Chapter 8: Momentum & Collisions

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8.1 linear momentum — mass times velocity

$$\vec{p} = m\vec{v} \qquad \text{unit: } kg \cdot m/s$$

momentum

e.g. a 110-kg football player moving at 8 m/s.

$$p = (110\ kg)(8\ m/s) = \boxed{880\ kg \cdot m/s}$$

e.g. a 0.41-kg football thrown at 25 m/s.

$$p = (0.41\ kg)(25\ m/s) = \boxed{10.25\ kg \cdot m/s}$$

- the 2nd Law ($\vec{F}_{net} = m\vec{a}$) is actually in terms of momentum.

$$\boxed{\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}} =$$

constant mass

$$\Rightarrow \Delta\vec{p} = \Delta(m\vec{v}) = m\Delta\vec{v}$$

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t} = m\vec{a}$$

e.g. hitting a tennis ball

time raquet in contact with ball.

$$m = 0.057\ kg \qquad v_f = 58\ m/s \qquad \Delta t = 5\ ms$$

$$p_f = mv_f = (0.057\ kg)(58\ m/s) = 3.31\ kg \cdot m/s$$

$$F_{net} = \frac{p_f - p_i}{\Delta t} = \frac{3.31\ kg \cdot m/s}{0.005\ s} = 661\ kg \cdot m/s^2 = \boxed{661\ N}$$

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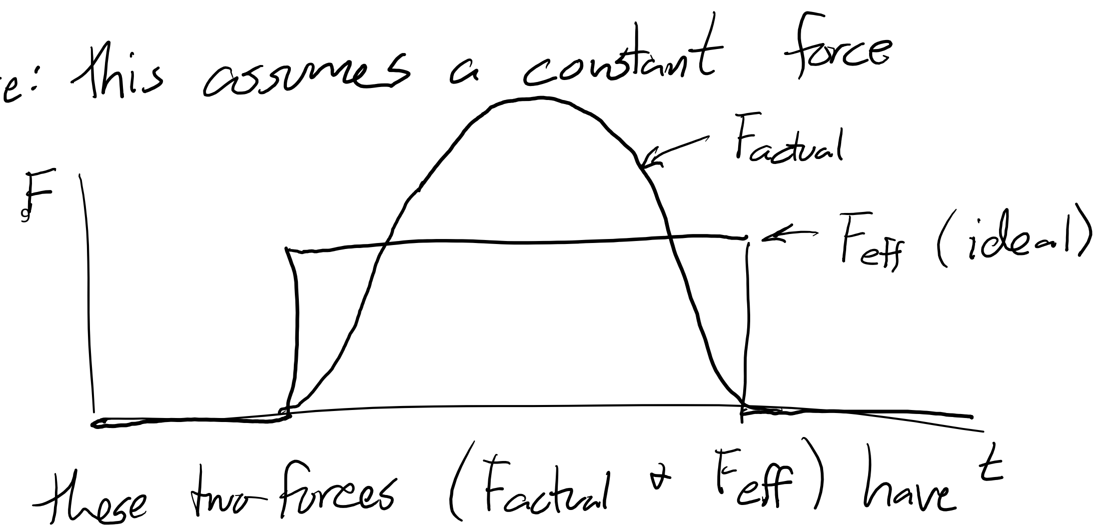
8.2) $\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} \implies \Delta\vec{p} = \underbrace{\vec{F}_{net}\,\Delta t}_{\text{called } \underline{\text{impulse}}}$

<u>impulse</u> - a force acting for some time $(\vec{F}_{net}\,\Delta t)$

Note: this assumes a constant force



these two forces (Factual & Feff) have the same impulse.

the same impulse can come from a weaker force if it acts over a longer time.

- e.g. a car crash.
  - the impulse acting on the person cannot be changed.
  - an air bag will make the crash take a longer amount of time.
    - $\implies$ means less force on the person
    - $\implies$ means the person gets less injured.

8.3) conservation of <u>momentum</u>

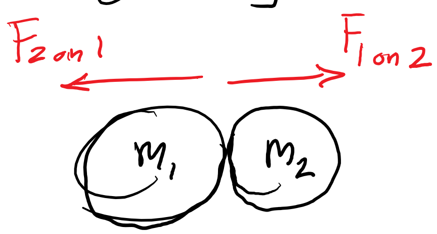
8.3 Conservation of momentum

For an isolated system, the total momentum is conserved.

isolated system → a system where the net external force is zero

total momentum → sum of the momentums of all of the objects in the system.

e.g. two balls colliding



before collision, mass 1 is moving + mass 2 is a rest.

