Chapter 9: Statics & Torque

Tuesday, July 6, 2021 12:41

HW: due on Sunday

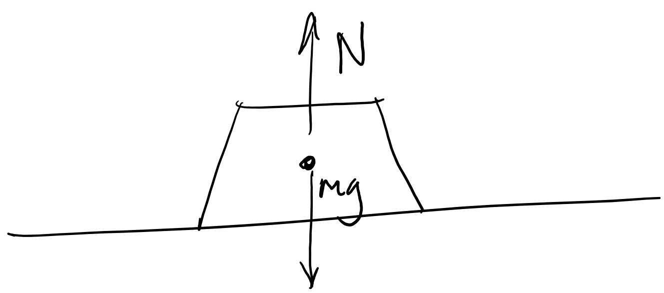
Quizzam 4 Regrade: due on Friday

statics - study of forces in equilibrium.

the forces "balanced" so the is no acceleration.

$\Rightarrow$ net $\vec{F} = 0$ (Newton's 2nd Law)

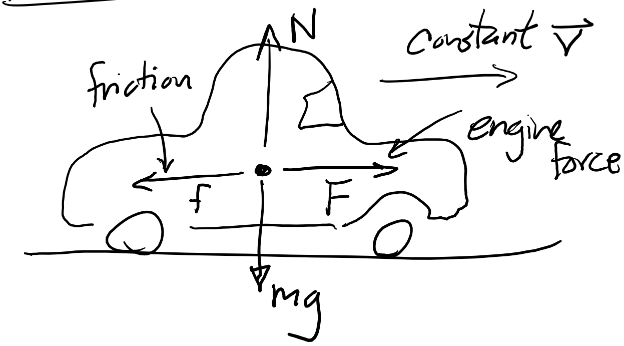
9.1 Equilibrium Condition 1: $\boxed{\text{net } \vec{F} = 0}$



need $N - mg = 0$

static equilibrium

↳ system is stationary



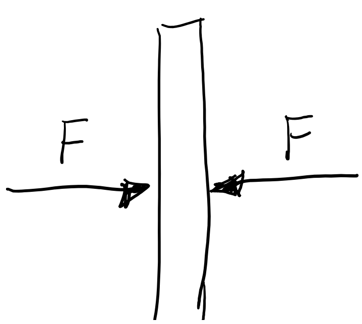
need $N - mg = 0$

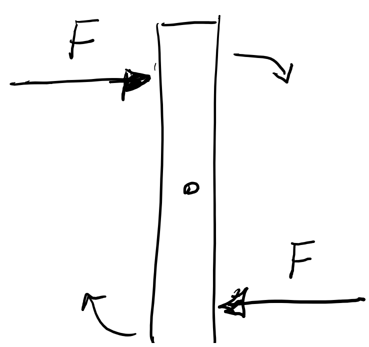
$F - f = 0$

dynamic equilibrium

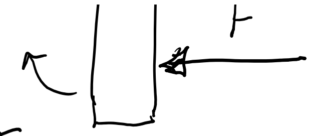
↳ system is moving with constant velocity

this condition is necessary but NOT sufficient





IN EQUILIBRIUM



NOT IN EQUILIBRIUM

in both cases net $\vec{F} = F - F = 0$, but only one is in equilibrium.

the case on the right will start to rotate.

angular acceleration - rate of change of angular velocity.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$\Delta\omega$ ← ang. vel.

α ← ang. accel.

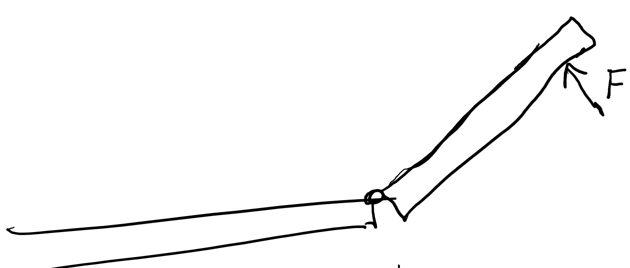
$$\alpha = \frac{a_\perp}{r}$$

$a_\perp$ ← tangential accel.

