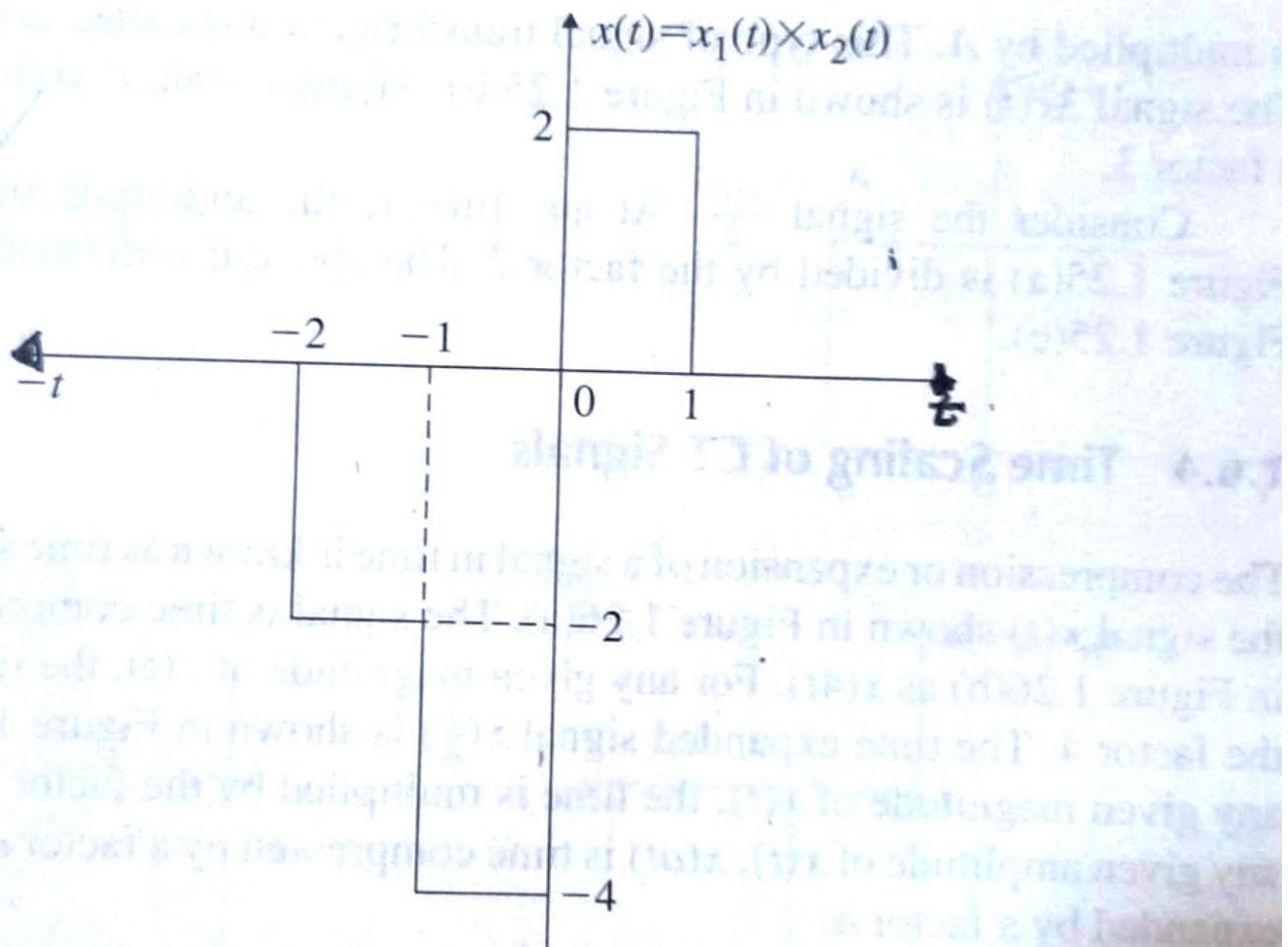


Figure 1.24 Multiplications of two CT signals.

1.6.6 Signal Reflection or Folding

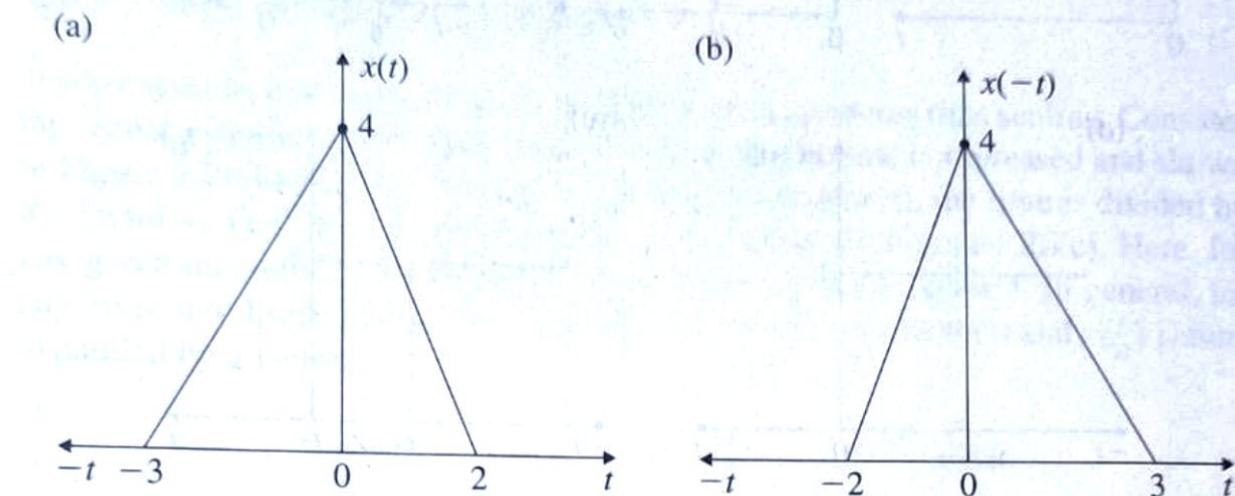


Figure 1.28 CT signal reflection or folding.

Consider the signal $x(t)$ shown in Figure 1.28(a). The signal $x(-t)$ is obtained by putting a mirror along the vertical axis. The signal to the right of the vertical axis gets reflected to the left and *vice versa*. Alternatively, if we make a folding across the vertical axis, the signal in the right of the vertical axis is printed in the left and *vice versa*. The signal so obtained is $x(-t)$.

1.6.7 Inverted CT Signal

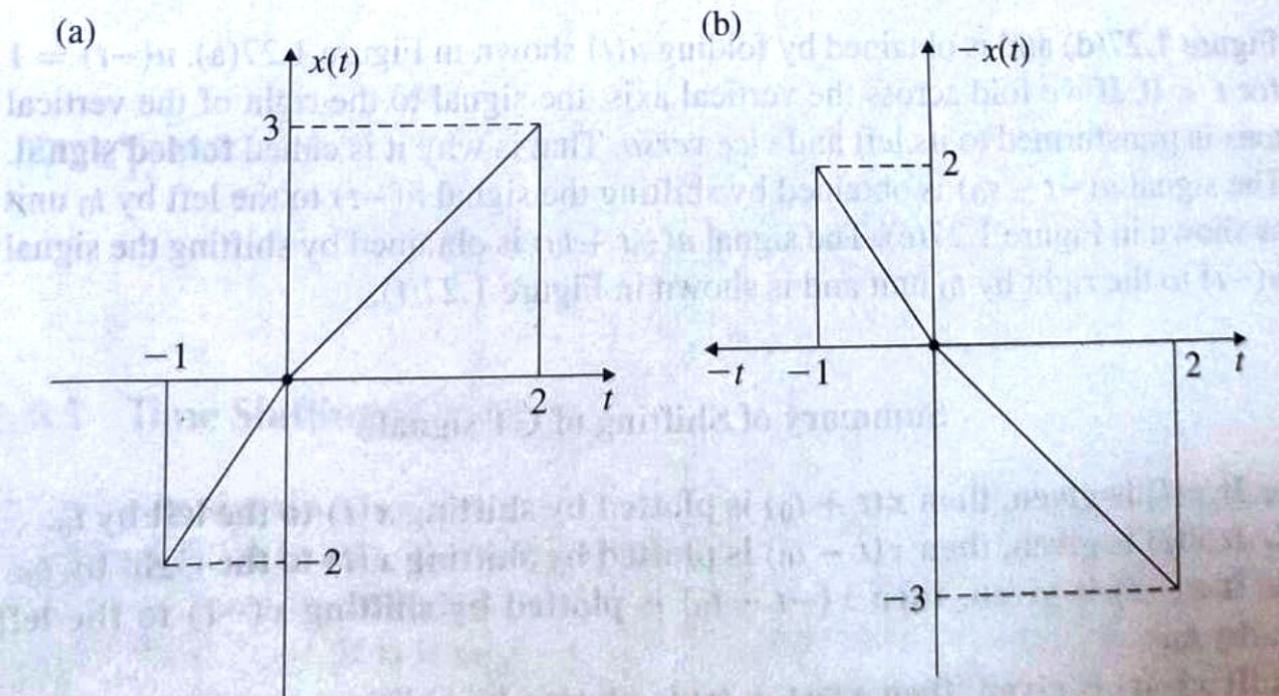


Figure 1.29 Inverted CT signal.

The following examples illustrate the above sequence of operations.

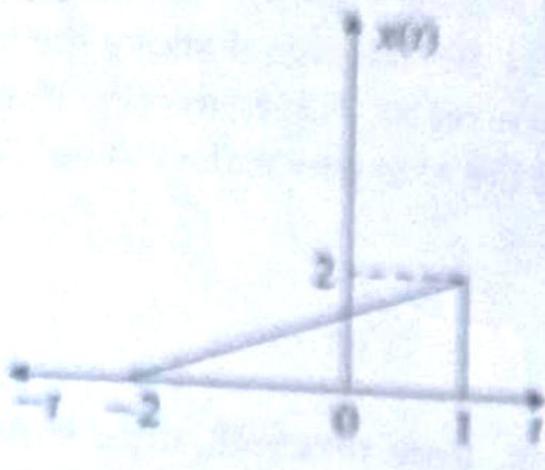
■ Example 1.3

Consider the signal $y(t) = 5x(-3t + 1)$ where $x(t)$ is shown in Figure 1.30(a). Plot $y(t)$ and $-y(t)$.

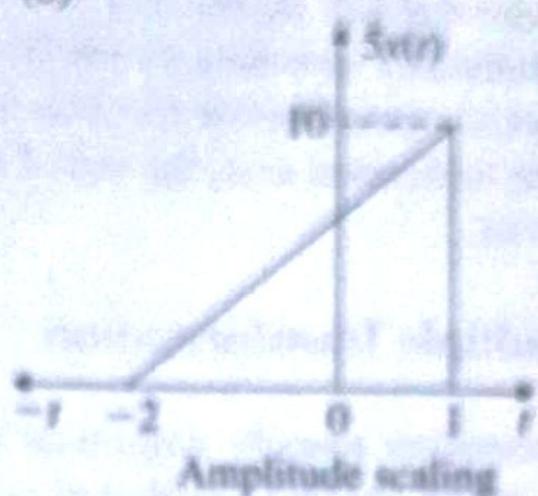
Solution:

1. The given signal $x(t)$ is represented in Figure 1.30(a).
2. The signal $x(t)$ is amplitude scaled and plotted in Figure 1.30(b).
3. $5x(-t)$ is obtained by folding $5x(t)$ in Figure 1.30(b) and is plotted in Figure 1.30(c).
4. $5x(-t)$ is time shifted by one unit to the right and $5x(-t + 1)$ is obtained and shown in Figure 1.30(d).
5. $5x(-t + 1)$ is time compressed by a factor 3 and $5x(-3t + 1)$ is obtained. This is shown in Figure 1.30(e).
6. $5x(-3t + 1)$ amplitude inverted to get $-5x(-3t + 1)$. This is shown in Figure 1.30(f).

