8

Network Functions

8.1 INTRODUCTION

A network function gives the relation between currents or voltages at different parts of the network. It is broadly classified as *driving point* and *transfer function*. It is associated with terminals and ports.

Any network may be represented schematically by a rectangular box. Terminals are needed to connect any network to any other network or for taking some measurements. Two such associated terminals are called *terminal pair* or *port*. If there is only one pair of terminals in the network, it is called a one-port network. If there are two pairs of terminals, it is called a two-port network. The port to which energy source is connected is called the *input port*. The port to which load is connected is known as the *output port*. One such network having only one pair of terminals (1 – 1′) is shown in Fig. 8.1 (a) and is called *one-port network*. Figure 8.1 (b) shows a two-port network with two pairs of terminals. The terminals 1 – 1′ together constitute a port. Similarly, the terminals 2 – 2′ constitute another port.

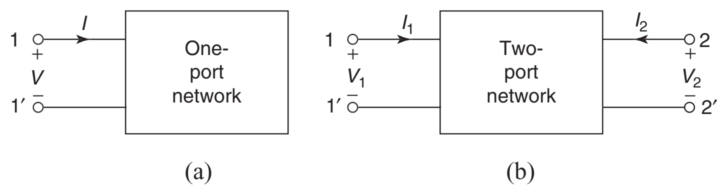


Fig. 8.1 *(a) One-port network (b) Two-port network*

A voltage and current are assigned to each of the two ports. V_1 and I_1 are assigned to the input port, whereas V_2 and I_2 are assigned to the output port. It is also assumed that currents I_1 and I_2 are entering into the network at the upper terminals 1 and 2 respectively.

8.2 DRIVING-POINT FUNCTIONS

If excitation and response are measured at the same ports, the network function is known as the driving-point function. For a one-port network, only one voltage and current are specified and hence only one network function (and its reciprocal) can be defined.

1. ***Driving-point Impedance Function*** It is defined as the ratio of the voltage transform at one port to the current transform at the same port. It is denoted by $Z(s)$.

$$Z(s) = \frac{V(s)}{I(s)}$$

2. ***Driving-point Admittance Function*** It is defined as the ratio of the current transform at one port to the voltage transform at the same port. It is denoted by $Y(s)$.

$$Y(s) = \frac{I(s)}{V(s)}$$

For a two-port network, the driving-point impedance function and driving-point admittance function at port 1 are

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

Similarly, at port 2,

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

$$Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

8.3 TRANSFER FUNCTIONS

The transfer function is used to describe networks which have at least two ports. It relates a voltage or current at one port to the voltage or current at another port. These functions are also defined as the ratio of a response transform to an excitation transform. Thus, there are four possible forms of transfer functions.

1. ***Voltage Transfer Function*** It is defined as the ratio of the voltage transform at one port to the voltage transform at another port. It is denoted by $G(s)$.

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)}$$

$$G_{21}(s) = \frac{V_1(s)}{V_2(s)}$$

2. ***Current Transfer Function*** It is defined as the ratio of the current transform at one port to the current transform at another port. It is denoted by $\alpha(s)$.

$$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)}$$

$$\alpha_{21}(s) = \frac{I_1(s)}{I_2(s)}$$

3. ***Transfer Impedance Function*** It is defined as the ratio of the voltage transform at one port to the current transform at another port. It is denoted by $Z(s)$.

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

$$Z_{21}(s) = \frac{V_1(s)}{I_2(s)}$$

4. ***Transfer Admittance Function*** It is defined as the ratio of the current transform at one port to the voltage transform at another port. It denoted by Y(s).

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)}$$

$$Y_{21}(s) = \frac{I_1(s)}{V_2(s)}$$

Example 8. 1 *Determine the driving-point impedance function of a one-port network shown in Fig. 8.2.*

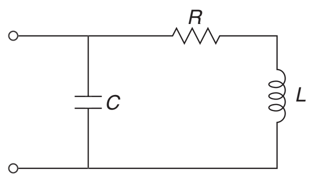


Fig. 8.2

Solution The transformed network is shown in Fig. 8.3.

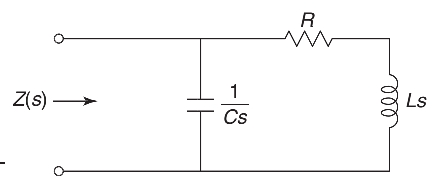


Fig. 8.3

$$Z(s) = \frac{\frac{1}{Cs}(R+Ls)}{\frac{1}{Cs}+(R+Ls)} = \frac{R+Ls}{LCs^2+RCs+1} = \frac{1}{C}\frac{s+\frac{R}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

Example 8.2 *Determine the driving-point impedance of the network shown in Fig. 8.4.*

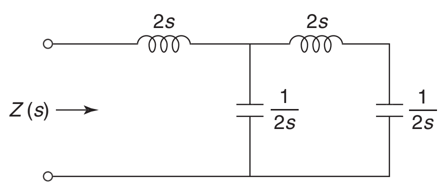


Fig. 8.4

Solution

$$Z(s) = 2s + \frac{\frac{1}{2s}\left(2s+\frac{1}{2s}\right)}{\frac{1}{2s}+2s+\frac{1}{2s}} = 2s + \frac{\frac{1}{2s}\left(2s+\frac{1}{2s}\right)}{\frac{2+4s^2}{2s}} = 2s + \frac{2s+\frac{1}{2s}}{2+4s^2} = \frac{4s+8s^3+2s+\frac{1}{2s}}{2+4s^2} = \frac{16s^4+12s^2+1}{8s^3+4s}$$

Example 8.3 *Determine the driving-point impedance of the network shown in Fig. 8.5.*

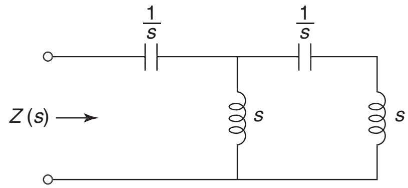


Fig. 8.5

Solution

$$Z(s) = \frac{1}{s} + \frac{s\left(\frac{1}{s}+s\right)}{s+\frac{1}{s}+s} = \frac{1}{s} + \frac{(1+s^2)s}{2s^2+1} = \frac{1}{s} + \frac{s+s^3}{2s^2+1} = \frac{s^4+3s^2+1}{2s^3+s}$$

Example 8.4 *Find the driving-point admittance function of the network shown in Fig. 8.6.*

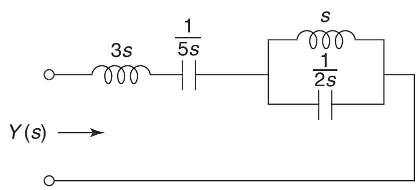


Fig. 8.6

Solution

$$Z(s) = 3s + \frac{1}{5s} + \frac{s\left(\frac{1}{2s}\right)}{s+\frac{1}{2s}} = 3s + \frac{1}{5s} + \frac{s}{2s^2+1} = \frac{30s^4+15s^2+2s^2+1+5s^2}{5s\,(2s^2+1)} = \frac{30s^4+22s^2+1}{5s\,(2s^2+1)}$$

$$Y(s) = \frac{1}{Z(s)} = \frac{5s\,(2s^2+1)}{30s^4+22s^2+1}$$

Example 8.5 *Find the transfer impedance function $Z_{12}(s)$ for the network shown in Fig. 8.7.*

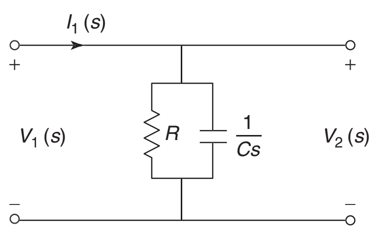


Fig. 8.7

Solution

$$V_2(s) = I_1(s)\frac{R\left(\dfrac{1}{Cs}\right)}{R+\dfrac{1}{Cs}}$$

$$\frac{V_2(s)}{I_1(s)} = \frac{R}{RCs+1}$$

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)} = \frac{1}{C\left(s+\dfrac{1}{RC}\right)}$$

Example 8.6 *Find voltage transfer function of the two-port network shown in Fig. 8.8.*

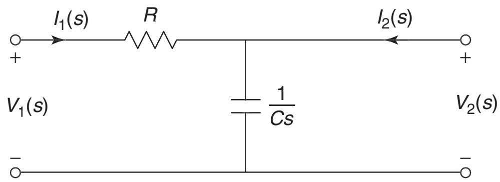


Fig. 8.8

Solution By voltage division rule,

$$V_2(s) = V_1(s)\frac{\dfrac{1}{Cs}}{R+\dfrac{1}{Cs}} = V_1(s)\frac{1}{RCs+1} = V_1(s)\frac{\dfrac{1}{RC}}{s+\dfrac{1}{RC}}$$

Voltage transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{\dfrac{1}{RC}}{s+\dfrac{1}{RC}}$$

8.4 ANALYSIS OF LADDER NETWORKS

The network functions of a ladder network can be obtained by a simple method. This method depends upon the relationships that exist between the branch currents and node voltages of the ladder network. Consider the network shown in Fig. 8.9 where all the impedances are connected in series branches and all the admittances are connected in parallel branches.

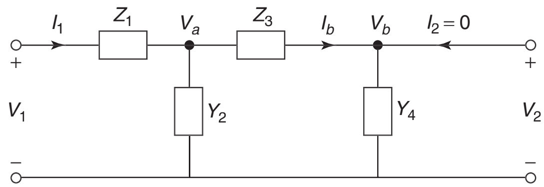


Fig. 8.9 *Ladder network*

Analysis is done by writing the set of equations. In writing these equations, we begin at the port 2 of the ladder and work towards the port 1.

$$V_b = V_2$$
$$I_b = Y_4\,V_2$$
$$V_a = Z_3\,I_b + V_2 = (Z_3Y_4 + 1)\,V_2$$
$$I_1 = Y_2\,V_a + I_b = [Y_2\,(Z_3\,Y_4 + 1) + Y_4]\,V_2$$
$$V_1 = Z_1\,I_1 + V_a = [Z_1\,\{Y_2\,(Z_3\,Y_4 + 1) + Y_4\} + (Z_3\,Y_4 + 1)]\,V_2$$

Thus, each succeeding equation takes into account one new impedance or admittance. Except the first two equations, each subsequent equation is obtained by multiplying the equation just preceding it by imittance (either impedance or admittance) that is next down the line and then adding to this product the equation twice preceding it. After writing these equations, we can obtain any network function.

Example 8.7 *For the network shown in Fig. 8.10, determine transfer function* $\dfrac{V_2}{V_1}$.

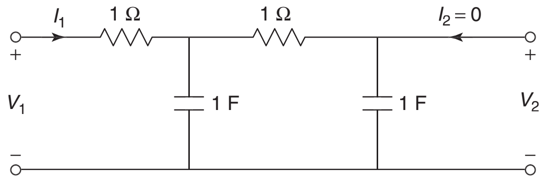


Fig. 8.10

Solution The transformed network is shown in Fig. 8.11.

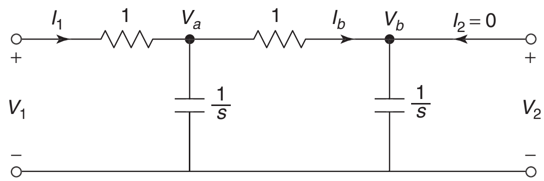


Fig. 8.11

$$V_b = V_2$$

$$I_b = \frac{V_2}{\frac{1}{s}} = sV_2$$

$$V_a = 1I_b + V_2$$
$$= sV_2 + V_2 = (s+1)V_2$$

$$I_1 = \frac{V_a}{\frac{1}{s}} + I_b = sV_a + I_b = s(s+1)V_2 + sV_2 = (s^2+2s)V_2$$

$$V_1 = 1I_1 + V_a = (s^2+2s)V_2 + (s+1)V_2 = (s^2+3s+1)V_2$$

Hence,
$$\frac{V_2}{V_1} = \frac{1}{s^2+3s+1}$$

Example 8.8 *For the network shown in Fig. 8.12, determine the voltage transfer function* $\dfrac{V_2}{V_1}$.

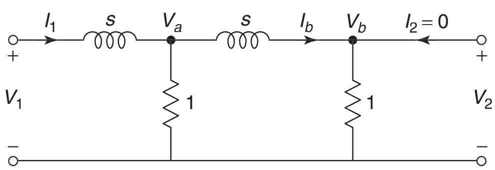


Fig. 8.12

Solution

$$V_b = V_2$$

$$I_b = \frac{V_2}{1} = V_2$$

$$V_a = sI_b + V_2 = sV_2 + V_2 = (s+1)V_2$$

$$I_1 = \frac{V_a}{1} + I_b = (s+1)V_2 + V_2 = (s+2)V_2$$

$$V_1 = sI_1 + V_a = s(s+2)V_2 + (s+1)V_2 = (s^2+3s+1)V_2$$

Hence,
$$\frac{V_2}{V_1} = \frac{1}{s^2 + 3s + 1}$$

Example 8.9 *Find the network functions* $\frac{V_1}{I_1}, \frac{V_2}{V_1}$ *and* $\frac{V_2}{I_1}$ *for the network shown in Fig. 8.13.*

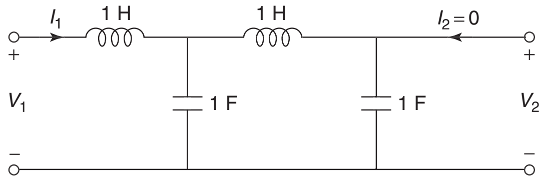


Fig. 8.13

Solution The transformed network is shown in Fig. 8.14.

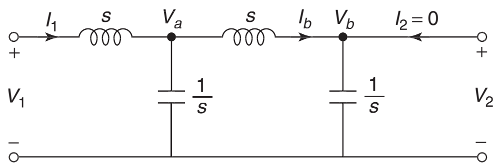


Fig. 8.14

$$V_b = V_2$$
$$I_b = \frac{V_2}{\frac{1}{s}} = sV_2$$
$$V_a = sI_b + V_2 = s(sV_2) + V_2 = (s^2 + 1)V_2$$
$$I_1 = \frac{V_a}{\frac{1}{s}} + I_b = sV_a + I_b = s(s^2 + 1)V_2 + sV_2 = (s^3 + 2s)V_2$$
$$V_1 = sI_1 + V_a = s(s^3 + 2s)V_2 + (s^2 + 1)V_2 = (s^4 + 2s^2 + s^2 + 1)V_2 = (s^4 + 3s^2 + 1)V_2$$

Hence,
$$\frac{V_1}{I_1} = \frac{s^4 + 3s^2 + 1}{s^3 + 2s}$$
$$\frac{V_2}{V_1} = \frac{1}{s^4 + 3s^2 + 1}$$
$$\frac{V_2}{I_1} = \frac{1}{s^3 + 2s}$$

Example 8.10 *Find the network functions* $\frac{V_1}{I_1}, \frac{V_2}{V_1}$*, and* $\frac{V_2}{I_1}$ *for the network in Fig. 8.15.*

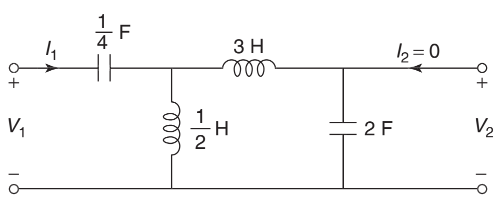


Fig. 8.15

Solution The transformed network is shown in Fig. 8.16.

