Module 3. Probability Distribution

* **Sample space:**
The set of all possible outcomes of an experiment is called sample space.
It is denoted by 'S'

Note that the element of set S is called sample points.

<u>for example</u>: In a throw of two coins,
the sample space $S = \{(H,H), (H,T), (T,H), (T,T)\}$

* **Event**: A Subset of sample space is called an event.

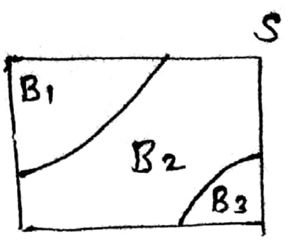
<u>for example</u>: ① If $S = \{(H,H), (H,T), (T,H), (T,T)\}$

then $A_1 = \{(H,H), (T,H)\}$
$A_2 = \{(H,T), (T,H), (T,T)\}$ are the events

② In a throw of a die,

The sample space $S = \{1, 2, 3, 4, 5, 6\}$

and $B_1 = \{1, 2\}$
$B_2 = \{3, 4, 5\}$
$B_3 = \{6\}$ are events



* probability:

If S is the sample space with n points which are mutually exclusive and A is the event (subset of S) with m points then the ratio $\frac{m}{n}$ is called probability of A and is denoted by '$P(A)$'

that is $P(A) = \frac{m}{n} = \frac{\text{number of points in } A}{\text{number of points in } S}$

Important Note: i) $P(A) \geq 0$, for any A

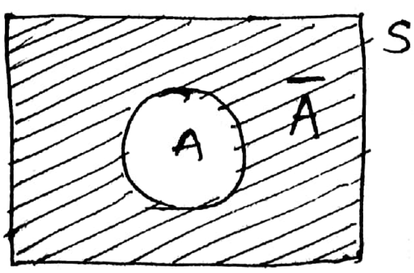
ii) $P(S) = 1$

iii) $P(A \cup B) = P(A) + P(B)$, for any exclusive events A, B

* Complement of the event :

If A be the event of a sample space S then the complement of A is denoted by $\bar{A}$ and is given by

$\bar{A} = S - A$



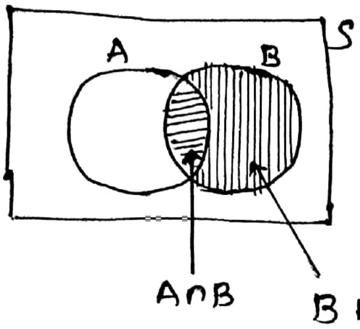
Note: $A \cup \bar{A} = S$

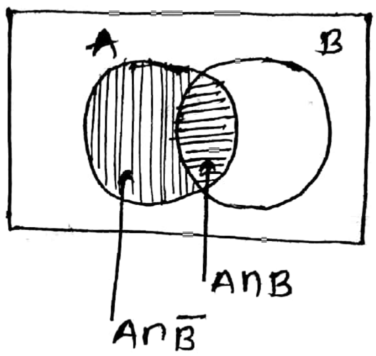
* Laws of probability :

(1) let A, B be any events then

* $P(B \cap \bar{A}) = P(B) - P(A \cap B)$

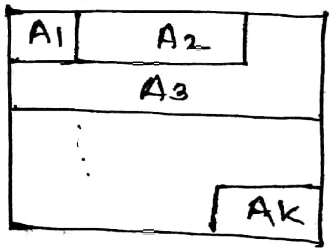
* $P(A \cap \bar{B}) = P(A) - P(A \cap B)$





(2) let $A_1, A_2, A_3, \cdots, A_k$ be the mutually exclusive events such that $A_1 \cup A_2 \cup A_3 \cdots \cup A_k = S$

then $P(A_1) + P(A_2) + \cdots + P(A_k) = 1$



(3) Addition theorem:

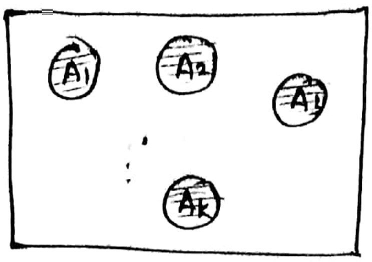
let A, B, C be any events then

* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

* $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

④ If $A_1, A_2, \cdots A_K$ are pairwise exclusive

then $P(A_1 \cup A_2 \cup \cdots \cup A_K) = P(A_1) + P(A_2) + \cdots + P(A_K)$



⑤ let A be any event

then $P(\bar{A}) = 1 - P(A)$

* De morgans law ;

* $P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$

* $P(\overline{A \cap B}) = P(\bar{A} \cup \bar{B})$

Note that :

① $^{n}C_{k} = \dfrac{n!}{(n-k)! \cdot k!}$, $n \geq k$

(combinations)

② $^{n}P_{k} = \dfrac{n!}{(n-k)!}$, $n \geq k$

(permutations)

* Conditional probability:

let A and B be two events in a sample space S then the conditional probability of an event A given that B has happed is denoted by $P(A/B)$ and is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Note that:
$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0$$

for ex. suppose there are 100 students in a class and the result of an examination of class is given in following table.

	passed	failed	Total
Boys	28	32	60
Girls	26	14	40
Total	54	46	100

let A = a student has passed

B = a student is a male (Boys)

If we selected a male student randomly then the probability of that this student has passed = $P(A/B)$

∴ clearly sample space of all male students has 60 point and out of these only 28 have passed

∴ $P(A \cap B) = 28$ and $P(B) = 60$

Hence, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{28}{60} = \dfrac{7}{15}$

* partition of sample space :-

let S be the sample space and $A_1, A_2, A_3, \dots, A_n$ be the events then $A_1, A_2, \dots, A_n$ are said to be partition of S

if i) $A_i \cap A_j = \phi$ for all $i \neq j$

ii) $A_1 \cup A_2 \cup A_3 \dots \cup A_n = S$