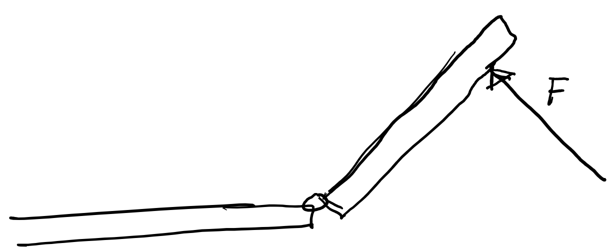
9.2 torque - basically, force but for rotation

↳ how much rotation depends on:

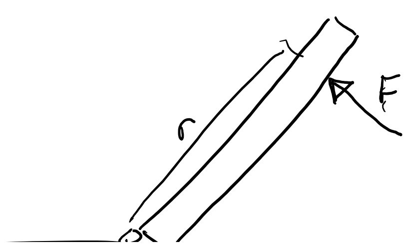
1. how much force is applied.
2. the direction the force is applied.
3. where the force is applied.

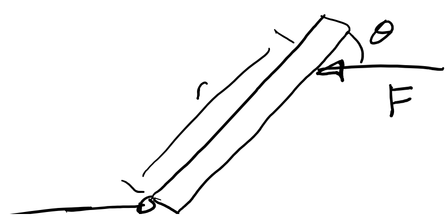


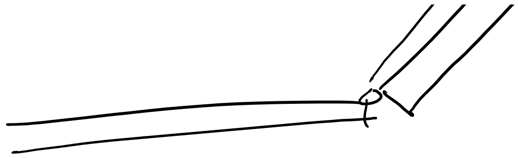
weak F → less torque



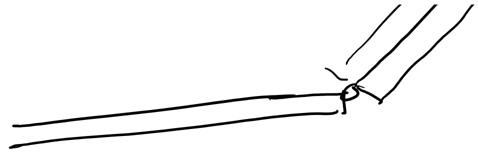
more F → more torque



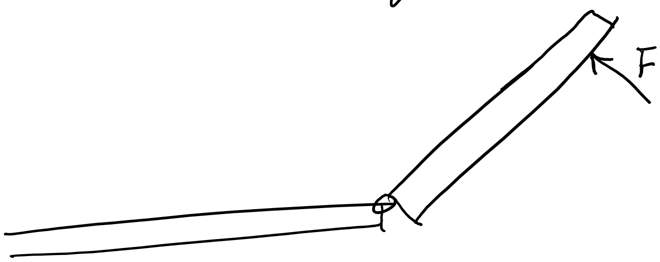




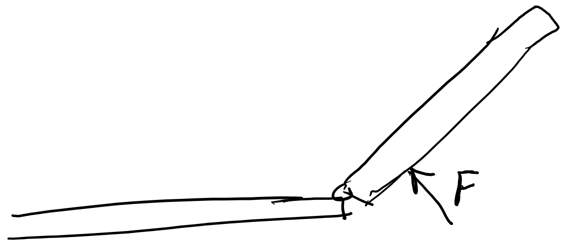
force at 90° to radius
→ maximum torque



force at some other angle $\theta \neq 90°$
→ less torque



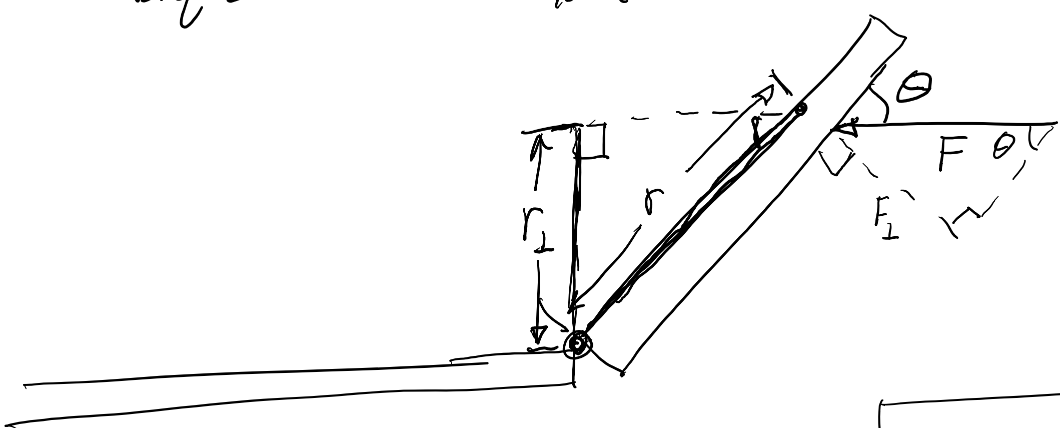
force far from pivot
→ more torque



force near pivot
→ less torque

$$\boxed{\tau = r F \sin\theta}$$

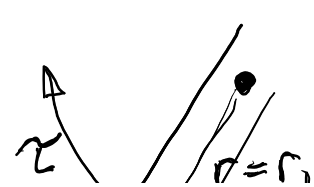
torque radius force angle between radius & force

