5

Graph Theory

5.1 INTRODUCTION

The purpose of network analysis is to find voltage across and current through all the elements. When the network is complicated and has a large number of nodes and closed paths, network analysis can be done conveniently by using 'Network Topology'. This theory does not make any distinction between different types of physical elements of the network but makes the study based on a geometric pattern of the network. The basic elements of this theory are nodes, branches, loops and meshes.

Node It is defined as a point at which two or more elements have a common connection.

Branch It is a line connecting a pair of nodes, the line representing a single element or series connected elements.

Loop Whenever there is more than one path between two nodes, there is a circuit or loop.

Mesh It is a loop which does not contain any other loops within it.

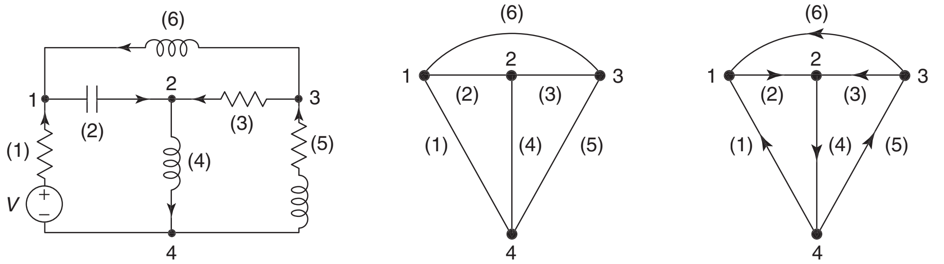
5.2 GRAPH OF A NETWORK

A linear graph is a collection of nodes and branches. The nodes are joined together by branches.

The graph of a network is drawn by first marking the nodes and then joining these nodes by lines which correspond to the network elements of each branch. All the voltage and current sources are replaced by their internal impedances. The voltage sources are replaced by short circuits as their internal impedances are zero whereas current sources are replaced by open circuits as their internal impedances are infinite. Nodes and branches are numbered. Figure 5.1 shows a network and its associated graphs.

Each branch of a graph may be given an orientation or a direction with the help of an arrow head which represents the assigned reference direction for current. Such a graph is then referred to as a directed or oriented graph.

Branches whose ends fall on a node are said to be incident at that node. Degree of a node is defined as the number of branches incident to it. Branches 2, 3 and 4 are incident at Node 2 in Fig. 5.1(c). Hence, the degree of Node 2 is 3.



(a) Network (b) Undirected graph (c) Directed or oriented graph

Fig. 5.1 *Network and its graphs*

5.3 GRAPH TERMINOLOGIES

1. Planar Graph A graph drawn on a two-dimensional plane is said to be planar if two branches do not intersect or cross at a point which is other than a node. Figure 5.2 shows such graphs.

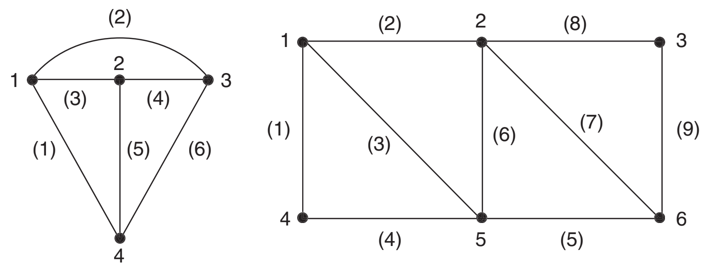


Fig. 5.2 *Planar graphs*

2. Non-planar Graph A graph drawn on a two-dimensional plane is said to be non-planar if there is intersection of two or more branches at another point which is not a node. Figure 5.3 shows non-planar graphs.

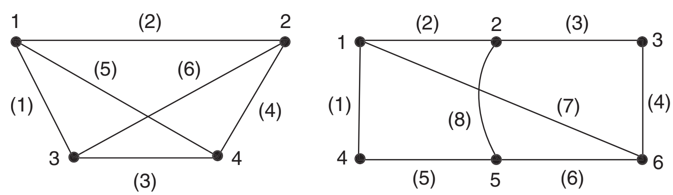


Fig. 5.3 *Non-planar graphs*

3. Sub-graph It is a subset of branches and nodes of a graph. It is a proper sub-graph if it contains branches and nodes less than those on a graph. A sub-graph can be just a node or only one branch of the graph. Figure 5.4 shows a graph and its proper sub-graph.

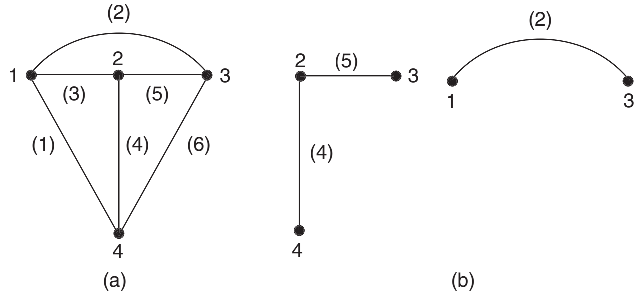


Fig. 5.4 *(a) Graph (b) Proper sub-graph*

4. Path It is an improper sub-graph having the following properties:

1. At two of its nodes called terminal nodes, there is incident only one branch of sub-graph.
2. At all remaining nodes called internal nodes, there are incident two branches of a graph.

In Fig. 5.5, branches 2, 5 and 6 together with all the four nodes, constitute a path.

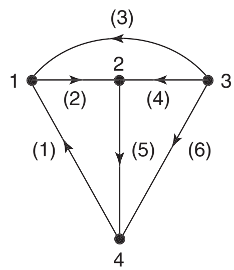


Fig. 5.5 *Path*

5. Connected Graph A graph is said to be connected if there exists a path between any pair of nodes. Otherwise, the graph is disconnected.

6. Rank of a Graph If there are n nodes in a graph, the rank of the graph is $(n - 1)$.

7. Loop or Circuit A loop is a connected sub-graph of a connected graph at each node of which are incident exactly two branches. If two terminals of a path are made to coincide, it will result in a loop or circuit. Figure 5.6 shows two loops.

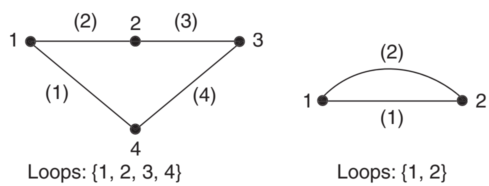


Fig. 5.6 *Loops*

Loops of a graph have the following properties:

1. There are at least two branches in a loop.
2. There are exactly two paths between any pair of nodes in a circuit.
3. The maximum number of possible branches is equal to the number of nodes.

8. Tree A tree is a set of branches with every node connected to every other node in such a way that removal of any branch destroys this property.

Alternately, a tree is defined as a connected sub-graph of a connected graph containing all the nodes of the graph but not containing any loops.

Branches of a tree are called twigs. A tree contains $(n - 1)$ twigs where n is the number of nodes in the graph. Figure 5.7 shows a graph and its trees.

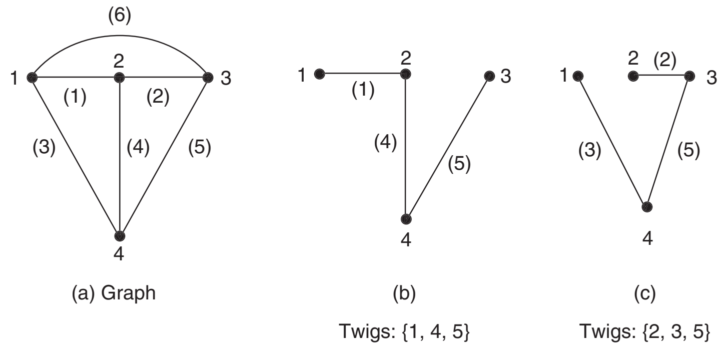


Fig. 5.7 *Graph and its trees*

Trees have the following properties:

1. There exists only one path between any pair of nodes in a tree.
2. A tree contains all nodes of the graph.
3. If n is the number of nodes of the graph, there are $(n - 1)$ branches in the tree.
4. Trees do not contain any loops.
5. Every connected graph has at least one tree.
6. The minimum terminal nodes in a tree are two.

9. Co-tree Branches which are not on a tree are called links or chords. All links of a tree together constitute the compliment of the corresponding tree and is called the co-tree.

A co-tree contains $b - (n - 1)$ links where b is the number of branches of the graph.
In Fig. 5.7 (b) and (c) the links are {2, 3, 6} and {1, 4, 6} respectively.

Example 5.1 *Draw directed graph of the networks shown in Fig. 5.8.*

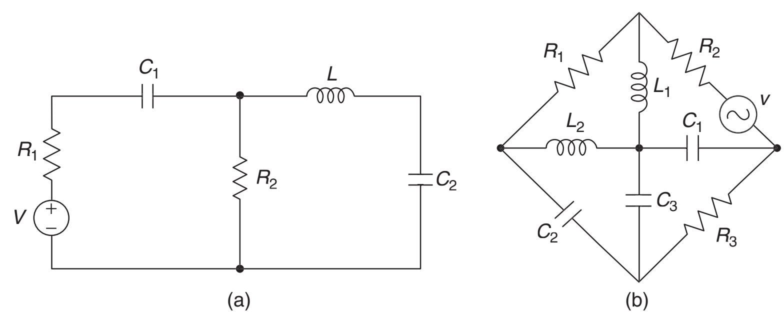


Fig. 5.8

Solution For drawing the directed graph,

1. replace all resistors, inductors and capacitors by line segments,
2. replace the voltage source by a short-circuit,
3. assume directions of branch currents, and
4. number all the nodes and branches.

The directed graph for the two networks are shown in Fig. 5.9.

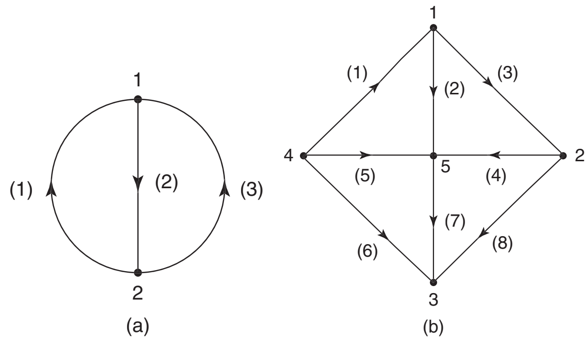


Fig. 5.9

Example 5.2 *Figure 5.10 shows a graph of the network. Show all the trees of this graph.*

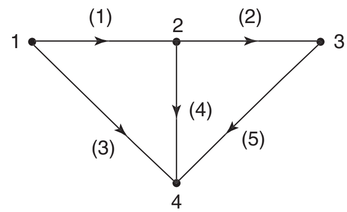


Fig. 5.10

Solution A graph has many trees. A tree is a connected sub-graph of a connected graph containing all the nodes of the graph but not containing any loops. Figure 5.11 shows various trees of the given graph.

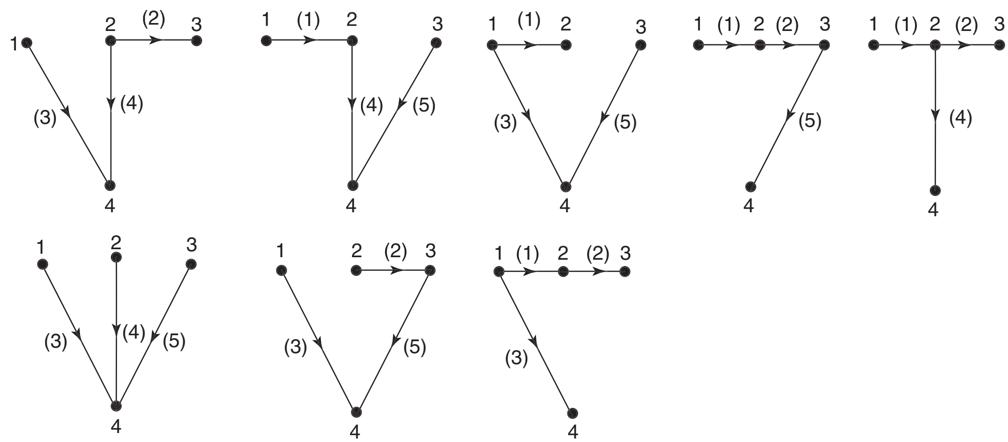


Fig. 5.11

5.4 INCIDENCE MATRIX

A linear graph is made up of nodes and branches. When a graph is given, it is possible to tell which branches are incident at which nodes and what are its orientations relative to the nodes.

5.4.1 Complete Incidence Matrix (A_a)

For a graph with n nodes and b branches, the complete incidence matrix is a rectangular matrix of order $n \times b$. Elements of this matrix have the following values:

$a_{ij} = 1$, if branch j is incident at node i and is oriented away from node i.
$= -1$, if branch j is incident at node i and is oriented towards node i.
$= 0$, if branch j is not incident at node i.

For the graph shown in Fig. 5.12, branch 1 is incident at nodes 1 and 4. It is oriented away from Node 1 and oriented towards Node 4. Hence, $a_{11} = 1$ and $a_{41} = -1$. Since branch 1 is not incident at nodes 2 and 3, $a_{21} = 0$ and $a_{31} = 0$. Similarly, other elements of the complete incidence matrix are written.

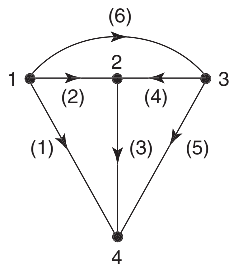


Fig. 5.12 *Graph*

Nodes ↓	Branches → 1	2	3	4	5	6
1	1	1	0	0	0	1
2	0	−1	1	−1	0	0
3	0	0	0	1	1	−1
4	−1	0	−1	0	−1	0

The complete incidence matrix is

$$A_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

It is seen from the matrix A_a that the sum of the elements in any column is zero. Hence, any one row of the complete incidence matrix can be obtained by the algebraic manipulation of other rows.

