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1.3.1 Articulated Manipulator (RRR)

The articulated manipulator is also called a **revolute**, **elbow**, or **anthropomorphic** manipulator. The KUKA 500 articulated arm is shown in Figure 1.11. In the anthropomorphic design the three links are designated as the body, upper arm, and forearm, respectively, as shown in Figure 1.11. The joint axes are designated as the **waist** (z_0), **shoulder** (z_1), and **elbow** (z_2). Typically, the joint axis z_2 is parallel to z_1 and both z_1 and z_2 are perpendicular to z_0. The workspace of the elbow manipulator is shown in Figure 1.12.

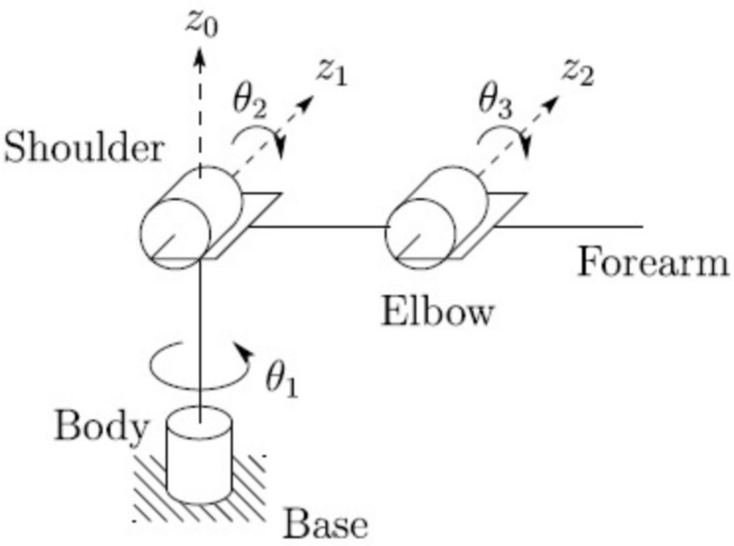

Figure 1.11 Symbolic representation of an RRR manipulator (left), and the KUKA 500 arm (right), which is a typical example of an RRR manipulator. The links and joints of the RRR configuration are analogous to human joints and limbs. (Photo courtesy of KUKA Robotics.)

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1.3.2 Spherical Manipulator (RRP)

By replacing the third joint, or elbow joint, in the revolute manipulator by a prismatic joint, one obtains the spherical manipulator shown in Figure 1.13. The term **spherical manipulator** derives from the fact that the joint coordinates coincide with the spherical coordinates of the end effector relative to a coordinate frame located at the shoulder joint. Figure 1.13 shows the Stanford Arm, one of the most well-known spherical robots.

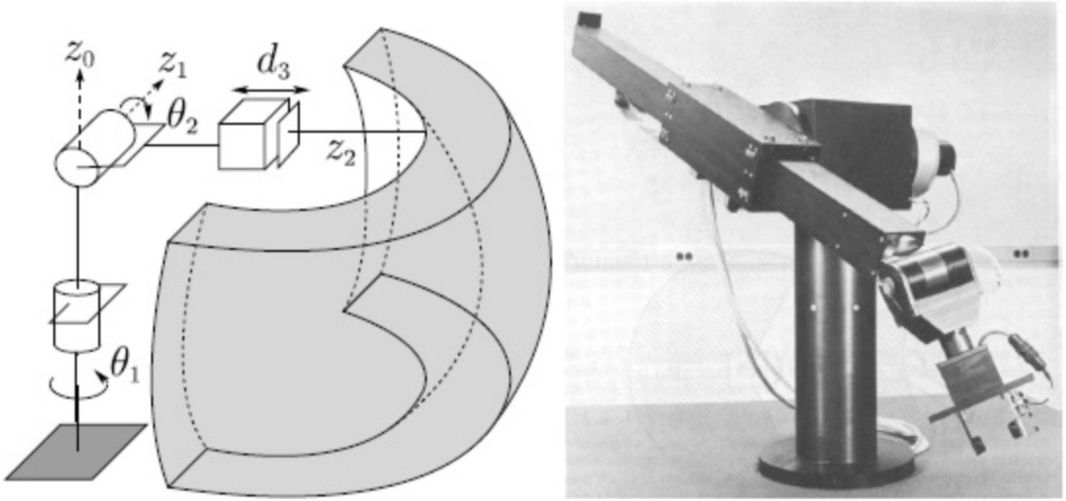

Figure 1.13 Schematic representation of an RRP manipulator, referred to as a spherical robot (left), and the Stanford Arm (right), an early example of a spherical arm. (Photo courtesy of the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign.)

