CALIFORNIA STATE UNIVERSITY
SACRAMENTO

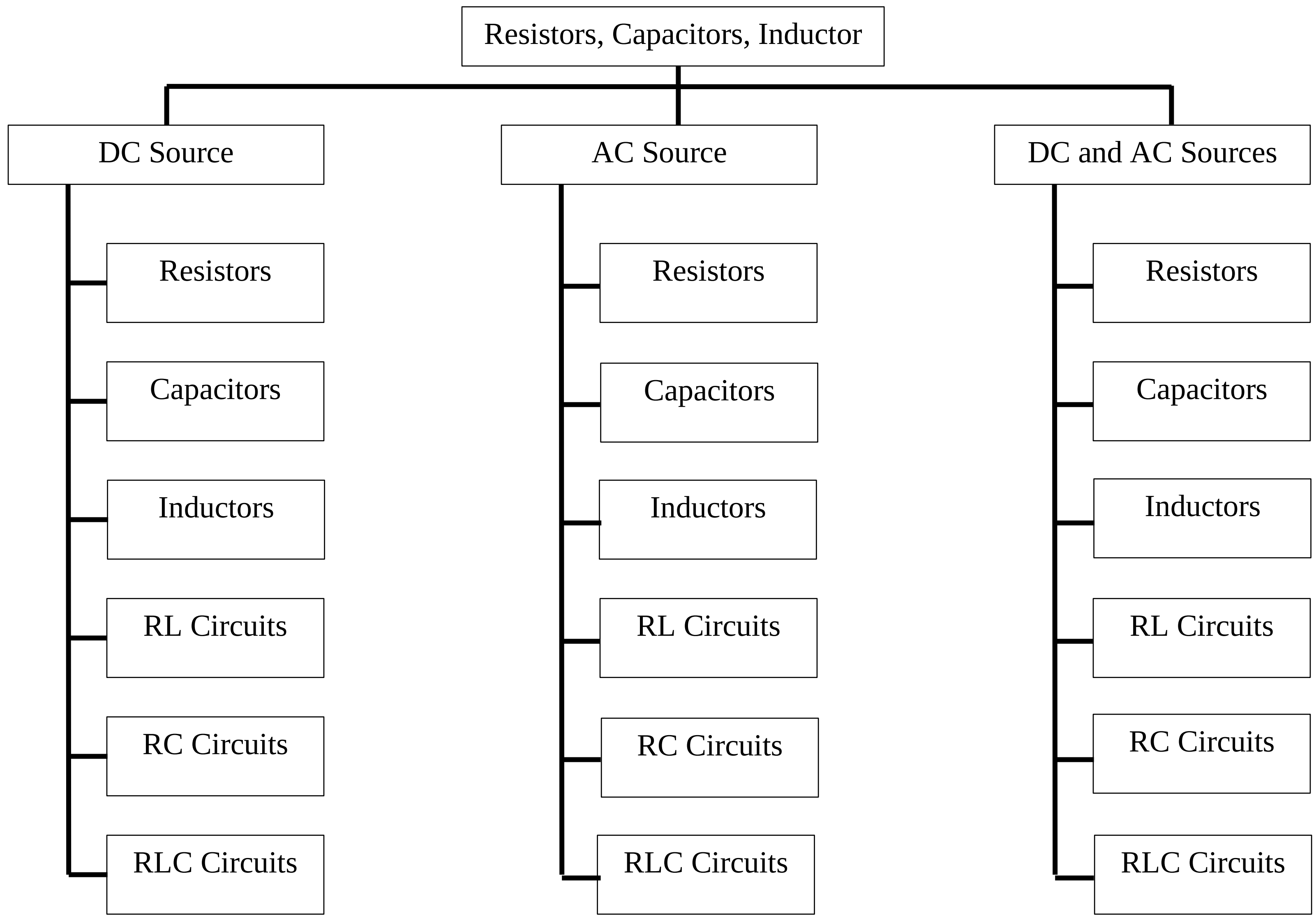
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EEE 117 Network Analysis

Text: Electric Circuits by J. Nilsson and S. Riedel Prentice Hall

Lecture Set 1: Sinusoidal Steady State Analysis

Instructor: Riaz Ahmad


Resistors, Capacitors, Inductor
DC Source
AC Source
DC and AC Sources
Resistors
Capacitors
Inductors
RL Circuits
RC Circuits
RLC Circuits
Resistors
Capacitors
Inductors
RL Circuits
RC Circuits
RLC Circuits
Resistors
Capacitors
Inductors
RL Circuits
RC Circuits
RLC Circuits

Sinusoidal Steady-State Analysis

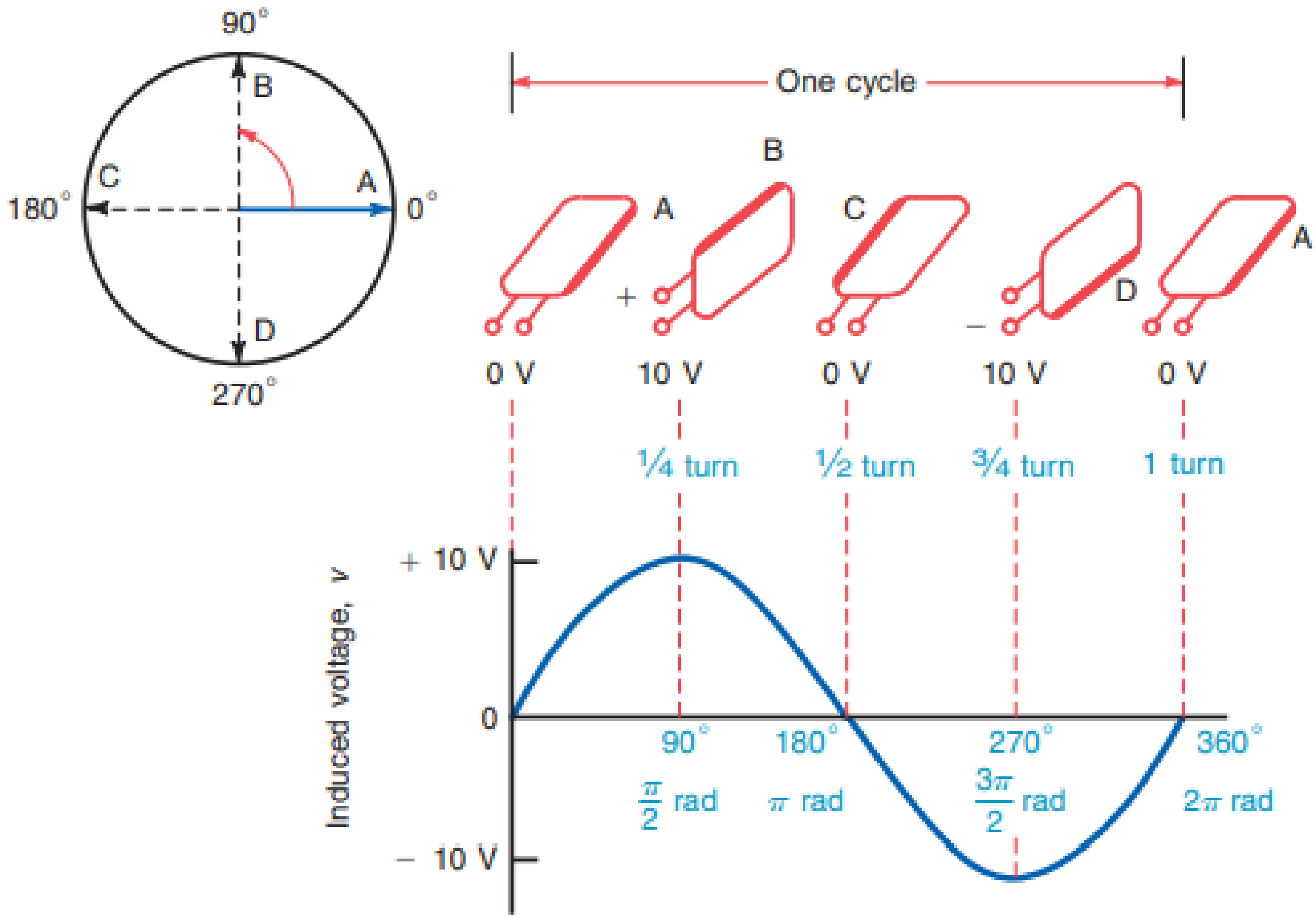
- Sinusoidal Source
- Phase Angle
- Phase Difference
- Phasor
- Representation of a Sinusoid
- Type of values of a Sinusoidal Signal
- Sinusoidal Response
- Behavior of Passive Circuit Elements with Sinusoid
- Good News AND Bad News
- Mutual Inductance
- Transformer
- Ideal Transformer

