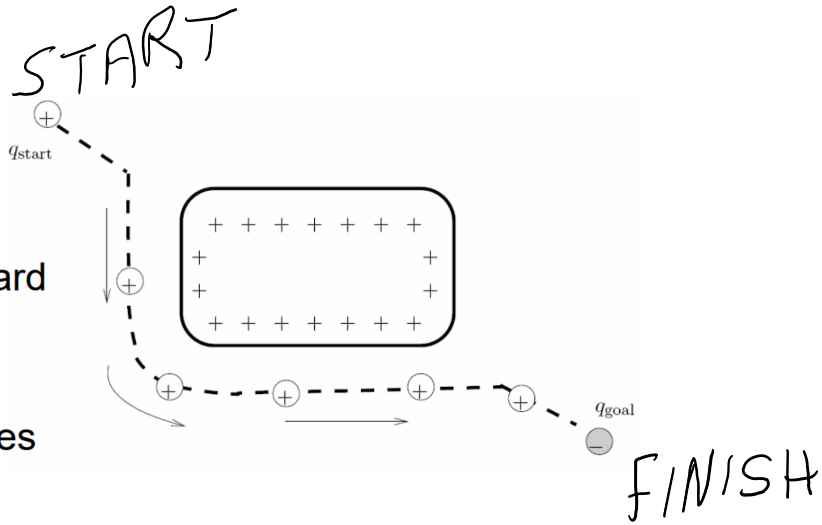
The Basic Idea

- A really simple idea:
  - Suppose the goal is a point $g \in \Re^2$
  - Suppose the robot is a point $r \in \Re^2$
  - Think of a "spring" drawing the robot toward the goal and away from obstacles:
  - Can also think of like and opposite charges



The General Idea

- Both the bowl and the spring analogies are ways of storing potential *energy*
- The robot moves to a lower energy configuration
- A *potential function* is a function U : $\Re^m \to \Re$
- Energy is minimized by following the negative *gradient* of the potential energy function:

$$\nabla U(q) = DU(q)^T = [\frac{\partial U}{\partial q_1}(q), \ldots, \frac{\partial U}{\partial q_m}(q)]^T$$

- We can now think of a *vector field* over the space of all q's ...
  - at every point in time, the robot looks at the vector at the point and goes in that direction

$$\nabla u = \frac{\partial u}{\partial x}\hat{x} + \frac{\partial u}{\partial y}\hat{y} + \frac{\partial u}{\partial z}\hat{z}$$

Attractive/Repulsive Potential Field

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

- U_{att} is the “attractive” potential --- move to the goal
- U_{rep} is the “repulsive” potential --- avoid obstacles

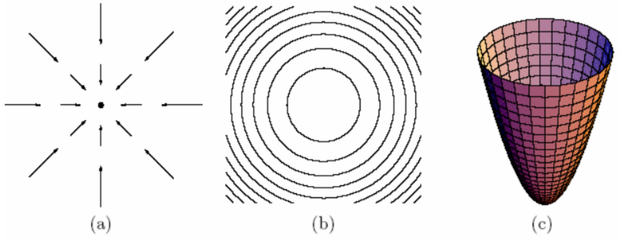
Artificial Potential Field Methods: Attractive Potential

Conical Potential

$$U(q) = \zeta d(q, q_{\text{goal}}).$$

$$\nabla U(q) = \frac{\zeta}{d(q,q_{\text{goal}})}(q - q_{\text{goal}}).$$

Quadratic Potential



$$U_{\text{att}}(q) = \frac{1}{2}\zeta d^2(q, q_{\text{goal}}),$$

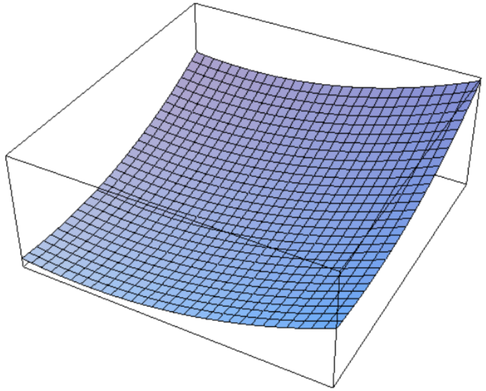
$$\begin{aligned} F_{\text{att}}(q) = \nabla U_{\text{att}}(q) &= \nabla\left(\frac{1}{2}\zeta d^2(q, q_{\text{goal}})\right), \\ &= \frac{1}{2}\zeta \nabla d^2(q, q_{\text{goal}}), \\ &= \zeta(q - q_{\text{goal}}), \end{aligned}$$