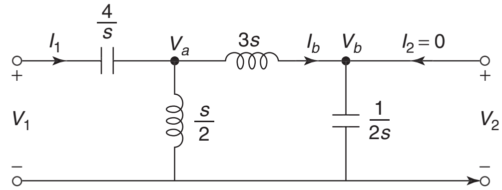


Fig. 8.16

$$V_b = V_2$$

$$I_b = \frac{V_2}{\frac{1}{2s}} = 2s V_2$$

$$V_a = 3s\, I_b + V_2 = 3s(2sV_2) + V_2 = (6s^2+1)V_2$$

$$I_1 = \frac{V_a}{\frac{s}{2}} + I_b = \frac{2}{s}(6s^2+1)V_2 + 2sV_2 = \left(\frac{14s^2+2}{s}\right)V_2$$

$$V_1 = \frac{4}{s}I_1 + V_a = \frac{4}{s}\left(\frac{14s^2+2}{s}\right)V_2 + (6s^2+1)V_2 = \left(\frac{6s^4+57s^2+8}{s^2}\right)V_2$$

Hence,

$$\frac{V_1}{I_1} = \frac{6s^4+57s^2+8}{14s^3+2s}$$

$$\frac{V_2}{V_1} = \frac{s^2}{6s^4+57s^2+8}$$

$$\frac{V_2}{I_1} = \frac{s}{14s^2+2}$$

Example 8.11 *For the ladder network of Fig. 8.17, find the driving point-impedance at the 1 – 1′ terminal with 2 – 2′ open.*

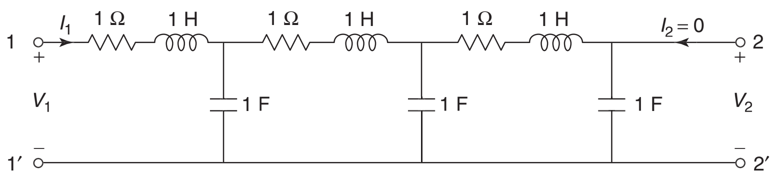


Fig. 8.17

Solution The transformed network is shown in Fig. 8.18.

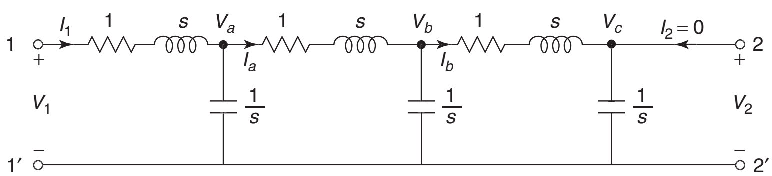


Fig. 8.18

$$V_c = V_2$$

$$I_b = \frac{V_2}{\frac{1}{s}} = s\, V_2$$

$$V_b = (s+1)\, I_b + V_2 = (s+1)\, sV_2 + V_2 = (s^2 + s + 1)V_2$$

$$I_a = \frac{V_b}{\frac{1}{s}} + I_b = sV_b + I_b = s(s^2 + s + 1)V_2 + sV_2 = (s^3 + s^2 + 2s)V_2$$

$$V_a = (s+1)\, I_a + V_b = (s+1)(s^3 + s^2 + 2s)V_2 + (s^2 + s + 1)V_2 = (s^4 + 2s^3 + 4s^2 + 3s + 1)V_2$$

$$I_1 = \frac{V_a}{\frac{1}{s}} + I_a = sV_a + I_a = s(s^4 + 2s^3 + 4s^2 + 3s + 1)V_2 + (s^3 + s^2 + 2s)V_2$$

$$= (s^5 + 2s^4 + 5s^3 + 4s^2 + 3s)V_2$$

$$V_1 = (s+1)\, I_1 + V_a = (s+1)(s^5 + 2s^4 + 5s^3 + 4s^2 + 3s)V_2 + (s^4 + 2s^3 + 4s^2 + 3s + 1)V_2$$

$$= (s^6 + 3s^5 + 8s^4 + 11s^3 + 11s^2 + 6s + 1)V_2$$

Hence, $$Z_{11} = \frac{V_1}{I_1} = \frac{s^6 + 3s^5 + 8s^4 + 11s^3 + 11s^2 + 6s + 1}{s^5 + 2s^4 + 5s^3 + 4s^2 + 3s}$$

Example 8.12 *Determine the voltage transfer function* $\frac{V_2}{V_1}$ *for the network shown in Fig. 8.19.*

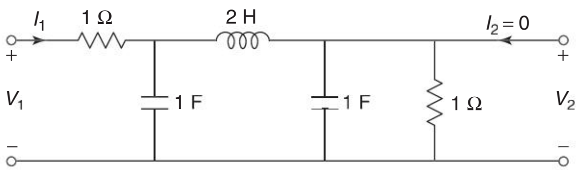


Fig. 8.19

Solution The transformed network is shown in Fig. 8.20.

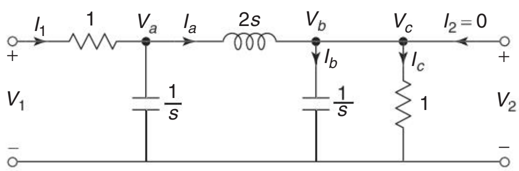


Fig. 8.20

$$V_c = V_b = V_2$$

$$I_a = I_b + I_c = \frac{V_2}{\frac{1}{s}} + \frac{V_2}{1} = s\, V_2 + V_2 = (s+1)V_2$$

$$V_a = 2s\, I_a + V_2 = 2s(s+1)V_2 + V_2$$

$$= (2s^2 + 2s + 1)V_2$$

$$I_1 = \frac{V_a}{\frac{1}{s}} + I_a = sV_a + I_a = s(2s^2 + 2s + 1)V_2 + (s+1)V_2 = (2s^3 + 2s^2 + 2s + 1)V_2$$

$$V_1 = 1I_1 + V_a = (2s^3 + 2s^2 + 2s + 1)V_2 + (2s^2 + 2s + 1)V_2 = (2s^3 + 4s^2 + 4s + 2)V_2$$

Hence, $$\frac{V_2}{V_1} = \frac{1}{2s^3 + 4s^2 + 4s + 2}$$

Example 8.13 *For the network shown in Fig. 8.21, determine the transfer function* $\dfrac{I_2}{V_1}$.

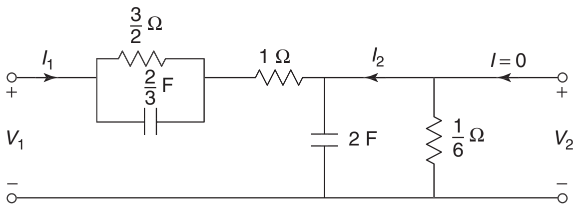


Fig. 8.21

Solution The transformed network is shown in Fig. 8.22.

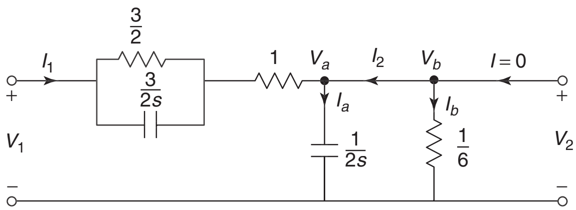


Fig. 8.22

$$V_b = V_a = V_2$$

$$I_1 = I_a + I_b = \frac{V_2}{\frac{1}{2s}} + \frac{V_2}{\frac{1}{6}} = 2sV_2 + 6V_2 = (2s+6)V_2$$

$$V_1 = \left(\frac{\frac{3}{2}\times\frac{3}{2s}}{\frac{3}{2}+\frac{3}{2s}} + 1\right) I_1 + V_2 = \left(\frac{9}{6s+6}+1\right) I_1 + V_2 = \frac{(6s+15)}{6s+6}(2s+6)V_2 + V_2 = \left[\frac{(2s+5)(s+3)}{(s+1)}+1\right]V_2$$

$$= \left[\frac{(2s+5)(s+3)+(s+1)}{s+1}\right]V_2 = \left(\frac{2s^2+6s+5s+15+s+1}{s+1}\right)V_2 = \left(\frac{2s^2+12s+16}{s+1}\right)V_2$$

$$= \frac{2(s^2+6s+8)}{s+1}V_2 = \frac{2(s+4)(s+2)}{s+1}V_2$$

Also, $\quad I_2 = -I_b = -6\,V_2$

$$\frac{I_2}{V_1} = -\frac{3(s+1)}{(s+4)(s+2)}$$

Example 8.14 *For the network shown in Fig. 8.23, compute* $\alpha_{12}(s) = \dfrac{I_2}{I_1}$ *and* $Z_{12}(s) = \dfrac{V_2}{I_1}$.

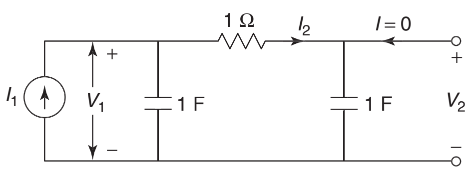


Fig. 8.23

Solution The transformed network is shown in Fig. 8.24.

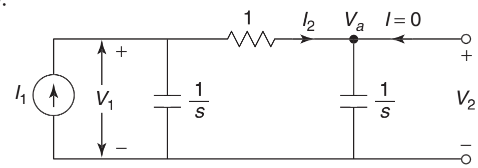


Fig. 8.24

$$V_a = V_2$$

$$I_2 = \frac{V_2}{\frac{1}{s}} = sV_2$$

$$V_1 = 1I_2 + V_a = sV_2 + V_2 = (s+1)V_2$$

$$I_1 = \frac{V_1}{\frac{1}{s}} + I_2 = sV_1 + I_2 = s(s+1)V_2 + sV_2 = (s^2 + 2s)V_2$$

Hence, $$\alpha_{12}(s) = \frac{I_2}{I_1} = \frac{1}{s+2}$$

and $$Z_{12}(s) = \frac{V_2}{I_1} = \frac{1}{s^2 + 2s}$$

Example 8.15 *Determine the voltage ratio $\frac{V_2}{V_1}$, current ratio $\frac{I_2}{I_1}$, transfer impedance $\frac{V_2}{I_1}$ and driving- point impedance $\frac{V_1}{I_1}$ for the network shown in Fig. 8.25.*

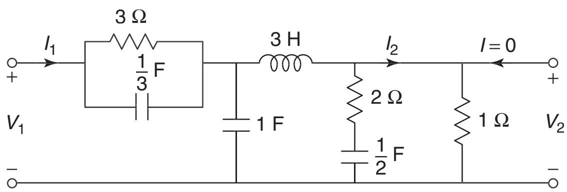


Fig. 8.25

Solution The transformed network is shown in Fig. 8.26.

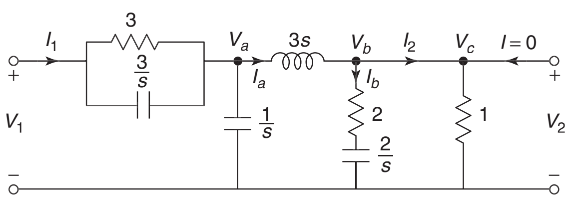


Fig. 8.26

$$V_c = V_b = V_2$$

$$I_2 = \frac{V_2}{1} = V_2$$

$$I_a = I_b + I_2 = \frac{V_2}{2 + \frac{2}{s}} + \frac{V_2}{1} = \frac{s}{2s+2}V_2 + V_2 = \left(\frac{3s+2}{2s+2}\right)V_2$$

$$V_a = 3s\,I_a + V_2 = \frac{3s(3s+2)}{2s+2}V_2 + V_2 = \left(\frac{9s^2 + 8s + 2}{2s+2}\right)V_2$$

$$I_1 = \frac{V_a}{\frac{1}{s}} + I_a = sV_a + I_a = \frac{s(9s^2+8s+2)}{2s+2}V_2 + \left(\frac{3s+2}{2s+2}\right)V_2 = \left(\frac{9s^3+8s^2+5s+2}{2s+2}\right)V_2$$

$$V_1 = \left(\frac{3\times\frac{3}{s}}{3+\frac{3}{s}}\right)I_1 + V_a = \left(\frac{3}{s+1}\right)I_1 + V_a = \left(\frac{3}{s+1}\right)\left(\frac{9s^3+8s^2+5s+2}{2s+2}\right)V_2 + \left(\frac{9s^2+8s+2}{2s+2}\right)V_2$$

$$= \left[\frac{27s^3+24s^2+15s+6+9s^3+8s^2+2s+9s^2+8s+2}{(s+1)(2s+2)}\right]V_2 = \frac{(36s^3+41s^2+25s+8)}{(s+1)(2s+2)}V_2$$

$$= \left(\frac{36s^3+41s^2+25s+8}{2s^2+4s+2}\right)V_2$$

Hence,

$$\frac{V_2}{V_1} = \frac{2s^2+4s+2}{36s^3+41s^2+25s+8}$$

$$\frac{I_2}{I_1} = \frac{2s+2}{9s^3+8s^2+5s+2}$$

$$\frac{V_2}{I_1} = \frac{2s+2}{9s^3+8s^2+5s+2}$$

$$\frac{V_1}{I_1} = \frac{36s^3+41s^2+25s+8}{(s+1)(9s^3+8s^2+5s+2)} = \frac{36s^3+41s^2+25s+8}{9s^4+17s^3+13s^2+7s+2}$$

Example 8.16 *For the resistive two-port network of Fig. 8.27, find* $\frac{V_2}{V_1}, \frac{V_2}{I_1}, \frac{I_2}{V_1}$ *and* $\frac{I_2}{I_1}$.

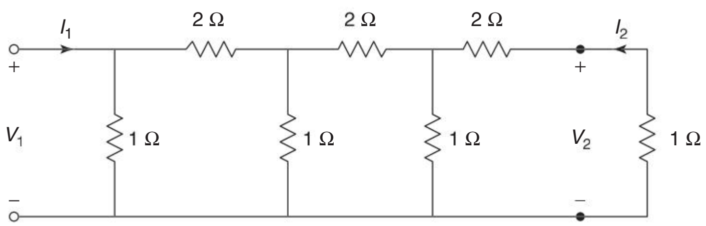


Fig. 8.27

Solution The network is redrawn as shown in Fig. 8.28.

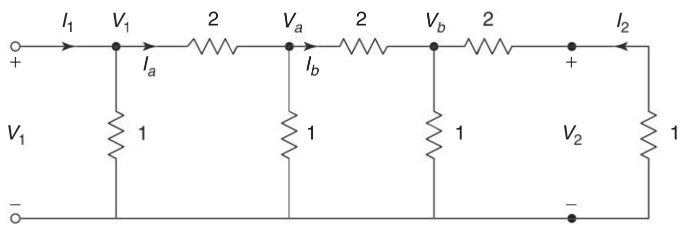


Fig. 8.28

$$I_2 = -\frac{V_2}{1} = -V_2$$
$$V_b = -3I_2 = 3V_2$$
$$I_b = \frac{V_b}{1} + \frac{V_b}{3} = \frac{4}{3}V_b = 4V_2$$
$$V_a = 2I_b + V_b = 8V_2 + 3V_2 = 11V_2$$
$$I_a = \frac{V_a}{1} + I_b = 11V_2 + 4V_2 = 15V_2$$
$$V_1 = 2I_a + V_a = 30V_2 + 11V_2 = 41V_2$$
$$I_1 = \frac{V_1}{1} + I_a = 41V_2 + 15V_2 = 56V_2$$

Hence,
$$\frac{V_2}{V_1} = \frac{1}{41}$$
$$\frac{V_2}{I_1} = \frac{1}{56}\,\Omega$$
$$\frac{I_2}{V_1} = -\frac{1}{41}\,\mho$$
$$\frac{I_2}{I_1} = -\frac{1}{56}$$

Example 8.17 *Find the network function* $\frac{V_2}{V_1}$ *for the network shown in Fig. 8.29.*

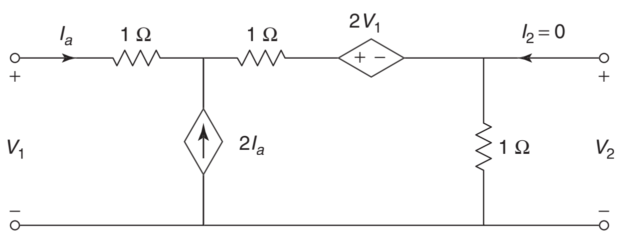


Fig. 8.29

Solution The network is redrawn as shown in Fig. 8.30.

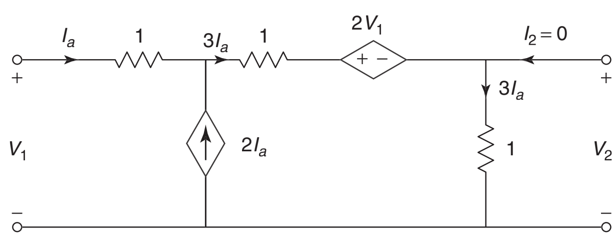


Fig. 8.30

From Fig. 8.30,

$$V_2 = 1\,(3\,I_a) = 3\,I_a$$

Applying KVL to the outermost loop,

$$V_1 - 1\,(I_a) - 1\,(3I_a) - 2\,V_1 - 1\,(3I_a) = 0$$

$$V_1 = -7\,I_a$$

Hence,

$$\frac{V_2}{V_1} = -\frac{3}{7}$$

Example 8.18 *Find the network function* $\frac{I_2}{I_1}$ *for the network shown in Fig. 8.31.*

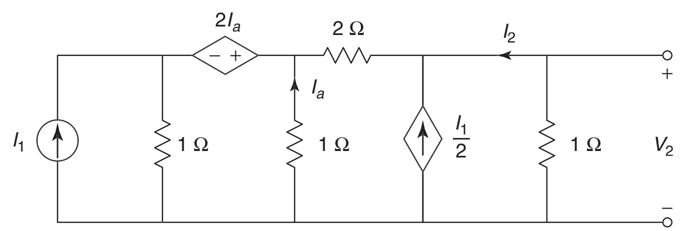


Fig. 8.31

Solution The network is redrawn as shown in Fig. 8.32.

