Operational Amplifiers

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* Most figures and examples are from the course textbook "Microelectronic Circuits" by Adel S. Sedra and Kenneth C. Smith, 6th Edition, © 2010 by Oxford University Press, Inc.

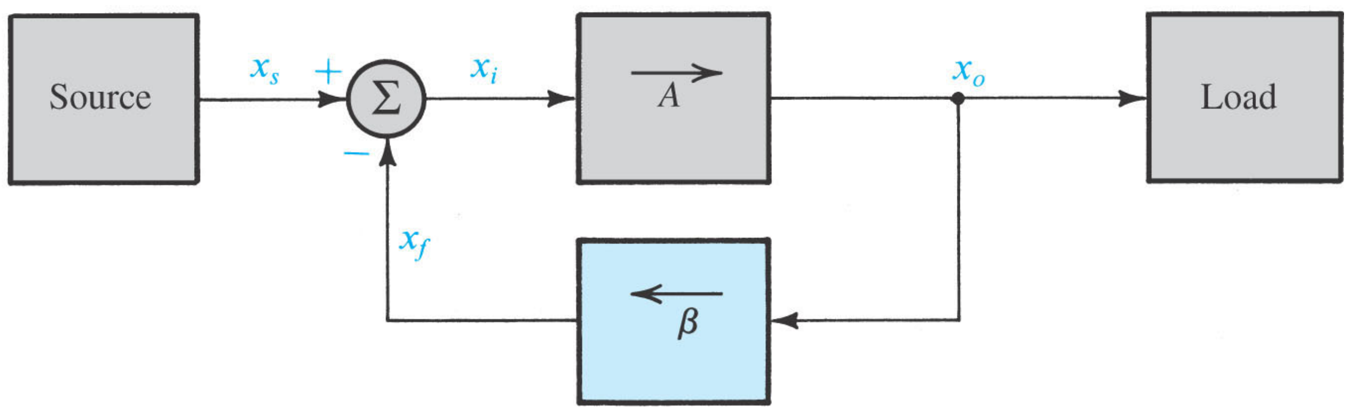
Outline

- Why use Operational Amplifiers? (“opamps”)
  - The wonderful effects of using negative feedback!
- Characteristics of ideal opamps
- Basic amplifiers using opamps
  - Inverting and non-inverting amplifiers, unity-gain buffers
  - Summing amplifiers
- Difference & Instrumentation amplifiers
- Integrators, Differentiators and Active Filters
- Non-ideal effects in real integrated circuit opamps
  - Finite DC gain, offset voltage, input bias and offset currents
  - Output voltage and current limits
  - Finite bandwidth, slew rate, and full-power bandwidth
- Summary of key concepts

Why use Operational Amplifiers?

- Opamps are a key building block in feedback systems
- Negative feedback makes everything better!
  - More accurate amplifier gain (closed-loop)
  - Higher or lower input & output resistance (whichever is needed)
  - Higher amplifier -3dB bandwidth (closed-loop)
- But, we have to pay for this!
  - Negative feedback trades high open-loop amplifier gain for everything else we need
- So, operational amplifiers are designed to have very high open-loop voltage gains of 100,000 (100dB) or more, with the intention of using them in closed-loop feedback circuits which trade away the extra gain for other desirable performance characteristics

Key Concepts for Negative Feedback



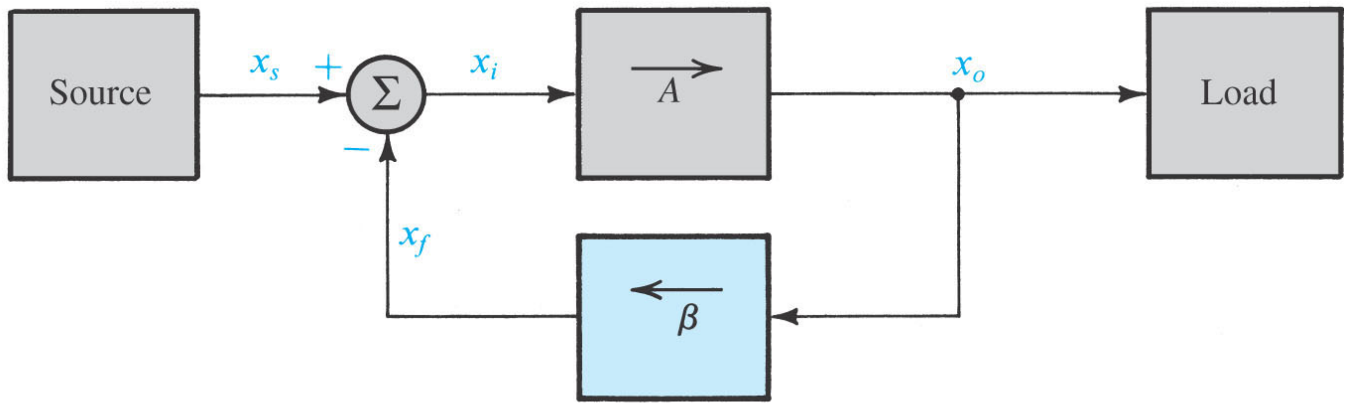
- Negative feedback systems sample the output signal of an amplifier and feed back a fraction of the output to the input, where it is subtracted from the source signal

A = X_o / X_i = the gain of the forward amplifier (the "open-loop" gain)

β = X_f / X_o = the gain of the feedback path (the "feedback factor")
= the fraction of the output signal fed back to the input

$\mathbf{A_f}$ = X_o / X_s = the gain of the feedback amplifier (the "closed-loop" gain)

Key Concepts for Negative Feedback



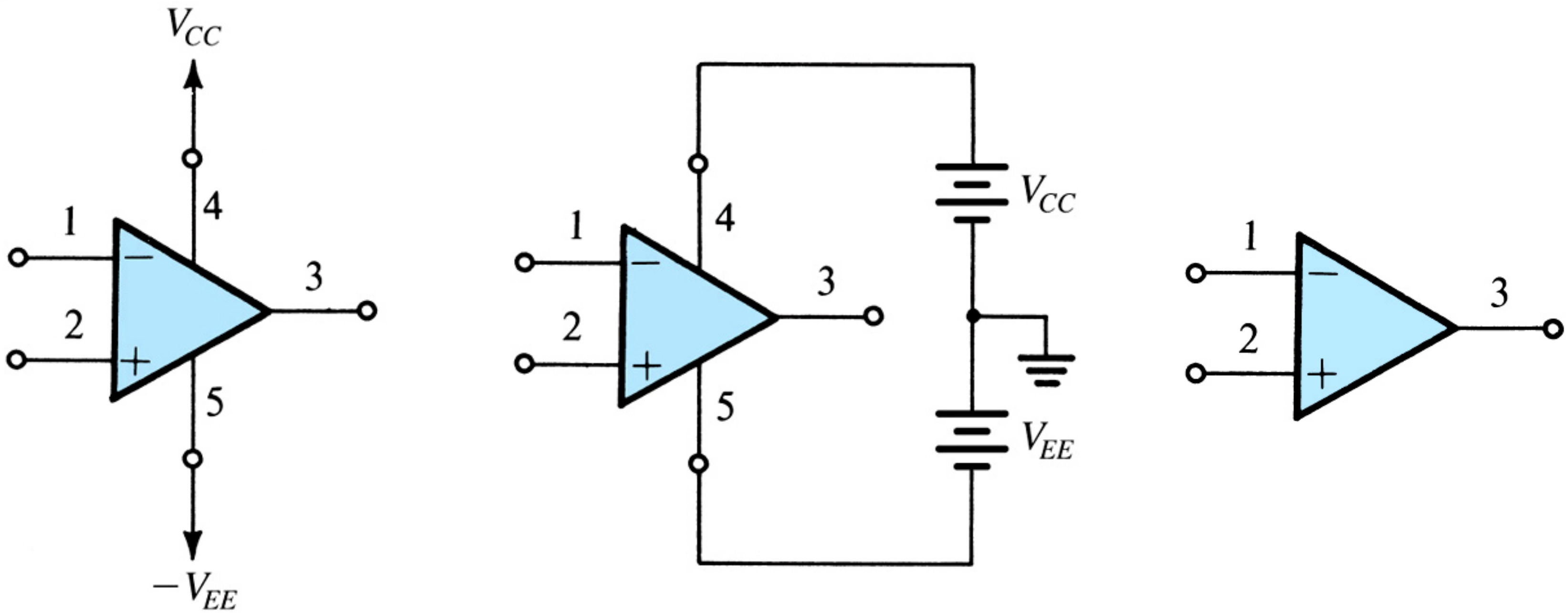
- Negative feedback makes the closed-loop gain with feedback insensitive to changes in the open loop gain!

→ Even if A varies with process, voltage and temperature (PVT variations), the gain with feedback stays ~ constant
  - β is usually set by a ratio of resistors, which stays ~ constant

$$\boxed{\frac{X_o}{X_s} = A_f = \frac{A}{1 + A\beta} \approx \frac{1}{\beta} \quad if \;\; A\beta >> 1}$$

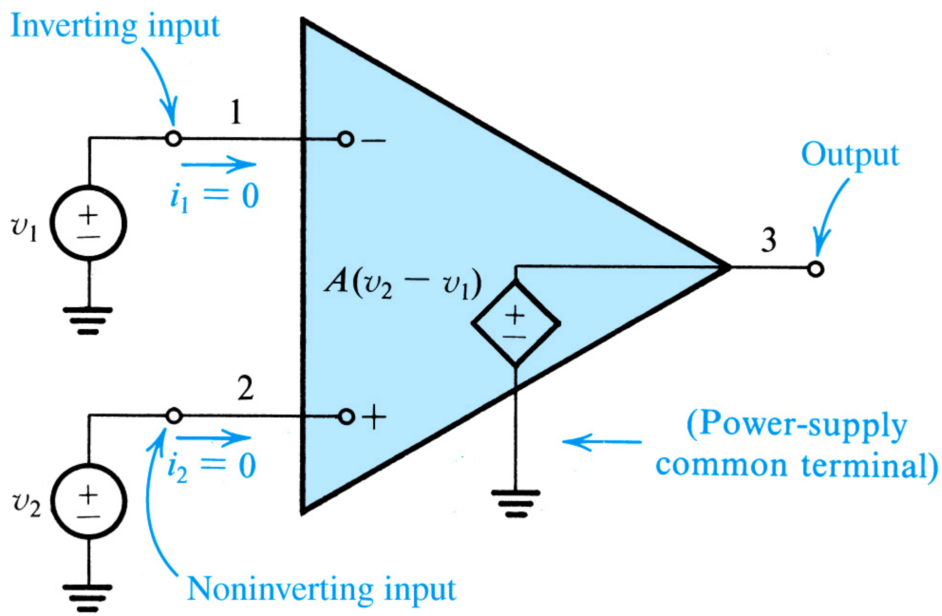
(For more on feedback, see Chapter 10 in the Sedra & Smith textbook)

Opamp symbols



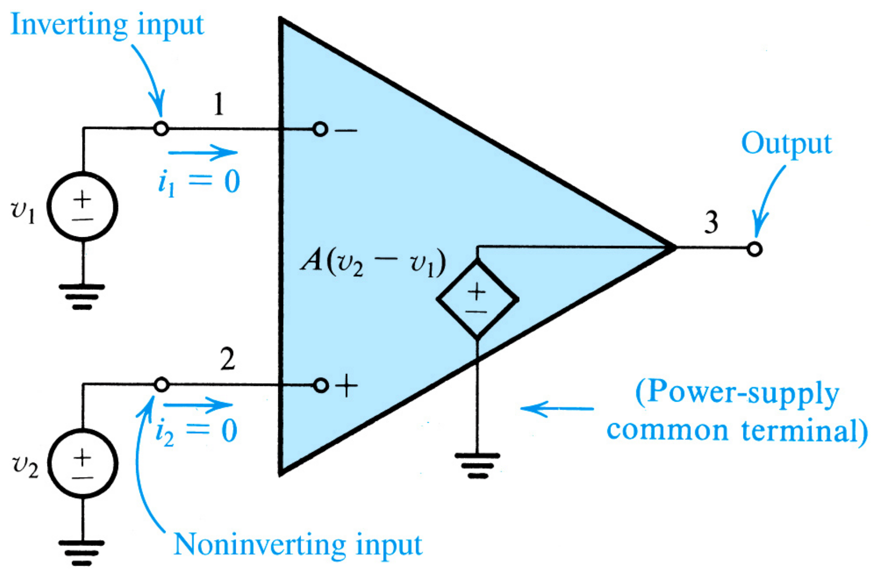
- Opamps require + and – power supplies to operate, but are often shown without these explicitly drawn to keep schematics simple and easy to understand
  - Everyone knows power supplies are required, so why draw them?
  - The purpose of a good schematic is to effectively communicate information, so reduce "clutter" to make them easier to read!