Artificial Potential Field Methods: Attractive Potential

Combined Potential

$$U_{\text{att}}(q) = \begin{cases} \frac{1}{2}\zeta d^2(q, q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ d^*_{\text{goal}}\zeta d(q, q_{\text{goal}}) - \frac{1}{2}\zeta (d^*_{\text{goal}})^2, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}. \end{cases}$$

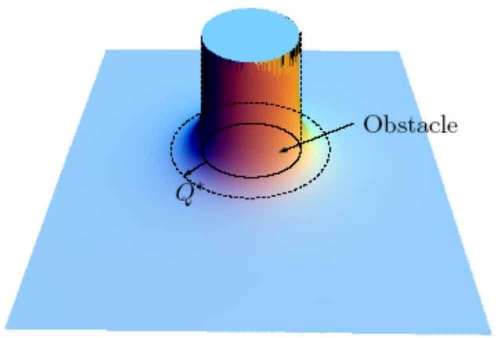
$$\nabla U_{\text{att}}(q) = \begin{cases} \zeta(q - q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ \dfrac{d^*_{\text{goal}}\zeta(q - q_{\text{goal}})}{d(q, q_{\text{goal}})}, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}, \end{cases}$$



In some cases, it may be desirable to have distance functions that grow more slowly to avoid huge velocities far from the goal

one idea is to use the quadratic potential near the goal (< d*) and the conic farther away
One minor issue: what?

The Repulsive Potential



$$U_{\text{rep}}(q) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{D(q)} - \frac{1}{Q^*}\right)^2, & D(q) \leq Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

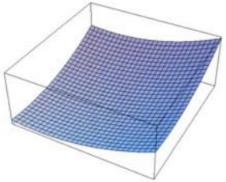
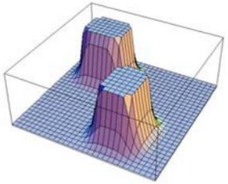
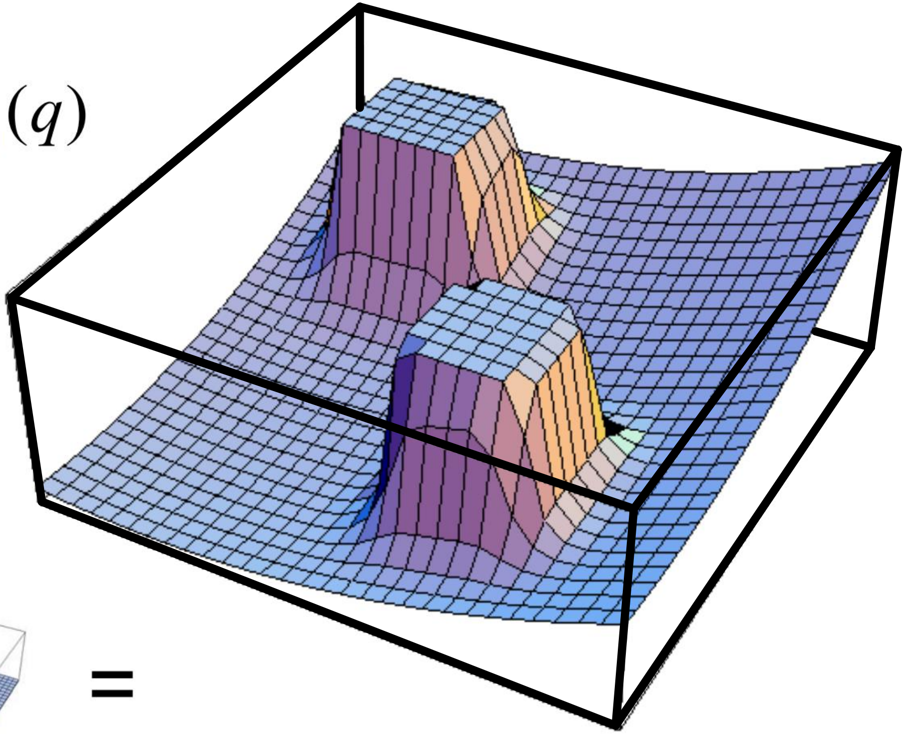
whose gradient is