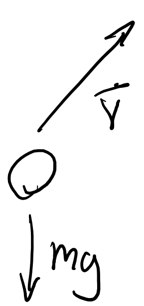
system = ball 1
→ net loss of momentum

system = ball 2
→ net gain of momentum

system = both ball 1 + ball 2
→ no change of momentum

Note: momentum is a vector + sometimes only some of the components are conserved.

e.g. projectile motion.



- the x-component of $\vec{F}_{net}$ is 0,
  so the x-component of $\vec{p}$ is conserved.
- the y-component of $\vec{F}_{net} \neq 0$,
  so the y-component of $\vec{p}$ is <u>not</u> conserved.

Conceptual Question: How can a small force impart the same change in momentum as a large force?

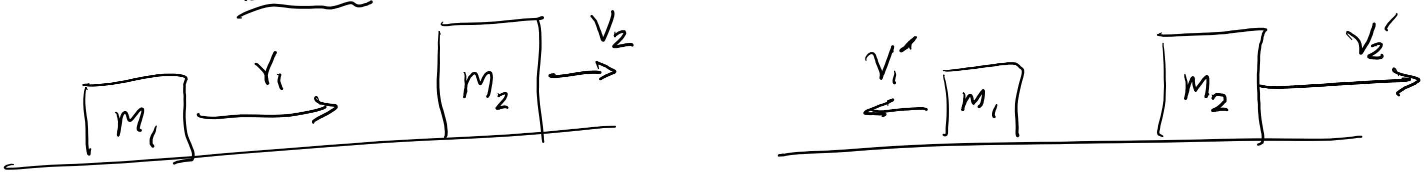
the same change in momentum as a large force?

Collisions

8.4| elastic collisions — a collision which conserves the total internal kinetic energy of the system.

1-dimension

before — afterwards



system is isolated → momentum is conserved.
collision is elastic → kinetic energy is conserved.

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2' \quad \leftarrow \quad P_{tot} = P_{tot}'$$

$$\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 V_2'^2 \quad \leftarrow \quad KE = KE'$$

e.g. $m_1 = 0.5\,kg$, $m_2 = 3.5\,kg$, $v_1 = 4\,m/s$, $V_2 = 0\,m/s$
want to find V_1' & V_2'.

$$m_1 V_1 = m_1 V_1' + m_2 V_2'$$

$$V_2' = \frac{m_1}{m_2}(V_1 - V_1')$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 V_2'^2$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2}(V_1 - V_1')\right)^2$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} \frac{m_1^2}{m_2}\left(V_1^2 - 2V_1V_1' + V_1'^2\right)$$

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}\frac{m_1^2}{m_2}\left(v_1^2 - 2v_1 v_1' + v_1'^2\right)$$

$$\left(\frac{1}{2}m_1 + \frac{1}{2}\frac{m_1^2}{m_2}\right)v_1'^2 - \frac{m_1^2}{m_2}v_1 v_1' + \left(\frac{1}{2}\frac{m_1^2}{m_2} - \frac{1}{2}m_1\right)v_1^2 = 0$$

plug into quadratic formula

$v_1' = 4\,m/s$ OR $\boxed{v_1' = -3\,m/s}$

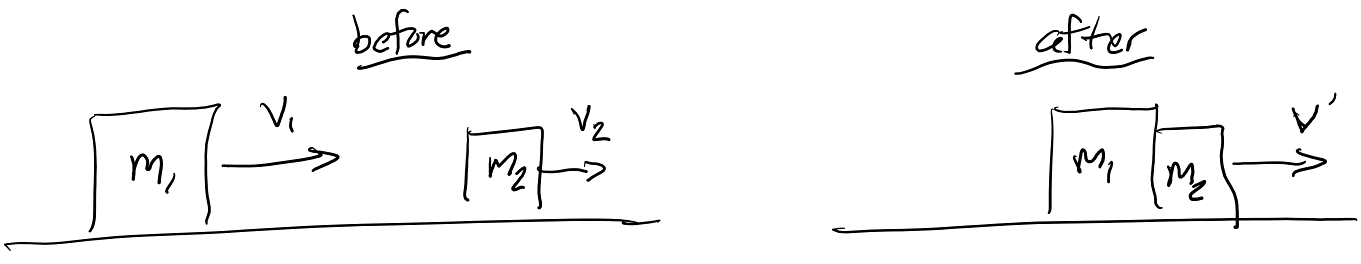
$$v_2' = \frac{0.5\,kg}{3.5\,kg}\left(4\,m/s - (-3\,m/s)\right) = \boxed{1\,m/s}$$

