

Homework-06 ENGR 117 Due date 05/09/2022

5 Questions 20 points each

Q-1 Design an op amp-based low-pass filter with a cutoff frequency of 2500 Hz and a passband gain of 5 using a 10 nF capacitor. The input to the low-pass filter is $3.5 \cos \omega t$ V.

- a) Draw your circuit, labeling the component values and output voltage.
- b) If the value of the feedback resistor in the filter is changed but the value of the resistor in the forward path is unchanged, what characteristic of the filter is changed?
- c) Find the output voltage when $\omega = \omega_c$.
- d) Find the output voltage when $\omega = 0.125 \omega_c$.
- e) Find the output voltage when $\omega = 8 \omega_c$.

Q-2 Write the relationship and draw phasor between phase and line quantities for a balanced 3 phase system if:

- a. Star Connection (phasor diagram for phase voltages and Line voltages).
- b. Delta Connection (phasor diagram for phase currents and Line currents).

Q-3 For each set of voltages, state whether or not the voltages form a balanced three-phase set. If the set is balanced, state whether the phase sequence is positive or negative. If the set is not balanced, explain why?

a) $v_a = 48 \cos(314t - 45^\circ) \text{ V},$
 $v_b = 48 \cos(314t - 165^\circ) \text{ V},$
 $v_c = 48 \cos(314t + 75^\circ) \text{ V}.$

b) $v_a = 188 \cos(250t + 60^\circ) \text{ V},$
 $v_b = -188 \cos 250t \text{ V},$
 $v_c = 188 \cos(250t - 60^\circ) \text{ V}.$

c) $v_a = 426 \cos 100t \text{ V},$
 $v_b = 462 \cos(100t + 120^\circ) \text{ V},$
 $v_c = 426 \cos(100t - 120^\circ) \text{ V}.$

d) $v_a = 1121 \cos(2000t - 20^\circ) \text{ V},$
 $v_b = 1121 \sin(2000t - 50^\circ) \text{ V},$
 $v_c = 1121 \cos(2000t + 100^\circ) \text{ V}.$

e) $v_a = 540 \sin 630t \text{ V},$
 $v_b = 540 \cos(630t - 120^\circ) \text{ V},$
 $v_c = 540 \cos(630t + 120^\circ) \text{ V}.$

f) $v_a = 144 \cos(800t + 80^\circ) \text{ V},$
 $v_b = 144 \sin(800t - 70^\circ) \text{ V},$
 $v_c = 144 \sin(800t + 50^\circ) \text{ V}.$

Q-4 A balanced three-phase circuit has the following characteristics.

- Y-Y connected;
- The line voltage at the source, V_{ab} , is $110\sqrt{3} \angle -60^\circ$ V;
- The phase sequence is positive;
- The line impedance is $3 + j2 \Omega/\phi$;
- The load impedance is $37 + j28 \Omega/\phi$;

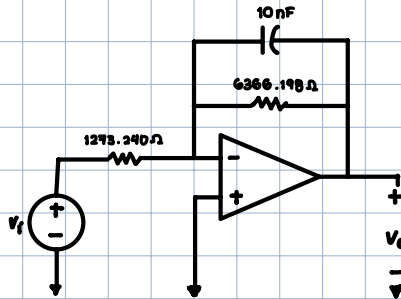
- a) Draw the single-phase equivalent circuit for the a-phase.
- b) Calculate the line currents for each phase.
- c) Calculate the line voltages at the load in each phase.

Q-5 A balanced, three-phase circuit is characterized as follows:

- Y- Δ connected;
- Source voltage in the b-phase is $150 \angle 135^\circ$ V;
- Source phase sequence is acb;
- Line impedance is $2 + j3 \Omega/\phi$;
- Load impedance is $129 + j171 \Omega/\phi$.

- a) Draw the single phase equivalent circuit for the a-phase.
- b) Calculate the a-phase line current.
- c) Calculate the a-phase line voltage for the three-phase load.