$$\nabla U_{\text{rep}}(q) = \begin{cases} \eta\left(\frac{1}{Q^*} - \frac{1}{D(q)}\right)\frac{1}{D^2(q)}\nabla D(q), & D(q) \leq Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

Total Potential Function

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

$$F(q) = -\nabla U(q)$$

 +  = 

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Path planning methods

Path planning

Path planning is one of the most fundamental problems in robotics. Path planning has several applications in robotics and many other areas, such as video game design and the study of biological molecules. This summary discusses some of the most popular planning algorithms.

Terminology

- q is the configuration variable
- Q is the configuration space.

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Path planning methods

The Potential Field Method

This is one of the most popular methods used for path planning. It was introduced in 1980s by Osama Khatib from Stanford University. In this method, the robot moves in an artificial potential field that is the sum of attractive and repulsive fields. The most important variables are U : the artificial potential field and F : the force resulting from U. A more detailed description is below.

Principle of the method

- The goal location generates an attractive potential pulling the robot towards the goal
- The obstacles generate a repulsive potential pushing the robot away from the obstacles
- *Artificial potential:*

$$U(q) = U_{att}(q) + U_{rep}(q) \tag{1}$$

  - $U_{att}(q)$: attractive potential.
  - $U_{rep}(q)$: repulsive potential.

- *Artificial force:*

$$F(q) = -\nabla U(q)$$

where

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}$$

Example: robot modeled as a point ($q = [x, y]^T$)

$$F(q) = -\nabla U(q) = \begin{bmatrix} \frac{-\partial U}{\partial x} \\ \frac{-\partial U}{\partial y} \end{bmatrix}$$

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- Example:
- We want to plan the path for the two-link planar manipulator in the figure using the potential field method. We will first calculate an attractive force felt between $O_1(q_f)$ and $O_1(q_0)$ and between $O_2(q_f)$ and $O_2(q_0)$. In each case the force will be proportion to the distance between each pair of points. We will then calculate the repulsive force experienced by $O_1(q_0)$ and $O_2(q_0)$ resulting from each of two obstacles [(1,1) and (2,0.5)]. Finally we will sum the attractive and repulsive forces together for $O_1(q_f)$ and $O_2(q_0)$ to obtain the net force on each point.

