Solution_Homework-01 EEE 117

Questions 1-5 2 points each **Questions 6-11 15 points each**

Q- 1 The rms value of $v(t) = V_{max}\cos(\omega t + \delta)$ is given by
(a) V_{max} (b) $V_{max}/\sqrt{2}$ (c) $2\ V_{max}$ (d) $\sqrt{2}\ V_{max}$

Q-2 If the rms phasor of a voltage is given by $V = 120\underline{/60^\circ}$ volts, then the corresponding $v(t)$ is given by
(a) $120\sqrt{2}\cos(\omega t + 60^\circ)$ (b) $120\cos(\omega t + 60^\circ)$
(c) $120\sqrt{2}\sin(\omega t + 60^\circ)$

Q-3 If a phasor representation of a current is given by $I = 70.7\underline{/45^\circ}$ A, it is equivalent to
(a) $100\ e^{j45^\circ}$ (b) $100 + j100$
(c) $50 + j50$

Q-4 With sinusoidal steady-state excitation, for a purely resistive circuit, the voltage and current phasors are
(a) in phase
(b) perpendicular with each other with V leading I
(c) perpendicular with each other with I leading V.

Q-5 For a purely inductive circuit, with sinusoidal steady-state excitation, the voltage and current phasors are
(a) in phase
(b) perpendicular to each other with V leading I
(c) perpendicular to each other with I leading V.

Q- 1	b
Q-2	a
Q-3	c
Q-4	a
Q-5	b

Q-6 Consider the sinusoidal voltage

$$v(t) = 25 \cos (400\pi t + 60^{\circ}) \text{ V}.$$

a) What is the maximum amplitude of the voltage?
b) What is the frequency in hertz?
c) What is the frequency in radians per second?
d) What is the phase angle in radians?
e) What is the phase angle in degrees?
f) What is the period in milliseconds?
g) What is the first time after $t = 0$ that $v = 0$ V?

a	Vm	=	25	V
b	f=w/2pi	=	200	Hz
c	w	=	1256.637	radian/s
d	Phase angle	=	1.047198	radian
e	Phase angle	=	60	degree
f	T	=	5	ms
g	t	=	416.67	us

Q-7 Use the concept of the phasor to combine the following sinusoidal functions into a single trigonometric expression:

a) $y = 30\cos(200t - 160^\circ) + 15\cos(200t + 70^\circ)$,

b) $y = 90\sin(50t - 20^\circ) + 60\cos(50t - 70^\circ)$,

c) $y = 50\cos(5000t - 60^\circ) + 25\sin(5000t + 110^\circ) - 75\cos(5000t - 30^\circ)$,

d) $y = 10\cos(\omega t + 30^\circ) + 10\sin\omega t + 10\cos(\omega t + 150^\circ)$.

a)

y =			-23.06048	+ j	3.83479	
y =			23.37715	L	170.559	
y = 23.377	cos	(	200	t +	170.559	)

b)

y =			-10.2606	+ j	-140.954	
y =			141.3269	L	265.837	
y = 141.33	cos	(	50	t +	265.837	)

c)

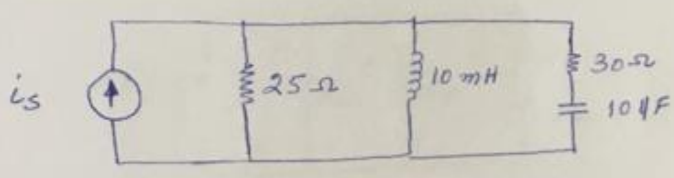
y =			-16.45959	+ j	2.74923	
y =			16.68761	L	170.517	
y = 16.688	cos	(	5000	t +	170.517	)

d)

y =			0	+ j	0	
y =			0	L	0	
y = 0						

Q-8 A 25 Ω resistor and a 10 mH inductor are connected in parallel. This parallel combination is also in parallel with the series combination of a 30 Ω resistor and a 10 μF capacitor. These three parallel branches are driven by a sinusoidal current source whose current is 125 sin(2500t + 60°) A.

a) Find total impedance.
b) Analyze the circuit and find currents for each component.



