Signals and Systems

Tutorial 6

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58, D9A

Q ①. Find the digital network.

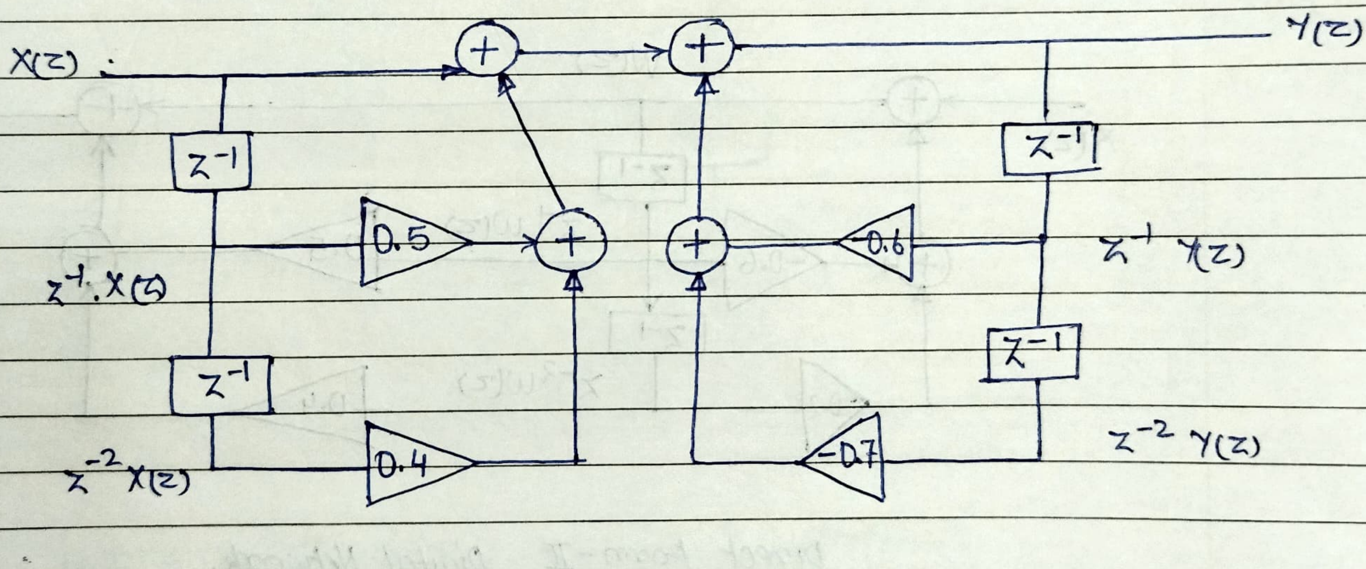
$$y(n) = x(n) + 0.5\,x(n-1) + 0.4\,x(n-2) - 0.6\,y(n-1) - 0.7\,y(n-2)$$

Soln. Direct Form I

Taking z-transform on both sides,

$$Y(z) = X(z) + 0.5\,z^{-1}X(z) + 0.4\,z^{-2}X(z) - 0.6\,z^{-1}Y(z) - 0.7\,z^{-2}Y(z) \qquad \text{—— ①}$$

From ①,



Direct Form-I Digital Network

Direct Form - II

From eq. ①,

$$Y(z)\,[1 + 0.6z^{-1} + 0.7z^{-2}] = [1 + 0.5z^{-1} + 0.4^{-2}]\,.X(z)$$

$$\therefore \frac{Y(z)}{X(z)} = \frac{1 + 0.05z^{-1} + 0.4^{-2}}{1 + 0.6z^{-1} + 0.7z^{-2}}$$

