Control systems

System : A system is a combination or an arrangement of different physical components which act together as a entire unit to achieve certain objective.

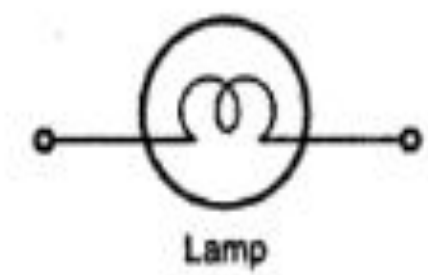
Every physical object is actually a system. A classroom is a good example of physical system. A room along with the combination of benches, blackboard, fans, lighting arrangement etc. can be called as a classroom which acts as elementary system.

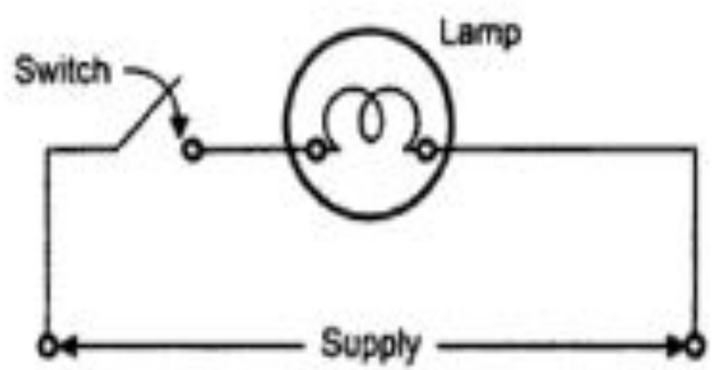
Control system : To control means to regulate, to direct or to command. Hence a control system is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system.

Control systems

Example

lamp is switched ON or OFF using a switch, the entire system can be called as a control system.





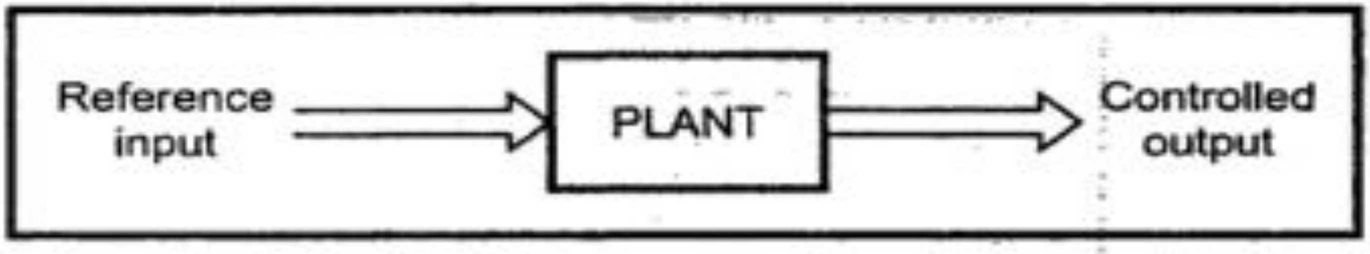
Control systems

Plant : The portion of a system which is to be controlled or regulated is called as the **plant or the Process.**

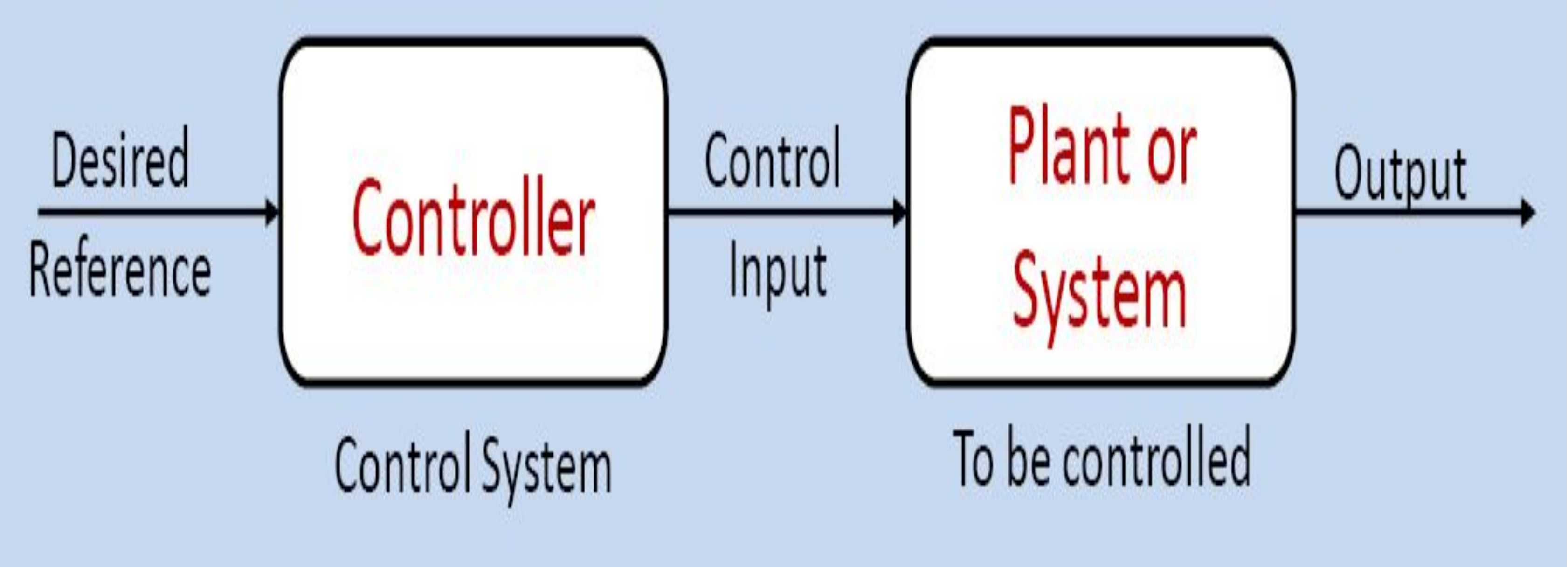
Controller : The element of the system itself or external to the system which controls the plant or the process is called as **controller.**

Input : It is an applied signal or an excitation signal applied to control system from an external energy source in order to produce a specified output.

Output : It is the particular signal of interest or the actual response obtained from a control system when input is applied to it.



Block Diagram

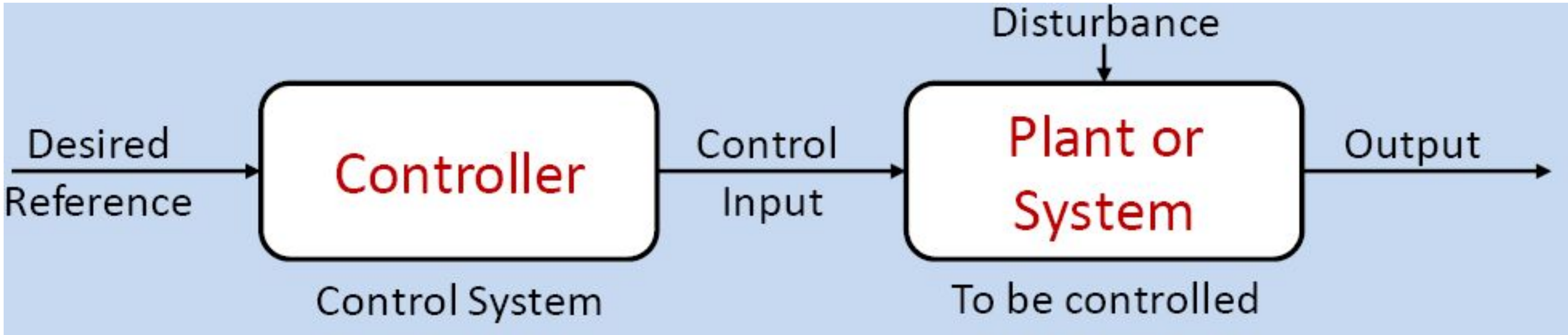


Disturbance

- Unwanted signals which affect the output of the system

E.g. People entering and leaving an AC room disturbs room temperature

- Controller has to eliminate the effects of disturbance



Classification of Systems

- Linear and non-linear systems
- Static and dynamic systems
- Time invariant and time variant systems
- Causal and non-causal systems

Classification of Systems

- **Linear systems**

  Output of the system varies linearly with input .e.g. Resistor

- **Non-linear systems**

  Output of the system does not vary linearly with input.e.g. Diode

- **Static systems**

  At any time, output of the system depends only on present input .e.g. Resistor

- **Dynamic systems**

  Output of the system depends on present as well as past inputs.e.g. Inductor

Classification of Systems

- **Time invariant systems**

  Output of the system is independent of the time at which the input is applied .e.g. Resistor

- **Time variant systems**

  Output of the system varies dependent on the time at which input is applied .e.g. Aircraft

- **Causal systems**

  Output is only dependent on inputs already received (present or past) .e.g. Motor or generator

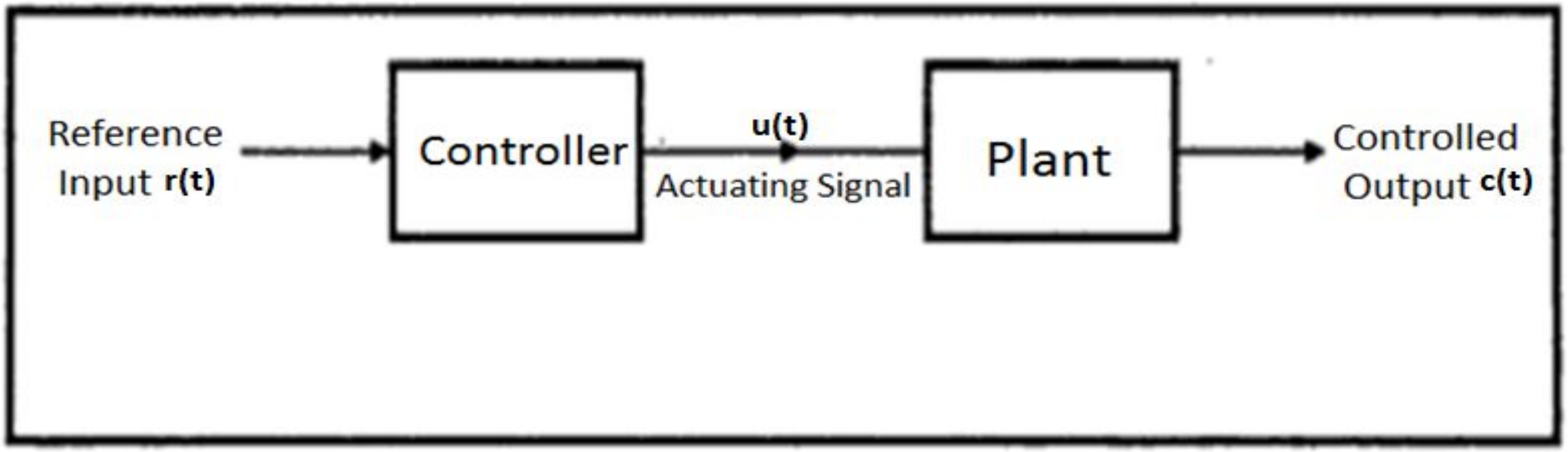
- **Non-causal systems**

  Output depends on future inputs as well .e.g. Weather forecasting system

Open Loop and Closed Loop Control Systems

Open Loop Control System

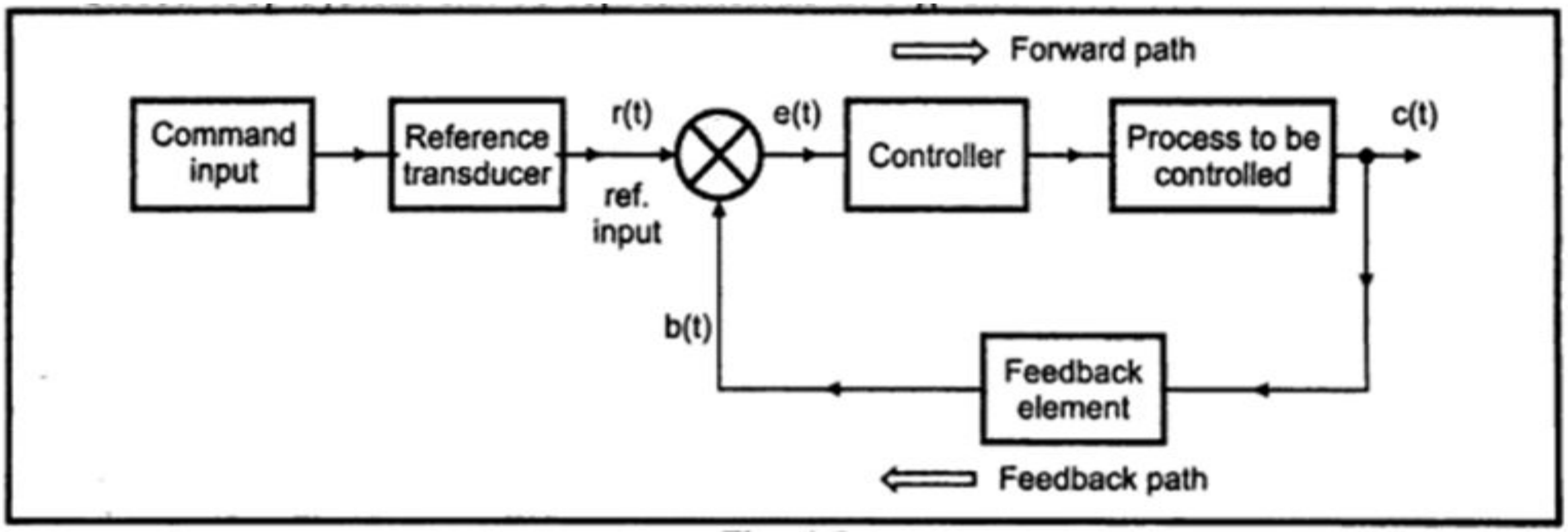
A system in which output is dependent on input but input is totally independent of the output or changes in output of the system.



