9

Two-Port Networks

9.1 INTRODUCTION

A two-port network has two pairs of terminals, one pair at the input known as *input port* and one pair at the output known as *output port* as shown in Fig. 9.1. There are four variables V_1, V_2, I_1 and I_2 associated with a two-port network. Two of these variables can be expressed in terms of the other two variables. Thus, there will be two dependent variables and two independent variables. The number of possible combinations generated by four variables taken two at a time is ${}^4C_2$, i.e., six. There are six possible sets of equations describing a two-port network.

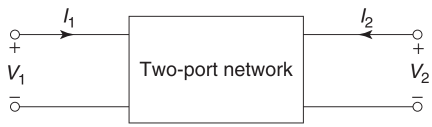

Fig. 9.1 *Two-port network*

Table 9.1 *Two–port parameters*

Parameter	*Variables*		*Equation*
	Express	*In terms of*	
Open-Circuit Impedance	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
Short-Circuit Admittance	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
Transmission	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Inverse Transmission	V_2, I_2	V_1, I_1	$V_2 = A' V_1 - B' I_1$ $I_2 = C' V_1 - D' I_1$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
Inverse Hybrid	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

9.2 OPEN-CIRCUIT IMPEDANCE PARAMETERS (Z PARAMETERS)

The Z parameters of a two-port network may be defined by expressing two-port voltages V_1 and V_2 in terms of two-port currents I_1 and I_2.

$$(V_1, V_2) = f(I_1, I_2)$$
$$V_1 = Z_{11}\, I_1 + Z_{12}\, I_2$$
$$V_2 = Z_{21}\, I_1 + Z_{22}\, I_2$$

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
$$[V] = [Z][I]$$

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero.

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

where Z_{11} is the driving-point impedance with the output port open-circuited. It is also called *open-circuit input impedance*.

Similarly,

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

where Z_{21} is the transfer impedance with the output port open-circuited. It is also called *open-circuit forward transfer impedance*.

Case 2 When input port is open-circuited, i.e., $I_1 = 0$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}$$

where Z_{12} is the transfer impedance with the input port open-circuited. It is also called *open-circuit reverse transfer impedance*.

Similarly,

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

where Z_{22} is the open-circuit driving-point impedance with the input port open-circuited. It is also called *open circuit output impedance*.

As these impedance parameters are measured with either the input or output port open-circuited, these are called *open-circuit impedance parameters*.

The equivalent circuit of the two-port network in terms of Z parameters is shown in Fig. 9.2.

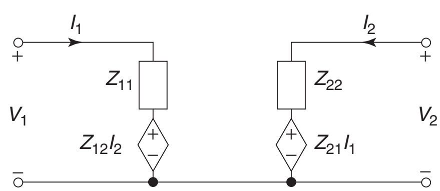

Fig. 9.2 *Equivalent circuit of the two-port network in terms of Z parameter*

9.2.1 Condition for Reciprocity

A network is said to be reciprocal if the ratio of excitation at one port to response at the other port is same if excitation and response are interchanged.

(a) As shown in Fig. 9.3, voltage V_s is applied at the input port with the output port short-circuited.

Fig. 9.3 *Network for deriving condition for reciprocity*

i.e.,
$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2 = -I_2'$$

From the Z-parameter equations,
$$V_s = Z_{11}\, I_1 - Z_{12}\, I_2'$$
$$0 = Z_{21}\, I_1 - Z_{22}\, I_2'$$
$$I_1 = \frac{Z_{22}}{Z_{21}}\, I_2'$$
$$V_s = Z_{11}\frac{Z_{22}}{Z_{21}} I_2' - Z_{12}\, I_2'$$
$$\frac{V_s}{I_2'} = \frac{Z_{11}\, Z_{22} - Z_{12}\, Z_{21}}{Z_{21}}$$

(b) As shown in Fig. 9.4, voltage V_s is applied at the output port with input port short-circuited.

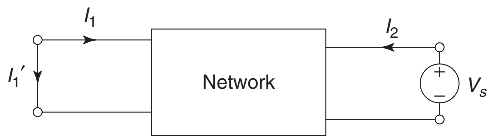

Fig. 9.4 *Network for deriving condition for reciprocity*

i.e.,
$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1 = -I_1'$$

From the Z-parameter equations,
$$0 = -Z_{11}\, I_1' + Z_{12}\, I_2$$
$$V_s = -Z_{21}\, I_1' + Z_{22}\, I_2$$
$$I_2 = \frac{Z_{11}}{Z_{12}}\, I_1'$$
$$V_s = -Z_{21} I_1' + Z_{22}\frac{Z_{11}}{Z_{12}}\, I_1'$$
$$\frac{V_s}{I_1'} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{12}}$$

Hence, for the network to be reciprocal,
$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

i.e.,
$$Z_{12} = Z_{21}$$

9.2.2 Condition for Symmetry

For a network to be symmetrical, the voltage-to-current ratio at one port should be the same as the voltage-to-current ratio at the other port with one of the ports open-circuited.

(a) When the output port is open-circuited, i.e., $I_2 = 0$
From the Z-parameter equation,
$$V_s = Z_{11}\, I_1$$
$$\frac{V_s}{I_1} = Z_{11}$$

(b) When the input port is open-circuited, i.e., $I_1 = 0$
From the Z-parameter equation,
$$V_s = Z_{22}\, I_2$$
$$\frac{V_s}{I_2} = Z_{22}$$

Hence, for the network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

i.e.,
$$Z_{11} = Z_{22}$$

Example 9.1 *Test results for a two-port network are (a) $I_1 = 0.1 \angle 0°$ A, $V_1 = 5.2 \angle 50°$ V, $V_2 = 4.1 \angle -25°$ V with Port 2 open-circuited (b) $I_2 = 0.1 \angle 0°$A, $V_1 = 3.1 \angle -80°$V, $V_2 = 4.2 \angle 60°$V, with Port 1 open-circuited. Find Z parameters.*

Solution

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{5.2\angle 50°}{0.1\angle 0°} = 52 \angle 50°\ \Omega, \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{3.1\angle -80°}{0.1\angle 0°} = 31 \angle -80°\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{4.1\angle -25°}{0.1\angle 0°} = 41 \angle -25°\ \Omega, \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{4.2\angle 60°}{0.1\angle 0°} = 42 \angle 60°\ \Omega$$

Hence, the *Z*-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 52\angle 50° & 31\angle -80° \\ 41\angle -25° & 42\angle 60° \end{bmatrix}$$

Example 9.2 *Find the Z parameters for the network shown in Fig. 9.5.*

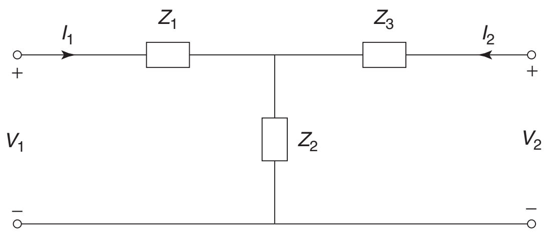

Fig. 9.5

Solution

First Method

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$.
Applying KVL to Mesh 1,

$$V_1 = (Z_1 + Z_2)\, I_1$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1 + Z_2$$

Also
$$V_2 = Z_2\, I_1$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_2$$

Case 2 When the input port is open-circuited, i.e., $I_1 = 0$.
Applying KVL to Mesh 2,

$$V_2 = (Z_2 + Z_3)\, I_2$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3$$

Also

$$V_1 = Z_2 I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_2$$

Hence, the Z-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

Second Method

The network is redrawn as shown in Fig. 9.6.

Applying KVL to Mesh 1,

$$V_1 = Z_1 I_1 + Z_2(I_1 + I_2)$$
$$= (Z_1 + Z_2)I_1 + Z_2 I_2 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = Z_3 I_2 + Z_2(I_1 + I_2)$$
$$= Z_2 I_1 + (Z_2 + Z_3)I_2 \quad \ldots\text{(ii)}$$

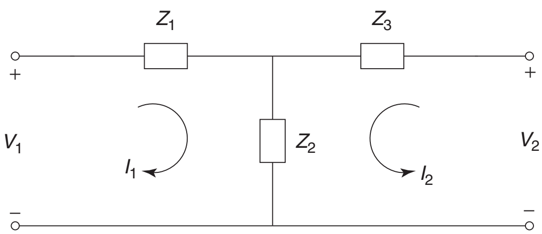

Fig. 9.6

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

Example 9.3 *Find Z-parameter for the network shown in Fig. 9.7.*

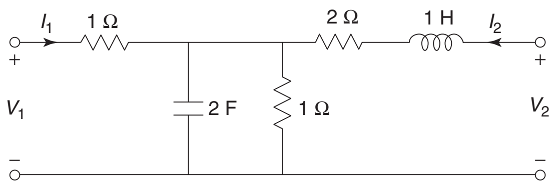

Fig. 9.7

Solution The transformed network is shown in Fig. 9.8.

$$Z_1 = 1$$

$$Z_2 = \frac{\left(\frac{1}{2s}\right)(1)}{\frac{1}{2s}+1} = \frac{1}{2s+1}$$

$$Z_3 = s + 2$$

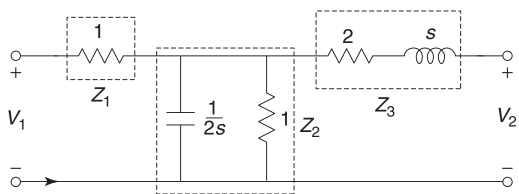

Fig. 9.8

From definition of Z-parameters,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1 + Z_2 = 1 + \frac{1}{2s+1} = \frac{2s+2}{2s+1}, \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_2 = \frac{1}{2s+1}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_2 = \frac{1}{2s+1}, \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3 = \frac{1}{2s+1} + s + 2 = \frac{2s^2+5s+3}{2s+1}$$

Example 9.4 *Find Z-parameters for the network shown in Fig. 9.9.*

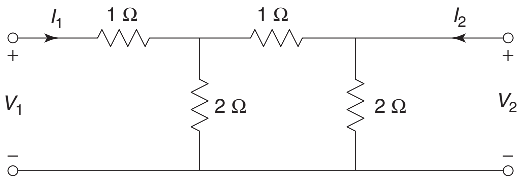

Fig. 9.9

Solution The network is redrawn as shown in Fig. 9.10.

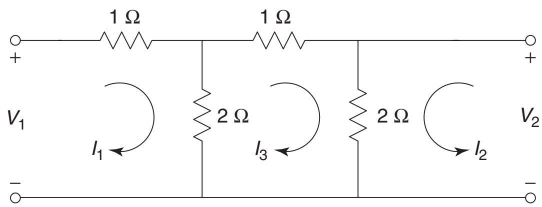

Fig. 9.10

Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-2I_1 + 2I_2 + 5I_3 = 0$$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2$$

$$= \frac{11}{5}I_1 + \frac{4}{5}I_2 \qquad \text{...(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 2I_2 + \frac{4}{5}I_1 - \frac{4}{5}I_2$$

$$= \frac{4}{5}I_1 + \frac{6}{5}I_2 \qquad \text{...(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{6}{5} \end{bmatrix}$$

Example 9.5 *Find the Z-parameters for the network shown in Fig. 9.11.*

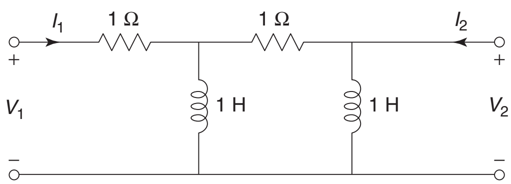

Fig. 9.11

Solution The transformed network is shown in Fig. 9.12.

Applying KVL to Mesh 1,

$$V_1 = (s+1)I_1 - sI_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = sI_2 + sI_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-sI_1 + sI_2 + (2s+1)I_3 = 0$$

$$I_3 = \frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2 \qquad \ldots\text{(iii)}$$

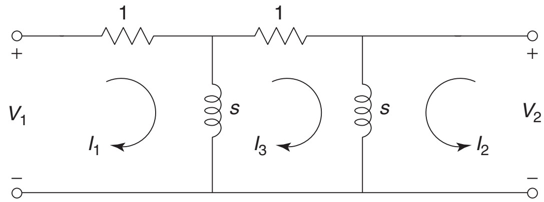

Fig. 9.12

Substituting Eq. (iii) in Eq. (i),

$$V_1 = (s+1)I_1 - s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$= \left(\frac{s^2+3s+1}{2s+1}\right)I_1 + \left(\frac{s^2}{2s+1}\right)I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = sI_2 + s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$= \left(\frac{s^2}{2s+1}\right)I_1 + \left(\frac{s^2+s}{2s+1}\right)I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{2s+1} & \dfrac{s^2}{2s+1} \\ \dfrac{s^2}{2s+1} & \dfrac{s^2+s}{2s+1} \end{bmatrix}$$

Example 9.6 *Find the open-circuit impedance parameters for the network shown in Fig. 9.13. Determine whether the network is symmetrical and reciprocal.*

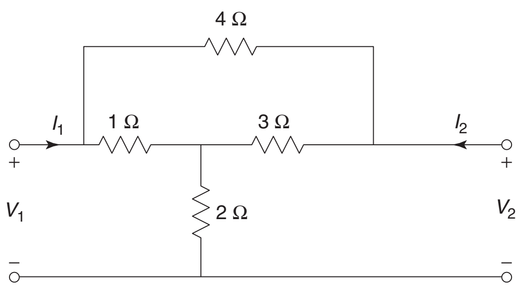

Fig. 9.13

Solution The network is redrawn as shown in Fig. 9.14.

Applying KVL to Mesh 1,

$$V_1 - 1(I_1 - I_3) - 2(I_1 + I_2) = 0$$

$$V_1 = 3I_1 + 2I_2 - I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 - 3(I_2 + I_3) - 2(I_1 + I_2) = 0$$

$$V_2 = 2I_1 + 5I_2 + 3I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-4I_3 - 3(I_2 + I_3) - 1(I_3 - I_1) = 0$$

$$I_1 - 3I_2 + 8I_3 = 0$$

$$I_3 = \frac{1}{8}I_1 - \frac{3}{8}I_2 \qquad \text{...(iii)}$$

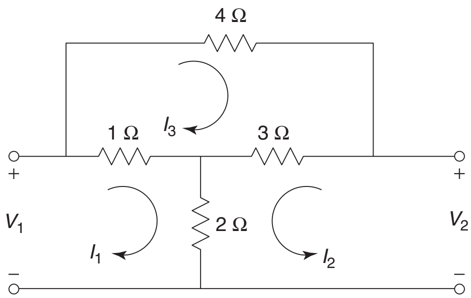

Fig. 9.14

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 + 2I_2 - \left(\frac{1}{8}I_1 - \frac{3}{8}I_2\right)$$

$$= \frac{23}{8}I_1 + \frac{19}{8}I_2 \qquad \text{...(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 2I_1 + 5I_2 + 3\left(\frac{1}{8}I_1 - \frac{3}{8}I_2\right)$$

$$= \frac{19}{8}I_1 + \frac{31}{8}I_2 \qquad \text{...(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{23}{8} & \frac{19}{8} \\ \frac{19}{8} & \frac{31}{8} \end{bmatrix}$$

Since $Z_{11} \neq Z_{22}$, the network is not symmetrical.
Since $Z_{12} = Z_{21}$, the network is reciprocal.

9.3 SHORT-CIRCUIT ADMITTANCE PARAMETERS (Y PARAMETERS)

The Y parameters of a two-port network may be defined by expressing the two-port currents I_1 and I_2 in terms of the two-port voltages V_1 and V_2.

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In matrix form, we can write

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual Y parameters for a given network can be defined by setting each of the port voltages equal to zero.

Case 1 When the output port is short-circuited, i.e., $V_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

where Y_{11} is the driving-point admittance with the output port short-circuited. It is also called *short-circuit input admittance*.

Similarly,

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

where Y_{21} is the transfer admittance with the output port short-circuited. It is also called *short-circuit forward transfer admittance*.

Case 2 When the input port is short-circuited, i.e., $V_1 = 0$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

where Y_{12} is the transfer admittance with the input port short-circuited. It is also called *short-circuit reverse transfer admittance*.

Similarly,

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

where Y_{22} is the short-circuit driving-point admittance with the input port short-circuited. It is also called the *short circuit output admittance*.

As these admittance parameters are measured with either input or output port short-circuited, these are called *short-circuit admittance parameters*.

The equivalent circuit of the two-port network in terms of Y parameters is shown in Fig. 9.15.

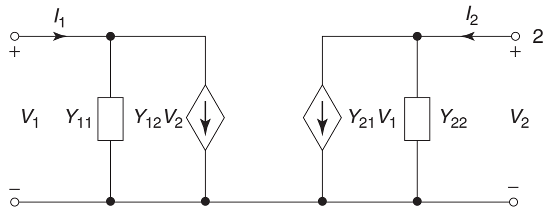

Fig. 9.15 *Equivalent circuit of the two-port network in terms of Y-parameters*

9.3.1 Condition for Reciprocity

(a) As shown in Fig. 9.16, voltage V_s is applied at input port with the output port short-circuited.

i.e,
$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2 = -I_2'$$

From the Y-parameter equation,

$$-I_2' = Y_{21}\,V_s$$
$$\frac{I_2}{V_s} = -Y_{21}$$

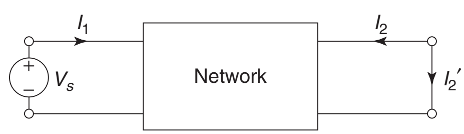

Fig. 9.16 *Network for deriving condition for reciprocity*

(b) As shown in Fig. 9.17, voltage V_s is applied at output port with the input port short-circuited.

i.e,
$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1 = -I_1'$$

From the Y-parameter equation,
$$-I_1' = Y_{12}\, V_s$$
$$\frac{I_1'}{V_s} = -Y_{12}$$

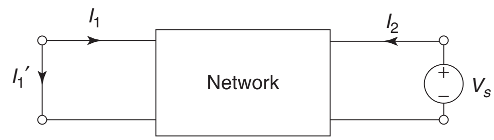

Fig. 9.17 *Network for deriving condition for reciprocity*

Hence, for the network to be reciprocal,
$$\frac{I_2'}{V_s} = \frac{I_1'}{V_s}$$

i.e,
$$Y_{12} = Y_{21}$$

9.3.2 Condition for Symmetry

(a) When the output port is short-circuited, i.e., $V_2 = 0$.
From the Y-parameter equation,
$$I_1 = Y_{11}\, V_s$$
$$\frac{V_s}{I_1} = \frac{1}{Y_{11}}$$

(b) When the input port is short-circuited, i.e., $V_1 = 0$.
From the Y-parameter equation,
$$I_2 = Y_{22}\, V_s$$
$$\frac{V_s}{I_2} = \frac{1}{Y_{22}}$$

Hence, for the network to be symmetrical,
$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

i.e.,
$$Y_{11} = Y_{22}$$

Example 9.7 *Test results for a two-port network are*

(a) Port 2 short-circuited: $V_1 = 50\,\angle 0°\,V,\, I_1 = 2.1\,\angle -30°\,A,\, I_2 = -1.1\,\angle -20°\,A$

(b) Port 1 short-circuited: $V_2 = 50\,\angle 0°\,V,\, I_2 = 3\,\angle -15°\,A,\, I_1 = -1.1\,\angle -20°\,A.$
Find Y-parameters.

Solution

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{2.1\angle -30°}{50\angle 0°} = 0.042\angle -30°\ \mho, \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-1.1\angle -20°}{50\angle 0°} = -0.022\angle -20°\ \mho$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-1.1\angle -20°}{50\angle 0°} = -0.022\angle -20°\ \mho, \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3\angle -15°}{50\angle 0°} = 0.06\angle -15°\ \mho$$

Hence, the Y-parameters are
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.042\angle -30° & -0.022\angle -20° \\ -0.022\angle -20° & 0.06\angle -15° \end{bmatrix}$$

Example 9.8 *Find Y-parameters for the network shown in Fig. 9.18.*

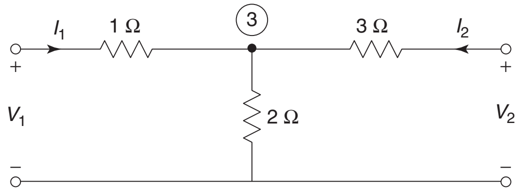

Fig. 9.18

Solution

First Method

Case 1 When the output port is short-circuited, i.e., $V_2 = 0$ as shown in Fig. 9.19,

$$R_{eq} = 1 + \frac{2\times 3}{2+3} = 1 + \frac{6}{5} = \frac{11}{5}\ \Omega$$

Now,

$$V_1 = \frac{11}{5} I_1$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{5}{11}\ \mho$$

Also,

$$I_2 = \frac{2}{5}(-I_1) = -\frac{2}{5}\times\frac{5}{11}V_1 = -\frac{2}{11}V_1$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{2}{11}\ \mho$$

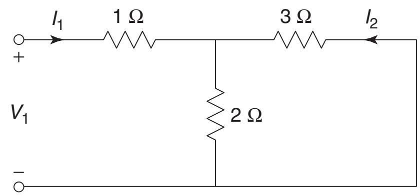

Fig. 9.19

Case 2 When the input port is short-circuited, i.e., $V_1 = 0$ as shown in Fig. 9.20,

$$R_{eq} = 3 + \frac{1\times 2}{1+2} = 3 + \frac{2}{3} = \frac{11}{3}\ \Omega$$

Now

$$V_2 = \frac{11}{3} I_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{11}\ \mho$$

Also

$$I_1 = \frac{2}{3}(-I_2) = -\frac{2}{3}\times\frac{3}{11}V_2 = -\frac{2}{11}V_2$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{2}{11}\ \mho$$

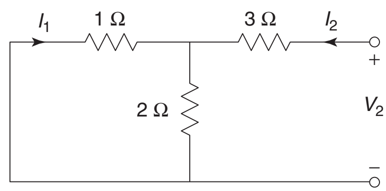

Fig. 9.20

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

Second Method (Refer Fig. 9.18)

$$I_1 = \frac{V_1 - V_3}{1}$$
$$= V_1 - V_3 \qquad \ldots\text{(i)}$$

$$I_2 = \frac{V_2 - V_3}{3}$$

$$= \frac{V_2}{3} - \frac{V_3}{3} \qquad \text{...(ii)}$$

Applying KCL at Node 3,

$$I_1 + I_2 = \frac{V_3}{2} \qquad \text{...(iii)}$$

Substituting Eqs (i) and (ii) in Eq. (iii),

$$V_1 - V_3 + \frac{V_2}{3} - \frac{V_3}{3} = \frac{V_3}{2}$$

$$V_1 + \frac{V_2}{3} = \frac{11}{6} V_3$$

$$V_3 = \frac{6}{11} V_1 + \frac{2}{11} V_2 \qquad \text{...(iv)}$$

Substituting Eq. (iv) in Eq. (i),

$$I_1 = V_1 - \frac{6}{11} V_1 - \frac{2}{11} V_2$$

$$= \frac{5}{11} V_1 - \frac{2}{11} V_2 \qquad \text{...(v)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_2 = \frac{V_2}{3} - \frac{1}{3}\left(\frac{6}{11} V_1 + \frac{2}{11} V_2\right)$$

$$= -\frac{2}{11} V_1 + \frac{3}{11} V_2 \qquad \text{...(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