5.4.2 Reduced Incidence Matrix (A)

The reduced incidence matrix A is obtained from the complete incidence matrix A_a by eliminating one of the rows. It is also called *incidence matrix*. It is of order $(n - 1) \times b$.

Eliminating the third row of matrix A_a,

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

When a tree is selected for the graph as shown in Fig. 5.13, the incidence matrix is obtained by arranging a column such that the first $(n - 1)$ column corresponds to twigs of the tree and the last $b - (n - 1)$ branches corresponds to the links of the selected tree.

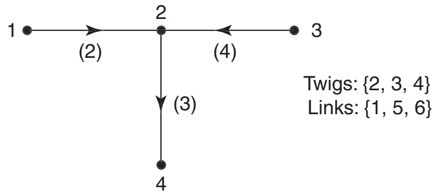


Fig. 5.13 *Tree*

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \multicolumn{3}{c}{\text{Twigs}} & \multicolumn{3}{c}{\text{Links}} \\ 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\
A = & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 \end{bmatrix}
\end{array}
$$

The matrix A can be subdivided into submatrices A_t and A_l.

$$A = [A_t : A_l]$$

Where A_t the is twig matrix and A_l is the link matrix.

5.4.3 Number of Possible Trees of a Graph

Let the transpose of the reduced incidence matrix A be A^T. It can be shown that the number of possible trees of a graph will be given by

$$\text{Number of possible trees} = |AA^T|$$

For the graphs shown in Fig. 5.12, the reduced incidence matrix is given by

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

Then transpose of this matrix will be

$$A^T = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Hence, number of all possible trees of the graph

$$AA^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$|AA^T| = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} = 3(9-1) + (1)(-3-1) - 1(1+3) = 16$$

Thus, 16 different trees can be drawn.

5.5 LOOP MATRIX OR CIRCUIT MATRIX

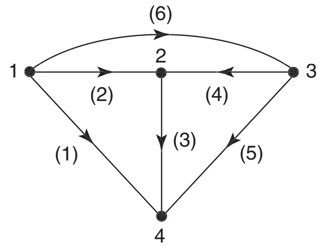
When a graph is given, it is possible to tell which branches constitute which loop or circuit. Alternately, if a loop matrix or circuit matrix is given, we can reconstruct the graph.

For a graph having n nodes and b branches, the loop matrix B_a is a rectangular matrix of order b columns and as many rows as there are loops.

Its elements have the following values:

$b_{ij} = 1$, if branch j is in loop i and their orientations coincide.
$= -1$, if branch j is in loop i and their orientations do not coincide.
$= 0$, if branch j is not in loop i.

A graph and its loops are shown in Fig. 5.14.



Loop 1: {1, 2, 3}
Loop 2: {3, 4, 5}
Loop 3: {2, 4, 6}
Loop 4: {1, 2, 4, 5}
Loop 5: {1, 5, 6}
Loop 6: {2, 3, 5, 6}
Loop 7: {1, 3, 4, 6}

Fig. 5.14 *Graph*

All the loop currents are assumed to be flowing in a clockwise direction.

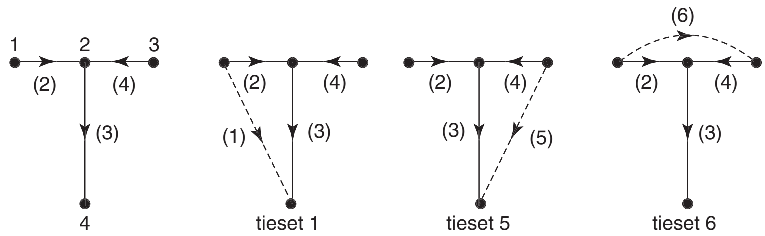
Loops ↓	Branches → 1	2	3	4	5	6
1	−1	1	1	0	0	0
2	0	0	−1	−1	1	0
3	0	−1	0	1	0	1
4	−1	1	0	−1	1	0
5	−1	0	0	0	1	1
6	0	−1	−1	0	1	1
7	−1	0	1	1	0	1

$$B_a = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \\ -1 & 1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & 0 & 1 & 1 \\ -1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

5.5.1 Fundamental Circuit (Tieset) and Fundamental Circuit Matrix

When a graph is given, first select a tree and remove all the links. When a link is replaced, a closed loop or circuit is formed. Circuits formed in this way are called fundamental circuits or f-circuits or tiesets.

Orientation of an f-circuit is given by the orientation of the connecting link. The number of f-circuits is same as the number of links for a graph. In a graph having b branches and n nodes, the number of f-circuits or tiesets will be $(b - n + 1)$. Figure 5.15 shows a tree and f-circuits (tiesets) for the graph shown in Fig. 5.14.



tieset 1: {1, 2, 3}
tieset 5: {5, 3, 4}
tieset 6: {6, 2, 4}

(a) Tree (b) *f*-circuits (tiesets)

Fig. 5.15 *Tree and f-circuits*

Here, $b = 6$ and $n = 4$.

$$\text{Number of tiesets} = b - n + 1 = 6 - 4 + 1 = 3$$

f-circuits are shown in Fig. 5.15. The orientation of each *f*-circuit is given by the orientation of the corresponding connecting link.

The branches 1, 2 and 3 are in the tieset 1. Orientation of tieset 1 is given by orientation of branch 1. Since the orientation of branch 1 coincides with orientation of tieset 1, $b_{11} = 1$. The orientations of branches 2 and 3 do not coincide with the orientation of tieset 1, Hence, $b_{12} = -1$ and $b_{13} = -1$. The branches 4, 5 and 6 are not in tieset 1. Hence, $b_{14} = 0$, $b_{15} = 0$ and $b_{16} = 0$. Similarly, other elements of the tieset matrix are written. Then, the tieset schedule will be written as

Tiesets	Branches →					
↓	1	2	3	4	5	6
1	1	−1	−1	0	0	0
5	0	0	−1	−1	1	0
6	0	−1	0	1	0	1

Hence, an *f*-circuit matrix or tieset matrix will be given as

$$B = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Usually, the *f*-circuit matrix B is rearranged so that the first $(n-1)$ columns correspond to the twigs and $b - (n-1)$ columns to the links of the selected tree.

$$\begin{array}{c} \quad \text{Twigs} \qquad\qquad \text{Links} \\ \begin{array}{cccccc} 2 & 3 & 4 & 1 & 5 & 6 \end{array} \end{array}$$

$$B = \begin{bmatrix} -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The matrix B can be partitioned into two matrices B_t and B_l.

$$B = [B_t : B_l] = [B_t : U]$$

where B_t is the twig matrix, B_l is the link matrix and U is the unit matrix.

5.5.2 Orthogonal Relationship between Matrix *A* and Matrix *B*

For a linear graph, if the columns of the two matrices A_a and B_a are arranged in the same order, it can be shown that

$$A_a B_a^T = 0$$

or

$$B_a A_a^T = 0$$

The above equations describe the orthogonal relationship between the matrices A_a and B_a.

If the reduced incidence matrix A and the *f*-circuit matrix B are written for the same tree, it can be shown that

$$A B^T = 0$$

or

$$B A^T = 0$$

These two equations show the orthogonal relationship between matrices A and B.

5.6 CUTSET MATRIX

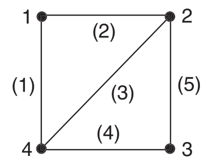


Fig. 5.16 *Graph*

Consider a linear graph. By removing a set of branches without affecting the nodes, two connected sub-graphs are obtained and the original graph becomes unconnected. The removal of this set of branches which results in cutting the graph into two parts are known as a *cutset*. The cutset separates the nodes of the graph into two groups, each being in one of the two groups.
Figure 5.16 shows a graph.

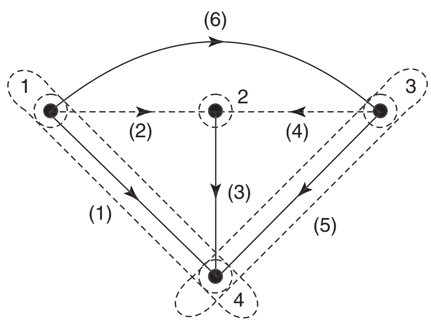
Branches 1, 3 and 4 will form a cutset. This set of branches separates the graph into two parts. One having an isolated node 4 and other part having branches 2 and 5 and nodes 1, 2 and 3.

Similarly, branches 1 and 2 will form a cutset. Each branch of the cutset has one of its terminals incident at a node in one part and its other end incident at other nodes in the other parts. The orientation of a cutset is made to coincide with orientation of defining branch.

For a graph having n nodes and b branches, the cutset matrix Q_a is a rectangular matrix of order b columns and as many rows as there are cutsets. Its elements have the following values:

q_{ij} = 1, if the branch j is in the cutset i and the orientation coincide.
= –1, if the branch j is in the cutset i and the orientations do not coincide.
= 0, if the branch j is not in the cutset i.

Figure 5.17 shows a directed graph and its cutsets.



Cutset 1: {1, 2, 6}
Cutset 2: {2, 3, 4}
Cutset 3: {3, 1, 5}
Cutset 4: {4, 5, 6}
Cutset 5: {5, 2, 3, 6}
Cutset 6: {6, 1, 3, 4}

Cutsets ↓	Branches → 1	2	3	4	5	6
1	1	1	0	0	0	1
2	0	1	–1	1	0	0
3	1	0	1	0	1	0
4	0	0	0	1	1	–1
5	0	–1	1	0	1	–1
6	1	0	1	–1	0	1

Fig. 5.17 *Directed graph*

For the cutset 2, which cuts the branches 2, 3 and 4 and is shown by a dotted circle, the entry in the cutset schedule for the branch 2 is 1, since the orientation of this cutset is given by the orientation of the branch 2 and hence it coincides. The entry for branch 3 is –1 as orientation of branch 3 is opposite to that of cutset 2,

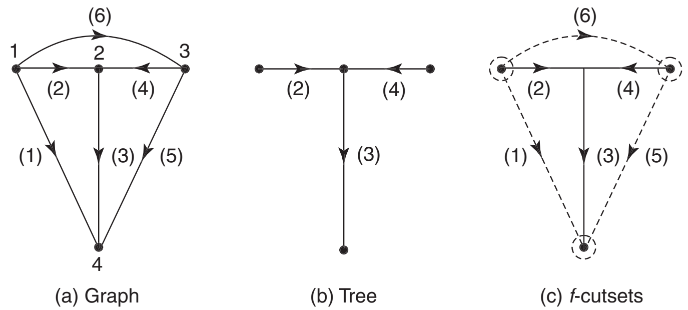
i.e., branch 2 goes into cutset while branch 3 goes out of cutset. The entry for branch 4 is 1 as the branch 2 and the branch 4 go into the cutset. Thus their orientations coincide.
Hence, the cutset matrix Q_a is given as

$$Q_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & -1 & 1 & 0 & 1 & -1 \\ 1 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

5.6.1 Fundamental Cutset and Fundamental Cutset Matrix

When a graph is given, first select a tree and note down its twigs. When a twig is removed from the tree, it separates a tree into two parts (one of the separated part may be an isolated node). Now, all the branches connecting one part of the disconnected tree to the other along with the twig removed, constitutes a cutset. This set of branches is called a fundamental cutset or *f*-cutset. A matrix formed by these *f*-cutsets is called an *f*-cutset matrix. The orientation of the *f*-cutset is made to coincide with the orientation of the defining branch, i.e., twig. The number of *f*-cutsets is the same as the number of twigs for a graph.

Figure 5.18 shows a graph, selected tree and *f*-cutsets corresponding to the selected tree.



f-cutset 2: {2, 1, 6}
f-cutset 3: {3, 1, 5}
f-cutset 4: {4, 5, 6}

Fig. 5.18 *Graph, selected tree and f-cutsets*

The branches 2, 1, and 6 are in the *f*-cutset 2. Orientation of *f*-cutset 2 is given by orientation of the branch 2 which is moving away from the *f*-cutset 2. Since the orientations of branches 2, 1 and 6 coincide with the orientation of the *f*-cutset 2, $q_{11} = 1$, $q_{12} = 1$ and $q_{16} = 0$. The branches 3, 4 and 5 are not in the *f*-cutset 2. Hence, $q_{13} = 0$, $q_{14} = 0$ and $q_{15} = 0$.

Similarly, other elements of the *f*-cutset matrix are written.

The cutset schedule is

f-cutsets ↓	Branches → 1	2	3	4	5	6
2	1	1	0	0	0	1
3	1	0	1	0	1	0
4	0	0	0	1	1	–1

Hence, the *f*-cutset matrix Q is given by

$$Q = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

The *f*-cutset matrix Q is rearranged so that the first $(n - 1)$ columns correspond to twigs and $b - (n - 1)$ columns to links of the selected tree.

$$Q = \begin{array}{c} \begin{array}{cccccc} \multicolumn{3}{c}{\text{Twigs}} & \multicolumn{3}{c}{\text{Links}} \\ 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

The matrix Q can be subdivided into matrices Q_t and Q_l.

$$Q = [Q_t : Q_l] = [U : Q_l]$$

where Q_t is the twig matrix, Q_l is the link matrix and U is the unit matrix.

