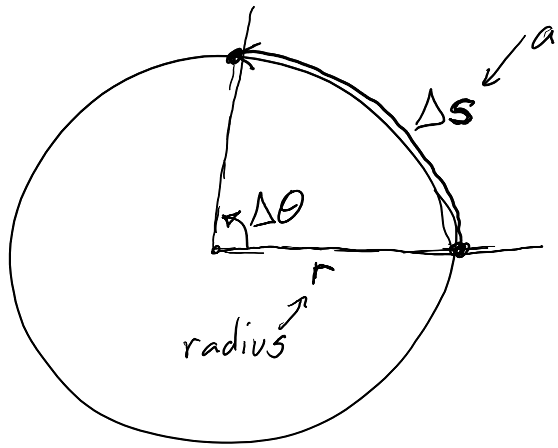
Chapter 6: Circular Motion & Gravity

Tuesday, June 22, 2021 12:58

<u>Uniform circular motion</u> - motion in a circle at constant speed.

<u>6.1</u> <u>rotation angle</u> - angle you have rotated through ($\Delta\theta$)



$$\Delta\theta = \frac{\Delta s}{r}$$

measured in radians (NOT degrees)

$180° = \pi$ radians

$360° = 2\pi$ radians $= 1$ rotation

$30° = \frac{\pi}{6}$ $\quad 45° = \frac{\pi}{4}$ $\quad 60° = \frac{\pi}{3}$ $\quad 90° = \frac{\pi}{2}$

$120° = \frac{2\pi}{3}$ $\quad 135° = \frac{3\pi}{4}$ $\quad 180° = \pi$

<u>angular velocity</u> - rate of change of angle.

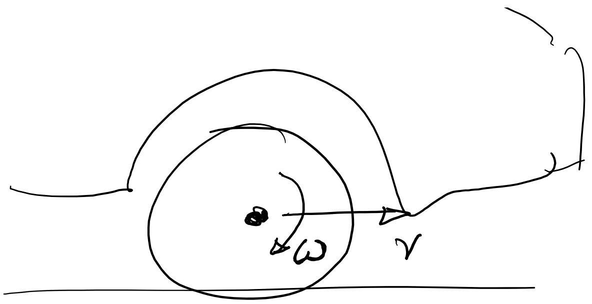
angular velocity $\rightarrow \omega = \frac{\Delta\theta}{\Delta t}$ ← change in angle (numerator), ← change in time (denominator)

(compare linear speed: $v = \frac{\Delta s}{\Delta t}$)

v: tangential speed

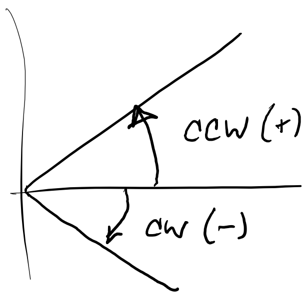
$$\boxed{v = r\omega} \qquad \boxed{\omega = \frac{v}{r}}$$

- it <u>does</u> have a direction



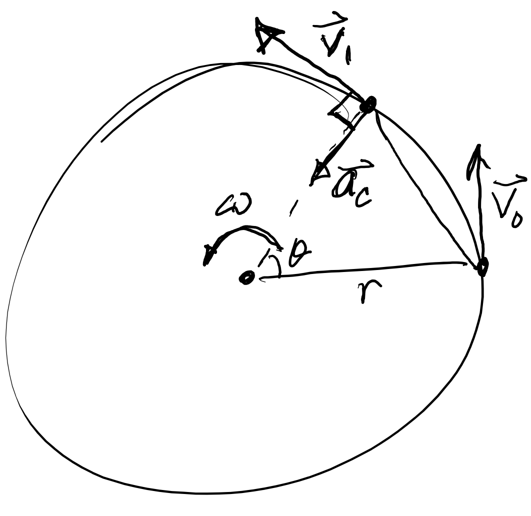
- it does have a direction
- Clockwise or counterclockwise about it axis

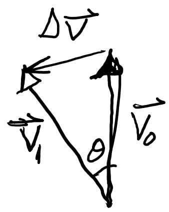
(CCW = +, CW = −)



6.2 centripetal acceleration (NOT CENTRIFUGAL)

- acceleration causing an object to have uniform circular motion.





