Introduction to Electrical Network Topology

Terms and Definitions

- Circuit elements
- Node
- Branch
- Path
- Closed path or Circuit or Loop or Mesh
- Topology, rather Electrical network topology
    - Graph and its types
        - Tree
        - Twigs
        - Co-tree
        - Links or Chords

Network Topology: Terms and Definitions - I

- **Circuit elements:**
  - The mathematical models of a two terminal electrical devices,
  - Completely characterized by its voltage-current relationship,
  - Can not be subdivided into other two-terminal devices.
- **Node:**
  - A point at which two or more circuit elements have a common connection,
  - The number of branches incident to a node is known as the degree of that node.
- **Branch:**
  - A single path, containing one circuit element, which connnects one node to any other node,
  - Represented by a line in the graph.
- **Path:**
  - A set of elements that may be traversed in order without passing through the same node twice.

Network Topology: Terms and Definitions - II

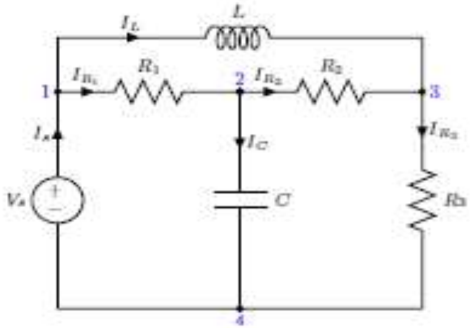
- **Loop:**
  - A close path or a closed contour selected in a network/circuit,
  - A path that may be started from a prticular node to other nodes through branches and comes to the original/starting node,
  - Also known as *closed path* or *circuit*.
- **Mesh[1] [2]:**
  - A loop that does not contain any other loops within it,
  - Any mesh is a circuit/loop but any loop/circuit may not be a mesh.
- **Network:**
  - The interconnection of two or more circuit elements forms an electical network.
- **Circuit:**
  - Network that contains at least one closed path,
  - Every circuit is a network, but not all networks are circuits.
- **Planar circuit:**
  - A circuit that may drawn on a plane surface in such a way that no branch passes over or under any other branch.

Network Topology: Terms and Definitions - III

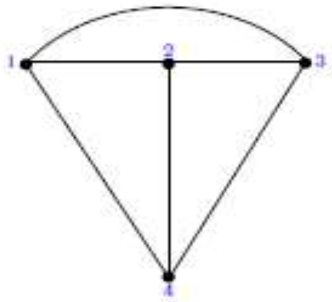
- **Topology:**
  - Deals with properties of networks which are unaffected when the network is stretched, twisted, or otherwise distorted the size and the shape,
  - Not concerned with the particular types of elements appearing in the circuit, but only with the way in which branches and nodes are arranged.
- **Graph:**
  - A graph corresponding to a given network is obtained by replacing all circuit elements with lines.
    - **Connected graph:** A graph in which at least one path exists between any two nodes of the graph. If the network has a transformer as one of the element, then the resulted graph is unconnected
    - **Directed or Oriented graph:** A graph that has all the nodes and branches numbered and also directions are given to the branches.
    - **Subgraph:** The subset of a graph. If the number of nodes and branches of a subgraph is less than that of the graph, the subgraph is said to be proper.

Network Topology: An example

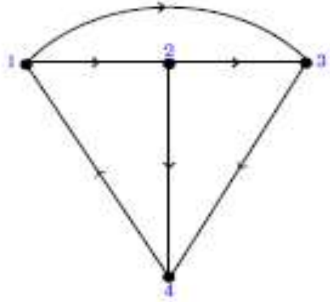
A circuit with topologically equivalent graphs:



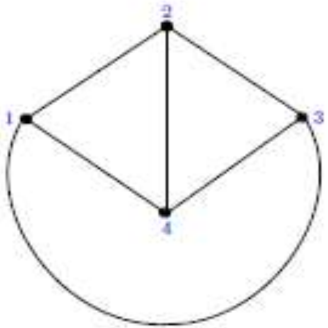
i) A Circuit

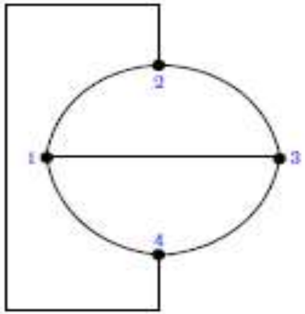


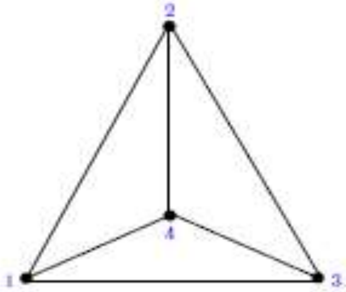
ii) its graph



iii) directed graph







An Electrical Network & its Graph - I

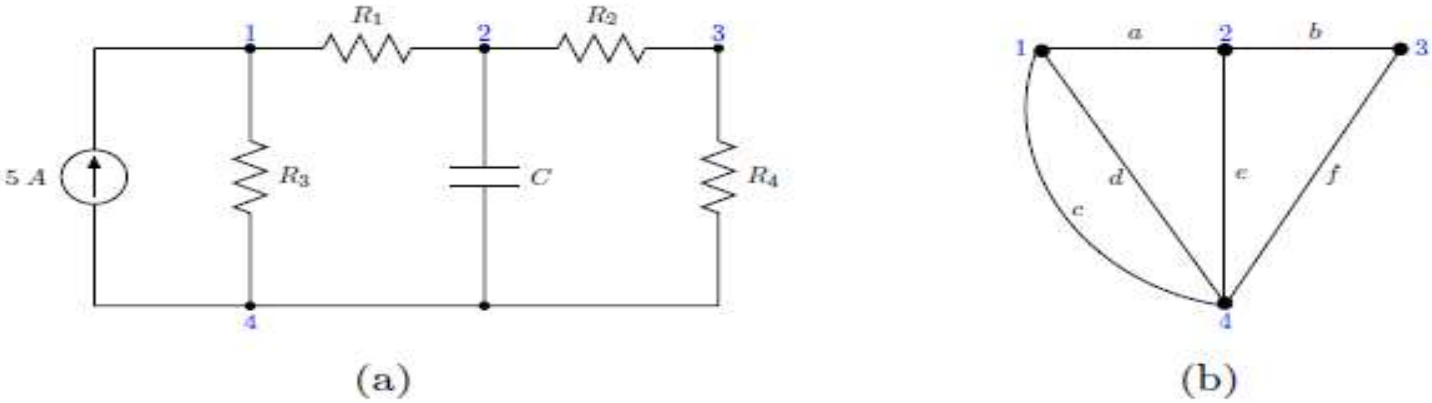
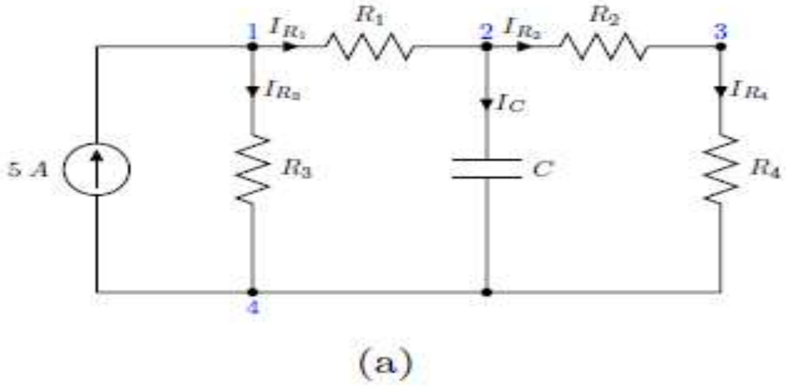