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1.3.4 Cylindrical Manipulator (RPP)

The cylindrical manipulator is shown in Figure 1.15. The first joint is revolute and produces a rotation about the base, while the second and third joints are prismatic. As the name suggests, the joint variables are the cylindrical coordinates of the end effector with respect to the base.

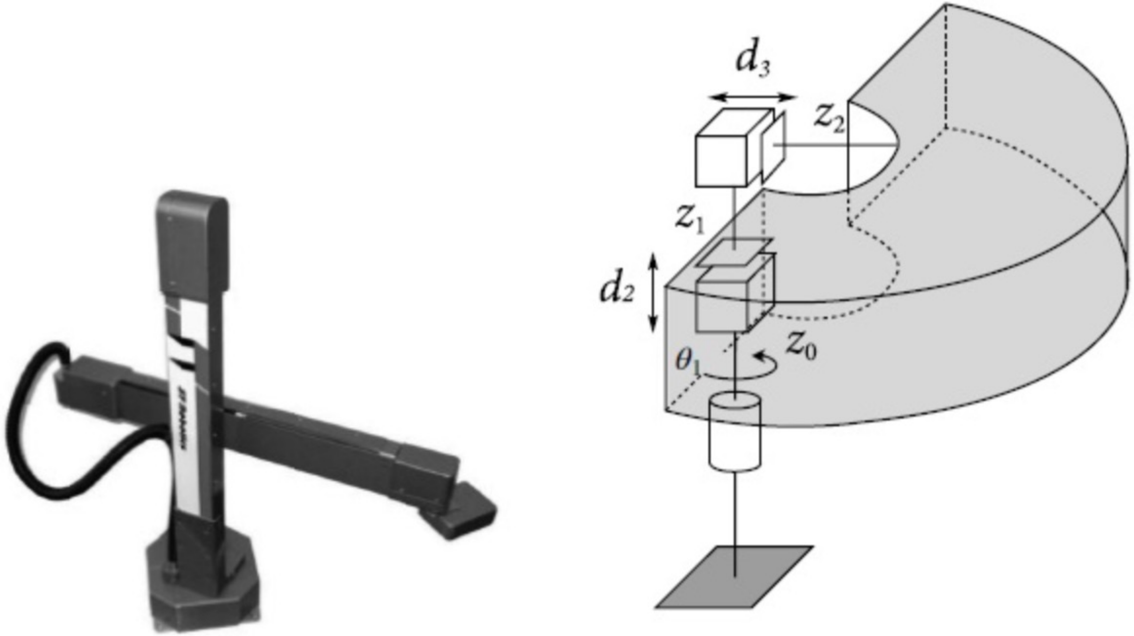

Figure 1.15 The ST Robotics R19 cylindrical robot (left) and the symbolic representation showing a portion of its workspace (right). Cylindrical robots are often used in materials transfer tasks. (Photo courtesy of ST Robotics.)

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1.3.5 Cartesian Manipulator (PPP)

A manipulator whose first three joints are prismatic is known as a Cartesian manipulator. The joint variables of the Cartesian manipulator are the Cartesian coordinates of the end effector with respect to the base. As might be expected, the kinematic description of this manipulator is the simplest of all manipulators. Cartesian manipulators are useful for table-top assembly applications and, as gantry robots, for transfer of material or cargo. The symbolic representation of a Cartesian robot is shown in Figure 1.16.

