EEE 117L Laboratory – Network Analysis
Lab #7: Low Pass Filters (Week II)

Lab Day and Time: Wednesday 1:30 pm - 4:10 pm

Group Number: # 03

Group Members: (Last Name, First Name)

Member #1: Algador, Vigomar Kim

Member #2: Chan, Casey

Member #3: Bon, Trinh

Total Score: /100

General Instructions:

1) Chapter 14 in the text on frequency selective circuits may be of use in the hand calculations and the general understanding of low pass filters and the cutoff frequency. Similarly, Chapter 15 on active, as opposed to passive, filter circuits will be useful.

2) You will need more pages in order to complete this worksheet. Make sure to include a title and page number for each section added. Make sure to show all your work and reasoning.

Work Breakdown Structure: It is important that every group member do their share of the work in these labs. Remember that you will receive no credit for the lab worksheet if you did not contribute. Write in the Table provided below, which group member(s) contributed to the solution of each problem in the lab worksheet. Also remember that only on lab worksheet per group will be turned in to Canvas. If there was any group member that did not contribute, then write their name in the space provided below.

Problem Number	Group member(s) that worked on the problem.
Part 1a	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 1b	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 1c	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 1d	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2a	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2b	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2c	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2d	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 3	Algador, Vigomar Kim Chan, Casey Trinh, Bon

Absent member(s):____

1. Unloaded Active Low Pass Filter Total Score: ____/45

a) Preliminary Measurements: Score: ____/5

This section of the lab involves the circuit shown in Figure 1 below. Before simulating or building the circuit, the values of the resistors, R_1 and R_2 , and capacitor, C, must be measured using the DMM. Do so and record the values in Table 1.

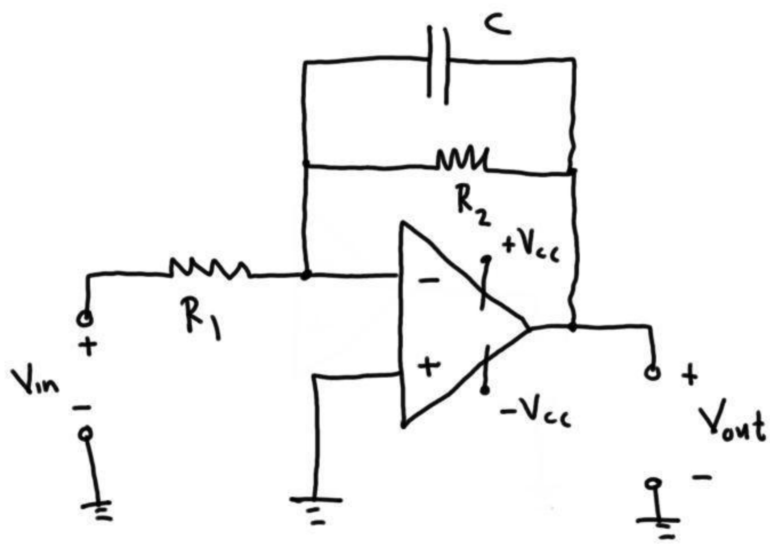


Figure 1. Unloaded Passive Low Pass Filter

	Theoretical Value	Experimental Value
R_1	1000 Ω	994
R_2	1000 Ω	994
C	100 nF	97.5

Table 1. Component Values for the Unloaded Active Low Pass Filter

b) Hand Calculations: Finding the Cutoff Frequency Score: ____/10

Using the resulting equation for the transfer function, $H(s)$, from the prelab calculations, follow the steps described below to calculate the cutoff frequency, f_c, for the circuit.

1) Change from Laplace space to frequency space using the relation $s = j\omega$ and find the magnitude of the transfer function, $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right|$, and the angle of the transfer function, $\angle H(j\omega) = \angle(\theta_{out} - \theta_{in})$. These will both be functions of ω.

$$H(s) = -\frac{1}{1+sRC}$$

$$H(j\omega) = -\frac{1}{1+j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad \angle H(j\omega) = 180 - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= 180 - \tan^{-1}(\omega RC)$$

Answers: $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$

$\angle H(j\omega) = \angle(\theta_{out} - \theta_{in}) = 180 - tan^{-1}(\omega RC)$

2) Using the magnitude of the transfer function, $|H(j\omega)|$, from the previous calculation, find the maximum value of the transfer function, H_{max}.

let $\omega = 0$:

$$H_{max} = \frac{1}{\sqrt{1+[(0)RC]^2}} = \frac{1}{\sqrt{1+0}} = 1$$

Answer: $H_{max} = 1$

3) Using the results from steps 1) and 2) above use the definition shown below to find the cutoff angular frequency, ω_c.

