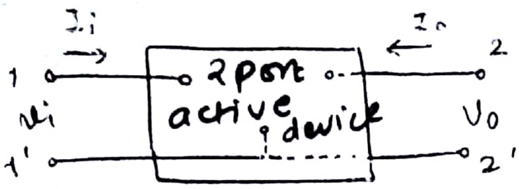
# The Hybrid Equivalent Model




two port system

$v_i = h_{11} I_i + h_{12} v_o$

$I_o = h_{21} I_i + h_{22} v_o$

h-parameter → hybrid →
(mixture of V & I)

① If we set $v_o = 0$ (short ckt o/p terminals)

$$\boxed{h_{11} = \left.\frac{v_i}{I_i}\right|_{v_o=0} \quad \Omega}$$

$$\boxed{h_{21} = \left.\frac{I_o}{I_i}\right|_{V_o=0} \quad \text{unitless}}$$

$h_{11}$ – short ckt i/p impedance parameter

$h_{21}$ – short ckt forward transfer current ratio parameter.

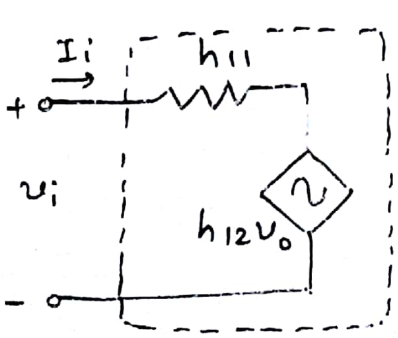
② If we set $I_i = 0$ (open the i/p terminals)

$$\boxed{h_{12} = \left.\frac{v_i}{v_o}\right|_{I_i=0} \quad \text{unitless}}$$

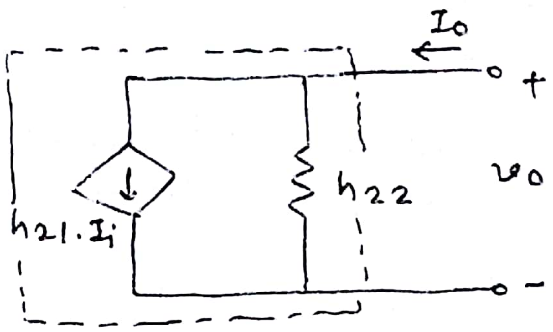
$$\boxed{h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i=0} \quad \text{siemens}}$$

$h_{12}$ = open ckt reverse transfer voltage ratio parameter

$h_{22}$ = open circuit output admittance parameter.




hybrid i/p Equivalent



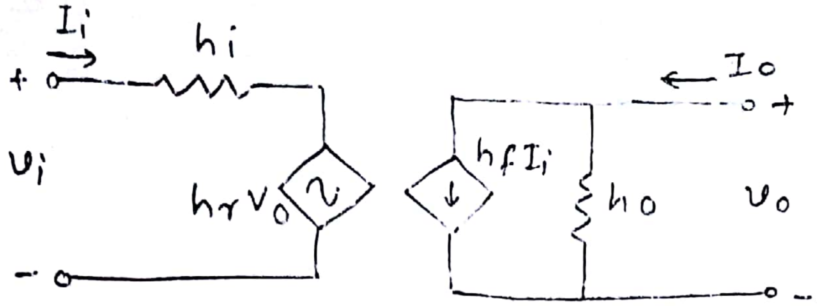

Hybrid o/p equivalent

$h_{11}$ → i/p resistance → $h_i$

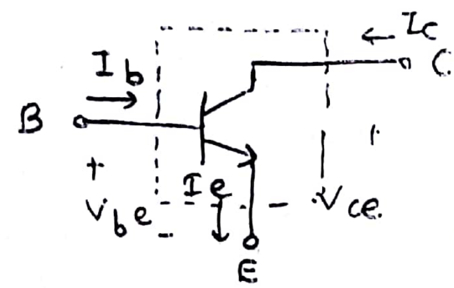
$h_{12}$ → reverse transfer vtg ratio → $h_r$

$h_{21}$ → forward transfer current ratio → $h_f$

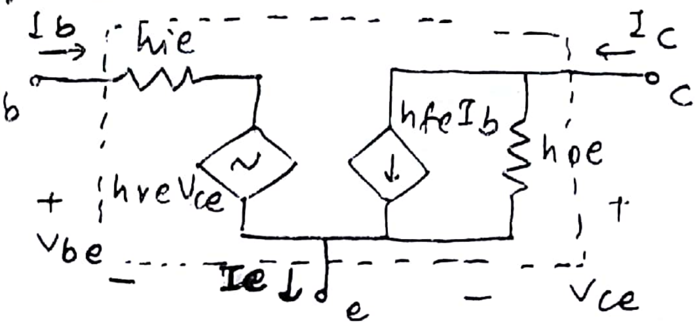
$h_{22}$ → o/p conductance → $h_o$




complete hybrid Equivalent Model

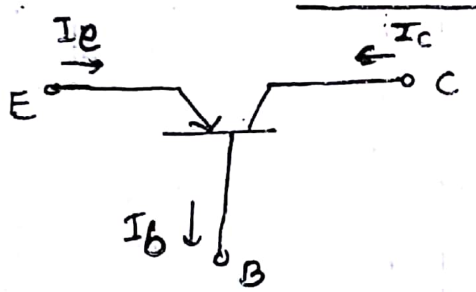


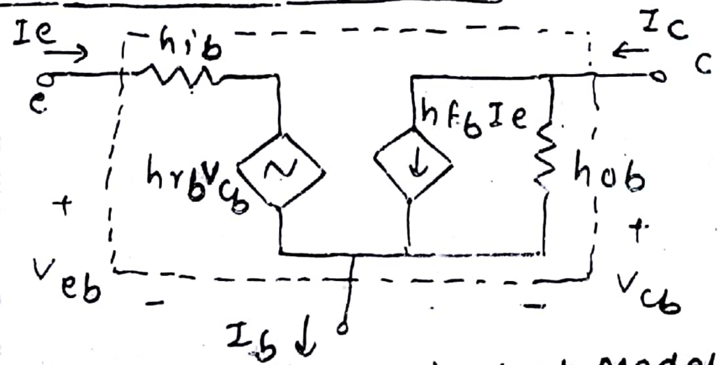

a) Graphical symbol




Hybrid Equivalent Model

Common Emitter Configuration

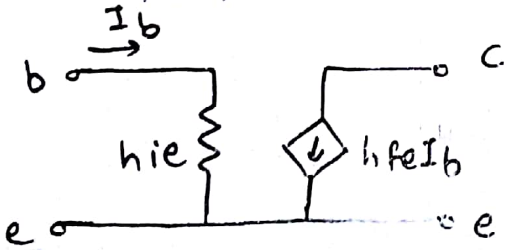


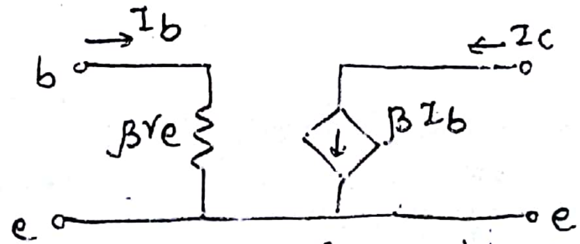




a) graphical symbol

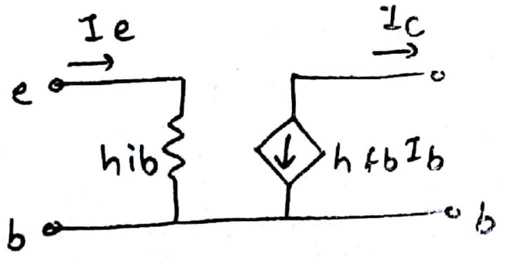
Hybrid Equivalent Model

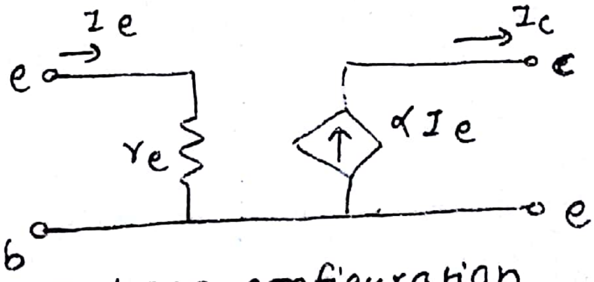
Common Base Configuration




common Emitter Configuration



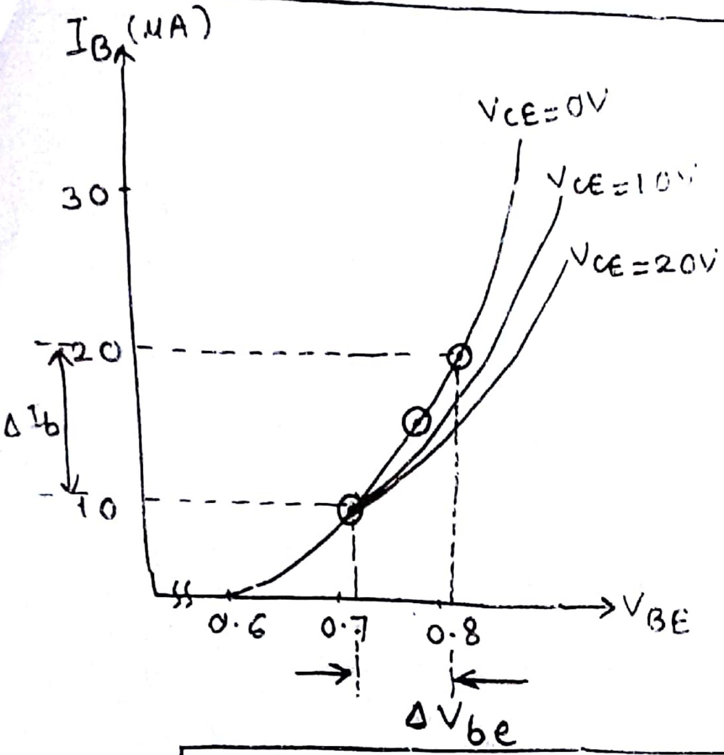

common base configuration

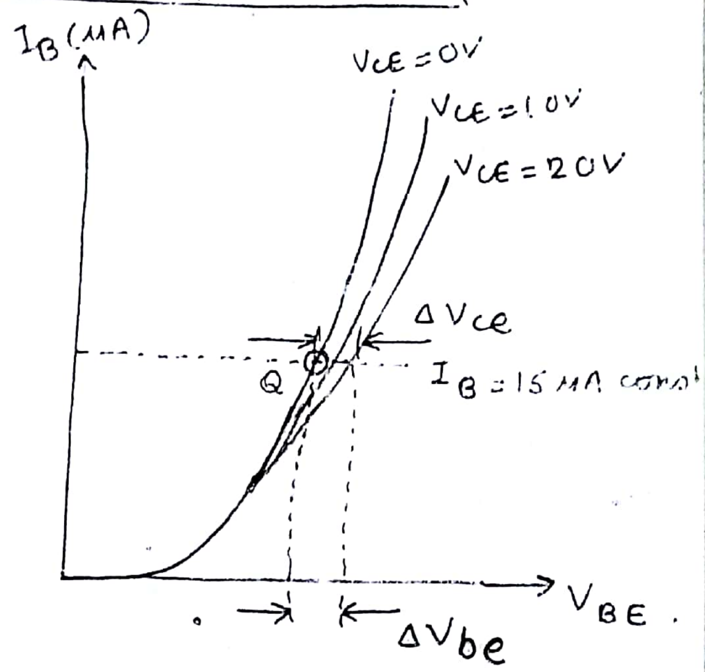
$\boxed{h_{ie} = \beta r_e \quad h_{fe} = \beta_{ac}}$

$\boxed{h_{ib} = r_e \quad h_{fb} = -\alpha \cong -1}$

Hybrid Vs 're' Model

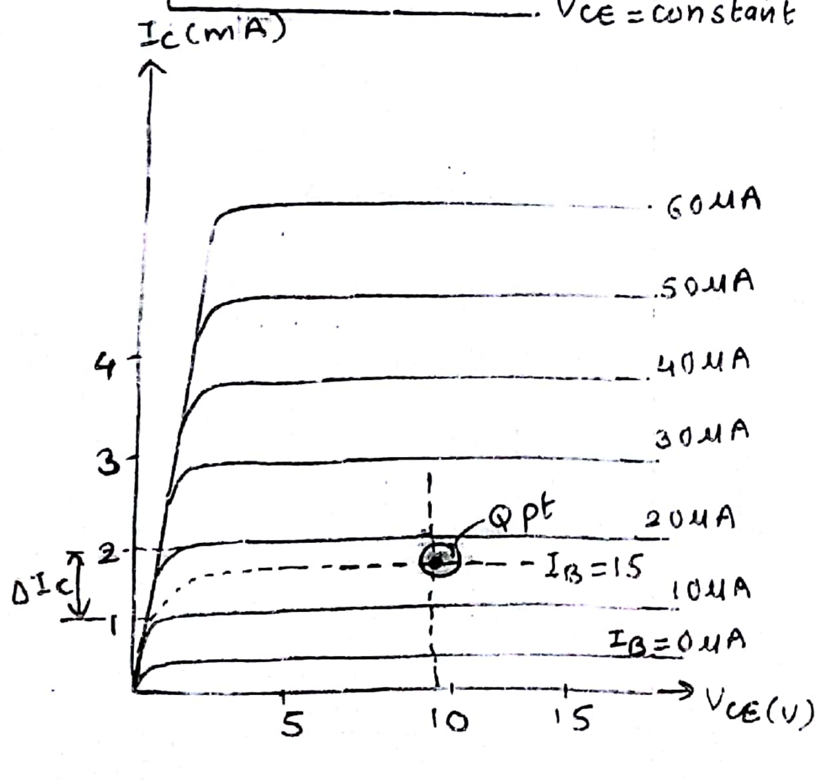
# Graphical Determination of h-parameters :-

