2

Analysis of DC Circuits

2.1 INTRODUCTION

In Chapter 1, we have studied basic circuit concepts. In network analysis, we have to find currents and voltages in various parts of networks. In this chapter, we will study elementary network theorems like Kirchhoff's laws, mesh analysis and node analysis. These methods are applicable to all types of networks. The first step in analyzing networks is to apply Ohm's law and Kirchhoff's laws. The second step is the solving of these equations by mathematical tools. There are some other methods also to analyse circuits. We will also study superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem, Reciprocity theorem and Millman's theorem. We can find currents and voltages in various parts of the circuits with these methods.

2.2 KIRCHHOFF'S LAWS

The entire study of electric network analysis is based mainly on Kirchhoff's laws. But before discussing this, it is essential to familiarise ourselves with the following terms:

Node A node is a junction where two or more network elements are connected together.

Branch An element or number of elements connected between two nodes constitute a branch.

Loop A loop is any closed part of the circuit.

Mesh A mesh is the most elementary form of a loop and cannot be further divided into other loops. All meshes are loops but all loops are not meshes.

1. ***Kirchhoff's Current Law (KCL)*** The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.

Consider five conductors, carrying currents I_1, I_2, I_3, I_4 and I_5 meeting at a point O as shown in Fig. 2.1. Assuming the incoming currents to be positive and outgoing currents negative, we have

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

$$I_1 + I_3 + I_5 = I_2 + I_4$$

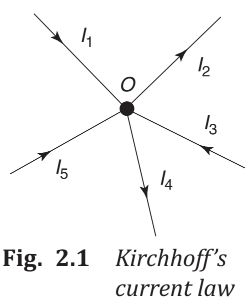


Fig. 2.1 *Kirchhoff's current law*

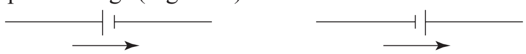
Thus, the above law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

2. ***Kirchhoff's Voltage Law (KVL)*** The algebraic sum of all the voltages in any closed circuit or mesh or loop is zero.

If we start from any point in a closed circuit and go back to that point, after going round the circuit, there is no increase or decrease in potential at that point. This means that the sum of emfs and the sum of voltage drops or rises meeting on the way is zero.

3. ***Determination of Sign*** A rise in potential can be assumed to be positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

(i) If we go from the positive terminal of the battery or source to the negative terminal, there is a fall in potential and so the emf should be assigned a negative sign (Fig. 2.2a). If we go from the negative terminal of the battery or source to the positive terminal, there is a rise in potential and so the emf should be given a positive sign (Fig. 2.2b).



(a) Fall in potential (b) Rise in potential

Fig. 2.2 *Sign convention*

(ii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in the potential and so the sign of this voltage drop is negative (Fig. 2.3a). If we go opposite to the direction of the current flow, there is a rise in potential and hence, this voltage drop should be given a positive sign (Fig. 2.3b).

(a) Fall in potential (b) Rise in potential

Fig. 2.3 *Sign convention*

2.3 MESH ANALYSIS

A mesh is defined as a loop which does not contain any other loops within it. Mesh analysis is applicable only for planar networks. A network is said to be planar if it can be drawn on a plane surface without crossovers. In this method, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. If a network has a large number of voltage sources, it is useful to use mesh analysis. Basically, this analysis consists of writing mesh equations by Kirchhoff's voltage law in terms of unknown mesh currents.

Steps to be Followed in Mesh Analysis

1. Identify the mesh, assign a direction to it and assign an unknown current in each mesh.
2. Assign the polarities for voltage across the branches.
3. Apply KVL around the mesh and use Ohm's law to express the branch voltages in terms of unknown mesh currents and the resistance.
4. Solve the simultaneous equations for unknown mesh currents.

Consider the network shown in Fig. 2.4 which has three meshes. Let the mesh currents for the three meshes be I_1, I_2, and I_3 and all the three mesh currents may be assumed to flow in the clockwise direction. The choice of direction for any mesh current is arbitrary.

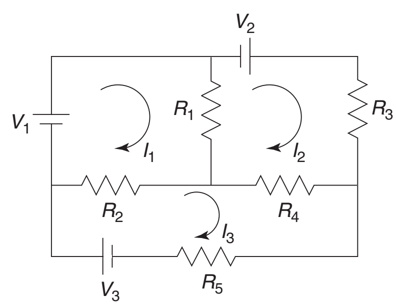


Fig. 2.4 *Circuit for mesh analysis*

Applying KVL to Mesh 1,

$$V_1 - R_1(I_1 - I_2) - R_2(I_1 - I_3) = 0$$

$$(R_1 + R_2)I_1 - R_1 I_2 - R_2 I_3 = V_1 \qquad \ldots(2.1)$$

Applying KVL to Mesh 2,

$$V_2 - R_3 I_2 - R_4(I_2 - I_3) - R_1(I_2 - I_1) = 0$$

$$-R_1 I_1 + (R_1 + R_3 + R_4)I_2 - R_4 I_3 = V_2 \qquad \ldots(2.2)$$

Applying KVL to Mesh 3,

$$-R_2(I_3 - I_1) - R_4(I_3 - I_2) - R_5 I_3 + V_3 = 0$$

$$-R_2 I_1 - R_4 I_2 + (R_2 + R_4 + R_5)I_3 = V_3 \qquad \ldots(2.3)$$

Writing Eqs (2.1), (2.2), and (2.3) in matrix form,

$$\begin{bmatrix} R_1 + R_2 & -R_1 & -R_2 \\ -R_1 & R_1 + R_3 + R_4 & -R_4 \\ -R_2 & -R_4 & R_2 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

In general,

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where, R_{11} = Self-resistance or sum of all the resistance of mesh 1
$R_{12} = R_{21}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 2
$R_{13} = R_{31}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 3
R_{22} = Self-resistance or sum of all the resistance of mesh 2
$R_{23} = R_{32}$ = Mutual resistance or sum of all the resistances common to meshes 2 and 3
R_{33} = Self-resistance or sum of all the resistance of mesh 3

If the directions of the currents passing through the common resistance are the same, the mutual resistance will have a positive sign, and if the direction of the currents passing through common resistance are opposite then the mutual resistance will have a negative sign. If each mesh current is assumed to flow in the clockwise direction then all self-resistances will always be positive and all mutual resistances will always be negative.

The voltages V_1, V_2 and V_3 represent the algebraic sum of all the voltages in meshes 1, 2 and 3 respectively. While going along the current, if we go from negative terminal of the battery to the positive terminal then its emf is taken as positive. Otherwise, it is taken as negative.

Example 2.1 *Find the current through the 5 Ω resistor is shown in Fig. 2.5.*

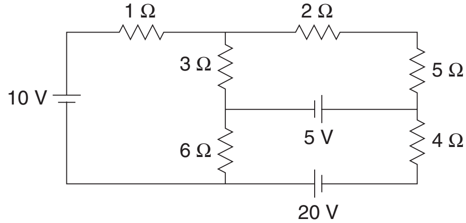


Fig. 2.5

Solution Assign clockwise currents in three meshes as shown in Fig. 2.6.

Applying KVL to Mesh 1,

$$10 - 1I_1 - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$

$$10I_1 - 3I_2 - 6I_3 = 10 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 5I_2 - 5 = 0$$

$$-3I_1 + 10I_2 = -5 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) + 5 - 4I_3 + 20 = 0$$

$$-6I_1 + 10I_3 = 25 \qquad \ldots\text{(iii)}$$

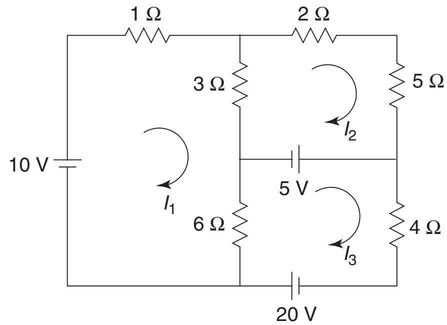


Fig. 2.6

Writing Eqs (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 10 & -3 & -6 \\ -3 & 10 & 0 \\ -6 & 0 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 25 \end{bmatrix}$$

We can write matrix equation directly from Fig. 2.6,

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where R_{11} = Self-resistance of Mesh 1 = 1 + 3 + 6 = 10 Ω

R_{12} = Mutual resistance common to meshes 1 and 2 = –3 Ω

Here, negative sign indicates that the current through common resistance are in opposite direction.

R_{13} = Mutual resistance common to meshes 1 and 3 = –6 Ω

Similarly,

$$R_{21} = -3\ \Omega$$

$$R_{22} = 3 + 2 + 5 = 10\ \Omega$$

$$R_{23} = 0$$

$$R_{31} = -6\ \Omega$$

$$R_{32} = 0$$

$$R_{33} = 6 + 4 = 10\ \Omega$$

For voltage matrix,

$$V_1 = 10\text{ V}$$

$$V_2 = -5\text{ V}$$

V_3 = algebraic sum of all the voltages in mesh 3 = 5 + 20 = 25 V

Solving Eqs (i), (ii) and (iii),

$$I_1 = 4.27\text{ A}$$

$$I_2 = 0.78\text{ A}$$

$$I_3 = 5.06\text{ A}$$

$$I_{5\Omega} = I_2 = 0.78\text{ A}$$

Example 2.2 *Determine the current through the 5 Ω resistor of the network shown in Fig. 2.7.*

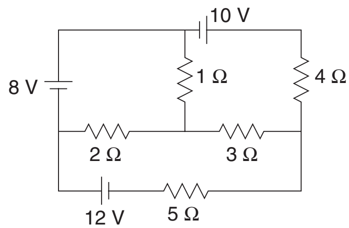


Fig. 2.7

Solution Assign clockwise currents in three meshes as shown in Fig. 2.8.

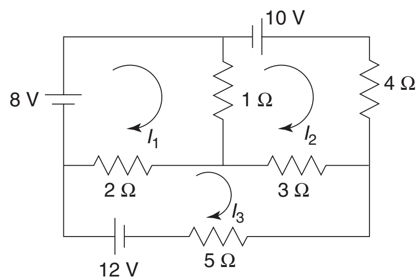


Fig. 2.8

Applying KVL to Mesh 1,

$$8 - 1(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$3I_1 - I_2 - 2I_3 = 8 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$10 - 4I_2 - 3(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-I_1 + 8I_2 - 3I_3 = 10 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 3(I_3 - I_2) - 5I_3 + 12 = 0$$

$$-2I_1 - 3I_2 + 10I_3 = 12 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$I_1 = 6.01 \text{ A}$$

$$I_2 = 3.27 \text{ A}$$

$$I_3 = 3.38 \text{ A}$$

$$I_{5\Omega} = I_3 = 3.38 \text{ A}$$

Example 2.3 *Find the current through the 2 Ω resistor in the network of Fig. 2.9.*

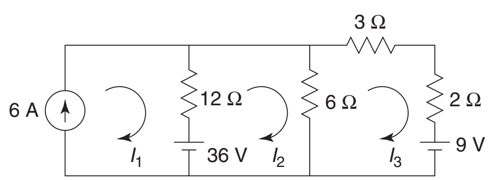


Fig. 2.9

Solution Mesh 1 contains a current source of 6 A. Hence, we can write current equation for Mesh 1. Since direction of current source and mesh current I_1 are same,

$$I_1 = 6 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$36 - 12(I_2 - I_1) - 6(I_2 - I_3) = 0$$
$$36 - 12(I_2 - 6) - 6I_2 + 6I_3 = 0$$
$$18I_2 - 6I_3 = 108 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_2) - 3I_3 - 2I_3 - 9 = 0$$
$$6I_2 - 11I_3 = 9 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_3 = 3\text{ A}$$
$$I_{2\Omega} = I_3 = 3\text{ A}$$

Example 2.4 *Find the current through the 5 Ω resistor in the network of Fig. 2.10.*

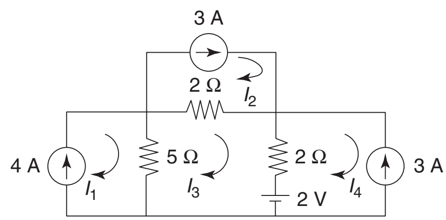


Fig. 2.10

Solution Writing current equations for Meshes 1, 2 and 4,

$$I_1 = 4 \qquad \ldots\text{(i)}$$
$$I_2 = 3 \qquad \ldots\text{(ii)}$$
$$I_4 = -3 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$-5(I_3 - I_1) - 2(I_3 - I_2) - 2(I_3 - I_4) - 2 = 0 \qquad \ldots\text{(iv)}$$

Substituting Eqs (i), (ii) and (iii) in Eq. (iv),

$$-5(I_3 - 4) - 2(I_3 - 3) - 2(I_3 + 3) - 2 = 0$$
$$I_3 = 2\text{ A}$$
$$I_{5\Omega} = I_1 - I_3 = 4 - 2 = 2\text{ A}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.5 *Obtain the branch currents in the network shown in Fig. 2.11.*

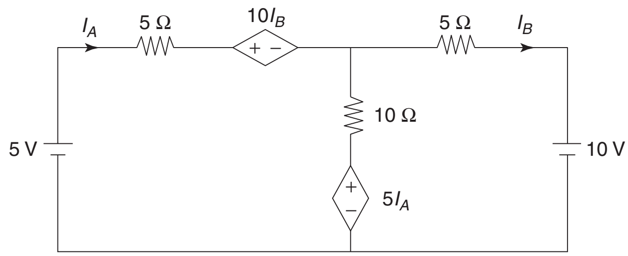


Fig. 2.11

Solution Assign clockwise currents in two meshes as shown in Fig. 2.12.
From Fig. 2.12,

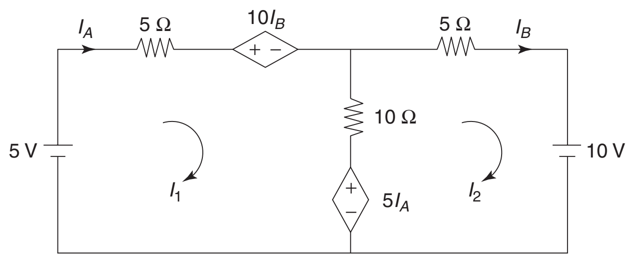


Fig. 2.12

$$I_A = I_1 \quad \ldots\text{(i)}$$
$$I_B = I_2 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$5-5I_1-10I_B-10(I_1-I_2)-5I_A=0$$
$$5-5I_1-10I_2-10I_1+10I_2-5I_1=0$$
$$-20I_1=-5$$
$$I_1=\frac{1}{4}=0.25\text{ A} \quad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$5I_A-10(I_2-I_1)-5I_2-10=0$$
$$5I_1-10I_2+10I_1-5I_2=10$$
$$15I_1-15I_2=10 \quad \ldots\text{(iv)}$$

Putting $I_1 = 0.25$ A in Eq. (iv),

$$15(0.25)-15I_2=10$$
$$I_2=-0.416\text{ A}$$

Example 2.6 *Find the mesh currents in the network shown in Fig. 2.13.*

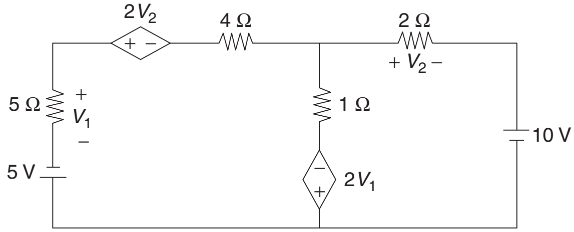


Fig. 2.13

Solution Assign clockwise currents in the two meshes as shown in Fig. 2.14.

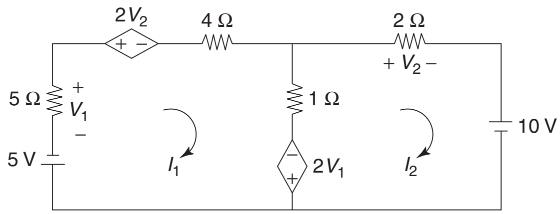


Fig. 2.14

From Fig. 2.14,

$$V_1 = -5I_1 \quad \ldots\text{(i)}$$

$$V_2 = 2I_2 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$-5 - 5I_1 - 2V_2 - 4I_1 - 1(I_1 - I_2) + 2V_1 = 0$$

$$-5 - 5I_1 - 2(2I_2) - 4I_1 - I_1 + I_2 + 2(-5I_1) = 0$$

$$20I_1 + 3I_2 = -5 \quad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$-2V_1 - 1(I_2 - I_1) - 2I_2 - 10 = 0$$

$$-2(-5I_1) - I_2 + I_1 - 2I_2 = 10$$

$$11I_1 - 3I_2 = 10 \quad \ldots\text{(iv)}$$

Solving Eqs (iii) and (iv),

$$I_1 = 0.161 \text{ A}$$

$$I_2 = -2.742 \text{ A}$$

Example 2.7 *Find currents I_x and I_y of the network shown in Fig. 2.15.*

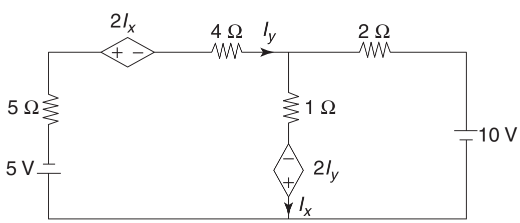


Fig. 2.15

Solution Assign clockwise currents in the two meshes as shown in Fig. 2.16.

From Fig. 2.16,

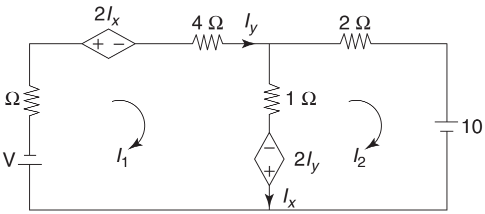


Fig. 2.16

$$I_y = I_1 \qquad \text{...(i)}$$

$$I_x = I_1 - I_2 \qquad \text{...(ii)}$$

Applying KVL to Mesh 1,

$$-5-5I_1-2I_x-4I_1-1(I_1-I_2)+2I_y=0$$

$$-5-5I_1-2(I_1-I_2)-4I_1-I_1+I_2+2I_1=0$$

$$-5-5I_1-2I_1+2I_2-4I_1-I_1+I_2+2I_1=0$$

$$-10I_1+3I_2=5 \qquad \text{...(iii)}$$

Applying KVL to Mesh 2,

$$-2I_y-1(I_2-I_1)-2I_2-10=0$$

$$-2I_1-I_2+I_1-2I_2=10$$

$$-I_1-3I_2=10 \qquad \text{...(iv)}$$

Solving Eqs (iii) and (iv),

$$I_1=-\frac{15}{11}=-1.364\text{ A}$$

$$I_2=-2.878\text{ A}$$

$$I_y=-1.364\text{ A}$$

$$I_x=I_1-I_2=-1.364+2.878=1.514\text{ A}$$

Example 2.8 *Find the currents in the three meshes of the network shown in Fig. 2.17.*

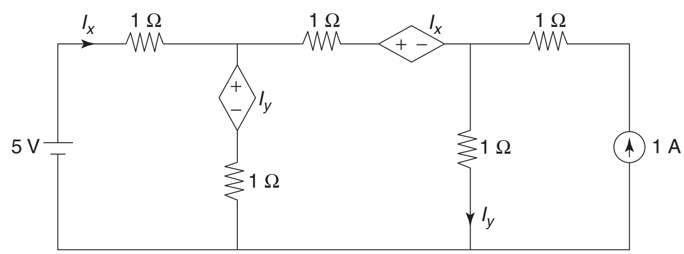


Fig. 2.17

Solution Assign clockwise currents in the three meshes as shown in Fig. 2.18.

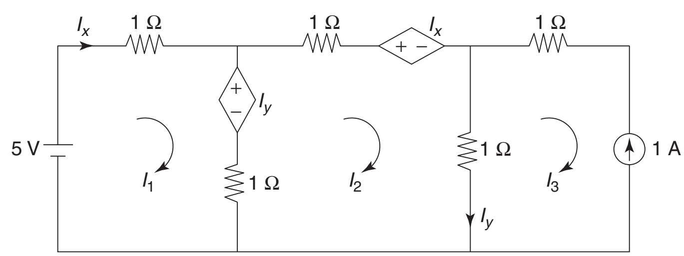


Fig. 2.18

From Fig. 2.18,

$$I_x = I_1 \qquad \text{...(i)}$$

$$I_x = I_1$$

$$I_y = I_2 - I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 1,

$$5 - 1I_1 - I_y - 1(I_1 - I_2) = 0$$

$$5 - I_1 - (I_2 - I_3) - (I_1 - I_2) = 0$$

$$-2I_1 + I_3 = -5 \qquad \text{...(iii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + I_y - 1I_2 - I_x - 1(I_2 - I_3) = 0$$

$$-(I_2 - I_1) + (I_2 - I_3) - I_2 - I_1 - (I_2 - I_3) = 0$$

$$-2I_2 = 0 \qquad \text{...(iv)}$$

For Mesh 3,

$$I_3 = -1 \qquad \text{...(v)}$$

Solving Eqs (iii), (iv) and (v),

$$I_1 = 2 \text{ A}$$

$$I_2 = 0$$

$$I_3 = -1 \text{ A}$$

Example 2.9 *For the network shown in Fig. 2.19, find the power supplied by the dependent voltage source.*

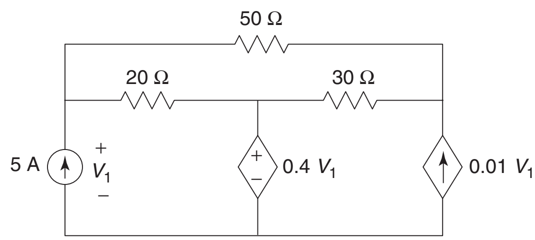


Fig. 2.19

Solution Assign clockwise currents in three meshes as shown in Fig. 2.20.

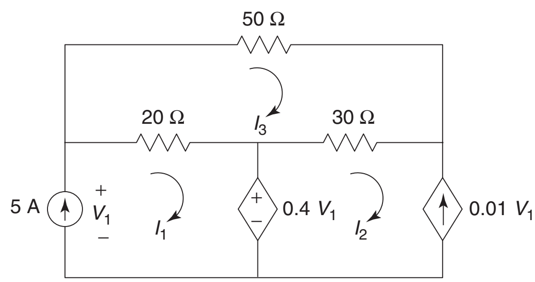


Fig. 2.20

From Fig. 2.20,

$$V_1 - 20(I_1 - I_3) - 0.4\,V_1 = 0$$

$$0.6\,V_1 = 20\,I_1 - 20\,I_3$$

$$V_1 = 33.33\,I_1 - 33.33\,I_3 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 5 \qquad \ldots\text{(ii)}$$

For Mesh 2,

$$I_2 = -0.01\,V_1 = -0.01(33.33\,I_1 - 33.33\,I_3)$$

$$0.33\,I_1 + I_2 - 0.33\,I_3 = 0 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$-50\,I_3 - 30(I_3 - I_2) - 20(I_3 - I_1) = 0$$

$$-20\,I_1 - 30\,I_2 + 100\,I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 5\text{ A}$$

$$I_2 = -1.47\text{ A}$$

$$I_3 = 0.56\text{ A}$$

$$V_1 = 33.33\,I_1 - 33.33\,I_3 = 33.33(5) - 33.33(0.56) = 148\text{ V}$$

Power supplied by the dependent voltage source $= 0.4\,V_1(I_1 - I_2) = 0.4\,(148)(5 + 1.47) = 383.02$ W

Example 2.10 *Find the voltage V_x in the network shown in Fig. 2.21.*

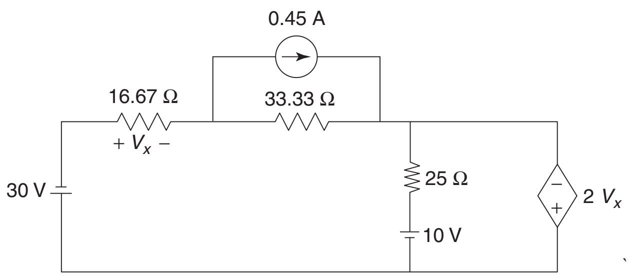


Fig. 2.21

Solution Assign clockwise currents in the three meshes as shown in Fig. 2.22.

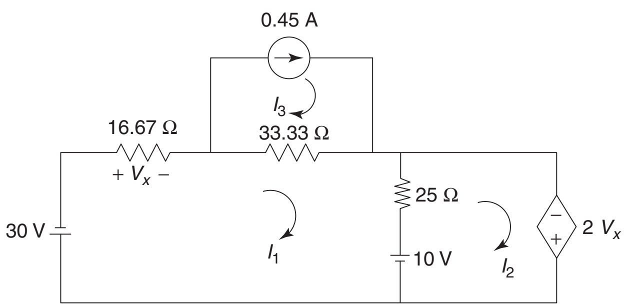


Fig. 2.22

From Fig. 2.22,

$$V_x = 16.67\,I_1 \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-30 - 16.67\,I_1 - 33.33(I_1 - I_3) - 25(I_1 - I_2) - 10 = 0$$
$$-30 - 16.67\,I_1 - 33.33\,I_1 + 33.33\,I_3 - 25\,I_1 + 25\,I_2 - 10 = 0$$
$$-75\,I_1 + 25\,I_2 + 33.33\,I_3 = 40 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$10 - 25(I_2 - I_1) + 2\,V_x = 0$$
$$10 - 25(I_2 - I_1) + 2(16.67\,I_1) = 0$$
$$10 - 25\,I_2 + 25\,I_1 + 33.34\,I_1 = 0$$
$$58.34\,I_1 - 25\,I_2 = -10 \qquad \text{...(iii)}$$

For Mesh 3,

$$I_3 = 0.45 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -0.9\text{ A}$$
$$I_2 = -1.7\text{ A}$$
$$I_3 = 0.45\text{ A}$$
$$V_x = 16.67\,I_1 = 16.67\,(-0.9) = -15\text{ V} \qquad \text{...(v)}$$

Example 2.11 *For the network shown in Fig. 2.23, find the mesh currents I_1, I_2 and I_3.*

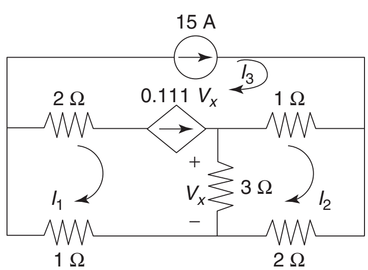


Fig. 2.23

Solution From Fig. 2.23,

$$V_x = 3(I_1 - I_2) \qquad \text{...(i)}$$

Writing current equation for the two current sources,

$$I_3 = 15 \qquad \text{...(ii)}$$

and

$$0.111\,V_x = I_1 - I_3$$
$$0.111\,[3\,(I_1 - I_2)] = I_1 - I_3$$

$$0.333\,I_1 - 0.333\,I_2 - I_1 + I_3 = 0$$
$$-0.667\,I_1 - 0.333\,I_2 + I_3 = 0 \quad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$-3\,(I_2 - I_1) - 1(I_2 - I_3) - 2\,I_2 = 0$$
$$-3\,I_1 + 6\,I_2 - I_3 = 0 \quad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 17 \text{ A}$$
$$I_2 = 11 \text{ A}$$
$$I_3 = 15 \text{ A}$$

Example 2.12 *For the network shown in Fig. 2.24, find the magnitude of V_0 and the current supplied by it, given that power loss in $R_L = 2\ \Omega$ resistor is 18 W.*

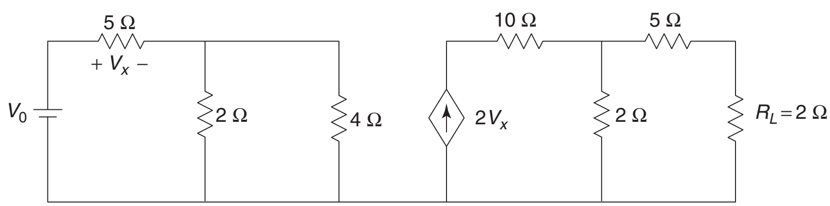


Fig. 2.24

Solution Assign clockwise currents in meshes as shown in Fig. 2.25.

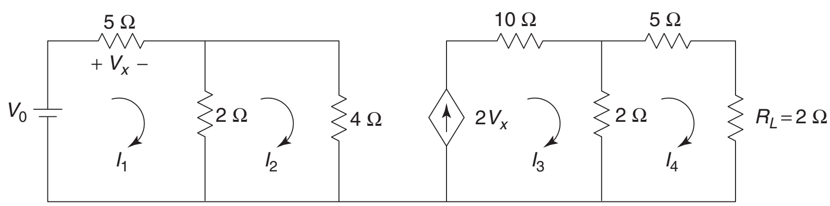


Fig. 2.25

From Fig. 2.25,

$$V_x = 5\,I_1 \quad \ldots\text{(i)}$$

Also,

$$I_4^{\,2}\,R_L = 18$$
$$I_4^{\,2}(2) = 18$$
$$I_4 = 3 \text{ A} \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$V_0 - 5\,I_1 - 2(I_1 - I_2) = 0$$
$$7\,I_1 - 2\,I_2 = V_0 \quad \ldots\text{(iii)}$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 4I_2 = 0$$

$$-2I_1 + 6I_2 = 0 \qquad \ldots\text{(iv)}$$

For Mesh 3,

$$I_3 = 2V_x = 2(5I_1) = 10I_1$$

$$10I_1 - I_3 = 0 \qquad \ldots\text{(v)}$$

Applying KVL to Mesh 4,

$$-2(I_4 - I_3) - 5I_4 - 2I_4 = 0$$

$$-2I_3 + 9I_4 = 0$$

$$-2I_3 + 9(3) = 0$$

$$I_3 = 13.5 \text{ A}$$

From Eq. (v),

$$I_1 = \frac{I_3}{10} = \frac{13.5}{10} = 1.35 \text{ A}$$

From Eq. (iv),

$$-2(1.35) + 6I_2 = 0$$

$$I_2 = 0.45 \text{ A}$$

From Eq. (iii),

$$7(1.35) - 2(0.45) = V_0$$

$$V_0 = 8.55 \text{ V}$$

Current supplied by voltage source $V_0 = I_1 = 1.35$ A

Example 2.13 *In the network shown in Fig. 2.26, find voltage V_2 such that $V_x = 0$.*

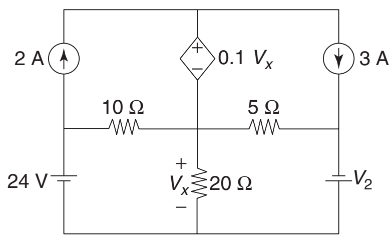


Fig. 2.26

Solution Assign clockwise currents in four meshes as shown in Fig. 2.27.

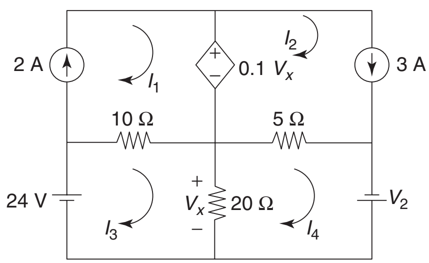


Fig. 2.27

From Fig. 2.27,

$$V_x = 20(I_3 - I_4) \qquad \ldots\text{(i)}$$

Writing current equations for Meshes 1 and 2,

$$I_1 = 2 \qquad \ldots\text{(ii)}$$

$$I_2 = 3 \qquad \ldots\text{(iii)}$$

Applying KVL to Mesh 3,

$$24 - 10(I_3 - I_1) - 20(I_3 - I_4) = 0$$
$$24 - 10(I_3 - 2) - 20(I_3 - I_4) = 0$$
$$-30 I_3 + 20 I_4 = -44 \qquad \ldots(iv)$$

Applying KVL to Mesh 4,

$$-20(I_4 - I_3) - 5(I_4 - I_2) + V_2 = 0$$
$$-20(I_4 - I_3) - 5(I_4 - 3) + V_2 = 0$$
$$20 I_3 - 25 I_4 = -V_2 - 15 \qquad \ldots(v)$$

But

$$V_x = 0$$
$$20(I_3 - I_4) = 0$$
$$I_3 = I_4$$

From Eq. (iv),

$$-30 I_3 + 20 I_3 = -44$$
$$I_3 = 4.4 \text{ A}$$
$$I_4 = 4.4 \text{ A}$$

From Eq. (v),

$$20\,(4.4) - 25\,(4.4) = -V_2 - 15$$
$$V_2 = 7 \text{ V}$$

2.4 SUPERMESH ANALYSIS

Meshes that share a current source with other meshes, none of which contains a current source in the outer loop, form a supermesh. A path around a supermesh doesn't pass through a current source. A path around each mesh contained within a supermesh passes through a current source. The total number of equations required for a supermesh is equal to the number of meshes contained in the supermesh. A supermesh requires one mesh current equation, that is, a KVL equation. The remaining mesh current equations are KCL equations.

Example 2.14 *Find the current in the 3 Ω resistor of the network shown in Fig. 2.28.*

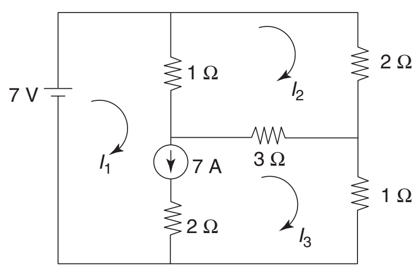


Fig. 2.28

Solution Meshes 1 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_1 - I_3 = 7 \qquad \ldots(i)$$

Applying KVL to the outer path of the supermesh,

$$7-1(I_1-I_2)-3(I_3-I_2)-1I_3=0$$

$$-I_1+4I_2-4I_3=-7 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2-I_1)-2I_2-3(I_2-I_3)=0$$

$$I_1-6I_2+3I_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=9\text{ A}$$

$$I_2=2.5\text{ A}$$

$$I_3=2\text{ A}$$

Current through the 3 Ω resistor = $I_2 - I_3$ = 2.5 – 2 = 0.5 A

Example 2.15 *Find the current in the 5 Ω resistor of the network shown in Fig. 2.29.*

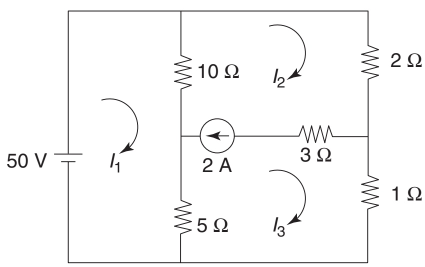


Fig. 2.29

Solution Applying KVL to Mesh 1,

$$50-10(I_1-I_2)-5(I_1-I_3)=0$$

$$15I_1-10I_2-5I_3=50 \qquad \text{...(i)}$$

Meshes 2 and 3 will form a supermesh as these two meshes share a common current source of 2 A. Writing current equation for the supermesh,

$$I_2-I_3=2 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-10(I_2-I_1)-2I_2-1I_3-5(I_3-I_1)=0$$

$$-15I_1+12I_2+6I_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=20\text{ A}$$

$$I_2=17.33\text{ A}$$

$$I_3=15.33\text{ A}$$

Current through the 5 Ω resistor $= I_1 - I_3 = 20 - 15.33 = 4.67$ A

Example 2.16 *Determine the power delivered by the voltage source and the current in the 10 Ω resistor of the network shown in Fig. 2.30.*

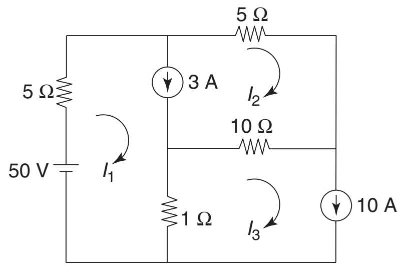


Fig. 2.30

Solution Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_1 - I_2 = 3 \qquad \ldots\text{(i)}$$

Applying KVL to the outer path of the supermesh,

$$50 - 5I_1 - 5I_2 - 10(I_2 - I_3) - 1(I_1 - I_3) = 0$$

$$-6I_1 - 15I_2 + 11I_3 = -50 \qquad \ldots\text{(ii)}$$

For Mesh 3,

$$I_3 = 10 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 9.76 \text{ A}$$

$$I_2 = 6.76 \text{ A}$$

$$I_3 = 10 \text{ A}$$

Power delivered by the voltage source $= 50\, I_1 = 50 \times 9.76 = 488$ W

$$I_{10\,\Omega} = I_3 - I_2 = 10 - 6.76 = 3.24 \text{ A}$$

Example 2.17 *For the network shown in Fig. 2.31, find current through the 8 Ω resistor.*

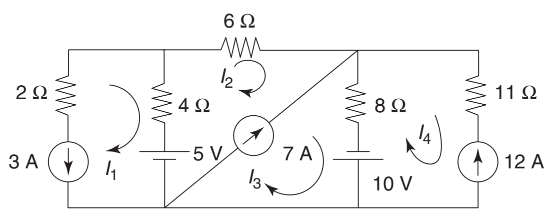


Fig. 2.31

Writing current equations for meshes 1 and 4,

$$I_1 = -3 \quad \text{...(i)}$$

$$I_4 = -12 \quad \text{...(ii)}$$

Meshes 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_3 - I_2 = 7 \quad \text{...(iii)}$$

Applying KVL to the outer path of the supermesh,

$$5 - 4(I_2 - I_1) - 6I_2 - 8(I_3 - I_4) + 10 = 0$$

$$5 - 4(I_2 + 3) - 6I_2 - 8(I_3 + 12) + 10 = 0$$

$$-10I_2 - 8I_3 = 93 \quad \text{...(iv)}$$

Solving Eqs (iii) and (iv),

$$I_2 = -8.28 \text{ A}$$

$$I_3 = -1.28 \text{ A}$$

$$I_{8\,\Omega} = I_3 - I_4 = -1.28 + 12 = 10.72 \text{ A}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.18 *In the network of Fig. 2.32, find currents I_1 and I_2.*

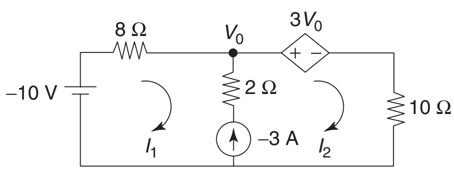


Fig. 2.32

Solution From Fig. 2.32,

$$-10 - 8I_1 - V_0 = 0$$

$$V_0 = -10 - 8I_1 \quad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equations for the supermesh,

$$I_2 - I_1 = -3 \quad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-10 - 8I_1 - 3V_0 - 10I_2 = 0$$

$$-10 - 8I_1 - 3(-10 - 8I_1) - 10I_2 = 0$$

$$16I_1 - 10I_2 = -20 \quad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -8.33 \text{ A}$$

$$I_2 = -11.33 \text{ A}$$

Example 2.19 *In the network of Fig. 2.33, find the current through the 3 Ω resistor.*

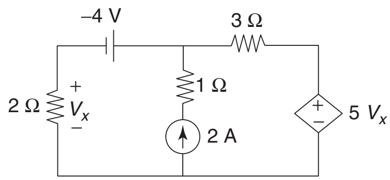


Fig. 2.33

Solution Assign clockwise currents in two meshes as shown in Fig. 2.34.
From Fig. 2.34,

$$V_x = -2\,I_1 \qquad \ldots\text{(i)}$$

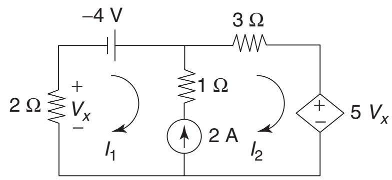


Fig. 2.34

Meshes 1 and 2 will form a supermesh.
Writing current equations for the supermesh,

$$I_2 - I_1 = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-2I_1 - 4 - 3I_2 - 5V_x = 0$$
$$-2I_1 - 4 - 3I_2 - 5(-2I_1) = 0$$
$$8\,I_1 - 3I_2 = 4 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 2\text{ A}$$
$$I_2 = 4\text{ A}$$
$$I_{3\,\Omega} = I_2 = 4\text{ A}$$

Example 2.20 *Find the currents I_1 and I_2 at the network shown in Fig. 2.35.*

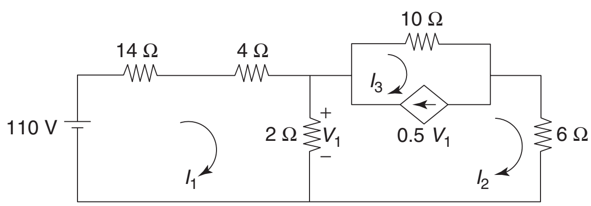


Fig. 2.35

Solution From Fig. 2.35,

$$V_1 = 2(I_1 - I_2)$$

Meshes 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_3 - I_2 = 0.5\text{ V}_1 = 0.5 \times 2(I_1 - I_2) = I_1 - I_2$$
$$I_3 = I_1$$

Applying KVL to outer path of the supermesh,

$$-2(I_2 - I_1) - 10 I_3 - 6 I_2 = 0$$
$$-2 I_2 + 2 I_1 - 10 I_1 - 6 I_2 = 0$$
$$I_1 = -I_2$$

Applying KVL to Mesh 1,

$$110 - 14 I_1 - 4 I_1 - 2(I_1 - I_2) = 0$$
$$110 - 20 I_1 + 2 I_2 = 0$$
$$110 + 20 I_1 + 2 I_2 = 0$$
$$I_2 = -5 \text{ A}$$
$$I_1 = -I_2 = 5 \text{ A}$$

Example 2.21 *For the network of Fig. 2.36, find current through the 8 Ω resistor.*

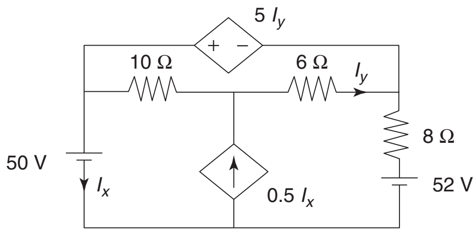


Fig. 2.36

Solution Assign clockwise currents to the three meshes as shown in Fig. 2.37.
From Fig. 2.37,

$$I_x = -I_1 \qquad \text{...(i)}$$
$$I_y = I_2 - I_3 \qquad \text{...(ii)}$$

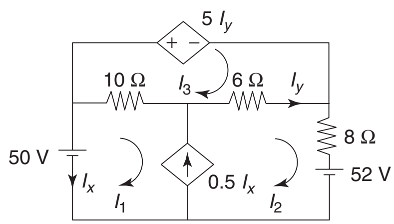


Fig. 2.37

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 0.5\, I_x = 0.5\,(-I_1)$$
$$-0.5\, I_1 + I_2 = 0 \qquad \text{...(iii)}$$

Applying KVL to the outer path of the supermesh,

$$50 - 10(I_1 - I_3) - 6(I_2 - I_3) - 8 I_2 - 52 = 0$$
$$-10 I_1 - 14 I_2 + 16 I_3 = 2 \qquad \text{...(iv)}$$

Applying KVL to Mesh 3,

$$-5\, I_y - 6\,(I_3 - I_2) - 10\,(I_3 - I_1) = 0$$
$$-5\,(I_2 - I_3) - 6\,(I_3 - I_2) - 10\,(I_3 - I_1) = 0$$
$$10\, I_1 + I_2 - 11\, I_3 = 0 \qquad \text{...(v)}$$

Solving Eqs (iii), (iv) and (v),

$$I_1 = -1.56 \text{ A}$$
$$I_2 = -0.58 \text{ A}$$
$$I_3 = -1.11 \text{ A}$$
$$I_{8\,\Omega} = I_2 = -0.58 \text{ A}$$

Example 2.22 *For the network shown in Fig. 2.38, find the current through the 10 Ω resistor.*

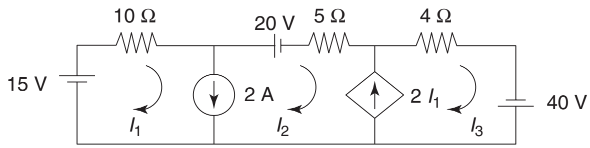


Fig. 2.38

Solution Meshes 1, 2 and 3 will form a supermesh.