Characteristics of Ideal Opamps



- Opamps always have both a + and – input to make it easy to apply negative feedback around them
- The output voltage, $V_o = A(v_2 - v_1)$ where $v_2 = v_+$, $v_1 = v_-$
- Outputs can be single-ended (measured with respect to ground) or differential with V_{op} , V_{on} ($V_{odm} = V_{op} - V_{on}$)

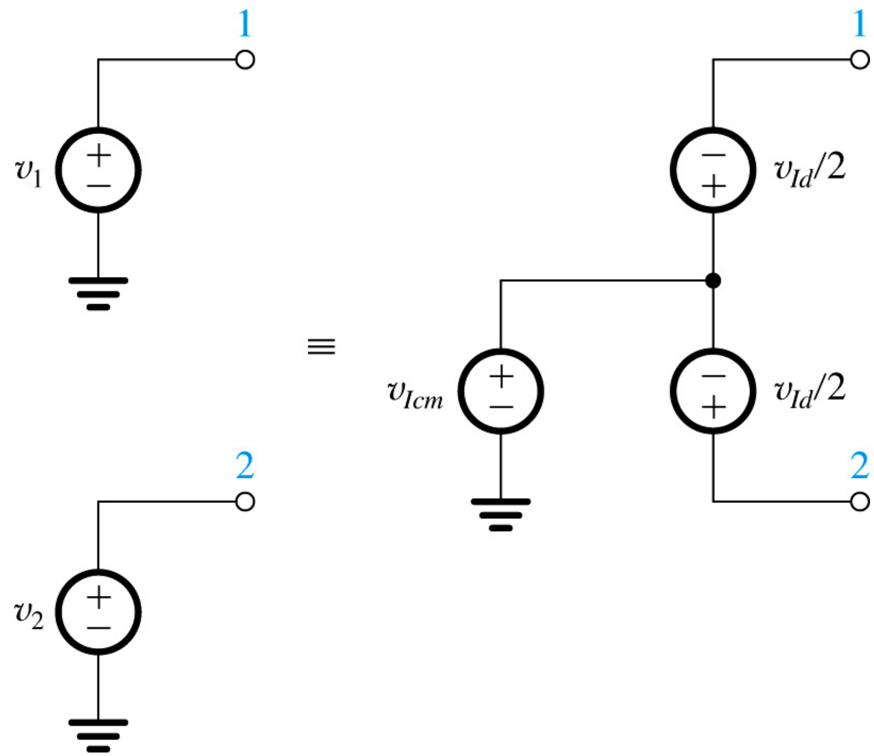
Characteristics of Ideal Opamps



- <u>Ideal opamps have :</u>
  - Infinite gain (A = ∞) → **there is 0 V between the + and – inputs!**
  - Infinite bandwidth ($f_{-3dB} = \infty$)
  - Infinite input resistance ($R_i = \infty$) → **i = 0 into the + and – inputs!**
  - Zero output resistance (no signal is lost, even for small loads)
  - Zero gain for common-mode inputs (only differences are amplified)

Differential & Common-mode signals

- Opamps are designed to amplify the difference between the + and – inputs, but NOT amplify any voltage common to both inputs (like noise)
- This is called **common-mode rejection**
- It is often useful to redraw the input signals as the combination of a **differential** signal and a **common-mode** signal



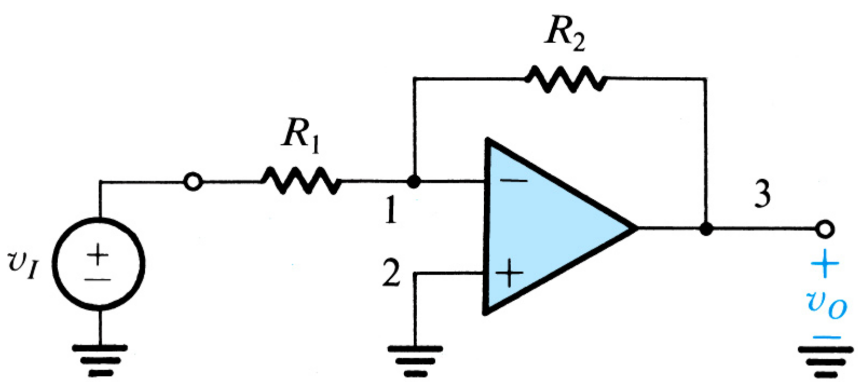
$$v_1 = v_{icm} - \frac{v_{idm}}{2}$$

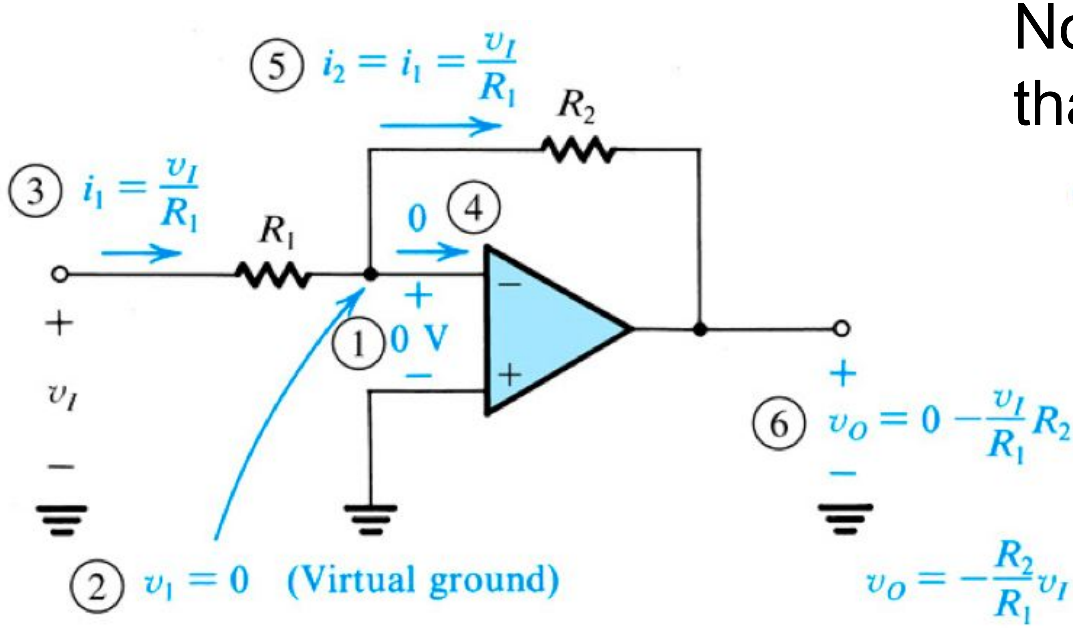
$$v_2 = v_{icm} + \frac{v_{idm}}{2}$$

$$v_{idm} = v_2 - v_1$$

$$v_{icm} = \frac{v_2 + v_1}{2}$$

Inverting Amplifier





Using the **Ideal Opamp assumptions** that the :

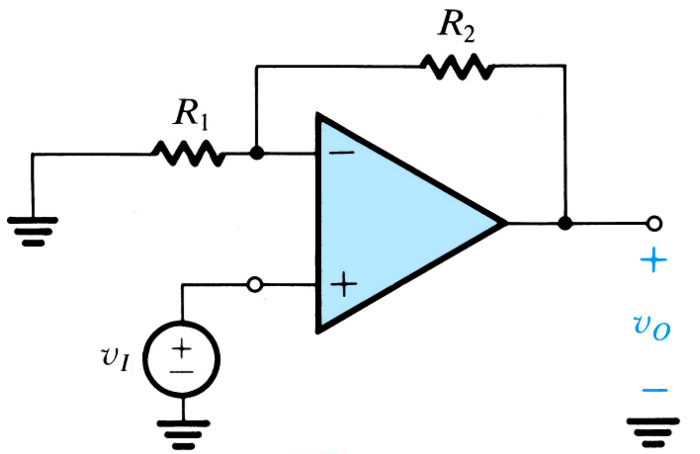
1. Voltage between the + and – input terminals = 0 (since A=∞)
2. Input currents = 0 (since R_i =∞)

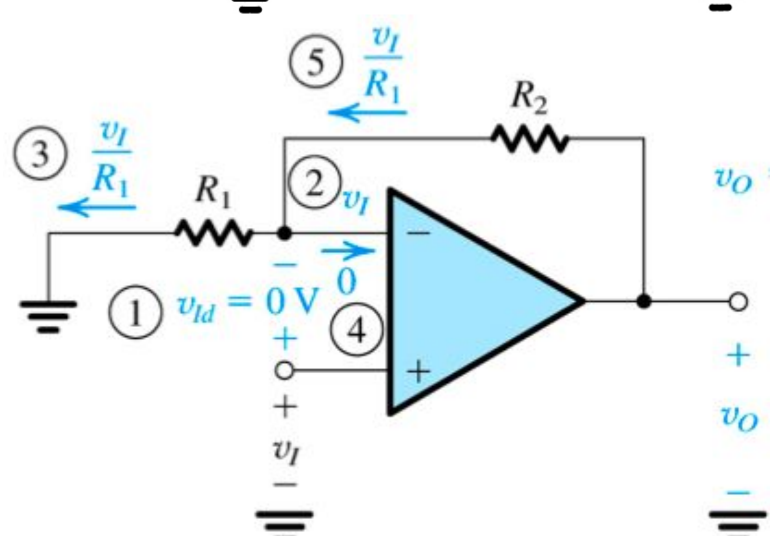
Nodal analysis shows that the voltage gain is :

$$\boxed{A_v = \frac{v_o}{v_i} = -\frac{R_2}{R_1}}$$

The – input here is a "**virtual ground**"

Non-Inverting Amplifier





$$v_O = v_I + \frac{v_I}{R_1}R_2 = v_I\left(1 + \frac{R_2}{R_1}\right) \quad (6)$$

Using the **Ideal Opamp assumptions** that the :

