Mathematical modeling of a control system

Mathematical modeling of a control system is the process of drawing the block diagrams for the systems in order to determine their performance and transfer functions.

Mathematical Modelling of Control System

There are various types of physical systems,

- Mechanical systems
- Electrical systems
- Electronic systems
- Thermal systems
- Hydraulic systems
- Chemical systems

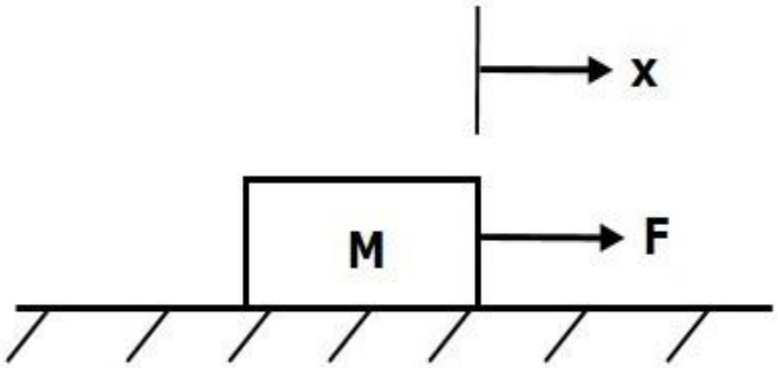
Mathematical Modelling of Mechanical Systems

- There are two types of mechanical systems based on the type of motion.
- Translational mechanical systems
- Rotational mechanical systems

- If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system.
- Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero.

Mass

- Mass is the property of a body, which stores **kinetic energy**.
- If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass.
- This opposing force is proportional to the acceleration of the body.



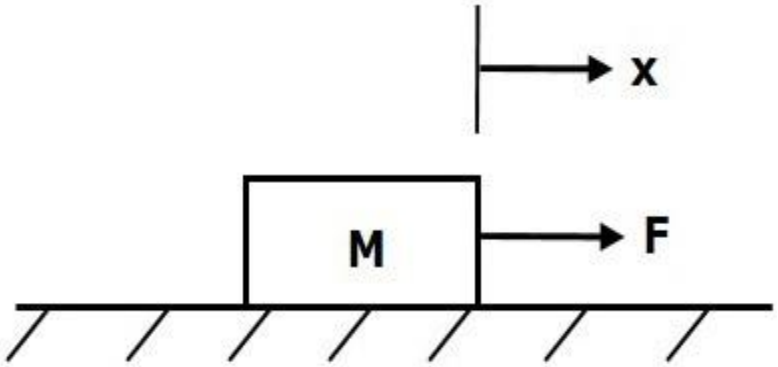
Mass

$$F_m \propto a$$

$$\Rightarrow F_m = Ma = M\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$$

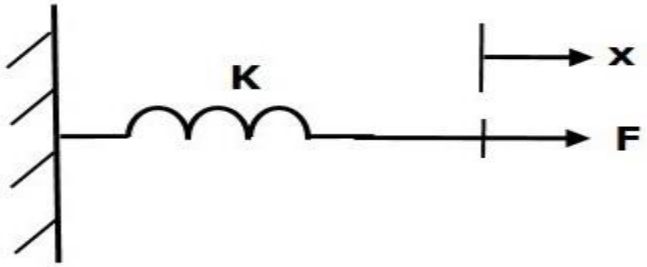
$$F = F_m = M\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$$

- **F** is the applied force
- F_m is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement



Spring

- Spring is an element, which stores **potential energy**.
- If a force is applied on spring **K**, then it is opposed by an opposing force due to elasticity of spring.
- This opposing force is proportional to the displacement of the spring.



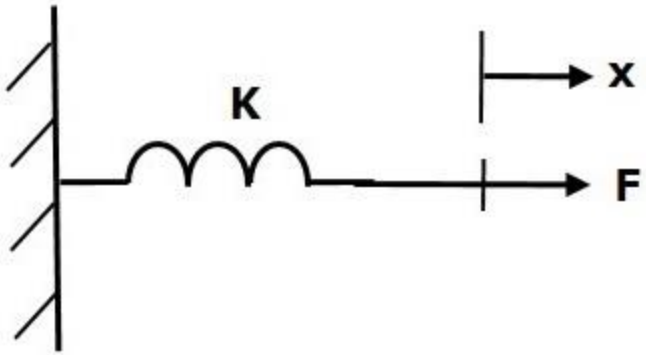
Spring

$$F \propto x$$

$$\Rightarrow F_k = Kx$$

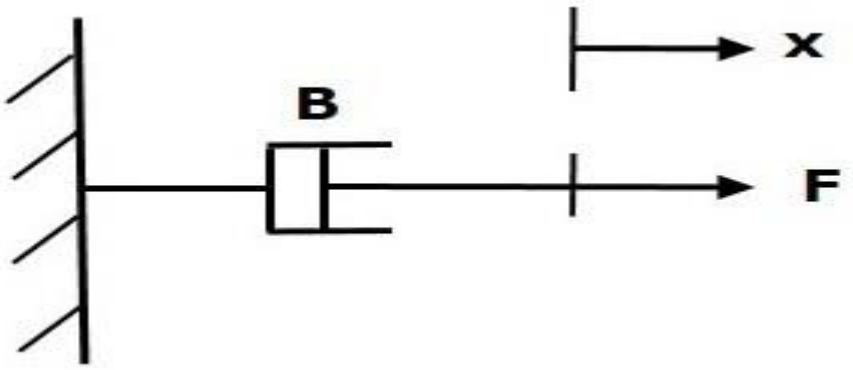
$$F = F_k = Kx$$

- **F** is the applied force
- F_k is the opposing force due to elasticity of spring
- **K** is spring constant
- **x** is displacement



Dashpot

- If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot.
- This opposing force is proportional to the velocity of the body.



Dashpot

$$F_b \propto \nu$$

$$\Rightarrow F_b = B\nu = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$F = F_b = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

- F_b is the opposing force due to friction of dashpot
- **B** is the frictional coefficient
- **v** is velocity
- **x** is displacement