Let, $\dfrac{Y(z)}{X(z)} = \dfrac{W(z)}{X(z)}\,,\ \dfrac{Y(z)}{W(z)}$

$$\frac{W(z)}{X(z)} = \frac{1}{1+0.6z^{-1}+0.7z^{-2}} \quad \text{--- (3)}$$

$$\frac{Y(z)}{W(z)} = 1+0.5z^{-1}+0.4z^{-2} \quad \text{--- (4)}$$

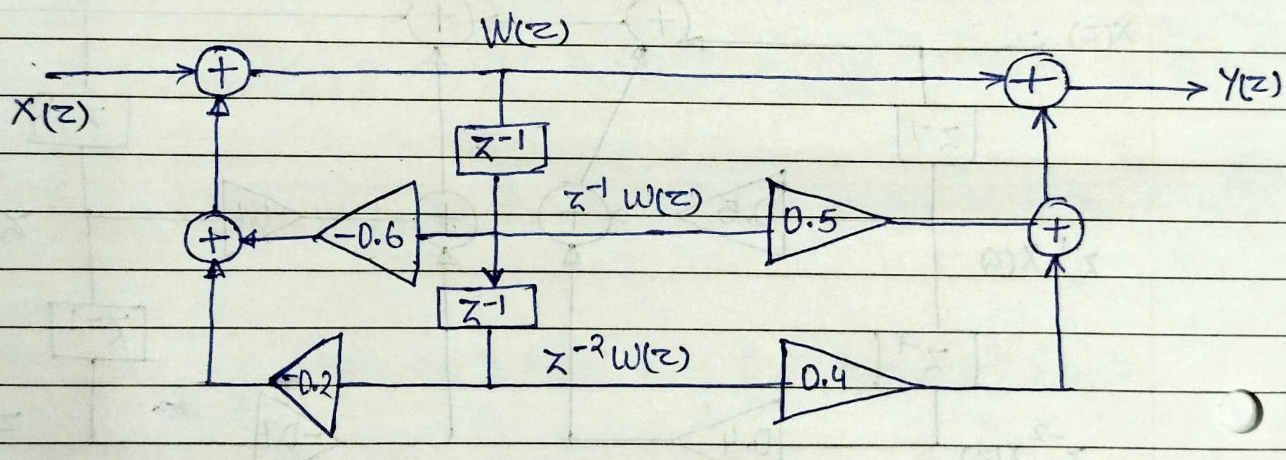
on multiplying eq. (3)

$$X(z) = W(z) + 0.6z^{-1}W(z) + 0.7z^{-2}W(z)$$

$$W(z) = X(z) - 0.6z^{-1} - 0.7z^{-2}W(z) \quad \text{--- (5)}$$

on multiplying eq. (4)

$$Y(z) = W(z) + 0.5z^{-1}W(z) + 0.4z^{-2}W(z) \quad \text{--- (6)}$$



Direct Form-II Digital Network.

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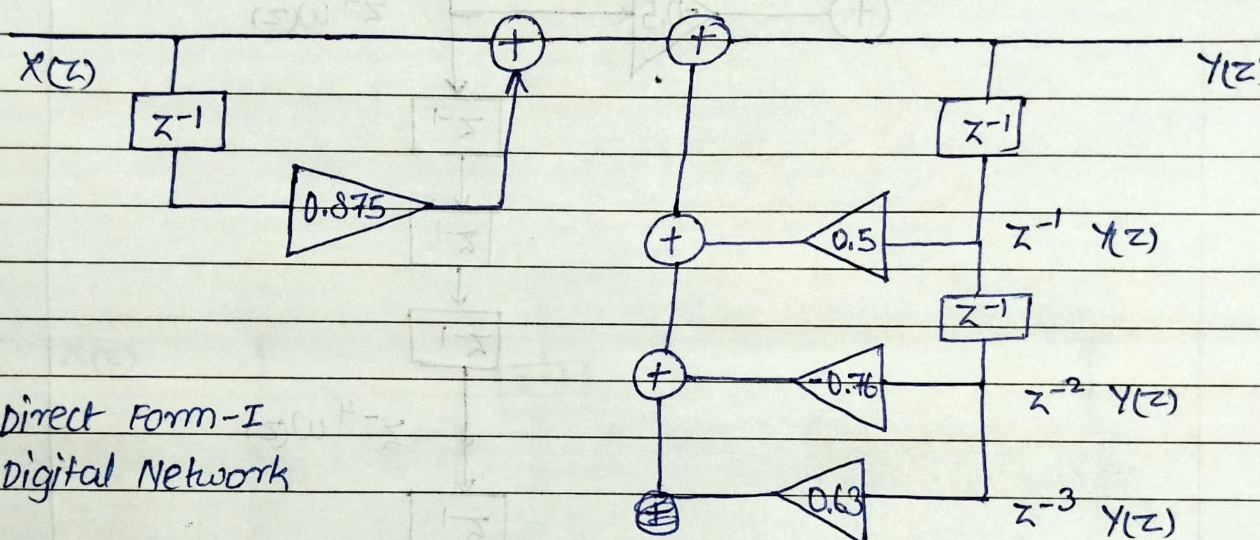
Q② Determine Direct-Form I.

