Math 285: Final **Name:** ______________________________
Spring 2020
5/4/2020

Directions

1. Do NOT open this exam booklet until you are instructed to do so!
2. You may use a TI-84/85 (or equivalent) calculator. Any other electronic devices or outside materials are not permitted.
3. This exam is 12 pages (including this cover page) and has 12 questions. Check that you have every page of the exam before handing it in.
4. Please write your answers in the space provided. If you need more space, continue on the back of a page (being sure to clearly label your work). Do not write any answers on scrap paper.
5. Work must be clearly written and organized. Please organize your work and write legibly! Circle your final answers.
6. If you have a question, please raise your hand.

Good luck!
Do not write in the tables or on the line below.

Question	Points	Score
1	5	
2	9	
3	8	
4	4	
5	10	
6	15	
7	10	
8	8	
9	10	
10	5	
11	10	
12	6	
Total:	100	

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1. (5 points) The size of a population of bacteria is given by $f(t) = 2300e^{0.3t}$, where t is the time measured in days.

   (a) What is the population after 27 days?

   (b) How many days will it take for the population to reach 6900?

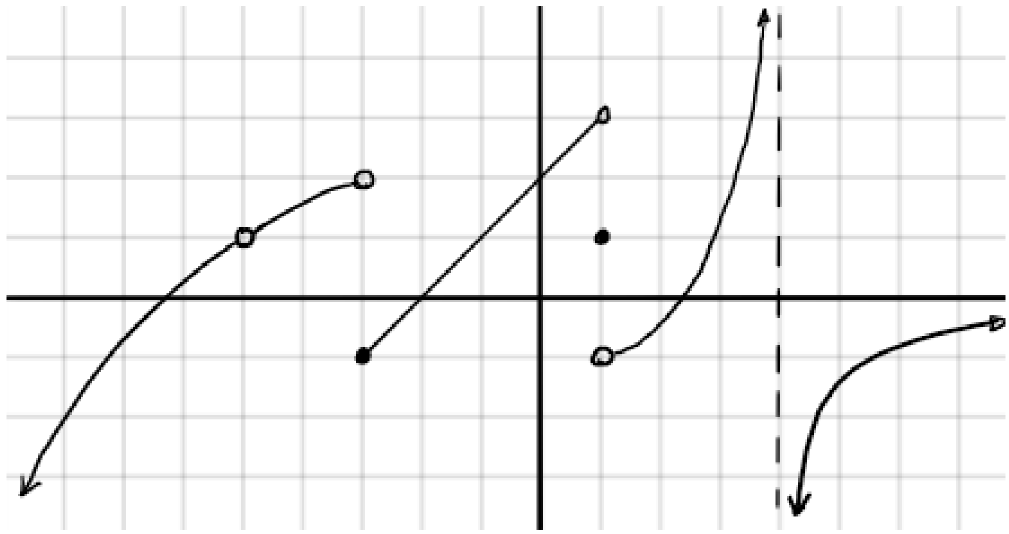
2. (9 points) Find the following limits.

(a) $\lim_{x \to -2} \dfrac{x^2 + 3x + 2}{x^2 - 4}$

(b) $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

(c) $\lim_{x \to \infty} \dfrac{x^4 + 3x^3 + 2x + 1}{x^3 - 2x + 7}$

The graph of the function $f(x)$, used in problems 3 and 4, is shown below.



3. (8 points) Find the following limits and function values. If a limit or function value does not exist, write 'does not exist'.

   (a) $\lim_{x \to -5} f(x) =$

   (b) $f(1) =$

   (c) $\lim_{x \to 1^-} f(x) =$

   (d) $f(-3) =$

4. (4 points) Is $f(x)$ continuous at the following points?

   (a) $x = -5$

   (b) $x = -3$

   (c) $x = 0$

   (d) $x = 1$

5. (10 points) Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^2 - 2x$. You *must* use the definition of the derivative to receive any credit for this problem.

6. (15 points) Find the derivative of each function. You do *not* need to simplify your answers.

   (a) $f(x) = x^4 \sin(x)$

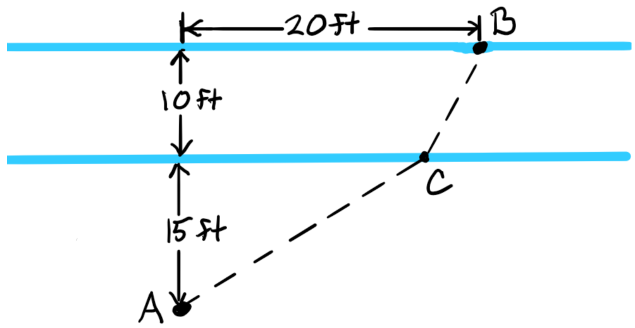
   (b) $f(x) = \ln(2x^2 + 3x)$

   (c) $f(x) = \dfrac{x^2 + 3x + 5}{2x + 1}$

   (d) $f(x) = x^7 + x^-3$

   (e) $f(x) = e^{x^3+1}$

7. (10 points) A woman is at point A 15 feet from the shore of a 10 foot wide river and wishes to reach point B which is 20 feet downstream on the other side. She can walk at a rate of 10 feet per minute and swim at a rate of 6 feet per minute. If the woman travels along the path which minimizes the total time taken, how far downstream of her starting position is the point C, where she reaches the near side of the river?



8. (8 points) Consider the function $f(x) = 2x^3 + 3x^2 - 36x + 15$.

   (a) Where is the function increasing or decreasing?

   (b) Where does the function have a relative maximum or minimum?

   (c) Where is the function concave up or concave down?

   (d) Where does the function have an inflection point?

9. (10 points) A spherical balloon is being inflated at a constant rate. If the radius is expanding at a rate of 4 centimeters per second, how fast (in cubic centimeters per second) must air be entering the balloon? [Note: the volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

10. (5 points) Consider the function defined implicitly by the equation

$$3x^2y + 2x + 1 = \ln(2x + y).$$

(a) Find the derivative dy/dx using implicit differentiation.

(b) Find the equation of the tangent line to this function at the point $(0, e)$.

11. (10 points) Consider the function $f(x, y) = x^3 + 2x^2y + y^2 + 7$. Find the following partial derivatives.

   (a) $f_x = \partial f/\partial x$

   (b) $f_y = \partial f/\partial y$

   (c) $f_{xx} = \partial^2 f/\partial x^2$

   (d) $f_{xy} = \partial^2 f/\partial x \partial y$

   (e) $f_{yy} = \partial^2 f/\partial y^2$

12. (6 points) Consider the function $f(x, y) = x^3 + 2x^2y + y^2 + 7$ as in the previous problem.

    (a) Find all critical points of f.

    (b) For each critical point, determine whether it is a relative minimum, relative maximum, or saddle point.