Example 9.9 *Find Y-parameters of the network shown in Fig. 9.21.*

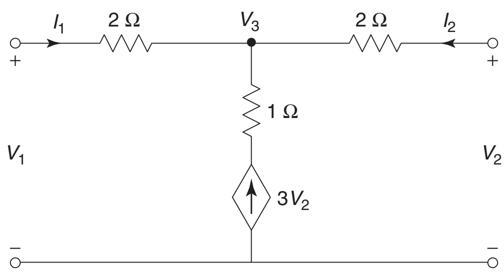

Fig. 9.21

Solution From Fig. 9.21,

$$I_1 = \frac{V_1 - V_3}{2}$$

$$= \frac{1}{2} V_1 - \frac{1}{2} V_3 \qquad \text{...(i)}$$

$$I_2 = \frac{V_2 - V_3}{2}$$

$$= \frac{1}{2}V_2 - \frac{1}{2}V_3 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$I_1 + I_2 + 3V_2 = 0 \qquad \ldots\text{(iii)}$$

Substituting Eqs (i) and (ii) in Eq. (iii),

$$\frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{2} + 3V_2 = 0$$

$$2V_3 = V_1 + 7V_2$$

$$V_3 = \frac{1}{2}V_1 + \frac{7}{2}V_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iv) in Eq. (i),

$$I_1 = \frac{1}{2}V_1 - \frac{1}{2}\left(\frac{1}{2}V_1 + \frac{7}{2}V_2\right)$$

$$= \frac{1}{4}V_1 - \frac{7}{4}V_2 \qquad \ldots\text{(v)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_2 = \frac{1}{2}V_2 - \frac{1}{2}\left(\frac{1}{2}V_1 + \frac{7}{2}V_2\right)$$

$$= -\frac{1}{4}V_1 - \frac{5}{4}V_2 \qquad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{7}{4} \\ -\frac{1}{4} & -\frac{5}{4} \end{bmatrix}$$

Example 9.10 *Determine Y-parameters for the network shown in Fig. 9.22. Determine whether the network is symmetrical and reciprocal.*

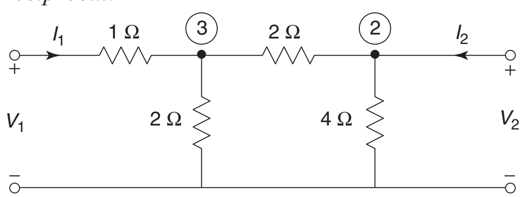

Fig. 9.22

Solution From Fig. 9.22,

$$I_1 = \frac{V_1 - V_3}{1}$$

$$= V_1 - V_3 \qquad \ldots\text{(i)}$$

Applying KCL at Node 3,

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_2}{2}$$

$$= V_3 - \frac{V_2}{2} \qquad \ldots\text{(ii)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{4} + \frac{V_2 - V_3}{2}$$

$$= \frac{3}{4}V_2 - \frac{V_3}{2} \quad \ldots\text{(iii)}$$

Substituting Eq. (i) in Eq. (ii),

$$V_1 - V_3 = V_3 - \frac{V_2}{2}$$

$$V_3 = \frac{V_1}{2} + \frac{V_2}{4} \quad \ldots\text{(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1 = \frac{V_1}{2} + \frac{V_2}{4} - \frac{V_2}{2}$$

$$= \frac{V_1}{2} - \frac{V_2}{4} \quad \ldots\text{(v)}$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2 = \frac{3}{4}V_2 - \frac{1}{2}\left(\frac{V_1}{2} + \frac{V_2}{4}\right)$$

$$= -\frac{V_1}{4} + \frac{5V_2}{8} \quad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{8} \end{bmatrix}$$

Since $Y_{11} \neq Y_{22}$, the network is not symmetrical.
Since $Y_{12} = Y_{21}$, the network is reciprocal.

Example 9.11 *Determine the short-circuit admittance parameters for the network shown in Fig. 9.23.*

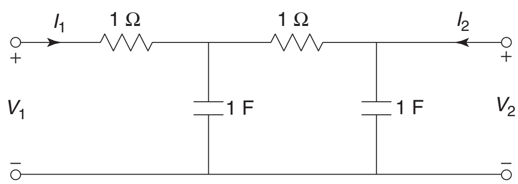

Fig. 9.23

Solution The transformed network is shown in Fig. 9.24.

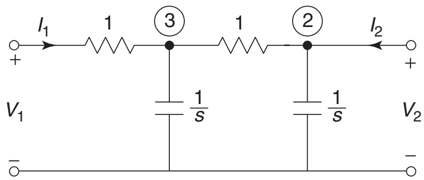

Fig. 9.24

From Fig. 9.24,

$$I_1 = \frac{V_1 - V_3}{1}$$

$$= V_1 - V_3 \quad \ldots\text{(i)}$$

Applying KCL at Node 3,

$$I_1 = \frac{V_3}{\frac{1}{s}} + \frac{(V_3 - V_2)}{1}$$

$$= (s+1)V_3 - V_2 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{\frac{1}{s}} + \frac{(V_2 - V_3)}{1}$$

$$= (s+1)V_2 - V_3 \qquad \text{...(iii)}$$

Substituting Eq. (i) in Eq. (ii),

$$V_1 - V_3 = (s+1)V_3 - V_2$$

$$(s+2)V_3 = V_1 + V_2$$

$$V_3 = \frac{1}{s+2}V_1 + \frac{1}{s+2}V_2 \qquad \text{...(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1 = (s+1)\left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right) - V_2$$

$$= \frac{s+1}{s+2}V_1 - \frac{1}{s+2}V_2 \qquad \text{...(v)}$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2 = (s+1)V_2 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= -\frac{1}{s+2}V_1 + \frac{s^2+3s+1}{s+2}V_2 \qquad \text{...(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{s+1}{s+2} & -\frac{1}{s+2} \\ -\frac{1}{s+2} & \frac{s^2+3s+1}{s+2} \end{bmatrix}$$

Example 9.12 *Determine Y-parameters for the network shown in Fig. 9.25.*

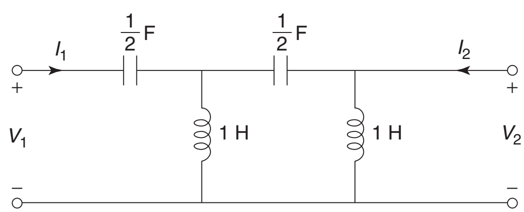

Fig. 9.25

Solution The transformed network is shown in Fig. 9.26.

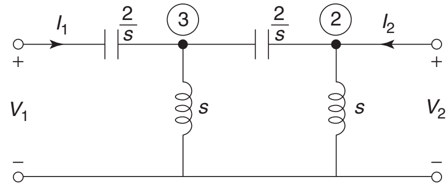

Fig. 9.26

From Fig. 9.26,

$$I_1 = \frac{V_1 - V_3}{\frac{2}{s}}$$

$$= \frac{s}{2}V_1 - \frac{s}{2}V_3 \qquad \ldots\text{(i)}$$

Applying KCL at Node 3,

$$\frac{s}{2}(V_1 - V_3) = \frac{V_3}{s} + \frac{s}{2}(V_3 - V_2)$$

$$\frac{s}{2}V_3 + \frac{1}{s}V_3 + \frac{s}{2}V_3 = \frac{s}{2}V_1 + \frac{s}{2}V_2$$

$$V_3 = \frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2 \qquad \ldots\text{(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$I_1 = \frac{s}{2}V_1 - \frac{s}{2}\left[\frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2\right]$$

$$= \left[\frac{s}{2} - \frac{s^3}{4(s^2+1)}\right]V_1 - \frac{s^3}{4(s^2+1)}V_2$$

$$= \frac{s^3+2s}{4(s^2+1)}V_1 - \frac{s^3}{4(s^2+1)}V_2 \qquad \ldots\text{(iii)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{s} + \frac{s}{2}(V_2 - V_3)$$

$$= \frac{s^2+2}{2s}V_2 - \frac{s}{2}V_3 \qquad \ldots\text{(iv)}$$

Substituting Eq. (ii) in Eq. (iv),

$$I_2 = \frac{s^2+2}{2s}V_2 - \frac{s}{2}\left[\frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2\right]$$

$$= -\frac{s^3}{4(s^2+1)}V_1 + \left[\frac{s^2+2}{2s} - \frac{s^3}{4(s^2+1)}\right]V_2$$

$$= -\frac{s^3}{4(s^2+1)}V_1 + \frac{s^4+6s^2+4}{4s(s^2+1)}V_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iii) and (v) with Y-parameter equation,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^3+2s}{4(s^2+1)} & -\frac{s^3}{4(s^2+1)} \\ -\frac{s^3}{4(s^2+1)} & \frac{s^4+6s^2+4}{4s(s^2+1)} \end{bmatrix}$$

Example 9.13 *Obtain Y-parameters of the network shown in Fig. 9.27.*

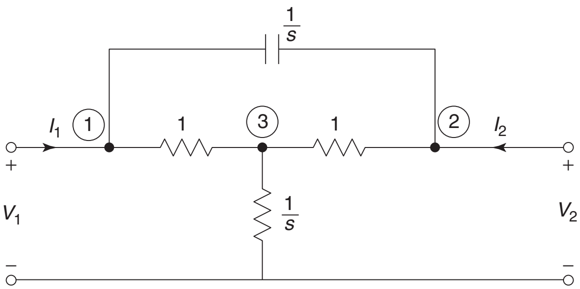

Fig. 9.27

Solution

Applying KCL at Node 1,

$$I_1 = \frac{V_1 - V_3}{1} + \frac{V_1 - V_2}{\frac{1}{s}}$$

$$= (s+1)\,V_1 - s\,V_2 - V_3 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2 - V_3}{1} + \frac{V_2 - V_1}{\frac{1}{s}}$$

$$= (s+1)\,V_2 - s\,V_1 - V_3 \qquad \text{...(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3}{\frac{1}{s}} + \frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{1} = 0$$

$$(s+2)\,V_3 - V_1 - V_2 = 0$$

$$V_3 = \frac{1}{s+2}V_1 + \frac{1}{s+2}V_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$I_1 = (s+1)\,V_1 - sV_2 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= \left[\frac{(s+1)(s+2)-1}{(s+2)}\right]V_1 - \left[\frac{s(s+2)+1}{(s+2)}\right]V_2$$

$$= \left(\frac{s^2+3s+1}{s+2}\right)V_1 - \left(\frac{s^2+2s+1}{s+2}\right)V_2 \qquad \text{...(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$I_2 = (s+1)\,V_2 - s\,V_1 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= -\left[\frac{s(s+2)+1}{(s+2)}\right]V_1 + \left[\frac{(s+1)(s+2)-1}{(s+2)}\right]V_2$$

$$= -\left(\frac{s^2+2s+1}{s+2}\right)V_1 + \left(\frac{s^2+3s+1}{s+2}\right)V_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{s+2} & -\dfrac{(s^2+2s+1)}{s+2} \\ -\dfrac{(s^2+2s+1)}{s+2} & \dfrac{s^2+3s+1}{s+2} \end{bmatrix}$$

9.4 TRANSMISSION PARAMETERS (ABCD PARAMETERS)

The transmission parameters or chain parameters or $ABCD$ parameters serve to relate the voltage and current at the input port to voltage and current at the output port. In equation form,

$$(V_1, I_1) = f(V_2, -I_2)$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Here, the negative sign is used with I_2 and not for parameters B and D. The reason the current I_2 carries a negative sign is that in transmission field, the output current is assumed to be coming out of the output port instead of going into the port.

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission matrix.

For a given network, these parameters are determined as follows:

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

where A is the reverse voltage gain with the output port open-circuited.

Similarly,
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

where C is the transfer admittance with the output port open-circuited.

Case 2 When output port is short-circuited, i.e., $V_2 = 0$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

where B is the transfer impedance with the output port short-circuited.

Similarly,
$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

where D is the reverse current gain with the output port short-circuited.

9.4.1 Condition for Reciprocity

(a) As shown in Fig. 9.28, voltage V_s is applied at the input port with the output port short-circuited.

i.e.,

$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2' = -I_2$$

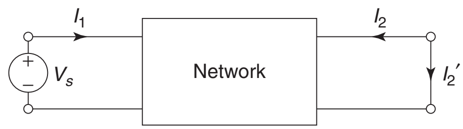

Fig. 9.28 *Network for deriving condition for reciprocity*

From the transmission parameter equations,

$$V_s = B\, I_2'$$
$$\frac{V_s}{I_2'} = B$$

(b) As shown in Fig. 9.29, voltage V_s is applied at the output port with the input port short-circuited.

i.e.,

$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1' = -I_1$$

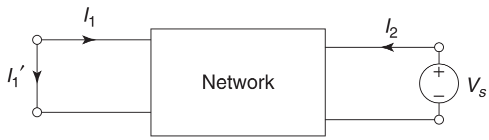

Fig. 9.29 *Network for deriving condition for reciprocity*

From the transmission parameter equations,

$$0 = AV_s - BI_2$$
$$-I_1' = CV_s - DI_2$$
$$I_2 = \frac{A}{B} V_s$$
$$-I_1' = CV_s - \frac{AD}{B} V_s$$
$$\frac{V_s}{I_1'} = \frac{B}{AD - BC}$$

Hence, for the network to be reciprocal,

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

i.e.,

$$B = \frac{B}{AD - BC}$$

i.e.,

$$AD - BC = 1$$

9.4.2 Condition for Symmetry

(a) When the output port is open-circuited, i.e., $I_2 = 0$.

From the transmission-parameter equations,

$$V_s = AV_2$$
$$I_1 = CV_2$$
$$\frac{V_s}{I_1} = \frac{A}{C}$$

(b) When the input port is open-circuited, i.e., $I_1 = 0$.

From the transmission parameter equation,

$$CV_s = DI_2$$
$$\frac{V_s}{I_2} = \frac{D}{C}$$

Hence, for network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

i.e.,

$$A = D$$

Example 9.14 *Find the transmission parameters for the network shown in Fig. 9.30.*

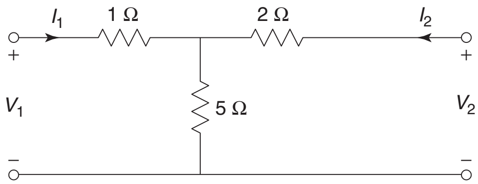

Fig. 9.30

Solution

First Method

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$.

$$V_1 = 6I_1$$

and

$$V_2 = 5I_1$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{6I_1}{5I_1} = \frac{6}{5}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{5}\ \mho$$

Case 2 When the output port is short-circuited, i.e., $V_2 = 0$, as shown in Fig. 9.31,

$$R_{\text{eq}} = 1 + \frac{5\times 2}{5+2} = 1 + \frac{10}{7} = \frac{17}{7}\ \Omega$$

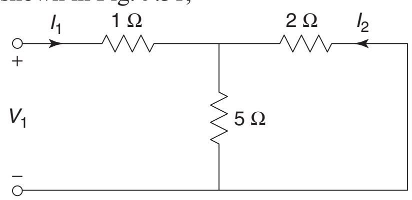

Fig. 9.31

Now

$$V_1 = \frac{17}{7} I_1$$

and

$$I_2 = \frac{5}{7}(-I_1) = -\frac{5}{7} I_1$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = -\frac{\frac{17}{7} I_1}{-\frac{5}{7} I_1} = \frac{17}{5}\ \Omega$$

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{7}{5}$$

Hence, the transmission parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

Second Method (Refer Fig. 9.37)
Applying KVL to Mesh 1,

$$V_1 = 6I_1 + 5I_2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 5I_1 + 7I_2 \qquad \text{...(ii)}$$

Hence,

$$5I_1 = V_2 - 7I_2$$

$$I_1 = \frac{1}{5}V_2 - \frac{7}{5}I_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 6\left(\frac{1}{5}V_2 - \frac{7}{5}I_2\right) + 5I_2$$

$$= \frac{6}{5}V_2 - \frac{17}{5}I_2 \qquad \text{...(iv)}$$

Comparing Eqs (iii) and (iv) with transmission parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

Example 9.15 *Obtain ABCD parameters for the network shown in Fig. 9.32.*

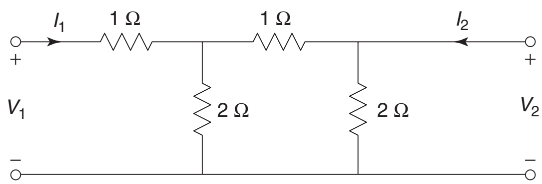

Fig. 9.32

Solution The network is redrawn as shown in Fig. 9.33.

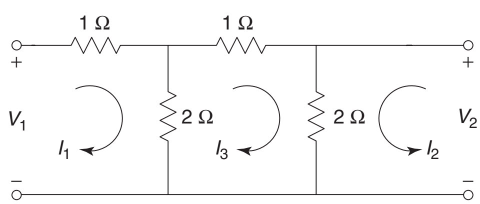

Fig. 9.33

Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - I_3 - 2(I_3 + I_2) = 0$$

$$5I_3 = 2I_1 - 2I_2$$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= \frac{11}{5}I_1 + \frac{4}{5}I_2 \qquad \text{...(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 2I_2 + 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= \frac{4}{5}I_1 + \frac{6}{5}I_2$$

$$\frac{4}{5}I_1 = V_2 - \frac{6}{5}I_2$$

$$I_1 = \frac{5}{4}V_2 - \frac{3}{2}I_2 \qquad \ldots\text{(v)}$$

Substituting Eq. (v) in Eq. (iv),

$$V_1 = \frac{11}{5}\left(\frac{5}{4}V_2 - \frac{3}{2}I_2\right) + \frac{4}{5}I_2$$

$$= \frac{11}{4}V_2 - \frac{5}{2}I_2 \qquad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with *ABCD* parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{11}{4} & \frac{5}{2} \\ \frac{5}{4} & \frac{3}{2} \end{bmatrix}$$

Example 9.16 *Determine the transmission parameters for the network shown in Fig. 9.34.*

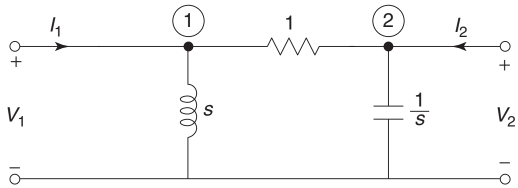

Fig. 9.34

Solution

Applying KCL at Node 1,

$$I_1 = \frac{V_1}{s} + (V_1 - V_2)$$

$$= \frac{s+1}{s}V_1 - V_2 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{\frac{1}{s}} + (V_2 - V_1)$$

$$= (s+1)\,V_2 - V_1$$

$$V_1 = (s+1)\,V_2 - I_2 \qquad \ldots\text{(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$I_1 = \frac{s+1}{s}[(s+1)V_2 - I_2] - V_2$$
$$= \left[\frac{(s+1)^2}{s} - 1\right]V_2 - \frac{s+1}{s}I_2$$
$$= \left(\frac{s^2+s+1}{s}\right)V_2 - \left(\frac{s+1}{s}\right)I_2 \quad \ldots\text{(iii)}$$

Comparing Eqs (ii) and (iii) with *ABCD* parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} s+1 & -1 \\ \dfrac{s^2+s+1}{s} & \dfrac{s+1}{s} \end{bmatrix}$$

Example 9.17 *Find transmission parameters for the two-port network shown in Fig. 9.35.*

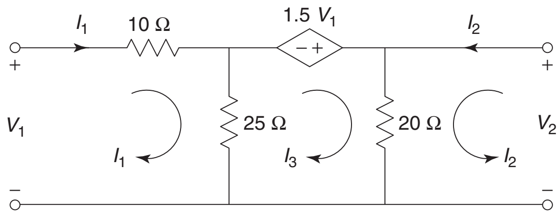

Fig. 9.35

Solution Applying KVL to Mesh 1,

$$V_1 = 10I_1 + 25(I_1 - I_3)$$
$$= 35I_1 - 25I_3 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 20(I_2 + I_3)$$
$$= 20I_2 + 20I_3 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-25(I_3 - I_1) + 1.5V_1 - 20(I_2 + I_3) = 0$$
$$-25I_3 + 25I_1 + 1.5(35I_1 - 25I_3) - 20I_2 - 20I_3 = 0$$
$$-25I_3 + 25I_1 + 52.5I_1 - 37.5I_3 - 20I_2 - 20I_3 = 0$$
$$82.5I_3 = 77.5I_1 - 20I_2$$
$$I_3 = 0.94I_1 - 0.24I_2 \quad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 35I_1 - 25(0.94I_1 - 0.24I_2)$$
$$= 11.5I_1 + 6I_2 \quad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 20I_2 + 20(0.94I_1 - 0.24I_2)$$
$$= 18.8I_1 + 15.2I_2 \quad \ldots\text{(v)}$$

From Eq. (v),

$$I_1 = 0.053\,V_2 - 0.81 I_2 \qquad \ldots\text{(vi)}$$

Substituting Eq. (vi) in Eq. (iv),

$$\begin{aligned} V_1 &= 11.5(0.053\,V_2 - 0.81 I_2) + 6 I_2 \\ &= 0.61\,V_2 - 3.32 I_2 \qquad \ldots\text{(vii)} \end{aligned}$$

Comparing Eqs (vi) and (vii) with *ABCD* parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.61 & -3.32 \\ 0.053 & -0.81 \end{bmatrix}$$

9.5 HYBRID PARAMETERS (*h* PARAMETERS)

The hybrid parameters of a two-port network may be defined by expressing the voltage of input port V_1 and current of output port I_2 in terms of current of input port I_1 and voltage of output port V_2.

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11}\, I_1 + h_{12}\, V_2$$

$$I_2 = h_{21}\, I_1 + h_{22}\, V_2$$

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual *h* parameters can be defined by setting $I_1 = 0$ and $V_2 = 0$.

Case 1 When the output port is short-circuited i.e., $V_2 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$$

where h_{11} is the short-circuit input impedance.

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$

where h_{21} is the short-circuit forward current gain.

Case 2 When the input port is open-circuited, i.e., $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$

where h_{12} is the open-circuit reverse voltage gain.

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$$

where h_{22} is the open-circuit output admittance.

Since *h* parameters represent dimensionally an impedance, an admittance, a voltage gain and a current gain, these are called *hybrid parameters*.

The equivalent circuit of a two-port network in terms of hybrid parameters is shown in Fig. 9.36.