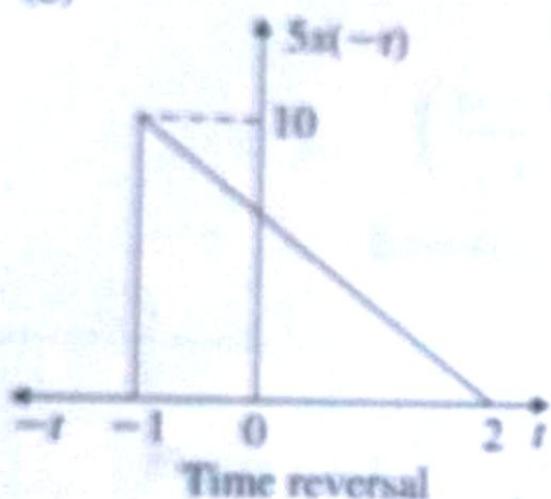
(a)



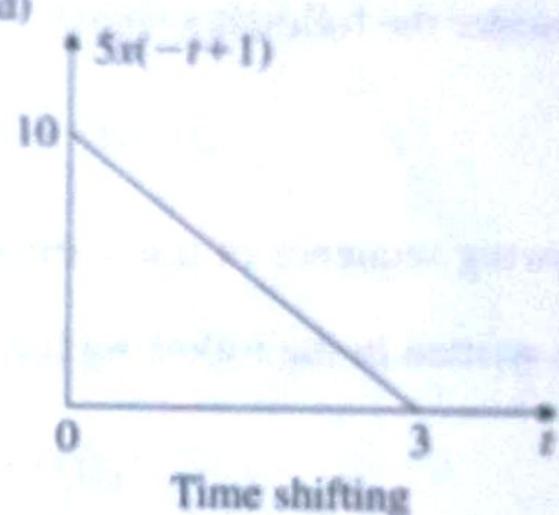
(b)



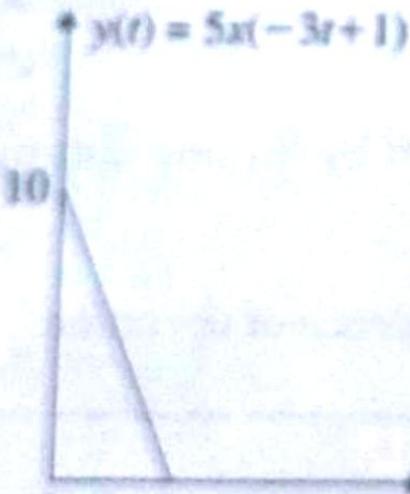
(c)



(d)



(e)



(f)

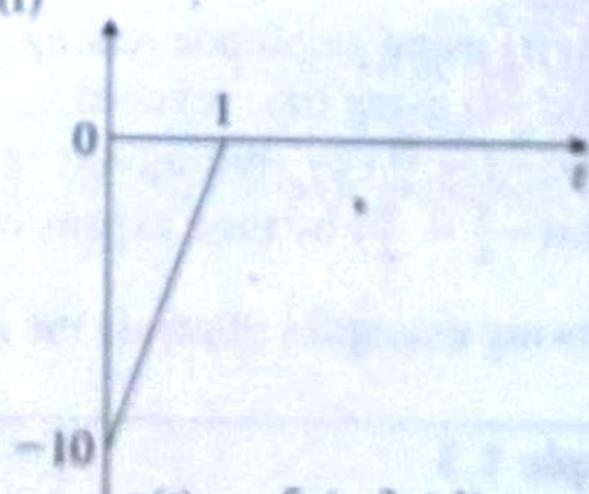


Figure 1.30 Basic operations on CT signal.

■ Example 1.4

For a signal $x(t)$ shown in Figure 1.31(a), sketch

$$(a) \quad x(3t + 2)$$

$$(b) \quad x\left(\frac{-t}{2} - 1\right)$$

(Anna Univers

Solution: To plot $x(3t + 2)$

1. $x(t)$ is represented in Figure 1.31(a). $x(t)$ is moved to the left by $t = 2$ and is shown in Figure 1.31(b).
2. By time compression by a factor 3, from Figure 1.31(b), $x(3t + 2)$ is obtained and is shown in Figure 1.31(c).

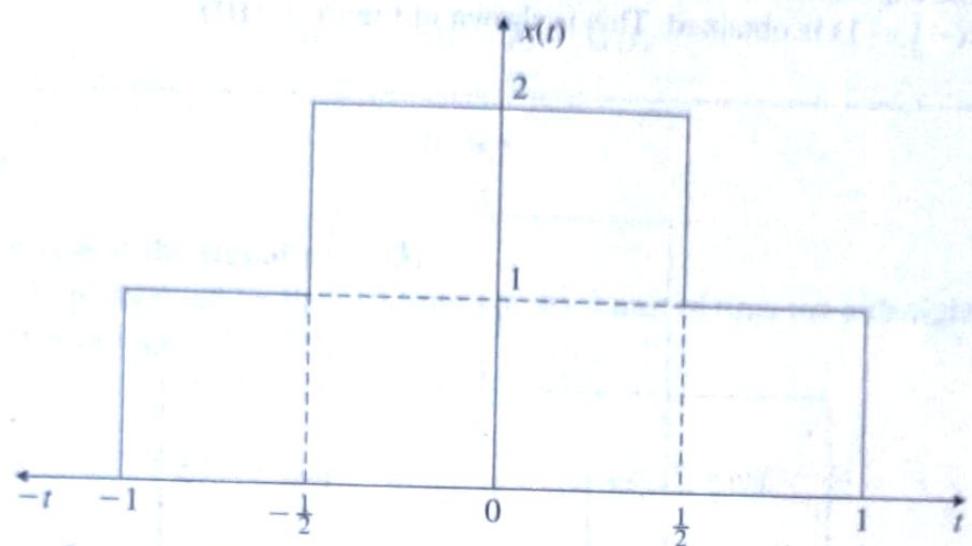


Figure 1.31 (a) Plot of $x(t)$.

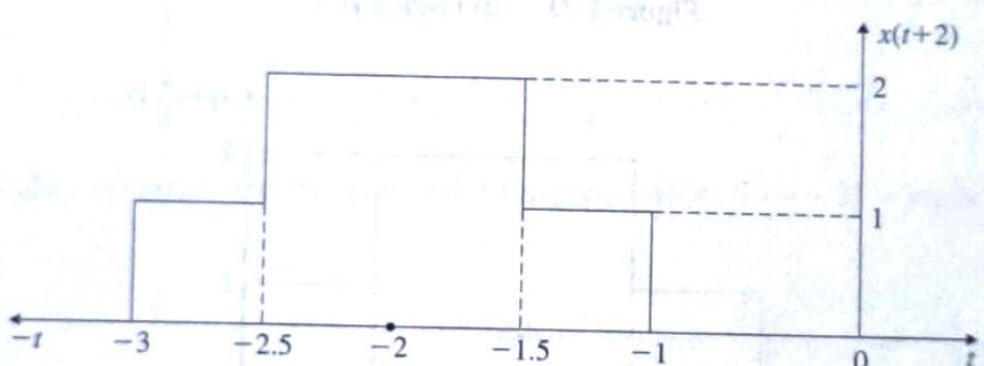


Figure 1.31 (b) Time shifted $x(t)$.

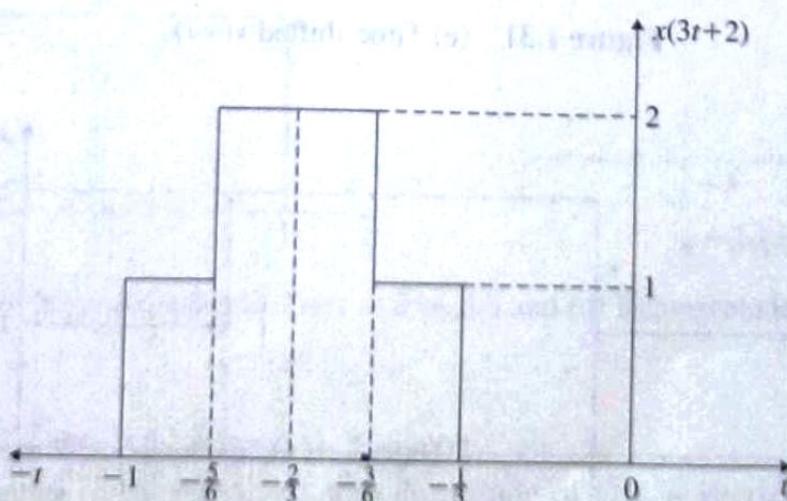


Figure 1.31 (c) Time compressed $x(t)$.

Solution: To plot $x(-(t/2) - 1)$

1. By folding $x(t)$ represented in Figure 1.31(a), $x(-t)$ is obtained and is shown in Figure 1.31(d).
2. $x(-t - 1)$ is obtained by shifting $x(-t)$ by $t = 1$ to the left. $x(-t - 1)$ is sketched as shown in Figure 1.31(e).
3. By time expansion, the time of the signal $x(-t - 1)$ is multiplied by the factor 2, and $x(-\frac{t}{2} - 1)$ is obtained. This is shown in Figure 1.31(f).

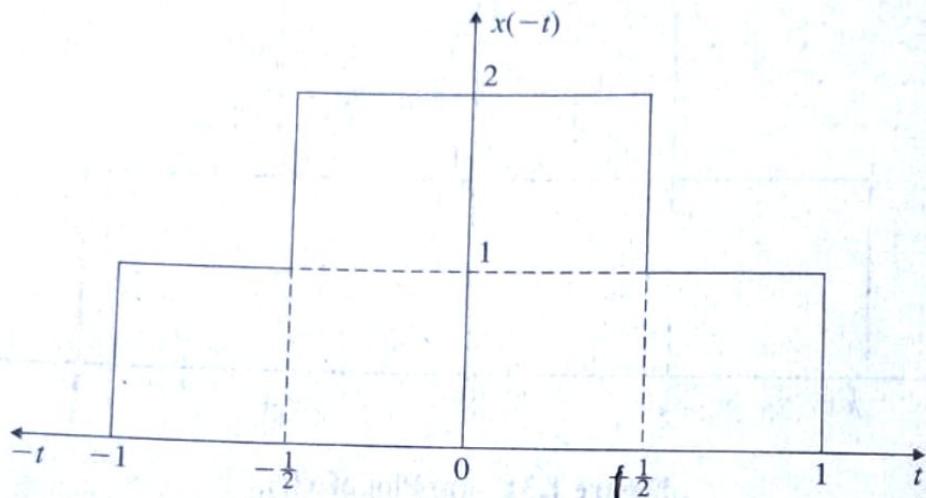


Figure 1.31 (d) Folded $x(t)$.

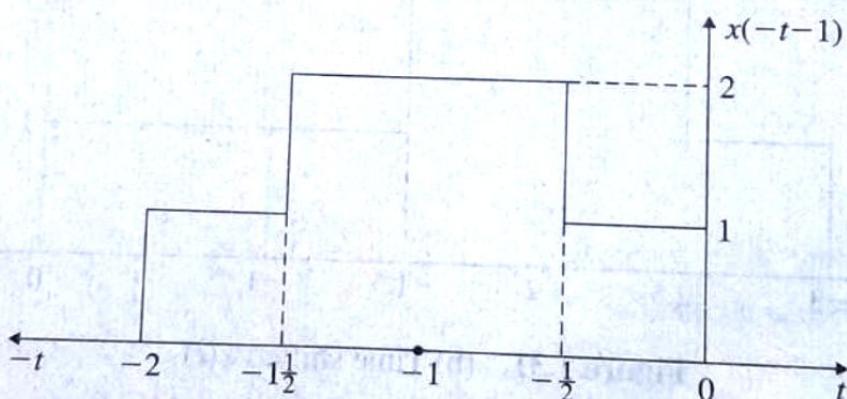


Figure 1.31 (e) Time shifted $x(-t)$.

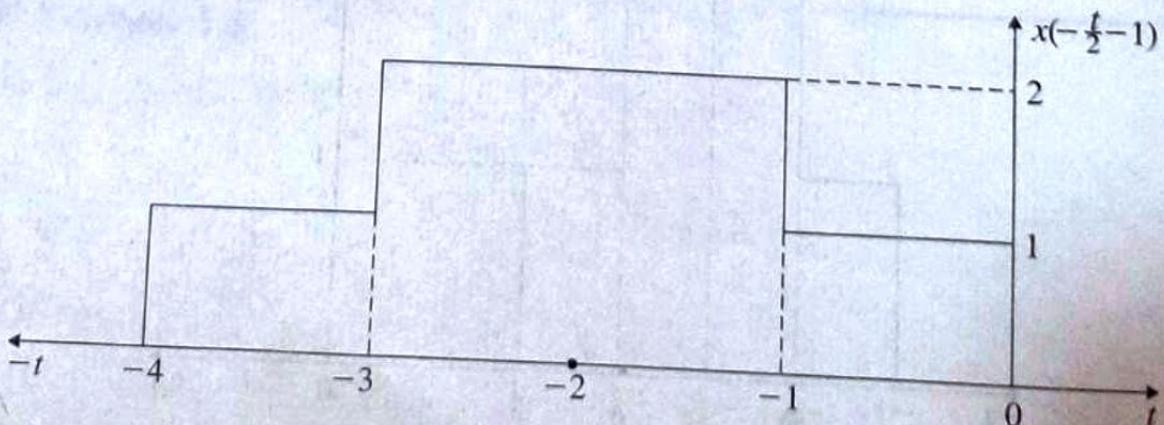


Figure 1.31 (f)

■ Example 1.5

The rectangular signal $x(t)$ is shown in Figure 1.32(a). Sketch the following signals:

- $x(t - 3)$
- $2x(t)$
- $-3x(t)$
- $x(t - 2) + 3x(t)$

Solution:

(a) **To represent the signal $x(t - 3)$**

$x(t - 3)$ is obtained by time shifting $x(t)$ by 3 unit of time towards right. This is shown in Figure 1.32(b).