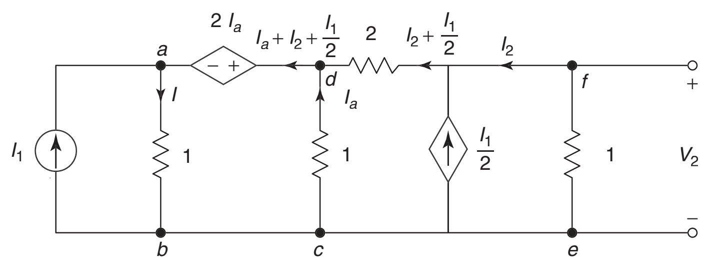


Fig. 8.32

From Fig. 8.32,

$$I = I_1 + I_a + I_2 + \frac{I_1}{2}$$

$$= \frac{3}{2}I_1 + I_a + I_2 \qquad \text{...(i)}$$

Applying KVL to the loop *abcda*,

$$-1I - 1I_a - 2I_a = 0$$

$$-I - 3I_a = 0$$

$$I + 3I_a = 0 \qquad \text{...(ii)}$$

Substituting Eq. (i) in Eq. (ii),

$$\frac{3}{2}I_1 + I_a + I_2 + 3I_a = 0$$

$$\frac{3}{2}I_1 + I_2 + 4I_a = 0 \qquad \text{...(iii)}$$

Applying KVL to the loop *dcefd,*

$$1I_a - 1I_2 - 2\left(I_2 + \frac{I_1}{2}\right) = 0$$

$$I_a - 3I_2 - I_1 = 0$$

$$I_a = 3I_2 + I_1 \qquad \text{...(iv)}$$

Substituting Eq. (iv) in Eq. (iii),

$$\frac{3}{2}I_1 + I_2 + 4(3I_2 + I_1) = 0$$

$$\frac{3}{2}I_1 + I_2 + 12I_2 + 4I_1 = 0$$

$$\frac{11}{2}I_1 + 13I_2 = 0$$

$$13I_2 = -\frac{11}{2}I_1$$

Hence,

$$\frac{I_2}{I_1} = -\frac{11}{26}$$

8.5 ANALYSIS OF NON-LADDER NETWORKS

The above method is applicable for ladder networks. There are other network configurations to which the technique described is not applicable. Figure 8.33 shows one such network.

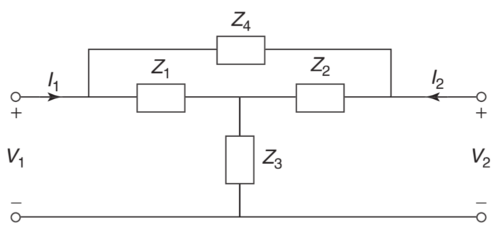


Fig. 8.33 *Non-ladder network*

For such a type of network, it is necessary to express the network functions as a quotient of determinants, formulated on KVL and KCL basis.

Example 8.19 *For the resistive bridged T network shown in Fig. 8.34, find* $\frac{V_2}{V_1}, \frac{V_2}{I_1}, \frac{I_2}{V_1}$ *and* $\frac{I_2}{I_1}$.

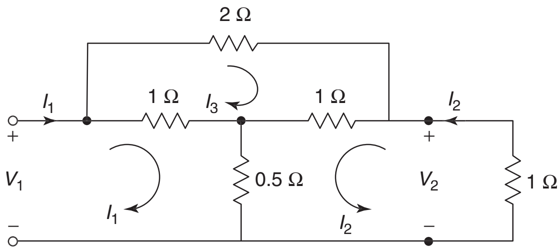


Fig. 8.34

Solution Applying KVL to Mesh 1,

$$V_1 = 1.5I_1 + 0.5I_2 - I_3 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$0 = 0.5I_1 + 2.5I_2 + I_3 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$0 = -I_1 + I_2 + 4I_3 \quad \text{...(iii)}$$

Writing these equations in matrix form,

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.5 & 0.5 & -1 \\ 0.5 & 2.5 & 1 \\ -1 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad \text{...(iv)}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} V_1 & 0.5 & -1 \\ 0 & 2.5 & 1 \\ 0 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 1.5 & 0.5 & -1 \\ 0.5 & 2.5 & 1 \\ -1 & 1 & 4 \end{vmatrix}} = \frac{V_1(10-1)}{9} = V_1 \quad \text{...(iv)}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 1.5 & V_1 & -1 \\ 0.5 & 0 & 1 \\ -1 & 0 & 4 \end{vmatrix}}{\begin{vmatrix} 1.5 & 0.5 & -1 \\ 0.5 & 2.5 & 1 \\ -1 & 1 & 4 \end{vmatrix}} = \frac{-V_1(2+1)}{9} = -\frac{1}{3}V_1 \quad \text{...(v)}$$

From Fig. 8.34, $V_2 = -1(I_2) = -I_2$

From Eq. (v), $V_1 = -3I_2$

From Eqs. (iv) and (v), $I_2 = -\frac{1}{3}V_1 = -\frac{1}{3}I_1$

$$I_1 = -3I_2$$

Hence, $\frac{I_2}{V_1} = -\frac{1}{3}\,\mho$

$$\frac{I_2}{I_1} = \frac{-\frac{1}{3}V_1}{V_1} = -\frac{1}{3}$$

$$\frac{V_2}{V_1} = \frac{-I_2}{-3I_2} = \frac{1}{3}$$

$$\frac{V_2}{I_1} = \frac{-I_2}{-3I_2} = \frac{1}{3}\,\Omega$$

Example 8.20 *For the network of Fig. 8.35, find Z_{11}, Z_{12} and G_{12}.*

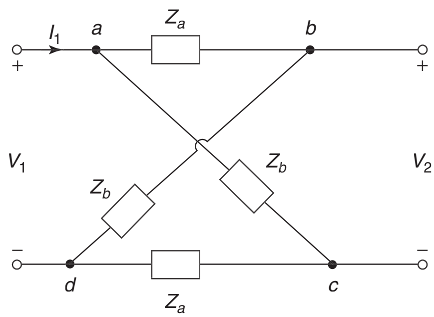


Fig. 8.35

Solution The network can be redrawn as shown in Fig. 8.36. Since the network consists of two identical impedances connected in parallel, the current in I_1 divides equally in each branch.

$$V_1 = (Z_a + Z_b)\frac{I_1}{2}$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{Z_a + Z_b}{2}$$

$$V_2 = Z_b\frac{I_1}{2} - Z_a\left(\frac{I_1}{2}\right) = (Z_b - Z_a)\frac{I_1}{2}$$

$$Z_{12} = \frac{V_2}{I_1} = \frac{Z_b - Z_a}{2}$$

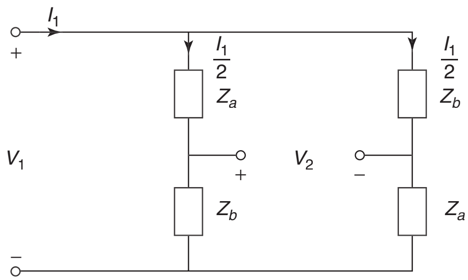


Fig. 8.36

By voltage-division rule,

$$V_2 = \frac{Z_b}{Z_a + Z_b}V_1 - \frac{Z_a}{Z_a + Z_b}V_1 = \frac{Z_b - Z_a}{Z_a + Z_b}V_1$$

$$G_{12} = \frac{V_2}{V_1} = \frac{Z_b - Z_a}{Z_a + Z_b}$$

Example 8.21 *For the network shown in Fig. 8.37, determine $Z_{11}(s)$, $G_{12}(s)$ and $Z_{12}(s)$.*

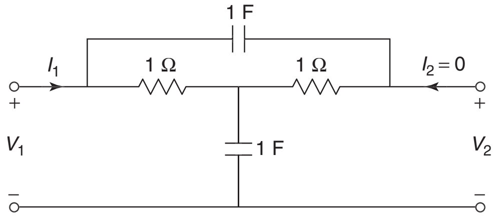


Fig. 8.37

Solution The transformed network is shown in Fig. 8.38.

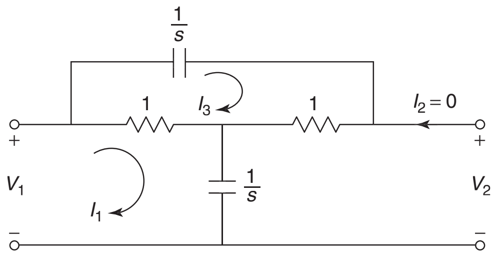


Fig. 8.38

Applying KVL to Mesh 1,

$$V_1 = \left(1 + \frac{1}{s}\right) I_1 - I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = \frac{1}{s} I_1 + I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$-I_1 + \left(2 + \frac{1}{s}\right) I_3 = 0$$

$$I_3 = \left(\frac{s}{2s+1}\right) I_1 \qquad \text{...(iii)}$$

Substituting Eq. (iii) in Eqs. (i) and (ii),

$$V_1 = \left(1 + \frac{1}{s}\right) I_1 - \left(\frac{s}{2s+1}\right) I_1 = \left(\frac{s+1}{s} - \frac{s}{2s+1}\right) I_1 = \left[\frac{s^2+3s+1}{s(2s+1)}\right] I_1$$

$$V_2 = \frac{1}{s} I_1 + \frac{s}{2s+1} I_1 = \left[\frac{s^2+2s+1}{s(2s+1)}\right] I_1$$

Hence,

$$Z_{11}(s) = \frac{V_1}{I_1} = \frac{s^2+3s+1}{s(2s+1)}$$

$$Z_{12}(s) = \frac{V_2}{I_1} = \frac{s^2+2s+1}{s(2s+1)}$$

$$G_{12}(s) = \frac{V_2}{V_1} = \frac{s^2+2s+1}{s^2+3s+1}$$

Example 8.22 *For the network shown in Fig.8.39, find the driving-point admittance Y_{11} and transfer admittance Y_{12}.*

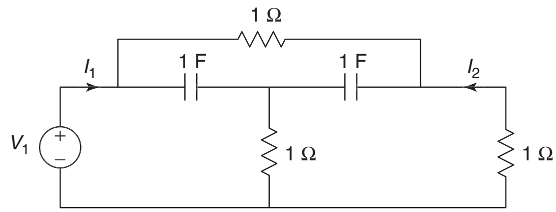


Fig. 8.39

Solution The transformed network is shown in Fig. 8.40.

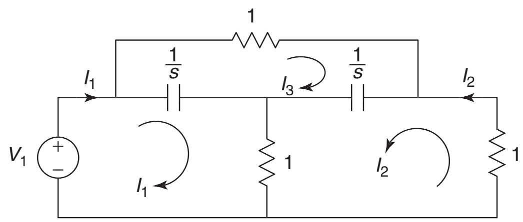


Fig. 8.40

Applying KVL to Mesh 1,

$$V_1 = \left(\frac{1}{s}+1\right)I_1 + I_2 - \frac{1}{s}I_3 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$0 = I_1 + \left(2+\frac{1}{s}\right)I_2 + \frac{1}{s}I_3 \quad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$0 = -\frac{1}{s}I_1 + \frac{1}{s}I_2 + \left(\frac{2}{s}+1\right)I_3 \quad \text{...(iii)}$$

Writing these equations in matrix from,

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s}+1 & 1 & -\frac{1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ -\frac{1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta = \begin{vmatrix} \frac{1}{s}+1 & 1 & -\frac{1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ -\frac{1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{vmatrix} = \left(\frac{1}{s}+1\right)\left[\left(2+\frac{1}{s}\right)\left(\frac{2}{s}+1\right)-\frac{1}{s^2}\right]-1\left[(1)\left(\frac{2}{s}+1\right)+\frac{1}{s^2}\right]-\frac{1}{s}\left[(1)\left(\frac{1}{s}\right)+\left(\frac{1}{s}\right)\left(2+\frac{1}{s}\right)\right]$$

$$= \frac{s^2+5s+2}{s^2}$$

$$\Delta_1 = \begin{vmatrix} V_1 & 1 & -\frac{1}{s} \\ 0 & 2+\frac{1}{s} & \frac{1}{s} \\ 0 & \frac{1}{s} & \frac{2}{s}+1 \end{vmatrix} = V_1\left[\left(2+\frac{1}{s}\right)\left(\frac{2}{s}+1\right)-\frac{1}{s^2}\right] = V_1\left(\frac{2s^2+5s+1}{s^2}\right)$$

$$I_1 = V_1\left(\frac{2s^2+5s+1}{s^2+5s+2}\right)$$

Hence, $$Y_{11} = \frac{I_1}{V_1} = \frac{2s^2+5s+1}{s^2+5s+2}$$

$$I_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{s}+1 & V_1 & -\frac{1}{s} \\ 1 & 0 & \frac{1}{s} \\ -\frac{1}{s} & 0 & \frac{2}{s}+1 \end{vmatrix} = -V_1\left(\frac{2}{s}+1+\frac{1}{s^2}\right) = -V_1\left(\frac{s^2+2s+1}{s^2}\right)$$

$$I_2 = -V_1\left(\frac{s^2+2s+1}{s^2+5s+2}\right)$$

Hence, $$Y_{12} = \frac{I_2}{V_1} = -\frac{s^2+2s+1}{s^2+5s+2}$$

8.6 POLES AND ZEROS OF NETWORK FUNCTIONS

The network function $F(s)$ can be written as ratio of two polynomials.

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1}\, s^{n-1} + \ldots + a_1\, s + a_0}{b_m s^m + b_{m-1}\, s^{m-1} + \ldots + b_1\, s + b_0}$$

where $a_0, a_1, \ldots, a_n$ and $b_0, b_1, \ldots, b_m$ are the coefficients of the polynomials $N(s)$ and $D(s)$. These are real and positive for networks of passive elements. Let $N(s) = 0$ have n roots as $z_1, z_2, \ldots\ldots, z_n$ and $D(s) = 0$ have m roots as $p_1, p_2, \ldots\ldots, p_m$. Then $F(s)$ can be written as

$$F(s) = H\frac{(s-z_1)(s-z_2)\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots(s-p_m)}$$

where $\frac{a_n}{b_m}$ is a constant called *scale factor* and $z_1, z_2, \ldots, z_n, p_1, p_2, \ldots, p_m$ are complex frequencies. When the variable s has the values $z_1, z_2, \ldots, z_n$, the network function becomes zero; such complex frequencies are known as the zeros of the network function. When the variable s has values $p_1, p_2, \ldots, p_m$, the network function becomes infinite; such complex frequencies are known as the poles of the network function. A network function is completely specified by its poles, zeros and the scale factor.

If the poles or zeros are not repeated, then the function is said to be having simple poles or simple zeros. If the poles or zeros are repeated, then the function is said to be having multiple poles or multiple zeros. When $n > m$, then $(n - m)$ zeros are at $s = \infty$, and for $m > n$, $(m - n)$ poles are at $s = \infty$.

The total number of zeros is equal to the total number of poles. For any network function, poles and zeros at zero and infinity are taken into account in addition to finite poles and zeros.

Poles and zeros are critical frequencies. The network function becomes infinite at poles, while the network function becomes zero at zeros. The network function has a finite, non-zero value at other frequencies.

Poles and zeros provide a representation of a network function as shown in Fig. 8.41. The zeros are shown by circles and the poles by crosses. This diagram is referred to as pole-zero plot.