Figure 1 : (a) A circuit and (b) its graph.

Note:

- The maximum number of branches possible, in a circuit, will be equal to the number of nodes or vertices.
- There are at least two branches in a circuit.

An Electrical Network & its Graph - II



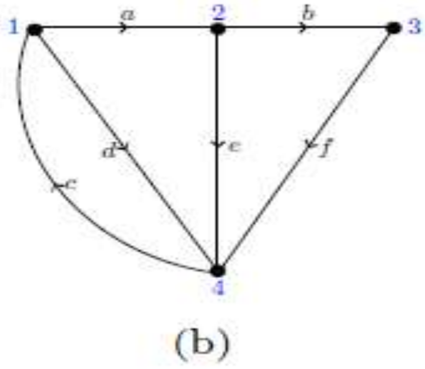


Figure 2 : (a) A circuit and (b) its directed graph.

Note:

- Each of the lines of the graph is indicated a reference direction by an arrow, and the resulted graph is called *oriented/directed graph*.

An Electrical Network & its Graph - III

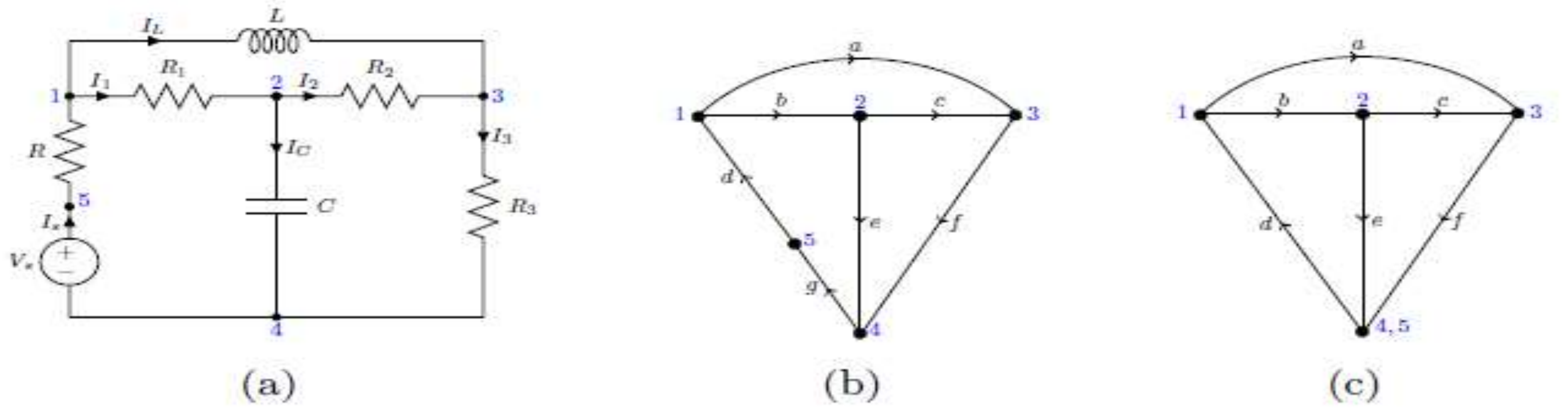


Figure 3 : (a) A circuit, (b) its directed graph and (c) simplified directed graph of (b).

Note:

- The active element branch is replaced by its internal resistance to simplify analysis and computation.

An Electrical Network & its Graph - IV

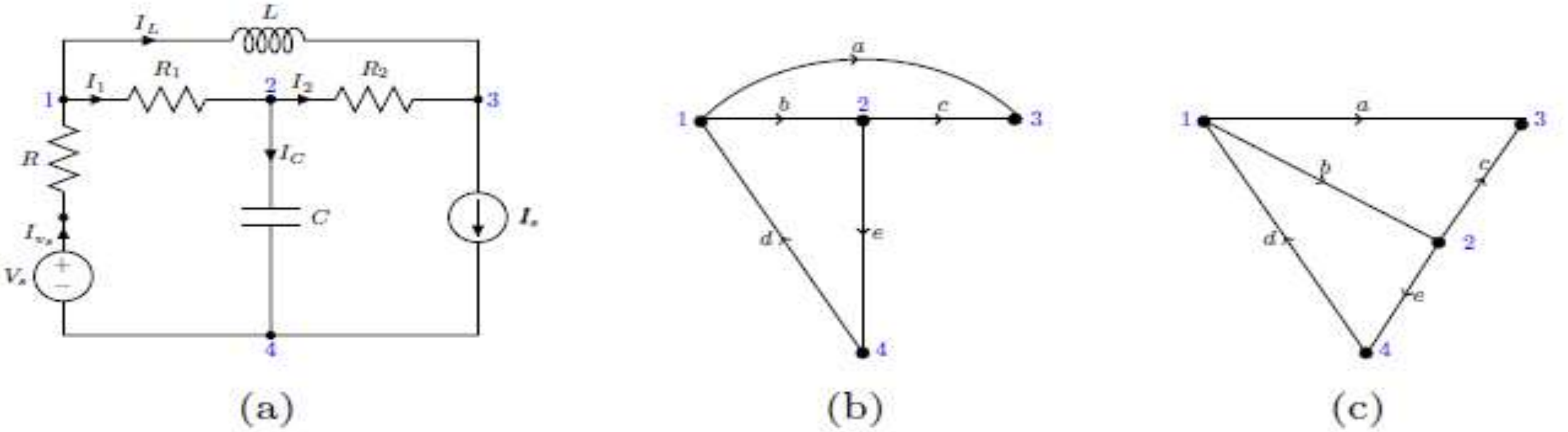


Figure 4 : (a) A circuit, and (b),(c) its directed graphs.