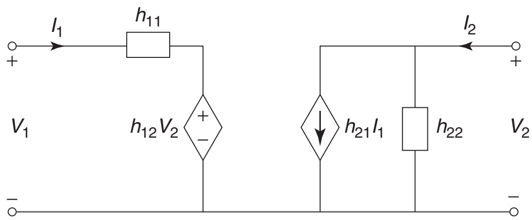

Fig. 9.36 *Equivalent circuit of the two-port network in terms of h-parameters*

9.5.1 Condition for Reciprocity

(a) As shown in Fig. 9.37, voltage V_s is applied at the input port and the output port is short-circuited.

i.e.,
$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2' = -I_2$$

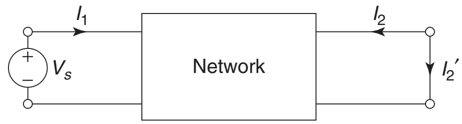

Fig. 9.37 *Network for deriving condition for reciprocity*

From the h-parameter equations,

$$V_s = h_{11}\, I_1$$
$$-I_2' = h_{21}\, I_1$$
$$\frac{V_s}{I_2'} = -\frac{h_{11}}{h_{21}}$$

(b) As shown in Fig. 9.38, voltage V_s is applied at the output port with the input port short-circuited.

i.e.,
$$V_1 = 0$$
$$V_2 = V_s$$
$$I_1 = -I_1'$$

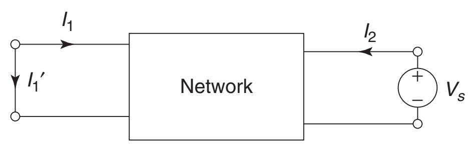

Fig. 9.38 *Network for deriving condition for reciprocity*

From the h-parameter equations,

$$0 = h_{11}I_1 + h_{12}\, V_s$$
$$h_{12}\, V_s = -h_{11}\, I_1 = h_{11}\, I_1'$$
$$\frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}}$$

Hence, for the network to be reciprocal,

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

i.e.,
$$h_{21} = -h_{12}$$

9.5.2 Condition for Symmetry

The condition for symmetry is obtained from the Z-parameters.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{h_{11}I_1 + h_{12}V_2}{I_1}\right|_{I_2=0} = h_{11} + h_{12}\frac{V_2}{I_1}$$

But with $I_2 = 0$,

$$0 = h_{21}\, I_1 + h_{22}\, V_2$$

$$\frac{V_2}{I_1} = -\frac{h_{21}}{h_{22}}$$

$$Z_{11} = h_{11} - \frac{h_{12}h_{21}}{h_{22}} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

where $\Delta h = h_{11}h_{22} - h_{12}h_{21}$

Similarly,

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

With $I_1 = 0$,

$$I_2 = h_{22}\, V_2$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$$

For a symmetrical network, $Z_{11} = Z_{22}$

i.e., $$\frac{\Delta h}{h_{22}} = \frac{1}{h_{22}}$$

i.e., $$\Delta h = 1$$

i.e., $$h_{11}\, h_{22} - h_{12}\, h_{21} = 1$$

Table 9.2 *Conditions for reciprocity and symmetry*

Parameter	Condition for Reciprocity	Condition for Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
T	$AD - BC = 1$	$A = D$
h	$h_{12} = -h_{21}$	$h_{11}\, h_{22} - h_{12}\, h_{21} = 1$

Example 9.18 *In the two-port network shown in Fig. 9.39, compute h-parameters from the following data:*

(a) With the output port short-circuited: $V_1 = 25\ V,\ I_1 = 1\ A,\ I_2 = 2\ A$

(b) With the input port open-circuited: $V_1 = 10\ V,\ V_2 = 50\ V,\ I_2 = 2\ A$

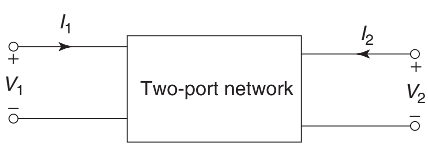

Fig. 9.39

Solution

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{25}{1} = 25\ \Omega, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{10}{50} = 0.2$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{2}{1} = 2, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{2}{50} = 0.04\ \mho$$

Hence, the h-parameters are

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 25 & 0.2 \\ 2 & 0.04 \end{bmatrix}$$

Example 9.19 *Determine hybrid parameters for the network of Fig. 9.40. Determine whether the network is reciprocal.*

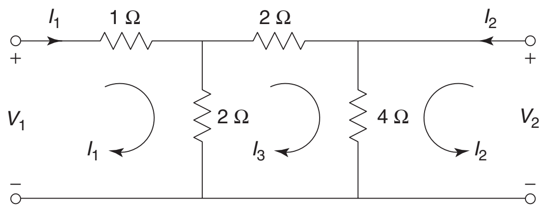

Fig. 9.40

Solution

First Method

Case 1 When Port 2 is short-circuited, i.e., $V_2 = 0$ as shown in Fig. 9.41,

$$R_{eq} = 1 + \frac{2\times 2}{2+2} = 2\ \Omega$$

Now,

$$V_1 = 2I_1$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 2\ \Omega$$

Also,

$$I_2 = -I_1 \times \frac{2}{2+2} = -\frac{I_1}{2}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{1}{2}$$

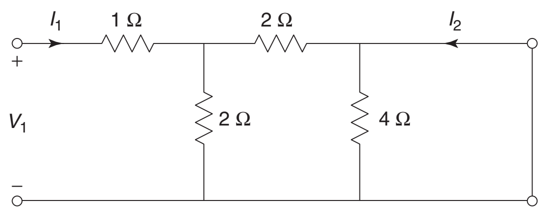

Fig. 9.41

Case 2 When Port 1 is open-circuited, i.e., $I_1 = 0$ as shown in Fig. 9.42,

$$R_{eq} = \frac{(2+2)\times 4}{2+2+4} = 2\ \Omega$$

$$V_1 = 2I_y$$

$$I_y = \frac{I_2}{2}$$

$$V_2 = 4I_x$$

$$I_x = \frac{I_2}{2}$$

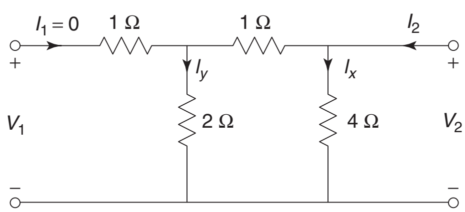

Fig. 9.42

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{2I_y}{4I_x} = \frac{2\times\frac{I_2}{2}}{4\times\frac{I_2}{2}} = \frac{1}{2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{2I_x}{4I_x} = \frac{1}{2}\ \mho$$

Hence, the h-parameters are

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Second Method (Refer Fig. 9.40)
Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 4I_2 + 4I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 2I_3 - 4(I_3 + I_2) = 0$$

$$8I_3 = 2I_1 - 4I_2$$

$$I_3 = \frac{I_1}{4} - \frac{I_2}{2} \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - 2\left(\frac{I_1}{4} - \frac{I_2}{2}\right)$$

$$= \frac{5}{2}I_1 + I_2 \qquad \text{...(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 4I_2 + 4\left(\frac{I_1}{4} - \frac{I_2}{2}\right)$$

$$= 4I_2 + I_1 - 2I_2$$

$$= I_1 + 2I_2$$

$$I_2 = -\frac{1}{2}I_1 + \frac{1}{2}V_2 \qquad \text{...(v)}$$

Substituting Eq. (v) in Eq. (iv),

$$V_1 = \frac{5}{2}I_1 - \frac{1}{2}I_1 + \frac{1}{2}V_2$$

$$= 2I_1 + \frac{1}{2}V_2 \qquad \text{...(vi)}$$

Comparing Eqs (v) and (vi) with h-parameter equations,

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Since $h_{12} = -h_{21}$, the network is reciprocal.

Example 9.20 *Find h-parameters for the network shown in Fig. 9.43.*

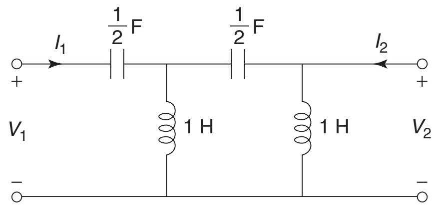

Fig. 9.43

Solution As solved in Example 9.12, derive the equations for I_1 and I_2 in terms of V_1 and V_2.

$$I_1 = \frac{s^3+2s}{4(s^2+1)}V_1 - \frac{s^3}{4(s^2+1)}V_2 \qquad \text{...(i)}$$

$$I_2 = -\frac{s^3}{4(s^2+1)}V_1 + \frac{s^4+6s^2+4}{4s(s^2+1)}V_2 \qquad \text{...(ii)}$$

From Eq. (i),

$$V_1 = \frac{4(s^2+1)}{s(s^2+2)}I_1 + \frac{s^2}{s^2+2}V_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq (ii),

$$I_2 = -\frac{s^3}{4(s^2+1)}\left[\frac{4(s^2+1)}{s(s^2+2)}I_1 + \frac{s^2}{s^2+2}V_2\right] + \frac{s^4+6s^2+4}{4s(s^2+1)}V_2$$

$$= -\frac{s^2}{s^2+2}I_1 + \frac{2(s^2+1)}{s(s^2+2)}V_2 \qquad \text{...(iv)}$$

Comparing Eqs (iii) and (iv) with h-parameter equations,

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{4(s^2+1)}{s(s^2+2)} & \dfrac{s^2}{s^2+2} \\ -\dfrac{s^2}{s^2+2} & \dfrac{2(s^2+1)}{s(s^2+2)} \end{bmatrix}$$

9.6 INTER-RELATIONSHIPS BETWEEN THE PARAMETERS

When it is required to find out two or more parameters of a particular network then finding each parameter will be tedious. But if we find a particular parameter then the other parameters can be found if the inter-relationship between them is known.

9.6.1 Z-parameters in Terms of Other Parameters

1. Z-parameters in Terms of Y-parameters We know that

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

By Cramer's rule,

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}I_1 - Y_{12}I_2}{Y_{11}Y_{22} - Y_{12}Y_{21}} = \frac{Y_{22}}{\Delta Y}I_1 - \frac{Y_{12}}{\Delta Y}I_2$$

where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

Comparing with $V_1 = Z_{11}I_1 + Z_{12}, I_2,$

$$Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

Also, $$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta Y} = \frac{Y_{11}}{\Delta Y}I_2 - \frac{Y_{21}}{\Delta Y}I_1$$

Comparing with $V_2 = Z_{21}\,I_1 + Z_{22}\,I_2,$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y}$$

2. Z-parameter in Terms of ABCD Parameters We know that

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Rewriting the second equation,

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$$

Comparing with $V_2 = Z_{21}\,I_1 + Z_{22}\,I_2,$

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

Also, $$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2 = \frac{A}{C}I_1 + \left[\frac{AD}{C} - B\right]I_2 = \frac{A}{C}I_1 + \left[\frac{AD - BC}{C}\right]I_2$$

Comparing with $V_1 = Z_{11}\,I_1 + Z_{12}\,I_2,$

$$Z_{11} = \frac{A}{C}$$

$$Z_{12} = \frac{AD - BC}{C}$$

3. Z-parameters in Terms of Hybrid Parameters We know that

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the second equation,

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

Comparing with

$$V_2 = Z_{21}\, I_1 + Z_{22}\, I_2,$$

$$Z_{21} = -\frac{h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Also, $V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right] = h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12}h_{21}}{h_{22}} I_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$

Comparing with $V_1 = Z_{11}\, I_1 + Z_{12}\, I_2,$

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

9.6.2 *Y*-parameters in Terms of Other Parameters

1. Y-parameters in terms of Z-parameters We know that

$$V_1 = Z_{11}\, I_1 + Z_{12}\, I_2$$
$$V_2 = Z_{21}\, I_1 + Z_{22}\, I_2$$

By Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{22}\, V_1 - Z_{12}\, V_2}{Z_{11}\, Z_{22} - Z_{12}\, Z_{21}} = \frac{Z_{22}}{\Delta Z} V_1 - \frac{Z_{12}}{\Delta Z} V_2$$

where $\Delta Z = Z_{11}\; Z_{22} - Z_{12}\; Z_{21}$

Comparing with $I_1 = Y_{11}\, V_1 + Y_{12}\, V_2,$

$$Y_{11} = \frac{Z_{22}}{\Delta Z}$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

Also,

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta Z} = \frac{Z_{11}V_2 - Z_{12}V_1}{\Delta Z} = -\frac{Z_{21}}{\Delta Z} V_1 + \frac{Z_{11}}{\Delta Z} V_2$$

Comparing with $I_2 = Y_{21}\, V_1 + Y_{22}\, V_2,$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z}$$

2. Y-parameters in Terms of ABCD Parameters We know that

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Rewriting the first equation,

$$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$,

$$Y_{21} = -\frac{1}{B}$$

$$Y_{22} = \frac{A}{B}$$

Also, $$I_1 = CV_2 - D\left[-\frac{1}{B}V_1 + \frac{A}{B}V_2\right] = \frac{D}{B}V_1 + \left[\frac{BC - AD}{B}\right]V_2$$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$,

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B} = -\frac{AD - BC}{B} = -\frac{\Delta T}{B}$$

3. Y-parameters in Terms of Hybrid Parameters We know that

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Rewriting the first equation,

$$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2$$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$,

$$Y_{11} = \frac{1}{h_{11}}$$

$$Y_{12} = -\frac{h_{12}}{h_{11}}$$

Also $$I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2 = \frac{h_{21}}{h_{11}}V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right]V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$,

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

$$Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} = \frac{\Delta h}{h_{11}}$$

9.6.3 *ABCD* Parameters in Terms of Other Parameters

1. ABCD Parameters in Terms of Z-parameters We know that

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Rewriting the second equation,

$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2$$

Comparing with $I_1 = CV_2 - DI_2$,

$$C = \frac{1}{Z_{21}}$$
$$D = \frac{Z_{22}}{Z_{21}}$$

Also,

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{22} Z_{11}}{Z_{21}} I_2 + Z_{12} I_2$$
$$= \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}\right] I_2$$

Comparing with $V_1 = AV_2 - BI_2$,

$$A = \frac{Z_{11}}{Z_{21}}$$
$$B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} = \frac{\Delta Z}{Z_{21}}$$

2. ABCD Parameters in terms of Y-parameters We know that

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Rewriting the second equation,

$$V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2$$

Comparing with $V_1 = AV_2 - BI_2$,

$$A = -\frac{Y_{22}}{Y_{21}}$$
$$B = -\frac{1}{Y_{21}}$$

Also,

$$I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2\right] + Y_{12} V_2 = \left[\frac{Y_{12} Y_{21} - Y_{11} Y_{22}}{Y_{21}}\right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

Comparing with $I_1 = CV_2 - DI_2$,

$$C = \frac{Y_{12} Y_{21} - Y_{11} Y_{22}}{Y_{21}} = -\frac{\Delta Y}{Y_{21}}$$
$$D = -\frac{Y_{11}}{Y_{21}}$$

3. *ABCD Parameters in Terms of Hybrid Parameters* We know that

$$V_1 = h_{11}\, I_1 + h_{12}\, V_2$$
$$I_2 = h_{21}\, I_1 + h_{22}\, V_2$$

Rewriting the second equation,

$$I_1 = -\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2$$

Comparing with $I_1 = CV_2 - DI_2$,

$$C = -\frac{h_{22}}{h_{21}}$$

$$D = -\frac{1}{h_{21}}$$

Also,

$$V_1 = h_{11}\left[\frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2\right] + h_{12}\, V_2 = \left[\frac{h_{12}\, h_{21} - h_{11}\, h_{22}}{h_{21}}\right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

Comparing with $V_1 = AV_2 - BI_2$,

$$A = \frac{h_{12}\, h_{21} - h_{11}\, h_{22}}{h_{21}} = -\frac{\Delta h}{h_{21}}$$

$$B = -\frac{h_{11}}{h_{21}}$$

9.6.4 Hybrid Parameters in Terms of Other Parameters

1. *Hybrid Parameters in terms of Z-parameters* We know that

$$V_1 = Z_{11}\, I_1 + Z_{12}\, I_2$$
$$V_2 = Z_{21}\, I_1 + Z_{22}\, I_2$$

Rewriting the second equation,

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2$$

Comparing with $I_2 = h_{21}\, I_1 + h_{22}\, V_2$,

$$h_{21} = -\frac{Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

Also,

$$V_1 = Z_{11}\, I_1 + Z_{12}\left[-\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right] = \left[\frac{Z_{11}\, Z_{22} - Z_{12}\, Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

Comparing with $V_1 = h_{11}\, I_1 + h_{12}\, V_2$,

$$h_{11} = \frac{Z_{11}\, Z_{22} - Z_{12}\, Z_{21}}{Z_{22}} = \frac{\Delta Z}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

2. *Hybrid Parameters in terms of Y-parameters* We know that

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Rewriting the first equation,

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2$$

Comparing with $V_1 = h_{11} I_1 + h_{12} V_2$,

$$h_{11} = \frac{1}{Y_{11}}$$

$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$

Also, $$I_2 = Y_{21}\left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} V_2 = \left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}}\right] V_2 + \frac{Y_{21}}{Y_{11}} I_1$$

Comparing with $I_2 = h_{21} I_1 + h_{22} V_2$,

$$h_{21} = \frac{Y_{22}}{Y_{11}}$$

$$h_{22} = \frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}} = \frac{\Delta Y}{Y_{11}}$$

3. *Hybrid Parameters in Terms of ABCD Parameters* We know that

$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$

Rewriting the second equation,

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2$$

Comparing with $I_2 = h_{21} I_1 + h_{22} V_2$,

$$h_{21} = -\frac{1}{D}$$

$$h_{22} = \frac{C}{D}$$

Also, $$V_1 = A V_2 - B\left[-\frac{1}{D} I_1 + \frac{C}{D} V_2\right] = \frac{B}{D} I_1 + \left[\frac{AD - BC}{D}\right] V_2$$

Comparing with $V_1 = h_{11} I_1 + h_{12} V_2$,

$$h_{11} = \frac{B}{D}$$

$$h_{12} = \frac{AD - BC}{D} = \frac{\Delta T}{D}$$

Table 9.3 *Inter-relationship between parameters*

$$\Delta X = X_{11}X_{22} - X_{12}X_{21}$$

	In terms of			
	[*Z*]	**[*Y*]**	**[*T*]**	**[*h*]**
[*Z*]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$
[*Y*]	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$
[*T*]	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$
[*h*]	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$

Example 9.21 *The Z parameters of a two-port network are $Z_{11} = 20\ \Omega$, $Z_{22} = 30\ \Omega$, $Z_{12} = Z_{21} = 10\ \Omega$. Find Y and ABCD parameters.*

Solution

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (20)(30) - (10)(10) = 500$$

***Y*-parameters**

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{30}{500} = \frac{3}{50}\ \mho, \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{10}{500} = -\frac{1}{50}\ \mho$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{10}{500} = -\frac{1}{50}\ \mho, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{20}{500} = \frac{2}{50}\ \mho$$

Hence, the *Y*-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{3}{50} & -\frac{1}{50} \\ -\frac{1}{50} & \frac{2}{50} \end{bmatrix}$$

***ABCD* parameters**

$$A = \frac{Z_{11}}{Z_{21}} = \frac{20}{10} = 2, \qquad B = \frac{\Delta Z}{Z_{21}} = \frac{500}{10} = 50$$

$$C = \frac{1}{Z_{21}} = \frac{1}{10} = 0.1, \qquad D = \frac{Z_{22}}{Z_{21}} = \frac{30}{10} = 3$$

Hence, the *ABCD* parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 50 \\ 0.1 & 3 \end{bmatrix}$$

Example 9.22 *Currents I_1 and I_2 entering at Port 1 and Port 2 respectively of a two-port network are given by the following equations:*

$$I_1 = 0.5V_1 - 0.2V_2$$
$$I_2 = -0.2V_1 + V_2$$

Find Y, Z and ABCD parameters for the network.

Solution

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0.5\ \mho, \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -0.2\ \mho$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -0.2\ \mho, \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 1\ \mho$$

Hence, the *Y*-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.2 \\ -0.2 & 1 \end{bmatrix}$$

***Z*-parameters**

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = (0.5)(1) - (-0.2)(-0.2) = 0.46$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{1}{0.46} = 2.174\ \Omega, \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} = -\frac{(-0.2)}{0.46} = 0.434\ \Omega$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{(-0.2)}{0.46} = 0.434\ \Omega, \qquad Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{0.5}{0.46} = 1.087\ \Omega$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2.174 & 0.434 \\ 0.434 & 1.087 \end{bmatrix}$$

***ABCD* parameters**

$$A = -\frac{Y_{22}}{Y_{21}} = -\frac{1}{-0.2} = 5, \qquad B = -\frac{1}{Y_{21}} = -\frac{1}{-0.2} = 5$$

$$C = -\frac{\Delta Y}{Y_{21}} = -\frac{0.46}{-0.2} = 2.3, \qquad D = -\frac{Y_{11}}{Y_{21}} = -\frac{0.5}{-0.2} = 2.5$$

Hence, the *ABCD* parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 2.3 & 2.5 \end{bmatrix}$$

Example 9.23 *Using the relation $Y = Z^{-1}$, show that $|Z| = \frac{1}{2}\left(\frac{Z_{22}}{Y_{11}} + \frac{Z_{11}}{Y_{22}}\right)$.*

Solution We know that

$$Y = Z^{-1}$$

i.e.,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$|Z| = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$\frac{1}{2}\left(\frac{Z_{22}}{Y_{11}} + \frac{Z_{11}}{Y_{22}}\right) = \frac{1}{2}\left(\frac{Z_{22}}{\frac{Z_{22}}{\Delta Z}} + \frac{Z_{11}}{\frac{Z_{11}}{\Delta Z}}\right) = \frac{1}{2}(\Delta Z + \Delta Z) = \frac{1}{2}(2\Delta Z) = \Delta Z = Z_{11} Z_{12} - Z_{12} Z_{21}$$

$$|Z| = \frac{1}{2}\left(\frac{Z_{22}}{Y_{11}} + \frac{Z_{11}}{Y_{22}}\right)$$

Example 9.24 *For the network shown in Fig. 9.44, find Z and Y-parameters.*

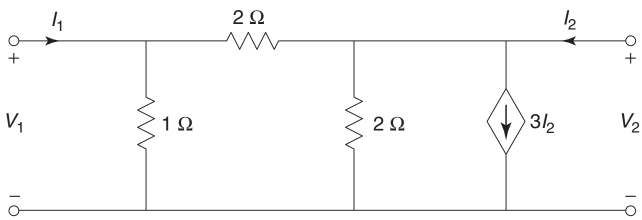

Fig. 9.44

Solution The network is redrawn by source transformation technique as shown in Fig. 9.45.

