Quizzam regrades - due Wednesday on blackboard
- redo missed problems for up to half points back.

Last week: 1D kinematics (constant a)

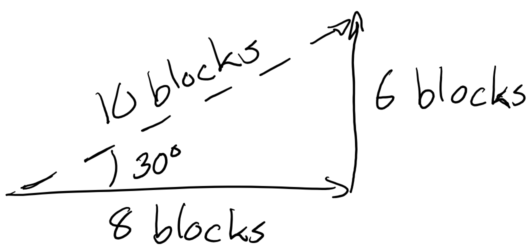
$$x = x_0 + \bar{v}t \qquad \bar{v} = \frac{v + v_0}{2}$$

$$v = v_0 + at \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

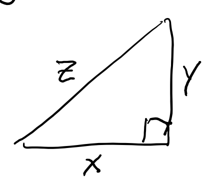
the real world is not 1 dimensional

- Consider moving in a city



distance traveled:
$8 + 6 = 14$ blocks
displacement:
10 blocks
in direction 30° north of east

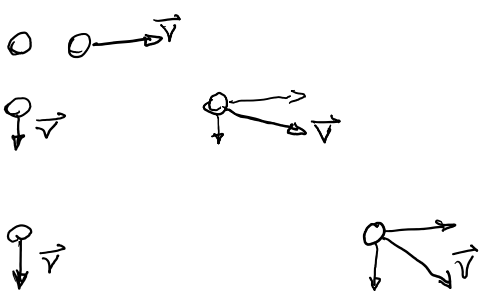
pythagorean theorem



$$z^2 = x^2 + y^2$$

$$\sqrt{8^2 + 6^2} = 10$$

- Consider throwing a ball horizontally.



- both fall vertically with the same acceleration.
- thrown ball moves horizontally with constant speed.

MOTION IN 2D CAN BE SPLIT UP INTO

- thrown ball moves horizontally with constant speed.

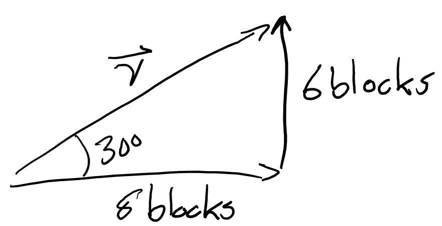
MOTION IN 2D CAN BE SPLIT UP INTO INDEPENDANT MOTION IN 2 PERPENDICULAR DIRECTIONS!

- ~~you can~~ pick the directions
  usually its easiest if one of the directions is the direction of acceleration (down in this case)

3.2 manipulating vectors graphically

notation: vectors will be written with an arrow above them $\vec{v}$ (in print, sometimes vectors are in <u>bold</u>)

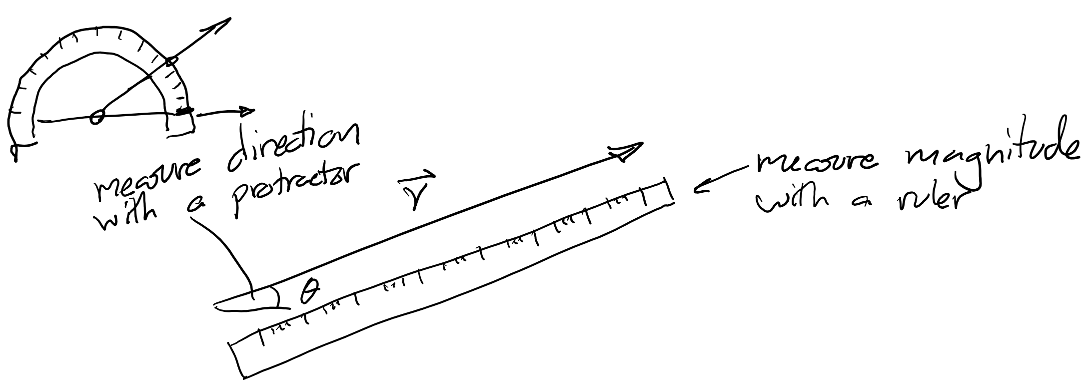
- their magnitude is given by the symbol without the arrow, v
- their direction is given by an angle from a reference direction (usually horizontal or east) and notated with a greek letter (usually θ)