Note:

- The active elements are excluded from the graph to simplify analysis and computation.

An Electrical Network & its Graph - V

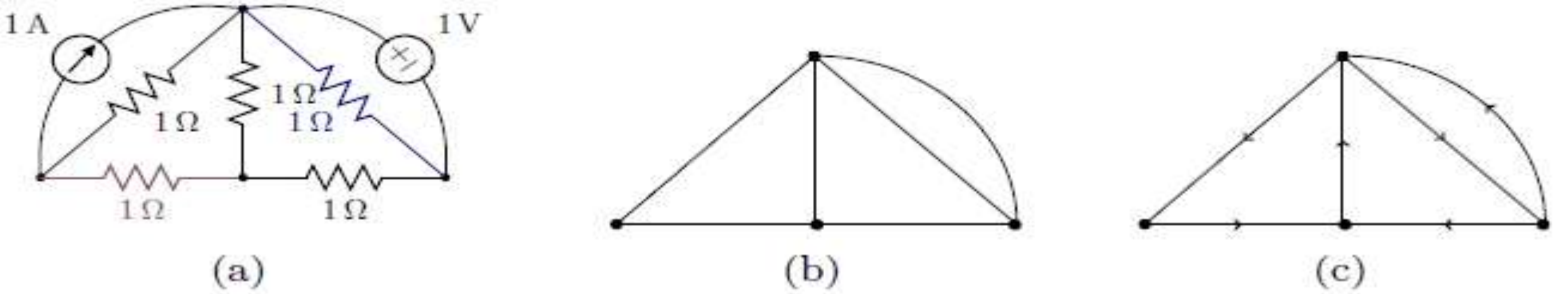


Figure 5 : (a) A circuit, and its- (b) simplified graph and (c) directed graph.

Note:

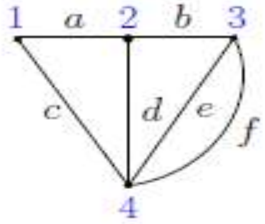
- When voltage source is not in series with any passive element in the given network, it is kept in the graph as a branch.

Network Topology: Terms and Definitions - IV

- **Tree:**
  - A connected subgraph having all the nodes of a graph without any loop.
  - Thus, a tree is a subgraph that has the following properties:
    - It must consist of all nodes of a complete graph.
    - For a graph having n number of nodes, the tree of the given graph will have $n-1$ branches.
    - There exists one and only one path between any pair of nodes.
    - A tree should not have any closed path.
    - The rank of a tree is $(n-1)$. This is also the rank of the graph to which the tree belongs.
- **Twigs:**
  - The branches of a tree are known as *twigs*,
- **Links or Chords:**
  - The branches that are removed from the graph while forming a tree are termed as *links or chords*,
  - Links are complement of twigs.
- **Co-tree:**
  - The graph constituted with links is known as *co-tree*.

Tree and Cotree

Given a Graph:



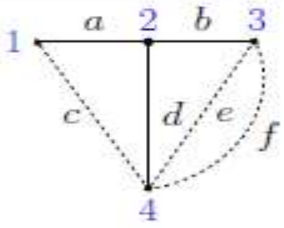
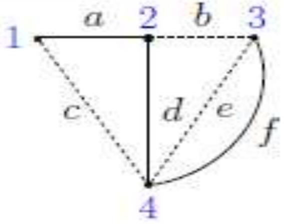
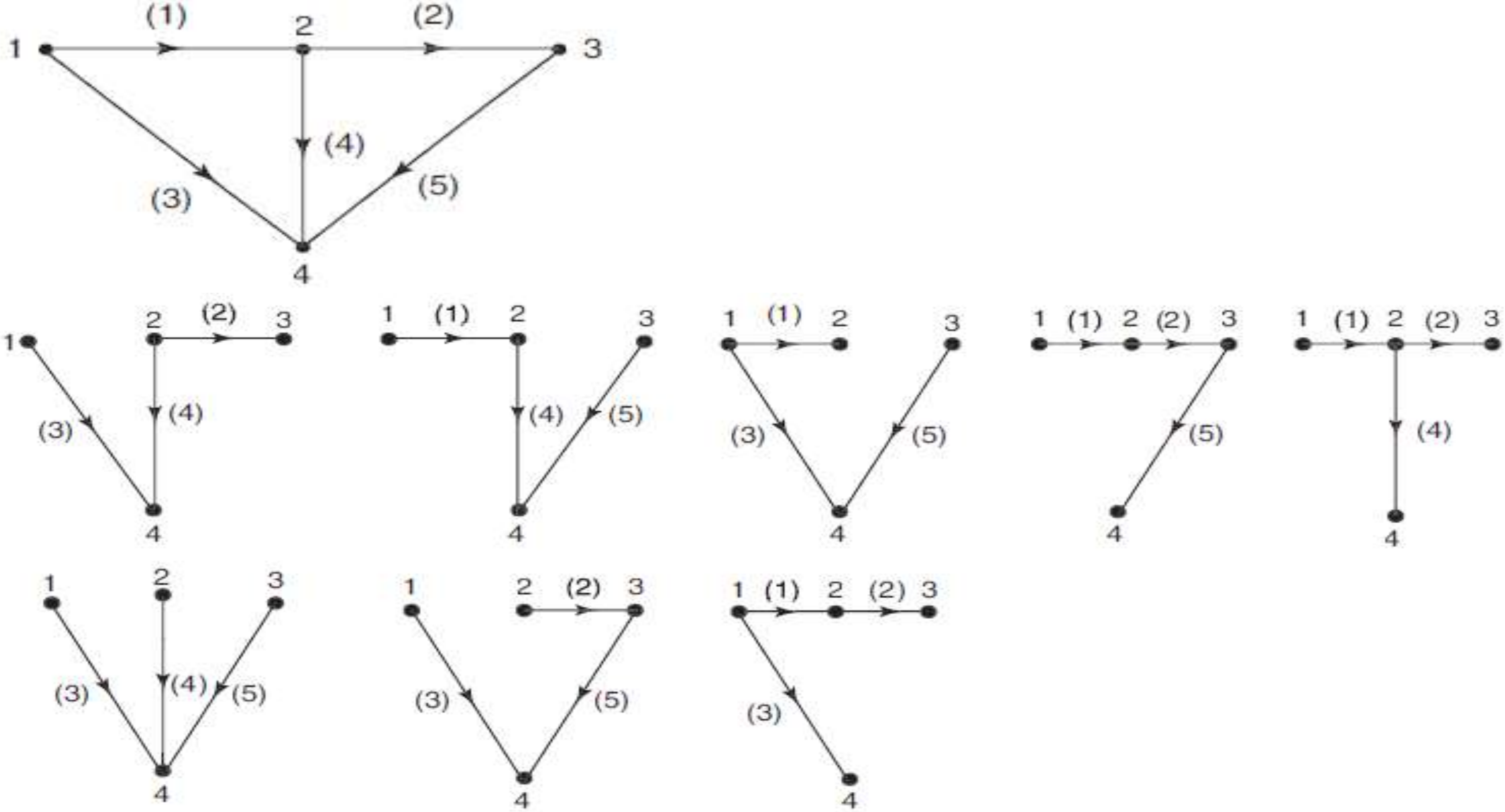
Tree	Twigs of tree	Links of cotree
	{a,b,d}	{c,e,f}
	{a,d,f}	{c,b,e}