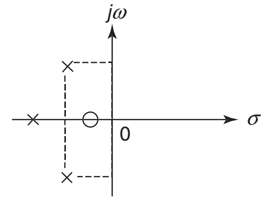


Fig. 8.41 *Pole-zero plot*

8.7 RESTRICTIONS ON POLE AND ZERO LOCATIONS FOR DRIVING-POINT FUNCTIONS [COMMON FACTORS IN *N*(*s*) AND *D*(*s*) CANCELLED]

(1) The coefficients in the polynomials $N(s)$ and $D(s)$ must be real and positive.
(2) The poles and zeros, if complex or imaginary, must occur in conjugate pairs.
(3) The real part of all poles and zeros, must be negative or zero, i.e., the poles and zeros must lie in left half of s plane.
(4) If the real part of pole or zero is zero, then that pole or zero must be simple.
(5) The polynomials $N(s)$ and $D(s)$ may not have missing terms between those of highest and lowest degree, unless all even or all odd terms are missing.
(6) The degree of $N(s)$ and $D(s)$ may differ by either zero or one only. This condition prevents multiple poles and zeros at $s = \infty$.
(7) The terms of lowest degree in $N(s)$ and $D(s)$ may differ in degree by one at most. This condition prevents multiple poles and zeros at $s = 0$.

8.8 RESTRICTIONS ON POLE AND ZERO LOCATIONS FOR TRANSFER FUNCTIONS [COMMON FACTORS IN *N*(S) AND *D*(S) CANCELLED]

(1) The coefficients in the polynomials $N(s)$ and $D(s)$ must be real, and those for $D(s)$ must be positive.
(2) The poles and zeros, if complex or imaginary, must occur in conjugate pairs.
(3) The real part of poles must be negative or zero. If the real part is zero, then that pole must be simple.
(4) The polynomial $D(s)$ may not have any missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.

(5) The polynomial $N(s)$ may have terms missing between the terms of lowest and highest degree, and some of the coefficients may be negative.
(6) The degree of $N(s)$ may be as small as zero, independent of the degree of $D(s)$.
(7) For voltage and current transfer functions, the maximum degree of $N(s)$ is the degree of $D(s)$.
(8) For transfer impedance and admittance functions, the maximum degree of $N(s)$ is the degree of $D(s)$ plus one.

Example 8.23 *Test whether the following represent driving-point immittance functions.*

(a) $\dfrac{5s^4+3s^2-2s+1}{s^3+6s+20}$ (b) $\dfrac{s^3+s^2+5s+2}{s^4+6s^3+9s^2}$ (c) $\dfrac{s^2+3s+2}{s^2+6s+2}$

Solution

(a) The numerator and denominator polynomials have a missing term between those of highest and lowest degree and one of the coefficient is negative in numerator polynomial. Hence, the function does not represent a driving-point immittance function.
(b) The term of lowest degree in numerator and denominator polynomials differ in degree by two. Hence, the function does not represent a driving-point immittance function.
(c) The function satisfies all the necessary conditions. Hence, it represents a driving-point immittance function.

Example 8.24 *Test whether the following represent transfer functions.*

(a) $G_{21}=\dfrac{3s+2}{5s^3+4s^2+1}$ (b) $\alpha_{12}=\dfrac{2s^2+5s+1}{s+7}$ (c) $Z_{21}=\dfrac{1}{s^3+2s}$

Solution

(a) The polynomial $D(s)$ has a missing term between that of highest and lowest degree. Hence, the function does not represent a transfer function.
(b) The degree of $N(s)$ is greater than $D(s)$. Hence the function does not represent a transfer function.
(c) The function satisfies all the necessary conditions. Hence, it represents a transfer function.

Example 8.25 *Obtain the pole-zero plot of the following functions.*

(a) $F(s)=\dfrac{s(s+2)}{(s+1)(s+3)}$ (b) $F(s)=\dfrac{s(s+1)}{(s+2)^2(s+3)}$

(c) $F(s)=\dfrac{s(s+2)}{(s+1+j1)(s+1-j1)}$ (d) $F(s)=\dfrac{(s+1)^2(s+5)}{(s+2)(s+3+j2)(s+3-j2)}$

(e) $F(s)=\dfrac{s^2+4}{(s+2)(s^2+9)}$

Solution

(a) The function $F(s)$ has zeros at $s = 0$ and $s = -2$ and poles at $s = -1$ and $s = -3$. The pole-zero plot is shown in Fig. 8.42.

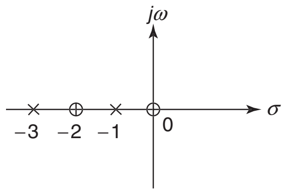


Fig. 8.42

(b) The function $F(s)$ has zeros at $s = 0$ and $s = -1$ and poles at $s = -2, -2$ and $s = -3$. The pole-zero plot is shown in Fig. 8.43.

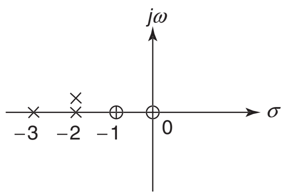


Fig. 8.43

(c) The function $F(s)$ has zeros at $s = 0$ and $s = -2$ and poles at $s = -1 - j1$ and $s = -1 + j1$. The pole-zero plot is shown in Fig. 8.44.

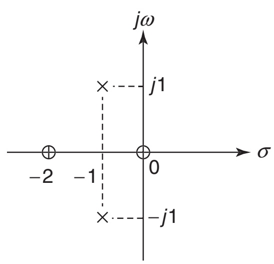


Fig. 8.44

(d) The function $F(s)$ has zeros at $s = -1, -1$ and $s = -5$ and poles at $s = -2$, $s = -3 + j2$ and $s = -3 - j2$. The pole-zero plot is shown in Fig. 8.45.

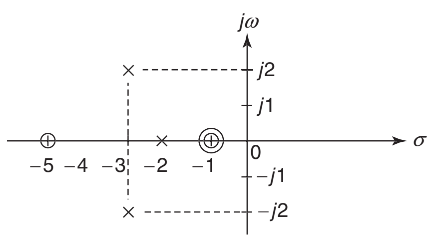


Fig. 8.45

(e) The function $F(s)$ has zeros at $s = j2$ and $s = -j2$ and poles at $s = -2$, $s = j3$ and $s = -j3$. The pole-zero plot is shown in Fig. 8.46.

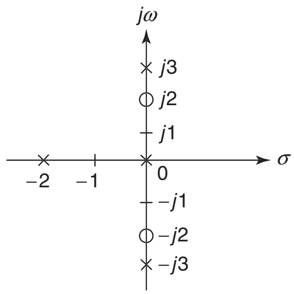


Fig. 8.46

Example 8.26 *Find poles and zeros of the impedance of the network shown in Fig. 8.47 and plot them on the s-plane.*

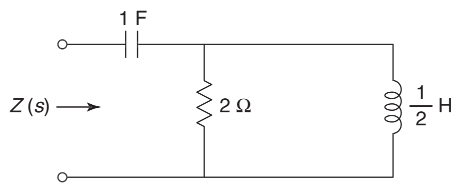


Fig. 8.47

Solution The transformed network is shown in Fig. 8.48.

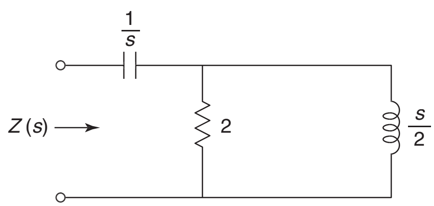


Fig. 8.48

$$Z(s) = \frac{1}{s} + \frac{\frac{s}{2}\times 2}{\frac{s}{2}+2} = \frac{1}{s} + \frac{2s}{s+4} = \frac{2s^2+s+4}{s(s+4)} = \frac{2(s^2+0.5s+2)}{s(s+4)}$$

$$= \frac{2(s+0.25+j1.4)(s+0.25-j1.4)}{s(s+4)}$$

The function $Z(s)$ has zeros at $s = -0.25 + j1.4$ and $s = -0.25 - j1.4$ and poles at $s = 0$ and $s = -4$ as shown in Fig. 8.49.

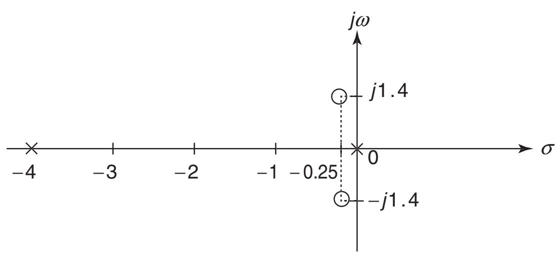


Fig. 8.49

Example 8.27 *Determine the poles and zeros of the impedance function Z (s) in the network shown in Fig. 8.50.*

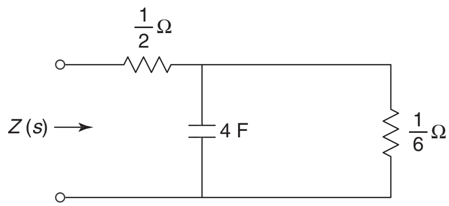


Fig. 8.50

Solution The transformed network is shown in Fig. 8.51.

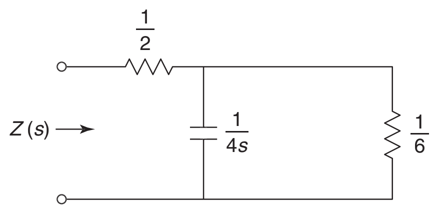


Fig. 8.51

$$Z(s)=\frac{1}{2}+\frac{\frac{1}{4s}\times\frac{1}{6}}{\frac{1}{4s}+\frac{1}{6}}=\frac{1}{2}+\frac{1}{4s+6}=\frac{4s+8}{2(4s+6)}=\frac{s+2}{2s+3}=\frac{0.5(s+2)}{s+1.5}$$

The function $Z(s)$ has zero at $s=-2$ and pole at $s=-1.5$.

Example 8.28 *Determine Z(s) in the network shown in Fig. 8.52. Find poles and zeros of Z(s) and plot them on s-plane.*

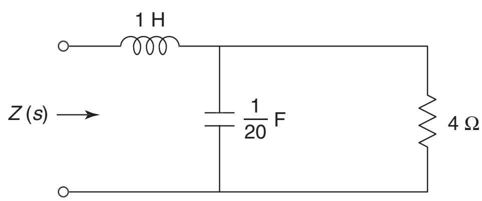


Fig. 8.52

Solution The transformed network is shown in Fig. 8.53.

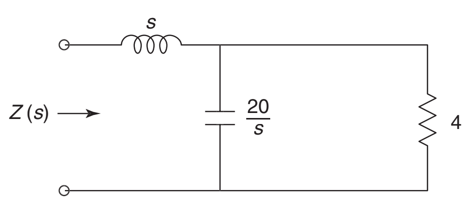


Fig. 8.53

$$Z(s) = s + \frac{\frac{20}{s} \times 4}{\frac{20}{s} + 4} = s + \frac{80}{4s + 20} = s + \frac{20}{s + 5} = \frac{s(s + 5) + 20}{s + 5} = \frac{s^2 + 5s + 20}{s + 5}$$

$$= \frac{(s + 2.5 + j3.71)(s + 2.5 - j3.71)}{s + 5}$$

The function $Z(s)$ has zeros at $s = -2.5 + j3.71$ and $s = -2.5 - j3.71$ and pole at $s = -5$. The pole-zero diagram is shown in Fig. 8.54.

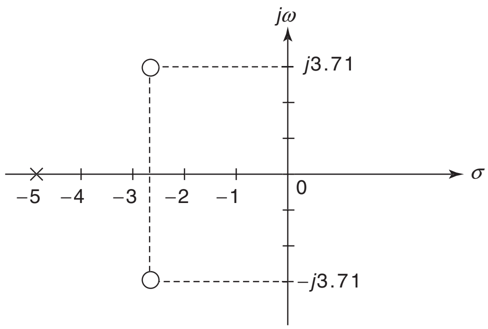


Fig. 8.54

Example 8.29 *For the network shown in Fig. 8.55, plot poles and zeros of function* $\frac{I_0}{I_i}$.

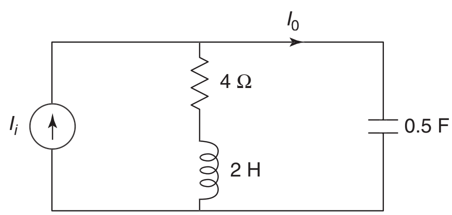


Fig. 8.55

Solution The transformed network is shown in Fig. 8.56.
By current-division rule,

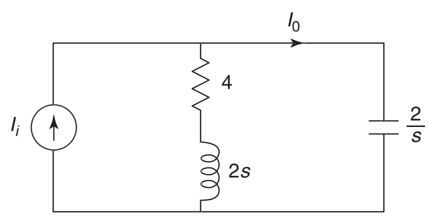


Fig. 8.56

$$I_0 = I_i \left(\frac{4 + 2s}{4 + 2s + \frac{2}{s}} \right)$$

$$\frac{I_0}{I_i} = \frac{s(4 + 2s)}{4s + 2s^2 + 2} = \frac{s(s + 2)}{s^2 + 2s + 1} = \frac{s(s + 2)}{(s + 1)(s + 1)}$$

The function has zeros at $s = 0$ and $s = -2$ and double poles at $s = -1$.
The pole-zero diagram is shown in Fig. 8.57.

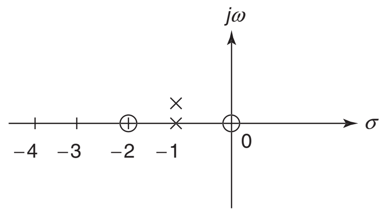


Fig. 8.57

Example 8.30 *Draw the pole-zero diagram of $\frac{I_2}{I_1}$ for the network shown in Fig. 8.58.*

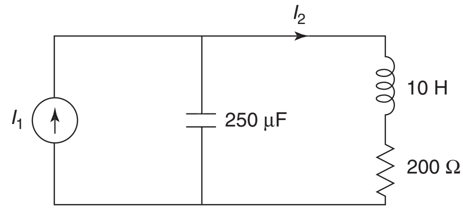


Fig. 8.58

Solution The transformed network is shown in Fig. 8.59.

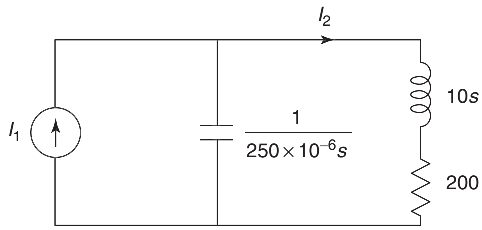


Fig. 8.59

By current-division rule,

$$I_2 = I_1 \frac{\dfrac{1}{250\times10^{-6}s}}{\dfrac{1}{250\times10^{-6}s}+10s+200}$$

$$\frac{I_2}{I_1} = \frac{400}{s^2+20s+400} = \frac{400}{(s+10-j17.32)(s+10+j17.32)}$$

The function has no zero and poles at $s = -10 + j17.32$ and $s = -10 - j17.32$.
The pole-zero diagram is shown in Fig. 8.60.

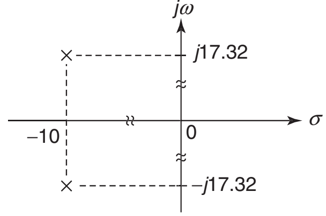


Fig. 8.60

Example 8.31 *For the network shown in Fig. 8.61, draw pole-zero plot of* $\frac{V_c}{V_1}$.

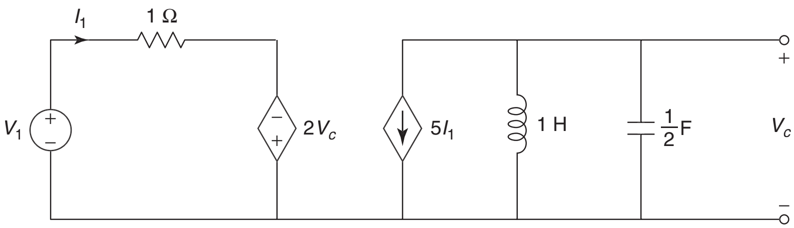


Fig. 8.61

Solution The transformed network is shown in Fig. 8.62.