Open Loop and Closed Loop Control Systems

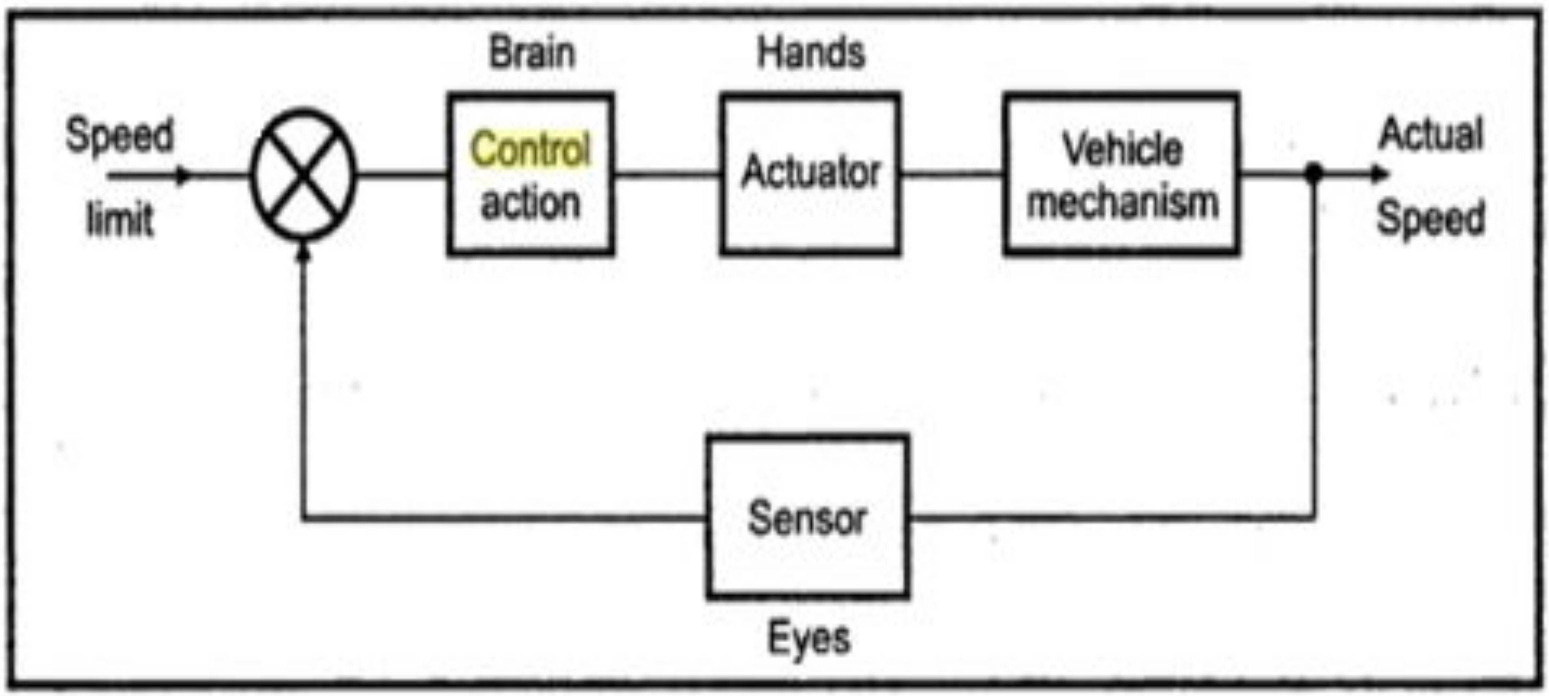
Closed Loop Control System

A system in which the input is dependent on the output or changes in output of the system



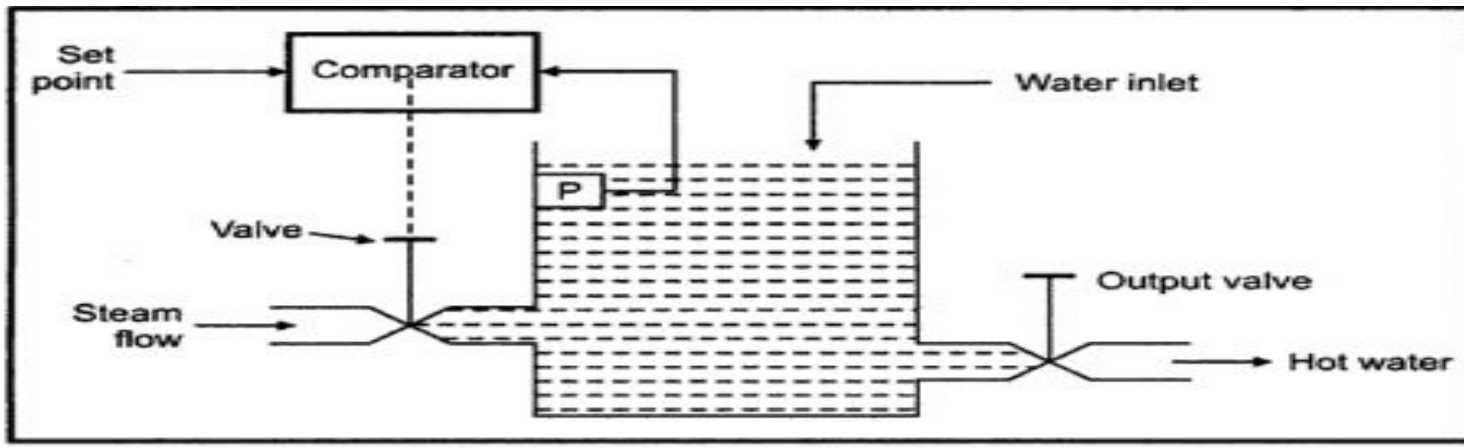
Examples of Closed Loop Control Systems

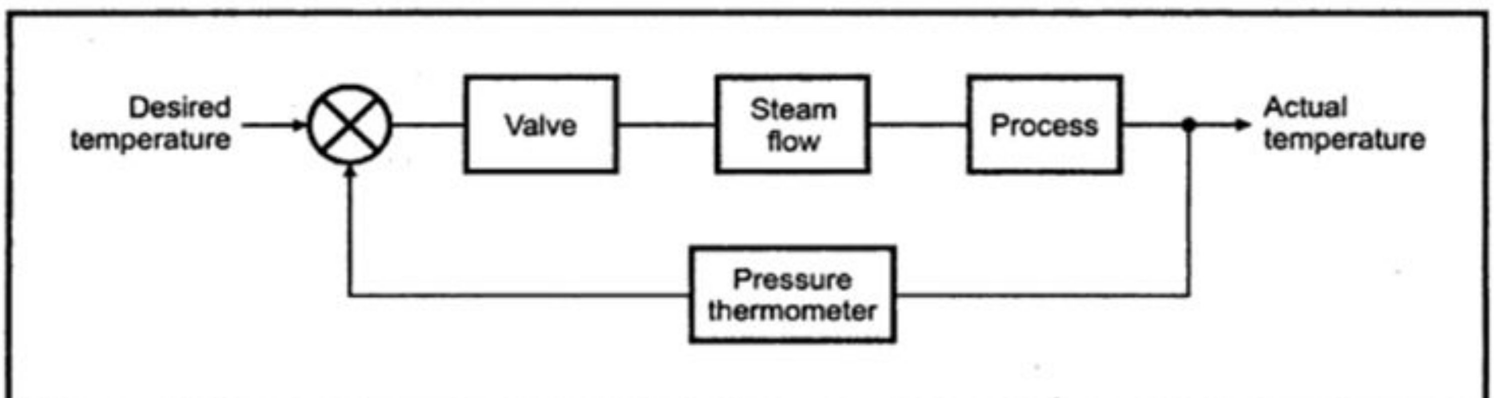
1. Manual speed control system



Examples of Closed Loop Control Systems

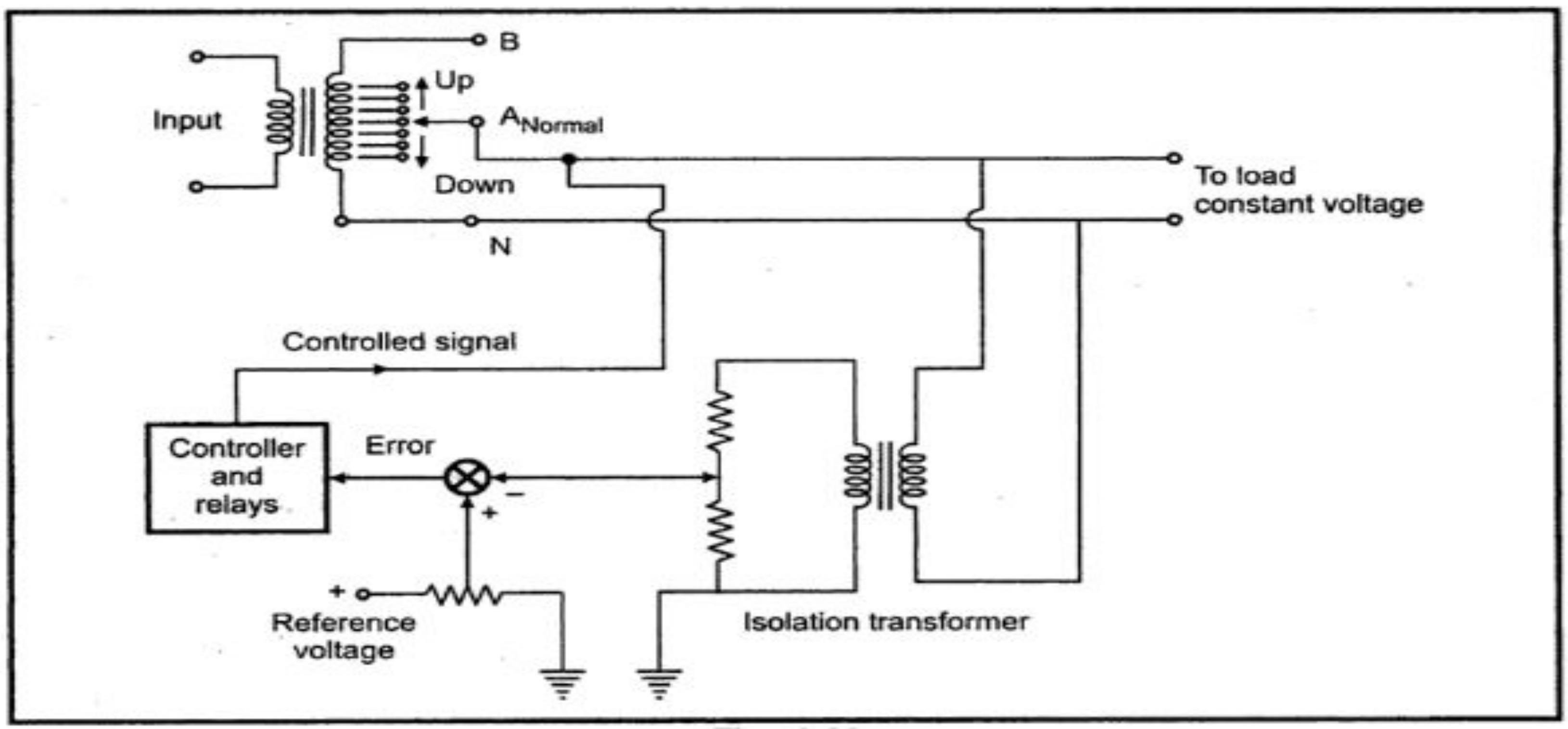
2. Temperature control system





Examples of Closed Loop Control Systems

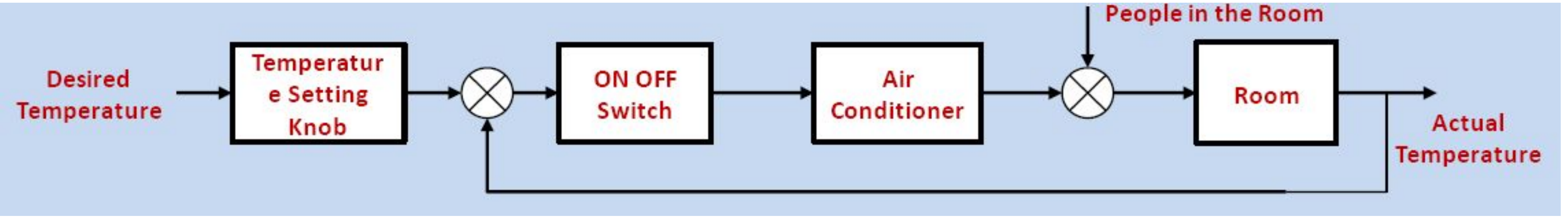
3. Voltage Stabilizer



Examples of Control Systems

- **Air conditioner maintaining desired temperature:**

Plant : Room
Control system : Air Conditioner
Reference : Desired temperature
Control Input : Compressor ON/OFF
Output : Output temperature
Disturbance : Factors affecting ambient temperature
Feedback : Measured temperature



Mathematical Modeling of Control Systems

- A mathematical model of a dynamic system is defined as a set of equations that represents the dynamics of the system accurately.