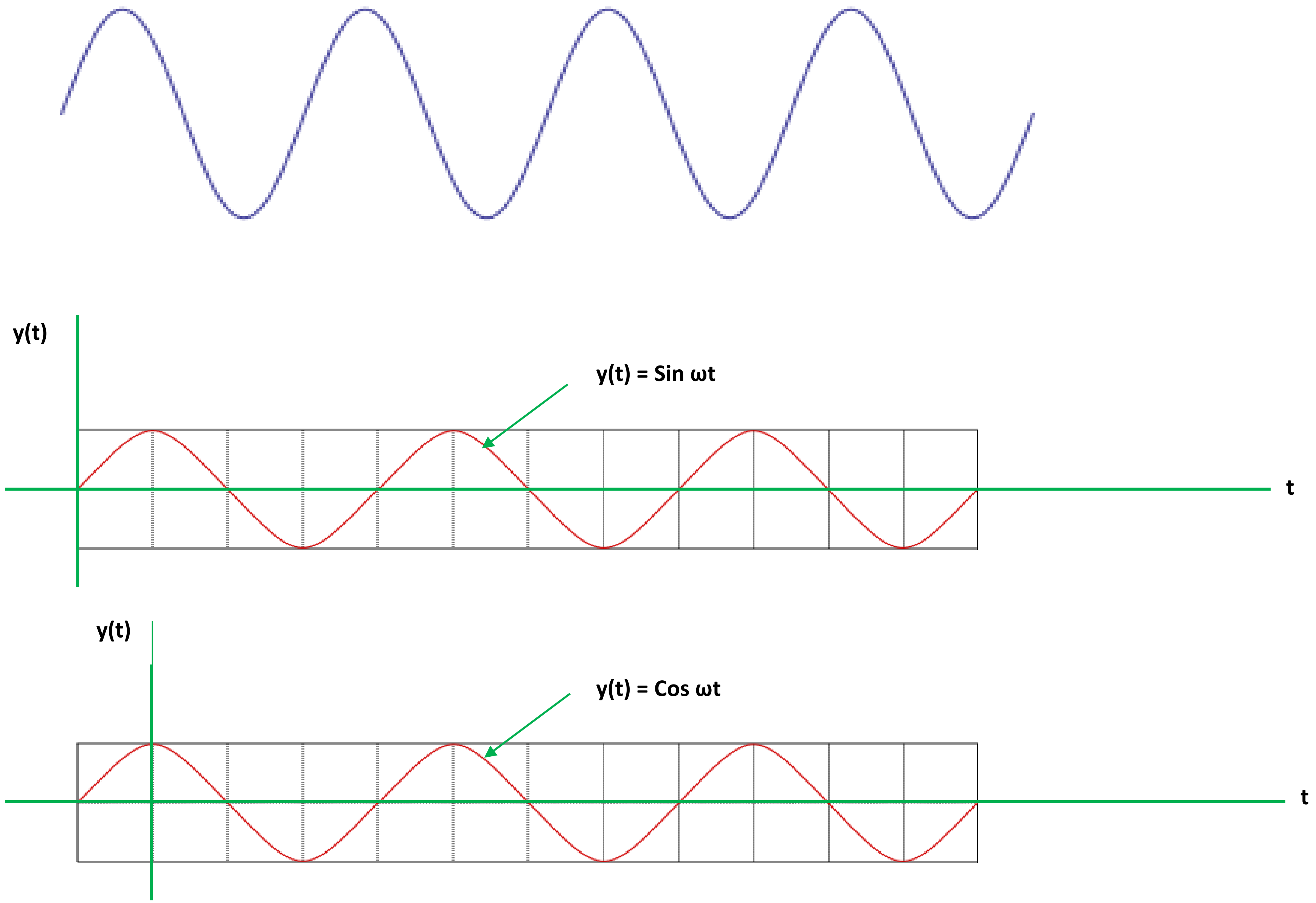
The Sinusoidal Source

- A sinusoidal voltage or current source produces a voltage or current that varies sinusoidally with respect to time.

Principle of Generation of sinusoidal Source:

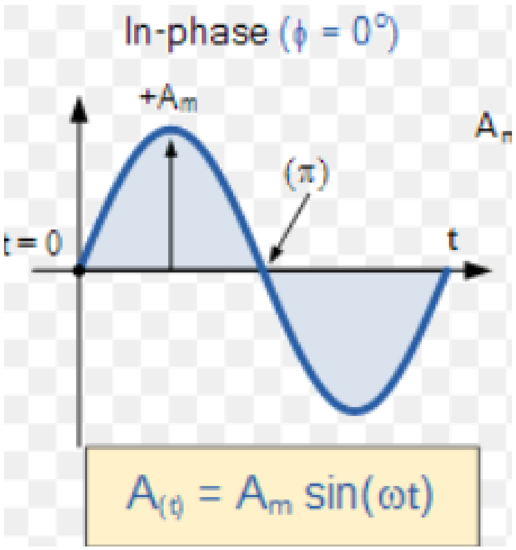
- A relative motion of magnetic field and a conductor inside the field induced emf.
- Induced emf = $-N\, d\Phi_B/dt$, which in turn creates voltage and current.
- A loop rotating in a magnetic field, produce induced voltage "v" with alternating polarities.
- Loop conductors moving parallel to magnetic field results in zero voltage
- Loop conductors cutting across magnetic field produce maximum induced voltage.


90°
B
C
A
180°
0°
D
270°
One cycle
B
A
C
A
+
D
−
0 V
10 V
0 V
10 V
0 V
¼ turn
½ turn
¾ turn
1 turn
Induced voltage, v
+ 10 V
0
− 10 V
90°
180°
270°
360°
π/2 rad
π rad
3π/2 rad
2π rad



y(t)
y(t) = Sin ωt
t
y(t)
y(t) = Cos ωt
t

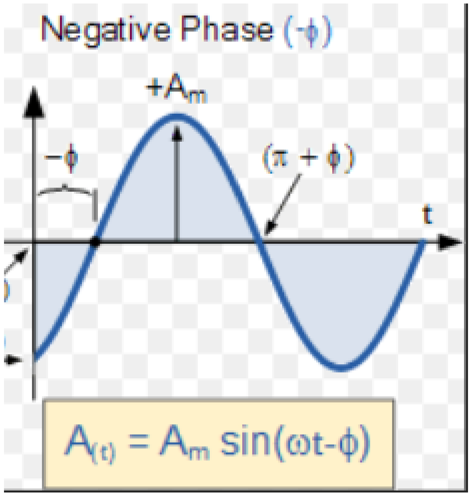
Case 1: (In Phase)



- Cycle starts at t=0, ωt = 0 and emf=0 at t=0
- emf values are constantly changing at every instant or at every angle from 0 to 2π.
- It attains three zero and two peak values in one complete cycle
- Angular frequency of rotation = ω rad/s
- Time period to complete one cycle (T) = 2π / ω
- The wave form generated are sinusoidal

$$\boxed{v(t) = V_m \text{ Sin } \omega t \qquad i(t) = I_m \text{ Sin } \omega t}$$

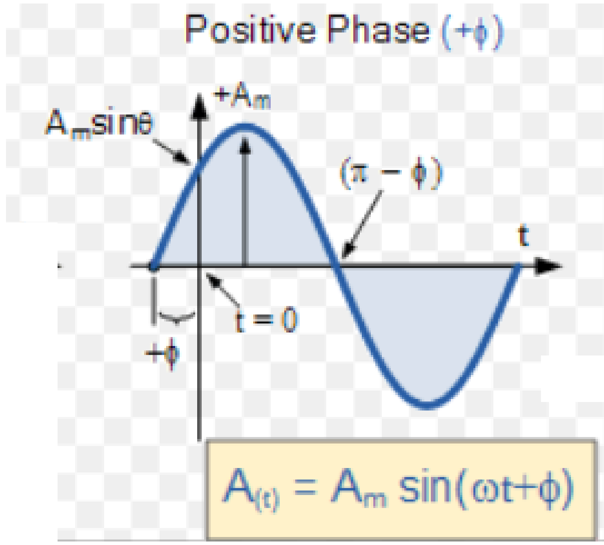
Case 2: (Negative Phase)



- At t=0, ωt = - ϕ and emf is not "0" at t=0.
- Same as case 1, emf values are constantly changing at every instant or at every angle.
- It attains three zero and two max values in one complete cycle
- Angular frequency of rotation = ω rad/s
- Time period to complete one cycle (T) = 2π / ω
- The wave form generated are sinusoidal

$$v(t) = V_m \, \text{Sin}(\omega t - \phi) \qquad i(t) = I_m \, \text{Sin}(\omega t - \phi)$$

Case 3: (Positive Phase)



- At t=0, $\omega t = \phi$ and emf is not "0" at t=0.
- Same as case 1 & 2, emf values are constantly changing at every instant or at every angle.
- It attains three zero and two max values in one complete cycle
- Angular frequency of rotation = ω rad/s
- Time period to complete one cycle (T) = $2\pi / \omega$
- The wave form generated are sinusoidal

$v(t) = V_m \,\mathrm{Sin}(\omega t + \phi)$ $\quad$ $i(t) = I_m \,\mathrm{Sin}(\omega t + \phi)$

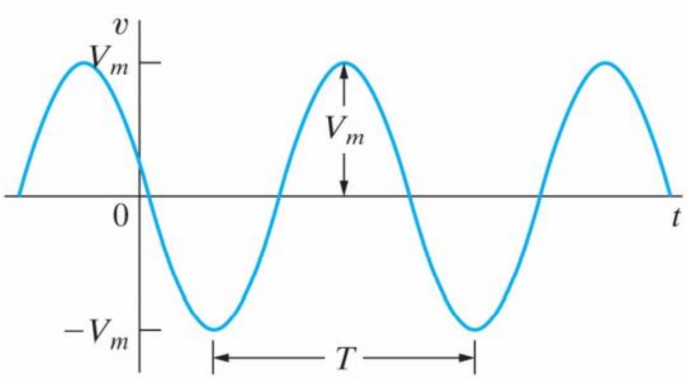
- Sinusoidal signal can be represented either by Sine or Cosine function.

$v(t) = V_m \, \mathrm{Sin}\,(\omega t \pm \phi)$ OR $v(t) = V_m \, \mathrm{Cos}\,(\omega t \pm \phi)$

$i(t) = I_m \, \mathrm{Sin}\,(\omega t \pm \phi)$ OR $i(t) = I_m \, \mathrm{Cos}\,(\omega t \pm \phi)$

- Some prefer Sine function, while other prefer Cosine function to represent sinusoids.
- In this course we will use cosine function.
- A typical sinusoidal voltage source is given as below:
- The period of the sinusoid function is T in seconds.
- The frequency is f (in Hz).
- The angular frequency is ω in radian/sec.

$$f = \frac{1}{T}$$



$$v = V_m \cos(\omega t + \phi)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Phase Angle

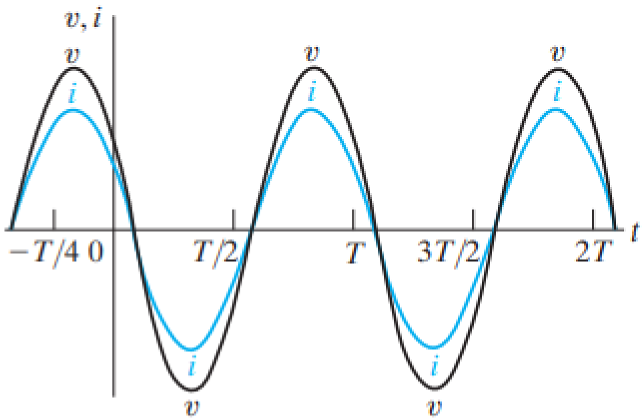
- Phase angle (φ) determine the value of sinusoidal function at $t=0$.
- Phase angle is a fixed point on the periodic wave at which we start measuring time.
- Changing the phase angle shifts the sinusoid along the time axis.
- Changing the phase angle has no effect on either the amplitude or frequency of sinusoid.
- Phase angle can be, 0, negative or positive.