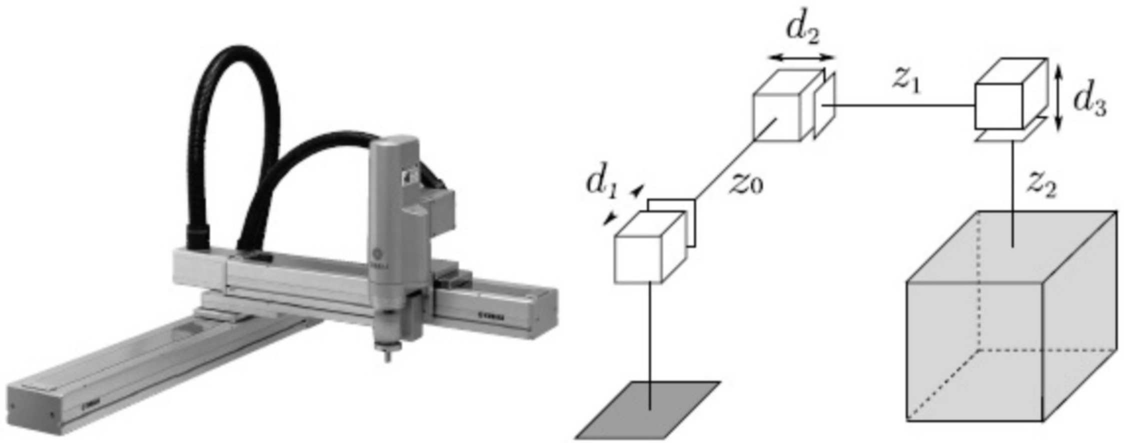

Figure 1.16 The Yamaha YK-XC Cartesian robot (left) and the symbolic representation showing a portion of its workspace (right). Cartesian robots are often used in pick-and-place operations. (Photo courtesy of Yamaha Robotics.)

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To perform the kinematic analysis, we attach a coordinate frame rigidly to each link. In particular, we attach $o_ix_iy_iz_i$ to link i. This means that, whatever motion the robot executes, the coordinates of each point on link i are constant when expressed in the i^{th} coordinate frame. Furthermore, when joint i is actuated, link i and its attached frame, $o_ix_iy_iz_i$, experience a resulting motion. The frame $o_0x_0y_0z_0$, which is attached to the robot base, is referred to as the **base frame**, **inertial frame** or **world frame**. Figure 3.1 illustrates the idea of attaching frames rigidly to links in the case of an elbow manipulator.

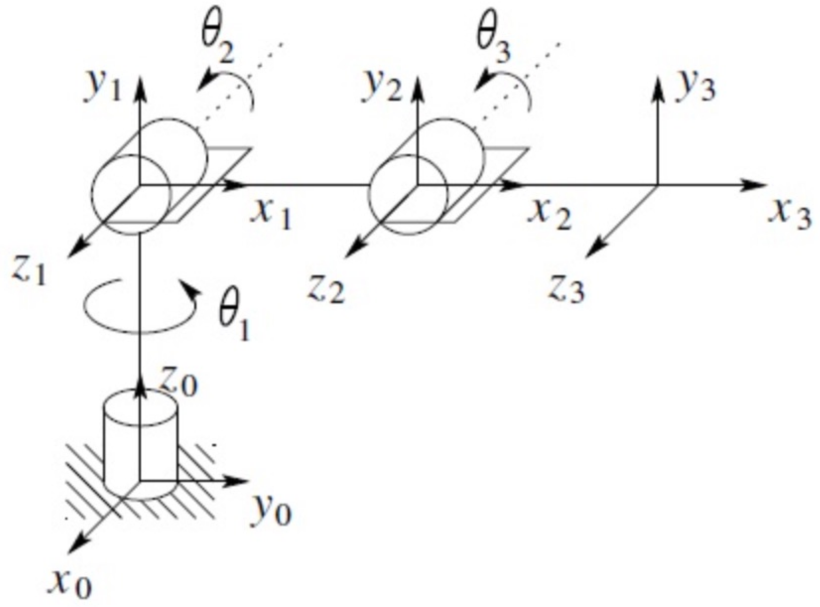

Figure 3.1 Coordinate frames attached to elbow manipulator.

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David-Hartenberg Convention

A commonly used convention for selecting frames of reference in robotics applications is the **Denavit and Hartenberg (D–H) convention** which was first introduced by Jacques Denavit and Richard S. Hartenberg. In this convention, coordinate frames are associated with each linked joint within a robot manipulator's robot "arm". The joints connecting the links are modeled as either hinged or sliding joints. The DH convention is used to represent the relationship between the individual joints and the end effector, in a manner that allows a standard homogeneous transformation (Ai) to be used while transitioning from each coordinate frame to the next. Under the DH convention, each homogeneous transformation Ai is represented by a product of four basic transformations.

