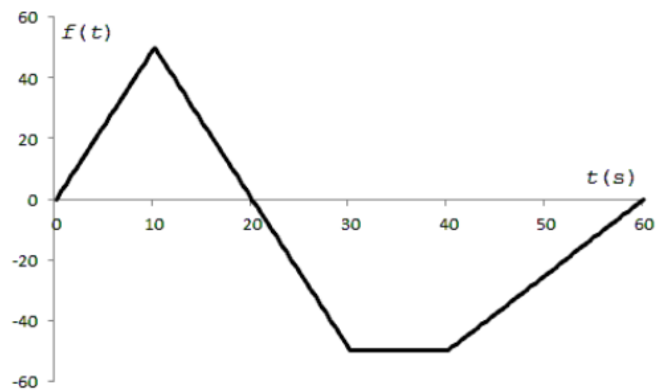
Solution HW-03 ENGR 117 Due date 03/14/2022

10 Questions 10 points each

Q-1 Write an expression for f(t)

A function $f(t)$ is defined as follows:

$$
\begin{aligned}
f(t) &= 0, && t \le 0 \\
&= 5t, && 0 \le t \le 10\,\text{s} \\
&= -5t + 100, && 10\,\text{s} \le t \le 30\,\text{s} \\
&= -50, && 30\,\text{s} \le t \le 40\,\text{s}; \\
&= 2.5t - 150 && 40\,\text{s} \le t \le 60\,\text{s} \\
&= 0, && 60\,\text{s} \le t < \infty.
\end{aligned}
$$

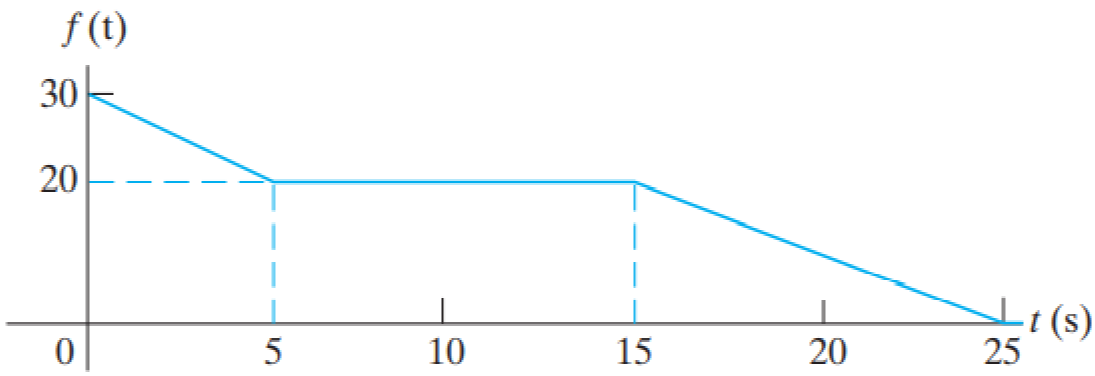


$$
\begin{aligned}
f(t) = {} & 5t[u(t) - u(t-10)] + (100-5t)[u(t-10) - u(t-30)] \\
& -50[u(t-30) - u(t-40)] \\
& +(2.5t - 150)[u(t-40) - u(t-60)]
\end{aligned}
$$

$$
\begin{aligned}
f(t) = {} & 5t\,u(t) - 5t\,u(t-10) + (100-5t)\,u(t-10) - (100-5t)\,u(t-30) \\
& - 50\,u(t-30) + 50\,u(t-40) + (2.5t-150)\,u(t-40) \\
& - (2.5t-150)\,u(t-60)
\end{aligned}
$$

$$
\begin{aligned}
f(t) = {} & 5t\,u(t) + (100-10t)\,u(t-10) + (5t-150)\,u(t-30) \\
& + (2.5t-100)\,u(t-40) - (2.5t-150)\,u(t-60)
\end{aligned}
$$

Q-2 Write an expression for f(t) for the following:



$$f(t) = (30 - 2t)[u(t) - u(t-5)] + 20[u(t-5) - u(t-15)]$$
$$+(50 - 2t)[u(t-15) - u(t-25)]$$

$$f(t) = (30-2t)u(t) + (2t-10)u(t-5) + (30-2t)u(t-15)$$
$$- u(t-25)$$

Q-3 Use the initial- and final-value theorems to find the initial and final values of f(t) for the following functions.