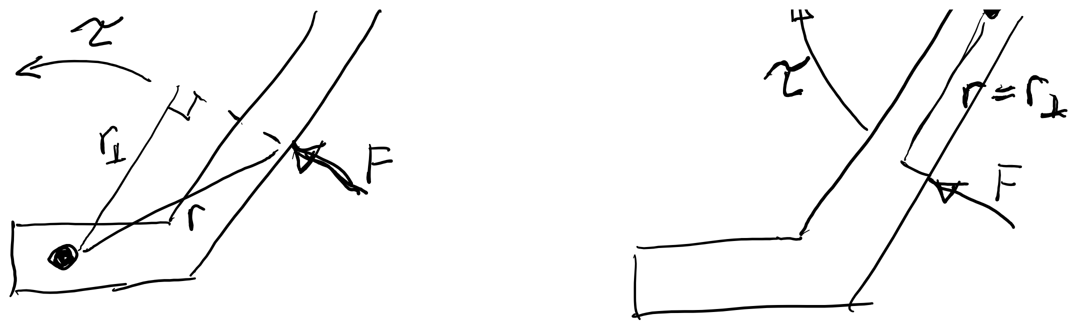


equivalently, let $r_\perp = r\sin\theta$ so $\boxed{\tau = r_\perp F}$

$F_\perp = F\sin\theta$ so $\boxed{\tau = r F_\perp}$

Note: the same force (at the same location) gives different torques for different pivot points.





Unit: newton-meter (N·m)

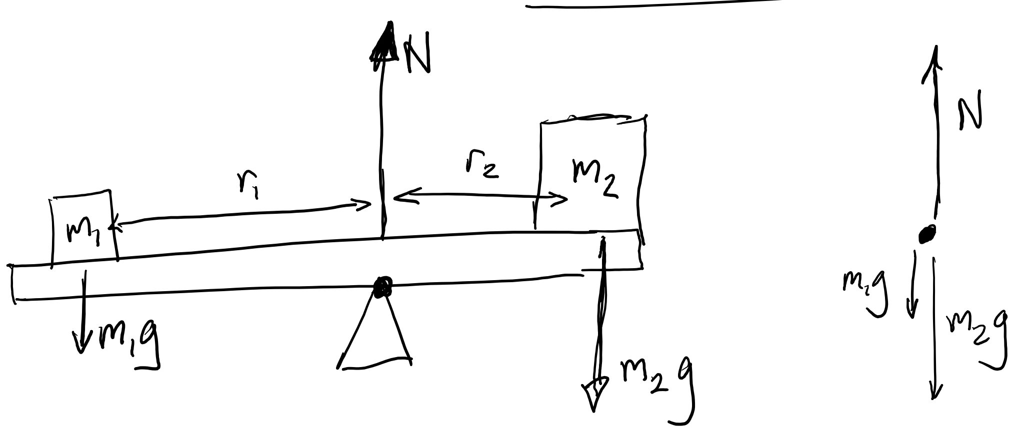
Note: torque has a direction (i.e. it is a vector)
- can be counterclockwise (+) or clockwise (−)

Note: you can calculate torque for any possible pivot point, even if it is not the actual center of rotation.

torque is related to angular acceleration similar to newton's second law. ($\vec{F} = m\vec{a}$)

if the radius is constant, $\tau = \underbrace{mr^2}_{\text{moment of inertia}}\alpha$

Equilibrium Condition 2: $\boxed{\text{net } \tau = 0}$



in equilibrium if:

$$\text{net } \vec{F} = N - m_1 g - m_2 g = 0$$

$$\text{net } \tau = r_1 m_1 g - r_2 m_2 g = 0$$

+ because ccw - because cw

e.g. if $m_1 = 26\,kg$, $r_1 = 1.6\,m$, & $m_2 = 32\,kg$, what is r_2?

torque eqn $\rightarrow$ $r_1 m_1 g = r_2 m_2 g$

$$r_2 = r_1 \frac{m_1}{m_2} = (1.6m)\frac{(26kg)}{(32kg)} \approx \boxed{1.3m}$$

e.g. in the same case, what is N?

force eqn $\rightarrow$ $N = m_1 g + m_2 g = (26kg)(9.8\,m/s^2) + (32kg)(9.8\,m/s^2)$

$$\approx \boxed{568N}$$

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so far - conditions for equilibrium