Figure shows a graph of the network. Show all the trees of this graph



INCIDENCE MATRIX

A linear graph is made up of nodes and branches. When a graph is given, it is possible to tell which branches are incident at which nodes and what are its orientations relative to the nodes.

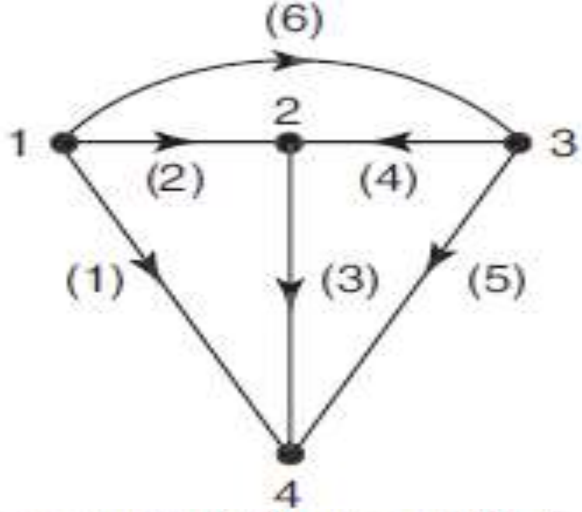
Complete Incidence Matrix (A_a)

For a graph with n nodes and b branches, the complete incidence matrix is a rectangular matrix of order $n \times b$. Elements of this matrix have the following values:

$a_{ij} = 1$, if branch j is incident at node i and is oriented away from node i.

$= -1$, if branch j is incident at node i and is oriented towards node i.

$= 0$, if branch j is not incident at node i.



Nodes ↓	Branches → 1	2	3	4	5	6
1	1	1	0	0	0	1
2	0	–1	1	–1	0	0
3	0	0	0	1	1	–1
4	–1	0	–1	0	–1	0

The complete incidence matrix is

$$A_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

It is seen from the matrix A_a that the sum of the elements in any column is zero. Hence, any one row of the complete incidence matrix can be obtained by the algebraic manipulation of other rows.

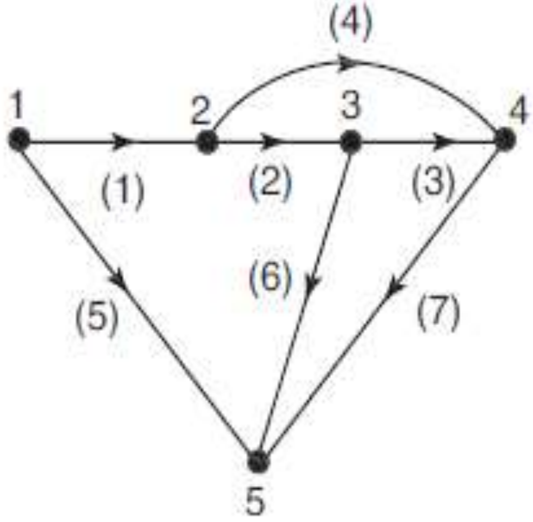
Reduced Incidence Matrix (A)

The reduced incidence matrix A is obtained from the complete incidence matrix A_a by eliminating one of the rows. It is also called *incidence matrix*. It is of order $(n-1) \times b$.

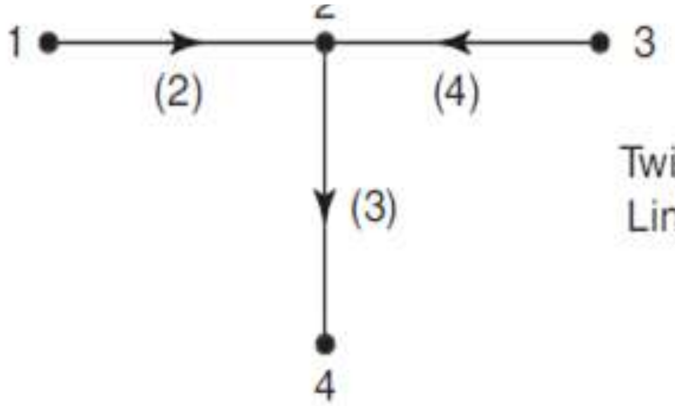
Eliminating the third row of matrix A_a,

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

Find the Incidence Matrix for the graph



When a tree is selected for the graph as shown in Fig. 5.13, the incidence matrix is obtained by arranging a column such that the first $(n - 1)$ column corresponds to twigs of the tree and the last $b - (n - 1)$ branches corresponds to the links of the selected tree.



Twigs: {2, 3, 4}
Links: {1, 5, 6}

$$\begin{array}{cc} & \begin{array}{cccccc} \multicolumn{3}{c}{\text{Twigs}} & \multicolumn{3}{c}{\text{Links}} \\ 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\ A = & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

The matrix A can be subdivided into submatrices A_t and A_l.

$$A = [A_t : A_l]$$

Where A_t the is twig matrix and A_l is the link matrix.