Q1 (a)



$$\omega_c = 2\pi f = 2\pi(2500 \text{ Hz}) = 5000\pi \text{ rad/s}$$

$$\omega_c = \frac{1}{R_2 C} \rightarrow R_2 = \frac{1}{(5000\pi)(10 \times 10^{-9})} = 6366.198 \Omega$$

$$K = \frac{R_2}{R_1} = 5 \rightarrow R_1 = \frac{R_2}{5} = \frac{6366.198}{5} = 1273.240 \Omega$$

(b) R_2 changed
 ω_c changed
 K changed

$$(c) |H(j\omega)| = \frac{R_2}{R_1} \cdot \frac{1/R_2 C}{\sqrt{(1/R_2 C)^2 + \omega^2}} \quad \omega = \omega_c = 5000\pi \text{ rad/s}$$

$$|H(j\omega_c)| = \left[\frac{6366.198}{1273.240} \right] \cdot \frac{1/(6366.198)(10 \text{ nF})}{\sqrt{[1/(6366.198)(10 \text{ nF})]^2 + (5000\pi)^2}} = 3.54$$

$$\begin{aligned} \angle H(j\omega_c) &= 180 - \tan^{-1}(\omega R_2 C) \\ &= 180 - \tan^{-1}[(5000\pi)(6366.198)(10 \times 10^{-9})] \\ &= 135^\circ \end{aligned}$$

$$\frac{V_o}{V_i} = 3.54 \angle 135^\circ$$

$$\begin{aligned} V_o &= 3.54 \angle 135^\circ V_i \\ &= (3.54 \angle 135^\circ)(3.5 \cos 5000\pi t) \\ &= 12.39 \cos(5000\pi t + 135^\circ) \text{ V} \end{aligned}$$

$$(d) |H(j\omega)| = \frac{R_2}{R_1} \cdot \frac{1/R_2 C}{\sqrt{(1/R_2 C)^2 + \omega^2}} \quad \omega = 0.125 \omega_c = 0.125(5000\pi) = 625\pi$$

$$|H(j\omega_c)| = \left[\frac{6366.198}{1273.240} \right] \cdot \frac{1/(6366.198)(10 \text{ nF})}{\sqrt{[1/(6366.198)(10 \text{ nF})]^2 + (625\pi)^2}} = 4.96$$

$$\begin{aligned} \angle H(j\omega_c) &= 180 - \tan^{-1}(\omega R_2 C) \\ &= 180 - \tan^{-1}[(625\pi)(6366.198)(10 \times 10^{-9})] \\ &= 172.87^\circ \end{aligned}$$

$$\frac{V_o}{V_i} = 4.96 \angle 172.87^\circ$$

$$\begin{aligned} V_o &= 4.96 \angle 172.87^\circ V_i \\ &= (4.96 \angle 172.87^\circ)(3.5 \cos 625\pi t) \\ &= 17.36 \cos(625\pi t + 172.87^\circ) \text{ V} \end{aligned}$$

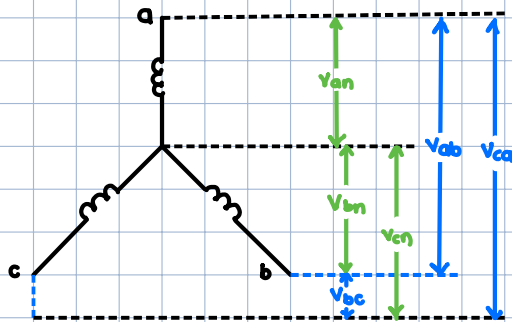
$$(c) |H(j\omega)| = \frac{R_2}{R_1} \cdot \frac{1/R_2 C}{\sqrt{(1/R_2 C)^2 + \omega^2}} \quad \omega = 8\omega_c = 8(5000\pi) = 40k\pi$$

$$|H(j\omega_c)| = \left[\frac{6366.198}{1273.240} \right] \cdot \frac{1/(6366.198)(10nF)}{\sqrt{[1/(6366.198)(10nF)]^2 + (40k\pi)^2}} = 0.62$$

$$\begin{aligned} \angle H(j\omega_c) &= 180 - \tan^{-1}(\omega R_2 C) \\ &= 180 - \tan^{-1}[(40k\pi)(6366.198)(10 \times 10^{-9})] \\ &= 97.125^\circ \end{aligned}$$