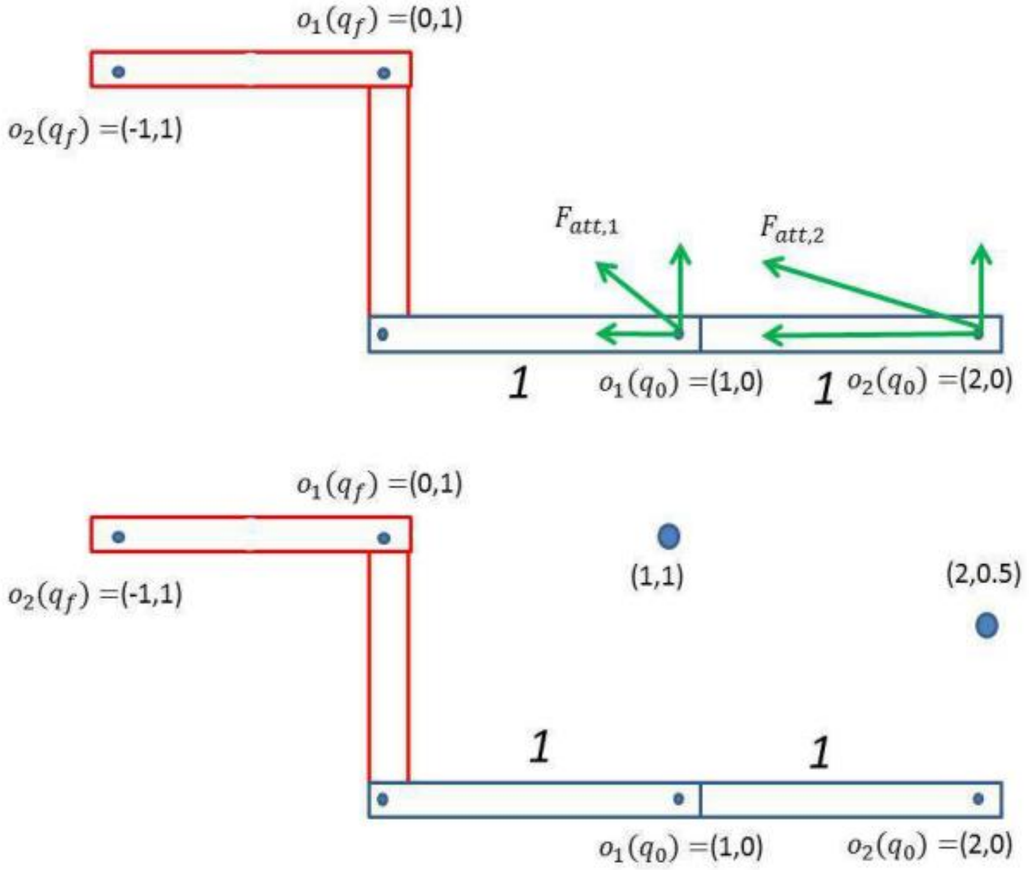

Fig. 3. Two link planar manipulator without (top) and with obstacles (bottom)

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Path planning methods

Calculating the attractive force

One choice for the potential field is

$$U_{att,i}(q) = 0.5\zeta_i \|o_i(q) - o_i(q_f)\|^2$$

This field is referred to as the parabolic well potential. Note that $o_i(q)$ is the origin of frame i attached to link i. q_f is the desired final configuration. ζ_i is a scaling parameter used to scale the effect of the attractive potential. The attractive force is given by

$$F_{att,i}(q) = -\nabla U_{att,i}(q) = -\zeta_i(o_i(q) - o_i(q_f))$$

Equation 6 is a quadratic fucntion of the distance. It is also possible to use a linear function of the distance. A quadratic function however presents several advantages, including simplicity of the calculations.

Calculating repulsive force

One choice for the potential field is

$$U_{rep,i}(q) = \begin{bmatrix} 0.5\eta_i \left(\frac{1}{\rho(o_i(q))} - \frac{1}{\rho_0}\right)^2 \ \texttt{if} \ \rho(o_i(q)) \leq \rho_0 \\ 0 \ \texttt{if} \ \rho(o_i(q)) > \rho_0 \end{bmatrix}$$

where ρ_0 is the distance of influence of the obstacle. $\rho(o_i(q))$ shortest distance between o_i and the obstacle. η_i is a scaling parameter. The repulsive force for $\rho(o_i(q)) \leq \rho_0$ is given by

$$F_{rep,i}(q) = \eta_i \left(\frac{1}{\rho(o_i(q))} - \frac{1}{\rho_0}\right) \frac{1}{\rho^2(o_i(q))} \nabla\rho(o_i(q))$$

where $\nabla\rho(o_i(q))$indicates the gradient calculated at $o_i(q)$. For a point obstacle b, $\rho(o_i(q))= \|o_i(q) - b\|$. Its gradient is

$$\nabla\rho(o_i(q))= \frac{o_i(q) - b}{\|o_i(q) - b\|}$$

The forces and the gradient are vectors. The potential is a scalar function.