1) net $\vec{F} = 0$

2) net $\tau = 0$

$\tau = rF\sin\theta$

torque

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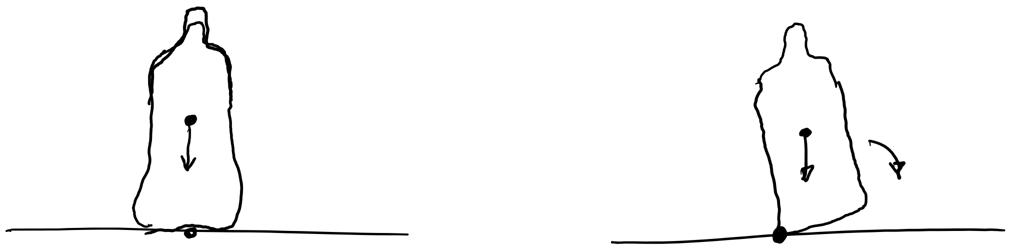
there are three types of equilibrium: stable, unstable, neutral

- what is the behavior if the system leaves equilibrium <u>slightly</u>

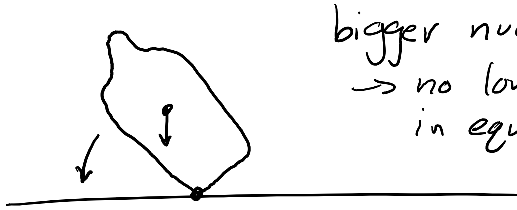
<u>stable equilibrium</u> - system will <u>return</u> to equilibrium when nudged.

e.g. a ball at the bottom of a bowl

e.g. a bottle standing upright



(Note: this is a small nudge, a bigger one might disrupt equilibrium.



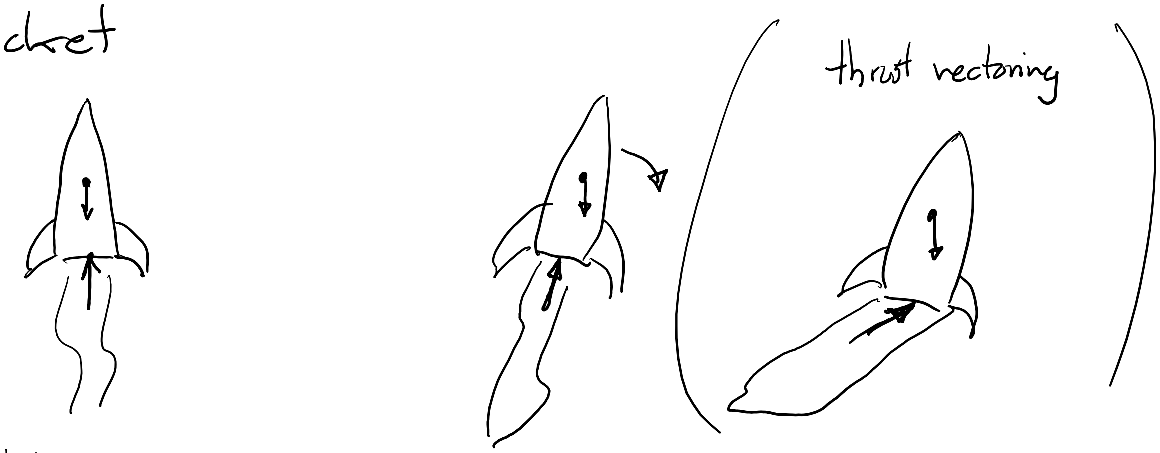
)

Unstable equilibrium – system will move further from equilibrium when nudged.

e.g. ball at the top of a hill

e.g. a bottle standing upside down

e.g. a rocket

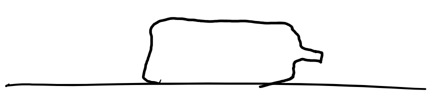


neutral equilibrium – the system remains in equilibrium even after a slight nudge.