5.6.2 Orthogonal Relationship between Matrix *B* and Matrix Q

For a linear graph, if the columns of two matrices B_a and Q_a are arranged in the same order, it can be shown that

$$Q_a\, B_a^T = 0$$

or

$$B_a\, Q_a^T = 0$$

If the *f*-circuit matrix B and the *f*-cutset matrix Q are written for the same selected tree, it can be shown that

$$B\, Q^T = 0$$

or

$$Q\, B^T = 0$$

These two equations show the orthogonal relationship between matrices A and B.

5.7 RELATIONSHIP AMONG SUBMATRICES OF A, B AND Q

Arranging the columns of matrices A, B and Q with twigs for a given tree first and then the links, we get the partitioned forms as

$$A = [A_t : A_l]$$
$$B = [B_t : B_l] = [B_t : U]$$
$$Q = [Q_t : Q_l] = [U : Q_l]$$

From the orthogonal relation, $AB^T = 0$,

$$AB^T = [A_t : A_l] \begin{bmatrix} B_t^T \\ \cdots \\ B_l^T \end{bmatrix}$$

$$A_t B_t^T + A_l B_l^T = 0$$

$$A_t B_t^T = -A_l B_l^T$$

Since A_t is non-singular, i.e., $|A| \neq 0$, A_t^{-1} exists.
Premultiplying with A_t^{-1},

$$B_t^T = -A_t^{-1} A_l B_l^T$$

$$B_t = -B_l (A_t^{-1} . A_l)^T$$

Since B_l is a unit matrix

$$B_t = -(A_t^{-1} . A_l)^T$$

Hence, matrix B is written as

$$B = [-(A_t^{-1} \cdot A_l)^T : U] \qquad \ldots(5.1)$$

We know that

$$AB^T = 0$$

$$A_l B_l^T = -A_t B_t^T$$

Postmultiplying with $(B_l^T)^{-1}$,

$$A_l = -A_t B_t^T (B_l^T)^{-1} = -A_t B_t^T (B_l^{-1})^T = -A_t (B_l^{-1} \cdot B_t)^T$$

Hence matrix A can be written as

$$\begin{aligned} A &= [A_l : -A_t (B_l^{-1} \cdot B_t)^T] \\ &= A_t [U : -(B_l^{-1} \cdot B_t)^T] \end{aligned} \qquad \ldots(5.2)$$

Similarly we can prove that

$$Q = [U : -(B_l^{-1} \cdot B_t)^T] \qquad \ldots(5.3)$$

From Eqs (5.2) and (5.3), we can write

$$A = A_t Q$$

$$Q = A_t^{-1} A = A_t^{-1} [A_t : A_l] = [U : A_t^{-1} A_l]$$

We have shown that

$$B_t = -(A_t^{-1} \cdot A_l)^T$$

$$B_t^T = -(A_t^{-1} \cdot A_l)$$

Hence, Q can be written as

$$Q = [U : -B_t^T]$$

$$Q_1 = -B_t^T$$

Example 5.3 *For the circuit shown in Fig. 5.19, draw the oriented graph and write the (a) incidence matrix, (b) tieset matrix, and (c) f-cutset matrix.*

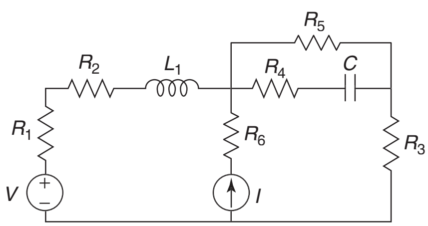


Fig. 5.19

Solution For drawing the oriented graph,

1. replace all resistors, inductors and capacitors by line segments,
2. replace the voltage source by short circuit and the current source by an open circuit,
3. assume the directions of branch currents arbitrarily, and
4. number all the nodes and branches.

The oriented graph is shown in Fig. 5.20.

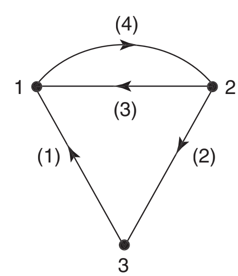


Fig. 5.20

(a) Incidence Matrix (A)

Nodes ↓	Branches → 1	2	3	4
1	−1	0	−1	1
2	0	1	1	−1
3	1	−1	0	0

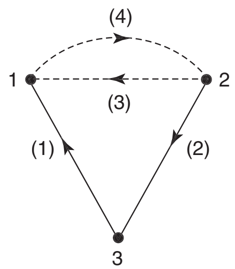
$$A_a = \begin{bmatrix} -1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

Eliminating the third row from the matrix A_a, we get the incidence matrix A.

$$A = \begin{bmatrix} -1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

(b) Tieset Matrix (B)

The oriented graph, selected tree and tiesets are shown in Fig. 5.21.



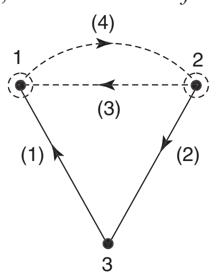
Links: {3, 4}
Tieset 3: {3, 1, 2}
Tieset 4: {4, 1, 2}

$$B = \begin{array}{c} \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Fig. 5.21

(c) f-cutset Matrix (Q)

The oriented graph, selected tree and f-cutsets are shown in Fig. 5.22.



Twigs: {1, 2}
f-cutset 1: {1, 3, 4}
f-cutset 2: {2, 3, 4}

$$Q = \begin{array}{c} \\ 1 \\ 2 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \end{array}$$

Fig. 5.22

Example 5.4 *For the network shown in Fig. 5.23, draw the oriented graph and write the (a) incidence matrix, (b) tieset matrix, and (c) f-cutset matrix.*

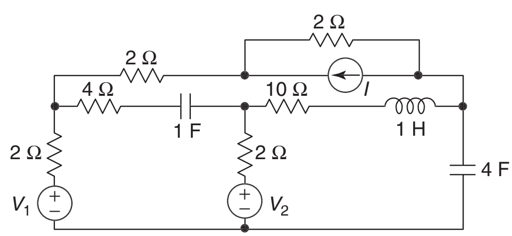


Fig. 5.23

Solution For drawing the oriented graph,

1. replace all resistors, inductors and capacitors by line segments,
2. replace all voltage sources by short circuits and current source by an open circuit,
3. assume directions of branch currents arbitrarily, and
4. number all the nodes and branches.

The oriented graph is shown in Fig. 5.24.

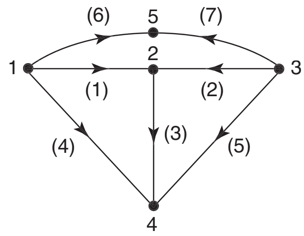


Fig. 5.24

(a) Incidence Matrix (A)

Nodes ↓	Branches → 1	2	3	4	5	6	7
1	1	0	0	1	0	1	0
2	−1	−1	1	0	0	0	0
3	0	1	0	0	1	0	1
4	0	0	−1	−1	−1	0	0
5	0	0	0	0	0	−1	−1

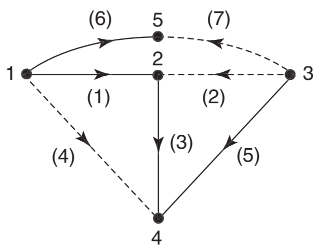
$$A_a = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

Eliminating the last row from the matrix A_a, we get the incidence matrix A.

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 \end{bmatrix}$$

(b) Tieset Matrix (B)

The oriented graph, selected tree and tiesets are shown in Fig. 5.25.



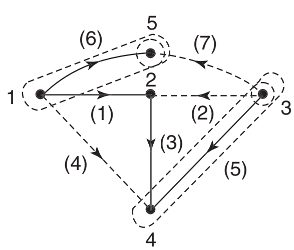
Links: {2, 4, 7}
Tieset 2: {2, 3, 5}
Tieset 4: {4, 1, 3}
Tieset 7: {7, 6, 1, 3, 5}

$$B = \begin{array}{c} \\ 2 \\ 4 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & -1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.25

(c) f-cutset Matrix (Q)

The oriented graph, selected tree and f-cutsets are shown in Fig. 5.26.



Twigs: {1, 3, 5, 6}
f-cutset 1: {1, 4, 7}
f-cutset 3: {3, 4, 7, 2}
f-cutset 5: {5, 2, 7}
f-cutset 6: {6, 7}

$$Q = \begin{array}{c} \\ 1 \\ 3 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.26

Example 5.5 *For the circuit shown in Fig. 5.27, draw the oriented graph and write (a) incidence matrix, (b) tieset matrix, and (c) cutset matrix.*

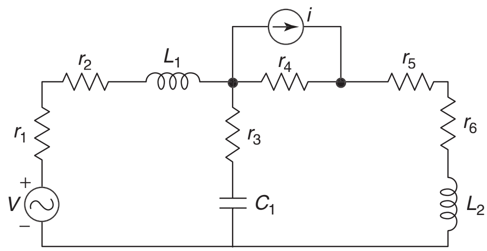


Fig. 5.27

Solution For drawing the oriented graph,

1. replace all resistors, inductors and capacitors by line segments,
2. replace voltage source by short circuit and current source by an open circuit,
3. assume directions of branch currents arbitrarily, and
4. number the nodes and branches.

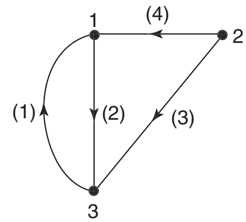


Fig. 5.28

The oriented graph is shown in Fig. 5.28.

(a) Incidence Matrix (A)

Nodes ↓	Branches → 1	2	3	4
1	–1	1	0	–1
2	0	0	1	1
3	1	–1	–1	0

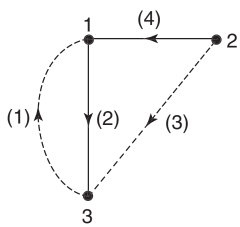
$$A_a = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$$

Eliminating the third row from the matrix A_a, we get the incidence matrix A.

$$A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(b) Tieset Matrix (B)

The oriented graph, selected tree and tiesets are shown in Fig. 5.29.



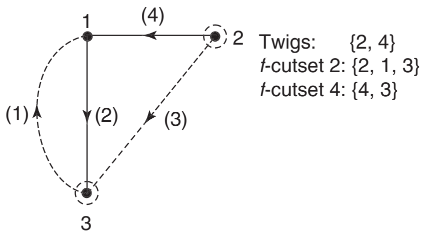
Links: {1, 3}
Tieset 1: {1, 2}
Tieset 3: {3, 2, 4}

$$B = \begin{array}{c} \\ 1 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

Fig. 5.29

(c) f-cutset Matrix (Q)

The oriented graph, selected tree and f-cutsets are shown in Fig. 5.30.



$$Q = \begin{array}{c} \\ 2 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.30

Example 5.6 *For the circuit shown in Fig. 5.31, (a) draw its graph, (b) draw its tree, and (c) write the fundamental cutset matrix.*

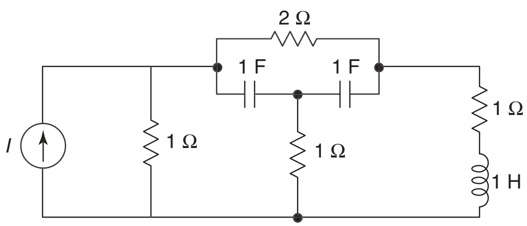


Fig. 5.31

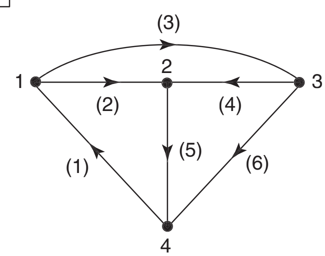


Fig. 5.32

Solution

(a) For drawing the oriented graph,

1. replace all resistors, inductors and capacitors by line segments,
2. replace the current source by an open circuit,
3. assume directions of branch currents, and
4. number all the nodes and branches.

The oriented graph is shown in Fig. 5.32.

(b) Tree

The oriented graph and its selected tree are shown in Fig. 5.33.

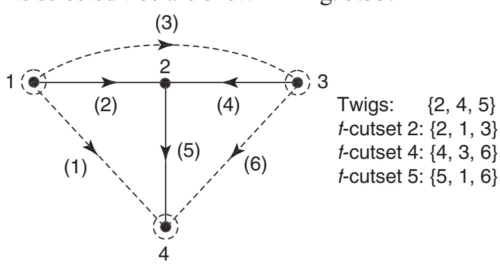


Fig. 5.33

(c) Fundamental Cutset Matrix (Q)

$$Q = \begin{array}{c} \\ 2 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Example 5.7 *The graph of a network is shown in Fig. 5.34. Write the (a) incidence matrix, (b) tieset matrix, and (c) f-cutset matrix.*

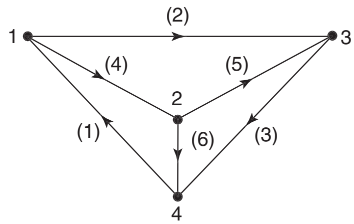


Fig. 5.34

Solution

(a) Incidence Matrix (A)

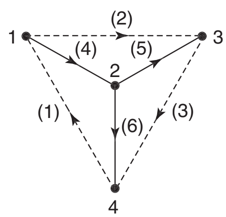
$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

The incidence matrix A is obtained by eliminating any row from the matrix A_a.

$$A = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

(b) Tieset, Matrix (B)

The oriented graph, selected tree and tiesets are shown in Fig. 5.35.



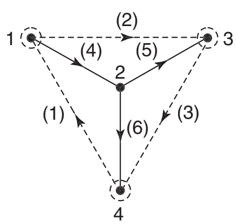
Links: {1, 2, 3}
Tieset 1: {1, 4, 6}
Tieset 2: {2, 4, 5}
Tieset 3: {3, 5, 6}

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

Fig. 5.35

(c) f-cutset Matrix (Q)

The oriented graph, selected tree and f-cutsets are shown in Fig. 5.36.