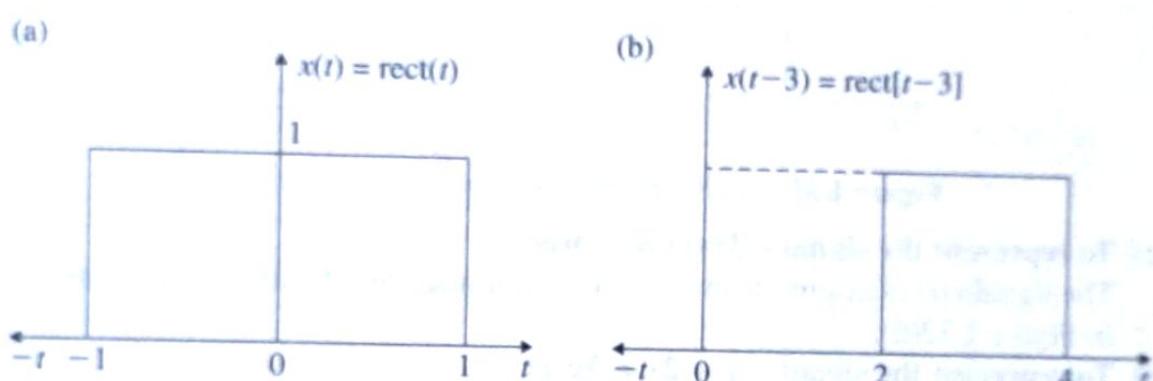


Figure 1.32 (a) $x(t) = \text{rect}(t)$ signal and (b) Representation of $x(t - 3) = \text{rect}[t - 3]$.

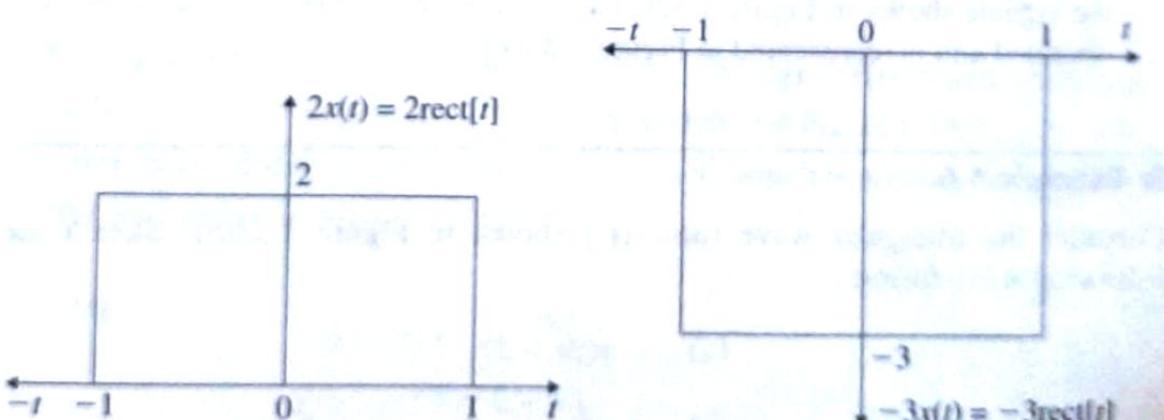


Figure 1.32 (c) Representation of $2x(t) = 2 \text{rect}[t]$ and (d) Representation of $-3x(t) = -3 \text{rect}[t]$.

(b) **To represent the signal $2x(t) = 2 \text{rect}[t]$**

This is amplitude scaled signal. The amplitude of $x(t) = \text{rect}[t]$ is multiplied by the factor 2 and is shown in Figure 1.32(c).

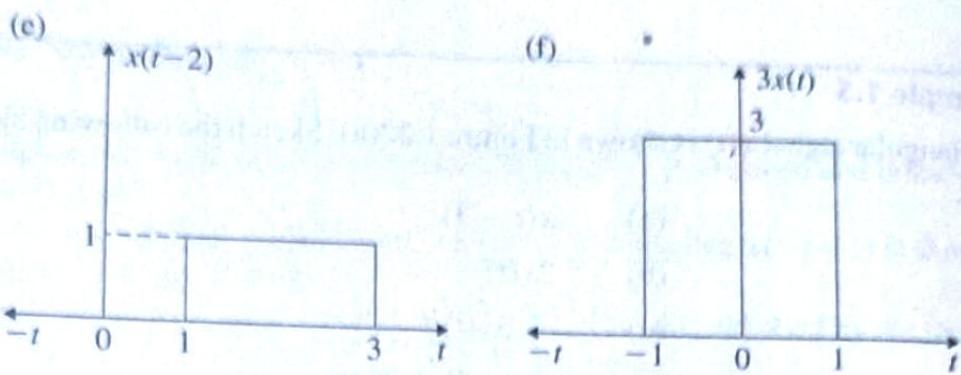


Figure 1.32 (e) Representation of $x(t - 2)$ and (f) Representation of $3x(t)$.

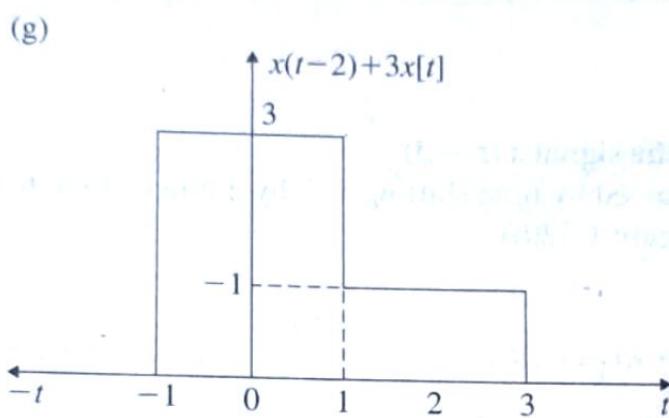


Figure 1.32 (g) Representation of $x(t - 2) + 3x(t)$.

(c) To represent the signal $-3x(t) = -3\text{rect}[t]$

The signal $x(t)$ is amplitude inverted and multiplied by a factor 3. This is shown in Figure 1.32(d).

(d) To represent the signal $x(t - 2) + 3x(t)$

The time delayed $x(t - 2)$ is obtained by shifting $x(t)$ to the right by a factor 2. This is represented in Figure 1.32(e). The signal $x(t)$ is amplitude multiplied by a factor 3 and $3x(t)$ is obtained. This is shown in Figure 1.32(f). By adding the signals shown in Figure 1.32(e) and in Figure 1.32(f), $x(t - 2) + 3x(t)$ is obtained and is represented in Figure 1.32(g).

■ Example 1.6

Consider the triangular wave form $x(t)$ shown in Figure 1.33(a). Sketch the following wave forms:

$$(a) \quad x(2t + 3)$$

$$(b) \quad x\left(\frac{t+3}{2}\right)$$

$$(c) \quad x\left(\frac{t}{2} - 3\right)$$

$$(d) \quad x(-2t + 3)$$

$$(e) \quad x(-2t - 3)$$

Solution:

(a) **To sketch $x(2t + 3)$**

Figure 1.33(a) shows $x(t) = \text{tri}(t)$. By time shifting by $t = 3$ towards left, $x(t + 3)$ is obtained and this is sketched in Figure 1.33(b). $x(t + 3)$ is time compressed by a factor of 2 to get $x(2t + 3)$. This is sketched in Figure 1.33(c).

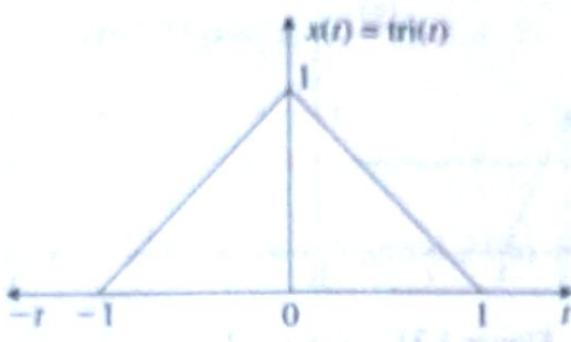


Figure 1.33 (a) $x(t) = \text{tri}(t)$.

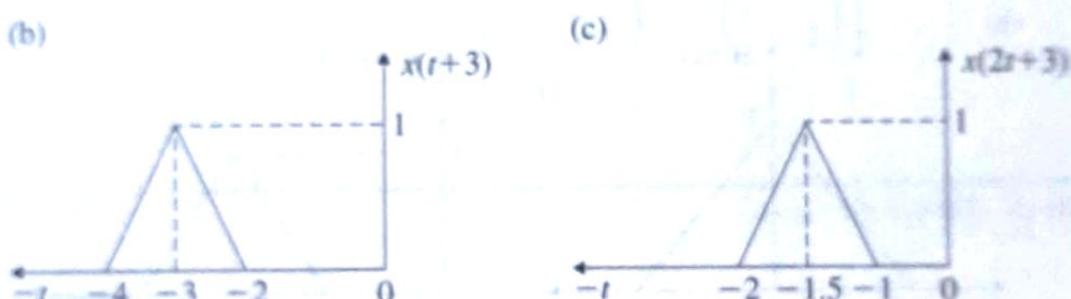


Figure 1.33 (b) $x(t + 3)$; (c) $x(2t + 3)$.

(b) **To sketch $x\left(\frac{t+3}{2}\right)$**

The signal $x\left(\frac{t+3}{2}\right)$ is written as $x\left(\frac{t}{2} + 1.5\right)$. The signal $x(t)$ is time shifted to the left by 1.5 unit to get $x(t + 1.5)$. This is sketched in Figure 1.33(d). $x(t + 1.5)$ is time expanded by a factor 2 to get $x\left(\frac{t}{2} + 1.5\right)$ which is nothing but $x\left(\frac{t+3}{2}\right)$. This is sketched in Figure 1.33(e).

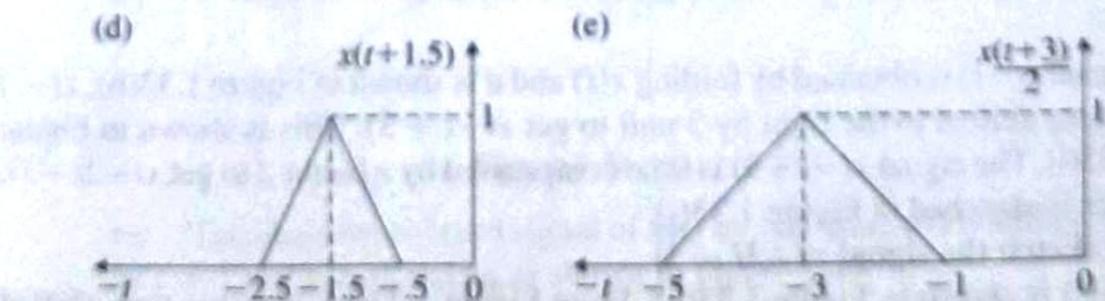
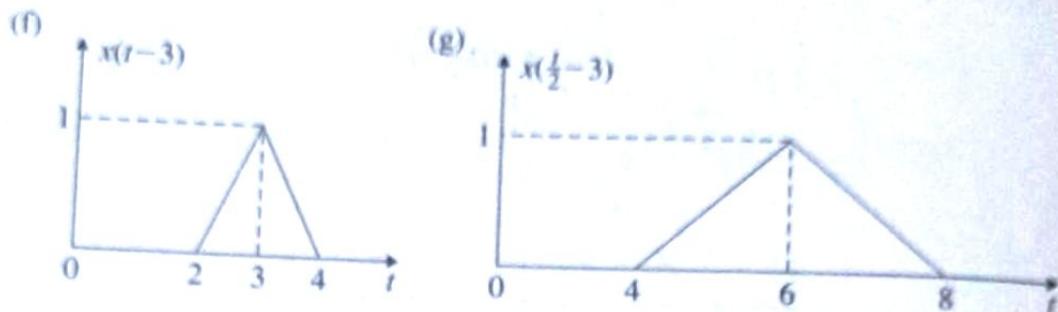
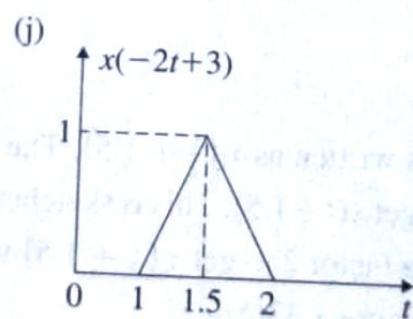
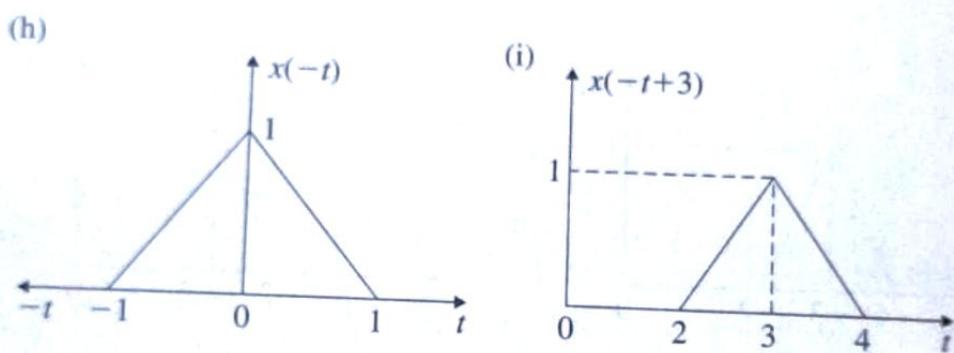


Figure 1.33 (d) $x(t + 1.5)$; (e) $x\left(\frac{t+3}{2}\right)$.