$$\left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}H_{max}$$

$$\left|H(j\omega_c)\right| = \frac{1}{\sqrt{1+(\omega_c RC)^2}}$$

$$\frac{1}{\sqrt{2}}H_{max} = \frac{1}{\sqrt{1+(\omega_c RC)^2}}$$

$$1+(\omega_c RC)^2 = \frac{1}{\left[\left(\frac{1}{\sqrt{2}}\right)(1)\right]^2}$$

$$\omega_c RC = \sqrt{2-1} = 1$$

$$\omega_c = \frac{1}{RC}$$

Answer: $\omega_c = \frac{1}{RC}$

4) Using the cutoff angular frequency found in step d) above, evaluate the phase shift at the cutoff angular frequency by calculating $\angle H(j\omega_c)$. Lastly, use the relation $\omega_c = 2\pi f_c$, calculate the cutoff frequency, f_c.

$$\omega_c = 2\pi f_c$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1/RC}{2\pi} = \frac{1}{2\pi RC}$$

$$\angle H(j\omega_c) = 180 - \tan^{-1}(\omega_c RC)$$

$$= 180 - \tan^{-1}\left[\frac{RC}{RC}\right]$$

$$= 180 - 45$$

$$= 135^\circ$$

Answers:

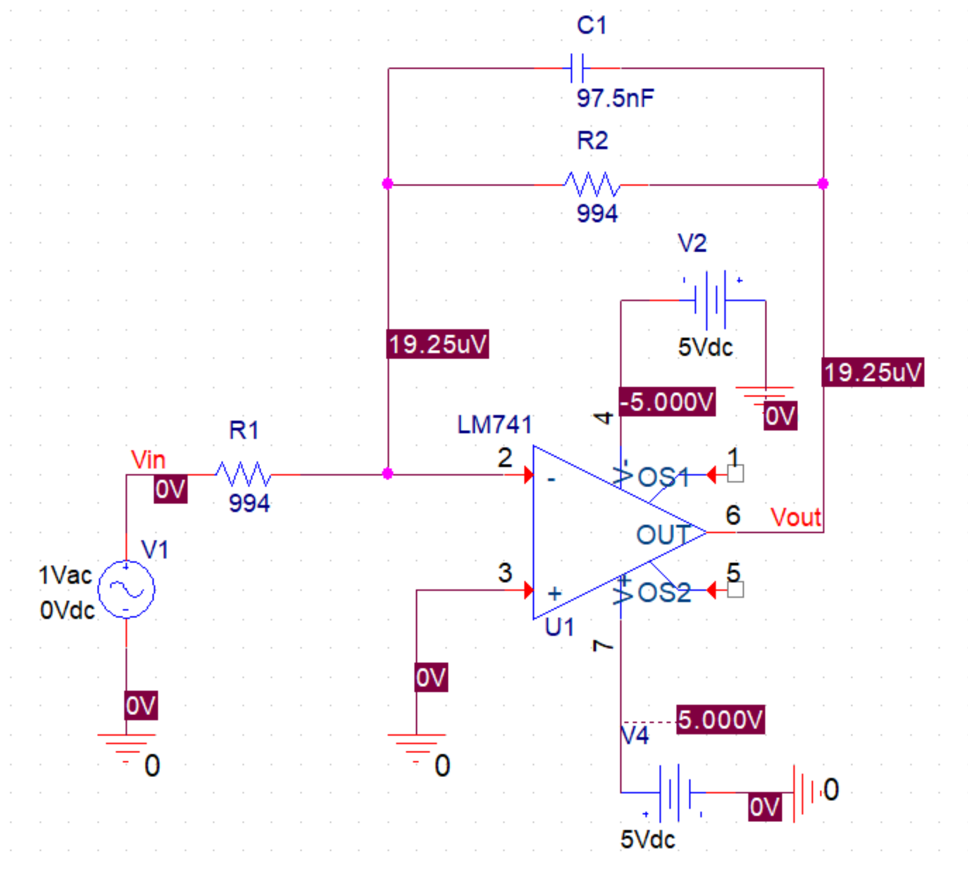
$$\angle H(j\omega_c) = \angle(\theta_{out} - \theta_{in}) = 135°$$