- A mathematical model is not unique to a given system.

- A system may be represented in many different ways and, therefore, may have many mathematical models, depending on one's perspective.

- The dynamics of many systems, whether they are mechanical, electrical,thermal, economic, biological, and so on, may be described in terms of differential equations.

- Such differential equations may be obtained by using physical laws governing a particular system.

- For example, Newton’s laws for mechanical systems and Kirchhoff’s laws for electrical systems.

- Deriving reasonable mathematical models is the most important part of the entire analysis of control systems.

Transfer Function

The *transfer function* of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

- The applicability of the concept of the transfer function is limited to linear, time-invariant, differential equation systems.
- The transfer function approach, however, is extensively used in the analysis and design of such systems.

Transfer Function. The *transfer function* of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Consider the linear time-invariant system defined by the following differential equation:

$$\begin{aligned} a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y \\ = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1}\dot{x} + b_m x \qquad (n \geq m) \end{aligned}$$

where y is the output of the system and x is the input. The transfer function of this system is the ratio of the Laplace transformed output to the Laplace transformed input when all initial conditions are zero, or

$$\begin{aligned} \text{Transfer function} = G(s) &= \left.\frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}\right|_{\text{zero initial conditions}} \\ &= \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \end{aligned}$$

By using the concept of transfer function, it is possible to represent system dynamics by algebraic equations in s. If the highest power of s in the denominator of the transfer function is equal to n, the system is called an *nth-order system*.

1. The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.

2. The transfer function is a property of a system itself, independent of the magnitude and nature of the input or driving function.

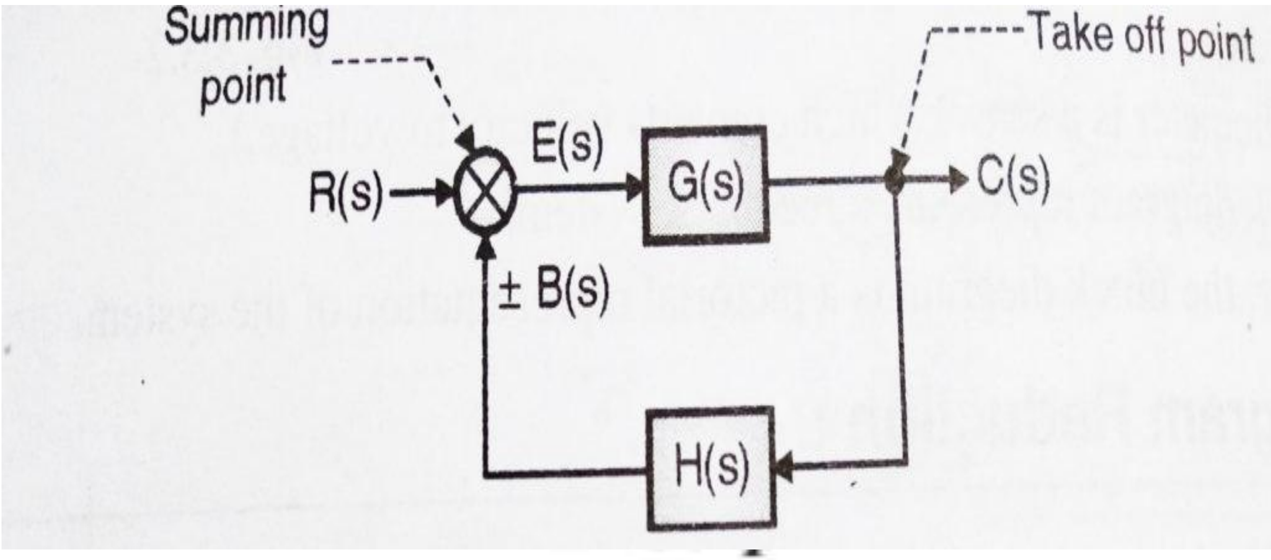
3. The transfer function includes the units necessary to relate the input to the output;
however, it does not provide any information concerning the physical structure of the system. (The transfer functions of many physically different systems can be identical.)

4. If the transfer function of a system is known, the output or response can be studied for various forms of inputs with a view toward understanding the nature of the system.

Block Diagram Reduction

- Output: It is defined as product of input and gain i.e Output = gain x input
- Summing point: More than one signal can be added or subtracted at summing point.
- Take off point: From this point output can be again fed back to input, thus this point will be used for feedback purpose.
- Forward path: This path represent direction of the signal flow in the system from input side to output side.
- Feedback path: This path represent direction of the signal flow in the system from output side to input side.

simple closed loop system



$$C(s) = E(s) \cdot G(s)$$
$$E(s) = R(s) \pm B(s)$$

R(s) = Laplace of input signal

C(s) = Laplace of output signal

E(s) = Laplace of error signal

B(s) = Laplace of feedback signal

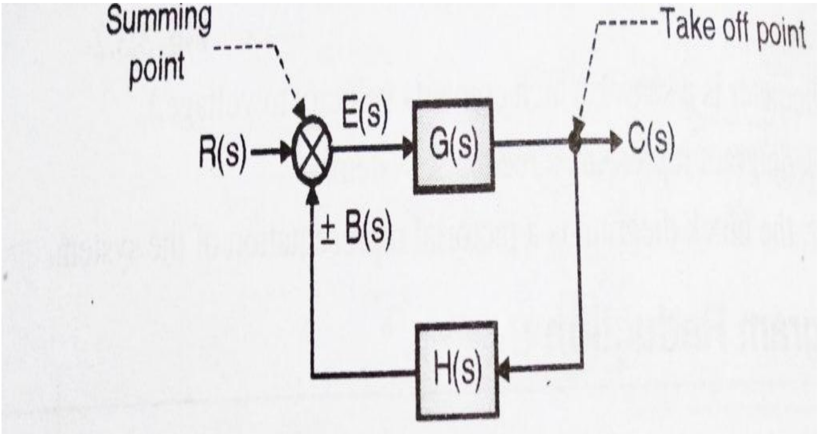
G(s) = forward transfer function

H(s) = feedback transfer function

Closed Loop (Feedback)System

$$C(s) = E(s) \cdot G(s)$$
$$E(s) = R(s) \pm B(s)$$

$$C(s) = [R(s) \pm B(s)]\, G(s)$$
$$\therefore C(s) = R(s)\, G(s) \pm B(s)\, G(s)$$
$$B(s) = C(s) \cdot H(s)$$



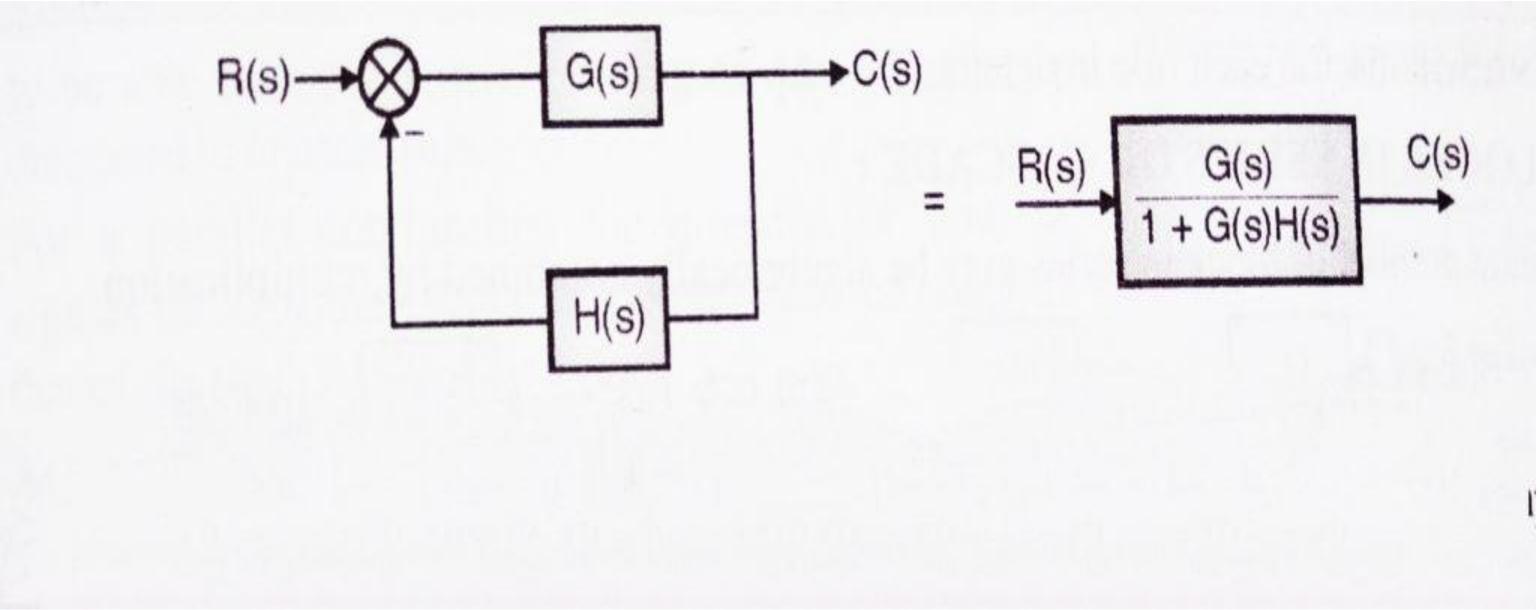
$$\therefore C(s) = R(s)\, G(s) \pm C(s)\, H(s)\, G(s)$$
$$\therefore C(s) \mp C(s)\, H(s)\, G(s) = R(s)\, G(s)$$
$$\therefore C(s)\, [1 \mp G(s)\, H(s)] = R(s)\, G(s)$$
$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)\, H(s)}$$

If the system is a negative feedback

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\, H(s)}$$

and if the system is a positive feedback

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\, H(s)}$$



If negative feedback is present in the system.