Writing current equations for the supermesh,

$$I_1 - I_2 = 2 \qquad \text{...(i)}$$
$$I_3 - I_2 = 2\,I_1$$

and
$$2\,I_1 + I_2 - I_3 = 0 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$15 - 10\,I_1 - 20 - 5\,I_2 - 4I_3 + 40 = 0$$
$$10\,I_1 + 5\,I_2 + 4\,I_3 = 35 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 1.96 \text{ A}$$
$$I_2 = -0.04 \text{ A}$$
$$I_3 = 3.89 \text{ A}$$
$$I_{10\,\Omega} = I_1 = 1.96 \text{ A}$$

Example 2.23 *In the network shown in Fig. 2.39, find the power delivered by the 4 V source and voltage across the 2 Ω resistor.*

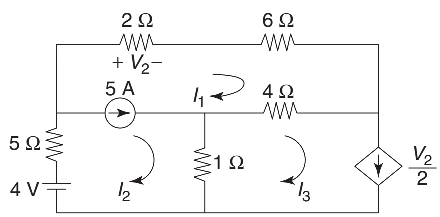


Fig. 2.39

Solution From Fig. 2.39,

$$V_2 = 2\,I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 5 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$4 - 5I_2 - 2I_1 - 6I_1 - 4(I_1 - I_3) - 1(I_2 - I_3) = 0$$

$$-12I_1 - 6I_2 + 5I_3 = -4 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = \frac{V_2}{2} = \frac{2I_1}{2} = I_1$$

$$I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -2 \text{ A}$$

$$I_2 = 3 \text{ A}$$

$$I_3 = -2 \text{ A}$$

Power delivered by the 4 V source $= 4I_2 = 4(3) = 12$ W

$$V_{2\,\Omega} = 2I_1 = 2(-2) = -4 \text{ V}$$

Example 2.24 *Find currents I_1, I_2, I_3, I_4 of the network shown in Fig. 2.40.*

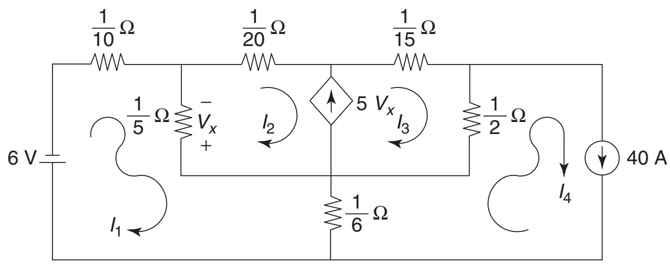


Fig. 2.40

From Fig. 2.40,

$$V_x = \frac{1}{5}(I_2 - I_1) \qquad \ldots\text{(i)}$$

For Mesh 4,

$$I_4 = 40 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$-6 - \frac{1}{10}I_1 - \frac{1}{5}(I_1 - I_2) - \frac{1}{6}(I_1 - I_4) = 0$$

$$-6 - \frac{1}{10}I_1 - \frac{1}{5}I_1 + \frac{1}{5}I_2 - \frac{1}{6}I_1 + \frac{1}{6}(40) = 0$$

$$-\frac{7}{15}I_1 + \frac{1}{5}I_2 = -\frac{2}{3} \qquad \ldots\text{(iii)}$$

Mesh 2 and 3 will form a supermesh.

Writing current equation for the supermesh,

$$I_3 - I_2 = 5\,V_x = 5\left[\frac{1}{5}(I_2 - I_1)\right] = I_2 - I_1$$

$$I_1 - 2I_2 + I_3 = 0 \qquad \text{...(iv)}$$

Applying KVL to the outer path of the supermesh,

$$-\frac{1}{5}(I_2 - I_1) - \frac{1}{20}I_2 - \frac{1}{15}I_3 - \frac{1}{2}(I_3 - I_4) = 0$$

$$-\frac{1}{5}I_2 + \frac{1}{5}I_1 - \frac{1}{20}I_2 - \frac{1}{15}I_3 - \frac{1}{2}I_3 + \frac{1}{2}(40) = 0$$

$$\frac{1}{5}I_1 - \frac{1}{4}I_2 - \frac{17}{30}I_3 = -20 \qquad \text{...(v)}$$

Solving Eqs (iii), (iv) and (v),

$$I_1 = 10\text{ A}$$
$$I_2 = 20\text{ A}$$
$$I_3 = 30\text{ A}$$
$$I_4 = 40\text{ A}$$

2.5 NODE ANALYSIS

Node analysis is based on Kirchhoff's current law which states that the algebraic sum of currents meeting at a point is zero. Every junction where two or more branches meet is regarded as a node. One of the nodes in the network is taken as *reference node* or *datum node*. If there are *n* nodes in any network, the number of simultaneous equations to be solved will be $(n - 1)$.

Steps to be followed in Node Analysis

1. Assuming that a network has *n* nodes, assign a reference node and the reference directions, and assign a current and a voltage name for each branch and node respectively.
2. Apply KCL at each node except for the reference node and apply Ohm's law to the branch currents.
3. Solve the simultaneous equations for the unknown node voltages.
4. Using these voltages, find any branch currents required.

Example 2.25 *Determine the current through the 5 Ω resistor for the network shown in Fig. 2.41.*

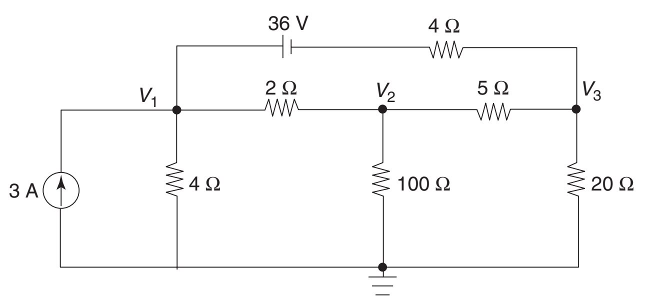


Fig. 2.41

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1}{4}+\frac{V_1-V_2}{2}+\frac{V_1-36-V_3}{4}=3$$

$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)V_1-\frac{1}{2}V_2-\frac{1}{4}V_3=3+\frac{36}{4}$$

$$V_1-0.5\,V_2-0.25\,V_3=12 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{2}+\frac{V_2}{100}+\frac{V_2-V_3}{5}=0$$

$$-\frac{1}{2}V_1+\left(\frac{1}{2}+\frac{1}{100}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=0$$

$$-0.5\,V_1+0.71\,V_2-0.2\,V_3=0 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{20}+\frac{V_3-(-36)-V_1}{4}=0$$

$$-\frac{1}{4}V_1-\frac{1}{5}V_2+\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)V_3=-9$$

$$-0.25\,V_1-0.2\,V_2+0.5\,V_3=-9 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=13.41\text{ V}$$
$$V_2=7.06\text{ V}$$
$$V_3=-8.47\text{ V}$$

$$\text{Current through the }5\,\Omega\text{ resistor}=\frac{V_2-V_3}{5}=\frac{7.06-(-8.47)}{5}=3.11\text{ A}$$

Example 2.26 *Find the power dissipated in the 6 Ω resistor for the network shown in Fig. 2.42.*

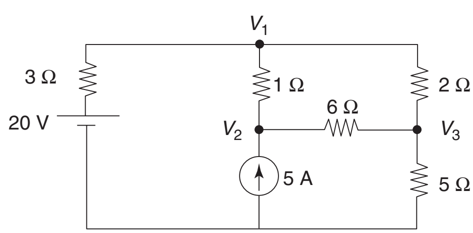


Fig. 2.42

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1-20}{3}+\frac{V_1-V_2}{1}+\frac{V_1-V_3}{2}=0$$

$$\left(\frac{1}{3}+1+\frac{1}{2}\right)V_1-V_2-\frac{1}{2}V_3=\frac{20}{3}$$

$$1.83\,V_1-V_2-0.5\,V_3=6.67 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{1}+\frac{V_2-V_3}{6}=5$$

$$-V_1+\left(1+\frac{1}{6}\right)V_2-\frac{1}{6}V_3=5$$

$$-V_1+1.17\,V_2-0.17\,V_3=5 \qquad \text{...(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3-V_1}{2}+\frac{V_3}{5}+\frac{V_3-V_2}{6}=0$$

$$-\frac{1}{2}V_1-\frac{1}{6}V_2+\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{6}\right)V_3=0$$

$$-0.5\,V_1-0.17\,V_2+0.87\,V_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=23.82\text{ V}$$

$$V_2=27.4\text{ V}$$

$$V_3=19.04\text{ V}$$

$$I_{6\,\Omega}=\frac{V_2-V_3}{6}=\frac{27.4-19.04}{6}=1.39\text{ A}$$

Power dissipated in the 6 Ω resistor $=(1.39)^2\times 6=11.59$ W

Example 2.27 *Find V_1 and V_2 for the network shown in Fig. 2.43.*

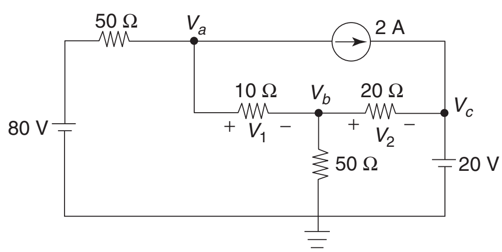


Fig. 2.43

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node *a*,

$$\frac{V_a - 80}{50} + \frac{V_a - V_b}{10} + 2 = 0$$

$$\left(\frac{1}{50} + \frac{1}{10}\right)V_a - \frac{1}{10}V_b = \frac{80}{50} - 2$$

$$0.12\,V_a - 0.1\,V_b = -0.4 \qquad \ldots\text{(i)}$$

Applying KCL at Node *b*,

$$\frac{V_b - V_a}{10} + \frac{V_b}{50} + \frac{V_b - V_c}{20} = 0$$

$$-\frac{1}{10}V_a + \left(\frac{1}{10} + \frac{1}{50} + \frac{1}{20}\right)V_b - \frac{1}{20}V_c = 0$$

$$-0.1\,V_a + 0.17\,V_b - 0.05\,V_c = 0 \qquad \ldots\text{(ii)}$$

Node *c* is directly connected to a voltage source of 20 V. Hence, we can write voltage equation at Node *c*.

$$V_c = 20 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$V_a = 3.08\text{ V}$$
$$V_b = 7.69\text{ V}$$
$$V_1 = V_a - V_b = 3.08 - 7.69 = -4.61\text{ V}$$
$$V_2 = V_b - V_c = 7.69 - 20 = -12.31\text{V}$$

Example 2.28 *Find the voltage across the 100 Ω resistor for the network shown in Fig. 2.44.*

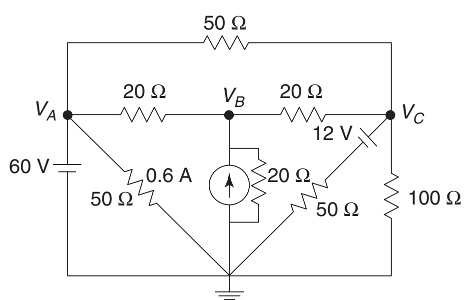


Fig. 2.44

Solution
Node *A* is directly connected to a voltage source of 60 V. Hence, we can write voltage equation at Node *A*.

$$V_A = 60 \qquad \ldots\text{(i)}$$

Assume that the currents are moving away from the nodes.

Applying KCL at Node B,

$$\frac{V_B - V_A}{20} + \frac{V_B - V_C}{20} + \frac{V_B}{20} = 0.6$$

$$-\frac{1}{20}V_A + \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{20}\right)V_B - \frac{1}{20}V_C = 0.6$$

$$-0.05\,V_A + 0.15\,V_B - 0.05\,V_C = 0.6 \qquad \ldots\text{(ii)}$$

Applying KCL at Node C,

$$\frac{V_C - V_A}{50} + \frac{V_C - V_B}{20} + \frac{V_C - 12}{50} + \frac{V_C}{100} = 0$$

$$-\frac{1}{50}V_A - \frac{1}{20}V_B + \left(\frac{1}{50} + \frac{1}{20} + \frac{1}{50} + \frac{1}{100}\right)V_C = \frac{12}{50}$$

$$-0.02\,V_A - 0.05\,V_B + 0.1\,V_C = 0.24 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$V_C = 31.68 \text{ V}$$

$$\text{Voltages across the } 100\ \Omega \text{ resistor} = 31.68 \text{ V}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.29 *Find the voltage across the 5 Ω resistor in the network shown in Fig. 2.45.*

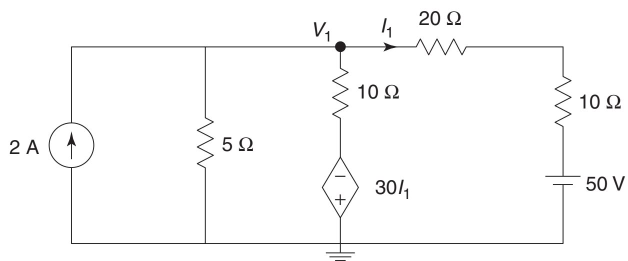


Fig. 2.45

Solution From Fig. 2.45,

$$I_1 = \frac{V_1 - 50}{20 + 10} = \frac{V_1 - 50}{30} \qquad \ldots\text{(i)}$$

Assume that the currents are moving away from the node.

Applying KCL at Node 1,

$$2 = \frac{V_1}{5} + \frac{V_1 + 30\,I_1}{10} + \frac{V_1 - 50}{30}$$

$$2 = \frac{V_1}{5} + \frac{V_1 + 30\left(\dfrac{V_1 - 50}{30}\right)}{10} + \frac{V_1 - 50}{30}$$

$$2 = \frac{V_1}{5} + \frac{2V_1 - 50}{10} + \frac{V_1 - 50}{30} \qquad \text{...(ii)}$$

Solving Eq. (ii),

$$V_1 = 20 \text{ V}$$

Voltage across the 5 Ω resistor = 20 V

Example 2.30 *For the network shown in Fig. 2.46, find the voltage V_x.*

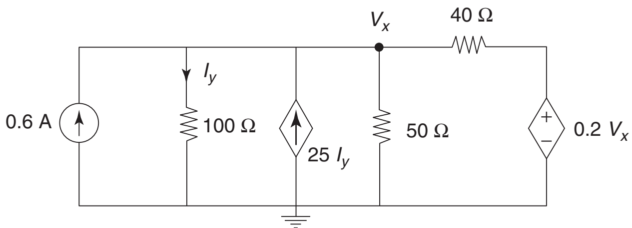


Fig. 2.46

Solution From Fig. 2.46,

$$I_y = \frac{V_x}{100} \qquad \text{...(i)}$$

Assume that the currents are moving away from the node.
Applying KCL at Node x,

$$25\,I_y + 0.6 = \frac{V_x}{100} + \frac{V_x}{50} + \frac{V_x - 0.2\,V_x}{40}$$

$$25\left(\frac{V_x}{100}\right) + 0.6 = \frac{V_x}{100} + \frac{V_x}{50} + \frac{0.8\,V_x}{40}$$

$$\left(\frac{1}{4} - \frac{1}{100} - \frac{1}{50} - \frac{0.8}{40}\right)V_x = -0.6$$

$$0.2\,V_x = -0.6$$

$$V_x = -3 \text{ V}$$

Example 2.31 *For the network shown in Fig. 2.47, find voltages V_1 and V_2.*

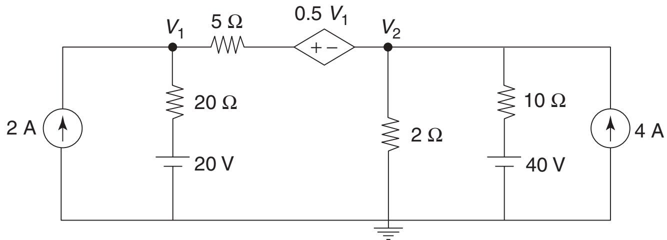


Fig. 2.47

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$2 = \frac{V_1 - 20}{20} + \frac{V_1 - 0.5\,V_1 - V_2}{5}$$

$$\left(\frac{1}{20} + \frac{1}{5} - \frac{0.5}{5}\right) V_1 - \frac{1}{5} V_2 = 3$$

$$0.15\,V_1 - 0.2\,V_2 = 3 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2 + 0.5\,V_1 - V_1}{5} + \frac{V_2}{2} + \frac{V_2 - 40}{10} = 4$$

$$\left(\frac{0.5}{5} - \frac{1}{5}\right) V_1 + \left(\frac{1}{5} + \frac{1}{2} + \frac{1}{10}\right) V_2 = 4 + \frac{40}{10}$$

$$-0.1\,V_1 + 0.8\,V_2 = 8 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$V_1 = 40 \text{ V}$$

$$V_2 = 15 \text{ V}$$

Example 2.32 *Determine the voltages V_1 and V_2 in the network of Fig. 2.48.*

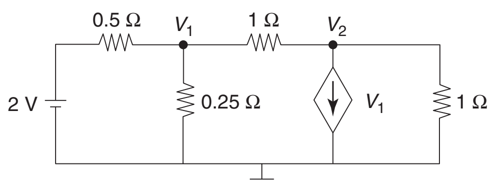


Fig. 2.48

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1 - 2}{0.5} + \frac{V_1}{0.25} + \frac{V_1 - V_2}{1} = 0$$

$$\left(\frac{1}{0.5} + \frac{1}{0.25} + 1\right)V_1 - V_2 = \frac{2}{0.5}$$

$$7\,V_7 - V_2 = 4 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} + V_1 = 0$$

$$2\,V_2 = 0$$

$$V_2 = 0 \qquad \text{...(ii)}$$

From Eq. (i),

$$V_1 = \frac{4}{7}\text{ V}$$

Example 2.33 *In the network of Fig. 2.49, find the node voltages V_1, V_2 and V_3.*

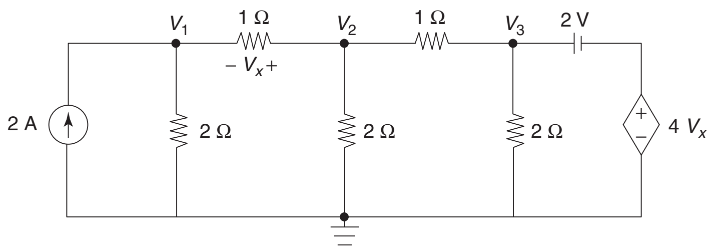


Fig. 2.49

Solution From Fig. 2.49,

$$V_x = V_2 - V_1 \qquad \text{...(i)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$2 = \frac{V_1}{2} + \frac{V_1 - V_2}{1}$$

$$\left(\frac{1}{2} + 1\right)V_1 - V_2 = 2$$

$$1.5\,V_1 - V_2 = 2 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0$$

$$-V_1 + \left(1 + \frac{1}{2} + 1\right) V_2 - V_3 = 0$$

$$-V_1 + 2.5\, V_2 - V_3 = 0 \qquad \ldots\text{(iii)}$$

At Node 3,

$$V_3 - 4\, V_x = 2$$

$$V_3 - 4(V_2 - V_1) = 2$$

$$4\, V_1 - 4\, V_2 + V_3 = 2 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$V_1 = -1.33 \text{ V}$$

$$V_2 = -4 \text{ V}$$

$$V_3 = -8.67 \text{ V}$$

Example 2.34 *For the network shown in Fig. 2.50, find the node voltages V_1 and V_2.*

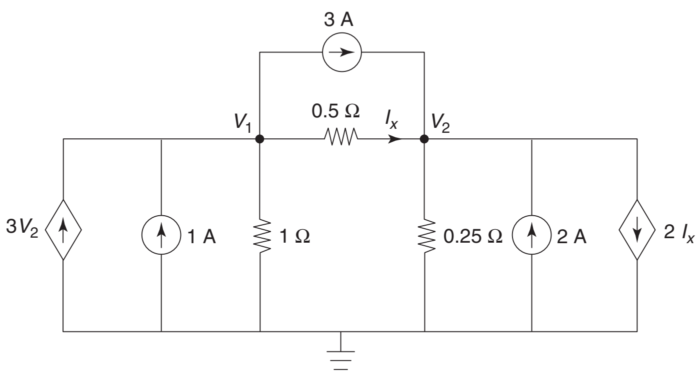


Fig. 2.50

Solution From Fig. 2.50,

$$I_x = \frac{V_1 - V_2}{0.5} \qquad \ldots\text{(i)}$$

Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$3V_2 + 1 = \frac{V_1}{1} + \frac{V_1 - V_2}{0.5} + 3$$

$$\left(1 + \frac{1}{0.5}\right)V_1 - \left(3 + \frac{1}{0.5}\right)V_2 = -2$$

$$3V_1 - 5V_2 = -2 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$3 + 2 = \frac{V_2 - V_1}{0.5} + \frac{V_2}{0.25} + 2I_x$$

$$5 = \frac{V_2 - V_1}{0.5} + \frac{V_2}{0.25} + 2\left(\frac{V_1 - V_2}{0.5}\right)$$

$$\left(-\frac{1}{0.5} + \frac{2}{0.5}\right)V_1 + \left(\frac{1}{0.5} + \frac{1}{0.25} - \frac{2}{0.5}\right)V_2 = 5$$

$$2V_1 + 2V_2 = 5 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$V_1 = 1.31 \text{ V}$$
$$V_2 = 1.19 \text{ V}$$

Example 2.35 *Find voltages V_1 and V_2 in the network shown in Fig. 2.51.*

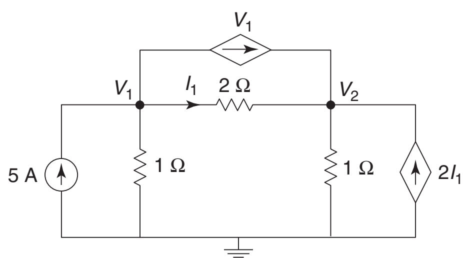


Fig. 2.51

Solution From Fig. 2.51,

$$I_1 = \frac{V_1 - V_2}{2} \qquad \text{...(i)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$5 = \frac{V_1}{1} + \frac{V_1 - V_2}{2} + V_1$$

$$\left(1 + \frac{1}{2} + 1\right)V_1 - \frac{1}{2}V_2 = 5$$

$$2.5V_1 - 0.5V_2 = 5 \qquad \text{...(ii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} = 2I_1 + V_1$$

$$V_2 - V_1 + V_2 = 2\left(\frac{V_1 - V_2}{2}\right) + V_1$$

$$3V_1 = 3V_2$$

$$V_1 = V_2 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$V_1 = 2.5\text{ V}$$

$$V_2 = 2.5\text{ V}$$

Example 2.36 *Find the power supplied by the 10 V source in the network shown in Fig. 2.52.*

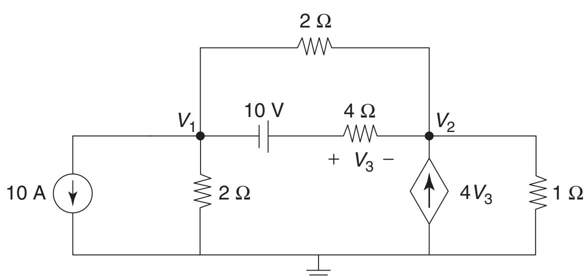


Fig. 2.52

Solution

From Fig. 2.52,

$$V_3 = V_1 + 10 - V_2 \qquad \ldots\text{(i)}$$

Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$10 + \frac{V_1}{2} + \frac{V_1 + 10 - V_2}{4} + \frac{V_1 - V_2}{2} = 0$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{2}\right)V_1 - \left(\frac{1}{4} + \frac{1}{2}\right)V_2 = -10 - \frac{10}{4}$$

$$1.25\,V_1 - 0.75\,V_2 = -12.5 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - 10 - V_1}{4} + \frac{V_2 - V_1}{2} + \frac{V_2}{1} = 4\,V_3$$

$$\frac{V_2 - 10 - V_1}{4} + \frac{V_2 - V_1}{2} + \frac{V_2}{1} = 4(V_1 + 10 - V_2)$$

$$\left(-\frac{1}{4}-\frac{1}{2}-4\right)V_1+\left(\frac{1}{4}+\frac{1}{2}+1+4\right)V_2=\frac{10}{4}+40$$

$$-4.75\,V_1+5.75\,V_2=42.5 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$V_1=-11.03\text{ V}$$

$$V_2=-1.72\text{ V}$$

$$I_{10\text{ V}}=\frac{V_1+10-V_2}{4}=\frac{-11.03+10-(-1.72)}{4}=0.173\text{ A}$$

Power supplied by the 10 V source = 10 × 0.173 = 1.73 W

Example 2.37 *For the network shown in Fig. 2.53, find voltages V_1 and V_2.*

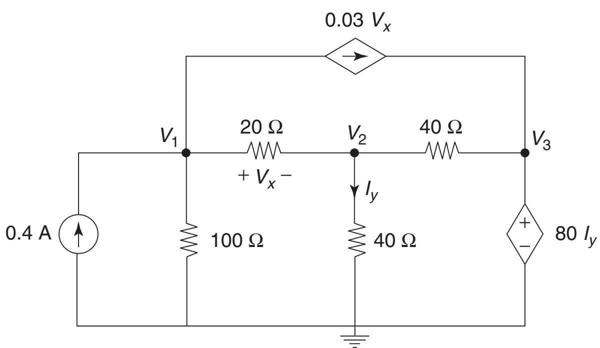


Fig. 2.53

Solution From Fig. 2.53,

$$V_x=V_1-V_2 \qquad \ldots\text{(i)}$$

$$I_y=\frac{V_2}{40} \qquad \ldots\text{(ii)}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$0.4=\frac{V_1}{100}+\frac{V_1-V_2}{20}+0.03\,V_x$$

$$0.4=\frac{V_1}{100}+\frac{V_1-V_2}{20}+0.03(V_1-V_2)$$

$$\left(\frac{1}{100}+\frac{1}{20}+0.03\right)V_1-\left(\frac{1}{20}+0.03\right)V_2=0.4$$

$$0.09\,V_1-0.08\,V_2=0.4 \qquad \ldots\text{(iii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{20} + \frac{V_2}{40} + \frac{V_2 - V_3}{40} = 0$$

$$-\frac{1}{20}V_1 + \left(\frac{1}{20} + \frac{1}{40} + \frac{1}{40}\right)V_2 - \frac{1}{40}V_3 = 0$$

$$-0.05\,V_1 + 0.1\,V_2 - 0.025\,V_3 = 0 \qquad \ldots\text{(iv)}$$

For Node 3,

$$V_3 = 80\,I_y = 80\left(\frac{V_2}{40}\right) = 2\,V_2$$

$$2\,V_2 - V_3 = 0 \qquad \ldots\text{(v)}$$

Solving Eqs (iii), (iv) and (v),

$$V_1 = 40\text{ V}$$
$$V_2 = 40\text{ V}$$
$$V_3 = 80\text{ V}$$

Example 2.38 *Find voltages V_a, V_b and V_c in the network shown in Fig. 2.54.*

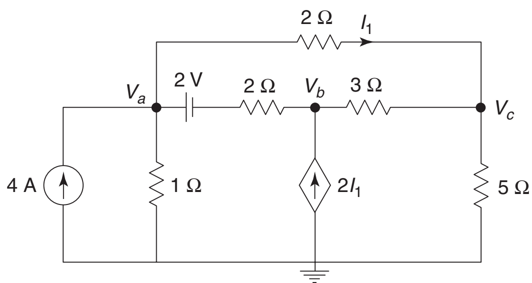


Fig. 2.54

Solution From Fig. 2.54,

$$I_1 = \frac{V_a - V_c}{2}$$

Assume that the currents are moving away from the nodes.
Applying KCL at Node a,

$$4 = \frac{V_a}{1} + \frac{V_a - V_c}{2} + \frac{V_a - 2 - V_b}{2}$$

$$\left(1 + \frac{1}{2} + \frac{1}{2}\right)V_a - \frac{1}{2}V_b - \frac{1}{2}V_c = 5$$

$$2\,V_a - 0.5\,V_b - 0.5\,V_c = 5 \qquad \ldots\text{(i)}$$

Applying KCL at Node b,

$$\frac{V_b + 2 - V_a}{2} + \frac{V_b - V_c}{3} = 2I_1$$

$$\frac{V_b + 2 - V_a}{2} + \frac{V_b - V_c}{3} = 2\left(\frac{V_a - V_c}{2}\right)$$

$$\frac{V_b + 2 - V_a}{2} + \frac{V_b - V_c}{3} = V_a - V_c$$

$$\left(-\frac{1}{2} - 1\right)V_a + \left(\frac{1}{2} + \frac{1}{3}\right)V_b + \left(1 - \frac{1}{3}\right)V_c = -1$$

$$-1.5\,V_a + 0.83\,V_b + 0.67\,V_c = -1 \qquad \text{...(ii)}$$

Applying KCL at Node c,

$$\frac{V_c - V_b}{3} + \frac{V_c}{5} = I_1$$

$$\frac{V_c - V_b}{3} + \frac{V_c}{5} = \frac{V_a - V_c}{2}$$

$$-\frac{1}{2}V_a - \frac{1}{3}V_b + \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{2}\right)V_c = 0$$

$$-0.5\,V_a - 0.33\,V_b + 1.033\,V_c = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii), and (iii),

$$V_a = 4.303\text{ V}$$

$$V_b = 3.88\text{ V}$$

$$V_c = 3.33\text{ V}$$

2.6 SUPERNODE ANALYSIS

Nodes that are connected to each other by voltage sources, but not to the reference node by a path of voltage sources, form a *supernode*. A supernode requires one node voltage equation, that is, a KCL equation. The remaining node voltage equations are KVL equations.