Number of Possible Trees of a Graph

Let the transpose of the reduced incidence matrix A be A^T. It can be shown that the number of possible trees of a graph will be given by

$$\text{Number of possible trees} = |AA^T|$$

the reduced incidence matrix is given by

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

Then transpose of this matrix will be

$$A^T = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Hence, number of all possible trees of the graph

$$AA^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$|AA^T| = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} = 3(9-1)+(1)(-3-1)-1(1+3)=16$$

Thus, 16 different trees can be drawn.

LOOP MATRIX OR CIRCUIT MATRIX

When a graph is given, it is possible to tell which branches constitute which loop or circuit. Alternately, if a loop matrix or circuit matrix is given, we can reconstruct the graph.

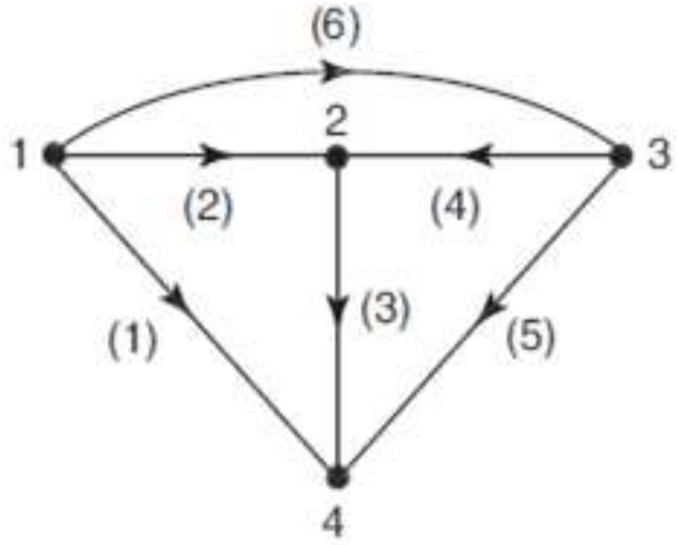
For a graph having n nodes and b branches, the loop matrix B_a is a rectangular matrix of order b columns and as many rows as there are loops.

Its elements have the following values:

b_{ij} = 1, if branch j is in loop i and their orientations coincide.

= – 1, if branch j is in loop i and their orientations do not coincide.

= 0, if branch j is not in loop i.



Loop 1: {1, 2, 3}
Loop 2: {3, 4, 5}
Loop 3: {2, 4, 6}
Loop 4: {1, 2, 4, 5}
Loop 5: {1, 5, 6}
Loop 6: {2, 3, 5, 6}
Loop 7: {1, 3, 4, 6}

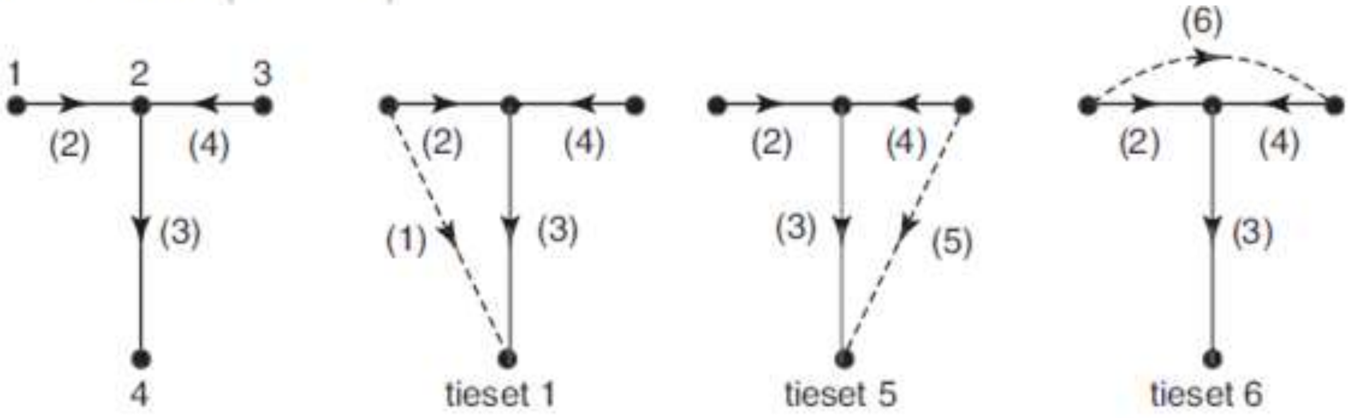
Loops ↓	Branches →					
	1	2	3	4	5	6
1	−1	1	1	0	0	0
2	0	0	−1	−1	1	0
3	0	−1	0	1	0	1
4	−1	1	0	−1	1	0
5	−1	0	0	0	1	1
6	0	−1	−1	0	1	1
7	−1	0	1	1	0	1

$$B_a = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \\ -1 & 1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & 0 & 1 & 1 \\ -1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Fundamental Circuit (Tieset) and Fundamental Circuit Matrix

When a graph is given, first select a tree and remove all the links. When a link is replaced, a closed loop or circuit is formed. Circuits formed in this way are called fundamental circuits or *f*-circuits or tiesets.

Orientation of an *f*-circuit is given by the orientation of the connecting link. The number of *f*-circuits is same as the number of links for a graph. In a graph having b branches and n nodes, the number of *f*-circuits or tiesets will be $(b - n + 1)$.



tieset 1: {1, 2, 3}
tieset 5: {5, 3, 4}
tieset 6: {6, 2, 4}

(a) Tree (b) *f*-circuits (tiesets)

Here, $b = 6$ and $n = 4$.

$$\text{Number of tiesets} = b - n + 1 = 6 - 4 + 1 = 3$$

The branches 1, 2 and 3 are in the tieset 1. Orientation of tieset 1 is given by orientation of branch 1. Since the orientation of branch 1 coincides with orientation of tieset 1, $b_{11} = 1$. The orientations of branches 2 and 3 do not coincide with the orientation of tieset 1, Hence, $b_{12} = -1$ and $b_{13} = -1$. The branches 4, 5 and 6 are not in tieset 1. Hence, $b_{14} = 0$, $b_{15} = 0$ and $b_{16} = 0$. Similarly, other elements of the tieset matrix are written. Then, the tieset schedule will be written as

Tiesets ↓	Branches → 1	2	3	4	5	6
1	1	−1	−1	0	0	0
5	0	0	−1	−1	1	0
6	0	−1	0	1	0	1

Hence, an *f*-circuit matrix or tieset matrix will be given as

$$B = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Usually, the f-circuit matrix B is rearranged so that the first $(n - 1)$ columns correspond to the twigs and $b - (n - 1)$ columns to the links of the selected tree.