- points towards center of rotation

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

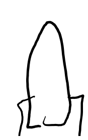
$$a_c = \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t} = \frac{v^2}{r}$$

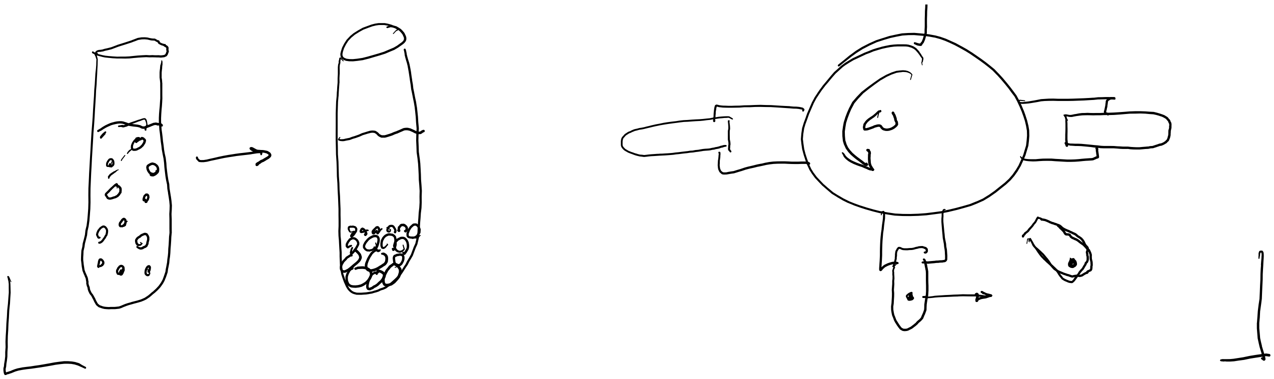
$$\boxed{a_c = v^2/r}$$

$$\boxed{a_c = r\omega^2}$$

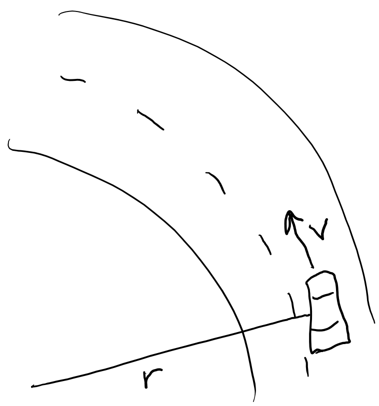
angular velocity

[Centrifuge - a device used to separate by density by rotating very fast.]





e.g. – how big is the centripetal acceleration of a turning car?



$r = 500\,m$

$v = 25\,m/s$

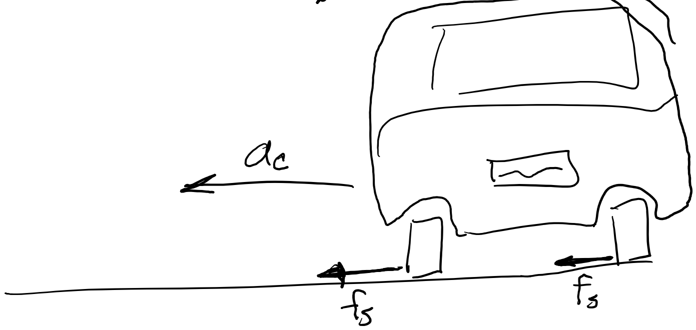
$$a_c = \frac{v^2}{r} = \frac{(25\,m/s)^2}{500\,m} = \boxed{1.25\,m/s^2}$$

6.3) centripetal force – any force(s) that cause centripetal acceleration.

$$F_c = m a_c = m\frac{v^2}{r} = m r \omega^2$$

(pointed towards center)

e.g. how much friction between the wheel + the road is needed?