Example 2.39 *Determine the current in the 5 Ω resistor for the network shown in Fig. 2.55.*

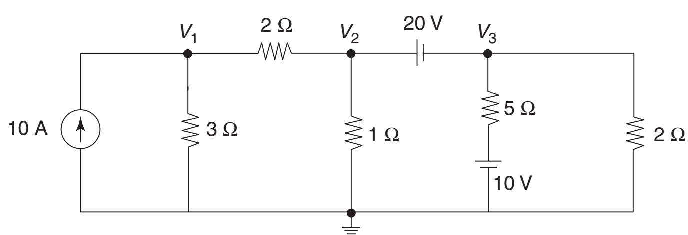


Fig. 2.55

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{3}+\frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 10$$

$$0.83\,V_1 - 0.5\,V_2 = 10 \qquad \ldots\text{(i)}$$

Nodes 2 and 3 will form a supernode.
Writing voltage equation for the supernode,

$$V_2 - V_3 = 20 \qquad \ldots\text{(ii)}$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{2}+1\right)V_2 + \left(\frac{1}{5}+\frac{1}{2}\right)V_3 = 2$$

$$-0.5\,V_1 + 1.5\,V_2 + 0.7\,V_3 = 2 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1 = 19.04 \text{ V}$$
$$V_2 = 11.6 \text{ V}$$
$$V_3 = -8.4 \text{ V}$$

$$I_{5\,\Omega} = \frac{V_3 - 10}{5} = \frac{-8.4 - 10}{5} = -3.68 \text{ A}$$

Example 2.40 *Find the power delivered by the 5 A current source in the network shown in Fig. 2.56.*

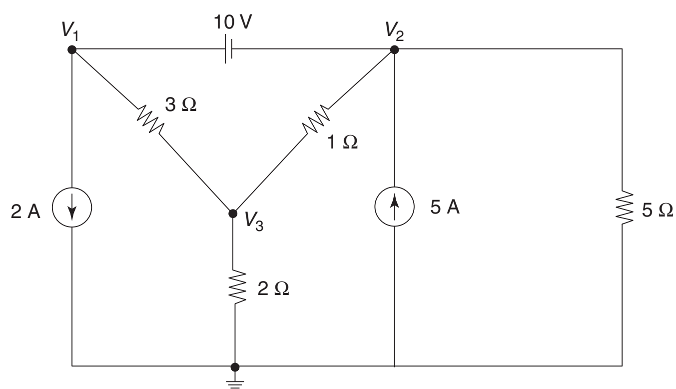


Fig. 2.56

Solution Assume that the currents are moving away from the nodes.
Nodes 1 and 2 will form a supernode.
Writing voltage equation for the supernode,

$$V_1 - V_2 = 10 \quad \ldots\text{(i)}$$

Applying KCL at the supernode,

$$2 + \frac{V_1 - V_3}{3} + \frac{V_2}{5} + \frac{V_2 - V_3}{1} = 5$$

$$\frac{1}{3}V_1 + \left(\frac{1}{5}+1\right)V_2 - \left(\frac{1}{3}+1\right)V_3 = 3$$

$$0.33\,V_1 + 1.2\,V_2 - 1.33\,V_3 = 3 \quad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$$

$$-\frac{1}{3}V_1 - V_2 + \left(\frac{1}{3}+1+\frac{1}{2}\right)V_3 = 0$$

$$-0.33\,V_1 - V_2 + 1.83\,V_3 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1 = 13.72 \text{ V}$$
$$V_2 = 3.72 \text{ V}$$
$$V_3 = 4.51 \text{ V}$$

Power delivered by the 5 A source = 5 V_2 = 5 × 3.72 = 18.6 W

Example 2.41 *In the network of Fig. 2.57, find the node voltages V_1, V_2 and V_3.*

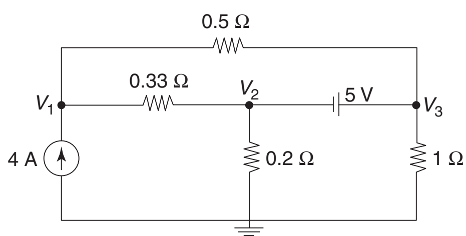


Fig. 2.57

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$4 = \frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.5}$$

$$\left(\frac{1}{0.33}+\frac{1}{0.5}\right)V_1 - \frac{1}{0.33}V_2 - \frac{1}{0.5}V_3 = 4$$

$$5.03\,V_1 - 3.03\,V_2 - 2\,V_3 = 4 \quad \ldots\text{(i)}$$

Nodes 2 and 3 will form a supernode.
Writing voltage equation for the supernode,

$$V_3 - V_2 = 5 \qquad \text{...(ii)}$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{0.33} + \frac{V_2}{0.2} + \frac{V_3}{1} + \frac{V_3 - V_1}{0.5} = 0$$

$$\left(-\frac{1}{0.33} - \frac{1}{0.5}\right)V_1 + \left(\frac{1}{0.33} + \frac{1}{0.2}\right)V_2 + \left(1 + \frac{1}{0.5}\right)V_3 = 0$$

$$-5.03\,V_1 + 8.03\,V_2 + 3\,V_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1 = 2.62 \text{ V}$$
$$V_2 = -0.17 \text{ V}$$
$$V_3 = 4.83 \text{ V}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.42 *For the network shown in Fig. 2.58, determine the voltage V_x.*

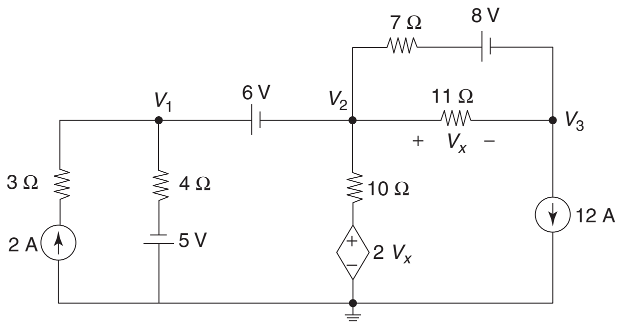


Fig. 2.58

Solution From Fig. 2.58,

$$V_x = V_2 - V_3$$

Assume that the currents are moving away from the nodes.
Node 1 and 2 will form a supernode.
Writing voltage equations for the supernode,

$$V_1 - V_2 = 6 \qquad \text{...(i)}$$

Applying KCL at the supernode,

$$2 = \frac{V_1 + 5}{4} + \frac{V_2 - 2V_x}{10} + \frac{V_2 - 8 - V_3}{7} + \frac{V_2 - V_3}{11}$$

$$2 = \frac{V_1 + 5}{4} + \frac{V_2 - 2(V_2 - V_3)}{10} + \frac{V_2 - 8 - V_3}{7} + \frac{V_2 - V_3}{11}$$

$$\frac{1}{4}V_1+\left(\frac{1}{10}-\frac{1}{5}+\frac{1}{7}+\frac{1}{11}\right)V_2+\left(\frac{1}{5}-\frac{1}{7}-\frac{1}{11}\right)V_3=2-\frac{5}{4}+\frac{8}{7}$$

$$0.25\,V_1+0.133\,V_2-0.033\,V_3=1.89 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{11}+\frac{V_3+8-V_2}{7}+12=0$$

$$\left(-\frac{1}{11}-\frac{1}{7}\right)V_2+\left(\frac{1}{11}+\frac{1}{7}\right)V_3=-12-\frac{8}{7}$$

$$-0.233\,V_2+0.233\,V_3=-13.14 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$V_1=1.8\text{ V}$$
$$V_2=-4.2\text{ V}$$
$$V_3=-60.6\text{ V}$$

Example 2.43 *Find the node voltages in the network shown in Fig. 2.59.*

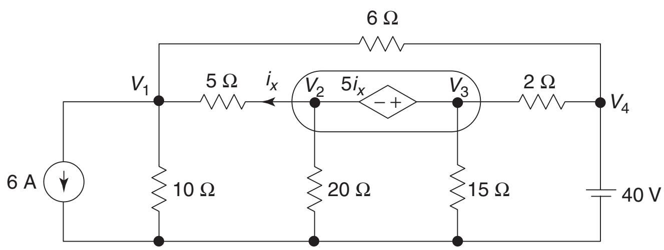


Fig. 2.59

Solution From Fig. 2.59,

$$I_x=\frac{V_2-V_1}{5} \qquad \ldots\text{(i)}$$

For Node 4,

$$V_4=40 \qquad \ldots\text{(ii)}$$

Applying KCL at Node 1,

$$6+\frac{V_1}{10}+\frac{V_1-V_2}{5}+\frac{V_1-V_4}{6}=0$$

$$6+\frac{V_1}{10}+\frac{V_1-V_2}{5}+\frac{V_1-40}{6}=0$$

$$\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{6}\right)V_1-\frac{1}{5}V_2=\frac{40}{6}-6$$

$$\frac{7}{15}V_1-\frac{1}{5}V_2=\frac{2}{3} \qquad \ldots\text{(iii)}$$

Nodes 2 and 3 will form a supernode,
Writing voltage equation for the supernode,

$$V_3 - V_2 = 5\,I_x = 5\left(\frac{V_2 - V_1}{5}\right) = V_2 - V_1$$

$$V_1 - 2V_2 + V_3 = 0 \qquad \text{...(iv)}$$

Applying KCL to the supernode,

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{V_3}{15} + \frac{V_3 - V_4}{2} = 0$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{V_3}{15} + \frac{V_3 - 40}{2} = 0$$

$$-\frac{1}{5}V_1 + \left(\frac{1}{5} + \frac{1}{20}\right)V_2 + \left(\frac{1}{15} + \frac{1}{2}\right)V_3 = 20$$

$$-\frac{1}{5}V_1 + \frac{1}{4}V_2 + \frac{17}{30}V_3 = 20 \qquad \text{...(v)}$$

Solving Eqs (iii), (iv) and (v),

$$V_1 = 10\text{ V}$$
$$V_2 = 20\text{ V}$$
$$V_3 = 30\text{ V}$$
$$V_4 = 40\text{ V}$$

Example 2.44 *Find the node voltages in the network shown in Fig. 2.60.*

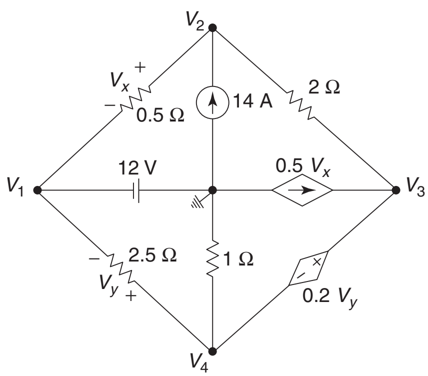


Fig. 2.60

Solution Selecting the central node as reference node,

$$V_1 = -12\text{ V} \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = 14$$

$$-\frac{1}{0.5}V_1 + \left(\frac{1}{0.5} + \frac{1}{2}\right)V_2 - \frac{1}{2}V_3 = 14$$

$$-2\,V_1 + 2.5\,V_2 - 0.5\,V_3 = 14 \qquad \ldots\text{(ii)}$$

Nodes 3 and 4 will form a supernode,

Writing voltage equation for the supernode,

$$V_3 - V_4 = 0.2V_y = 0.2(V_4 - V_1)$$

$$0.2V_1 + V_3 - 1.2V_4 = 0 \qquad \ldots\text{(iii)}$$

Applying KCL to the supernode,

$$\frac{V_3 - V_2}{2} - 0.5V_x + \frac{V_4}{1} + \frac{V_4 - V_1}{2.5} = 0$$

$$\frac{V_3 - V_2}{2} - 0.5(V_2 - V_1) + V_4 + \frac{V_4 - V_1}{2.5} = 0$$

$$\left(0.5 - \frac{1}{2.5}\right)V_1 - \left(\frac{1}{2} + 0.5\right)V_2 + \frac{1}{2}V_3 + \left(1 + \frac{1}{2.5}\right)V_4 = 0$$

$$0.1\,V_1 - V_2 + 0.5\,V_3 + 1.4\,V_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (i), (ii), (iii) and (iv),

$$V_1 = -12\text{ V}$$

$$V_2 = -4\text{ V}$$

$$V_3 = 0$$

$$V_4 = -2\text{ V}$$

2.7 SUPERPOSITION THEOREM

It states that '*in a linear network containing more than one independent source and dependent source, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned. The independent current sources are represented by infinite resistances, i.e., open circuits.

The dependent sources are not sources but dissipative components—hence they are active at all times. A dependent source has zero value only when its control voltage or current is zero.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the network shown in Fig. 2.61. Suppose we have to find current I_4 through resistor R_4.

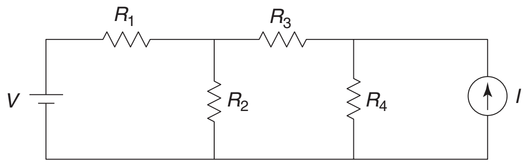


Fig. 2.61 *Network to illustrate superposition theorem*

The current flowing through resistor R_4 due to constant voltage source V is found to be say I_4' (with proper direction), representing constant current source with infinite resistance, i.e., open circuit.

The current flowing through resistor R_4 due to constant current source I is found to be say I_4'' (with proper direction), representing the constant voltage source with zero resistance or short circuit.

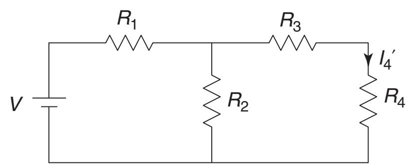


Fig. 2.62 *When voltage source V is acting alone*

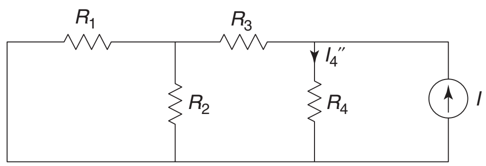


Fig. 2.63 *When current source I is acting alone*

The resultant current I_4 through resistor R_4 is found by superposition theorem.

$$I_4 = I_4' + I_4''$$

Steps to be followed in Superposition Theorem

1. Find the current through the resistance when only one independent source is acting, replacing all other independent sources by respective internal resistances.
2. Find the current through the resistance for each of the independent sources.
3. Find the resultant current through the resistance by the superposition theorem considering magnitude and direction of each current.

Example 2.45 *Find the current through the 4 Ω resistor in Fig. 2.64.*

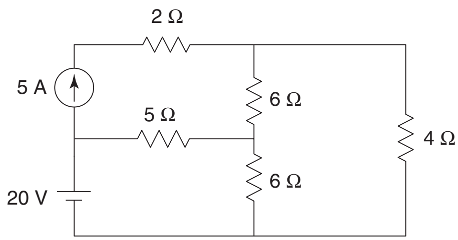


Fig. 2.64

Solution

Step I When the 5 A source is acting alone (Fig. 2.65)

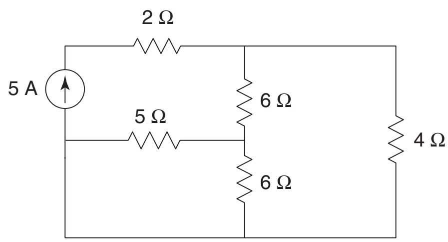


Fig. 2.65

By series-parallel reduction technique (Fig. 2.66),

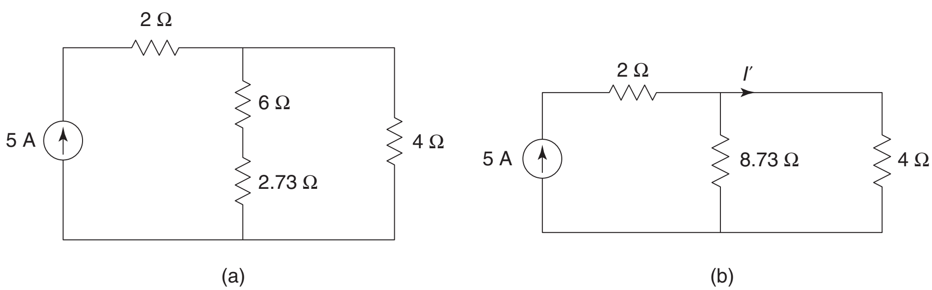


Fig. 2.66

$$I' = 5 \times \frac{8.73}{8.73+4} = 3.43 \text{ A}(\downarrow)$$

Step II When the 20 V source is acting alone (Fig. 2.67)

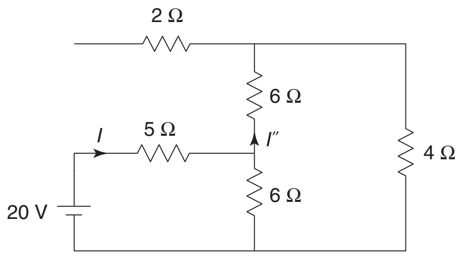


Fig. 2.67

By series-parallel reduction technique (Fig. 2.68),

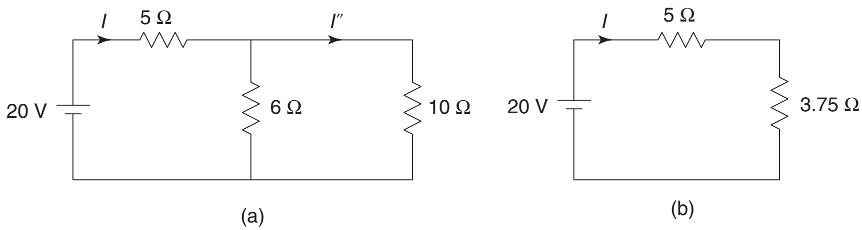


Fig. 2.68

$$I = \frac{20}{5+3.75} = 2.29 \text{ A}$$

From Fig. 2.68(a), by current-division rule,

$$I'' = 2.29 \times \frac{6}{6+10} = 0.86 \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.43 + 0.86 = 4.29 \text{ A } (\downarrow)$$

Example 2.46 *Find the current through the 3 Ω resistor in Fig. 2.69.*

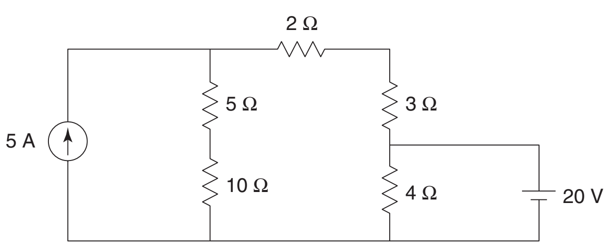


Fig. 2.69

Solution

Step I When the 5 A source is acting alone (Fig. 2.70)

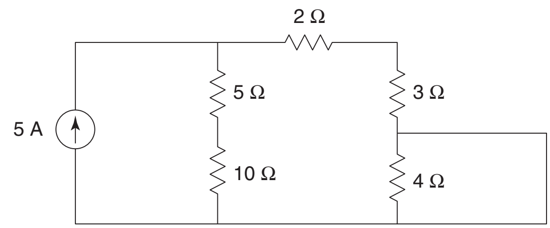


Fig. 2.70

By series-parallel reduction technique (Fig. 2.71),

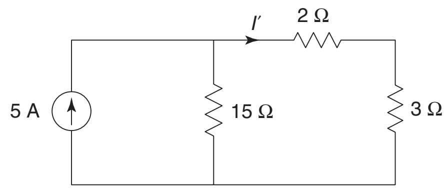


Fig. 2.71

$$I' = 5 \times \frac{15}{15+2+3} = 3.75 \text{ A}(\downarrow)$$

Step II When the 20 V source is acting alone (Fig. 2.72)

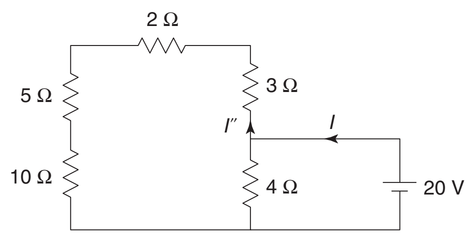


Fig. 2.72

By series-parallel reduction technique (Fig. 2.73),

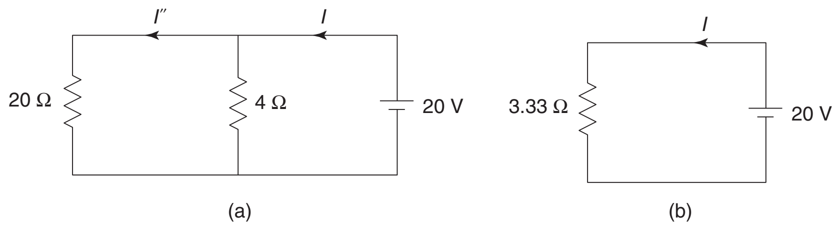


Fig. 2.73

$$I = \frac{20}{3.33} = 6 \text{ A}$$

From Fig. 2.73(a), by current-division rule,

$$I'' = 6 \times \frac{4}{20+4} = 1 \text{ A}(\uparrow) = -1 \text{ A}(\downarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.75 - 1 = 2.75 \text{ A } (\downarrow)$$

Example 2.47 *Find the current in the 1 Ω resistors in Fig. 2.74.*

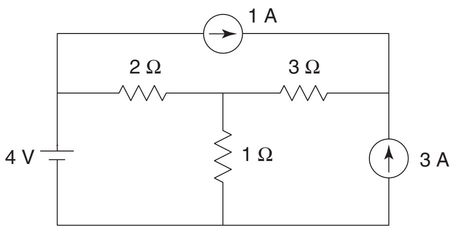


Fig. 2.74

Solution

Step I When the 4 V source is acting alone (Fig. 2.75)

$$I' = \frac{4}{2+1} = 1.33 \text{ A}(\downarrow)$$

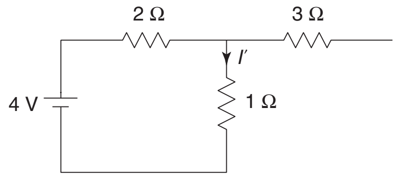


Fig. 2.75

Step II When the 3 A source is acting alone (Fig. 2.76)
By current-division rule,

$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ A} (\downarrow)$$

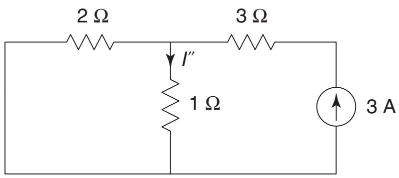


Fig. 2.76

Step III When the 1 A source is acting alone (Fig. 2.77)

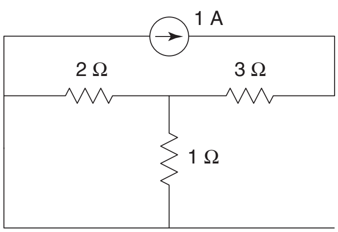


Fig. 2.77

Redrawing the network (Fig. 2.78),
By current-division rule,

$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A}(\downarrow)$$

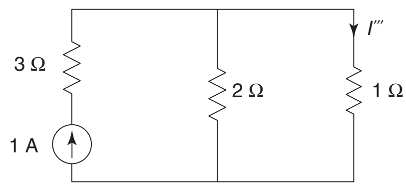


Fig. 2.78

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 1.33 + 2 + 0.66 = 4 \text{ A} (\downarrow)$$

Example 2.48 *Find the voltage V_{AB} in Fig. 2.79.*

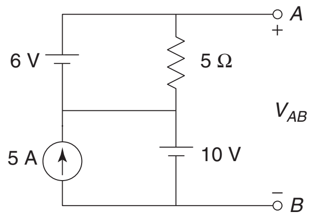


Fig. 2.79

Solution

Step I When the 6 V source is acting alone (Fig. 2.80)

$$V'_{AB} = 6 \text{ V}$$

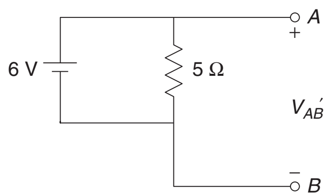


Fig. 2.80

Step II When the 10 V source is acting alone (Fig. 2.81)
Since the resistor of 5 Ω is shorted, the voltage across it is zero.

$$V''_{AB} = 10 \text{ V}$$

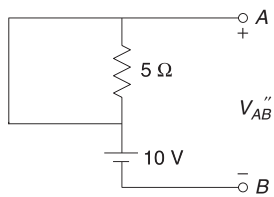


Fig. 2.81

Step III When the 5 A source is acting alone (Fig. 2.82)
Due to short circuit in both the parts,

$$V'''_{AB} = 0$$

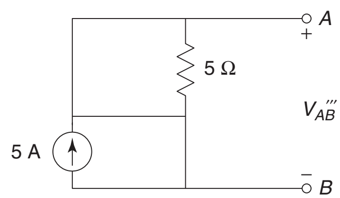


Fig. 2.82

Step IV By superposition theorem,

$$V_{AB} = V'_{AB} + V''_{AB} + V'''_{AB} = 6 + 10 + 0 = 16 \text{ V}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.49 *Find the current through the 6 Ω resistor in Fig. 2.83.*

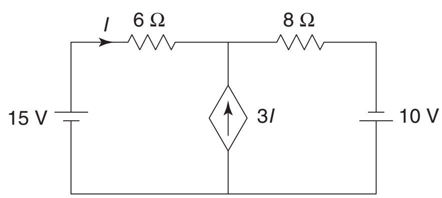


Fig. 2.83

Solution

Step I When the 15 V source is acting alone (Fig. 2.84)

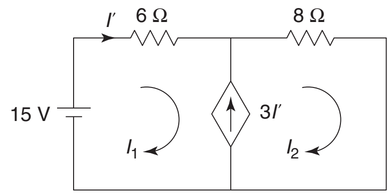


Fig. 2.84

From Fig. 2.84,

$$I' = I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 3I' = 3I_1$$

$$4I_1 - I_2 = 0 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$15 - 6I_1 - 8I_2 = 0$$

$$6I_1 + 8I_2 = 15 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.39\text{A}$$
$$I_2 = 1.57\text{A}$$
$$I' = I_1 = 0.39 \text{ A } (\rightarrow)$$

Step II When the 10 V source is acting alone (Fig. 2.85)

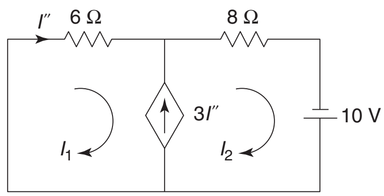


Fig. 2.85

From Fig. 2.85,

$$I'' = I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 3I'' = 3I_1$$
$$4I_1 - I_2 = 0 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-6I_1 - 8I_2 + 10 = 0$$
$$6I_1 + 8I_2 = 10 \qquad \text{.....(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.26 \text{ A}$$
$$I_2 = 1.05 \text{ A}$$
$$I'' = I_1 = 0.26 \text{ A} \quad (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 0.39 + 0.26 = 0.65 \text{ A } (\rightarrow)$$

Example 2.50 *Find the current I_x in Fig. 2.86.*

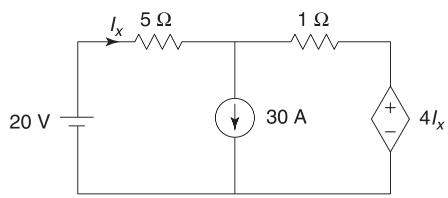


Fig. 2.86

Solution

Step I When the 30 A source is acting alone (Fig. 2.87)
From Fig. 2.87,

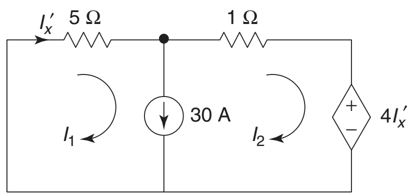


Fig. 2.87

$$I_x' = I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_1 - I_2 = 30 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-5I_1 - 1I_2 - 4I_x' = 0$$
$$-5I_1 - I_2 - 4I_1 = 0$$
$$9I_1 + I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 3\text{ A}$$
$$I_2 = -27\text{ A}$$
$$I_x' = I_1 = 3\text{ A }(\rightarrow)$$

Step II When the 20 V source is acting alone (Fig. 2.88)
Applying KVL to the mesh,

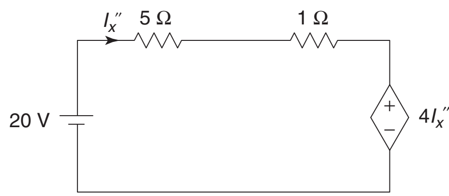


Fig. 2.88

$$20 - 5I_x'' - 1I_x'' - 4I_x'' = 0$$
$$I_x'' = 2\text{ A}(\rightarrow)$$

Step III By superposition theorem,

$$I_x = I_x' + I_x'' = 3 + 2 = 5\text{ A}(\rightarrow)$$

Example 2.51 *Find the current I_1 in Fig. 2.89.*

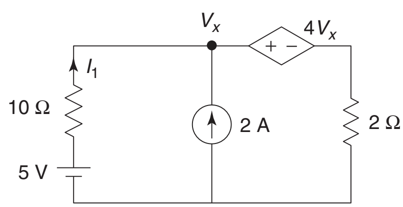


Fig. 2.89

Solution

Step I When the 5 V source is acting alone (Fig. 2.90)
From Fig. 2.90,

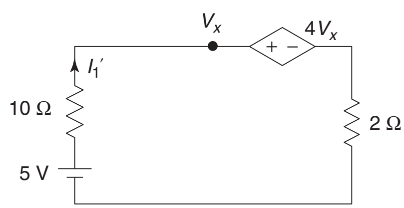


Fig. 2.90

$$V_x = 5 - 10I_1'$$

Applying KVL to the mesh,

$$5 - 10I_1' - 4V_x - 2I_1' = 0$$

$$5 - 10I_1' - 4(5 - 10I_1') - 2I_1' = 0$$
$$5 - 10I_1' - 20 + 40I_1' - 2I_1' = 0$$
$$I_1' = \frac{15}{28} = 0.54 \text{ A}(\uparrow)$$

Step II When the 2 A source is acting alone (Fig. 2.91)

From Fig. 2.91,

$$V_x = -10I_1' \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1' = 2 \qquad \ldots\text{(ii)}$$

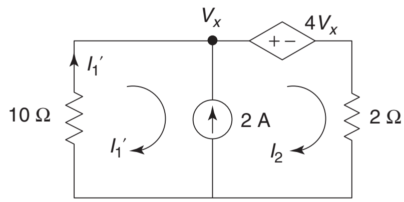


Fig. 2.91

Applying KVL to the outer path of the supermesh,

$$-10I_1' - 4V_x - 2I_2 = 0$$
$$-10I_1' - 4(-10I_1') - 2I_2 = 0$$
$$30I_1' - 2I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.14 \text{ A } (\uparrow)$$
$$I_2 = 2.14 \text{ A}$$

Step III By superposition theorem,

$$I_1 = I_1' + I_1'' = 0.54 + 0.14 = 0.68 \text{ A } (\uparrow)$$

Example 2.52 *Determine the current through the 10 Ω resistor in Fig. 2.92.*

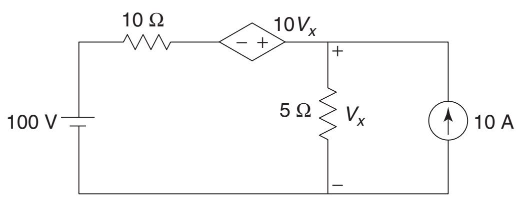


Fig. 2.92

Solution

Step I When the 100 V source is acting alone (Fig. 2.93)

From Fig. 2.93,

$$V_x = 5I'$$

Applying KVL to the mesh,

$$100 - 10I' + 10V_x - 5I' = 0$$
$$100 - 10I' + 10(5I') - 5I' = 0$$

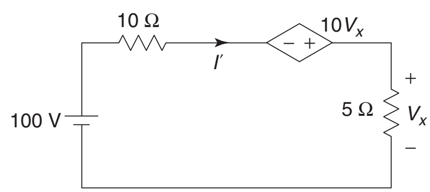


Fig. 2.93

$$I' = -2.86 \text{ A} (\rightarrow)$$

Step II When the 10 A source is acting alone (Fig. 2.94)

From Fig. 2.94,

$$V_x = 5(I_1 - I_2) \quad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$-10I_1 + 10V_x - 5(I_1 - I_2) = 0$$

$$-10I_1 + 10\{5(I_1 - I_2)\} - 5(I_1 - I_2) = 0$$

$$35I_1 - 45I_2 = 0 \quad \ldots\text{(ii)}$$

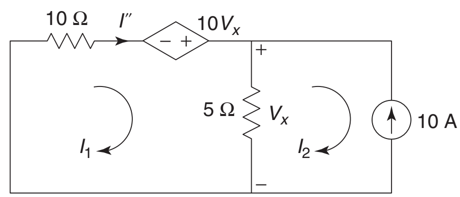


Fig. 2.94

For Mesh 2,

$$I_2 = -10 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -12.86 \text{ A}$$

$$I_2 = -10 \text{ A}$$

$$I'' = I_1 = -12.86 \text{ A} (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = -2.86 - 12.86 = -15.72 \text{ A} (\rightarrow)$$

Example 2.53 *Find the current I in the network of Fig. 2.95.*

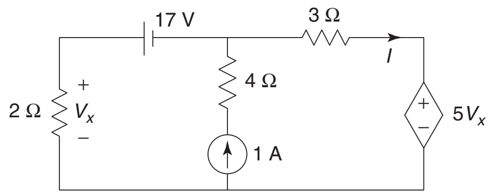


Fig. 2.95

Solution

Step I When the 17 V source is acting alone (Fig. 2.96)

From Fig. 2.96,

$$V_x = -2I'$$

Applying KVL to the mesh,

$$-2I' - 17 - 3I' - 5V_x = 0$$

$$-2I' - 17 - 3I' - 5(-2I') = 0$$

$$I' = 3.4 \text{ A} (\rightarrow)$$

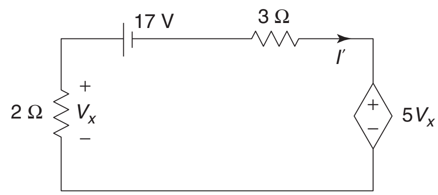


Fig. 2.96

Step II When the 1 A source is acting alone (Fig. 2.97)

From Fig. 2.97,

$$V_x = -2I_1 \quad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 1 \quad \ldots\text{(ii)}$$

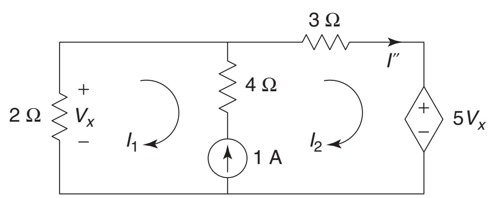


Fig. 2.97

Applying KVL to the outer path of the supermesh,

$$-2I_1 - 3I_2 - 5V_x = 0$$
$$-2I_1 - 3I_2 - 5(-2I_1) = 0$$
$$8I_1 - 3I_2 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.6 \text{ A}$$
$$I_2 = 1.6 \text{ A}$$
$$I'' = I_2 = 1.6 \text{ A } (\rightarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.4 + 1.6 = 5 \text{ A } (\rightarrow)$$

Example 2.54 *Find the voltage V_1 in Fig. 2.98.*

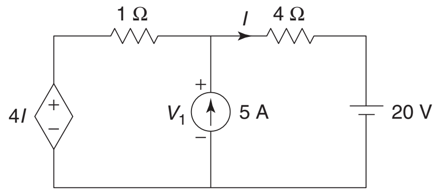


Fig. 2.98

Solution

Step I When the 5 A source is acting alone (Fig. 2.99)

From Fig. 2.99,

$$I = \frac{V_1'}{4}$$

Applying KCL at Node 1,

$$\frac{V_1' - 4I}{1} + \frac{V_1'}{4} = 5$$

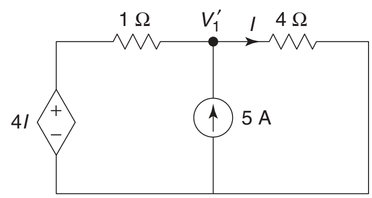


Fig. 2.99

$$V_1' - 4\left(\frac{V_1'}{4}\right) + \frac{V_1'}{4} = 5$$

$$V_1' = 20 \text{ V}$$

Step II When the 20 V source is acting alone (Fig. 2.100)

Applying KVL to the mesh,

$$4I - I - 4I - 20 = 0$$
$$I = -20 \text{ A}$$
$$V_1'' = 4I - 1(I) = 3I = 3\,(-20) = -60 \text{ V}$$

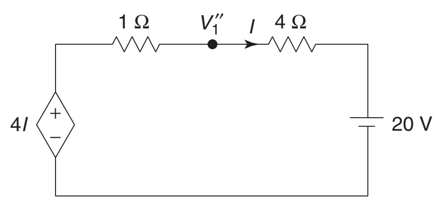


Fig. 2.100

Step III By superposition theorem,

$$V_1 = V_1' + V_1'' = 20 - 60 = -40 \text{ V}$$

Example 2.55 *Find the current in the 6 Ω resistor in Fig. 2.101.*

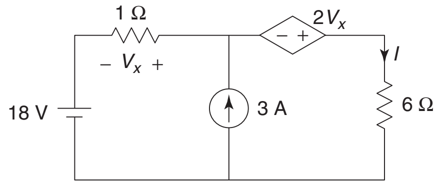


Fig. 2.101

Solution

Step I When the 18 V source is acting alone (Fig. 2.102)
From Fig. 2.102,

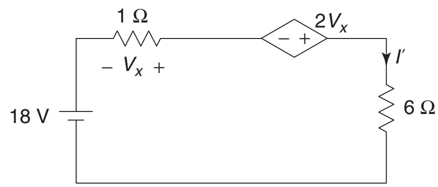


Fig. 2.102

$$V_x = -I'$$

Applying KVL to the mesh,

$$18 - I' + 2V_x - 6I' = 0$$
$$18 - I' - 2I' - 6I' = 0$$
$$I' = 2 \text{ A } (\downarrow)$$

Step II When the 3 A source is acting alone (Fig. 2.103)
From Fig. 2.103,

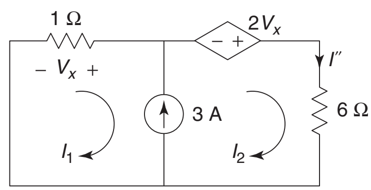


Fig. 2.103

$$V_x = -1\,I_1 = -I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing current equation for the supermesh,

$$I_2 - I_1 = 3 \qquad \text{...(ii)}$$

Applying KVL to the outerpath of the supermesh,

$$-1I_1 + 2V_x - 6I_2 = 0$$
$$-I_1 + 2(-I_1) - 6I_2 = 0$$
$$3I_1 + 6I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -2 \text{ A}$$
$$I_2 = 1 \text{ A}$$
$$I'' = I_2 = 1 \text{ A } (\downarrow)$$

Step III By superposition theorem,

$$I_{6\,\Omega} = I' + I'' = 2 + 1 = 3 \text{ A } (\downarrow)$$

Example 2.56 *Find the current I_y in Fig. 2.104.*

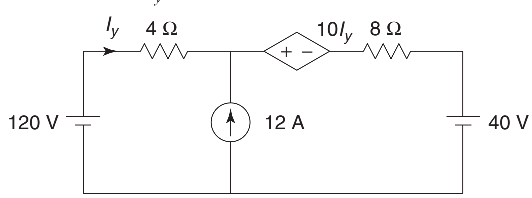


Fig. 2.104

Solution

Step I When the 120 V source is acting alone (Fig. 2.105)

Applying KVL to the mesh,

$$120 - 4I_y' - 10I_y' - 8I_y' = 0$$

$$I_y' = 5.45 \text{ A } (\rightarrow)$$

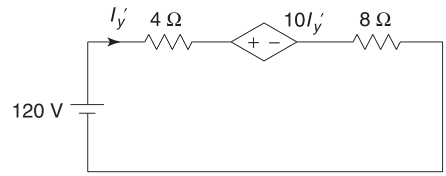


Fig. 2.105

Step II When the 12 A source is acting alone (Fig. 2.106)

From Fig. 2.106,

$$I_y'' = I_1 \quad \ldots\text{(i)}$$

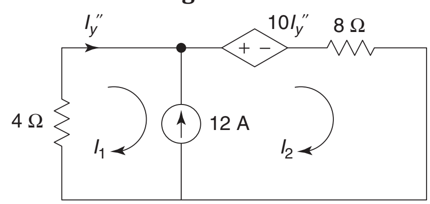


Fig. 2.106

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 12 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-4I_1 - 10I_y'' - 8I_2 = 0$$

$$-4I_1 - 10I_1 - 8I_2 = 0$$

$$14I_1 + 8I_2 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -4.36 \text{ A}$$

$$I_2 = 7.64 \text{ A}$$

$$Iy'' = I_1 = -4.36 \text{ A } (\rightarrow)$$

Step III When the 40 V source is acting alone (Fig. 2.107)

Applying KVL to the mesh,

$$-4\, I_y''' - 10I_y''' - 8I_y''' - 40 = 0$$

$$I_y''' = -\frac{40}{22} = -1.82 \text{ A } (\rightarrow)$$

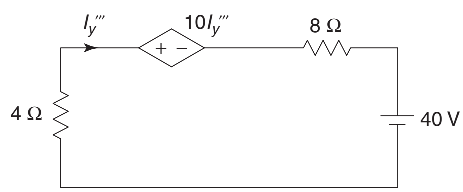


Fig. 2.107

Step IV By superposition theorem,

$$I_y = I_y' + I_y'' + I_y''' = 5.45 - 4.36 - 1.82 = -0.73 \text{ A } (\rightarrow)$$

Example 2.57 *Find the voltage V_x in Fig. 2.108.*

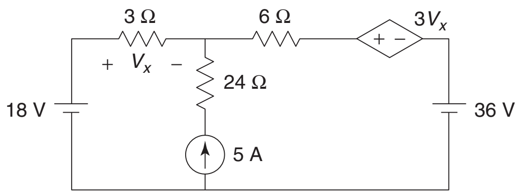


Fig. 2.108

Solution

Step I When the 18 V source is acting alone (Fig. 2.109)