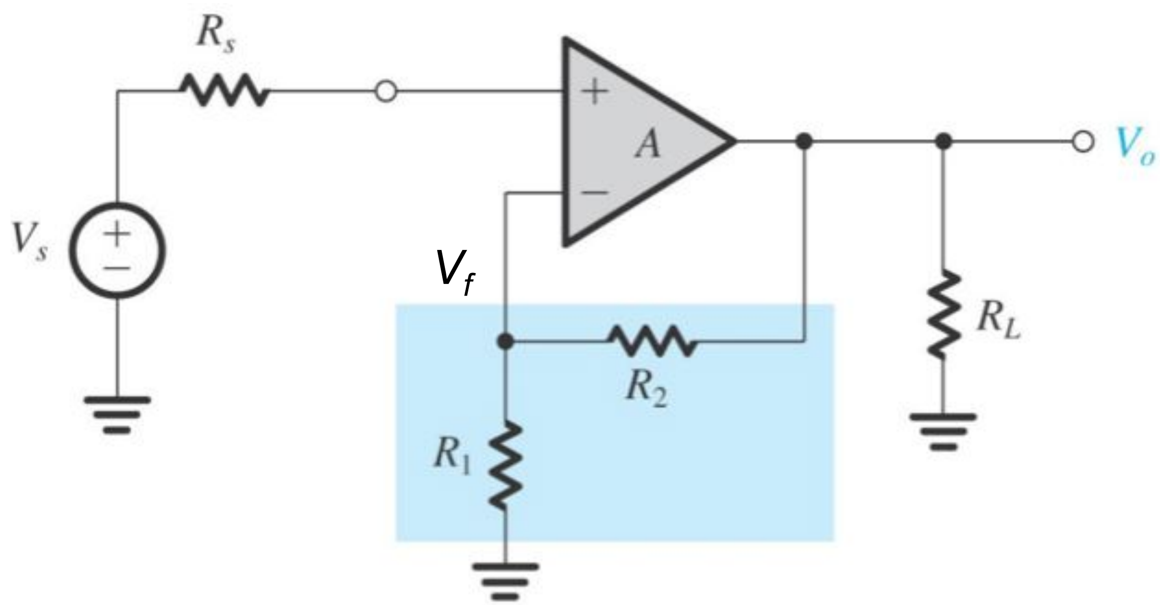
1. Voltage between the + and – input terminals = 0 (since A=∞)
2. Input currents = 0 (since R_i =∞)

Nodal analysis shows that the voltage gain is :

$$\boxed{A_v = \frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}}$$

Here the voltages at both the + and – inputs = v_i since A = ∞ causes v_{id} = 0 V

Non-inverting Amp as a Feedback Amp



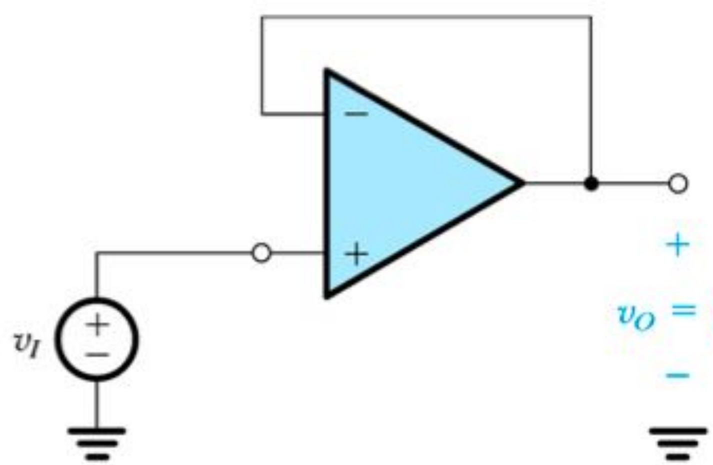
Feedback is provided by the R_1- R_2 voltage divider circuit :

$$Feedback\ \ factor = \beta = \frac{V_f}{V_o} = \frac{R_1}{R_1 + R_2} \Rightarrow A_f = \frac{A}{1 + A\beta} \approx \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Which is the same gain found using ideal opamp analysis!

(But this more exact equation can be used to find A_f when $A \neq \infty$!)

Unity-gain Buffers



- Unity-gain buffers are a special case of the non-inverting amp, with a voltage gain = 1

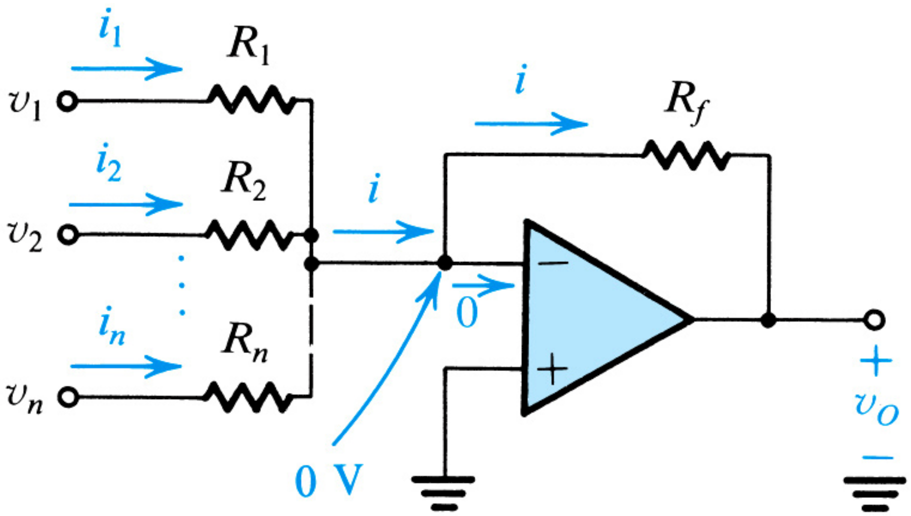
For a non-inverting amp:

$$\boxed{A_v = \frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}}$$

So if $R_2 = 0$ and $R_1 = \infty$ (R_1 is an open circuit) then :

$$\boxed{A_v = \frac{v_o}{v_i} = 1}$$

Inverting Summing Amplifiers



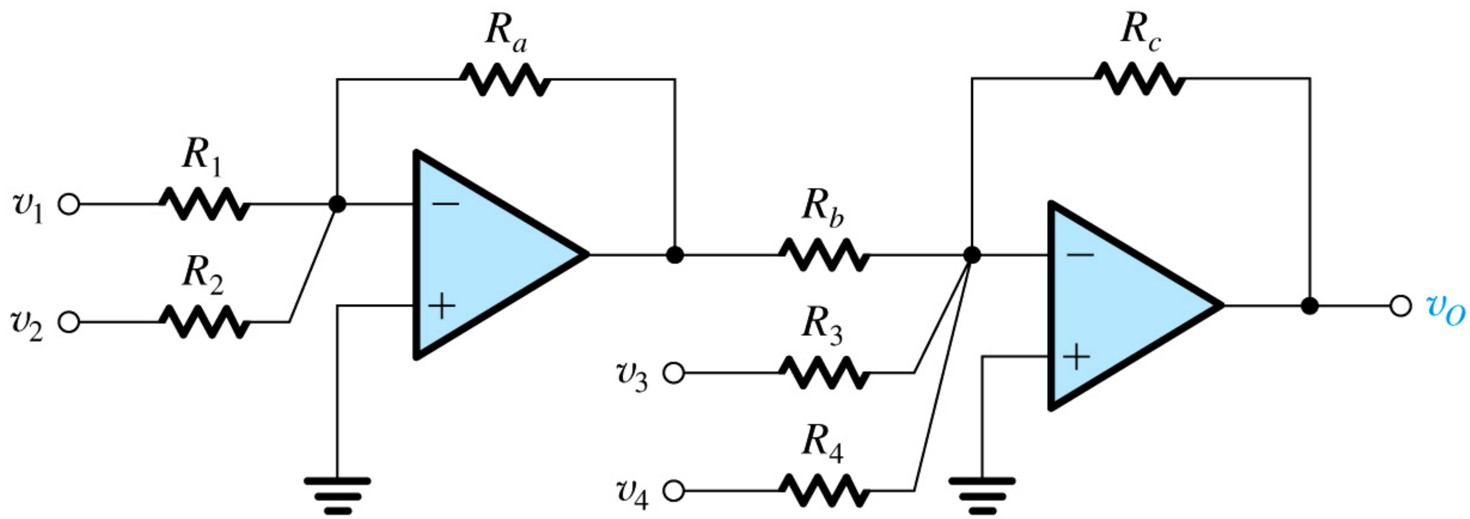
$$i_1 = \frac{v_1}{R_1},\ i_2 = \frac{v_2}{R_2},\ \ldots\ i_n = \frac{v_n}{R_n}$$

$$and\ \ i = i_1 + i_2 + \ldots + i_n$$

- Additional inputs can be added to an inverting amp to create a **Weighted Summing Amplifier**
  - V_o is the sum of all the inputs, each with their own separate gain
  - All input currents combine at the **summing node** and flow into R_f

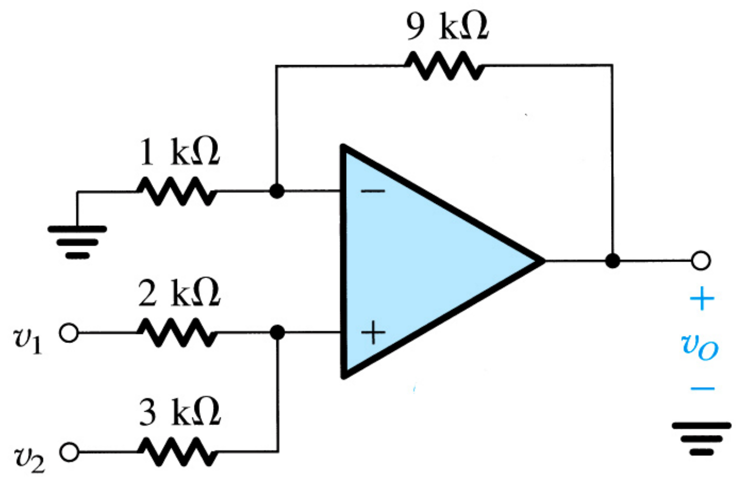
$$\boxed{v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \ldots + \frac{R_f}{R_n}v_n\right)}$$

Summing Amps with + and - inputs



- Multiple amplifier stages can also be cascaded to provide for both positive and negative gains
  - Note that the output signal from the 1st stage sees an additional gain of $–R_c / R_b$ as it passes through the 2nd stage, resulting in an overall positive gain
  - This does require multiple opamps → higher cost, power, etc.