$V_1(t) = V_m \cos(\omega t + 0)$ =====➔ Phase angle = 0

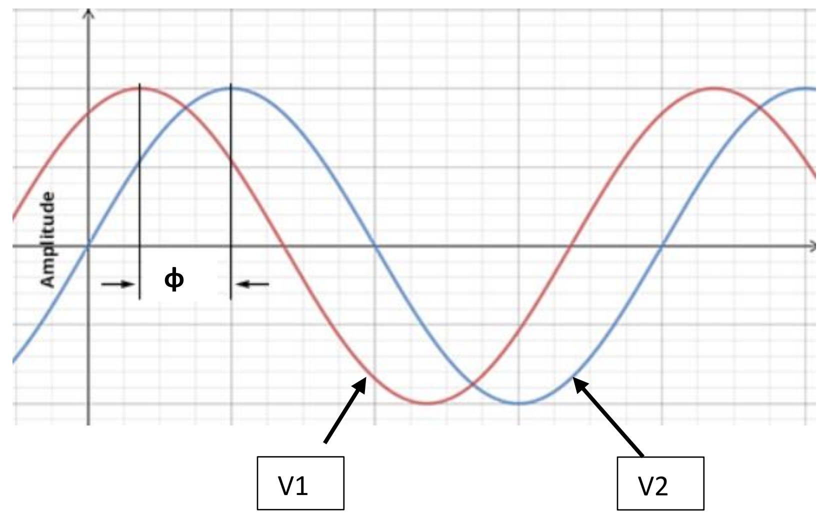
$V_2(t) = V_m \cos(\omega t - \phi)$ =====➔ Phase angle = $-\phi$

$V_3(t) = V_m \cos(\omega t + \phi)$ =====➔ Phase angle = $+\phi$

Phase Difference



No Phase Difference



- Phase difference is the angular difference b/w the +ve peak values of two sinusoidal functions having same frequency.

  $V_1(t) = V_m \cos(\omega t + 0)$ =====→ Phase angle = 0

  $V_2(t) = V_m \cos(\omega t - \varphi)$ =====→ Phase angle = $-\varphi$
- Peak and Zero values of sinusoids do not reach at same time.
- V_2 reaches at its +ve peak value first, then V_1.
- V_2 leads V_1 OR V_1 lags V_2
- PD b/w V_1 & $V_2 = \varphi$

Phasor

- Phasor is a complex number that represents the amplitude and the initial phase at t=0 of a sinusoidal function.
- Phasor is a representation of ONLY a sinusoidal quantity in the form of vector.
- Phasor is nothing but an arrow whose height represent Sine and its width represent Cosine of phase angle.
- Phasor must have Sin(ωt) or Cos(ωt) term.
- Phasor is usually not used for resistive network but can be used.
- Phasor is very useful for a complex network consisting of R, L and C.
- A sinusoidal function is represented in the form of phasor as shown below.

$$\boxed{V_m \cos(\omega t + \phi) \Rightarrow V_m \angle \phi}$$

From Euler identity:

$e^{\pm j\theta}$= Cos Θ ± j Sin Θ

$\boxed{\text{Cos } \Theta = \text{Real } \{ e^{j\theta} \}}$ and $\boxed{\text{Sin } \Theta = \text{Imaginary } \{ e^{j\theta} \}}$

So, if we have a sinusoidal signal

$V(t) = V_m \, \text{Cos}(\omega t+\varphi)$ we can write it as:

$V(t) = V_m \, \text{Real } \{ e^{j(\omega t+\varphi)} \} = V_m \, \text{Real } \{ e^{j\omega t} e^{j\varphi} \} = \text{Real } \{ V_m \, e^{j\omega t} e^{j\varphi} \}$

The quantity $V_m \, e^{j\varphi}$ is a complex number which carries the information about amplitude and the phase angle of the sinusoid. This quantity is phasor representation, phasor transform or just phasor. Now we can represent any sinusoidal quantity as phasor.

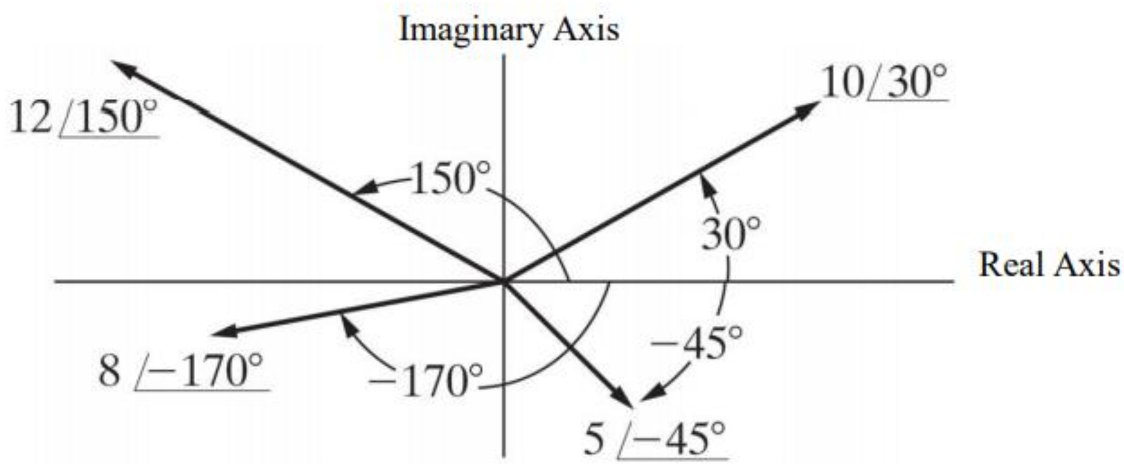
$\boxed{V_m \, \text{Cos}(\omega t+\varphi) = V_m \, e^{j\varphi} = V_m \, \{ \text{Cos } \varphi \pm j \text{ Sin } \varphi \} = V_m \, L \, \varphi}$

Conclusion: Phasor representation is very easy to manipulate.

Phasor transforms the sinusoidal function from time domain to complex # domain.

Phasor Diagrams:

- A phasor diagram shows the magnitude and phase of a quantity on the complex number plane.
- Phase angles are measured counter-clockwise from the positive real axis.
- Magnitude is measured from the origin of the axes.



Representation of a Sinusoid

- A sinusoid can be represented in the following ways:

Graphical form:

$v(t) = V_m \, Sin\,(\omega t \pm \Phi)$ OR $v(t) = V_m \, Cos\,(\omega t \pm \Phi)$

$i(t) = I_m \, Sin\,(\omega t \pm \Phi)$ OR $i(t) = I_m \, Cos\,(\omega t \pm \Phi)$

Phasor Form:

$v(t) = V_m \angle \underline{\pm\Phi}$ OR $i(t) = I_m \angle \underline{\pm\Phi}$

Trigonometric form:

$v(t) = V_m \, Cos\,\Phi \pm j\, V_m \, Sin\,\Phi$

$i(t) = I_m \, Cos\,\Phi \pm j\, I_m \, Sin\,\Phi$

Rectangular form :

$v(t) = a \pm jb$ $\quad \Phi = \tan^{-1}\left(\frac{b}{a}\right)$

$i(t) = a \pm jb$ $\quad \Phi = \tan^{-1}\left(\frac{b}{a}\right)$

Type of values of a Sinusoidal Signal

- **Instantaneous Value:** It is the value of an alternating quantity at a particular instant of time.

$v(t) = V_m \text{ Sin } (\omega t \pm \Phi)$ or $v(t) = V_m \text{ Cos } (\omega t \pm \Phi)$

$i(t) = I_m \text{ Sin } (\omega t \pm \Phi)$ or $i(t) = I_m \text{ Cos } (\omega t \pm \Phi)$

- **Peak values:** It is the maximum value +ve or –ve in sinusoid. It is represented by symbols:

V_m, V_0, V and I_m, I_0, I

- **Average Values:** It is the average of all the instantaneous values of an alternating quantity over half cycle. WHY?

$V_{av} = (2/\pi) V_m = 0.636 V_m$ and $I_{av} = (2/\pi) I_m = 0.636 I_m$

- **RMS Values:** RMS or Root-Mean-Square value generate the same amount of heat in a particular component for a particular period of time as it will produce by DC for the same duration.

$V_{rms} = V_m / \sqrt{2} = 0.707 V_m$ $I_{rms} = I_m / \sqrt{2} = 0.707 I_m$

Derivation of RMS value: (Root of Mean of Square)

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)\, dt}$$

For one time period ie T=2π

$$= V_m \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} \cos^2(\omega t)\, dt} \qquad = V_m \sqrt{\frac{1}{2\pi}\left[\frac{t}{2} + \frac{1}{4\omega}\sin(2\omega t)\right]_0^{2\pi}}$$

$$= V_m \sqrt{\frac{1}{2\pi}\left[\frac{2\pi}{2} - \frac{0}{2} + 0\right]} \quad = V_m\sqrt{\frac{1}{2}} \quad = \frac{V_m}{\sqrt{2}}$$

Note: The result is only valid for a sinusoid.

The Sinusoidal Response

- When we input any signal into a circuit, we expect to see an initial response and then, after a long time, a response that persists forever (until the input signal goes away).
- The initial immediate response to the signal input is called the transient response, AND, the long-lasting response to the signal input is called the steady-state response.
- The total response to the signal input is a combination of the transient and the steady-state responses.
- Here we will be discussing the steady-state response.
- The steady-state response of a circuit is also a sinusoid function.
- The frequency of the circuit's response is equal to the frequency of the input signal.
  The amplitude of the response depends on the circuit and most likely will NOT be the amplitude of the sinusoidal input.
- The phase of the circuit's response will most likely NOT be the phase of the sinusoidal input.

Behavior of Passive Circuit Elements with AC Source

Purely Resistive Circuit:

In a purely resistive circuit, voltage and current are in phase

Let assume:

$i(t) = I_m \, Cos(\omega t + \Theta_i)$ Then $v(t) = i(t) * R$

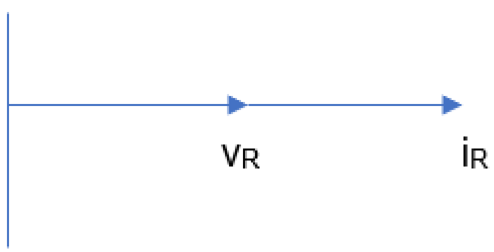
$v(t) = I_m R \, Cos(\omega t + \Theta_i)$ $v(t) = V_m \, Cos(\omega t + \Theta_i)$

For both current and voltage, phase angle is same.