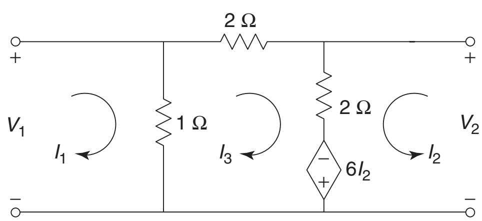

Fig. 9.45

Applying KVL to Mesh 1,

$$V_1 = I_1 - I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2(I_2 + I_3) - 6I_2$$
$$= -4I_2 + 2I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-(I_3 - I_1) - 2I_3 - 2(I_2 + I_3) + 6I_2 = 0$$
$$5I_3 = I_1 + 4I_2$$
$$I_3 = \frac{1}{5}I_1 + \frac{4}{5}I_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = I_1 - \frac{1}{5}I_1 - \frac{4}{5}I_2$$

$$= \frac{4}{5}I_1 - \frac{4}{5}I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = -4I_2 + 2\left(\frac{1}{5}I_1 + \frac{4}{5}I_2\right)$$

$$= \frac{2}{5}I_1 - \frac{12}{5}I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{4}{5} \\ \frac{2}{5} & -\frac{12}{5} \end{bmatrix}$$

***Y*-parameters**

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = \left(\frac{4}{5}\right)\left(-\frac{12}{5}\right) - \left(-\frac{4}{5}\right)\left(\frac{2}{5}\right) = -\frac{40}{25} = -\frac{8}{5}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{-\frac{12}{5}}{-\frac{8}{5}} = \frac{3}{2}\,\mho, \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = \frac{-\frac{4}{5}}{-\frac{8}{5}} = -\frac{1}{2}\,\mho$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = \frac{-\frac{2}{5}}{-\frac{8}{5}} = \frac{1}{4}\,\mho, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{\frac{4}{5}}{-\frac{8}{5}} = -\frac{1}{2}\,\mho$$

Hence, the *Y*-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

Example 9.25 *Find Z and h-parameters for the network shown in Fig. 9.46.*

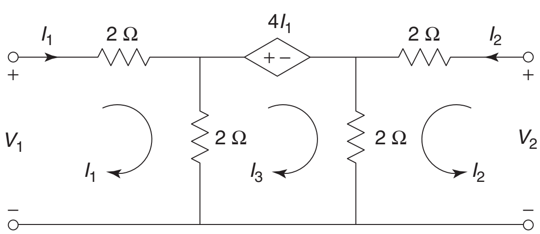

Fig. 9.46

Solution Applying KVL to Mesh 1,

$$V_1 = 2I_1 + 2(I_1 - I_3)$$

$$= 4I_1 - 2I_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2(I_2 + I_3)$$

$$= 4I_2 + 2I_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 4I_1 - 2(I_3 + I_2) = 0$$

$$I_1 + I_2 = -2I_3 \qquad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 4I_1 + I_1 + I_2$$

$$= 5I_1 + I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 4I_2 - I_1 - I_2$$

$$= -I_1 + 3I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

***h*-parameters**

$$\Delta Z = Z_{11}\,Z_{22} - Z_{12}\,Z_{21} = (5)(3) - (1)(-1) = 15 + 1 = 16$$

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{16}{3}\,\Omega, \qquad h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{1}{3}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = \frac{1}{3}, \qquad h_{22} = \frac{1}{Z_{22}} = \frac{1}{3}\,\mho$$

Hence, the *h*-parameters are

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{22} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{16}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

Example 9.26 *Find Y and Z-parameters for the network shown in Fig. 9.47.*

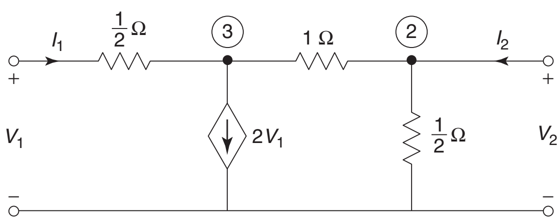

Fig. 9.47

Solution Applying KCL at Node 3,

$$2(V_1 - V_3) = 2V_1 + (V_3 - V_2)$$

$$V_3 = \frac{V_2}{3} \qquad \ldots\text{(i)}$$

Now,

$$I_1 = 2V_1 + (V_3 - V_2)$$

$$= 2V_1 + \frac{V_2}{3} - V_2$$

$$= 2V_1 - \frac{2}{3}V_2 \qquad \ldots\text{(ii)}$$

$$\begin{aligned} I_2 &= 2V_2 + (V_2 - V_3) \\ &= 3V_2 - \frac{V_2}{3} \\ &= \frac{8}{3}V_2 \end{aligned} \qquad \text{...(iii)}$$

Comparing Eqs (ii) and (iii) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 2 & -\frac{2}{3} \\ 0 & \frac{8}{3} \end{bmatrix}$$

***Z*-parameters**

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = (2)\left(\frac{8}{3}\right) - 0 = \frac{16}{3}$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{\frac{8}{3}}{\frac{16}{3}} = \frac{1}{2}\ \Omega, \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} = -\frac{\left(-\frac{2}{3}\right)}{\frac{16}{3}} = \frac{1}{8}\ \Omega$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{0}{\frac{16}{3}} = 0, \qquad Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{2}{\frac{16}{3}} = \frac{3}{8}\ \Omega$$

Hence, the Z-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{8} \\ 0 & \frac{3}{8} \end{bmatrix}$$

Example 9.27 *For the network shown in Fig. 9.48, find Y and Z-parameters.*

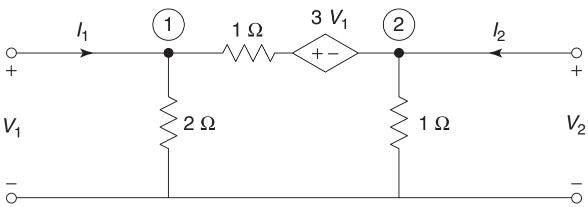

Fig. 9.48

Solution Applying KCL at Node 1,

$$\begin{aligned} I_1 &= \frac{V_1}{2} + \frac{V_1 - 3V_1 - V_2}{1} \\ &= -\frac{3}{2}V_1 - V_2 \end{aligned} \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\begin{aligned} I_2 &= \frac{V_2}{1} + \frac{V_2 + 3V_1 - V_1}{1} \\ &= 2V_1 + 2V_2 \end{aligned} \qquad \text{...(ii)}$$

Comparing Eqs (i) and (ii) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & -1 \\ 2 & 2 \end{bmatrix}$$

***Z*-parameters**

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = \left(-\frac{3}{2}\right)(2) - (-1)(2) = -3 + 2 = -1$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{2}{-1} = -2\ \Omega, \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} = -\frac{(-1)}{(-1)} = -1\ \Omega$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{2}{(-1)} = 2\ \Omega, \qquad Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{-\frac{3}{2}}{-1} = \frac{3}{2}\ \Omega$$

Hence, the Z-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 2 & \frac{3}{2} \end{bmatrix}$$

Example 9.28 *Find Z-parameters for the network shown in Fig. 9.49. Hence, find Y and h-parameters.*

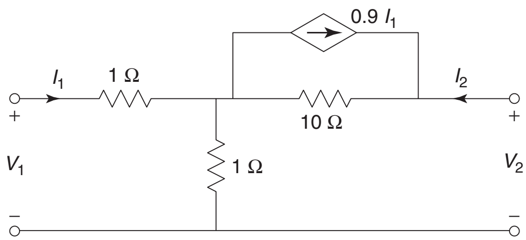

Fig. 9.49

Solution The network is redrawn by source transformation technique as shown in Fig. 9.50.

Applying KVL to Mesh 1,

$$V_1 = 2I_1 + I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$\begin{aligned} V_2 &= 9I_1 + 10I_2 + 1(I_1 + I_2) \\ &= 10I_1 + 11I_2 \qquad \ldots\text{(ii)} \end{aligned}$$

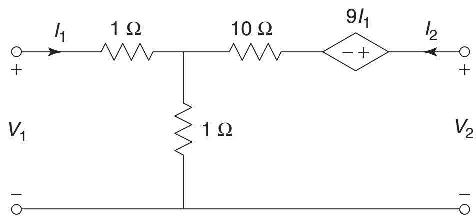

Fig. 9.50

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 10 & 11 \end{bmatrix}$$

***Y*-parameters**

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (2)(11) - (1)(10) = 22 - 10 = 12$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{11}{12}\ \mho, \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{1}{12}\ \mho$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{10}{12} = -\frac{5}{6}\ \mho, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{2}{12} = \frac{1}{6}\ \mho$$

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{12} & -\frac{1}{12} \\ -\frac{5}{6} & \frac{1}{6} \end{bmatrix}$$

***h*-parameters**

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{12}{11}\,\Omega, \qquad h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{1}{11}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = -\frac{10}{11}, \qquad h_{22} = \frac{1}{Z_{22}} = \frac{1}{11}\,\mho$$

Hence, h-parameters are

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{12}{11} & \frac{1}{11} \\ -\frac{10}{11} & \frac{1}{11} \end{bmatrix}$$

Example 9.29 *Find Y and Z-parameters of the network shown in Fig. 9.51.*

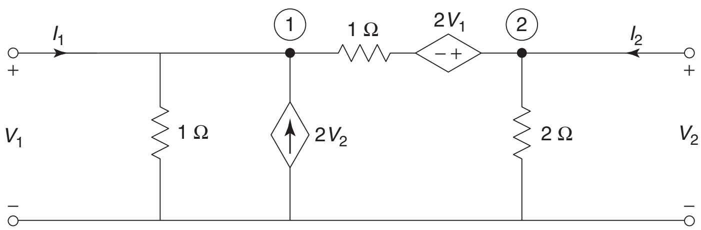

Fig. 9.51

Solution Applying KCL at Node 1,

$$I_1 + 2V_2 = \frac{V_1}{1} + \frac{3V_1 - V_2}{1}$$

$$I_1 = 4V_1 - 3V_2 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{2} + \frac{V_2 - 2V_1 - V_1}{1}$$

$$= -3V_1 + 1.5V_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -3 & 1.5 \end{bmatrix}$$

***Z*-parameters**

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = (4)(1.5) - (-3)(-3) = -3$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = -\frac{1.5}{3} = -0.5\,\Omega, \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} = -\frac{(-3)}{-3} = -1\,\Omega$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{(-3)}{-3} = -1\,\Omega, \qquad Z_{22} = \frac{Y_{11}}{\Delta Y} = -\frac{4}{3} = -\frac{4}{3}\,\Omega$$

Hence, the Z-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -0.5 & -1 \\ -1 & -\frac{4}{3} \end{bmatrix}$$

Example 9.30 *Determine Y and Z-parameters for the network shown in Fig. 9.52.*

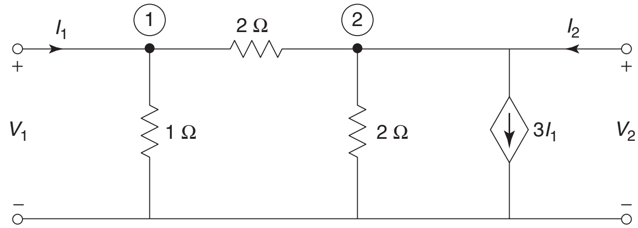

Fig. 9.52

Solution Applying KCL at Node 1,

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$$
$$= 1.5V_1 - 0.5V_2 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{2} + 3I_1 + \frac{V_2 - V_1}{2}$$
$$= \frac{V_2}{2} + 3(1.5V_1 - 0.5V_2) + \frac{V_2 - V_1}{2}$$
$$= 0.5V_2 + 4.5V_1 - 1.5V_2 + 0.5V_2 - 0.5V_1$$
$$= 4V_1 - 0.5V_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with the Y-parameter equation,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix}$$

***Z*-parameters**

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = (1.5)(-0.5) - (-0.5)(4) = 1.25$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = -\frac{0.5}{1.25} = -0.4\ \Omega, \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} = \frac{0.5}{1.25} = 0.4\ \Omega$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{4}{1.25} = -3.2\ \Omega, \qquad Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{1.5}{1.25} = 1.2\ \Omega$$

Hence, the Z-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$$

Example 9.31 *Determine the Y and Z-parameters for the network shown in Fig. 9.53.*

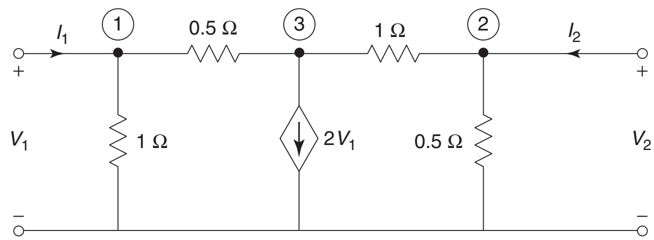

Fig. 9.53

Solution Applying KCL at Node 1,

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_3}{0.5}$$
$$= 3V_1 - 2V_3 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{0.5} + \frac{V_2 - V_3}{1}$$
$$= 3V_2 - V_3 \qquad \text{...(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3 - V_1}{0.5} + 2V_1 + \frac{V_3 - V_2}{1} = 0$$
$$V_3 = \frac{1}{3}V_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eqs (i) and (ii),

$$I_1 = 3V_1 - \frac{2}{3}V_2 \qquad \text{...(iv)}$$
$$I_2 = 0V_1 + \frac{8}{3}V_2 \qquad \text{...(v)}$$

Comparing Eqs (iv) and (v) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 3 & -\frac{2}{3} \\ 0 & \frac{8}{3} \end{bmatrix}$$

***Z*-parameters**

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = (3)\left(\frac{8}{3}\right) - 0 = 8$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{\frac{8}{3}}{8} = \frac{1}{3}\,\Omega, \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} = \frac{\frac{2}{3}}{8} = \frac{1}{12}\,\Omega$$

$$Z_{21} = -\frac{Y_{12}}{\Delta Y} = \frac{\frac{2}{3}}{8} = \frac{1}{12}\,\Omega, \qquad Z_{22} = \frac{Y_{11}}{Y\Delta} = \frac{3}{8}\,\Omega$$

Hence, the Z-parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{12} \\ 0 & \frac{3}{8} \end{bmatrix}$$

Example 9.32 *Determine Z and Y-parameters of the network shown in Fig. 9.54.*

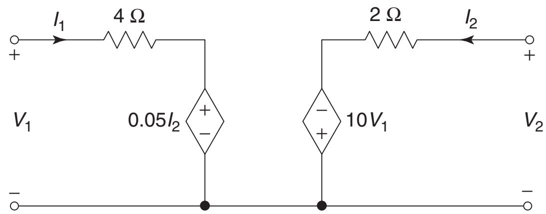

Fig. 9.54

Solution Applying KVL to Mesh 1,

$$V_1 - 4I_1 - 0.05\, I_2 = 0$$

$$V_1 = 4I_1 + 0.05I_2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 - 2I_2 + 10V_1 = 0$$

$$V_2 = 2I_2 - 10V_1 \qquad \text{...(ii)}$$

Substituting Eq. (i) in Eq. (ii),

$$V_2 = 2I_2 - 40I_1 - 0.5I_2$$

$$= -40I_1 + 1.5I_2 \qquad \text{...(iii)}$$

Comparing Eqs (i) and (iii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.05 \\ -40 & 1.5 \end{bmatrix}$$

***Y*-parameters**

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (4)(1.5) - (0.05)(-40) = 8$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{1.5}{8}\ \mho, \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{0.05}{8}\ \mho$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{(-40)}{8} = \frac{40}{8}\ \mho, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{4}{8}\ \mho$$

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1.5}{8} & -\frac{0.05}{8} \\ \frac{40}{8} & \frac{4}{8} \end{bmatrix}$$

Example 9.33 *Determine Z and Y-parameters of the network shown in Fig. 9.55.*

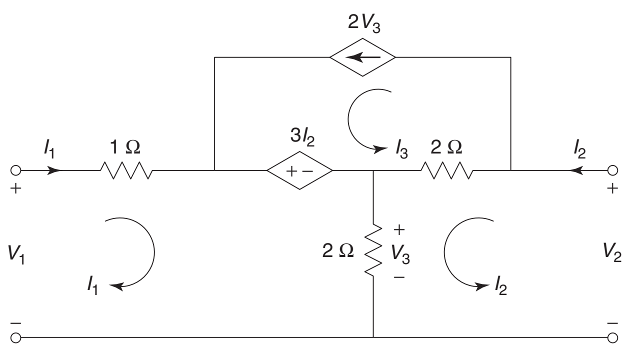

Fig. 9.55

Solution Applying KVL to Mesh 1,

$$V_1 - 1I_1 - 3I_2 - 2(I_1 + I_2) = 0$$

$$V_1 = 3I_1 + 5I_2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 - 2(I_2 - I_3) - 2(I_1 + I_2) = 0$$
$$V_2 - 2I_2 + 2I_3 - 2I_1 - 2I_2 = 0$$
$$V_2 = 2I_1 + 4I_2 - 2I_3 \qquad \ldots\text{(ii)}$$

Writing equation for Mesh 3,

$$I_3 = 2V_3 \qquad \ldots\text{(iii)}$$

From Fig. 9.55,

$$V_3 = 2(I_1 + I_2)$$
$$I_3 = 2V_3 = 4I_1 + 4I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$V_2 = -6I_1 - 4I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (i) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix}$$

***Y*-parameters**

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (3)(-4) - (5)(-6) = 18$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = -\frac{4}{18} = -\frac{2}{9}\ \mho, \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{5}{18}\ \mho$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{(-6)}{18} = \frac{1}{3}\ \mho, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{3}{18}\ \mho$$

Hence, Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -\dfrac{2}{9} & -\dfrac{5}{18} \\ \dfrac{1}{3} & \dfrac{3}{18} \end{bmatrix}$$

9.7 INTERCONNECTION OF TWO-PORT NETWORKS

We shall now discuss the various types of interconnections of two-port networks, namely, cascade, parallel, series, series-parallel and parallel-series. We shall derive the relation between the input and output quantities of the combined two-port networks.

9.7.1 Cascade Connection

Transmission Parameter Representation Figure 9.56 shows two-port networks connected in cascade. In the cascade connection, the output port of the first network becomes the input port of the second network. Since it is assumed that input and output currents are positive when they enter the network, we have

$$I_1' = -I_2$$

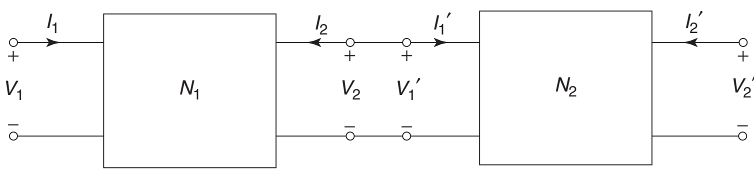

Fig. 9.56 *Cascade Connection*

Let A_1, B_1, C_1, D_1 be the transmission parameters of the network N_1 and A_2, B_2, C_2, D_2 be the transmission parameters of the network N_2.
For the network N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad ...(9.1)$$

For the network N_2,

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \qquad ...(9.2)$$

Since $V_1' = V_2$ and $I_2' = -I_2$, we can write

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \qquad ...(9.3)$$

Combining Eqs (9.1) and (9.3),

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

Hence,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \qquad ...(9.4)$$

Equation 9.4 shows that the resultant *ABCD* matrix of the cascade connection is the product of the individual *ABCD* matrices.

Example 9.34 *Two identical sections of the network shown in Fig. 9.57 are connected in cascade. Obtain the transmission parameters of the overall connection.*

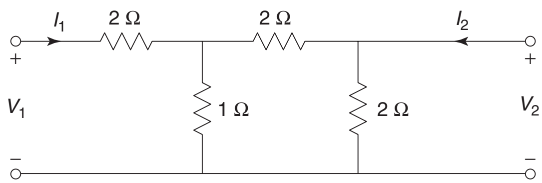

Fig. 9.57

Solution The network is redrawn as shown in Fig. 9.58.

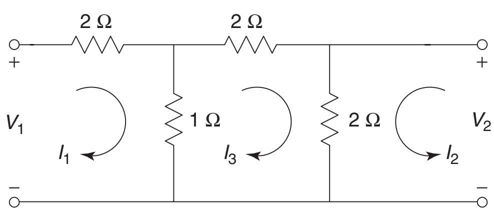

Fig. 9.58

Applying KVL to Mesh 1,

$$V_1 = 3I_1 - I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-I_1 + 2I_2 + 5I_3 = 0$$

$$I_3 = \frac{1}{5}I_1 - \frac{2}{5}I_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - \left(\frac{1}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= \frac{14}{5}I_1 + \frac{2}{5}I_2 \qquad \text{...(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 2I_2 + 2\left(\frac{1}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= \frac{2}{5}I_1 + \frac{6}{5}I_2$$

$$I_1 = \frac{5}{2}V_2 - 3I_2 \qquad \text{...(v)}$$

Substituting Eq. (v) in Eq. (iv),

$$V_1 = \frac{14}{5}\left(\frac{5}{2}V_2 - 3I_2\right) + \frac{2}{5}I_2$$

$$= 7V_2 - 8I_2 \qquad \text{...(vi)}$$

Comparing the Eqs (vi) and (v) with *ABCD* parameter equations,

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 2.5 & 3 \end{bmatrix}$$

Hence, transmission parameters of the overall cascaded network are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 2.5 & 3 \end{bmatrix}\begin{bmatrix} 7 & 8 \\ 2.5 & 3 \end{bmatrix} = \begin{bmatrix} 69 & 80 \\ 25 & 29 \end{bmatrix}$$

Example 9.35 *Determine ABCD parameters for the ladder network shown in Fig. 9.59.*

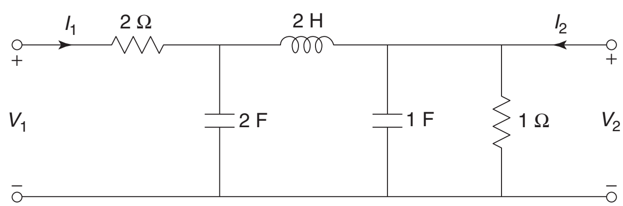

Fig. 9.59

Solution The above network can be considered as a cascade connection of two networks N_1 and N_2. The network N_1 is shown in Fig. 9.60.

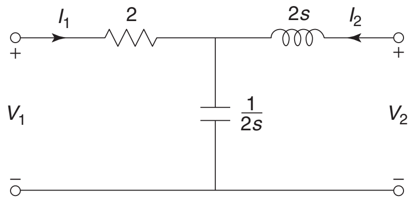

Fig. 9.60

Applying KVL to Mesh 1,

$$V_1 = \left(2 + \frac{1}{2s}\right) I_1 + \frac{1}{2s} I_2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = \frac{1}{2s} I_1 + \left(2s + \frac{1}{2s}\right) I_2 \qquad \text{...(ii)}$$

From Eq. (ii),

$$I_1 = 2s\, V_2 - (4s^2 + 1)\, I_2 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = \left(2 + \frac{1}{2s}\right)[2s\, V_2 - (4s^2 + 1) I_2] + \frac{1}{2s} I_2$$

$$= (4s + 1) V_2 - (8s^2 + 2s + 2) I_2 \qquad \text{...(iv)}$$

Comparing Eqs (iv) and (iii) with *ABCD* parameter equations,

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 4s+1 & 8s^2 + 2s + 2 \\ 2s & 4s^2 + 1 \end{bmatrix}$$

The network N_2 is shown in Fig. 9.61.

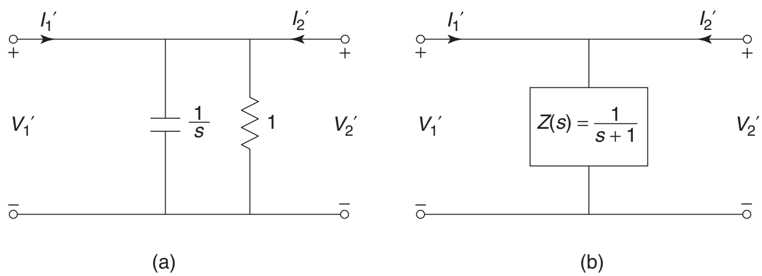

Fig. 9.61

Applying KVL to Mesh 1,

$$V_1' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \qquad \text{...(ii)}$$

From Eq. (ii),

$$I_1' = (s+1)V_2' - I_2' \qquad \ldots\text{(iii)}$$

Also, $$V_1' = V_2' \qquad \ldots\text{(iv)}$$

Comparing Eqs (iv) and (iii) with *ABCD* parameter equations,

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

Hence, overall *ABCD* parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4s+1 & 8s^2+2s+2 \\ 2s & 4s^2+1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} = \begin{bmatrix} 8s^3+10s^2+8s+3 & 8s^2+2s+2 \\ 4s^3+4s^2+3s+1 & 4s^2+1 \end{bmatrix}$$

9.7.2 Parallel Connection

Figure 9.62 shows two-port networks connected in parallel. In the parallel connection, the two networks have the same input voltages and the same output voltages.