The following four transformation parameters are known as D–H parameters:.[4]

- d : offset along previous z to the common normal
- θ : angle about previous z, from old x to new x
- r : length of the common normal (aka a, but if using this notation, do not confuse with α). Assuming a revolute joint, this is the radius about previous z.
- α : angle about common normal, from old z axis to new z axis

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A commonly used convention for selecting frames of reference in robotic applications is the **Denavit–Hartenberg**, or **DH convention**. In this convention, each homogeneous transformation A_i is represented as a product of four basic transformations

$$A_i = \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i} \tag{3.10}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where the four quantities a_i α_i d_i θ_i are parameters associated with link i and joint j. These four parameters are given the names **link length** (a_i) **link twist** (α_i), **link offset** (d_i) and **joint angle** (θ_i)

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The coordinate systems of the SCARA robot established with DH method are shown as Fig. 2.

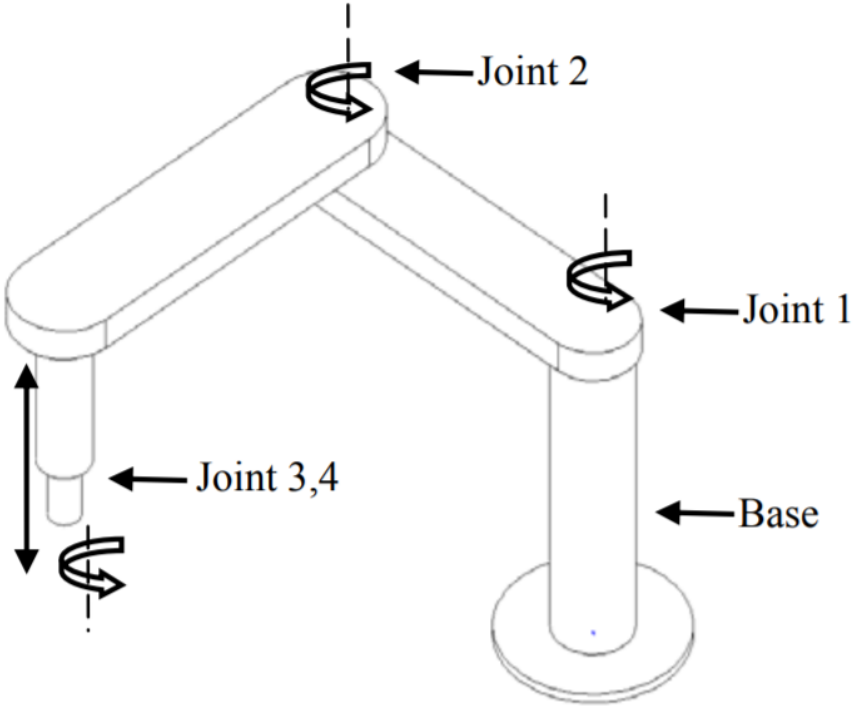

Fig. 1 The structure of the SCARA robot

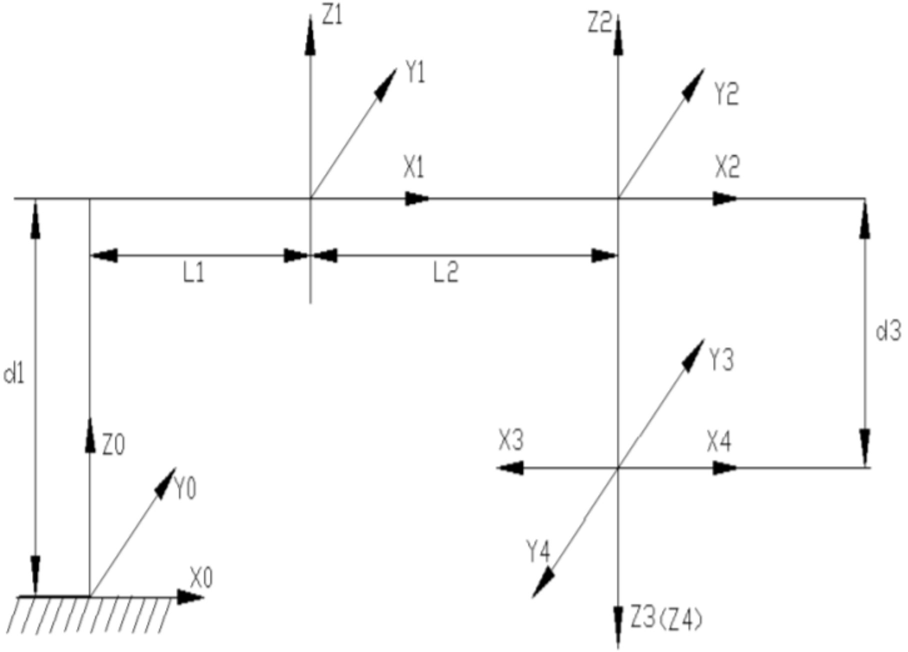

Fig. 2 The coordinate systems of the SCARA robot

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(DH1) The axis x_1 is perpendicular to the axis z_0.

(DH2) The axis x_1 intersects the axis z_0.

These two properties are illustrated in Figure 3.2. Under these conditions, we claim that there exist unique numbers a, d, θ, α such that

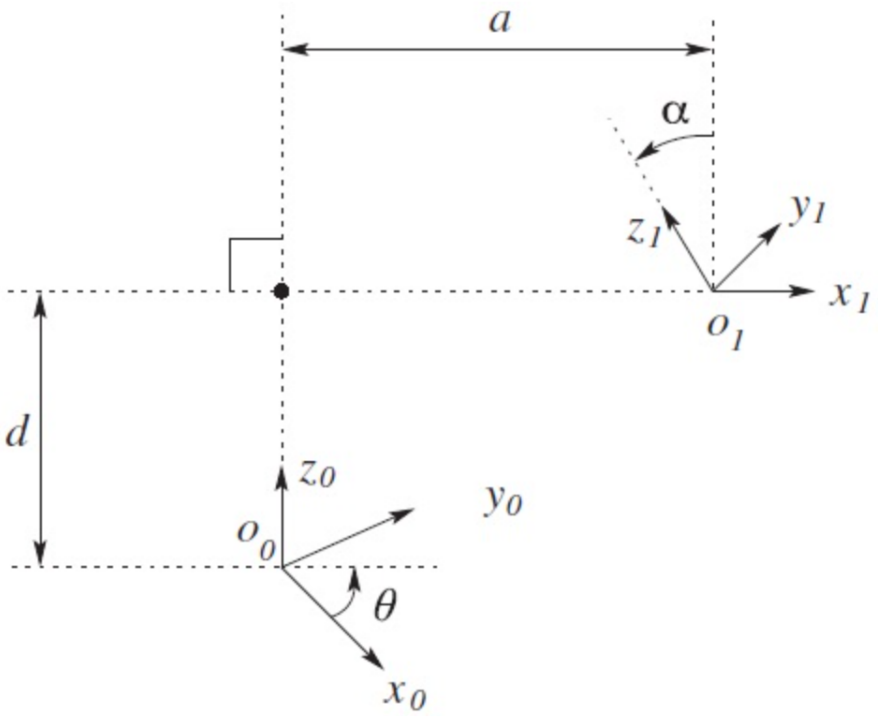

Figure 3.2 Coordinate frames satisfying assumptions DH1 and DH2.

$$A = \text{Rot}_{z,\theta}\text{Trans}_{z,d}\text{Trans}_{x,a}\text{Rot}_{x,\alpha}$$

