CALIFORNIA STATE UNIVERSITY
SACRAMENTO

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

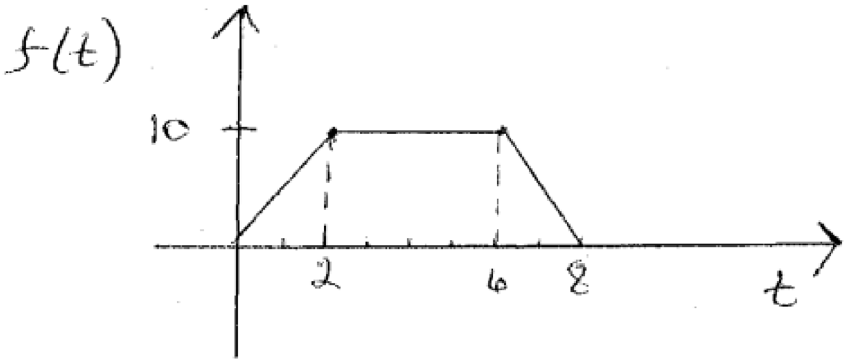
EEE 117 Network Analysis

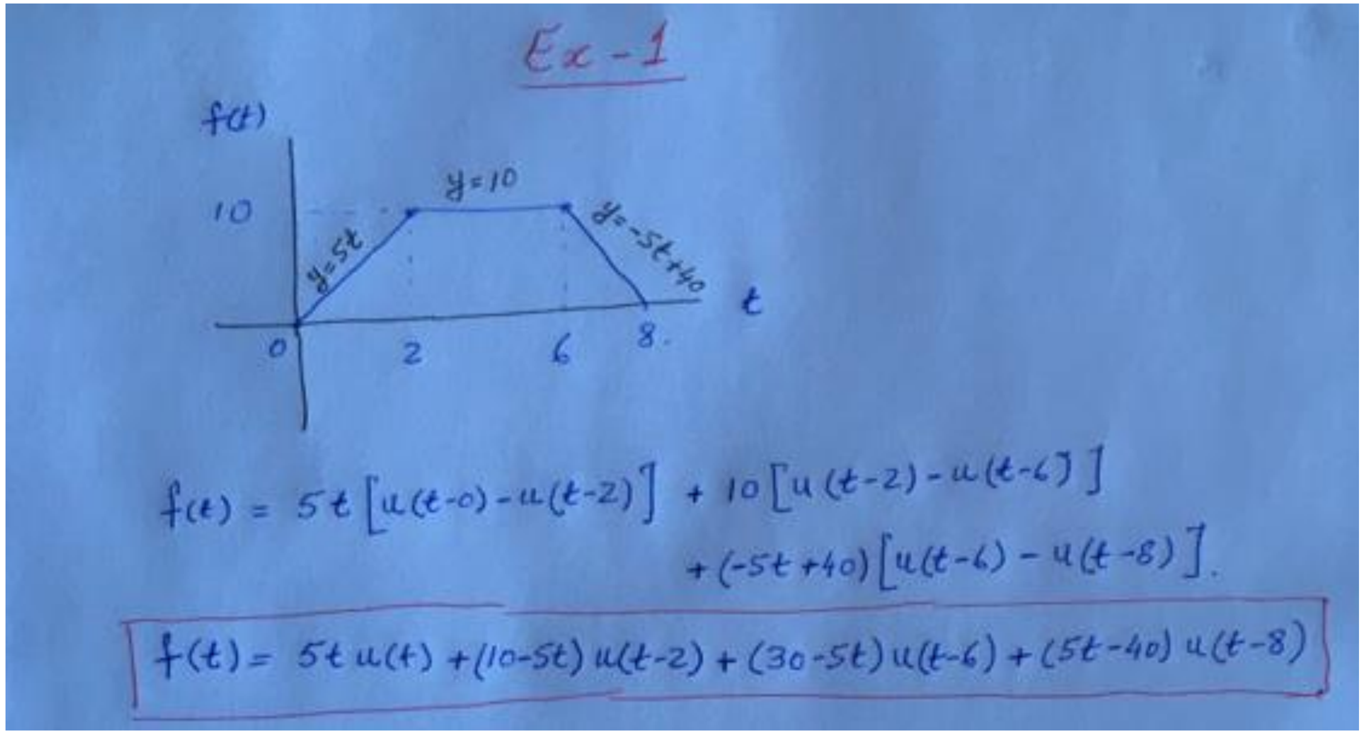
Text: Electric Circuits by J. Nilsson and S. Riedel Prentice Hall