(c) To sketch $x\left(\frac{t}{2} - 3\right)$

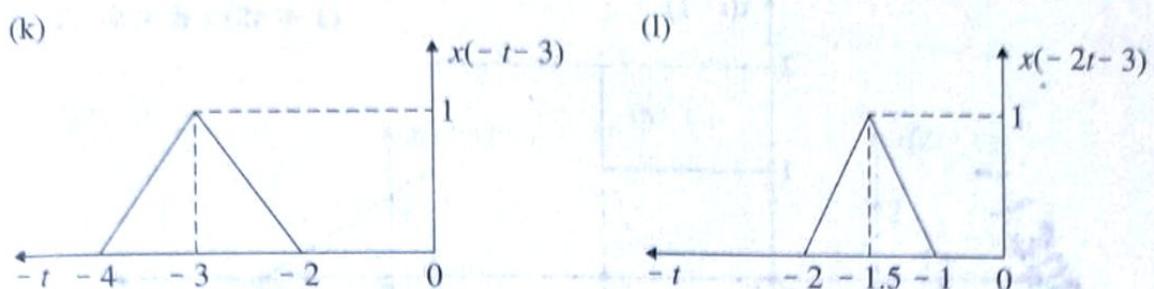
$x(t - 3)$ is obtained from $x(t)$ by time shifting the signal $x(t)$ to the right by 3 unit and is shown in Figure 1.33(f). By time expansion of $x(t - 3)$ by a factor 2, $x\left(\frac{t}{2} - 3\right)$ is obtained and sketched as shown in Figure 1.33(g).

Figure 1.33 (f) $x(t - 3)$; (g) $x\left(\frac{t}{2} - 3\right)$.(d) To sketch the signal $x(-2t + 3)$ Figure 1.33 (h) $x(-t)$; (i) $x(-t+3)$; (j) $x(-2t+3)$.

Signal $x(-t)$ is obtained by folding $x(t)$ and it is shown in Figure 1.33(h). $x(-t)$ is time shifted to the right by 3 unit to get $x(-t+3)$. This is shown in Figure 1.33(i). The signal $x(-t+3)$ is time compressed by a factor 2 to get $x(-2t+3)$. This is sketched in Figure 1.33(j).

(e) To sketch the signal $x(-2t - 3)$

$x(-t)$ is shown in Figure 1.33(h). From Figure 1.33(h), $x(-t)$ is time shifted towards left by 3 units to get $x(-t - 3)$. This is shown in Figure 1.33(k). $x(t - 3)$ is time compressed by a factor 2 to get $x(-2t - 3)$. This is sketched in Figure 1.33(l).

Figure 1.33 (k) $x(-t - 3)$; (l) $x(-2t - 3)$.**■ Example 1.7**

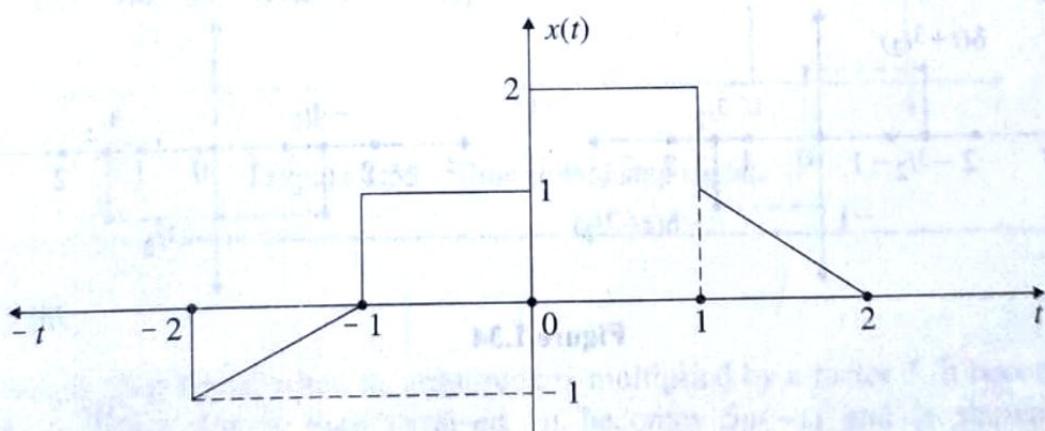
A continuous time signal $x(t)$ is shown in Figure 1.34(a). Sketch and label carefully each of the following signals:

- (a) $x(t - 1)$
- (b) $x(2 - t)$
- (c) $x(t) \left[\delta\left(t + \frac{3}{2}\right) - \delta\left(t - \frac{3}{2}\right) \right]$
- (d) $\underline{x(2t + 1)}$

(Anna University, April, 2008)

Solution:

- (a) To sketch
- $x(t - 1)$

Figure 1.34 (a) $x(t)$ plot.

$x(t - 1)$ is the time delayed signal of $x(t)$ by one unit. $x(t)$ is shifted to the right by $t = 1$ and it is sketched as shown in Figure 1.34(b).

- (b) To sketch
- $x(2 - t)$

The folded signal of $x(t)$ is $x(-t)$ and is shown in Figure 1.34(c). $x(-t)$ is right shifted by 2 unit to get $x(2 - t)$ and is shown in Figure 1.34(d).

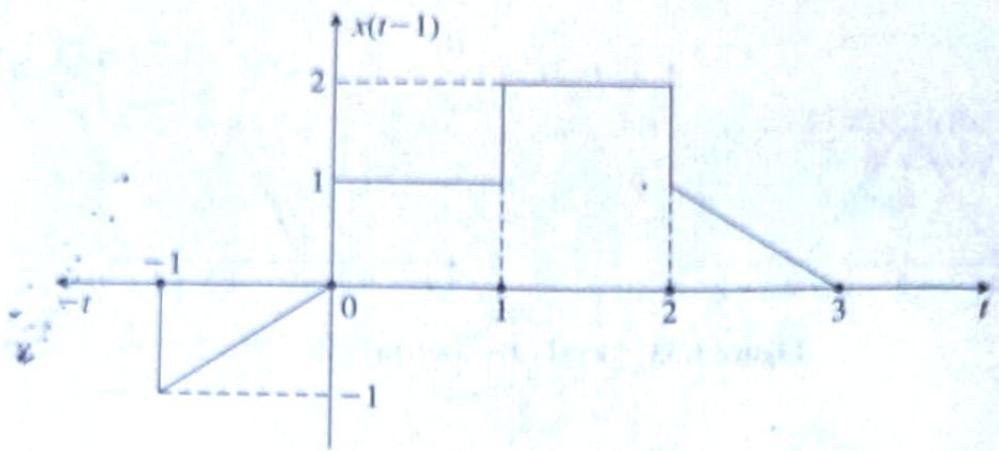


Figure 1.34 (b) $x(t - 1)$ plot.

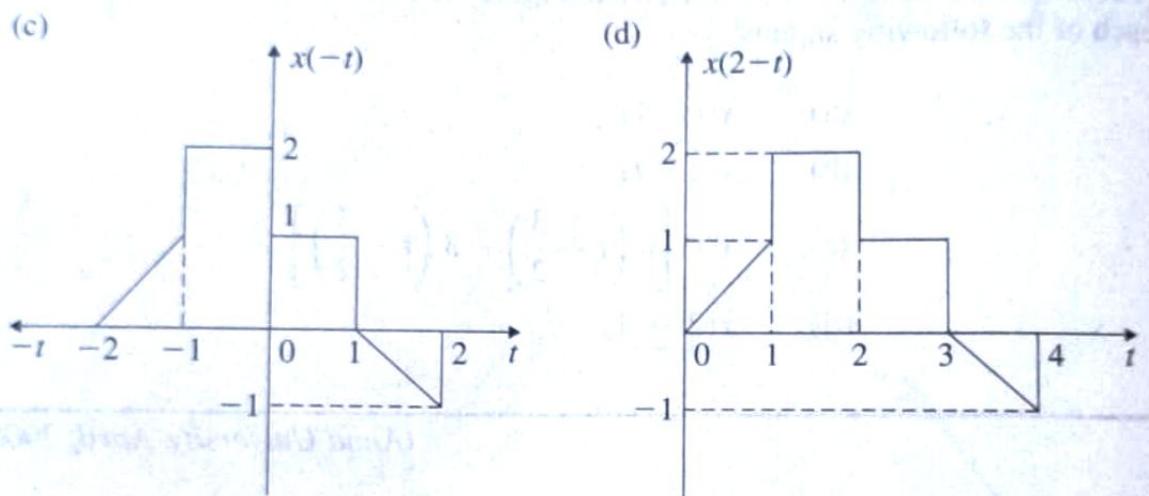


Figure 1.34

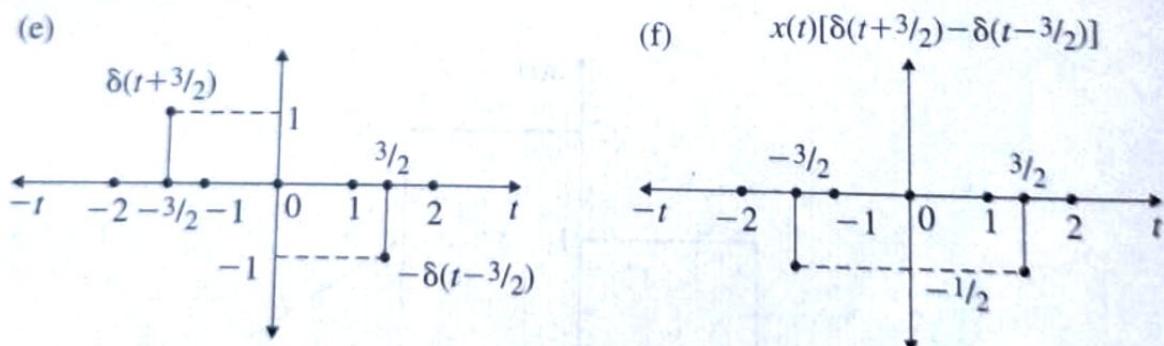


Figure 1.34

(c) To sketch $x(t)[\delta(t + \frac{3}{2}) - \delta(t - \frac{3}{2})]$

$\delta(t + \frac{3}{2})$ and $\delta(t - \frac{3}{2})$ are shown in Figure 1.34(e), which occur as unit impulses at $t = -\frac{3}{2}$ and $t = \frac{3}{2}$ respectively. At $t = -\frac{3}{2}$, $x(t) = -\frac{1}{2}$ and $\delta(t + \frac{3}{2}) = 1$. Using the property of impulse $x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$, we get $x(t)\delta(t + \frac{3}{2}) = -\frac{1}{2}$. Similarly at $t = \frac{3}{2}$, $x(t) = \frac{1}{2}$ and $-\delta(t - \frac{3}{2}) = -1$. Hence, $x(t)\delta(t - \frac{3}{2}) = -\frac{1}{2}$. This is sketched as shown in Figure 1.34(f).

(d) To sketch $x(2t + 1)$

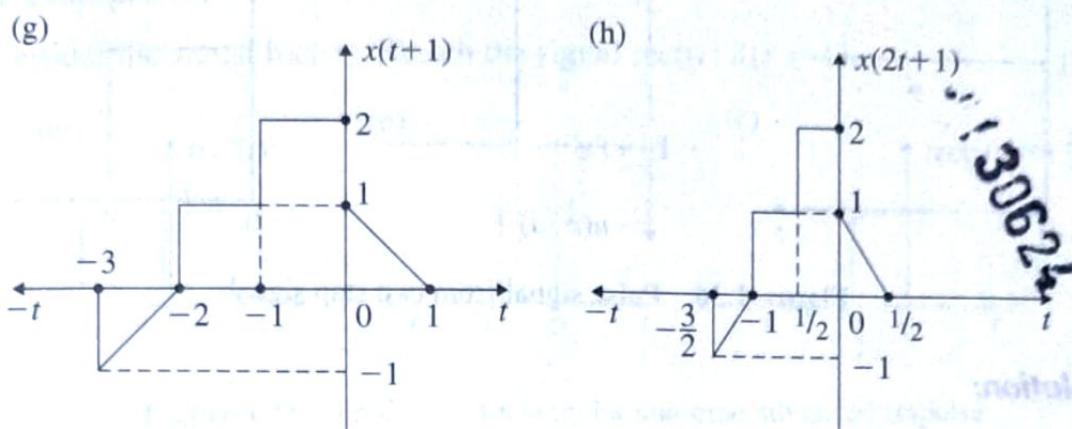


Figure 1.34

From Figure 1.34(a), $x(t + 1)$ is derived by shifting $x(t)$ to the left by $t = 1$. This is shown in Figure 1.34(g). By time compression of $x(t + 1)$ by a factor 2, $x(2t + 1)$ is obtained and sketched as shown in Figure 1.34(h).