8.5 inelastic collisions – collisions that do not conserve internal kinetic energy.

- maybe they stick together
- maybe they bounce (but not as much) ← most collisions
- maybe they somehow gain energy while colliding

• in any case, you still have conservation of momentum

perfectly inelastic collision – a collision in which the objects stick together. (reduces KE by as much as possible)



eg. a 70 kg hockey goalie catches a 0.15 kg puck initially going at 35 m/s. Assuming no friction, what is his final velocity?

isolated system $\longrightarrow$ conservation of momentum

perfectly inelastic $\longrightarrow$ $v_1' = v_2' = v'$

$m_1 = 70\ kg$ $\qquad$ $m_2 = 0.15\ kg$

$v_1 = 0\ m/s$ $\qquad$ $v_2 = 35\ m/s$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' = (m_1 + m_2) v'$$

$$\boxed{v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}} = \boxed{0.075\ m/s}$$

final velocity for <u>any</u> perfectly inelastic collision.

e.g. an inelastic collision which <u>gains</u> energy

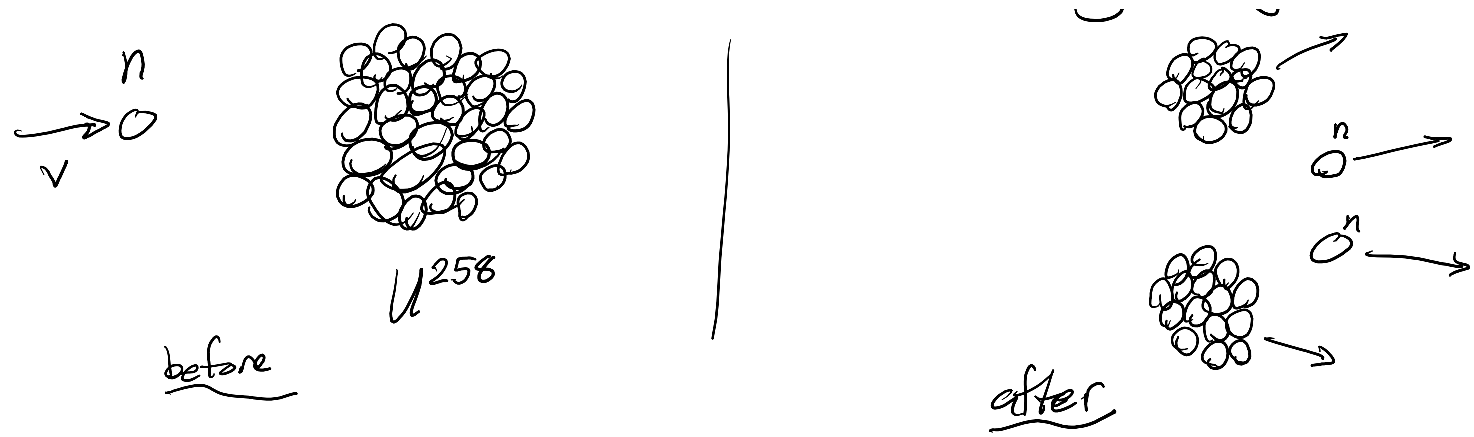
compressed spring with a latch

when the collision occurs, the latch will release and be allowed to expand

PE of the spring turns into KE of the carts

- a less contrived example, nuclear fission.