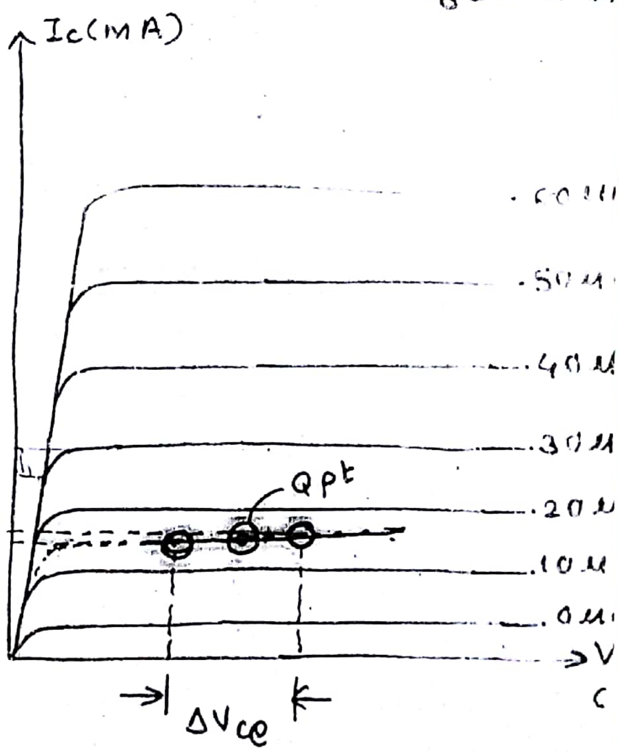



$$|h_{ie}| = \left.\frac{\Delta V_{be}}{\Delta I_b}\right|_{V_{CE} = \text{constant}}$$

$$|h_{re}| = \left.\frac{\Delta V_{be}}{\Delta V_{ce}}\right|_{I_B = \text{constant}}$$




$$|h_{fe}| = \left.\frac{\Delta I_c}{\Delta I_b}\right|_{V_{CE} = \text{constant}}$$

$$|h_{oe}| = \left.\frac{\Delta I_c}{\Delta V_{CE}}\right|_{I_B = \text{constant}}$$

| Parameter | CE | CC | CB |
|---|---|---|---|
| $h_i$ | $1K\Omega$ | $1K\Omega$ | $20\Omega$ |
| $h_r$ | $2.5\times10^{-4}$ | $\cong 1$ | $3\times10^{-4}$ |
| $h_f$ | 50 | $-50$ | $-0.98$ |
| $h_o$ | $25\mu A/V$ | $25\mu A/V$ | $0.5\mu A/V$ |
| $1/h_o$ | $40K\Omega$ | $40K\Omega$ | $2M\Omega$ |

## Parameter Relationship :-

| CE to CC | CE to CB |
|---|---|
| $h_{ic} = h_{ie}$ | $h_{ib} \approx h_{ie}/(1+h_{fe})$ |
| $h_{rc} = 1 + h_{re}$ | $h_{rb} = \dfrac{h_{ie}h_{oe}}{1+h_{fe}} - h_{re}$ |
| $h_{fc} = -(1+h_{fe})$ | $h_{fb} = \dfrac{-h_{fe}}{1+h_{fe}}$ |
| $h_{oc} = h_{oe}$ | $h_{ob} = \dfrac{h_{oe}}{1+h_{fe}}$ |

| CE configuration | CB configuration |
|---|---|
| $h_{ie} = \beta r_e$ | $h_{ib} = r_e$ |
| $h_{fe} = \beta_{ac}$ | $h_{fb} = -\alpha \cong -1$ |

## Graphical determination of h-parameters -

| | |
|---|---|
| $h_{ie} \cong \left.\dfrac{\Delta V_{be}}{\Delta I_b}\right\vert_{V_{CE}=const}$ (ohms) | $h_{fe} \cong \left.\dfrac{\Delta I_c}{\Delta I_b}\right\vert_{V_{CE}=constant}$ (unitless) |
| $h_{re} \cong \left.\dfrac{\Delta V_{be}}{\Delta V_{ce}}\right\vert_{I_B=constant}$ (unitless) | $h_{oe} \cong \left.\dfrac{\Delta I_c}{\Delta V_{ce}}\right\vert_{I_B=constant}$ (siemens) |

## * Small signal Amplifier Analysis using Hybrid - π Model -

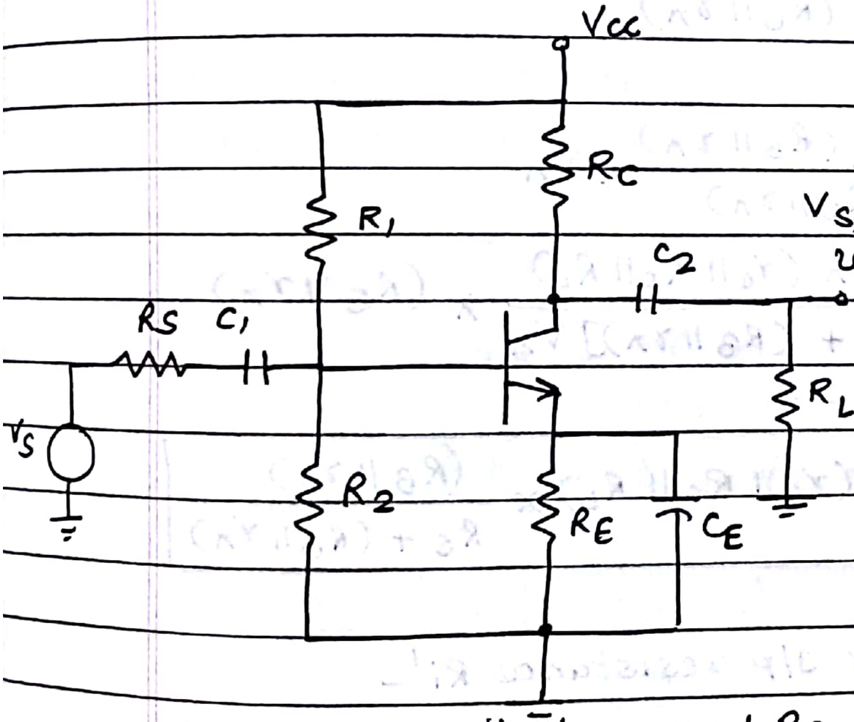
### ① CE Amplifier with Bypassed $R_E$ - (May11, Dec14)

*** Type of Questions**

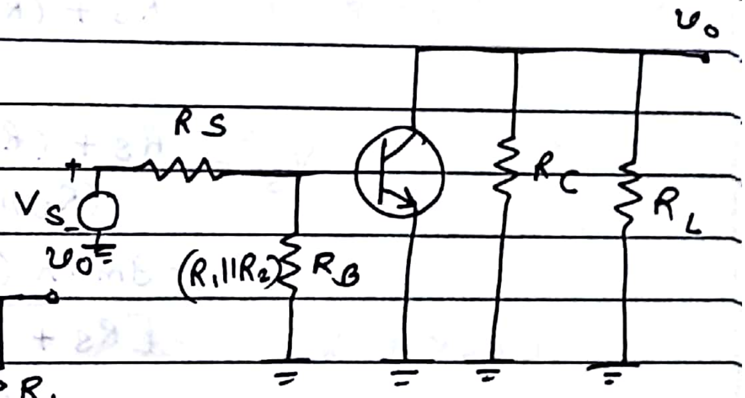
① For a CE amplifier derive the expressions for AV, Ai, Zi & Zo (May 11, 10)

② Draw circuit diagram of CE amplifier with voltage divider bias with bypassed $R_E$ & derive expression for voltage gain, current gain, i/p resistance, o/p resistance using hybrid π Model which includes early effect. (Dec14, 10)

**Step ①** Draw the ac equivalent circuit -



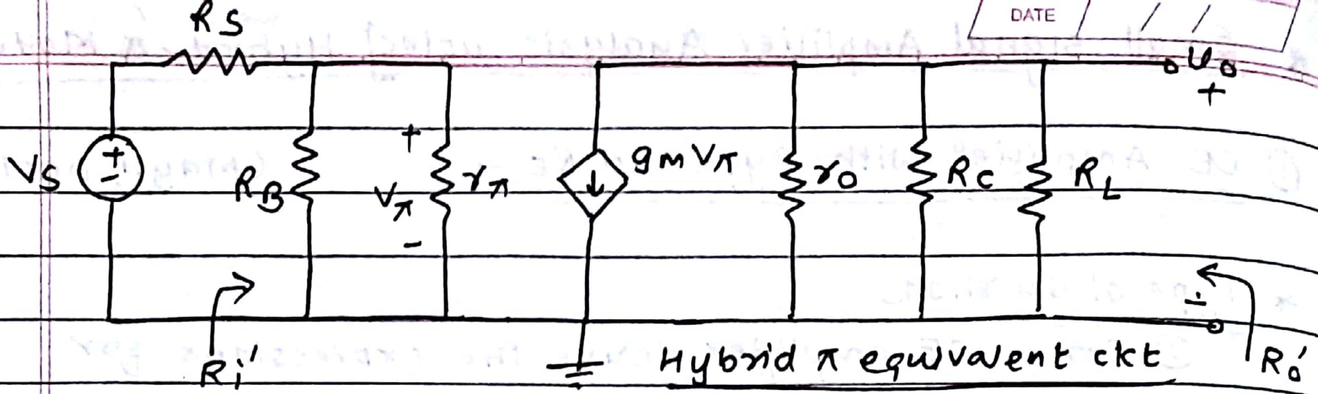

CE Amplifier with bypassed $R_E$




AC equivalent circuit

**Step ②** Draw hybrid π equivalent ckt -

Replace the transistor in ac equivalent ckt by the hybrid π equivalent ckt.




Hybrid π equivalent ckt

**Step ③ - Expression for voltage gain ($A_{vs}$)-**

$$A_{vs} = \frac{V_o}{V_s}$$

$$v_o = -g_m V_\pi (r_o \parallel R_C \parallel R_L)$$

$$V_\pi = \frac{(R_B \parallel r_\pi)}{R_S + (R_B \parallel r_\pi)} \times V_s \qquad (R_B = R_1 \parallel R_2)$$

$$V_s = \frac{R_S + (R_B \parallel r_\pi)}{(R_B \parallel r_\pi)} \times V_\pi$$

$$= \frac{-g_m V_\pi (r_o \parallel R_C \parallel R_L)}{[R_S + (R_B \parallel r_\pi)] V_\pi} \times (R_B \parallel r_\pi)$$

$$\boxed{A_{vs} = -g_m (r_o \parallel R_C \parallel R_L) \times \frac{(R_B \parallel r_\pi)}{R_S + (R_B \parallel r_\pi)}}$$

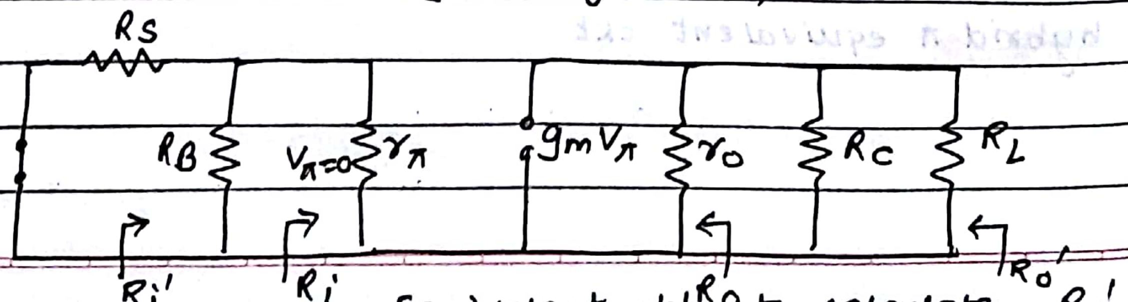
**Step ④ Expression for i/p resistance $R_i'$-**

$$R_i' = (R_B \parallel r_\pi) = (R_1 \parallel R_2 \parallel r_\pi)$$

also $R_i = r_\pi$

**Step ⑤ Expression for output Resistance $R_o'$ -**

$R_o'$ is obtained by setting $V_s = 0$, As $V_s = 0$, $V_\pi = 0$, $\therefore g_m V_\pi =$