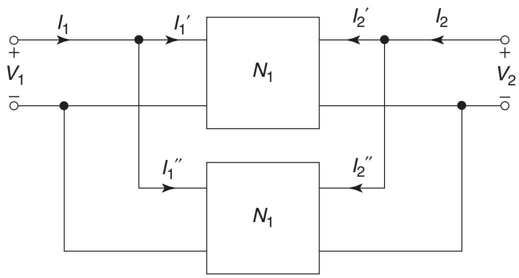

Fig. 9.62 *Parallel connection*

Let $Y_{11}', Y_{12}', Y_{21}', Y_{22}'$ be the *Y*-parameters of the network N_1 and $Y_1'', Y_{12}'', Y_{21}'', Y_{22}''$ be the *Y*-parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For the network N_2,

$$\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For the combined network, $\quad I_1 = I_1' + I_1''$ and $I_2 = I_2' + I_2''$.

Hence, $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' + I_1'' \\ I_2' + I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus, the resultant *Y*-parameter matrix for parallel connected networks is the sum of *Y* matrices of each individual two-port networks.

Example 9.36 *Determine Y-parameters for the network shown in Fig. 9.63.*

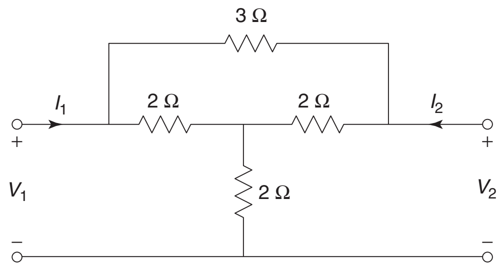

Fig. 9.63

Solution The above network can be considered as a parallel connection of two networks, N_1 and N_2. The network N_1 is shown in Fig. 9.64.

Applying KCL at Node 3,

$$I_1' + I_2' = \frac{V_3}{2} \qquad \ldots\text{(i)}$$

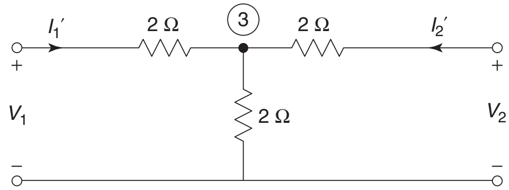

Fig. 9.64

From Fig. 9.64,

$$I_1' = \frac{V_1 - V_3}{2} \qquad \ldots\text{(ii)}$$

$$I_2' = \frac{V_2 - V_3}{2} \qquad \ldots\text{(iii)}$$

Substituting Eqs (ii) and (iii) in Eq (i),

$$\frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{2} = \frac{V_3}{2}$$

$$3V_3 = V_1 + V_2$$

$$V_3 = \frac{V_1}{3} + \frac{V_2}{3} \qquad \ldots\text{(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1' = \frac{V_1}{2} - \frac{1}{2}\left(\frac{V_1}{3} + \frac{V_2}{3}\right)$$

$$= \frac{1}{3}V_1 - \frac{1}{6}V_2 \qquad \ldots\text{(v)}$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2' = \frac{V_2}{2} - \frac{1}{2}\left(\frac{V_1}{3} + \frac{V_2}{3}\right)$$

$$= -\frac{1}{6}V_1 + \frac{1}{3}V_2 \qquad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

The network N_2 is shown in Fig. 9.65.

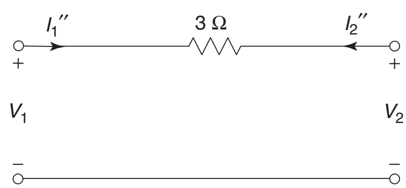

Fig. 9.65

$$I_1'' = -I_2'' = \frac{V_1 - V_2}{3} = \frac{1}{3}V_1 - \frac{1}{3}V_2$$

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

The overall Y-parameters of the network are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{1}{3} + \frac{1}{3} & -\frac{1}{6} - \frac{1}{3} \\ -\frac{1}{6} - \frac{1}{3} & \frac{1}{3} + \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{2}{3} \end{bmatrix}$$

Example 9.37 *Find Y-parameters for the network shown in Fig. 9.66.*

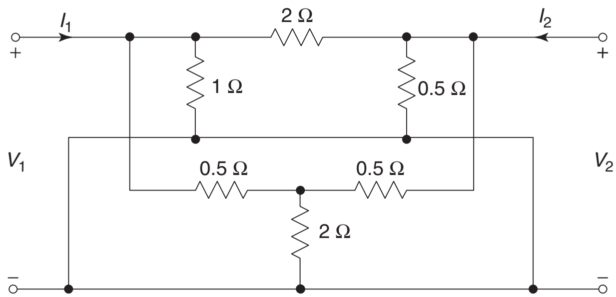

Fig. 9.66

Solution The above network can be considered as a parallel combination of two networks N_1 and N_2. The network N_1 is shown in Fig. 9.67.

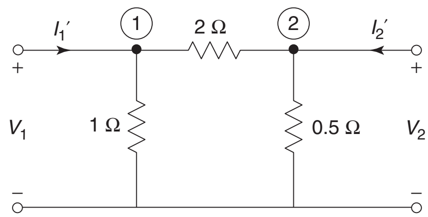

Fig. 9.67

Applying KCL at Node 1,

$$I_1' = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$$

$$= \frac{3}{2}V_1 - \frac{1}{2}V_2 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$I_2' = \frac{V_2}{0.5} + \frac{V_2 - V_1}{2}$$

$$= -\frac{1}{2}V_1 + \frac{5}{2}V_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with Y-parameter equation,

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

The network N_2 is shown in Fig. 9.68.

Applying KCL at Node 3,

$$I_1'' + I_2'' = \frac{V_3}{2} \qquad \ldots\text{(i)}$$

where
$$I_1'' = \frac{V_1 - V_3}{0.5} = 2V_1 - 2V_3$$

$$I_2'' = \frac{V_2 - V_3}{0.5} = 2V_2 - 2V_3$$

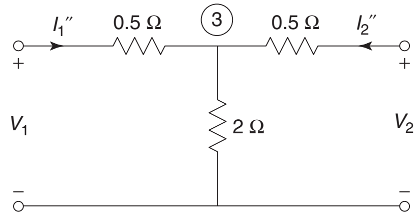

Fig. 9.68

Substituting I_1'' and I_2'' in Eq (i),

$$2V_1 - 2V_3 + 2V_2 - 2V_3 = 0.5V_3$$

$$4.5V_3 = 2V_1 + 2V_2$$

$$V_3 = \frac{4}{9}V_1 + \frac{4}{9}V_2 \qquad \ldots\text{(ii)}$$

$$I_1'' = 2V_1 - 2V_3 = 2V_1 - 2\left(\frac{4}{9}V_1 + \frac{4}{9}V_2\right) = \frac{10}{9}V_1 - \frac{8}{9}V_2 \qquad \ldots\text{(iii)}$$

and
$$I_2'' = 2V_2 - 2V_3 = 2V_2 - 2\left(\frac{4}{9}V_1 + \frac{4}{9}V_2\right)$$

$$= -\frac{8}{9}V_1 + \frac{10}{9}V_2 \qquad \ldots\text{(iv)}$$

Comparing Eqs (iii) and (iv) with Y-parameter equations,

$$\begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{10}{9} & -\frac{8}{9} \\ -\frac{8}{9} & \frac{10}{9} \end{bmatrix}$$

Hence, overall Y-parameters of the network are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{3}{2} + \frac{10}{9} & -\frac{1}{2} - \frac{8}{9} \\ -\frac{1}{2} - \frac{8}{9} & \frac{5}{2} + \frac{10}{9} \end{bmatrix} = \begin{bmatrix} \frac{47}{18} & -\frac{25}{18} \\ -\frac{25}{18} & \frac{65}{18} \end{bmatrix}$$

Example 9.38 *Find Y-parameters for the network shown in Fig. 9.69.*

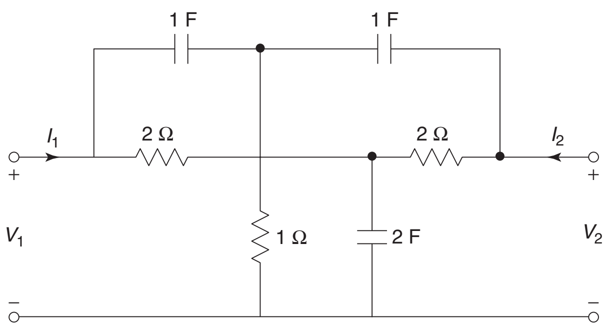

Fig. 9.69

Solution The above network can be considered as a parallel connection of two networks, N_1 and N_2. The network N_1 is shown in Fig. 9.70.

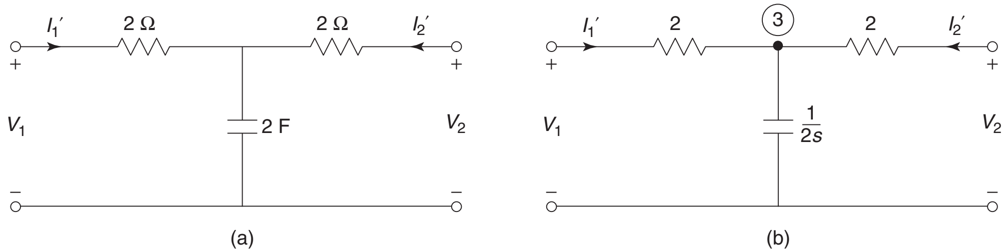

Fig. 9.70

Applying KCL at Node 3,

$$I_1' + I_2' = 2s\, V_3 \qquad \ldots\text{(i)}$$

From Fig. 9.70,

$$I_1' = \frac{V_1 - V_3}{2}$$

$$= \frac{1}{2}V_1 - \frac{1}{2}V_3 \qquad \ldots\text{(ii)}$$

$$I_2' = \frac{V_2 - V_3}{2}$$

$$= \frac{1}{2}V_2 - \frac{1}{2}V_3 \qquad \ldots\text{(iii)}$$

Substituting Eq. (ii) and Eq. (iii) in Eq. (i),

$$\frac{V_1}{2} - \frac{V_3}{2} + \frac{V_2}{2} - \frac{V_3}{2} = (2s)\, V_3$$

$$(2s+1)V_3 = \frac{V_1}{2} + \frac{V_2}{2}$$

$$V_3 = \frac{1}{2(2s+1)}V_1 + \frac{1}{2(2s+1)}V_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1' = \frac{V_1}{2} - \frac{1}{2}\left[\frac{1}{2(2s+1)}V_1 + \frac{1}{2(2s+1)}V_2\right]$$

$$= \left(\frac{4s+1}{8s+4}\right)V_1 - \left(\frac{1}{8s+4}\right)V_2 \qquad \ldots\text{(v)}$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2' = \frac{V_2}{2} - \frac{1}{2}\left[\frac{1}{2(2s+1)}V_1 - \frac{1}{2(2s+1)}V_2\right]$$

$$= -\left(\frac{1}{8s+4}\right)V_1 + \left(\frac{4s+1}{8s+4}\right)V_2 \qquad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} \dfrac{4s+1}{8s+4} & -\dfrac{1}{8s+4} \\ -\dfrac{1}{8s+4} & \dfrac{4s+1}{8s+4} \end{bmatrix}$$

The network N_2 is shown in Fig. 9.71.

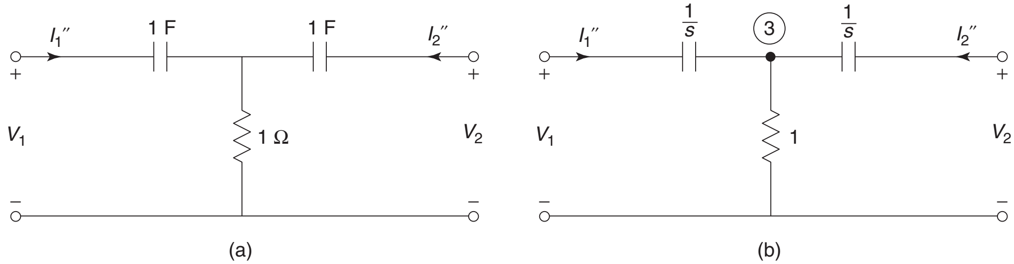

Fig. 9.71

Applying KCL at Node 3,

$$I_1'' + I_2'' = V_3 \quad \ldots\text{(i)}$$

From Fig. 9.71,

$$I_1'' = \frac{V_1 - V_3}{\frac{1}{s}}$$

$$= sV_1 - sV_3 \quad \ldots\text{(ii)}$$

$$I_2' = \frac{V_2 - V_3}{\frac{1}{s}}$$

$$= sV_2 - sV_3 \quad \ldots\text{(iii)}$$

Substituting Eqs (ii) and (iii) in Eq. (i),

$$sV_1 - sV_3 + sV_2 - sV_3 = V_3$$

$$(2s+1)V_3 = sV_1 + sV_2$$

$$V_3 = \left(\frac{s}{2s+1}\right)V_1 + \left(\frac{s}{2s+1}\right)V_2 \quad \ldots\text{(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1'' = sV_1 - s\left[\left(\frac{s}{2s+1}\right)V_1 + \frac{s}{(2s+1)}V_2\right]$$

$$= \left[\frac{s(s+1)}{2s+1}\right]V_1 - \left(\frac{s^2}{2s+1}\right)V_2 \quad \ldots\text{(v)}$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2'' = sV_2 - s\left[\left(\frac{s}{2s+1}\right)V_1 + \left(\frac{s}{2s+1}\right)V_2\right]$$

$$= -\left(\frac{s^2}{2s+1}\right)V_1 + \left[\frac{s(s+1)}{2s+1}\right]V_2 \quad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} = \begin{bmatrix} \frac{s(s+1)}{2s+1} & -\left(\frac{s^2}{2s+1}\right) \\ -\left(\frac{s^2}{2s+1}\right) & \frac{s(s+1)}{2s+1} \end{bmatrix}$$

Hence, the overall Y-parameters of the network are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21}' & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}'+Y_{11}'' & Y_{12}'+Y_{12}'' \\ Y_{21}'+Y_{21}'' & Y_{22}'+Y_{22}'' \end{bmatrix} = \begin{bmatrix} \dfrac{4s^2+8s+1}{4(2s+1)} & -\dfrac{(4s^2+1)}{4(2s+1)} \\ -\dfrac{(4s^2+1)}{4(2s+1)} & \dfrac{4s^2+8s+1}{4(2s+1)} \end{bmatrix}$$

9.7.3 Series Connection

Figure 9.72 shows two-port networks connected in series. In a series connection, both the networks carry the same input current. Their output currents are also equal.

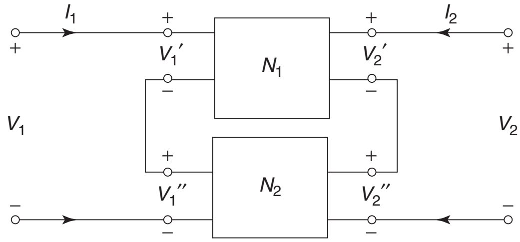

Fig. 9.72 *Series connection*

Let $Z_{11}', Z_{12}', Z_{21}', Z_{22}'$ be the Z-parameters of the network N_1 and $Z_{11}'', Z_{12}'', Z_{21}'', Z_{22}''$ be the Z-parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}'' \\ Z_{21}' & Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the network N_2,

$$\begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}'' \\ Z_{21}' & Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the combined network $\quad V_1 = V_1' + V_1''$ and $V_2 = V_2' + V_2''$.

Hence,
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1'+V_1'' \\ V_2'+V_2'' \end{bmatrix} = \begin{bmatrix} Z_{11}'+Z_{11}'' & Z_{12}'+Z_{12}'' \\ Z_{21}'+Z_{21}'' & Z_{22}'+Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus, the resultant Z-parameter matrix for the series-connected networks is the sum of Z matrices of each individual two-port network.

Example 9.39 *Two identical sections of the network shown in Fig. 9.73 are connected in series. Obtain Z-parameters of the overall connection.*

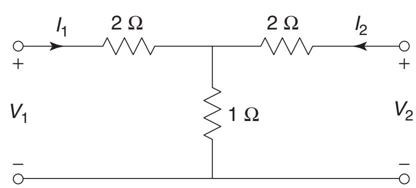

Fig. 9.73

Solution

Applying KVL to Mesh 1,

$$V_1 = 3I_1 + I_2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = I_1 + 3I_2 \qquad \text{...(ii)}$$

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Hence, Z-parameters of the overall connection are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$

Example 9.40 *Determine Z-parameters for the network shown in Fig. 9.74.*

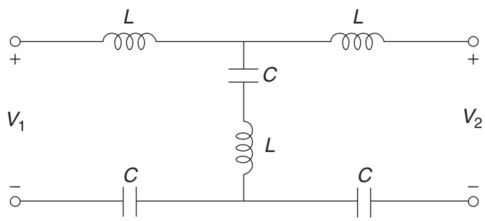

Fig. 9.74

Solution The above network can be considered as a series connection of two networks, N_1 and N_2. The network N_1 is shown in Fig. 9.75.

Applying KVL to Mesh 1,

$$V_1' = \left(Ls + \frac{1}{Cs}\right) I_1 + \left(\frac{1}{Cs}\right) I_2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2' = \left(\frac{1}{Cs}\right) I_1 + \left(Ls + \frac{1}{Cs}\right) I_2 \qquad \text{...(ii)}$$

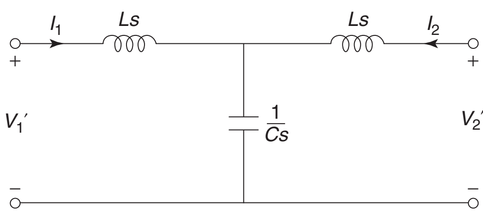

Fig. 9.75

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} = \begin{bmatrix} Ls + \dfrac{1}{Cs} & \dfrac{1}{Cs} \\ \dfrac{1}{Cs} & Ls + \dfrac{1}{Cs} \end{bmatrix}$$

The network N_2 is shown in Fig. 9.76.

Applying KVL to Mesh 1,

$$V_1'' = \left(Ls + \frac{1}{Cs}\right) I_1 + (Ls) I_2 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2'' = (Ls) I_1 + \left(Ls + \frac{1}{Cs}\right) I_2 \qquad \text{...(ii)}$$

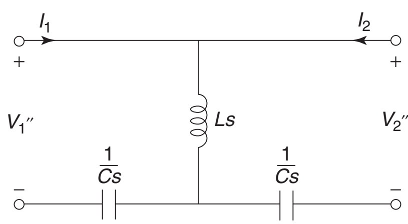

Fig. 9.76

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix} = \begin{bmatrix} Ls + \dfrac{1}{Cs} & Ls \\ Ls & Ls + \dfrac{1}{Cs} \end{bmatrix}$$

Hence, the overall Z-parameters of the network are,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}' + Z_{11}'' & Z_{12}' + Z_{12}'' \\ Z_{21}' + Z_{21}'' & Z_{22}' + Z_{22}'' \end{bmatrix} = \begin{bmatrix} 2Ls + \dfrac{2}{Cs} & Ls + \dfrac{1}{Cs} \\ Ls + \dfrac{1}{Cs} & 2Ls + \dfrac{2}{Cs} \end{bmatrix} = \left(Ls + \frac{1}{Cs}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

9.7.4 Series-Parallel Connection

Figure 9.77 shows two networks connected in series-parallel. Here, the input ports of two networks are connected in series and the output ports are connected in parallel.

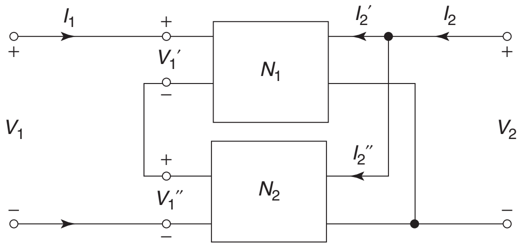

Fig. 9.77 *Series-parallel connection*

Let $h_{11}', h_{12}', h_{21}', h_{22}'$ be the h-parameters of the network N_1 and $h_{11}'', h_{12}'', h_{21}'', h_{22}''$ be the h-parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

For the network N_2,

$$\begin{bmatrix} V_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

For the combined network, $V_1 = V_1' + V_1''$ and $I_2 = I_2' + I_2''$

Hence,
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1' + V_1'' \\ I_2' + I_2'' \end{bmatrix} = \begin{bmatrix} h_{11}' + h_{11}'' & h_{12}' + h_{12}'' \\ h_{21}' + h_{21}'' & h_{22}' + h_{22}'' \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Thus, the resultant h-parameter matrix is the sum of h-parameter matrices of each individual two-port networks.

Example 9.41 *Determine h-parameters for the network shown in Fig. 9.78.*

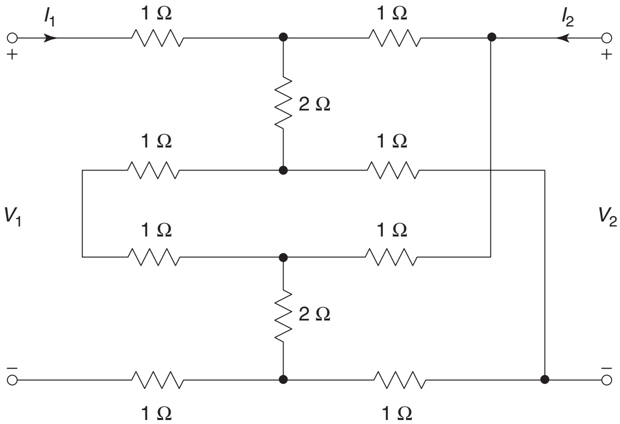

Fig. 9.78

Solution The above network can be considered as a series-parallel connection of two networks N_1 and N_2. The network N_1 is shown in Fig. 9.79.