e.g. ball on a flat plane

still in equilibrium

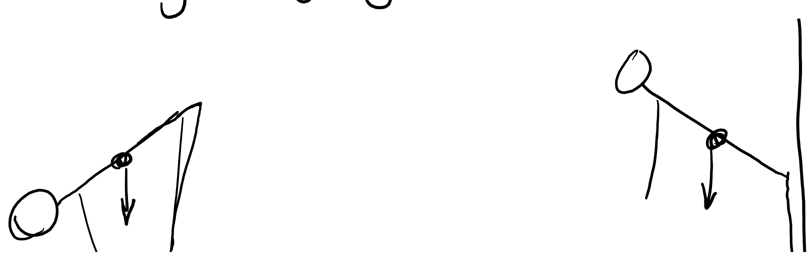
e.g. bottle on its side

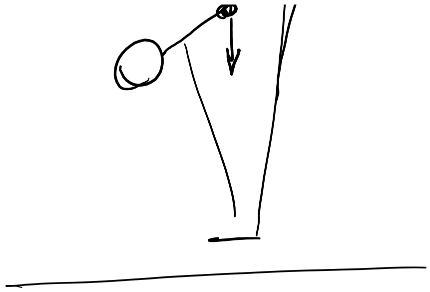


still be in equilibrium when rolled slightly

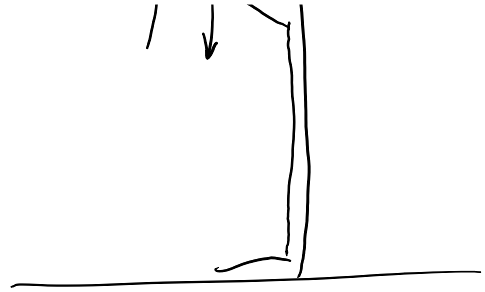
NEUTRAL – in equilibrium IN THE NEW POSITION.
STABLE – RETURN TO ORIGINAL POSITION to be in equilibrium.
UNSTABLE – LEAVES EQUILIBRIUM.

Note: different nudges might give different equilibrium types.

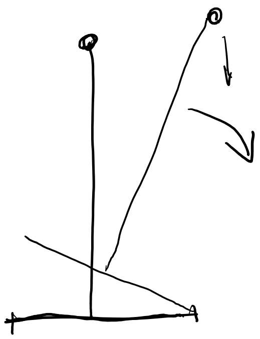




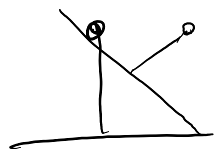
CM stays above feet



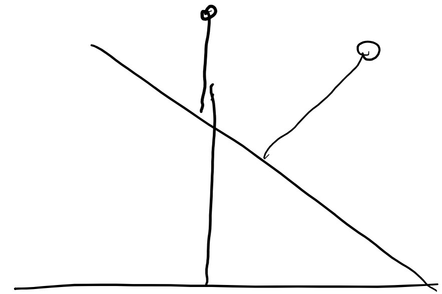
CM not above feet (you fall over)



higher CM → less able to tilt



lower CM → more able to tilt



wider base → more able to tilt

9.4 problem solving steps

1.) is it in equilibrium (i.e. is $\vec{a} = 0$ & $\alpha = 0$)

accel. ang. accel.

2.) DRAW A FREE BODY DIAGRAM!
(note <u>where</u> the forces are applied)

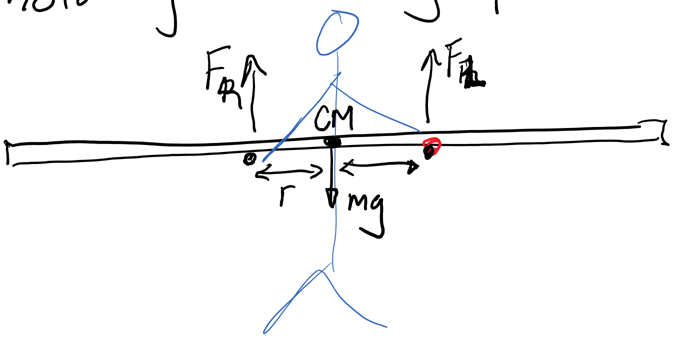
3) apply the equilibrium conditions (net $\vec{F} = 0$ & net $\tau = 0$)
(remember <u>you</u> can choose the pivot point for torque, choose one that makes it easy.)

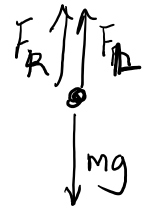
4) CHECK IF YOUR ANSWER MAKES SENSE!