$$f_c = \frac{1}{2\pi RC}$$

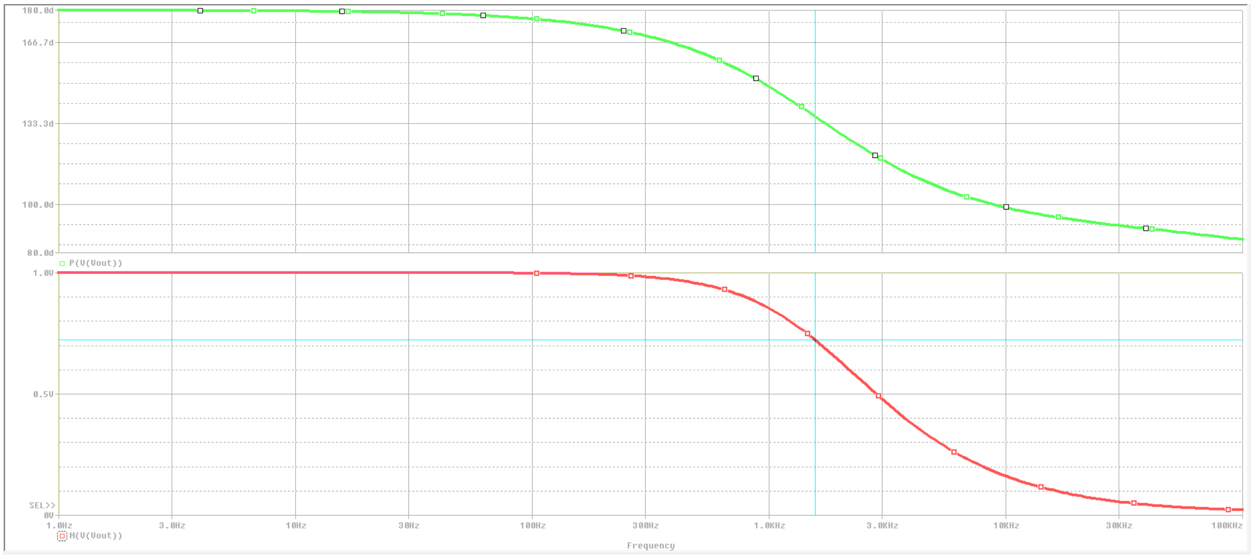
c) Prediction: Simulation Score: ____/15

Using the experimentally found values of the resistors, R_1 and R_2, and capacitor, C in part a), run an AC Sweep simulation using PSpice of the circuit shown in Figure 1 in order to find the cutoff frequency. The external power supplies should be set to $+V_{CC} = 5\,V$ and $-V_{CC} =- 5\,V$. Adjust the source for 1 Volt with 0° phase shift. For sweep type, select decades. The sweep parameters should be 50-points/decade, with a start frequency of 1Hz and an end frequency of 100kHz. For the first trace, use the magnitude of the output voltage. Select M (for magnitude) from the functions and V(Vout) from the variables. The next trace will be the phase. Since the values of the magnitude and phase are so different, a different plot should be used for the phase. Use ADD new plot (under PLOT on the tool bar.) In the new plot add P[V(Vout)]. Note that PSPICE plots the graphs in Hertz and not radians per second. Include pictures of 1) the circuit schematic and 2) the output graph of the simulation below. What according to your plots, what is the cutoff frequency and what is the phase shift at the cutoff frequency? You can use the cursor button to find these parameters.

1) Circuit Schematic:



2) Output:



	Trace Color	Trace Name	Y1	Y2	Y1 - Y2
		X Values	1.5665K	1.0000	1.5655K
	CURSOR 1,2	M(V(Vout))	723.042m	1.0000	-276.946m
		P(V(Vout))	136.206	179.965	-43.759

Answers: Cutoff frequency, $f_C = 1.5665\text{k}$

Phase shift at cutoff frequency, $\angle H(j\omega_c) = \angle(\theta_{out} - \theta_{in}) = 136.206$

d) Data Collection and Analysis: Find the Cutoff Frequency Score: ____/15

Build the circuit shown in Figure 1 and fill in Tables 2a-c below. Show your work for the calculations of the theoretical values on a separate sheet of paper. Here you will be obtaining the value of H_{max} experimentally by assuming that the maximum value of the transfer function occurs at the low frequency value of $f = 100\ Hz$. Keep in mind that this is a valid assumption only for this specific circuit. (Can you figure out why based on your hand calculations?) You then will be changing the frequency of the input signal until the following condition is met: $\left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}H_{max}$. Recall this is the the definition of the cutoff frequency.