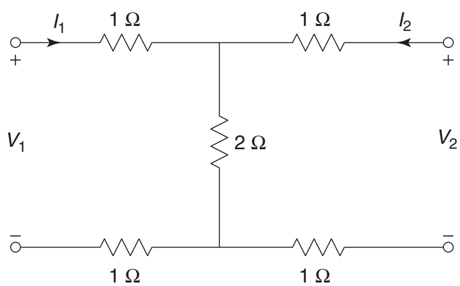

Fig. 9.79

Applying KVL to Mesh 1,

$$V_1 = 4I_1 + 2I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_1 + 4I_2 \qquad \ldots\text{(ii)}$$

Rewriting Eq. (ii)

$$4I_2 = -2I_1 + V_2$$

$$I_2 = -\frac{1}{2}I_1 + \frac{1}{4}V_2 \qquad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 4I_1 + 2\left(-\frac{1}{2}I_1 + \frac{1}{4}V_2\right)$$

$$= 3I_1 + \frac{1}{2}V_2 \qquad \ldots\text{(iv)}$$

Comparing Eqs (iii) and (iv) with h-parameters equations,

$$\begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

For network N_2, h-parameters will be same as the two networks are identical.

$$\begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

Hence, the overall h-parameters of the network are

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix} + \begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} + \begin{bmatrix} 3 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ -1 & \frac{1}{2} \end{bmatrix}$$

9.8 *T*-NETWORK

Any two-port network can be represented by an equivalent *T* network as shown in Fig. 9.80. The elements of the equivalent *T* network may be expressed in terms of *Z*-parameters.

Applying KVL to Mesh 1,

$$V_1 = Z_A I_1 + Z_C (I_1 + I_2)$$
$$= (Z_A + Z_C) I_1 + Z_C I_2 \qquad \text{...(9.9)}$$

Applying KVL to Mesh 2,

$$V_2 = Z_B I_2 + Z_C (I_2 + I_1)$$
$$= Z_C I_1 + (Z_B + Z_C) I_2 \qquad \text{...(9.10)}$$

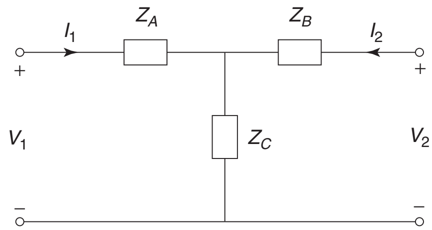

Fig. 9.80 *T-Network*

Comparing Eqs (9.9) and (9.10) with *Z*-parameter equations,

$$Z_{11} = Z_A + Z_C$$
$$Z_{12} = Z_C$$
$$Z_{21} = Z_C$$
$$Z_{22} = Z_B + Z_C$$

Solving the above equations,

$$Z_A = Z_{11} - Z_{12} = Z_{11} - Z_{21}$$
$$Z_B = Z_{22} - ZZ_{21} = Z_{22} - Z_{12}$$
$$Z_C = Z_{12} = Z_{21}$$

9.9 *PI* (π)-NETWORK

Any two-port network can be represented by an equivalent pi (π) network as shown in Fig. 9.81.

Applying KCL at Node 1,

$$I_1 = Y_A V_1 + Y_B (V_1 - V_2)$$
$$= (Y_A + Y_B) V_1 - Y_B V_2 \qquad \text{...(9.11)}$$

Applying KCL at Node 2,

$$I_2 = Y_C V_2 + Y_B (V_2 - V_1)$$
$$= -Y_B V_1 + (Y_B + Y_C) V_2 \qquad \text{...(9.12)}$$

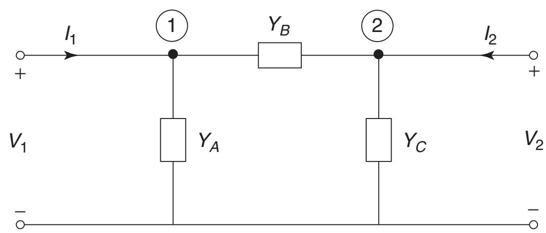

Fig. 9.81 *π-network*

Comparing Eqs (9.11) and (9.12) with *Y*-parameter equations,

$$Y_{11} = Y_A + Y_B$$
$$Y_{12} = -Y_B$$
$$Y_{21} = -Y_B$$
$$Y_{22} = Y_B + Y_C$$

Solving the above equations,

$$Y_A = Y_{11} + Y_{12} = Y_{11} + Y_{21}$$
$$Y_B = -Y_{12} = -Y_{21}$$
$$Y_C = Y_{22} + Y_{12} = Y_{22} + Y_{21}$$

Example 9.42 *The Z-parameters of a two-port network are: $Z_{11} = 10\ \Omega$, $Z_{12} = Z_{21} = 5\ \Omega$, $Z_{22} = 20\ \Omega$. Find the equivalent T-network.*

Solution The *T*-network is shown in Fig. 9.82.

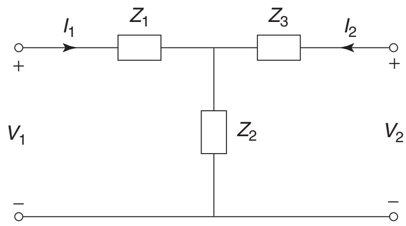

Fig. 9.82

Applying KVL to Mesh 1,

$$V_1 = (Z_1 + Z_2)I_1 + Z_2 I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = Z_2 I_1 + (Z_2 + Z_3)I_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with Z parameter equations,

$$Z_{11} = Z_1 + Z_2 = 10$$
$$Z_{12} = Z_2 = 5$$
$$Z_{21} = Z_2 = 5$$
$$Z_{22} = Z_2 + Z_3 = 20$$

Solving the above equations,

$$Z_1 = 5\ \Omega$$
$$Z_2 = 5\ \Omega$$
$$Z_3 = 15\ \Omega$$

Example 9.43 *Admittance parameters of a pi network are $Y_{11} = 0.09$ ℧, $Y_{12} = Y_{21} = -0.05$ ℧ and $Y_{22} = 0.07$ ℧. Find the values of R_a, R_b and R_c.*

Solution The pi network is shown in Fig. 9.83.

Applying KCL at Node, 1,

$$I_1 = \frac{V_1}{R_a} + \frac{V_1 - V_2}{R_b}$$
$$= \left(\frac{1}{R_a} + \frac{1}{R_b}\right)V_1 - \frac{1}{R_b}V_2 \qquad \ldots\text{(i)}$$

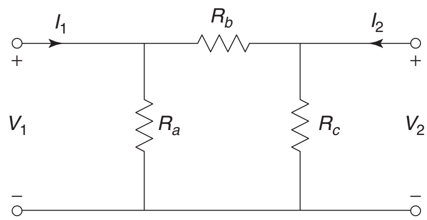

Fig. 9.83

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{R_c} + \frac{V_2 - V_1}{R_b}$$
$$= -\frac{1}{R_b}V_1 + \left(\frac{1}{R_B} + \frac{1}{R_c}\right)V_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with *Y*-parameter equations,

$$Y_{11} = \frac{1}{R_a} + \frac{1}{R_b} = 0.09$$
$$Y_{12} = -\frac{1}{R_b} = -0.05$$
$$Y_{21} = -\frac{1}{R_b} = -0.05$$
$$Y_{22} = \frac{1}{R_b} + \frac{1}{R_c} = 0.07$$

Solving the above equations,

$$R_a = 25\ \Omega$$
$$R_b = 20\ \Omega$$
$$R_c = 50\ \Omega$$

Example 9.44 *Find the parameters YA, YB and YC of the equivalent p network as shown in Fig. 9.84 to represent a two-terminal pair network for which the following measurements were taken:*

(a) With terminal 2 short-circuited, a voltage of 10 ∠ 0° V applied at terminal pair I resulted in $I_1 = 2.5 \angle 0°$ A and $I_2 = -0.5 \angle 0°$ A.

(b) With terminal 1 short-circuited, the same voltage at terminal pair 2 resulted in $I_2 = 1.5 \angle 0°$ A and $I_1 = -1.1 \angle -20°$ A.

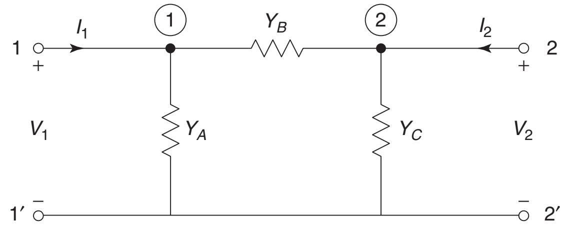

Fig. 9.84

Solution Since measurements were taken with either of the terminal pairs short-circuited, we have to calculate Y-parameters first.

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{2.5\angle 0°}{10\angle 0°} = 0.25\ ℧$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-0.5\angle 0°}{10\angle 0°} = -0.05\ ℧$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1.5\angle 0°}{10\angle 0°} = 0.15\ ℧$$

Applying KCL at Node 1,

$$I_1 = Y_A V_1 + Y_B(V_1 - V_2)$$
$$= (Y_A + Y_B)V_1 - Y_B V_2 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$I_2 = Y_C V_2 + Y_B(V_2 - V_1)$$
$$= -Y_B V_1 + (Y_B + Y_C)V_2 \qquad \text{...(ii)}$$

Comparing Eqs (i) and (ii) with the Y-parameter equation,

$$Y_{11} = Y_A + Y_B = 0.25$$
$$Y_{12} = Y_{21} = -Y_B = -0.05$$
$$Y_{22} = Y_B + Y_C = 0.15$$

Solving the above equation,

$$Y_A = 0.20\ ℧$$
$$Y_B = 0.05\ ℧$$
$$Y_C = 0.10\ ℧$$

Example 9.45 *A network has two input terminals (a, b) and two output terminals (c, d) as shown in Fig. 9.85. The input impedance with c and d open-circuited is (250 + j100) ohms and with c and d short-circuited is (400 + j 300) ohms. The impedance across c and d with a and b open-circuited is 200 ohms. Determine the equivalent T-network parameters.*

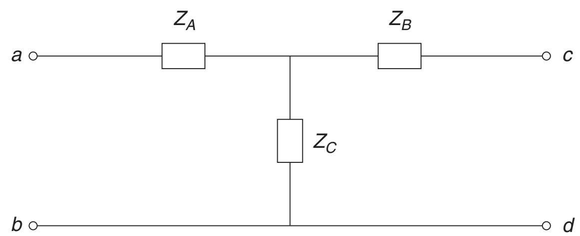

Fig. 9.85

Solution The input impedance with c and d open-circuited is

$$Z_A + Z_C = 250 + j100 \qquad \ldots\text{(i)}$$

The input impedance with c and d short-circuited is,

$$Z_A + \frac{Z_B Z_C}{Z_B + Z_C} = 400 + j300 \qquad \ldots\text{(ii)}$$

The impedance across c and d with a and b open-circuited is

$$Z_B + Z_C = 200 \qquad \ldots\text{(iii)}$$

Subtracting Eq. (i) from (ii),

$$\frac{Z_B Z_C}{Z_B + Z_C} - Z_C = 150 + j200 \qquad \ldots\text{(iv)}$$

From Eq. (iii),

$$Z_B = 200 - Z_C \qquad \ldots\text{(v)}$$

Subtracting the value of Z_B in the equation (iv) and simplifying,

$$Z_C = (100 - j200)\ \Omega \qquad \ldots\text{(vi)}$$

From Eqs (i) and (vi),

$$Z_A = (150 + j300)\ \Omega$$

From Eqs (iii) and (vi),

$$Z_B = (100 + j200)\ \Omega$$

Example 9.46 *Find the equivalent π-network for the T-network shown in Fig. 9.86.*

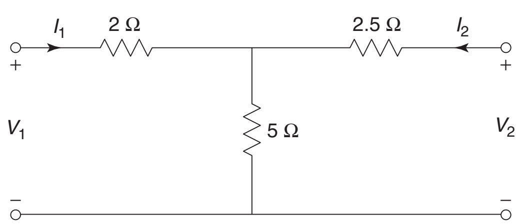

Fig. 9.86

Solution Figure 9.87 shows T-network and π-network.

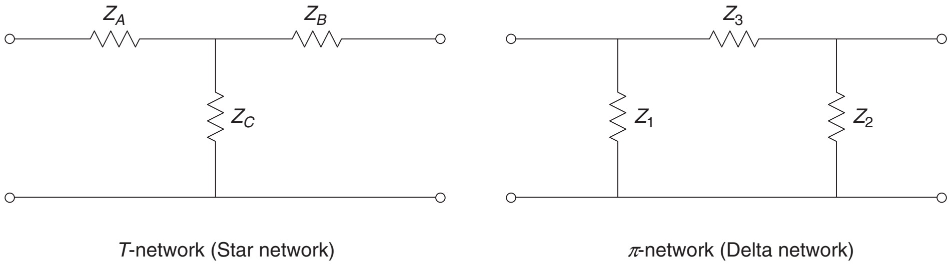

Fig. 9.87

For converting a T-network (star network) into an equivalent π-network (delta network), we can use star-delta transformation technique.

$$Z_1 = Z_A + Z_C + \frac{Z_A Z_C}{Z_B} = 2 + 5 + \frac{2\times 5}{2.5} = 11\,\Omega$$

$$Z_3 = Z_A + Z_B + \frac{Z_A Z_B}{Z_C} = 2 + 2.5 + \frac{2\times 2.5}{5} = 5.5\,\Omega$$

$$Z_2 = Z_B + Z_C + \frac{Z_B Z_C}{Z_A} = 2.5 + 5 + \frac{2.5\times 5}{2} = 13.75\,\Omega$$

The equivalent π-network is shown in Fig. 9.88.

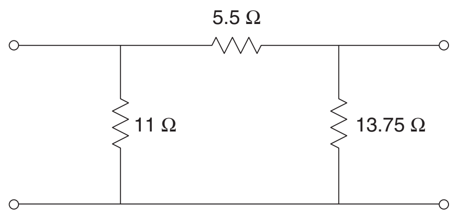

Fig. 9.88

Example 9.47 *For the network shown in Fig. 9.89. Find the equivalent T-network.*

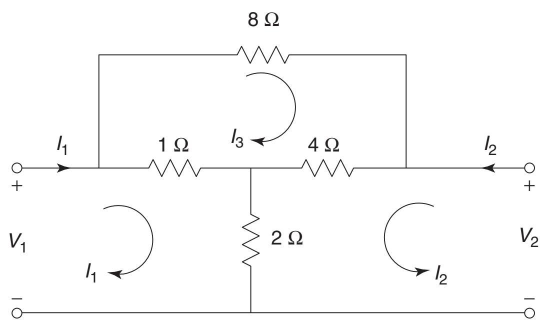

Fig. 9.89

Solution Applying KVL to Mesh 1,

$$V_1 = 3I_1 + 2I_2 - I_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_1 + 6I_2 + 4I_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$13I_3 - I_1 + 4I_2 = 0$$

$$I_3 = \frac{1}{13}I_1 - \frac{4}{13}I_2 \qquad \ldots\text{(iii)}$$

Substituting the Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 + 2I_2 - \frac{1}{13}I_1 + \frac{4}{13}I_2$$

$$= \frac{38}{13}I_1 + \frac{30}{13}I_2 \qquad \ldots\text{(iv)}$$

Substituting the Eq. (iii) in Eq. (ii),

$$V_2 = 2I_1 + 6I_2 + 4\left(\frac{1}{13}I_1 - \frac{4}{13}I_2\right)$$

$$= \frac{30}{13}I_1 + \frac{62}{13}I_2 \qquad \ldots\text{(v)}$$

The T-network is shown in Fig. 9.90.

Applying KVL to Mesh 1,

$$V_1 = (Z_A + Z_C)I_1 + Z_C I_2 \qquad \ldots\text{(vi)}$$

Applying KVL to Mesh 2,

$$V_2 = Z_C I_1 + (Z_B + Z_C)I_2 \qquad \ldots\text{(vii)}$$

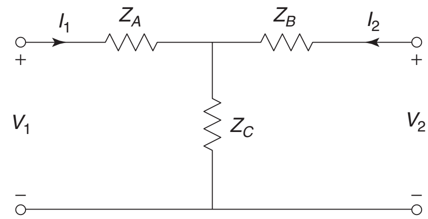

Fig. 9.90

Comparing Eqs (iv) and (v) with Eqs (vi) and (vii),

$$Z_A + Z_C = \frac{38}{13}$$

$$Z_C = \frac{30}{13}$$

$$Z_B + Z_C = \frac{62}{13}$$

Solving the above equations,

$$Z_A = \frac{8}{13}\,\Omega$$

$$Z_B = \frac{32}{13}\,\Omega$$

$$Z_C = \frac{30}{13}\,\Omega$$

9.10 LATTICE NETWORKS

A lattice network is one of the common two-port networks, shown in Fig. 9.91. It is used in filter sections and is also used as attenuator. This network can be represented in terms of z-parameters.

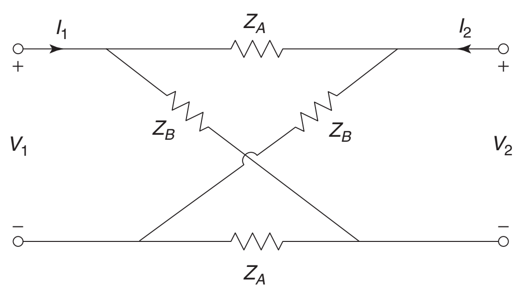

Fig. 9.91 *Lattice network*

The lattice network can be redrawn as a bridge network as shown in Fig. 9.92. This lattice network is symmetric and reciprocal. The current I_1 divides equally between the two arms of the bridge.

When the output port is open-circuited, i.e., $I_2 = 0$

$$V_1 = \frac{I_1}{2}(Z_A + Z_B)$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{Z_A + Z_B}{2}$$

Also

$$V_2 = \frac{I_1}{2}Z_B - \frac{I_1}{2}Z_A = \frac{I_1}{2}(Z_B - Z_A)$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{Z_B - Z_A}{2}$$

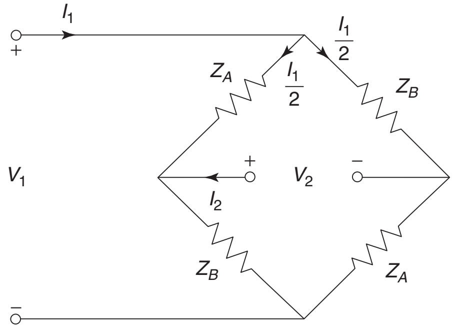

Fig. 9.92 *Bridge network*

Since the network is symmetric,

$$Z_{11} = Z_{22} = \frac{Z_A + Z_B}{2}$$

$$Z_{12} = Z_{21} = \frac{Z_B - Z_A}{2}$$

Solving the above equations,

$$Z_A = Z_{11} - Z_{12}$$

$$Z_B = Z_{11} + Z_{12}$$

The lattice network can be represented in terms of other two-port network parameters, with the help of inter-relationship formulae of various parameters.

Example 9.48 *Find the lattice equivalent of a symmetrical T network shown in Fig. 9.93.*

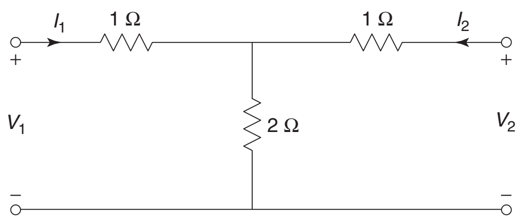

Fig. 9.93

Solution Applying KVL to Mesh 1,

$$V_1 = 3I_1 + 2I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_1 + 3I_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$Z_{11} = 3\ \Omega$$

$$Z_{12} = 2\ \Omega$$

$$Z_{21} = 2\ \Omega$$

$$Z_{22} = 3\ \Omega$$

Since $Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$, the network is symmetric and reciprocal. The parameters of lattice network are

$$Z_A = Z_{11} - Z_{12} = 3 - 2 = 1\,\Omega$$
$$Z_B = Z_{11} + Z_{12} = 3 + 2 = 5\,\Omega$$

The lattice network is shown in Fig. 9.94.

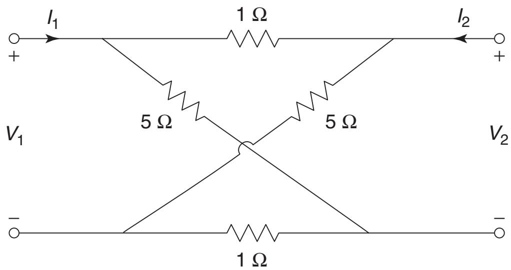

Fig. 9.94

Example 9.49 *Find the lattice equivalent of a symmetric π-network shown in Fig. 9.95.*

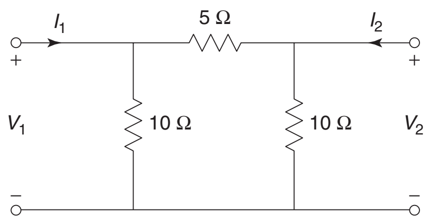

Fig. 9.95

Solution The network is redrawn as shown in Fig. 9.96.

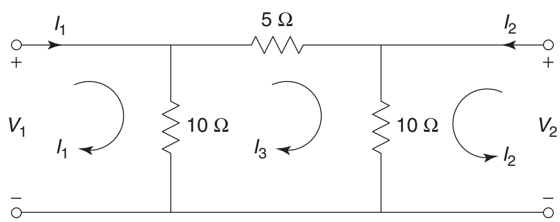

Fig. 9.96

Applying KVL to Mesh 1,

$$V_1 = 10\,I_1 - 10\,I_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 10\,I_2 + 10\,I_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-10\,I_1 + 10\,I_2 + 25\,I_3 = 0$$

$$I_3 = \frac{2}{5}I - \frac{2}{5}I_2 \qquad \ldots\text{(iii)}$$

Substituting Eq (iii) in Eq (i),

$$V_1 = 10\,I_1 - 10\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= 6\,I_1 + 4\,I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq (iii) in Eq (ii),

$$V_2 = 10\,I_2 + 10\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= 4\,I_1 - 6\,I_2 \qquad \ldots\text{(v)}$$

Comparing the Eqs (iv) and (v) with Z-parameter equations,

$$Z_{11} = 6\,\Omega$$
$$Z_{12} = 4\,\Omega$$
$$Z_{21} = 4\,\Omega$$
$$Z_{22} = 6\,\Omega$$

Since $Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$, the network is symmetric and reciprocal. The parameters of lattice network are

$$Z_A = Z_{11} - Z_{12} = 6 - 4 = 2\,\Omega$$
$$Z_B = Z_{11} + Z_{12} = 6 + 4 = 10\,\Omega$$

The lattice network is shown in Fig. 9.97.

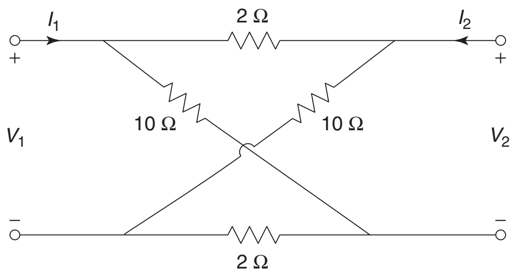

Fig. 9.97

9.11 TERMINATED TWO-PORT NETWORKS

9.11.1 Driving-Point Impedance at Input Port

A two-port network is shown in Fig. 9.98. The output port of the network is terminated in load impedance Z_L. The input impedance of this network can be expressed in terms of parameters of two-port network parameters.