Non-Inverting Summing Amplifiers

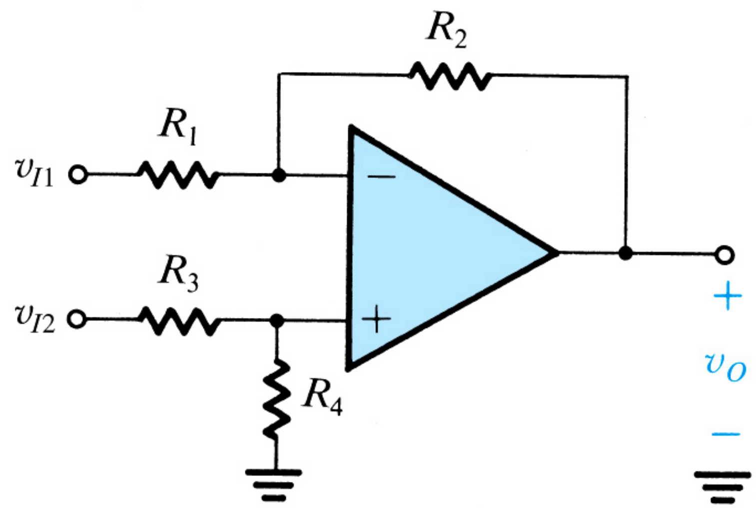


Exercise 2.9, on Page 70 in Sedra & Smith 6th Edition

- Non-inverting summing amps can also be built by adding multiple resistors to the + opamp input, but since this node isn't a virtual ground the gains for v_1, v_2, etc., are not independent of each other! Using superposition :

$$v_o = \left(1 + \frac{9k}{1k}\right)\left(\frac{3k}{2k+3k}v_1 + \frac{2k}{2k+3k}v_2\right) = 6v_1 + 4v_2$$

Difference Amplifiers



- Difference amplifiers are used to amplify the difference between 2 inputs, and not amplify the common-mode

$$v_{idm} = v_{I2} - v_{I1}$$

$$v_{icm} = \frac{v_{I2} + v_{I1}}{2}$$

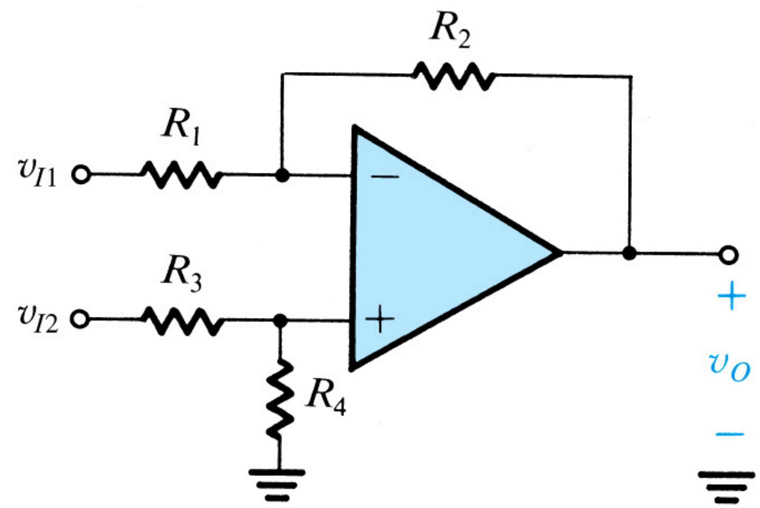
$$A_{dm} = \frac{v_o}{v_{idm}}$$

$$A_{cm} = \frac{v_o}{v_{icm}}$$

$$v_o = A_{dm} v_{idm} + A_{cm} v_{icm}$$

Ideally $A_{cm} = 0$. In practice this isn't possible, but we can make $A_{dm} >> A_{cm}$

Difference Amplifiers



Analyze using Superposition :

$$v_{o1} = -v_{I1}\left(\frac{R_2}{R_1}\right)$$

$$v_{o2} = v_{I2}\left(\frac{R_4}{R_4 + R_3}\right)\left(1 + \frac{R_2}{R_1}\right)$$

And if $R_3 = R_1$ and $R_4 = R_2$ then :

$$v_{o2} = v_{I2}\left(\frac{R_2}{R_2 + R_1}\right)\left(\frac{R_1 + R_2}{R_1}\right) = v_{I2}\left(\frac{R_2}{R_1}\right)$$

$$\Rightarrow v_o = v_{o1} + v_{o2} = \left(\frac{R_2}{R_1}\right)(v_{I2} - v_{I1}) = \left(\frac{R_2}{R_1}\right)v_{idm}$$

$$A_{dm} = \frac{v_o}{v_{idm}} = \frac{R_2}{R_1}$$

$$A_{cm} = \frac{v_o}{v_{icm}} = 0$$

Note the dependence on matching resistors!

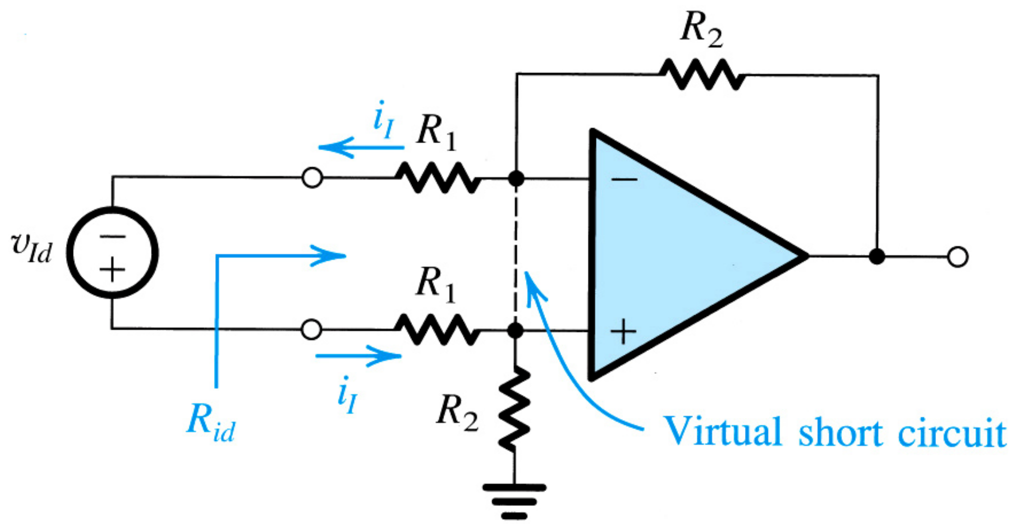
Common-mode Rejection Ratio

- **Common-mode Rejection Ratio** or **CMRR** is a measure of how well we achieve our goal of making the gain for differential inputs >> the gain for common-mode inputs

$$CMRR \equiv \frac{|A_{dm}|}{|A_{cm}|} \quad or \quad CMRR_{dB} \equiv 20\log_{10}\left(\frac{|A_{dm}|}{|A_{cm}|}\right)$$

- Ideally $A_{cm} = 0$, so CMRR = ∞
- For practical circuits CMRR can be very high (60-120dB), but not infinite
- CMRR is typically limited by how well devices match
  - In this Difference Amplifier the ratio R_4/R_3 must match to R_2/R_1
  - CMRR for opamps typically depends on the matching of devices inside of the operational amplifier (e.g., transistors, resistors)

Differential Input Resistance



The **Differential Input Resistance** is found by summing voltages around the input loop, with 0V between + and - :

$$v_{idm} = i_I R_1 + 0 + i_I R_1$$

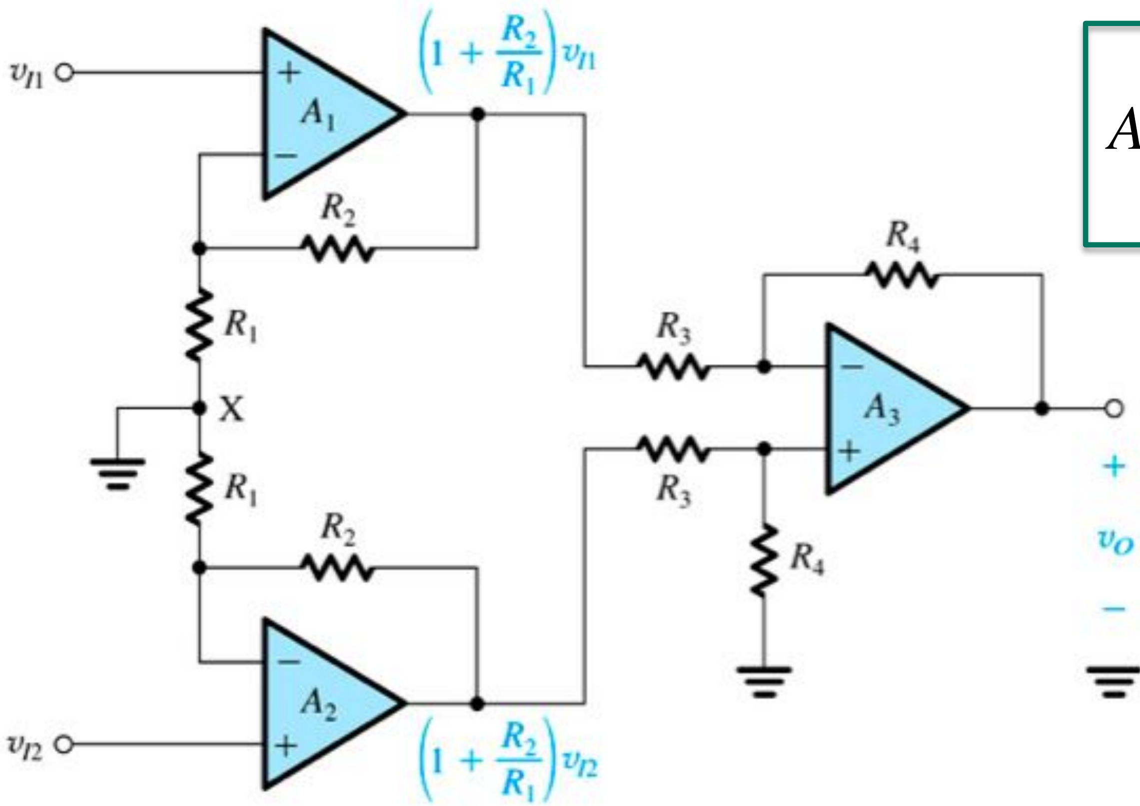
$$\Rightarrow \boxed{R_{idm} \equiv \frac{v_{idm}}{i_I} = 2R_1}$$

Note the tradeoff between gain and R_{idm} :

- R_1 needs to be small for high A_{dm}
- R_1 needs to be large for high R_{idm}

Hard to get both high at the same time!

Instrumentation Amplifiers



$$A_{dm} = \frac{v_o}{v_{idm}} = \frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right)$$

<u>For the 1st stage :</u>

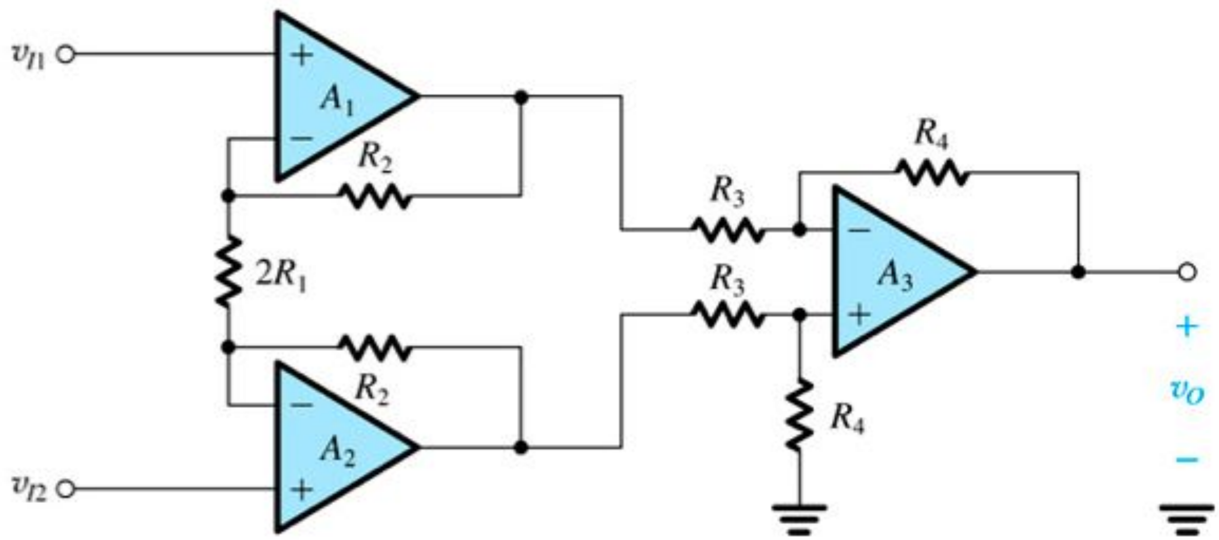
$$A_{dm} = A_{cm} = \left(1+\frac{R_2}{R_1}\right)$$