$$y(n) = 0.5y(n-1) - 0.76y(n-2) + 0.63y(n-3) + x(n) + 0.875x(n-1)$$

Soln. Taking z-transform on both sides,

$$Y(z) = 0.5z^{-1}Y(z) - 0.76z^{-2}Y(z) + 0.63z^{-3}Y(z) + X(z) + 0.875z^{-1}X(z)$$



Direct Form-I
Digital Network

Q③. Determine Direct Form-II

(i). $2y(n) - y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$

$$2Y(z) - z^{-1}Y(z) - 4z^{-3}Y(z) = X(z) + 3z^{-5}X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1+3z^{-5}}{2+z^{-1}-4z^{-3}} = \frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{2+z^{-1}-4z^{-3}} \quad \text{—①} \quad \&$$

$$\frac{Y(z)}{W(z)} = 1+3z^{-5} \quad \text{—②}$$

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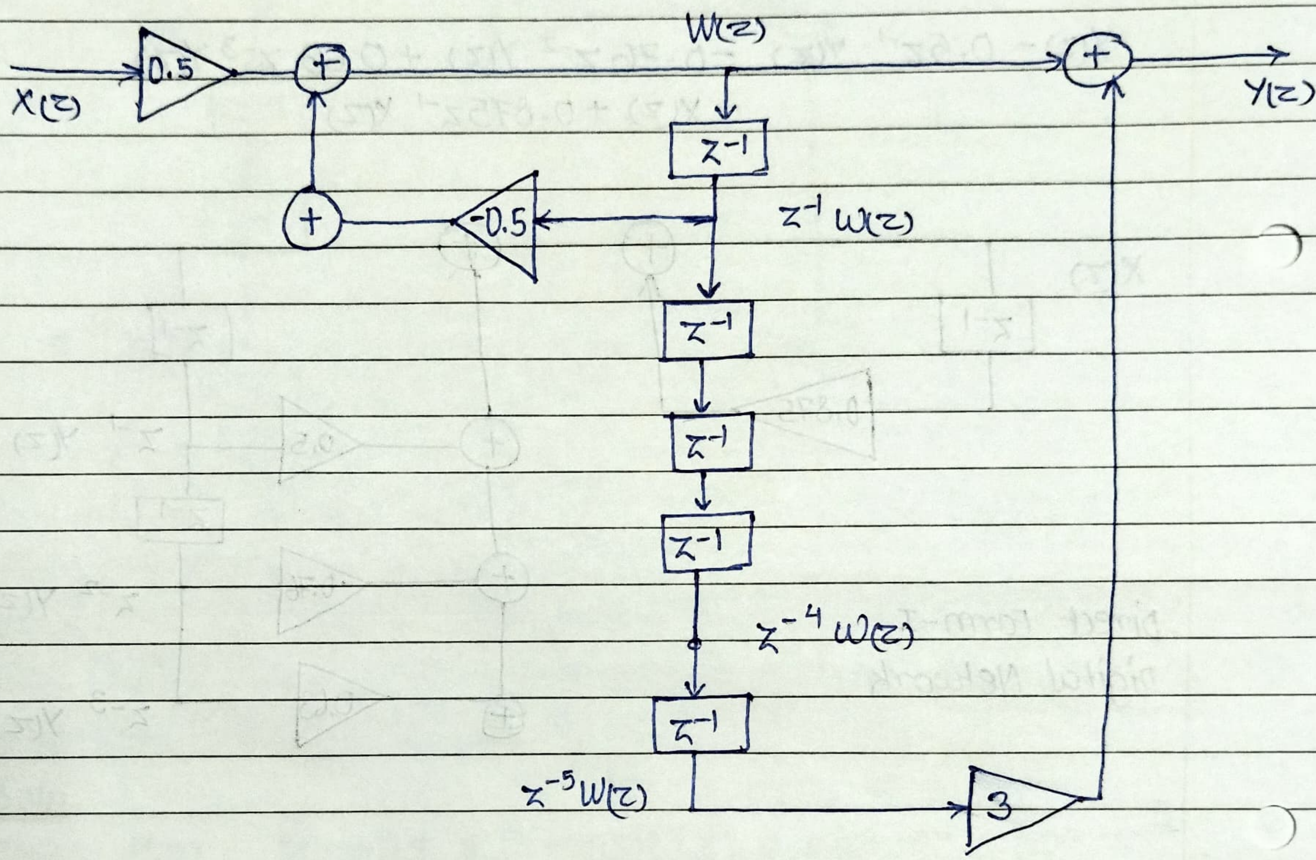
From ①, $W(z) = 0.5X(z) - 0.5z^{-1}W(z) + 2z^{-3}W(z)$ — ③