■ Example 1.8

Represent the signal $x(t) = 5u(4 - t)$.

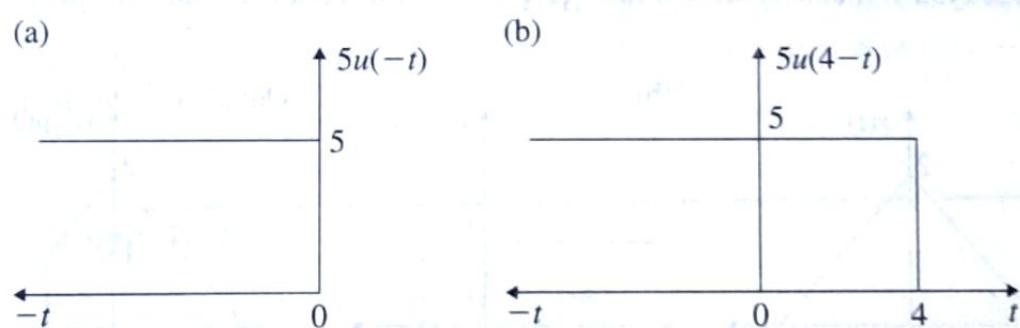


Figure 1.35 Time shifted step signal.

Solution:

1. The unit step signal when its amplitude is multiplied by a factor 5, it becomes $5u(t)$. When this is time reversed, it becomes $5u(-t)$ and is shown in Figure 1.35(a).
2. $5u(-t)$ is time shifted to the right by $t = 4$ and is sketched as $5u(4 - t)$ in Figure 1.35(b).

■ Example 1.9

Sketch the signal $x(t) = [u(t) - u(t - a)]$ where $a > 0$.

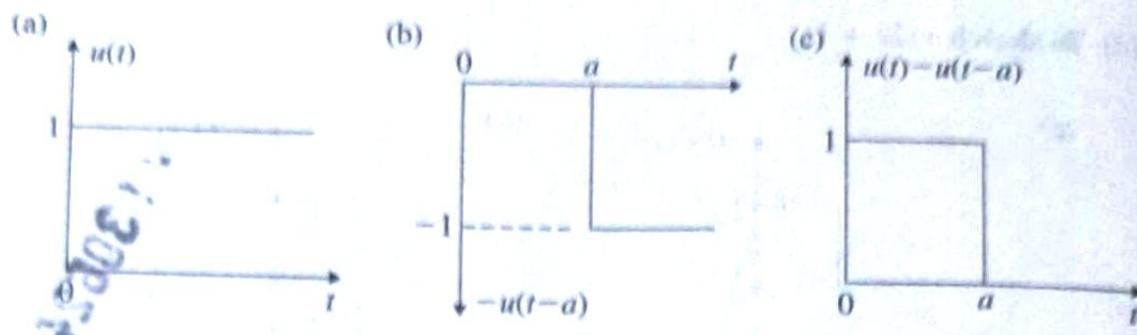


Figure 1.36 Pulse signal from two step signals.

Solution:

- (1) The unit step signal $u(t)$ is shown in Figure 1.36(a).
- (2) The unit step signal with a time delay a and amplitude inverted is shown in Figure 1.36(b).
- (3) If the above two step signals are added, a pulse signal is obtained and is sketched as shown in Figure 1.36(c) which gives $u(t) - u(t - a)$. The above signal is defined as

$$x(t) = 1 \quad 0 \leq t \leq a$$

■ Example 1.10

Consider the signal $x(t)$ shown in Figure 1.37(a). Sketch the signal $x(t)u(1-t)$.

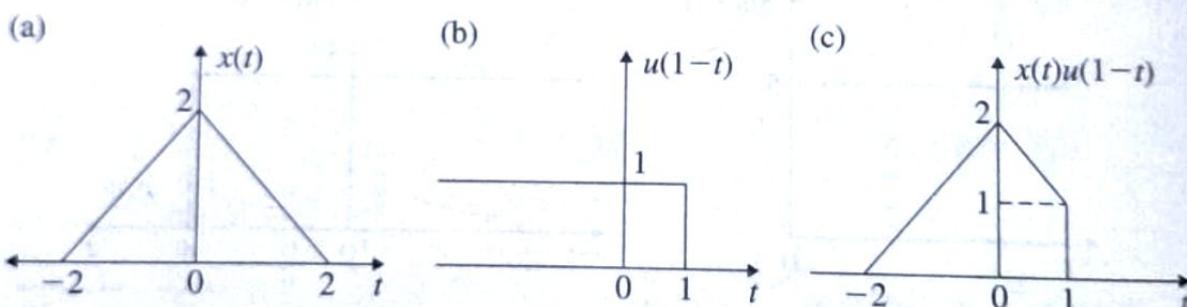


Figure 1.37 Product of triangular and time delayed step signals.

Solution:

1. The signal $x(t)$ is shown in Figure 1.37(a). The signal $u(1-t)$ is shown in Figure 1.37(b).
2. The signal $x(t)$ is multiplied by the factor 1 for the intervals $-2 \leq t \leq 0$ and $0 \leq t \leq 1$. During these time intervals, the slope of the straight lines of the triangles are +1 and -1 respectively. Hence, $x(t)$ is retained as it is. At $t = 1$, $x(t) = 1$ and $u(1-t) = 1$. Hence, $x(t)u(1-t) = 1$.
3. For $t > 1$, $u(1-t) = 0$ and hence $x(t)u(1-t) = 0$. This is sketched in Figure 1.37(c).

■ Example 1.11

Consider the signal $\text{rect}(t)$. Sketch the signal $\text{rect}(t)\delta(t + \frac{1}{2})$.

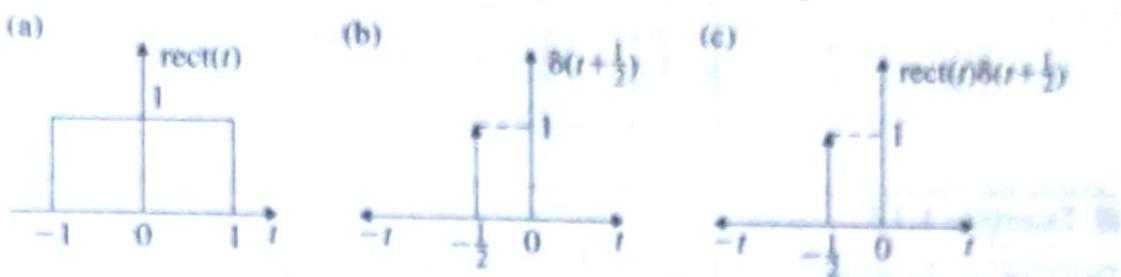


Figure 1.38 Product of rectangular and time advanced impulse.

Solution:

1. The rectangular pulse $\text{rect}(t)$ is shown in Figure 1.38(a).
2. The time advanced impulse $\delta(t + \frac{1}{2})$ is defined as follows:

$$\begin{aligned}\delta\left(t + \frac{1}{2}\right) &= 1 && \text{if } t = -\frac{1}{2} \\ &= 0 && \text{otherwise}\end{aligned}$$

This is sketched in Figure 1.38(b).

3. At $t = -\frac{1}{2}$, the magnitude of $\text{rect}(t) = 1$. Hence using the property $x(t)\delta(t + t_0) = x(t_0)$, we sketch $x(t)\delta(t + t_0)$ as an impulse at $t = -\frac{1}{2}$ which is shown in Figure 1.38(c).

■ Example 1.12

$$x(t) = 10e^{-3t+4}$$

Determine $x(t + 2)$, $x(-t + 2)$ and $x(\frac{t}{4} - 5)$.

Solution:

$$x(t) = 10e^{-3t+4}$$

1. For $t = t + 2$,

$$x(t + 2) = 10e^{-3(t+2)+4}$$

$$x(t + 2) = 10e^{-3t-2}$$

2. For $t = -t + 2$,

$$x(-t + 2) = 10e^{-3(-t+2)+4}$$

$$x(-t + 2) = 10e^{3t-2}$$

3. For $t = (\frac{t}{4} - 5)$,

$$x\left(\frac{t}{4} - 5\right) = 10e^{-3(\frac{t}{4}-5)+4}$$

$$\boxed{x\left(\frac{t}{4} - 5\right) = 10e^{-\frac{3}{4}t+19}}$$

■ Example 1.13

Decompose the signal $x(t)$ shown in Figure 1.39(a) in terms of basic signals such as delta, step and ramp.

(Anna University, December, 2007)

Solution:

1. The given signal $x(t)$ is shown in Figure 1.39(a).
2. The signals $u(t) + u(t - 1) - 3u(t - 2)$ are shown in Figure 1.39(b) and their sum is shown in Figure 1.39(c).

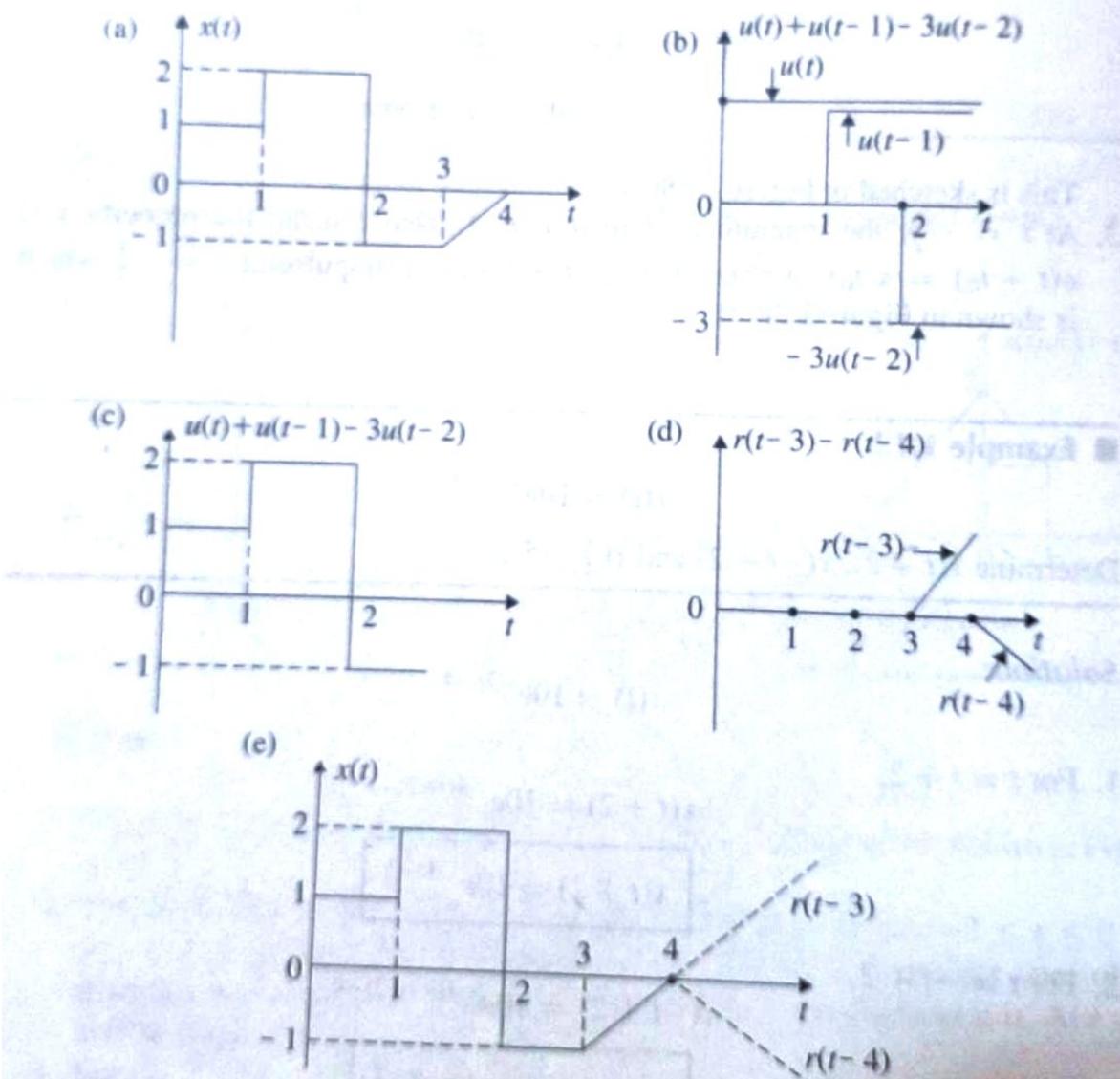


Figure 1.39 Composite signal expressed in terms of basic signals.



Fig. P. 1.6.4(f)

1.6.5 : (a) Plot the signal with respect to time

$$x(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$$

(b) State whether this signal is energy or power signal ? Find corner power as the case may be.

(c) Find even and odd parts of this signal.

Soln. : The given signal is,

$$x(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5) \quad \dots(1)$$

Let us draw this waveform step wise.

