3D Rotation

Tuesday, July 13, 2021 12:37

HW 6: due Thursday

Review: Rotational Motion

translational	rotational
displacement $\vec{x}$	angle $\theta = \frac{x_t}{r}$
velocity $\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$	angular velocity $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v_t}{r}$
acceleration $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$	angular acceleration $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{a_t}{r}$
$x = \bar{v} t$	$\theta = \bar{\omega} t$
$\bar{v} = \frac{v_0 + v}{2}$	$\bar{\omega} = \frac{\omega_0 + \omega}{2}$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = v_0 t + \frac{1}{2} a t^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
force $\vec{F}$	torque $\tau = rF\underbrace{\sin\theta}_{\text{assume } \theta = 90^\circ}$
mass m (gives inertia)	moment of inertia $I = \underbrace{mr^2}_{\text{for a point mass}}$
$\vec{F} = m\vec{a}$	$\tau = I\alpha$

work-energy theorem

work-energy theorem

$$\text{net } W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$
$$= \Delta KE$$

$$KE = \frac{1}{2}mv^2$$

$$W = Fd\underbrace{\cos\theta}_{\text{assume } \theta = 0^\circ}$$

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2$$
$$= \Delta KE$$

$$KE = \frac{1}{2}I\omega^2$$

$$W = \tau\theta$$ (assuming the F creating the torque is perp. to the radius)

momentum $\vec{p}$

$$\vec{p} = m\vec{v}$$

$$\text{net } \vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

conserved if net $\vec{F} = 0$

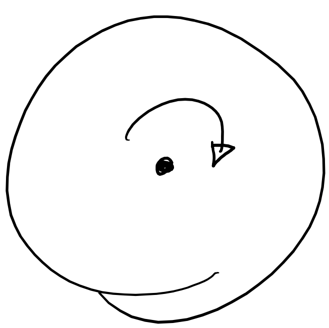
angular momentum L

$$L = I\omega$$

$$\text{net } \tau = \frac{\Delta L}{\Delta t}$$

conserved if net $\tau = 0$

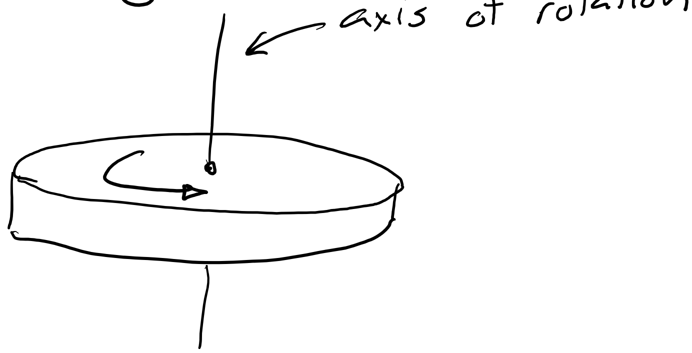
so far we've been working in 2 dimensions.
- the axis of rotation is out/into the "page"

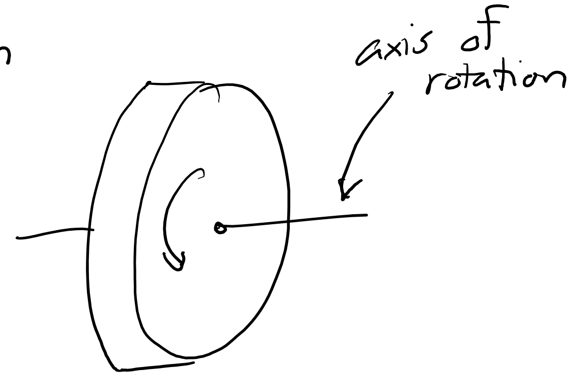


- rotation is either clockwise (−) or counterclockwise (+)

- in 3 dimensions, the axis of rotation could be anything.

- in 3 dimensions, the axis of ro...
anything.



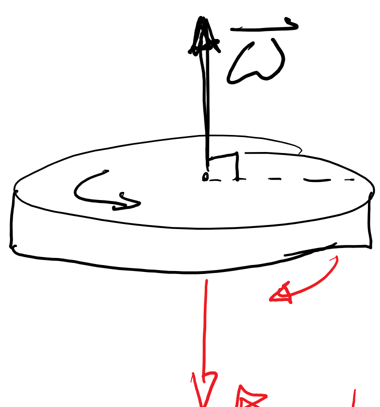


- all of the rotational quantaties defined before have both a magnitude <u>and</u> a direction.

THEY <u>ARE</u> VECTORS

- in 3d, its not enough to say cw or ccw we need to know what the axis of rotation is.

we represent these quantaties as a vector (arrow) directed along the axis of rotation.