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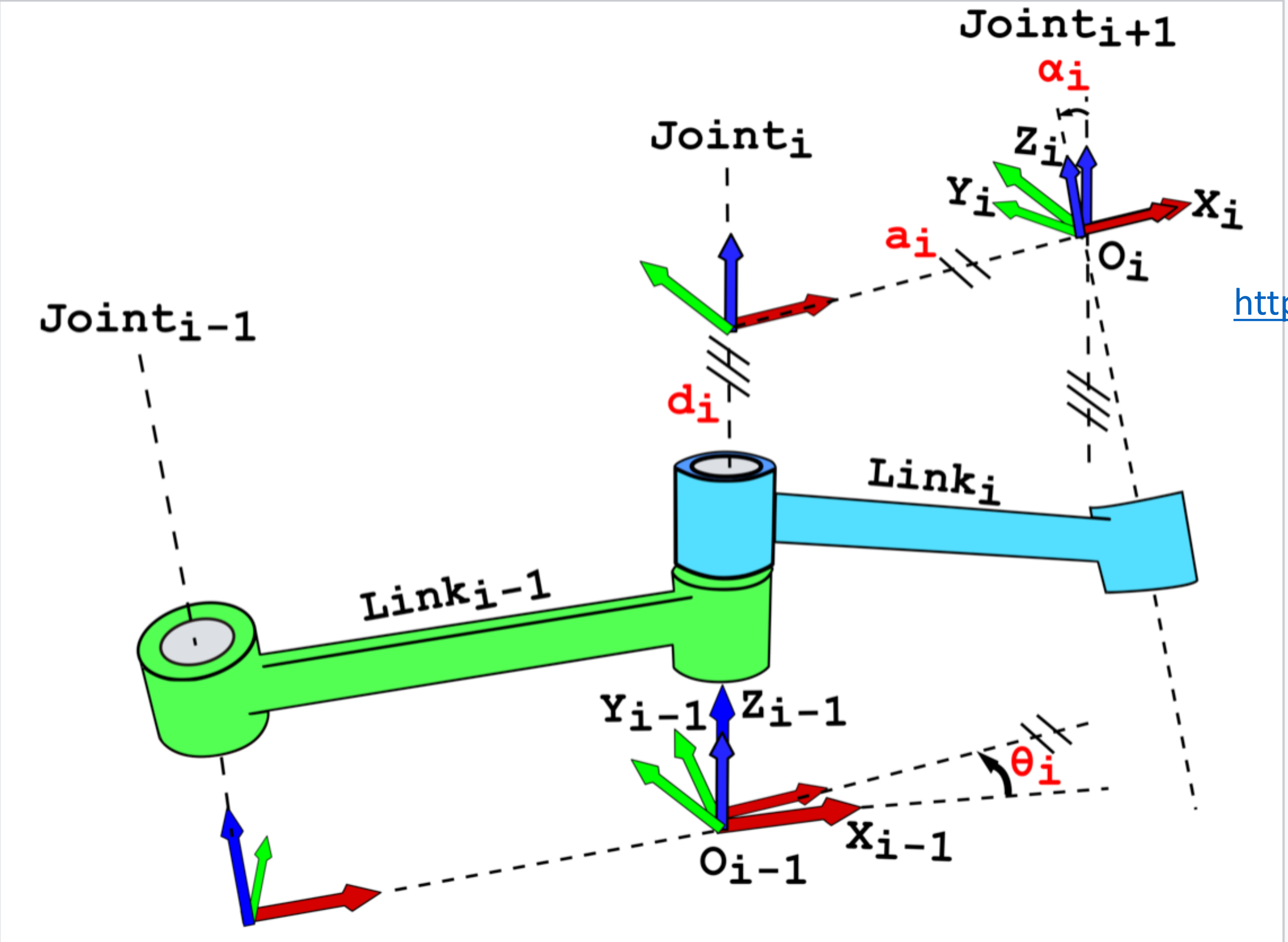

The four parameters of classic DH convention are shown in red text, which are $\theta_i, d_i, a_i, \alpha_i$. With those four parameters, we can translate the coordinates from $O_{i-1}X_{i-1}Y_{i-1}Z_{i-1}$ to $O_iX_iY_iZ_i$.

Here's a Wiki link that provides an excellent description of the four D-H parameters

https://www.youtube.com/watch?v=rA9tm0gTln8

Summary of the DH Procedure

We may summarize the procedure based on the DH convention in the following algorithm for deriving the forward kinematics for any manipulator.

Step l: Locate and label the joint axes $z_0, \ldots, z_{n-1}$.

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-handed frame.

For $i = 1, \ldots, n-1$ perform Steps 3 to 5.

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-handed frame.

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Assuming the n^{th} joint is revolute, set $z_n = a$ parallel to z_{n-1}. Establish the origin o_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = s$ in the direction of the gripper closure and set $x_n = n$ as $s \times a$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-handed frame.

Step 7: Create a table of DH parameters $a_i, d_i, \alpha_i, \theta_i$.

a_i = distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.

d_i = distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. If joint i is prismatic, d_i is variable.