From Fig. 2.109,

$$V_x' = 3I$$

Applying KVL to the mesh,

$$18 - 3I - 6I - 3V_x' = 0$$
$$18 - 3I - 6I - 3\,(3I) = 0$$
$$I = 1\text{ A}$$
$$V_x' = 3\text{ V}$$

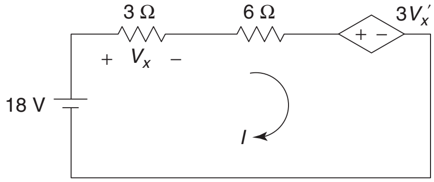


Fig. 2.109

Step II When the 5 A source is acting alone (Fig. 2.110)

From Fig. 2.110,

$$V_x'' = -3I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 5 \qquad \text{...(ii)}$$

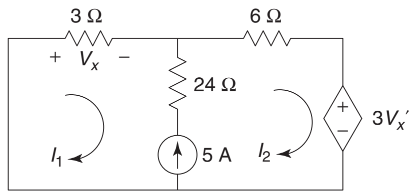


Fig. 2.110

Applying KVL to the outer path of the supermesh,

$$-3I_1 - 6I_2 - 3V_x'' = 0$$
$$-3I_1 - 6I_2 - 3(3I_1) = 0$$
$$12I_1 + 6I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -1.67\text{A}$$
$$I_2 = 3.33\text{ A}$$
$$V_x'' = 3I_1 = 3(-1.67) = -5\text{ V}$$

Step III When the 36 V source is acting alone (Fig. 2.111)

From Fig. 2.111,

$$V_x''' = -3I$$

Applying KVL to the mesh,

$$36 + 3V_x''' - 6I - 3I = 0$$
$$36 + 3V_x''' - 6\left(\frac{-V_x'''}{3}\right) - 3\left(\frac{-V_x'''}{3}\right) = 0$$
$$36 + 3V_x''' + 2V_x''' + V_x''' = 0$$
$$V_x''' = -6\text{ V}$$

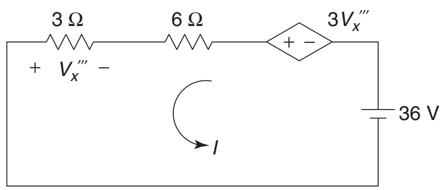


Fig. 2.111

Step IV By superposition theorem,

$$V_x = V_x' + V_x'' + V_x''' = 3 - 5 - 6 = -8\text{ V}$$

Example 2.58 *Find the voltage V in the network of Fig. 2.112.*

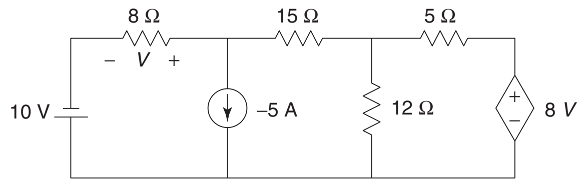


Fig. 2.112

Solution

Step I When the 10 V source is acting alone (Fig. 2.113)

From Fig. 2.113,

$$V' = -8I_1 \qquad \text{...(i)}$$

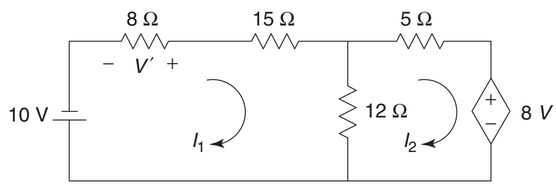


Fig. 2.113

Applying KVL to Mesh 1,

$$-10 - 8I_1 - 15\,I_1 - 12\,(I_1 - I_2) = 0$$

$$35I_1 - 12I_2 = -10 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-12(I_2 - I_1) - 5I_2 - 8V' = 0$$

$$-12I_2 + 12I_1 - 5I_2 - 8(-8I_1) = 0$$

$$76I_1 - 17I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.54 \text{ A}$$

$$I_2 = 2.4 \text{ A}$$

$$V' = -8I_1 = -8(0.54) = -4.32 \text{ V}$$

Step II When the –5 A source is acting alone (Fig. 2.114)

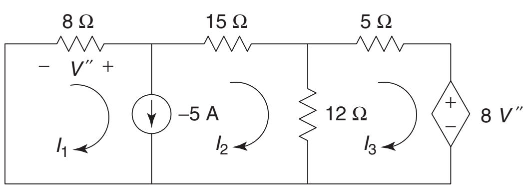


Fig. 2.114

From Fig. 2.114,

$$V'' = -8I_1 \qquad \text{...(i)}$$

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_1 - I_2 = -5 \qquad \text{...(ii)}$$

Applying KVL to the outer path of the supermesh,

$$-8I_1 - 15I_2 - 12(I_2 - I_3) = 0$$
$$-8I_1 - 27I_2 + 12I_3 = 0 \quad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-12(I_3 - I_2) - 5I_3 - 8V'' = 0$$
$$-12I_3 + 12I_2 - 5I_3 - 8(-8I_1) = 0$$
$$64I_1 + 12I_2 - 17I_3 = 0 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 4.97 \text{ A}$$
$$I_2 = 9.97 \text{ A}$$
$$I_3 = 25.74 \text{ A}$$
$$V'' = -8I_1 = -8(-4.97) = -39.76 \text{ V}$$

Step III By superposition theorem,

$$V = V' + V'' = -4.32 - 39.76 = -44.08 \text{ V}$$

Example 2.59 *For the network shown in Fig. 2.115, find the voltage V_0.*

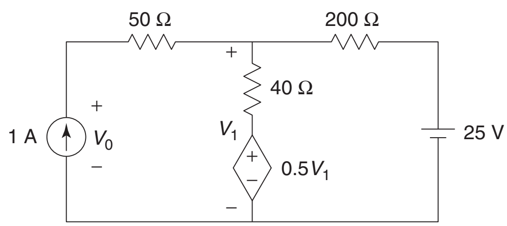


Fig. 2.115

Solution

Step I When the 1 A source is acting alone (Fig. 2.116)

From Fig. 2.116,

$$V_1 = 200\, I_2 \quad \text{...(i)}$$

For Mesh 1,

$$I_1 = 1 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$0.5V_1 - 40(I_2 - I_1) - 200\, I_2 = 0$$
$$0.5(200I_2) - 40I_2 + 40I_1 - 200I_2 = 0$$
$$40I_1 - 140\, I_2 = 0 \quad \text{...(iii)}$$

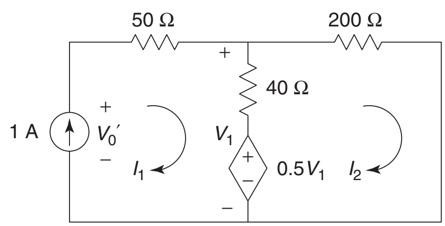


Fig. 2.116

Solving Eqs (ii) and (iii),

$$I_1 = 1 \text{ A}$$
$$I_2 = 0.29 \text{ A}$$
$$V_0' - 50\, I_1 - 200I_2 = 0$$
$$V_0' - 50(1) - 200(0.29) = 0$$
$$V_0' = 108 \text{ V}$$

Step II When the 25 V source is acting alone (Fig. 2.117)
From Fig. 2.117,

$$V_1 - 200I - 25 = 0$$

$$V_1 = 200I + 25 \qquad \text{...(i)}$$

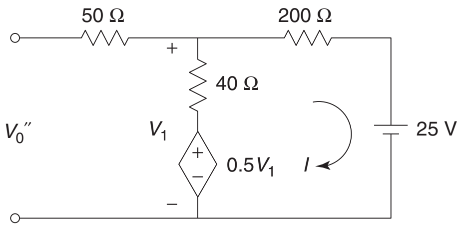


Fig. 2.117

Applying KVL to Mesh 1,

$$0.5V_1 - 40I - 200I - 25 = 0$$

$$0.5(200I + 25) - 40I - 200I - 25 = 0$$

$$I = -0.09 \text{ A}$$

$$V_0'' = V_1 = 200I + 25 = 200(-0.09) + 25 = 7 \text{ V}$$

Step III By superposition theorem,

$$V_0 = V_0' + V_0'' = 108 + 7 = 115 \text{ V}$$

Example 2.60 *For the network shown in Fig. 2.118, find the voltage V_x.*

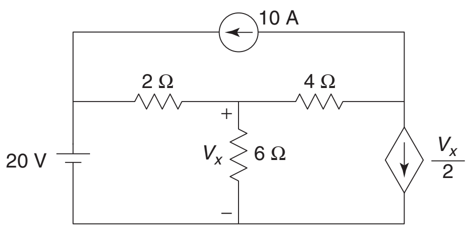


Fig. 2.118

Solution

Step I When the 20 V source is acting alone (Fig. 2.119)
From Fig. 2.119,

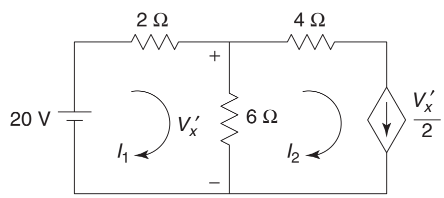


Fig. 2.119

$$V_x' = 6(I_1 - I_2) \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$20 - 2I_1 - 6(I_1 - I_2) = 0$$

$$8I_1 - 6I_2 = 20 \qquad \text{...(ii)}$$

For Mesh 2,

$$I_2 = \frac{V_x'}{2} = \frac{6(I_1 - I_2)}{2} = 3I_1 - 3I_2$$

$$3I_1 - 4I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 5.71 \text{ A}$$

$$I_2 = 4.29 \text{ A}$$

$$V_x' = 6(I_1 - I_2) = 6(5.71 - 4.29) = 8.52 \text{ V}$$

Step II When the 10 A source is acting alone (Fig. 2.120)
From Fig. 2.120,

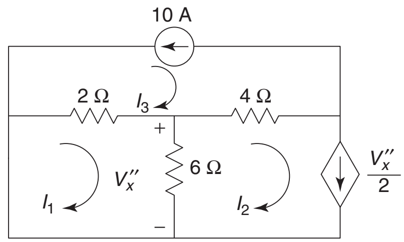


Fig. 2.120

$$V_x'' = 6(I_1 - I_2) \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-2(I_1 - I_3) - 6(I_1 - I_2) = 0$$

$$8I_1 - 6I_2 - 2I_3 = 0 \quad \text{...(ii)}$$

For Mesh 2,

$$I_2 = \frac{V_x''}{2} = \frac{6(I_1 - I_2)}{2} = 3I_1 - 3I_2$$

$$3I_1 - 4I_2 = 0 \quad \text{...(iii)}$$

For Mesh 3,

$$I_3 = -10 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -5.71 \text{ A}$$

$$I_2 = -4.29 \text{ A}$$

$$I_3 = -10 \text{ A}$$

$$V_x'' = 6(I_1 - I_2) = 6(-5.71 + 4.29) = -8.52 \text{ V}$$

Step III By superposition theorem,

$$V_x = V_x' + V_x'' = 8.52 - 8.52 = 0$$

Example 2.61 *Calculate the current I in the network shown in Fig. 2.121.*

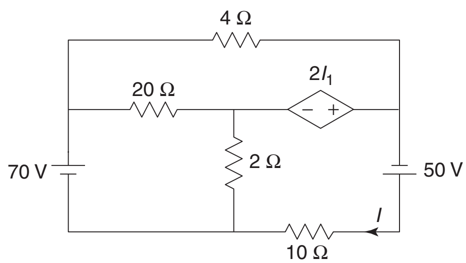


Fig. 2.121

Solution

Step I When the 70 V source is acting alone (Fig. 2.122)
From Fig. 2.122,

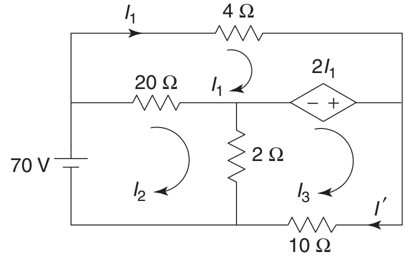


Fig. 2.122

$$I' = I_3 \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 20(I_1 - I_2) = 0$$

$$26I_1 - 20I_2 = 0 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$70 - 20(I_2 - I_1) - 2(I_2 - I_3) = 0$$

$$-20I_1 + 22I_2 - 2I_3 = 70 \quad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_2) + 2I_1 - 10I_3 = 0$$

$$2I_1 + 2I_2 - 12I_3 = 0 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 8.94\ \text{A}$$
$$I_2 = 11.62\ \text{A}$$
$$I_3 = 3.43\ \text{A}$$
$$I' = I_3 = 3.43\ \text{A}\ (\leftarrow)$$

Step II When the 50 V source is acting alone (Fig. 2.123)
From Fig. 2.123,

$$I'' = I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 20(I_1 - I_2) = 0$$
$$26I_1 - 20I_2 = 0 \qquad \text{...(ii)}$$

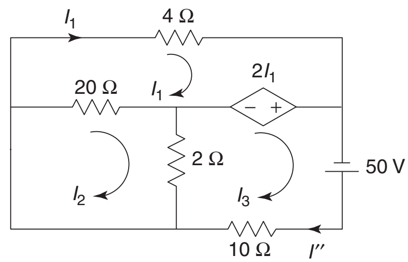


Fig. 2.123

Applying KVL to Mesh 2,

$$-20(I_2 - I_1) - 2(I_2 - I_3) = 0$$
$$-20I_1 + 22I_2 - 2I_3 = 0 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_2) + 2I_1 + 50 - 10I_3 = 0$$
$$2I_1 + 2I_2 - 12I_3 = -50 \qquad \text{...(iv)}$$

Solving Eqs (ii), (iii), and (iv),

$$I_1 = 1.06\ \text{A}$$
$$I_2 = 1.38\ \text{A}$$
$$I_3 = 4.57\ \text{A}$$
$$I'' = I_3 = 4.57\ \text{A}\ (\leftarrow)$$

Step III By superposition theorem,

$$I = I' + I'' = 3.43 + 4.57 = 8\ \text{A}\ (\leftarrow)$$

Example 2.62 *Find the voltage V_0 in the network of Fig. 2.124.*

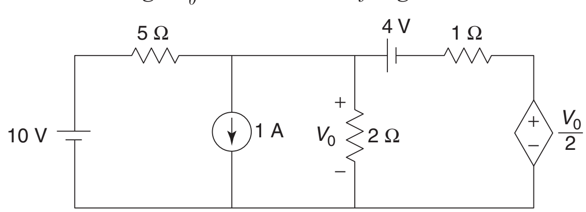


Fig. 2.124

Solution

Step I When the 10 V source is acting alone (Fig. 2.125)

Applying KCL at the node,

$$\frac{V_0'-10}{5}+\frac{V_0'}{2}+\frac{V_0'-\dfrac{V_0'}{2}}{1}=0$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0'=2$$

$$V_0'=1.67 \text{ V}$$

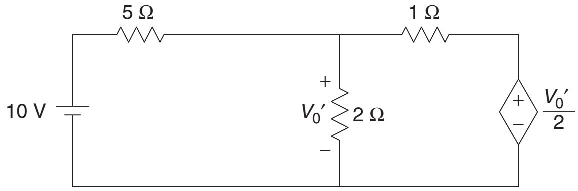


Fig. 2.125

Step II When the 1A current source is acting alone (Fig. 2.126)

Applying KCL at the node,

$$\frac{V_0''}{5}+1+\frac{V_0''}{2}+\frac{V_0''-\dfrac{V_0''}{2}}{1}$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0''=-1$$

$$V_0''=-0.83 \text{ V}$$

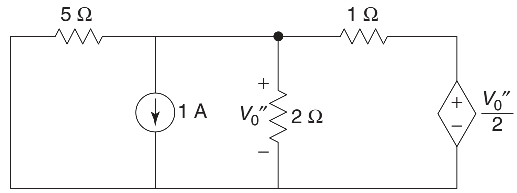


Fig. 2.126

Step III When the 4 V source is acting alone (Fig. 2.127)

Applying KCL at the node,

$$\frac{V_0'''}{5}+\frac{V_0'''}{2}+\frac{V_0'''-4-\dfrac{V_0'''}{2}}{1}=0$$

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)V_0'''=4$$

$$V_0'''=3.33 \text{ V}$$

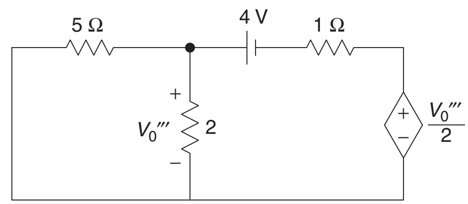


Fig. 2.127

Step IV By superposition theorem,

$$V_0 = V_0'+V_0''+V_0'''=1.67-0.83+3.33=4.17 \text{ V}$$

2.8 THEVENIN'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.*'

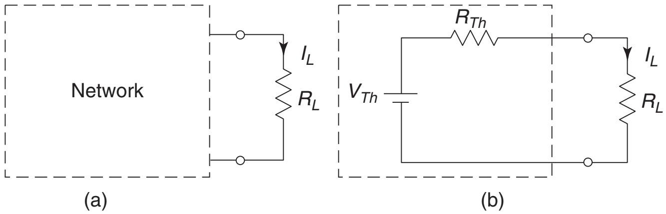


Fig. 2.128 *Network illustrating Thevenin's theorem*

Explanation Consider a simple network as shown in Fig. 2.129.

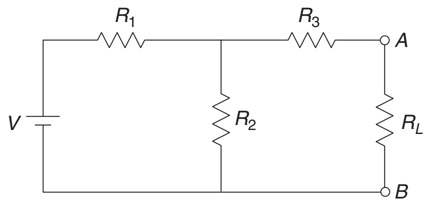


Fig. 2.129 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate open circuit voltage V_{Th} across points A and B as shown in Fig. 2.130.

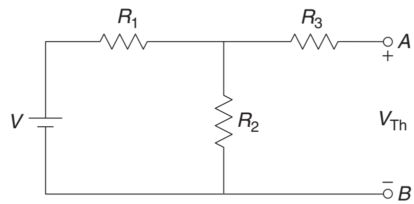


Fig. 2.130 *Calculation of V_{Th}*

$$V_{\text{Th}} = \frac{R_2}{R_1 + R_2} \text{V}$$

For finding series resistance R_{Th}, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 2.131.

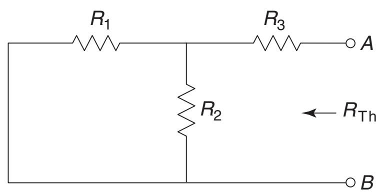


Fig. 2.131 *Calculation of R_{Th}*

$$R_{\text{Th}} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Thevenin's equivalent network is shown in Fig. 2.132.

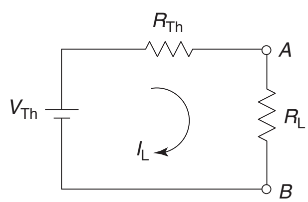


Fig. 2.132 *Thevenin's equivalent network*

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

If the network contains both independent and dependent sources, Thevenin's resistance R_{Th} is calculated as,

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N}$$

where I_N is the short-circuit current which would flow in a short circuit placed across the terminals A and B. Dependent sources are active at all times. They have zero values only when the control voltage or current is zero. R_{Th} may be negative in some cases which indicates negative resistance region of the device, i.e., as voltage increases, current decreases in the region and vice-versa.

If the network contains only dependent sources then

$$V_{Th} = 0$$
$$I_N = 0$$

For finding R_{Th} in such a network, a known voltage V is applied across the terminals A and B and current is calculated through the path AB.

$$R_{Th} = \frac{V}{I}$$

or a known current source I is connected across the terminals A and B and voltage is calculated across the terminals A and B.

$$R_{Th} = \frac{V}{I}$$

Thevenin's equivalent network for such a network is shown in Fig. 2.133.

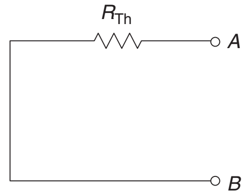


Fig. 2.133 *Thevenin's equivalent network*

Steps to be Followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Example 2.63 *Determine the current through the 24 Ω resistor in Fig. 2.134.*

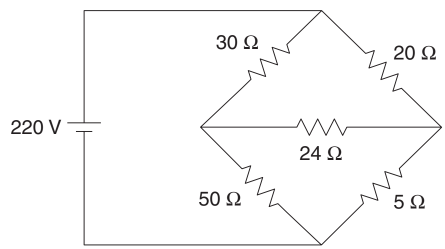


Fig. 2.134

Solution

Step I Calculation of V_{Th} (Fig. 2.135)

$$I_1 = \frac{220}{30 + 50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20 + 5} = 8.8 \text{ A}$$

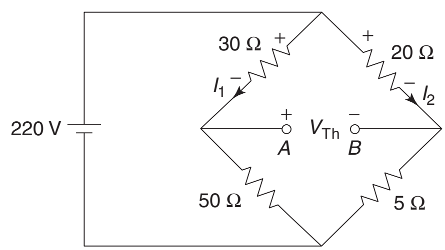


Fig. 2.135

Writing the V_{Th} equation,

$$V_{\text{Th}} + 30I_1 - 20I_2 = 0$$
$$V_{\text{Th}} = 20I_2 - 30I_1 = 20(8.8) - 30(2.75) = 93.5 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.136)

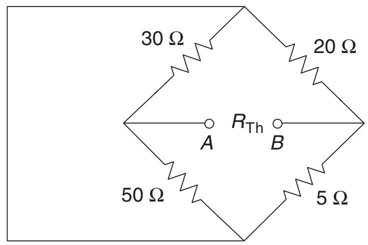


Fig. 2.136

Redrawing the circuit (Fig. 2.137),

$$R_{\text{Th}} = (30 \,||\, 50) + (20 \,||\, 5) = 22.75\ \Omega$$

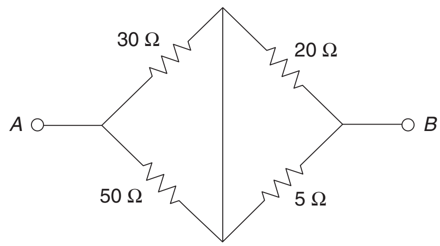


Fig. 2.137

Step III Calculation of I_L (Fig. 2.138)

$$I_L = \frac{93.5}{22.75 + 24} = 2 \text{ A}$$

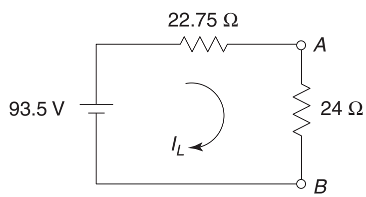


Fig. 2.138

Example 2.64 *Find the current through the 20 Ω resistor in Fig. 2.139.*

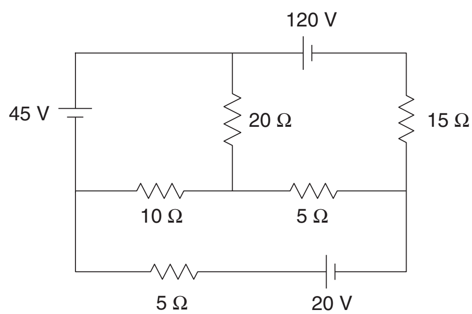


Fig. 2.139

Solution

Step I Calculation of V_{Th} (Fig. 2.140)

Applying KVL to Mesh 1,

$$45 - 120 - 15I_1 - 5(I_1 - I_2) - 10(I_1 - I_2) = 0 \qquad \text{...(i)}$$
$$30I_1 - 15I_2 = -75$$

Applying KVL to Mesh 2,

$$20 - 5I_2 - 10(I_2 - I_1) - 5(I_2 - I_1) = 0 \qquad \text{...(ii)}$$
$$-15I_1 + 20I_2 = 20$$

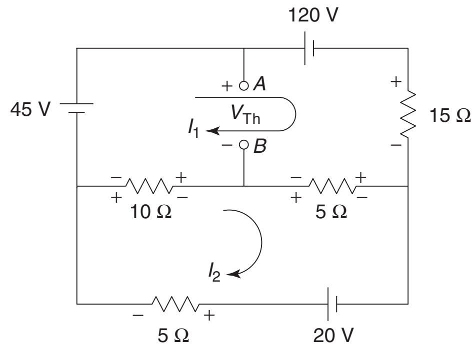


Fig. 2.140

Solving Eqs (i) and (ii),

$$I_1 = -3.2 \text{ A}$$
$$I_2 = -1.4 \text{ A}$$

Writing the V_{Th} equation,

$$45 - V_{\text{Th}} - 10(I_1 - I_2) = 0$$
$$V_{\text{Th}} = 45 - 10(I_1 - I_2) = 45 - 10[-3.2 - (-1.4)] = 63 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.141)

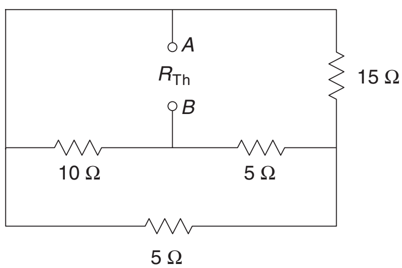


Fig. 2.141

Converting the delta formed by resistors of 10 Ω, 5 Ω and 5 Ω into an equivalent star network (Fig. 2.142),

$$R_1 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_2 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_3 = \frac{5 \times 5}{20} = 1.25\ \Omega$$

Simplifying the network (Fig. 2.143 and Fig. 2.144),

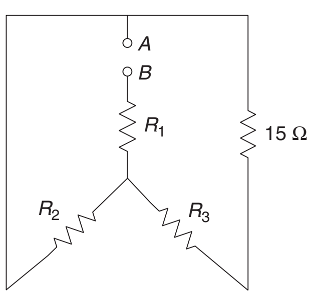


Fig. 2.142

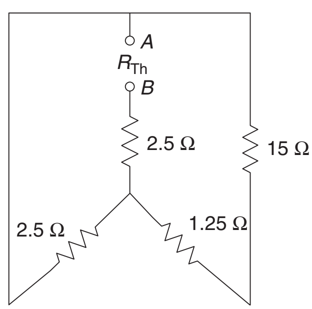


Fig. 2.143

$$R_{Th} = (16.25 || 2.5) + 2.5 = 4.67\ \Omega$$

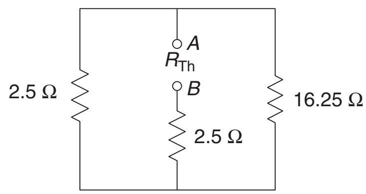


Fig. 2.144

Step III Calculation of I_L (Fig. 2.145)

$$I_L = \frac{63}{4.67 + 20} = 2.55\ \text{A}$$

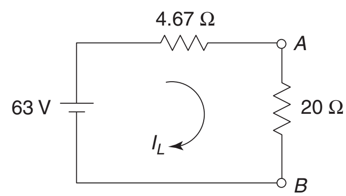


Fig. 2.145

Example 2.65 *Find the current through the 10 Ω resister in Fig. 2.146.*

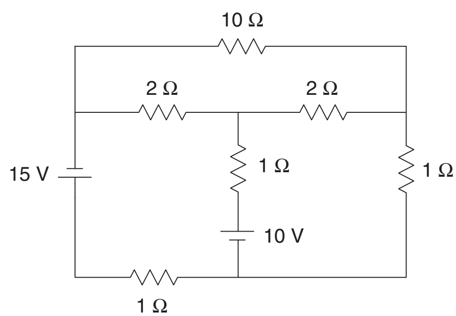


Fig. 2.146

Solution

Step I Calculation of V_{Th} (Fig. 2.147)

Applying KVL to Mesh 1,

$$-15 - 2I_1 - 1(I_1 - I_2) - 10 - 1I_1 = 0$$
$$4I_1 - I_2 = -25 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$10 - 1(I_2 - I_1) - 2I_2 - 1I_2 = 0$$
$$-I_1 + 4I_2 = 10 \quad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = -6\ \text{A}$$
$$I_2 = 1\ \text{A}$$

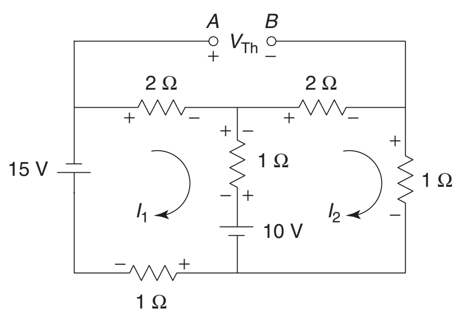


Fig. 2.147

Writing the V_{Th} equation,

$$-V_{\text{Th}} + 2I_2 + 2I_1 = 0$$

$$V_{\text{Th}} = 2I_1 + 2I_2 = 2(-6) + 2(1) = -10\text{ V}$$

$$= 10\text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II Calculation of R_{Th} (Fig. 2.148)

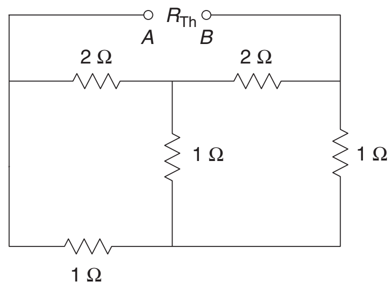


Fig. 2.148

Converting the star network formed by resistors of $2\,\Omega$, $2\,\Omega$ and $1\,\Omega$ into an equivalent delta network (Fig. 2.149),

$$R_1 = 2 + 2 + \frac{2\times 2}{1} = 8\,\Omega$$

$$R_2 = 2 + 1 + \frac{2\times 1}{2} = 4\,\Omega$$

$$R_3 = 2 + 1 + \frac{2\times 1}{2} = 4\,\Omega$$

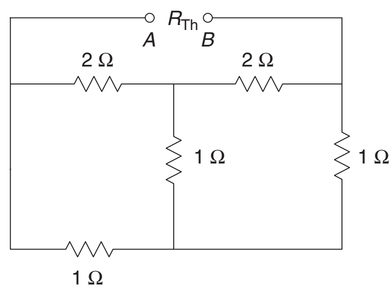


Fig. 2.149

Simplifying the network (Fig. 2.150),

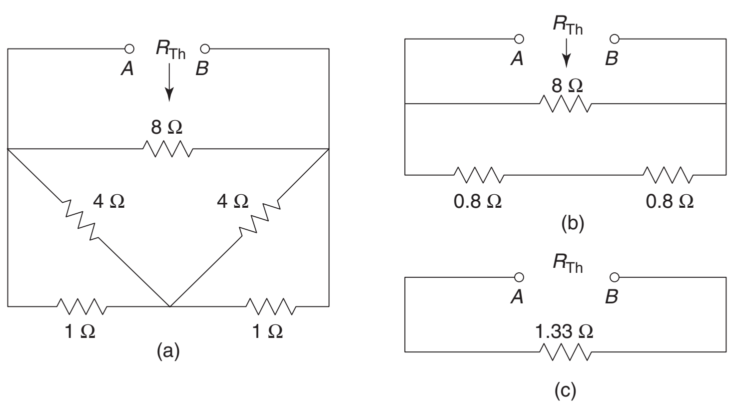


Fig. 2.150

$$R_{\text{Th}} = 1.33\ \Omega$$

Step III Calculation of I_L (Fig. 2.151)

$$I_L = \frac{10}{1.33+10} = 0.88\ \text{A}\ (\uparrow)$$

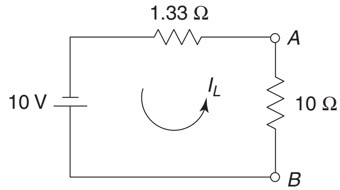


Fig. 2.151

Example 2.66 *Find the current through the 1 Ω resistor in Fig. 2.152.*

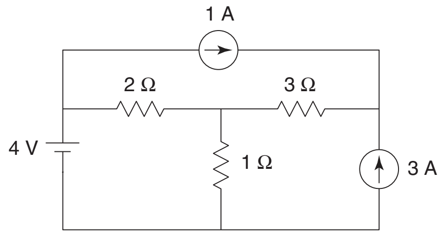


Fig. 2.152

Solution

Step I Calculation of V_{Th} (Fig. 2.153)

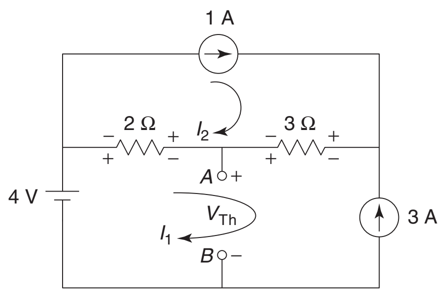


Fig. 2.153

Writing the current equations for Meshes 1 and 2,

$$I_1 = -3$$

$$I_2 = 1$$

Writing the V_{Th} equation,

$$4 - 2(I_1 - I_2) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 4 - 2(I_1 - I_2) = 4 - 2(-4) = 12\ \text{V}$$

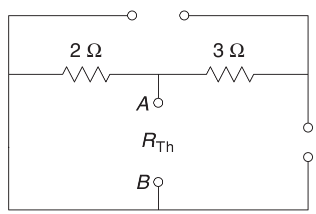


Fig. 2.154

Step II Calculation of R_{Th} (Fig. 2.154)

$$R_{\text{Th}} = 2\ \Omega$$

Step III Calculation of I_L (Fig. 2.155)

$$I_L = \frac{12}{2+1} = 4\ \text{A}$$

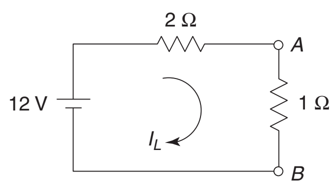


Fig. 2.155

EXAMPLES WITH DEPENDENT SOURCES

Example 2.67 *Obtain the Thevenin equivalent network for the given network of Fig. 2.156 at terminals A and B.*

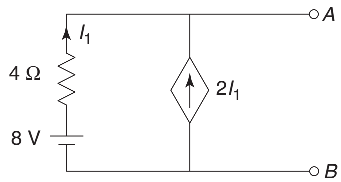


Fig. 2.156

Solution

Step I Calculation of V_{Th} (Fig. 2.157)

From Fig. 2.157,

$$I_1 = -2I_1$$
$$3I_1 = 0$$
$$I_1 = 0$$

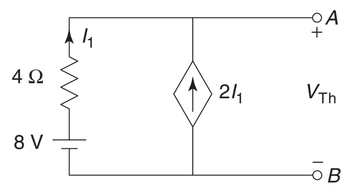


Fig. 2.157

Writing the V_{Th} equation,

$$8 - 0 - V_{Th} = 0$$
$$V_{Th} = 8 \text{ V}$$

Step II Calculation of I_N (Fig. 2.158),

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 2I_1$$
$$3I_1 - I_2 = 0 \qquad \text{...(i)}$$

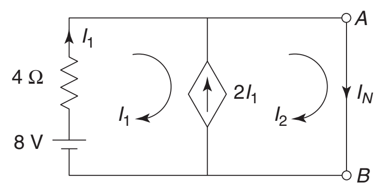


Fig. 2.158

Applying KVL to the outer path of the supermesh,

$$8 - 4I_1 = 0$$
$$I_1 = 2 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = 6 \text{ A}$$
$$I_N = I_2 = 6 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{8}{6} = 1.33\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.159)

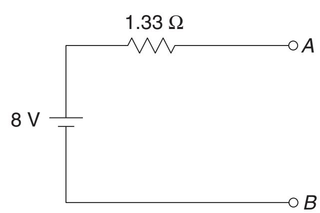


Fig. 2.159

Example 2.68 *Find Thevenin's equivalent network of Fig. 2.160.*

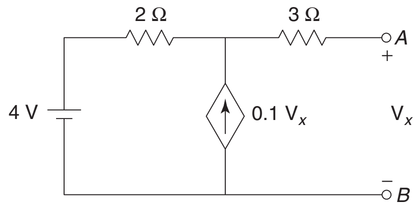


Fig. 2.160

Solution

Step I Calculation of V_{Th} (Fig. 2.161)

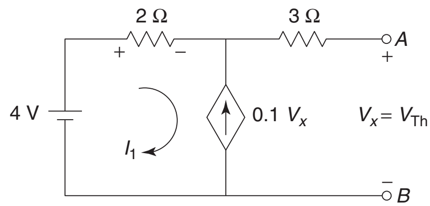


Fig. 2.161

$$V_x = V_{\text{Th}}$$

$$I_1 = -0.1V_x$$

Writing the V_{Th} equation,

$$4 - 2I_1 - V_x = 0$$

$$4 - 2(-0.1V_x) - V_x = 0$$

$$4 - 0.8V_x = 0$$

$$V_x = 5 \text{ V}$$

$$V_x = V_{\text{Th}} = 5 \text{ V}$$

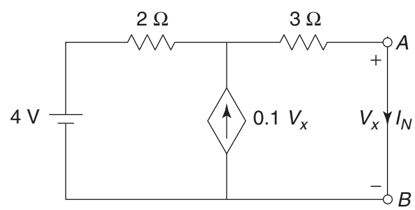


Fig. 2.162

Step II Calculation of I_N (Fig. 2.162)
From Fig. 2.162,

$$V_x = 0$$

The dependent source 0.1 V_x depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e., 0.1 $V_x = 0$ as shown in Fig. 2.163.