	R_1 (Ω)	R_2 (Ω)	C (μF)
Theoretical Values	1000	1000	0.1
Experimental Values	994	994	0.0975
% Error	0.6	0.6	2.5

Table 2a. Preliminary Measurements for Unloaded Active LPF

<u>For Low Frequency Values:</u> $f = 100\ Hz$

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	1	0	176.52
Experimental Values	1.015	1.015	0	177.62
% Error	1.5	1.5	0	0.62

Table 2b. Low Frequency Measurements for Unloaded Active LPF

<u>At the Cutoff Frequency:</u> f_C = **1642.21 Hz**

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	0.7071	0.7071	135
Experimental Values	1.015	0.735	0.7241	136.78
% Error	1.5	3.95	2.40	1.32

Table 2c. Measurements at the Cutoff Frequency for Unloaded Active LPF

CALCULATIONS:

$$\% \text{ Error} = \frac{|\text{Experimental} - \text{Theoretical}|}{\text{Theoretical}} \times 100$$

$$\text{RESISTOR: } R \ \% \text{ Error} = \frac{|994\Omega - 1000\Omega|}{1000\Omega} \times 100 = 0.6\%$$

$$\text{CAPACITOR: } \% \text{ Error} = \frac{|0.0975 - 0.1|}{0.1} \times 100 = 2.5\%$$

FOR LOW FREQUENCY VALUES: $f = 100 \text{ Hz}$

$$\omega = 2\pi f = 2\pi(100) = 200\pi$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + [(200\pi)(999)(97.5 \times 10^{-9})]^2}}$$

$$\left|\frac{V_{out}}{1}\right| = 0.998$$

$$V_{out} = 0.998 \text{ V} \approx 1 \text{ V}$$

$$\text{GAIN: } \frac{V_{out}}{V_{in}} = \frac{1}{1} = 1$$

$$\text{PHASE SHIFT: } \angle H(j\omega) = 180 - \tan^{-1}(\omega RC)$$

$$= 180 - \tan^{-1}[(200\pi)(999)(97.5 \times 10^{-9})]$$

$$= 176.52$$

AT THE CUTOFF FREQUENCY:

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(994)(97.5 \times 10^{-9})} = 1642.21 \text{ Hz}$$

$$\omega_c = \frac{1}{RC} = \frac{1}{(994)(97.5 \times 10^{-9})} = 10318.32 \text{ rad/s}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega_c RC)^2}} = \frac{1}{\sqrt{1 + [(10318.32)(999)(97.5 \times 10^{-9})]^2}}$$

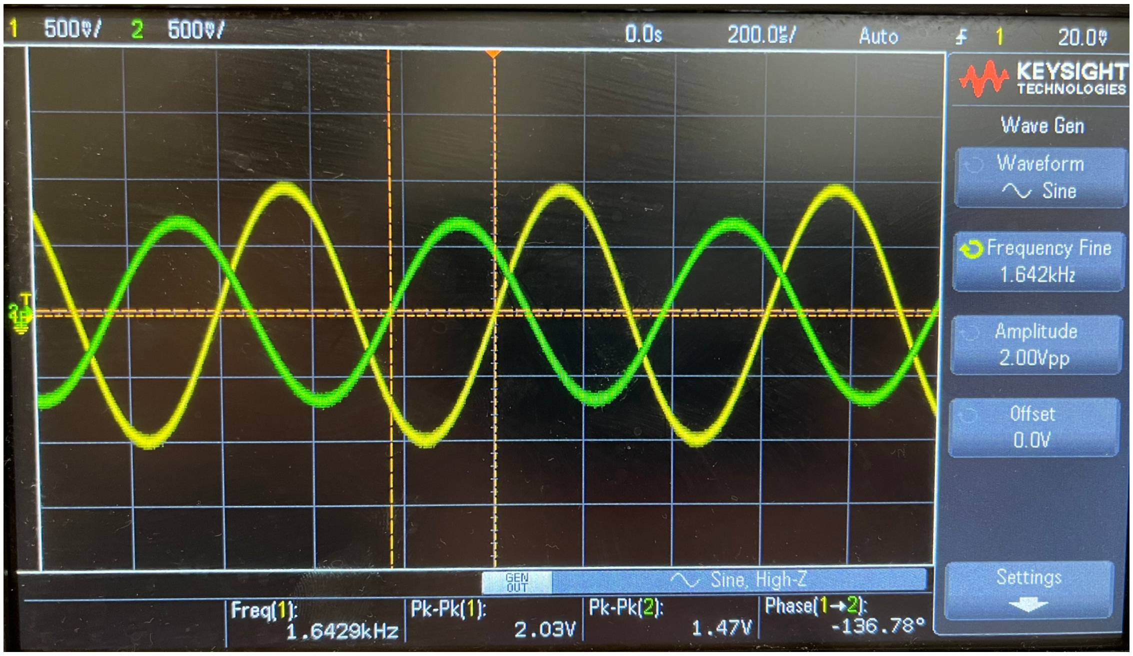
$$\left|\frac{V_{out}}{1}\right| = 0.7071$$

$$V_{out} = 0.7071$$

$$\text{GAIN: } \frac{V_{out}}{V_{in}} = \frac{0.7071}{1} = 0.7071$$

$$\text{PHASE SHIFT: } \angle H(j\omega_c) = 135^\circ$$

Experimental output: Include an image both channels of the oscilloscope at the cutoff frequency



Question: How does this compare to the simulated results?