$i_s = 125 \sin(2500t + 60°)$ A

$X_L = \cancel{j}\omega L = 2500 \times \frac{10}{1000}$

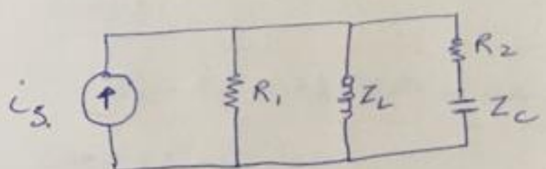
$X_L = 25$

$Z_L = 0 + j25 = 25\angle 90$

$X_C = \frac{1}{\omega C} = \frac{1}{2500 \times 10 \times 10^{-6}}$

$X_C = 40$

$Z_C = 0 - j40 = 40\angle -90$



$R_2 + Z_C = (30 + j0) + (0 - j40)$

$= 30 - j40$

$= 50\angle -53.13°$

a) $\frac{1}{Z_{eq}} = \frac{1}{R_1} + \frac{1}{Z_L} + \frac{1}{(R_2 + Z_c)}$

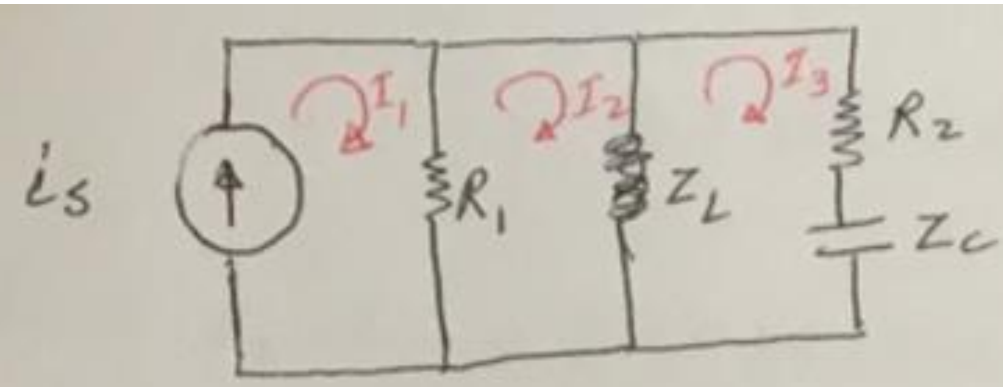
$\frac{1}{Z_{eq}} = \frac{1}{25\angle 0} + \frac{1}{25\angle 90} + \frac{1}{50\angle -53.13}$

$\frac{1}{Z_{eq}} = (0.04\angle 0) + (0.04\angle -90) + 0.02\angle 53.13$

$\frac{1}{Z_{eq}} = (0.04 + j0) + (0 - j0.04) + (0.012 + j0.016)$

$\frac{1}{Z_{eq}} = 0.052 - j0.024 = 0.05727\angle -24.775$

$\boxed{Z_{eq} = 17.46\angle 24.775}$



Loop 1: $i_s = 125 \sin(2500t + 60) = 125 \cos(2500t - 30)$

$i_s = 125 \angle -30^\circ = 108.25 - j62.5$

$\boxed{I_1 = i_s}$

Loop 2: $R_1(I_2 - I_1) + Z_L(I_2 - I_3) = 0$

$R_1 I_2 - R_1 I_1 + Z_L I_2 - Z_L I_3 = 0$

$(R_1 + Z_L) I_2 - Z_L I_3 = R_1 I_1$

$(25 + j25) I_2 - (0 + j25) I_3 = (2706.33 - j1562.5)$ — (1)

Loop 3: $Z_L(I_3 - I_2) + (R_2 + Z_c) I_3 = 0$

$-Z_L I_2 + (R_2 + Z_L + Z_c) I_3 = 0$

$(0 - j25) I_2 + (30 - j15) I_3 = 0$ —— (2)

Solve eq (1) & (2) we get

$$I_2 = 21.31 - j54.55$$
$$I_3 = 29.26 + j32.39$$

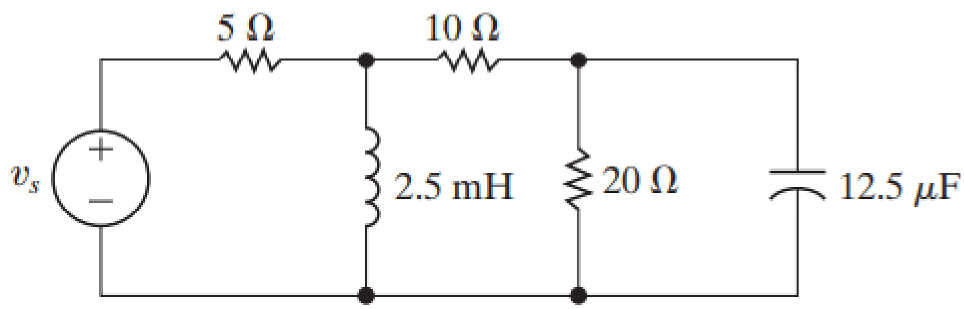
$$I_{R_1} = I_1 - I_2 = 86.94 - j7.95 = 87.303 \angle -5.224^\circ \text{ A}$$