The phasor form is (**V** and **I** are phasors)

$\mathbf{V} = R \, I_m \angle \Theta_i = R\mathbf{I}$

$\boxed{\mathbf{V} = R \, \mathbf{I}}$



Purely Inductive Circuit:

In a purely inductive circuit, voltage leads the current by 90°
OR Current lags voltage by 90°.

Assume $i_L = I_m \cos(\omega t+\theta_i)$, the inductor voltage is found by

$$v = L\frac{di}{dt} = L\frac{d}{dt}[I_m \cos(\omega t + \theta_i)] \quad = LI_m(-\omega)\sin(\omega t + \theta_i)$$

$$v = -\omega L I_m \cos(\omega t + \theta_i - 90^\circ)$$

Write in the polar form

$$\mathrm{V} = -\omega L I_m e^{j(\theta_i - 90^\circ)} = -\omega L I_m e^{j\theta_i} e^{-j90^\circ}$$

$$\mathrm{V} = -\omega L I_m e^{j\theta_i)}\left(-j1\right) = j\omega L I_m e^{j\theta_i}$$

$$\mathrm{V} = j\omega L I_m e^{j\theta_i} = \left(\omega L\angle 90^\circ\right) I_m \angle\theta_i$$

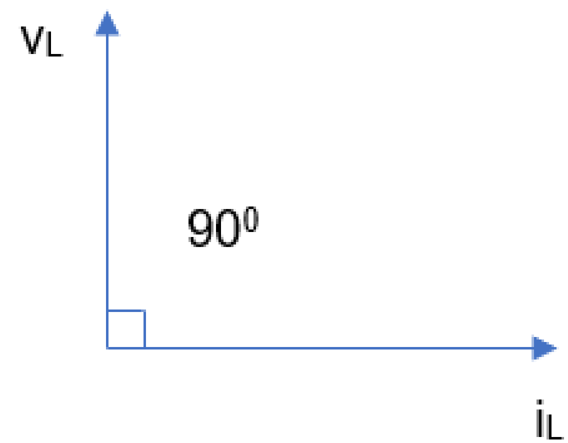
Another way of looking at $e^{-j90°}$ is to use Euler's Identity.

$$e^{-j90^\circ} = \cos 90^\circ - j\sin 90^\circ = 0 - j1 = -j$$

$$V = j\omega L\, I$$

Z_L (inductive Impedance) = $j\omega L$ = jX_L in ohm
X_L (inductive Reactance) = ωL in ohm

$$\boxed{\mathbf{V_L} = Z_L\,\mathbf{I_L}}$$



Purely Capacitive Circuit:

In a purely capacitive circuit, voltage lags the current by 90°
OR Current leads voltage by 90°

Assume $v_C = V_m \cos(\omega t+\theta_v)$, the capacitor current is found by

$$i = C\frac{dv}{dt} = C\frac{d}{dt}[V_m \cos(\omega t + \theta_v)] \quad = CV_m(-\omega)\sin(\omega t + \theta_v)$$

$$i = -\omega C V_m \cos(\omega t + \theta_v - 90^\circ)$$

The phasor representation is

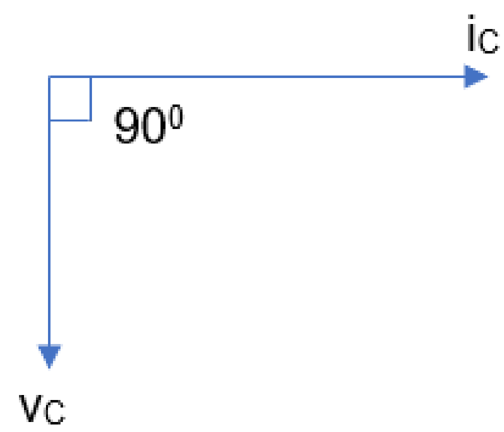
$$I = -\omega C V_m e^{j\theta_v} e^{-j90^\circ} = -\omega C V_m e^{j\theta_v}(-j1) = j\omega C \underbrace{V_m e^{j\theta_v}}_{\mathrm{V}}$$

Rewrite the last result to resemble Ohm's Law then

$$\mathrm{V} = \frac{1}{j\omega C}\,\mathrm{I}$$

Z_C (Capacitive Impedance) = $(1/j\omega C)$ = $-jX_C$ in ohm
X_C (Capacitive Reactance) = $(1/\omega C)$ in ohm

$$\boxed{\mathbf{V_C} = Z_c\,\mathbf{I_C}}$$



VERY important concept:

- The resistor, capacitor and inductor all have a very similar looking form:

$$\boxed{V = I Z}$$

- The "Z" represents the impedance of the circuit element at the frequency of the driving source. The imaginary part is called the reactance.
- In other words, the capacitor and inductor have a specific response (impedance) once we know the frequency of the input signal.

$$\boxed{\begin{aligned} V &= IR \\ V &= I(j\omega L) \\ V &= I\left(\frac{1}{j\omega C}\right) \end{aligned}}$$

Good News AND Bad News

- Good news is that almost all the laws and theorems you are familiar will also be applicable here. (Note: for the steady state response)
- The bad news is that you now have to solve the circuits and apply the laws in terms of complex numbers and phasors form.

- ***Suggestion:*** From now, start treating all the quantities, NOT numbers BUT the complex number.
- Be very comfortable representing the complex number in rectangular and polar form.

$$\text{rectangular form} \rightarrow \mathrm{x} + \mathrm{j}\ \mathrm{y} \qquad A\angle\theta = \sqrt{x^2 + y^2}\angle\tan^{-1}\frac{y}{x}$$

$$\text{polar form} \rightarrow A\angle\pm\theta = A\left[\cos\theta \pm j\sin\theta\right]$$

You must absolutely know in which quadrant the angle θ belongs!

Ex-1: A sinusoidal current has a maximum amplitude of 20 A. The magnitude of the current at t = 0 is 10 A. The period of the current sinusoidal is 1 ms.

a. What is the frequency of the current?
b. What is the expression of i(t)?

Given: Im=20 A   I(t=0) =10 A   T = 1.0 ms = 0.001 s

$f = \frac{1}{T} = \frac{1}{0.001} = 1000$ Hz ======➔ $\omega = 2\pi f = 2000\pi$ rad/s

$i(t) = I_m \cos(\omega t + \phi) = 20 \cos(2000\pi t + \phi)$

At t=0

$10 = 20 \cos(\phi)$ =====➔ $\phi = 60^{o}$

$i(t) = 20 \cos(2000\pi t + 60^{o})$ Amp

Ex-2: Given: -3 – j4. What is the polar form?

$$A=\sqrt{9+16}=\sqrt{25}=5$$

$$\tan^{-1}\frac{-4}{-3}=53.13^{\circ}$$

The angle is in the 3rd quadrant (-x and –y).

So Angle = 53.13+180 = 233.13° = 5 L 233.13

Ex-3: Given: 5L -126.87°. What is the rectangular form?

$$5\angle-126.87^{\circ} = 5\cos(126.87^{\circ}) - j5\sin(126.87^{\circ})$$

$$= 5(-0.6) - j5(0.8) = -3 - j4$$

Ex-4: If $y_1 = 20 \cos(\omega t - 30^\circ)$ and $y_2 = 40 \cos(\omega t + 60^\circ)$

By using the phasor concept, find:

1) $y_1 + y_2$
2) $y_1 - y_2$
3) $y_1 \times y_2$
4) y_1 / y_2

Solution

$y_1 = 20 \cos(\omega t - 30^\circ) = (\,20 \angle -30^\circ\,) = (\,17.32 - j10\,)$

$y_2 = 40 \cos(\omega t + 60^\circ) = (\,40 \angle 60^\circ\,) = (\,20 + j34.64\,)$

$\mathbf{y_1 + y_2} = (17.32 - j10) + (20 + j34.64) = (37.32 + j24.64) = (44.72 \angle 33.43^\circ)$

$\mathbf{y_1 - y_2} = (17.32 - j10) - (20 + j34.64) = (-2.68 - j44.64) = (44.7 \angle 266.56^\circ)$

$\mathbf{y_1 \times y_2} = (\,20 \angle -30^\circ\,) \times (\,40 \angle 60^\circ\,) = (\,800 \angle 30^\circ\,) = (692.8 + j400)$

$\mathbf{y_1 / y_2} = (\,20 \angle -30^\circ\,) / (\,40 \angle 60^\circ\,) = (\,0.5 \angle -90^\circ\,) = (0 - j0.5)$

Ex-5: Find the phasor transform of each trigonometric function:

a) $v = 170 \cos(377t - 40^o)$ V
b) $i = 10 \sin(1000t + 20^o)$ A
c) $i = [5 \cos(\omega t + 36.87^o) + 10 \cos(\omega t - 53.13^o)]$ A
d) $v = [300 \cos(20000\pi t + 45^o) - 100 \sin(20000\pi t + 30^o)]$ mV

[a] $\mathbf{V} = 170\underline{/-40^\circ}$ V

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ} \text{ A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ} \text{ A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ} \text{ mV}$$

Ex-6: Find the time-domain expression corresponding to each phasor:

a) $\mathbf{V} = 18.6 \angle -54^{o}$ V
b) $\mathbf{I} = [20 \angle 45^{o} - 50 \angle -30^{o}]$ mA
c) $\mathbf{V} = [(20 + j80) - (30 \angle 15^{o})]$ V

[a] $v = 18.6\cos(\omega t - 54^{\circ})\text{ V}$

[b] $\mathbf{I} = 20\angle 45^{\circ} - 50\angle -30^{\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\angle 126.68^{\circ}$$

Therefore $i = 48.81\cos(\omega t + 126.68^{\circ})\text{ mA}$

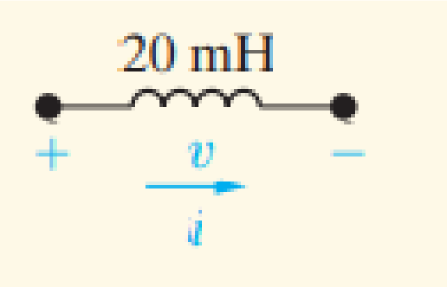
[c] $\mathbf{V} = 20 + j80 - 30\angle 15^{\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\angle 97.08^{\circ}$$

$$v = 72.79\cos(\omega t + 97.08^{\circ})\text{ V}$$

Ex-7: The current in 20 mH inductor is 10 cos (10,000t + 30º) mA.