The simulation and the experimental output showed very similar results. The simulated values are also similar to the theoretical values.

2. Loaded Active Low Pass Filter Total Score: ____/45

a) Preliminary Measurements: Score: ____/5

This section of the lab involves the circuit shown in Figure 2 below. Before simulating or building the circuit, the values of the resistors, R_1, R_2, and R_L, and capacitor, C, must be measured using the DMM. Do so and record the values in Table 3.

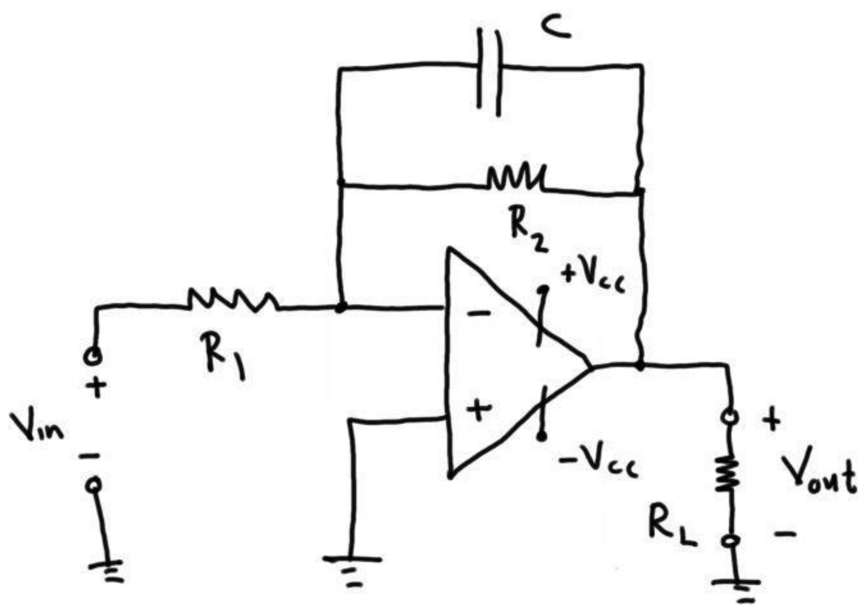


Figure 2. Loaded Active Low Pass Filter

	Theoretical Value	Experimental Value
R_1	1000 Ω	994
R_2	1000 Ω	994
R_L	1000 Ω	992
C	100 nF	97.5

Table 3. Component Values for the Loaded Active Low Pass Filter

b) Hand Calculations: Finding the Cutoff Frequency Score: ____/10

Using the resulting equation for the transfer function, $H(s)$, from the prelab calculations, follow the steps described below to calculate the cutoff frequency, f_c, for the circuit.

1) Change from Laplace space to frequency space using the relation $s = j\omega$ and find the magnitude of the transfer function, $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right|$, and the angle of the transfer function, $\angle H(j\omega) = \angle\left(\theta_{out} - \theta_{in}\right)$. These will both be functions of ω.

$$H(s) = -\frac{1}{1+sRC}$$

$$H(j\omega) = -\frac{1}{1+j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad \angle H(j\omega) = 180 - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= 180 - \tan^{-1}(\omega RC)$$

Answers: $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$

$\angle H(j\omega) = \angle\left(\theta_{out} - \theta_{in}\right) = 180 - tan^{-1}(\omega RC)$

2) Using the magnitude of the transfer function, $|H(j\omega)|$, from the previous calculation, find the maximum value of the transfer function, H_{max}.

let $\omega = 0$:

$$H_{max} = \frac{1}{\sqrt{1+[(0)RC]^2}} = \frac{1}{\sqrt{1+0}} = 1$$

Answer: $H_{max} = 1$

3) Using the results from steps 1) and 2) above use the definition shown below to find the cutoff angular frequency, ω_c.

$$\left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}H_{max}$$

$$|H(j\omega_c)| = \frac{1}{\sqrt{1+(\omega_c RC)^2}}$$

$$\frac{1}{\sqrt{2}}H_{max} = \frac{1}{\sqrt{1+(\omega_c RC)^2}}$$

$$1+(\omega_c RC)^2 = \frac{1}{\left[\left(\frac{1}{\sqrt{2}}\right)(1)\right]^2}$$

$$\omega_c RC = \sqrt{2-1} = 1$$

$$\omega_c = \frac{1}{RC}$$

Answer: $\omega_c = \frac{1}{RC}$

4) Using the cutoff angular frequency found in step d) above, evaluate the phase shift at the cutoff angular frequency by calculating $\angle H\left(j\omega_c\right)$. Lastly, use the relation $\omega_c = 2\pi f_c$, calculate the cutoff frequency, f_c.