Step 1 : Consider first two terms of Equation (1).

$$\text{Let } x_1(t) = u(t) - r(t-1) \quad \dots(2)$$

Here $u(t)$ indicates unit step. While $r(t-1)$ is delayed ramp. It is obtained by shifting unit ramp towards right by '1' position. Then this signal is subtracted from unit step. The complete operation is shown in Fig. P. 1.6.5(a). The calculations are done as follows.

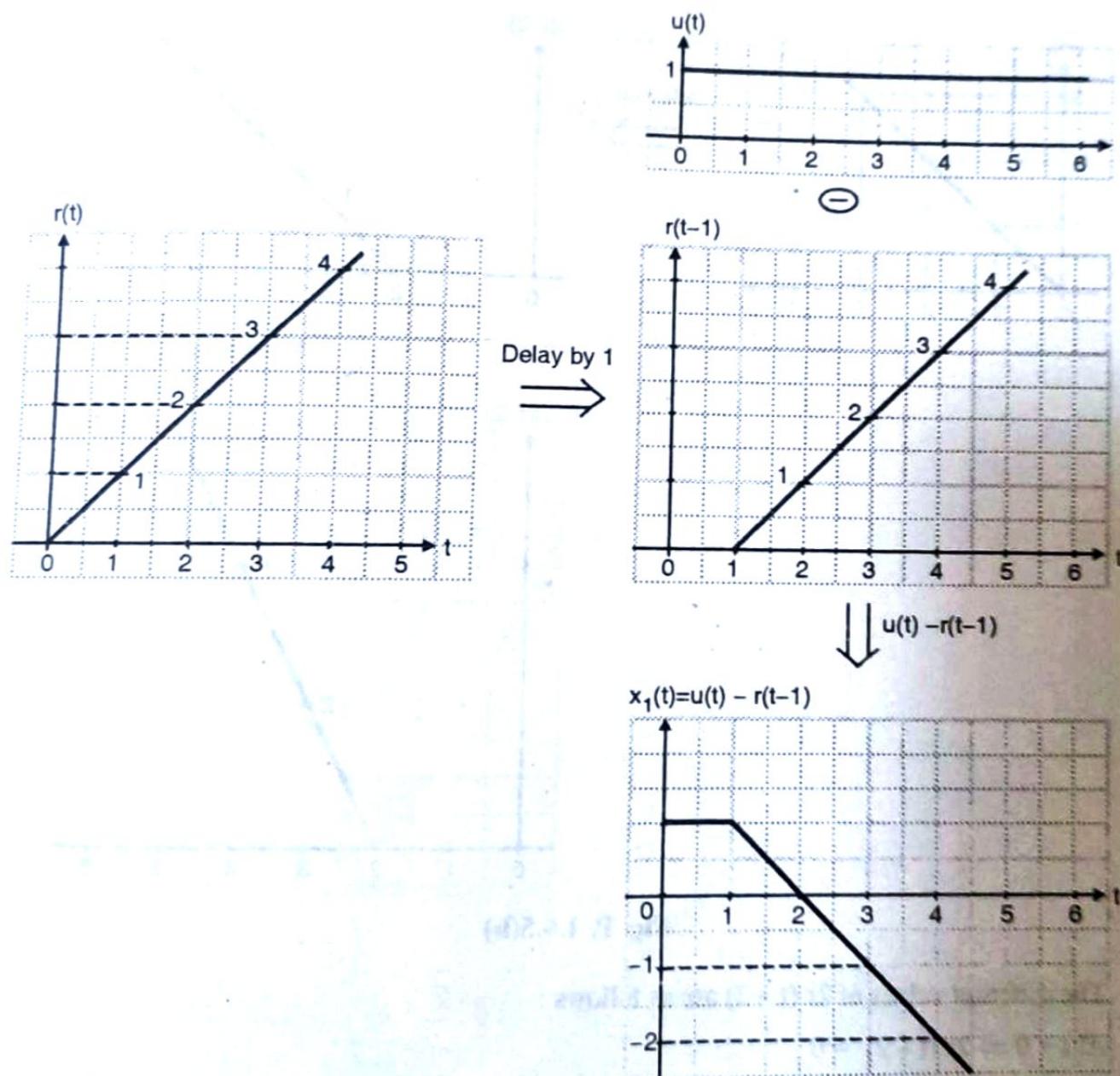


Fig. P. 1.6.5(a)

$$\text{For } t = 0 \Rightarrow x_1(0) = 1 - 0 = 1$$

$$\text{For } t = 1 \Rightarrow x_1(1) = 1 - 0 = 1$$

$$\text{For } t = 2 \Rightarrow x_1(2) = 1 - 1 = 0$$

$$\text{For } t = 3 \Rightarrow x_1(3) = 1 - 2 = -1$$

$$\text{For } t = 4 \Rightarrow x_1(4) = 1 - 3 = -2$$

Step 2 : Now consider the term $u(t) - r(t-1) + 2r(t-2)$

$$\text{It can be expressed as } x_2(t) = x_1(t) + 2r(t-2)$$

Here $x_1(t)$ is the signal obtained in step I.

Now the signal $2r(t-2)$ is obtained as follows.

First delay $r(t)$ by 2 position to obtain $r(t-2)$. Then its amplitude is multiplied by '2' to get signal $2r(t-2)$. This operation is shown in Fig. P. 1.6.5(b).

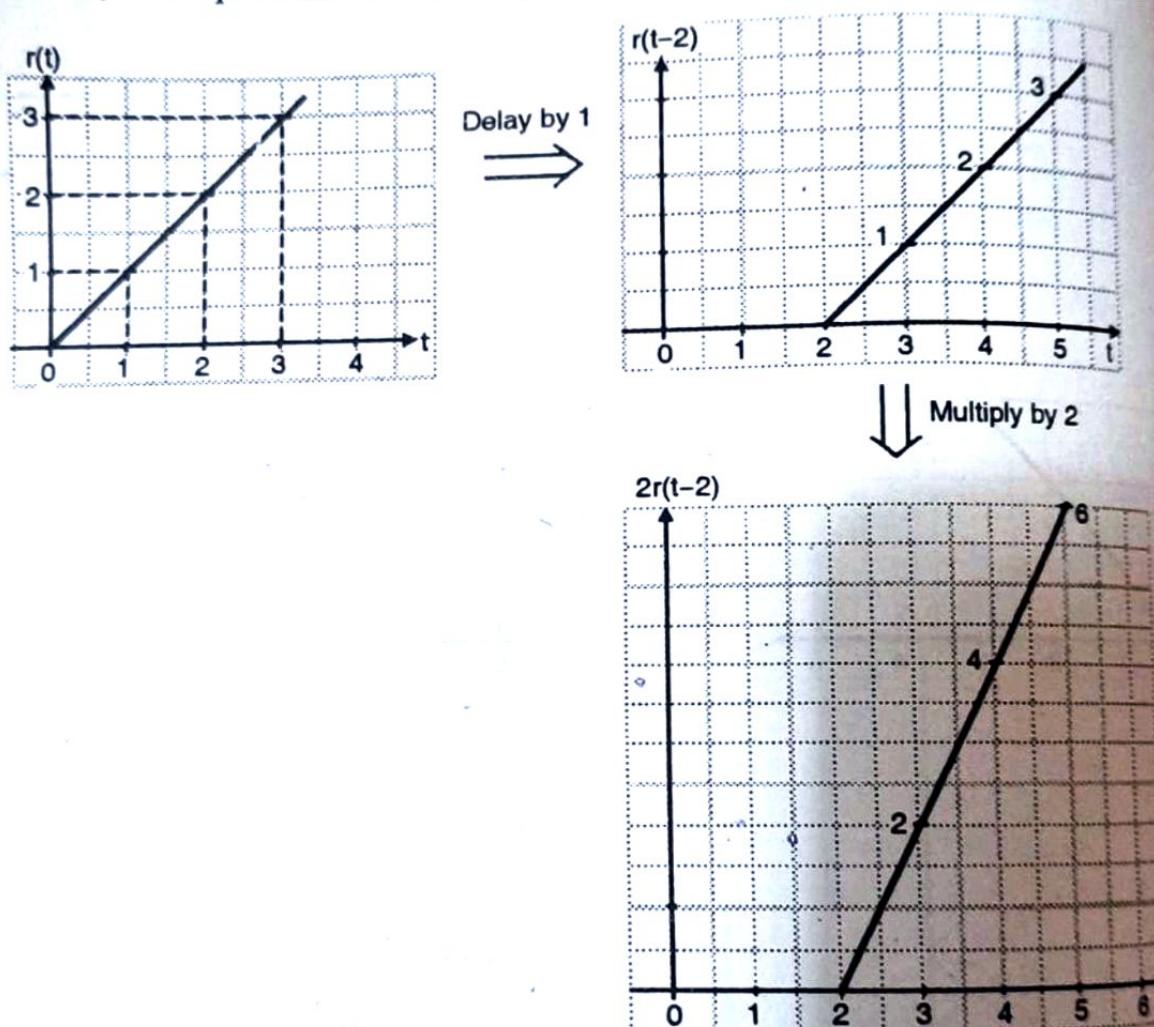


Fig. P. 1.6.5(b)

The different values of $2r(t-2)$ are as follows :

$$\text{At } t = 0 \Rightarrow 2r(t-2) = 0$$

$$\text{At } t = 1 \Rightarrow 2r(t-2) = 0$$

$$\text{At } t = 2 \Rightarrow 2r(t-2) = 0$$

$$\text{At } t = 3 \Rightarrow 2r(t-2) = 2$$

$$\text{At } t = 4 \Rightarrow 2r(t-2) = 4$$

Now the signal $x_2(t)$ is obtained by adding $x_1(t)$ and $2r(t-1)$. That means by adding P. 1.6.5(a) and Fig. P. 1.6.5(b). This operation is shown in Fig. P. 1.6.5(c).

Using Fig. P. 1.6.5(a) and Fig. P. 1.6.5(b); the calculations are done as follows

$$\text{For } t = 0 \Rightarrow x_2(0) = 1 + 0 = 1 \quad \text{For } t = 1 \Rightarrow x_2(1) = 1 + 0 = 1$$

$$\text{For } t = 2 \Rightarrow x_2(2) = 0 + 0 = 0 \quad \text{For } t = 3 \Rightarrow x_2(3) = -1 + 2 = 1$$

$$\text{For } t = 4 \Rightarrow x_2(4) = -2 + 4 = 2 \quad \text{For } t = 5 \Rightarrow x_2(5) = -3 + 6 = 3$$

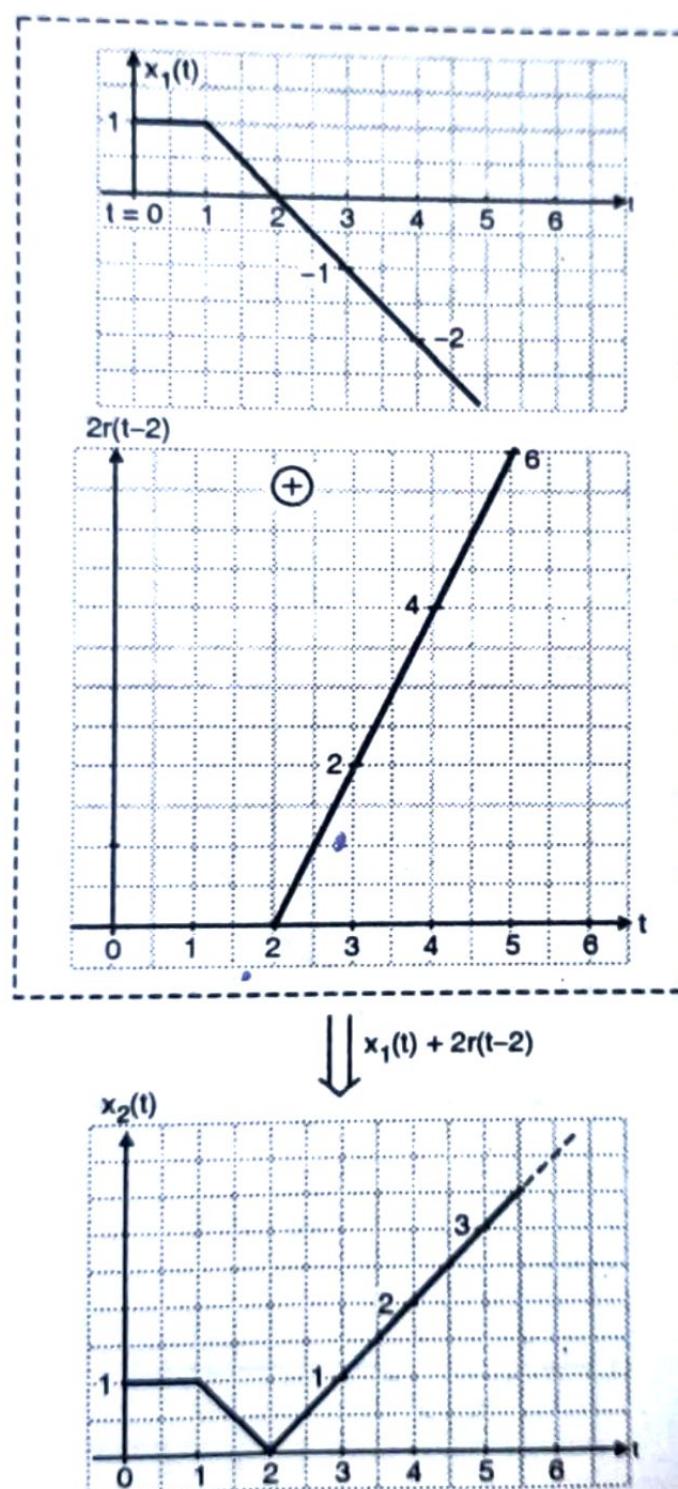


Fig. P. 1.6.5(c)

Step 3: Consider the term $u(t) = r(t-1) + 2r(t-2) - r(t-3)$
It can be expressed as,

$$x_3(t) = x_2(t) - r(t-3)$$

Here $r(t-3)$ is unit ramp delayed by 3 units as shown in Fig. P. 1.6.5(d).

