Chapter 7: Work & Energy

Monday, June 28, 2021 12:53

energy – the ability to do work

- comes in many forms.
- Conserved – it can change form, but not be created or destroyed.

7.1

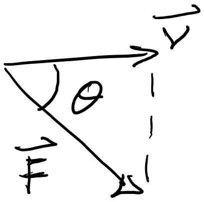
work – force applied over a distance

(the component of force in the direction of motion times the distance traveled)

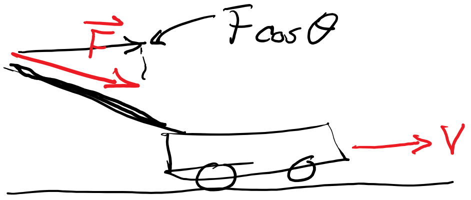
$$W = (|\vec{F}|\cos\theta)|\vec{d}| = \boxed{Fd\cos\theta}$$

work ↗

$$= \vec{F}\cdot\vec{d}$$ ← dot product

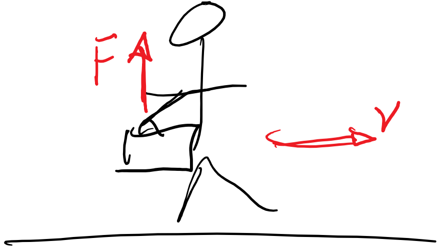


- pushing a lawnmower



WORK IS DONE

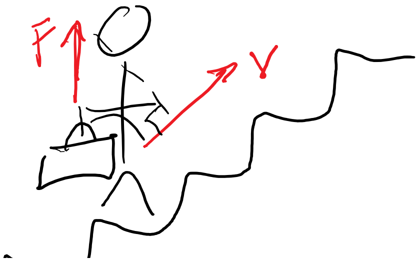
- carrying a briefcase



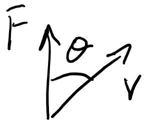
NO WORK DONE!

force + direction of motion are perpendicular

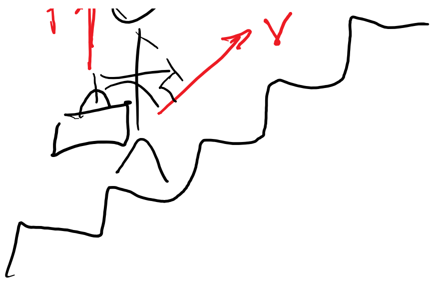
-



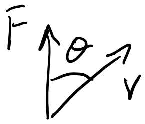
WORK DONE



$W = Fd\cos\theta$

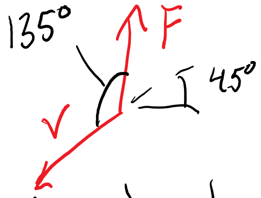


WORK DONE



$W = Fd\cos\theta$

- carrying it downstairs



$W = Fd\cos\theta < 0$

(NEGATIVE WORK DONE ON THE BREIFCASE)

this is what we've been talking about

↳ WORK DONE ON AN OBJECT vs. WORK DONE BY AN OBJECT

UNIT: $1\,\text{N}\cdot\text{m}$ (newton-meter) $= 1\,\text{J}$ (joule) $= \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}$

1 kilocalorie (kcal) = 1 Calorie (Cal) = $\underbrace{4186\,\text{J}}$ ↖ food unit

1 calorie (cal) = $\underbrace{4.186\,\text{J}}$

7.2 net work - work done by the net force.

$$W_{net} = F_{net}\cdot d\cdot\cos\theta$$

$W_{net} = F_{net}\,d\cos\theta$ (assume $\theta = 0°$, i.e. net force in direction of motion)

$= F_{net}\,d = mad$ (2nd Law)

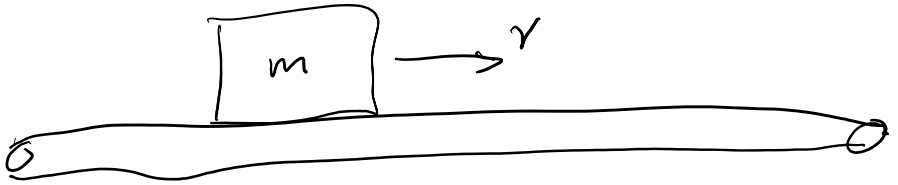
$$= m\left(\frac{v^2 - v_0^2}{2}\right) \qquad (v^2 = v_0^2 + 2ad)$$

$$\boxed{W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = KE_f - KE_i = \Delta KE}$$