Calculate (a) the inductive reactance; (b) the impedance of the inductor; (c) the phasor voltage **V**; and (d) the steady-state expression for *v*(t).



i= 10 cos(10000t+30º) = 10 ***L*** 30º mA

[a] $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

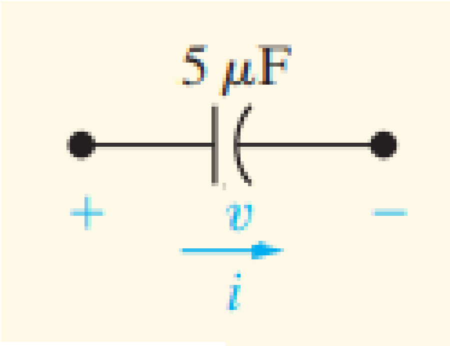
[b] $Z_L = j\omega L =$ 0+j200 = 200 ***L*** 90º Ω

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\ \mathrm{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\ \mathrm{V}$

Ex-8: The voltage across the terminals of the 5 uF capacitor is 30 cos (4000t + 25º) V.

Calculate (a) the capacitive reactance; (b) the impedance of the capacitor; (c) the phasor current **I**; and (d) the steady-state expression for *i*(t).



$$\boldsymbol{v} = 30\cos(4000t+25^\circ) = 30\ \angle\ 25^\circ\ \text{V}$$

[a] $X_C = \frac{1}{\omega C} = \frac{1}{4000(5\times10^{-6})} = 50\,\Omega$

[b] $Z_C = jX_C =$ $0\text{-}j50 = 50\ \angle\ {-90^\circ}\ \Omega$

[c] $\mathbf{I} = \frac{\mathbf{V}}{Z_C} = \frac{30\angle 25^\circ}{50\angle -90^\circ} = 0.6\angle 115^\circ\ \text{A}$

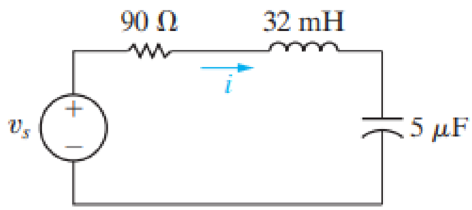
[d] $i = 0.6\cos(4000t + 115^\circ)\ \text{A}$

Ex-9: A 90 Ω resistor, a 32 mH inductor, and a 5uF capacitor are connected in series across the terminals of a sinusoidal voltage source, as shown in the figure. The steady-state expression for the source voltage is 750 cos (5000t + 30°) V.

Calculate the steady-state current i(t) by the phasor method.

ω	=	5000	rad/sec
Vi	=	750	V
Phase	=	30	deg.
R	=	90	ohm
L	=	32	mH
XL	=	160	ohm
C	=	5	uf
XC	=	40	0hm

Vs	649.5191	+ j	375	=	750	L	30		
R	90	+ j	0	=	90	L	0		
ZL	9.8E-15	+ j	160	=	160	L	90		
ZC	2.45E-15	+ j	-40	=	40	L	-90		
ZT	90	+ j	120	=	150	L	53.1301		
I	4.598076	+ j	-1.9641	=	5	L	-23.1301	=	5 Cos (5000 t + -23.13 °)

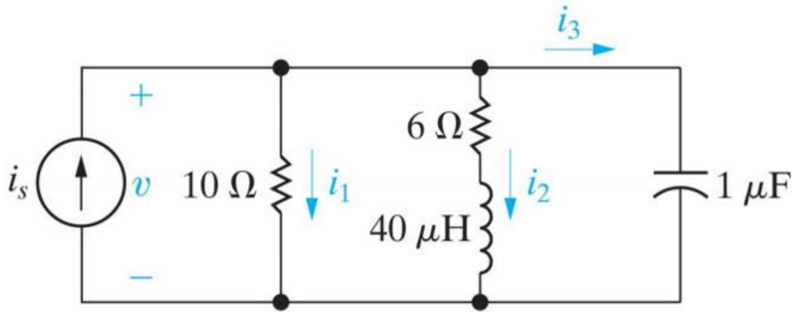


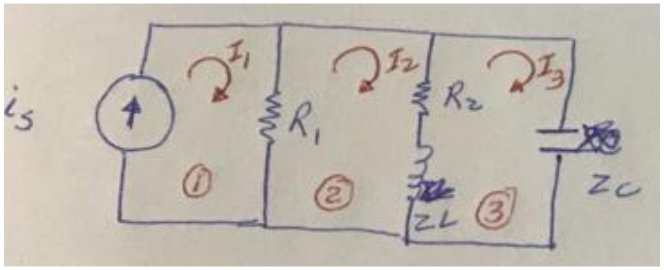
Ans:

$\boldsymbol{i}$ = 5 cos (5000t - 23.13°) A

Ex-10: Find i_1, i_2, i_3.

$i_s = 8 \cos(200000t)$ A





$\boldsymbol{i} = 8 \cos(200000t) = 8 \angle 0^o$ A

$X_L = \omega L = 200000 \times 40 \times 10^{-6} = 8\ \Omega$

$X_C = 1/\omega C = 1/(200000 \times 1 \times 10^{-6}) = 5\ \Omega$

$i_s = 8 \text{ Cos}(200000t) = 8\angle 0 = 8 + j0$ Amp

$R_1 = 10 = 10\angle 0 = 10 + j0$ ohm

$R_2 = 6 = 6\angle 0 = 6 + j0$ ohm

$Z_L = 8\angle 90 = 0 + j8$ ohm

$Z_C = 5\angle -90 = 0 - j5$ ohm

Answer:

$i_1 = 4 \angle -36.87^o$

$i_2 = 4 \angle -90^o$

$i_3 = 8 \angle -53.13$

Apply Mesh Analysis:

Loop 1:

$I_1 = i_S = 8\angle 0 = 8 + j0$ ------------ (1)

Loop 2:

$R_1(I_2-I_1) + R_2(I_2-I_3) + Z_L (I_2-I_3) = 0$

$-R_1I_1 + (R_1+R_2+Z_L)I_2 – (R_2+Z_L)I_3 = 0$

$(R_1+R_2+Z_L)I_2 – (R_2+Z_L)I_3 = R_1I_1$

$(10+j0+6+j0+0+j8)I_2 – (6+j0+0+j8)I_3 = (10\angle 0)(8\angle 0)$

$(16+j8)I_2 – (6+j8)I_3 = 80\angle 0$ ------------ (2)

<u>Loop 3:</u>

$Z_L(I_3-I_2) + R_2(I_3-I_2) + Z_c\ I_3 = 0$

$Z_L I_3 - Z_L I_2 + R_2 I_3 – R_2 I_2 + Z_C I_3 = 0$

$(-Z_L-R_2)I_2 + (Z_L+R_2+Z_C)I_3 = 0$

$(0-j8-6+j0)I_2 + (0+j8+6+j0+0-j5)I_3 = 0$

$(-6-j8)I_2 + (6+j3)I_3 = 0$ ------------ (3)

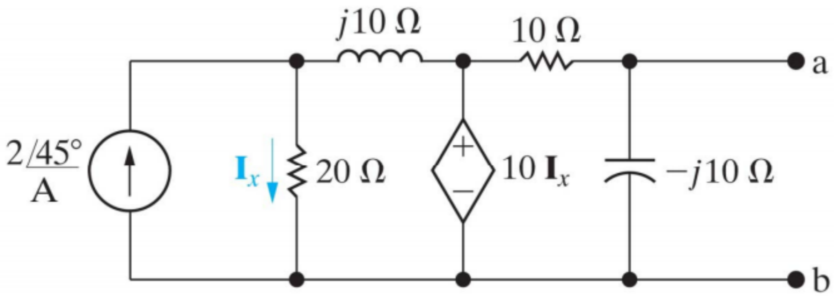
Solving eq(2) and eq(3) we get

$I_2 = 4.8 + j2.4$

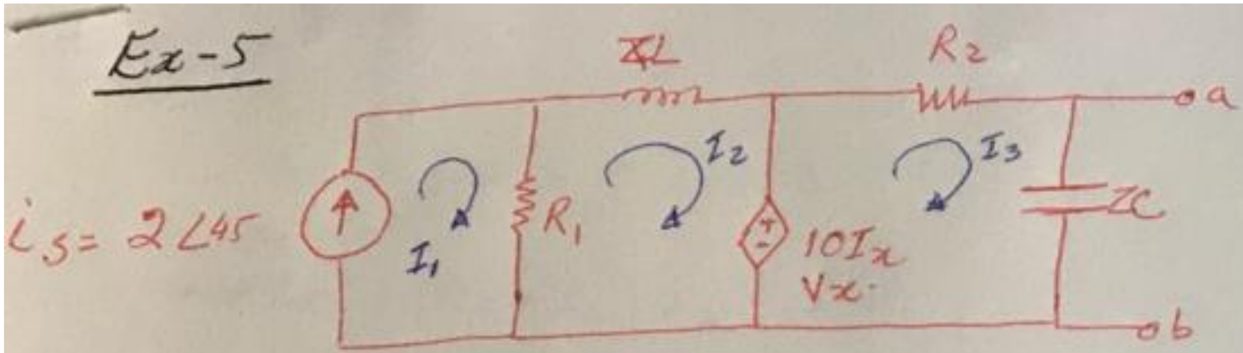
$I_3 = 4.8 + j6.4$ and we already know $I_1 = 8 + j0$ So

<table>
<tr><td>$i_1 = I_1-I_2 = (8+j0) - (4.8+j2.4) = 3.2-j2.4 = 4\ L\ -36.87^o$</td><td rowspan="3"><u>Answer:</u>
$i_1 = 4\ L\ -36.87\ ^o$
$i_2 = 4\ L\ -90\ ^o$
$i_3 = 8\ L\ -53.13$</td></tr>
<tr><td>$i_2 = I_2-I_3 = (4.8+j2.4) - (4.8+j6.4) = 0-j4 = 4\ L\ -90^o$</td></tr>
<tr><td>$i_1 = I_3 \quad = 4.8-j6.4 = 8\ L\ -53.13^o$</td></tr>
</table>

Ex-11: Analyze the following circuit.