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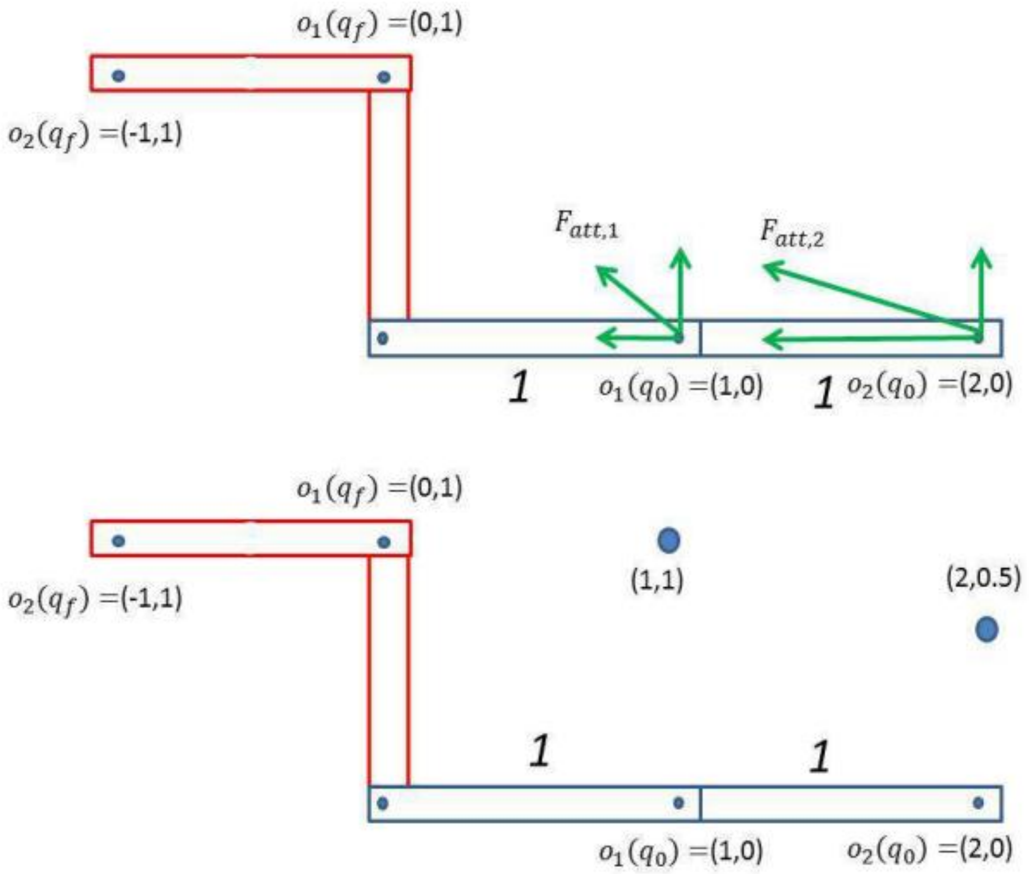

Fig. 3. Two link planar manipulator without (top) and with obstacles (bottom)

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From the figure, we have

$$o_1(q_0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$o_1(q_f) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

