# HW 7

1) Problem 6.1.
2) Problem 6.5.
3) Problem 6.19.
4) Problem 6.24.
5) A fair coin is tossed 400 times. Let the random variable $X_i$ represent the outcome of the $i$th toss where

$$X_i = \begin{cases} 1, & \text{if } i\text{th toss is heads} \\ 0, & \text{otherwise} \end{cases}.$$

Use the central limit theorem to find an approximation for the probability of at most 190 heads.

*Hint: if* $S = \sum_{i=1}^{400} X_i$, *then the question asks for P(S≤190)?*

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HOMEWORK #07

1. PROBLEM 6.1
THE MAGNITUDE OF THE VOLTAGE V ACROSS A COMPONENT IN AN ELECTRONIC CIRCUIT HAS A MEAN VALUE OF 0.45 VOLTS. GIVEN ONLY THIS INFORMATION, FIND A BOUND ON THE PROBABILITY THAT $V \geq 1.35$.

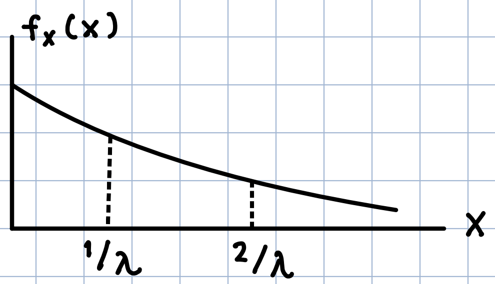
$$\Pr[V \geq 1.35] \leq 0.45/1.35 = 1/3$$

2. PROBLEM 6.5
LET X BE AN EXPONENTIAL RANDOM VARIABLE WITH PARAMETER $\lambda$.

(a) APPLY THE MARKOV INEQUALITY TO BOUND $\Pr[X \geq 2/\lambda]$

$$\Pr[X \geq 2/\lambda] \leq \frac{E[X]}{2/\lambda} = \frac{1/\lambda}{2/\lambda} = 1/2$$

(b) USE THE CHEBYSHEV INEQUALITY TO COMPUTE THE BOUND




$$\Pr[X \geq 2/\lambda] = \Pr[|X - 1/\lambda| \geq 1/\lambda]$$
$$\leq \frac{\sigma_\lambda^2}{(1/\lambda)^2} = \frac{1/\lambda^2}{(1/\lambda)^2} = 1$$

(c) USE THE ONE-SIDED CHEBYSHEV INEQUALITY TO COMPUTE THE BOUND.

$$\Pr[(X - 1/\lambda) \geq 1/\lambda] \leq \frac{\sigma_X^2}{\sigma_X^2 + (1/\lambda)^2} = \frac{1/\lambda^2}{1/\lambda^2 + 1/\lambda^2} = 1/2$$

(d) COMPUTE THE ACTUAL PROBABILITY OF THIS EVENT

$$\Pr[X \geq 2/\lambda] = \int_{2/\lambda}^{\infty} \lambda e^{-\lambda x} dx = \left[e^{-\lambda x}\right]_{\infty}^{2/\lambda} = e^{-2} \approx 0.135$$

3. PROBLEM 6.19

LET X BE A BERNOULLI RANDOM VARIABLE TAKING ON VALUES OF 0 AND 1 WITH PROBABILITY $p$ AND $1-p$. GIVEN $n$ INDEPENDENT SAMPLES $X_1, X_2 \dots X_n$

(a) SUGGEST AN ESTIMATE FOR THE PARAMETER $p$.

$$\hat{p} = \frac{1}{n}\sum_{K=1}^{n} X_K$$

(b) FIND THE MEAN AND VARIANCE OF THIS ESTIMATE.

$$E\{\hat{p}\} = \frac{1}{n}\sum_{K=1}^{n} E\{X_K\} = \frac{1}{n}\cdot np = p$$

$$\mathrm{Var}[\hat{p}] = \frac{1}{n}\sum_{K=1}^{n} \mathrm{Var}[X_K] = \frac{1}{n}\cdot np(1-p) = p(1-p)$$

4. PROBLEM 6.24

FIVE HUNDRED OBSERVATIONS OF A RANDOM VARIABLE X WITH VARIANCE $\sigma_X^2 = 25$ ARE TAKEN. THE SAMPLE MEAN BASED ON 500 SAMPLES IS COMPUTED TO BE $M_{500} = 3.25$. FIND 95% AND 98% CONFIDENCE INTERVALS FOR THIS ESTIMATE.

THE VARIANCE OF THE SAMPLE MEAN IS

$$\sigma_{500}^2 = \frac{25}{500} = 0.05 \qquad \sigma_{500} = \sqrt{\frac{25}{500}} = 0.2236$$

- FOR THE 95% CI:

  $\alpha = 0.05$, $z = 1.960$ (FIG 6.6)

  CONFIDENCE INTERVAL:

  $$(\hat{\theta} - z\sigma, \hat{\theta} + z\sigma)$$
  $$= (3.25 - (1.960)(0.2236), 3.25 + (1.960)(0.2236))$$
  $$= (2.81, 3.69)$$

- FOR THE 98% CI:

  USE THE TABLE Q FUNCTIONS: $Q(2.325) = 0.01$

  HALF THE PROBABILITY OF THE CRITICAL REGION.

  $z = 2.325$

  CONFIDENCE INTERVAL:

  $$(\hat{\theta} - z\sigma, \hat{\theta} + z\sigma)$$
  $$= (3.25 - (2.325)(0.2236), 3.25 + (2.325)(0.2236))$$
  $$= (2.73, 3.77)$$

5. A FAIR COIN IS TOSSED 400 TIMES. LET THE RANDOM VARIABLE $X_i$ REPRESENT THE OUTCOME OF THE iTH TOSS WHERE

$$X_i = \begin{cases} 1, & \text{IF iTH TOSS IS HEADS} \\ 0, & \text{OTHERWISE} \end{cases}$$

USE THE CENTRAL LIMIT THEOREM TO FIND AN APPROXIMATION FOR THE PROBABILITY OF AT MOST 190 HEADS.

$$\mu = EX_i = np = (1)(1/2) = 1/2$$
$$\sigma^2 = var(X_i) = np(1-p) = (1)(1/2)(1-1/2) = 1/4$$

LET $S_{400} = \sum_{i=1}^{400} X_i$ COUNT THE NUMBER OF HEADS WE GET OUT OF 400 COIN TOSSES.

BY CENTRAL LIMIT THEOREM:

$$P(S_{400} \leq 190) = P\left(\frac{S_{400} - 400\mu}{\sqrt{400\sigma^2}} \leq \frac{190-200}{10}\right)$$
$$= P\left(\frac{S_{400} - 400\mu}{\sqrt{400\sigma^2}} \leq -1\right)$$
$$\approx 1 - \Phi(1) = 1 - 0.8413 = 0.1587$$