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \multicolumn{3}{c}{\text{Twigs}} & \multicolumn{3}{c}{\text{Links}} \\ 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\
B = & \begin{bmatrix} -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

The matrix B can be partitioned into two matrices B_t and B_l.

$$B = [B_t : B_l] = [B_t : U]$$

where B_t is the twig matrix, B_l is the link matrix and U is the unit matrix.

Orthogonal Relationship between Matrix *A* and Matrix *B*

For a linear graph, if the columns of the two matrices A_a and B_a are arranged in the same order, it can be shown that

$$A_a B_a^T = 0$$

or

$$B_a A_a^T = 0$$

The above equations describe the orthogonal relationship between the matrices A_a and B_a.

If the reduced incidence matrix A and the *f*-circuit matrix B are written for the same tree, it can be shown that

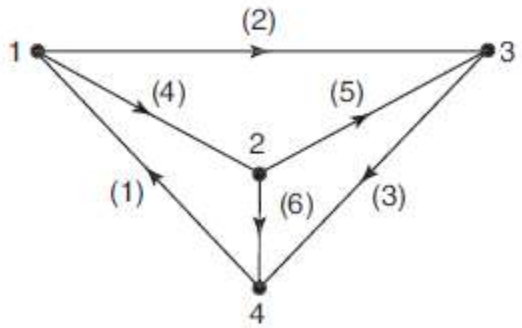
$$A B^T = 0$$

or

$$B A^T = 0$$

These two equations show the orthogonal relationship between matrices A and B.

The graph of a network is shown in Fig.Write the (a) incidence matrix, (b) tieset matrix,



Solution

(a) Incidence Matrix (A)

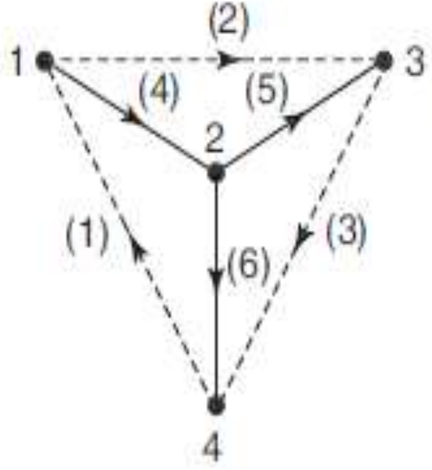
$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

The incidence matrix A is obtained by eliminating any row from the matrix A_a.

$$A = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

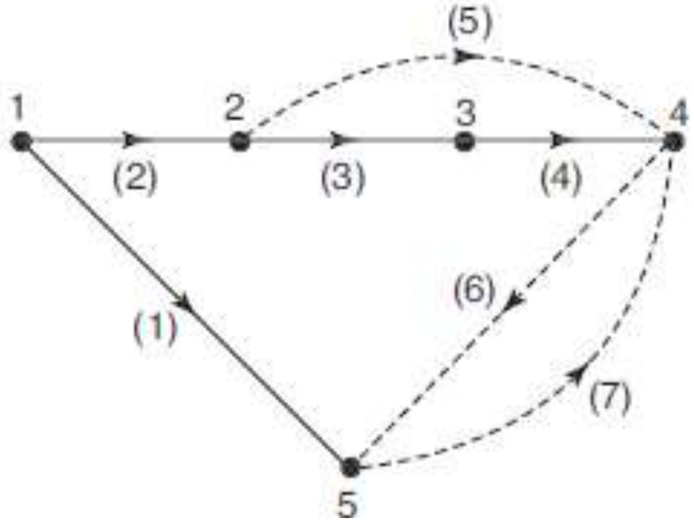
(b) Tieset, Matrix (B)

The oriented graph, selected tree and tiesets are shown in Fig. 5.35.



Links: {1, 2, 3}
Tieset 1: {1, 4, 6}
Tieset 2: {2, 4, 5}
Tieset 3: {3, 5, 6}

$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix} \end{array}$$



(5)
1
2
3
4
(2)
(3)
(4)
(6)
(1)
(7)
5

CUTSET MATRIX

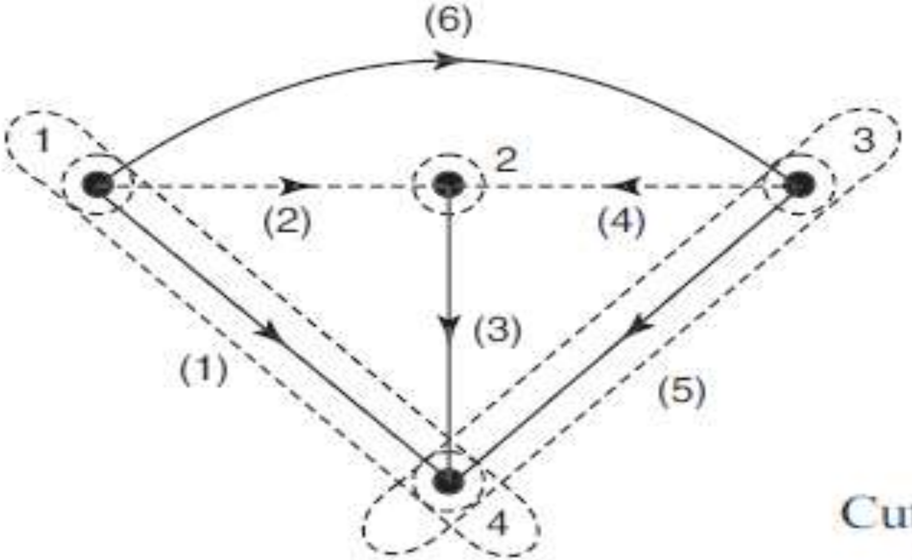
Consider a linear graph. By removing a set of branches without affecting the nodes, two connected sub-graphs are obtained and the original graph becomes unconnected. The removal of this set of branches which results in cutting the graph into two parts are known as a *cutset*. The cutset separates the nodes of the graph into two groups, each being in one of the two groups.

For a graph having n nodes and b branches, the cutset matrix Q_a is a rectangular matrix of order b columns and as many rows as there are cutsets. Its elements have the following values:

q_{ij} = 1, if the branch j is in the cutset i and the orientation coincide.

= –1, if the branch j is in the cutset i and the orientations do not coincide.

= 0, if the branch j is not in the cutset i.