e.g. holding a long pole

e.g. holding a long pole

$m = 5\,kg$.
distances are equal



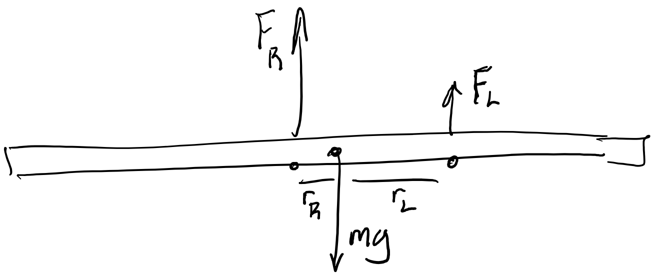


net $F = 0 \rightarrow F_L + F_R - mg = 0$

net $\tau = 0 \rightarrow rF_L - rF_R = 0$ ← pivot at CM

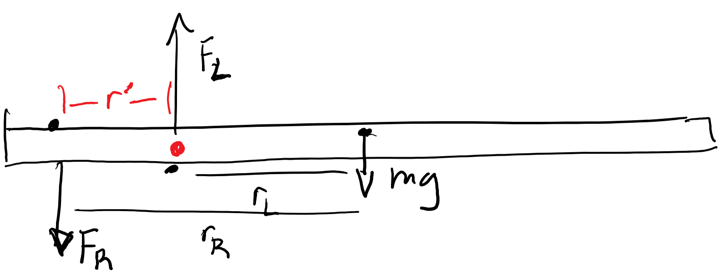
$rmg - 2rF_R = 0$ ← pivot at left hand

$F_L = F_R = \frac{1}{2}mg \approx 24.8\,N$



$r_L F_L - r_R F_R = 0$

$F_L + F_R - mg = 0$



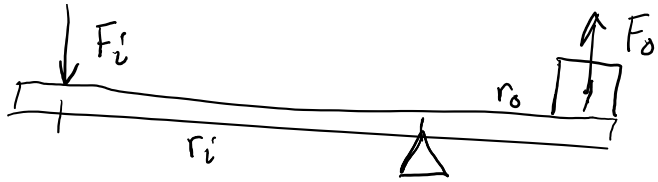
$r_R F_R - r_L F_L = 0$

$r' F_R - r_L mg = 0$

$F_L - F_R - mg = 0$

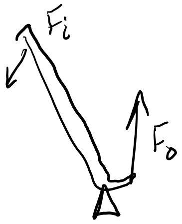
9.5 simple machines - any device used to multiply or redirect force.

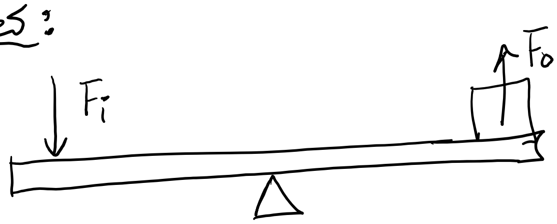
mechanical advantage $(MA) = \dfrac{F_o}{F_i}$ ← force out / ← force in __lever__ – a rigid bar which pivots around a point called the __fulcrum__.



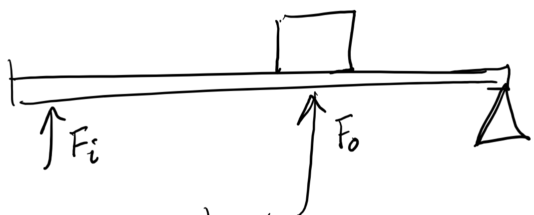
$$\text{net } \tau = 0 \longrightarrow r_i F_i = r_o F_o \longrightarrow \boxed{\frac{F_o}{F_i} = \frac{r_i}{r_o} = MA}$$