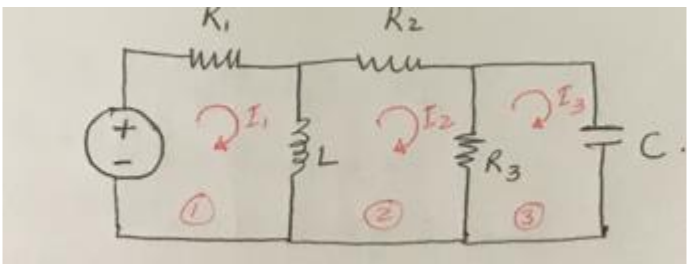
$$I_L = I_2 - I_3 = -7.95 - j86.94 = 87.303 \angle 264.775^\circ \text{ A}$$

$$I(R_2 + C) = I_3 = 29.26 + j32.39 = 43.65 \angle 47.91^\circ \text{ A}$$

Q-9 For the circuit shown below.

c) Find total impedance.
d) Analyze the circuit and find currents for each component.
if vs = 25 Cos 4000t V.





$V_s = 25\angle 0 = 25 + j0$

$R_1 = 5\,\Omega = 5\angle 0 = 5 + j0$

$R_2 = 10\,\Omega = 10\angle 0 = 10 + j0$

$R_3 = 20\,\Omega = 20\angle 0 = 20 + j0$

$L = 2.5\,mH \Rightarrow X_L = \omega L = 4000 \times 2.5 \times 10^{-3} = 10\,\Omega.$

$Z_L = 10\angle 90 = 0 + j10.$

$C = 12.5\,\mu F \Rightarrow X_C = \frac{1}{\omega C} = \frac{1 \times 10^6}{4000 \times 12.5} = 20\,\Omega$

$Z_C = 20\angle -90 = 0 - j20.$

Total Impedance:

$$Z_x = \left(\frac{Z_C \times R_3}{Z_C + R_3}\right) + R_2 = \left[\frac{(20\angle -90)(20\angle 0)}{(20-j20)}\right] + (10+j0)$$

$$Z_x = \left(\frac{400\angle -90}{28.28\angle -45}\right) + (10+j\varnothing) = (14.14\angle -45) + (10+j0)$$

$$Z_x = (10-j10) + (10+j0) = (20-j10) = 22.36\angle -26.56$$

$$Z_T = \left(\frac{Z_L * Z_x}{Z_L + Z_x}\right) + R_1 = \left[\frac{(10\angle 90)(22.36\angle -26.56)}{(0+j10)+(20-j10)}\right] + (5+j0)$$

$$Z_T = \left(\frac{220.36\angle 63.44}{20+j0}\right) + (5+j\phi) = (11.02\angle 63.44) + (5+j0)$$

$$Z_T = (4.927 + j9.857) + (5+j0)$$

$$Z = \boxed{9.927 + j9.857 \approx 13.99\angle 44.8^\circ}$$

Loop 1:

$$R_1 I_1 + Z_L(I_1 - I_2) = V_S$$

$$(R_1 + Z_L)I_1 - Z_L I_2 = V_S.$$

$$(5 + j10)I_1 - (0 + j10)I_2 = 25 + j0$$

$$(5 + j10)I_1 + (0 - j10)I_2 = 25 + j0 \quad \text{--- (1)}$$

Loop 2:

$$Z_L(I_2 - I_1) + R_2 I_2 + R_3(I_2 - I_3) = 0$$

$$-Z_L I_1 + (R_2 + R_3 + Z_L)I_2 - R_3 I_3 = 0$$

$$-(0 + j10)I_1 + (30 + j10)I_2 - (20 + j0)I_3 = 0$$

$$(0 - j10)I_1 + (30 + j10)I_2 + (-20 + j0)I_3 = 0 \quad \text{--- (2)}$$

Loop3:

$$R_3(I_3 - I_2) + Z_C I_3 = 0$$

$$-R_3 I_2 + (R_3 + Z_C) I_3 = 0$$

$$-(20 + j0) I_2 + (20 - j20) I_3 = 0$$

$$(-20 - j0) I_2 + (20 - j20) I_3 = 0 \qquad \text{———} \quad (3)$$

Solve these equations we get

$$I_1 = 1.25 - j1.25 = 1.767\angle -45$$

$$I_2 = 0.625 + j0.625 = 0.883\angle 45$$

$$I_3 = 0 + j0.625 = 0.625\angle 90$$

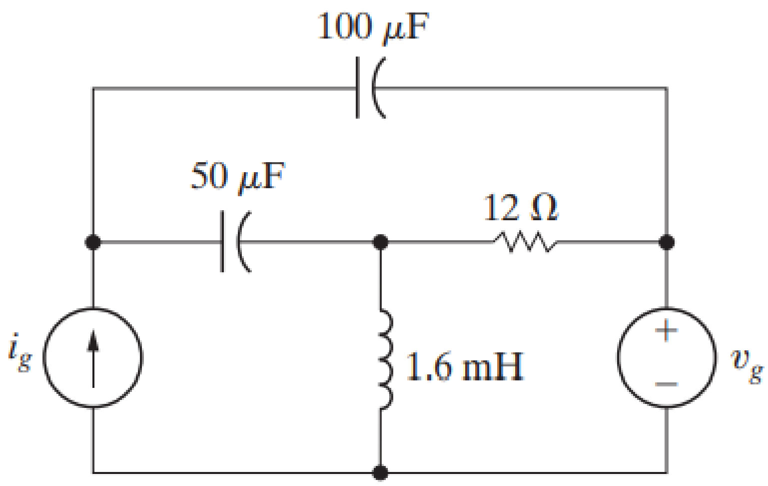
$$I_{R_1} = I_1, \quad I_{R_2} = I_2 \quad I_C = I_3.$$

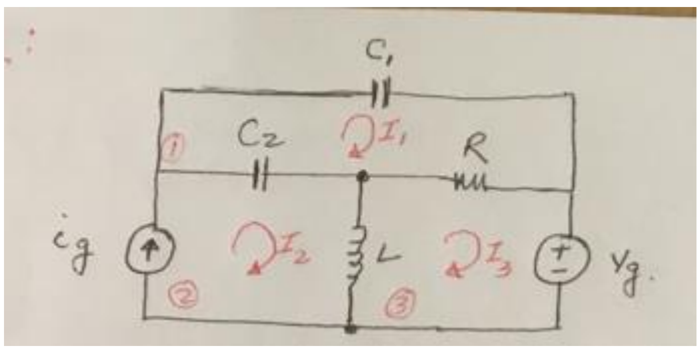
$I_L = I_1 - I_2 =$ **0.625 – j 1.875 = 1.976 L -71.56**

$$I_{R3} = I_2 - I_3 = 0.625 + j0 = 0.625\angle 0$$

Q-10 Analyze the circuit and find currents for each component.

if $i_g = 5\cos 2500t$ A and $v_g = 20\cos(2500t + 90°)$ V.





$i_g = 5\cos 2500t = 5\angle 0 = 5 + j0$ A

$V_g = 20\cos(2500t + 90) = 20\angle 90 = 0 + j20$ V

$R = 12\,\Omega = 12\angle 0 = 12 + j0$

$C_1 = 100\,\mu F \Rightarrow X_{C_1} = \frac{1}{\omega C} = \frac{1 \times 10^6}{2500 \times 100} = 4\,\Omega$

$Z_{C_1} = 4\angle -90 = 0 - j4$

$C_2 = 50\,\mu F \Rightarrow X_{C_2} = \frac{1}{\omega C} = \frac{1 \times 10^6}{2500 \times 50} = 8\,\Omega$

$Z_{C_2} = 8\angle -90 = 0 - j8$

$L = 1.6\,mH \Rightarrow X_L = \omega L = 1.6 \times 10^{-3} \times 2500 = 4\,\Omega$

$Z_L = 4\angle 90 = 0 + j4$

Loop: 1

$$Z_{C_1} I_1 + R(I_1 - I_3) + Z_{C_2}(I_1 - I_2) = 0$$

$$(R + Z_{C_1} + Z_{C_2}) I_1 - Z_{C_2} I_2 - R I_3 = 0$$

$$[(12+j0)+(0-j4)+(0-j8)] I_1 - (0-j8) I_2 - (12+j0) I_3 = 0$$

$$(12 - j12) I_1 + (0+j8) I_2 + (-12 - j0) I_3 = 0 \quad \text{——} \; (1)$$

Loop 2:

$$\boxed{I_2 = i_g = 5\angle 0 = 5 + j0}$$

Loop 3:

$$Z_L(I_3 - I_2) + R(I_3 - I_1) = -V_g$$

$$-R I_1 - Z_L I_2 + (R + Z_L) I_3 = -V_g$$

$$R I_1 + Z_L I_2 - (R + Z_L) I_3 = V_g$$

Since $I_2 = i_g$ so

$$(12+j0) I_1 + (4\angle 90)(5\angle 0) - (12 + j0 + 0 + j4) I_3 = 0 + j20$$

$$\ldots I_3 = 0 + j20$$

$$(12+j0) I_1 + 20\angle 90 - (12 + j4) I_3 = (0 + j20)$$

$$(12+j0) I_1 + (-12 - j4) I_3 = (0 + j20) - (0 + j20)$$

$$(12+j0) I_1 + (-12 - j4) I_3 = 0 \quad \text{——} \; (2)$$

Put the value of I_2 in eq (1) we get.