α_i = the angle from z_{i-1} to z_i measured about x_i.

θ_i = the angle from x_{i-1} to x_i measured about z_{i-1}. If joint i is revolute, θ_i is variable.

Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into Equation (3.10).

Step 9: Form $T_n^0 = A_1 \cdots A_n$. This then gives the position and orientation of the tool frame expressed in base coordinates.

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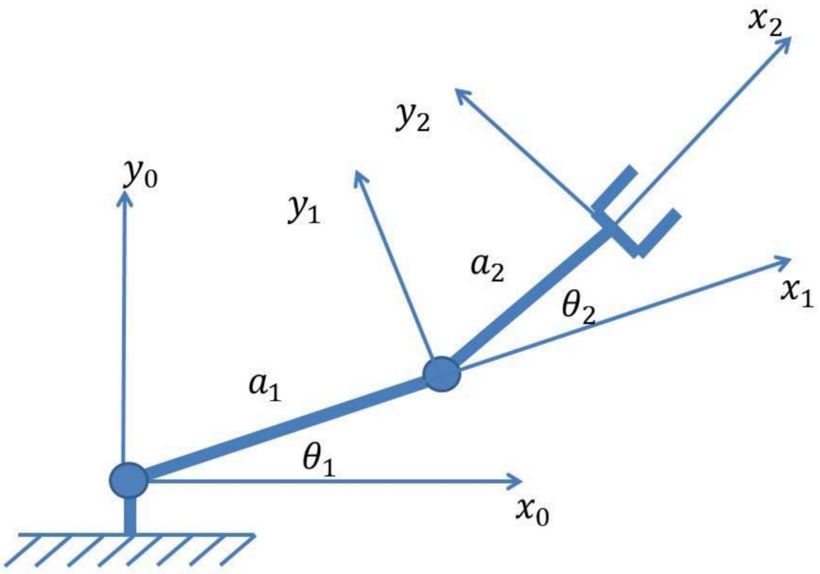
Using the DH convention can obtain the Ai transformation for a very basic two link system



Under the DH convention, each homogeneous transformation A_i is represented by a product of four basic transformations as follows

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

The DH parameters for the basic two link system are;

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2

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A commonly used convention for selecting frames of reference in robotic applications is the **Denavit–Hartenberg**, or **DH convention**. In this convention, each homogeneous transformation A_i is represented as a product of four basic transformations

$$
\begin{aligned}
A_i &= \text{Rot}_{z,\theta_i}\text{Trans}_{z,d_i}\text{Trans}_{x,a_i}\text{Rot}_{x,\alpha_i} \qquad (3.10)\\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Where the four quantities a_i α_i d_i θ_i are parameters associated with link i and joint j. These four parameters are given the names **link length** (a_i) **link twist** (α_i), **link offset** (d_i) and **joint angle** (θ_i)

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(Basic Two Link System)

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Notice that the first two entries of the last column of T_2^0 are the x and y components of the origin o_2 in the base frame; that is,

$$x = a_1c_1 + a_2c_{12}$$
$$y = a_1s_1 + a_2s_{12}$$

are the coordinates of the end effector in the base frame. The rotational part of T_2^0 gives the orientation of the frame $o_2x_2y_2z_2$ relative to the base frame.