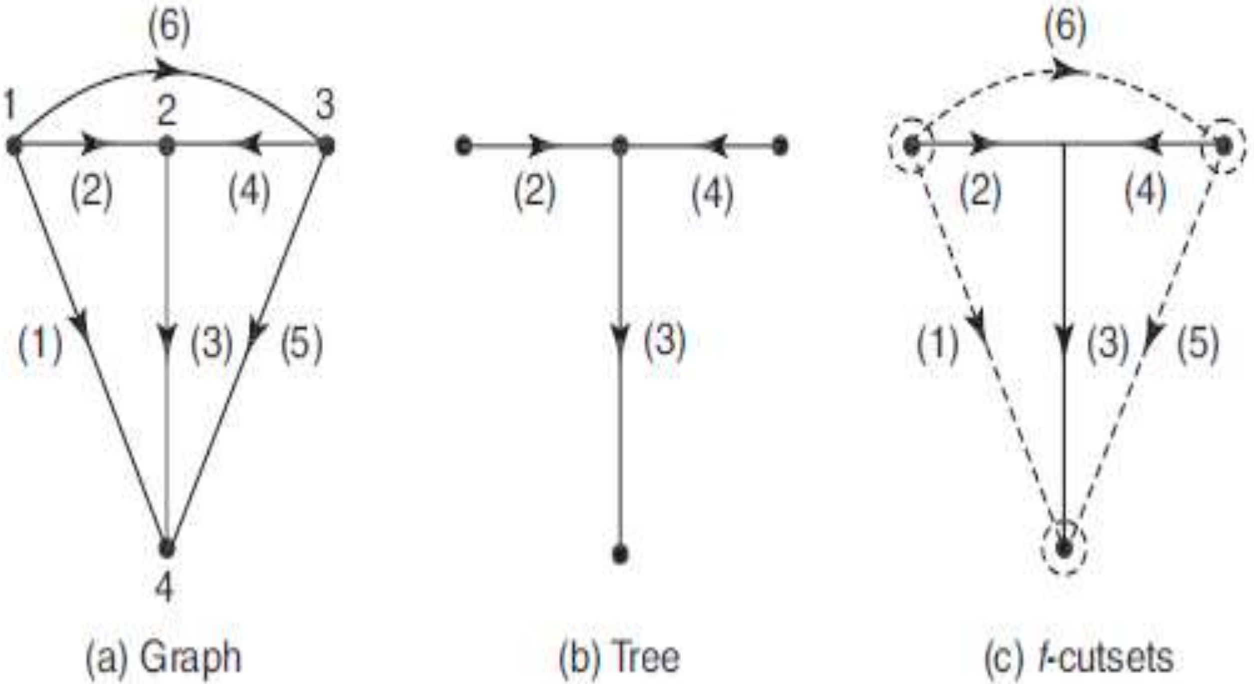
Cutset 1: {1, 2, 6}
Cutset 2: {2, 3, 4}
Cutset 3: {3, 1, 5}
Cutset 4: {4, 5, 6}
Cutset 5: {5, 2, 3, 6}
Cutset 6: {6, 1, 3, 4}

Cutsets ↓	Branches → 1	2	3	4	5	6
1	1	1	0	0	0	1
2	0	1	−1	1	0	0
3	1	0	1	0	1	0
4	0	0	0	1	1	−1
5	0	−1	1	0	1	−1
6	1	0	1	−1	0	1

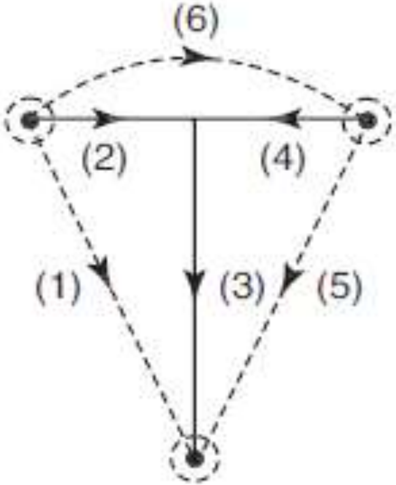
Hence, the cutset matrix Q_a is given as

$$Q_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & -1 & 1 & 0 & 1 & -1 \\ 1 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

Fundamental Cutset and Fundamental Cutset Matrix



f-cutset 2: {2, 1, 6}
f-cutset 3: {3, 1, 5}
f-cutset 4: {4, 5, 6}



f-cutset 2: {2, 1, 6}
f-cutset 3: {3, 1, 5}
f-cutset 4: {4, 5, 6}

f-cutsets ↓	Branches → 1	2	3	4	5	6
2	1	1	0	0	0	1
3	1	0	1	0	1	0
4	0	0	0	1	1	–1

Hence, the *f*-cutset matrix Q is given by

$$Q = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

The *f*-cutset matrix Q is rearranged so that the first $(n-1)$ columns correspond to twigs and $b-(n-1)$ columns to links of the selected tree.

$$
\begin{array}{c} \\ \\ Q = \end{array}
\begin{array}{c}
\begin{array}{cccccc} \multicolumn{3}{c}{\text{Twigs}} & \multicolumn{3}{c}{\text{Links}} \\ 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\
\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}
\end{array}
$$

The matrix Q can be subdivided into matrices Q_t and Q_l.

$$Q = [Q_t : Q_l] = [U : Q_l]$$

where Q_t is the twig matrix, Q_l is the link matrix and U is the unit matrix.

Orthogonal Relationship between Matrix *B* and Matrix Q

If the *f*-circuit matrix B and the *f*-cutset matrix Q are written for the same selected tree, it can be shown that

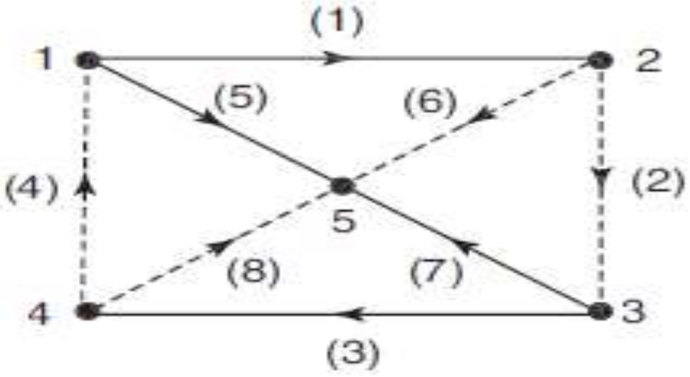
$$B\,Q^T = 0$$

or

$$Q\,B^T = 0$$

These two equations show the orthogonal relationship between matrices A and B.