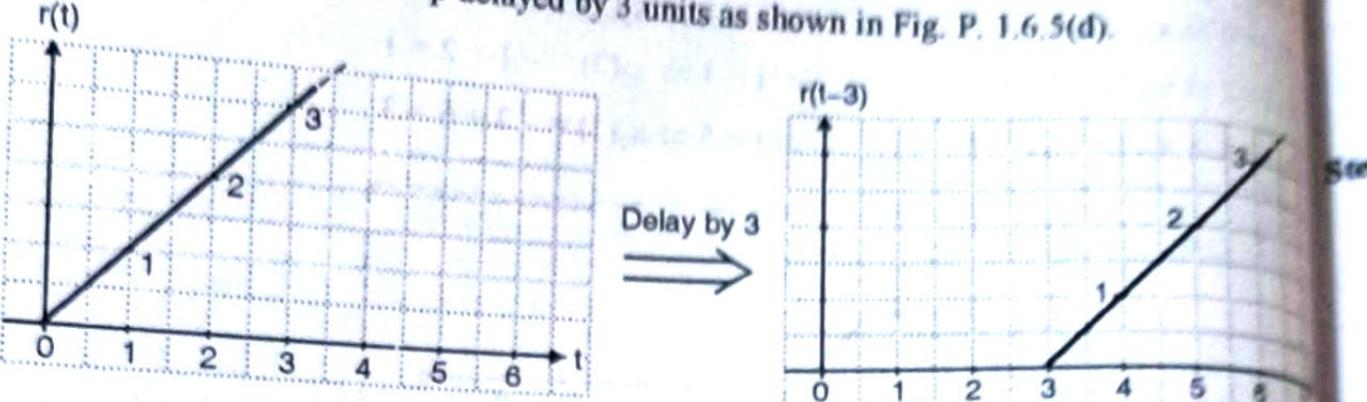


Fig. P. 1.6.5(d)

Now $x_3(t)$ is obtained by subtracting $r(t-3)$ from $x_2(t)$ as shown in Fig. P. 1.6.5(e).

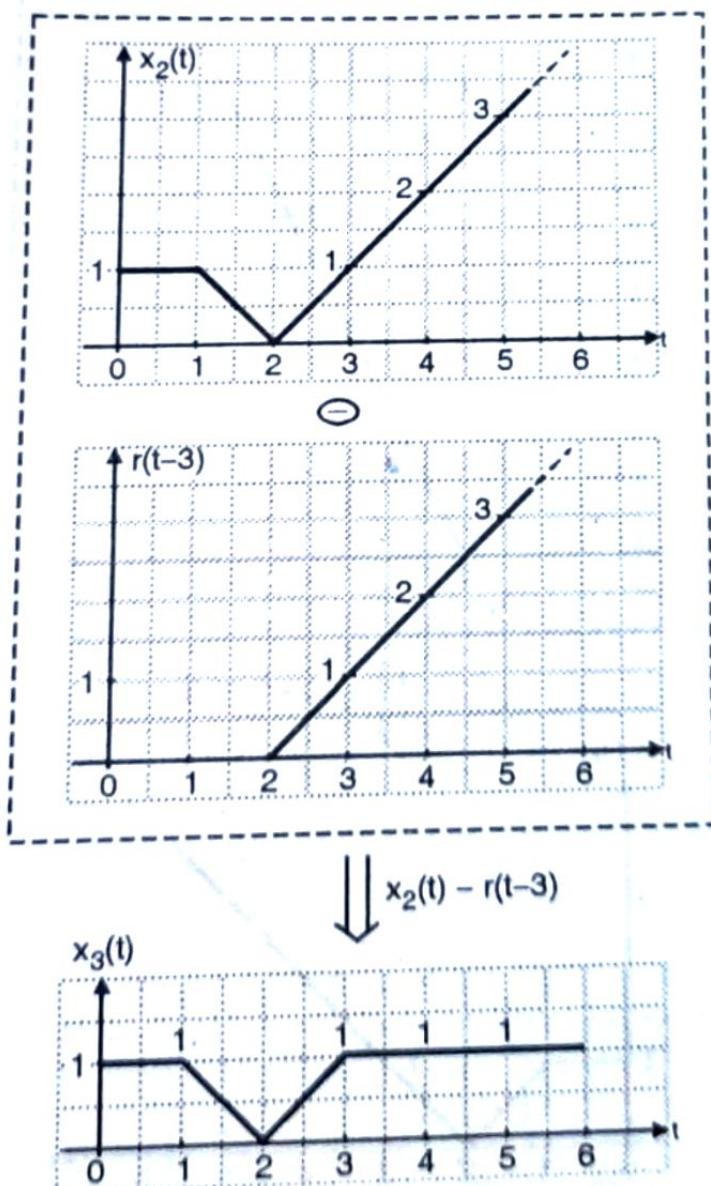


Fig. P. 1.6.5(e)

The calculations are done as follows

$$\text{At } t = 0 \Rightarrow x_3(t) = 1 - 0 = 1$$

$$\text{At } t = 2 \Rightarrow x_3(t) = 0 - 0 = 0$$

$$\text{At } t = 4 \Rightarrow x_3(t) = 2 - 1 = 1$$

$$\text{At } t = 1 \Rightarrow x_3(t) = 1 - 0 = 1$$

$$\text{At } t = 3 \Rightarrow x_3(t) = 1 - 0 = 1$$

$$\text{At } t = 5 \Rightarrow x_3(t) = 3 - 2 = 1$$

Step 4 : Consider the term

$$u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4)$$

It can be expressed as,

$$x_4(t) = x_3(t) + u(t-4) \quad \dots(5)$$

Here $u(t-4)$ is unit step delayed by 4 units. It is shown in Fig. P. 1.6.5(f).

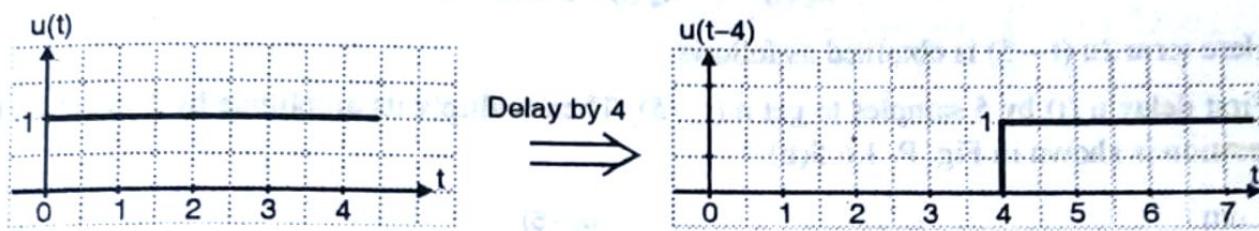


Fig. P. 1.6.5(f)

The signal $x_4(t)$ is obtained by adding $x_3(t)$ and $u(t-4)$ as shown in Fig. P. 1.6.5(g). The different calculations are done as follows

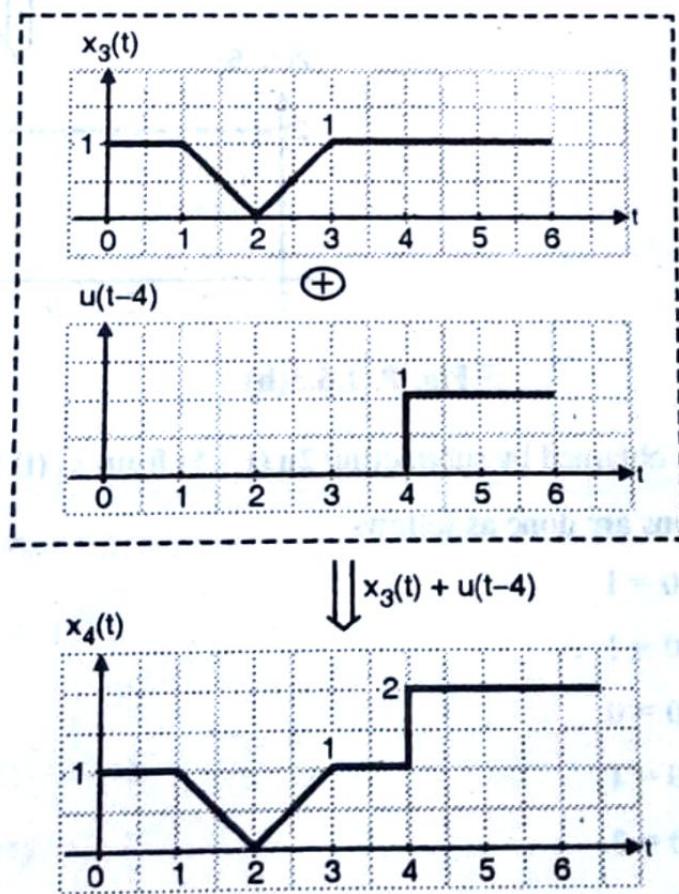


Fig. P. 1.6.5(g)

- At $t = 0 \Rightarrow x_4(t) = 1 + 0 = 1$
 At $t = 1 \Rightarrow x_4(t) = 1 + 0 = 1$
 At $t = 2 \Rightarrow x_4(t) = 0 + 0 = 0$
 At $t = 3 \Rightarrow x_4(t) = 1 + 0 = 1$
 At $t = 4 \Rightarrow x_4(t) = 1 + 1 = 2$
 At $t = 5 \Rightarrow x_4(t) = 1 + 1 = 2$

Step 5: Now consider the term

$$u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$$

It can be expressed as

$$x_5(t) = x_4(t) - 2u(t-5)$$

Here term $2u(t-5)$ is obtained as follows

First delay $u(t)$ by 5 samples to get $u(t-5)$. Then multiply its amplitude by 2 to get $2u(t-5)$. This operation is shown in Fig. P. 1.6.5(h).

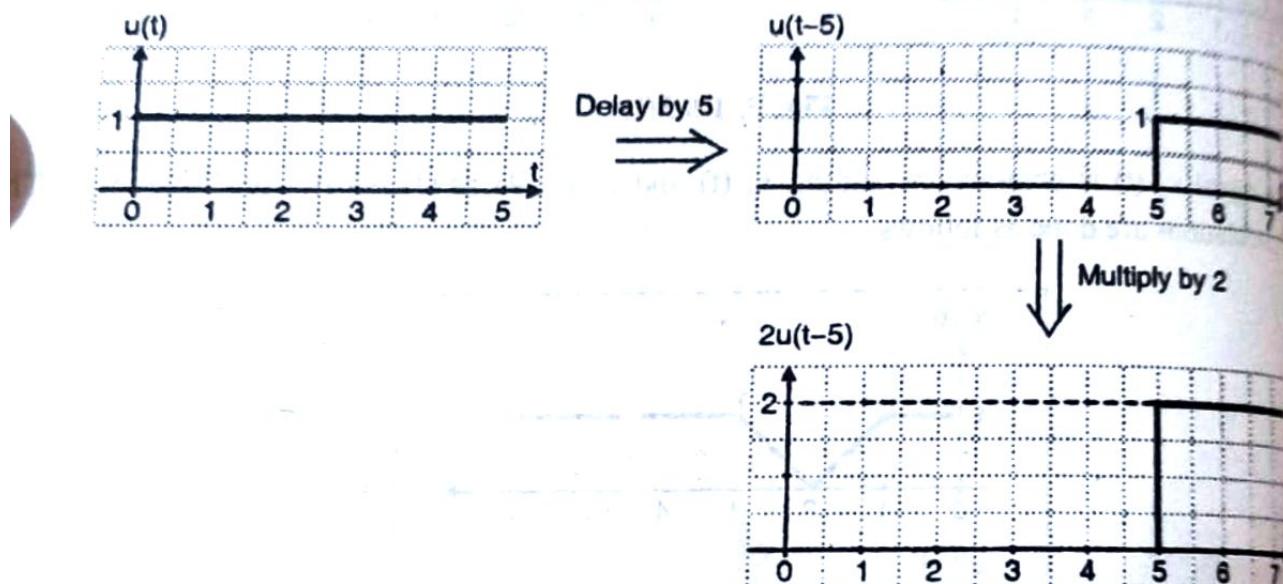


Fig. P. 1.6.5(h)

Now the signal $x_5(t)$ is obtained by subtracting $2u(t-5)$ from $x_4(t)$ as shown in Fig. P. 1.6.5(i).