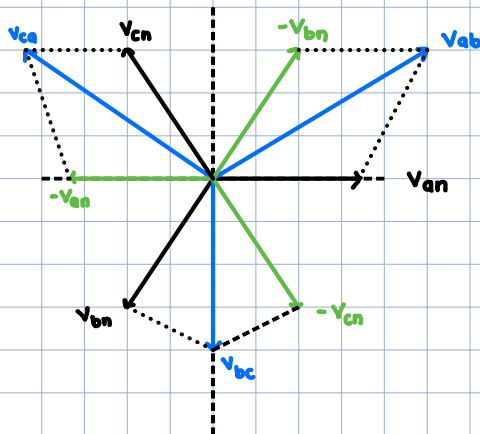
$$\begin{aligned} \frac{V_o}{V_i} &= 0.62 \angle 97.125^\circ \quad V_o = 0.62 \angle 97.125^\circ V; \\ &= (0.62 \angle 97.125^\circ)(3.5 \cos 40k\pi t) \\ &= \boxed{2.17 \cos(40000\pi t + 97.125^\circ) V} \end{aligned}$$

Q2 (a)



Phase voltage : V_{an}, V_{bn}, V_{cn}

Line voltage : V_{ab}, V_{bc}, V_{ca}



$$|V_{ab}|^2 = |V_{an}|^2 + |V_{bn}|^2 + 2|V_{an}||V_{bn}|\cos 60^\circ$$

$$\text{balanced system : } |V_{an}| = |V_{bn}| = |V_{cn}| = V_\phi$$

$$V_{ab} = \sqrt{3} V_\phi \quad \text{angle } +30^\circ \text{ from ref. axis and } +30^\circ \text{ from } V_{an}$$

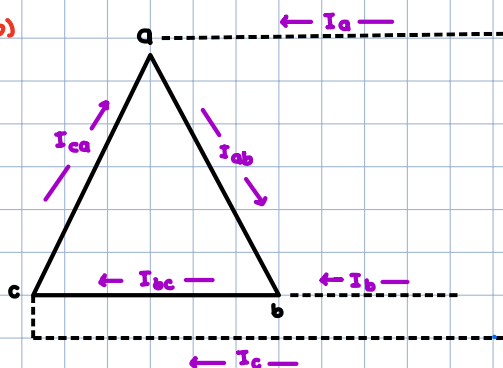
$$V_{bc} = \sqrt{3} V_\phi \quad \text{angle } -90^\circ \text{ from ref. axis and } +30^\circ \text{ from } V_{bn}$$

$$V_{ca} = \sqrt{3} V_\phi \quad \text{angle } +150^\circ \text{ from ref. axis and } +30^\circ \text{ from } V_{cn}$$

$$V_{line} = \sqrt{3} V_{phase}$$

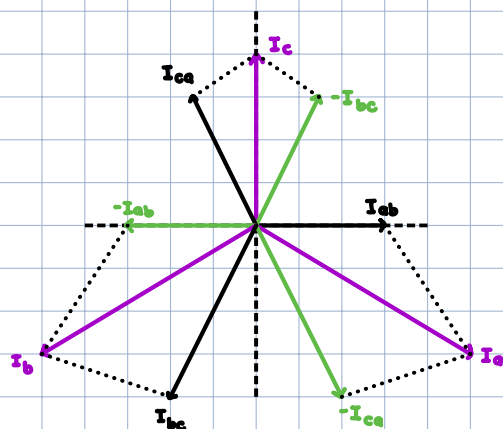
Line voltage leads phase voltage by 30°

(b)



Phase current : I_{ab}, I_{bc}, I_{ca}

Line current : I_a, I_b, I_c



$$|I_a|^2 = |I_{ab}|^2 + |I_{ca}|^2 + 2|I_{ab}||I_{ca}|\cos 60^\circ$$

$$\text{balanced system: } |I_{ab}| = |I_{bc}| = |I_{ca}| = I_\phi$$

$$I_a = \sqrt{3} I_\phi \quad \text{angle } -30^\circ \text{ from ref. axis and } -30^\circ \text{ from } I_{ab}$$

$$I_b = \sqrt{3} I_\phi \quad \text{angle } -150^\circ \text{ from ref. axis and } -30^\circ \text{ from } I_{bc}$$

$$I_c = \sqrt{3} I_\phi \quad \text{angle } +90^\circ \text{ from ref. axis and } -30^\circ \text{ from } I_{ca}$$