Twigs: {4, 5, 6}
f-cutset 4: {4, 1, 2}
f-cutset 5: {5, 2, 3}
f-cutset 6: {6, 1, 3}

$$Q = \begin{array}{c} \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Fig. 5.36

Example 5.8 *For the graph shown in Fig. 5.37, write the incidence matrix, tieset matrix and f-cutset matrix.*

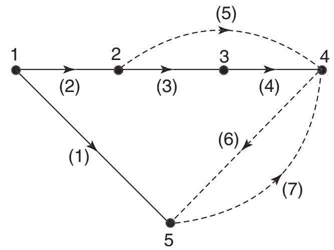


Fig. 5.37

Solution

(a) Incidence Matrix (A)

$$A_a = \begin{array}{cc} & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

The incidence matrix A is obtained by eliminating any row from the matrix A_a.

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & -1 \end{bmatrix}$$

(b) Tieset Matrix (B)

Links: {5, 6, 7}
Tieset 5: {5, 3, 4}
Tieset 6: {6, 1, 2, 3, 4}
Tieset 7: {7, 1, 2, 3, 4}

$$B = \begin{array}{cc} & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 5 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(c) f-cutset Matrix (Q)

The oriented graph, selected tree and f-cutsets are shown in Fig. 5.38.

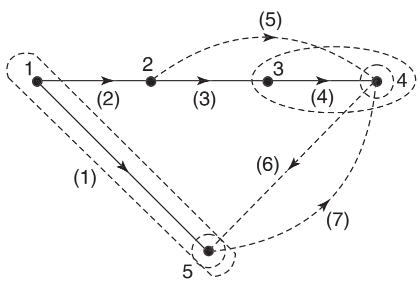


Fig. 5.38

Twigs: {1, 2, 3, 4}
f-cutset 1: {1, 6, 7}
f-cutset 2: {2, 6, 7}
f-cutset 3: {3, 5, 6, 7}
f-cutset 4: {4, 5, 6, 7}

$$Q = \begin{array}{cc} & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 \end{bmatrix} \end{array}$$

Example 5.9 *For the graph shown in Fig. 5.39, write the incidence matrix, tieset matrix and f-cutset matrix.*

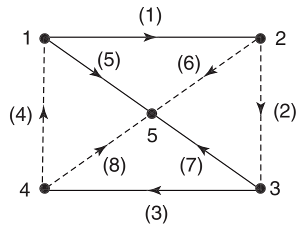


Fig. 5.39

Solution

(a) Incidence Matrix (A)

$$A_a = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

The incidence matrix is obtained by eliminating any one row.

$$A = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Tieset Matrix (B)

Links: $\{2, 4, 6, 8\}$
Tieset 2: $\{2, 7, 5, 1\}$
Tieset 4: $\{4, 5, 7, 3\}$
Tieset 6: $\{6, 5, 1\}$
Tieset 8: $\{8, 7, 3\}$

$$B = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \\ 8 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

(c) *f*-cutset Matrix (Q)

The oriented graph, selected tree and *f*-cutsets are shown in Fig. 5.40.

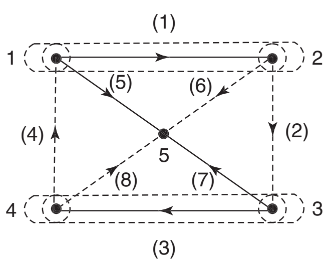


Fig. 5.40

Twigs: {1, 3, 5, 7}
f-cutset 1: {1, 6, 2}
f-cutset 3: {3, 4, 8}
f-cutset 5: {5, 4, 6, 2}
f-cutset 7: {7, 2, 8, 4}

$$Q = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Example 5.10 *How many trees are possible for the graph of the network of Fig. 5.41.*

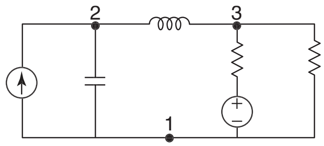


Fig. 5.41

Solution To draw the graph,

1. replace all resistors, inductors and capacitors by line segments,
2. replace voltage source by short circuit and current source by an open circuit,
3. assume directions of branch currents arbitrarily, and
4. number all the nodes and branches.

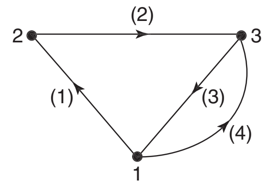


Fig. 5.42

The oriented graph is shown in Fig. 5.42.
The complete Incidence Matrix (A_a) is written as

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

The reduced incidence matrix A is obtained by eliminating the last row from matrix A_a.

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

The number of possible trees = $\left|AA^T\right| = \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} = 6 - 1 = 5.$

Example 5.11 *Draw the oriented graph from the complete incidence matrix given below;*

Nodes ↓ \ Branches →	1	2	3	4	5	6	7	8
1	1	0	0	0	1	0	0	1
2	0	1	0	0	–1	1	0	0
3	0	0	1	0	0	–1	1	–1
4	0	0	0	1	0	0	–1	0
5	–1	–1	–1	–1	0	0	0	0

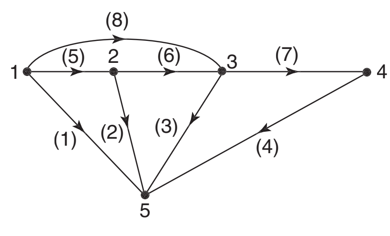


Fig. 5.43

Solution First, note down the nodes 1, 2, 3, 4, 5 as shown in Fig. 5.43. From the complete incidence matrix, it is clear that the branch number 1 is between nodes 1 and 5 and it is going away from node 1 and towards node 5 as the entry against node 1 is 1 and that against 5 is –1. Hence, connect the nodes 1 and 5 by a line, point the arrow towards 5 and call it branch 1 as shown in Fig. 5.43. Similarly, draw the other oriented branches.

Example 5.12 *The reduced incidence matrix of an oriented graph is given below. Draw the graph.*

$$A = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution First, writing the complete incidence matrix from the matrix A such that the sum of all entries in each column of A_a will be zero, we have

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

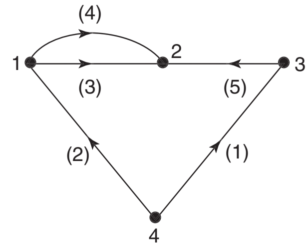


Fig. 5.44

Now, the oriented graph can be drawn with matrix A_a as shown in Fig. 5.44.

Example 5.13 *The reduced incidence matrix of an oriented graph is*

$$A = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Draw the graph. (b) How many trees are possible for this graph? (c) Write the tieset and cutset matrices.

Solution

(a) First, writing the complete incidence matrix A_a such that the sum of all the entries in each column of A_a is zero, we have

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

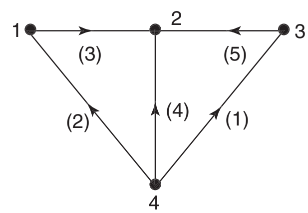


Fig. 5.45

Now, the oriented graph can be drawn with the matrix A_a, as shown in Fig. 5.45.

(b) The number of possible trees $= |AA^T|$

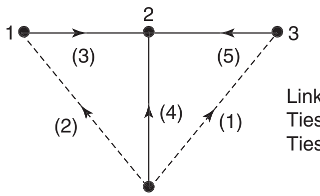
$$AA^T = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\left|AA^T\right| = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 2(6-1)+1(-2) = 8$$

The number of possible trees = 8.

(c) Tieset Matrix (B)

The oriented graph, selected tree and tiesets are shown in Fig. 5.46.



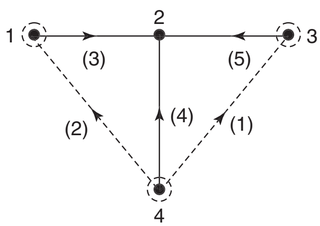
Links: {1, 2}
Tieset 1: {1, 4, 5}
Tieset 2: {2, 3, 4}

$$B = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 & 0 \end{bmatrix} \end{array}$$

Fig. 5.46

f-cutset Matrix (Q)

The oriented graph, selected tree and f-cutsets are shown in Fig. 5.47



Twigs: {3, 4, 5}
f-cutset 3: {3, 2}
f-cutset 4: {4, 2, 1}
f-cutset 5: {5, 1}

$$Q = \begin{array}{c} \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Fig. 5.47

Example 5.14 *The fundamental cutset matrix of a network is given as follows;*

Twigs			*Links*		
a	*c*	*e*	*b*	*d*	*f*
1	0	0	1	0	1
0	1	0	0	1	1
0	0	1	1	1	1

Draw the oriented graph.

Solution No. of links $l = b - n + 1$

No. of nodes $n = b - l + 1 = 6 - 3 + 1 = 4$

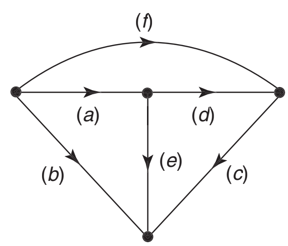
f-cutsets are written as,

f-cutsets a: $\{a, b, f\}$

f-cutsets c: $\{c, d, f\}$

f-cutsets e: $\{e, b, d, f\}$

The oriented graph is drawn as shown in Fig. 5.48.



Twigs: {a, c, e}
Links: {b, d, f}

Fig. 5.48

Example 5.15 *Draw the oriented graph of a network with the f-cutset matrix as shown below:*

Twigs				*Links*		
1	2	3	4	5	6	7
1	0	0	0	–1	0	0
0	1	0	0	1	0	1
0	0	1	0	0	1	1
0	0	0	1	0	1	0

Solution No. of links $l = b - n + 1$

No. of nodes $n = b - l + 1 = 7 - 3 + 1 = 5$

f-cutsets are written as

f-cutset 1: {1, 5}

f-cutset 2: {2, 5, 7}

f-cutset 3: {3, 6, 7}

f-cutset 4: {4, 6}

Then oriented graph can be drawn as shown in Fig. 5.49.

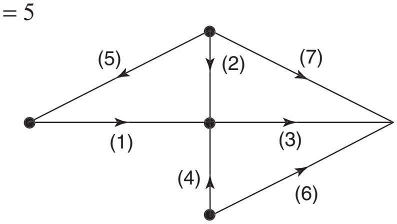


Fig. 5.49

5.8 KIRCHHOFF'S VOLTAGE LAW

KVL states that if v_k is the voltage drop across the k^{th} branch, then

$$\sum v_k = 0$$

the sum being taken over all the branches in a given loop. If l is the number of loops or *f*-circuits, then there will be l number of KVL equations, one for each loop. The KVL equation for the *f*-circuit or loop 'l' can be written as

$$\sum_{k=1}^{b} b_{ik}\, v_k = 0 \qquad (k = 1, 2, \ldots, l)$$

where b_{ik} is the elements of the tieset matrix B, b being the number of branches. The set of l KVL equations can be written in matrix form.

$$BV_b = 0$$

where $V_b = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_b \end{bmatrix}$ is a column vector of branch voltages.

and B is the fundamental circuit matrix.

5.9 KIRCHHOFF'S CURRENT LAW

KCL states that if i_k is the current in the k^{th} branch then at a given node

$$\sum i_k = 0$$

the sum being taken over all the branches incident at a given node. If there are 'n' nodes, there will 'n' such equations, one for each node

$$\sum_{k=1}^{b} a_{ik}\, i_k = 0 \qquad (k = 1, 2, \ldots , n)$$

so that set of n equations can be written in matrix form.

$$A_a\, I_b = 0 \qquad \ldots(5.4)$$

where $I_b = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_b \end{bmatrix}$ is a column vector of branch currents.

and A_a is the complete incidence matrix.

If one node is taken as reference node or datum node, we can write the Eq. (5.4) as,

$$AI_b = 0 \qquad \ldots(5.5)$$

where A is the incidence matrix of order $(n - 1) \times b$.

We know that $A = A_t\, Q$

Equation (5.5) can be written as

$$A_t\, QI_b = 0$$

Premultiplying with A_t^{-1},

$$A_t^{-1} A_t\, Q\, I_b = A_t^{-1} \cdot 0$$

$$I\, Q\, I_b = 0$$

$$Q\, I_b = 0$$

where Q is the f-cutset matrix.

5.10 RELATION BETWEEN BRANCH VOLTAGE MATRIX V_b, TWIG VOLTAGE MATRIX V_t AND NODE VOLTAGE MATRIX V_n

We know that $B\, V_b = 0$

$$[B_t : B_l]\begin{bmatrix} V_t \\ \cdots \\ V_l \end{bmatrix} = 0$$

$$B_t\, V_t + B_l\, V_l = 0$$

$$B_l\, V_l = -B_t\, V_t$$

Premultiplying with B_l^{-1}.

$$V_l = -B_l^{-1} B_t\, V_t = -(B_l^{-1} B_t) V_t$$

Now $V_b = \begin{bmatrix} V_t \\ \cdots \\ V_l \end{bmatrix}$

$$= \begin{bmatrix} V_t \\ \cdots \\ -\left(B_l^{-1} B_t\right) V_t \end{bmatrix} = \begin{bmatrix} U \\ \cdots \\ -\left(B_l^{-1} B_t\right) \end{bmatrix} \cdot V_t \qquad \ldots(5.6)$$

Also,
$$V_b = Q^T V_t$$
$$Q = A_t^{-1} A$$
$$Q^T = A^T (A_t^{-1})^T = A^T (A_t^T)^{-1}$$

Hence Eq. (5.6) can be written as
$$V_b = A^T (A_t^T)^{-1} V_t = A^T \left\{(A_t^T)^{-1} V_t\right\} = A^T V_n$$

where $\quad V_n = (A_t^T)^{-1} V_t$ is node voltage matrix.