$$\Rightarrow CMRR = 1 = 0dB$$

$$\textit{for the 1st stage!}$$

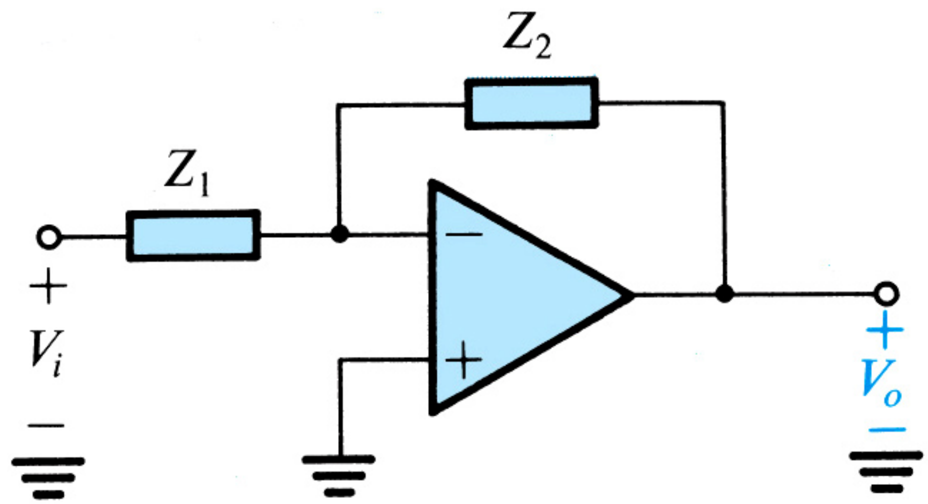
- Instrumentation Amplifiers provide much higher CMRR and input resistance, but require additional opamps
  - In this version, the 1st stage gain is the same for v_{idm} and v_{icm}

Instrumentation Amplifiers



- Much lower common-mode gain, A_{cm} , can be achieved in the 1st stage (and therefore much higher CMRR) just by removing the wire between node X and ground !
  - In this new version the 1st stage gain for v_{idm} is the same as before
  - But the 1st stage gain for common-mode v_{icm} is reduced to only 1 !
  - Note that **zero current** flows in both R_1 and R_2 due to v_{icm} !

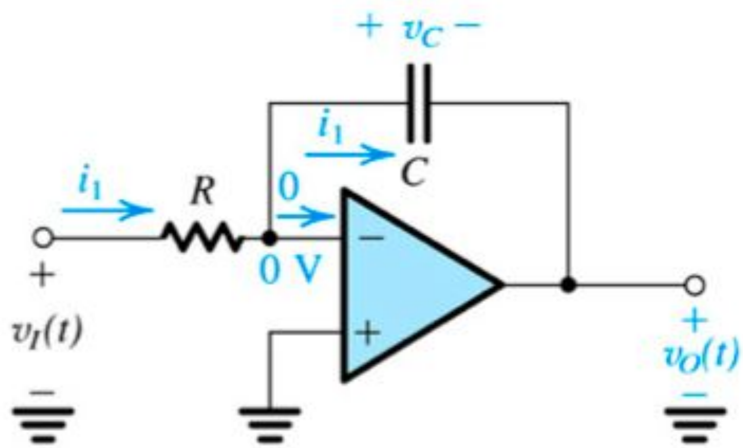
Amplifiers with General Impedances



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

- All the opamp circuits we've been studying work just as well if we replace the resistors with impedances !
  - Z_1, Z_2 don't have to be just resistors, these elements can be frequency dependant elements like capacitors or inductors too!
  - The Ideal Opamp Assumptions still hold as long as the loop gain $A\beta >> 1$ (starts to fail at high frequencies, as $A(\omega)$ drops off)
- Can use this to build integrators, differentiators, and filters

Integrators

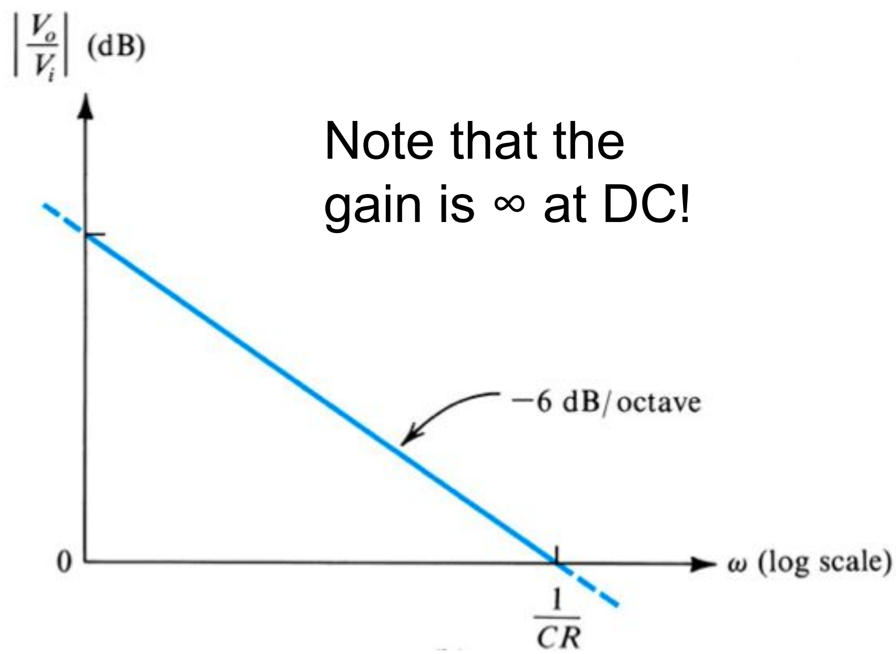


By replacing the resistor in feedback around the opamp with a capacitor we get an **Inverting Integrator** !

$$v_c(t) = \frac{1}{C}\int_0^t i_1(t)dt \; + \; V_C\Big|_{t=0}$$

$$i_1(t) = v_i(t)/R \;\; and \;\; v_O(t) = -v_c(t)$$

$$\Rightarrow \boxed{v_O(t) = -\frac{1}{RC}\int_0^t v_i(t)dt \; - \; V_C\Big|_{t=0}}$$

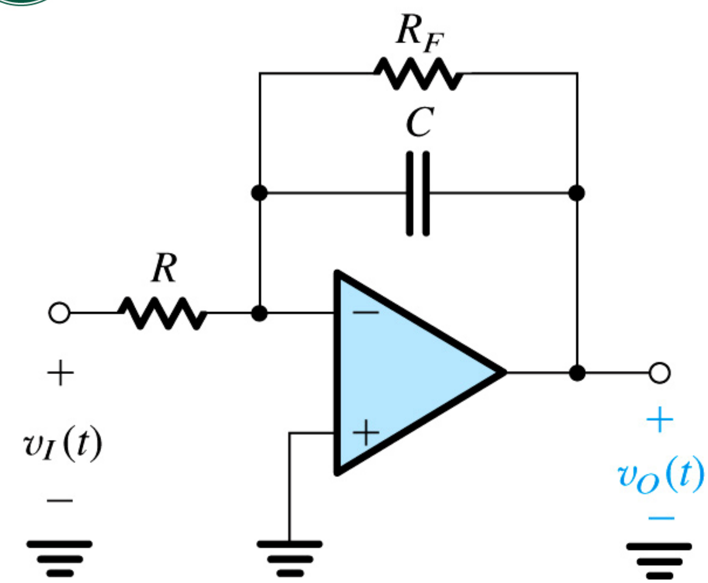


Note that the gain is ∞ at DC!

<u>Or in the frequency domain :</u>

$$\boxed{\frac{v_O(s)}{v_i(s)} = -\frac{1}{sRC} \Rightarrow \frac{v_O(j\omega)}{v_i(j\omega)} = -\frac{1}{j\omega RC}}$$

Integrator with finite DC gain



By adding a resistor in feedback in parallel with the capacitor we can limit the low frequency gain:

$$\frac{v_O(s)}{v_i(s)} = -\frac{Z_F}{R} \quad where \quad Z_F = R_F \parallel \left(\frac{1}{sC}\right)$$

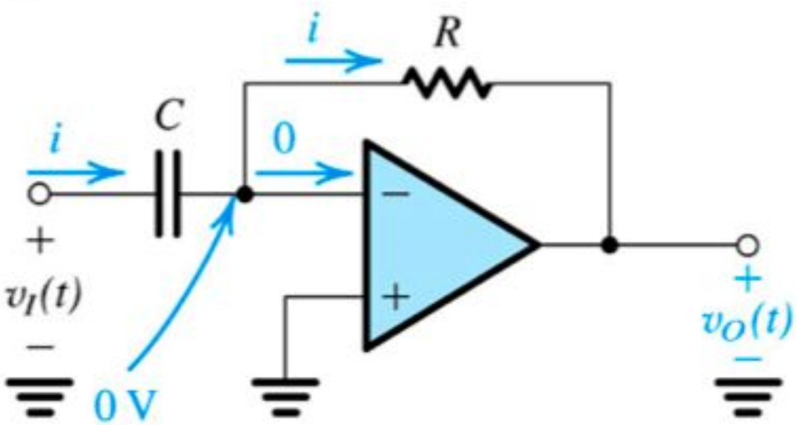
$$Since: \; Z_F = \frac{R_F\left(\frac{1}{sC}\right)}{R_F + \left(\frac{1}{sC}\right)} = \frac{R_F}{1 + sCR_F}$$

$$\Rightarrow \boxed{\frac{v_O(s)}{v_i(s)} = -\left(\frac{R_F}{R}\right)\left(\frac{1}{1 + sCR_F}\right)}$$

Integrators with ∞ DC gain are usually avoided since small DC voltages cause V_o to ramp to V_{DD} or V_{SS}

– Since C = an open ckt at DC, no negative feedback exists!