For the graph shown in Fig. write the incidence matrix, tieset matrix and f-cutset matrix.



Incidence Matrix (A)

$$A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

(b) Tieset Matrix (B)

Links: $\{2, 4, 6, 8\}$

Tieset 2: $\{2, 7, 5, 1\}$

Tieset 4: $\{4, 5, 7, 3\}$

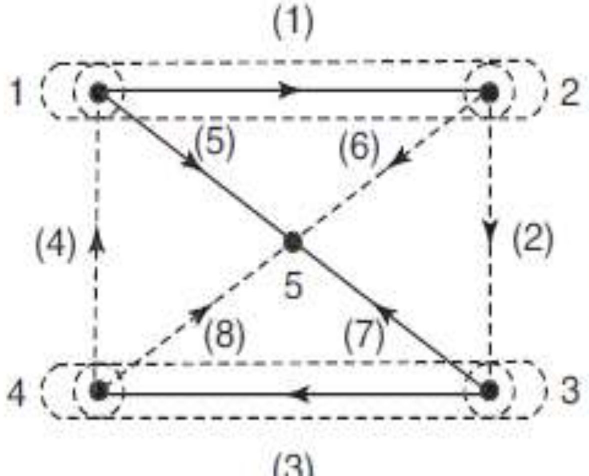
Tieset 6: $\{6, 5, 1\}$

Tieset 8: $\{8, 7, 3\}$

$$B = \begin{array}{cc} & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{array}{c} 2 \\ 4 \\ 6 \\ 8 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

(c) f-cutset Matrix (Q)

The oriented graph, selected tree and f-cutsets are shown in Fig. 5.40.



Twigs: $\{1, 3, 5, 7\}$

f-cutset 1: $\{1, 6, 2\}$

f-cutset 3: $\{3, 4, 8\}$

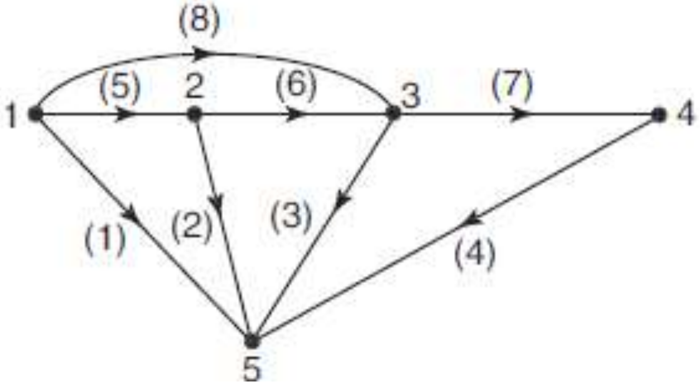
f-cutset 5: $\{5, 4, 6, 2\}$

f-cutset 7: $\{7, 2, 8, 4\}$

$$Q = \begin{array}{cc} & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 5 \\ 7 \end{array} & \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Draw the oriented graph from the complete incidence matrix given below;

Nodes ↓ \ Branches →	1	2	3	4	5	6	7	8
1	1	0	0	0	1	0	0	1
2	0	1	0	0	−1	1	0	0
3	0	0	1	0	0	−1	1	−1
4	0	0	0	1	0	0	−1	0
5	−1	−1	−1	−1	0	0	0	0



The fundamental cutset matrix of a network is given as follows;

Twigs			Links		
a	c	e	b	d	f
1	0	0	1	0	1
0	1	0	0	1	1
0	0	1	1	1	1

Draw the oriented graph.

Solution No. of links $l = b - n + 1$

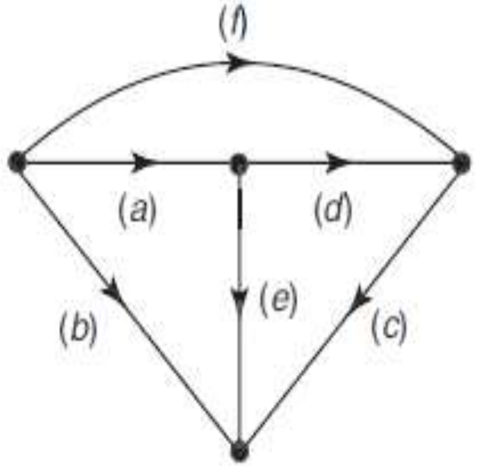
No. of nodes $n = b - l + 1 = 6 - 3 + 1 = 4$

f-cutsets are written as,

f-cutsets a: $\{a, b, f\}$

f-cutsets c: $\{c, d, f\}$

f-cutsets e: $\{e, b, d, f\}$



Twigs: $\{a, c, e\}$
Links: $\{b, d, f\}$

Draw the oriented graph of a network with the f-cutset matrix as shown below:

$$\begin{array}{cccc|ccc} \multicolumn{4}{c}{\text{Twigs}} & \multicolumn{3}{c}{\text{Links}} \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{array}$$

Solution No. of links $l = b - n + 1$

No. of nodes $n = b - l + 1 = 7 - 3 + 1 = 5$

f-cutsets are written as

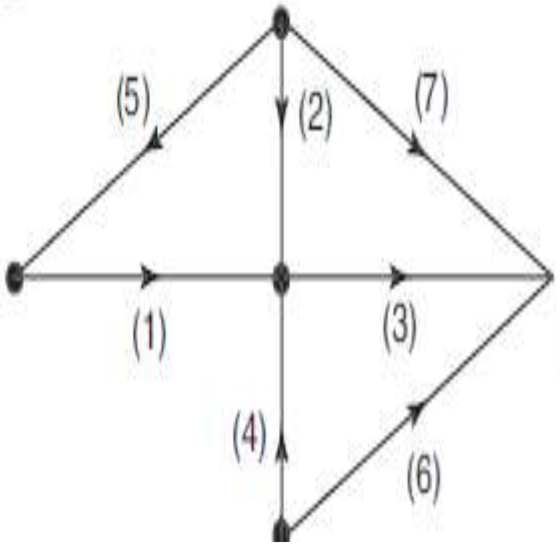
f-cutset 1: $\{1, 5\}$

f-cutset 2: $\{2, 5, 7\}$

f-cutset 3: $\{3, 6, 7\}$

f-cutset 4: $\{4, 6\}$

Then oriented graph can be drawn as shown in Fig. 5.40.



Twigs: $\{1, 2, 3, 4\}$
Links: $\{5, 6, 7\}$