$m = 900\,kg$

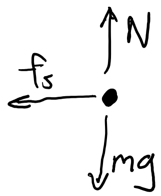
$r = 500\,m$

$v = 25\,m/s$

$$F_c = m\frac{v^2}{r} = (900\,kg)\frac{(25\,m/s)^2}{(500\,m)} = \boxed{1125\,N}$$

e.g. what is the smallest possible coefficient of static friction between the wheels + the road.

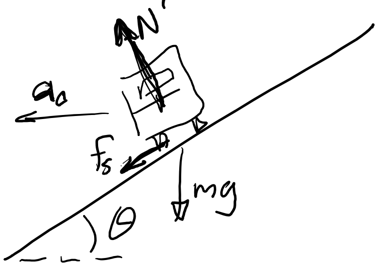
$$F_c = f_s = \mu_s N = \mu_s mg$$



$$N - mg = 0$$

$$\mu_s = \frac{F_c}{mg} = \boxed{0.127}$$

<u>banked curve</u> - tilt the curved road so less friction is necessary.



<u>ideally banked curve</u> - no friction necessary

$$N\cos\theta = mg$$
$$N\sin\theta = F_c$$

e.g. what is the ideal banking angle for a 900kg car taking a 500m radius curve at 25.0m/s ?

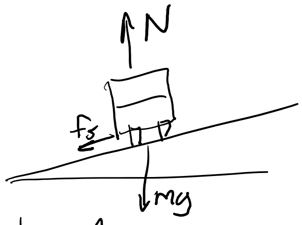
$$N\cos\theta = mg \qquad ma_c = F_c = N\sin\theta = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta$$

$$N = \frac{mg}{\cos\theta} \qquad a_c = g\tan\theta \Rightarrow \tan\theta = \frac{a_c}{g}$$

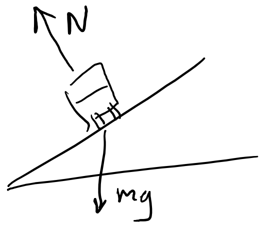
$$\boxed{\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad \tan\theta = \frac{r\omega^2}{g}}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{(25\,\mathrm{m/s})^2}{(500\,\mathrm{m})(9.8\,\mathrm{m/s^2})}\right) = \boxed{7.27^\circ = 0.126\text{ radians}}$$

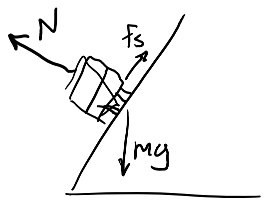
(if too steep: friction must be other direction)



too shallow



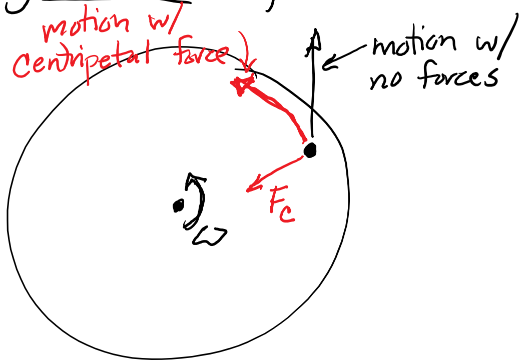
ideal



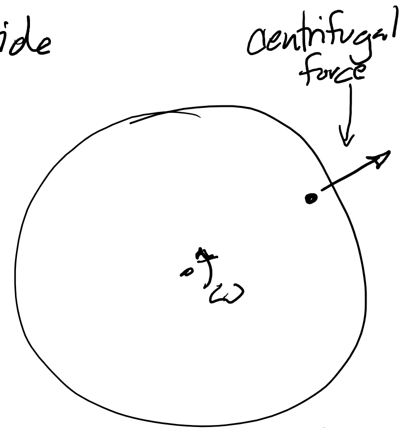
too steep

6.4) fictitious forces - "Forces" that aren't really there (due to inertia in non-inertial (accelerating) reference frames)