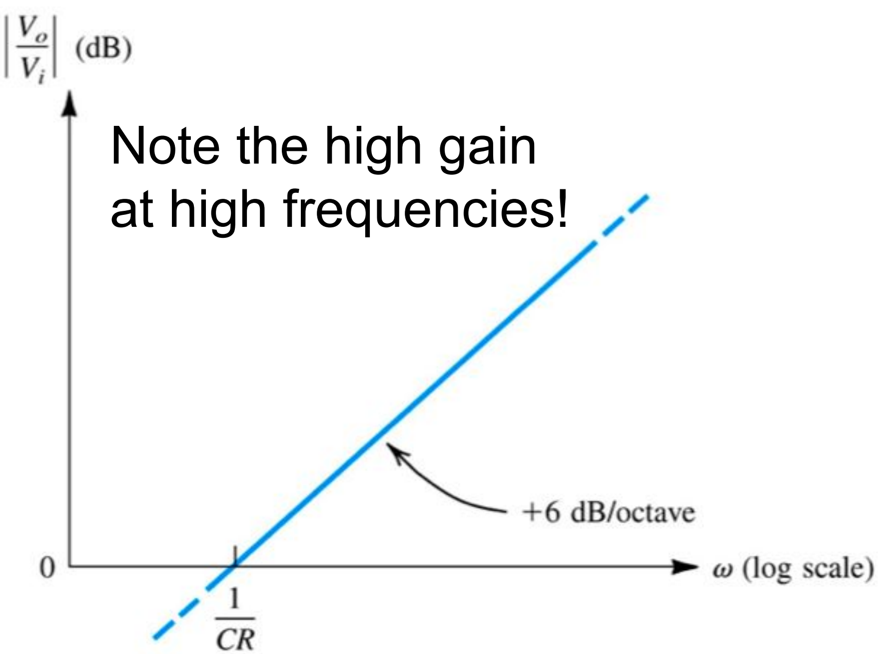
Differentiators



By replacing the resistor at the input with a capacitor we get an **Inverting Differentiator** !

$$i(t) = C\frac{\partial v_I(t)}{\partial t}$$

$$\Rightarrow \boxed{v_O(t) = -RC\frac{\partial v_I(t)}{\partial t}}$$



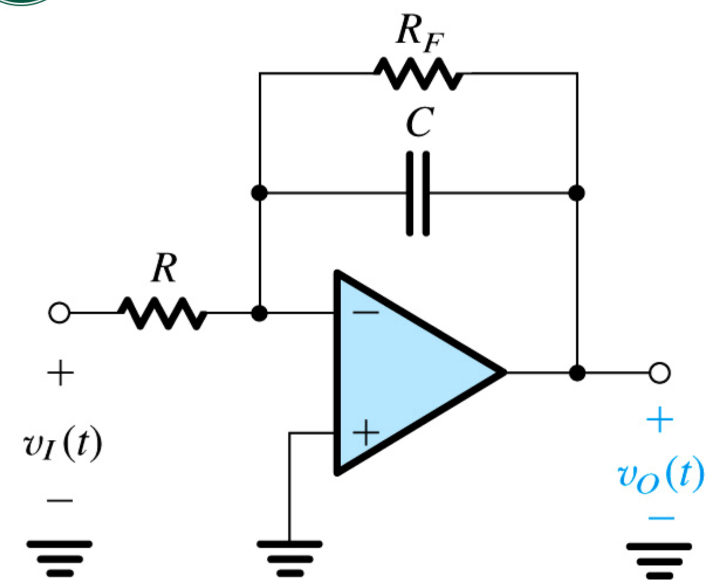
Note the high gain at high frequencies!

<u>Or in the frequency domain :</u>

$$\boxed{\frac{v_O(s)}{v_i(s)} = -sRC \Rightarrow \frac{v_O(j\omega)}{v_i(j\omega)} = -j\omega RC}$$

Note that differentiators are very sensitive to high frequency noise

Active Lowpass Filter



An integrator with R_F in parallel with C can also be used as an **Active Lowpass Filter**

$$\frac{v_O(s)}{v_i(s)} = -\left(\frac{R_F}{R}\right)\left(\frac{1}{1+sCR_F}\right) \Rightarrow$$

$$\boxed{\frac{v_O(j\omega)}{v_i(j\omega)} = -\left(\frac{R_F}{R}\right)\left(\frac{1}{1+j\omega CR_F}\right)}$$

<u>Gain at low frequencies is:</u>

$$\frac{v_O}{v_i} \approx -\left(\frac{R_F}{R}\right) \quad for \quad \omega << \frac{1}{CR_F}$$

<u>Gain is reduced by -3dB at :</u>

$$\omega_{-3dB} = \frac{1}{CR_F}$$

(where the $|\omega CR_F| = 1$)

At $\omega > \omega_{-3dB}$ the gain magnitude rolls off at -20dB/decade

Non-ideal effects in IC opamps

- Actual integrated circuit opamps have a number of non-ideal effects that affect performance and must be considered in real-world applications
- DC non-idealities include :
  - Finite DC gain (the gain is not really infinite!)
  - Offset voltage
  - Input bias and offset currents
  - Output voltage and current limits
- AC non-idealities include :
  - Finite bandwidth (the bandwidth is not really infinite!)
  - Slew rate and full-power bandwidth

Effect of finite DC open-loop gain

- We assume that ideal opamps have infinite DC gain. Real IC opamps have high, but not infinite gain. This non-ideal effect causes small errors in the actual gains we get in our amplifiers. Recall for a feedback amp :

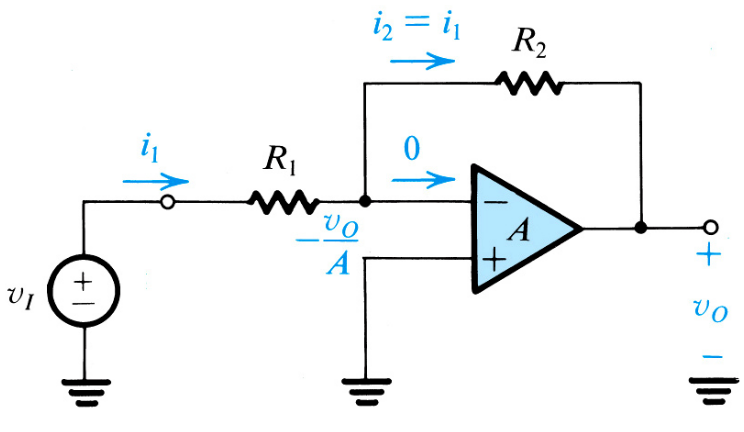
$$A_f = \frac{A}{1+A\beta} \approx \frac{1}{\beta} \quad if\ \ A\beta >> 1 \quad but\ \ A_f\ drops\ as\ A\beta\ gets\ smaller!$$

Example: Assume β = 0.1 is used to set A_f to ~ 10
What is the value of A_f as A gets small?

- For A = 1000 and β = 0.1 → A_f = 9.90, an error of ~ -1%
- For A = 100 and β = 0.1 → A_f = 9.09, an error of ~ -10%
- For A = 50 and β = 0.1 → A_f = 8.33, an error of ~ -17%
- For A = 10 and β = 0.1 → A_f = 5.00, an error of ~ -50%

Effect of finite DC open-loop gain

<u>For an inverting amplifier :</u>



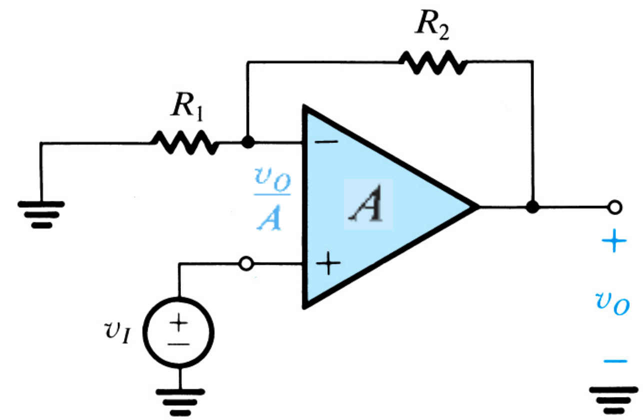
$$v_O = A(v_+ - v_-) \Rightarrow v_- = -v_O/A$$

$$\Rightarrow i_1 = \left(v_I - \left(-v_O/A\right)\right)/R_1$$

$$\Rightarrow v_O = -v_O/A + R_2\left(\left(v_I + v_O/A\right)/R_1\right)$$

$$\Rightarrow \boxed{\frac{v_O}{v_I} = \frac{-R_2/R_1}{1 + \left(1 + R_2/R_1\right)/A}}$$

<u>For a non-inverting amplifier :</u>



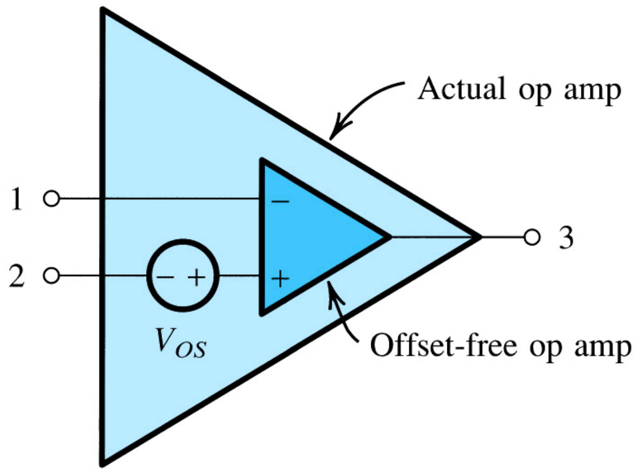
$$v_O = A(v_+ - v_-) \Rightarrow v_- = v_I - v_O/A$$

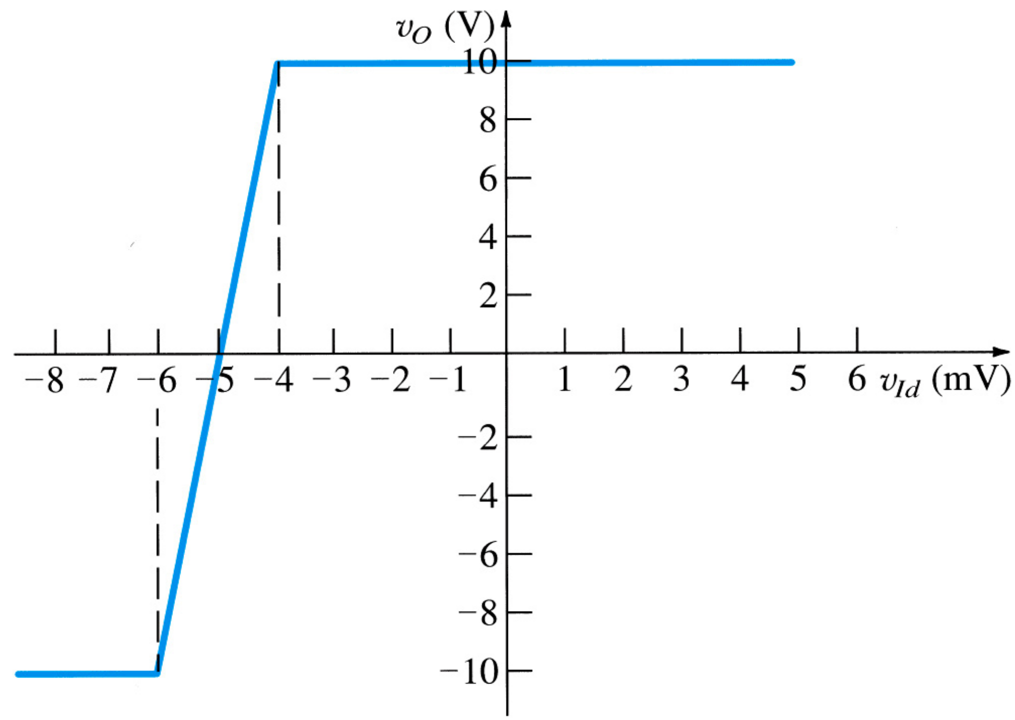
$$\Rightarrow i_1 = \left(v_I - v_O/A\right)/R_1$$

$$\Rightarrow v_O = \left(v_I - v_O/A\right) + R_2\left(\left(v_I - v_O/A\right)/R_1\right)$$

$$\Rightarrow \boxed{\frac{v_O}{v_I} = \frac{\left(1 + R_2/R_1\right)}{1 + \left(1 + R_2/R_1\right)/A}}$$

