* Mean and Variance :-

we know that $E(X) = \sum_i p_i x_i$ or $\int_{-\infty}^{\infty} x f(x)\,dx$

$$\Rightarrow \quad E(X^2) = \sum_i p_i x_i^2 \text{ or } \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

Note that : ① $\boxed{\text{Mean} = E(X)}$

② Variance of X is denoted as Var.(X) and is defined as

$$\boxed{\text{Var.}(X) = E(X^2) - [E(X)]^2}$$

* Examples on Discrete probability distribution :

Example 1. A discrete random variable has the probability density function given below.

X	:	−2	−1	0	1	2	3
P(X = x)	:	0.2	k	0.1	2k	0.1	2k

find k, the mean and variance.

Solution: Note that the Total probability is 1

$$\therefore \sum p_i = 1$$

$$\Rightarrow 0.2 + k + 0.1 + 2k + 0.1 + 2k = 1$$

$$\Rightarrow 5k + 0.4 = 1$$

$$\Rightarrow 5k = 0.6 \Rightarrow k = \frac{0.6}{5} = \frac{6}{50} = \frac{3}{25}$$

$$\Rightarrow \boxed{k = \frac{3}{25}}$$

Therefore The probability distribution is

X	:	-2	-1	0	1	2	3
$P(X=x)$	:	$\frac{2}{10}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{6}{25}$	$\frac{1}{10}$	$\frac{6}{25}$

* Mean $= E(X) = \sum_{i=1}^{6} p_i x_i$

$$= \left(\frac{2}{10}\right)(-2) + \left(\frac{3}{25}\right)(-1) + \left(\frac{1}{10}\right)(0) + \left(\frac{6}{25}\right)(1) + \left(\frac{1}{10}\right)(2) + \left(\frac{6}{25}\right)(3)$$

$$= -\frac{4}{10} - \frac{3}{25} + 0 + \frac{6}{25} + \frac{2}{10} + \frac{18}{25}$$

$$= \frac{6}{25}$$

* Variance $=$ Var(X) $= E(X^2) - [E(X)]^2$ —— ①

first to find $E(x^2)$:

$$E(x^2) = \sum_{i=1}^{6} p_i x_i^2$$

$$= \left(\frac{2}{10}\right)(-2)^2 + \left(\frac{3}{25}\right)(-1)^2 + \left(\frac{1}{10}\right)(0)^2 + \left(\frac{6}{25}\right)(1)^2 + \left(\frac{1}{10}\right)(2)^2 + \left(\frac{6}{25}\right)(3)^2$$

$$= \frac{8}{10} + \frac{3}{25} + 0 + \frac{6}{25} + \frac{4}{10} + \frac{54}{25}$$

$$= \frac{73}{250}$$

Also we have $E(X) = \frac{6}{25}$

therefore,

$$\text{Var.}(x) = E(X^2) - [E(X)]^2$$

$$= \frac{73}{250} - \left(\frac{6}{25}\right)^2$$

$$= \frac{73}{250} - \frac{36}{625}$$

$$= \frac{293}{625}$$

$\therefore$ mean $= \frac{6}{25}$ and variance $= \frac{293}{625}$

Example 2 If the mean of the following distribution is 16 find m, n and variance

X :	8	12	16	20	24
$P(X=x)$:	$\frac{1}{8}$	m	n	$\frac{1}{4}$	$\frac{1}{12}$

Solution: Note that the total probability is 1

$$\therefore \quad \sum_i P_i = 1$$

$$\Rightarrow \quad \frac{1}{8} + m + n + \frac{1}{4} + \frac{1}{12} = 1$$

$$\Rightarrow \quad m+n = \frac{13}{24} \quad \text{———} \quad (1)$$

since, mean $= 16$

$$\Rightarrow \quad \sum P_i x_i = 16$$

$$\Rightarrow \quad \frac{1}{8}(8) + m(12) + n(16) + \frac{1}{4}(20) + \frac{1}{12}(24) = 16$$

$$\Rightarrow \quad 12m + 16n = 8 \quad \text{———} \quad (2)$$

from (1), $\quad m = \frac{13}{24} - n$

$\therefore$ equation (2) becomes

$$12\left(\frac{13}{24} - n\right) + 16n = 8 \quad \Rightarrow \quad \frac{13}{2} - 12n + 16n = 8$$

$$\Rightarrow \quad 4n = 8 - \frac{13}{2} \quad \Rightarrow \quad n = \frac{3}{8}$$

$\therefore$ equation (1) becomes.

$$m + \frac{3}{8} = \frac{13}{24} \quad \Rightarrow \quad m = \frac{4}{24} = \frac{1}{6}$$

$$\therefore \quad \boxed{m = \frac{1}{6} \quad \text{and} \quad n = \frac{3}{8}}$$

