

CSC 28 HW

Counting HW, Part 1

1. How many 3-permutations are there of the letters a, b, c, d?
2. How many permutations are there of 11 objects?
3. In how many ways can we select a chair person, vice president, secretary and treasurer from a group of 12 people?
4. Determine how many strings can be formed by ordering the letters ABCDE subject to the following conditions:
 - a. Contains the letters ACE together in any order
 - b. contains the substring DB and AE
 - c. contains neither of the substrings AB, CD
5. In how many ways can we select a committee of four from a group of 12 persons?
6. At one point in the Illinois State lottery lotto game, a person was required to choose six numbers (in any order) among 44 numbers. In how many ways this can be done?
7. in a club consisting of six distinct men and seven distinct women
 - a. In how many ways can we select a committee of three men and four women?
 - b. In how many ways can we select a committee of four persons that has at least one woman?
 - c. in how many ways can we select a committee of four persons that has persons of both sexes?
8. A shipment of 50 microprocessors of which 4 are defective
 - a. In how many ways can we select a set of four non-defective micro processors?
 - b. In how many ways can we select a set of four micro-processors containing exactly two defective micro-processors?
 - C. In how many ways can we select a set of four micro-processors containing at least one defective micro-processor?
9. Eighteen persons have first names Alfie, Ben, Cissi and last names Dumont, Elem. Show that at least three persons have the same first and last names.
10. Professor Euclid is paid every other week on Friday. Show that in some months she is paid three times.
11. Suppose that each person in a group of 32 people receives a check in January. Prove that at least two people receive checks on the same day.
12. prove that among 35 students in a class, at least two have the first names that starts with the same letter

1. $p(n, r) = \frac{n!}{(n-r)!}$

$$p(4, 3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \boxed{24} \text{ ways}$$

2. $p(n, n) = n!$ $p(11, 11) = \boxed{11!}$ ways

3. $p(12, 4) = \frac{12!}{8!} = \boxed{12 \times 11 \times 10 \times 9} = 11880 \text{ ways}$

4. (a) ACE, B, D

$$P(3, 3) = 3! \quad \text{since ACE is in any order : } p(3, 3) = 3!$$

$$\text{Answer: } \boxed{3! \times 3!} \text{ ways}$$

(b) DB, AE, C

$$P(3, 3) = \boxed{3!} \text{ ways}$$

(c) neither AB, CD

$$AB, C, D \rightarrow 4!$$

$$A, B, CD \rightarrow 4!$$

$$AB, CD, C \rightarrow 3!$$

$$\boxed{4! + 4! - 3!} = 42 \text{ ways}$$

5. $c(n, r) = \frac{12!}{4! 8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 495 \text{ ways}$

6. $c(44, 6) = \frac{44!}{6! 38!}$

7. 6 distinct men, 7 distinct women

(a) $C(6,3) \times C(7,4)$

(b) $C(13,4) - C(6,4)$

(c) All women : $C(6,4)$

All men : $C(7,4)$

$$C(13,4) - [C(6,4) + C(7,4)]$$

Total : $C(12,4)$

8. 50 microprocessors, 4 defective

(a) $50 - 4 = 46$ $C(46,4)$

(b) $C(46 \times 4) \times C(4,2)$

(c) $C(50,4) - C(46,4)$

9. 18 persons

First names: Alfie, Ben, Cissi

$$\text{First} \times \text{last} = 3 \times 2 = 6$$

Last names: Dumont, Elem

$$\left\lceil \frac{18}{6} \right\rceil = 3 \text{ will have the same first and last name}$$

10. 52 Fridays per year

$$\rightarrow 52/2 = 26 \text{ Fridays}$$

Professor Euclid get

$$\left\lceil \frac{\# \text{ time paid per year}}{\# \text{ months in a year}} \right\rceil = \left\lceil \frac{26}{12} \right\rceil = 3$$

3 time get paid in same month.

paid 26 times per year.

11. $\left\lceil \frac{\# \text{ of people}}{\# \text{ of days in January}} \right\rceil = \left\lceil \frac{32}{31} \right\rceil = 2$ at least 2 people received check on the same day

12. $\left\lceil \frac{\# \text{ of students}}{\# \text{ of letters in Alphabet}} \right\rceil = \left\lceil \frac{35}{26} \right\rceil = 2$ At least 2 students will have first name starts with the same letter