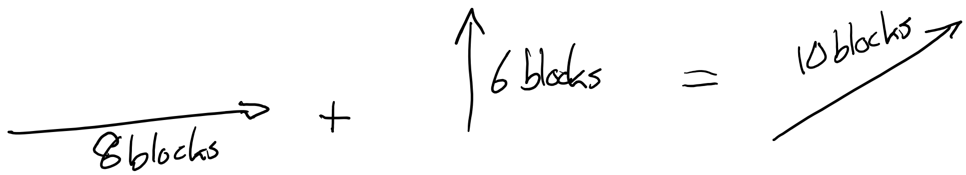
$\vec{v}$ = 10 blocks at 30° N of E

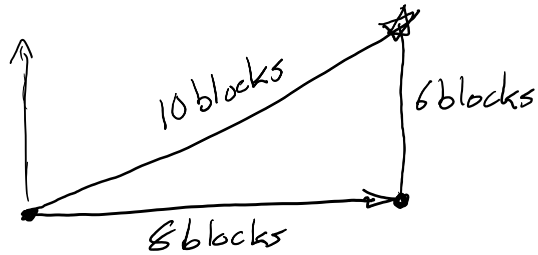
v = 10 blocks

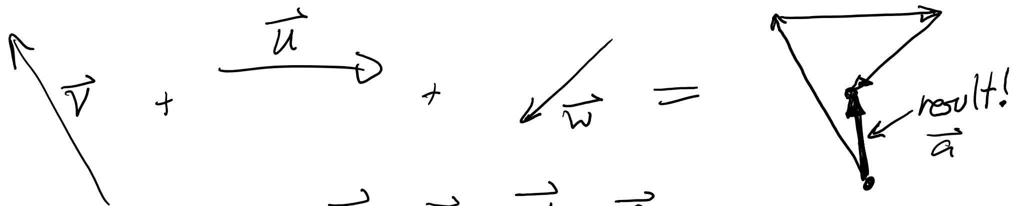
$\theta = 30°$



- Add two Vectors







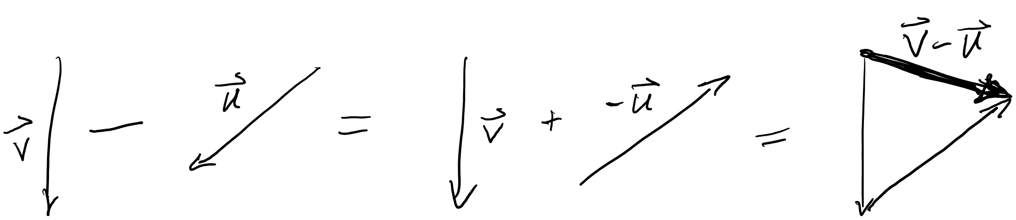
$$\vec{v} + \vec{u} + \vec{w} = \vec{a}$$

<u>subtract two vectors</u>

negative of a vector = vector of the same magnitude that is in the opposite direction

subtract two vectors by adding the negative

$$\vec{v} - \vec{u} = \vec{v} + (-\vec{u})$$



multiply (or divide) a vector by a scalar.

- scale the magnitude, keeping direction the same
  (if scalar is negative, flip scaled vector)

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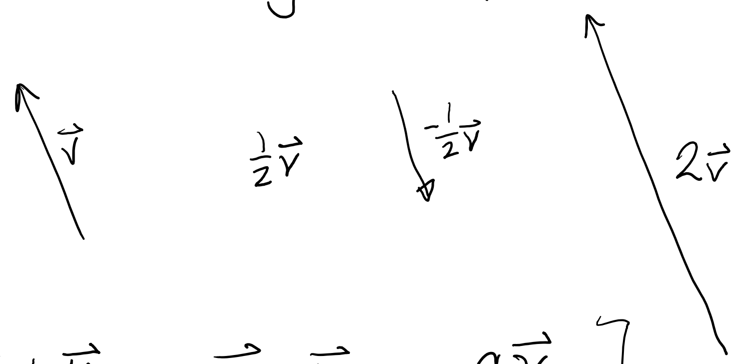
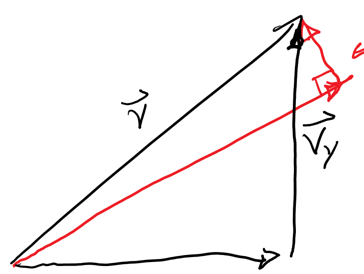
(if scalar is negative, flip scaled vector)



[$\vec{v} + \vec{u}$, $\vec{v} - \vec{u}$, $c\vec{v}$]