(traslational) kinetic energy: $\boxed{KE = \frac{1}{2}mv^2}$

- net work is the change in kinetic energy
- WORK-ENERGY THEOREM $W_{net} = \Delta KE$
- applies in general for any movement.

e.g. a box on a conveyor belt



$m = 30 kg$

$v = 0.5 m/s$

$KE = \frac{1}{2}(30kg)(0.5 \frac{m}{s})^2$

$= \boxed{3.75 J}$

e.g. accelerating the box

$m = 30kg$ $F = 120N$ applied friction $f = 5N$

$v_0 = 0.5 m/s$ $d = 0.8m$

← 0.8m

$$W_{net} = F_{net}\, d = (120N - 5N)(0.8m) = \boxed{92J}$$

what is its final velocity? $W_{net} = KE_f - KE_0$

$$KE_f = W_{net} + KE_0 = 92J + 3.75J = 95.75J = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2(KE)}{m}} = \sqrt{\frac{2(95.75J)}{30kg}} = \boxed{2.5\, m/s}$$

---

work - force applied over a distance

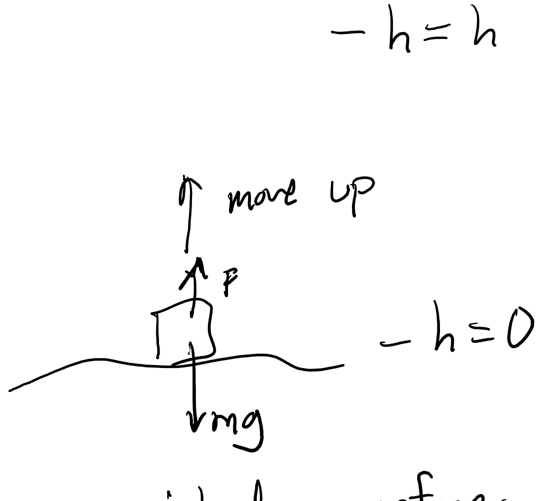
energy - measure of the ability to do work

↳ kinetic energy - energy due to movement

work-energy thm: $W_{net} = \Delta KE$

7.3) gravitational potential energy - energy due to elevation (i.e. due to its position in the gravitational field)

lift an object



some force F applies to oppose gravity.

$$W = \boxed{mgh = PE_g}$$

- we picked a reference height with PE=0

- we only really care about work - i.e. change in energy

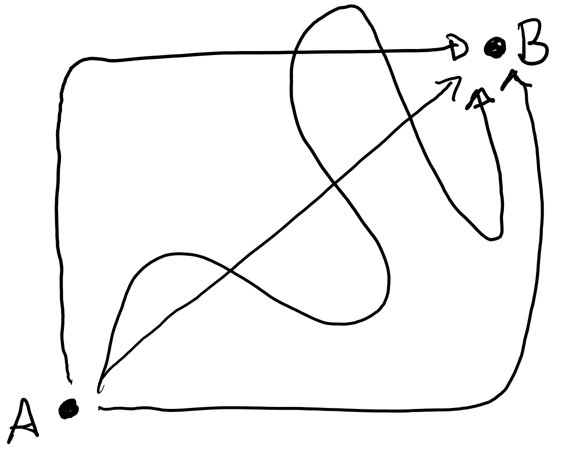
$$W_{net} = mgh = -(\underset{\substack{\uparrow \\ PE_f}}{0} - \underset{\substack{\uparrow \\ PE_i}}{mgh})$$

$$\underset{\substack{\uparrow \\ \text{work energy} \\ \text{theorem}}}{W_{net}} = \Delta KE = -\Delta PE$$

$$\Longrightarrow \boxed{KE + PE = \text{constant}}$$

7.4) Conservative force

Conservative force - a force where the work done only depends on the starting + ending positions, and not on the path taken.