$$\omega_c = 2\pi f_c$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1/RC}{2\pi} = \frac{1}{2\pi RC}$$

$$\angle H(j\omega_c) = 180 - \tan^{-1}(\omega_c RC)$$

$$= 180 - \tan^{-1}\left[\frac{RC}{RC}\right]$$

$$= 180 - 45$$

$$= 135^\circ$$

Answers:

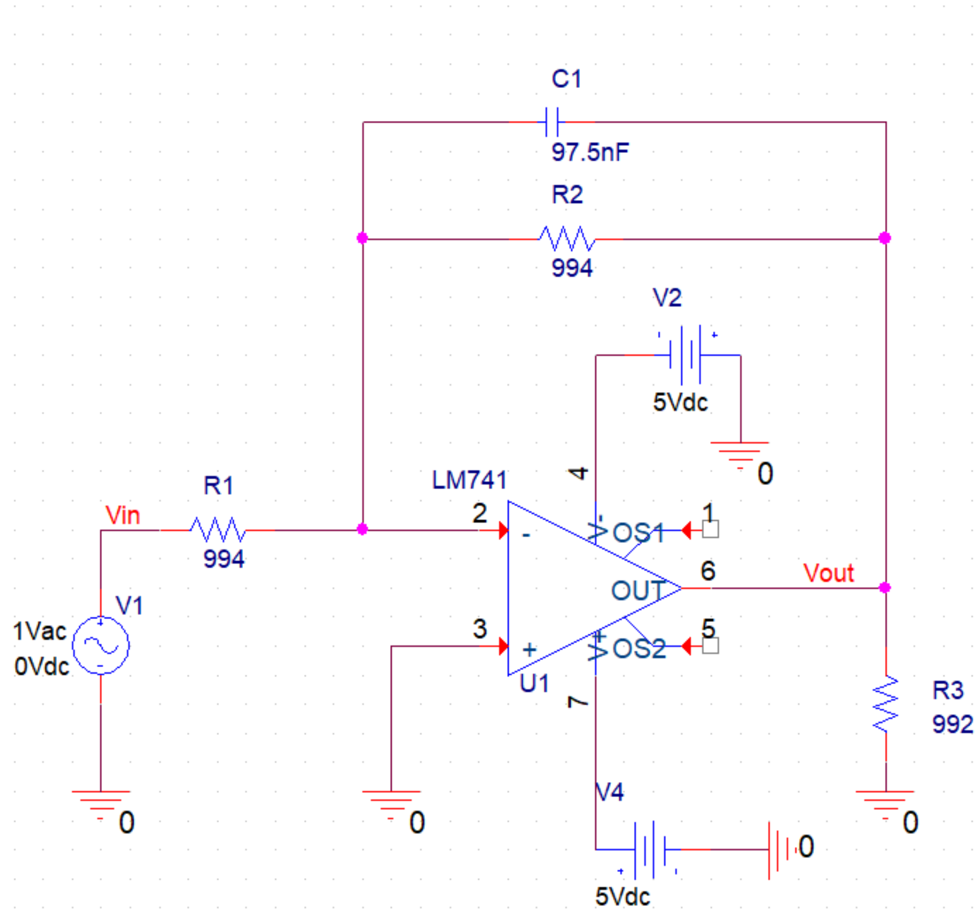
$$\angle H\left(j\omega_c\right) = \angle\left(\theta_{out} - \theta_{in}\right) = 135°$$

$$f_c = \frac{1}{2\pi RC}$$

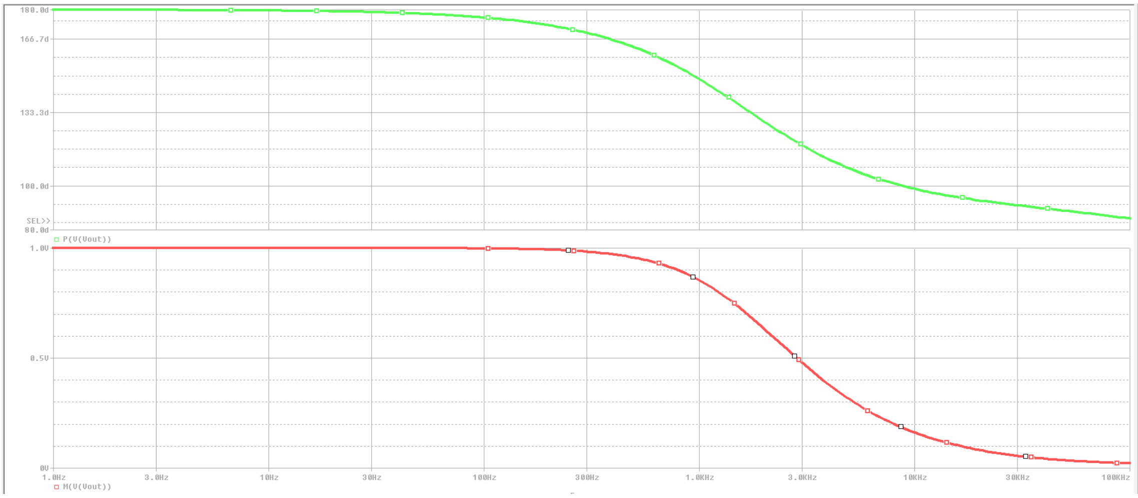
c) Prediction: Simulation Score: ____/15