T.F=C(S)/R(S)=G(S)/(1+G(S)H(S))

If positive feedback is present in the system

T.F=C(S)/R(S)=G(S)/(1-G(S)H(S))

- For open loop transfer function, as no feedback present, transfer function can be given by:

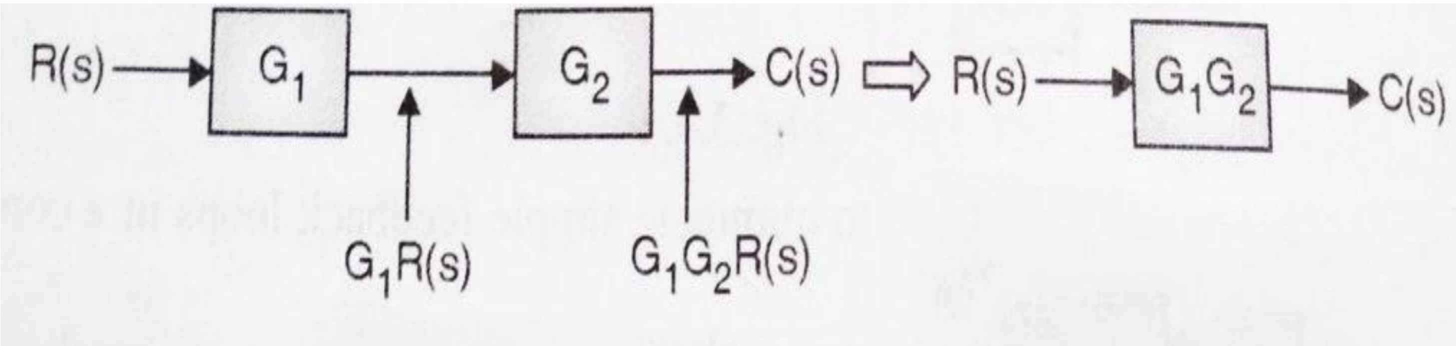
Open loop transfer function = G(s).H(s)

Block Diagram Reduction Rules

- **Rule 1: Blocks in cascade/series**
- **Rule 2: Blocks in parallel**
- **Rule 3: Feedback loop elimination**
- **Rule 4: Associative law for summing**
- **Rule 5: Shifting a summing point before a block**
- **Rule 6: Shifting a summing point after a block**
- **Rule 7: Shifting take-off point before a block**
- **Rule 8: Shifting take-off point after a block**
- **Rule 9 : Shifting a Take-off point after a Summing Point**
- **Rule 10 : Shifting a Take-off point before a Summing Point**

Rule 1 -Blocks in cascade/series

finite number of blocks in series can be combined together by multiplication

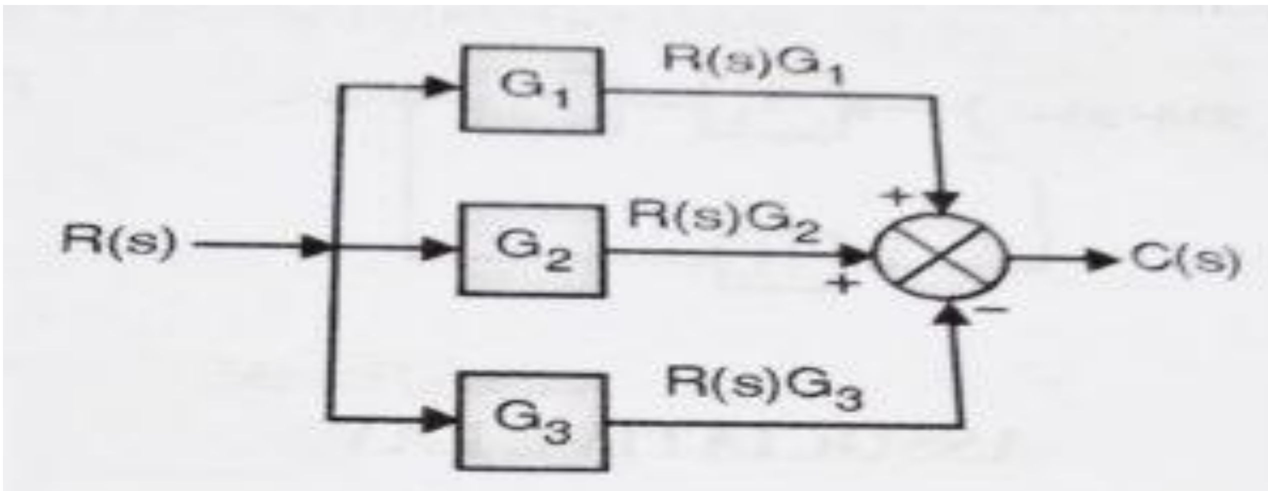


Output C(s) = G1 x G2 x R(s)

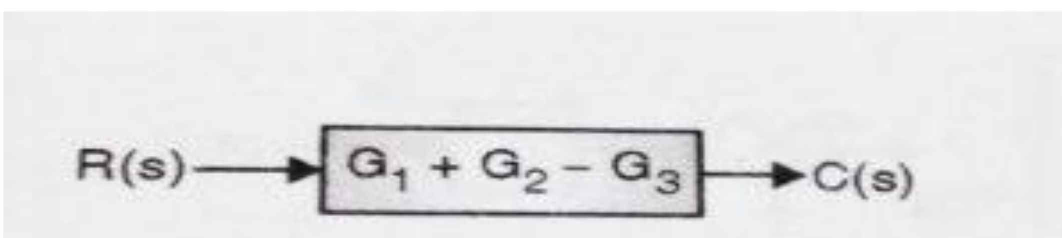
If there is a take-off point or summing point between the blocks, the blocks cannot be said to be in cascade/series. (The take-off / summing point has to be shifted before or after the block using another rule)

Rule 2 - Blocks in parallel

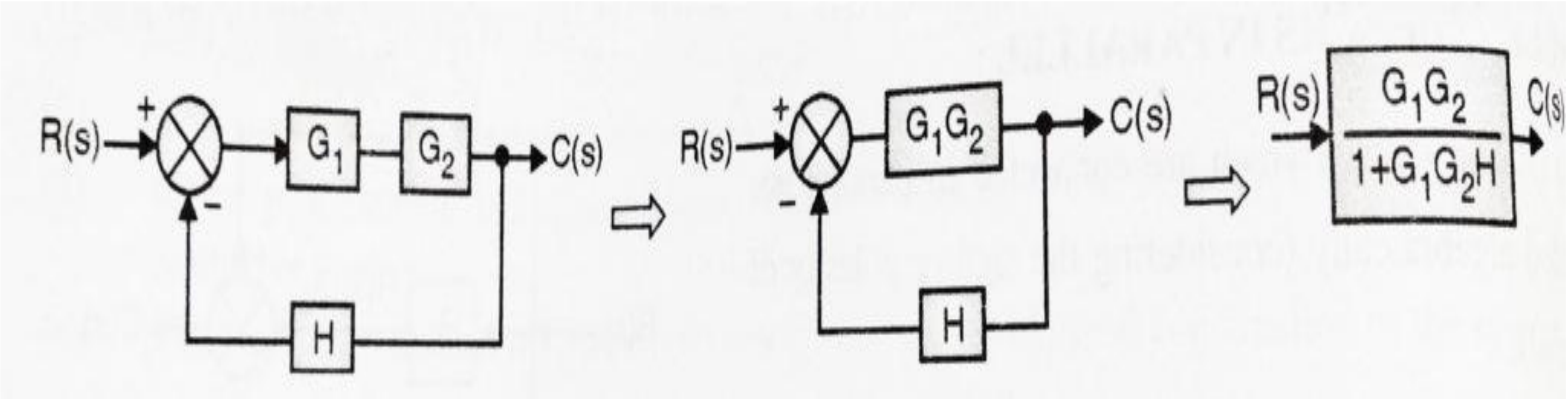
When the blocks are connected in parallel combination, they get added algebraically (considering the sign of the signal)



C(s) = R(s)G1 + R(s)G2 – R(s)G3

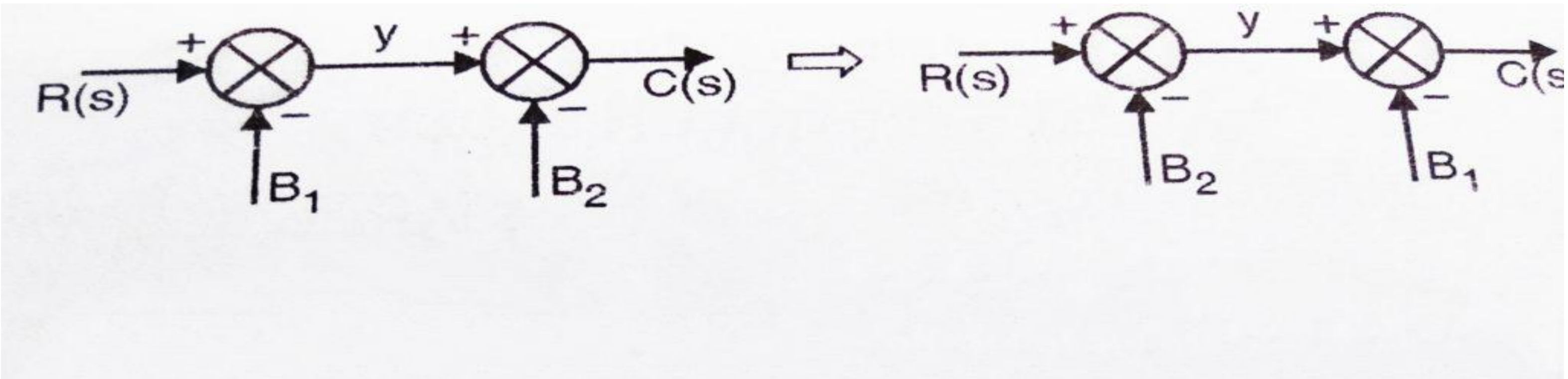


Rule 3: Elimination of feedback Loop



$$T.F = C(S)/R(S) = \frac{G(S)}{1+G(S)H(S)}$$

Rule 4: Associative Law For Summing Point

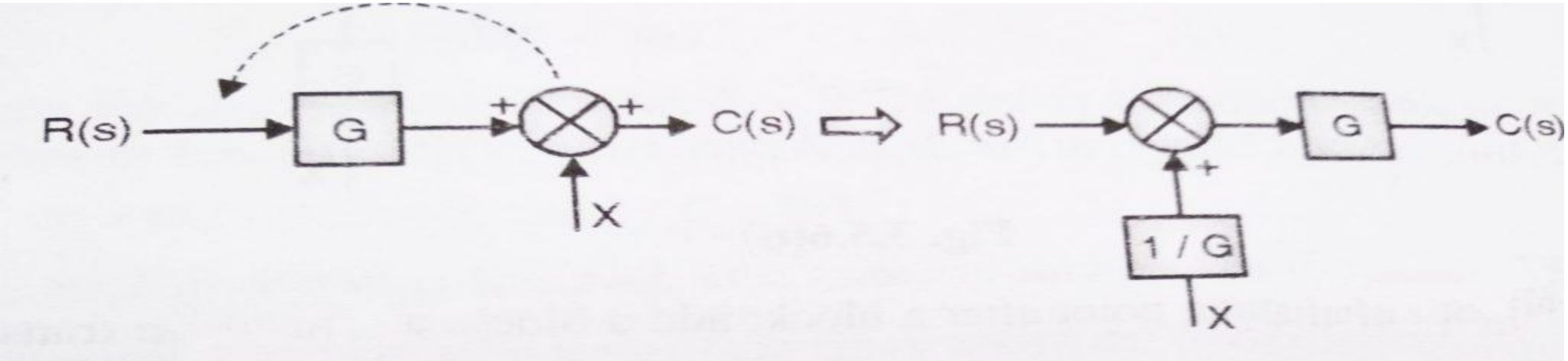


Y= R(s) – B1

C(s) = y – B2 = R(s) – B1 – B2

This law is applicable only to summing points which are connected directly to each other.
Note: if there is a block present between two summing points then this rule cannot be applied.

Rule 5: Shifting of a Summing Point before a block



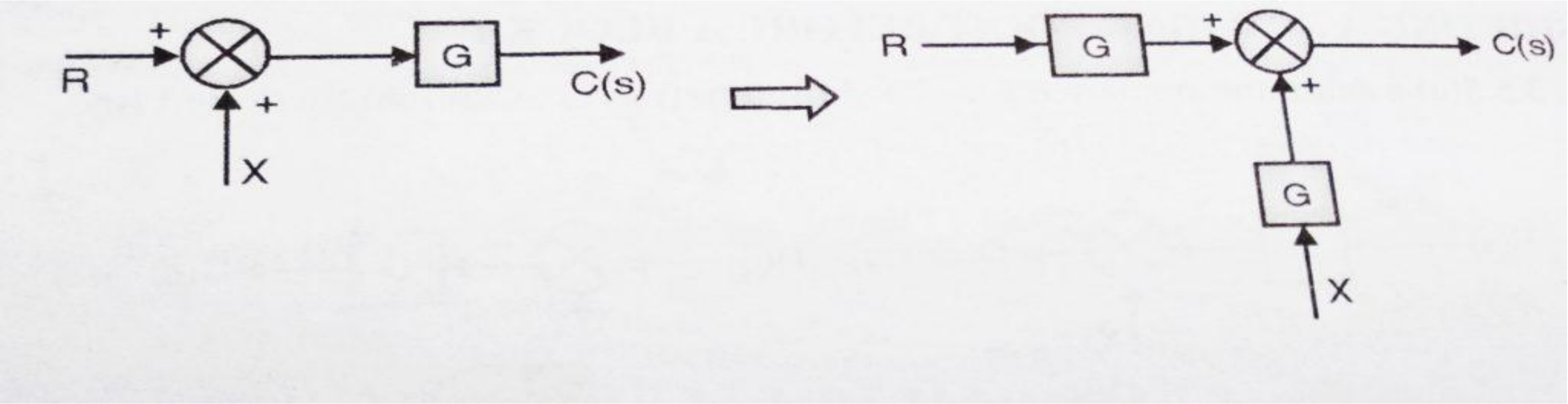
C(s) = GR(s) + X

After shifting the summing point, we will get

C(s) = [R +(X/G)] G = GR + X which is same as output in the first case.

Hence to shift a summing point before a block, we need o add another block of transfer function '1/G' before the summing point .