Equivalent ckt to calculate $R_o'$

o/p Resistance back into o/p terminal,

$R_o' = (r_o \| R_C \| R_L)$ also $R_o = r_o$

**step ⑥** Expression for current gain ($A_{IS}$) -

$$A_{IS} = \frac{I_o}{I_i}$$

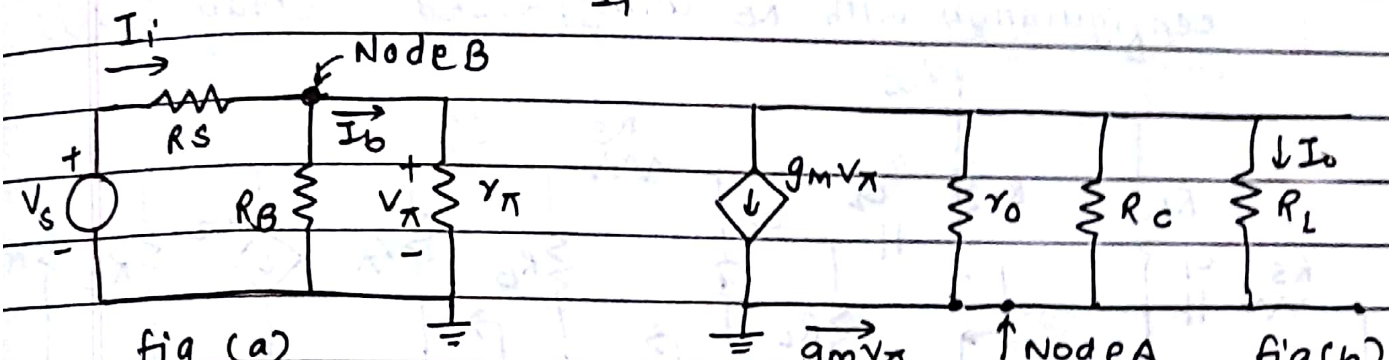



fig (a) fig (b)

using current division Rule, $I_o = \frac{-g_m V_\pi (r_o \| R_C)}{(r_o \| R_C) + R_L}$ — ①

from fig a) $V_\pi = I_b r_\pi$ — ②

using current division Rule, $I_b = \frac{R_B}{R_B + r_\pi} \times I_i$ Put in ②

$$\therefore V_\pi = \frac{R_B r_\pi}{(R_B + r_\pi)} \times I_i = (R_B \| r_\pi) I_i$$

Put value of $V_\pi$ in eq$^n$ ①

$$I_o = \frac{-g_m (r_o \| R_C)}{R_L + (r_o \| R_C)} \times (R_B \| r_\pi) I_i$$

current gain

$$\boxed{A_{IS} = \frac{I_o}{I_i} = \frac{-g_m (r_o \| R_C)(R_B \| r_\pi)}{R_L + (r_o \| R_C)}}$$

**Advantage** - provides high volltage gain (due to low $R_i$)

**Disadvantage** - low i/p Resistance

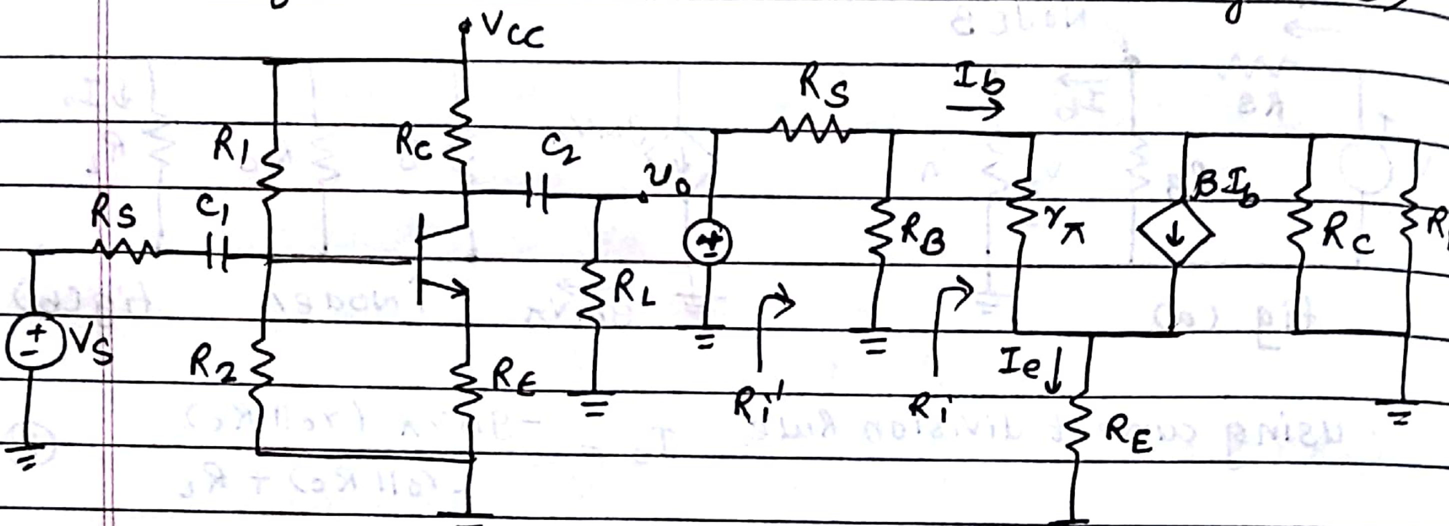
$\boxed{R_i = r_\pi}$ & $\boxed{R_i' = R_1 \| R_2 \| r_\pi}$

# * CE Amplifier with Unbypassed $R_E$

DATE / /

Type of Questions -

1. Derive the equations for $A_V$, $A_i$, $R_i$ & $R_o$ for NPN transistor in CE mode voltage divider bias configuration with $R_E$ unbypassed. (May 15, 10)




CE amplifier with unbypassed $R_E$ — Hybrid $\pi$ equivalent ckt

**Step ①** obtain expression for $R_i$ & $R_i'$

$$R_i = \frac{V_b}{I_b} \quad \text{--- (1)}$$

But $V_b = I_b r_\pi + I_e R_E$

$$= I_b r_\pi + (1+\beta) I_b R_E$$

$V_b = I_b [r_\pi + (1+\beta) R_E]$ Put in eqn (1)

$$R_i = \frac{I_b [r_\pi + (1+\beta) R_E]}{I_b}$$

$$\boxed{R_i = r_\pi + (1+\beta) R_E}$$

$$R_i' = R_B \parallel R_i = (R_1 \parallel R_2) \parallel [r_\pi + (1+\beta) R_E]$$

**Step ②** obtain expression of voltage gain $A_{Vs}$ -

$$A_{Vs} = \frac{V_o}{V_s}$$

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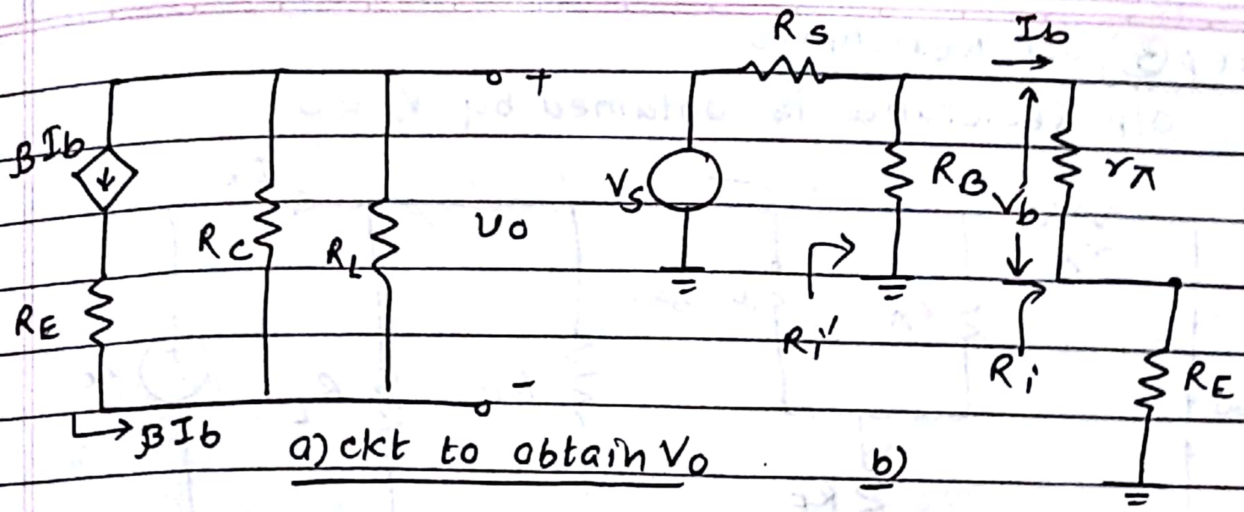



a) ckt to obtain $V_o$ b)

$$V_o = -\beta I_b (R_C \parallel R_L)$$

from fig b), $V_b = I_b R_i$ or $I_b = V_b / R_i$

$$V_o = \frac{-\beta V_b (R_C \parallel R_L)}{R_i} \quad \text{But } V_b = \frac{R_i'}{R_i' + R_s} \times V_s$$

$$V_o = \frac{-\beta R_i' V_s}{(R_i' + R_s)} \times \frac{(R_C \parallel R_L)}{R_i}$$

$$\frac{V_o}{V_s} = A_{V_s} = \frac{-\beta R_i'}{(R_i' + R_s)} \times \frac{(R_C \parallel R_L)}{R_i}$$

But $R_i = r_\pi + (1+\beta) R_E$

$$\boxed{A_{V_s} = \frac{-\beta (R_C \parallel R_L)}{r_\pi + (1+\beta) R_E} \times \frac{R_i'}{(R_i' + R_s)}}$$ Exact expression

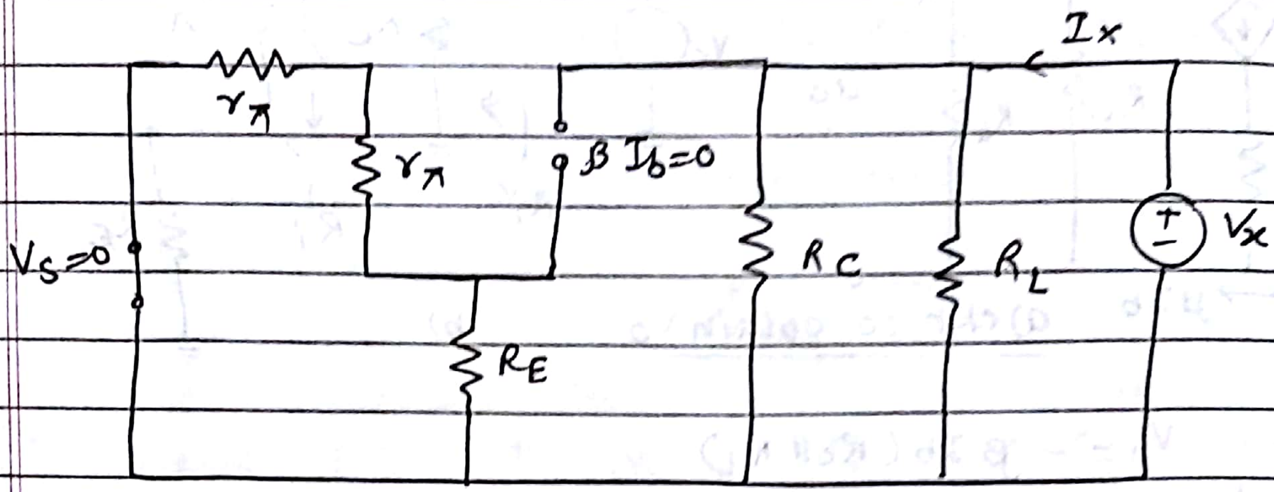
But if $R_i' >> R_s$ & $(1+\beta) >> r_\pi$ then

$$A_{V_s} \approx \frac{-\beta (R_C \parallel R_L)}{(1+\beta) R_E}$$ if $\beta = (1+\beta)$

$$\boxed{A_{V_s} = \frac{-(R_C \parallel R_L)}{R_E}}$$ Approximate voltage gain

## Step ③ o/p Resistance

o/p Resistance is obtained by $V_s = 0$




Circuit to obtain output Resistance

As $V_s = 0$, $I_b = 0$ $\therefore \beta I_b = 0$

$$\boxed{R_o' = \frac{V_x}{I_x} = R_C \parallel R_L}$$

**Advantages -**

- It increases i/p impedance ($R_i'$)
- stability of Av increases w.r.t $\beta$

**Disadvantage -**

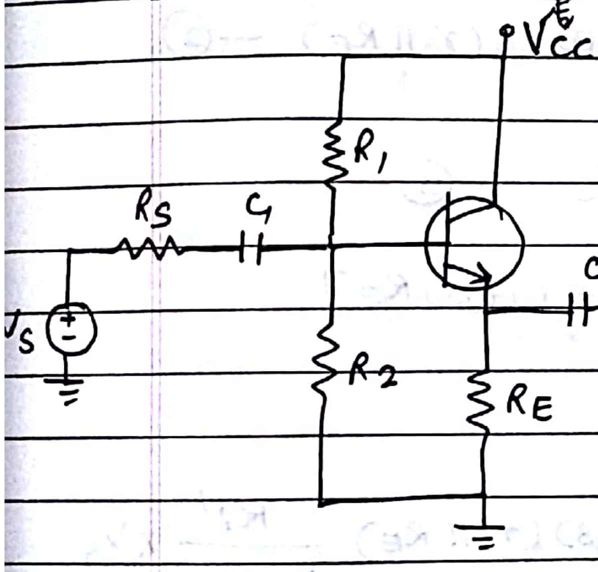
Reduction in voltage gain (Av)