Now probability distribution is

X :	8	12	16	20	24
$P(X=x)$:	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

$\therefore \quad E(x^2) = \sum p_i x_i^2$

$= \frac{1}{8}(8)^2 + \frac{1}{6}(12)^2 + \frac{3}{8}(16)^2 + \frac{1}{4}(20)^2 + \frac{1}{12}(24)^2$

$= 276$

$\therefore$ * Variance $= var.(X) = E(x^2) - [E(X)]^2$

$= E(x^2) - (\text{mean})^2$

$= 276 - (16)^2$

$= \underline{\underline{20}}$

* Examples on contineous distribution probability:

Example 3. If X is a contineous random variable with probability density function given by

$$f(x) = \begin{cases} k(x - x^3), & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

find k, mean and variance.

Solution: Note that the Total probability is 1

$\Rightarrow \int_0^1 f(x)\,dx = 1$

$\Rightarrow \int_0^1 k(x - x^3)\,dx = 1 \quad \Rightarrow \quad k\int_0^1 (x - x^3)\,dx = 1$

$\Rightarrow k\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 1$

$\Rightarrow k\left[\left(\frac{1}{2} - \frac{1}{4}\right) - (0 - 0)\right] = 1$

$\Rightarrow k = 4$

therefore,

$$f(x) = \begin{cases} 4(x - x^3), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

* Mean $= E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$= \int_0^1 4x(x - x^3)\, dx = 4\int_0^1 (x^2 - x^4)\, dx$$

$$= 4\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = 4\left[\left(\frac{1}{3} - \frac{1}{5}\right) - (0-0)\right]$$

$$= \frac{8}{15}$$

Now, $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

$$= \int_0^1 4x^2(x - x^3)\, dx = 4\int_0^1 (x^3 - x^5)\, dx$$

$$= 4\left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^1 = 4\left[\left(\frac{1}{4} - \frac{1}{6}\right) - (0-0)\right]$$

$$= \frac{1}{3}$$

* Variance $= \text{var.}(X) = E(X^2) - [E(X)]^2$

$$= \frac{1}{3} - \left(\frac{8}{15}\right)^2$$

$$= \frac{33}{225} = 0.1467$$

Example 4: Find the value of k, if the function

$$f(x) = kx^2(1 - x^3), \quad 0 \le x \le 1$$
$$= 0, \quad \text{otherwise}$$

is a probability density function. Also find $p(0 \le x \le \frac{1}{2})$ and the mean and variance.

Solution: Note that the total probability is 1

$$\Rightarrow \int_0^1 kx^2(1 - x^3)\, dx = 1$$

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$\Rightarrow \quad k \int_0^1 (x^2 - x^5)\,dx = k\left[\frac{x^3}{3} - \frac{x^6}{6}\right]_0^1 = 1$

$\Rightarrow \quad k\left[\left(\frac{1}{3} - \frac{1}{6}\right) - (0-0)\right] = 1 \quad \Rightarrow \quad \boxed{k = 6}$

Now, $P(0 \le x \le \frac{1}{2}) = 6\int_0^{\frac{1}{2}} (x^2 - x^5)\,dx = 6\left[\frac{x^3}{3} - \frac{x^6}{6}\right]_0^{\frac{1}{2}}$

$= 6\left[\left(\frac{(0.5)^3}{3} - \frac{(0.5)^6}{6}\right) - (0-0)\right] = \frac{15}{84}$

i.e $\quad P(0 \le x \le \frac{1}{2}) = \frac{15}{84}$

* Mean : $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = 6\int_0^1 x\, x^2(1-x^3)\,dx$

$= 6\int_0^1 (x^3 - x^6)\,dx = 6\left[\frac{x^4}{4} - \frac{x^7}{7}\right]_0^1$

$= 6\left[\left(\frac{1}{4} - \frac{1}{7}\right) - (0-0)\right] = 6\left(\frac{3}{28}\right)$

$= \frac{9}{14}$

Now, $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = 6\int_0^1 x^2 \cdot x^2(1-x^3)\,dx$

$= 6\int_0^1 (x^4 - x^7)\,dx = 6\left[\frac{x^5}{5} - \frac{x^8}{8}\right]_0^1$

$= 6\left[\left(\frac{1}{5} - \frac{1}{8}\right) - (0-0)\right] = \frac{9}{20}$

* variance = $\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{9}{20} - \left(\frac{9}{14}\right)^2$

$= \frac{9}{245}$

Homework
Example 5) A contineous random variable has probability density function $f(x) = k(x - x^2)$, $0 \le x \le 1$ find k, mean and variance

(Ans: $k = 6$, $E(X) = \frac{1}{2}$, $E(X^2) = \frac{3}{10}$, $\text{var}(X) = \frac{1}{20}$)