__3 types__:

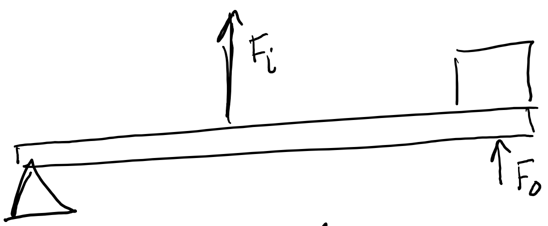




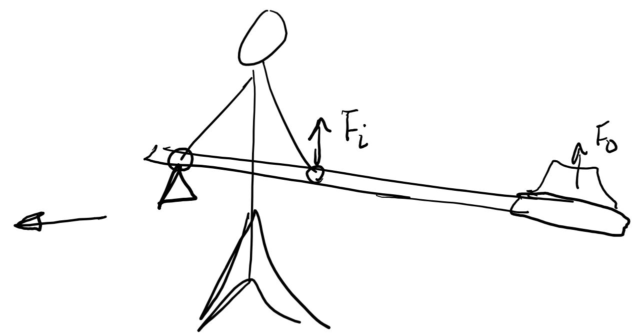
e.g. see-saw, crowbar



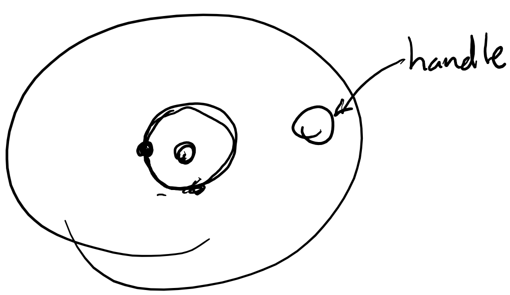
e.g. wheelbarrow
$MA > 1$



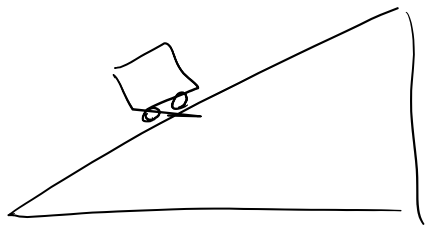
$MA < 1$
e.g. shovel



__crank__ – a lever that can rotate 360°

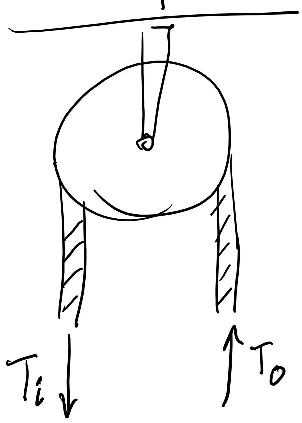


__inclined plane__ – a ramp

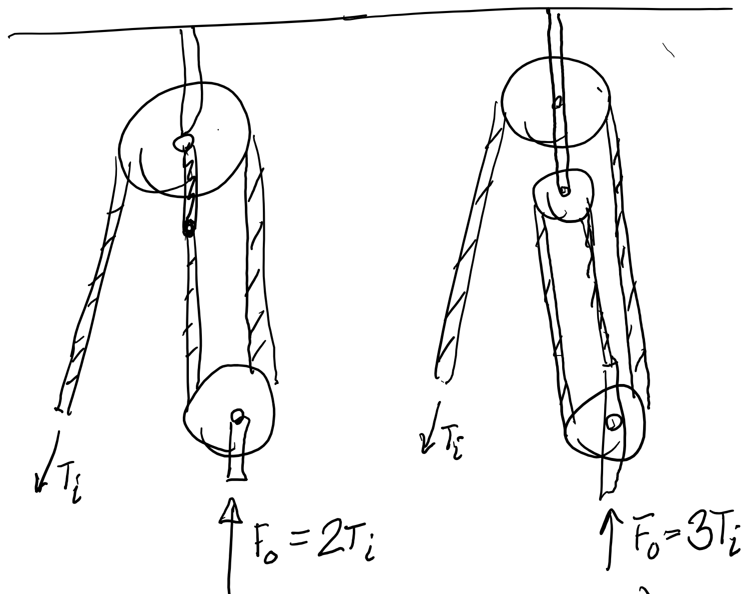


easier to push a cart up a ramp then to lift it the same height vertically.

pulley - a rope over a wheel



ordinary pulley
(MA = 1)



(MA = 2)

(MA = 3)

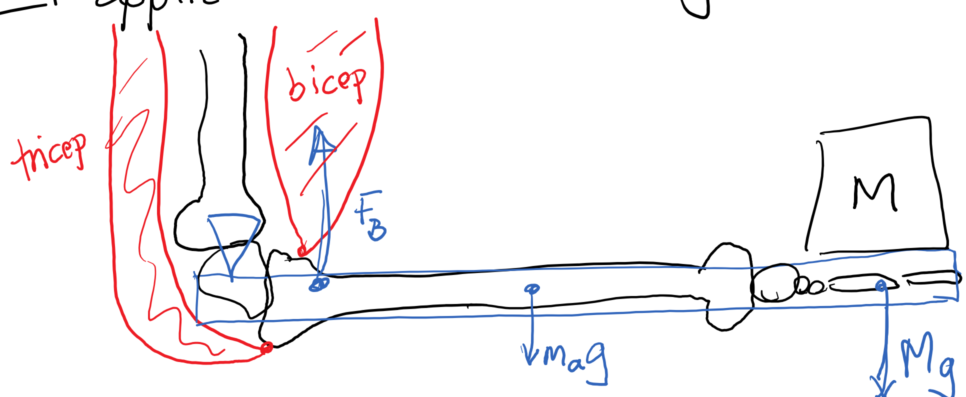
compound pulley

"block + tackle"

block: set of pulleys at the top (stationary)