<u>Components</u> - split a vector into 2 perpendicular vectors



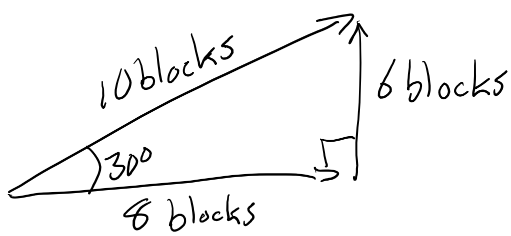
$$\vec{v} = \vec{v}_x + \vec{v}_y$$

usually use x- & y-directions as horizontal & vertical
or east & north.

<u>3.3</u> vectors analytically

specify a vector (in 2D) with 2 numbers

either: DIRECTION & MAGNITUDE
OR x- & y- COMPONENTS



a) 10 blocks at 30° N of E
b) 8 blocks E & 6 blocks N

either give v and θ or v_x and v_y

either give v and θ or v_x and v_y

magnitude direction components

you can convert between these representations.

$v_x = v\cos\theta \qquad v = \sqrt{v_x^2 + v_y^2}$

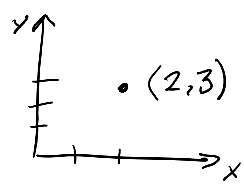
$v_y = v\sin\theta \qquad \theta = \tan^{-1}(v_y/v_x)$

ASIDE

$$\left[\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \theta = \tan^{-1} x,\ x = \tan\theta\right]$$

analytic methods are usually done with components

notations: ordered pair (8 blocks, 6 blocks)



Using unit vectors $\hat{v}$

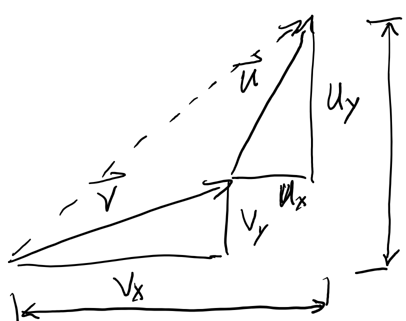
↳ a vector of length 1

specify a direction (without caring about magnitude)

$\vec{v} = 8 \text{ blocks } \hat{\imath} + 6 \text{ blocks } \hat{\jmath} = 8 \text{ blocks } \hat{x} + 6 \text{ blocks } \hat{y}$

x-direction y-direction

$\boxed{\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath}}$ ← most common notation



$\vec{v} + \vec{u}$ has components $v_x + u_x$ and $v_y + u_y$

add two vectors by adding their components.

$\vec{w} = \vec{v} + \vec{u}$ means $w_x = v_x + u_x$

$$W_y = V_y + U_y$$

negate a vector by negating its components.

$\vec{W} = -\vec{V}$ means $W_x = -V_x$

$W_y = -V_y$

subtraction is still adding negative $\vec{V} - \vec{U} = \vec{V} + (-\vec{U})$

(subtract the components)

multiply or divide by a scalar by scaling components

$\vec{W} = C\vec{V}$ mean $W_x = C\,V_x$

$W_y = C\,V_y$

$$(5m\,\hat{\imath} + 4m\,\hat{\jmath}) + (3m\,\hat{\imath} - 6m\,\hat{\jmath}) = \boxed{8m\,\hat{\imath} - 2m\,\hat{\jmath}}$$

$$(8\,m/s\,\hat{\imath} - 2\,m/s\,\hat{\jmath}) - (2\,m/s\,\hat{\imath} + 1\,m/s\,\hat{\jmath}) = \boxed{6\,m/s\,\hat{\imath} - 3\,m/s\,\hat{\jmath}}$$

$$(4\,m/s\,\hat{\imath} + 3\,m/s\,\hat{\jmath})(6\,s) = \boxed{24m\,\hat{\imath} + 18m\,\hat{\jmath}}$$

$(4\,m/\cancel{s})(6\,\cancel{s}) = 24m$ $\qquad \Delta x = v\Delta t$

3.4 projectile motion - motion of a object that has been thrown into the air.