Ex-5



$i_s = 1.414 + j1.414 = 2\angle 45$

$R_1 = 20 + j\phi = 20\angle 0$

$R_2 = 10 + j\phi = 10\angle 0$

$Z_L = 0 + j10 = 10\angle 90$.

$Z_C = 0 - j10 = 10\angle -90$.

$V_x = 10\,I_x$.

Loop 1: $I_1 = i_s$

$I_x = I_1 - I_2$

Loop 2: $R_1(I_2 - I_1) + Z_L I_2 + V_x = 0$

$R_1(I_2 - I_1) + Z_L I_2 + 10 I_x = 0$

$R_1(I_2 - I_1) + Z_L I_2 + 10(I_1 - I_2) = 0$

$R_1 I_2 - R_1 I_1 + Z_L I_2 + 10 I_1 - 10 I_2 = 0$

$(10 - R_1) I_1 + (R_1 + Z_L - 10) I_2 = 0$

$(10 - 20 + j0) I_1 + (20 + j0 + 0 + j10 - 10) I_2 = 0$

$-10 I_1 + (10 + j10) I_2 = 0$

$$I_2 = \frac{10 I_1}{10 + j10} = \frac{10\, i_s}{14.14\angle 45} = \frac{10 \times 2\angle 45}{14.14\angle 45}$$

$I_2 = 1.414\angle 0$

$V_x = 10 I_x = 10(I_1 - I_2) = 10(i_s - I_2)$

$V_x = 10[(1.414 + j1.414) - (1.414 + j0)] = 0 + j14.14$

$V_x = 14.14\angle 90$.

Loop 3:

$$-V_x + R_2 I_3 + Z_c I_3 = 0$$

$$-V_x + (R_2 + Z_c) I_3 = 0$$

$$(R_2 + Z_c) I_3 = V_x.$$

$$\{(10 + j\phi) + (0 - j10)\} I_3 = 14.14 \angle 90.$$

$$(10 - j10) I_3 = 14.14 \angle 90.$$

$$(14.14 \angle -45) I_3 = 14.14 \angle 90$$

$$I_3 = \frac{14.14 \angle 90}{14.14 \angle -45} = 1 \angle 135$$

$$I_x = \{(1.414 + j1.414) - (1.414 + j\phi)\}$$

$$I_x = 0 + j1.414 = 1.414 \angle 90$$

$$V_{R_1} = (1.414 \angle 90) \times (20 \angle 0) = 28.28 \angle 90$$

$$= (0 + j28.28)$$

Ex-12: A sinusoidal current has a maximum amplitude of 40 A. The current passes through one complete cycle in 3 ms. The magnitude of the current at zero time is 8 A.

a) What is the frequency of the current in hertz?
b) What is the frequency in radians per second?
c) Write the expression for i(t), using the cosine function.
d) What is the rms value of the current?

Ex-13: Find the phasor transform of each trigonometric function.

a) $v = 170\cos(377t - 40^\circ)$ V.

b) $i = 10\sin(1000t + 20^\circ)$ A.

c) $i = [5\cos(\omega t + 36.87^\circ) + 10\cos(\omega t - 53.13^\circ)]$ A.

d) $v = [300\cos(20{,}000\pi t + 45^\circ) - 100\sin(20{,}000\pi t + 30^\circ)]$ mV.

(a) $170\angle{-40^\circ}$ V;

(b) $10\angle{-70^\circ}$ A;

(c) $11.18\angle{-26.57^\circ}$ A;

(d) $339.90\angle{61.51^\circ}$ mV.

Ex-14: Find the time-domain expression of each phasor.

a) $\mathbf{V} = 18.6\angle -54^\circ$ V.

b) $\mathbf{I} = (20\angle 45^\circ - 50\angle -30^\circ)$ mA.

c) $\mathbf{V} = (20 + j80 - 30\angle 15^\circ)$ V.

(a) $18.6\cos(\omega t - 54^\circ)$ V;

(b) $48.81\cos(\omega t + 126.68^\circ)$ mA;

(c) $72.79\cos(\omega t + 97.08^\circ)$ V.

Ex-15: The current in the 20 mH inductor is 10 Cos(10000t+30º).

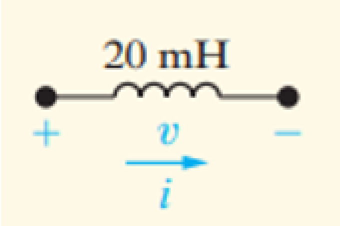
Calculate:

a) The inductive reactance.

b) The inductive impedance.

c) The Phasor voltage **V**.

d) The steady state expression for *v(t)*.



(a) $200\ \Omega$;

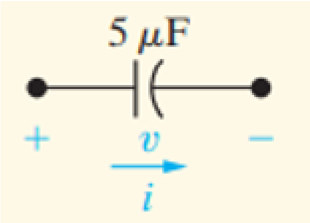
(b) $j200\ \Omega$;

(c) $2\angle 120^\circ$ V;

(d) $2\cos(10{,}000t + 120^\circ)$ V.

Ex-16: The voltage across the terminals of the 5 μF capacitor is $30\ Cos(4000t+25^{o})$ V.

Calculate:



a) The capacitive reactance.

b) The capacitive impedance.

c) The Phasor current **I**.

d) The steady state expression for *i(t)*.

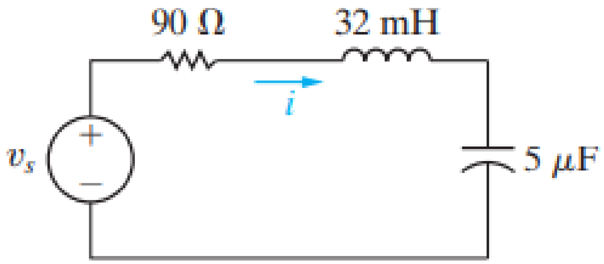
(a) $-50\ \Omega$;

(b) $-j50\ \Omega$;

(c) $0.6\angle 115^{\circ}$ A;

(d) $0.6\cos(4000t + 115^{\circ})$ A.

Ex-17: A resistor 90 Ω, a 32 mH inductor, and a capacitor 5 μF are connected in series across the terminals of a sinusoidal voltage source, as shown in the figure. The steady-state expression for the source voltage vs is 750 cos(5000t + 30°) V.



a) Make frequency-domain equivalent circuit.

b) Calculate the steady-state current i.

Ex-18: A 20 Ω resistor is connected in parallel with a 5 mH inductor. This parallel combination is connected in series with a resistor 5 Ω and a 25μF capacitor.

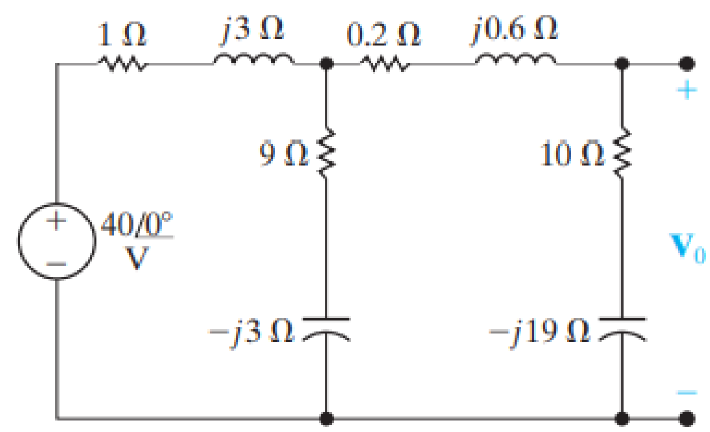
a) Calculate the impedance of this interconnection if the frequency is 2 krad/s.

b) Repeat (a) for a frequency of 8 krad/s.

c) At what finite frequency does the impedance of the interconnection become purely resistive?

d) What is the impedance at the frequency found in (c)?

Ex-19: Analyze the Circuit and find v_o.

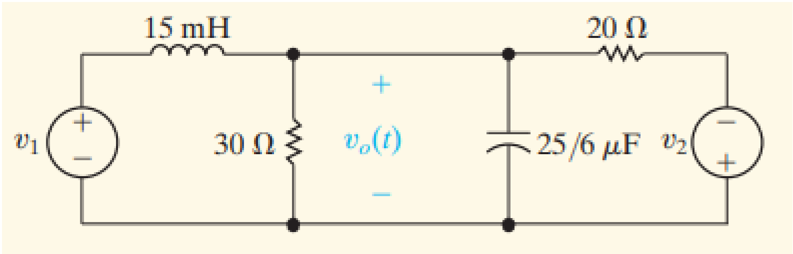


Ex-20: Find the steady-state expression for v_o for the circuit shown below.