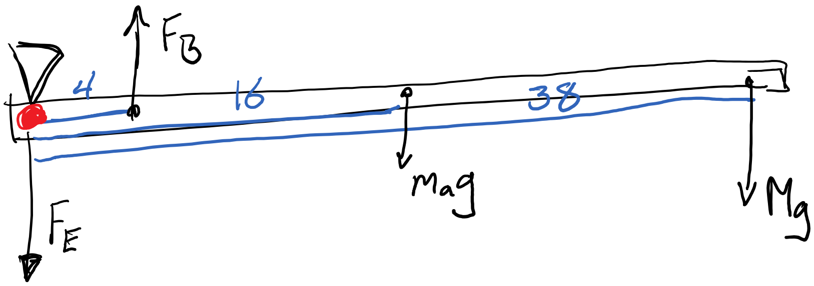
tackle: set of pulleys at the bottom (moving)

9.6 application to muscles + joints



muscles can only contract. → they operate in pairs

your arm is a lever.



mass of arm = 2.5 kg
mass of object held = 4.0 kg

CM of arm = 16 cm from elbow
length of arm = 38 cm
location of bicep = 4 cm

net $\tau = 0 \rightarrow$ $(4\text{cm})F_B - (16\text{cm})(2.5\text{kg})(9.8\,\text{m/s}^2) - (38\text{cm})(4.0\text{kg})(9.8\,\text{m/s}^2) = 0\ \text{N}\cdot\text{m}$

$$\boxed{F_B = 470\text{N}}$$

for comparison: weight of object ≈ 39.2 N
weight of arm ≈ 24.5 N
total weight 63.7 N

how much does your upper arm compress?

first → find F_E

net $F = 0 \rightarrow$ $F_B - F_E - (2.5\text{kg})(9.8\,\text{m/s}^2) - (4\text{kg})(9.8\,\text{m/s}^2) = 0\text{N}$

$$F_E = 470\text{N} - 63.7\text{N} = \underline{406.3\text{N}}$$

$$\Delta L = \frac{1}{Y}\frac{F_E}{A}L_0$$

← initial length of upper arm

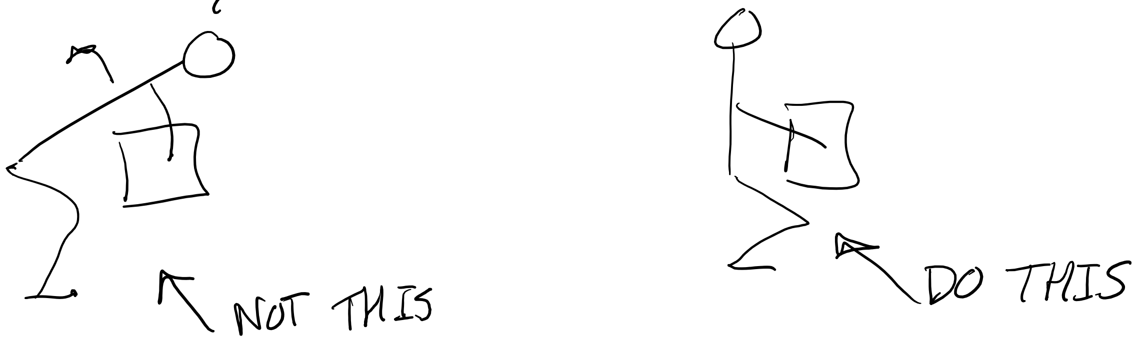
cross-sectional area of the upper arm bone

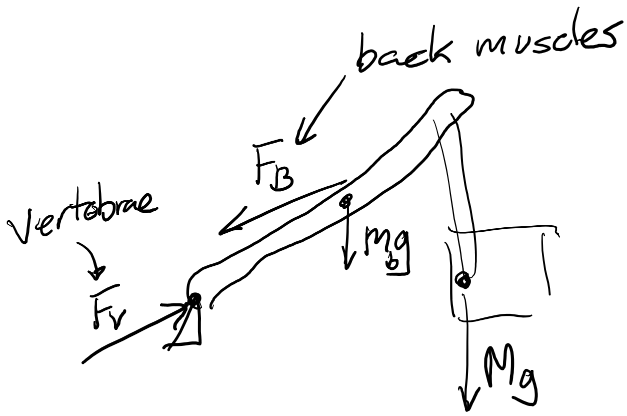
Young's modulus for ~~bone~~ under compression

8

<u>application</u>
don't lift with your back

don't lift with your back





55kg upper body lifting a 30kg box

$$F_B = \underline{4.2 \times 10^3\ N}$$

$$F_V = 4.66 \times 10^3 N$$