centrifugal force - push towards outside

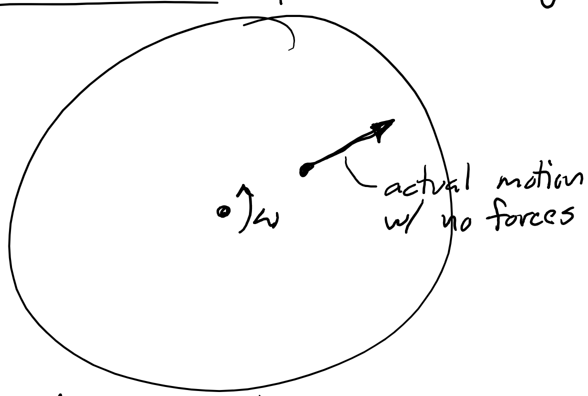


viewed from stationary frame

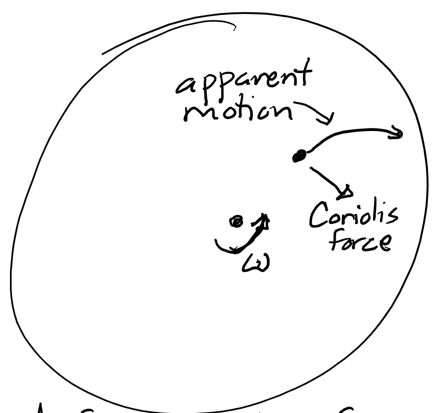


viewed from rotating frame

Coriolis force - push to the right (in ccw rotation)



viewed from stationary frame



viewed from rotating frame

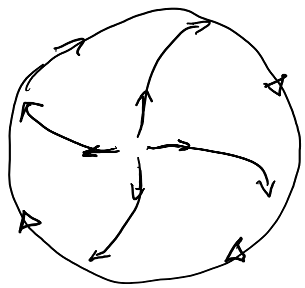
- we've been considering Earth to be stationary but it is actually rotating. You can see the Coriolis effect.

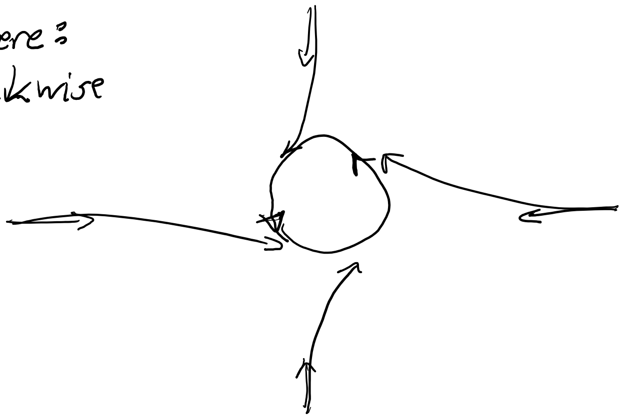
in the Northern hemisphere:

- hurricanes always rotate counterclockwise

- flushing your toilet will swirl counterclockwise

in the Southern hemisphere:
- rotation is clockwise

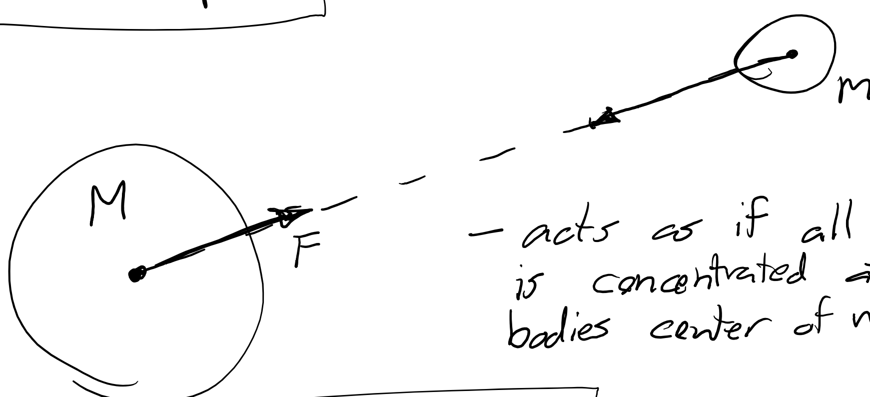




6.3 Gravity

Newton's Law of Universal Gravitation

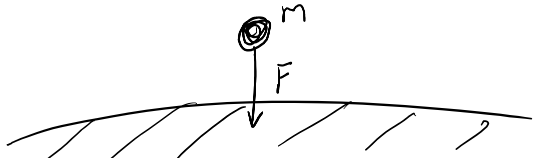
$$\boxed{F = G\frac{mM}{r^2}}$$ in direction towards center of other body.



- acts as if all mass is concentrated at the bodies center of mass.

$$\boxed{G = 6.674 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}}$$ ← Gravitational constant

gravity of earth

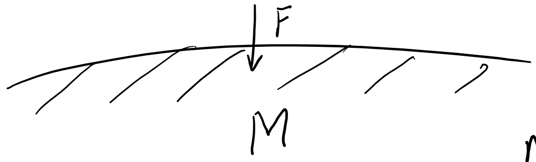


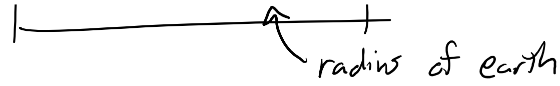
$$mg = F = G\frac{Mm}{r^2}$$

$$\boxed{g = \frac{GM}{r^2}}$$

mass of earth

radius of earth





$$M_\oplus = 5.98 \times 10^{24}\ \text{kg}$$
$$R_\oplus = 6.38 \times 10^{6}\ \text{m}$$

why don't we think about the force you exert on the Earth? acceleration of Earth is very small

$$Ma = F = G\frac{Mm}{r^2} \Rightarrow a = \frac{Gm}{r^2}$$

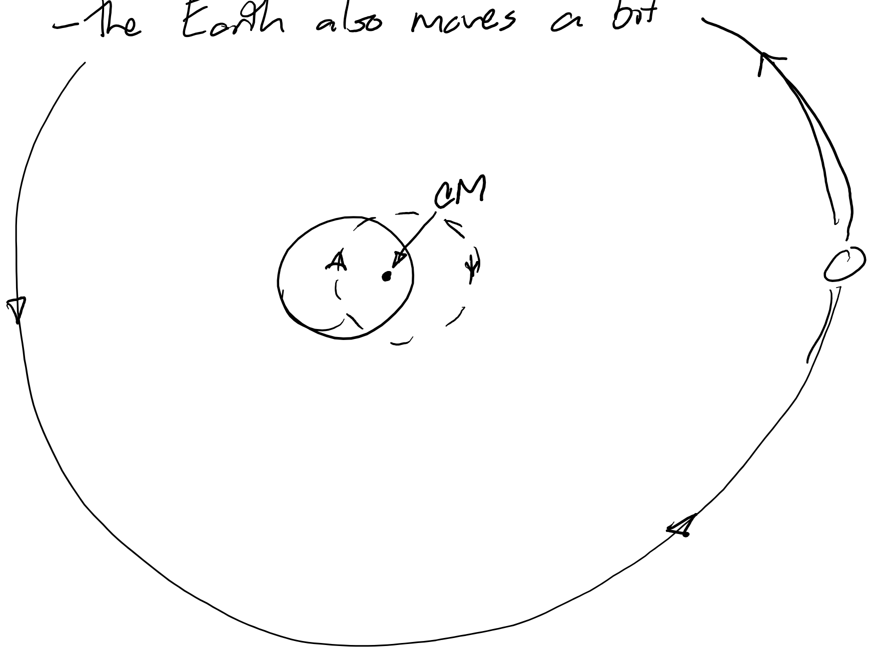
How does gravity relate to uniform circular motion?