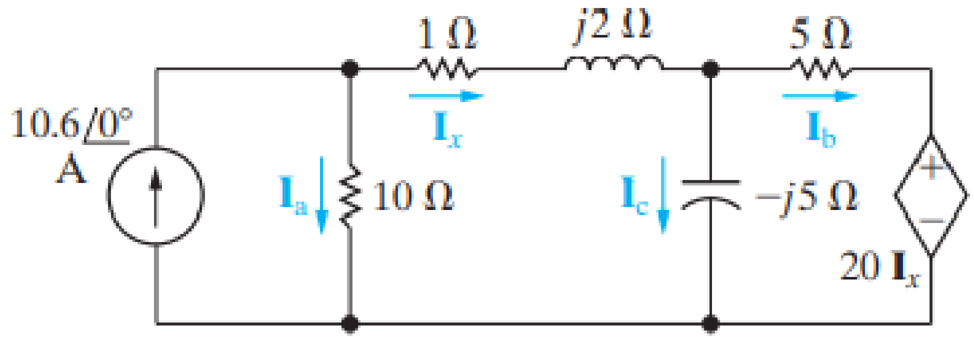
The sinusoidal voltage sources are

$v_1 = 240 \cos (4000t + 53.13^o)$ V

$v_2 = 96 \sin 4000t$ V



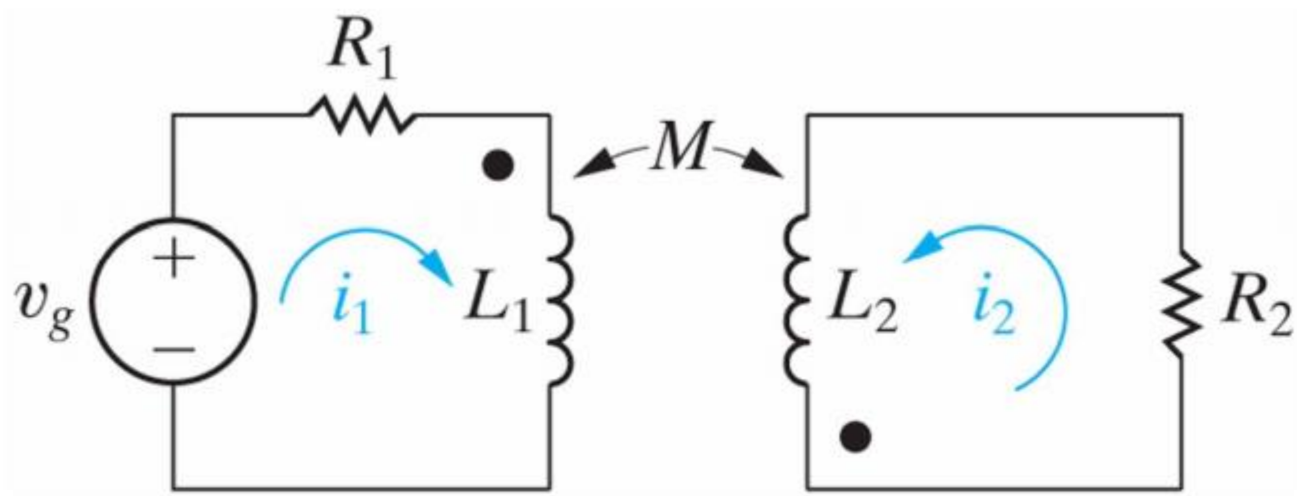
<u>Ex-21:</u> Use the node-voltage method to find the branch currents in the circuit.



Mutual Inductance

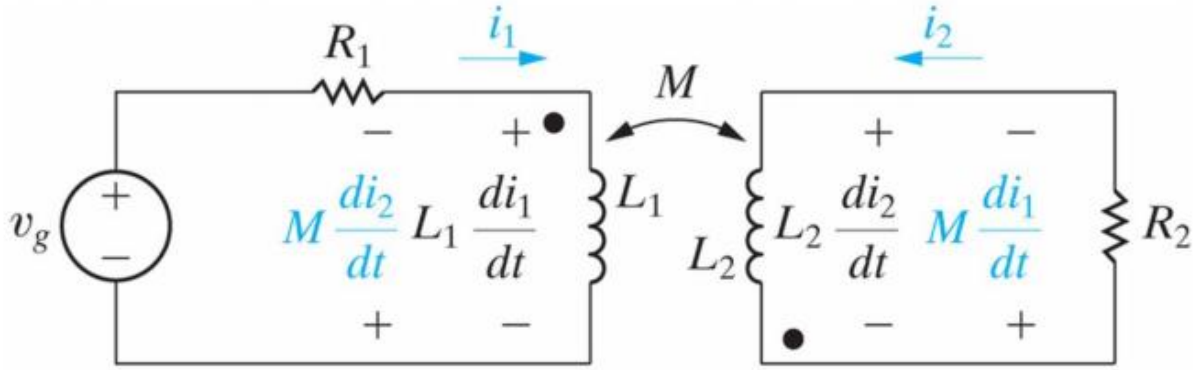
- We previously looked at the effect of a moving charge (current) creating a magnetic field which is called inductance and since the effect was the current in a single circuit, it should be properly called **self-inductance**.
- Here in this section, we will investigate the situation where the magnetic field created by a time varying current in one circuit is linked with the other circuit and induce voltage. This phenomenon is known as **mutual inductance**.
- The polarity of mutually induced voltage depends on the way the coils are wound in relation to the reference direction of coil currents.
- Dots are placed on the terminals to carry the polarity information schematically (rather than knowing righty or lefty directions).
- Hence for mutual inductance, we also rely on the dot convention. we will discuss it later in following section.

R_1
v_g
i_1
L_1
L_2
R_2



R_1
M
v_g
i_1
L_1
L_2
i_2
R_2

- Let's analyze the following mutual inductance circuit.



KVL on the left hand side gives us

$$-v_g + i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0$$

KVL on the right hand side gives us

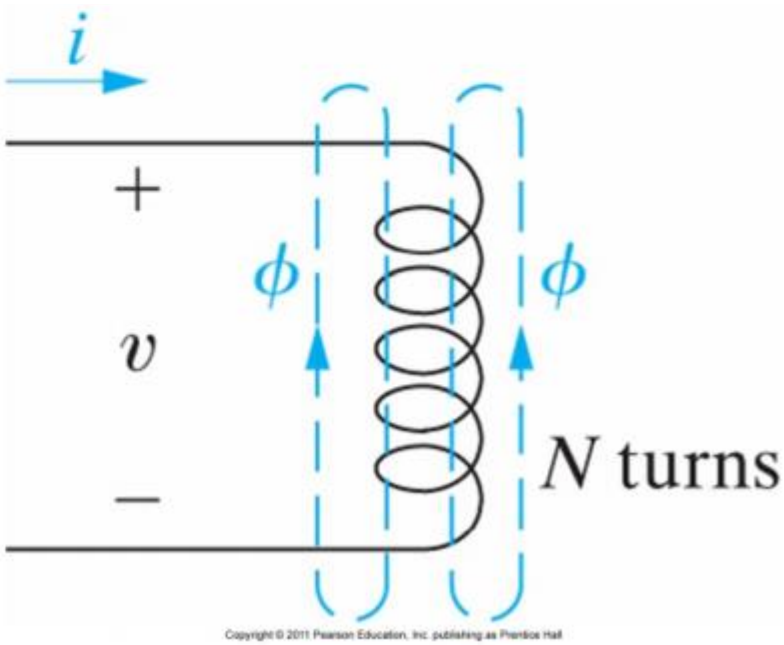
$$-L_2 \frac{di_2}{dt} - i_2 R_2 + M \frac{di_1}{dt} = 0$$

Or equally (multiply last result by -1)

$$L_2 \frac{di_2}{dt} + i_2 R_2 - M \frac{di_1}{dt} = 0$$

Note: M12 = M21

The voltage induced by the magnetic field surrounding a current carrying conductor is described by Faraday's Law:



$$v = \frac{d\lambda}{dt} = \frac{d(N\phi)}{dt}$$

$$= N\frac{d\phi}{dt} = N\frac{d}{dt}(\mathcal{P}Ni)$$

$$= N^2\mathcal{P}\frac{di}{dt} = L\frac{di}{dt},$$

- Where λ is called the magnetic flux linkage – measured in weber-turns.
- The flux linkage λ is the product of the magnetic field (ϕ) and the number of turns linked (N).

$$\lambda = N\phi$$

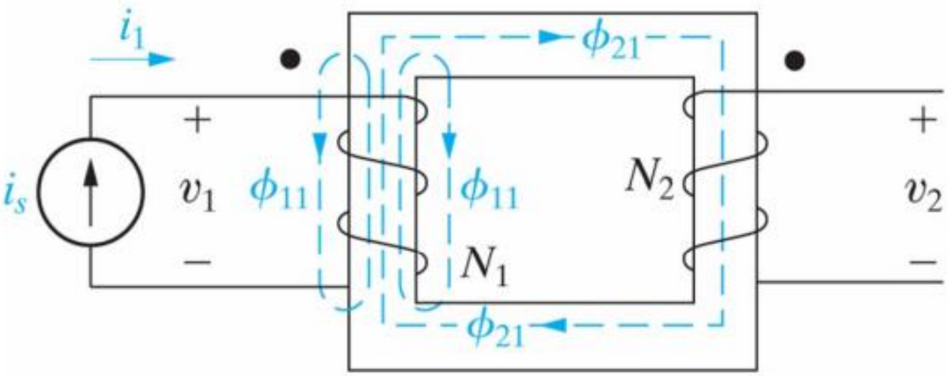
- The magnetic field strength per unit volume depends on the material in that space - described by the permeance of the material.
- The text has more details, but the important message is the magnetic coupling M may not be perfect.
- Thus, we introduce a coefficient of coupling k which can be used to better model a specific physical circumstance.
- The value of k depends on what fraction of flux is linking to the other coil.

$$M = k\sqrt{L_1 L_2}$$

Where $0 \le k \le 1$.

In general, k is found by experimental measurement.

- We thus have the following view of a mutually coupled circuit:



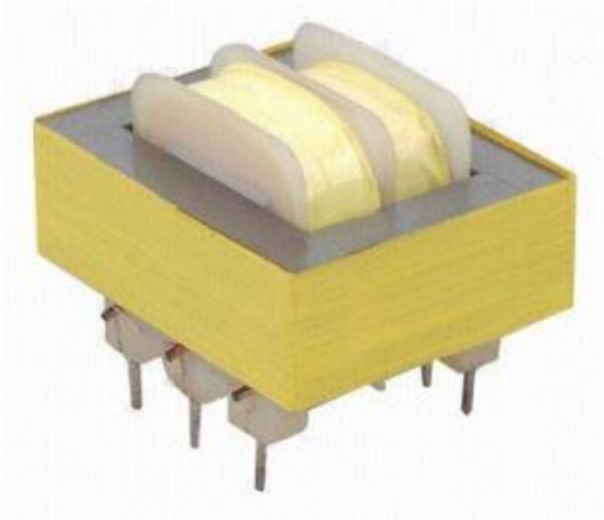
$$M = k\sqrt{L_1 L_2}$$

The energy stored in these two coils (self-inductance) and the energy stored in the coupled magnetic fields (mutual inductance) has the form:

$$w(t) = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

Transformer

- A transformer is a device that is based on magnetic coupling.
- In communications, a transformer can be used to match impedance between two circuits and to eliminate dc signals.
- A very common use of transformers is in power systems to raise or lower voltage levels.