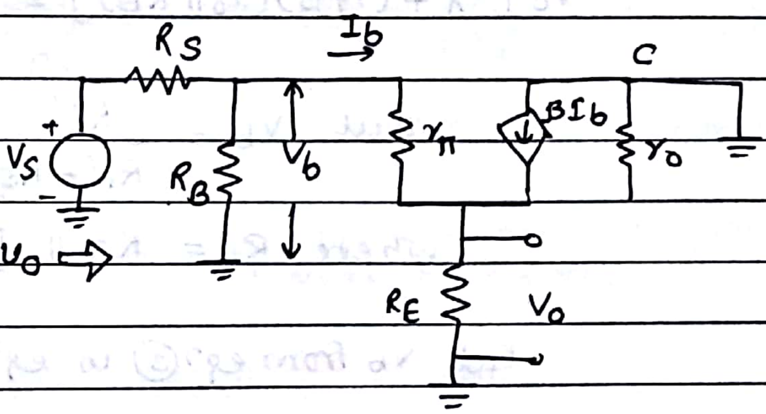
# * Common Collector (Emitter Follower) Amplifier –

**Type of Questions –**

1. Write short note on small signal equivalent circuit of CC ampl$^r$. (May 15, 5)
2. Draw emitter follower circuit & derive an expression for voltage gain Av. (Dec 15, 10)
3. Draw a neat circuit diagram of emitter follower configuration & its hybrid-π Model. (Dec 16, 04)

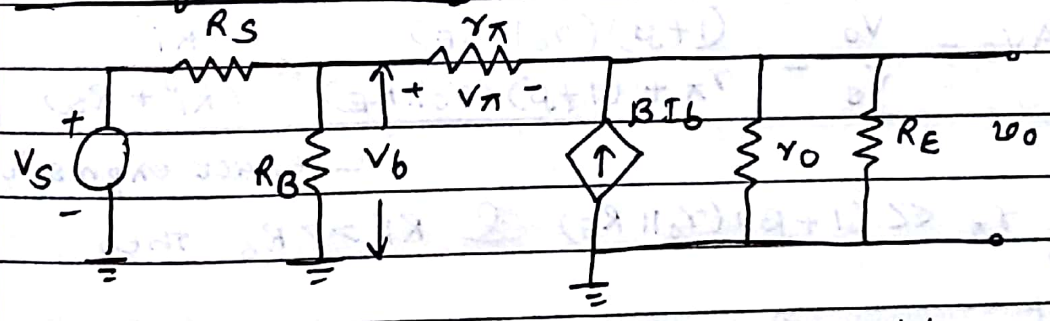
**Step ①** Redraw the hybrid π circuit




b) Hybrid-π Equivalent ckt

a) Emitter follower amplifier




c) simplified hybrid-π Equivalent ckt

**Step ②** Expression for AVs –

$$V_o = I_b (r_o \parallel R_E) + \beta I_b (r_o \parallel R_E)$$

$\because$ (current through $(r_o \parallel R_E)$ is $I_b + \beta I_b$)

$$V_o = (1+\beta) I_b (r_o \parallel R_E) \quad \text{—①}$$

from fig c) $I_b = \frac{V_\pi}{r_\pi} = \frac{V_b - V_e}{r_\pi}$ But $V_e = V_o$

$$\therefore I_b = \frac{V_b - V_o}{r_\pi} \quad \text{Put in eq}^n \text{ (1)}$$

$$V_o = (1+\beta)\left[\frac{V_b - V_o}{r_\pi}\right](r_o \| R_E)$$

$$= \frac{(1+\beta) V_b}{r_\pi}(r_o \| R_E) - \frac{(1+\beta) V_o}{r_\pi}(r_o \| R_E)$$

$$V_o\left[1 + (1+\beta)\frac{(r_o \| R_E)}{r_\pi}\right] = \frac{(1+\beta) V_b (r_o \| R_E)}{r_\pi}$$

$$V_o\left[r_\pi + (1+\beta)(r_o \| R_E)\right] = (1+\beta) V_b (r_o \| R_E) \quad \text{— (2)}$$

$$\text{But } V_b = \frac{R_i'}{(R_i' + R_S)} V_S \quad \text{— (3)}$$

where $R_i' = R_B \| [r_\pi + (1+\beta) R_E]$

Put $V_b$ from eq$^n$ (3) in eq$^n$ (2)

$$V_o\left[r_\pi + (1+\beta)(r_o \| R_E)\right] = (1+\beta)(r_o \| R_E)\frac{R_i'}{(R_i' + R_S)} \times V_S$$

$$\boxed{A_{VS} = \frac{V_o}{V_S} = \frac{(1+\beta)(r_o \| R_E)}{r_\pi + (1+\beta)(r_o \| R_E)} \times \frac{R_i'}{(R_i' + R_S)}}$$

— exact expression.

If $r_\pi << (1+\beta)(r_o \| R_E)$ & $R_i' >> R_S$ then

$$\boxed{A_{VS} = \frac{(1+\beta)(r_o \| R_E)}{(1+\beta)(r_o \| R_E)} \simeq 1}$$

— Approximate expression

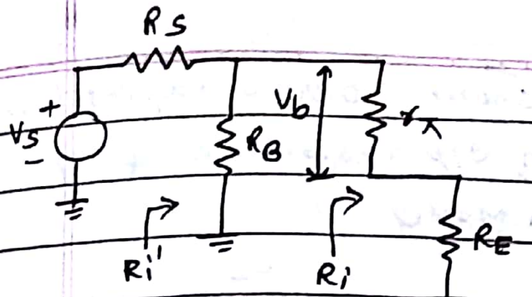
**step (3)** Input Resistance :-

Apply KVL to i/p loop,

$V_b = I_b r_\pi + I_e R_E$ But $I_e = (1+\beta) I_b$

$= I_b r_\pi + (1+\beta) I_b R_E$

$= I_b [r_\pi + (1+\beta) R_E]$

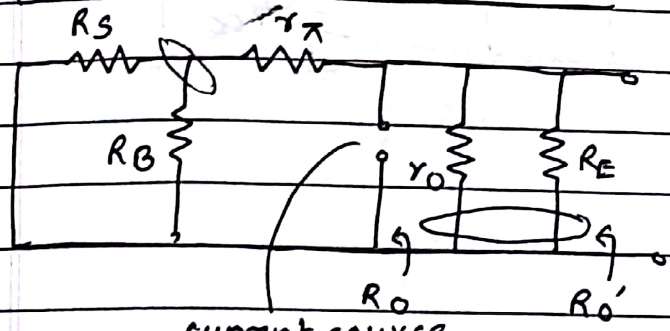



circuit for obtaining $R_i$ & $R_i'$

$$\boxed{R_i = \frac{V_b}{I_b} = r_\pi + (1+\beta) R_E}$$

$$\boxed{\begin{aligned} R_i' &= R_B \parallel R_i \\ &= R_B \parallel [r_\pi + (1+\beta) R_E] \end{aligned}}$$

## step (4) output Resistance -




Equivalent circuit to calculate $R_o$ & $R_o'$

set $V_s = 0$,

$$R_o = (R_s \parallel R_B) + r_\pi$$

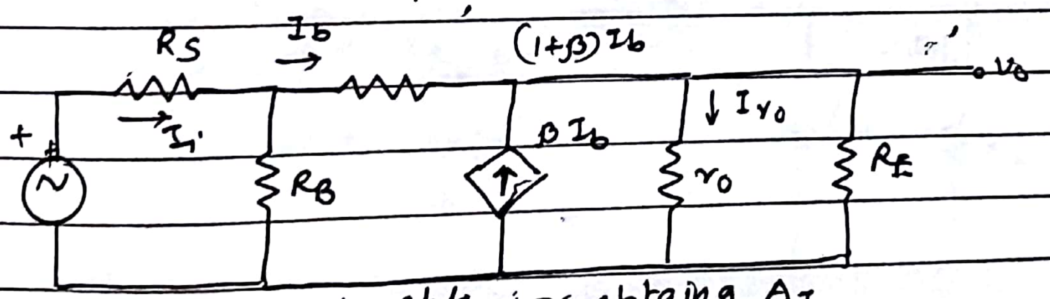
$$\& \quad R_o' = \frac{R_o}{(1+\beta)} \parallel (r_o \parallel R_E)$$

where $\frac{R_o}{(1+\beta)}$ represents reflected resistance from base to emitter.

$$\boxed{R_o' = \frac{(R_s \parallel R_B) + r_\pi}{(1+\beta)} \parallel (r_o \parallel R_E)}$$

## step (5) small signal current gain

$$A_I = \frac{I_e}{I_i'} \quad \text{where } I_e = \text{o/p current}$$




Equivalent ckt for obtaing $A_I$

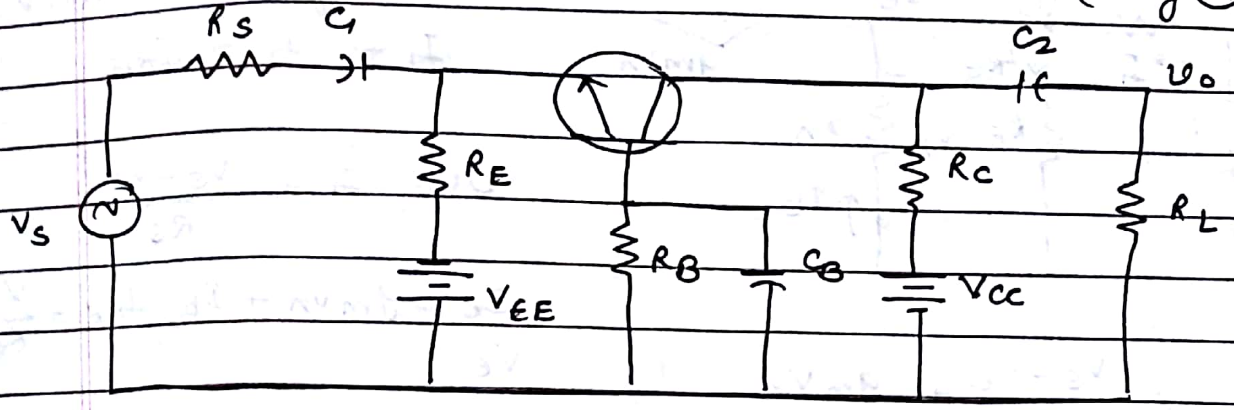
$$I_e = (1+\beta) I_b \qquad I_b = \frac{R_B}{(R_B + R_E)} I_i$$

$$I_i' = (1+\beta) \frac{R_B}{(R_B + R_i)} \times I_i$$

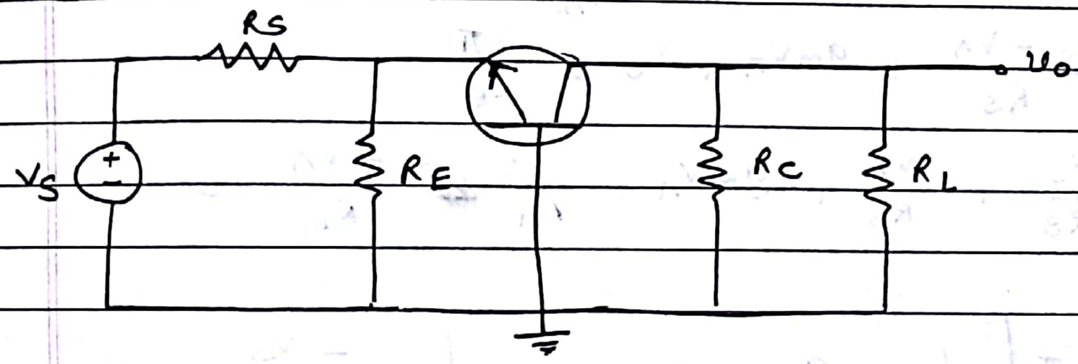
$$A_I = \frac{I_o'}{I_i} = \frac{(1+\beta) R_B}{R_B + R_i} \qquad \text{if } R_i >> R_B \quad \boxed{A_I = (1+\beta)}$$

## * Common Base Amplifier -

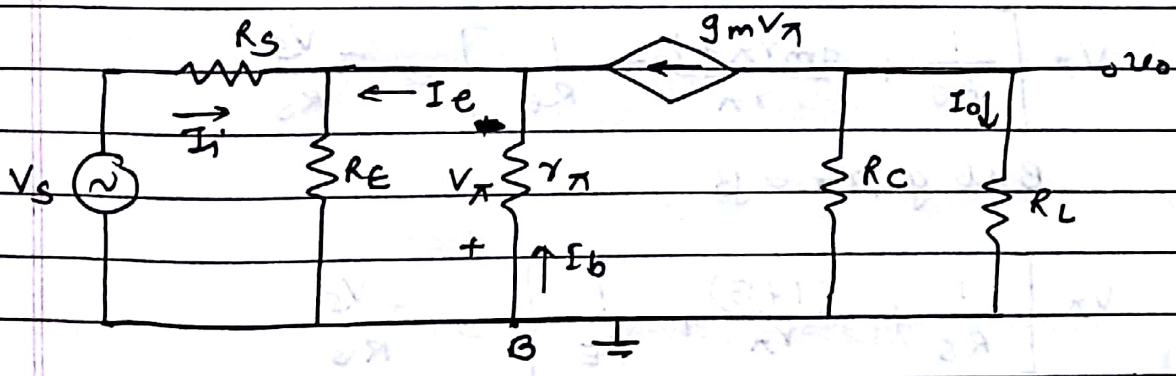
Question - For the common Base amplifier shown, derive expression for voltage gain, current gain, input resistance & output resistance using hybrid $\pi$ Model. (May 14, 12)