5.11 RELATION BETWEEN BRANCH CURRENT MATRIX I_b AND LOOP CURRENT MATRIX I_l

We know that, $A I_b = 0$
$$[A_t : A_l]\begin{bmatrix} I_t \\ \cdots \\ I_l \end{bmatrix} = 0$$
$$A_t I_t + A_l I_l = 0$$
$$A_t I_t = -A_l I_l$$

Premultiplying with A_t^{-1},
$$I_t = -A_t^{-1} A_l I_l = -(A_t^{-1} A_l) I_l$$

Now
$$I_b = \begin{bmatrix} I_t \\ \cdots \\ I_l \end{bmatrix} = \begin{bmatrix} -(A_t^{-1} A_l) I_l \\ \cdots \\ I_l \end{bmatrix} = \begin{bmatrix} -(A_t^{-1} A_l) \\ \cdots \\ U \end{bmatrix} \cdot I_l$$
$$I_b = B^T I_l$$

5.12 NETWORK EQUILIBRIUM EQUATION

5.12.1 KVL Equation

1. If there is a voltage source v_{sk} in the branch k having impedance z_k and carrying current i_k, as shown in Fig. 5.50,
$$v_k = z_k i_k - v_{sk} \qquad (k = 1, 2, \ldots, b)$$
In matrix form,
$$V_b = Z_b I_b - V_s$$
where Z_b is the branch impedance matrix, I_b is the column vector of branch currents and V_s is the column vector of source voltages. Hence, KVL equation can be written as
$$B V_b = 0$$
$$B(Z_b I_b - V_s) = 0$$
$$B Z_b I_b = B V_s$$

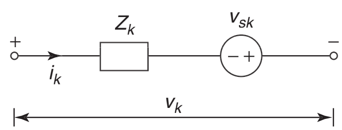


Fig. 5.50 *Circuit diagram*

Also,
$$I_b = B^T I_l$$
$$B Z_b B^T I_l = B V_s$$
$$Z I_l = E$$
where
$$E = B V_s$$
and
$$Z = B Z_b B^T$$
The matrix Z is called *loop impedance matrix*.

2. If there is a voltage source in series with an impedance and a current source in parallel with the combination as shown in Fig. 5.51,

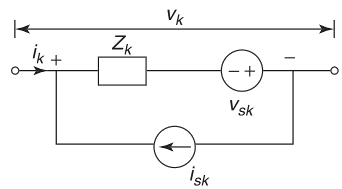


Fig. 5.51 *Circuit diagram*

$$i_k = \frac{(v_k + v_{sk})}{zk} - i_{sk}$$
$$v_k = z_k i_k + z_k i_{sk} - v_{sk}$$
In matrix form,
$$V_b = Z_b I_b + Z_b I_s - V_s$$
KVL equation is $B V_b = 0$.
$$B V_b = B(Z_b I_b + Z_b I_s - V_s) = 0$$
$$B Z_b I_b = B V_s - B Z_b I_s$$
Now
$$I_b = B^T I_l$$
$$B Z_b B^T I_l = B V_s - B Z_b I_s$$
$$Z I_l = B V_s - B Z_b I_s$$
where $Z = B Z_b B^T$ is the loop impedance matrix. This is the generalised KVL equation.

5.12.2 KCL Equation

1. If the branch k contains an input current source i_{sk} and an admittance y_k as shown in Fig. 5.52,

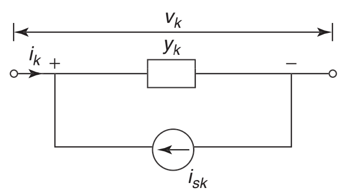


Fig. 5.52 *Circuit diagram*

$$i_k = y_k v_k - i_{sk} \qquad (k = 1, 2, \ldots, b)$$
In the matrix form,
$$I_b = Y_b V_b - I_s$$
where Y_b is the branch admittance matrix.
Hence KCL equation is given by,
$$A I_b = 0$$
$$A(Y_b V_b - I_s) = 0$$
$$A Y_b V_b = A I_s$$
Also
$$V_b = A^T V_n$$
$$A Y_b A^T V_n = A I_s$$
$$Y V_n = I$$
where
$$Y = A Y_b A^T$$
and
$$I = A I_s$$

The matrix Y is called admittance matrix. This is the KCL equation in matrix form.

In terms of f-cutset matrix, the KCL equation can be written as

$$Q\,I_b = 0$$
$$Q(Y_b V_b - V_s) = 0$$
$$Q\,Y_b V_b = Q\,I_s$$

Also $$V_b = Q^T V_t$$

$$Q\,Y_b\,Q^T V_t = Q\,I_s$$
$$Y\,V_t = I$$

where $$Y = Q\,Y_b\,Q^T$$

and $$I = Q\,I_s$$

This is the KCL equation in matrix form.

2. If there is a voltage source in series with an impedance and a current source in parallel with the combination as shown in Fig. 5.53,

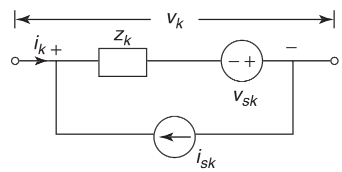


Fig. 5.53 *Circuit diagram*

$$y_k = \frac{1}{z_k}$$
$$i_k = y_k\,v_k + y_k\,v_{sk} - i_{sk}$$

In matrix form,

$$I_b = Y_b V_b + Y_b V_s - I_s$$

KCL equation will be given by,

$$A\,I_b = 0$$
$$A(Y_b V_b + Y_b V_s - I_s) = 0$$
$$A\,Y_b V_b = A\,I_s - A\,Y_b V_s$$

Also $$V_b = A^T V_n$$

$$A\,Y_b\,A^T V_n = A\,I_s - A\,Y_b V_s$$
$$Y\,V_n = A\,I_s - A V_b V_s$$

where $Y = A\,Y_b\,A^T$ is the node admittance matrix. This is a generalised KCL equation.

In terms of f-cutset matrix, the KCL equation can be written as

$$Q\,I_b = 0$$
$$Q(Y_b V_b + Y_b V_s - I_s) = 0$$
$$Q\,Y_b V_b = Q\,I_s - Q\,Y_b V_s$$

Also $$V_b = Q^T V_t$$

$$Q\,Y_b\,Q^T V_t = Q\,I_s - Q\,Y_b V_s$$
$$Y\,V_t = Q\,I_s - Q\,Y_b V_s$$

This is a generalised KCL equation.

Note

(i) For a graph having b branches, the branch impedance matrix Z_b is a square matrix of order b, having branch impedances as diagonal elements and the mutual impedances between the branches as non-diagonal elements. For a network having no mutual impedances, only diagonal elements will be present in the branch impedance matrix.

(ii) For a graph having b branches, the branch admittance matrix Y_b is a square matrix of order b, having branch admittances as diagonal elements and the mutual admittances between the branches as non-diagonal elements. For a network having no mutual admittances, only diagonal elements will be present in the branch admittance matrix.

(iii) For a graph having b branches, the voltage source matrix or vector V_s is a rectangular matrix of order $b \times 1$, having the value of the voltage source in the particular branch. The value will be positive if there is a voltage rise in the direction of current and will be negative if there is a voltage fall in the direction of current.

(iv) For a graph having b branches, the current source matrix or vector I_s is a rectangular matrix of order $b \times 1$, having the value of the current source in the particular parallel branch. The value will be positive if the direction of the current source and the corresponding parallel branch current are not same. The value will be negative if the directions of the current source and corresponding parallel branch current are same.

Example 5.16 *Write the incidence matrix of the graph of Fig. 5.54 and express branch voltages in terms of node voltages. Write the tieset matrix and express branch currents in terms of loop currents.*

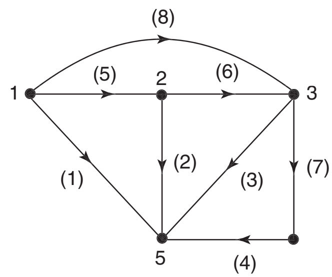


Fig. 5.54

Solution

(a) Incidence Matrix

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The incidence matrix is obtained by eliminating any one row.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

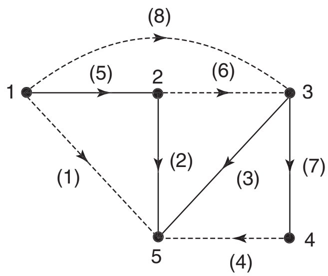
(b) Branch voltages in terms of node voltages

$$V_b = A^T V_n$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_{n_1} \\ V_{n_2} \\ V_{n_3} \\ V_{n_4} \end{bmatrix}$$

(c) Tieset Matrix

Selected tree and tiesets are shown in Fig. 5.55.



Links: {1, 4, 6, 8}
Tieset 1: {1, 2, 5}
Tieset 4: {4, 3, 7}
Tieset 6: {6, 2, 3}
Tieset 8: {8, 2, 3, 5}

$$B = \begin{array}{c} \\ 1 \\ 4 \\ 6 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Fig. 5.55

(d) Branch currents in terms of loop currents

$$I_b = B^T I_l$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_4} \\ I_{l_6} \\ I_{l_8} \end{bmatrix}$$

Example 5.17 *Branch current and loop-current relationships are expressed in matrix form as*

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \\ I_{l_3} \\ I_{l_4} \end{bmatrix}$$

Draw the oriented graph.

Solution Writing the equation in matrix form,

$$I_b = B^T I_l$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \therefore B = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

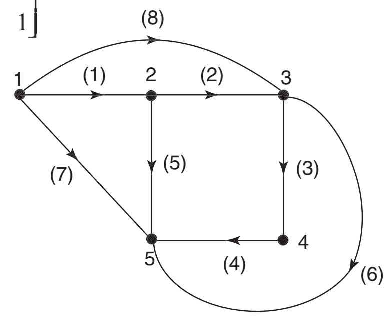


Fig. 5.56

From tieset matrix,

No. of links $l = 4$

No of branches $b = 8$

No of nodes $n = b - l + 1 = 8 - 4 + 1 = 1$

The oriented graph is shown in Fig. 5.56.

Example 5.18

For the given graph shown in Fig. 5.57, write down the basic tieset matrix and taking a tree of branches 2, 4, 5, write down KVL equations from the matrix.

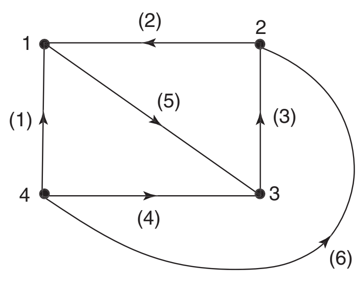


Fig. 5.57

Solution Selecting branches 2, 4, and 5 as the tree as shown in Fig. 5.58,

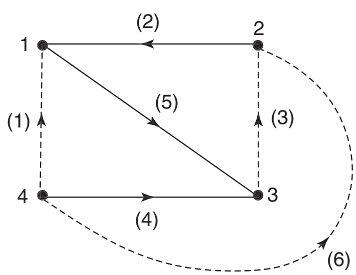


Fig. 5.58

Links: {1, 3, 6}
Tieset 1: {1, 5, 4}
Tieset 3: {3, 2, 5}
Tieset 6: {6, 2, 5, 4}

$$B = \begin{array}{c} \\ 1 \\ 3 \\ 6 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

The KVL equation in matrix form is given by

$$BV_b = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = 0$$

$$V_1 - V_4 + V_5 = 0$$

$$V_2 + V_3 + V_5 = 0$$

$$V_2 - V_4 + V_5 + V_6 = 0$$

Example 5.19 *Obtain the f-cutset matrix for the graph shown in Fig. 5.59 taking 1, 2, 3, 4 as tree branches. Write down the network equations from the f-cutset matrix.*

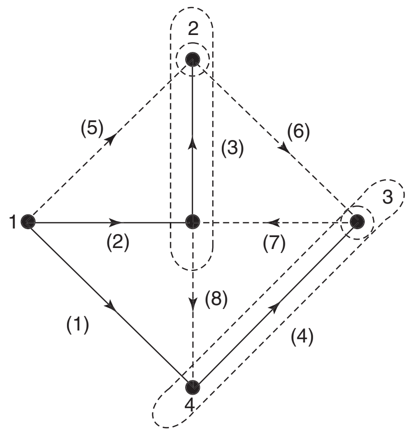


Fig. 5.59

Solution Twigs: $\{1, 2, 3, 4\}$

f-cutset 1 : $\{1, 6, 7, 8\}$

f-cutset 2 : $\{2, 5, 6, 7, 8\}$

f-cutset 3 : $\{3, 5, 6\}$

f-cutset 4 : $\{4, 6, 7\}$

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \end{bmatrix} \end{array}$$

The KCL equation in matrix form is given by

$$QI_b = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = 0$$

$$I_1 + I_6 - I_7 + I_8 = 0$$
$$I_2 + I_5 - I_6 + I_7 - I_8 = 0$$
$$I_3 + I_5 - I_6 = 0$$
$$I_4 + I_6 - I_7 = 0$$

Example 5.20 *The reduced incidence matrix of a graph is given as*

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Express branch voltages in terms of node voltages.

Solution For the given graph,

No of branches $b = 5$

No of nodes $n = 3$

Branch voltages can be expressed in terms of node voltages by

$$V_b = A^T V_n$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{n_1} \\ V_{n_2} \\ V_{n_3} \end{bmatrix}$$

$$V_1 = V_{n_1} - V_{n_2}$$
$$V_2 = -V_{n_2}$$
$$V_3 = -V_{n_2} + V_{n_3}$$
$$V_4 = -V_{n_3}$$
$$V_5 = -V_{n_1}$$

Example 5.21 *The fundamental cutset matrix of a graph is given as*

$$Q = \begin{bmatrix} -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Express branch voltages in terms of twig voltages.