From ②, $Y(z) = W(z) + 3z^{-5}W(z)$ — ④

From ③ and ④,

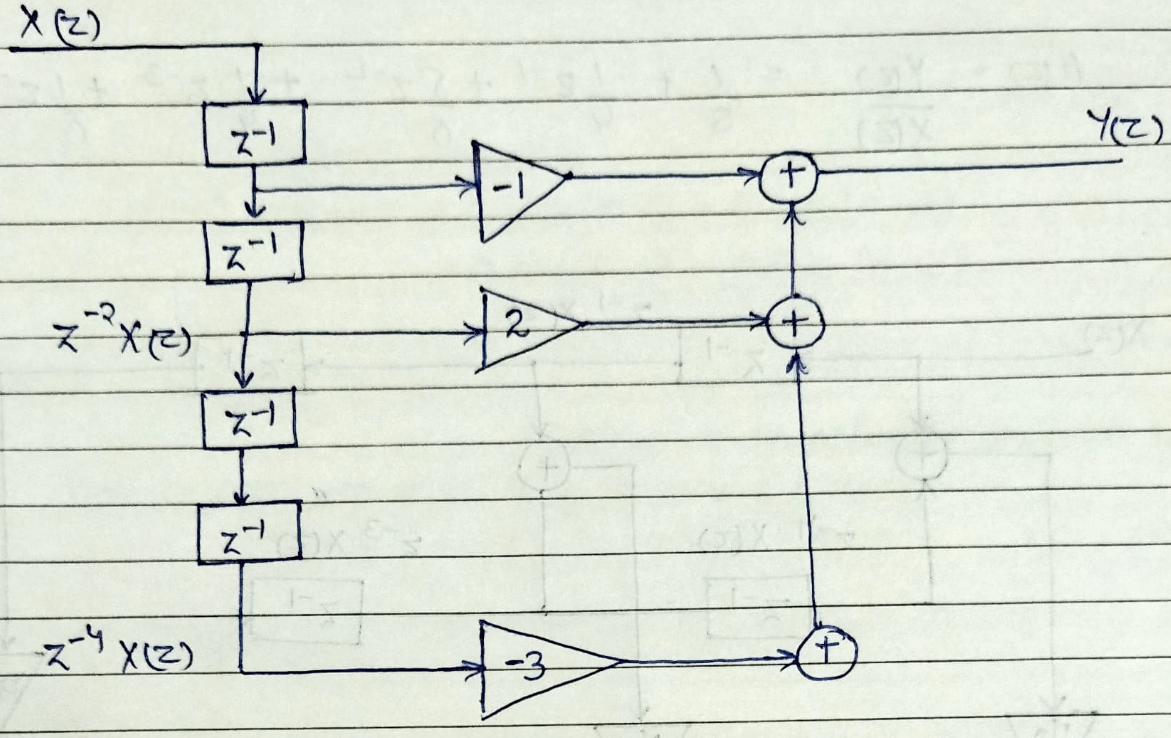


(ii). $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$

Taking z-transform on both sides,

$$Y(z) = X(z) - z^{-1}X(z) + 2z^{-2}X(z) - 3z^{-4}X(z)$$

$$\frac{Y(z)}{X(z)} = 1 - z^{-1} + 2z^{-2} - 3z^{-4} \quad \therefore W(z) = X(z)$$



Q.④ Realise the following FIR filter using minimum number of multipliers.