- necessary centripetal ~~force~~ force for the moon to go in a circular orbit with its average radius is <u>very close</u> to the force of gravity between the Earth + the moon.

$$a = \frac{GM_\oplus}{r^2} \approx \underline{a_c = \frac{v^2}{r}}$$

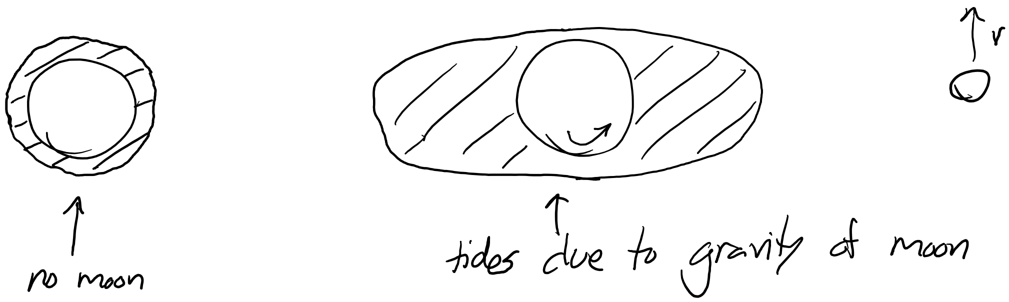
not exact:

- the orbit is actually an ellipse
- the Earth also moves a bit

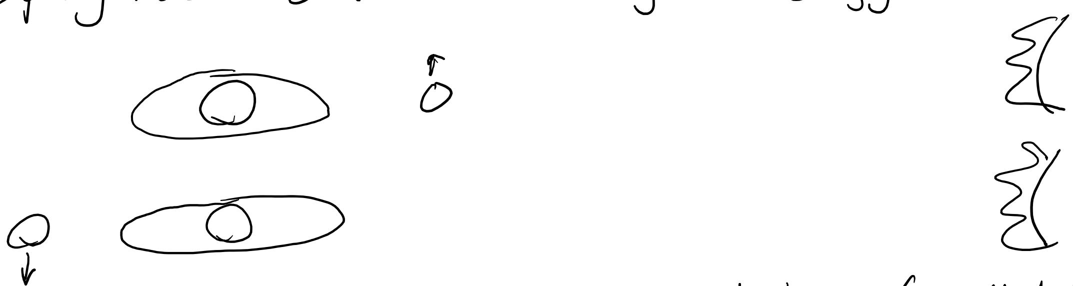


Note: this wiggle in the sun is evidence for the existence of exoplanets! (other stars wiggle like this too!)

tides – the moon pulls on the Earth + its oceans



spring tide: sun + moon aligned (biggest tide)



neap tide: sun + moon are perpendicular (smallest tide)

"weightless ness" – in free-fall
(there is still a gravitational force + thus weight)

"microgravity" – apparent net acceleration due ~~to~~ gravity is small
(the actual acceleration is still pretty large)

health effects
- muscle atrophy
- lack blood pressure differential

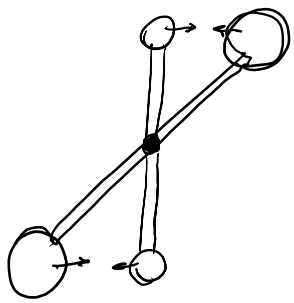
G must be measured experimentally (this is hard)

- we know GM for celestial bodies more accurately then G or M by themselves.