(only affected by gravity, for now neglect air resistance)

(small, heavy, slow)

notation: $\vec{S}$ total displacement

x, y are its components (S_x, S_y)

Recall: 1D equations

$$x = x_0 + \bar{v}t \qquad \bar{v} = \frac{v + v_0}{2} \qquad v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v^2 = v_0^2 + 2a(x - x_0)$$

$$\vec{S} = \vec{S_0} + \left(\frac{\vec{V} + \vec{V_0}}{2}\right)t \qquad \vec{V} = \vec{V_0} + \vec{a}t$$

$$\boxed{\vec{s} = \vec{s}_0 + \left(\frac{\vec{v} + \vec{v}_0}{2}\right)t \qquad \vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{s} = \vec{s}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2}$$

- treat horizontal + vertical motion independently
  i.e. two separate 1D motions, with the same time
- only gravity: acceleration is $\vec{a} = 0\,\mathrm{m/s^2}\,\hat{\imath} - g\,\hat{\jmath}$

$$\boxed{\vec{a} = -9.81\,\mathrm{m/s^2}\,\hat{\jmath}}$$

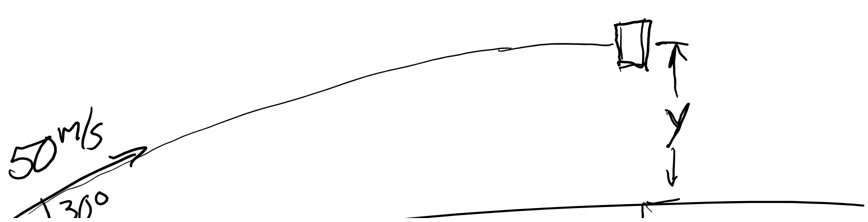
Horizontal: $a_x = 0\,\mathrm{m/s^2} \Rightarrow v_x = v_{0,x} = \text{constant}$

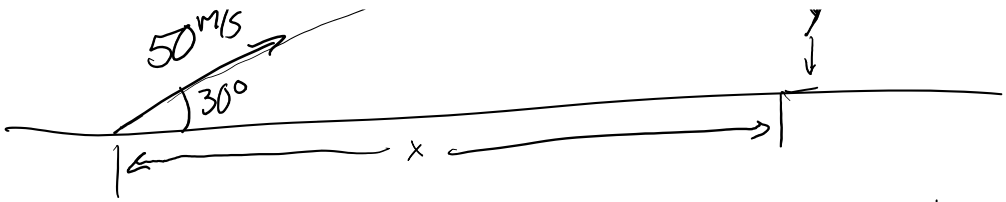
$$\boxed{x = x_0 + v_x t}$$

Vertical: $a_y = -g = -9.81\,\mathrm{m/s^2}$

$$\boxed{\begin{aligned} y &= y_0 + \frac{v_y + v_{0,y}}{2}t & v_y &= v_{0,y} - gt \\ y &= y_0 + v_{0,y}t - \tfrac{1}{2}gt^2 & v_y^2 &= v_{0,y}^2 - 2g(y - y_0) \end{aligned}}$$

a ball is launched with an initial speed of 50 m/s at an angle of 30° above the horizontal. It strikes a target above the ground 3s later. What are the x and y positions of this target.





$$v_{0,x} = v_0 \cos\theta = 50\,\text{m/s} \cos 30^\circ = 43.3\,\text{m/s}$$

$$v_{0,y} = v_0 \sin\theta = 50\,\text{m/s} \sin 30^\circ = 25\,\text{m/s}$$

$t = 3\,\text{s}$ $\quad x = ?$ $\quad y = ?$ $\quad x_0 = \cancel{y_0} = 0\,\text{m}$

$$x = x_0 + v_{0,x} t = 0\,\text{m} + (43.3\,\text{m/s})(3\,\text{s}) = \boxed{129.9\,\text{m}}$$

$$y = y_0 + v_{0,y} t - \frac{1}{2} g t^2 = 0\,\text{m} + (25\,\text{m/s})(3\,\text{s}) - \frac{1}{2}(9.81\,\text{m/s}^2)(3\,\text{s})^2 = \boxed{30.9\,\text{m}}$$

a ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction.

a) at what speed does it hit the ground?

$v_{0,x} = 16\,\text{m/s}$ $\quad v_x = 16\,\text{m/s}$

$v_{0,y} = 12\,\text{m/s}$ $\quad v_y = ?$ $\quad y = y_0 = 0\,\text{m}$

$$v_y^2 = v_{0,y}^2 - \cancelto{0}{2g(y - y_0)}$$

$$v_y^2 = (12\,\text{m/s})^2 - 0 \implies v_y = 12\,\text{m/s} \text{ OR } \boxed{-12\,\text{m/s}}$$

start of motion $\quad$ end of motion

$$v = \sqrt{v_x^2 + v_y^2} = \boxed{20\,\text{m/s}}$$

b) for how long does the ball remain in the air?