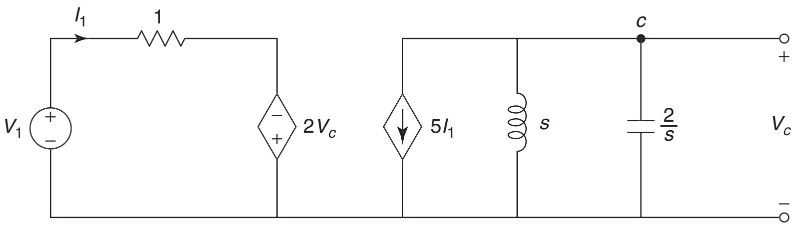


Fig. 8.62

Applying KVL to the left loop,

$$V_1 - 1I_1 + 2V_c = 0$$

$$I_1 = V_1 + 2V_c$$

Applying KCL at Node C,

$$5I_1 + \frac{V_c}{s} + \frac{V_c}{\frac{2}{s}} = 0$$

$$5(V_1 + 2V_c) + \frac{V_c}{s} + \frac{s}{2}V_c = 0$$

$$5V_1 + 10V_c + \frac{V_c}{s} + \frac{s}{2}V_c = 0$$

$$V_c\left(\frac{20s + 2 + s^2}{2s}\right) = -5V_1$$

$$\frac{V_c}{V_1} = -\frac{10s}{s^2 + 20s + 2} = -\frac{10s}{(s+0.1)(s+19.9)}$$

The function has zero at $s = 0$ and poles at $s = -0.1$ and $s = -19.9$.
The pole-zero diagram is shown in Fig. 8.63.

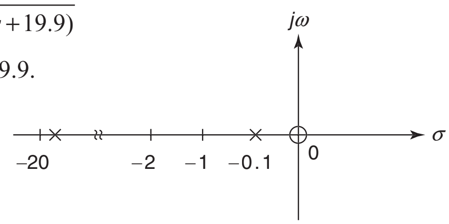


Fig. 8.63

Example 8.32 *Find the driving point admittance function and draw pole-zero plot for the network shown in Fig. 8.64.*

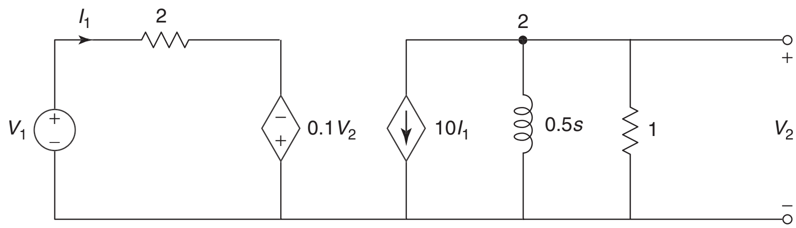


Fig. 8.64

Solution Applying KVL to the left loop,

$$V_1 - 2\,I_1 + 0.1\,V_2 = 0$$

$$I_1 = \frac{V_1 + 0.1\,V_2}{2} \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$10\,I_1 + \frac{V_2}{0.5s} + \frac{V_2}{1} = 0$$

$$10\,I_1 + \frac{2}{s}V_2 + V_2 = 0$$

$$10\,I_1 + \left(\frac{2}{s} + 1\right)V_2 = 0$$

$$10\,I_1 + \left(\frac{2+s}{s}\right)V_2 = 0$$

$$\left(\frac{s+2}{s}\right)V_2 = -10\,I_1$$

$$V_2 = -\frac{10s}{s+2}\,I_1 \qquad \text{...(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$I_1 = \frac{V_1 + 0.1\left(-\dfrac{10s}{s+2}\right)I_1}{2} = 0.5V_1 + 0.05\left(-\frac{10s}{s+2}\right)I_1$$

$$I_1\left(1 + \frac{0.5s}{s+2}\right) = 0.5V_1$$

Hence,

$$Y_{11} = \frac{I_1}{V_1} = \frac{0.5}{1 + \dfrac{0.5s}{s+2}} = \frac{0.5\,(s+2)}{s + 0.5s + 2} = \frac{0.5s + 1}{1.5s + 2}$$

The function has zero at $s = -2$ and pole at $s = -1.33$. The pole-zero diagram is shown in Fig 8.65.

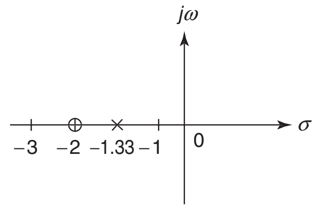


Fig. 8.65

Example 8.33 *For the network shown in Fig. 8.66, determine* $\frac{V_2}{I_g}$. *Plot the pole-zero diagram of* $\frac{V_2}{I_g}$.

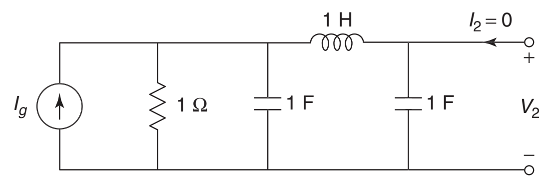


Fig. 8.66

Solution The transformed network is shown in Fig. 8.67.

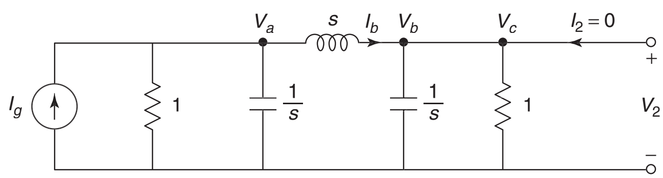


Fig. 8.67

$$V_c = V_b = V_2$$

$$I_b = \frac{V_b}{\frac{1}{s}} + \frac{V_c}{1} = sV_2 + V_2 = (s+1)V_2$$

$$V_a = sI_b + V_b = s(s+1)V_2 + V_2 = (s^2+s+1)V_2$$

$$I_g = \frac{V_a}{1} + \frac{V_a}{\frac{1}{s}} + I_b = (s^2+s+1)V_2 + s(s^2+s+1)V_2 + (s+1)V_2 = (s^3+2s^2+3s+2)V_2$$

Hence,
$$\frac{V_2}{I_g} = \frac{1}{s^3+2s^2+3s+2}$$

The function has no zeros. It has poles at $s = -1$, $s = -0.5 + j1.32$ and $s = -0.5 - j1.32$,
The pole-zero diagram is shown in Fig. 8.68.

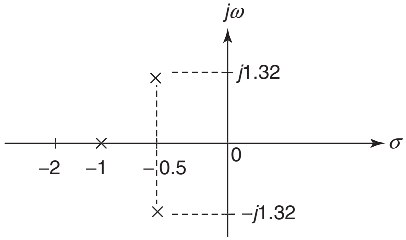


Fig. 8.68

Example 8.34 *For the transfer function* $H(s) = \dfrac{V_0}{V_i} = \dfrac{10}{s^2 + 3s + 10}$, *realise the function using the network shown in Fig. 8.69. Find L and C when R = 5 Ω.*

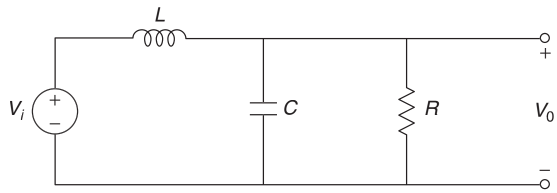


Fig. 8.69

Solution The transformed network is shown in Fig. 8.70.

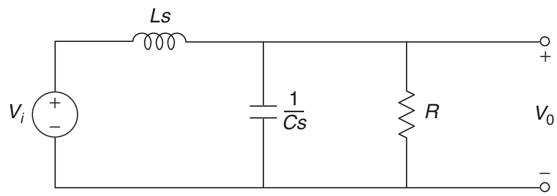


Fig. 8.70

Simplifying the network as shown in Fig. 8.71,

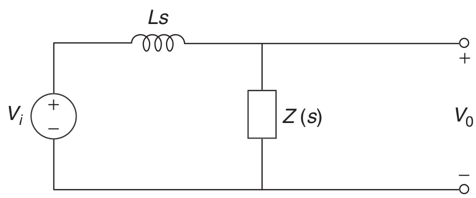


Fig. 8.71

$$Z(s) = \frac{R \times \dfrac{1}{Cs}}{R + \dfrac{1}{Cs}} = \frac{R}{RCs + 1}$$

$$V_0 = V_i \frac{\dfrac{R}{RCs+1}}{Ls + \dfrac{R}{RCs+1}}$$

$$\frac{V_0}{V_i} = \frac{R}{RLC\,s^2 + Ls + R} = \frac{\dfrac{1}{LC}}{s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}} \qquad \text{...(i)}$$

But
$$\frac{V_0}{V_i} = \frac{10}{s^2 + 3s + 10} \qquad \text{...(ii)}$$

and
$$R = 5\ \Omega$$

Comparing Eq. (ii) with Eq. (i),

$$\frac{1}{RC} = 3$$

$$\frac{1}{LC} = 10$$

Solving the above equations,

$$L = 1.5\ \text{H}$$

$$C = \frac{1}{15}\ \text{F}$$

Example 8.35 *Obtain the impedance function Z(s) for which pole-zero diagram is shown in Fig. 8.72.*

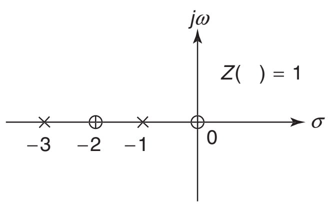


Fig. 8.72

Solution The function $Z(s)$ has poles at $s = -1$ and $s = -3$ and zeros at $s = 0$ and $s = -2$.

$$Z(s) = H\frac{s(s+2)}{(s+1)(s+3)} = H\frac{s^2\left(1+\dfrac{2}{s}\right)}{s^2\left(1+\dfrac{1}{s}\right)\left(1+\dfrac{3}{s}\right)}$$

For $s = \infty$,

$$Z(\infty) = H\frac{1}{(1)(1)} = H$$

When
$$Z(\infty) = 1,$$

$$H = 1$$

$$Z(s) = \frac{s\,(s+2)}{(s+1)(s+3)}$$

Example 8.36 *Obtain the admittance function Y(s) for which the pole-zero diagram is shown in Fig. 8.73.*

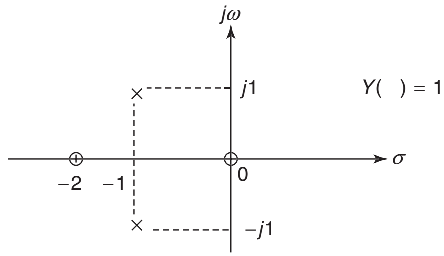


Fig. 8.73

Solution The function $Y(s)$ has poles at $s = -1 + j1$ and $s = -1 - j1$ and zeros at $s = 0$ and $s = -2$.

$$Y(s) = H\frac{s(s+2)}{(s+1+j1)(s+1-j1)} = H\frac{s(s+2)}{(s+1)^2-(j1)^2} = H\frac{s(s+2)}{s^2+2s+2} = H\frac{s^2\left(1+\frac{2}{s}\right)}{s^2\left(1+\frac{2}{s}+\frac{2}{s^2}\right)}$$

For $s = \infty$,

$$Y(\infty) = H\frac{(1)}{(1)} = H$$

When $Y(\infty) = 1$,

$$H = 1$$

$$Y(s) = \frac{s(s+2)}{s^2+2s+2}$$

Example 8.37 *A network and its pole-zero configuration are shown in Fig. 8.74. Determine the values of R, L and C if Z (j0) = 1.*

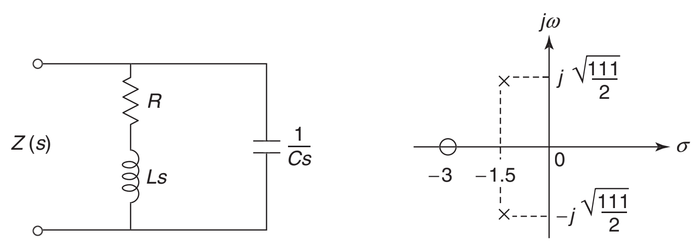


Fig. 8.74

Solution

$$Z(s) = \frac{(Ls+R)\frac{1}{Cs}}{(Ls+R)+\frac{1}{Cs}} = \frac{Ls+R}{LCs^2+RCs+1} = \frac{\frac{1}{C}\left(s+\frac{R}{L}\right)}{s^2+\frac{R}{L}s+\frac{1}{LC}} \quad \text{...(i)}$$

From the pole-zero diagram, zero is at $s = -3$ and poles are at $s = -1.5 + j\dfrac{\sqrt{111}}{2}$ and $s = -1.5 - j\dfrac{\sqrt{111}}{2}$

$$Z(s) = H\frac{s+3}{\left(s+1.5+j\dfrac{\sqrt{111}}{2}\right)\left(s+1.5-j\dfrac{\sqrt{111}}{2}\right)}$$

$$Z(s) = H\frac{s+3}{(s+1.5)^2 - \left(j\dfrac{\sqrt{111}}{2}\right)^2} = H\frac{s+3}{s^2+3s+30}$$

When $\qquad Z(j0) = 1,$

$$1 = H\left(\frac{3}{30}\right)$$

$$H = 10$$

$$Z(s) = \frac{10(s+3)}{s^2+3s+30} \qquad \text{...(ii)}$$

Comparing Eq. (ii) with Eq. (i),

$$\frac{R}{L} = 3$$

$$\frac{1}{C} = 10$$

$$\frac{1}{LC} = 30$$

Solving the above equations,

$$C = \frac{1}{10}\text{ F}$$

$$L = \frac{1}{3}\text{ H}$$

$$R = 1\,\Omega$$

Example 8.38 *A network is shown in Fig. 8.75. The poles and zeros of the driving-point function Z(s) of this network are at the following places:*

Poles at $-\dfrac{1}{2} \pm j\dfrac{\sqrt{3}}{2}$

Zero at –1

If Z (j0) = 1, determine the values of R, L and C.

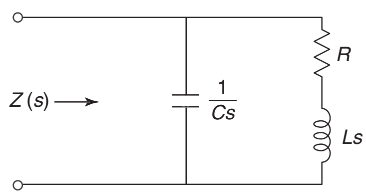


Fig. 8.75

Solution

$$Z(s)=\frac{(Ls+R)\dfrac{1}{Cs}}{Ls+R+\dfrac{1}{Cs}}=\frac{Ls+R}{LCs^2+RCs+1}=\frac{\dfrac{1}{C}\left(s+\dfrac{R}{L}\right)}{s^2+\dfrac{R}{L}s+\dfrac{1}{LC}} \qquad \text{...(i)}$$

The poles are at $-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$ and zero is at –1.

$$Z(s)=H\frac{s+1}{\left(s+\dfrac{1}{2}+j\dfrac{\sqrt{3}}{2}\right)\left(s+\dfrac{1}{2}-j\dfrac{\sqrt{3}}{2}\right)}=H\frac{s+1}{\left(s+\dfrac{1}{2}\right)^2-\left(j\dfrac{\sqrt{3}}{2}\right)^2}=H\frac{s+1}{s^2+s+1}$$

When $Z(j0)=1$,

$$1=H\frac{(1)}{(1)}$$

$$H=1$$

$$Z(s)=\frac{s+1}{s^2+s+1} \qquad \text{...(ii)}$$

Comparing Eq. (ii) with Eq. (i),

$$C=1$$

$$\frac{R}{L}=1$$

$$\frac{1}{LC}=1$$

Solving the above equations,

$$C=1\text{ F}$$
$$L=1\text{ H}$$
$$R=1\ \Omega$$

Example 8.39 *The pole-zero diagram of the driving-point impedance function of the network of Fig. 8.76 is shown below. At dc, the input impedance is resistive and equal to 2 W. Determine the values of R, L and C.*

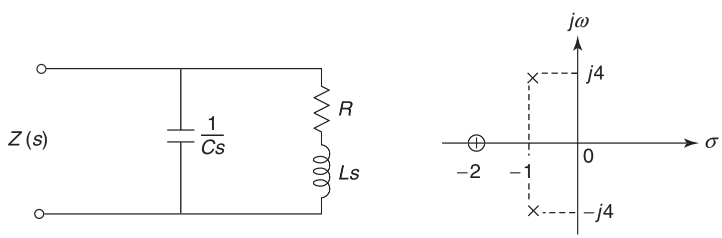


Fig. 8.76

Solution

$$Z(s)=\frac{(Ls+R)\dfrac{1}{Cs}}{Ls+R+\dfrac{1}{Cs}}=\frac{Ls+R}{LCs^2+RCs+1}=\frac{\dfrac{1}{C}\left(s+\dfrac{R}{L}\right)}{s^2+\dfrac{R}{L}s+\dfrac{1}{LC}} \qquad \text{...(i)}$$