Ideal Transformer:

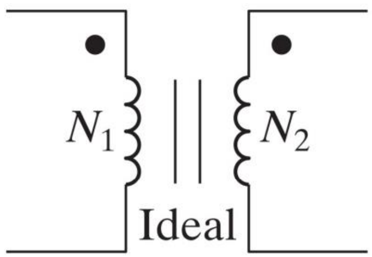
- An ideal transformer is an imaginary transformer, consists of two magnetically coupled coils having:
- N1 turns around the coil on the primary side.
- N2 turns around the coil on the secondary side.
- Coefficient of coupling k = 1. Perfect coupling.
- The self-inductance of each coil is infinite. $L_1 = L_2 = \infty$
- The resistance of each coil is negligible. Parasitic resistance $\rightarrow$ zero.
- There are no copper losses (no winding resistance).
- There is no iron loss in core.
- There is no leakage flux.
- an ideal transformer gives output power exactly equal to the input power.
- Efficiency of an idea transformer is 100%.
- It is impossible to have such a transformer in practice, but ideal transformer model makes problems easier.

Dot Convention:

Dot convention is a method to define the polarity of voltage and current in a mutually coupled circuit and can be summarized as:

- If the coil voltages V_1 and V_2 are both positive or both negative at the dot-marked terminal, then use a plus sign. Otherwise, use a minus sign.
- If the coil currents I_1 and I_2 are both directed into or both directed out of the dot-marked terminal, then use a minus sign. Otherwise, use a plus sign.

Let's consider the following four cases of an ideal transformer to determine the correct polarity of voltages and currents.

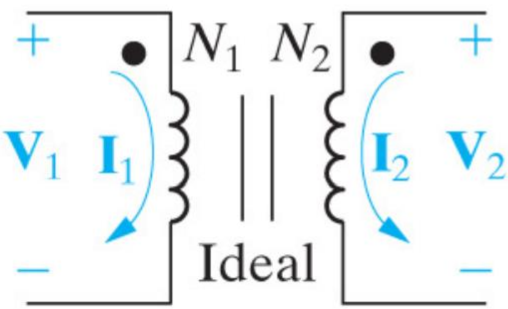


$$\frac{V_1}{N_1} = (\pm)\frac{V_2}{N_2}$$

$$I_1 N_1 = (\pm) I_2 N_2$$

Case 1: Both voltages are positive at the dot-marked terminal.

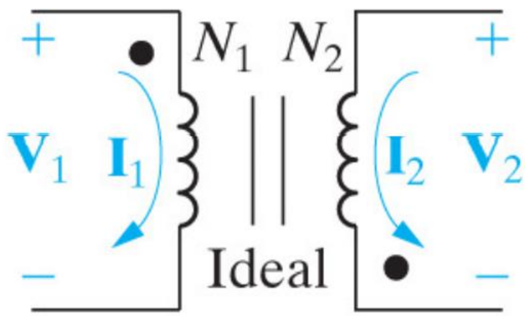
Both currents are into the dot-marked terminal.



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

Case 1



$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

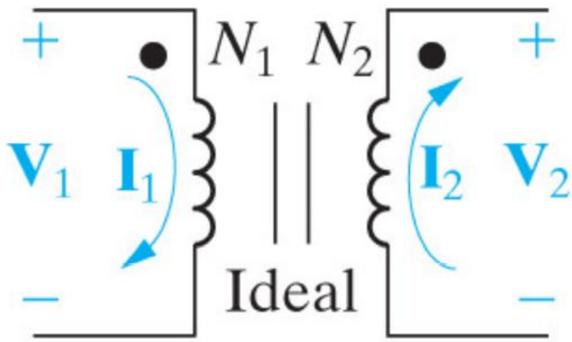
Case 2

Case 2: Only one voltage is positive at the dot-marked terminal.

Only one current is into the dot-marked terminal.

Case 3: Both voltages are positive at the dot-marked terminal.

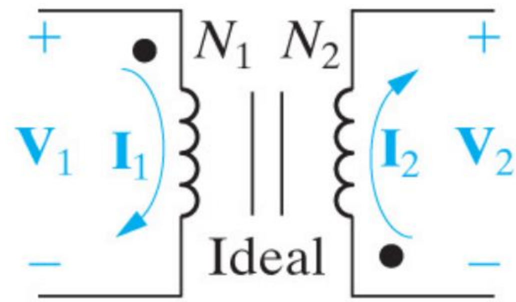
Only one current is into the dot-marked terminal.



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

Case 3



$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

Case 4

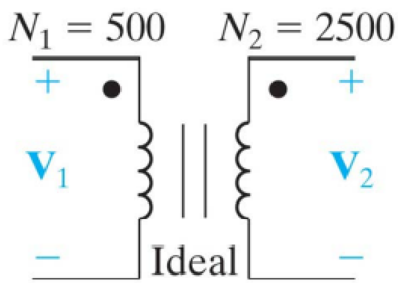
Case 4: Only one voltage is positive at the dot-marked terminal.

Both currents are into the dot-marked terminal.

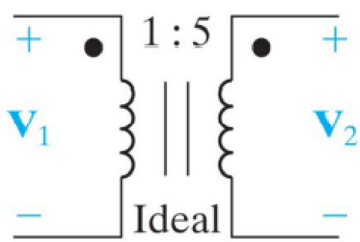
The ratio of turns on the two windings may be defined as either

N_1/N_2 or as N_2/N_1 Here we will use $a = \dfrac{N_2}{N_1}$

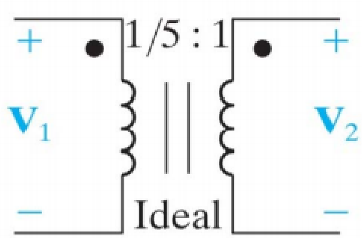
Equivalent ways to show the turns ratio.



$$a = \frac{N_2}{N_1} = \frac{2,500}{500} = 5$$



$$1 : a$$

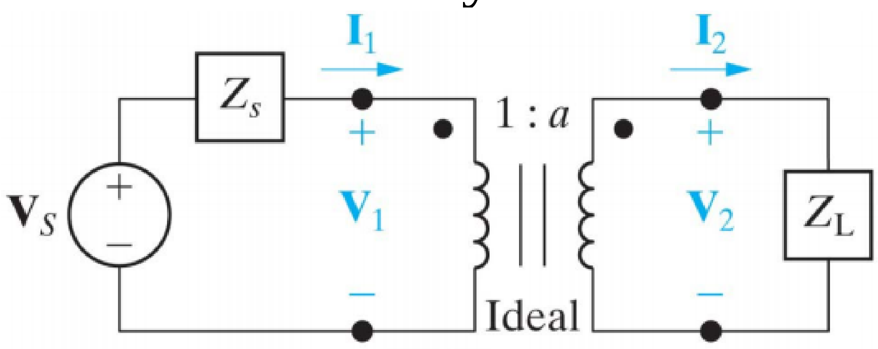


$$\frac{1}{a} : 1$$

Impedance Matching:

- Ideal transformers can be used to raise or lower the impedance of load as seen by the source.

$$\frac{V_1}{1} = (+)\frac{V_2}{a}$$



$$I_1(1) = (+)I_2(a)$$

Thus the input impedance is (leave out the source impedance Z_s)

$$Z_{in} \equiv \frac{V_1}{I_1} \qquad Z_L \equiv \frac{V_2}{I_2}$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{\frac{V_2}{a}}{aI_2} = \frac{1}{a^2}\frac{V_2}{I_2} = \frac{1}{a^2}Z_L$$

- Thus, the ideal transformer's secondary coil reflects the load impedance back to the primary coil with a scaling factor.
- The magnitude is scaled, but the phase induced by the load is not altered!
- If the phase of Z_L cannot be changed, the greatest power is delivered to the load is when the magnitudes of the Thévenin and load impedances are equal.

$$\left|Z_{TH}\right| = \left|Z_L\right|$$

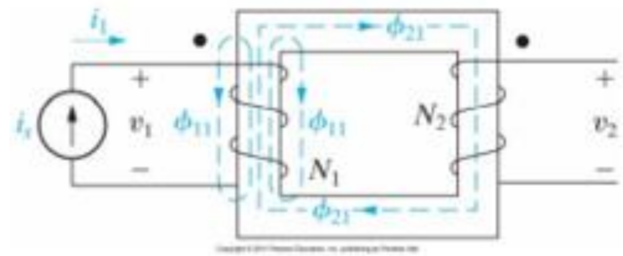
- Impedance matching with ideal transformers allow us to create the above condition.

Ex-22: Two magnetically coupled coils have:

L_1=60 mH $\quad$ L_2 = 9.6 mH $\quad$ M= 22.8 mH

What is the coefficient of coupling?

What is the largest value of M?



Find K :

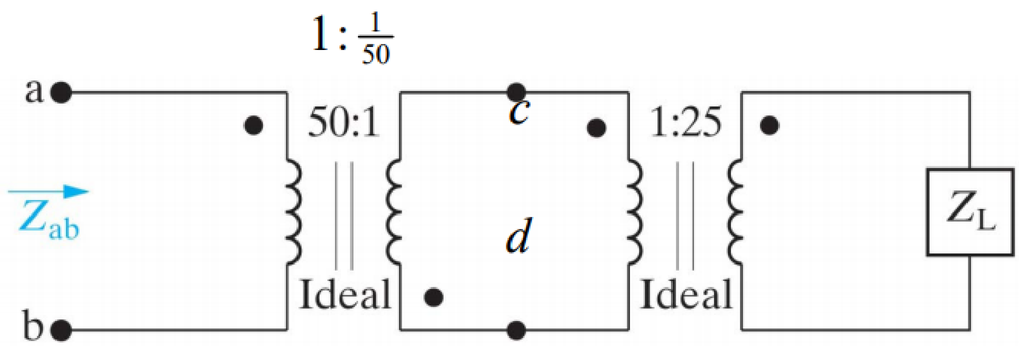
$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{22.8\times10^{-3}}{\sqrt{60\times10^{-3}\times 9.6\times10^{-3}}} = 0.95$$

Find largest value of M:

M is max^m when K = 1.

$$M = 1\sqrt{60\times10^{-3}\times 9.6\times10^{-3}} = 24.0 \text{ mH}$$

Ex-23: Find the impedance Zab?



$Z_L = 200 + j150\,\Omega$

$$Z_{cd} = \left(\frac{N_{cd}}{N_L}\right)^2 \cdot Z_L = \left(\frac{1}{25}\right)^2 \cdot (200 + j150).$$

$$Z_{ab} = \left(\frac{N_{ab}}{N_{cd}}\right)^2 \cdot Z_{cd} = \left(\frac{50}{1}\right)^2 \left[\frac{1}{25^2}(200 + j150)\right]$$

$$\boxed{Z_{ab} = 800 + j600 = 1000\angle 36.87^\circ}$$