Examples Set 3: Laplace and Inverse Laplace Transformations

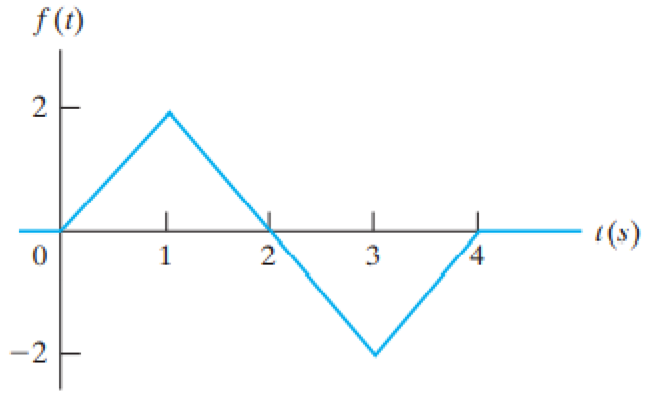
Instructor: Riaz Ahmad

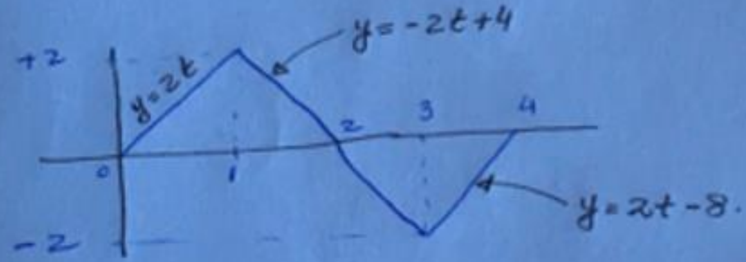
Ex-1: Use step functions to write an expression for the following function.





Ex-2: Use step functions to write an expression for the following function.



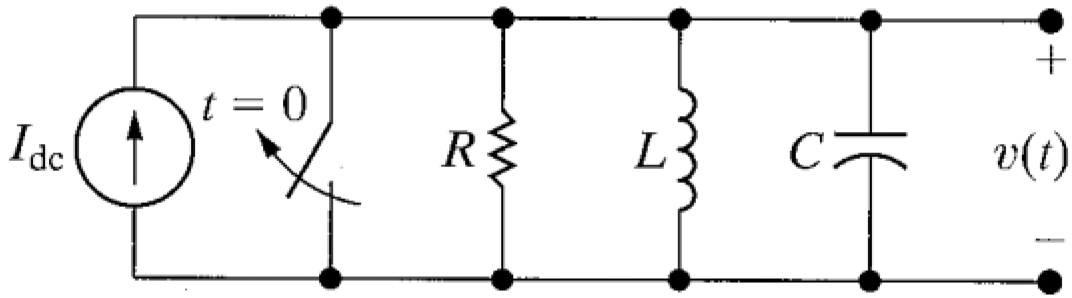


$$f(t) = 2t\,[u(t-0) - u(t-1)] + (-2t+4)\,[u(t-1) - u(t-3)] + (2t-8)\,[u(t-3) - u(t-4)]$$

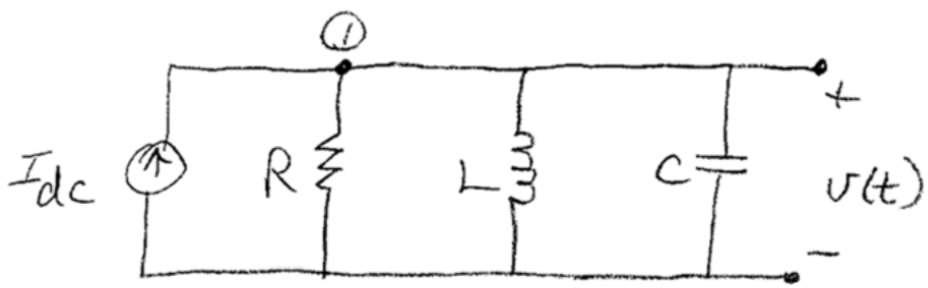
$$f(t) = 2t\,u(t) - 2t\,u(t-1) + (-2t+4)\,u(t-1) - (-2t+4)\,u(t-3) + (2t-8)\,u(t-3) - (2t-8)\,u(t-4)$$

$$\boxed{f(t) = 2t\,u(t) + (-4t+4)\,u(t-1) + (4t-12)\,u(t-3) - (2t-8)\,u(t-4)}$$

Ex-3: Apply the Laplace transform to the following circuit.