From the pole-zero diagram, zero is at $s=-2$ and poles are at $s=-1+j4$ and $s=-1-j4$.

$$Z(s) = H\frac{s+2}{(s+1+j4)(s+1-j4)} = H\frac{s+2}{(s+1)^2-(j4)^2} = H\frac{s+2}{s^2+2s+17}$$

At dc, i.e., $\omega = 0,\ Z(j0) = 2$

$$2 = H\frac{2}{17}$$

$$H = 17$$

$$Z(s) = 17\frac{s+2}{s^2+2s+17} \qquad \text{...(ii)}$$

Comparing Eq. (ii) with Eq. (i),

$$\frac{1}{C} = 17$$

$$\frac{R}{L} = 2$$

$$\frac{1}{LC} = 17$$

Solving the above equations,

$$C = \frac{1}{17}\text{ F}$$

$$L = 1\text{ H}$$

$$R = 2\ \Omega$$

Example 8.40 *The network shown in Fig. 8.77 has the driving-point admittance Y (s) of the form*

$$Y(s) = H\frac{(s-s_1)(s-s_2)}{(s-s_3)}$$

(*a*) *Express s_1, s_2, s_3 in terms of R, L and C.*
(*b*) *When $s_1 = -10 + j10^4$, $s_2 = -10 - j10^4$ and $Y(j0) = 10^{-1}$ mho, find the values of R, L and C and determine the value of s_3.*

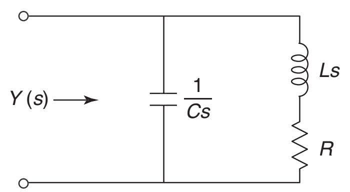


Fig. 8.77

Solution

(a)

$$Y(s) = Cs + \frac{1}{Ls+R} = \frac{(Ls+R)\,Cs+1}{Ls+R} = \frac{LCs^2+RCs+1}{Ls+R} = \frac{C\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}{s+\frac{R}{L}} \qquad \text{...(i)}$$

But

$$Y(s) = \frac{H(s-s_1)(s-s_2)}{(s-s_3)}$$

where
$$s_1, s_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$
$$s_3 = -\frac{R}{L}$$

(b) When
$$s_1 = -10 + j10^4$$
$$s_2 = -10 - j10^4$$
$$Y(s) = H\frac{(s+10-j10^4)(s+10+j10^4)}{s-s_3} = H\frac{s^2+20s+10^8}{s-s_3} \qquad \text{...(ii)}$$

Comparing Eq. (ii) with Eq. (i),
$$\frac{R}{L} = 20$$
$$s_3 = -20$$
$$Y(s) = H\frac{(s^2+20s+10^8)}{(s+20)}$$

At $s = j0$,
$$Y(j0) = H\frac{(10^8)}{20} = 10^{-1}$$
$$H = 0.02\times10^{-6}$$
$$Y(s) = 0.02\times10^{-6}\,\frac{(s^2+20s+10^8)}{(s+20)} \qquad \text{...(iii)}$$

Comparing Eq. (iii) with Eq. (i),
$$C = 0.02\times10^{-6}\,\text{F} = 0.02\,\mu\text{F}$$
$$\frac{1}{LC} = 10^8$$
$$L = \frac{1}{2}\,\text{H}$$
$$\frac{R}{L} = 20$$
$$R = 10\,\Omega$$

Example 8.41 *A network and pole-zero diagram for driving-point impedance Z(s) are shown in Fig. 8.78. Calculate the values of the parameters R, L, G and C if Z(j0) = 1.*

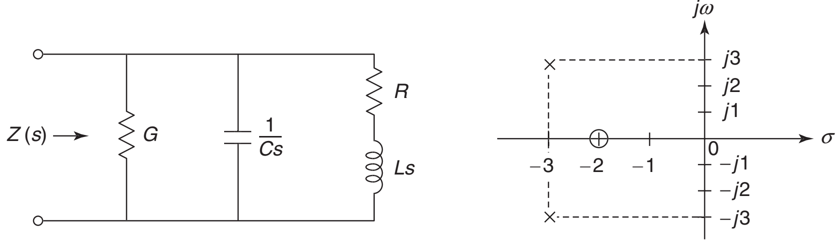


Fig. 8.78

Solution It is easier to calculate $Y(s)$ and then invert it to obtain $Z(s)$.

$$Y(s) = G + Cs + \frac{1}{Ls+R} = \frac{(G+Cs)(Ls+R)+1}{Ls+R} = \frac{LCs^2+(GL+RC)\,s+1+GR}{Ls+R}$$

$$Z(s) = \frac{1}{Y(s)} = \frac{Ls+R}{LCs^2+(GL+RC)\,s+1+GR} = \frac{\frac{1}{C}\left(s+\frac{R}{L}\right)}{s^2+\left(\frac{G}{C}+\frac{R}{L}\right)s+\left(\frac{1+GR}{LC}\right)} \qquad \text{...(i)}$$

From the pole-zero diagram, zero is at $s = -2$ and poles are at $s = -3 \pm j3$.

$$Z(s) = H\frac{(s+2)}{(s+3-j3)(s+3+j3)} = H\frac{(s+2)}{(s+3)^2-(j3)^2} = H\frac{s+2}{s^2+6s+18}$$

When $\qquad Z(j0) = 1,$

$$1 = H\frac{2}{18}$$

$$H = 9$$

$$Z(s) = \frac{9\,(s+2)}{(s^2+6s+18)} \qquad \text{...(ii)}$$

Comparing Eq. (ii) with Eq. (i),

$$\frac{1}{C} = 9$$

$$\frac{R}{L} = 2$$

$$\frac{G}{C}+\frac{R}{L} = 6$$

$$\frac{1+GR}{LC} = 18$$

Solving the above equation,

$$C = \frac{1}{9}\text{ F}$$

$$L = \frac{9}{10}\text{ H}$$

$$G = \frac{4}{9}\ \mho$$

$$R = \frac{9}{5}\ \Omega$$

Example 8.42 *A series R-L-C circuit has its driving-point admittance and pole-zero diagram is shown in Fig. 8.79. Find the values of R, L and C.*

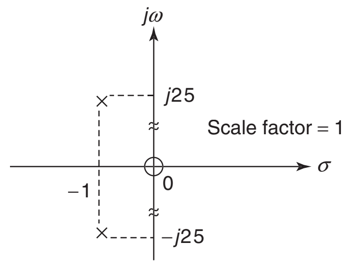


Fig. 8.79

Solution The function $Y(s)$ has poles at $s = -1 + j25$ and $s = -1 - j25$ and zero at $s = 0$.

$$Y(s) = H\frac{s}{(s+1+j25)(s+1-j25)} = H\frac{s}{(s+1)^2-(j25)^2} = H\frac{s}{s^2+2s+626}$$

Scale factor $\qquad H = 1$

$$Y(s) = \frac{s}{s^2+2s+626} \qquad \text{...(i)}$$

For a series *RLC* circuit,

$$Z(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs} = \frac{L\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}{s}$$

$$Y(s) = \frac{1}{Z(s)} = \frac{s}{L\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)} \qquad \text{...(ii)}$$

Comparing Eq. (i) with Eq. (ii),

$$L = 1\text{ H}$$

$$\frac{1}{LC} = 626$$

$$C = \frac{1}{626}\text{F}$$

$$\frac{R}{L} = 2$$

$$R = 2\,\Omega$$

8.9 TIME-DOMAIN BEHAVIOUR FROM THE POLE-ZERO PLOT

The time-domain behaviour of a system can be determined from the pole-zero plot. Consider a network function

$$F(s) = H\frac{(s-z_1)(s-z_2)\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots(s-p_m)}$$

The poles of this function determine the time-domain behaviour of $f(t)$. The function $f(t)$ can be determined from the knowledge of the poles, the zeros and the scale factor H. Figure 8.80 shows some pole locations and the corresponding time-domain response.

(i) When pole is at origin, i.e., at $s = 0$, the function $f(t)$ represents steady-state response of the circuit i.e., dc value. (Fig. 8.80)

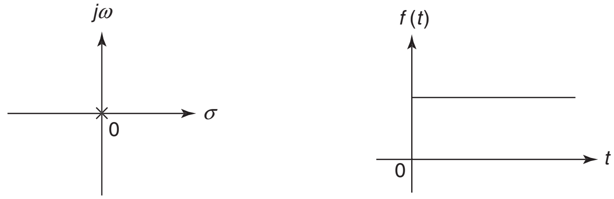


Fig. 8.80 *Pole at origin*

(ii) When pole lies in the left half of the s-plane, the response decreases exponentially. (Fig. 8.81)

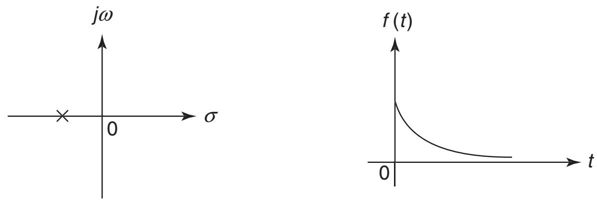


Fig. 8.81 *Pole in left half of the s-plane*

(iii) When pole lies in the right half of the s-plane, the response increases exponentially. A pole in the right-half plane gives rise to unbounded response and unstable system. (Fig. 8.82)

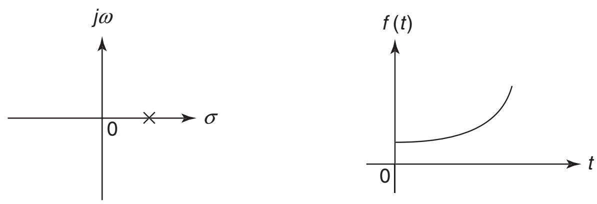


Fig. 8.82 *Pole in right half of the s-plane*

(iv) For $s = 0 + j\omega_n$, the response becomes $f(t) = Ae^{\pm j\omega_n t} = \text{A}(\cos \omega_n t \pm j \sin \omega_n t)$. The exponential response $e^{\pm j\omega_n t}$ may be interpreted as a rotating phasor of unit length. A positive sign of exponential $e^{j\omega_n t}$ indicates counterclockwise rotation, while a negative sign of exponential $e^{-j\omega_n t}$ indicates clockwise rotation. The variation of exponential function $e^{j\omega_n t}$ with time is thus sinusoidal and hence constitutes the case of sinusoidal steady state. (Fig. 8.83)

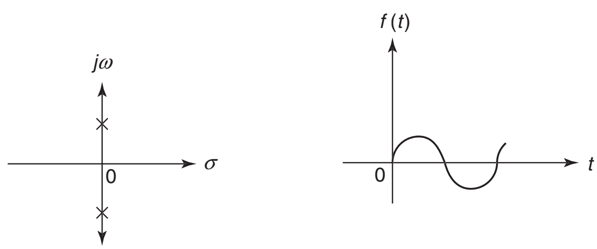
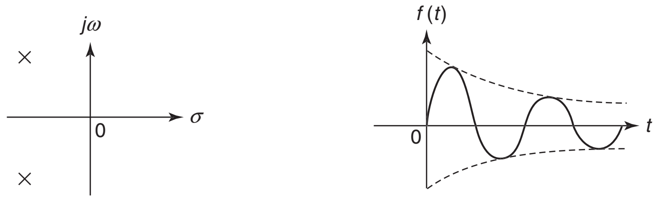
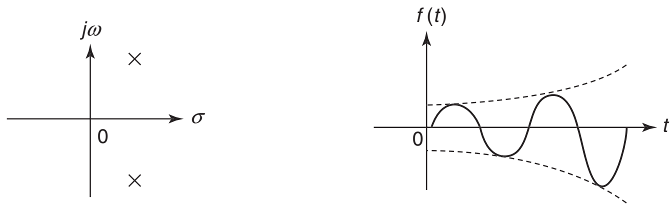


Fig. 8.83 *Poles on jω-axis*

(v) For $s = \sigma_n + j\omega_n$, the response becomes $f(t) = Ae^{st} = Ae^{(\sigma_n + j\omega_n t)} = Ae^{\sigma_n t}\, e^{j\omega_n t}$. The response $e^{\sigma_n t}$ is an exponentially increasing or decreasing function. The response $e^{j\omega_n t}$ is a sinusoidal function. Hence, the response of the product of these responses will be over damped sinusoids or under damped sinusoids (Fig. 8.84).



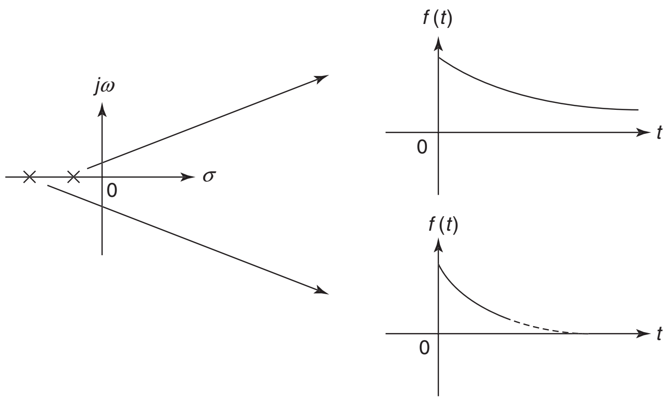
(a)



(b)

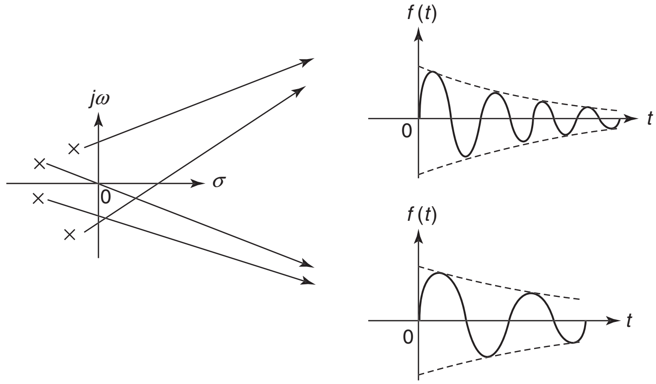
Fig. 8.84 *(a) Complex conjugate poles in left half of the S-plane*
(b) Complex conjugate poles in right half of the S-plane

(vi) The real part s of the pole is the displacement of the pole from the imaginary axis. Since σ is also the damping factor, a greater value of σ (i.e., a greater displacement of the pole from the imaginary axis) means that response decays more rapidly with time. The poles with greater displacement from the real axis correspond to higher frequency of oscillation (Fig. 8.85).



(a)

Fig. 8.85 *Nature of response with different positions of poles*



(b)

Fig.8.85 *(Continued)*

8.9.1 Stability of the Network

Stability of the network is directly related to the location of poles in the s-plane.

(i) When all the poles lie in the left half of the s-plane, the network is said to be stable.
(ii) When the poles lie in the right half of the s-plane, the network is said to be unstable.
(iii) When the poles lie on the $j\omega$ axis, the network is said to be marginally stable.
(iv) When there are multiple poles on the $j\omega$ axis, the network is said to be unstable.
(v) When the poles move away from $j\omega$ axis towards the left half of the s-plane, the relative stability of the network improves.

8.10 GRAPHICAL METHOD FOR DETERMINATION OF RESIDUE

Consider a network function,

$$F(s) = H\frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

By partial fraction expansion,

$$F(s) = \frac{K_1}{(s-p_1)} + \frac{K_2}{(s-p_2)} + \cdots + \frac{K_m}{(s-p_m)}$$

The residue K_i is given by

$$K_i = (s-p_i)F(s)\big|_{s\to p_i} = H\frac{(p_i-z_1)(p_i-z_2)\cdots(p_i-z_n)}{(p_i-p_1)(p_i-p_2)\cdots(p_i-p_m)}$$

Each term $(p_i - z_i)$ represents a phasor drawn from zero z_i to pole p_i.
Each term $(p_i - p_k)$, $i \neq k$, represents a phasor drawn from other poles to the pole p_i.

$$K_i = H\frac{\text{Product of phasors (polar form) from each zero to } p_i}{\text{Product of phasors (polar form) from other poles to } p_i}$$

The residues can be obtained by graphical method in the following way:

(1) Draw the pole-zero diagram for the given network function.
(2) Measure the distance from each of the other poles to a given pole.
(3) Measure the distance from each of the other zeros to a given pole.
(4) Measure the angle from each of the other poles to a given pole.
(5) Measure the angle from each of the other zeros to a given pole.
(6) Substitute these values in the required residue equation.