$$(12-j12)I_1 + (8\angle 90)(5\angle 0) + (-12-j0)I_3 = 0$$

$$(12-j12)I_1 + (-12-j0)I_3 = -40\angle 90$$

$$(12-j12)I_1 + (-12-j0)I_3 = 0 - j40 \quad \text{—— (3)}$$

Solve eq (2) & eq (3) for I_1 & I_3 we get

$$\boxed{\begin{aligned} I_1 &= 4.66 - j0.666 = 4.713\angle -8.125 \\ I_3 &= 4 - j2 = 4.472\angle -26.56 \end{aligned}}$$

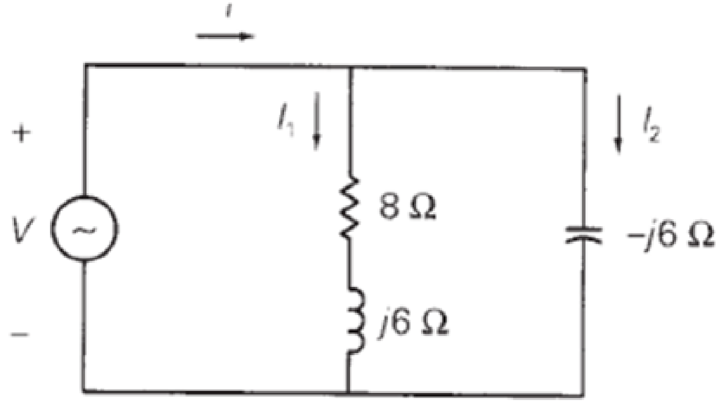
$$\boxed{IC_1 = I_1 = 4.66 - j0.666 = 4.713\angle -8.125}$$

$$\boxed{\begin{aligned} IC_2 &= -I_2 + I_1 = (5+j0) + (4.66 - j0.666) \\ &= -0.34 - j0.666 = 0.747\angle 242.95 \end{aligned}}$$

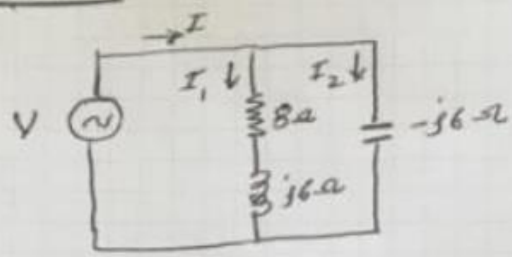
$$\boxed{\begin{aligned} I_R &= I_1 - I_3 = (4.66 - j0.666) - (4 - j2) \\ &= 0.66 + j1.334 = 1.488\angle 63.67 \end{aligned}}$$

$$\boxed{\begin{aligned} I_L &= I_2 - I_3 = (5+j0) - (4-j2) \\ &= 1 + j2 = 2.236\angle 63.43 \end{aligned}}$$

Q-11 In the figure shown below, $\bar{I} = 10\angle 0\ A$, compute the phasors $\bar{I_1}$, $\bar{I_2}$ and $\bar{V}$.



Q-11



$I = 10 + 0j = 10\angle 0^\circ$ A

$R = 8 + 0j = 8\angle 0^\circ\ \Omega$

$Z_L = 0 + j6 = 6\angle 90^\circ\ \Omega$

$Z_C = 0 - j6 = 6\angle -90^\circ\ \Omega$

$Z_1 = R + Z_L = 8 + j6 = 10\angle 36.87^\circ\ \Omega$

$$Z_T = \frac{Z_1 \times Z_C}{Z_1 + Z_C} = \frac{(10\angle 36.87)(6\angle -90)}{(8+j6)+(0-j6)} = 7.5\angle -53.13\ \Omega$$

$$\boxed{\bar{V}} = I Z_T = (10\angle 0)(7.5\angle -53.13) = \boxed{75\angle -53.13\ \text{V}}$$

$$\boxed{I_1} = \frac{V}{Z_1} = \frac{75\angle -53.13}{10\angle 36.87} = \boxed{7.5\angle -90\ \text{A}}$$

$$\boxed{\bar{I_2}} = \frac{V}{Z_C} = \frac{75\angle -53.13}{6\angle -90} = \boxed{12.5\angle 36.87\ \text{A}}$$