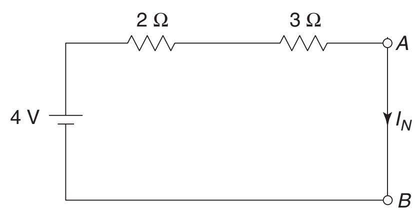


Fig. 2.163

$$I_N = \frac{4}{2+3} = 0.8 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{5}{0.8} = 6.25\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.164)

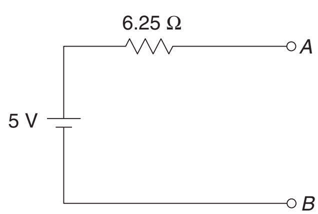


Fig. 2.164

Example 2.69 *Obtain the Thevenin equivalent network of Fig. 2.165 for the terminals A and B.*

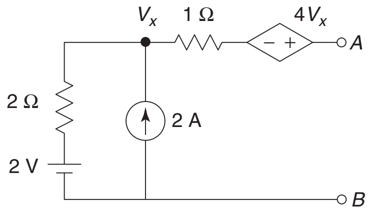


Fig. 2.165

Solution

Step I Calculation of V_{Th} (Fig. 2.166)
From Fig. 2.166,

$$2 - 2I_1 - V_x = 0$$
$$V_x = 2 - 2I_1$$

For Mesh 1,

$$I_1 = -2 \text{ A}$$
$$V_x = 2 - 2(-2) = 6 \text{ V}$$

Writing the V_{Th} equation,

$$2 - 2I_1 - 0 + 4V_x - V_{Th} = 0$$
$$2 - 2(-2) - 0 + 4(6) - V_{Th} = 0$$
$$V_{Th} = 30 \text{ V}$$

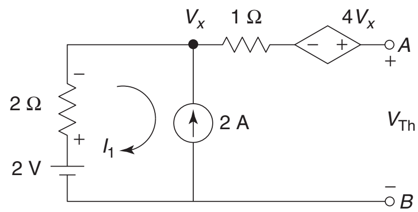


Fig. 2.166

Step II Calculation of I_N (Fig. 2.167)
From Fig. 2.167,

$$V_x = 2 - 2I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh,
Writing current equation for the supermesh

$$I_2 - I_1 = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$2 - 2I_1 - 1I_2 + 4V_x = 0$$
$$2 - 2I_1 - I_2 + 4(2 - 2I_1) = 0$$
$$10I_1 + I_2 = 10 \qquad \ldots\text{(iii)}$$

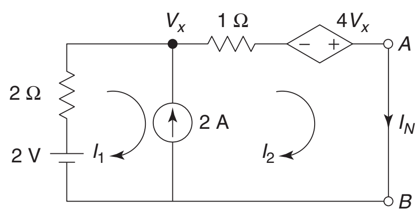


Fig. 2.167

Solving Eqs (ii) and (iii),

$$I_1 = 0.73 \text{ A}$$
$$I_2 = 2.73 \text{ A}$$
$$I_N = I_2 = 2.73 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{30}{2.73} = 10.98\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.168)

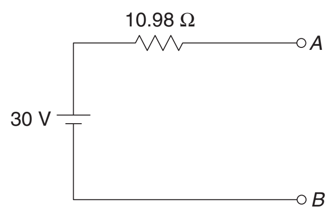


Fig. 2.168

Example 2.70 *Find the Thevenin equivalent network of Fig. 2.169 for the terminals A and B.*

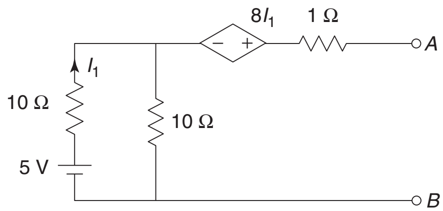


Fig. 2.169

Solution

Step I Calculation of V_{Th} (Fig. 2.170)
Applying KVL to the mesh,

$$5 - 10I_1 - 10I_1 = 0$$

$$I_1 = \frac{5}{20} = 0.25 \text{ A}$$

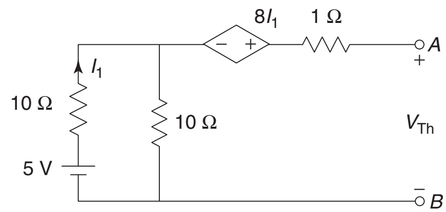


Fig. 2.170

Writing the V_{Th} equation,

$$5 - 10I_1 + 8I_1 - 0 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 5 - 2I_1 = 5 - 2(0.25) = 4.5 \text{ V}$$

Step II Calculation of I_N (Fig. 2.171)
Applying KVL to Mesh 1,

$$5 - 10I_1 - 10(I_1 - I_2) = 0$$

$$20I_1 - 10I_2 = 5 \qquad \text{...(i)}$$

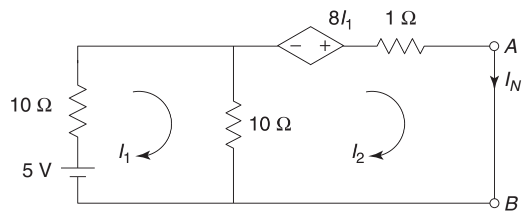


Fig. 2.171

Applying KVL to Mesh 2,

$$-10(I_2 - I_1) + 8I_1 - 1I_2 = 0$$

$$18I_1 - 11I_2 = 0 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 1.375 \text{ A}$$

$$I_2 = 2.25 \text{ A}$$

$$I_N = I_2 = 2.25 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{4.5}{2.25} = 2\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.172)

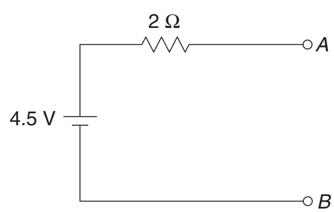


Fig. 2.172

Example 2.71 *Find V_{Th} and R_{Th} between terminals A and B of the network shown in Fig. 2.173.*

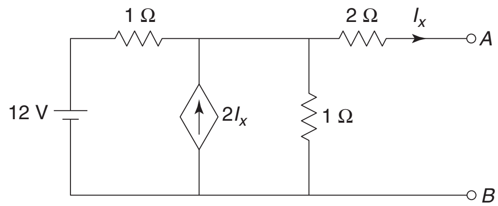


Fig. 2.173

Solution

Step I Calculation of V_{Th} (Fig. 2.174)

$$I_x = 0$$

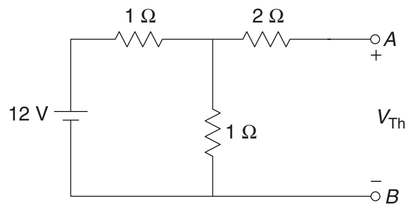


Fig. 2.174

The dependent source $2I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e., $2I_x = 0$ as shown in Fig. 2.174.

Writing the V_{Th} equation,

$$V_{\text{Th}} = 12 \times \frac{1}{1+1} = 6 \text{ V}$$

Step II Calculation of I_N (Fig. 2.175)
From Fig. 2.175,

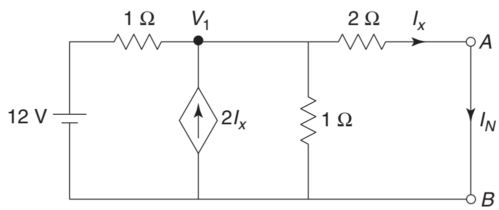


Fig. 2.175

$$I_x = \frac{V_1}{2}$$

Applying KCL at Node 1,

$$\frac{V_1 - 12}{1} + \frac{V_1}{1} + \frac{V_1}{2} = 2I_x$$

$$V_1 + V_1 + \frac{V_1}{2} - 12 = 2\left(\frac{V_1}{2}\right)$$

$$V_1 = 8 \text{ V}$$

$$I_N = \frac{V_1}{2} = \frac{8}{2} = 4 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{6}{4} = 1.5\ \Omega$$

Example 2.72 *Obtain the Thevenin equivalent network of Fig. 2.176 for the given network at terminals a and b .*

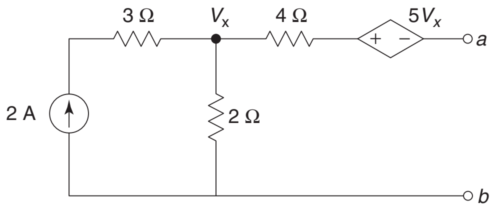


Fig. 2.176

Solution

Step I Calculation of V_{Th} (Fig. 2.177)
Applying KCL at Node x,

$$2 = \frac{V_x}{2}$$

$$V_x = 4 \text{ V}$$

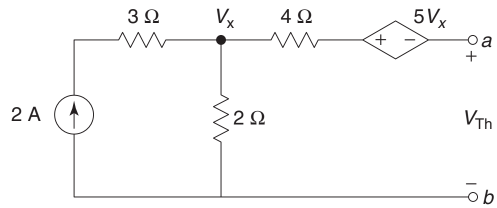


Fig. 2.177

Writing the V_{Th} equation,

$$V_{Th} = V_x - 5V_x = -4V_x$$

$$= -16 \text{ V} \quad (\text{the terminal } a \text{ is negative w.r.t. } b)$$

Step II Calculation of I_N (Fig. 2.178)
Applying KCL at Node x,

$$2 = \frac{V_x}{2} + \frac{V_x - 5V_x}{4}$$

$$2 = \frac{V_x}{2} - V_x = -\frac{V_x}{2}$$

$$V_x = -4 \text{ V}$$

$$I_N = \frac{V_x - 5V_x}{4} = -V_x = 4 \text{ A}$$

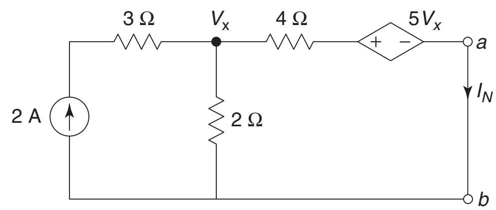


Fig. 2.178

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{-16}{4} = -4\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.179)

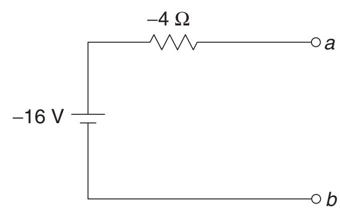


Fig. 2.179

Example 2.73 *Obtain the Thevenin equivalent network of Fig. 2.180 for the given network.*

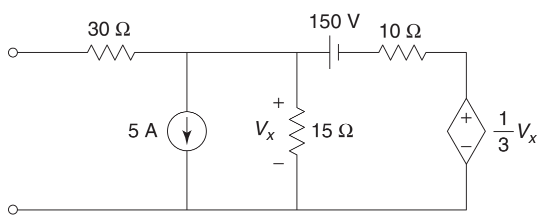


Fig. 2.180

Solution

Step I Calculation of V_{Th} (Fig. 2.181)
From Fig. 2.181,

$$V_x = V_{Th}$$

Applying KCL at the node,

$$\frac{V_x - 150 - \frac{1}{3}V_x}{10} + \frac{V_x}{15} + 5 = 0$$

$$V_x = 75 \text{ V}$$

$$V_{Th} = 75 \text{ V}$$

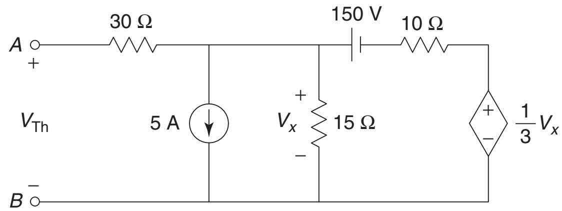


Fig. 2.181

Step II Calculation of I_N (Fig. 2.182)
Applying KCL at Node x,

$$\frac{V_x}{30} + 5 + \frac{V_x}{15} + \frac{V_x - 150 - \frac{1}{3}V_x}{10} = 0$$

$$\frac{V_x}{30} + \frac{V_x}{15} + \frac{V_x}{10} - \frac{V_x}{30} = 15 - 5$$

$$V_x = 60 \text{ V}$$

$$I_N = \frac{V_x}{30} = \frac{60}{30} = 2 \text{ A}$$

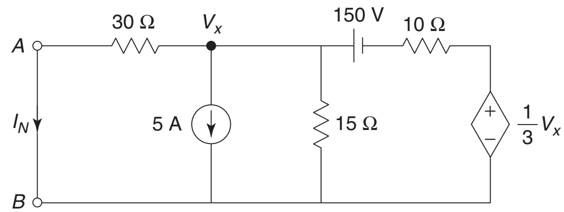


Fig. 2.182

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{75}{2} = 37.5\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.183)

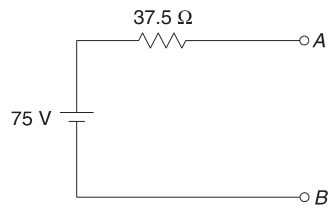


Fig. 2.183

Example 2.74 *Find the Thevenin's equivalent network of the network to the left of A-B in the Fig. 2.184.*

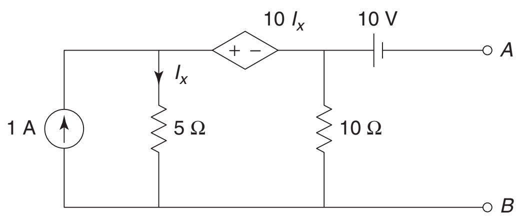


Fig. 2.184

Solution

Step I Calculation of V_{Th} (Fig. 2.185)
From Fig. 2.185,

$$I_x = I_1 - I_2 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 1 \qquad \ldots\text{(ii)}$$

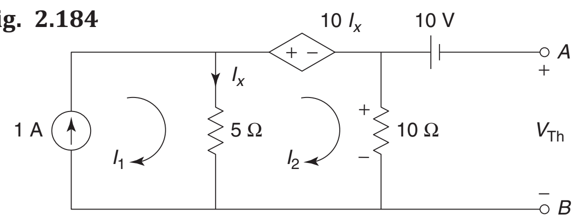


Fig. 2.185

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 10I_x - 10I_2 = 0$$

$$-5(I_2 - I_1) - 10(I_1 - I_2) - 10I_2 = 0$$

$$5I_1 + 5I_2 = 0 \quad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 1 \text{ A}$$

$$I_2 = -1 \text{ A}$$

$$I_x = I_1 - I_2 = 1 - (-1) = 2 \text{ A}$$

Writing the V_{Th} equation,

$$10I_2 - 10 - V_{\text{Th}} = 0$$

$$10(-1) - 10 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -20 \text{ V}$$

Step II Calculation of I_N (Fig. 2.186)
From Fig. 2.186,

$$I_x = I_1 - I_2 \quad \text{...(i)}$$

For Mesh 1,

$$I_1 = 1 \quad \text{...(ii)}$$

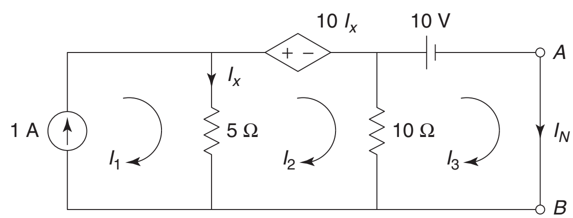


Fig. 2.186

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 10I_x - 10(I_2 - I_3) = 0$$

$$-5(I_2 - I_1) - 10(I_1 - I_2) - 10(I_2 - I_3) = 0$$

$$-5I_1 - 5I_2 + 10I_3 = 0 \quad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-10(I_3 - I_2) - 10 = 0$$

$$10I_2 - 10I_3 = 10 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 1 \text{ A}$$

$$I_2 = 3 \text{ A}$$

$$I_3 = 2 \text{ A}$$

$$I_N = I_3 = 2 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{-20}{2} = -10\ \Omega$$

Step IV Thevenin's Equivalent Network (Fig. 2.187)

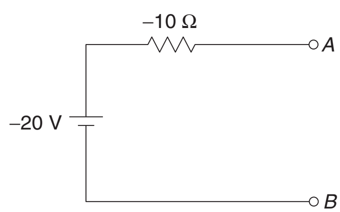


Fig. 2.187

Example 2.75 *Find Thevenin's equivalent network at terminals A and B in the network of Fig. 2.188.*

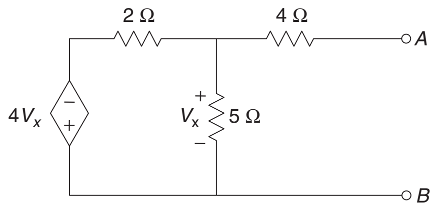


Fig. 2.188

Solution

Since the network does not contain any independent source,

$$V_{\text{Th}} = 0$$

$$I_N = 0$$

But the R_{Th} can be calculated by applying a known voltage source of 1 V at the terminals A and B as shown in Fig. 2.189.

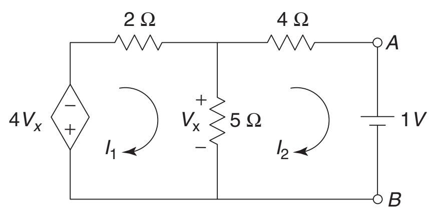


Fig. 2.189

$$R_{\text{Th}} = \frac{V}{I} = \frac{1}{I}$$

From Fig. 2.189,

$$V_x = 5(I_1 - I_2) \qquad \text{...(i)}$$

Applying KVL to Mesh 1,

$$-4V_x - 2I_1 - 5(I_1 - I_2) = 0$$

$$-4[5(I_1 - I_2] - 2I_1 - 5I_1 + 5I_2 = 0$$

$$-27I_1 + 25I_2 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 4I_2 - 1 = 0$$

$$5I_1 - 9I_2 = 1 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -0.21 \text{ A}$$

$$I_2 = -0.23 \text{ A}$$

Hence, current supplied by voltage source of 1 V is 0.23 A.

$$R_{\text{Th}} = \frac{1}{0.23} = 4.35\ \Omega$$

Hence, Thevenin's equivalent network is shown in Fig. 2.190.

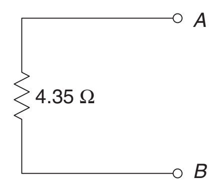


Fig. 2.190

Example 2.76 *Find the current in the 9 Ω resistor in Fig. 2.191.*

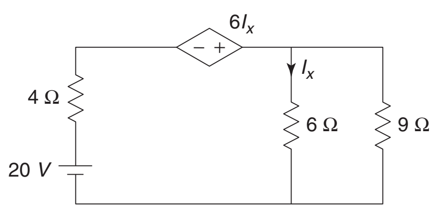


Fig. 2.191

Solution

Step I Calculation of V_{Th} (Fig. 2.192)

Applying KVL to the mesh,

$$20 - 4I_x + 6I_x - 6I_x = 0$$
$$I_x = 5\text{ A}$$

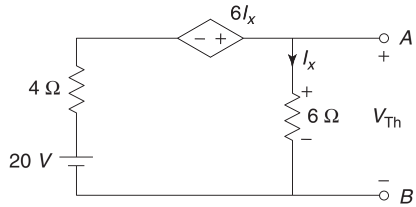


Fig. 2.192

Writing the V_{Th} equation,

$$6I_x - V_{Th} = 0$$
$$6(5) - V_{Th} = 0$$
$$V_{Th} = 30\text{ V}$$

Step II Calculation of I_N (Fig. 2.193).

From Fig. 2.193,

$$I_x = 0$$

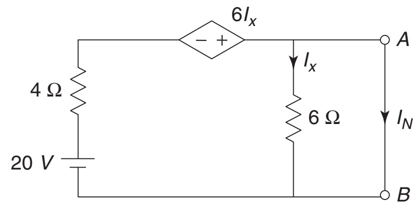


Fig. 2.193

The dependent source $6I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e., $6I_x = 0$ as shown in Fig. 2.194.

$$I_N = \frac{20}{4} = 5\text{ A}$$

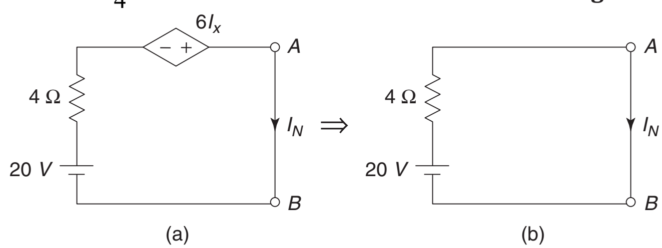


Fig. 2.194

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{30}{5} = 6\ \Omega$$

Step IV Calculation of I_L (Fig. 2.195)

$$I_L = \frac{30}{6+9} = 2\text{ A}$$

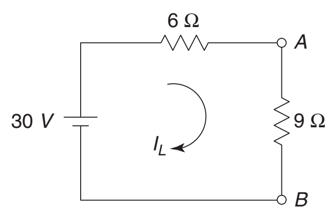


Fig. 2.195

Example 2.77 *Determine the current in the 16 Ω resistor in Fig. 2.196.*

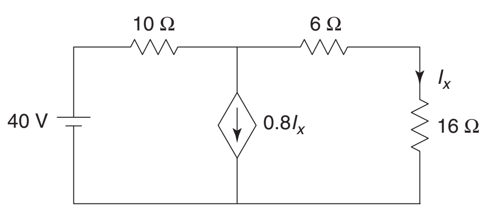


Fig. 2.196

Solution

Step I Calculation of V_{Th} (Fig. 2.197)

From Fig. 2.197,

$$I_x = 0$$

The dependent source $0.8I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, as shown in Fig. 2.198.

i.e., $$0.8I_x = 0$$

$$V_{Th} = 40 \text{ V}$$

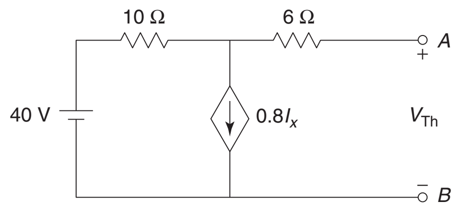


Fig. 2.197

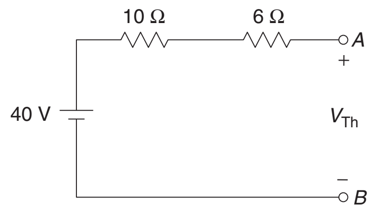


Fig. 2.198

Step II Calculation of I_N (Fig. 2.199)

From Fig. 2.199,

$$I_x = I_2 \quad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh,

Writing current equation for the supermesh,

$$I_1 - I_2 = 0.8\, I_x = 0.8\, I_2$$

$$I_1 - 1.8\, I_2 = 0 \quad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$40 - 10\, I_1 - 6\, I_2 = 0$$

$$10\, I_1 + 6\, I_2 = 40 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 3 \text{ A}$$

$$I_2 = \frac{5}{3} \text{ A}$$

$$I_N = I_2 = \frac{5}{3} \text{ A}$$

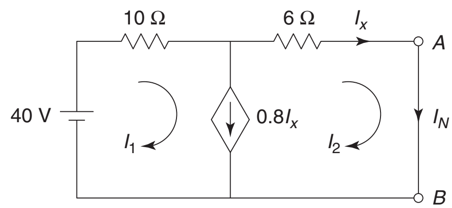


Fig. 2.199

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{40}{\frac{5}{3}} = 24\ \Omega$$

Step IV Calculation of I_L (Fig. 2.200)

$$I_L = \frac{40}{24 + 16} = 1 \text{ A}$$

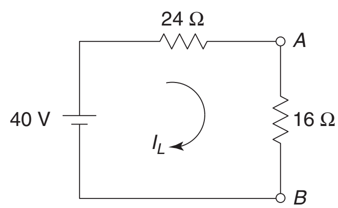


Fig. 2.200

Example 2.78 *Find the current in the 6 Ω resistor in Fig. 2.201.*

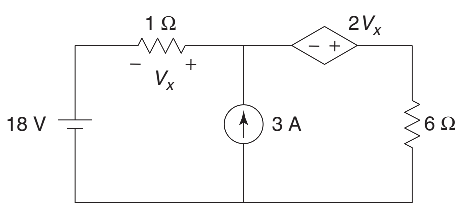


Fig. 2.201

Solution

Step I Calculation of V_{Th} (Fig. 2.202)

From Fig. 2.202,

$$V_x = -1I_1 = -I_1 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = -3\text{ A} \qquad \ldots\text{(ii)}$$

$$V_x = 3\text{ V}$$

Writing the V_{Th} equation,

$$18 - 1\,I_1 + 2\,V_x - V_{Th} = 0$$

$$18 + 3 + 2(3) - V_{Th} = 0$$

$$V_{Th} = 27\text{ V}$$

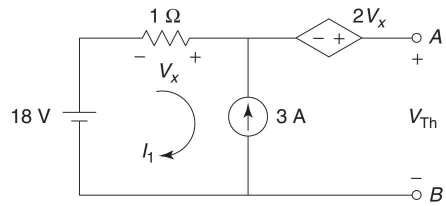


Fig. 2.202

Step II Calculation of I_N (Fig. 2.203)

From Fig. 2.203,

$$V_x = -I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh,

Writing current equation for supermesh,

$$I_2 - I_1 = 3 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$18 - 1I_1 + 2\,V_x = 0$$

$$18 - I_1 + 2(-I_1) = 0 \qquad \ldots\text{(iii)}$$

$$I_1 = 6\text{ A}$$

Solving Eqs (ii) and (iii),

$$I_2 = 9\text{ A}$$

$$I_N = I_2 = 9\text{ A}$$

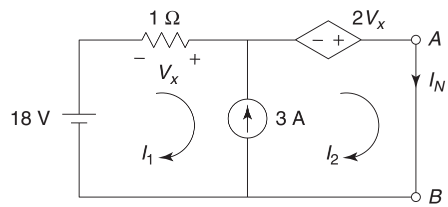


Fig. 2.203

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{27}{9} = 3\,\Omega$$

Step IV Calculation of I_L (Fig. 2.204)

$$I_L = \frac{27}{3+6} = 3\text{ A}$$

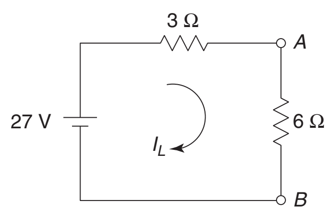


Fig. 2.204

Example 2.79 *Find the current in the 10 Ω resistor.*

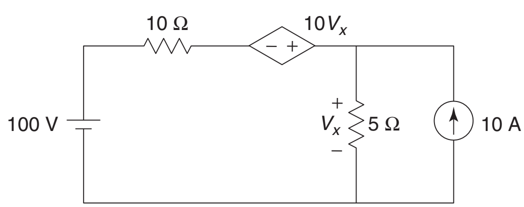


Fig. 2.205

Solution

Step I Calculation of V_{Th} (Fig. 2.206)

From Fig. 2.206,

$$V_x = 10 \times 5 = 50\text{ V}$$

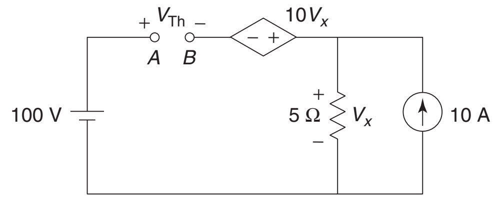


Fig. 2.206

Writing the V_{Th} equation,

$$100 - V_{\text{Th}} + 10V_x - V_x = 0$$

$$100 - V_{\text{Th}} + 9V_x = 0$$

$$100 - V_{\text{Th}} + 9(50) = 0$$

$$V_{\text{Th}} = 550\text{ V}$$

Step II Calculation of I_N (Fig. 2.207)

From Fig. 2.207,

$$V_x = 5(I_N + 10)$$

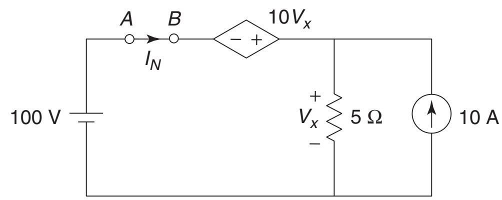


Fig. 2.207

Applying KVL to Mesh 1,

$$100 + 10V_x - V_x = 0$$

$$V_x = -\frac{100}{9}$$

$$-\frac{100}{9} = 5I_N + 50$$

$$I_N = -\frac{550}{45}\text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{550}{-\frac{550}{45}} = -45\ \Omega$$

Step IV Calculation of I_L (Fig. 2.208)

$$I_L = \frac{550}{-45+10} = -\frac{110}{7}\text{ A}$$

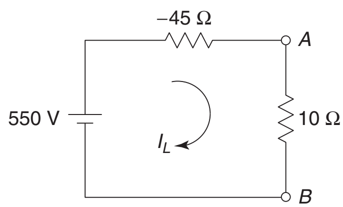


Fig. 2.208

2.9 NORTON'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.*' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances.

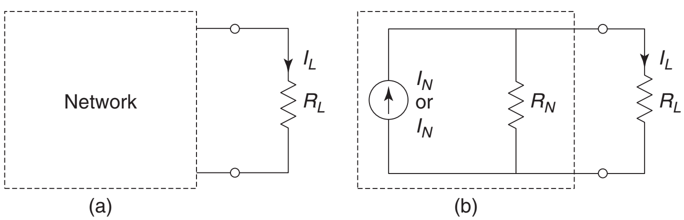


Fig. 2.209 *Network illustrating Norton's theorem*

Explanation Consider a simple network as shown in Fig. 2.210.

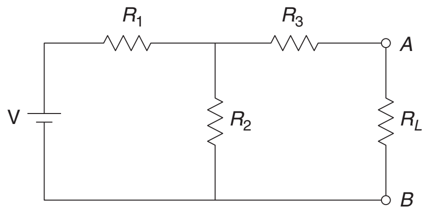


Fig. 2.210 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate short circuit current I_{SC} or I_N which would flow in a short circuit placed across terminals A and B as shown in Fig. 2.211.

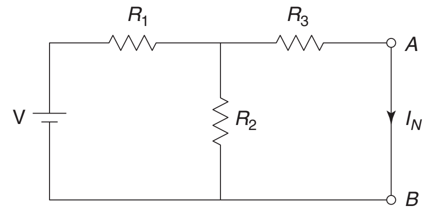


Fig. 2.211 *Calculation of I_N*

For finding parallel resistance R_N, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 2.212.

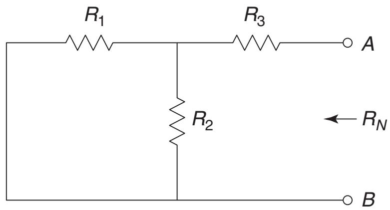


Fig. 2.212 *Calculation of R_N*

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Norton's equivalent network is shown in Fig. 2.213.

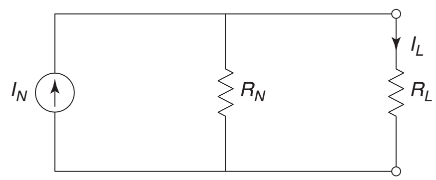


Fig. 2.213 *Norton's equivalent network*

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

If the network contains both independent and dependent sources, Norton's resistances R_N is calculated as

$$R_N = \frac{V_{\text{Th}}}{I_N}$$

where V_{Th} is the open-circuit voltage across terminals A and B. If the network contains only dependent sources, then

$$V_{\text{Th}} = 0$$

$$I_N = 0$$

To find R_{Th} in such network, a known voltage V or current I is applied across the terminals A and B, and the current I or the voltage V is calculated respectively.

$$R_N = \frac{V}{I}$$

Norton's equivalent network for such a network is shown in Fig. 2.214.

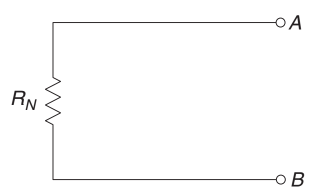


Fig. 2.214 *Norton's equivalent network*

Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{SC} or I_N.
3. Find the resistance R_N as seen from points A and B.
4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_L by current–division rule.

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 2.80 *Find the current through the 10 Ω resistor in Fig. 2.215.*

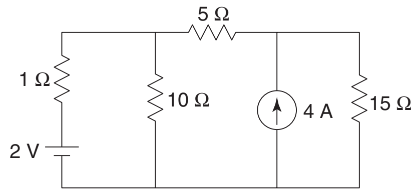


Fig. 2.215

Solution

Step I Calculation of I_N (Fig. 2.216)

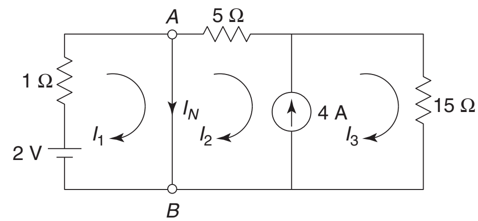


Fig. 2.216

Applying KVL to Mesh 1,

$$2 - 1I_1 = 0$$

$$I_1 = 2 \quad \text{...(i)}$$

Meshes 2 and 3 will form a supermesh.
Writing the current equation for the supermesh,

$$I_3 - I_2 = 4 \quad \text{...(ii)}$$

Applying KVL to the supermesh,

$$-5I_2 - 15I_3 = 0 \quad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 2\text{ A}$$

$$I_2 = -3\text{ A}$$

$$I_3 = 1\text{ A}$$

$$I_N = I_1 - I_2 = 2 - (-3) = 5\text{ A}$$

Step II Calculation of R_N (Fig. 2.217)

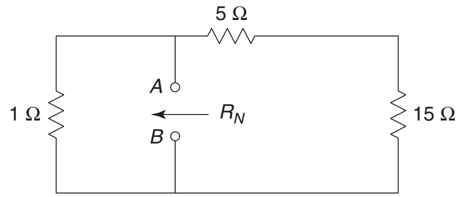


Fig. 2.217

$$R_N = 1 \,||\, (5 + 15) = 0.95\ \Omega$$

Step III Calculation of I_L (Fig. 2.218)

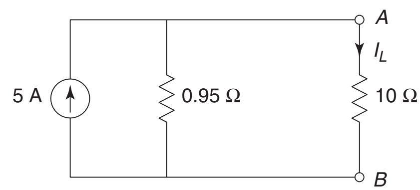


Fig. 2.218

$$I_L = 5 \times \frac{0.95}{0.95 + 10} = 0.43\text{ A}$$

Example 2.81 *Find the current through the 10 Ω resistor in Fig. 2.219.*

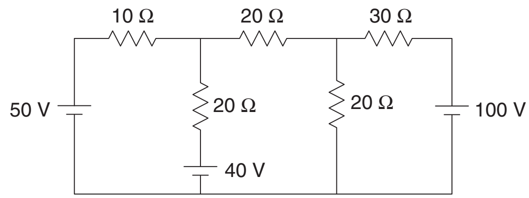


Fig. 2.219

Solution

Step I Calculation of I_N (Fig. 2.220)

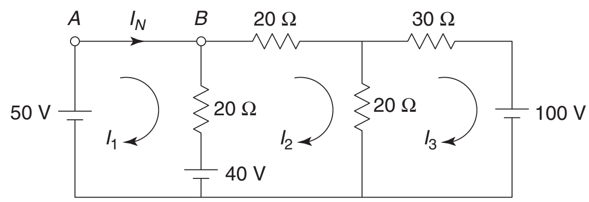


Fig. 2.220

Applying KVL to Mesh 1,

$$50 - 20(I_1 - I_2) - 40 = 0$$
$$20I_1 - 20I_2 = 10 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$40 - 20(I_2 - I_1) - 20I_2 - 20(I_2 - I_3) = 0$$
$$-20I_1 + 60I_2 - 20I_3 = 40 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-20(I_3 - I_2) - 30I_3 - 100 = 0$$
$$-20I_2 + 50I_3 = -100 \quad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.81\,\text{A}$$
$$I_N = I_1 = 0.81\,\text{A}$$

Step II Calculation of R_N (Fig. 2.221)

$$R_N = [(20 \,||\, 30) + 20] \,||\, 20 = 12.3\,\Omega$$

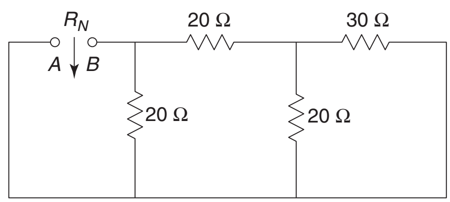


Fig. 2.221

Step III Calculation of I_L (Fig. 2.222)

$$I_L = 0.81 \times \frac{12.3}{12.3 + 10} = 0.45\,\text{A}$$

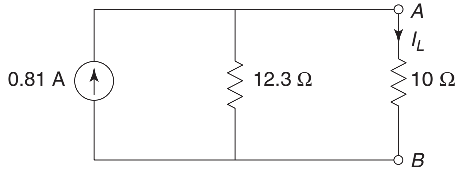


Fig. 2.222

Example 2.82 *Find the current through the 8 Ω resistor in Fig. 2.223.*

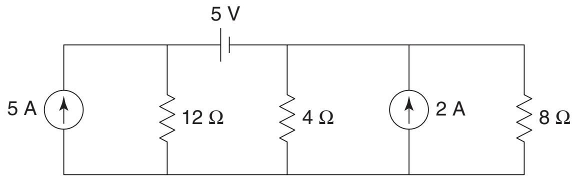


Fig. 2.223

Solution

Step I Calculation of I_N (Fig. 2.224)

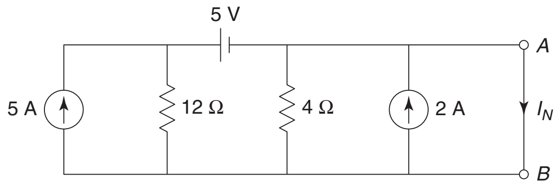


Fig. 2.224

The resistor of the 4 Ω gets shorted as it is in parallel with the short circuit. Simplifying the network by source transformation (Fig. 2.225),

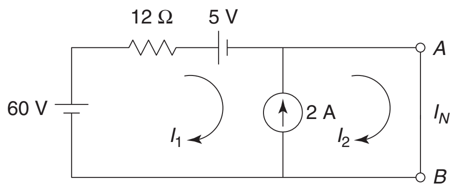


Fig. 2.225

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 2 \quad \text{...(i)}$$

Applying KVL to the supermesh,

$$60 - 12I_1 - 5 = 0$$
$$12I_1 = 55 \quad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 4.58\,\text{A}$$
$$I_2 = 6.58\,\text{A}$$
$$I_N = I_2 = 6.58\,\text{A}$$

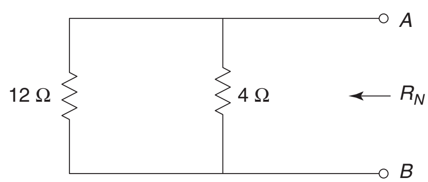


Fig. 2.226

Step II Calculation of R_N (Fig. 2.226)

$$R_N = 12 \,||\, 4 = 3\,\Omega$$

Step III Calculation of I_L (Fig. 2.227)

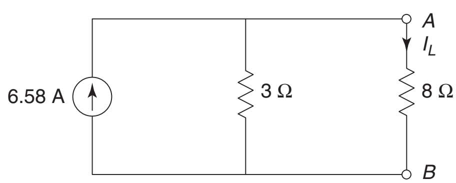


Fig. 2.227

$$I_L = 6.58 \times \frac{3}{3+8} = 1.79\,\text{A}$$

Example 2.83 *Find the current through the 1 Ω resistor in Fig. 2.228.*

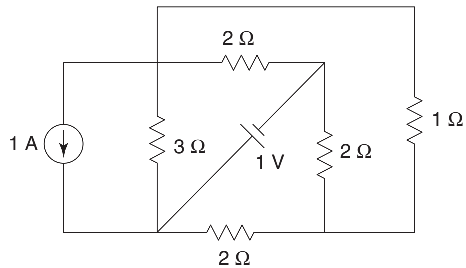


Fig. 2.228

Solution

Step I Calculation of I_N (Fig. 2.229)