Solution For the given graph,

No. of branches $b = 5$

No. of twigs = 3

Branch voltages are expressed in terms of twig voltages by

$$V_b = Q^T V_t$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_{t_1} \\ V_{t_2} \\ V_{t_3} \end{bmatrix}$$

$$V_1 = -V_{t_1} + V_{t_3}$$
$$V_2 = V_{t_1}$$
$$V_3 = V_{t_2}$$
$$V_4 = V_{t_3}$$
$$V_5 = -V_{t_1} - V_{t_2}$$

Example 5.22 *For this network shown in Fig. 5.60, write down the tieset matrix and obtain the network equilibrium equation in matrix form using KVL. Calculate the loop currents and branch currents.*

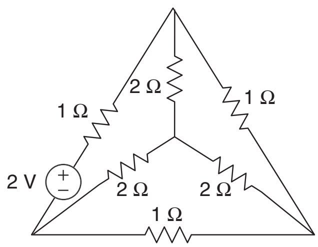
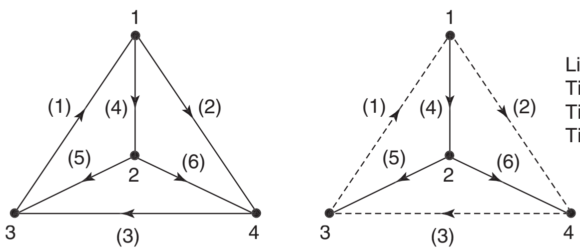


Fig. 5.60

Solution The oriented graph and one of its trees are shown in Fig. 5.61.



Links: {1, 2, 3}
Tieset 1: {1, 4, 5}
Tieset 2: {2, 4, 6}
Tieset 3: {3, 5, 6}

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.61

The KVL equation in matrix form is given by

$$B\,Z_b\,B^T\,I_l = B\,V_s - B\,Z_b\,I_s$$

Here, $$I_s = 0,$$

$$B\,Z_b\,B^T\,I_l = B\,V_s$$

$$Z_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix};\ B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix};\ V_s = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 & 2 \end{bmatrix}$$

$$B\,Z_b\,B^T = \begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & -2 \\ -2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix}$$

$$B\,V_s = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

The KVL equation in matrix form is given by

$$\begin{bmatrix} 5 & -2 & -2 \\ -2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \\ I_{l_3} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

Solving this matrix equation,

$$I_{l_1} = \frac{6}{7}\,\text{A}$$

$$I_{l_2} = \frac{4}{7}\,\text{A}$$

$$I_{l_3} = \frac{4}{7}\,\text{A}$$

The branch currents are given by

$$I_b = B^T\, I_l$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{6}{7} \\ \frac{4}{7} \\ \frac{4}{7} \end{bmatrix} = \begin{bmatrix} \frac{6}{7} \\ \frac{4}{7} \\ \frac{4}{7} \\ \frac{2}{7} \\ \frac{2}{7} \\ 0 \end{bmatrix}$$

Example 5.23 *For the network shown in Fig. 5.62, write down the tieset matrix and obtain the network equilibrium equation in matrix form using KVL. Calculate loop currents.*

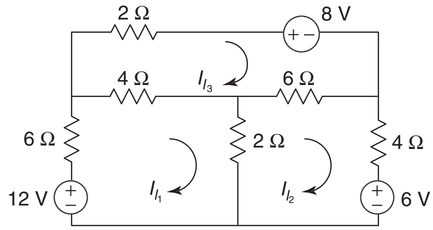
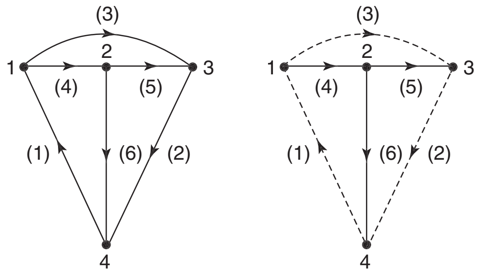


Fig. 5.62

Solution The oriented graph and its selected tree are shown in Fig. 5.63.



Links: {1, 2, 3}
Tieset 1: {1, 4, 6}
Tieset 2: {2, 5, 6}
Tieset 3: {3, 5, 4}

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

Fig. 5.63

The KVL equation in matrix form is given by

$$B\,Z_b\,B^T\,I_l = BV_s - B\,Z_b\,I_s$$

Here, $I_s = 0,$

$$B\,Z_b\,B^T I_l = BV_s$$

$$Z_b = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix};\ B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix};\ V_s = \begin{bmatrix} 12 \\ -6 \\ -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 & 4 & 0 & 2 \\ 0 & 4 & 0 & 0 & 6 & -2 \\ 0 & 0 & 2 & -4 & -6 & 0 \end{bmatrix}$$

$$BZ_b\,B^T = \begin{bmatrix} 6 & 0 & 0 & 4 & 0 & 2 \\ 0 & 4 & 0 & 0 & 6 & -2 \\ 0 & 0 & 2 & -4 & -6 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -2 & -4 \\ -1 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix}$$

$$BV_s = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ -6 \\ -8 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix}$$

Hence, the KVL equation in matrix form is given by

$$\begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \\ I_{l_3} \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix}$$

Solving this matrix equation,

$$I_{l_1} = 0.55\text{A}$$
$$I_{l_2} = -0.866\text{A}$$
$$I_{l_3} = -0.916\text{A}$$

Example 5.24 *For the network shown in Fig. 5.64, draw the oriented graph. Write the tieset schedule and hence obtain the equilibrium equation on loop basis. Calculate the values of branch currents and branch voltages.*

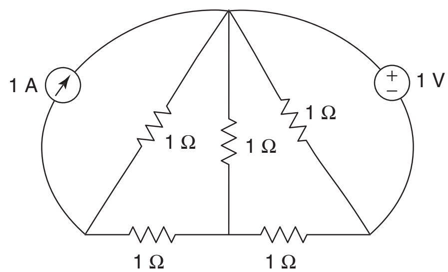


Fig. 5.64

Solution The oriented graph and one of its trees are shown in Fig. 5.65.

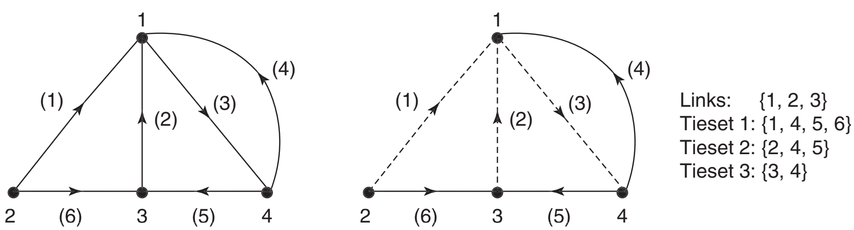


Fig. 5.65

$$\begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ B = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The KVL equation in matrix form is given by

$$B\,Z_b\,B^T\,I_l = B V_s - B\,Z_b\,I_s$$

$$Z_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};\; B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix};\; V_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix};\; I_s = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$BZ_b\,B^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B V_s = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$B\,Z_b\,I_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the KVL equation in matrix form is given by

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{l_1} \\ I_{l_2} \\ I_{l_3} \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Solving this matrix equation,

$$I_{l_1} = \frac{1}{5}\text{A}$$

$$I_{l_2} = -\frac{3}{5}\text{A}$$

$$I_{l_3} = 1\text{A}$$

The branch currents are given by

$$I_b = B^T I_l + I_s$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{5} \\ -\frac{3}{5} \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} \\ -\frac{3}{5} \\ 1 \\ \frac{7}{5} \\ -\frac{2}{5} \\ -\frac{1}{5} \end{bmatrix}$$

The branch voltages are given by

$$V_b = Z_b I_b - V_s$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{4}{5} \\ -\frac{3}{5} \\ 1 \\ \frac{7}{5} \\ -\frac{2}{5} \\ -\frac{1}{5} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} \\ -\frac{3}{5} \\ 1 \\ -1 \\ -\frac{2}{5} \\ -\frac{1}{5} \end{bmatrix}$$

Example 5.25 *For the network shown in Fig. 5.66, obtain the loop equation in matrix form.*

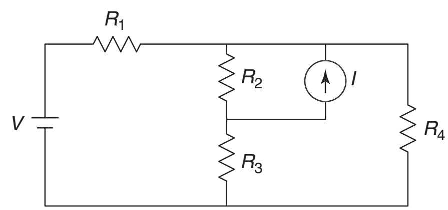
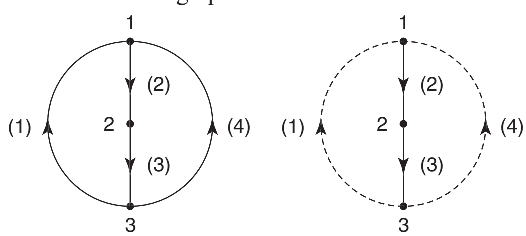


Fig. 5.66

Solution The oriented graph and one of its trees are shown in Fig. 5.67.



Links: {1, 4}
Tieset 1: {1, 2, 3}
Tieset 4: {4, 2, 3}

$$B = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.67

The KVL equation in matrix form is given by

$$B\,Z_b\,B^T\,I_l = BV_s - B\,Z_b I_s$$

$$Z_b = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & R_4 \end{bmatrix};\; B^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix};\; V_s = \begin{bmatrix} V \\ 0 \\ 0 \\ 0 \end{bmatrix};\; I_s = \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & R_4 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & R_3 & 0 \\ 0 & R_2 & R_3 & R_4 \end{bmatrix}$$

$$B\,Z_b\,B^T = \begin{bmatrix} R_1 & R_2 & R_3 & 0 \\ 0 & R_2 & R_3 & R_4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1+R_2+R_3 & R_2+R_3 \\ R_2+R_3 & R_2+R_3+R_4 \end{bmatrix}$$

$$BV_s = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix}$$

$$B\,Z_b\,I_s = \begin{bmatrix} R_1 & R_2 & R_3 & 0 \\ 0 & R_2 & R_3 & R_4 \end{bmatrix}\begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_2 I \\ R_2 I \end{bmatrix}$$

Hence, the KVL equation in matrix form is given by

$$\begin{bmatrix} R_1+R_2+R_3 & R_2+R_3 \\ R_2+R_3 & R_2+R_3+R_4 \end{bmatrix}\begin{bmatrix} I_{l_1} \\ I_{l_4} \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix} - \begin{bmatrix} R_2 & I \\ R_2 & I \end{bmatrix} = \begin{bmatrix} V - R_2 I \\ -R_2 I \end{bmatrix}$$

Example 5.26 *For the network shown in Fig. 5.68, write down the tieset matrix and obtain the network equilibrium equation in matrix form using KVL.*

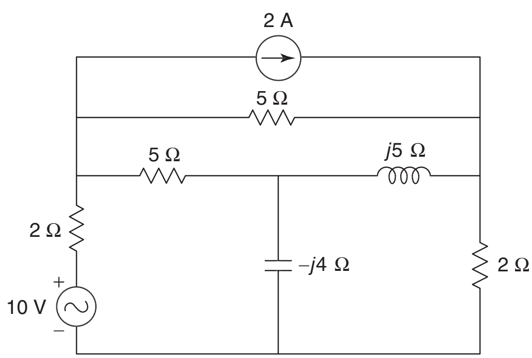
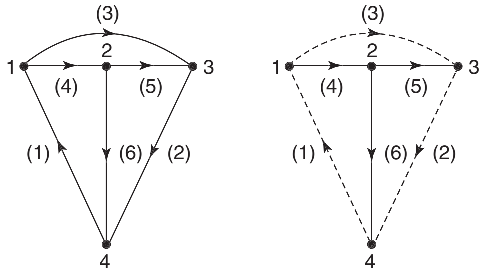


Fig. 5.68

Solution The oriented graph and its selected tree are shown in Fig. 5.69.



Links: {1, 2, 3}
Tieset 1: {1, 4, 6}
Tieset 2: {2, 5, 6}
Tieset 3: {3, 5, 4}

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

Fig. 5.69

The KVL equation in matrix form is given by

$$B\,Z_b\,B^T\,I_l = B V_s - B\,Z_b I_s$$

$$Z_b = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & j5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j4 \end{bmatrix};\ B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix};\ V_s = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};\ I_s = \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & j5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j4 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 5 & 0 & -j4 \\ 0 & 2 & 0 & 0 & j5 & j4 \\ 0 & 0 & 5 & -5 & -j5 & 0 \end{bmatrix}$$

$$B\,Z_b\,B^T = \begin{bmatrix} 2 & 0 & 0 & 5 & 0 & -j4 \\ 0 & 2 & 0 & 0 & j5 & j4 \\ 0 & 0 & 5 & -5 & -j5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 7-j4 & j4 & -5 \\ j4 & 2+j1 & -j5 \\ -5 & -j5 & 10+j5 \end{bmatrix}$$

$$B V_s = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b\,I_s = \begin{bmatrix} 2 & 0 & 0 & 5 & 0 & -j4 \\ 0 & 2 & 0 & 0 & j5 & j4 \\ 0 & 0 & 5 & -5 & -j5 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Hence, the KCL equation in matrix form is given by

$$\begin{bmatrix} 7-j4 & j4 & -5 \\ j4 & 2+j1 & -j5 \\ -5 & -j5 & 10+j5 \end{bmatrix}\begin{bmatrix} I_{l_1} \\ I_{l_2} \\ I_{l_3} \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 10 \end{bmatrix}$$

Example 5.27 *For the network shown in Fig. 5.70, write down the tieset matrix and obtain the network equilibrium equation in matrix form using KVL.*

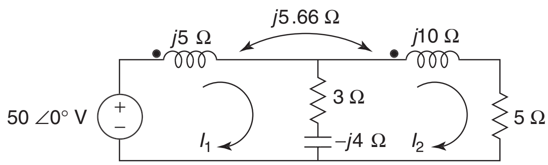
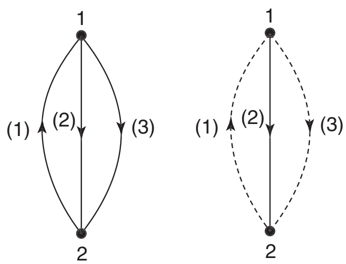


Fig. 5.70

Solution The branch currents are so chosen that they assume the direction out of the dotted terminals. Because of this choice of current direction, the mutual inductance is positive. The oriented graph and its selected tree are shown in Fig. 5.71.