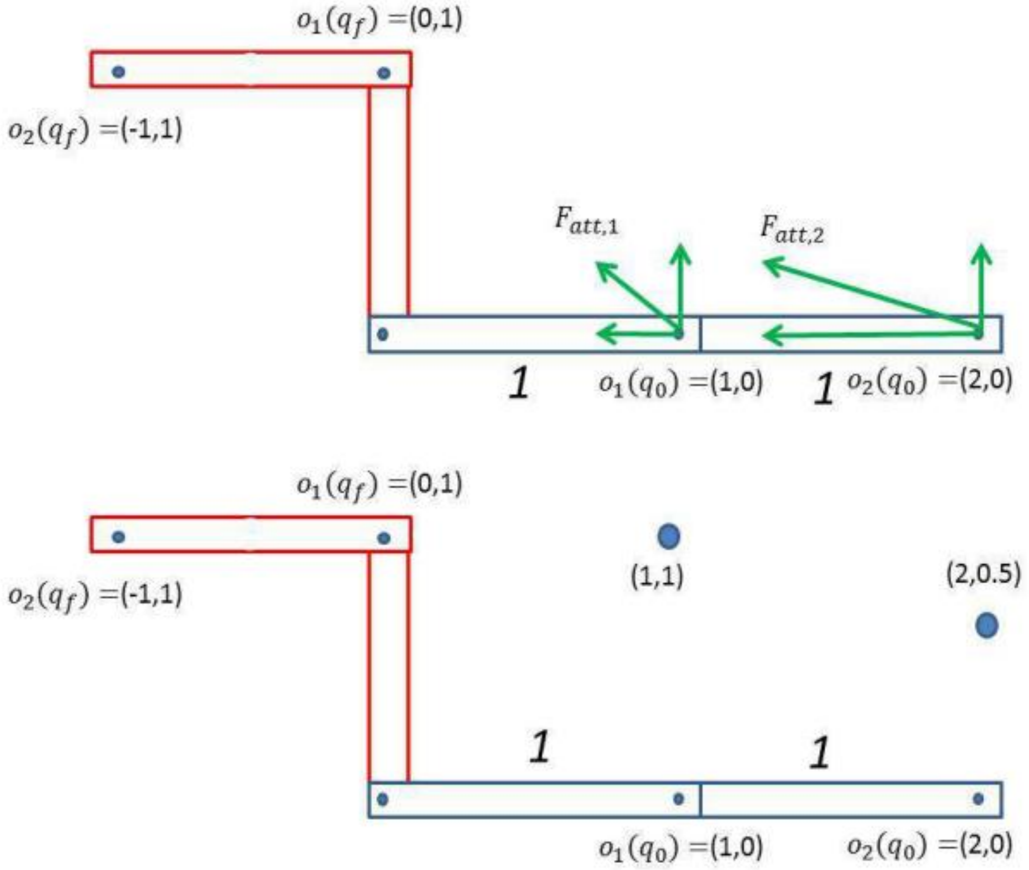
$$o_2(q_0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$o_2(q_f) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Using the equations for the attractive force, we get:

$$F_{att,1} = -\zeta[o_1(q_0) - o_1(q_f)] = \zeta \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$F_{att,2} = -\zeta[o_2(q_0) - o_2(q_f)] = \zeta \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$



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Calculating repulsive force between $O_2(q_0)$ and obstacle at (2,0)

Calculating repulsive force

One choice for the potential field is

$$U_{rep,i}(q) = \left[\begin{array}{c} 0.5\eta_i \left(\frac{1}{\rho(o_i(q))} - \frac{1}{\rho_0}\right)^2 \ \texttt{if} \ \rho(o_i(q)) \leq \rho_0 \\ 0 \ \texttt{if} \ \rho(o_i(q)) > \rho_0 \end{array} \right] \tag{7}$$

where ρ_0 is the distance of influence of the obstacle. $\rho(o_i(q))$ shortest distance between o_i and the obstacle. η_i is a scaling parameter. The repulsive force for $\rho(o_i(q)) \leq \rho_0$ is given by

$$F_{rep,i}(q) = \eta_i \left(\frac{1}{\rho(o_i(q))} - \frac{1}{\rho_0}\right) \frac{1}{\rho^2(o_i(q))} \nabla\rho(o_i(q)) \tag{8}$$

where $\nabla\rho(o_i(q)$ indicates the gradient calculated at $o_i(q)$. For a point obstacle b, $\rho(o_i(q) = \|o_i(q) - b\|$. Its gradient is

$$\nabla\rho(o_i(q) = \frac{o_i(q) - b}{\|o_i(q) - b\|} \tag{9}$$

The forces and the gradient are vectors. The potential is a scalar function.

