HW: Propositional function

1. Determine the truth value of each of the following statements. Domain of the function is R
   a. $\forall x, |x| = x$;
   b. $\exists x, x^2 = x$

2. Let A = {1,2,3,4,5} Determine the truth value of the following statements
   a. $\exists x \in A(x + 3 = 10)$
   b. $\forall x \in A(x + 3 <=7)$

3. Negate the following statements and write them symbolically using the quantifiers:
   a. All students live in the dormitories
   b. All mathematician majors are male
   c. Some students are 25 years or older

4. Negate each of the following statements:
   a. $\forall x p(x) \wedge \exists y q(y)$
   b. $\exists x p(x) \vee \forall y p(y)$

5. Find a counter example for each of the following statements, B = {2,3,…8,9}
   a. $\forall x \in B, x + 5 < 12$
   b. $\forall x \in B, x$ is prime

6. Determine the truth value of the following statements , where A = {1,2,3}
   a. $\exists x \forall y, x^2 < y + 1$
   b. $\forall x \exists y, x^2 + y^2 < 12$

7. Let p(x) : "X is taking a math course." Domain: all the students. Write each of the following propositions in words:
   a. $\forall x\, p(x)$
   b. $\exists x \sim p(x)$
   c. $\sim(\forall x p(x))$

8. Let p(x): "X is a professional athlete", q(x): "X plays soccer". Domain = all people. Write each proposition in words
   a. $\forall x(p(x) \rightarrow q(x))$
   b. $\exists x(q(x) \rightarrow p(x))$

9. Let the statements "X is an accountant" and "X owns a Porsha" write each of the following statements symbolically
   a. All accountants own Porsha
   b. All accountants own Porsha
   c. Someone who owns a Porsha is an accountant

10. Write each stamen symbolically
    a. All men do not cheat on their wives
    b. Every environmental problem is not a tragedy

11. The proposition function "if x and y are distinct people", then "x is taller than y". Write each proposition in words
    a. $\forall x \forall y T(x,y)$
    b. $\exists x \exists x T(x,y)$

1. a. $\forall x, |x| = x$;   Domain : Real Numbers

      • $|-3| = -3$

        The statement is false for negative numbers.

  b. $\exists x, \quad x^2 = x$ Domain : Real Numbers

       $1^2 = 1$

        The statement is true

2. $A = \{1, 2, 3, 4, 5\}$

  a. $\exists x \in A \ (x+3 = 10)$

     $x + 3 = 10$ showing $x = 7$, we verified that there is

     $x = 7$ no equivalent in the domain.

        The statement is false.

  b. $\forall x \in A \ (x+3 <= 7)$

     $x + 3 <= 7$ Any number less than or equal to 4.

     $x <= 4$ Since the domain included 5,

               the statement is false.

3. a. NEGATION: some students doesn't live in dormitories

     D : All students

     $p(x)$: lives in dormitories

      $\sim(\forall x \in D \ p(x)) \equiv \exists x \in D \ \sim p(x)$

  b. NEGATION: some mathematicians are not male

     D : All mathematician majors

     $p(x)$: male

    $\sim(\forall x \in D \ p(x)) \equiv \exists x \in D \ \sim p(x)$

C. NEGATION: All students are not 25 years or older

D: students

p(x): 25 years or older

$\sim(\exists x \in D\ p(x)) \equiv \forall x \in D\ \sim p(x)$

4. a. $\sim(\forall x\ p(x) \wedge \exists y\ q(y))$

$\exists x \sim p(x) \vee \forall y \sim q(y)$

b. $\sim(\exists x\ p(x) \vee \forall y\ p(y))$

$\forall x \sim p(x) \wedge \exists y \sim p(y)$

5. $B = \{2, 3, \ldots, 8, 9\}$

a. $\forall x \in B,\ x + 5 < 12$

$x < 7$ B includes 7, 8, 9 as counter example

b. $\forall x \in B,\ x$ is prime

4, 6, 8, 9 are not prime as counter example

6. $A = \{1, 2, 3\}$

a. $\exists x\ \forall y,\ x^2 < y + 1$

For $x = 1$, every y satisfy the statement

$1^2 < 1 + 1 = 1 < 2$

The statement is correct.

b. $\forall x\ \exists y,\ x^2 + y^2 < 12$

For $y = 1$, every x satisfy the condition

$3^2 + 1^2 < 12 = 10 < 12$

The statement is correct.

7. p(x): x is taking a math course   Domain: all the students

a. $\forall x\ p(x)$

All of the students are taking a math course.

b. $\exists x \sim p(x)$

   Some of the students are not taking a math course.

c. $\sim (\forall x p(x))$

   Not all of the students are taking a math course.

8. $p(x):$ x is a professional athlete   Domain : all people

   $q(x):$ x plays soccer

   a. $\forall x (p(x) \rightarrow q(x))$

      All professional athletes play soccer

   b. $\exists x (q(x) \rightarrow p(x))$

      Everyone who plays soccer are professional athlete

9. $p(x):$ x is an accountant

   $q(x):$ x owns a Porsha

   a. All accountant owns Porsha

      Domain: All accountant

      $\forall x \, q(x)$

   b. All accountant owns Porsha

      Domain: All people

      $\forall x \in D \quad p(x) \rightarrow q(x)$

   c. Someone who owns Porsha is an accountant

      $\exists x \; p(x) \wedge q(x)$

10. a. All men do not cheat on their wives

      $p(x):$ men   Domain: All men

      $q(x):$ cheat on their wives

         $\forall x \sim q(x)$

b. Every environmental problem is not a tragedy.

    p(x): environmental problem

    q(x): tragedy

      Domain: All environmental problem

    $\forall x \sim q(x)$

11. a. $\forall x \forall y \, T(x,y)$

    If all x and y are distinct people, then x is taller than y.

b. $\exists x \exists y \, T(x,y)$

    If some x and some y are distinct people,
    then x is taller than y.