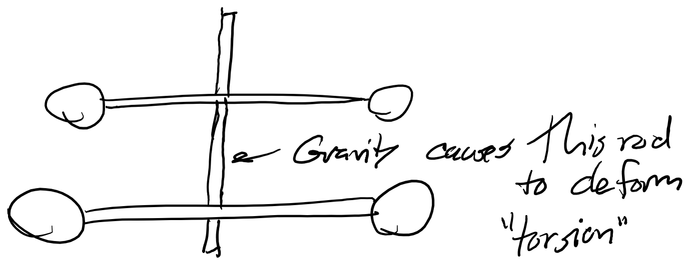
Cavendish experiment (measures G)

- two rods have (large) masses on the ends.
- attach them to another rod + see how much it twists.

top view



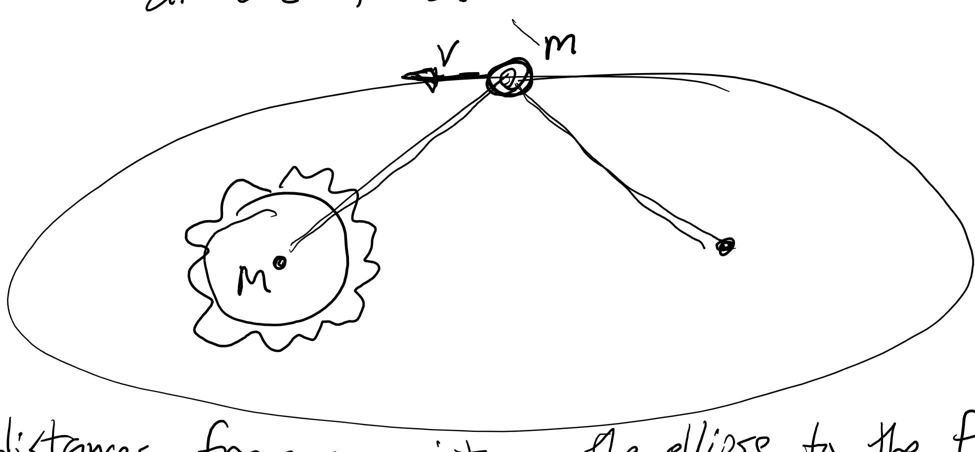
side view



6.6 Kepler's Laws of Planetary Motion

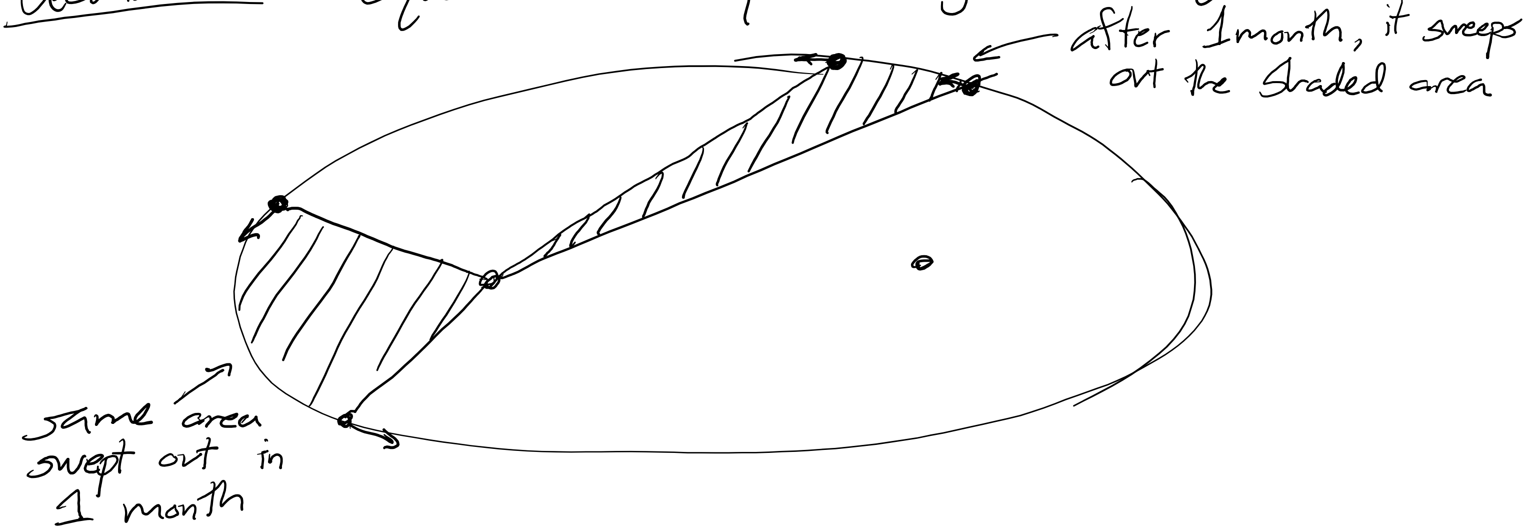
- one object is much smaller then the other (treat larger object as stationary)
- system is isolated from other masses