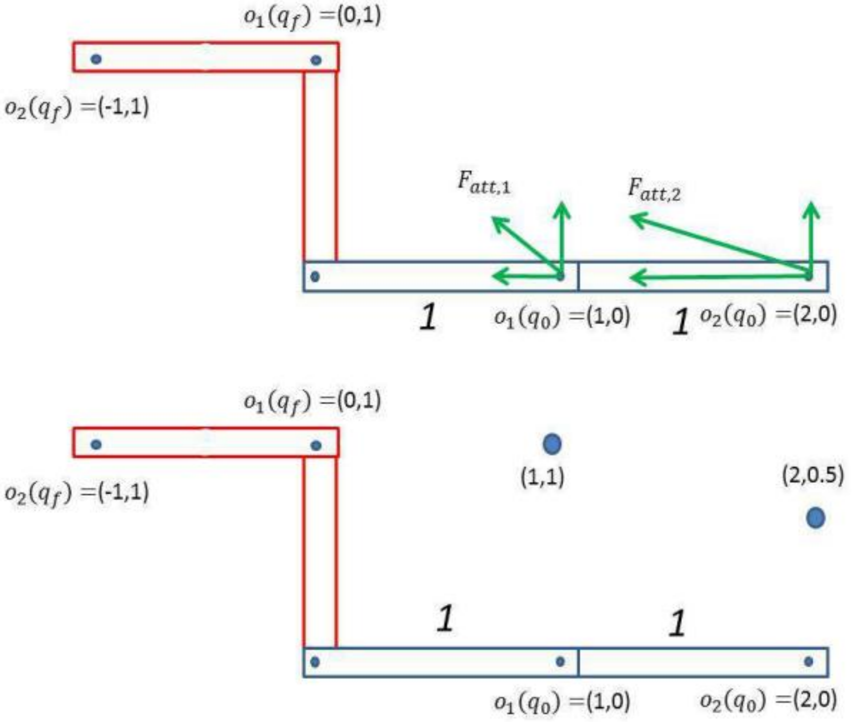

Fig. 3. Two link planar manipulator without (top) and with obstacles (bottom)

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Calculating repulsive force between $O_2(q_0)$ and obstacle at (2,0)

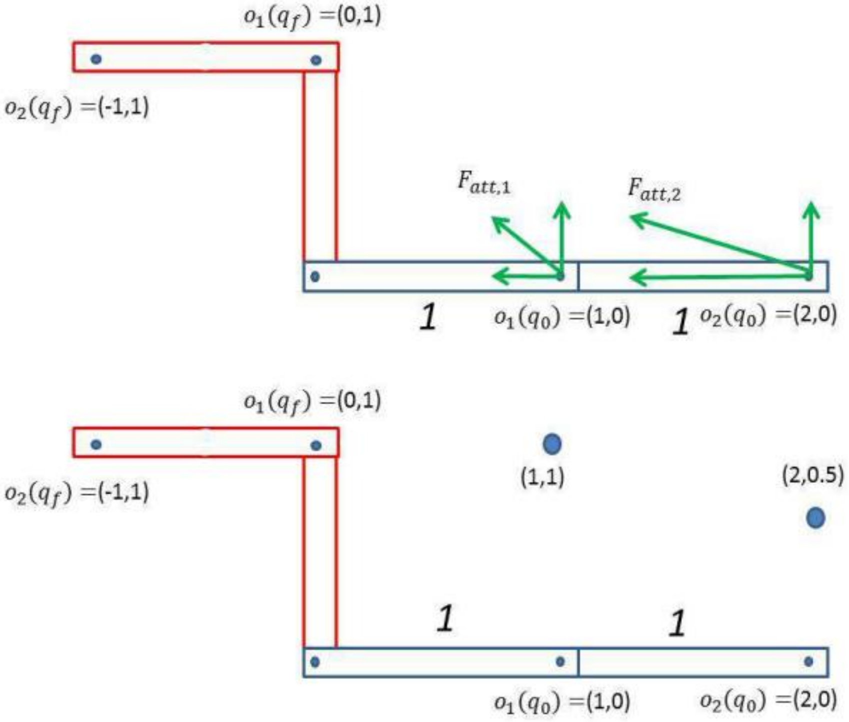
We take the threshold $\rho_0 = 1$. For this value, o_1 is not in the zone of influence of the obstacle, however, o_2 is. We calculate the repulsive force applied to o_1 and o_2.