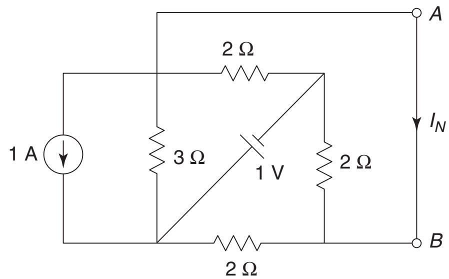


Fig. 2.229

By source transformation (Fig. 2.230),

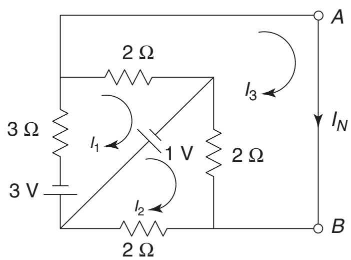


Fig. 2.230

Applying KVL to Mesh 1,

$$-3-3I_1-2(I_1-I_3)+1=0$$

$$5I_1-2I_3=-2 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-1-2(I_2-I_3)-2I_2=0$$
$$4I_2-2I_3=-1 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3-I_1)-2(I_3-I_2)=0$$
$$-2I_1-2I_2+4I_3=0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1=-0.64\text{ A}$$
$$I_2=-0.55\text{ A}$$
$$I_3=-0.59\text{ A}$$
$$I_N=I_3=-0.59\text{ A}$$

Step II Calculation of R_N (Fig. 2.231)

$$R_N=2.2\ \Omega$$

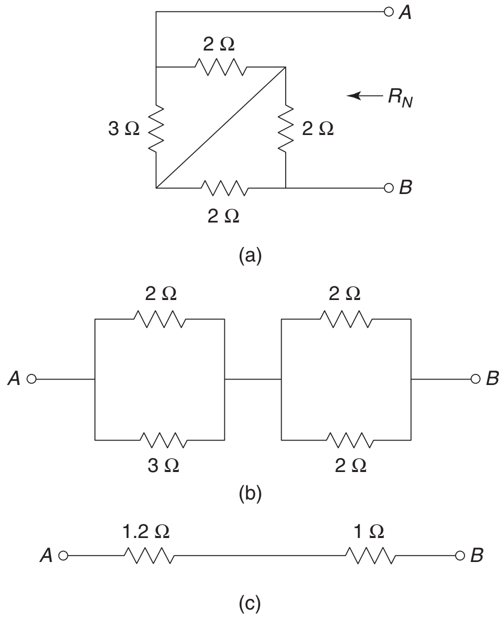


Fig. 2.231

Step III Calculation of I_L (Fig. 2.232)

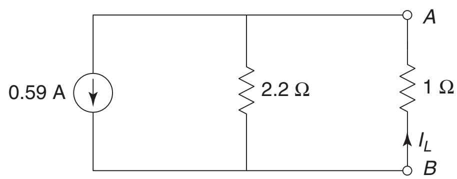


Fig. 2.232

$$I_L=0.59\times\frac{2.2}{2.2+1}=0.41\text{ A}$$

EXAMPLES WITH DEPENDENT SOURCES

Example 2.84 *Find Norton's equivalent network across terminals A and B of Fig. 2.233.*

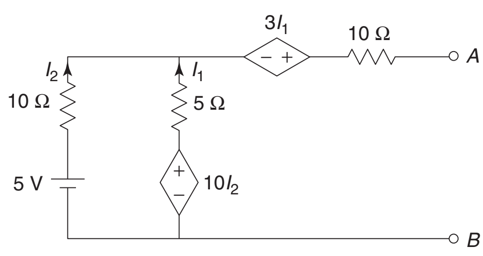


Fig. 2.233

Solution

Step I Calculation of V_{Th} (Fig. 2.234)

From Fig. 2.234,

$$I_2 = I_x$$
$$I_1 = -I_x$$

Applying KVL to the mesh,

$$5 - 10I_x - 5I_x - 10I_2 = 0$$
$$5 - 10I_x - 5I_x - 10I_x = 0$$
$$I_x = 0.2 \text{ A}$$
$$I_1 = -0.2 \text{ A}$$

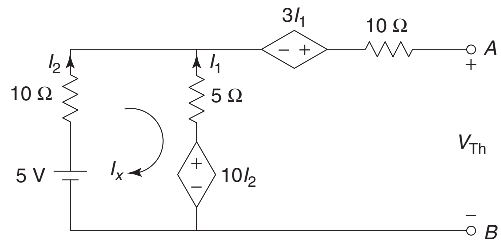


Fig. 2.234

Writing the V_{Th} equation,

$$5 - 10I_x + 3I_1 - V_{Th} = 0$$
$$5 - 10(0.2) + 3(-0.2) - V_{Th} = 0$$
$$V_{Th} = 2.4 \text{ V}$$

Step II Calculation of I_N (Fig. 2.235)

From Fig. 2.235,

$$I_2 = I_x \qquad \ldots\text{(i)}$$
$$I_1 = I_y - I_x \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 1,

$$5 - 10I_x - 5(I_x - I_y) - 10I_2 = 0$$
$$5 - 10I_x - 5I_x + 5I_y - 10I_x = 0$$
$$25I_x - 5I_y = 5 \qquad \ldots\text{(iii)}$$

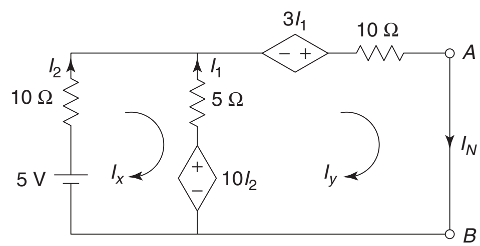


Fig. 2.235

Applying KVL to Mesh 2,

$$10I_2 - 5(I_y - I_x) + 3I_1 - 10I_y = 0$$
$$10I_x - 5I_y + 5I_x + 3(I_y - I_x) - 10I_y = 0$$
$$12I_x - 12I_y = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (iii) and (iv),

$$I_x = 0.25 \text{ A}$$
$$I_y = 0.25 \text{ A}$$
$$I_N = I_y = 0.25 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{2.4}{0.25} = 9.6\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.236)

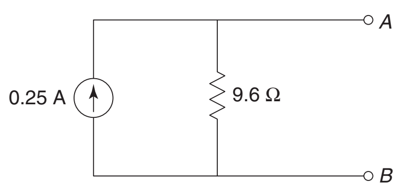


Fig. 2.236

Example 2.85 *For the network shown in Fig. 2.237, find Norton's equivalent network.*

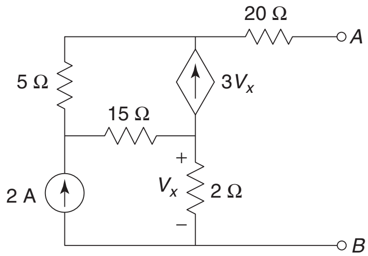


Fig. 2.237

Solution

Step I Calculation of V_{Th} (Fig. 2.238)

From Fig. 2.238,

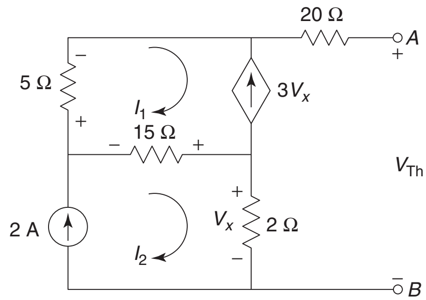


Fig. 2.238

$$V_x = 2I_2 \quad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = -3V_x = -3(2I_2) = -6I_2 \quad \ldots\text{(ii)}$$

For Mesh 2,

$$I_2 = 2 \quad \ldots\text{(iii)}$$

$$I_1 = -6I_2 = -6(2) = -12 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 0 + 5I_1 + 15(I_1 - I_2) - 2I_2 = 0$$
$$V_{Th} + 5(-12) + 15(-12 - 2) - 2(2) = 0$$
$$V_{Th} = 274 \text{ V}$$

Step II Calculation of I_N (Fig. 2.239)

From Fig. 2.239,

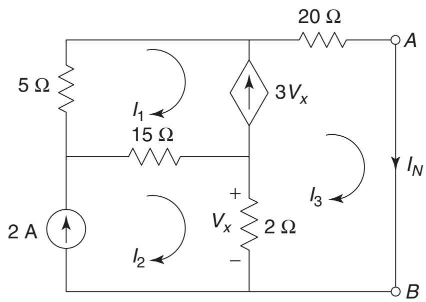


Fig. 2.239

$$V_x = 2(I_2 - I_3) \quad \ldots\text{(i)}$$

For Mesh 2,

$$I_2 = 2 \quad \ldots\text{(ii)}$$

Meshes 1 and 3 will form a supermesh.

Writing the current equation for the supermesh,

$$I_3 - I_1 = 3V_x = 3[2(I_2 - I_3)] = 6I_2 - 6I_3$$

$$I_1 + 6I_2 - 7I_3 = 0 \quad \ldots\text{(iii)}$$

Applying KVL to the outer path of the supermesh,

$$-5I_1 - 20I_3 - 2(I_3 - I_2) - 15(I_1 - I_2) = 0$$
$$-20I_1 + 17I_2 - 22I_3 = 0 \quad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -0.16 \text{ A}$$
$$I_2 = 2 \text{ A}$$
$$I_3 = 1.69 \text{ A}$$
$$I_N = I_3 = 1.69 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{274}{1.69} = 162.13\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.240)

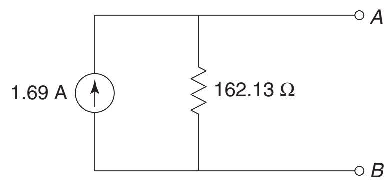


Fig. 2.240

Example 2.86 *Obtain Norton's equivalent network across A-B in the network of Fig. 2.241.*

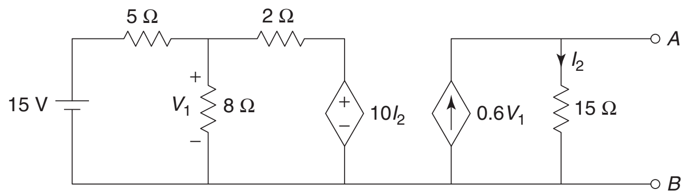


Fig. 2.241

Solution

Step I Calculation of V_{Th} (Fig. 2.242)

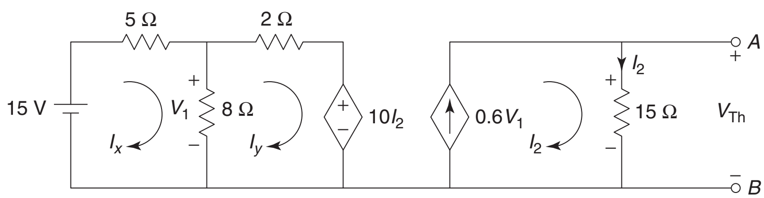


Fig. 2.242

From Fig. 2.242,

$$V_1 = 8(I_x - I_y) \quad \text{...(i)}$$

Applying KVL to Mesh 1,

$$15 - 5I_x - 8(I_x - I_y) = 0$$
$$13I_x - 8I_y = 15 \quad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-8(I_y - I_x) - 2I_y - 10I_2 = 0$$
$$8I_x - 10I_y - 10I_2 = 0 \quad \text{...(iii)}$$

For Mesh 3,

$$I_2 = 0.6V_1 = 0.6\left[8(I_x - I_y)\right]$$

$$4.8I_x - 4.8I_y - I_2 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_x = 3.28 \text{ A}$$
$$I_y = 3.45 \text{ A}$$
$$I_2 = -0.83 \text{ A}$$

Writing the V_{Th} equation,

$$15I_2 - V_{\text{Th}} = 0$$
$$15(-0.83) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = -12.45 \text{ V}$$

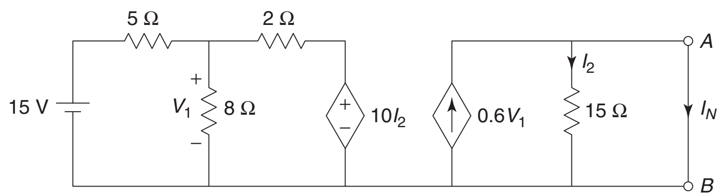


Fig. 2.243

Step II Calculation of I_N (Fig. 2.243)
From Fig. 2.243,

$$I_2 = 0$$

The dependent source of 10 I_2 depends on the controlling variable I_2. When $I_2 = 0$, the dependent source vanishes, i.e. $10I_2 = 0$ as shown in Fig. 2.244.

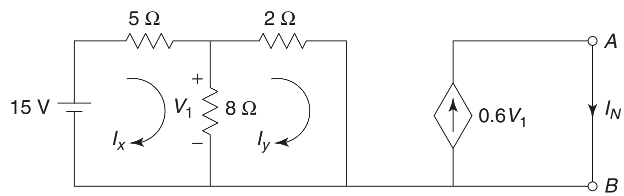


Fig. 2.244

From Fig. 2.244,

$$V_1 = 8(I_x - I_y) \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$15 - 5I_x - 8(I_x - I_y) = 0$$
$$13I_x - 8I_y = 15 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-8(I_y - I_x) - 2I_y = 0$$
$$-8I_x + 10I_y = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_x = 2.27 \text{ A}$$
$$I_y = 1.82 \text{ A}$$
$$V_1 = 8(I_x - I_y) = 8(2.27 - 1.82) = 3.6 \text{ V}$$

For Mesh 3,

$$I_N = 0.6V_1 = 0.6(3.6) = 2.16 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-12.45}{2.16} = -5.76\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.245)

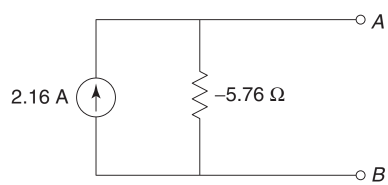


Fig. 2.245

Example 2.87 *Find Norton's equivalent network of Fig. 2.246.*

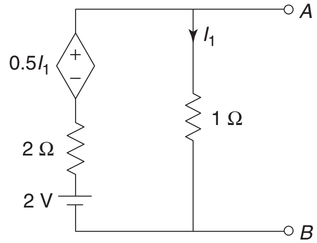


Fig. 2.246

Solution

Step I Calculation of V_{Th} (Fig. 2.247)

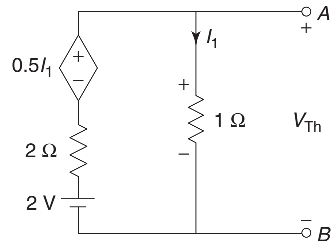


Fig. 2.247

Applying KVL to the mesh,

$$2 - 2I_1 + 0.5I_1 - 1I_1 = 0$$

$$2 - 2.5I_1 = 0$$

$$I_1 = 0.8 \text{ A}$$

Writing the V_{Th} equation,

$$1I_1 - V_{Th} = 0$$

$$1(0.8) - V_{Th} = 0$$

$$V_{Th} = 0.8 \text{ V}$$

Step II Calculation of I_N (Fig. 2.248)

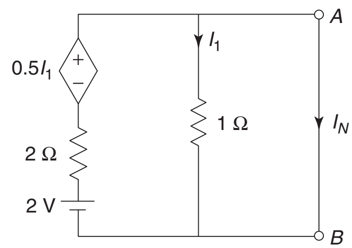


Fig. 2.248

When a short circuit is placed across the 1 Ω resistor, it gets shorted.

$$I_1 = 0$$

The dependent source of $0.5I_1$ depends on the controlling variable I_1. When $I_1 = 0$, the dependent source vanishes, i.e. $0.5\, I_1 = 0$ as shown in Fig. 2.249.

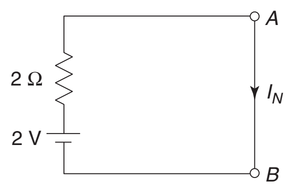


Fig. 2.249

$$I_N = \frac{2}{2} = 1 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{0.8}{1} = 0.8\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.250)

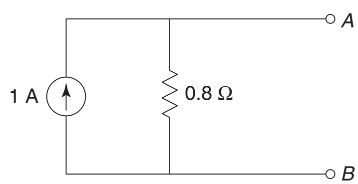


Fig. 2.250

Example 2.88 *Find Norton's equivalent network at the terminals A and B of Fig. 2.251.*

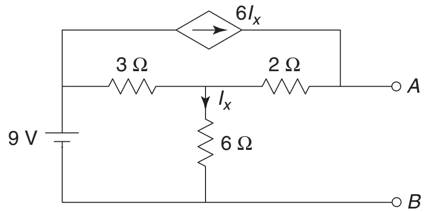


Fig. 2.251

Solution

Step I Calculation of V_{Th} (Fig. 2.252)

From Fig. 2.252,

$$I_x = I_1 \quad \text{...(i)}$$

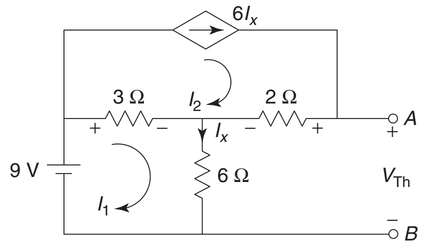


Fig. 2.252

Applying KVL to Mesh 1,

$$9 - 3(I_1 - I_2) - 6I_1 = 0$$
$$9I_1 - 3I_2 = 9 \quad \text{...(ii)}$$

For Mesh 2,

$$I_2 = 6I_x = 6I_1$$
$$6I_1 - I_2 = 0 \quad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = -1 \text{ A}$$
$$I_2 = -6 \text{ A}$$

Writing the V_{Th} equation,

$$9 - 3(I_1 - I_2) + 2I_2 - V_{Th} = 0$$
$$9 - 3(-1+6) + 2(-6) - V_{Th} = 0$$
$$V_{Th} = -18 \text{ V}$$

Step II Calculation of I_N (Fig. 2.253)

From Fig. 2.253,

$$I_x = I_1 - I_3 \quad \text{...(i)}$$

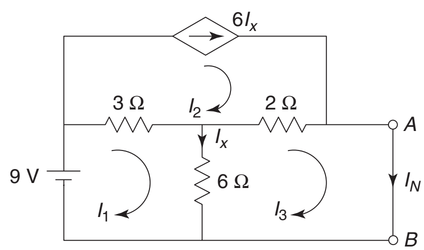


Fig. 2.253

Applying KVL to Mesh 1,

$$9 - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$
$$9I_1 - 3I_2 - 6I_3 = 9 \quad \text{...(ii)}$$

For Mesh 2,

$$I_2 = 6I_x = 6(I_1 - I_3)$$
$$6I_1 - I_2 - 6I_3 = 0 \quad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) - 2(I_3 - I_2) = 0$$
$$-6I_1 - 2I_2 + 8I_3 = 0 \quad \text{...(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 5 \text{ A}$$
$$I_2 = 3 \text{ A}$$
$$I_3 = 4.5 \text{ A}$$
$$I_N = I_3 = 4.5 \text{ A}$$

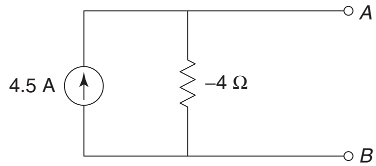


Fig. 2.254

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-18}{4.5} = -4\ \Omega$$

Step IV Norton's Equivalent Network (Fig. 2.254)

Example 2.89 *Find Norton's equivalent network to the left of terminal A-B in Fig. 2.255.*

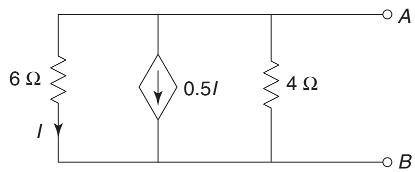


Fig. 2.255

Solution Since the network does not contain any independent source,

$$V_{\text{Th}} = 0$$
$$I_N = 0$$

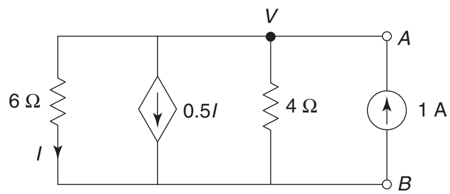


Fig. 2.256

But R_N can be calculated by applying a known current source of 1 A at the terminals A and B as shown in Fig. 2.256.

From Fig. 2.256,

$$I = \frac{V}{6}$$

Applying KCL at the node,

$$\frac{V}{6} + 0.5I + \frac{V}{4} = 1$$

$$\frac{V}{6} + 0.5\left(\frac{V}{6}\right) + \frac{V}{4} = 1$$

$$\left(\frac{1}{6} + \frac{0.5}{6} + \frac{1}{4}\right)V = 1$$

$$V = 2$$

$$R_N = \frac{V}{1} = \frac{2}{1} = 2\ \Omega$$

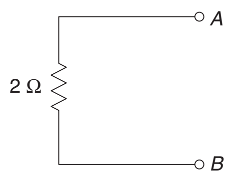


Fig. 2.257

Hence, Norton's equivalent network is shown in Fig. 2.257.

Example 2.90 *Find the current through the 2 Ω resistor in the network shown in Fig. 2.258.*

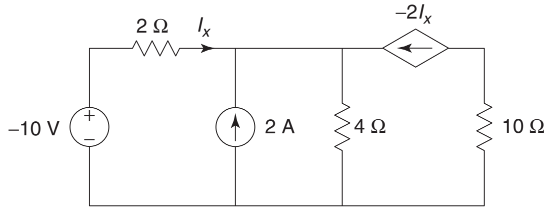


Fig. 2.258

Solution

Step I Calculation of V_{Th} (Fig. 2.259)

From Fig. 2.259,

$$I_x = 0$$

The dependent source of $-2\ I_x$ depends on the controlling variable I_x. When $I_x = 0$, the dependent source vanishes, i.e. $-2I_x = 0$ as shown in Fig. 2.260.

$$I_1 = 2$$

Writing the V_{Th} equation,

$$-10 - V_{Th} - 4I_1 = 0$$
$$-10 - V_{Th} - 4(2) = 0$$
$$V_{Th} = -18\text{ V}$$

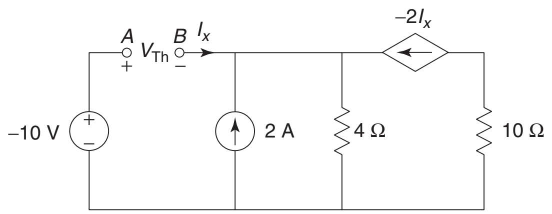


Fig. 2.259

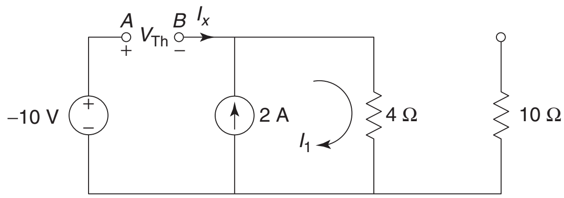


Fig. 2.260

Step II Calculation of I_N (Fig. 2.261)

From Fig. 2.261,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Mesh 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

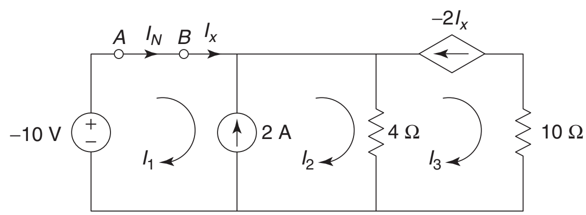


Fig. 2.261

$$-10 - 4(I_2 - I_3) = 0$$
$$-4I_2 + 4I_3 = 10 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = -(-2I_x) = 2I_x = 2I_1$$
$$2I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 4.5 \text{ A}$$
$$I_2 = 6.5 \text{ A}$$
$$I_3 = 9 \text{ A}$$
$$I_N = I_1 = 4.5 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-18}{4.5} = -4\ \Omega$$

Step IV Calculation of I_L (Fig. 2.262)

$$I_L = 4.5 \times \frac{-4}{-4+2} = 9 \text{ A}$$

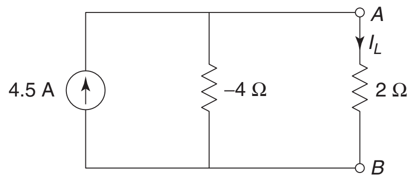


Fig. 2.262

Example 2.91 *Find the current through the 2Ω resistor in the network of Fig. 2.263.*

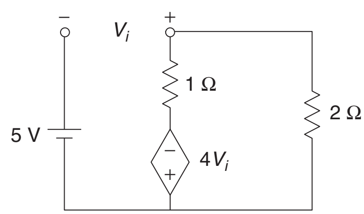


Fig. 2.263

Solution

Step I Calculation of V_{Th} (Fig. 2.264)

From Fig. 2.264,

$$5 + V_i + 4V_i = 0$$
$$V_i = -1 \text{ V}$$

Writing the V_{Th} equation,

$$-4V_i - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = -4V_i = -4(-1) = 4 \text{ V}$$

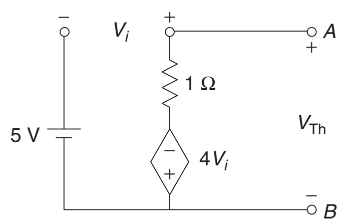


Fig. 2.264

Step II Calculation of I_N (Fig. 2.265)

From Fig. 2.265,

$$5 + V_i = 0$$
$$V_i = -5 \text{ V}$$

Applying KVL to the mesh,

$$-4V_i - 1I_N = 0$$
$$I_N = -4V_i = -4(-5) = 20 \text{ A}$$

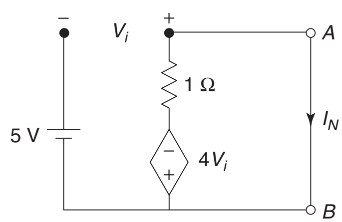


Fig. 2.265

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{4}{20} = 0.2\ \Omega$$

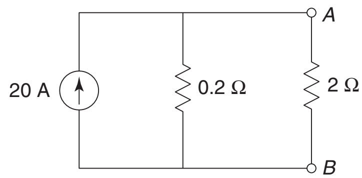


Fig. 2.266

Step IV Calculation of I_L (Fig. 2.266)

$$I_L = 20 \times \frac{0.2}{0.2+2} = 1.82\ \text{A}$$

Example 2.92 *Find the current in the 2 Ω resistor in the network of Fig. 2.267.*

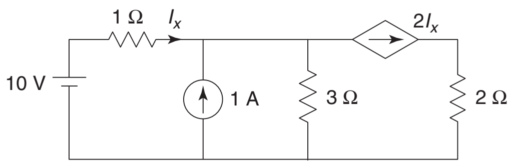


Fig. 2.267

Solution

Step I Calculation of V_{Th} (Fig. 2.268)

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \ldots\text{(i)}$$

Applying KVL to the outer path of the supermesh,

$$10 - 1I_1 - 3I_2 = 0$$

$$I_1 + 3I_2 = 10 \qquad \ldots\text{(ii)}$$

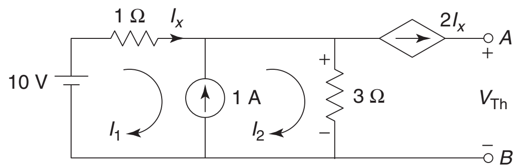


Fig. 2.268

Solving Eqs (i) and (ii),

$$I_1 = 1.75\ \text{A}$$

$$I_2 = 2.75\ \text{A}$$

Writing the V_{Th} equation,

$$3I_2 - V_{Th} = 0$$

$$3(2.75) - V_{Th} = 0$$

$$V_{Th} = 8.25\ \text{V}$$

Step II Calculation of I_N (Fig. 2.269)

From Fig. 2.269,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

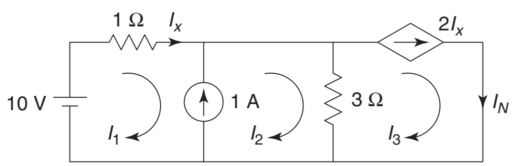


Fig. 2.269

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$10 - 1I_1 - 3(I_2 - I_3) = 0$$

$$I_1 + 3I_2 - 3I_3 = 10 \qquad \ldots\text{(iii)}$$

For Mesh 3,

$$I_3 = 2I_x = 2I_1$$

$$2I_1 - I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = -3.5 \text{ A}$$

$$I_2 = -2.5 \text{ A}$$

$$I_3 = -7 \text{ A}$$

$$I_N = I_3 = -7 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{8.25}{-7} = -1.18\ \Omega$$

Step IV Calculation of I_L (Fig. 2.270)

$$I_L = -7 \times \frac{-1.18}{-1.18 + 2} = 10.07 \text{ A}$$

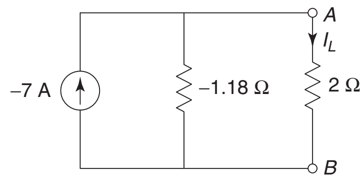


Fig. 2.270

Example 2.93 *Find the current through the 10 Ω resistor for the network of Fig. 2.271.*

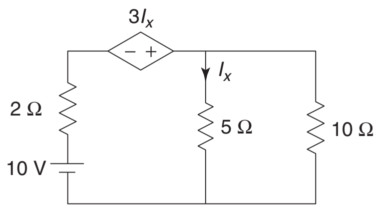


Fig. 2.271

Solution

Step I Calculation of V_{Th} (Fig. 2.272)
Applying KVL to the mesh,

$$10 - 2I_x + 3I_x - 5I_x = 0$$

$$I_x = 2.5 \text{ A}$$

Writing the V_{Th} equation,

$$5I_x - V_{\text{Th}} = 0$$

$$5(2.5) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 12.5 \text{ V}$$

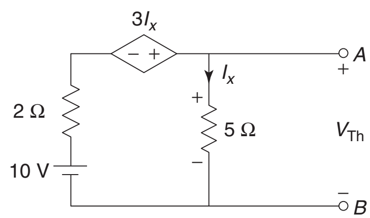


Fig. 2.272

Step II Calculation of I_N (Fig. 2.273)
From Fig. 2.273,

$$I_x = 0$$

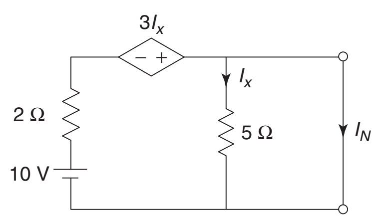


Fig. 2.273

The dependent source of 3 I_x depends on the controlling variable I_x. When $I_x = 0$, the dependent source 3 I_x vanishes, i.e. 3 $I_x = 0$ as shown in Fig. 2.274.

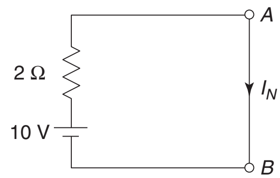


Fig. 2.274

$$I_N = \frac{10}{2} = 5 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{12.5}{5} = 2.5\ \Omega$$

Step IV Calculation of I_L (Fig. 2.275)

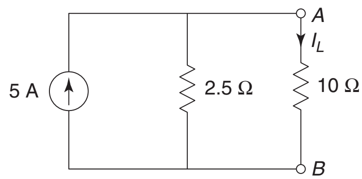


Fig. 2.275

$$I_L = 5 \times \frac{2.5}{2.5+10} = 1 \text{ A}$$

Example 2.94 *Find the current through the 5 Ω resistor in the network of Fig. 2.276.*

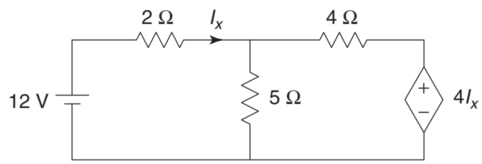


Fig. 2.276

Solution

Step I Calculation of V_{Th} (Fig. 2.277)

Applying KVL to the mesh,

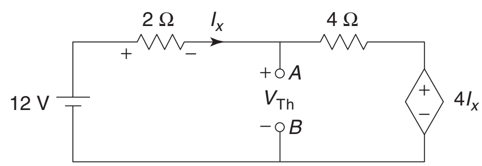


Fig. 2.277

$$12 - 2I_x - 4I_x - 4I_x = 0$$
$$12 - 10I_x = 0$$
$$I_x = 1.2 \text{ A}$$

Writing the V_{Th} equation,

$$12 - 2I_x - V_{Th} = 0$$
$$12 - 2(1.2) - V_{Th} = 0$$
$$V_{Th} = 9.6 \text{ V}$$

Step II Calculation of I_N (Fig. 2.278)

From Fig. 2.278,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

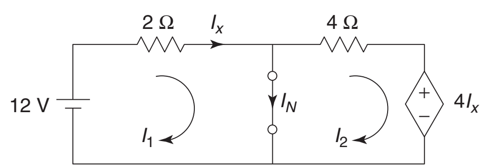


Fig. 2.278

$$12 - 2I_1 = 0$$
$$I_1 = 6 \text{ A} \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-4I_2 - 4I_x = 0$$
$$-4I_2 - 4I_1 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_2 = -6 \text{ A}$$
$$I_N = I_1 - I_2 = 6 - (-6) = 12 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{9.6}{12} = 0.8\ \Omega$$

Step IV Calculation of I_L (Fig. 2.279)

$$I_L = 12 \times \frac{0.8}{0.8+5} = 1.66\ \text{A}$$

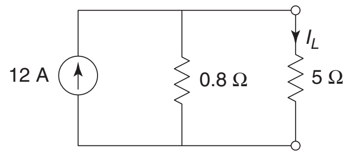


Fig. 2.279

Example 2.95 *Find the current through the 10 Ω resistor for the network of Fig. 2.280.*

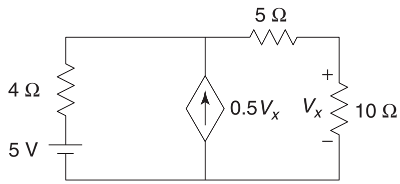


Fig. 2.280

Solution

Step I Calculation of V_{Th} (Fig. 2.281)
For the mesh,

$$I = -0.5V_x = -0.5V_{\text{Th}}$$

Writing the V_{Th} equation,

$$5 - 4I - 0 - V_{\text{Th}} = 0$$

$$5 - 4(-0.5V_{\text{Th}}) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -5\ \text{V}$$

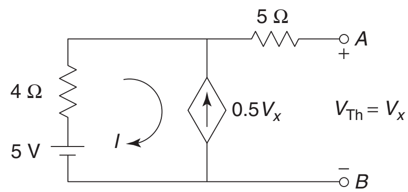


Fig. 2.281

Step II Calculation of I_N (Fig. 2.282)
From Fig. 2.282,

$$V_x = 0$$

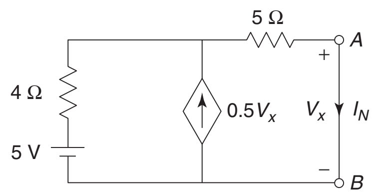


Fig. 2.282

The dependent source of 0.5 V_x depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e. 0.5 $V_x = 0$ as shown in Fig. 2.283.

$$I_N = \frac{5}{4+5} = \frac{5}{9}\ \text{A}$$

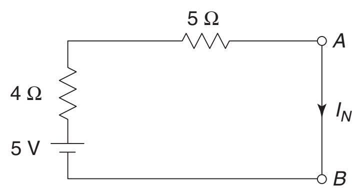


Fig. 2.283

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{-5}{\frac{5}{9}} = -9\ \Omega$$

Step IV Calculation of I_L (Fig. 2.284)

$$I_L = \frac{5}{9} \times \frac{-9}{-9+10} = -5\ \text{A}$$

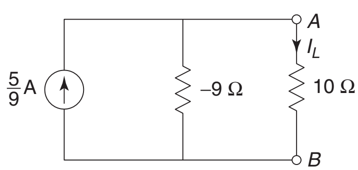


Fig. 2.284

Example 2.96 *Find the current through the 10 Ω resistor in the network shown in Fig. 2.285.*

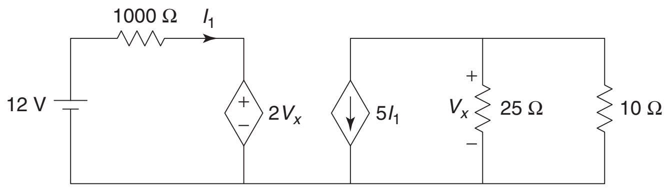


Fig. 2.285

Solution

Step I Calculation of V_{Th} (Fig. 2.286)

From Fig. 2.286,

$$V_x = -25(5I_1) = -125\,I_1 \quad \ldots\text{(i)}$$

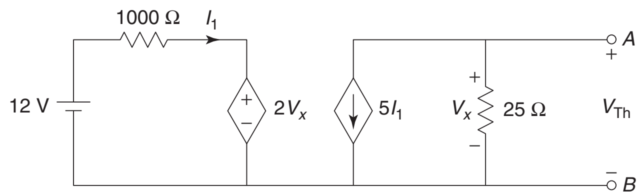


Fig. 2.286

Applying KVL to Mesh 1,

$$12 - 1000I_1 - 2V_x = 0$$

$$12 - 1000I_1 - 2(-125I_1) = 0 \quad \ldots\text{(ii)}$$

$$I_1 = 0.016 \text{ A}$$

$$V_x = -125I_1 = -125(0.016) = -2 \text{ V}$$

Writing the V_{Th} equation,

$$V_{Th} = V_x = -2 \text{ V}$$

Step II Calculation of I_N (Fig. 2.287)

From Fig. 2.287,

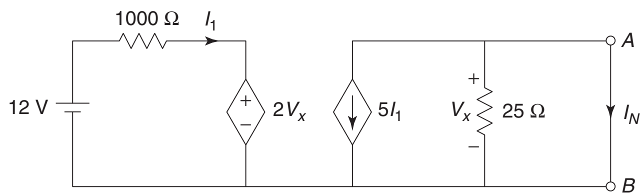


Fig. 2.287

$$V_x = 0$$

The dependent source of $2V_x$ depends on the controlling variable V_x. When $V_x = 0$, the dependent source vanishes, i.e. $2\,V_x = 0$, as shown in Fig. 2.288.