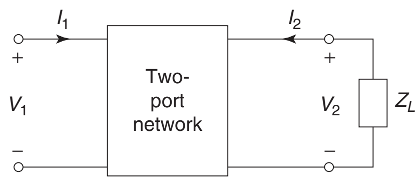

Fig. 9.98 *Terminated two-port network*

1. Input Impedance in Terms of Z-parameters We know that

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

From Fig. 9.98,

$$V_2 = -Z_L I_2$$
$$-I_2 Z_L = Z_{21} I_1 + Z_{22} I_2$$
$$I_2 = -\frac{Z_{21}}{Z_{22} + Z_L} I_1$$
$$Z_{\text{in}} = \frac{V_1}{I_1} = Z_{11} + Z_{12}\left(-\frac{Z_{21}}{Z_{22} + Z_L}\right) = \frac{Z_{11}Z_{22} + Z_{11}Z_L - Z_{12}Z_{21}}{Z_{22} + Z_L}$$

If the output port is open-circuited, i.e., $Z_L = \infty$,

$$Z_{\text{in}} = \lim_{Z_L \to \infty} \frac{\dfrac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_L} + Z_{11}}{\dfrac{Z_{22}}{Z_L} + 1} = Z_{11}$$

If the output port is short-circuited, i.e., $Z_L = 0$,

$$Z_{\text{in}} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}$$

2. Input Impedance in Terms of Y-parameters We know that

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

From Fig. 9.98,

$$V_2 = -Z_L I_2$$

$$I_2 = -\frac{V_2}{Z_L} = -Y_L V_2 \qquad \text{where } Y_L = \frac{1}{Z_L}$$

$$-Y_L V_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V_2 = -\frac{Y_{21}}{Y_{22} + Y_L} V_1$$

$$I_1 = Y_{11}V_1 + Y_{12}\left(-\frac{Y_{21}}{Y_{22} + Y_L}\right)V_1 = Y_{11}V_1 - \frac{Y_{21}Y_{12}}{Y_{22} + Y_L}V_1 = \frac{Y_{11}Y_{22} - Y_{12}Y_{21} + Y_{11}Y_L}{Y_{22} + Y_L}V_1$$

$$Z_{\text{in}} = \frac{V_1}{I_1} = \frac{Y_{22} + Y_L}{Y_{11}Y_{22} - Y_{12}Y_{21} + Y_{11}Y_L}$$

When output port is open-circuited, i.e., $Y_L = 0$

$$Z_{\text{in}} = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

When output port is short-circuited, i.e., $Y_L = \infty$,

$$Z_{\text{in}} = \lim_{Y_L \to \infty} \frac{\dfrac{Y_{22}}{Y_L} + 1}{\dfrac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_L} + Y_{11}} = \frac{1}{Y_{11}}$$

3. Input Impedance in Terms of Transmission Parameters We know that

$$V_1 = AV_2 - BV_2$$
$$I_1 = CV_2 - DI_2$$

From Fig. 9.98,

$$V_2 = -Z_L I_2$$

$$I_1 = -CZ_L I_2 - DI_2 = -(CZ_L + D)I_2$$

$$I_2 = -\frac{I_1}{CZ_L + D}$$

$$V_1 = AZ_L I_2 - B = \left(-\frac{I_1}{CZ_L + D}\right) = \left(\frac{AZ_L + B}{CZ_L + D}\right)I_1$$

$$Z_{\text{in}} = \frac{V_1}{I_1} = \frac{AZ_L + B}{CZ_L + D}$$

If the output port is open-circuited, i.e., $Z_L = \infty$,

$$Z_{\text{in}} = \frac{A}{C}$$

If the output port is short-circuited, i.e., $Z_L = 0$,

$$Z_{\text{in}} = \frac{B}{D}$$

4. *Input Impedance in Terms of Hybrid Parameters* We know that

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$
$$V_2 = -Z_L I_2$$
$$I_2 = h_{21}I_1 - h_{22}Z_L I_2$$
$$I_2 = \frac{h_{21}}{1+h_{22}Z_L} I_L$$
$$V_2 = -\frac{h_{21}Z_L}{1+h_{22}Z_L} I_1$$

Substituting the value of V_2 in V_1,

$$V_1 = h_{11}I_1 + h_{12}\left[\frac{-h_{21}Z_L}{1+h_{22}Z_L} I_L\right] = \left[\frac{(h_{11}h_{22} - h_{12}h_{21})Z_L + h_{11}}{1+h_{22}Z_L}\right] I_1$$

$$Z_{\text{in}} = \frac{V_1}{I_1} = \frac{(h_{11}h_{22} - h_{12}h_{21})Z_L + h_{11}}{1+h_{22}Z_L}$$

If the output port is open-circuited, i.e., $Z_L = \infty$,

$$Z_{\text{in}} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

If the output port is short-circuited, i.e., $Z_L = 0$,

$$Z_{\text{in}} = h_{11}$$

9.11.2 Driving-Point Impedance at Output Port

A two-port network is shown in Fig. 9.99. The input port is terminated in load impedance Z_L. The output impedance of this network can be expressed in terms of two port network parameters.

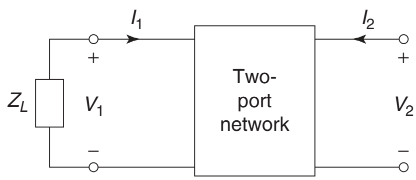

Fig. 9.99 *Terminated two-port network*

1. *Output Impedance in terms of Z-parameters* We know that

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

From Fig. 9.99,

$$V_1 = -Z_L I_1$$

$$-I_1 Z_1 = Z_{11} I_1 + Z_{12} I_2$$

$$I_1 = \left(-\frac{Z_{12}}{Z_L + Z_{11}}\right) I_2$$

$$V_2 = Z_{21}\left(-\frac{Z_{12}}{Z_L + Z_{11}}\right) I_2 + Z_{22} I_2 = I_2\left(Z_{22} - \frac{Z_{21} Z_{12}}{Z_L + Z_{11}}\right) = \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21} + Z_{22} Z_L}{Z_{11} + Z_L}\right) I_2$$

$$Z_0 = \frac{V_2}{I_2} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21} + Z_{22} Z_L}{Z_{11} + Z_L}$$

If the input port is open-circuited, i.e., $Z_L = \infty$,

$$Z_0 = Z_{22}$$

If the input port is short-circuited, i.e., $Z_L = 0$,

$$Z_0 = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{11}}$$

2. *Output Impedance in Terms of Y-parameters* We know that

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

From Fig. 9.99,

$$V_1 = -Z_L I_1$$

$$I_1 = -\frac{V_1}{Z_L} = -Y_L V_1$$

$$-Y_L V_1 = Y_{11} V_1 + Y_{12} V_2$$

$$V_1 = \left(-\frac{Y_{12}}{Y_L + Y_{11}}\right) V_2$$

$$I_2 = Y_{21}\left(-\frac{Y_{12}}{Y_L + Y_{11}}\right) V_2 + Y_{22} V_2 = V_2\left[Y_{22} - \frac{Y_{21} Y_{12}}{Y_L + Y_{11}}\right] = V_2\left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21} + Y_L Y_{22}}{Y_L + Y_{11}}\right]$$

$$Z_0 = \frac{V_2}{I_2} = \frac{Y_L + Y_{11}}{Y_{11} Y_{22} - Y_{12} Y_{21} + Y_L Y_{22}}$$

If input port is open-circuited, i.e., $Y_L = 0$,

$$Z_0 = \frac{Y_{11}}{Y_{11} Y_{22} - Y_{12} Y_{21}}$$

If input port is short-circuited, i.e., $Y_L = \infty$,

$$Z_0 = \frac{1}{Y_{22}}$$

3. *Output Impedance in Terms of ABCD Parameters* We know that

$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

From Fig. 9.99,

$$V_1 = -Z_L I_1$$

$$\frac{V_1}{I_1} = -Z_L = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$V_2(CZ_L + A) = I_2(DZ_L + B)$$

$$Z_0 = \frac{V_2}{I_2} = \frac{DZ_L + B}{CZ_L + A}$$

If input port is open-circuited, i.e., $Z_L = \infty$,

$$Z_0 = \frac{D}{C}$$

If input port is short-circuited, i.e., $Z_L = 0$,

$$Z_0 = \frac{B}{A}$$

4. *Output Impedance in Terms of h-parameters* We know that

$$V_1 = h_{11}I_1 + h_{12}V_{12}$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

From Fig. 9.99,

$$V_1 = -Z_L I_1$$

$$-I_1 Z_L = h_{11}I_1 + h_{22}V_2$$

$$I_1 = \left(-\frac{h_{12}}{h_{11} + Z_L}\right)V_2$$

$$I_2 = h_{21}\left(-\frac{h_{12}}{h_{11} + Z_L}\right)V_2 + h_{22}V_2 = V_2\left[\frac{h_{11}h_{22} - h_{12}h_{21} + h_{22}Z_L}{h_{11} + Z_L}\right]$$

$$Z_0 = \frac{V_2}{I_2} = \frac{h_{11} + Z_L}{h_{11}h_{22} - h_{12}h_{21} + h_{22}Z_L}$$

If input port is open-circuited, i.e., $Z_L = \infty$,

$$Z_0 = \frac{1}{h_{22}}$$

If input port is short-circuited i.e., $Z_L = 0$,

$$Z_0 = \frac{h_{11}}{h_{11}h_{22} - h_{12}h_{21}}$$

Example 9.50 *Measurements were made on a two-terminal network shown in Fig. 9.100.*

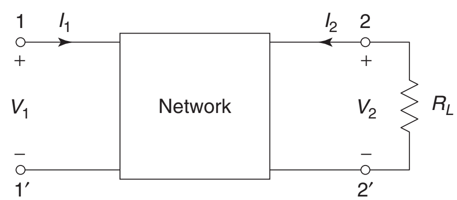

Fig. 9.100

(a) With terminal pair 2 open, a voltage of 100 ∠0° V applied to terminal pair 1 resulted in

$$I_1 = 10\angle 0°\ A, \quad V_2 = 25\angle 0°\ V$$

(b) With terminal pair 1 open, the same voltage applied to terminal pair 2 resulted in

$$I_2 = 20\angle 0°\ A, \quad V_1 = 50\angle 0°\ V$$

Write mesh equations for this network. What will be the voltage across a 10-Ω resistor connected across Terminal pair 2 if a 100 ∠0° V is connected across terminal pair 1?

Solution Since measurements were done with either of the terminal pairs open-circuited, we have to calculate Z-parameters first.

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{100\angle 0°}{10\angle 0°} = 10\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{25\angle 0°}{10\angle 0°} = 2.5\ \Omega$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = \frac{100\angle 0°}{20\angle 0°} = 5\ \Omega$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{50\angle 0°}{20\angle 0°} = 2.5\ \Omega$$

Putting these values in Z-parameter equations,

$$V_1 = 10I_1 + 2.5I_2 \qquad \text{...(i)}$$

$$V_2 = 2.5I_1 + 5I_2 \qquad \text{...(ii)}$$

When a 10-Ω resistor is connected across terminal pair 1,

$$V_1 = 100\angle 0°\ V$$

$$V_2 = -R_L I_2 = -10I_2$$

Substituting values of V_1 and V_2 in Eqs (i) and (ii),

$$100 = 10I_1 + 2.5I_2$$

and

$$-10I_2 = 2.5I_1 + 5I_2$$

$$2.5I_1 = -15I_2$$

$$I_1 = -6I_2$$

$$100 = -60I_2 + 2.5I_2$$

$$I_2 = -\frac{100}{57.5} = -1.74\ \text{A}$$

$$\text{Voltage across the resistor} = -I_2R_L = -10(-1.74) = 17.4\ \text{V}$$

Example 9.51 *The Z-parameters of a two-port network shown in Fig. 9.101 are $Z_{11} = Z_{22} = 10\ \Omega$, $Z_{21} = Z_{22} = 4\ \Omega$. If the source voltage is 20 V, determine I_1, I_2, V_2 and input impedance.*

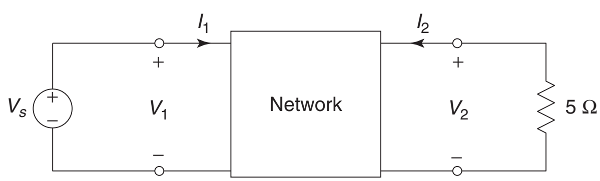

Fig. 9.101

Solution

$$V_1 = V_s = 20 \text{ V}$$
$$V_2 = -20I_2$$

The two-port network can be represented in terms of Z-parameters.

$$V_1 = 10I_1 + 4I_2 \qquad \ldots\text{(i)}$$
$$V_2 = 4I_1 + 10I_2 \qquad \ldots\text{(ii)}$$
$$-20I_2 = 4I_1 + 10I_2$$
$$4I_1 = -30I_2$$
$$I_1 = -7.5I_2$$

Substituting the value of I_1 in Eq. (i),

$$V_1 = 10(-7.5I_2) + 4I_2 = -71I_2$$
$$20 = -71I_2$$
$$I_2 = -0.28 \text{ A}$$
$$I_1 = -7.5(-0.28) = 2.1 \text{ A}$$
$$V_2 = -20(-0.28) = 56 \text{ V}$$

$$\text{Input impedance } Z_i = \frac{V_1}{I_1} = \frac{20}{2.1} = 9.52 \ \Omega$$

Example 9.52 *The Z-parameters of a two-port network shown in Fig. 9.102 are, $Z_{11} = 2 \ \Omega$, $Z_{12} = 1 \ \Omega$, $Z_{21} = 2 \ \Omega$, $Z_{22} = 5 \ \Omega$. Calculate the voltage ratio $\frac{V_2}{V_s}$, current ratio $-\frac{I_2}{I_1}$ and input impedance $\frac{V_1}{I_1}$.*

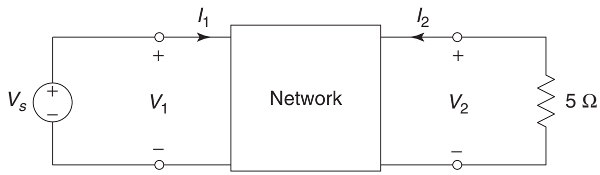

Fig. 9.102

Solution The two-port network can be represented in terms of Z-parameters.

$$V_1 = 2I_1 + I_2 \qquad \ldots\text{(i)}$$
$$V_2 = 2I_1 + 5I_2 \qquad \ldots\text{(ii)}$$

When the 5 Ω resistor is connected across port-2,

$$V_2 = -5I_2 \qquad \ldots\text{(iii)}$$

Applying KVL to the input port,

$$V_s - 1I_1 - V_1 = 0$$
$$V_1 = V_s + I_1 \qquad \ldots\text{(iv)}$$

Substituting values of V_1 and V_2 in Eqs (i) and (ii),

$$V_s - I_1 = 2I_1 + I_2$$
$$V_s = 3I_1 + I_2 \qquad \ldots\text{(v)}$$

and
$$-5I_2 = 2I_1 + 5I_2$$
$$0 = 2I_1 + 10I_2 \qquad \text{...(vi)}$$

Solving Eqs (v) and (vi), we get

$$I_1 = \frac{\begin{vmatrix} V_s & 1 \\ 0 & 10 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 2 & 10 \end{vmatrix}} = \frac{5}{14}V_s$$

$$I_2 = \frac{\begin{vmatrix} 3 & V_s \\ 0 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 2 & 10 \end{vmatrix}} = -\frac{1}{14}V_s$$

$$-\frac{I_2}{I_1} = \frac{1}{5}$$

$$V_2 = 2I_1 + 5I_2 = 2\left(\frac{5}{14}V_s\right) + 5\left(-\frac{1}{14}V_s\right) = \frac{5}{14}V_s$$

$$\frac{V_2}{V_s} = \frac{5}{14}$$

$$V_1 = 2I_1 + I_2 = 2\left(\frac{5}{14}V_s\right) - \frac{1}{14}V_s = \frac{9}{14}V_s$$

$$\frac{V_1}{I_1} = \frac{9}{5}\,\Omega$$

Example 9.53 *The following equations give the voltages V_1 and V_2 at the two ports of a two-port network shown in Fig. 9.103.*

$$V_1 = 5I_1 + 2I_2$$
$$V_2 = 2I_1 + I_2$$

A load resistor of 3 Ω is connected across port 2. Calculate the input impedance.

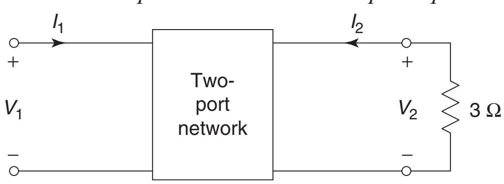

Fig. 9.103

Solution From Fig. 9.103,

$$V_2 = -3I_2 \qquad \text{...(i)}$$

Substituting Eq. (i) in the given equation,

$$-3I_2 = 2I_1 + I_2$$
$$I_2 = -\frac{I_1}{2} \qquad \text{(ii)}$$

Substituting the Eq. (ii) in the given equation.

$$V_1 = 5I_1 - I_1 = 4I_1$$

Input impedance
$$Z_i = \frac{V_1}{I_1} = 4\,\Omega$$

Example 9.54 *The y-parameters for a two-port network shown in Fig. 9.104 are given as,* $Y_{11} = 4\ \mho$, $Y_{22} = 5\ \mho$, $Y_{12} = Y_{21} = 4\ \mho$. *If a resistor of 1 Ω is connected across port-1 of the network then find the output impedance.*

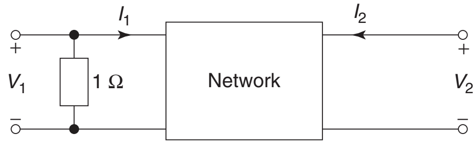

Fig. 9.104

Solution The two-port network can be represented in terms of Y-parameters.

$$I_1 = 4V_1 + 4V_2 \quad \ldots\text{(i)}$$
$$I_2 = 4V_1 + 5V_2 \quad \ldots\text{(ii)}$$

When the 1-Ω resistor is connected across port-1 of the network,

$$V_1 = -1I_1 = -I_1$$
$$I_1 = -V_1$$

Substituting value of I_1 in Eq (i),

$$-V_1 = 4V_1 + 4V_2$$
$$-5V_1 = 4V_2$$
$$V_1 = -\frac{4}{5}V_2$$

Substituting value of V_1 in Eq (ii),

$$I_2 = 4\left(-\frac{4}{5}V_2\right) + 5V_2 = \frac{9}{5}V_2$$

Output impedance $Z_0 = \dfrac{V_2}{I_2} = \dfrac{5}{9}\ \Omega$

Example 9.55 *The following equation gives the voltage and current at the input port of a two-port network shown in Fig. 9.105.*

$$V_1 = 5V_2 - 3I_2$$
$$I_1 = 6V_2 - 2I_2$$

A load resistance of 5 Ω is connected across the output port. Calculate the input impedance.

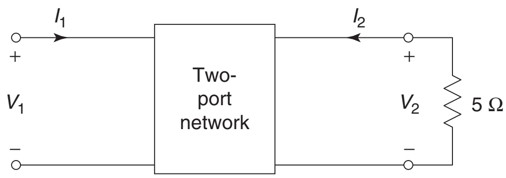

Fig. 9.105

Solution From Fig. 9.105,

$$V_2 = -5I_2$$

Substituting the value of V_2 in the given equations,

$$V_1 = 5(-5I_2) - 3I_2 = -28I_2$$
$$I_1 = 6(-5I_2) - 2I_2 = -32I_2$$

Input impedance $\quad Z_i = \dfrac{V_1}{I_1} = \dfrac{-28I_2}{-32I_2} = \dfrac{7}{8}\ \Omega$

Example 9.56 *The ABCD parameters of a two-port network shown in Fig. 9.106 are $A = 2.5$, $B = 4\ \Omega$, $C = 1\ \mho$, $D = 2$. What must be the input voltage V_1 applied for the output voltage V_2 to be 10 V across the load of 10 Ω connected at Port 2?*

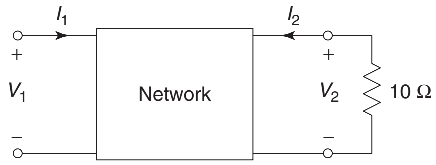

Fig. 9.106

Solution The two-port network can be represented in terms of *ABCD* parameters.

$$V_1 = 2.5\,V_2 - 4I_2 \qquad \text{...(i)}$$

$$I_1 = V_2 - 2I_2 \qquad \text{...(ii)}$$

When the 10 Ω resistor is connected across Port 2,

$$V_2 = -10I_2 = 10 \qquad \text{...(iii)}$$

$$I_2 = -1\text{A}$$

$$V_1 = 2.5(10) - 4(-1) = 29\text{ V}$$

Example 9.57 *The h-parameters of a two-port network shown in Fig. 9.107 are $h_{11} = 4\ \Omega$, $h_{12} = 1$, $h_{21} = 1$, $h_{22} = 0.5\ \mho$. Calculate the output voltage V_2 when the output port is terminated in a 3 Ω resistance and a 1 V is applied at the input port.*

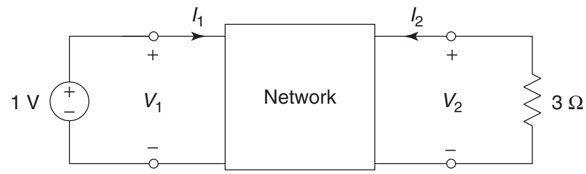

Fig. 9.107

Solution

$$V_1 = 1\text{ V}$$

$$V_2 = -3I_2$$

The two-port network can be represented in terms of *h*-parameters.

$$V_1 = 4I_1 + V_2 \qquad \text{...(i)}$$

$$I_2 = I_1 + 0.5\,V_2 \qquad \text{...(ii)}$$

$$I_2 = I_1 + 0.5(-3I_2)$$

$$2.5I_2 = I_1$$

Substituting the value of V_1 and I_1 in Eq (i),

$$1 = 4(2.5I_2) - 3I_2$$

$$1 = 7I_2$$

$$I_2 = \frac{1}{7}\text{A}$$

$$V_2 = -3\left(\frac{1}{7}\right) = -\frac{3}{7}\text{V}$$

Example 9.58 *The h-parameters of a two-port network shown in Fig. 9.108 are $h_{11} = 1\ \Omega$, $h_{12} = -h_{21} = 2$, $h_{22} = 1\ \mho$. The power absorbed by a load resistance of 1 Ω connected across port-2 is 100W. The network is excited by a voltage source of generated voltage V_s and internal resistance 2 Ω. Calculate the value of V_s.*

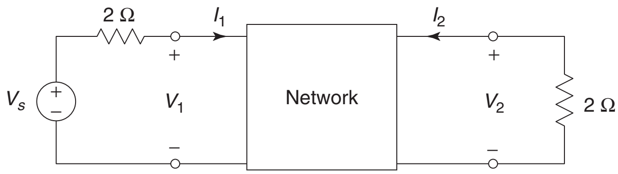

Fig. 9.108

Solution The two-port network can be represented in terms of *h*-parameters.