First Law - orbits are ellipses with the larger mass at one focus.



sum of distances from a point on the ellipse to the foci is constant.

Second Law - equal time sweeps out regions of equal area.



- objects move faster when closer + slower when further away.

Third Law: ratio of squares of periods is the same as the ratio of cubes of average radius for two objects orbiting the same mass.

period $\rightarrow$ $$\frac{T_1^{\,2}}{T_2^{\,2}} = \frac{r_1^{\,3}}{r_2^{\,3}}$$

derivation from Newton:

$$G\frac{mM}{r^2} = m\frac{v^2}{r} \implies v^2 = \frac{GM}{r}$$

speed $$v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{T} \implies \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$\implies \boxed{T^2 = \frac{4\pi^2}{GM} r^3}$$

there is a table of orbital data for various moons + planets

(table 6.2)

e.g. the moon orbits every 27.3 days at average distance of 3.84×10^8 m from the center of the Earth. what is the period of a space station at average altitude of 1500 km above Earth's surface?

$$\frac{T_{moon}^2}{T_{sat}^2} = \frac{r_{moon}^3}{r_{sat}^3} \Rightarrow T_{sat} = T_{moon}\sqrt{\frac{r_{sat}^3}{r_{moon}^3}}$$

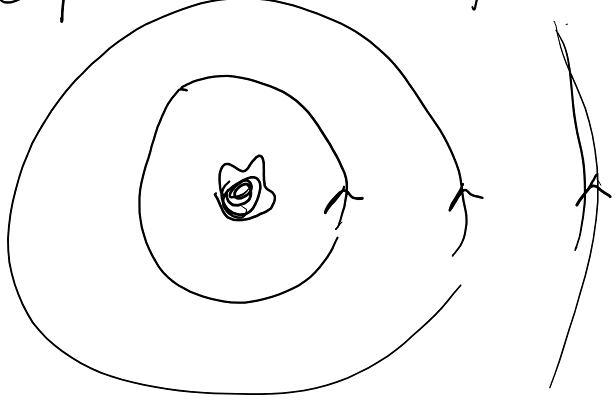
$$r_{sat} = (6.38\times10^6\,m) + (1.5\times10^6\,m) = 7.88\times10^6\,m$$

$$T_{sat} = (27.3\text{ days})\sqrt{\left(\frac{7.88\times10^6\,m}{3.84\times10^8\,m}\right)^3} = 0.08\text{ days}$$
$$= 1.926\text{ hours}$$
$$\boxed{= 115\text{ minutes}}$$

---

orbital models & simplicity

Copernican model - planets orbit the sun



simple (how physics works (mostly))

Ptolemaic model - planets & the sun orbit the earth

complex (epicycles)

Complex