Using the experimentally found values of the resistors, R_1, R_2, and R_L, and capacitor, C in part a), run an AC Sweep simulation using PSpice of the circuit shown in Figure 2 in order to find the cutoff frequency. The external power supplies should be set to $+V_{CC} = 5\,V$ and $-V_{CC} = -5\,V$. Adjust the source for 1 Volt with 0° phase shift. For sweep type, select decades. The sweep parameters should be 50-points/decade, with a start frequency of 1Hz and an end frequency of 100KHz. For the first trace, use the magnitude of the output voltage. Select M (for magnitude) from the functions and V(VOUT) from the variables. The next trace will be the phase. Since the values of the magnitude and phase are so different, a different plot should be used for the phase. Use ADD new plot (under PLOT on the tool bar.) In the new plot add P[V(VOUT)]. Note that PSPICE plots the graphs in Hertz and not radians per second. Include pictures of 1) the circuit schematic and 2) the output graph of the simulation below. What according to your plots, what is the cutoff frequency and what is the phase shift at the cutoff frequency?

1) Circuit Schematic:



2) Output:



	Trace Color	Trace Name	Y1	Y2	Y1 - Y2
		X Values	1.5665K	1.0000	1.5655K
		M(V(Vout))	723.013m	1.0000	-276.974m
	CURSOR 1,2	P(V(Vout))	136.200	179.965	-43.765

Answer: Cutoff frequency, $f_C = 1.5665\text{k}$

Phase shift at cutoff frequency, $\angle H\left(j\omega_c\right) = \angle\left(\theta_{out} - \theta_{in}\right) = 136.2$

d) Data Collection and Analysis Score: ____/15

Build the circuit shown in Figure 2 and fill in Tables 4a-c below. Show your work for the calculations of the theoretical values on a separate sheet of paper. Here you will be obtaining the value of H_{max} experimentally by assuming that the maximum value of the transfer function occurs at the low frequency value of $f = 100\ Hz$. Keep in mind that this is a valid assumption only for this specific circuit. You then will be changing the frequency of the input signal until the following condition is met: $\left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}H_{max}$. Recall this is the the definition of the cutoff frequency.

	R_1 (Ω)	R_2 (Ω)	R_L (Ω)	C (μF)
Theoretical Values	1000	1000	1000	0.1
Experimental Values	994	994	992	0.0975
% Error	0.6	0.6	0.8	2.5

Table 4a. Preliminary Measurements for Loaded Active LPF

For Low Frequency Values: $f = 100\ Hz$

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	1	0	180
Experimental Values	1.015	1.015	0	177.38
% Error	1.5	1.5	0	1.45

Table 4b. Low Frequency Measurements for Loaded Active LPF

At the Cutoff Frequency: f_C = **1642.21 Hz**

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	0.7071	0.7071	135
Experimental Values	1.015	0.735	0.724	134.87
% Error	1.5	3.95	2.39	0.096

Table 4c. Measurements at the Cutoff Frequency for Loaded Active LPF

CALCULATIONS:

$$\% \text{ Error} = \frac{|\text{Experimental} - \text{Theoretical}|}{\text{Theoretical}} \times 100$$

RESISTOR : $R\ \% \text{ Error} = \frac{|994\Omega - 1000\Omega|}{1000\Omega} \times 100 = 0.6\%$

$R_L\ \% \text{ Error} = \frac{|992\Omega - 1000\Omega|}{1000\Omega} \times 100 = 0.8\%$

CAPACITOR : $\% \text{ Error} = \frac{|0.0975 - 0.1|}{0.1} \times 100 = 2.5\%$

FOR LOW FREQUENCY VALUES : $f = 100\text{ Hz}$

$$\omega = 2\pi f = 2\pi(100) = 200\pi$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + [(200\pi)(999)(97.5\times10^{-9})]^2}}$$

$$\left|\frac{V_{out}}{1}\right| = 0.998$$

$$V_{out} = 0.998\text{ V} \approx 1\text{ V}$$

GAIN: $\frac{V_{out}}{V_{in}} = \frac{1}{1} = 1$

PHASE SHIFT: $\angle H(j\omega) = 180 - \tan^{-1}(\omega RC)$

$$= 180 - \tan^{-1}[(200\pi)(999)(97.5\times10^{-9})]$$

$$= 176.52$$

AT THE CUTOFF FREQUENCY :

