

## Counting Homework part 2

*Show the steps used in doing each of the conversions*

### Counting Principles:

1. Suppose a code consists of five characters, two letters followed by three digits. Find the number of a) codes; b) codes with distinct letter; c) codes with the same letters.

### Permutations:

2. Find the number of automobile license plates where: a) each plate contains 2 different letters followed by 3 different digits; b) the first digit cannot be 0.

### Permutations with repetitions:

3. Find the number  $n$  of distinct permutations that can be formed from all the letters of each word:  
a) THOSE; b) UNUSUAL; c) SOCIOLOGICAL

### Inclusion-Exclusion Principle:

4. Find the number  $m$  of elements in the union of sets A, B, C, D with the following 4 conditions:  
(i) A, B, C, D have 50, 60, 70, 80 elements, respectively.  
(ii) Each pair of sets has 20 elements in common.  
(iii) Each three of the sets has 10 elements in common.  
(iv) All four of the sets have 5 elements in common.

### Combinations:

5. Suppose that there are 9 faculty members in the math department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete math course if the committee is to consist of three faculty from the math department and four from the computer science department?

### Pigeonhole Principle:

6. Find the minimum number of students needed to guarantee that 3 of them:  
a) have last names which begin with the same first letter  
b) were born on the same day of a month with 31 days.

1. code : 5 char , 2 letters followed by 3 digits

(a) codes :  $26^2 \times 10^3$

(b) codes with distinct letters :  $26 \times 25 \times 10^3$

(c) codes with the same letter :  $26 \times 10^3$

2. (a) 2 different letters followed by 3 different digits :  $26 \times 25 \times 10 \times 9 \times 8$

(b) the first digit cannot be 0 :  $26 \times 25 \times 9 \times 9 \times 8$

3. (a) THOSE :  $n = 5! = 120$

(b) UNUSUAL :  $n = 7! / 3! = 840$

(c) SOCIOLOGICAL :  $n = \frac{12!}{3!2!2!2!}$

4. 1.  $A + B + C + D = 260$

2.  $(A \cap B) + (A \cap C) + (A \cap D) +$   
 $(B \cap C) + (B \cap D) + (C \cap D) = 120$

3.  $(A \cap B \cap C) + (A \cap B \cap D) +$   $260 - 120 + 30 + 5 = 175$   
 $(B \cap C \cap D) = 30$

4.  $A \cap B \cap C \cap D = 5$

5.  $C(9, 3) \times C(11, 4)$

6. Let  $n$  = number of students

(a)  $\left\lceil \frac{n}{\substack{\text{\# of letters} \\ \text{in Alphabet}}} \right\rceil = \left\lceil \frac{n}{26} \right\rceil = 3 \rightarrow 26(3-1) + 1 = 53$

minimum of 53 students

(b)  $\left\lceil \frac{n}{\substack{\text{\# of days} \\ \text{in month}}} \right\rceil = \left\lceil \frac{n}{31} \right\rceil = 3 \rightarrow 31(3-1) + 1 = 63$

minimum of 63 students