Links: {1, 3}
Tieset 1: {1, 2}
Tieset 3: {3, 2}

$$B = \begin{array}{c} \begin{matrix} 1 & \;\;2 & \;\;3 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.71

The KVL equation in matrix form is given by

$$B Z_b B^T I_l = BV_s - B Z_b I_s$$

Here, $\quad I_s = 0,$

$$B Z_b B^T I_l = BV_s$$

$$Z_b = \begin{bmatrix} j5 & 0 & j5.66 \\ 0 & 3-j4 & 0 \\ j5.66 & 0 & 5+j10 \end{bmatrix}; B^T = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}; V_s = \begin{bmatrix} 50\angle 0° \\ 0 \\ 0 \end{bmatrix}$$

$$B Z_b = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} j5 & 0 & j5.66 \\ 0 & 3-j4 & 0 \\ j5.66 & 0 & 5+j10 \end{bmatrix} = \begin{bmatrix} j5 & 3-j4 & j5.66 \\ j5.66 & -3+j4 & 5+j10 \end{bmatrix}$$

$$B Z_b B^T = \begin{bmatrix} j5 & 3-j4 & j5.66 \\ j5.66 & -3+j4 & 5+j10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3+j1 & -3+j9.66 \\ -3+j9.66 & 8+j6 \end{bmatrix}$$

$$BV_s = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 50\angle 0° \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 50\angle 0° \\ 0 \end{bmatrix}$$

Hence, the KVL equation in matrix form is given by

$$\begin{bmatrix} 3+j1 & -3+j9.66 \\ -3+j9.66 & 8.j6 \end{bmatrix}\begin{bmatrix} I_{l_1} \\ I_{l_2} \end{bmatrix} = \begin{bmatrix} 50\angle 0^\circ \\ 0 \end{bmatrix}$$

Example 5.28 *For the network shown in Fig. 5.72, write down the tieset matrix and obtain the network equilibrium equation in matrix form using KVL.*

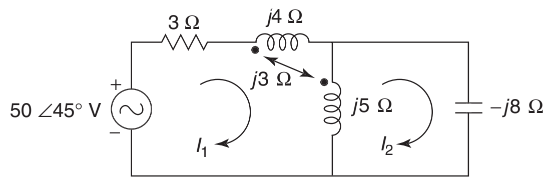
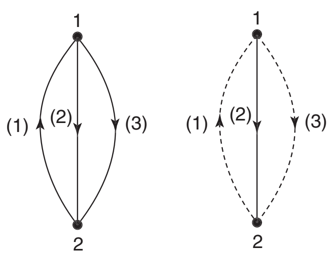


Fig. 5.72

Solution The branch currents are so chosen that they assume the direction out of the dotted terminals. Because of this choice of current direction, the mutual inductance is positive. The oriented graph and its selected tree are shown in Fig. 5.73.



Links: {1, 3}
Tieset 1: {1, 2}
Tieset 3: {3, 2}

$$B = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.73

The KVL equation in matrix form is given by

$$B\,Z_b\,B^T\,I_l = B\,V_s - B\,Z_b\,I_s$$

Here, $I_s = 0,$

$$B\,Z_b\,B^T\,I_l = B\,V_s$$

$$Z_b = \begin{bmatrix} 3+j4 & j3 & 0 \\ j3 & j5 & 0 \\ 0 & 0 & -j8 \end{bmatrix}; B^T = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}; V_s = \begin{bmatrix} 50\angle 45^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$B\,Z_b = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3+j4 & j3 & 0 \\ j3 & j5 & 0 \\ 0 & 0 & -j8 \end{bmatrix} = \begin{bmatrix} 3+j7 & j8 & 0 \\ -j3 & -j5 & -j8 \end{bmatrix}$$

$$B\,Z_b\,B^T = \begin{bmatrix} 3+j7 & j8 & 0 \\ -j3 & -j5 & -j8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3+j15 & -j8 \\ -j8 & -j3 \end{bmatrix}$$

$$B\,V_s = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 50\angle 45^\circ \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 50\angle 45^\circ \\ 0 \end{bmatrix}$$

Hence, the KVL equation in matrix form is given by,

$$\begin{bmatrix} 3+j15 & -j8 \\ -j8 & -j3 \end{bmatrix}\begin{bmatrix} I_{l_1} \\ I_{l_3} \end{bmatrix} = \begin{bmatrix} 50\angle 45^\circ \\ 0 \end{bmatrix}$$

Example 5.29 *For the network shown in Fig. 5.74, obtain branch voltages using KCL equation on node basis.*

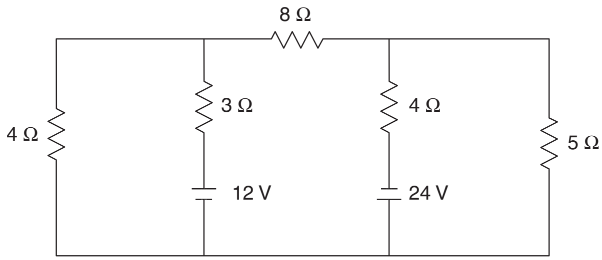


Fig. 5.74

Solution The oriented graph is shown in Fig. 5.75.

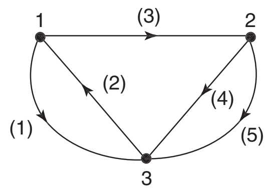


Fig. 5.75

The complete incidence matrix for the graph is

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ -1 & 1 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

Eliminating the last row from the matrix A_a, we get the incidence matrix A.

$$A = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

The KCL equation in matrix form is given by

$$A Y_b A^T V_n = A I_s - A Y_b V_s$$

$$I_s = 0\,,$$

Here,

$$AY_b\,A^T V_n = -AY_b\,V_s$$

$$Y_b = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} \end{bmatrix};\ A^T = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix};\ V_s = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 24 \\ 0 \end{bmatrix}$$

$$AY_b = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{3} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$AY_bA^T = \begin{bmatrix} \frac{1}{4} & -\frac{1}{3} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{17}{24} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{23}{40} \end{bmatrix}$$

$$AY_b\,V_s = \begin{bmatrix} \frac{1}{4} & -\frac{1}{3} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 0 \\ 12 \\ 0 \\ 24 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

Hence, the KCL equation in matrix form is given by

$$\begin{bmatrix} \frac{17}{24} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{23}{40} \end{bmatrix} \begin{bmatrix} V_{n_1} \\ V_{n_2} \end{bmatrix} = -\begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

Solving this matrix equation,

$$V_{n_1} = 3.96\,\text{V}$$

$$V_{n_2} = -9.57\,\text{V}$$

Branch voltages are given by,

$$V_b = A^T V_n$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3.96 \\ -9.57 \end{bmatrix} = \begin{bmatrix} 3.96 \\ -3.96 \\ 13.53 \\ -9.57 \\ -9.57 \end{bmatrix}$$

Example 5.30 *For the network shown in Fig. 5.76, write down the f-cutset matrix and obtain the network equilibrium equation in matrix form using KCL.*

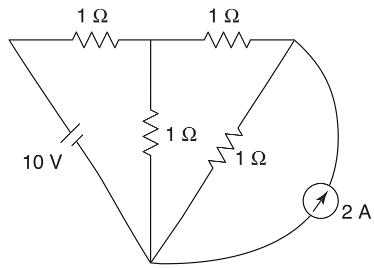
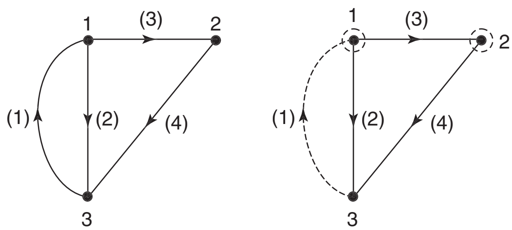


Fig. 5.76

Solution The oriented graph and its selected tree are shown in Fig. 5.77.



Twigs: {2, 4}
f-cutset 2: {2, 1, 3}
f-cutset 4: {4, 3}

$$Q = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 2 \\ 4 \end{array} & \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.77

The KCL equation in matrix form is given by

$$Q Y_b Q^T V_t = Q I_s - Q Y_b V_s$$

$$Y_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; I_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}; V_s = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}; Q^T = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$Q Y_b = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$Q Y_b Q^T = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

$$Q\,I_s = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$Q\,Y_b\,V_s = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

Hence, the KCL equation is given by

$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} V_{t_2} \\ V_{t_4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} -10 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

Solving this matrix equation,

$$V_{t_2} = 4.4\,\text{V}$$
$$V_{t_4} = 3.2\,\text{V}$$

Example 5.31 *Calculate the twig voltages using KCL equation for the network shown in Fig. 5.78.*

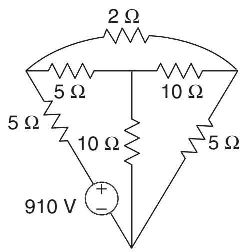
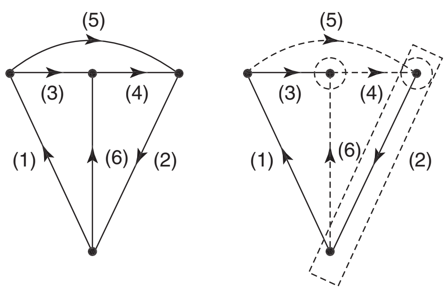


Fig. 5.78

Solution The oriented graph and one of the trees are shown in Fig. 5.79.



Twigs: {1, 2, 3}
f-cutset 1: {1, 4, 5, 6}
f-cutset 2: {2, 4, 5}
f-cutset 3: {3, 4, 6}

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

Fig. 5.79

The network equilibrium equation on node basis can be written as

$$Q\,Y_b\,Q^T\,V_t = QI_s - QY_b\,V_s$$

Here, $I_s = 0,$

$$Q\,Y_b\,Q^T V_t = -Q\,Y_b\,V_s$$

$$Y_b = \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}; Q^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}; V_s = \begin{bmatrix} 910 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$QY_b = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.2 & 0 & 0 & -0.1 & -0.5 & 0.1 \\ 0 & 0.2 & 0 & -0.1 & -0.5 & 0 \\ 0 & 0 & 0.2 & -0.1 & 0 & 0.1 \end{bmatrix}$$

$$QY_b Q^T = \begin{bmatrix} 0.2 & 0 & 0 & -0.1 & -0.5 & 0.1 \\ 0 & 0.2 & 0 & -0.1 & -0.5 & 0 \\ 0 & 0 & 0.2 & -0.1 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \\ -1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.6 & 0.2 \\ 0.6 & 0.8 & 0.1 \\ 0.2 & 0.1 & 0.3 \end{bmatrix}$$

$$QY_b V_s = \begin{bmatrix} 0.2 & 0 & 0 & -0.1 & -0.5 & 0.1 \\ 0 & 0.2 & 0 & -0.1 & -0.5 & 0 \\ 0 & 0 & 0.2 & -0.1 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} 910 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 182 \\ 0 \\ 0 \end{bmatrix}$$

Hence, KCL equation can be written as,

$$\begin{bmatrix} 0.9 & 0.6 & 0.2 \\ 0.6 & 0.8 & 0.1 \\ 0.2 & 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} v_{t_1} \\ v_{t_2} \\ v_{t_3} \end{bmatrix} = \begin{bmatrix} -182 \\ 0 \\ 0 \end{bmatrix}$$

Solving this matrix equation,

$$v_{t_1} = -460 \text{ V}$$
$$v_{t_2} = 320 \text{ V}$$
$$v_{t_3} = 200 \text{ V}$$

Example 5.32 *For the network shown in Fig. 5.80, obtain equilibrium equation on node basis.*

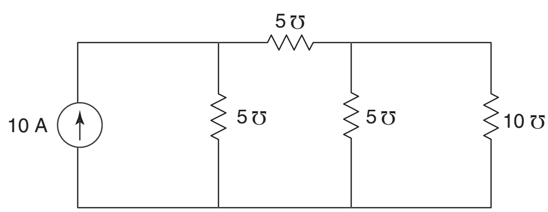
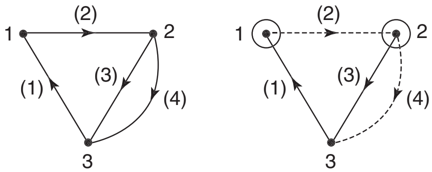


Fig. 5.80

Solution The oriented graph and its selected tree are shown in Fig. 5.81.