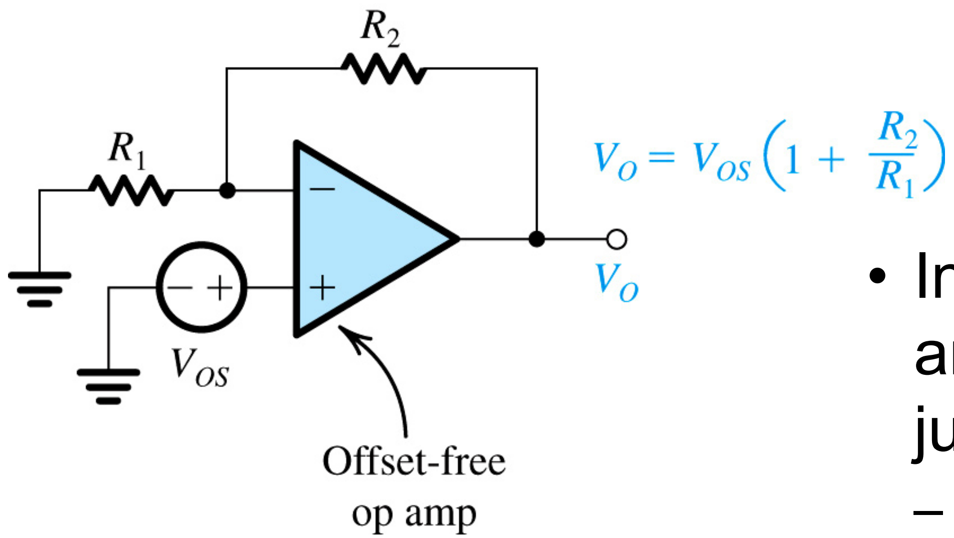
Opamp Input Offset Voltage

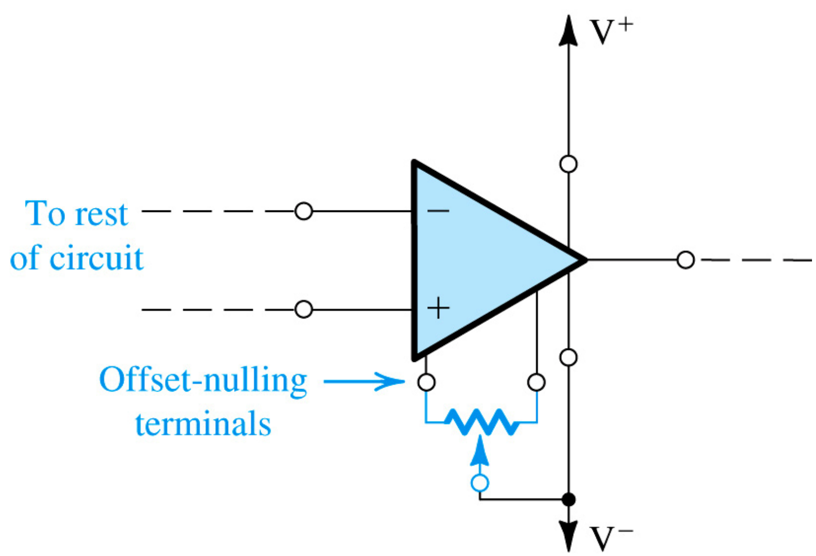




- The opamp's **input offset voltage** is the voltage which must be applied between the + and – inputs to set $V_o = 0$
  - Appears as a shift in the V_O vs V_{idm} DC transfer curve
  - Can be modeled as an extra V_{OS} source at the opamp input
  - V_{OS} is caused by mismatch between transistors inside the opamp

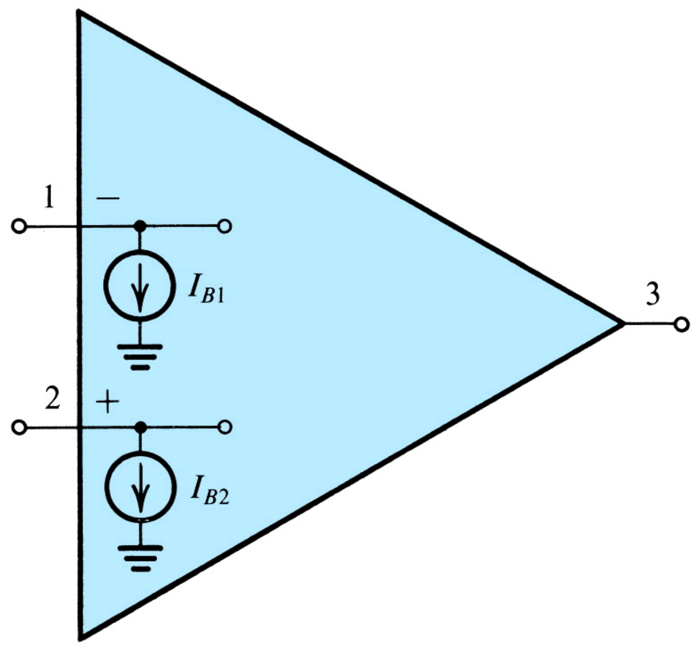
Opamp Input Offset Voltage





- Input offset voltages are amplified by the gain, just like any input
  - **This can cause errors at the amplifier output!**
- Many opamps have extra terminals which allow the user to null out (cancel) the input offset voltage
- Can also buy low V_{OS} opamps when needed

Opamp input bias and offset currents

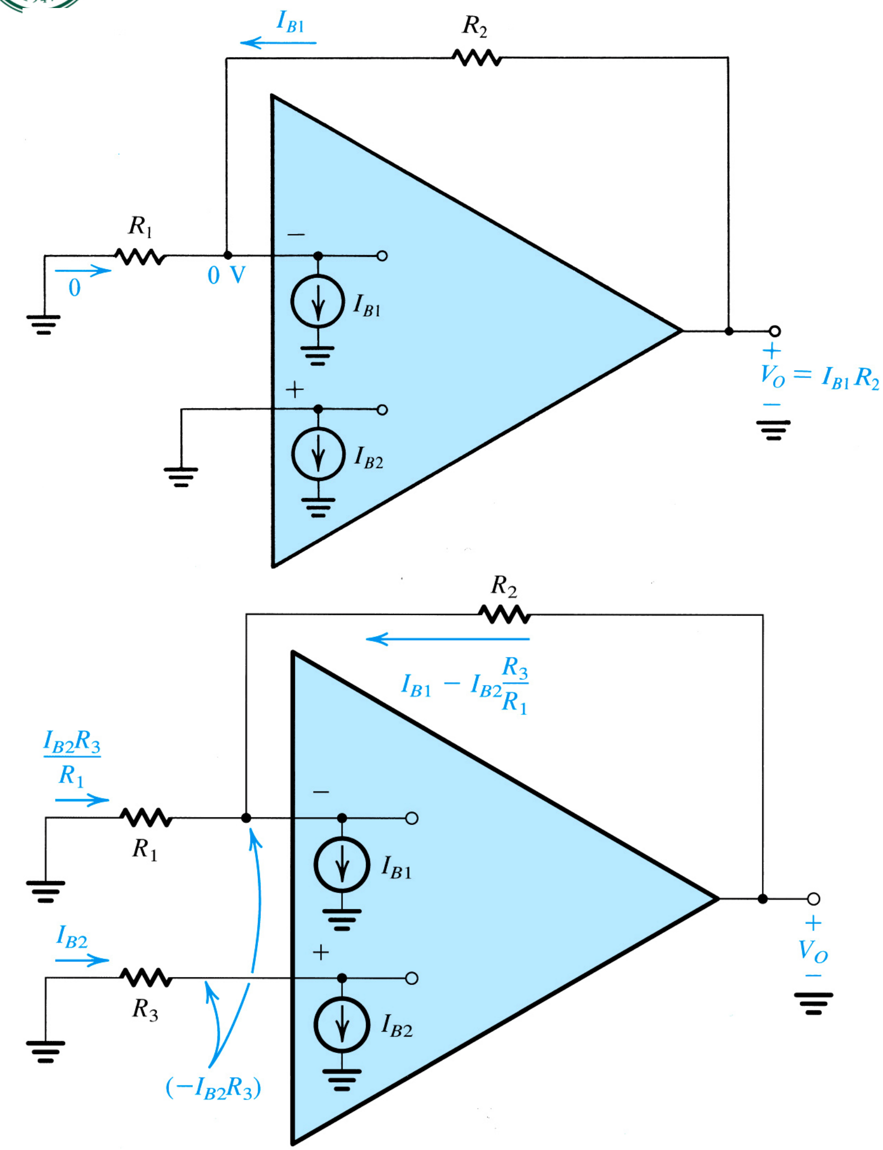


$$I_B = \frac{I_{B1} + I_{B2}}{2} = \text{Input bias current}$$

$$I_{OS} = \left| I_{B1} - I_{B2} \right| = \text{Input offset current}$$

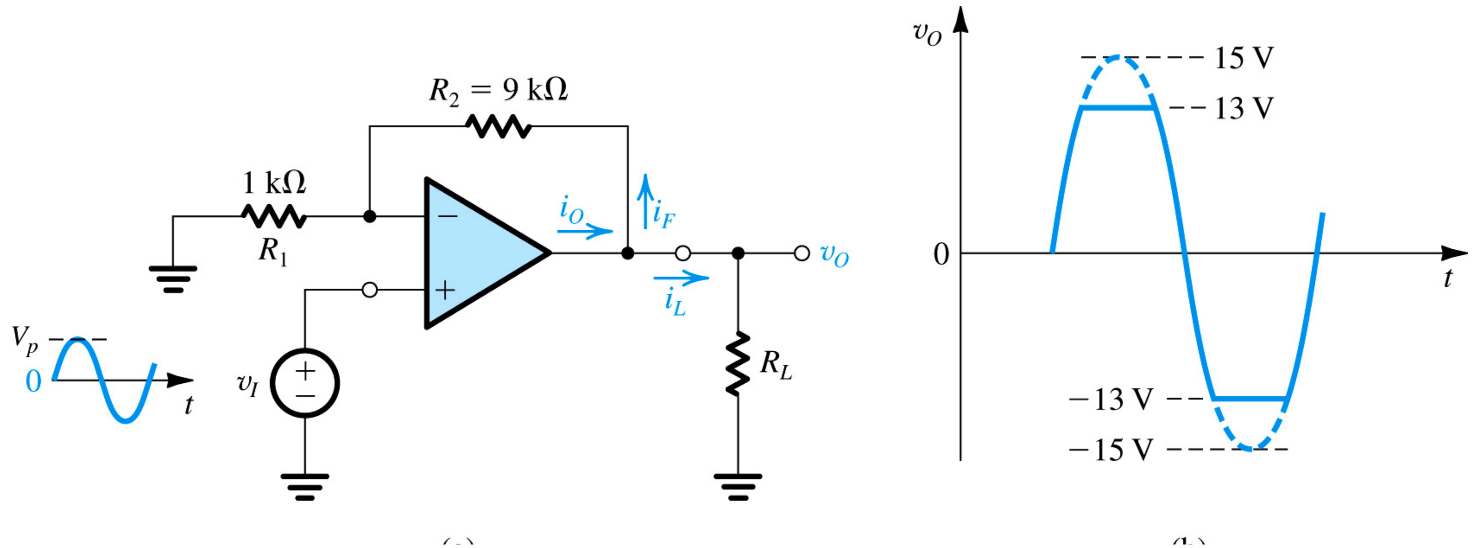
- Opamps built with BJTs have **input bias currents** and **input offset currents** which can cause errors in an opamp's output voltage
- Not a problem for opamps which use MOSFET inputs, since MOSFETs typically have zero gate current
  - Some newer "nanoscale" MOS processes do have small gate leakage currents which must be considered