**Step 1** Draw AC equivalent circuit




**Step 2** Draw the hybrid $\pi$ equivalent ckt




small signal equivalent ckt for CB configuration

**Step 3** Expression for small signal voltage gain -

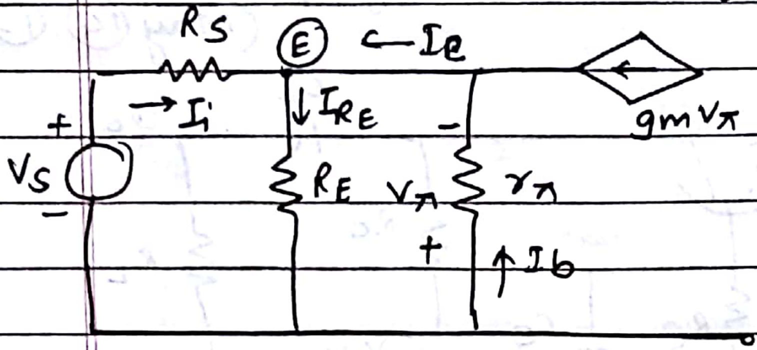
$$v_o = I_o R_L$$

But $I_o = \dfrac{-R_C}{R_C + R_L} \times g_m V_\pi$

$$\therefore V_o = \frac{-R_C R_L}{(R_C + R_L)} g_m V_\pi$$

But $\frac{R_C R_L}{R_C + R_L} = (R_C \parallel R_L)$

$$\therefore \; V_o = -(R_C \parallel R_L)\, g_m V_\pi \quad \text{—(1)}$$




Apply KCL at node E,

$$I_i + I_e = I_{RE}$$

But $I_i = \frac{V_S - V_e}{R_S}$

$I_e = g_m V_\pi + I_b$, $I_{RE} = \frac{V_e}{R_E}$

$$\therefore \frac{V_S - V_e}{R_S} + g_m V_\pi + I_b = \frac{V_e}{R_E}$$

But $V_e = -V_\pi$ & $I_b = \frac{V_\pi}{R_\pi}$

$$\frac{V_S + V_\pi}{R_S} + g_m V_\pi + I_b = \frac{-V_\pi}{R_E}$$

$$\frac{V_S}{R_S} + \frac{V_\pi}{R_S} + g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{+V_\pi}{R_E} = 0$$

$$V_\pi\left[\frac{1}{R_S} + g_m + \frac{1}{r_\pi} + \frac{1}{R_E}\right] = \frac{-V_S}{R_S}$$

$$V_\pi\left[\frac{1}{R_S} + \frac{g_m r_\pi + 1}{r_\pi} + \frac{1}{R_E}\right] = \frac{-V_S}{R_S}$$

But $g_m r_\pi = \beta$

$$V_\pi\left[\frac{1}{R_S} + \frac{(1+\beta)}{r_\pi} + \frac{1}{R_E}\right] = \frac{-V_S}{R_S}$$

$$\therefore V_\pi = \frac{-V_S}{R_S\left[\frac{1}{R_S} + \frac{(1+\beta)}{r_\pi} + \frac{1}{R_E}\right]}$$

Let $\frac{1}{R} = \frac{1}{R_S} + \frac{(1+\beta)}{r_\pi} + \frac{1}{R_E}$ Hence $R = R_S \parallel \left(\frac{1+\beta}{r_\pi}\right) \parallel R_E$

$$\therefore V_\pi = -\frac{V_S}{R_S} \times R = -\frac{V_S}{R_S}\left[R_S \parallel \frac{(1+\beta)}{r_\pi} \parallel R_E\right] \quad \text{--- (2)}$$

Put in eqⁿ (1)

$$V_o = -(R_C \parallel R_L)\, g_m \times -\frac{V_S}{R_S}\left[R_S \parallel \frac{(1+\beta)}{r_\pi} \parallel R_E\right]$$

$$\boxed{\frac{v_o}{v_s} = A_{VS} = g_m \frac{(R_C \parallel R_L)}{R_S}\left[R_S \parallel \frac{(1+\beta)}{r_\pi} \parallel R_E\right]}$$

└ Exact Expression

If we assume $R_S = 0$, parallel combination of $R_S$, $(1+\beta)/r_\pi$ & $R_E$ approaches to $R_S$.

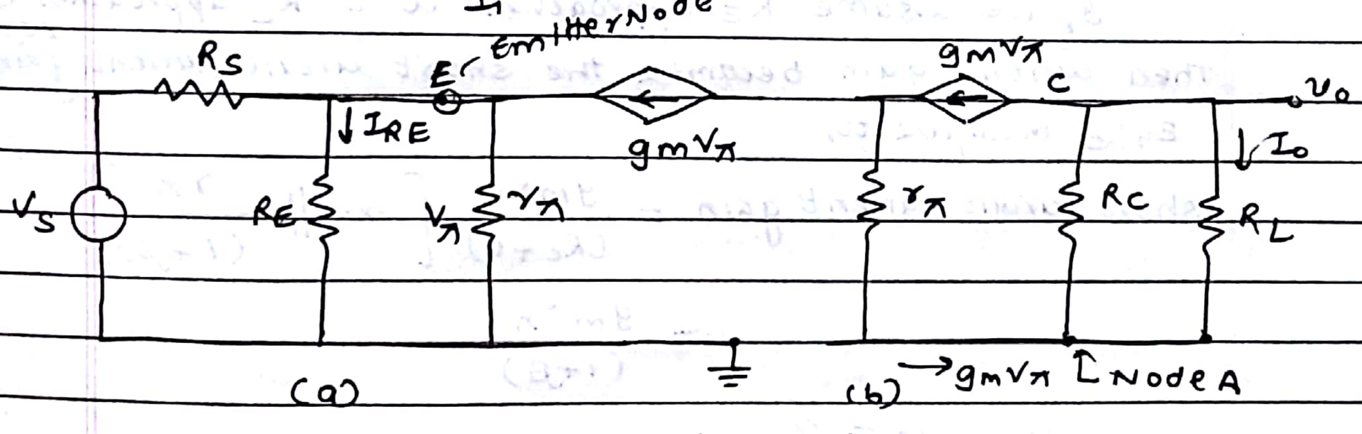
$$\therefore R_S \parallel \frac{(1+\beta)}{r_\pi} \parallel R_E = R_S$$

$$\therefore A_{VS} = \frac{g_m (R_C \parallel R_L)}{R_S} \times R_S$$

$\boxed{A_{VS} = g_m (R_C \parallel R_L)}$ — Approximate expression

Step (4) Expression for small signal current gain

$$A_{IS} = \frac{I_o}{I_i}$$




Apply KVL to emitter Node

$$I_i + I_b + g_m V_\pi - I_{RE} = 0$$

$$\therefore I_i + \frac{V_\pi}{r_\pi} + g_m V_\pi + \frac{V_\pi}{R_E} = 0$$

$$V_\pi\left[\frac{1}{r_\pi} + g_m + \frac{1}{R_E}\right] = -I_i$$

DATE / / /

$$\therefore V_\pi = \frac{-I_i}{\left[\frac{1}{r_\pi} + g_m + \frac{1}{R_E}\right]} = \frac{-I_i}{\left[\frac{(1+g_m r_\pi)}{r_\pi} + \frac{1}{R_E}\right]}$$

But $g_m r_\pi = \beta$

$$V_\pi = \left[\frac{-I_i}{\left[\frac{(1+\beta)}{r_\pi} + \frac{1}{R_E}\right]}\right] = -I_i\left[R_E \parallel \frac{r_\pi}{(1+\beta)}\right] \quad \text{--- (1)}$$

Write the expression for current $I_o$ using the principle of current division.

current ($g_m V_\pi$) get divided between $R_C$ & $R_L$ at node A.

$$I_o = \frac{-R_C}{R_C + R_L} \times g_m V_\pi$$ Put value of $V_\pi$ from eqn (1)

$$I_o = \frac{-g_m R_C}{R_C + R_L} \times -I_i\left[R_E \parallel \frac{r_\pi}{(1+\beta)}\right]$$

$$\boxed{A_{IS} = \frac{I_o}{I_i} = \frac{g_m R_C}{(R_C + R_L)}\left[R_E \parallel \frac{r_\pi}{(1+\beta)}\right]} \quad \text{--- (2)}$$ — exact expression

Approximate expression for current gain,

If we assume $R_E$ approaches $\infty$ & $R_L$ approaches 0 zero. then current gain becomes the short circuit current gain.

Eqn (2) modifies to,

$$\text{short circuit current gain} = \frac{g_m R_C}{(R_C + 0)}\left[\infty \parallel \frac{r_\pi}{(1+\beta)}\right]$$

$$= \frac{g_m r_\pi}{(1+\beta)}$$

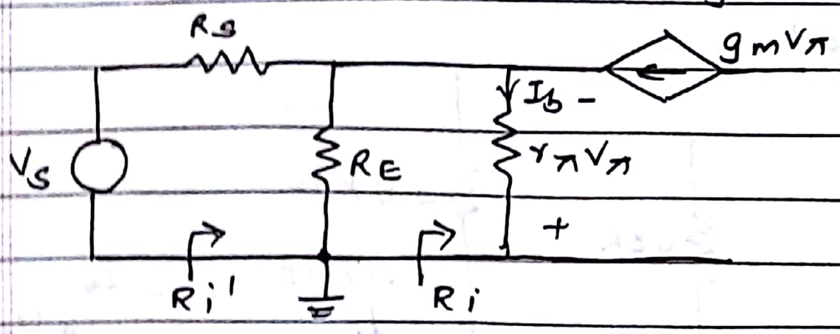
But $g_m r_\pi = \beta$

$$\therefore \boxed{A_{IS(S.C)} = \frac{\beta}{(1+\beta)} = \alpha}$$ — Approximate expression

step ⑤ Expression for i/p Resistance

Let $R_i$ = Resistance seen between emitter & ground &

$R_i'$ = Resistance seen by the voltage source.




$R_i$ = Resistance between emitter & ground

= $r_\pi$ reflected from base side to emitter.

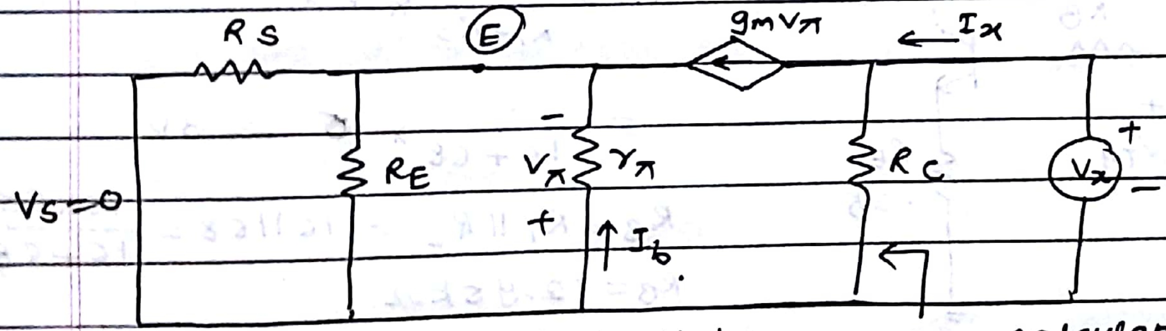
$$\boxed{R_i = \frac{r_\pi}{(1+\beta)} = r_e}$$

$r_e$ is very small as $\beta$ is in denominator.

$$\boxed{R_i' = R_E \| R_i = R_E \| \frac{r_\pi}{(1+\beta)}}$$

step ⑥ Expression for output Resistance –

set $V_s = 0$ & assume that a source $V_x$ is connected between o/p terminals as shown in next fig.