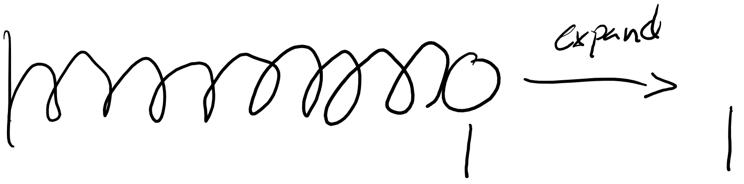


any of these paths will give the same work done on the object.

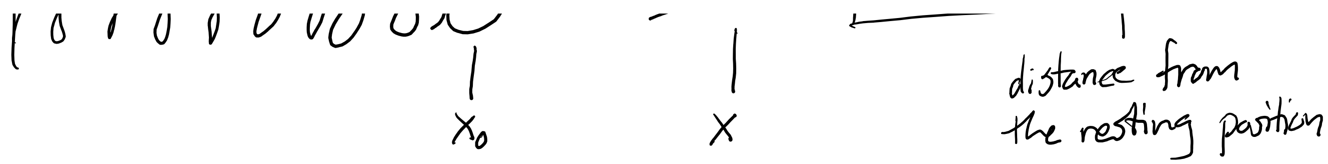
→ you can define a potential energy for any conservative force.

- Hooke's law $(F = k\Delta L)$ for springs is conservative

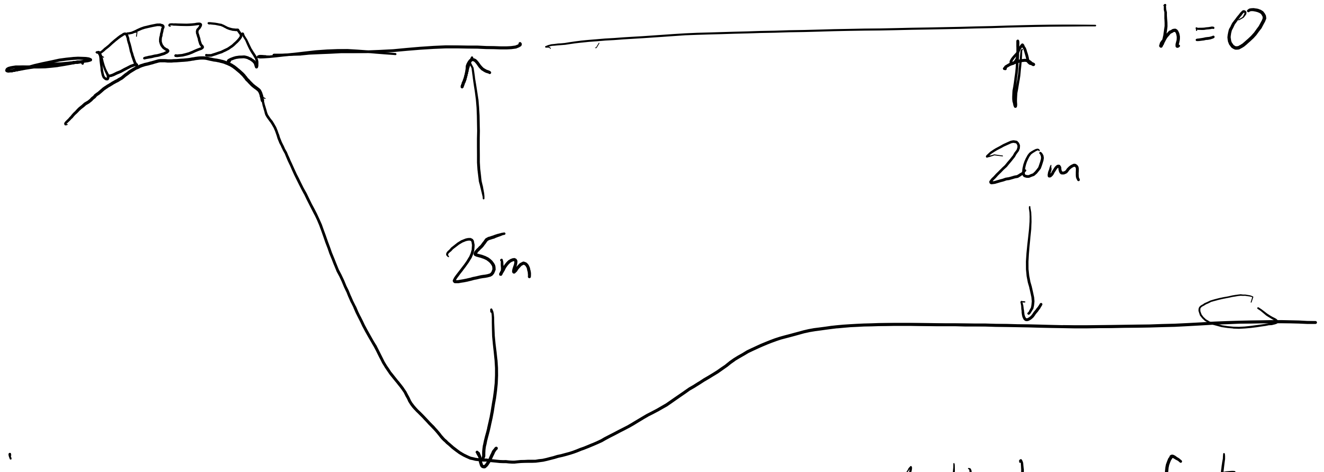
work done is $W = \frac{1}{2}kx_0^2 - \frac{1}{2}kx^2 = -\Delta PE$



$$\boxed{PE_s = \frac{1}{2}k\underset{\substack{\uparrow \\ \text{distance from}}}{x^2}}$$



e.g. rollercoaster



if it starts at rest at the top of the hill, how fast is it going at the end?

$$PE_0 = mgh_0 = 0\,J$$

$$PE_f = mg(-20m)$$

$$KE_0 = 0\,J$$

$$KE_f = \frac{1}{2}mv_f^2$$

$$-\Delta PE = \Delta KE$$

$$-(-20mg - 0) = \frac{1}{2}mv^2 - 0$$

$$(20m)\,mg = \frac{1}{2}mv^2$$

$$v = \sqrt{2(20m)(9.8\,m/s^2)} = \boxed{19.8\,m/s}$$

e.g. toy car pushed by a spring

spring starts compressed by 4cm w/ $k = 250\,N/m$.
mass of car is $m = 0.1\,kg$. no friction.