Rule 6: Shifting of a Summing Point after a block



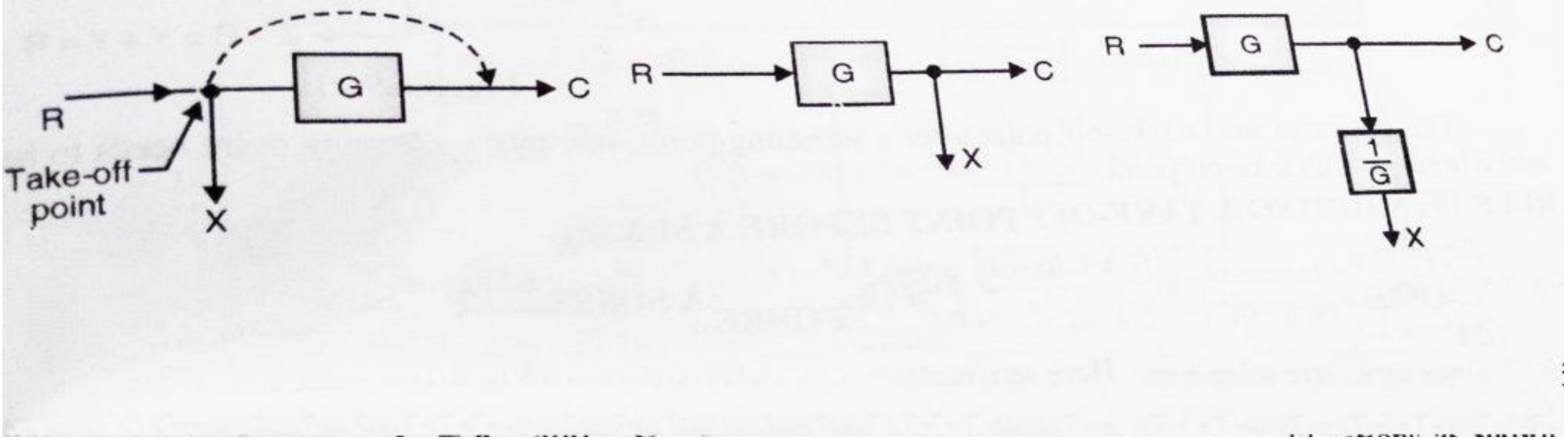
C(s) = (R + X)G

After shifting the summing point,

C(s) = (R +X) G = GR + XG which is same as output in the first case.

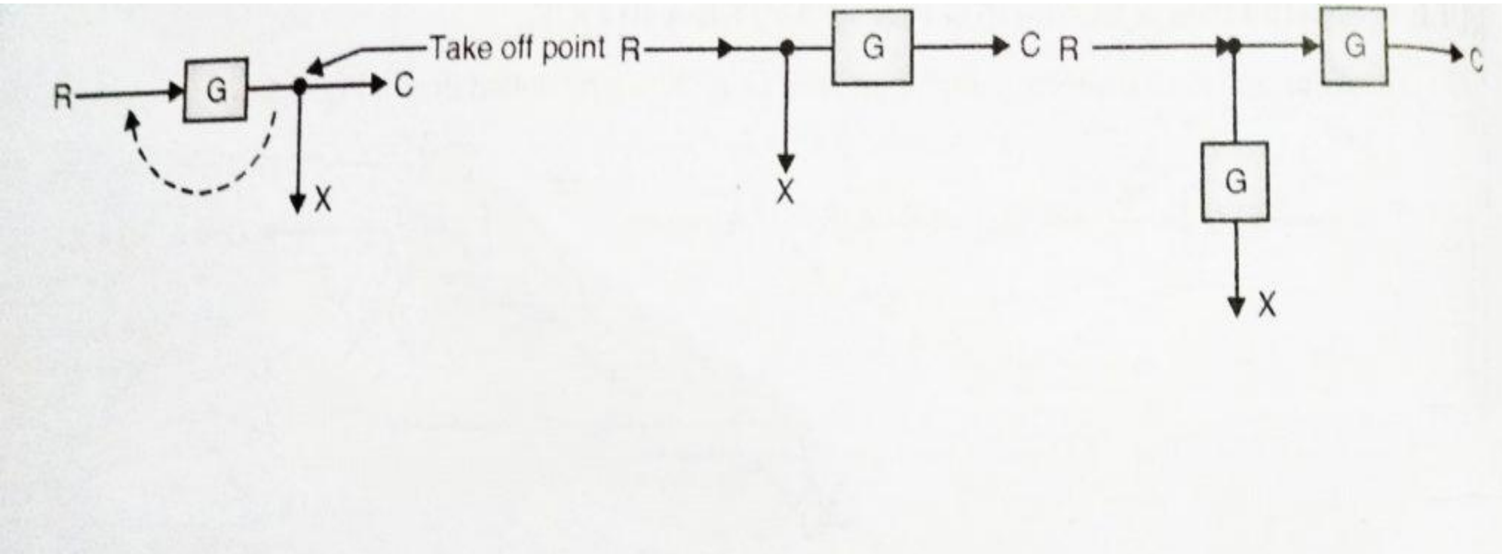
Hence to shift a summing point before a block, add another block having same transfer function at the summing point.

Rule 7: Shifting of Take-off point after a block



X = R and C = RG (initially)…….(I)
X = RG and C = RG
But as per equation (I) ,X=R
In order to achieve this, we need to add a block of transfer function ‘1/G’ in series with X.
X=(RG)*(1/G)=R and C=RG

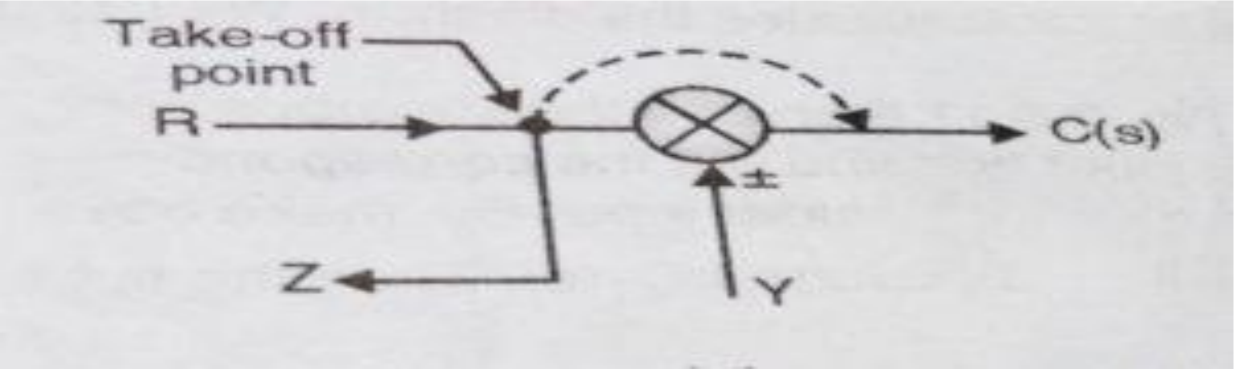
Rule 8 : Shifting of Take-off point before a block



Here we have X = RG and C = RG (initially)….(I)
X=R ,C=RG
But as per equation (I) ,X=RG
In order to achieve this we need to add a block of transfer function ‘G’ in series with X signal taking off from that point.
X=RG ,C=RG

CRITICAL RULES

Rule 9 : Shifting a Take-off point after a Summing Point



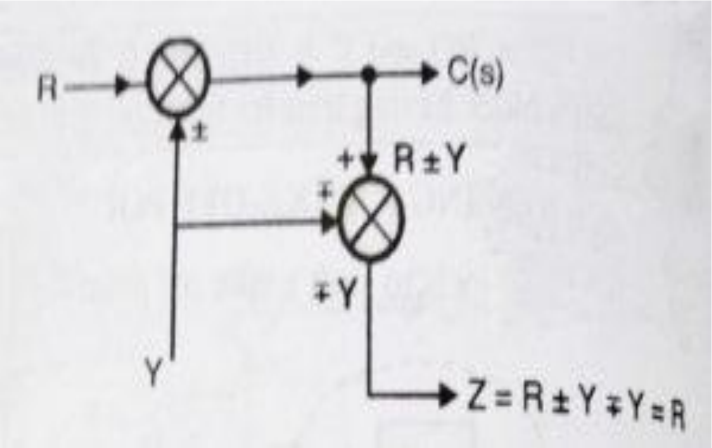
$Z = R$ and $C=R\pm Y$ (initially).......(I)

If we directly shift take off point after summing point,then

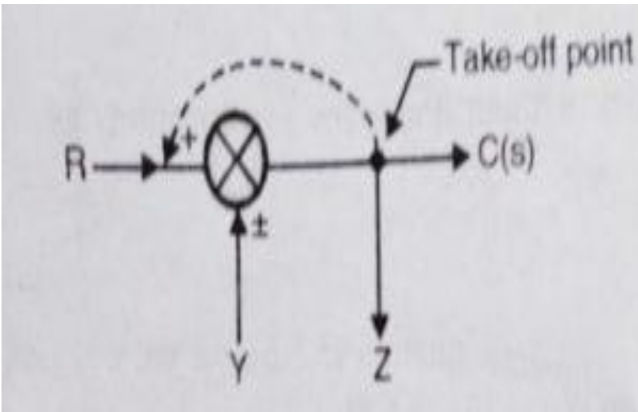
$Z = R\pm Y$ and $C=R\pm Y$

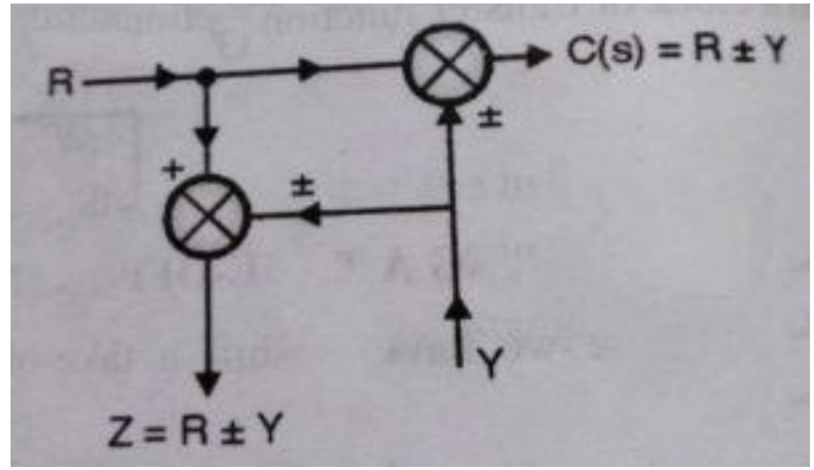
But as per equation (I) ,Z=R

In order to achieve this, we need to add one more summing point in series with take off point



Rule 10: Shifting a take-off point before a summing point





$C(s) = R \pm Y$

and $Z = R \pm Y$ (initially)….(I)

In order to satisfy equation (I), we need to add a summing point in series with the take-off point.

Procedure to solve Block Diagram Reduction Techniques

Step 1 : Reduce the blocks connected in series.

Step 2 : Reduce the blocks connected in parallel.

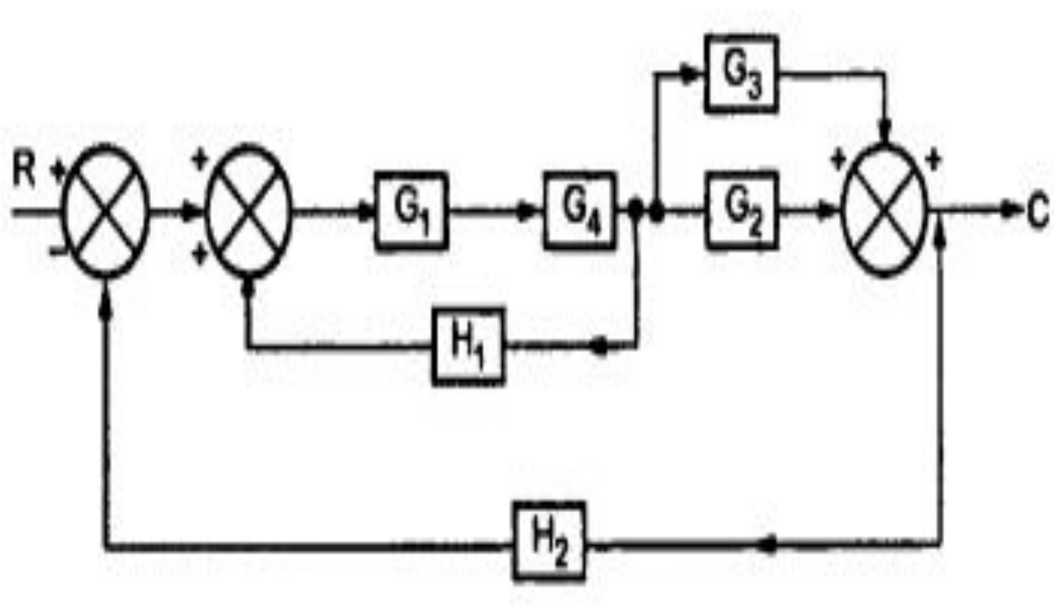
Step 3 : Reduce the minor internal feedback loops.

Step 4 : As far as possible **try to shift takeoff points towards right and summing points to the left.** Unless and until it is the requirement of problem do not use rule 10 and 11.