The graphical method can be used if poles are simple and complex. But it cannot be used when there are multiple poles.

Example 8.43 *The current I(s) in a network is given by* $I(s) = \dfrac{2s}{(s+1)(s+2)}$. *Plot the pole-zero pattern in the s-plane and hence obtain i(t).*

Solution Poles are at $s = -1$ and $s = -2$ and zero is at $s = 0$. The pole-zero plot is shown in Fig. 8.86.

By partial-fraction expansion,

$$I(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

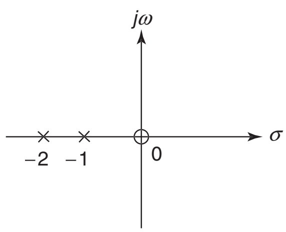


Fig. 8.86

The coefficients K_1 and K_2, often referred as residues, can be evaluated from he pole-zero diagram. From Fig. 8.87,

$$K_1 = H\frac{\text{Phasor from zero at origin to pole at } A}{\text{Phasor from pole at } B \text{ to pole at } A} = 2\left(\frac{1\angle 180^\circ}{1\angle 0^\circ}\right) = 2\angle 180^\circ = -2$$

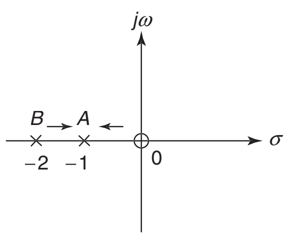


Fig. 8.87

From Fig. 8.88,

$$K_2 = H\frac{\text{Phasor from zero at origin to pole at } B}{\text{Phasor from pole at } A \text{ to pole at } B} = 2\left(\frac{2\angle 180^\circ}{1\angle 180^\circ}\right) = 4$$

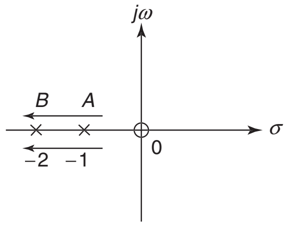


Fig. 8.88

$$I(s) = -\frac{2}{s+1} + \frac{4}{s+2}$$

Taking inverse Laplace transform,

$$i(t) = -2e^{-t} + 4e^{-2t}$$

Example 8.44 *The voltage V(s) of a network is given by*

$$V(s) = \frac{3s}{(s+2)(s^2+2s+2)}$$

Plot its pole-zero diagram and hence obtain v (t).

Solution

$$V(s) = \frac{3s}{(s+2)(s^2+2s+2)} = \frac{3s}{(s+2)(s+1+j1)(s+1-j1)}$$

Poles are at $s = -2$ and $s = -1 \pm j1$ and zero is at $s = 0$ as shown in Fig. 8.89.

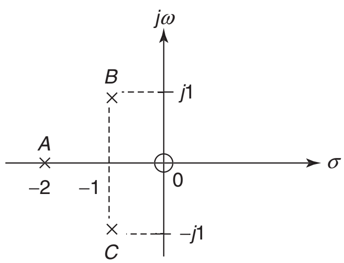


Fig. 8.89

By partial-fraction expansion,

$$V(s) = \frac{K_1}{s+2} + \frac{K_2}{s+1-j1} + \frac{K_2^*}{s+1+j1}$$

The coefficients K_1, K_2 and K_2^* can be evaluated from the pole-zero diagram.
From Fig. 8.90,

$$K_1 = \frac{3\,(\overline{OA})}{(\overline{BA})\,(\overline{CA})} = 3\left[\frac{2\angle 180^\circ}{(\sqrt{2}\angle -135^\circ)\,(\sqrt{2}\angle 135^\circ)}\right] = 3\sqrt{180^\circ} = -3$$

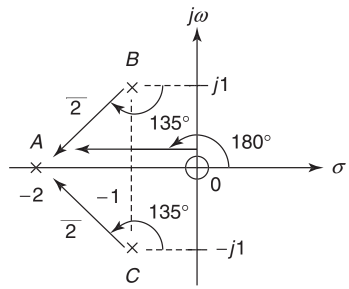


Fig. 8.90

From Fig. 8.91,

$$K_2 = \frac{3\,(\overline{OB})}{(\overline{AB})\,(\overline{CB})} = 3\left[\frac{(\sqrt{2}\angle 135°)}{(\sqrt{2}\angle 45°)\,(2\angle 90°)}\right] = \frac{3}{2}$$

$$K_2^* = \frac{3}{2}$$

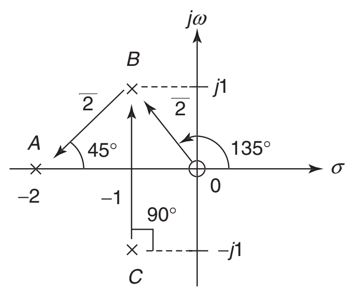


Fig. 8.91

$$V(s) = -\frac{3}{(s+2)} + \frac{\frac{3}{2}}{(s+1-j1)} + \frac{\frac{3}{2}}{(s+1+j1)}$$

Taking inverse Laplace transform,

$$v(t) = -3e^{-2t} + \frac{3}{2}\left[e^{(-1+j1)t} + e^{(-1-j1)t}\right] = -3e^{-2t} + 2\times\frac{3}{2}e^{-t}\left(\frac{e^{j1}+e^{-j1}}{2}\right) = -3e^{-2t} + 3e^{-t}\cos t$$

Example 8.45 *Find the function v(t) using the pole-zero plot of following function:*

$$V(s) = \frac{(s+2)(s+6)}{(s+1)(s+5)}$$

Solution If the degree of the numerator is greater or equal to the degree of the denominator, we have to divide the numerator by the denominator such that the remainder can be expanded into partial fractions.

$$V(s)=\frac{s^2+8s+12}{s^2+6s+5}=1+\frac{2s+7}{s^2+6s+5}=1+\frac{2(s+3.5)}{(s+1)(s+5)}$$

By partial fraction expansion,

$$V(s)=1+\frac{K_1}{s+1}+\frac{K_2}{s+5}$$

K_1 and K_2 can be evaluated from the pole-zero diagram shown in Fig. 8.92 and Fig. 8.93.

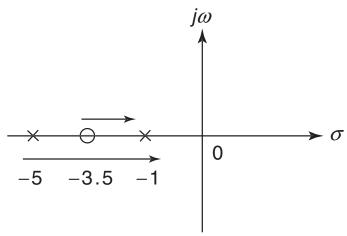


Fig. 8.92

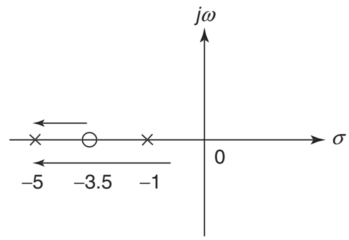


Fig. 8.93

From Fig. 8.92

$$K_1=2\left(\frac{2.5\angle 0°}{4\angle 0°}\right)=\frac{5}{4}$$

From Fig. 8.93

$$K_2=2\left(\frac{1.5\angle 180°}{4\angle 180°}\right)=\frac{3}{4}$$

$$V(s)=1+\frac{\frac{5}{4}}{s+1}+\frac{\frac{3}{4}}{s+5}$$

Taking inverse Laplace transform,

$$v(t)=\delta(t)+\frac{5}{4}e^{-t}+\frac{3}{4}e^{-5t}$$

Example 8.46 *The pole-zero plot of the driving-point impedance of a network is shown in Fig. 8.94. Find the time-domain response.*

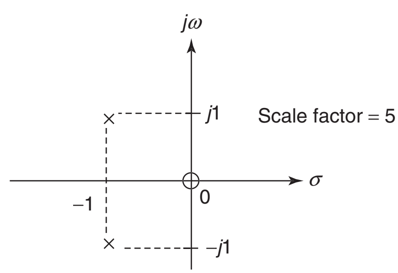


Fig. 8.94

Solution The function $Z(s)$ has poles at $s=-1+j1$ and $s=-1-j1$ and zero at $s=0$.

$$Z(s) = H\frac{s}{(s+1+j1)(s+1-j1)}$$

Scale factor $H = 5$

$$Z(s) = \frac{5s}{(s+1+j1)(s+1-j1)}$$

By partial fraction expansion,

$$Z(s) = \frac{K_1}{s+1+j1} + \frac{K_1^*}{s+1-j1}$$

The coefficients K_1 and K_1^* can be evaluated from the pole-zero diagram. From Fig. 8.95,

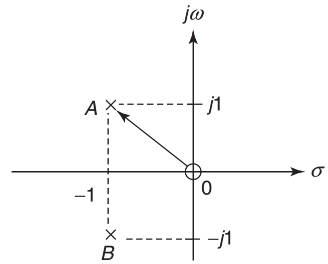


Fig. 8.95

$$K_1 = \frac{5\,(\overline{OA})}{(\overline{BA})} = \frac{5\,(\sqrt{2}\angle 135°)}{2\angle 90°} = 3.54\angle 45°$$

$$K_1^* = 3.54\angle -45°$$

$$Z(s) = \frac{3.54\angle 45°}{s+1+j1} + \frac{3.54\angle -45°}{s+1-j1}$$

Taking inverse Laplace transform,

$$z(t) = 3.54\ \angle\ 45°\ e^{(-1-j1)t} + 3.54\ \angle{-45°}e^{(-1+j1)t}$$

Example 8.47 *Evaluate amplitude and phase of the network function* $F(s) = \dfrac{4s}{s^2+2s+2}$ *from the pole-zero plot at* $s = j2$.

Solution

$$F(s) = \frac{4s}{s^2+2s+2} = \frac{4s}{(s+1+j1)(s+1-j1)}$$

The pole-zero plot is shown in Fig. 8.96.

At $s = j2$,

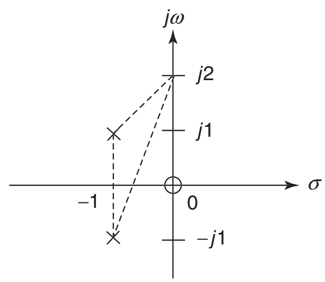


Fig. 8.96

$$|F(j2)| = \frac{\text{Product of phasor magnitudes from all zero to } j2}{\text{Product of phasor magnitudes from all poles to } j2} = \frac{2}{(\sqrt{2})\,(\sqrt{10})} = 0.447$$

$$\phi(\omega) = \tan^{-1}\left(\frac{2}{0}\right) - \tan^{-1}\left(\frac{3}{1}\right) - \tan^{-1}\left(\frac{1}{1}\right) = 90° - 71.56° - 45° = -26.56°$$

Example 8.48 *Using the pole-zero plot, find magnitude and phase of the function*

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)} \text{ at } s = j4.$$

Solution

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)}$$

The pole-zero plot is shown in Fig. 8.97.

At $s = j4$,

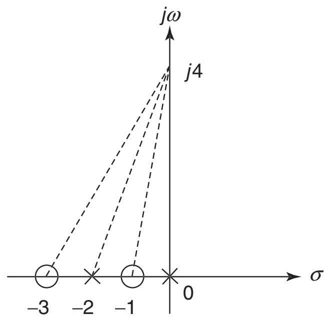


Fig. 8.97

$$|F(j4)| = \frac{\text{Product of phasor magnitudes from all zeros to } j4}{\text{product of phasor magnitudes from all poles to } j4} = \frac{(5)(\sqrt{17})}{(\sqrt{20})(4)} = 1.15$$

$$\phi(\omega) = \tan^{-1}\left(\frac{4}{1}\right) + \tan^{-1}\left(\frac{4}{3}\right) - \tan^{1}\left(\frac{4}{0}\right) - \tan^{-1}\left(\frac{4}{2}\right) = 75.96° + 53.13° - 90° - 63.43° = -24.34°$$

Example 8.49 *Plot amplitude and phase response for*

$$F(s) = \frac{s}{s+10}$$

Solution

$$F(j\omega) = \frac{j\omega}{j\omega+10}$$

$$|F(j\omega)| = \frac{\omega}{\sqrt{\omega^2+100}}$$

ω	$\lvert F(j\omega)\rvert$
0	0
10	0.707
100	0.995
1000	1

The amplitude response is shown in Fig. 8.98.

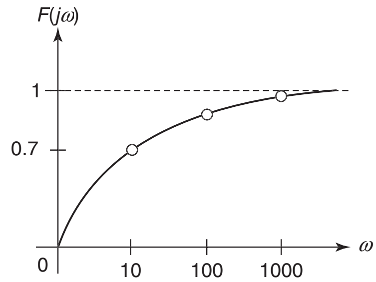


Fig. 8.98

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) = 90° - \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω	$\phi(\omega)$
0	90°
10	45°
100	5.7°
1000	0°

The phase response is shown in Fig. 8.99.

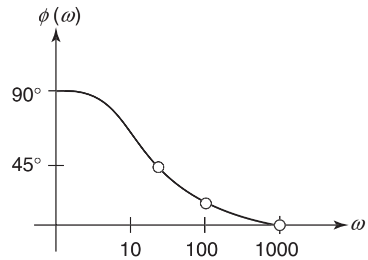


Fig. 8.99

Example 8.50 *Sketch amplitude and phase response for* $F(s) = \dfrac{s+10}{s-10}$

Solution

$$F(j\omega) = \frac{j\omega + 10}{j\omega - 10}$$

$$|F(f\omega)| = \frac{\sqrt{\omega^2 + 100}}{\sqrt{\omega^2 + 100}}$$

For all ω, magnitude is unity.
The amplitude response is shown in Fig. 8.100.

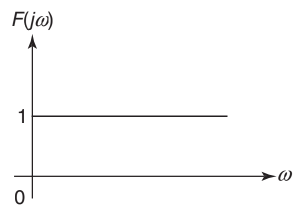


Fig. 8.100

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(-\frac{\omega}{10}\right) = 2\tan^{-1}\left(\frac{\omega}{10}\right)$$

The phase response is shown in Fig. 8.101.

ω	$\phi(\omega)$
0	0°
10	90°
100	168.6°
1000	178.9°

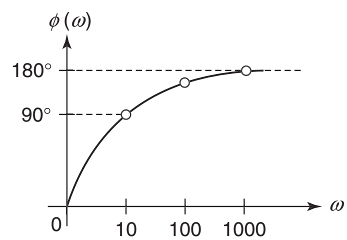


Fig. 8.101

Exercises

8.1 Determine the driving-point impedance $\dfrac{V_1}{I_1}$, transfer impedance $\dfrac{V_2}{I_1}$ and voltage transfer ratio $\dfrac{V_2}{V_1}$ for the network shown in Fig. 8.102.

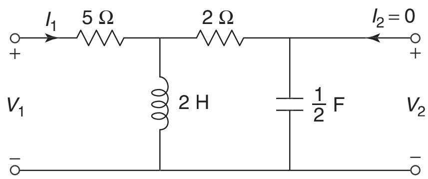


Fig. 8.102

$$\left[\frac{V_1}{I_1} = \frac{7s^2+7s+5}{s2+s+1};\ \frac{V_2}{I_1} = \frac{2s}{s^2+s+1};\ \frac{V_2}{V_1} = \frac{2s}{7s^2+7s+5}\right]$$

8.2 For the network shown in Fig. 8.103, determine $\dfrac{V_2}{V_1}$ and $\dfrac{V_2}{I_1}$.

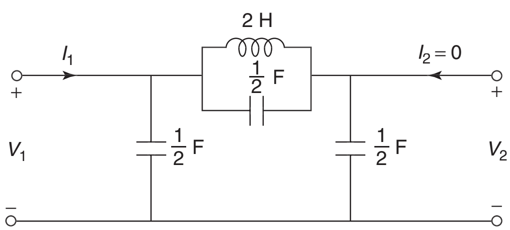


Fig. 8.103

$$\left[\frac{V_2}{V_1} = \frac{s^2+1}{2s^2+1};\ \frac{V_2}{I_1} = \frac{2s^2+2}{s(3s^2+2)}\right]$$

8.3 Find the open-circuit transfer impedance Z_{21} and open-circuit voltage ratio G_{21} for the ladder network shown in Fig. 8.104.