$t = ?$ $\quad v_y = v_{0,y} - gt$

$$t = \frac{v_{0,y} - v_y}{g} = \frac{12\,\text{m/s} + (+12\,\text{m/s})}{9.81\,\text{m/s}^2} = \boxed{2.4\,\text{s}}$$

$$t = \frac{v_{0,y} - v_y}{g} = \frac{12\,m/s - (-12\,m/s)}{9.81\,m/s^2} = \boxed{2.4\,s}$$

c) what is the maximum height obtained by the ball?

$v_{0,y} = 12\,m/s \qquad y_0 = 0\,m \qquad v_y = 0\,m/s \qquad y = ?$

$$\cancel{v_y^2}^{\,0} = v_{0,y}^2 - 2g(y - \cancel{y_0}^{\,0})$$

$$v_{0,y}^2 = 2gy \implies y = \frac{v_{0,y}^2}{2g} = \frac{(12\,m/s)^2}{2(9.81\,m/s^2)} = \boxed{7.34\,m}$$

MAXIMUM HEIGHT

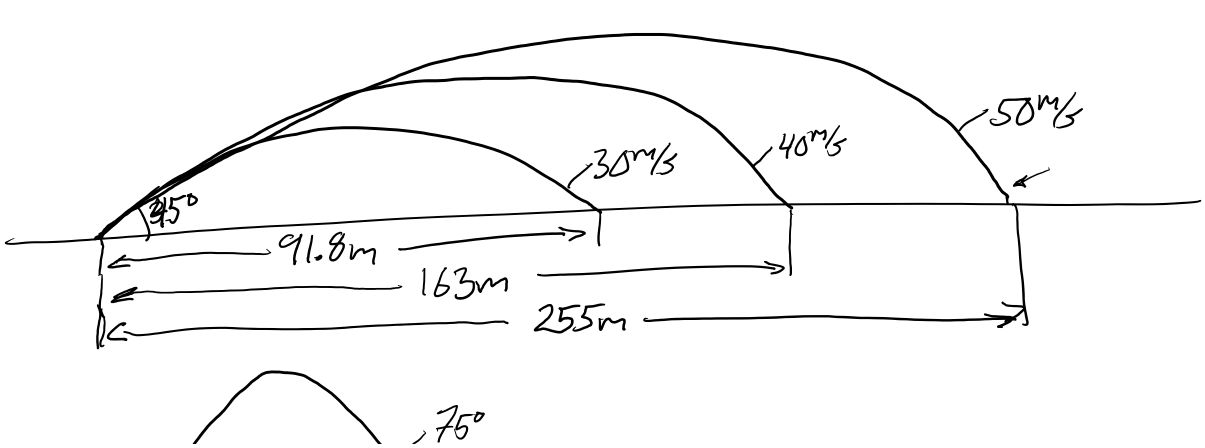
$$\boxed{h = \frac{v_{0,y}^2}{2g}}$$

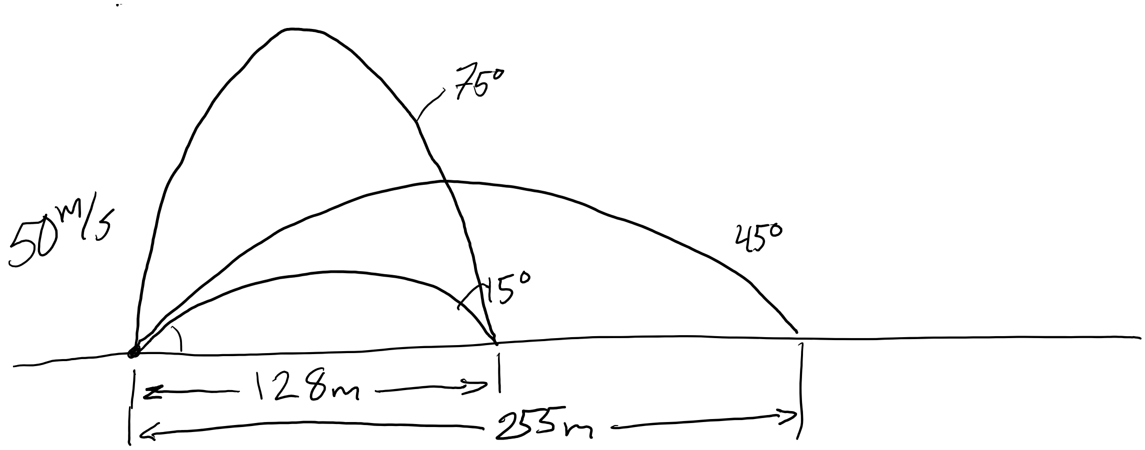
RANGE (how far horizontally did it go?)