$$I_{\text{line}} = \sqrt{3} I_{\text{phase}}$$

Q3. (a) $V_a = 48 \angle -45^\circ$

$$V_b = 48 \angle -165^\circ$$

$$V_c = 48 \angle 75^\circ$$

$$V_a + V_b + V_c = 0$$

$$(48 \angle -45^\circ) + (48 \angle -165^\circ) + (48 \angle 75^\circ) = 0$$

$$0 + j0 = 0 \quad \text{Three-phase set voltages}$$

is **balanced**

balanced form:

$$V_a = 48 \cos(314t - 45^\circ) \text{ V}$$

$$V_b = 48 \cos(314t - 45^\circ - 120^\circ) \text{ V}$$

$$V_c = 48 \cos(314t - 45^\circ + 120^\circ) \text{ V}$$

phase sequence: abc

positive

(b) $V_a = 188 \cos(250t + 60^\circ) \text{ V} = 188 \angle 60^\circ \text{ V}$

$$V_b = 188 \cos(250t + 180^\circ) \text{ V} = 188 \angle 180^\circ \text{ V}$$

$$V_c = 188 \cos(250t - 60^\circ) \text{ V} = 188 \angle -60^\circ \text{ V}$$

$$V_a + V_b + V_c = 0$$

$$(94 + j94\sqrt{3}) + (-188 + j0) + (94 - j94\sqrt{3}) = 0$$

$$0 + j0$$

Three-phase set voltages

is **balanced**

balanced form:

$$V_a = 188 \cos(250t + 60^\circ) \text{ V}$$

$$V_b = 188 \cos(250t + 60^\circ + 120^\circ) \text{ V}$$

$$V_c = 188 \cos(250t + 60^\circ - 120^\circ) \text{ V}$$

phase sequence: acb

negative

(c) magnitude of phase voltage V_b is not equal to V_a and V_c .

Therefore, three-phase voltage are **unbalanced**

(d) $V_a = 1121 \cos(2000t - 20^\circ) \text{ V} = 1121 \angle -20^\circ \text{ V}$

$$V_b = 1121 \cos(2000t - 50^\circ - 90^\circ) \text{ V} = 1121 \angle -140^\circ \text{ V}$$

$$V_c = 1121 \cos(2000t + 100^\circ) \text{ V} = 1121 \angle 100^\circ \text{ V}$$

$$V_a + V_b + V_c = 0$$

$$(1121 \angle -20^\circ) + (1121 \angle -140^\circ) + (1121 \angle 100^\circ) = 0$$

$$0 + j0 = 0$$

Three-phase set voltages

is **balanced**

balanced form:

$$V_a = 1121 \cos(2000t - 20^\circ) \text{ V}$$

$$V_b = 1121 \cos(2000t - 20^\circ - 120^\circ) \text{ V}$$

$$V_c = 1121 \cos(2000t - 20^\circ + 120^\circ) \text{ V}$$

phase sequence: abc

positive

(e) $V_a = 540 \angle -90^\circ \text{ V}$

$$V_b = 540 \angle -120^\circ \text{ V}$$

$$V_c = 540 \angle 120^\circ \text{ V}$$

$$V_a + V_b + V_c = 0$$

$$(540 \angle -90^\circ) + (540 \angle -120^\circ) + (540 \angle 120^\circ) = 0$$

$$-540 - j540 \neq 0$$

Three-phase set voltages is **unbalanced**

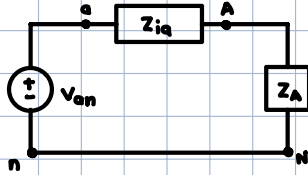
$$\begin{aligned}
 (f) \quad V_a &= 144 \cos(800t + 80^\circ) \text{ V} = 144 \angle 80^\circ \text{ V} \\
 V_b &= 144 \cos(800t - 70^\circ - 90^\circ) \text{ V} = 144 \angle -160^\circ \text{ V} \\
 V_c &= 144 \cos(800t + 50^\circ - 90^\circ) \text{ V} = 144 \angle -40^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V_a + V_b + V_c &= 0 \\
 (144 \angle 80^\circ) + (144 \angle -160^\circ) + (144 \angle -40^\circ) &= 0 \\
 0 + j0 &= 0 \quad \text{Three-phase set voltages} \\
 &\text{is balanced}
 \end{aligned}$$

balanced form:

$$\begin{aligned}
 V_a &= 144 \cos(800t + 80^\circ) \text{ V} \\
 V_b &= 144 \cos(800t + 80^\circ - 240^\circ) \text{ V} \\
 V_c &= 144 \cos(800t + 80^\circ - 120^\circ) \text{ V} \\
 \text{phase sequence: } &\text{acb} \\
 &\text{negative}
 \end{aligned}$$