Equivalent circuit to calculate Ro

As $V_s = 0$, $V_\pi = 0$ $\therefore g_m V_\pi = 0$ so,

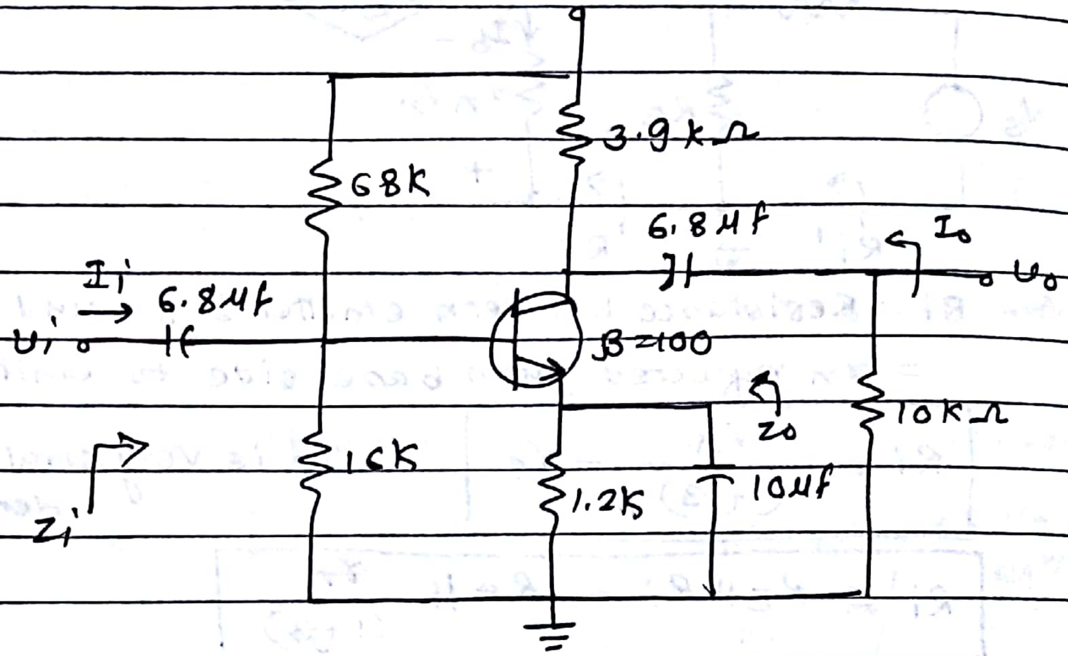
$\underline{R_o = R_C}$ $\qquad$ $\underline{R_o' = R_o \| R_L = R_C \| R_L}$

## * Numericals on Hybrid π Model (CE, CB, CC)

CE ampl^r

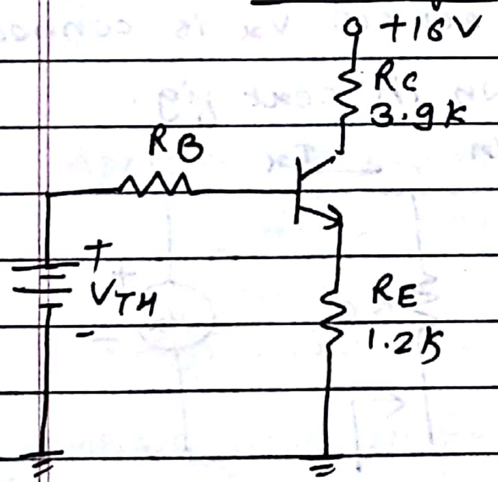
**Ex 1)** For the given circuit find,

1. Determine $Z_i$, $Z_o$ & $A_v$ no load
2. $A_v$ with load
3. $A_i$




Sol^n - ckt - CE amplifier with bypassed $R_E$

Step ① DC analysis -




Thevenin's DC equivalent

$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{cc}$$

$$= \frac{16}{16+68} \times 16 = 3V$$

$$R_B = R_1 \parallel R_2 = 16 \parallel 68 = \frac{16 \times 68}{16+68}$$

$$R_B = 12.95\,k\Omega$$

$$I_B = \frac{V_{Th} - V_{BE}}{R_B + (1+\beta) R_E} = \frac{3 - 0.7}{12.95k + (101 \times 1.2k)} = 17.15\,\mu A$$

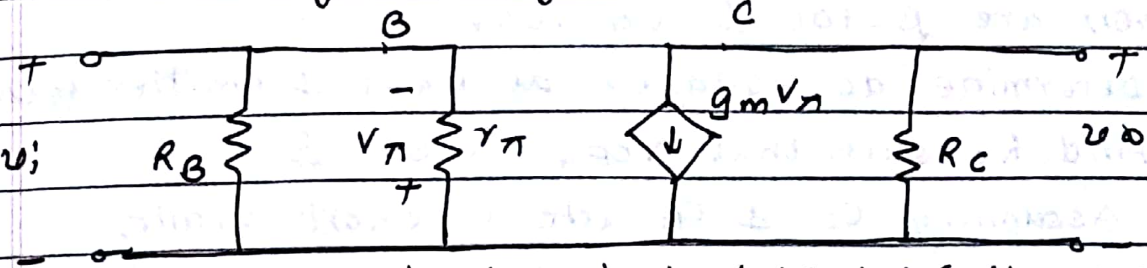
$$I_{CQ} = \beta I_B = 100 \times 17.15\,\mu A = \underline{\underline{1.715\,mA}}$$

Let $V_T = 26\,mV$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{26 \times 100}{1.715} = 1.516\,k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.715}{26 \times 10^{-3}} = 65.96\,mA/V, \quad r_o = \frac{V_A}{I_{CQ}} = \infty$$

step ② Small signal Analysis -




small signal equivalent Model (without load)

$R_i = r_\pi = 1.516 k\Omega$

$R_i' = R_B \| R_i = 1.516k \| 12.95k = 1.357 k\Omega$

$A_V = \frac{V_o}{V_i} = \frac{-g_m V_\pi R_C}{V_\pi} = -g_m R_C = -65.96 \times 3.9$

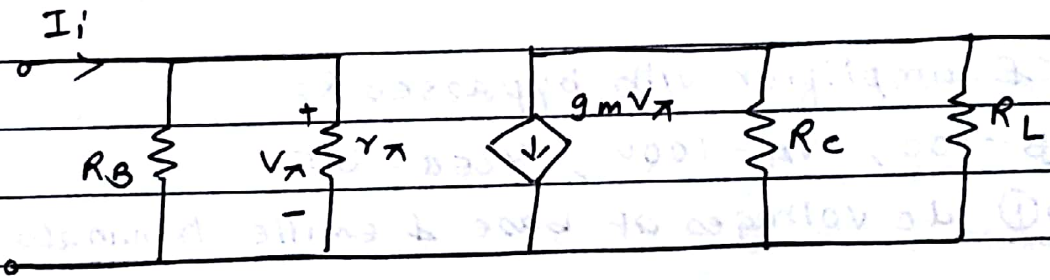
$= -257.244$

$R_o = \infty$

$R_o' = R_C = 3.9 k\Omega$

$A_V = -g_m(R_C \| R_L) = -65.96(3.9 \| 10)$

$= -65.96 \times 2.805 = \underline{-185}$




$A_i = \frac{I_o}{I_b} = \frac{I_o}{I_c} \times \frac{I_c}{I_b}$ But $I_o = \frac{-R_C}{R_C + R_L} \times I_c$

$\frac{I_o}{I_c} = \frac{-R_C}{R_C + R_L}$ , $\frac{I_C}{I_b} = \frac{g_m V_\pi}{V_\pi / r_\pi} = g_m r_\pi$

$A_i = \frac{-R_C}{R_C + R_L} \times g_m r_\pi$

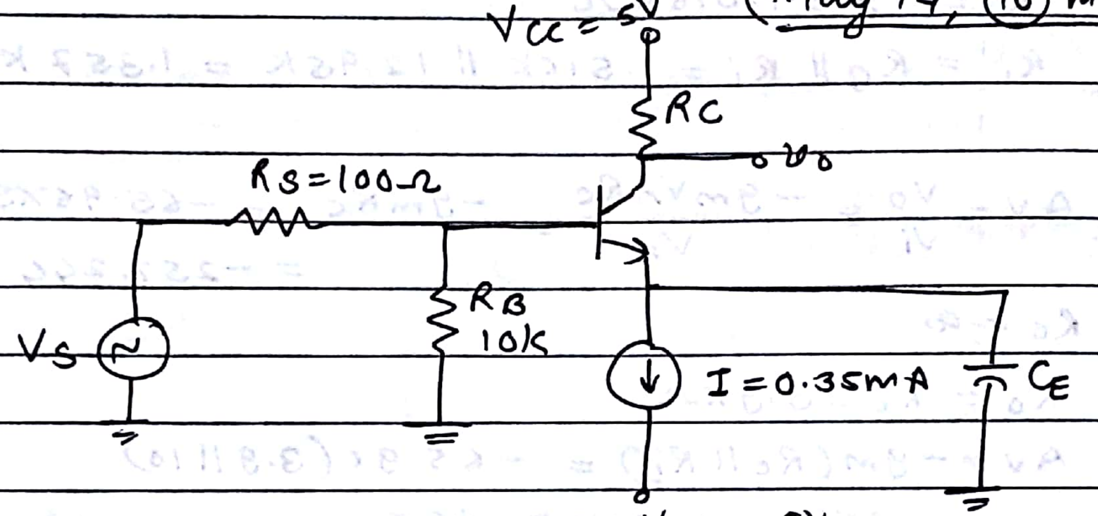
$A_i = (-65.96 \times 1.516) \times \frac{3.9}{3.9 + 10} = \underline{-28.05}$

Ex(2) The parameters of the 'transistor' in the circuit given are $\beta = 100$ & $V_A = 100V$

1. Determine dc voltages at base & emitter terminals.
2. Find $R_C$ such that $V_{CEQ} = 3.5V$ &
3. Assuming $C_C$ & $C_E$ acts as short circuit, Determine small signal voltage gain $A_V = \frac{V_o}{V_s}$

(May 14, 10 marks)

Soln –

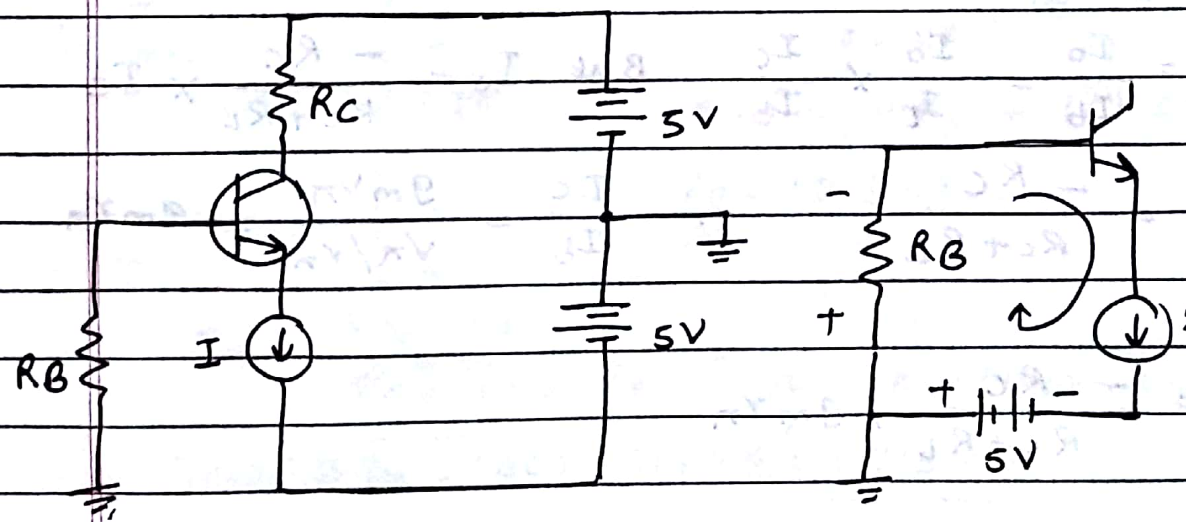


given circuit

Soln – CE amplifier with bypassed $R_E$

$\beta = 100$, $V_A = 100V$, $V_{CEQ} = 3.5V$

Step (1) dc voltages at base & emitter terminals –

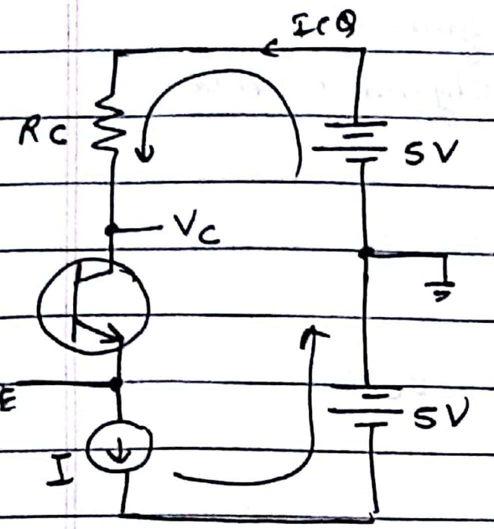



$$I_B = \frac{I_E}{1+\beta} = \frac{I}{1+\beta} = \frac{0.35 \times 10^{-3}}{101} = 3.47\mu A$$

$$V_B = -I_B R_B = -3.47 \times 10^{-6} \times 10 \times 10^{3} = -0.0347V$$

$$V_E = V_B - 0.7 = -0.0347 - 0.7 = -0.7347V$$

**step ②** find $R_C$




$V_{CEQ} = 3.5V$

$V_{CEQ} = V_C - V_E$

$3.5V = V_C - (-0.7347)$

$\boxed{V_C = 2.77V}$

$R_C = \frac{5V - V_C}{I_{CQ}}$

assuming $I_{CQ} = I = 0.35mA$

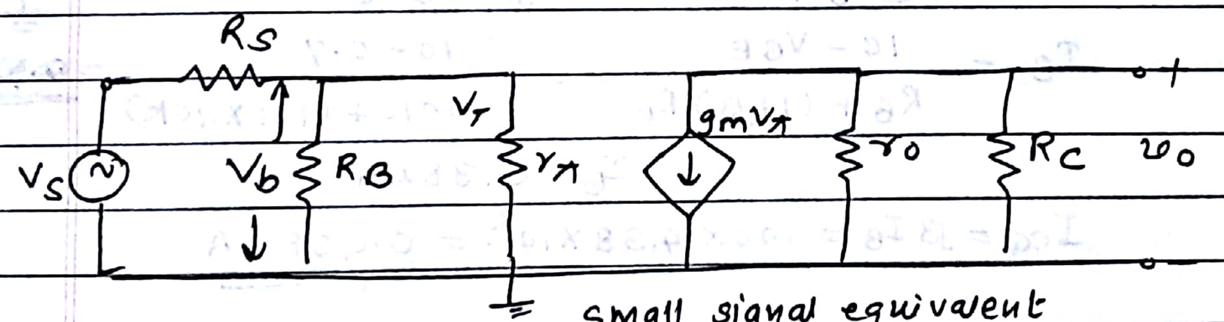
$R_C = \frac{5 - 2.77}{0.35 \times 10^{-3}} = 6.37k\Omega$