$$\boxed{R = \frac{v_0^2 \sin 2\theta_0}{g}}$$ ← angle of initial velocity

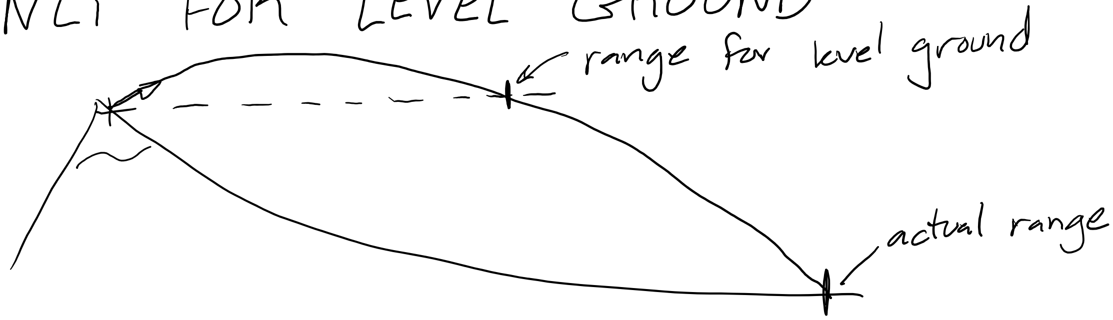
(on level ground)

- first find time from vertical motion
- then find range from horizontal motion

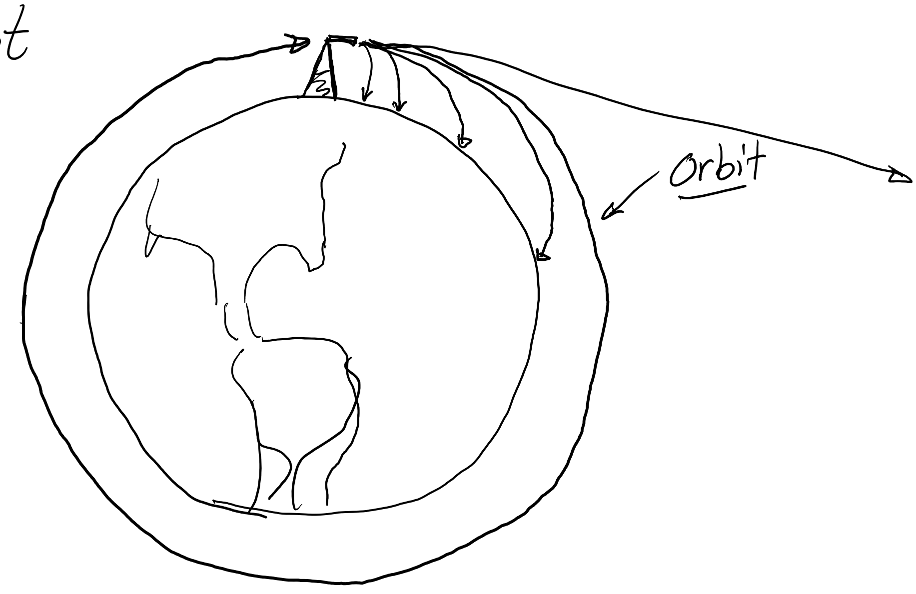




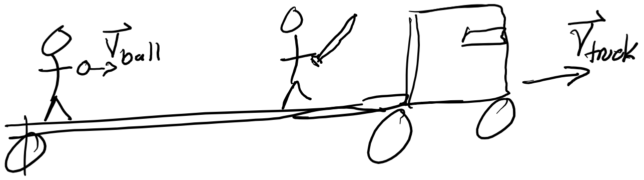
ONLY FOR LEVEL GROUND

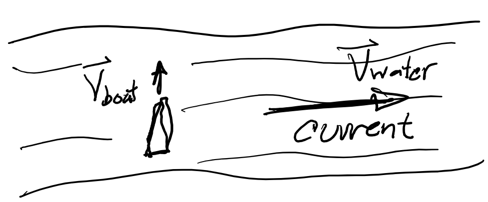


thought experiment



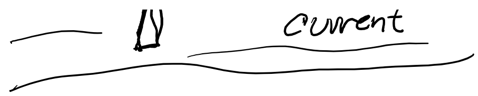
3.5 | relative velocity (addition of velocity)