Step 5 : Repeat steps 1 to 4 till simple form is obtained.

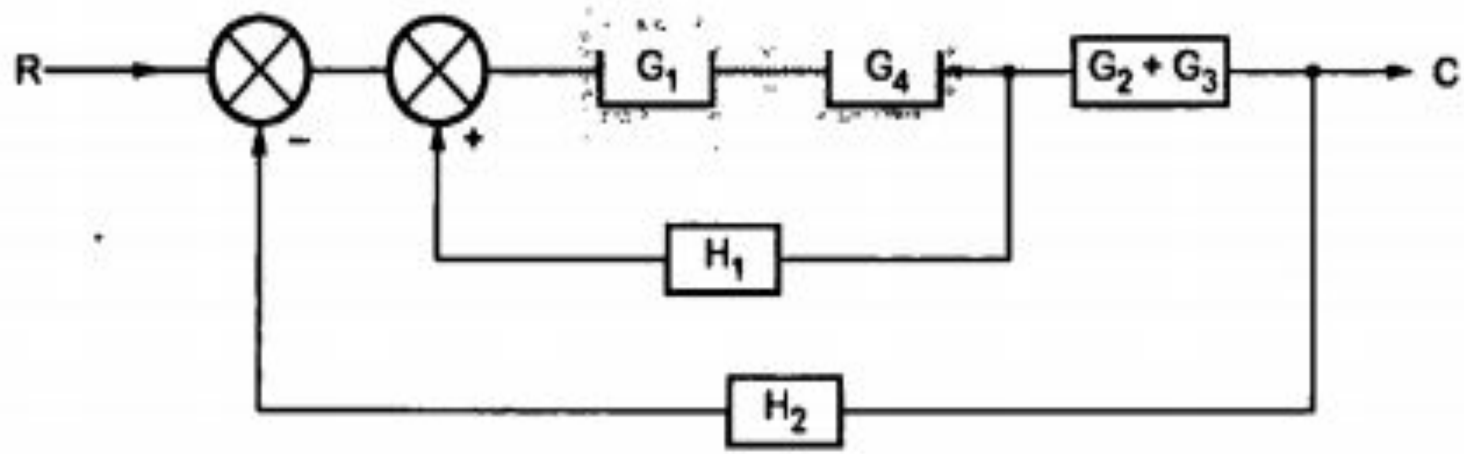
Step 6 : Using standard T.F. of simple closed loop system, obtain the closed loop T.F. $\frac{C(s)}{R(s)}$ of the overall system.

Example 1
Determine transfer function C(S)/R(S)

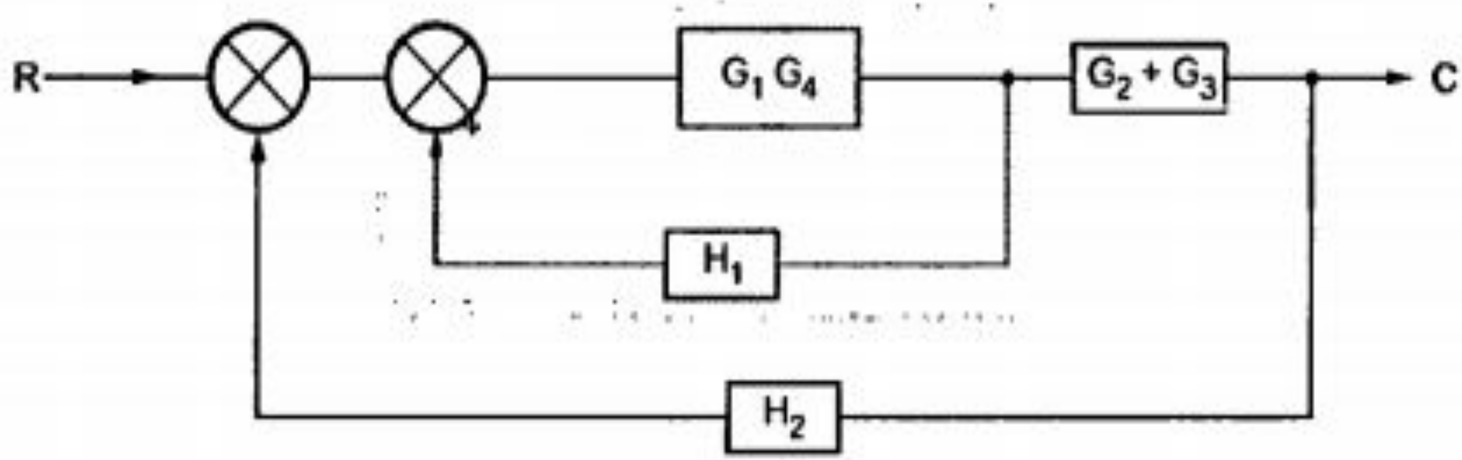


The blocks G_2 and G_3 are in parallel so combining them as (G_2+G_3)

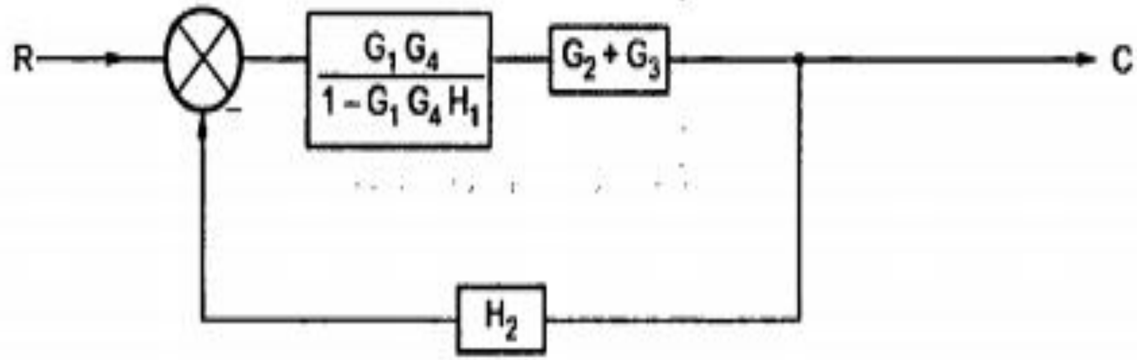
Blocks in series

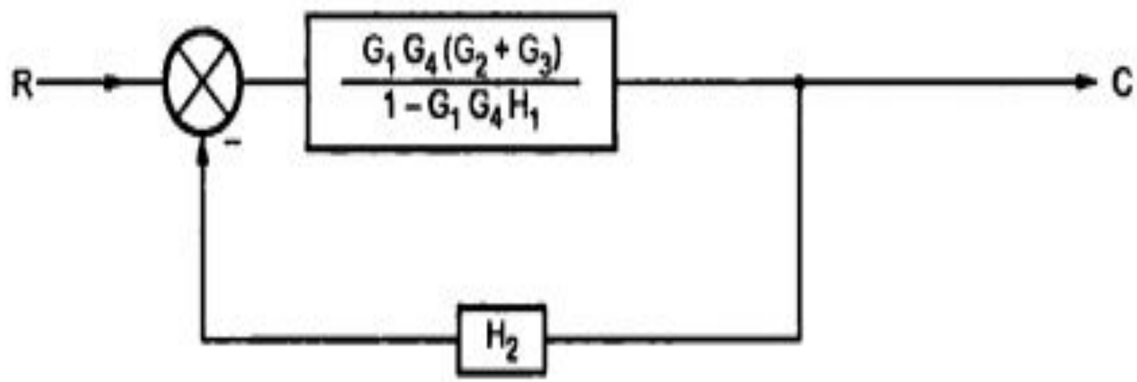


Minor feedback loop



Blocks in series



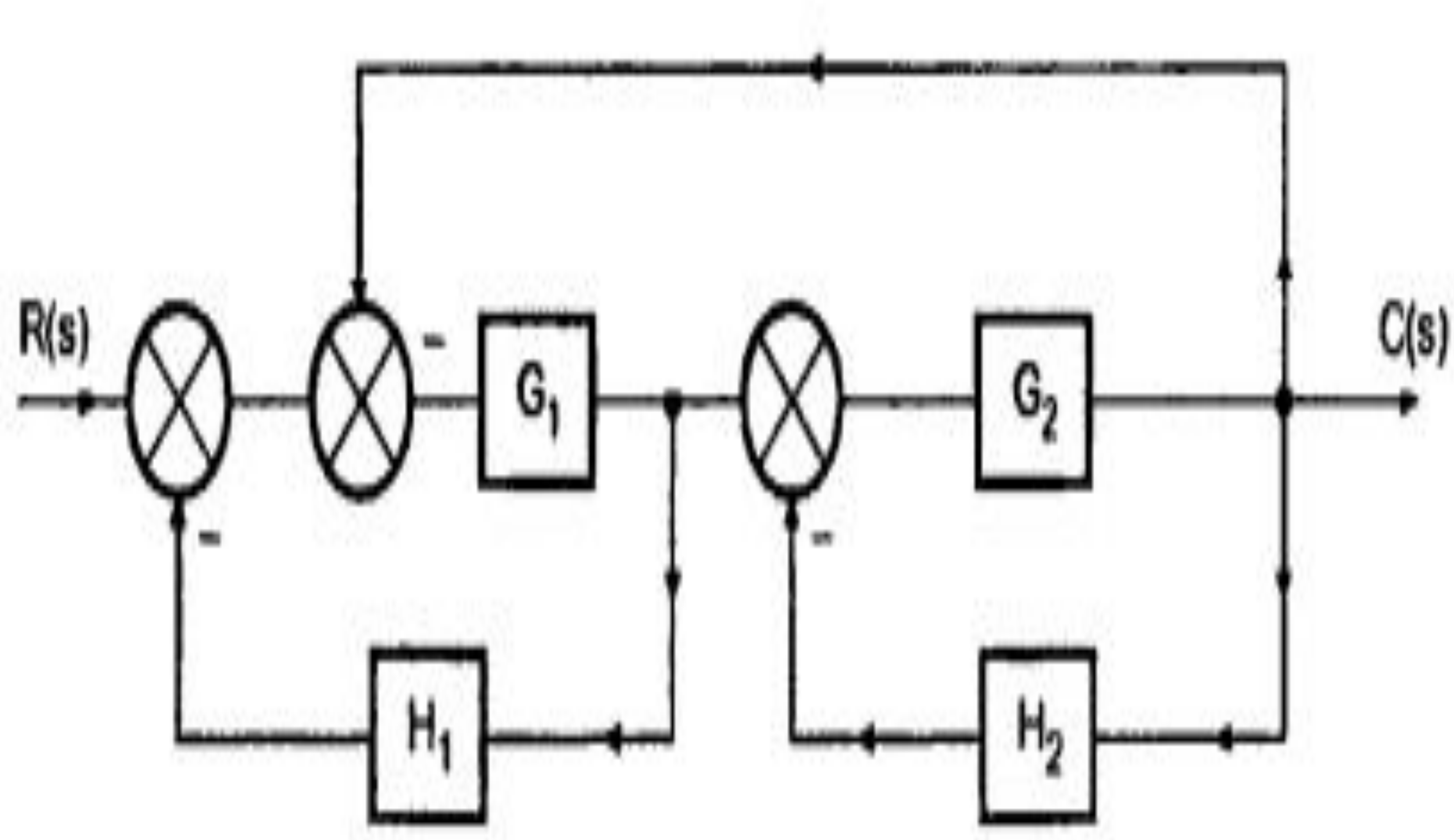


Feedback loop

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_4 (G_2 + G_3)}{1 - G_1 G_4 H_1}}{1 + \dfrac{G_1 G_4 (G_2 + G_3) H_2}{1 - G_1 G_4 H_1}}$$