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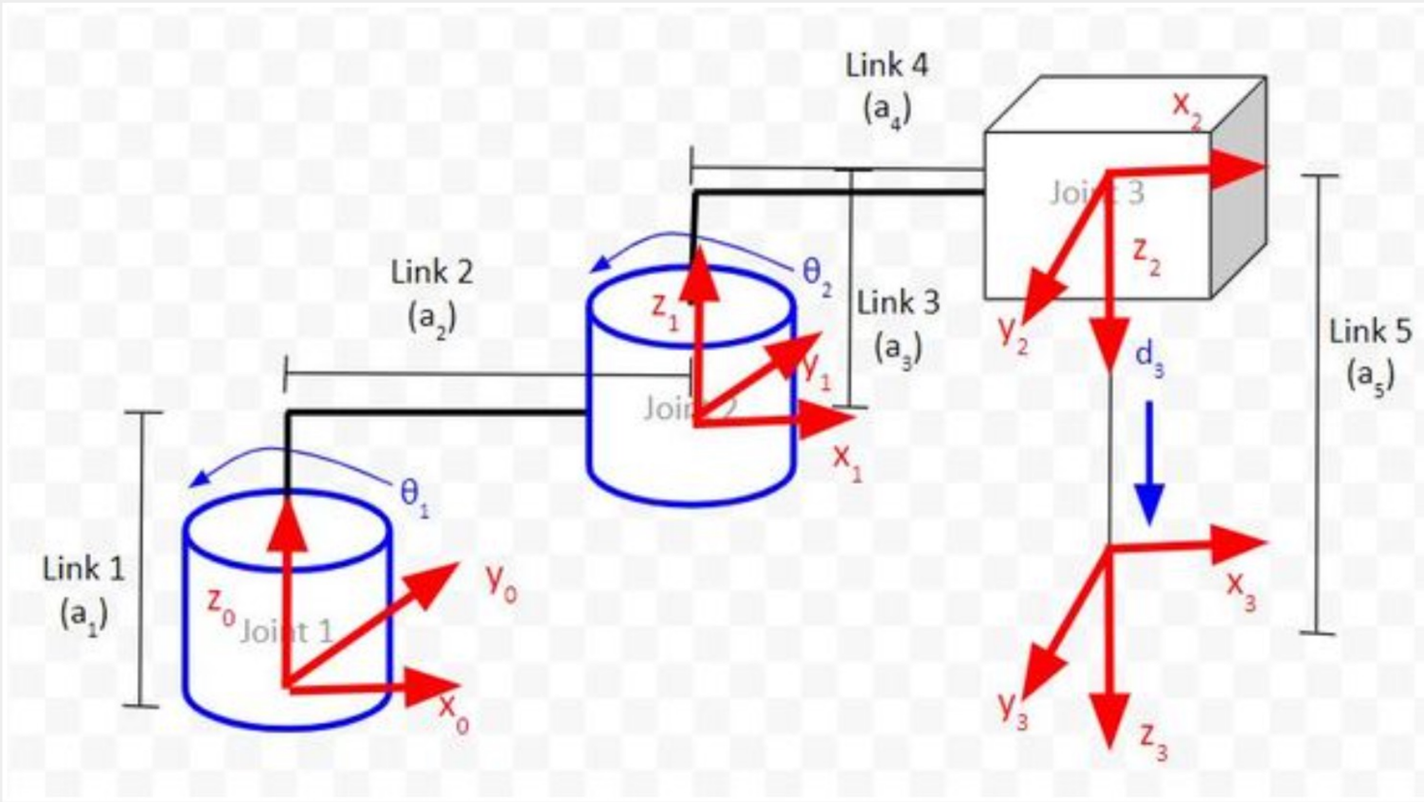

We then use that diagram to find the Denavit–Hartenberg parameters:

https://automaticaddison.com/homogeneous-transformation-matrices-using-denavit-hartenberg/

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Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0	a_2	a_1
2	θ_2	180	a_4	a_3
3	0	0	0	$a_5 + d_3$

homgen_0_1 =

$$\begin{bmatrix} \cos\Theta_1 & -\sin\Theta_1\cos\alpha_1 & \sin\Theta_1\sin\alpha_1 & r_1\cos\Theta_1 \\ \sin\Theta_1 & \cos\Theta_1\cos\alpha_1 & -\cos\Theta_1\sin\alpha_1 & r_1\sin\Theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

homgen_1_2 =

$$\begin{bmatrix} \cos\Theta_2 & -\sin\Theta_2\cos\alpha_2 & \sin\Theta_2\sin\alpha_2 & r_2\cos\Theta_2 \\ \sin\Theta_2 & \cos\Theta_2\cos\alpha_2 & -\cos\Theta_2\sin\alpha_2 & r_2\sin\Theta_2 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

homgen_2_3 =

$$\begin{bmatrix} \cos\Theta_3 & -\sin\Theta_3\cos\alpha_3 & \sin\Theta_3\sin\alpha_3 & r_3\cos\Theta_3 \\ \sin\Theta_3 & \cos\Theta_3\cos\alpha_3 & -\cos\Theta_3\sin\alpha_3 & r_3\sin\Theta_3 \\ 0 & \sin\alpha_3 & \cos\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

https://automaticaddison.com/homogeneous-transformation-matrices-using-denavit-hartenberg/