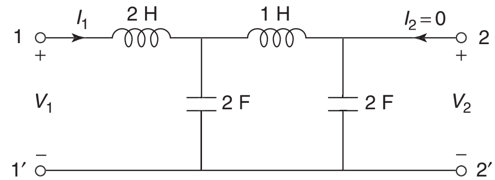


Fig. 8.104

$$\left[Z_{21} = \frac{1}{2s^3+3s},\ G_{21} = \frac{1}{4s^4+7s^2+1}\right]$$

8.4 For the two-port network shown in Fig. 8.105, determine Z_{11}, Z_{21} and voltage transfer ratio $G_{21}(s)$.

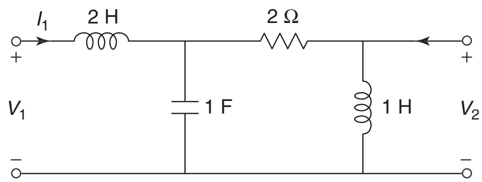


Fig. 8.105

$$\left[Z_{11} = \frac{2s^3+4s^2+3s+2}{s^2+2s+1},\ Z_{21} = \frac{s}{s^2+2s+1},\ G_{21} = \frac{s}{2s^3+4s^2+3s+2}\right]$$

8.5 Draw the pole-zero diagram of the following network functions:

(i) $F(s) = \dfrac{s^2+4}{s^2+6s+4}$

(ii) $F(s) = \dfrac{5s-12}{s^2+4s+13}$

(iii) $F(s) = \dfrac{s+1}{(s^2+2s+2)^2}$

(iv) $F(s) = \dfrac{s\,(s^2+5)}{s^4+2s^2+1}$

(v) $F(s) = \dfrac{s^2 + s + 2}{s^4 + 5s^3 + 6s^2}$

(vi) $F(s) = \dfrac{s^2 - s}{s^3 + 2s^2 - s - 2}$

(vii) $F(s) = \dfrac{s^2 + 3s + 2}{s^2 + 3s}$

(viii) $F(s) = \dfrac{(s^2 + 4)(s + 1)}{(s^2 + 1)(s^2 + 2s + 5)}$

8.6 For the network shown in Fig. 8.106, draw the pole-zero plot of the impedance function $Z(s)$.

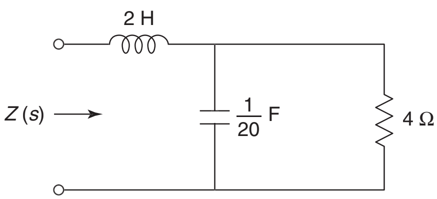


Fig. 8.106

$$Z(s) = \left[\frac{(s + 2.5 - j1.94)(s + 2.5 + j1.94)}{s + 5}\right]$$

8.7 For the network shown in Fig. 8.107, draw the pole-zero plot of driving-point impedance function $Z(s)$.

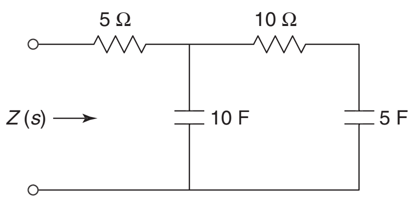


Fig. 8.107

$$\left[Z(s) = \frac{5(s + 0.01)(s + 0.04)}{s(s + 0.03)}\right]$$

8.8 Find the driving-point impedance of the network shown in Fig. 8.108. Also, find poles and zeros.

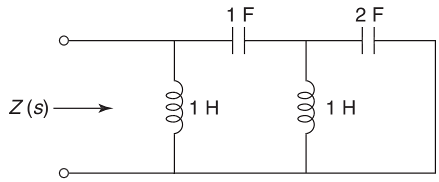


Fig. 8.108

$$\left[Z(s) = \frac{1.5s(s^2 + 0.33)}{(s^2 + 1.707)(s^2 + 0.293)}\right]$$

8.9 Find network functions $\dfrac{V_2}{V_1}$ and $\dfrac{V_1}{I_1}$ for the network shown in Fig. 8.109 and plot poles and zeros of $\dfrac{V_2(s)}{V_1(s)}$.

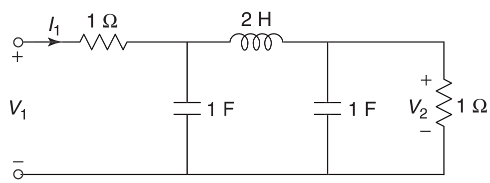


Fig. 8.109

$$\left[\frac{V_2}{V_1} = \frac{1}{2(s^3 + 2s^2 + 2s + 1)}, \frac{V_1}{I_1} = \frac{2(s^3 + 2s^2 + 2s + 1)}{2s^3 + 2s^2 + 2s + 1}\right]$$

8.10 For the network shown in Fig. 8.110, determine $\dfrac{V_1}{I_1}$ *and* $\dfrac{V_2}{I_1}$. Plot the poles and zeros of $\dfrac{V_2}{I_1}$.

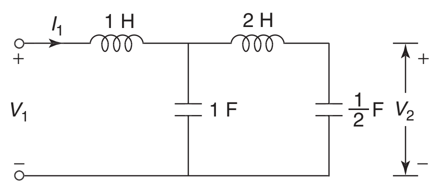


Fig. 8.110

$$\left[\frac{V_1}{I_1} = \frac{2s^4 + 5s^2 + 2}{2s^3 + 3s}, \frac{V_2}{I_1} = \frac{2}{2s^3 + 3s}\right]$$

8.11 For the network shown in Fig. 8.111, determine $\frac{V_1}{I_1}$ and $\frac{V_2}{V_1}$. Plot the pole and zeros for $\frac{V_2}{V_1}$.

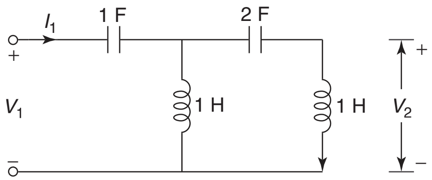


Fig. 8.111

$$\left[\frac{V_1}{I_1} = \frac{s^4 + 3s^2 + 1}{2s^3 + s}\right]$$

8.12 For the network shown in Fig. 8.112, plot the poles and zeros of transfer impedance function.

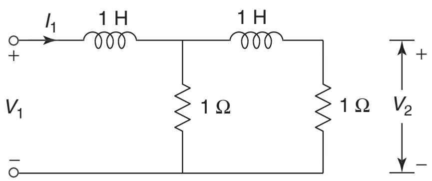


Fig. 8.112

$$\left[\frac{V_2}{I_1} = \frac{1}{s+2}\right]$$

8.13 For the network shown in Fig. 8.113, determine $\frac{V_1}{I_1}$ and $\frac{V_2}{V_1}$. Plot the poles and zeros of transfer impedance function.

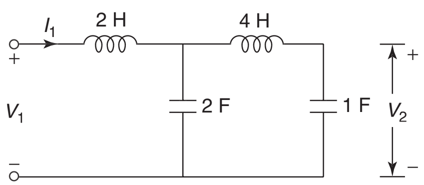


Fig. 8.113

$$\left[\frac{V_1}{I_1} = \frac{16s^4 + 10s^2 + 1}{8s^3 + 3s}, \frac{V_2}{I_1} = \frac{1}{8s^3 + 3s}, \frac{V_2}{V_1} = \frac{1}{16s^4 + 10s^2 + 1}\right]$$

8.14 Obtain the impedance function for which the pole-zero diagram is shown in Fig. 8.114.

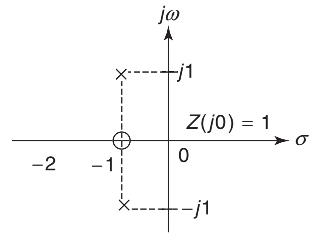


Fig. 8.114

$$\left[Z(s) = \frac{2(s+1)}{s^2 + 2s + 2}\right]$$

8.15 For the network shown in Fig. 8.115, poles and zeros of driving point function $Z(s)$ are,

Poles: $(-1 \pm j4)$; zero: -2

If $Z(j0) = 1$, find the values of R, L and C.

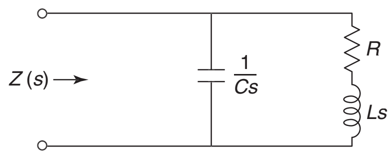


Fig. 8.115

$$\left[1\,\Omega, 0.5\text{ H}, \frac{2}{17}\text{ F}\right]$$

8.16 For the two-port network shown in Fig. 8.116, find R_1, R_2 and C. $\frac{V_2}{V_1} = \frac{0.2}{s^2 + 3s + 2}$

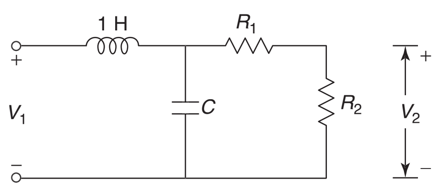


Fig. 8.116

$$\left[\frac{3}{5}\,\Omega, \frac{1}{15}\,\Omega, 0.5\text{ F}\right]$$

8.17 For the given network function, draw the pole-zero diagram and hence obtain the time domain voltage.

$$V(s)=\frac{5(s+5)}{(s+2)(s+7)}$$

$[v(t)=3e^{-2t}+2e^{-7t}]$

8.18 A transfer function is given by $Y(s)=\dfrac{10s}{(s+5+j15)(s+5-j15)}$. Find time-domain response using graphical method.

$\left[5.26\angle 18.4°\, e^{-(5+j15)t}+5.26\angle -18.4°\, e^{-(5-j15)t}\right]$

Objective-Type Questions

8.1 Of the four networks N_1, N_2, N_3 and N_4 of Fig. 8.117, the networks having identical driving-point functions are

(a) N_1 and N_2 (b) N_2 and N_4
(c) N_1 and N_3 (d) N_1 and N_4

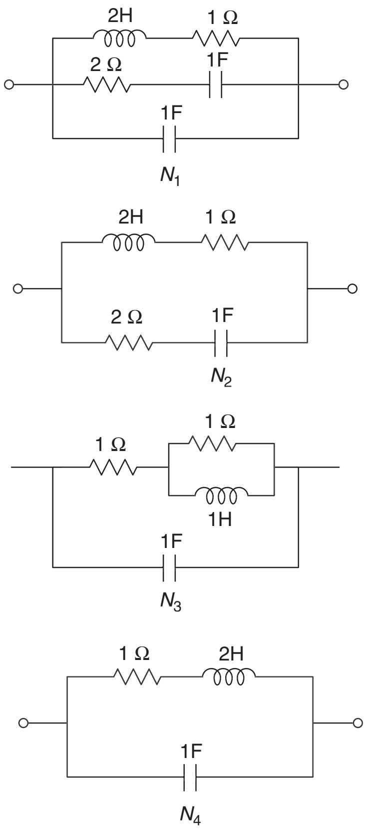


Fig. 8.117

8.2 The driving-point impedance $Z(s)$ of a network has the pole-zero locations as shown in Fig. 8.118. If $Z(0) = 3$, then $Z(s)$ is

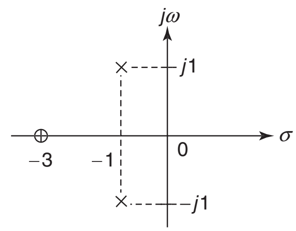


Fig. 8.118

(a) $\dfrac{3(s+3)}{s^2+2s+3}$ (b) $\dfrac{2(s+3)}{s^2+2s+2}$

(c) $\dfrac{3(s-3)}{s^2-2s-2}$ (d) $\dfrac{2(s-3)}{s^2-2s-3}$

8.3 For the circuit shown in Fig. 8.119, the initial conditions are zero. Its transfer function $H(s)=\dfrac{V_0(s)}{V_1(s)}$ is

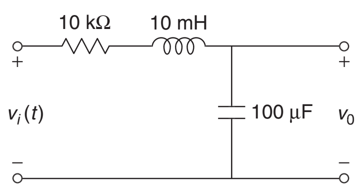


Fig. 8.119

(a) $\dfrac{1}{s^2+10^6 s+10^6}$ (b) $\dfrac{10^6}{s^2+10^3 s+10^6}$

(c) $\dfrac{10^3}{s^2+10^3 s+10^6}$ (d) $\dfrac{10^6}{s^2+10^6 s+10^6}$

8.4 In Fig. 8.120, assume that all the capacitors are initially uncharged. If $v_i(t) = 10\,u(t)$, then $v_0(t)$ is given by

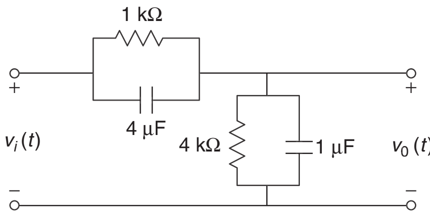


Fig. 8.120

(a) $8\,e^{-0.004\,t}$ (b) $8\,(1-e^{-0.004\,t})$

(c) $8\,u(t)$ (d) 8

8.5 A system is represented by the transfer function $\dfrac{10}{(s+1)(s+2)}$. The dc gain of this system is

(a) 1 (b) 2

(c) 5 (d) 10

8.6 Which one of the following is the ratio $\dfrac{V_{24}}{V_{13}}$ of the network shown in Fig. 8.121.

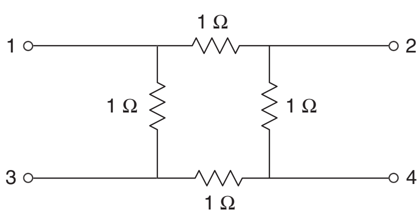


Fig. 8.121

(a) $\dfrac{1}{3}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{3}{4}$ (d) $\dfrac{4}{3}$

8.7 A network has response with time as shown in Fig. 8.122. Which one of the following diagrams represents the location of the poles of this network?

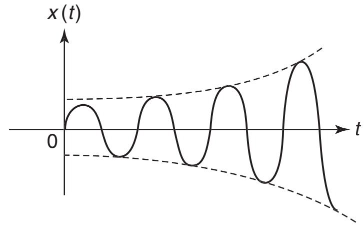


Fig. 8.122

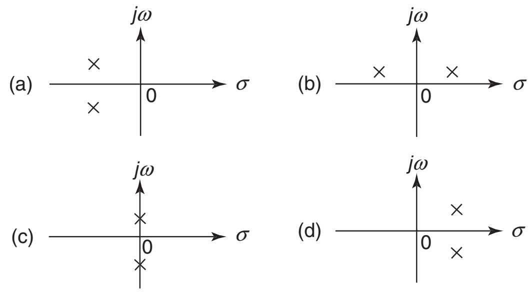


Fig. 8.123

8.8 The transfer function of a low-pass *RC* network is

(a) $(RCs)\,(1 + RCs)$ (b) $\dfrac{1}{1+RCs}$

(c) $\dfrac{RCs}{1+RCs}$ (d) $\dfrac{s}{1+RCs}$

8.9 The driving-point admittance function of the network shown in Fig. 8.124 has a

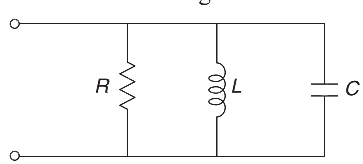


Fig. 8.124

(a) pole at $s = 0$ and zero at $s = \infty$
(b) pole at $s = 0$ and pole at $s = \infty$
(c) pole at $s = \infty$ and zero at $s = 0$
(d) pole at $s = \infty$ and zero at $s = \infty$

8.10 The transfer function $Y_{12}(s) = \dfrac{I_2(s)}{V_1(s)}$ for the network shown in Fig. 8.125 is

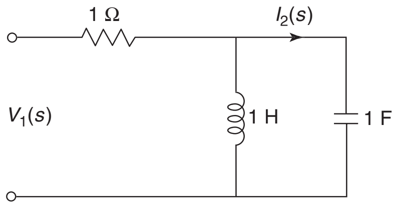


Fig. 8.125

(a) $\dfrac{s^2}{s^2+s+1}$ (b) $\dfrac{s}{s+1}$

(c) $\dfrac{1}{s+1}$ (d) $\dfrac{s+1}{s^2+1}$

8.11 As the poles of a network shift away from the *x* axis, the response

(a) remains constant

(b) becomes less oscillating

(c) becomes more oscillating

(d) none of these

Answers to Objective-Type Questions

8.1. (c)	8.2. (b)	8.3. (d)	8.4. (c)	8.5. (c)	8.6. (a)	8.7. (d)
8.8. (b)	8.9. (a)	8.10. (a)	8.11. (c)			