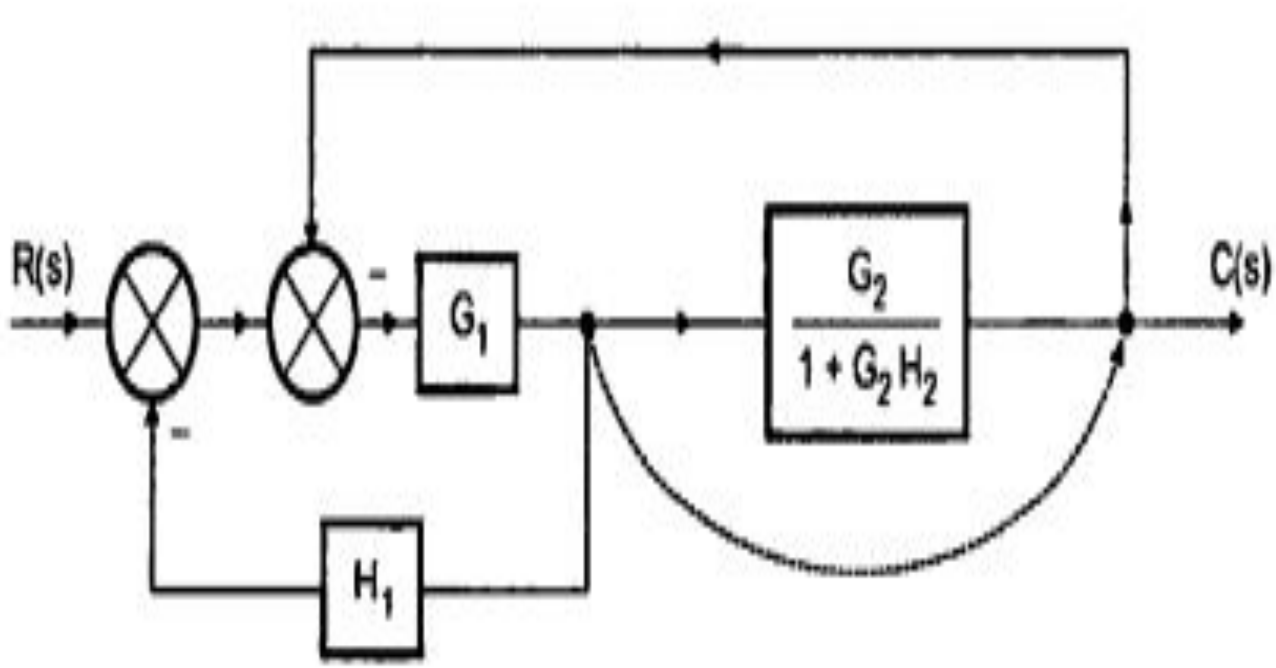
$$\frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2 + G_3)}{1 - G_1 G_4 H_1 + G_1 G_4 (G_2 + G_3) H_2}$$

Example 2



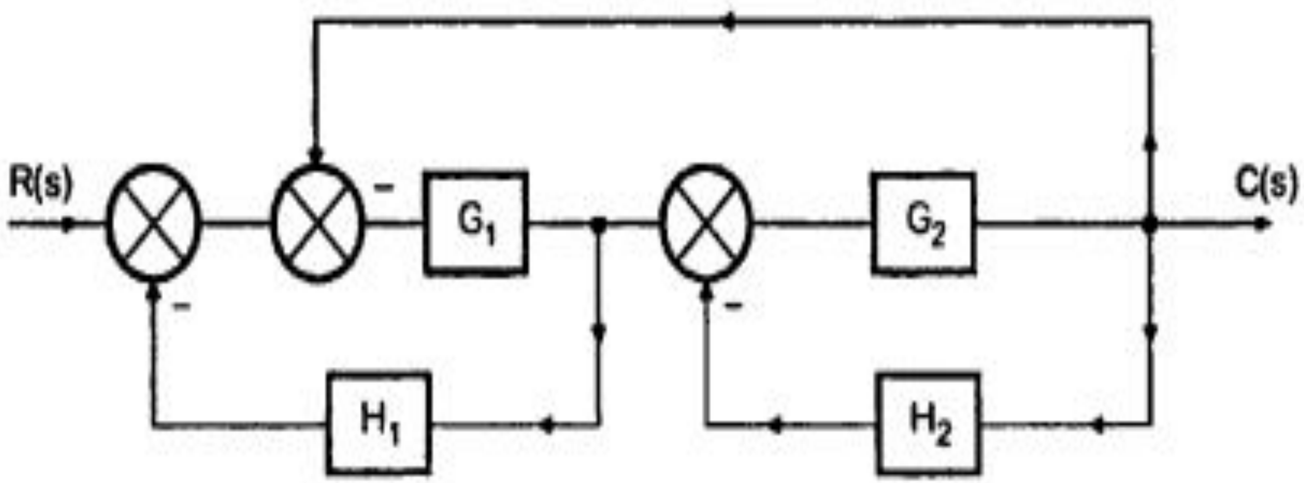
Example 2

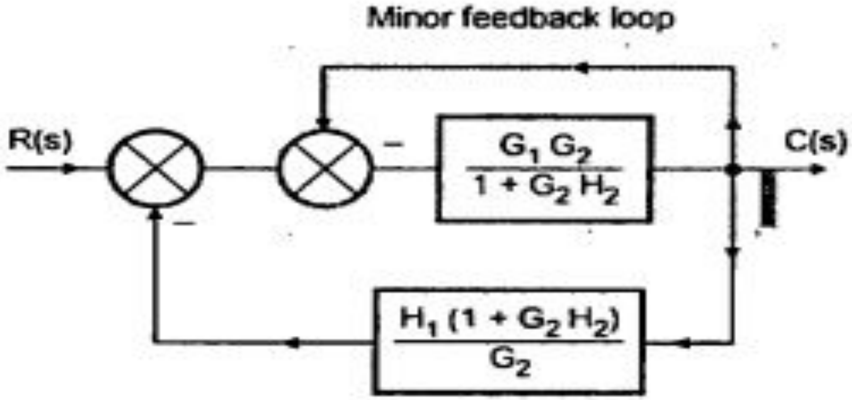
No blocks are connected in series or parallel. Blocks having transfer functions G_2 and H_2 form minor feedback loop so eliminating that loop

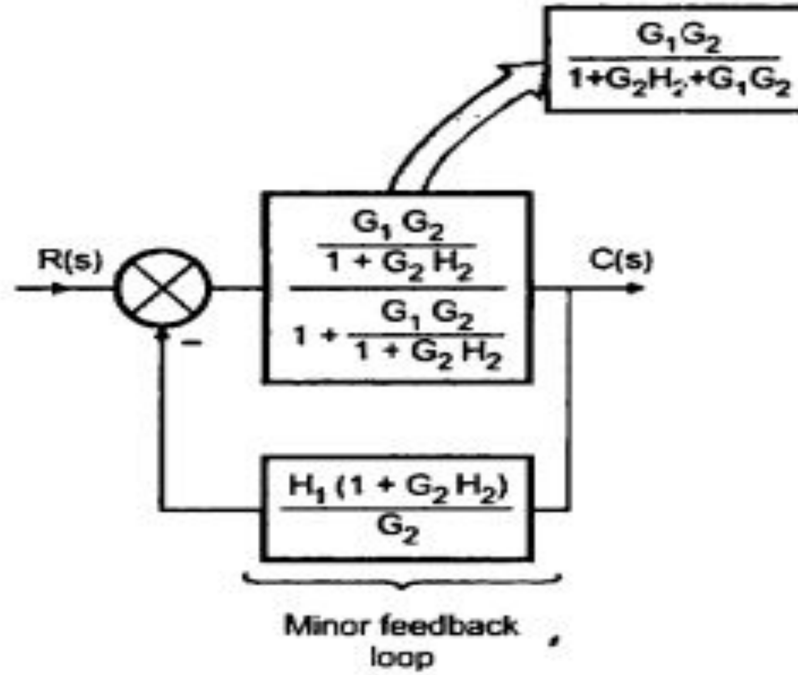


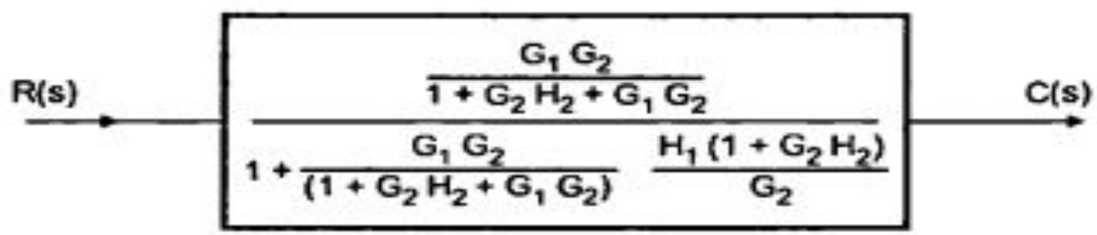
Key Point: *Always try to shift takeoff point towards right i.e. output side and summing point towards left i.e. input side.*

So shift takeoff point after G_1 to the right. While doing so, it is necessary to add a block having T.F. equal to reciprocal of the T.F. of the block after which takeoff point is to be shifted, in series with signal at that takeoff point. So in series with H_1 we get a block of $1/\left(\frac{G_2}{1+G_2H_2}\right)$ i.e $\frac{1+G_2H_2}{G_2}$ after shifting takeoff point.