$$H(z) = \frac{1}{8} + \frac{1}{4}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{8}z^{-4} \quad \text{— ①}$$

Soln.

$$H(z) = \sum_{n=0}^{\infty} h(n).z^{-n} = h(0).z^{0} + h(1).z^{-1} + h(2).z^{-2} + h(3).z^{-3} + h(4).z^{-4} \quad \text{— ②}$$

on comparing ① and ②

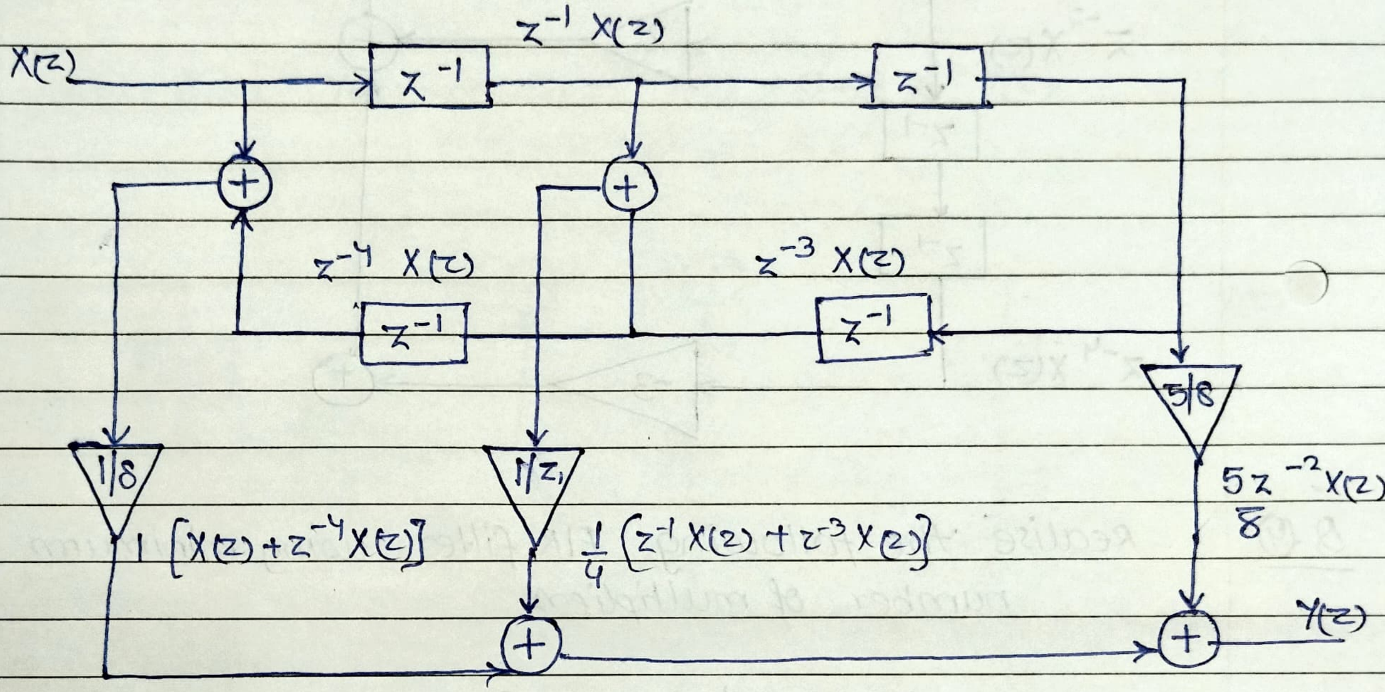
$$h(n) = \left\{ \frac{1}{8}, \frac{1}{4}, \frac{5}{8}, \frac{1}{4}, \frac{1}{8} \right\}$$

h(n) satisfies the condition $h(n) = h(N-1-n)$

∴ impulse response in symmetrical

Hence the system has linear phase and can be realised with minimum no. of multipliers.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{8} + \frac{1}{4}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{8}z^{-4}$$



$$Y(z) = \frac{1}{8}\left[X(z) + z^{-4}X(z)\right] + \frac{1}{4}\left[z^{-1}X(z) + z^{-3}X(z)\right]$$

$$+ \frac{5}{8}\left[z^{-3}.\,X(z)\right]$$