Q4 (a)



$$V_{an} = \frac{V_{ab}}{\sqrt{3} \angle 30^\circ} = \frac{110\sqrt{3} \angle -60^\circ}{\sqrt{3} \angle 30^\circ} = 110 \angle -90^\circ \text{ V}$$

$$Z_{ia} = 3 + j2 \Omega = \sqrt{13} \angle 33.69^\circ \Omega / \phi$$

$$Z_A = 37 + j28 \Omega = 46.4 \angle 37.12^\circ \Omega / \phi$$

$$\begin{aligned}
 (b) \quad I &= \frac{V}{Z} \rightarrow I_{aA} = \frac{V_{an}}{Z_{ia} + Z_A} = \frac{0 - j110}{(3 + j2) + (37 + j28)} \\
 &= -1.32 - j1.76 = 2.2 \angle -126.87^\circ \text{ A}
 \end{aligned}$$

$$I_a = 2.2 \angle -126.87^\circ \text{ A}$$

$$\begin{aligned}
 I_b &= 2.2 \angle -126.87^\circ + 120^\circ \\
 &= 2.2 \angle -6.87^\circ \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 I_c &= 2.2 \angle -126.87^\circ - 120^\circ \\
 &= 2.2 \angle -246.87^\circ \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad V_{AN} &= I_{aA} Z_A \\
 &= (2.2 \angle -126.87^\circ)(46.4 \angle 37.12^\circ) \\
 &= 102.08 \angle -89.75^\circ \text{ V}
 \end{aligned}$$

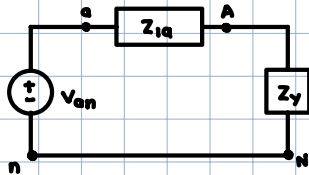
$$\begin{aligned}
 V_{ab} &= (\sqrt{3} \angle 30^\circ) V_{AN} \\
 &= (\sqrt{3} \angle 30^\circ)(102.08 \angle -89.75^\circ) \\
 &= 176.81 \angle -59.75^\circ \text{ V}
 \end{aligned}$$

$$\text{LOAD: } V_{ab} = 176.81 \angle -59.75^\circ \text{ V}$$

$$\begin{aligned}
 V_{ac} &= 176.81 \angle -59.75^\circ + 120^\circ \\
 &= 176.81 \angle 60.25^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V_{bc} &= 176.81 \angle -59.75^\circ - 120^\circ \\
 &= 176.81 \angle -179.75^\circ \text{ V}
 \end{aligned}$$

Q5 (a)



$$\begin{aligned}
 V_{an} &= 150 \angle 135^\circ - 120^\circ \\
 &= 150 \angle 15^\circ \text{ V}
 \end{aligned}$$

$$Z_Y = \frac{Z_\Delta}{3} = \frac{129 + j171}{3} = 43 + j57 \Omega / \phi$$

$$Z_{ia} = 2 + j3 \Omega / \phi$$

$$\begin{aligned}
 (b) \quad I &= \frac{V}{Z} \rightarrow I_{aA} = \frac{V_{an}}{Z_{ia} + Z_Y} = \frac{150 \angle 15^\circ}{(43 + j57) + (2 + j3)} \\
 &= 1.57 - j1.24 = 2 \angle -38.30^\circ \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad V_{AN} &= Z_Y I_{aA} = (43 + j57)(2 \angle -38.30^\circ) \\
 &= 142.80 \angle 14.67^\circ \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 V_{AB} &= (\sqrt{3} \angle 30^\circ) V_{AN} \\
 &= (\sqrt{3} \angle 30^\circ)(142.80 \angle 14.67^\circ) \\
 &= 247.34 \angle 44.67^\circ \text{ V}
 \end{aligned}$$