- how fast is it going when it leaves the spring?

$$PE_i = \frac{1}{2}kx^2 = \frac{1}{2}(250\tfrac{N}{m})(0.04m)^2 = 0.2J$$

$$PE_f = 0J$$

$$KE_i = 0J$$

$$KE_f = \frac{1}{2}mv^2$$

$$PE_i + \cancelto{0}{KE_i} = \cancelto{0}{PE_f} + KE_f$$

$$0.2J = \frac{1}{2}mv^2 \Rightarrow$$

$$v = \sqrt{\frac{2(0.2J)}{0.1kg}} = \boxed{2\,m/s}$$

- how fast is it going at the end of the track?

$$PE_i = 0.2J \qquad PE_f = mgh = (0.1kg)(9.8\,m/s^2)(0.18m) = 0.176J$$

$$KE_i = 0J \qquad KE_f = \frac{1}{2}mv^2$$

$$0.2J = 0.176J + \frac{1}{2}mv^2 \Rightarrow \boxed{v = 0.68\,m/s}$$

7.5| non-conservative force is a force where the work done depends on The path taken.

↳ add or remove mechanical energy

↳ kinetic or potential energy

e.g. friction converts mech. energy to thermal energy

$$W_{net} = \underbrace{W_{nc}}_{\text{non-conservative work}} + \underbrace{W_c}_{\text{conservative work } (-\Delta PE)} = \Delta KE \Rightarrow W_{nc} = \Delta KE + \Delta PE$$

7.6| Conservation of Energy - energy cannot be created or destroyed, it just changes form.

or destroyed, it just changes form.

$$E_i = KE_i + PE_i + W_{nc} + OE_i$$

$$\|$$

$$E_f = KE_f + PE_f + OE_f$$

other energy (i.e. chemical
thermal
electrical)

<u>efficiency</u> of energy conversion

$$Eff = \frac{\text{useful work}}{\text{total energy in}} = \frac{W_{out}}{E_{in}}$$

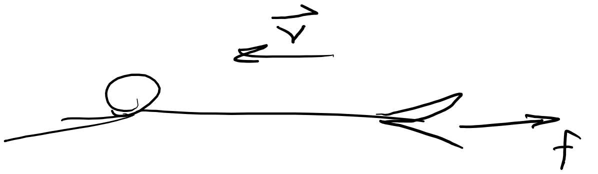
e.g. power plant

chemical energy (coal) → electrical energy (useful)
\+
→ thermal energy (wasted)

efficiency ~ <u>42%</u>

e.g. baseball player sliding

What is the distance a 65 kg player slides, given an initial speed of 6 m/s + a constant force of friction of 450N?



$$W_{nc} = -fd$$

$$KE_i + W_{nc} = KE_f$$

$$\frac{1}{2}mv_i^2 - fd = 0$$

$$d = \frac{\frac{1}{2}mv_i^2}{f} = \frac{(65kg)(6\,m/s)^2}{2(450N)} = \boxed{2.6m}$$

7.7 | power - rate at which work is done

$$\boxed{P = W/t}$$

power

work

time

Unit: watt (W) $\quad 1\,W = 1\,\frac{J}{s}$

horsepower (hp) $\quad 1\,hp = 746\,W$

$\boxed{W = Pt}$ ⟶ a unit for energy is $W \cdot s$

- kilowatt-hour ($kW \cdot h$) is a common unit for measuring electrical energy

e.g. running up the stairs

$m = 60\,kg$

start at rest

Final speed of $v = 2\,m/s$

climb 3m up in 3.5s.

what is their power output?

$$W = \Delta KE + \Delta PE = \tfrac{1}{2}mv^2 + mgh$$

$$= \tfrac{1}{2}(60\,kg)(2\,m/s)^2 + (60\,kg)(9.8\,m/s^2)(3\,m) = 1884\,J$$

$$P = \frac{W}{t} = \frac{1884\,J}{3.5\,s} = \boxed{538\,W}$$

7.8) basal metabolic rate (BMR) - rate at which the body (at rest) consumes energy.