a)

$$F(s) = \frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)}.$$

$$f(t) = (4e^{-3t} + 6e^{-4t} - 3e^{-5t})u(t).$$

b)

$$F(s) = \frac{(4s^2 + 7s + 1)}{s(s+1)^2}$$

$$f(t) = (1 + 2te^{-t} + 3e^{-t})u(t).$$

a)

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \left[\frac{7s^3[1 + (9/s) + (134/(7s^2))]}{s^3[1 + (3/s)][1 + (4/s)][1 + (5/s)]}\right] = 7$$

$$\therefore \quad f(0^+) = 7$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0} \left[\frac{7s^3 + 63s^2 + 134s}{(s+3)(s+4)(s+5)}\right] = 0$$

$$\therefore \quad f(\infty) = 0$$

b)

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \left[\frac{s^3[4 + (7/s) + (1/s)^2]}{s^3[1 + (1/s)]^2}\right] = 4$$

$$\therefore \quad f(0^+) = 4$$

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0} \left[\frac{4s^2 + 7s + 1}{(s+1)^2}\right] = 1$$

$$\therefore \quad f(\infty) = 1$$

Q-4 There is no energy stored in the circuit shown in Fig. at the time the switch is opened.

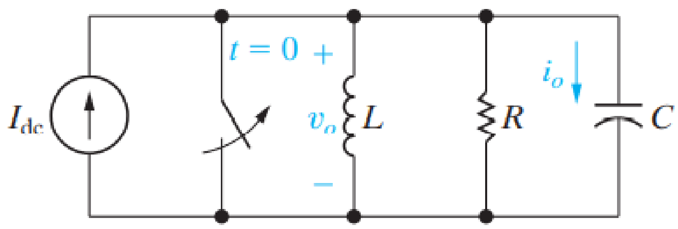
a) Derive the integrodifferential equation that governs the behavior of the voltage v_o.

b) Show that

$$V_o(s) = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}.$$

c) Show that

$$I_o(s) = \frac{sI_{dc}}{s^2 + (1/RC)s + (1/LC)}.$$



[a] $$\boxed{I_{dc} = \frac{1}{L}\int_0^t v_o\,dx + \frac{v_o}{R} + C\frac{dv_o}{dt}}$$

[b] $$\frac{I_{dc}}{s} = \frac{V_o(s)}{sL} + \frac{V_o(s)}{R} + sCV_o(s)$$

$$\therefore \boxed{V_o(s) = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}}$$

[c] $$i_o = C\frac{dv_o}{dt}$$

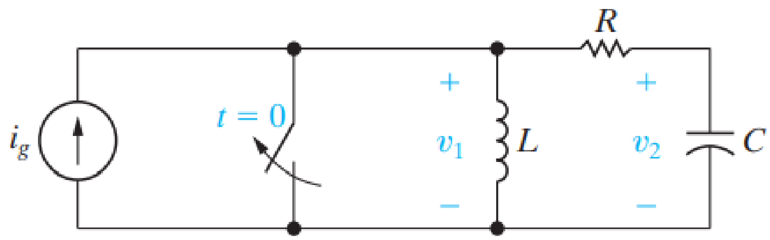
$$\therefore \boxed{I_o(s) = sCV_o(s) = \frac{sI_{dc}}{s^2 + (1/RC)s + (1/LC)}}$$

Q-5 There is no energy stored in the circuit shown in Fig. at the time the switch is opened.

a) Derive the integrodifferential equations that govern the behavior of the node voltages v_1 and v_2.

b) Show that

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}.$$



$$\boxed{[a]\ \frac{1}{L}\int_0^t v_1\, d\tau + \frac{v_1 - v_2}{R} = i_g}$$

Note "ig" is not dc current (Constant). It is sinusoidal function.