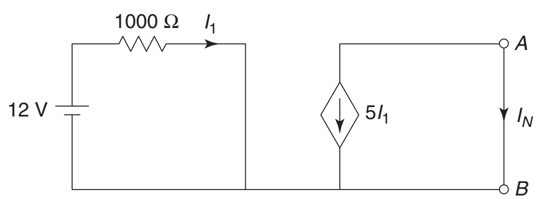


Fig. 2.288

$$I_1 = \frac{12}{1000} = 0.012 \text{ A}$$

$$I_N = -5I_1 = -5(0.012) = -0.06 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{Th}}{I_N} = \frac{-2}{-0.06} = 33.33\ \Omega$$

Step IV Calculation of I_L (Fig. 2.289)

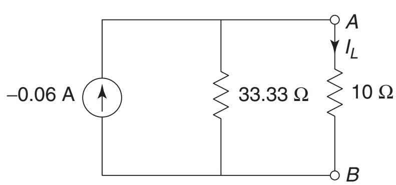


Fig. 2.289

$$I_L = -0.06 \times \frac{33.33}{33.33 + 10} = -0.046 \text{ A}$$

Example 2.97 *Find the current through the 5 Ω resistor for the network of Fig. 2.290.*

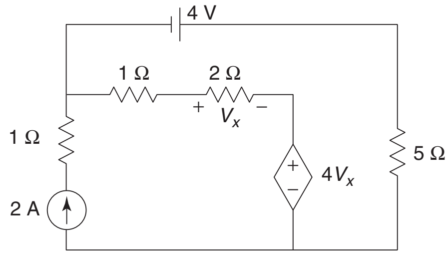


Fig. 2.290

Solution

Step I Calculation of V_{Th} (Fig. 2.291)

From Fig. 2.291,

$$V_x = 2I \qquad \ldots\text{(i)}$$

For the mesh,

$$I = 2 \qquad \ldots\text{(ii)}$$

$$V_x = 2(2) = 4 \text{ V}$$

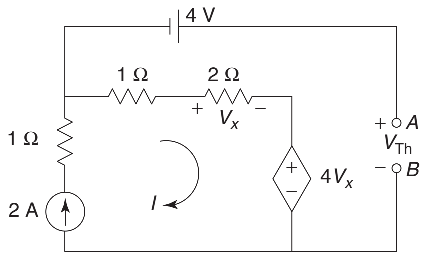


Fig. 2.291

Writing the V_{Th} equation,

$$4V_x + 2I + 1I + 4 - V_{\text{Th}} = 0$$

$$4(4) + 2(2) + 2 + 4 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 26 \text{ V}$$

Step II Calculation of I_N (Fig. 2.292)

From Fig. 2.292,

$$V_x = 2(I_1 - I_2) \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 2 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$4V_x - 2(I_2 - I_1) - 1(I_2 - I_1) + 4 = 0$$

$$4[2(I_1 - I_2)] - 2I_2 + 2I_1 - I_2 + I_1 + 4 = 0$$

$$11I_1 - 11I_2 = -4 \qquad \ldots\text{(iii)}$$

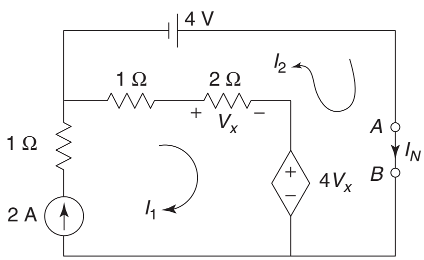


Fig. 2.292

Solving Eqs (ii) and (iii),

$$I_1 = 2 \text{ A}$$

$$I_2 = 2.36 \text{ A}$$

$$I_N = I_2 = 2.36 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{26}{2.36} = 11.02\ \Omega$$

Step IV Calculation of I_L (Fig. 2.293)

$$I_L = 2.36 \times \frac{11.02}{11.02 + 5} = 1.62 \text{ A}$$

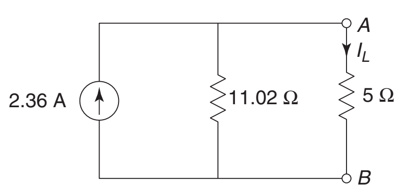


Fig. 2.293

Example 2.98 *Find the current through the 1 Ω resistor in the network of Fig. 2.294.*

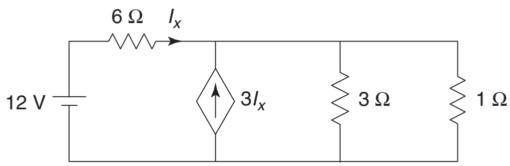


Fig. 2.294

Solution

Step I Calculation of V_{Th} (Fig. 2.295)

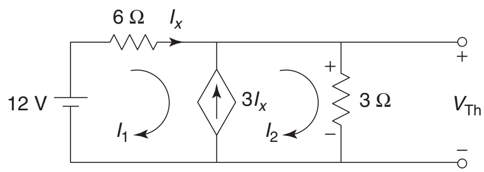


Fig. 2.295

From Fig. 2.295,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing the current equation for the supermesh,

$$I_2 - I_1 = 3I_x = 3I_1$$

$$4I_1 - I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$12 - 6I_1 - 3I_2 = 0$$

$$6I_1 + 3I_2 = 12 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.67 \text{ A}$$

$$I_2 = 2.67 \text{ A}$$

Writing the V_{Th} equation,

$$3I_2 - V_{\text{Th}} = 0$$

$$3(2.67) - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 8 \text{ V}$$

Step II Calculation of I_N (Fig. 2.296)

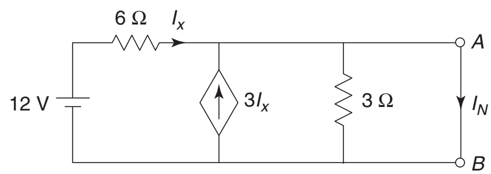


Fig. 2.296

When a short circuit is placed across a 3 Ω resistor, it gets shorted as shown in Fig. 2.297.

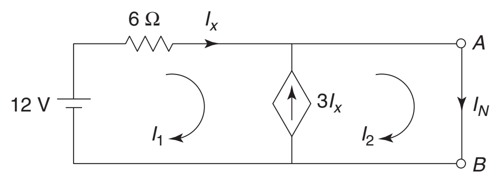


Fig. 2.297

From Fig. 2.297,

$$I_x = I_1 \qquad \ldots\text{(i)}$$

Meshes 1 and 2 will form a supermesh.
Writing the current equation for the supermesh,

$$I_2 - I_1 = 3I_x = 3I_1$$

$$4I_1 - I_2 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$12 - 6I_1 = 0$$

$$I_1 = 2 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 2 \text{ A}$$
$$I_2 = 8 \text{ A}$$
$$I_N = I_2 = 8 \text{ A}$$

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{8}{8} = 1\ \Omega$$

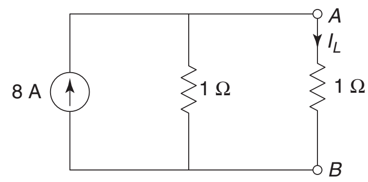


Fig. 2.298

Step IV Calculation of I_L (Fig. 2.298)

$$I_L = 8 \times \frac{1}{1+1} = 4 \text{ A}$$

Example 2.99 *Find the current through the 1.6 Ω resistor in the network of Fig. 2.299.*

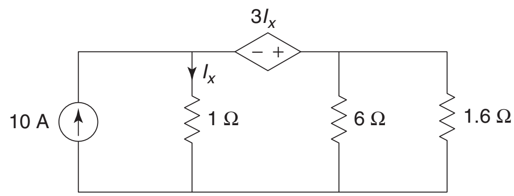


Fig. 2.299

Solution

Step I Calculation of V_{Th} (Fig. 2.300)

From Fig. 2.300,

$$I_x = I_1 - I_2 \quad \ldots\text{(i)}$$

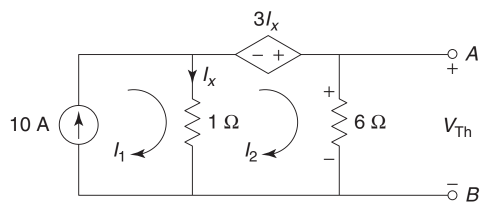


Fig. 2.300

For Mesh 1,

$$I_1 = 10 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + 3I_x - 6I_2 = 0$$
$$-I_2 + I_1 + 3(I_1 - I_2) - 6I_2 = 0$$
$$4I_1 - 10I_2 = 0 \quad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 4 \text{ A}$$

Writing the V_{Th} equation,

$$6I_2 - V_{\text{Th}} = 0$$
$$6(4) - V_{\text{Th}} = 0$$
$$V_{\text{Th}} = 24 \text{ V}$$

Step II Calculation of I_N (Fig. 2.301)

When a short circuit is placed across the 3 Ω resistor, it gets shorted as shown in Fig. 2.302.

From Fig. 2.302,

$$I_x = I_1 - I_2 \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I_1 = 10 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) + 3I_x = 0$$
$$-I_2 + I_1 + 3(I_1 - I_2) = 0$$
$$4I_1 - 4I_2 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 10 \text{ A}$$
$$I_N = I_2 = 10 \text{ A}$$

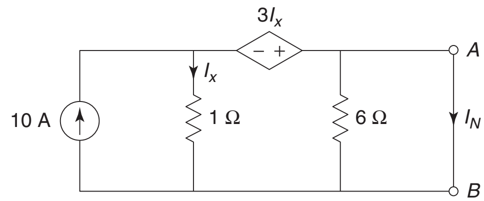


Fig. 2.301

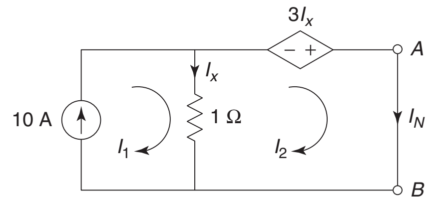


Fig. 2.302

Step III Calculation of R_N

$$R_N = \frac{V_{\text{Th}}}{I_N} = \frac{24}{10} = 2.4\ \Omega$$

Step IV Calculation of I_L (Fig. 2.303)

$$I_L = 10 \times \frac{2.4}{2.4 + 1.6} = 6 \text{ A}$$

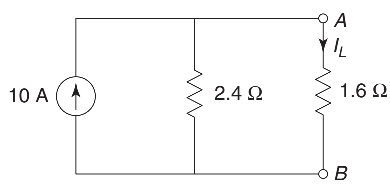


Fig. 2.303

2.10 MAXIMUM POWER TRANSFER THEOREM

It states that '*the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.*'

Proof From Fig. 2.304,

$$I = \frac{V}{R_s + R_L}$$

Power delivered to the load $R_L = P = I^2\ R_L = \dfrac{V^2 R_L}{(R_s + R_L)^2}$

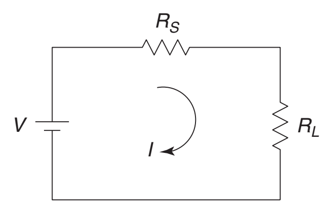


Fig. 2.304 *Network illustrating maximum power transfer theorem*

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_s + R_L)^2} R_L$$

$$= \frac{V^2[(R_s + R_L)^2 - (2R_L)(R_s + R_L)]}{(R_s + R_L)^4}$$

$$(R_s + R_L)^2 - 2\,R_L(R_s + R_L) = 0$$

$$R_s^{\,2} + R_L^{\,2} + 2R_sR_L - 2R_LR_s - 2R_L^2 = 0$$

$$R_s = R_L$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{\text{Th}}$$

5. Find the maximum power (Fig. 2.305).

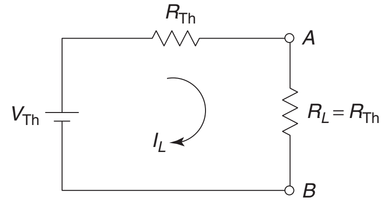


Fig. 2.305 *Thevenin's equivalent network*

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{V_{\text{Th}}}{2R_{\text{Th}}}$$

$$P_{\max} = I_L^2\,R_L = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}^2} \times R_{\text{Th}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}$$

Example 2.100 *For the value of resistance R_L in Fig. 2.306 for maximum power transfer and calculate the maximum power.*

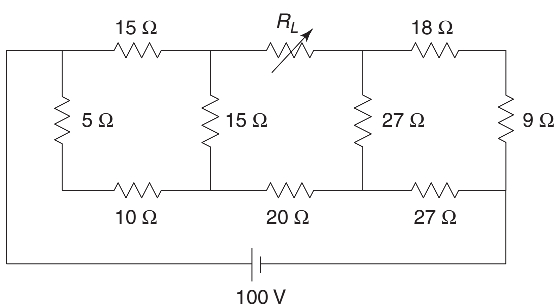


Fig. 2.306

Solution

Step I Calculation of V_{Th} (Fig. 2.307)

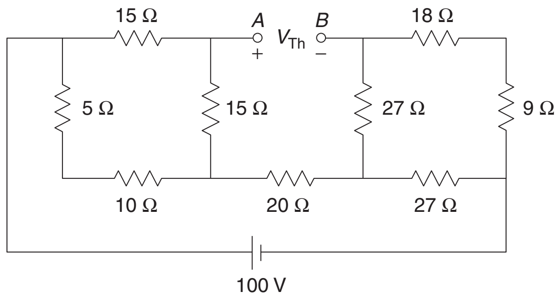


Fig. 2.307

By star-delta transformation (Fig. 2.308),

$$I = \frac{100}{5+5+20+9+9} = 2.08 \text{ A}$$

Writing the V_{Th} equation,

$$100 - 5I - V_{Th} - 9I = 0$$

$$V_{Th} = 100 - 14I$$

$$= 100 - 14(2.08)$$

$$= 70.88 \text{ V}$$

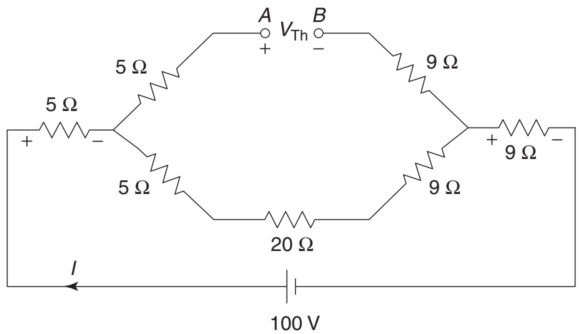


Fig. 2.308

Step II Calculation of R_{Th} (Fig. 2.309)

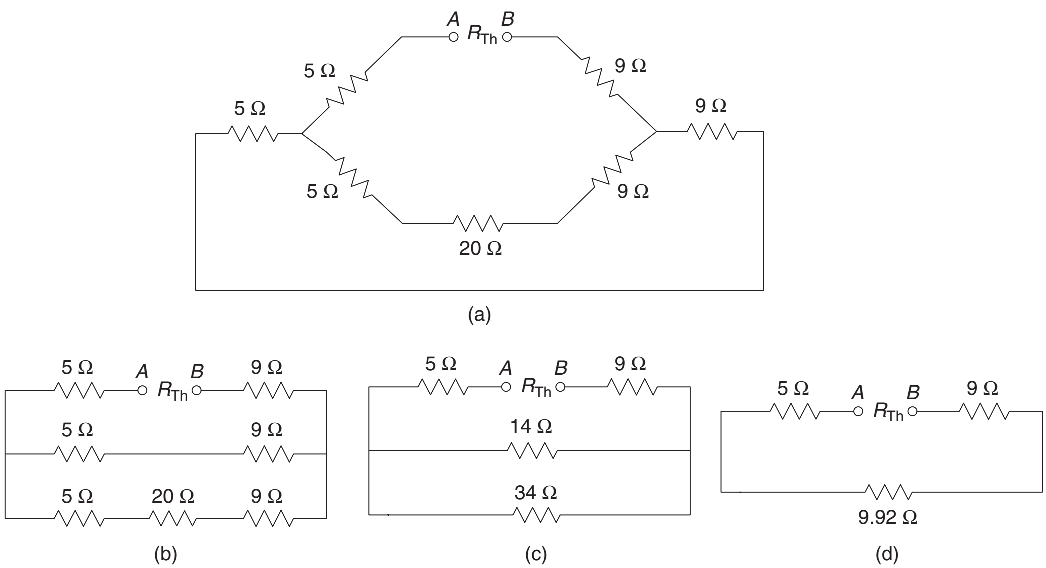


Fig. 2.309

$$R_{Th} = 23.92\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 23.92\ \Omega$$

Step IV Calculation of P_{max} (Fig. 2.310)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(70.88)^2}{4\times 23.92} = 52.51\ \text{W}$$

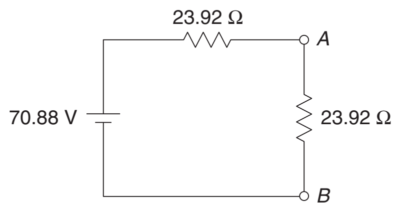


Fig. 2.310

Example 2.101 *For the value of resistance R_L in Fig. 2.311 for maximum power transfer and calculate the maximum power.*

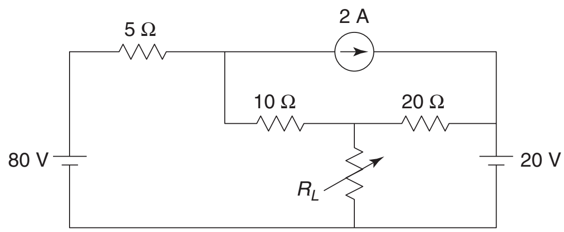


Fig. 2.311

Solution

Step I Calculation of V_{Th} (Fig. 2.312)

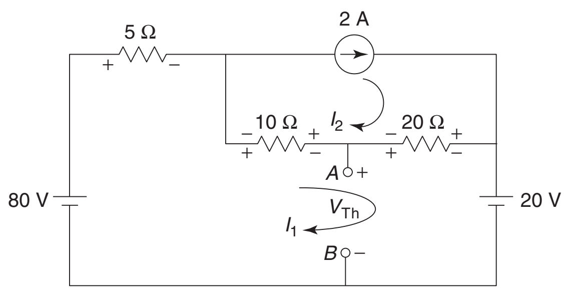


Fig. 2.312

Applying KVL to Mesh 1,

$$80-5I_1-10(I_1-I_2)-20(I_1-I_2)-20=0$$

$$35I_1-30I_2=60 \qquad \text{...(i)}$$

Writing the current equation for Mesh 2,

$$I_2 = 2 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 3.43 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 20(I_1 - I_2) - 20 = 0$$

$$V_{Th} = 20(3.43 - 2) + 20 = 48.6 \text{ V}$$

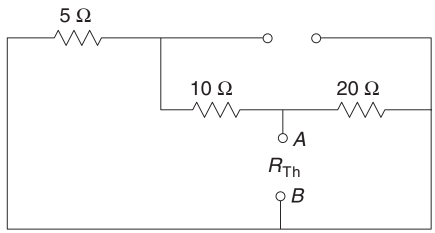


Fig. 2.313

Step II Calculation of R_{Th} (Fig. 2.313)

$$R_{Th} = 15 \parallel 20 = 8.57\ \Omega$$

Step III Calculation of R_L
For maximum power transfer,

$$R_L = R_{Th} = 8.57\ \Omega$$

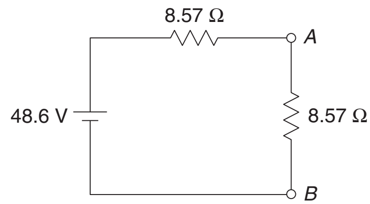


Fig. 2.314

Step IV Calculation of P_{max} (Fig. 2.314)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(48.6)^2}{4 \times 8.57} = 68.9 \text{ W}$$

Example 2.102 *For the value of resistance R_L in Fig. 2.315 for maximum power transfer and calculate the maximum power.*

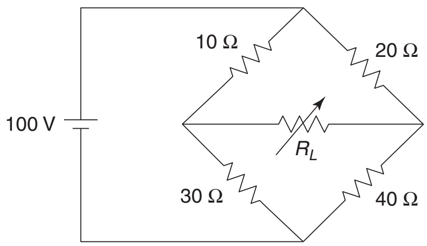


Fig. 2.315

Solution

Step I Calculation of V_{Th} (Fig. 2.316)

$$I_1 = \frac{100}{10+30} = 2.5 \text{ A}$$

$$I_2 = \frac{100}{20+40} = 1.66 \text{ A}$$

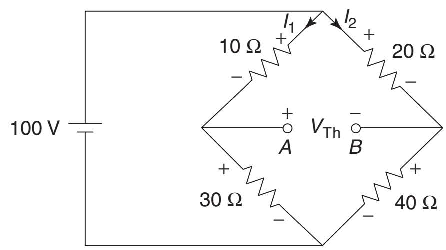


Fig. 2.316

Writing the V_{Th} equation,

$$V_{Th} + 10 I_1 - 20 I_2 = 0$$

$$V_{Th} = 20I_2 - 10I_1 = 20(1.66) - 10(2.5) = 8.2 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.317)

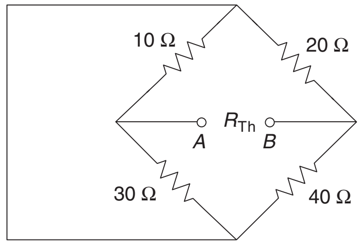


Fig. 2.317

Redrawing the network (Fig. 2.318),

$$R_{Th} = (10 \| 30) + (20 \| 40) = 20.83\ \Omega$$

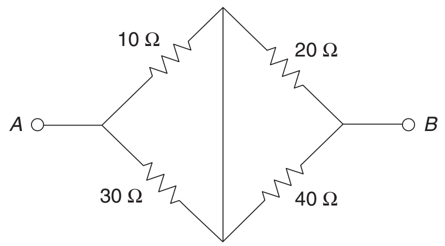


Fig. 2.318

Step III Value of R_L
For maximum power transfer,

$$R_L = R_{Th} = 20.83\ \Omega$$

Step IV Calculation of P_{max} (Fig. 2.319)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8.2)^2}{4 \times 20.83} = 0.81\ \text{W}$$

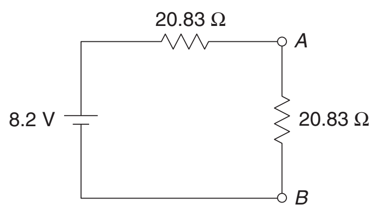


Fig. 2.319

Example 2.103 *For the value of resistance R_L in Fig. 2.320 for maximum power transfer and calculate the maximum power.*

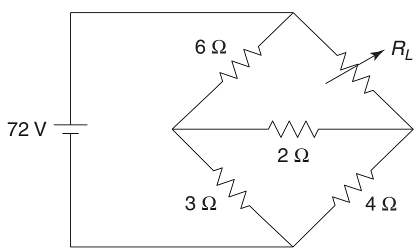


Fig. 2.320

Solution

Step I Calculation of V_{Th} (Fig. 2.321)

Applying KVL to Mesh 1,

$$72 - 6I_1 - 3(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 72 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 4I_2 = 0$$

$$-3I_1 + 9I_2 = 0 \quad \ldots\text{(ii)}$$

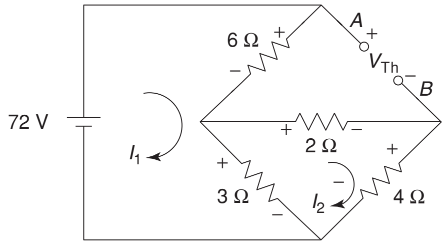


Fig. 2.321

Solving Eqs (i) and (ii),

$$I_1 = 9 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 6I_1 - 2I_2 = 0$$

$$V_{Th} = 6I_1 + 2I_2 = 6(9) + 2(3) = 60 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 2.322)

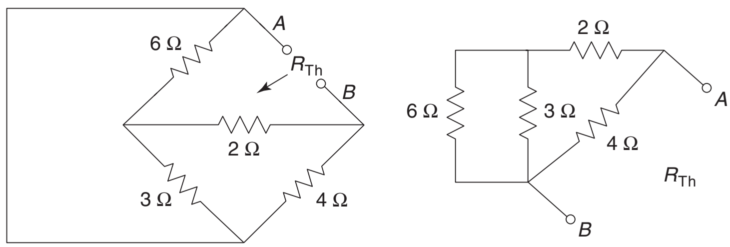


Fig. 2.322

$$R_{Th} = [(6 \parallel 3) + 2] \parallel 4 = 2\ \Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 2\ \Omega$$

Step IV Calculation of P_{max} (Fig. 2.323)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 2} = 450 \text{ W}$$

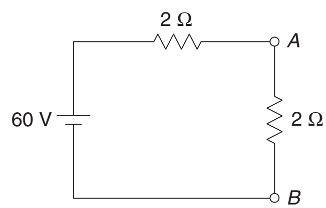


Fig. 2.323

EXAMPLES WITH DEPENDENT SOURCES

Example 2.104 *For the network shown in Fig. 2.324, find the value of R_L for maximum power transfer. Also, calculate maximum power.*

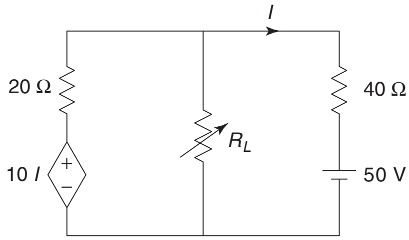


Fig. 2.324

Solution

Step I Calculation of V_{Th} (Fig. 2.325)

Applying KVL to the mesh,

$$10I - 20I - 40I - 50 = 0$$
$$I = -1 \text{ A}$$

Writing the V_{Th} equation,

$$V_{Th} - 40I - 50 = 0$$
$$V_{Th} - 40(-1) - 50 = 0$$
$$V_{Th} = 10 \text{ V}$$

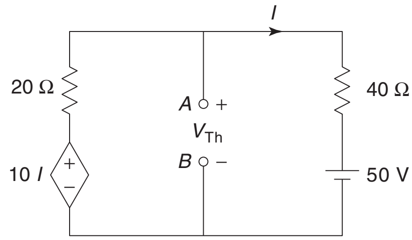


Fig. 2.325

Step II Calculation of I_N (Fig. 2.326)

From Fig. 2.326,

$$I = I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 1,

$$10I - 20I_1 = 0$$
$$10I_2 - 20I_1 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 2,

$$-40I_2 - 50 = 0$$
$$I_2 = -1.25 \text{ A} \qquad \ldots\text{(iii)}$$

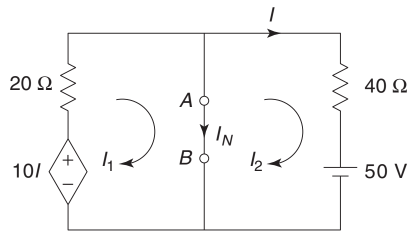


Fig. 2.326

Solving Eqs (i), (ii) and (iii),

$$I_1 = -0.625 \text{ A}$$
$$I_N = I_1 - I_2 = -0.625 + 1.25 = 0.625 \text{ A}$$

Step III Calculation of R_N

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{10}{0.625} = 16\ \Omega$$

Step IV Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 16\ \Omega$$

Step V Calculation of P_{max} (Fig. 2.327)

$$P_{max} = \frac{V_{Th}}{4R_{Th}} = \frac{(10)^2}{4 \times 16} = 1.56 \text{ W}$$

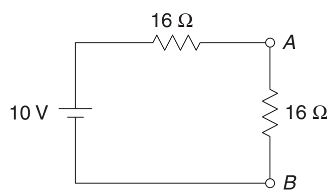


Fig. 2.327

Example 2.105 *For the network shown in Fig. 2.328, calculate the maximum power that may be dissipated in the load resistor R_L.*

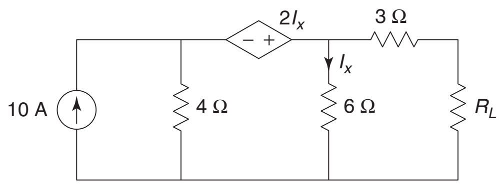


Fig. 2.328

Solution

Step I Calculation of V_{Th} (Fig. 2.329)

From Fig. 2.329,

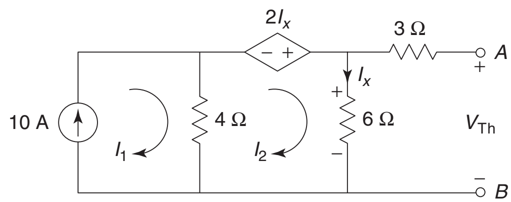


Fig. 2.329

$$I_x = I_2 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = 10 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) + 2I_x - 6I_2 = 0$$
$$-4I_2 + 4I_1 + 2I_2 - 6I_2 = 0$$
$$4I_1 - 8I_2 = 0 \qquad \text{...(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 5 \text{ A}$$

Writing the V_{Th} equation,

$$6I_2 - 0 - V_{Th} = 0$$
$$V_{Th} = 6I_2 = 6(5) = 30 \text{ V}$$

Step II Calculation of I_N (Fig. 2.330)

From Fig. 2.330,

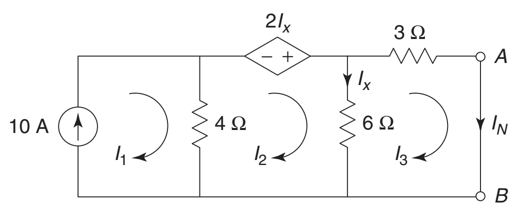


Fig. 2.330

$$I_x = I_2 - I_3 \qquad \text{...(i)}$$

For Mesh 1,

$$I_1 = 10 \qquad \text{...(ii)}$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) + 2I_x - 6(I_2 - I_3) = 0$$
$$-4I_2 + 4I_1 + 2(I_2 - I_3) - 6I_2 + 6I_3 = 0$$
$$4I_1 - 8I_2 + 4I_3 = 0 \qquad \text{...(iii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_2) - 3I_3 = 0$$

$$6I_2 - 9I_3 = 0 \qquad \ldots\text{(iv)}$$

Solving Eqs (ii), (iii) and (iv),

$$I_1 = 10 \text{ A}$$

$$I_2 = 7.5 \text{ A}$$

$$I_3 = 5 \text{ A}$$

$$I_N = I_3 = 5 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{30}{5} = 6\ \Omega$$

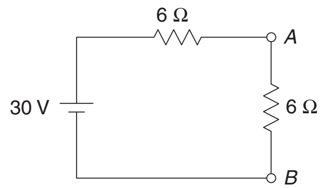


Fig. 2.331

Step IV Calculation of R_L
For maximum power transfer,

$$R_L = R_{\text{Th}} = 6\ \Omega$$

Step V Calculation of P_{max} (Fig. 2.331)

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(30)^2}{4 \times 16} = 37.5 \text{ W}$$

Example 2.106 *For the network shown in Fig. 2.332, find the value of R_L for maximum power transfer. Also, find maximum power.*

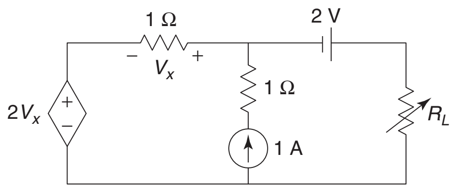


Fig. 2.332

Solution

Step I Calculation of V_{Th} (Fig. 2.333)

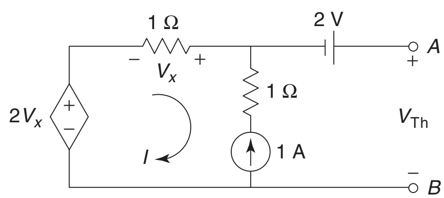


Fig. 2.333

From Fig. 2.333,

$$V_x = -1I = -I \qquad \ldots\text{(i)}$$

For Mesh 1,

$$I = -1 \qquad \ldots\text{(ii)}$$

$$V_x = 1 \text{ V}$$

Writing the V_{Th} equation,

$$2V_x - 1I + 2 - V_{Th} = 0$$
$$2(1) - (-1) + 2 - V_{Th} = 0$$
$$V_{Th} = 5 \text{ V}$$

Step II Calculation of I_N (Fig. 2.334)

From Fig. 2.334,

$$V_x = -1I_1 = -I_1 \qquad \ldots\text{(i)}$$

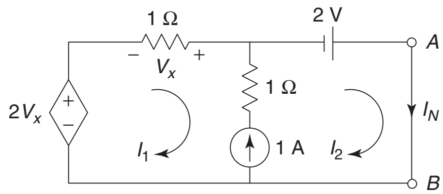


Fig. 2.334

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \qquad \ldots\text{(ii)}$$

Applying KVL to the outer path of the supermesh,

$$2V_x - 1I_1 + 2 = 0$$
$$2(-I_1) - I_1 + 2 = 0$$
$$3I_1 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (ii) and (iii),

$$I_1 = 0.67 \text{ A}$$
$$I_2 = 1.67 \text{ A}$$
$$I_N = I_2 = 1.67 \text{ A}$$

Step III Calculation of R_{Th}

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{5}{1.67} = 3\ \Omega$$

Step IV Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 3\ \Omega$$

Step V Calculation of P_{max} (Fig. 2.335)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(5)^2}{4\times 3} = 2.08 \text{ W}$$

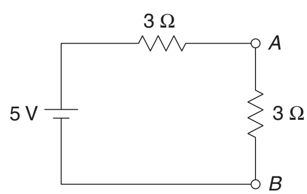


Fig. 2.335

Example 2.107 *What will be the value of R_L in Fig. 2.336 to get maximum power delivered to it? What is the value of this power?*

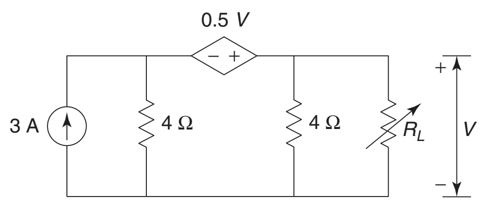


Fig. 2.336

Solution

Step I Calculation of V_{Th} (Fig. 2.337)
By source transformation,
From Fig. 2.337,

$$V_{\text{Th}} = 4I$$

Applying KVL to the mesh,

$$12 - 4I + 0.5\,V_{\text{Th}} - 4I = 0$$

$$12 - V_{\text{Th}} + 0.5\,V_{\text{Th}} - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 8\text{ V}$$

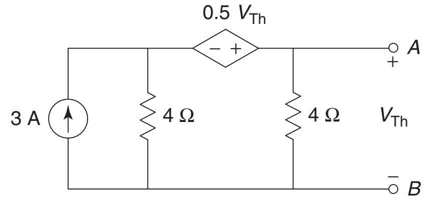


Fig. 2.337

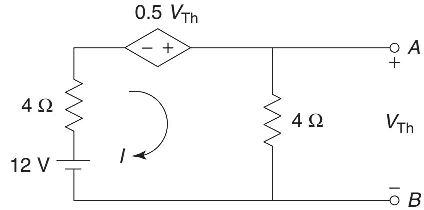


Fig. 2.338

Step II Calculation of I_N (Fig. 2.338)

If two terminals A and B are shorted, the 4 Ω resistor gets shorted.

$$V = 0$$

Dependent source 0.5 V depends on the controlling variable V. When $V = 0$, the dependent source vanishes, i.e. 0.5 $V = 0$ as shown in Fig. 2.339 and Fig. 2.340.

$$I_N = \frac{12}{4} = 3\text{ A}$$

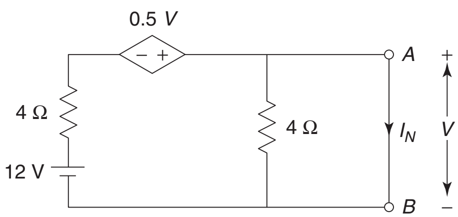


Fig. 2.339

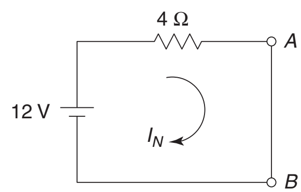


Fig. 2.340

Step III Calculation of R_{Th}

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{8}{3} = 2.67\ \Omega$$

Step IV *Calculation of R_L*
For maximum power transfer,

$$R_L = R_{\text{Th}} = 2.67\ \Omega$$

Step V Calculation of $P_{\max}$ (Fig. 2.341)

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(8)^2}{4 \times 2.67} = 6\text{ W}$$

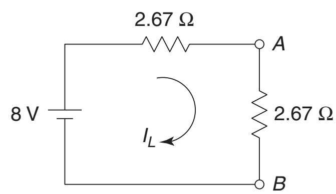


Fig. 2.341

2.11 RECIPROCITY THEOREM

It states that '*in a linear, bilateral, active, single source network, the ratio of excitation to response remains same when the positions of excitation and response are interchanged.*'

In other words, it may be stated as 'if a single voltage source V_a in the branch 'a' produces a current I_b in the branch 'b' then if the voltage source V_a is removed and inserted in the branch 'b', it will produce a current I_b in branch 'a''.

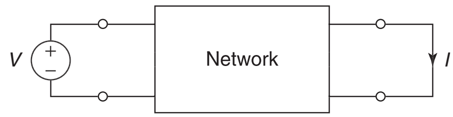


Fig. 2.342 *Network*

Explanation Consider a network shown in Fig. 2.342.

When the voltage source V is applied at the port 1, it produces a current I at the port 2. If the positions of the excitation (source) and response are interchanged, i.e., if the voltage source is applied at the port 2 then it produces a current I at the port 1 (Fig. 2.343).

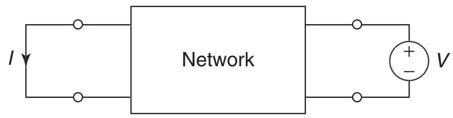


Fig. 2.343 *Network when excitation and response are interchanged*

The limitation of this theorem is that it is applicable only to a single-source network. This theorem is not applicable in the network which has a dependent source. This is applicable only in linear and bilateral networks. In the reciprocity theorem, position of any passive element (R, L, C) do not change. Only the excitation and response are interchanged.

Steps to be followed in Reciprocity Theorem

1. Identify the branches between which reciprocity is to be established.
2. Find the current in the branch when excitation and response are not interchanged.
3. Find the current in the branch when excitation and response are interchanged.

Example 2.108 *Calculate current I and verify the reciprocity theorem for the network shown in Fig. 2.344.*

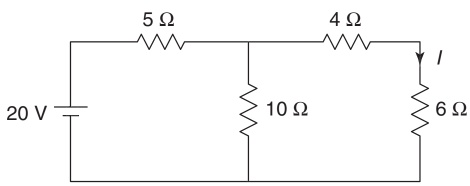


Fig. 2.344

Solution

Case I Calculation of current I when excitation and response are not interchanged (Fig. 2.345)

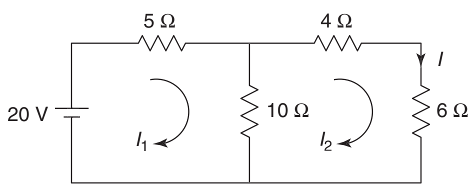


Fig. 2.345

Applying KVL to Mesh 1,

$$20 - 5I_1 - 10(I_1 - I_2) = 0$$

$$15I_1 - 10I_2 = 20 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-10(I_2 - I_1) - 4I_2 - 6I_2 = 0$$

$$-10I_1 + 20I_2 = 0 \qquad \ldots\text{(ii)}$$

Solving. Eqs (i) and (ii),

$$I_1 = 2 \text{ A}$$
$$I_2 = 1 \text{ A}$$
$$I = I_2 = 1 \text{ A}$$

Case II Calculation of current I when excitation and response are interchanged (Fig. 2.346).