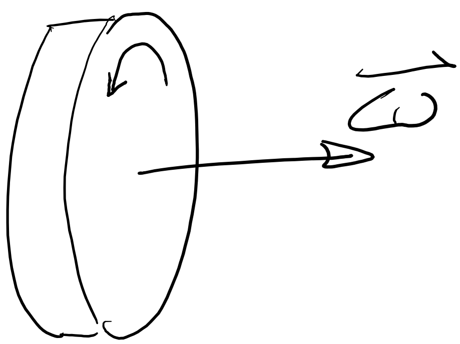
- the length (magnitude) of the arrow says how fast it is rotating
- the direction specifies the axis of rotation.

why doesn't this arrow work
it gives the same axis of rotation
(it represents motion in the <u>other</u> direction)

WHICH ARROW IS RIGHT?

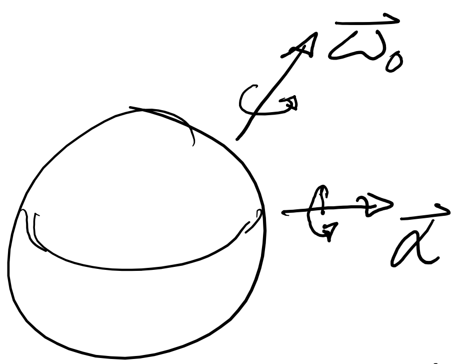
use the RIGHT HAND RULE

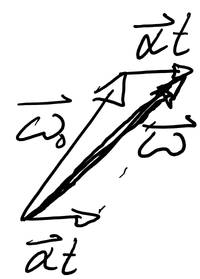
- take your <u>right hand</u> and point your thumb in the direction of the arrow
- then curl your fingers, the way they curl is the direction of rotation



you can do this "backwards" to find the direction of the arrow.

adding vectors works the same way as for translational vectors.





angular acceleration around a different axis.

$$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha} t$$

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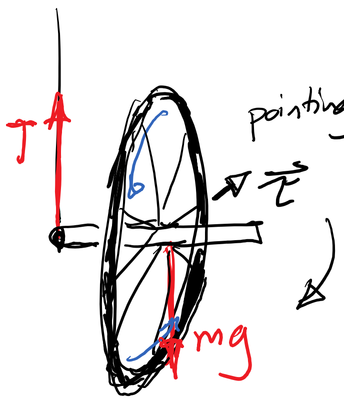
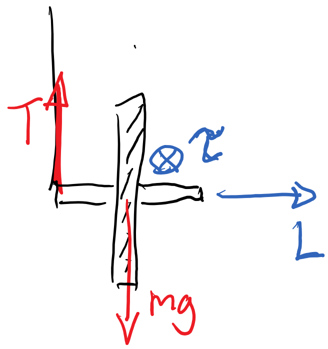
a surprising consequence - the <u>gyroscopic</u> effect

- setup - a spinning bicycle wheel suspended by one end of its axle pointing into page



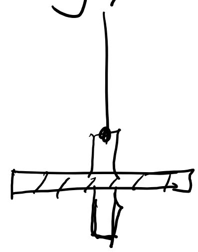
torque wants to rotate it like this

side view



if not spinning, the wheel fall

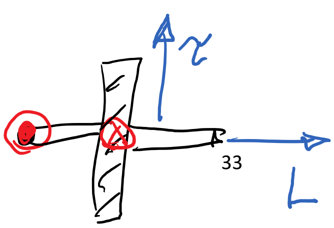
picture afterwards.

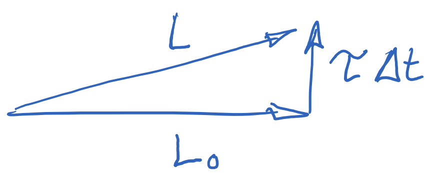


if the wheel <u>is</u> spinning it has <u>angular momentum</u>.

$\vec{\tau} = \dfrac{\Delta \vec{L}}{\Delta t}$, i.e. angular momentum will change <u>in the direction</u> of the torque.

top view



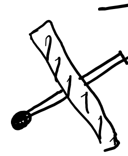


<u>still horizontal</u> just in a different direction

i.e. afterwards

side

top



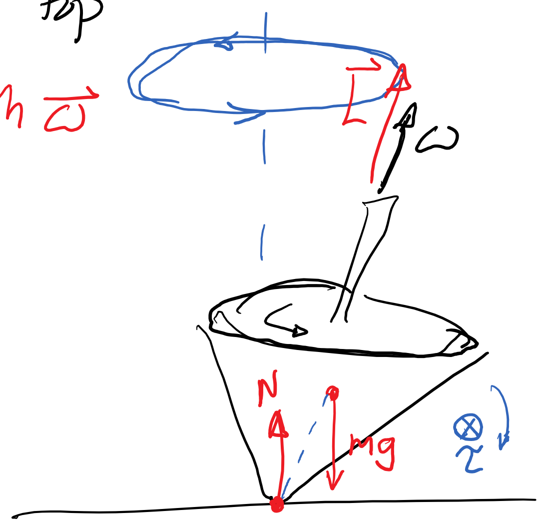
IT ROTATED AROUND THE STRING INSTEAD OF <u>FALLING</u>.

INSTEAD OF FALLING.

e.g. a spinning top

$\vec{L}$ is aligned with $\vec{\omega}$

$\vec{\tau}$ is horizontal



instead of falling over, the motion "precesses"

the axis of rotation will rotate