Here is my circuit for t > 0.



Recall $i_L = \frac{1}{L}\int_0^t v(x)dx$ and $i_C = C\frac{dv(t)}{dt}$

$I_{dc} = 0$ for $t < 0$

At node 1, the node equation is

$$-I_{DC}u(t) + \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(x)dx + C\frac{dv(t)}{dt} = 0$$

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(x)dx + C\frac{dv(t)}{dt} = I_{DC}u(t)$$

This node equation in the frequency domain is

$$\frac{V(s)}{R}+\frac{1}{L}\frac{V(s)}{s}+C\left[sV(s)-v(0^-)\right]=I_{DC}\left(\frac{1}{s}\right)$$

$v(t = 0^-)$ = zero as we said in the time domain preview.

Thus we can write

$$V(s)\left(\frac{1}{R}+\frac{1}{sL}+Cs\right)=\frac{I_{DC}}{s}$$

$$V(s)=\frac{I_{DC}}{s\left(\frac{1}{R}+\frac{1}{sL}+Cs\right)} \qquad =\frac{\frac{I_{DC}}{C}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}$$

<u>Ex-4:</u> Find f(t) for $F(s) = \dfrac{s+6}{s(s+3)(s+1)^2}$

Check rationality

m>n

already rational.

$$F(s) = \frac{s+6}{s(s+3)(s+1)^2} = \frac{A}{s} + \frac{B}{(s+3)} + \frac{C}{(s+1)^2} + \frac{D}{s+1}$$

Already rational

$$\frac{s+6}{s(s+3)(s+1)^2} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+1)^2} + \frac{D}{s+1}$$

$$s+6 = A(s+3)(s+1)^2 + B(s+1)^2 \cdot s + C(s+3) \cdot s + D\,s(s+3)(s+1).$$

If $s = 0 \Rightarrow 6 = 3A \Rightarrow \boxed{A = 2}$

If $s = -3 \Rightarrow 3 = -12B \Rightarrow \boxed{B = -\frac{1}{4}}$

If $s = -1 \Rightarrow 5 = -2C \Rightarrow \boxed{C = -\frac{5}{2}}$

If $s = 1$

$$7 = 2 \cdot 4 \cdot 4 + \left(-\tfrac{1}{4}\right) \cdot 4 \cdot 1 + \left(-\tfrac{5}{2}\right) \cdot 4 \cdot 1 + D \cdot 1 \cdot 4 \cdot 2$$

$$7 = 32 - 1 - 10 + 8D \Rightarrow \boxed{D = -\frac{7}{4}}$$

Hence

$$F(s) = \frac{2}{s} - \frac{1}{4(s+3)} - \frac{5}{2} \cdot \frac{1}{(s+1)^2} - \frac{7}{4} \cdot \frac{1}{s+1}$$

$$f(t) = 2 - \frac{1}{4} e^{-3t} - \frac{5}{2} t e^{-t} - \frac{7}{4} \cdot e^{-t}$$

$$\boxed{f(t) = 2 - \frac{1}{4} e^{-3t} - \left(\frac{5}{2} t + \frac{7}{4}\right) e^{-t}}$$

$$\boxed{f(t) = \left[2 - \frac{1}{4} e^{-3t} - \left(\frac{5}{2} t + \frac{7}{4}\right) e^{-t}\right] u(t).}$$

Ex-5: Find f(t) for $F(s)=\dfrac{5s^2+29s+32}{(s+2)(s+4)}$

$$F(s)=\frac{5s^2+29s+32}{(s+2)(s+4)}$$

Note Here $m=n$.
So divide neumerator by denominator to make it proper rational function

$$F(s)=\frac{5s^2+29s+32}{s^2+6s+8}$$

$$F(s)=5+\frac{(-s-8)}{(s+2)(s+4)}=5+\frac{A}{s+2}+\frac{B}{s+4}$$

$$\Rightarrow \frac{-s-8}{(s+2)(s+4)}=\frac{A}{(s+2)}+\frac{B}{s+4}$$

$$-s-8=A(s+4)+B(s+2)$$

For $s=-2 \Rightarrow -6 = 2A \Rightarrow \boxed{A=-3}$.