Opamp input bias and offset currents



- Fortunately, it's usually easy to minimize errors due to input bias currents by adding a resistor in the + opamp terminal
  - Choose $R_3 = R_1 \,||\, R_2$ to set $V_O = 0$ due to I_B
- Still left with a small error due to I_{OS} :
  - $V_o = I_{OS} R_2$
- This typically reduces the output voltage error by approximately 10x

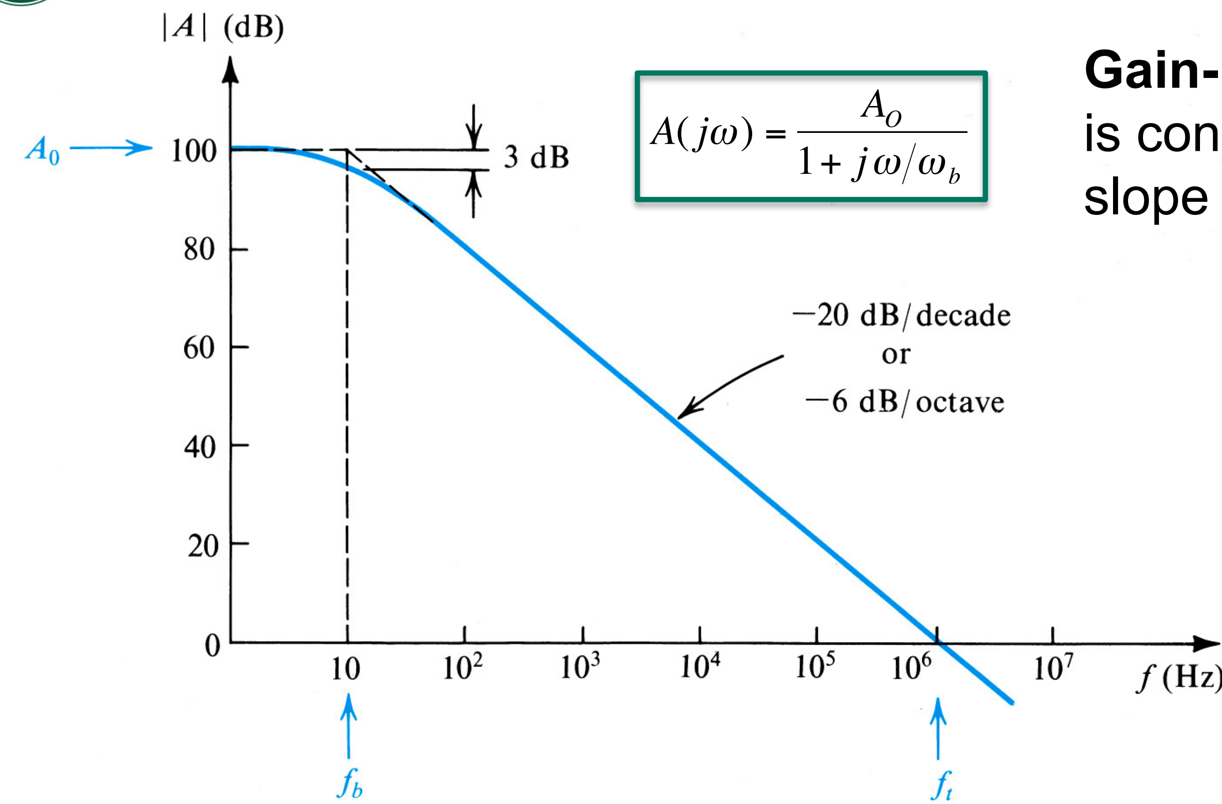
Opamp output voltage & current limits



- **The opamp output voltage is limited by the supply!**
- In the example shown, the amplifier gain was set to 10. If V_I peaks at 1.5V this means V_o should peak at 15V. BUT, if +/- 13V power supplies are used clipping occurs!
- Clipping can also occur if the opamp's output current i_o needs to be larger than the max to reach the desired V_o

  $V_o = i_L R_L$ → must be able to supply enough current for small R_L!

Finite Bandwidth effects



$$A(j\omega) = \frac{A_O}{1 + j\omega/\omega_b}$$

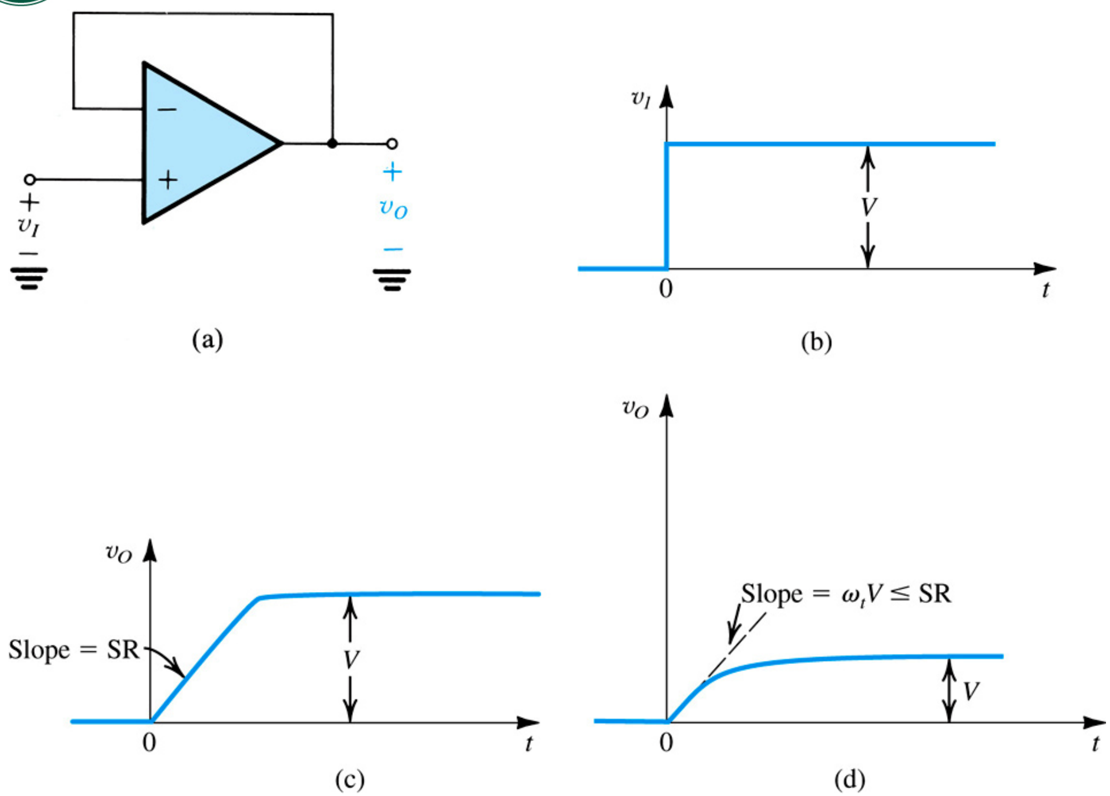
Gain-Bandwidth Product is constant anywhere on the slope between f_b and f_T!

$$\begin{aligned} GBW &= 1\,x\,f_T \\ &= A_O\,x\,f_b \\ &= A(f)\,x\,f \end{aligned}$$

Note -20dB = 10x drop in |A|, and a decade increase in frequency = 10x

- Opamps typically have high gains at low frequency, which roll off to lower values as frequency increases
  - Internal compensation causes gain to drop at -20db/decade
  - Lower gain can cause frequency dependant errors in Vo

Slew rate errors



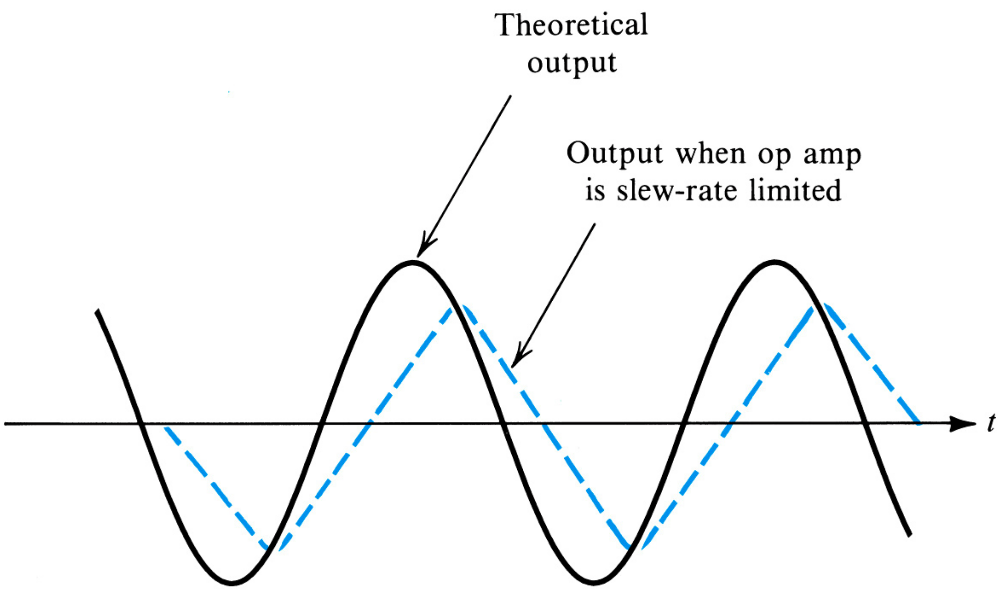
$$SR = \max \frac{\partial V_O}{\partial t}$$

- **Slew Rate** is the maximum slope at the output voltage for fast inputs
- For slower inputs, V_o is exponential :

$$V_O(t) = V\left(1 - e^{(-t/\tau)}\right)$$

- If the input changes too fast, the opamp's output voltage may not be able to keep up with the input!
- How fast V_o can change is limited by the max **Slew Rate**

Full-power Bandwidth



<u>For a sine wave input :</u>

$$V_I(t) = V\sin(\omega t)$$

$$\Rightarrow \frac{\partial V_I}{\partial t} = \omega V\cos(\omega t)$$

$$\Rightarrow \frac{\partial V_I}{\partial t}\Big|\max = \omega V$$

- The max slope for a sine wave is when it goes through zero
- If this is > Slew Rate, V_o is **slew-rate limited!**
- **Full-power bandwidth** is the max frequency sine wave that an opamp can output at full amplitude without hitting slew rate limits

$$\boxed{f_M = \frac{SR}{2\pi Vo\max}}$$

Summary of Key Concepts

- Operational Amplifiers ("opamps") are a key building block used in negative feedback systems!
- Ideal opamps have ∞ gain, bandwidth, R_i and zero R_o
  - Key "Ideal Opamp Assumptions" include the facts that there is zero volts between the + and - inputs, and zero input current
- Many useful functions can be built using opamps!
  - Inverting and non-inverting amplifiers, unity-gain buffers, summing amplifiers, difference & instrumentation amplifiers, integrators, differentiators, active filters, plus many more!
- The performance of real integrated circuit opamps is limited by both DC and AC non-ideal effects
  - Finite DC gain, offset voltage, input bias and offset currents
  - Output voltage and current limits
  - Finite bandwidth, slew rate, and full-power bandwidth