Applying KVL to Mesh 1,

$$-5I_1 - 10(I_1 - I_2) = 0$$
$$15I_1 - 10I_2 = 0 \quad \ldots\text{(i)}$$

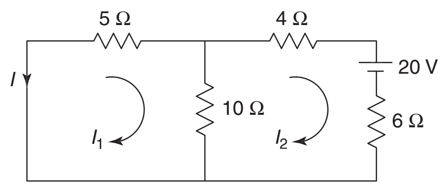


Fig. 2.346

Applying KVL to Mesh 2,

$$-10(I_2 - I_1) - 4I_2 - 20 - 6I_2 = 0$$
$$-10I_1 + 20I_2 = -20 \quad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = -1 \text{ A}$$
$$I_2 = -1.5 \text{ A}$$
$$I = -I_1 = 1 \text{ A}$$

Since the current I remains the same in both the cases, reciprocity theorem is verified.

Example 2.109 *Find the current I and verify reciprocity theorem for the network shown in Fig. 2.347.*

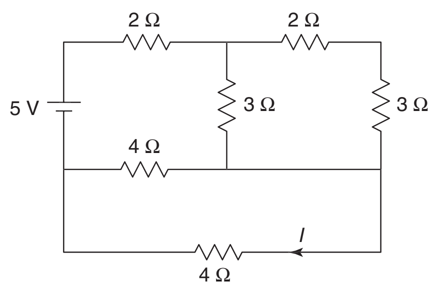


Fig. 2.347

Solution

Case I Calculation of the current I when excitation and response are not interchanged (Fig. 2.348)

Applying KVL to Mesh 1,

$$5 - 2I_1 - 3(I_1 - I_2) - 4(I_1 - I_3) = 0$$
$$9I_1 - 3I_2 - 4I_3 = 5 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 3I_2 = 0$$
$$-3I_1 + 8I_2 = 0 \quad \ldots\text{(ii)}$$

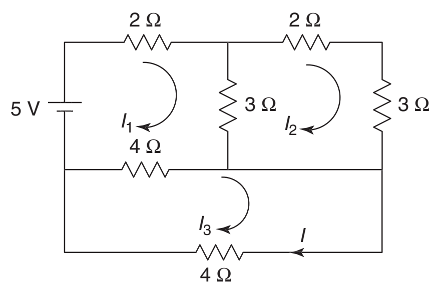


Fig. 2.348

Applying KVL to Mesh 3,

$$-4(I_3 - I_1) - 4I_3 = 0$$

$$-4I_1 + 8I_3 = 0 \qquad \text{...(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.85 \text{ A}$$

$$I_2 = 0.32 \text{ A}$$

$$I_3 = 0.43 \text{ A}$$

$$I = I_3 = 0.43 \text{ A}$$

Case II Calculation of current I when excitation and response are interchanged (Fig. 2.349).

Applying KVL to Mesh 1,

$$-2I_1 - 3(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$9I_1 - 3I_2 - 4I_3 = 0 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-3(I_2 - I_1) - 2I_2 - 3I_2 = 0$$

$$-3I_1 + 8I_2 = 0 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-4(I_3 - I_1) + 5 - 4I_3 = 0$$

$$-4I_1 + 8I_3 = 5 \qquad \text{...(iii)}$$

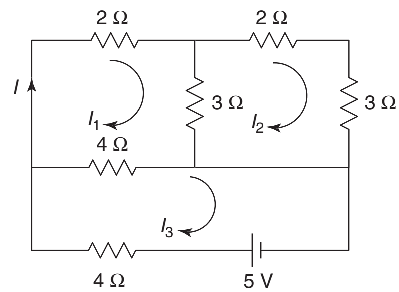


Fig. 2.349

Solving Eqs (i), (ii) and (iii),

$$I_1 = 0.43 \text{ A}$$

$$I_2 = 0.16 \text{ A}$$

$$I_3 = 0.84 \text{ A}$$

$$I = I_1 = 0.43 \text{ A}$$

Since the current I remains the same in both the cases, reciprocity theorem is verified.

Example 2.110 *Find the voltage V and verify reciprocity theorem for the network shown in Fig. 2.350.*

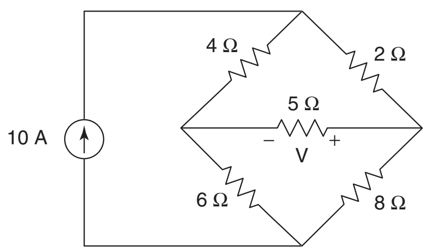


Fig. 2.350

Solution

Case I Calculation of the voltage V when excitation and response are not interchanged (Fig. 2.351)

For Mesh 1,

$$I_1 = 10 \qquad \ldots\text{(i)}$$

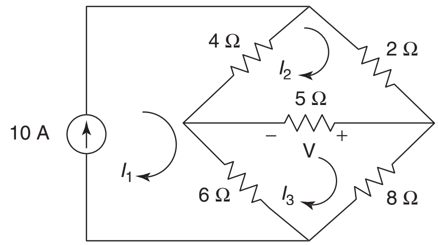


Fig. 2.351

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) - 2I_2 - 5(I_2 - I_3) = 0$$

$$-4I_1 + 11I_2 - 5I_3 = 0 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_1) - 5(I_3 - I_2) - 8I_3 = 0$$

$$-6I_1 - 5I_2 + 19I_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 10 \text{ A}$$
$$I_2 = 5.76 \text{ A}$$
$$I_3 = 4.67 \text{ A}$$
$$V = 5(I_2 - I_3) = 5(5.76 - 4.67) = 5.45 \text{ V}$$

Case II Calculation of voltage V when excitation and response are interchanged (Fig. 2.352).

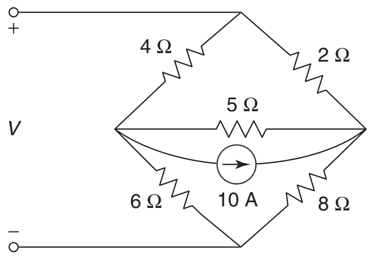


Fig. 2.352

By source transformation (Fig. 2.353),

Applying KVL to Mesh 1,

$$-4I_1 - 2I_1 - 50 - 5(I_1 - I_2) = 0$$

$$11I_1 - 5I_2 = -50 \qquad \ldots\text{(i)}$$

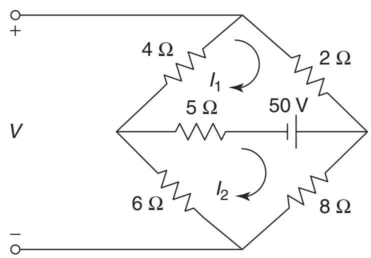


Fig. 2.353

Applying KVL to Mesh 2,

$$-6I_2 - 5(I_2 - I_1) + 50 - 8I_2 = 0$$

$$-5I_1 + 19I_2 = 50 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = -3.8 \text{ A}$$
$$I_2 = 1.63 \text{ A}$$

From Fig. 2.353,

$$V + 4I_1 + 6I_2 = 0$$

$$V + 4(-3.8) + 6(1.63) = 0$$

$$V = 5.42 \text{ V}$$

Since the voltage V is same in both the cases, the reciprocity theorem is verified.

2.12 MILLMAN'S THEOREM

It states that '*if there are n voltage sources $V_1, V_2, \ldots, V_n$ with internal resistances $R_1, R_2, \ldots, R_n$ respectively connected in parallel then these voltage sources can be replaced by a single voltage source V_m and a single series resistance R_m*, '(Fig. 2.354).

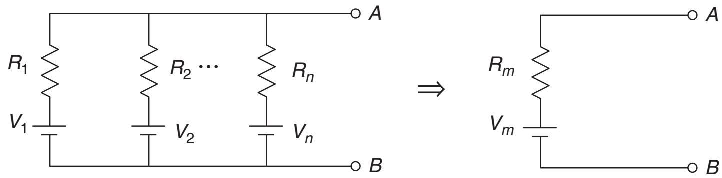


Fig. 2.354 *Millman's network*

where $$V_m = \frac{V_1 G_1 + V_2 G_2 + \ldots + V_n G_n}{G_1 + G_2 + \ldots + G_n}$$

and $$R_m = \frac{1}{G_m} = \frac{1}{G_1 + G_2 + \ldots + G_n}$$

Explanation By source transformation, each voltage source in series with a resistance can be converted to a current source in parallel with a resistance as shown in Fig. 2.355.

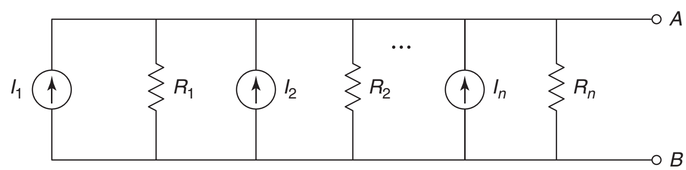


Fig. 2.355 *Equivalent network*

Let I_m be the resultant current of the parallel current sources and R_m be the equivalent resistance as shown in Fig. 2.356.

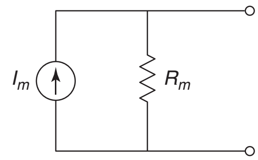


Fig. 2.356 *Equivalent network*

$$I_m = I_1 + I_2 + \ldots + I_n = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_n}{R_n} = V_1\,G_1 + V_2\,G_2 + \ldots V_n\,G_n$$

$$\frac{1}{R_m} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

$$G_m = G_1 + G_2 + \ldots + G_n$$

By source transformation, the parallel circuit can be converted into a series circuit as shown in Fig. 2.357.

$$V_m = I_m R_m = \frac{I_m}{G_m} = \frac{V_1 G_1 + V_2 G_2 + \ldots + V_n G_n}{G_1 + G_2 + \ldots + G_n}$$

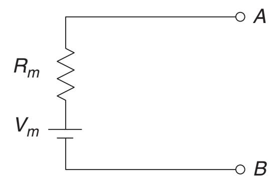


Fig. 2.357 *Millman's equivalent network*

Dual of Millman's Theorem

It states that '*if there are n current sources $I_1, I_2, \ldots, I_n$ with internal resistances $R_1, R_2, \ldots, R_n$ respectively, connected in series then these current sources can be replaced by a single current source I_m and a single parallel resistance R_m*' (Fig. 2.358).

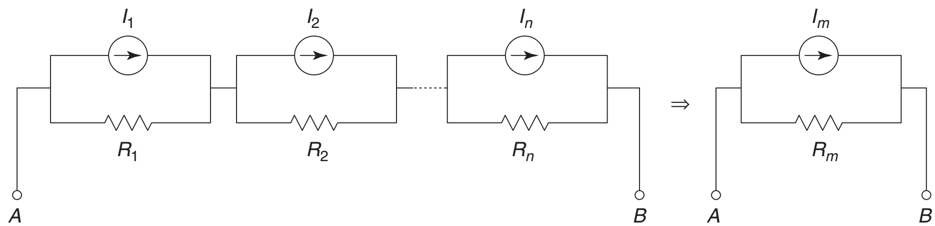


Fig. 2.358 *Millman's network*

where

$$I_m = \frac{I_1 R_1 + I_2 R_2 + \ldots + I_n R_n}{R_1 + R_2 + \ldots + R_n}$$

$$R_m = R_1 + R_2 + \ldots + R_n$$

Steps to be followed in Millman's Theorem

1. Remove the load resistance R_L.
2. Find Millman's voltage across points *A* and *B*.

$$V_m = \frac{V_1 G_1 + V_2 G_2 + \ldots + V_n G_n}{G_1 + G_2 + \ldots + G_n}$$

3. Find the resistance R_m between points *A* and *B*.

$$R_m = \frac{1}{G_1 + G_2 + \ldots + G_n}$$

4. Replace the network by a voltage source V_m in series with the resistance R_m.
5. Find the current through R_L using ohm's law.

$$I_L = \frac{V_m}{R_m + R_L}$$

Example 2.111 *Find Millman's equivalent for the left of the terminals A-B in Fig. 2.359.*

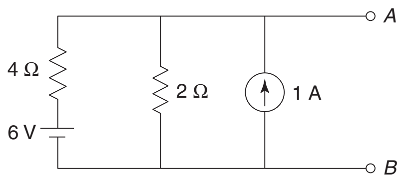


Fig. 2.359

Solution By source transformation, the network is redrawn as shown in Fig. 2.360.

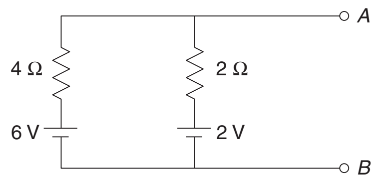


Fig. 2.360

Step I Calculation of V_m

$$V_m = \frac{V_1 G_1 + V_2 G_2}{G_1 + G_2} = \frac{6\left(\frac{1}{4}\right) + 2\left(\frac{1}{2}\right)}{\frac{1}{4} + \frac{1}{2}} = 3.33 \text{ V}$$

Step II Calculation of R_m

$$R_m = \frac{1}{G_m} = \frac{1}{G_1 + G_2} = \frac{1}{\frac{1}{4} + \frac{1}{2}} = 1.33 \ \Omega$$

Step III Millman's Equivalent Network (Fig. 2.361)

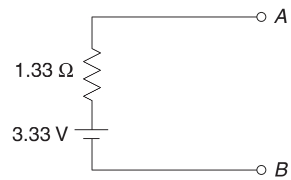


Fig. 2.361

Example 2.112 *Find the current through the 10 Ω resistor in the network of Fig. 2.362.*

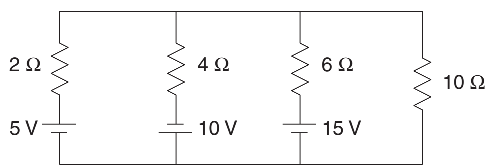


Fig. 2.362

Solution

Step I Calculation of V_m

$$V_m = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3} = \frac{5\left(\frac{1}{2}\right) - 10\left(\frac{1}{4}\right) + 15\left(\frac{1}{6}\right)}{\frac{1}{2} + \frac{1}{4} + \frac{1}{6}} = 2.73 \text{ V}$$

Step II Calculation of R_m

$$R_m = \frac{1}{G_m} = \frac{1}{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}} = 1.09\ \Omega$$

Step III Calculation of I_L (Fig. 2.363)

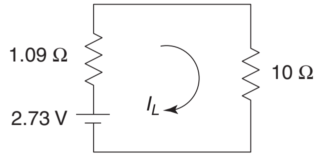


Fig. 2.363

$$I_L = \frac{2.73}{1.09+10} = 0.25\ \text{A}$$

Example 2.113 *Find the current through the 10 Ω resistor in the network of Fig. 2.364.*

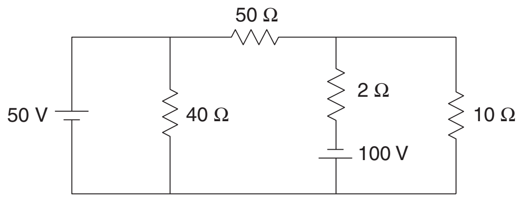


Fig. 2.364

Solution

Since the 40 Ω resistor is connected in parallel with the 50 V source, it becomes redundant. The network can be redrawn as shown in Fig. 2.365.

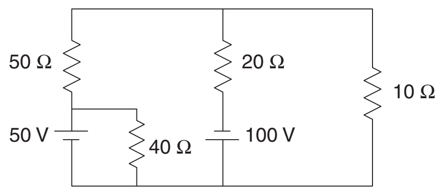


Fig. 2.365

Step I Calculation of V_m

$$V_m = \frac{V_1 G_1 + V_2 G_2}{G_1 + G_2} = \frac{50\left(\frac{1}{50}\right) - 100\left(\frac{1}{20}\right)}{\frac{1}{50}+\frac{1}{20}} = -57.15\ \text{V}$$

Step II Calculation of R_m

$$R_m = \frac{1}{G_m} = \frac{1}{G_1 + G_2} = \frac{1}{\frac{1}{50}+\frac{1}{20}} = 14.29\ \Omega$$

Step III Calculation of I_L (Fig. 2.366)

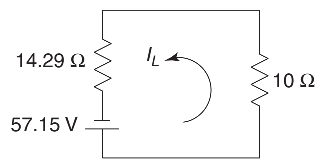


Fig. 2.366

$$I_L = \frac{57.15}{14.29+10} = 2.35\ \text{A}$$

Example 2.114 *Draw Millman's equivalent network across terminals AB in the network of Fig. 2.367.*

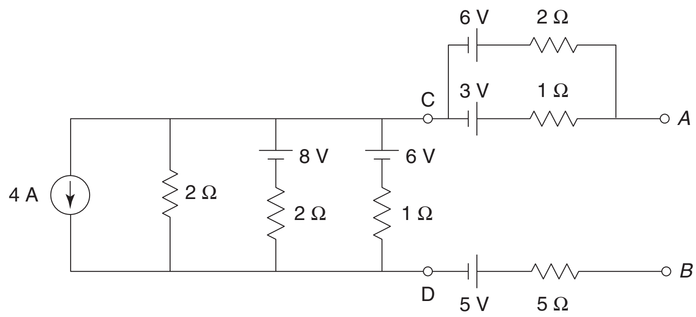


Fig. 2.367

Step I By source transformation, the network is redrawn as shown in Fig. 2.368.

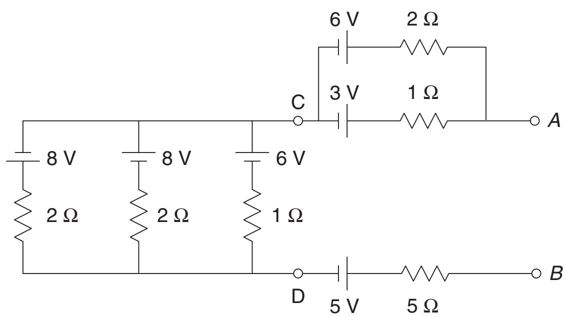


Fig. 2.368

Step II Applying Millman's theorem at terminals *CD,*

$$V_{m_1} = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3} = \frac{-8\left(\frac{1}{2}\right) + 8\left(\frac{1}{2}\right) + 6(1)}{\frac{1}{2} + \frac{1}{2} + 1} = 3 \text{ V}$$

$$R_{m_1} = \frac{1}{G_{m_1}} = \frac{1}{G_1 + G_2 + G_3} = \frac{1}{\frac{1}{2} + \frac{1}{2} + 1} = 0.5\ \Omega$$

Step III Applying Millman's theorem at terminals *CA*,

$$V_{m_2} = \frac{V_4 G_4 + V_5 G_5}{G_4 + G_5} = \frac{6\left(\frac{1}{2}\right) + 3(1)}{\frac{1}{2} + 1} = 4 \text{ V}$$

$$R_{m_2} = \frac{1}{G_{m_2}} = \frac{1}{G_4 + G_4} = \frac{1}{\frac{1}{2} + 1} = 0.67\ \Omega$$

Step IV Millman's Equivalent Network (Fig. 2.369)

Simplifying Fig. 2.369 further, the Millman's equivalent network is shown in Fig. 2.370.

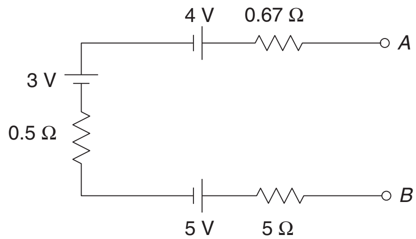


Fig. 2.369

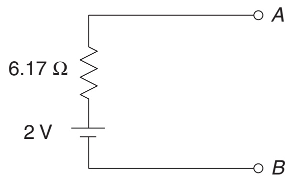


Fig. 2.370

Exercises

Mesh Analysis

2.1 Find currents I_x and I_y in the network shown in Fig. 2.371.

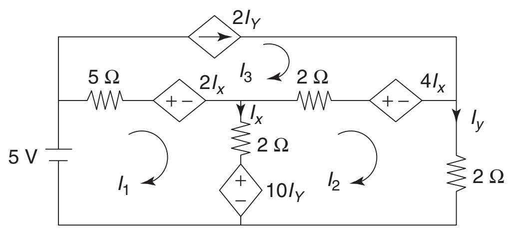


Fig. 2.371

[0.5 A, 0.1 A]

2.2 In the network shown in Fig. 2.372, find V_3 if element A is a

(i) short circuit
(ii) 5 Ω resistor
(iii) 20 V independent voltage source, positive reference on the right
(iv) dependent voltage source of 1.5 i_1, with positive reference on the right
(v) dependent current source 5 i_1, arrow directed to the right

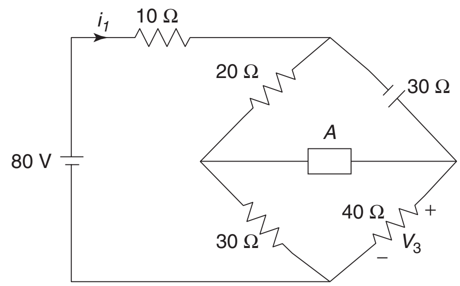


Fig. 2.372

[69.4 V, 72.38 V, 73.68 V, 70.71 V, 97.39 V]

2.3 Find currents I_1, I_2, and I_3 in the network shown in Fig. 2.373.

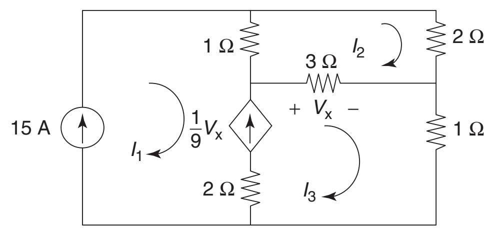


Fig. 2.373

[15 A, 11 A, 17 A]

2.4 Find currents I_x in the network shown in Fig. 2.374.

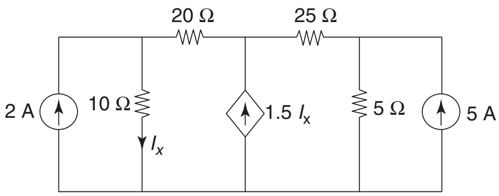


Fig. 2.374

[8.33 A]

2.5 Find currents I_1 in the network shown in Fig. 2.375.

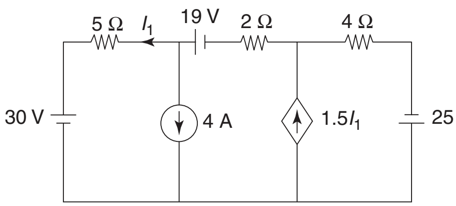


Fig. 2.375

[–12 A]

Node Analysis

2.6 Find the voltage V_x in the network shown in Fig. 2.376.

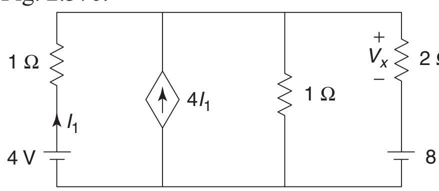


Fig. 2.376

[–4.31 V]

2.7 Find the currents V_x in the network shown in Fig. 2.377.

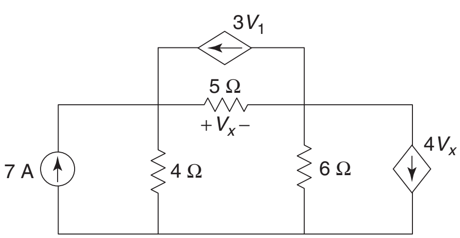


Fig. 2.377

[2.09 V]

2.8 Find the voltage V_x in the network shown in Fig. 2.378.

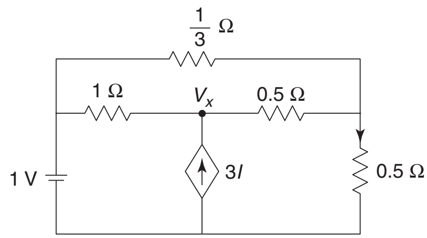


Fig. 2.378

[6.2 V]

2.9 Determine V_1 in the network shown in Fig. 2.379.

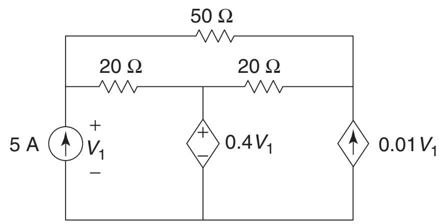


Fig. 2.379

[140 V]

2.10 Find the voltage V_y in the network shown in Fig. 2.380.

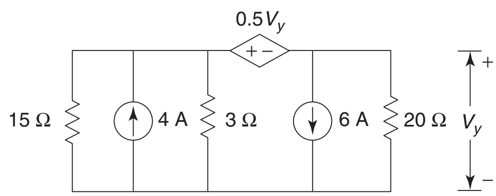


Fig. 2.380

[–10 V]

2.11 Find the voltage V_2 in the network shown in Fig. 2.381.

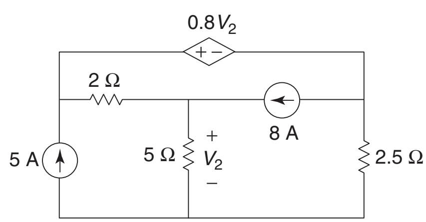


Fig. 2.381

[25.9 V]

Superposition Theorem

2.12 Find the voltage V_x in Fig. 2.382.

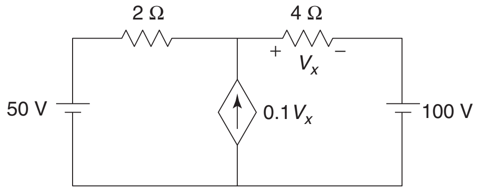


Fig. 2.382

[−38.5 V]

2.13 Determine the voltages V_1 and V_2 in Fig. 2.383.

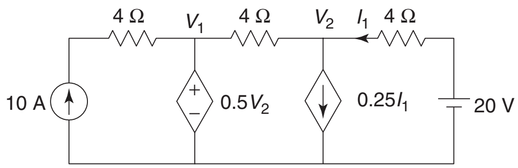


Fig. 2.383

[6 V,12 V]

2.14 Find the voltage V_x in Fig. 2.384.

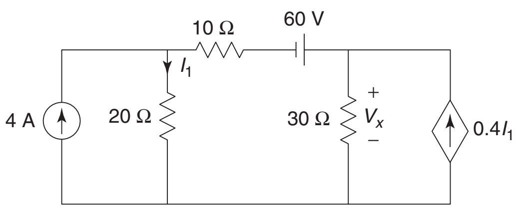


Fig. 2.384

[7.5 V]

Thevenin's Theorem

2.15 Determine Thevenin's equivalent network for figures 2.385 to 2.388 shown below.

(i)

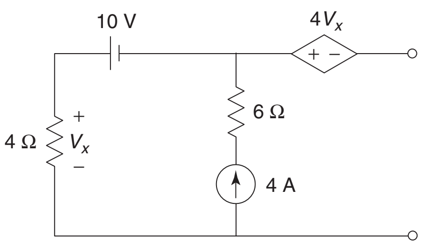


Fig. 2.385

[−58 V,12 Ω]

(ii)

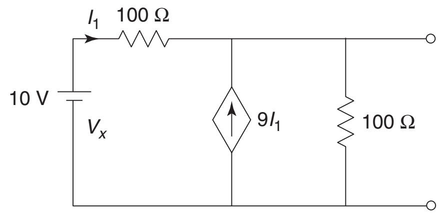


Fig. 2.386

[9.09 V,9.09 Ω]

(iii)

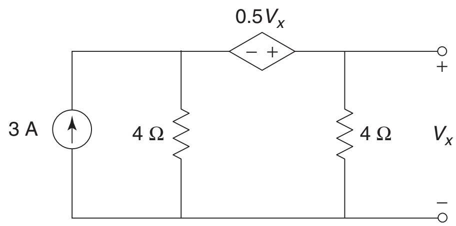


Fig. 2.387

[8 V,2.66 Ω]

(iv)

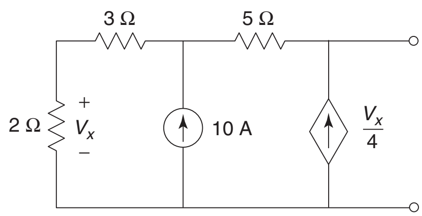


Fig. 2.388

[150 V,20 Ω]

2.16 Find the current I_x in Fig. 2.389.

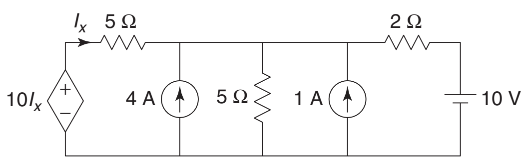


Fig. 2.389

[4 A]

2.17 Find the current in the 24 Ω resistor in Fig. 3.390.

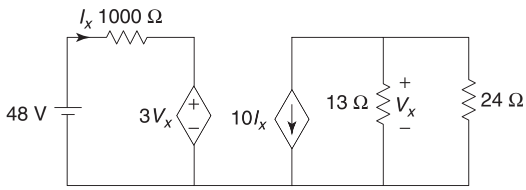


Fig. 3.390

[0.225 A]

Norton's Theorem

2.18 Find Norton's equivalent network and hence find the current in the 10 Ω resistor in Fig. 2.391.

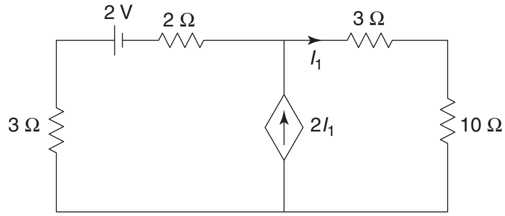


Fig. 2.391

[0.25 A]

2.19 Find Norton's equivalent network in Fig. 2.392.

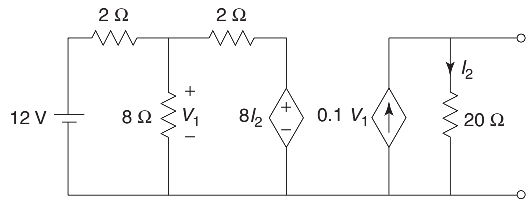


Fig. 2.392

[0.533 A, 31 Ω]

Objective-Type Questions

2.1 Two electrical sub-networks N_1 and N_2 are connected through three resistors as shown in Fig. 2.393. The voltages across the 5 Ω resistor and 1 Ω resistor are given to be 10 V and 5 V respectively. Then the voltage across the 15 Ω resistor is

(a) −105 V (b) 105 V
(c) −15 V (d) 15 V

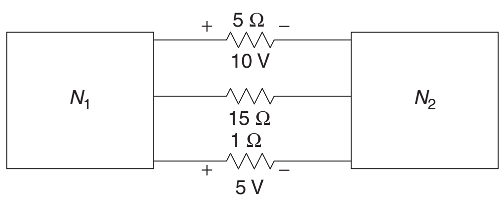


Fig. 2.393

2.2 The nodal method of circuit analysis is based on

(a) KVL and Ohm's law
(b) KCL and Ohm's law
(c) KCL and KVL
(d) KCL, KVL and Ohm's law

2.3 The voltage across terminals *a* and *b* in Fig. 2.394 is

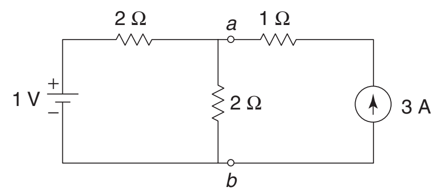


Fig. 2.394

(a) (a) 0.5 V (b) 3 V
(c) 3.5 V (d) 4 V

2.4 The voltage V_0 in Fig. 2.395 is

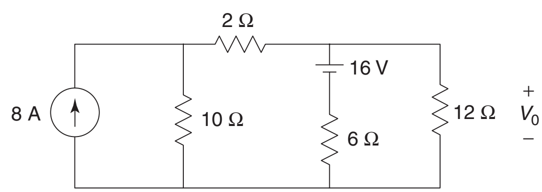


Fig. 2.395

(a) 48 V (b) 24 V
(c) 36 V (d) 28 V

2.5 The dependent current source shown in Fig. 2.396.

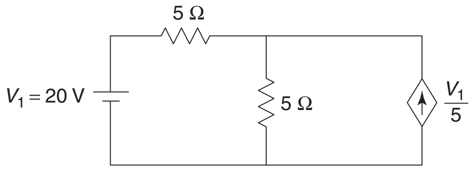


Fig. 2.396

(a) delivers 80 W (b) absorbs 80 W
(c) delivers 40 W (d) absorbs 40 W

2.6 If $V = 4$ in Fig. 2.397, the value of I_s is given by

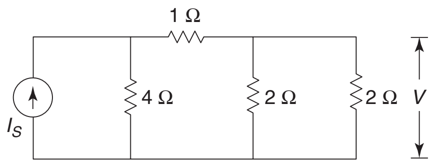


Fig. 2.397

(a) 6 A (b) 2.5 A
(c) 12 A (d) none of these

2.7 The value of V_x, V_y and V_z in Fig. 2.398 shown are

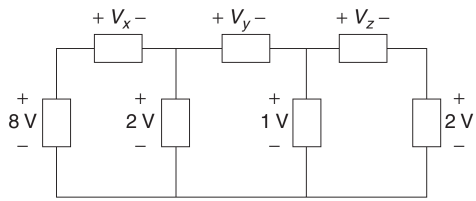


Fig. 2.398

(a) –6, 3, –3 (b) –6, –3, 1
(c) 6, 3, 3 (d) 6, 1, 3

2.8 The circuit shown in Fig. 2.399 is equivalent to a load of

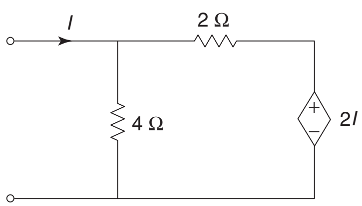


Fig. 2.399

(a) $\frac{4}{3}$ Ω (b) $\frac{8}{3}$ Ω
(c) 4 Ω (d) 2 Ω

2.9 In the network shown in Fig. 2.400, the effective resistance faced by the voltage source is

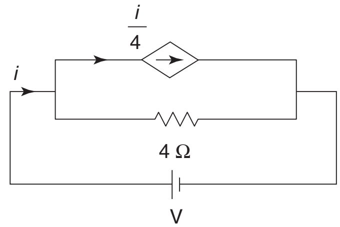


Fig. 2.400

(a) 4 Ω (b) 3 Ω
(c) 2 Ω (d) 1 Ω

2.10 A network contains only an independent current source and resistors. If the values of all resistors are doubled, the value of the node voltages will
(a) become half
(b) remain unchanged
(c) become double
(d) none of these

2.11 The value of the resistance *R* connected across the terminals *A* and *B* in Fig. 2.401, which will absorb the maximum power is

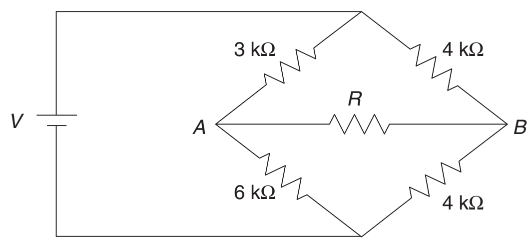


Fig. 2.401

(a) 4 kΩ (b) 4.11 kΩ
(c) 8 kΩ (d) 9 k Ω

2.12 Superposition theorem is not applicable to networks containing
(a) nonlinear elements
(b) dependent voltage source

(c) dependent current source
(d) transformers

2.13 The value of R required for maximum power transfer in the network shown in Fig. 2.402 is

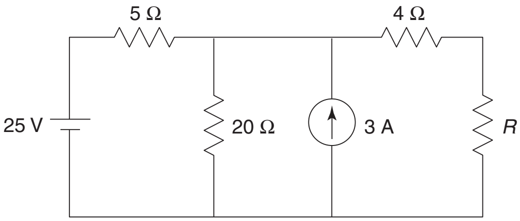


Fig. 2.402

(a) 2 Ω (b) 4 Ω
(c) 8 Ω (d) 16 Ω

2.14 In the network of Fig. 2.403, the maximum power is delivered to R_L if its value is

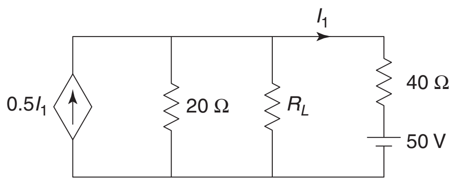


Fig. 2.403

(a) 16 Ω (b) $\frac{40}{3}$ Ω
(c) 60 Ω (d) 20 Ω

2.15 The maximum power that can be transferred to the load R_L from the voltage source in Fig. 2.404 is

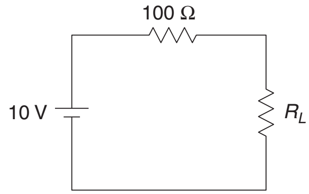


Fig. 2.404

(a) 1 W (b) 10 W
(c) 0.25 W (d) 0.5 W

2.16 For the circuit shown in Fig. 2.405, Thevenin's voltage and Thevenin's equivalent resistance at terminals *a*-*b* is

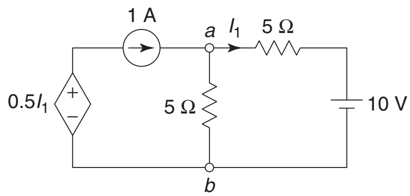


Fig. 2.405

(a) 5 V and 2 Ω (b) 7.5 V and 2.5 Ω
(c) 4 V and 2 Ω (d) 3 V and 2.5 Ω

2.17 The value of R_L in Fig. 2.406 for maximum power transfer is

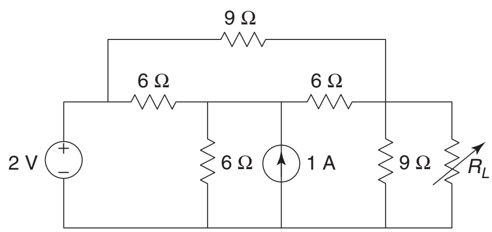


Fig. 2.406

(a) 3 Ω (b) 1.125 Ω
(c) 4.1785 Ω (d) none of these

Answers to Objective-Type Questions

2.1. (a)	2.2. (b)	2.3. (c)	2.4. (d)	2.5. (a)	2.6. (d)	2.7. (a)
2.8. (a)	2.9. (d)	2.10. (b)	2.11. (a)	2.12. (a)	2.13. (c)	2.14. (a)
2.15. (c)	2.16. (b)	2.17. (a)				