and

$$C\frac{dv_2}{dt} + \frac{v_2}{R} - \frac{v_1}{R} = 0$$

[b] $$\frac{V_1}{sL} + \frac{V_1 - V_2}{R} = I_g$$

$$\frac{V_2 - V_1}{R} + sCV_2 = 0$$

or

$$(R + sL)V_1(s) - sLV_2(s) = RLsI_g(s)$$

$$-V_1(s) + (RCs + 1)V_2(s) = 0$$

Solving,

$$\boxed{V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}}$$

Q-6 Find f(t) for the following function:

$$F(s) = \frac{6(s+10)}{(s+5)(s+8)}.$$

$$F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+8}$$

$$K_1 = \frac{6(s+10)}{(s+8)}\bigg|_{s=-5} = 10$$

$$K_2 = \frac{6(s+10)}{(s+5)}\bigg|_{s=-8} = -4$$

$$\boxed{f(t) = [10e^{-5t} - 4e^{-8t}]u(t)}$$

Q-7 Find f(t) for the following function:

$$F(s) = \frac{15s^2 + 112s + 228}{(s+2)(s+4)(s+6)}.$$

$$F(s) = \frac{K_1}{s+2} + \frac{K_2}{s+4} + \frac{K_3}{s+6}$$

$$K_1 = \frac{15s^2 + 112s + 228}{(s+4)(s+6)}\bigg|_{s=-2} = 8$$

$$K_2 = \frac{15s^2 + 112s + 228}{(s+2)(s+6)}\bigg|_{s=-4} = -5$$

$$K_3 = \frac{15s^2 + 112s + 228}{(s+2)(s+4)}\bigg|_{s=-6} = 12$$

$$\boxed{f(t) = [8e^{-2t} - 5e^{-4t} + 12e^{-6t}]u(t)}$$

Q-8 Find f(t) for the following function:

$$F(s) = \frac{14s^2 + 56s + 152}{(s + 6)(s^2 + 4s + 20)}$$

$$F(s) = \frac{K_1}{s+6} + \frac{K_2}{s+2-j4} + \frac{K_2^*}{s+2+j4}$$

$$K_1 = \frac{14s^2 + 56s + 152}{s^2 + 4s + 20}\bigg|_{s=-6} = 10$$

$$K_2 = \frac{14s^2 + 56s + 152}{(s+6)(s+2+j4)}\bigg|_{s=-2+j4} = 2 + j2 = 2.83\underline{/45^\circ}$$

$$\boxed{f(t) = [10e^{-6t} + 5.66e^{-2t}\cos(4t + 45^\circ)]u(t)}$$

Q-9 Find f(t) for the following function:

$$F(s) = \frac{60(s + 5)}{(s + 1)^2(s^2 + 6s + 25)}$$

$$F(s) = \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1} + \frac{K_3}{s+3-j4} + \frac{K_3^*}{s+3+j4}$$

$$K_1 = \frac{60(s+5)}{s^2+6s+25}\bigg|_{s=-1} = 12$$

$$K_2 = \frac{d}{ds}\left[\frac{60(s+5)}{s^2+6s+25}\right] = \left[\frac{60}{s^2+6s+25} - \frac{60(s+5)(2s+6)}{(s^2+6s+25)^2}\right]_{s=-1} = 0.6$$

$$K_3 = \frac{60(s+5)}{(s+1)^2(s+3+j4)}\bigg|_{s=-3+j4} = 1.68\underline{/100.305^\circ}$$

$$\boxed{f(t) = [12te^{-t} + 0.6e^{-t} + 3.35e^{-3t}\cos(4t + 100.305^\circ)]u(t)}$$

Q-10 Find f(t) for the following function:

$$F(s) = \frac{5s^3 + 20s^2 - 49s - 108}{s^2 + 7s + 10}$$

$$F(s) = \begin{array}{r|l} & \quad\quad\quad\quad\quad\quad 5s - 15 \\ \hline s^2 + 7s + 10 & 5s^3 + 20s^2 - 49s - 108 \\ & 5s^3 + 35s^2 + 50s \\ \hline & \quad -15s^2 - 99s - 108 \\ & \quad -15s^2 - 105s - 150 \\ \hline & \quad\quad\quad\quad\quad\quad 6s + 42 \end{array}$$

$$F(s) = 5s - 15 + \frac{K_1}{s+2} + \frac{K_2}{s+5}$$

$$K_1 = \frac{6s+42}{s+5}\bigg|_{s=-2} = 10$$

$$K_2 = \frac{6s+42}{s+2}\bigg|_{s=-5} = -4$$

$$\boxed{f(t) = 5\delta'(t) - 15\delta(t) + [10e^{-2t} - 4e^{-5t}]u(t)}$$