CSUS College of Engineering and Computer Science
Electrical & Electronic Engineering
ENGR 120 Probability and Random Signals

HW 4

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1) Problem 3.12
2) Problem 3.16
3) Problem 3.22
4) A random variable *X* is Gaussian with zero mean and unit variance. Find
    a. $P(|X|>2)$
    b. $P(X>2)$
5) A production line manufactures 1000 Ω resistors that must satisfy a10% tolerance. Let the random variable *X* represent the resistance of a resistor. Assuming that *X* is Gaussian with mean 1000 Ω and standard deviation σ=40 Ω, what fraction of resistors is expected to be rejected?

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HOMEWORK # 4

1.

$$f(x) \begin{cases} Cx^2 & 0 \le x \le 1 \\ C(2-x) & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) $$c\int_0^1 x^2 dx + c\int_1^2 (2-x)dx = 1$$

$$c\left[\frac{x^3}{3}\right]_0^1 + c\left[2x - \frac{x^2}{2}\right]_1^2 = 1$$

$$\frac{c}{3} + c\left[4-2-2+\tfrac{1}{2}\right] = 1$$

$$c/3 + c/2 = 1 \longrightarrow \boxed{c = 6/5}$$

(b) $$Pr[0.75 < x \le 1.5] = c\int_{0.75}^1 x^2 dx + c\int_1^{1.5} (2-x)dx$$

$$= 6/5\left[x^3/3\right]_{0.75}^1 + 6/5\left[2x - x^2/2\right]_1^{1.5} = \boxed{0.6813}$$

(c) $$Pr[x \le \alpha] = 0.9$$

$$c\int_0^1 x^2 dx + c\int_1^{\alpha} (2-x)dx = 0.9$$

$$6/5(1/3) + 6/5\left[2x - x^2/2\right]_1^{\alpha} = 0.9$$

$$2/5 + 6/5(2\alpha - \alpha^2/2 - 2 + 1/2) = 0.9$$

$$2/5 + 6/5(2\alpha - \alpha^2/2 - 3/2) = 0.9$$

$$-\alpha^2/2 + 2\alpha - 3/2 = 5/12$$

$$\alpha^2 - 4\alpha + 23/6 = 0 \longrightarrow \alpha = 2.4082, 1.5918$$

$$\boxed{\alpha = 1.5918}$$

2. $$f_X(x) = \begin{cases} 0.4 + Cx & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) $$\int_0^5 (0.4 + Cx)dx = 1 \longrightarrow \left[0.4x + \frac{Cx^2}{2}\right]_0^5 = 1$$

$$2 + 12.5c = 1 \longrightarrow \boxed{c = -0.08 = -\frac{2}{25}}$$

(b) $$Pr[x > 3] = \int_3^5 (0.4 - 0.08x)dx = \left[0.4x - 0.08x^2/2\right]_3^5$$

$$= \boxed{0.16}$$

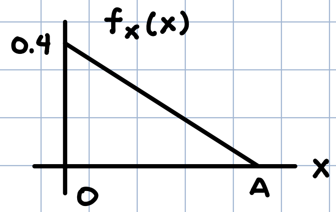
$$Pr[1 < x \le 4] = \int_1^4 (0.4 - 0.08x)dx = \left[0.4x - 0.08x^2/2\right]_1^4$$

$$\boxed{= 0.6}$$

(c) Find the CDF, $F_X(x)$

for $x<0$, $F_X(x)=0$
for $0\le x\le 5$, $F_X(x)=\int_0^x (0.4-0.08x)\,dx = 0.4x-0.04x^2$
for $x>5$, $F_X(x)=1$

3.



$$f_X(x)=\begin{cases}0.4(1-x/A) & 0\le x\le A\\ 0 & \text{otherwise}\end{cases}$$

(a) $\int_0^A 0.4(1-x/A)\,dx=1 \longrightarrow 0.4\left[x-\frac{x^2}{2A}\right]_0^A=1$

$A-\frac{A^2}{2A}=\frac{5}{2} \longrightarrow \boxed{A=5}$

(b) $0\le x\le 5$

$F_X(x)=\int_0^x f_X(z)\,dz=\int_0^x 0.4(1-z/5)\,dz$

$=0.4\left[z-\frac{z^2}{10}\right]_0^x=\boxed{0.4x-0.04x^2}$

4. (a) $P(|x|>2)=P(2<x)+P(x<-2)=1-P(x\le 2)+P(x<-2)$
$=1-F_X(2)+F_X(-2)$
$=\boxed{0.0456}$

(b) $P(x>2)=1-P(x\le 2)=1-F_X(2)=1-0.9772=\boxed{0.0228}$

5. $P_{reject}=P(x<900)+P(x>1100)$
$=2P(x>1100)=2Q[(1100-100)/40]$
$=2Q(2.5)=\boxed{0.1 \text{ or } 1.2\%}$