

**Directions**

1. Do NOT open this exam booklet until you are instructed to do so!
2. You may use a TI-84/85 (or equivalent) calculator. Any other electronic devices or outside materials are not permitted.
3. This exam is 7 pages (including this cover page) and has 6 questions. Check that you have every page of the exam before handing it in.
4. Please write your answers in the space provided. If you need more space, continue on the back of a page (being sure to clearly label your work). Do not write any answers on scrap paper.
5. Work must be clearly written and organized. Please organize your work and write legibly! Circle your final answers.
6. If you have a question, please raise your hand.

**Good luck!**

Do not write in the tables or on the line below.

Question	Points	Score
1	20	
2	15	
3	20	
4	10	
5	15	
6	20	
Total:	100	

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1. (20 points) A farmer wishes to construct a feeding trough with a triangular cross section for his pigs. The distance across the top of the trough is twice the height of the trough. The trough should have a volume of 20 cubic feet. If lumber costs \$2.50 per square foot, what are the dimensions of the trough that will minimize the cost. (The surface area of such a trough is  $SA = 2\sqrt{2}\ell h + 2h^2$ , where  $\ell$  is the length and  $h$  is the height.)

2. (15 points) Let  $f(x) = x^3 - 3x + 1$  be defined on the closed interval  $-3 \leq x \leq 4$ .

(a) What is the absolute maximum of  $f(x)$  and at what point  $x$  does it occur?

(b) What is the absolute minimum of  $f(x)$  and at what point  $x$  does it occur?

(c) Are there any relative maximums or minimums which are not absolute extrema?  
If so, at what value of  $x$  do they occur?

3. (20 points) Two cars start driving away from a house at the same time, one traveling north at 30 miles per hour and one traveling east at 40 miles per hour. At what rate is the distance between them changing after they have been traveling for 2 hours?

4. (10 points) Consider the curve defined by  $3x^2y + \ln y + 1 = \sin x$ .

(a) What is  $dy/dx$ ?

(b) Find the equation of a line tangent to this curve at the point  $(0, 1/e)$ . c cv

5. (15 points) Let  $f(x, y, z) = 3x^2y + \sin(xz) - 2yz^2$ .

(a) Find  $f(1, 2, 3)$ .

(b) Find  $f_x = \frac{\partial f}{\partial x}$ .

(c) Find  $f_z = \frac{\partial f}{\partial z}$ .

(d) Find  $f_{xy} = \frac{\partial^2 f}{\partial y \partial x}$ .

(e) Find  $f_{zz} = \frac{\partial^2 f}{\partial z^2}$ .

6. (20 points) Let  $f(x, y) = x^3 - 3x^2y + \frac{3}{2}y^2$ . We find

$$\begin{aligned} f_x(x, y) &= 3x^2 - 6xy, & f_y(x, y) &= -3x^2 + 3y \\ f_{xx}(x, y) &= 6x - 6y, & f_{xy}(x, y) &= -6x, & f_{yy}(x, y) &= 3. \end{aligned}$$

(a) Where are the critical points of  $f$ ?

(b) What is the discriminant  $D$  of the function.

(c) For each critical point, is it a relative maximum, relative minimum, saddle point, or are you unable to tell. Explain your reasoning.