**step ③** $r_\pi = \frac{V_T \cdot \beta}{I_{CQ}}$ assume $V_T = 26mV$

$r_\pi = \frac{26 \times 10^{-3} \times 100}{0.35 \times 10^{-3}} = 7.43k\Omega$

$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.35 \times 10^{-3}} = 285.7k\Omega$

$g_m = \frac{I_{CQ}}{V_T} = \frac{0.35 mA}{26 \times 10^{-3} V} = 13.46 mA/V$




small signal equivalent

Voltage gain $A_V = \frac{V_o}{V_b} = \frac{-g_m V_\pi (r_o \| R_C)}{V_b}$ But $V_b = V_\pi$

$\therefore A_V = -g_m (r_o \| R_C) = -13.46 [285.7K \| 6.37K] = \underline{-83.87}$

$A_{VS} = \frac{V_o}{V_S} = \frac{R_i}{R_i + R_S} \cdot A_V$ where $R_i = R_B \| r_\pi$

$= 10k \| 7.43K$

$= 4.26K\Omega$

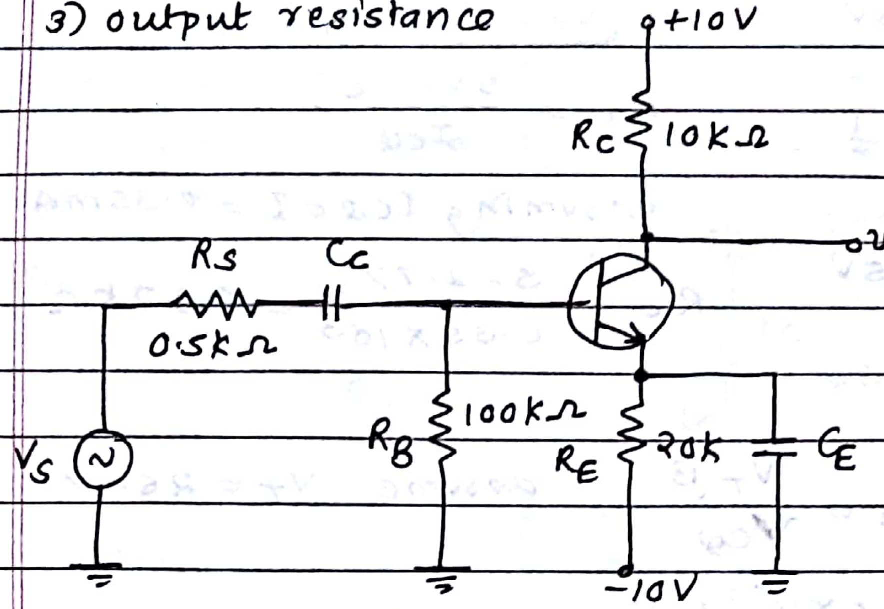
$A_{VS} = \frac{v_o}{v_i} = \frac{4.26}{4.26 + 0.1} \times -83.87 = \underline{-81.94}$

Ex ③ For the circuit shown in fig, let $\beta = 100$, $V_A = 100V$, $V_{BE(ON)} = 0.7V$

Determine 1) small signal voltage gain

2) Input resistance seen by the signal source

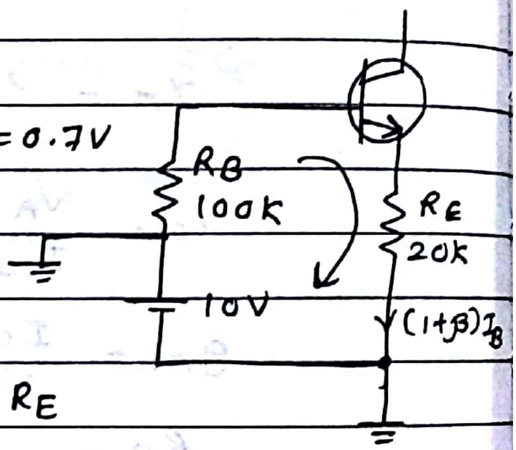
3) output resistance




Soln:- given $\beta = 100$, $V_A = 100V$, $V_{BE(ON)} = 0.7V$

DC Analysis -

① find $I_{CQ}$ -




Apply KVL to Base loop,

$$10V = I_B R_B + V_{BE} + (1+\beta) I_B R_E$$

$$I_B = \frac{10 - V_{BE}}{R_B + (1+\beta) R_E} = \frac{10 - 0.7}{100k + (101 \times 20k)} = \cancel{0.438\,mA}$$

$$I_B = 4.38\,\mu A$$

$$I_{CQ} = \beta I_B = 100 \times 4.38 \times 10^{-6} = 0.438\,mA$$

step ② Calculate $g_m$, $r_\pi$ & $r_o$ -

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{26 \times 100}{0.438} = 5.94\,k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.438}{26 \times 10^{-3}} = 16.85\,mA/V$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{0.438 \times 10^{-3}} = 228.3\,k\Omega$$

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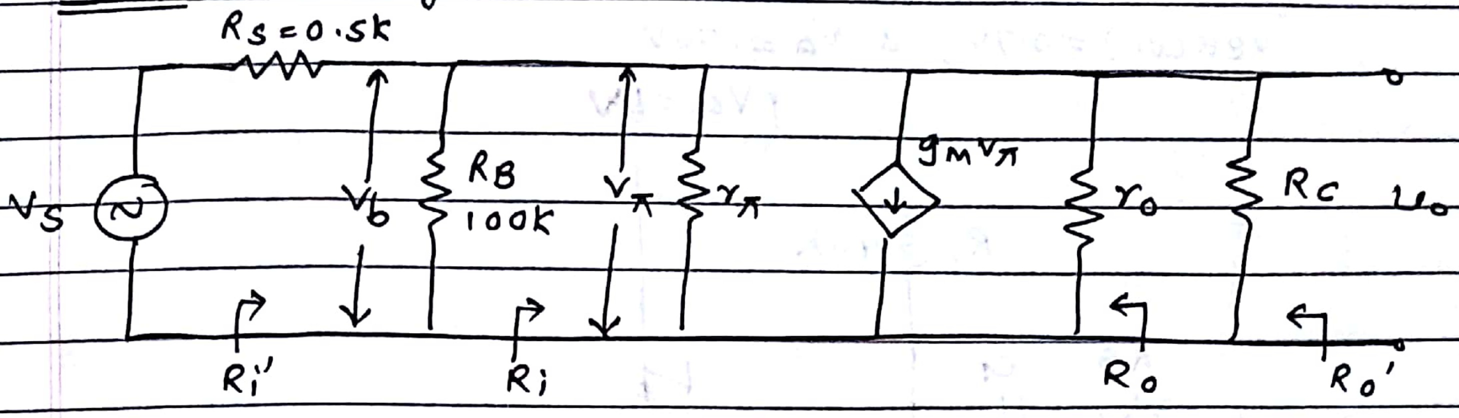
## AC Analysis -

**Step ③ Draw hybrid π equivalent ckt**




**i/p resistance** - $R_i = r_\pi = 5.94\,k\Omega$

$R_i' = R_i \| R_B = 5.94k \| 100k = \underline{5.6\,k\Omega}$

**Step ④ Voltage gain ($A_{Vs}$) -**

$$A_{Vs} = \frac{V_o}{V_s} = \frac{V_o}{V_\pi} \times \frac{V_\pi}{V_s}$$

$$V_o = -g_m V_\pi (r_o \| R_c)$$

$$\frac{V_o}{V_\pi} = -g_m (r_o \| R_c)$$

$$V_\pi = \frac{R_i'}{R_s + R_i'} \cdot V_s$$

$$\frac{V_\pi}{V_s} = \frac{R_i'}{R_s + R_i'}$$

$$A_{Vs} = -g_m (r_o \| R_c) \times \frac{R_i'}{R_i' + R_s}$$

$$= -16.85(228.3K \| 10k) \frac{5.6}{(5.6+0.5)}$$

$$\boxed{A_{Vs} = -148.2}$$

**Step ⑤ output Resistance -**

$R_o = r_o = 228.3\,k\Omega$

$R_o' = r_o \| R_c = 228.3k \| 10k$

$$\boxed{R_o' = 9.58\,k\Omega}$$

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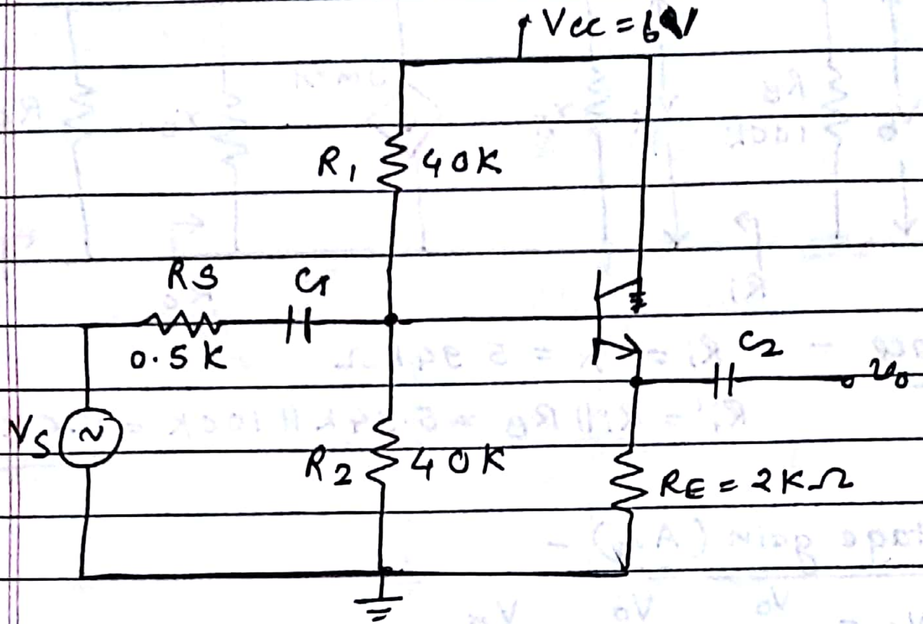
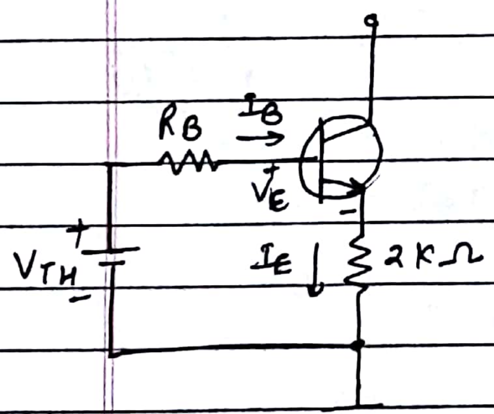
## CC Ampl^r

**Ex (4)** calculate the small signal voltage gain for the emitter follower circuit shown in fig. Assume that $\beta = 100$, $V_{BE}(ON) = 0.7V$, & $V_A = 100V$




**Sol^n** DC analysis

$$R_B = R_1 \parallel R_2 = 40k \parallel 40k = \underline{20k}$$




$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{cc}$$

$$= \frac{40k}{40k + 40k} \times 6V$$

$$\underline{V_{TH} = 3V}$$

Thevenin's equivalent ckt

$$I_B = \frac{V_{TH} - V_{BE}}{R_B + (1+\beta) R_E} = \frac{3 - 0.7}{20k + (101 * 2k)} = \underline{\underline{10.36 \mu A}}$$

$$\& \ I_{CQ} = \beta I_B = 100 \times 10.36 \times 10^{-6} = \underline{1.036 \, mA}$$