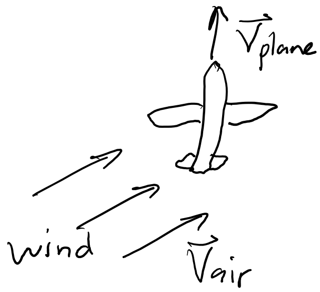


throw ball from a moving

throw ball from a moving truck



row a boat across a fast river

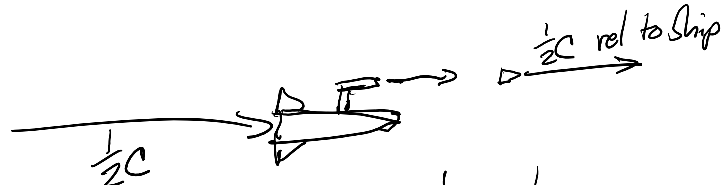


wind pushing an airplane

- velocity given relative to something moving (medium).
- velocity of medium relatave to ground.
- want velocity of object relative to ground.

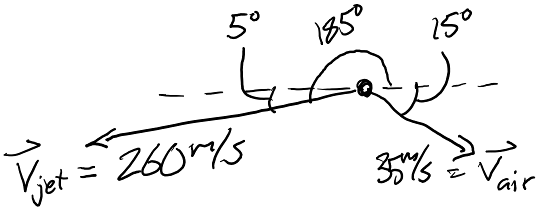
$$\vec{V}_{tot} = \vec{V}_{boat} + \vec{V}_{water}$$
$$= \vec{V}_{plane} + \vec{V}_{air}$$
$$= \vec{V}_{ball} + \vec{V}_{truck}$$

- just add like a normal vector
- this only works in the classical realm i.e. "slow" speeds



bullet <u>not</u> going at $\frac{1}{2}c + \frac{1}{2}c = c$

a jet has an airspeed of 260 m/s in a direction 5° S of W. it is in the jet stream, blowing at 35 m/s in a direction 15° S of E. What is the velocity of the jet relative to the earth?

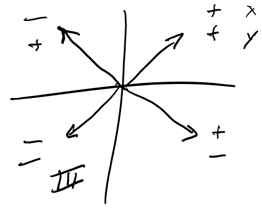


$$\vec{V}_{tot} = \vec{V}_{jet} + \vec{V}_{air}$$

$V_x = V\cos\theta$ ← from E

$$V_{j,x} = -260 \text{ m/s} \cos 5° = (260 \text{ m/s}) \cos 185° = -259 \text{ m/s}$$

$$v_{j,y} = 260\,\text{m/s}\ \sin 185^\circ = -22.7\,\text{m/s}$$



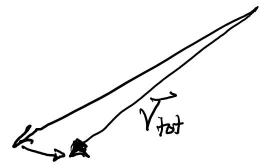
$$v_{a,x} = 35\,\text{m/s}\ \cos(-15^\circ) = 33.8\,\text{m/s}$$

$$v_{a,y} = 35\,\text{m/s}\ \sin(-15^\circ) = -9.06\,\text{m/s}$$

$$\boxed{\begin{aligned} v_{tot,x} &= -259\,\text{m/s} + 33.8\,\text{m/s} = -225\,\text{m/s} \\ v_{tot,y} &= -22.7\,\text{m/s} + (-9.06\,\text{m/s}) = -31.8\,\text{m/s} \end{aligned}}$$

$$v_{tot} = \sqrt{(-225\,\text{m/s})^2 + (-31.8\,\text{m/s})^2} = \boxed{227\,\text{m/s}}$$

$$\theta = \tan^{-1}\left(\frac{-31.8\,\text{m/s}}{-225\,\text{m/s}}\right) = \boxed{8^\circ \text{ S of W}}$$



$$\boxed{227\,\text{m/s at } 8^\circ \text{ S of W}}$$ ✓