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(994)(97.5\times10^{-9})} = 1642.21\text{ Hz}$$

$$\omega_c = \frac{1}{RC} = \frac{1}{(994)(97.5\times10^{-9})} = 10318.32\text{ rad/s}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega_c RC)^2}} = \frac{1}{\sqrt{1 + [(10318.32)(999)(97.5\times10^{-9})]^2}}$$

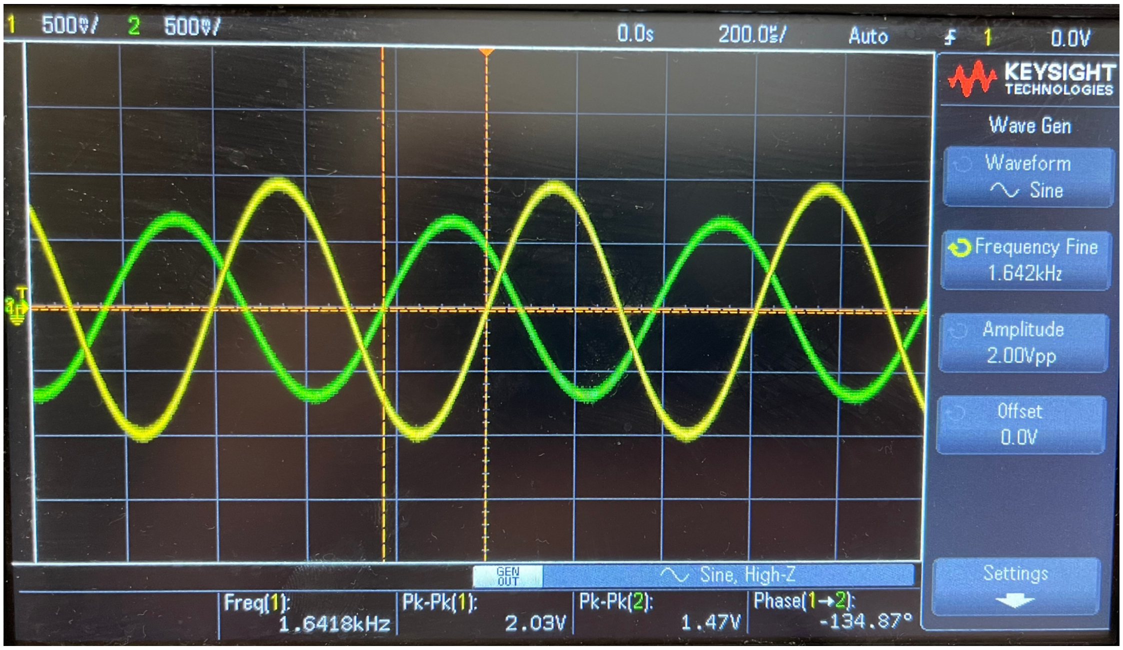
$$\left|\frac{V_{out}}{1}\right| = 0.7071$$

$$V_{out} = 0.7071$$

GAIN: $\frac{V_{out}}{V_{in}} = \frac{0.7071}{1} = 0.7071$

PHASE SHIFT : $\angle H(j\omega_c) = 135°$

Experimental output: Include an image both channels of the oscilloscope at the cutoff frequency



Question: How does this compare to the simulated results?

The experimental output is very similar to the simulated results. The experimental values are also similar to the theoretical values.

3. Conclusions Total Score: /10

Explain in a few paragraphs the purpose of the lab, the experimental set up and methodology, and central results of the lab and these experiments. **You should be quantitative** in this summary. Include any important equations used and explain their significance. Write the conclusion as if you were writing an English essay. This is an important portion of the lab, so make sure to do a good and thorough job.

In this lab, we continued working with low pass filters. To find the magnitude, we used the transfer function, $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right|$. To find the angle of the transfer function, the formula used is $\angle H(j\omega) = \angle\left(\theta_{out} - \theta_{in}\right)$. Once the magnitude was found we were then able to solve for H_{max}. Once we found these values, we were then able to solve for the cutoff angular frequency. Finally, we were able to find the cutoff frequency. We noticed that in the data collection, the numbers were almost the same in both circuits. We learned that the load resistor does not affect the overall op amp circuit. The cutoff frequency is the same for both circuits. Since we had done the hand calculations in the previous lab, we were much more familiar with solving the calculations.