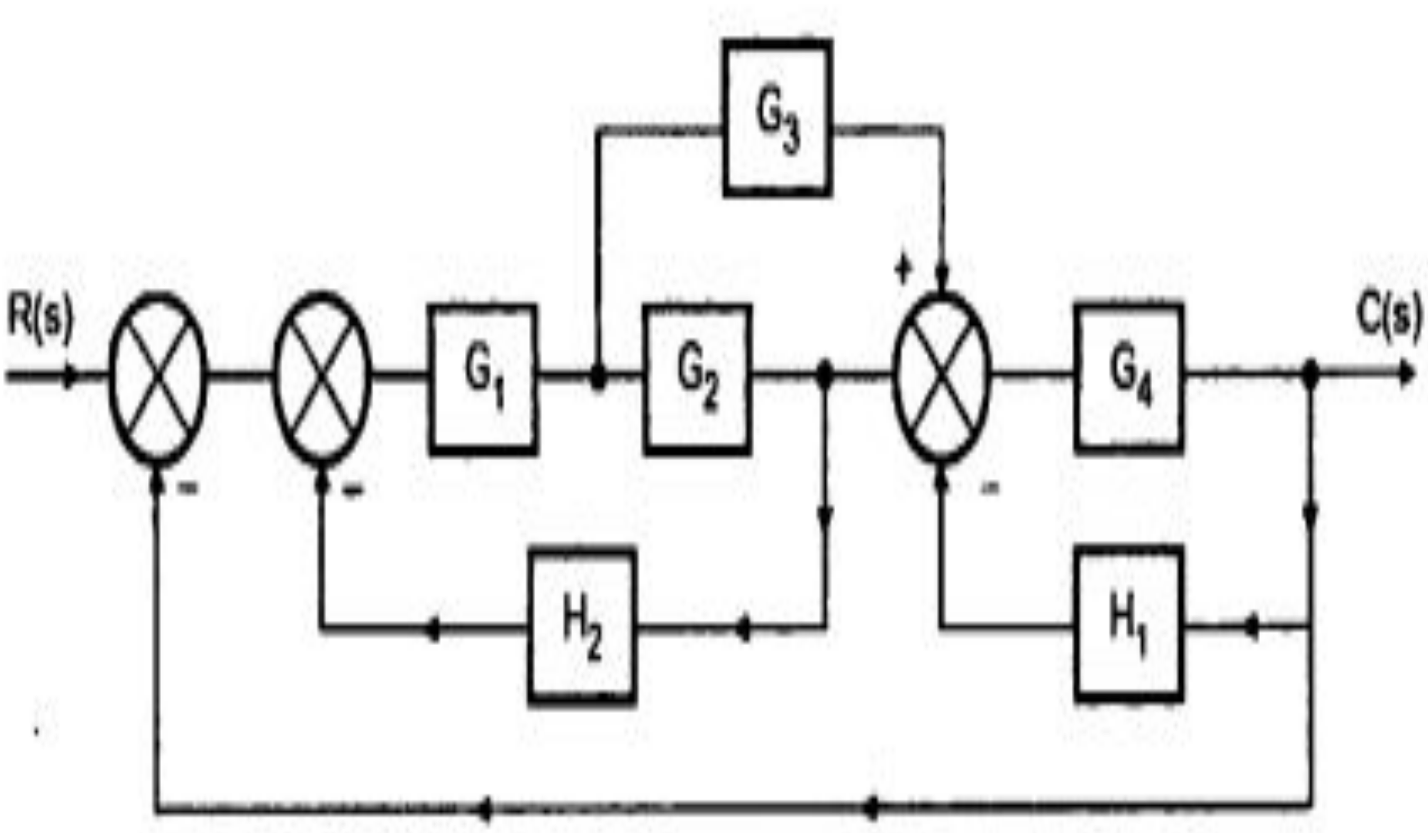




Simplifying,

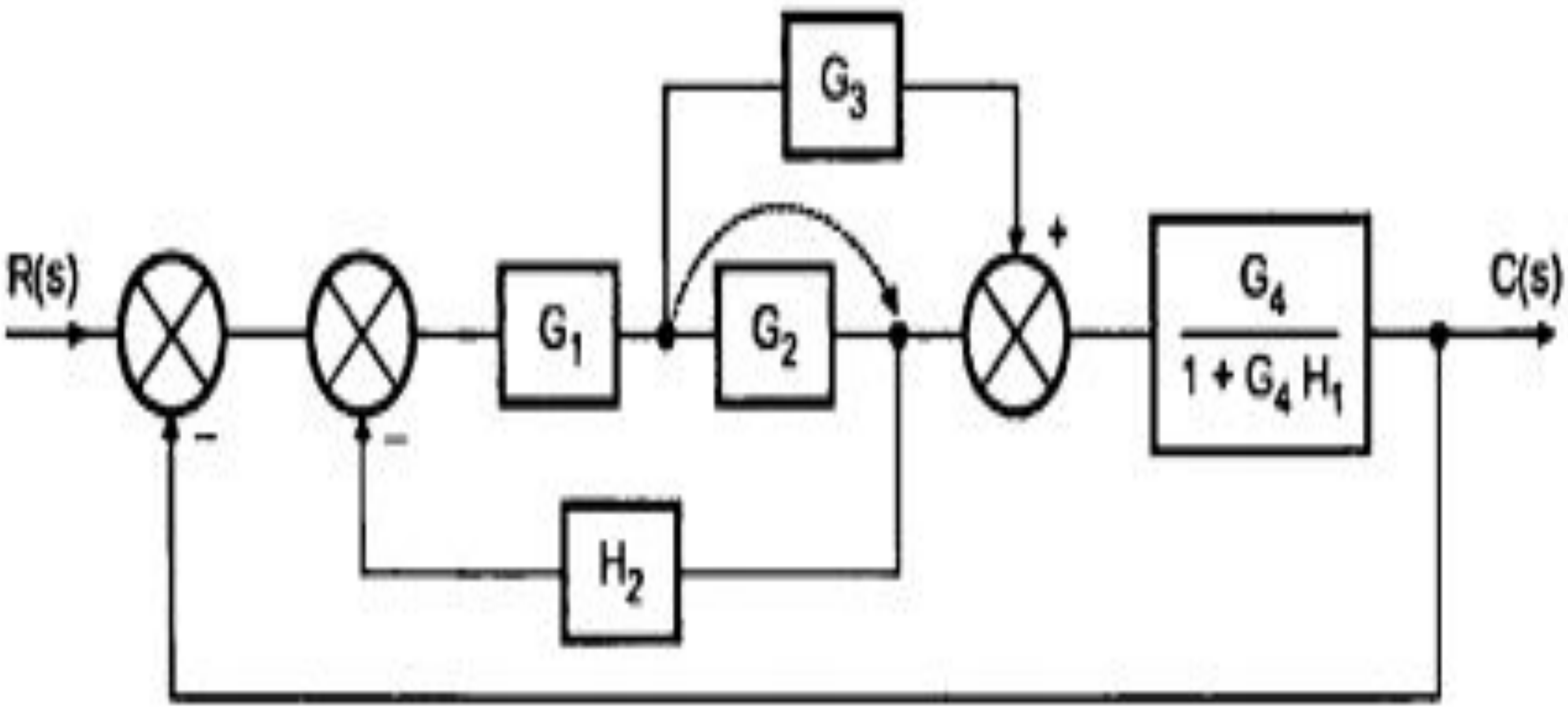
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 + G_2 H_2 + G_1 H_1 + G_1 G_2 H_1 H_2}}$$

Example 3

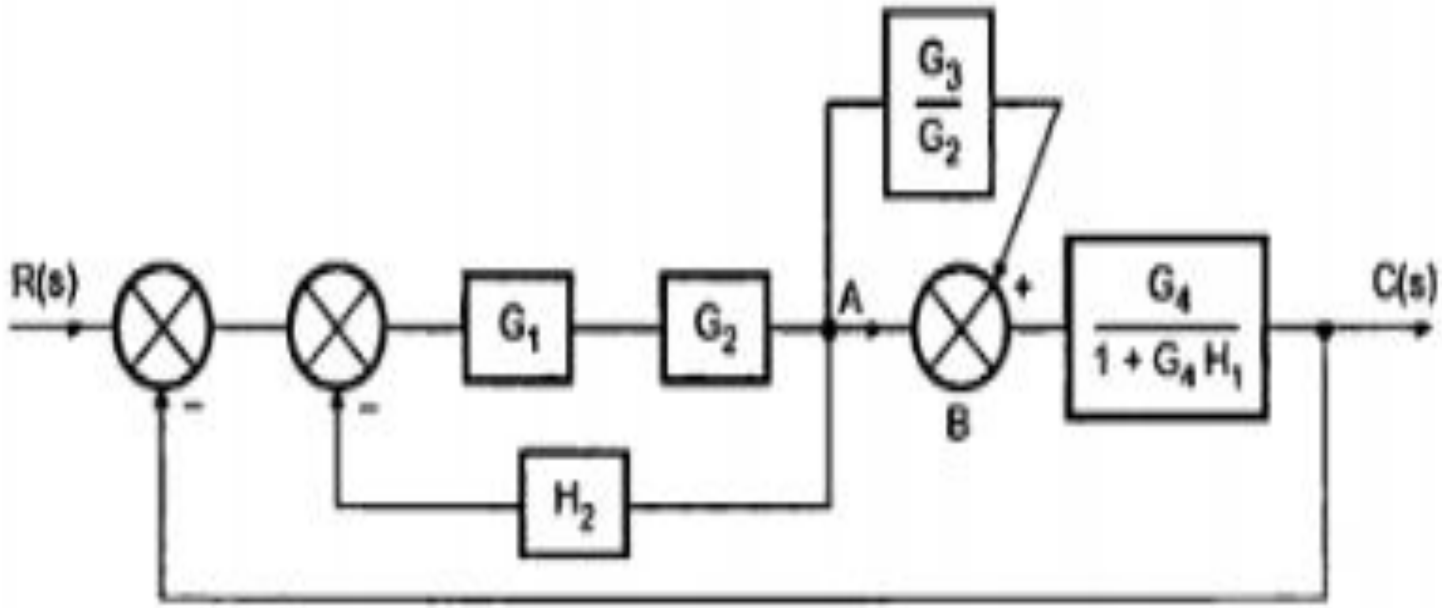


Example 3

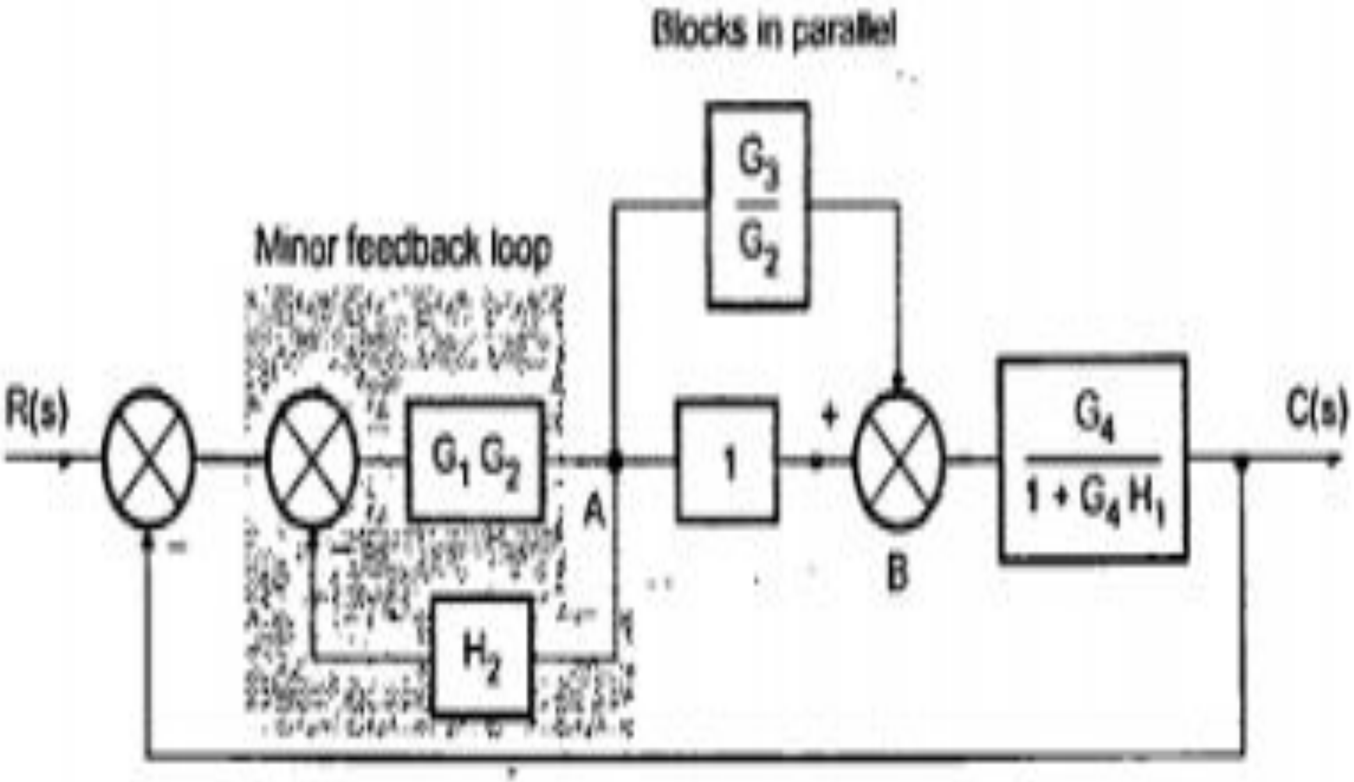
The blocks G_4 and H_1 form minor feedback loop,

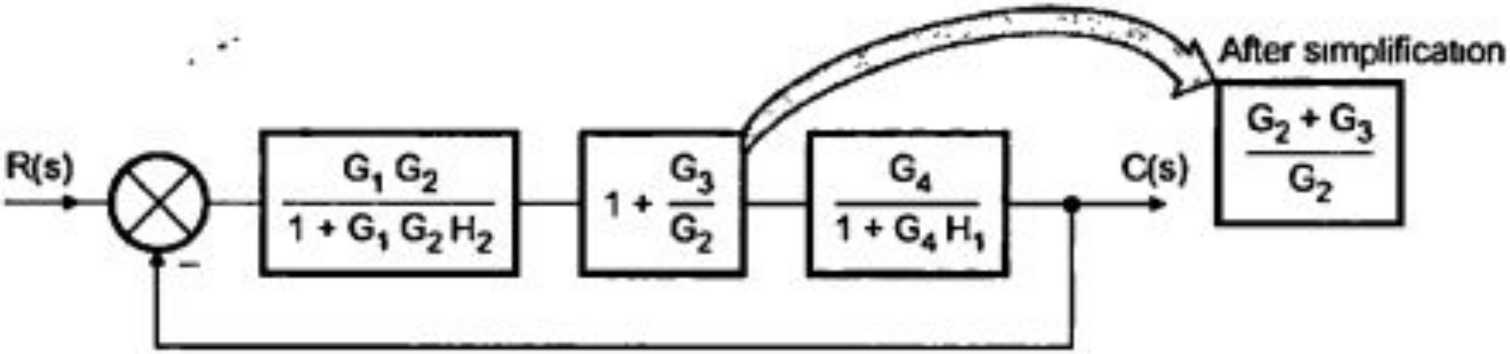


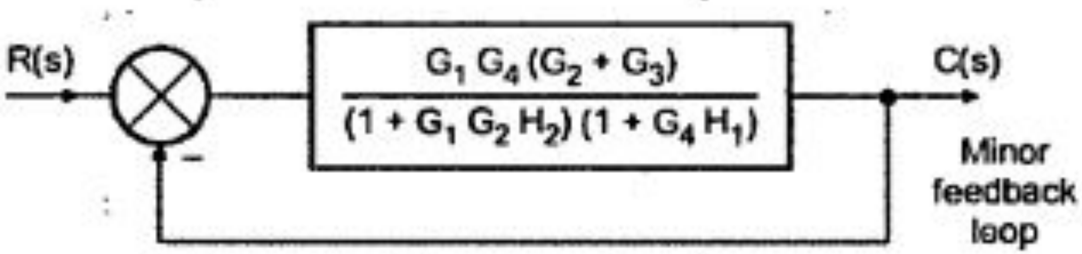
Shifting takeoff point

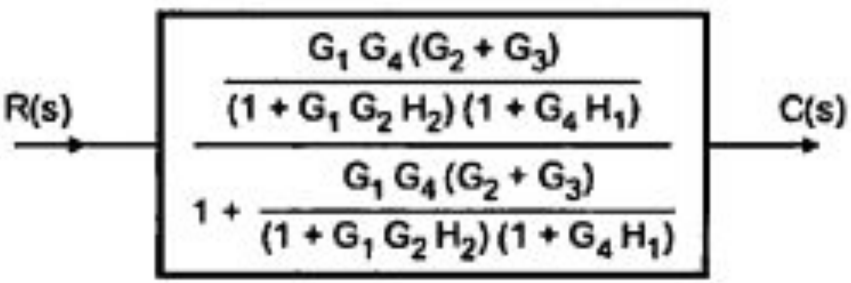


The signal from takeoff point A reaching to summing point B is without any block i.e. gain of that branch joining A to B is one. So introduce block of T.F. '1' in between A and B to avoid further confusion as shown below.









$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2 + G_3)}{1 + G_1 G_2 H_2 + G_4 H_1 + G_1 G_2 G_4 H_1 H_2 + G_1 G_4 (G_2 + G_3)}}$$