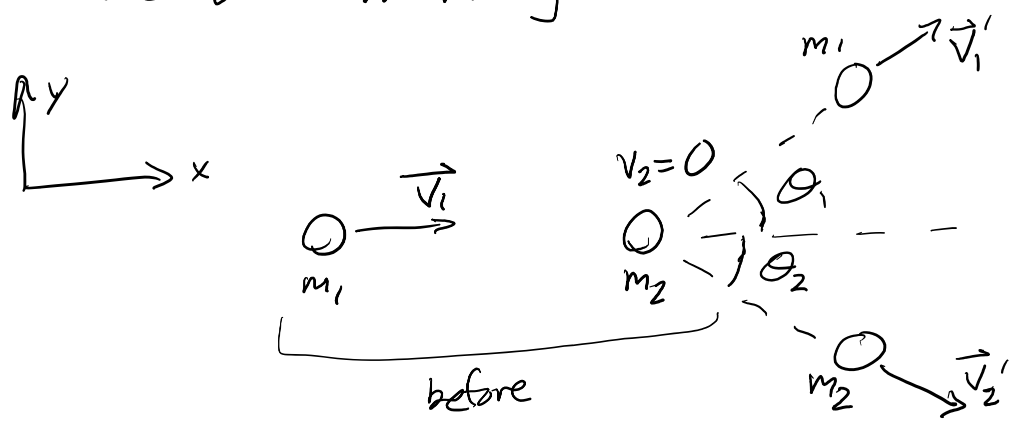
a neutron hits uranium + then the uranium breaks apart releasing energy



8.6 | 2D collisions

simplifications → point masses don't consider rotation
- choosing good coordinates

"lab frame" – one of the objects is initially stationary and the other is moving in the +x direction.



conserve momentum

$$m_1\vec{v_1} + \cancel{m_2\vec{v_2}}^{\,0} = m_1\vec{v_1}' + m_2\vec{v_2}'$$

$$m_1 v_{1,x} = m_1 v_{1,x}' + m_2 v_{2,x}' = \boxed{m_1 v_1'\cos\theta_1 + m_2 v_2'\cos\theta_2 = m_1 v_1}$$

$$m_1 v_{1,y} = m_1 v_{1,y}' + m_2 v_{2,y}' = \boxed{m_1 v_1'\sin\theta_1 + m_2 v_2'\sin\theta_2 = 0}$$

- need to know at least 2 of v_1', v_2', θ_1, and θ_2

- if perfectly elastic $\rightarrow$ also have conservation of KE

$$\boxed{\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2}$$

(only need 1 quantity to solve)

- if perfectly inelastic $\rightarrow$ $v_1' = v_2'$ + $\theta_1 = \theta_2$

  (can solve w/ only initial velocity)

  (will be a 1D collision)

- perfectly elastic + same mass (i.e. $m_1 = m_2 = m$)

  we will get the equation (conservation of momentum)

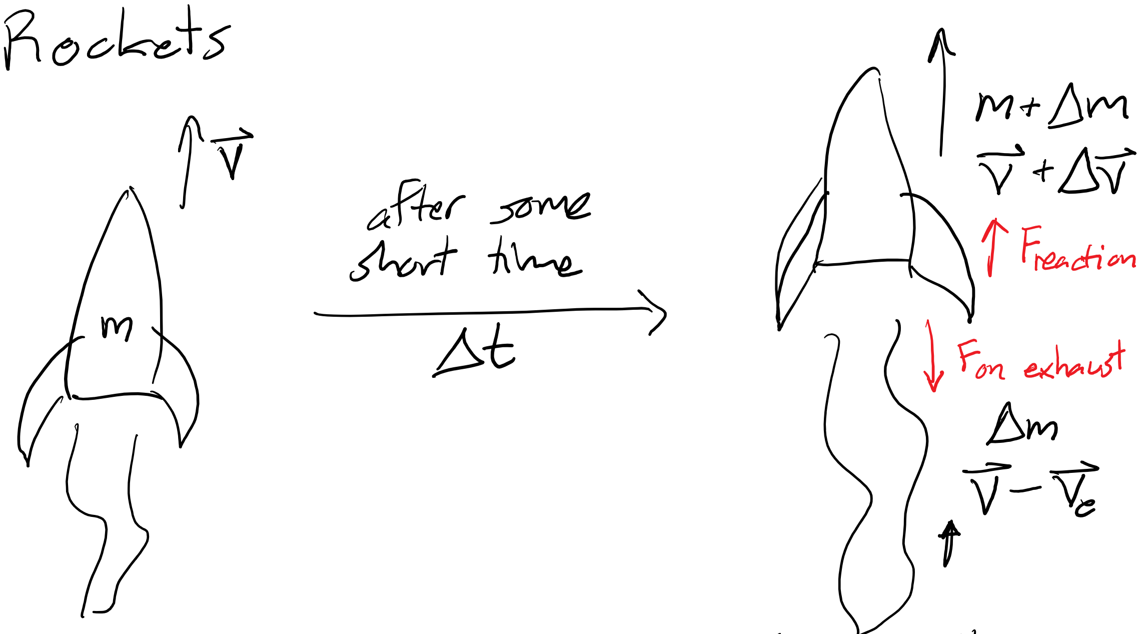
$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 \underbrace{+ mv_1'v_2'\cos(\theta_1 - \theta_2)}$$