1) For o_1, we have
$$F_{rep,1} = 0$$
2) To calculate $F_{rep,2}$, we need to know $\rho(o_2(q))$ $\nabla\rho(o_2(q))$. We have:

Distance from repulsive obstacle = $o_2(q) - b = \begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$

and
$$||o_2(q) - b|| = \sqrt{(2-2)^2 + (0-0.5)^2} = \rho(o_2(q))$$

Therefore,
$$\nabla\rho(o_2(q)) = \frac{\begin{bmatrix} 0 \\ -0.5 \end{bmatrix}}{\sqrt{(2-2)^2 + (0-0.5)^2}} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$



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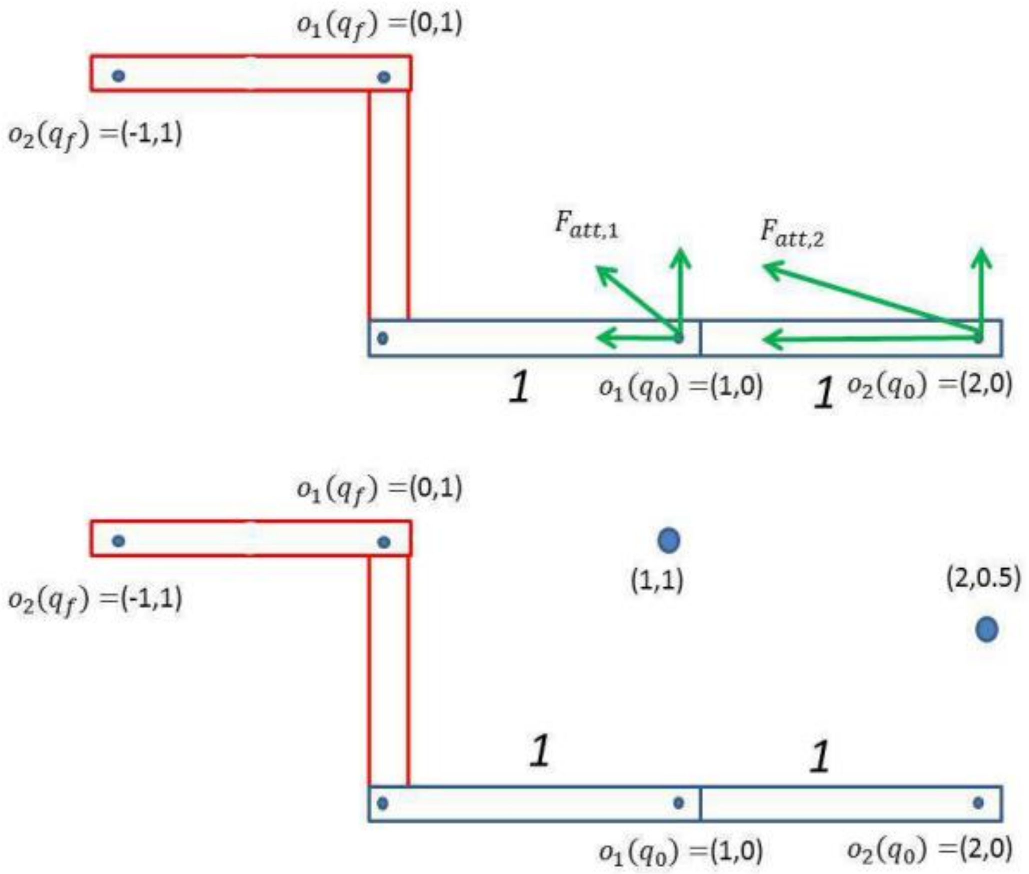
and
$$F_{rep,2} = \eta_2\left(\frac{1}{0.5} - 1\right)\left(\frac{1}{0.25}\right)\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \eta_2 \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$
For $\eta_2 = 1$ and $\zeta_2 = 1$, the total force is
$$F_{total,2} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -4 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$



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$$F_{att,1} = -\zeta[o_1(q_0) - o_1(q_f)] = \zeta \begin{bmatrix} -1 \\ 1 \end{bmatrix} = F_{total,1}$$

$$F_{att,2} = -\zeta[o_2(q_0) - o_2(q_f)] = \zeta \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Calculating repulsive force between $O_2(q_0)$ and obstacle at (2,0)

We take the threshold $\rho_0 = 1$. For this value, o_1 is not in the zone of influence of the obstacle, however, o_2 is. We calculate the repulsive force applied to o_1 and o_2.

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