$$V_1 = I_1 + 2V_2 \quad \ldots\text{(i)}$$

$$I_2 = -2I_1 + V_2 \quad \ldots\text{(ii)}$$

When the 1 Ω resister is connected across port-2,

$$\frac{V_2^2}{1} = 100$$

$$V_2 = 10\text{ V}$$

$$V_2 = -1I_2 = 10$$

$$I_2 = -10\text{ A}$$

Substituting values of I_2 and V_2 in Eq (ii),

$$-10 = -2I_1 + 10$$

$$I_1 = 10\text{ A}$$

Applying KVL to the input port,

$$V_s - 2I_1 - V_1 = 0$$

$$V_s - 2I_1 - (I_1 + 2V_2) = 0$$

$$V_s - 3I_1 - 2V_2 = 0$$

$$V_s = 3I_1 + 2V_2 = 3(10) + 2(10) = 50\text{ V}$$

Exercises

9.1 Determine *Z*-parameters for the network shown in Fig. 9.109.

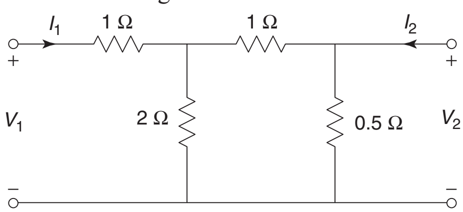

Fig. 9.109

$$Z = \begin{bmatrix} \frac{13}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} \end{bmatrix}$$

9.2 Find *Z*-parameters for the network shown in Fig. 9.110.

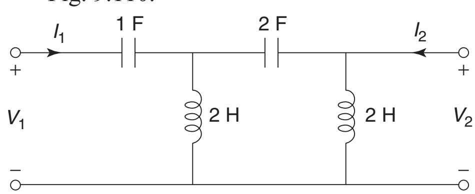

Fig. 9.110

$$Z = \begin{bmatrix} \frac{4s^4 + 6s^2 + 1}{4s^3 + s} & \frac{4s^3}{4s^2 + 1} \\ \frac{4s^3}{4s^2 + 1} & \frac{4s^3 + 2s}{4s^2 + 1} \end{bmatrix}$$

9.3 Find Y-parameters of the network shown in Fig. 9.111.

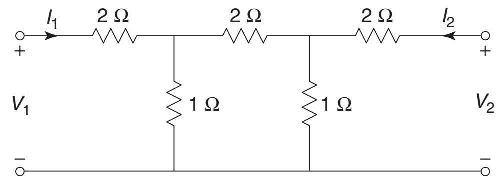

Fig. 9.111

$$Y = \begin{bmatrix} 0.36 & -0.033 \\ -0.033 & -0.36 \end{bmatrix}$$

9.4 Find Y-parameters for the network shown in Fig. 9.112.

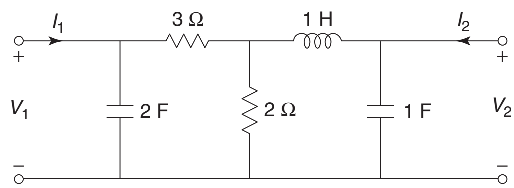

Fig. 9.112

$$Y = \begin{bmatrix} \dfrac{10s^2+13s+2}{5s+6} & -\dfrac{2}{5s+6} \\ -\dfrac{2}{5s+6} & \dfrac{5s^2+6s+5}{5s+6} \end{bmatrix}$$

9.5 Find Y-parameters for the network shown in Fig. 9.113.

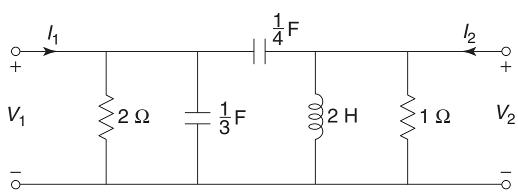

Fig. 9.113

$$Y = \begin{bmatrix} \dfrac{7s+6}{12} & -\dfrac{s}{4} \\ -\dfrac{s}{4} & \dfrac{s^2+4s+2}{4s} \end{bmatrix}$$

9.6 Find Y-parameters for the network shown in Fig. 9.114.

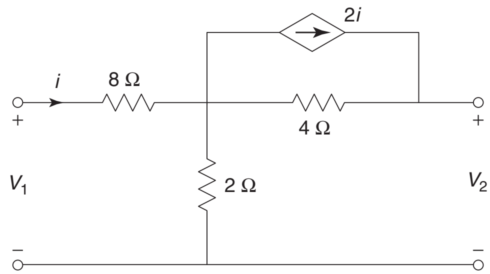

Fig. 9.114

$$Y = \begin{bmatrix} \dfrac{3}{20} & -\dfrac{1}{20} \\ -\dfrac{1}{4} & \dfrac{1}{4} \end{bmatrix}$$

9.7 Show the $ABCD$ parameters of the network shown in Fig. 9.115.

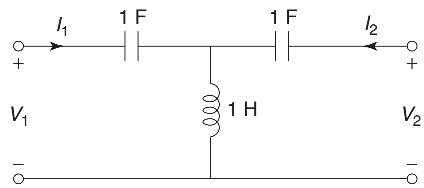

Fig. 9.115

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{1+s^2}{s^2} & \dfrac{1+2s^2}{s^3} \\ \dfrac{1}{s} & \dfrac{1+s^2}{s^2} \end{bmatrix}$$

9.8 Find $ABCD$ parameters for the network shown in Fig. 9.116.

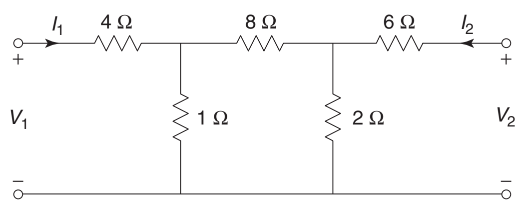

Fig. 9.116

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 27 & 206 \\ \dfrac{11}{2} & 42 \end{bmatrix}$$

9.9 For the network shown in Fig. 9.117, determine parameter h_{21}.

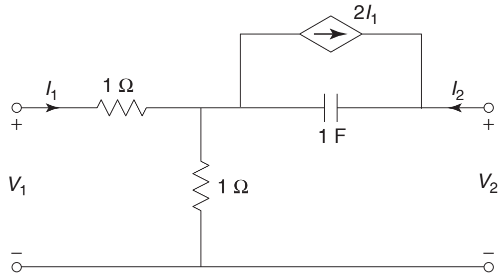

Fig. 9.117

$$\left[h_{21} = \frac{-(2+s)}{1+s}\right]$$

9.10 Determine Y and Z-parameters for the network shown in Fig. 9.118.

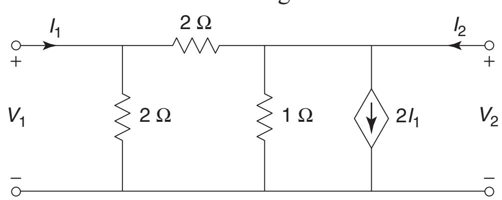

Fig. 9.118

$$\left[Y_{11} = 1\,℧, Y_{12} = -0.5\,℧, Y_{21} = 1.5\,℧, Y_{22} = 0.5\,℧\right.$$
$$\left.Z_{11} = \frac{2}{5}\Omega, Z_{12} = \frac{2}{5}\Omega, Z_{21} = -\frac{6}{5}\Omega, Z_{22} = \frac{4}{5}\Omega\right]$$

9.11 For the bridged T, R-C network shown in Fig. 9.119 determine Y-parameters using interconnections of two-port networks.

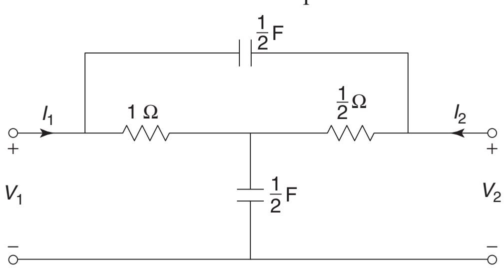

Fig. 9.119

$$Y = \begin{bmatrix} \dfrac{s^2+8s+8}{2(s+6)} & -\dfrac{(s^2+6s+8)}{2(s+6)} \\ -\dfrac{(s^2+6s+8)}{2(s+6)} & \dfrac{s^2+10s+8}{2(s+6)} \end{bmatrix}$$

9.12 For the network of Fig. 9.120, find Y-parameters using interconnection of two-port networks.

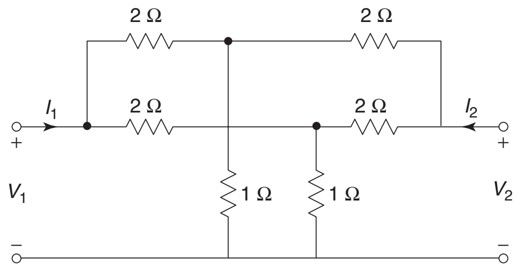

Fig. 9.120

$$Y = \begin{bmatrix} \dfrac{3}{4} & -\dfrac{1}{4} \\ -\dfrac{1}{4} & \dfrac{3}{4} \end{bmatrix}$$

9.13 Two identical sections of the network shown in Fig. 9.121 are connected in parallel. Obtain Y-parameters of the connection.

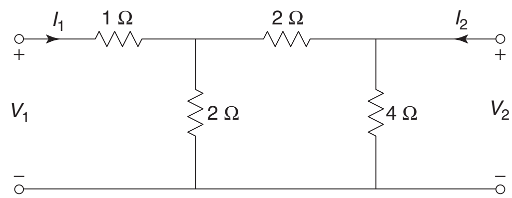

Fig. 9.121

$$Y = \begin{bmatrix} 1 & -\dfrac{1}{2} \\ -\dfrac{1}{2} & \dfrac{5}{4} \end{bmatrix}$$

9.14 Determine Y-parameters using interconnection of two-port networks for the network shown in Fig. 9.122.

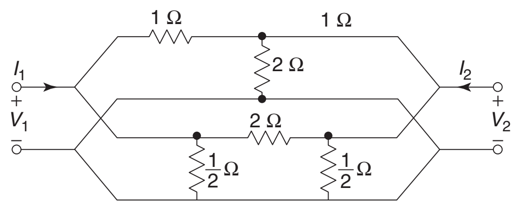

Fig. 9.122

$$Y = \begin{bmatrix} 3.1 & -0.9 \\ -0.9 & 3.1 \end{bmatrix}$$

9.15 Determine the transmission parameters of the network shown in Fig. 9.123 using the concept of interconnection of two two-port networks.

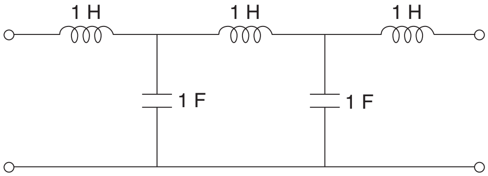

Fig. 9.123

$$\begin{bmatrix} 1+3s^2+s^4 & 3s+4s^3+s^5 \\ 2s+s^3 & 1+3s^2+s^4 \end{bmatrix}$$

9.16 Two networks shown in Fig. 9.124 are connected in series. Obtain the Z-parameters of the resulting network.

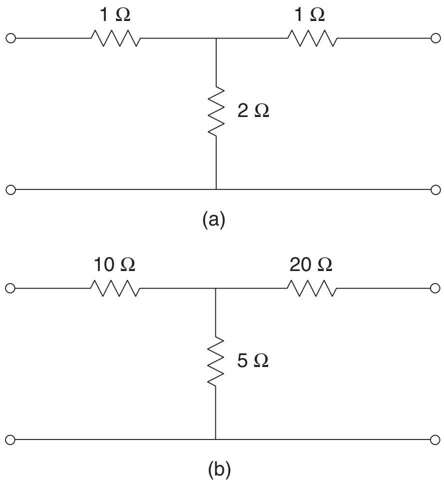

Fig. 9.124

$$\begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$$

9.17 Two identical sections of the network shown in Fig. 9.125 are connected in series-parallel. Determine the h-parameters of the overall network.

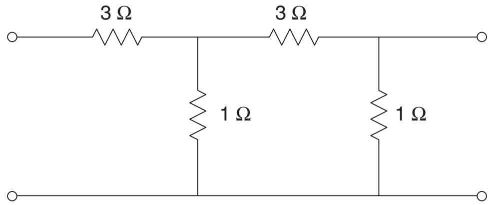

Fig. 9.125

$$\begin{bmatrix} \frac{15}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

9.18 The h-parameters of a two-port network shown in Fig. 9.126 are $h_{11} = 2\ \Omega$, $h_{12} = 4$, $h_{21} = -5$, $h_{22} = 2\ \mho$. Determine the supply voltage V_s if the power dissipated in the load resistor of 4 Ω is 25 W and $R_s = 2\ \Omega$.

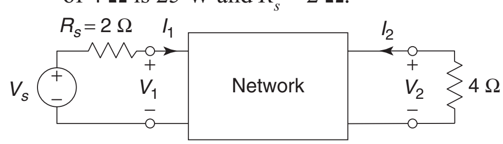

Fig. 9.126

[58 V]

9.19 The Z-parameters of a two-port network are $Z_{11} = 2.1\ \Omega$, $Z_{12} = Z_{21} = 0.6\ \Omega$, $Z_{22} = 1.6\ \Omega$. A resistor of 2 Ω is connected across port 2. What voltage must be applied at port 1 to produce a current of 0.5 A in the 2 Ω resistor.

[6 V]

9.20 If a two-port network has $Z_{11} = 25\ \Omega$, $Z_{12} = Z_{21} = 20\ \Omega$, $Z_{22} = 50\ \Omega$, find the equivalent T-network.

[10 Ω, 30 Ω, 20 Ω]

Objective-Type Questions

9.1 The open-circuit impedance matrix of the two-port network shown in Fig. 9.127 is

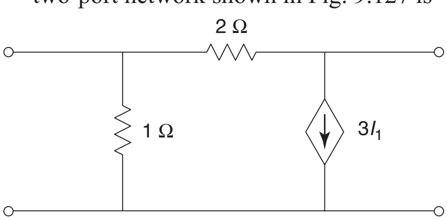

Fig. 9.127

(a) $\begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & -8 \\ 1 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

9.2 Two two-port networks are connected in cascade. The combination is to be represented as a single two-port network. The parameters are obtained by multiplying the individual

(a) z-parameter matrix
(b) h-parameter matrix
(c) y-parameter matrix
(d) $ABCD$ parameter matrix

9.3 For a two-port network to be reciprocal

(a) $z_{11} = z_{22}$ (b) $y_{21} = y_{12}$
(c) $h_{21} = -h_{12}$ (d) $AD - BC = 0$

9.4 The short-circuit admittance matrix of a two-port network is $\begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$. The two-port network is

(a) non-reciprocal and passive
(b) non-reciprocal and active
(c) reciprocal and passive
(d) reciprocal and active

9.5 A two-port network is shown in Fig. 9.128. The parameter h_{21} for this network can be given by

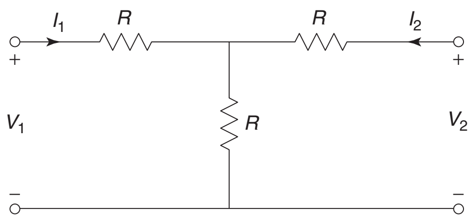

Fig. 9.128

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

9.6 The admittance parameter Y_{12} in the two-port network in Fig. 9.129.

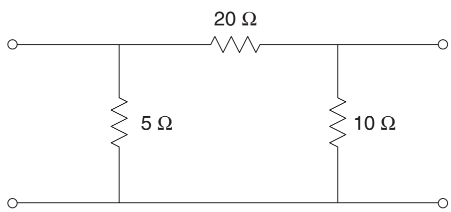

Fig. 9.129

(a) –0.2 mho (b) 0.1 mho
(c) –0.05 mho (d) 0.05 mho

9.7 The Z-parameters Z_{11} and Z_{21} for the two-port network in Fig. 9.130 are,

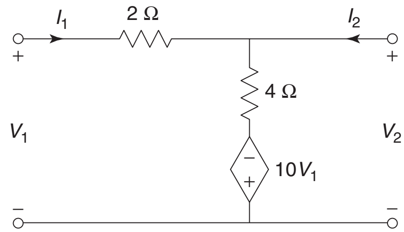

Fig. 9.130

(a) $-\frac{6}{11}\Omega, \frac{16}{11}\Omega$ (b) $\frac{6}{11}\Omega, \frac{4}{11}\Omega$
(c) $\frac{6}{11}\Omega, -\frac{16}{11}\Omega$ (d) $\frac{4}{11}\Omega, \frac{4}{11}\Omega$

9.8 The impedance parameters Z_{11} and Z_{12} of a two-port network in Fig. 9.131.

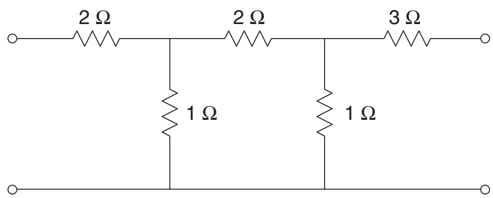

Fig. 9.131

(a) 2.75 Ω, 0.25 Ω (b) 3 Ω, 0.5 Ω
(c) 3 Ω, 0.25 Ω (d) 2.25 Ω, 0.5 Ω

9.9 The h parameters of the circuit shown in Fig. 9.132.

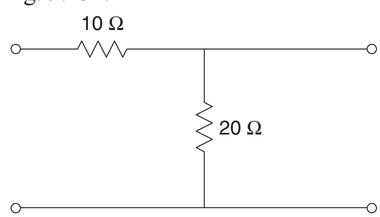

Fig. 9.132

(a) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$

(c) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

9.10 A two-port network is represented by $ABCD$ parameters given by $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$. If port 2 is terminated by R_L, then the input impedance seen at port 1 is given by

(a) $\dfrac{A + BR_L}{C + DR_L}$ (b) $\dfrac{AR_L + C}{BR_L + D}$

(c) $\dfrac{DR_L + A}{BR_L + C}$ (d) $\dfrac{B + AR_L}{D + CR_L}$

9.11 In the two-port network shown in Fig. 9.133, Z_{12} and Z_{21} are respectively

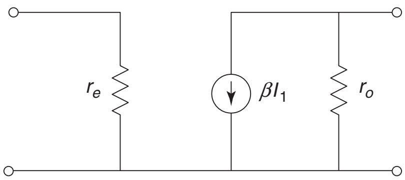

Fig. 9.133

(a) r_e and βr_0 (b) 0 and $-\beta r_0$
(c) 0 and βr_0 (d) r_e and $-\beta r_0$

9.12 If a two-port network is passive, then we have, with the usual notation, the following relationship for symmetrical network

(a) $h_{12} = h_{21}$
(b) $h_{12} = -h_{21}$
(c) $h_{11} = h_{22}$
(d) $h_{11}h_{22} - h_{12}h_{21} = 1$

9.13 A two-port network is defined by the following pair of equations $I_1 = 2V_1 + V_2$ and $I_2 = V_1 + V_2$. Its impedance parameters (Z_{11}, Z_{12}, Z_{21}, Z_{22}) are given by

(a) 2, 1, 1, 1 (b) 1, –1, –1, 2
(c) 1, 1, 1, 2 (d) 2, –1, –1, 1

9.14 A two-port network has transmission parameters $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$. The input impedance of the network at port 1 will be

(a) $\dfrac{A}{C}$ (b) $\dfrac{AD}{BC}$

(c) $\dfrac{AB}{DC}$ (d) $\dfrac{D}{C}$

9.15 A two-port network is symmetrical if

(a) $Z_{11}\,Z_{22} - Z_{12}\,Z_{21} = 1$
(b) $AD - BC = 1$
(c) $h_{11}\,h_{22} - h_{12}\,h_{21} = 1$
(d) $Y_{11}\,Y_{22} - Y_{12}\,Y_{21} = 1$

9.16 For the network shown in Fig. 9.134 admittance parameters are $Y_{11} = 8$ mho, $Y_{12} = Y_{21} = -6$ mho and $Y_{22} = 6$ mho. The value of Y_A, Y_B and Y_C (in mho) will be respectively

(a) 2, 6, –6 (b) 2,6,0
(c) 2,0,6 (d) 2,6,8

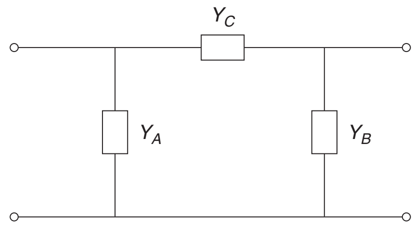

Fig. 9.134

9.17 The impedance matrices of two two-port networks are given by $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 15 & 5 \\ 3 & 25 \end{bmatrix}$. If these two networks are connected in series, the impedance matrix of the resulting two-port network will be

(a) $\begin{bmatrix} 3 & 5 \\ 2 & 25 \end{bmatrix}$ (b) $\begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$

(c) $\begin{bmatrix} 15 & 2 \\ 5 & 3 \end{bmatrix}$ (d) inderminate

9.18 If the π network and T network are equivalent, then the values of R_1, R_2 and R_3 (in ohms) will be respectively

(a) 6, 6, 6 (b) 6, 6, 9
(c) 9, 6, 9 (d) 6, 9, 6

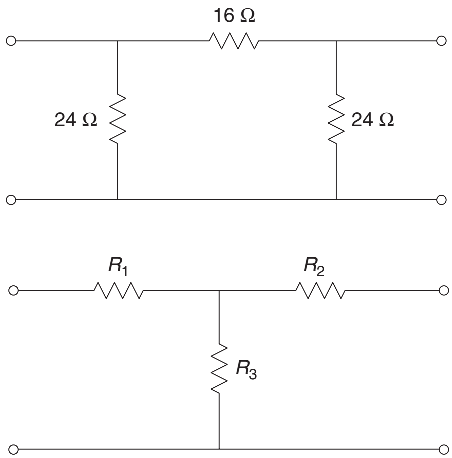

Fig. 9.135

9.19 For a two-port symmetrical bilateral network, if $A = 3$ and $B = 1$, the value of the parameter C will be

(a) 4 (b) 6
(c) 8 (d) 16

9.20 The impedance matrix for the network shown in Fig. 9.136 is

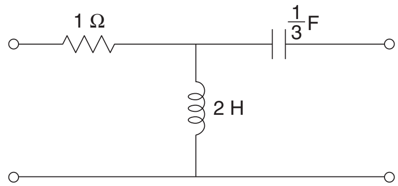

Fig. 9.136

(a) $\begin{bmatrix} 2s+1 & 2s \\ 2s & 2s+\frac{3}{s} \end{bmatrix}$ (b) $\begin{bmatrix} 2s+1 & -2s \\ -2s & 2s+\frac{3}{s} \end{bmatrix}$

(c) $\begin{bmatrix} 2s+1 & 2s \\ -2s & 2s+\frac{3}{s} \end{bmatrix}$ (d) $\begin{bmatrix} 2s+\frac{3}{2} & -2s \\ 2s & 2s+\frac{3}{s} \end{bmatrix}$

9.21 With the usual notations, a two-port resistive network satisfies the conditions $A = D = \frac{3}{2}B = \frac{4}{3}C$. The Z_{11} of the network is

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Answers to Objective-Type Questions

9.1. (a)	9.2. (d)	9.3. (b) , (c)	9.4. (b)	9.5. (a)	9.6. (c)	9.7. (c)
9.8. (a)	9.9. (d)	9.10. (d)	9.11. (b)	9.12. (d)	9.13. (b)	9.14. (a)
9.15. (c)	9.16. (c)	9.17. (b)	9.18. (a)	9.19. (c)	9.20. (a)	9.21. (b)