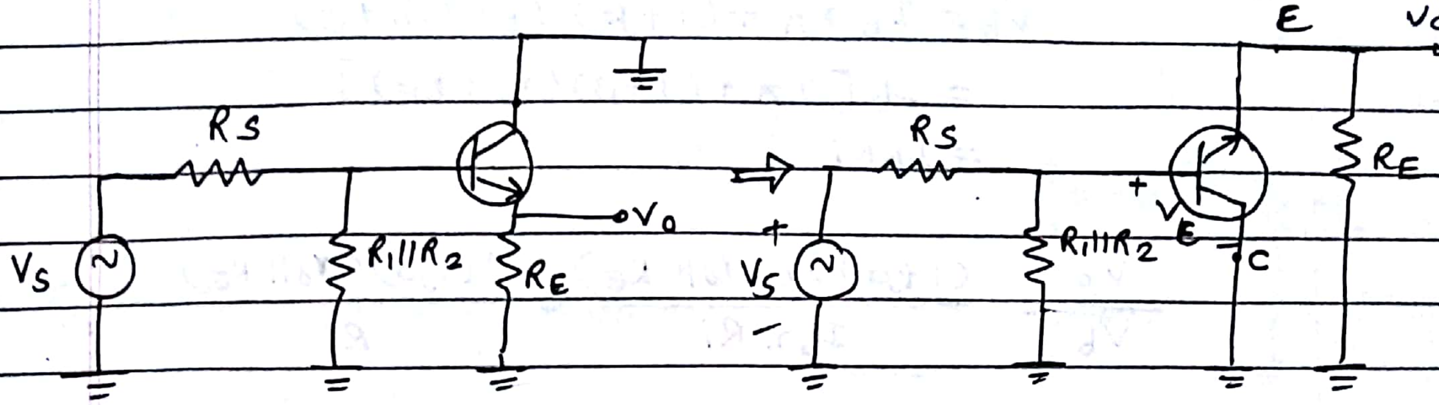
**AC Analysis**

calculate $r_\pi$ & $r_o$,

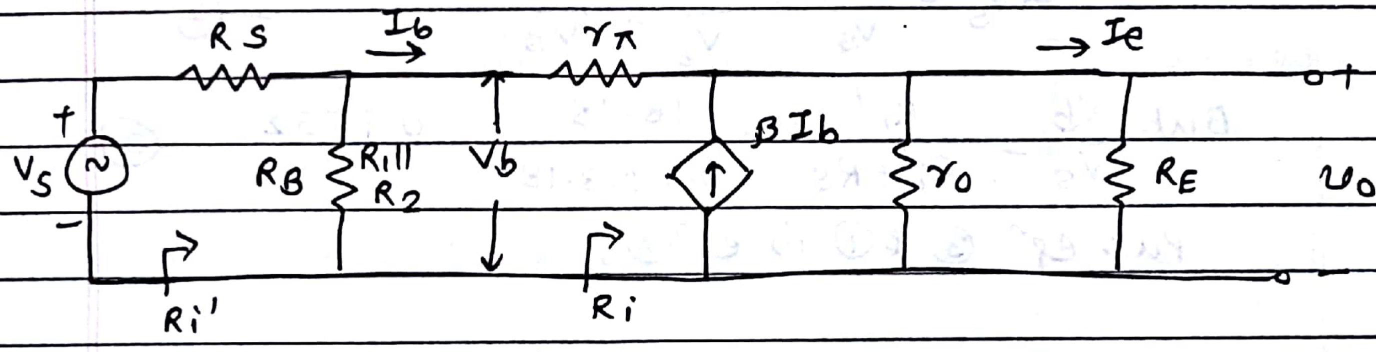
$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 26mV}{1.036 \, mA} = \underline{\underline{2.509 \, k\Omega}}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{1.036 \times 10^{-3}} = \underline{\underline{96.525 \times 10^3 \, k\Omega}}$$

Draw AC equivalent ckt




Draw the hybrid π equivalent ckt-




calculate Ri & Ri'-

$$R_i = \frac{V_o}{I_b}$$ Apply KVL to o/p loop,

$$V_b = I_b r_\pi + (1+\beta) I_b (r_o \| R_E)$$

$$= I_b [r_\pi + (1+\beta)(r_o \| R_E)]$$

$$R_i = \frac{\cancel{I_b}[r_\pi + (1+\beta)(r_o \| R_E)]}{\cancel{I_b}} = r_\pi + (1+\beta)(r_o \| R_E)$$

$$= 2.509k + 101(96.525k \| 2k)$$

$$\boxed{R_i = 200.408\,k\Omega}$$

$$R_i' = (R_1 \| R_2) \| R_i = 20k \| 200.408k = \underline{\underline{18.18\,k\Omega}}$$

calculate o/p voltage -

$$V_o = (\beta I_b + I_b) \times (r_o \| R_E)$$

$$= (1+\beta) I_b (r_o \| R_E)$$

calculate $V_b$ – Apply KVL to outer loop

$$V_b = I_b r_\pi + (1+\beta) I_b (r_o \| R_E)$$
$$= I_b [r_\pi + (1+\beta)(r_o \| R_E)]$$
$$= I_b R_i$$

$$\frac{V_o}{V_b} = \frac{(1+\beta) I_b (r_o \| R_E)}{I_b \times R_i} = \frac{(1+\beta)(r_o \| R_E)}{R_i} \quad \text{— (1)}$$

calculate voltage gain

$$A_{V_S} = \frac{V_o}{V_S} = \frac{V_o}{V_b} \times \frac{V_b}{V_S} \quad \text{— (2)}$$

But $$\frac{V_b}{V_S} = \frac{R_i'}{R_i' + R_S} = \frac{18.18}{0.5 + 18.18} = 0.9732 \quad \text{— (3)}$$

Put eqⁿ (3) & (1) in eqⁿ (2),

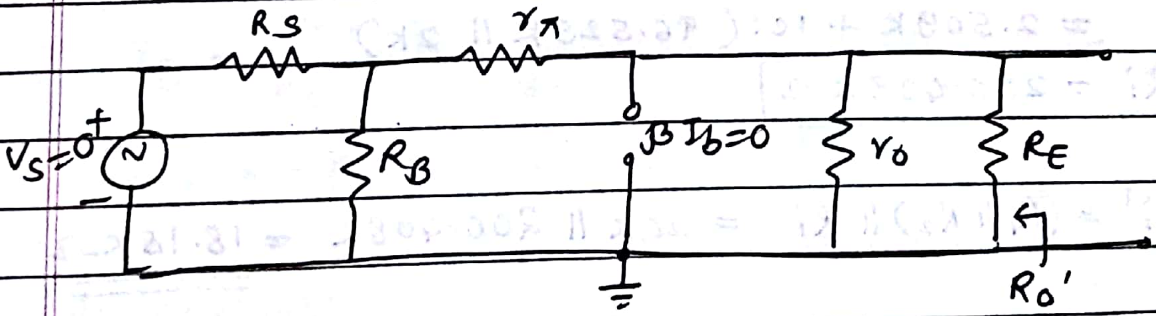
$$A_{V_S} = \frac{(1+\beta)(r_o \| R_E)}{R_i} \times 0.9732$$

$$= \frac{101 \times (96.525k \| 2k)}{200.408k} \times 0.9732$$

$$\boxed{A_{V_S} = 0.961}$$

o/p Resistance –

Put $V_S = 0$



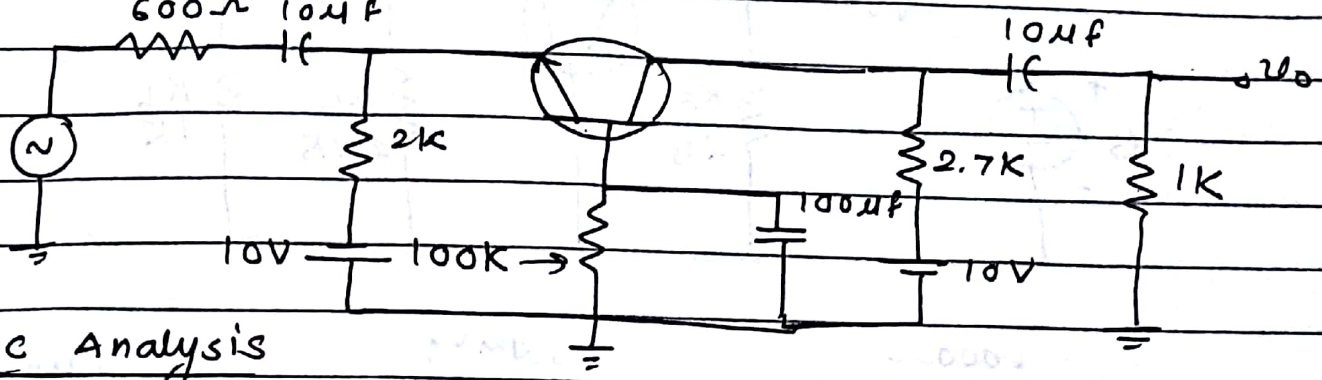

$$R_o' = \frac{[(R_S \| R_B) + r_\pi]}{(1+\beta)} \| (r_o \| R_E)$$

$$= \frac{[(0.5 \| 20) + 2.509]}{101} \| (96.525 \| 2) = \boxed{29.22\ \Omega}$$

$$\boxed{R_o' = 29.22\ \Omega}$$

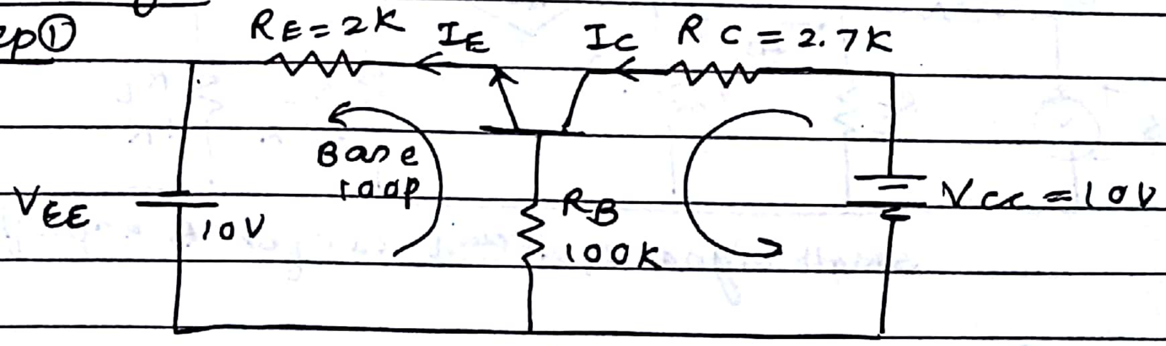
CB Amplifier

(Ex 1) Determine Ri, Ro, Av & gm for amplifier shown in given figure.




Soln DC Analysis

Step ①




Apply KVL to base loop.

$$V_{BE} + I_E R_E - 10 + I_B R_B = 0$$

$$V_{BE} + (1+\beta) I_B R_E + I_B R_B = 10$$

$$I_B = \frac{10 - V_{BE}}{[(1+\beta)R_E + R_B]} = \frac{10-0.7}{(101 \times 2000) + 10000} = 30.79\,\mu A$$

$$I_{CQ} = \beta I_B = 100 \times (30.79\,\mu A) = \underline{\underline{3.079\,mA}}$$

Step ② calculate $r_\pi$, $r_o$ & $g_m$ -

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{26 \times 10^{-3} \times 100}{3.079 \times 10^{-3}} = 844.43\,\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{3.079\,mA}{26 \times 10^{-3}} = 118.42\,mA/V$$

$$r_o = \frac{V_A}{I_{CQ}} = \infty \quad (\because V_A = \infty)$$

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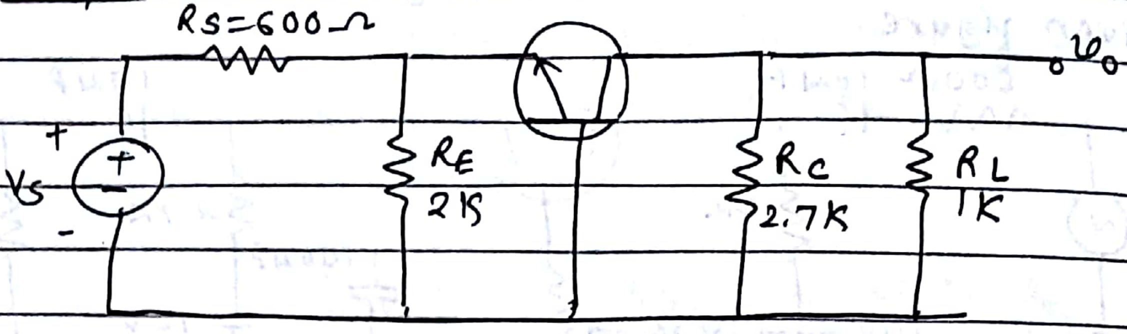
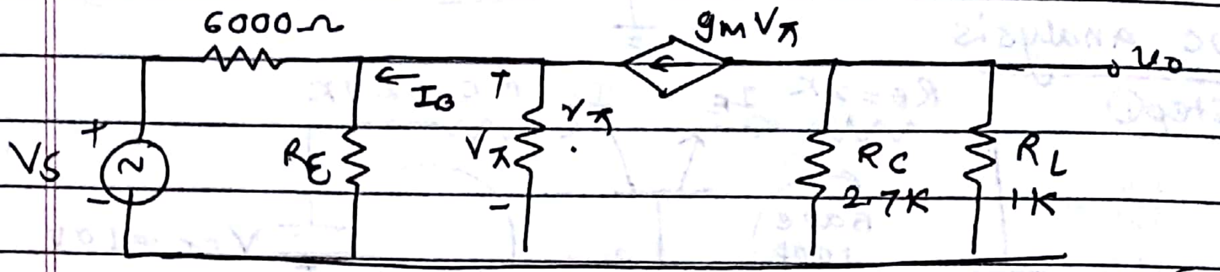
## AC Analysis

**Step ③ Draw AC equivalent circuit.**




small signal equivalent ckt for CB amplifier

**Step ④ calculate Ri, Ro, AV**

$$R_i = \frac{r_\pi}{(1+\beta)} = \frac{844.43}{101} = 8.36\,\Omega$$

$$R_o = R_C = 2.7K\,\Omega$$

$$A_V = g_m(R_C \| R_L) = 0.118(2.7K \| 1K) = 86.108$$

**Ex 2** For the common Base circuit shown, the transistor has parameters $\beta = 120$ & $V_A = \infty$

① Determine the quiescent $V_{CEQ}$

② Determine the small signal voltage gain & o/p resistance

(May 2016, 10)