- that means that $mv_1'v_2'\cos(\theta_1 - \theta_2) = 0$.

  - either:

    1. $v_1' = 0$ (incoming ball stops)
    2. $v_2' = 0$ (no collision actually happens)
    3. $\cos(\theta_1 - \theta_2) = 0 \Rightarrow \theta_1 - \theta_2 = 90°$

8.7 | Rockets



- some mass Δm expelled as exhaust (w/ velocity v_e relative to the rocket)
- the rest of the mass (rocket, $m-\Delta m$) is now going faster $(v+\Delta\vec{v})$
- the entire system decreases momentum due to gravity.

$$\Delta\vec{p} \simeq -mg\,\Delta t$$

want acceleration. ~~find~~ find using momentum.

$$\underbrace{(m-\Delta m)(\vec{v}+\Delta\vec{v})}_{\text{final momentum of rocket}} + \underbrace{\Delta m(\vec{v}-\vec{v}_e)}_{\text{final momentum of exhaust}} - \underbrace{m\vec{v}}_{\text{initial momentum}} = \underbrace{-mg\,\Delta t}_{\text{impulse}}$$

$$\cancel{mv} + m\Delta v - \cancel{v\Delta m} - \cancel{\Delta m\Delta v}^{\,\to 0} + \cancel{v\Delta m} - v_e\Delta m - \cancel{mv} = -mg\,\Delta t$$

$$m\Delta v - v_e\Delta m = -mg\,\Delta t$$

$$m\frac{\Delta v}{\Delta t} - v_e\frac{\Delta m}{\Delta t} = -mg$$

$$\boxed{a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g}$$

acceleration of rocket

acceleration depends on:

1. velocity of exhaust gas, v_e (practical limit of $\sim 2.5\times10^3$ m/s)
2. rate at which the mass is expeled $\left(\frac{\Delta m}{\Delta t}\right)$

"thrust" $= v_e\frac{\Delta m}{\Delta t}$ unit: N

$$\text{"thrust"} = v_e \frac{\Delta m}{\Delta t} \qquad \underline{\text{unit}}\text{: N}$$

3. mass of the rocket (m)

Note: as a rocket flies it loses mass
→ it accelerates faster.

e.g. Saturn V
mass at liftoff $= 2.8 \times 10^6$ kg
fuel burn rate $= 1.4 \times 10^4$ kg/s
exhaust velocity $= 2.4 \times 10^3$ m/s
what was it's initial acceleration.

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g = \frac{(2.4\times10^3 \text{ m/s})}{(2.8\times10^6 \text{ kg})}\left(1.4\times10^4 \frac{\text{kg}}{\text{s}}\right) - 9.8 \text{ m/s}^2$$
$$= \boxed{2.2 \text{ m/s}^2}$$

Can also get a formula for the final velocity of the rocket.

$$\boxed{v = v_e \ln \frac{m_0}{m_r}}$$

initial mass of rocket + fuel

mass of just the rocket

final velocity

(can use for a portion of the flight
$$\Delta v = v_e \ln \frac{m_i}{m_f}$$
)

e.g. how much fuel does it take to get to escape velocity.

escape velocity $= 11.2 \times 10^3$ m/s
exhaust velocity maximized $\rightarrow$ $V_e = 2.5 \times 10^3$ m/s

$$11.2 \times 10^3 \text{ m/s} = (2.5 \times 10^3 \text{ m/s}) \ln \frac{m_0}{m_r} \rightarrow \boxed{\frac{m_0}{m_r} = e^{4.4} = 88}$$

$\rightarrow \frac{87}{88} = 97\%$ of the rocket is fuel!