For $s=-4 \Rightarrow -4 = -2B \Rightarrow \boxed{B=2}$

Hence

$$F(s) = 5 - \frac{3}{s+2} + \frac{2}{s+4}$$

$$\boxed{f(t) = 5\,\delta(t) - 3e^{-2t} + 2e^{-4t}}$$

$$\boxed{f(t) = 5\,\delta(t) - \left(3e^{-2t} - 2e^{-4t}\right)u(t).}$$

Ex-6: Find f(t) for $F(s)=\dfrac{10(s^2+119)}{(s+5)(s^2+10s+169)}$

$$F(s)=\frac{10(s^2+119)}{(s+5)(s^2+10s+169)}=\frac{10(s^2+119)}{(s+5)(s+5-j12)(s+5+j12)}$$

$$=\frac{a}{(s+5)}+\frac{b}{(s+5-j12)}+\frac{b^*}{(s+5+j12)}$$

Note: complex roots ALWAYS appear as complex conjugate pairs in real circuits with physically realizable sources.

Using the same procedure as before, we find

a = 10 $\quad$ b = $4.17\langle 90^\circ$ $\quad$ b^* = $4.17\langle -90^\circ$

$$F(s)=\frac{10(s^2+119)}{(s+5)(s^2+10s+169)}=\frac{10}{(s+5)}+\frac{4.17\langle 90^\circ}{(s+5-j12)}+\frac{4.17\langle -90^\circ}{(s+5+j12)}$$

Use Tables 12.1 and 12.3 $\quad \dfrac{K\langle\theta}{s+\alpha-j\beta}+\dfrac{K\langle-\theta}{s+\alpha+j\beta}\Leftrightarrow 2|K|e^{-\alpha t}\cos(\beta t+\theta)$

$$f(t)=\left[10e^{-5t}+2(4.17)e^{-5t}\cos(12t+90^\circ)\right]u(t)$$

$$=\underline{\underline{\left[10e^{-5t}-2(4.17)e^{-5t}\sin(12t)\right]u(t)}}$$

Ex-7: Plot the poles and zeros of the following rational function.

$$F(s) = \frac{8s^2 + 120s + 400}{2s^4 + 20s^3 + 70s^2 + 100s + 48}$$

$$= \frac{4(s+5)(s+10)}{(s+1)(s+2)(s+3)(s+4)}$$

Poles : -1, -2, -3, -4

Zeros : -5, -10

Ex-8: Prove Initial-value and Final-value Theorems for the following:

Given: $$f(t) = \left[-12e^{-6t} + 20e^{-3t}\cos(4t - 53.13^\circ)\right]u(t)$$

$$F(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)}$$

The initial-value theorem states $\boxed{\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)}$

Find: $\lim_{s\to\infty} sF(s)$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s\frac{100(s+3)}{(s+6)(s^2+6s+25)} \approx \lim_{s\to\infty}\frac{s^2}{s^3} = 0$$

Evaluate f(t) at t= 0^+ $$f(t=0^+) = \left[-12e^0 + 20e^0\cos(-53.13^\circ)\right]$$

$$= -12 + 20(0.6) \quad = -12 + 12 \quad = 0$$

Initial value theorem is proved.

The final-value theorem states $\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$

Find: $\lim_{s\to 0} sF(s)$

$$\lim_{s\to 0} sF(s) \;=\lim_{s\to 0} s\frac{100(s+3)}{(s+6)(s^2+6s+25)} \;=\frac{0}{6(25)} \qquad =0$$

Check this with the known f(t).

$$f(t\to\infty)=\left[-12e^{-\infty}+20e^{-\infty}\cos(\infty-53.13^\circ)\right]$$

$$=0+0(0.6) \qquad =0$$

Answer checks!

Final value theorem is proved.