Twigs: {1, 3}
f-cutset 1: {1, 2}
f-cutset 3: {3, 2, 4}

$$Q = \begin{array}{c} \\ 1 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.81

The KCL equation in matrix form is given by

$$Q Y_b Q^T V_t = Q I_s - Q Y_b V_s$$

Here,
$$V_s = 0,$$

$$Q Y_b Q^T V_t = Q I_s$$

$$Y_b = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}; Q^T = \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}; I_s = \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q Y_b = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 0 & 0 \\ 0 & -5 & 5 & 10 \end{bmatrix}$$

$$Q Y_b Q^T = \begin{bmatrix} 5 & -5 & 0 & 0 \\ 0 & -5 & 5 & 10 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix}$$

$$Q I_s = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

Hence, KCL equation will be written as

$$\begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix} \begin{bmatrix} v_{t_1} \\ v_{t_3} \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

Solving this matrix equation,

$$v_{t_1} = -\frac{8}{7}\text{V}$$

$$v_{t_3} = \frac{2}{7}\text{V}$$

Example 5.33 *For the network shown in Fig. 5.82, write down the f-cutset matrix and obtain the network equilibrium equation in matrix form using KCL and calculate v.*

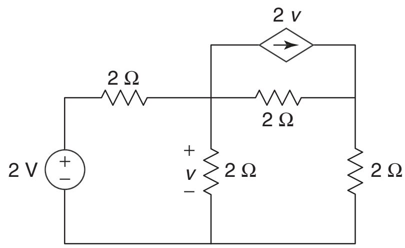
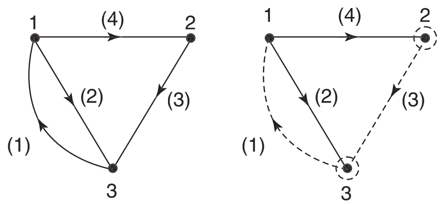


Fig. 5.82

Solution The oriented graph and its selected tree are shown in Fig. 5.83
Since voltage v is to be determined, Branch 2 is chosen as twig,



Twigs: {2, 4}
f-cutset 2: {2, 1, 3}
f-cutset 3: {4, 3}

$$Q = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

Fig. 5.83

The KCL equation in matrix form is given by

$$Q\,Y_b\,Q^T\,V_t = Q\,I_s - Q\,Y_b\,V_s$$

$$Y_b = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}; Q^T = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}; I_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2v \end{bmatrix}; V_s = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q\,Y_b = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 \end{bmatrix}$$

$$Q\,Y_b\,Q^T = \begin{bmatrix} -0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

$$Q\,I_s = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2v \end{bmatrix} = \begin{bmatrix} 0 \\ -2v \end{bmatrix}$$

$$Q\,Y_b\,V_s = \begin{bmatrix} -0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$Q\,I_s - Q\,Y_b\,V_s = \begin{bmatrix} 1 \\ -2v \end{bmatrix}$$

Hence, the KCL equation can be written as

$$\begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} v_{t_2} \\ v_{t_4} \end{bmatrix} = \begin{bmatrix} 1 \\ -2v \end{bmatrix}$$

From Fig. 5.82, $v_{t_2} = v$

Solving this matrix equation,

$$v_{t_2} = 0.44 \text{ V}$$
$$v_{t_4} = 0.66 \text{ V}$$
$$v = v_{t_2} = 0.44 \text{ V}$$

Example 5.34 *For the network shown in Fig. 5.84, write down the f-cutset matrix and obtain the network equilibrium equation in matrix form using KCL and calculate v.*

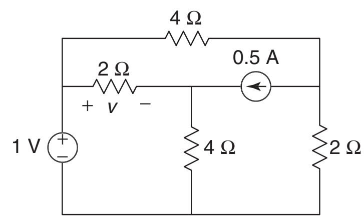


Fig. 5.84

Solution The voltage and current sources are converted into accompanied sources by source–shifting method as shown in Fig. 5.85.

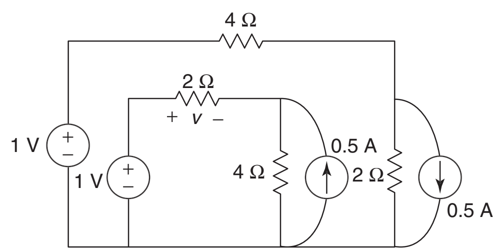
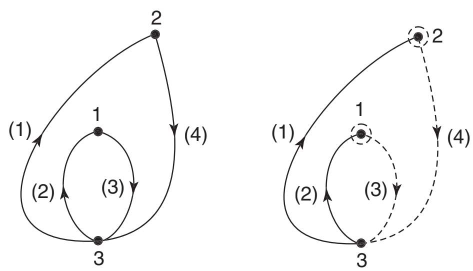


Fig. 5.85

The oriented graph and its selected tree are shown in Fig. 5.86.



Twigs: {1, 2}
f-cutset 1: {1, 4}
f-cutset 2: {2, 3}

$$Q = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \end{array}$$

Fig. 5.86

The KCL equation in the matrix form is given by

$$Q\,Y_b\,Q^T\,V_t = Q\,I_s - Q\,Y_b\,V_s$$

$$Y_b = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}; Q^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix}; \; I_s = \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix}; \; V_s = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$Q\,Y_b = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.25 & 0 & 0 & -0.5 \\ 0 & 0.5 & -0.25 & 0 \end{bmatrix}$$

$$Q\,Y_b\,Q^T = \begin{bmatrix} 0.25 & 0 & 0 & -0.5 \\ 0 & 0.5 & -0.25 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix}$$

$$Q\,I_s = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

$$Q\,Y_b\,V_s = \begin{bmatrix} 0.25 & 0 & 0 & -0.5 \\ 0 & 0.5 & -0.25 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$$

$$Q\,I_s - Q\,Y_b\,V_s = \begin{bmatrix} 0.25 \\ -1 \end{bmatrix}$$

Hence, the KCL equation can be written as

$$\begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix} \begin{bmatrix} v_{t_1} \\ v_{t_2} \end{bmatrix} = \begin{bmatrix} 0.25 \\ -1 \end{bmatrix}$$

Solving this matrix equation,

$$v_{t_1} = 0.33 \text{ V}$$
$$v_{t_2} = -1.33 \text{ V}$$

From Fig. 5.85, $\quad v = 1 + v_{t_2} = -0.33 \text{ V}$

Exercises

5.1 For the networks shown in Fig. 5.87–5.90, write the incidence matrix, tieset matrix and *f*-cutset matrix.

(i)

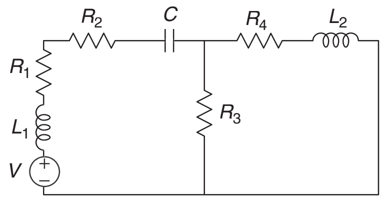


Fig. 5.87

(ii)

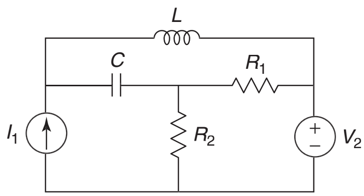


Fig. 5.88

(iii)

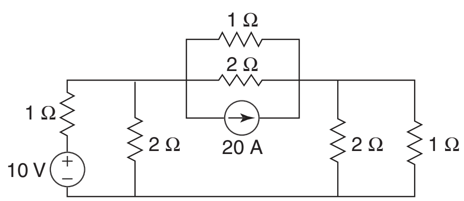


Fig. 5.89

(iv)

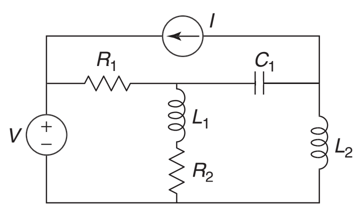


Fig. 5.90

5.2 For the graph shown in Fig. 5.91, write the incidence matrix, tieset matrix and *f*-cutset matrix.

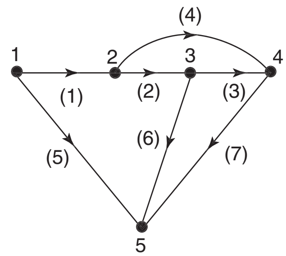


Fig. 5.91

5.3 The incidence matrix is given as follows:

Branches →

1	2	3	4	5	6	7	8
–1	–1	0	0	0	0	1	0
0	1	1	0	1	0	0	0
0	0	–1	–1	0	1	0	0
1	0	0	1	0	0	0	1

Draw oriented graph and write tieset matrix.

5.4 The incidence matrix is given below:

Branches →

1	2	3	4	5	6	7	8	9	10
0	0	1	1	1	1	0	1	0	0
0	–1	–1	0	0	0	–1	0	0	–1
–1	1	0	0	0	0	0	–1	–1	1
1	0	0	0	–1	–1	1	0	0	0

Draw the oriented graph.

5.5 For the network shown in Fig. 5.92, draw the oriented graph and obtain the tieset matrix. Use this matrix to calculate the current *i*.

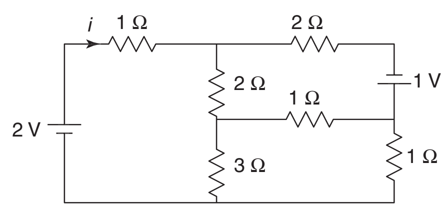


Fig. 5.92

[0.91 A]

5.6 Using the principles of network topology, write the loop/node equation in matrix form for the network shown in Fig. 5.93.

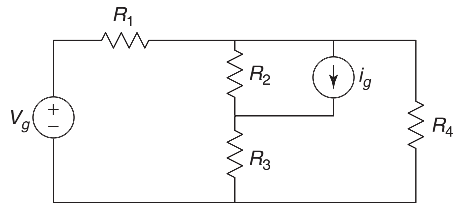


Fig. 5.93

Objective-Type Questions

5.1 The number of independent loops for a network with n nodes and b branches is
(a) n − 1
(b) b − n
(c) b − n + 1
(d) independent of the number of nodes

5.2 A network has 7 nodes and 5 independent loops. The number of branched in the network is
(a) 13 (b) 12
(c) 11 (d) 10

5.3 Identify which of the following is NOT a tree of the graph shown in Fig. 5.94.

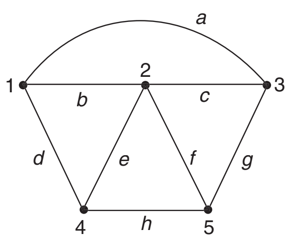


Fig. 5.94

(a) *begh* (b) *defg*
(c) *adfg* (d) *aegh*

5.4 The minimum number of equations required to analyze the circuit shown in Fig. 5.95 is

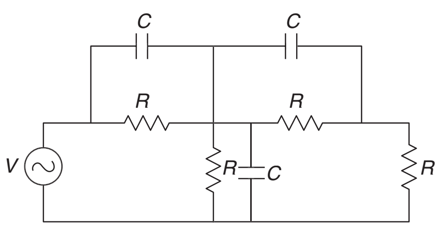


Fig. 5.95

(a) 3 (b) 4
(c) 6 (d) 7

5.5 Consider the network graph shown in Fig. 5.96.

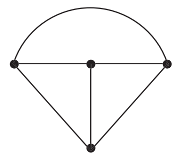
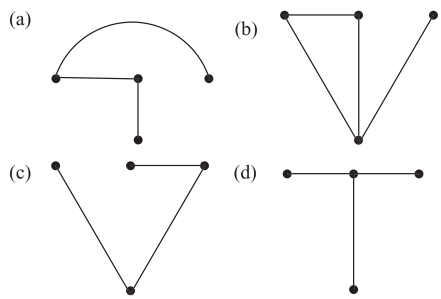


Fig. 5.96

Which one of the following is NOT a tree of this graph?



5.6 Figure 5.97 below shows a network and its graph is drawn aside. A proper tree chosen for analyzing the network will contain the edges.

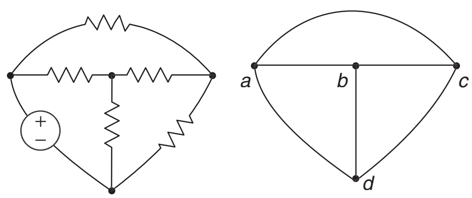


Fig. 5.97

(a) *ab, bc, ad* (b) *ab, bc, ca*
(c) *ab, bd, ca?* (d) *ac, bd, ad*

5.7 The graph of an electrical network has n nodes and b branches. The number of links with respect to the choice of a tree is given by

(a) $b - n + 1$ (b) $b + n$
(c) $n - b + 1$ (d) $n - 2b - 1$.

5.8 In the graph shown in Fig. 5.98, one possible tree is formed by the branches 4, 5, 6, 7. Then one possible fundamental cutset is

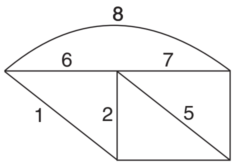


Fig. 5.98

(a) 1, 2, 3, 8 (b) 1, 2, 5, 6
(c) 1, 5, 6, 8 (d) 1, 2, 3, 7, 8

5.9 Which one of the following represents the total number of trees in the graph given in Fig. 5.99?

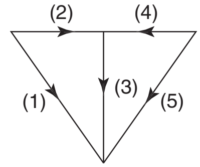


Fig. 5.99

(a) 4 (b) C
(c) 5 (d) 8

5.10 Which one of the following is a cutset of the graph shown in Fig. 5.100?

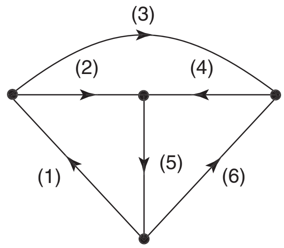


Fig. 5.100

(a) 1, 2, 3 and 4 (b) 2, 3, 4 and 6
(c) 1, 4, 5 and 6 (d) 1, 2, 4 and 5

Answers to Objective-Type Questions

5.1. (c)	5.2. (c)	5.3. (c)	5.4. (b)	5.5. (b)
5.6. (d)	5.7. (a)	5.8. (d)	5.9. (d)	5.10. (d)