The different calculations are done as follows

- At $t = 0 \Rightarrow x_5(t) = 1 - 0 = 1$
 At $t = 1 \Rightarrow x_5(t) = 1 - 0 = 1$
 At $t = 2 \Rightarrow x_5(t) = 0 - 0 = 0$
 At $t = 3 \Rightarrow x_5(t) = 1 - 0 = 1$
 At $t = 4 \Rightarrow x_5(t) = 2 - 0 = 2$
 At $t = 5 \Rightarrow x_5(t) = 2 - 2 = 0$
 At $t = 0 \Rightarrow x_5(t) = 2 - 2 = 0$

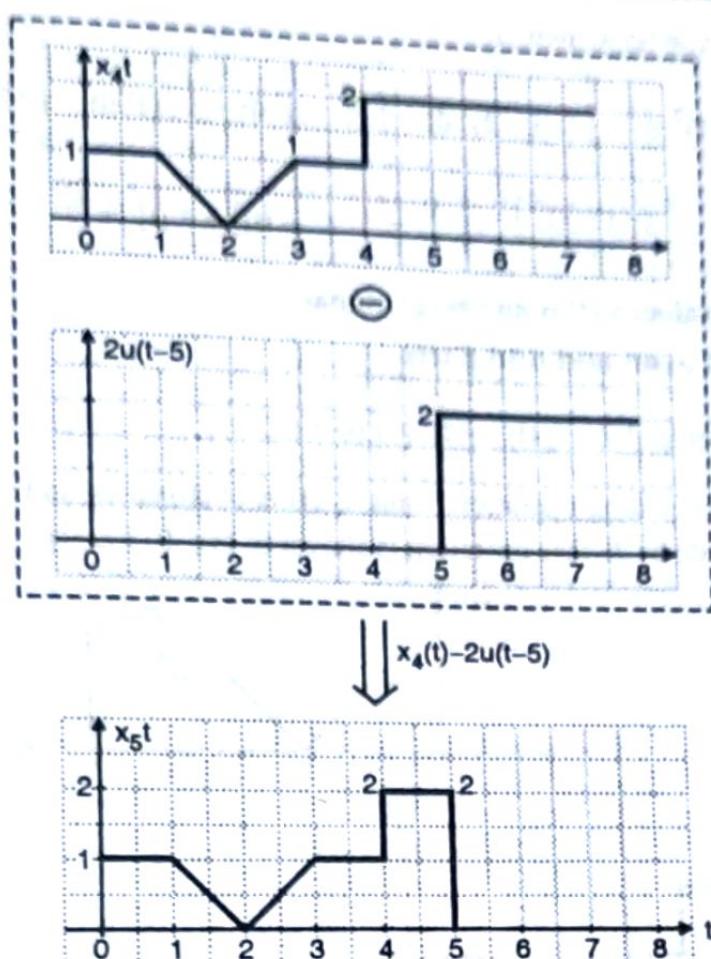


Fig. P. 1.6.5(i)

Part (b) : Calculation of energy or power :

The resultant signal $x(t)$ is as shown in Fig. P. 1.6.5(i). It is non-periodic signal. We know that a non-periodic signal is an energy signal. Now energy as signal is given by,

$$E = \int_{-\infty}^{\infty} x^2(t) dt \quad \text{---(7)}$$

The signal $x(t)$ exists between the range $t = 0$ to $t = 5$. Referring Fig. P. 1.6.5(i), we can write different expressions of $x(t)$ as follows

$$\text{For } 0 \leq t \leq 1 \Rightarrow x(t) = 1$$

$$\text{For } 1 \leq t \leq 2 \Rightarrow x(t) = -t + 2$$

$$\text{For } 2 \leq t \leq 3 \Rightarrow x(t) = t - 2$$

$$\text{For } 3 \leq t \leq 4 \Rightarrow x(t) = 1$$

$$\text{For } 4 \leq t \leq 5 \Rightarrow x(t) = 2$$

Thus Equation (7) can be written as,

$$E = \int_0^1 (1)^2 dt + \int_1^2 (-t+2)^2 dt + \int_2^3 (t-2)^2 dt + \int_3^4 (1)^2 dt + \int_4^5 (2)^2 dt$$

$$\therefore E = [t]_0^1 + \left[\frac{(-t+2)^3}{3} \right]_1^2 + \left[\frac{(t-2)^3}{3} \right]_2^3 + [t]_3^4 + 4[t]_4^5 = \frac{20}{3}$$

Since this is finite value; $x(t)$ is an energy signal.

Part (c) : Calculation of even and odd parts :

Even part of $x(t)$ is given by, $x_e(t) = \frac{1}{2} \{ x(t) + x(-t) \}$

Here $x(-t)$ indicates folding operation. Thus $x_e(t)$ is obtained by adding $x(t)$ and $x(-t)$, then by dividing its amplitude by 2. This operation is shown in Fig. P. 1.6.5(j).

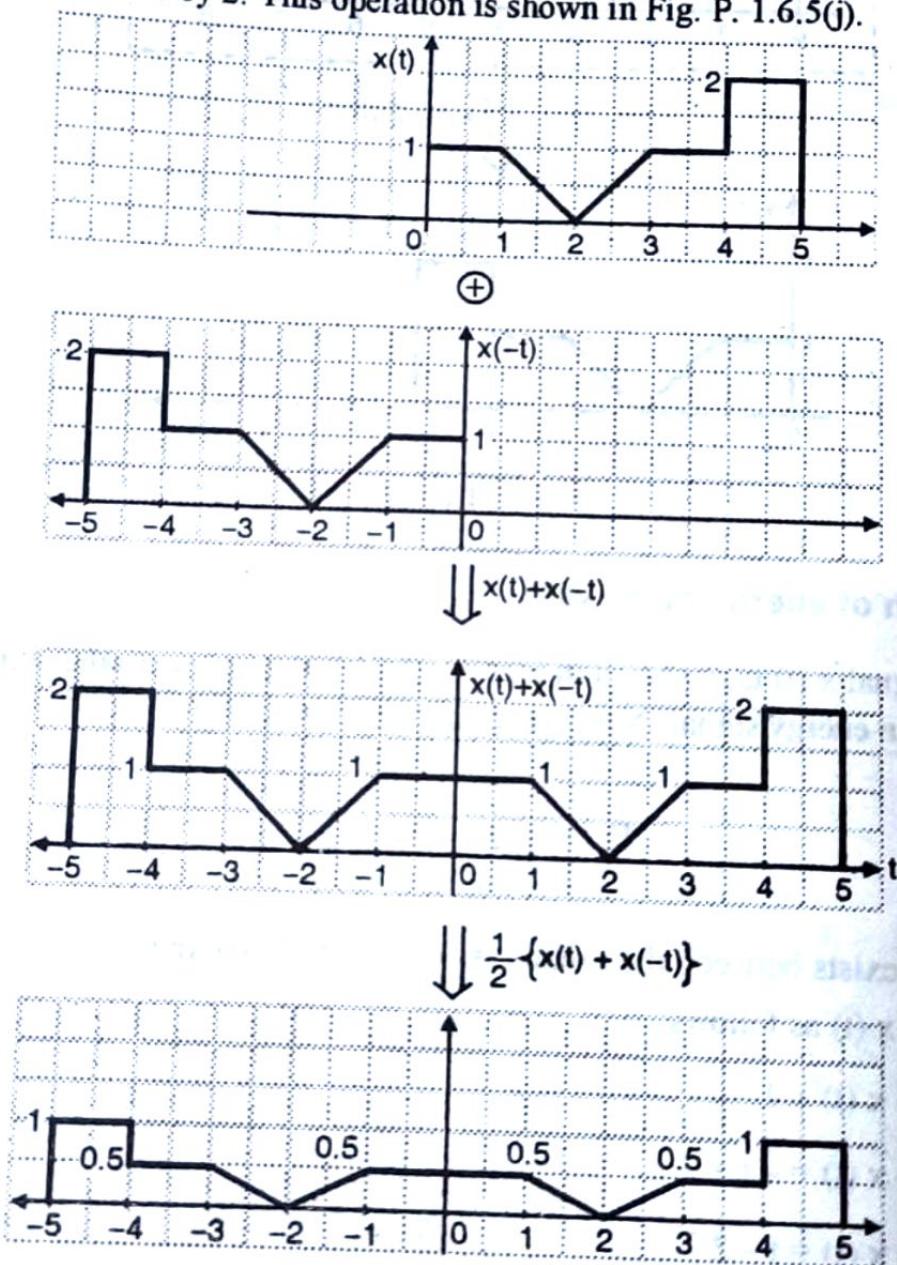


Fig. P. 1.6.5(j)

Odd part of $x(t)$ is given by, $x_o(t) = \frac{1}{2} \{ x(t) - x(-t) \}$

This signal is obtained by subtracting $x(-t)$ and $x(t)$ and then by dividing its amplitude by 2. This operation is shown in Fig. P. 1.6.5(k).

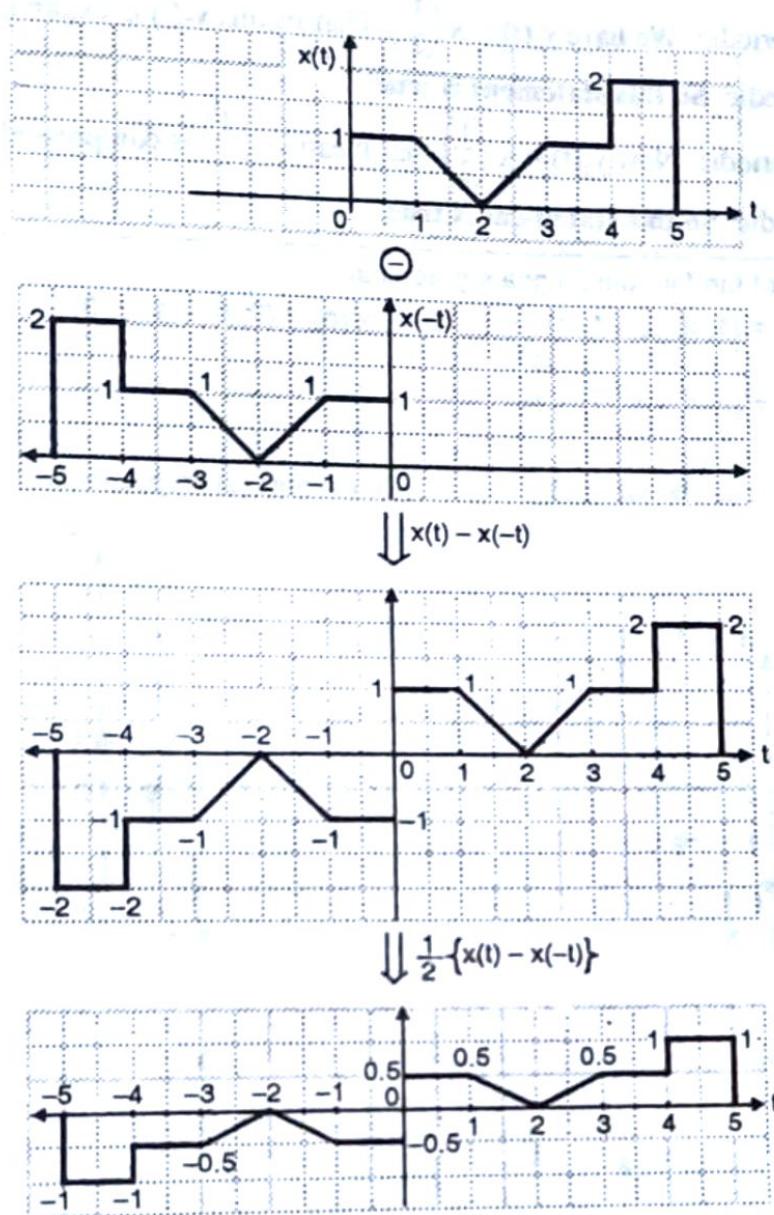


Fig. P. 1.6.5(k)

Ex. 1.6.6 : Let $x(t)$ be a continuous time signal and let $y_1(t) = x(2t)$ and $y_2(t) = x\left(\frac{t}{2}\right)$. Are the following statements true

1. If $x(t)$ is periodic then $y_1(t)$ is periodic
2. If $y_1(t)$ is periodic then $x(t)$ is periodic
3. If $x(t)$ is periodic then $y_2(t)$ is periodic
4. If $y_2(t)$ is periodic then $x(t)$ is periodic.

Soln. :

1. Here $x(t)$ is periodic. Let its period be T_0 . Now $y_1(t) = x\left(\frac{t}{2}\right)$. It indicates that $y_1(t)$ is obtained by compressing $x(t)$ in time domain. That means the period of $y_1(t)$ is $\frac{T_0}{2}$. Thus $y_1(t)$ is also periodic. So this statement is true.