CALIFORNIA STATE UNIVERSITY
SACRAMENTO

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EEE 117 Network Analysis

Text: Electric Circuits by J. Nilsson and S. Riedel Prentice Hall

Lecture Set 2: Sinusoidal Steady State Power Analysis

Instructor: Riaz Ahmad

Sinusoidal Steady-State Power Analysis

- Instantaneous Power
- Average, Active or Real power
- Reactive power
- Apparent Power
- Complex Power
- Effective Power
- Power factor and reactive factor
- Power for Circuit elements
- Maximum Power Transfer
- Mutual Inductance
- Transformer
- Ideal Transformer

Instantaneous Power

- Instantaneous power is the power at any one moment.

  Let's say at any instance "t"

  $v(t) = V_m \cos(\omega t + \Theta v)$ and $i(t) = I_m \cos(\omega t + \Theta i)$

  Θv = phase angle of voltage

  Θi = phase angle of current

- Instantaneous power p(t) can be defined as:
  $p(t) = V_m \cos(\omega t + \Theta v) * I_m \cos(\omega t + \Theta i)$

  After using the trig. identities, we get

$$\boxed{p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)}$$

EQ-1

Derivation of eq-1:

$$p = vi = V_m \cos(\omega t + \theta_v)\, I_m \cos(\omega t + \theta_i)$$

$$\begin{aligned} p &= V_m \cos(\omega t + \theta_v - \theta_i)\, I_m \cos(\omega t + \theta_i - \theta_i) \\ &= V_m \cos(\omega t + \theta_v - \theta_i)\, I_m \cos(\omega t) \\ &= V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos(\omega t) \end{aligned}$$

Now use a trig identity

$$\cos\alpha \cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

And we get

$$p = V_m I_m \left[\tfrac{1}{2}\cos\left(\underbrace{\omega t + \theta_v - \theta_i}_{\alpha} - \underbrace{\omega t}_{\beta} \right) + \tfrac{1}{2}\cos(\underbrace{\omega t + \theta_v - \theta_i}_{\alpha} + \underbrace{\omega t}_{\beta}) \right]$$

$$= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

We will again use another trig identity to simplify the 2nd term.

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$p=\underbrace{\frac{V_m I_m}{2}\cos(\theta_v-\theta_i)}_{P_{avg}=p\ \ at\ DC} \qquad \underbrace{+\frac{V_m I_m}{2}\cos(\underbrace{\theta_v-\theta_i}_{\alpha})\cos(\underbrace{2\omega t}_{\beta})}_{p\ \ at\ 2\omega}$$

$$\underbrace{-\frac{V_m I_m}{2}\sin(\underbrace{\theta_v-\theta_i}_{\alpha})\sin(\underbrace{2\omega t}_{\beta})}_{\text{Reactive power from C or L}}$$

Some observations:

- First term is always less than V*I because of Cos Φ.
- First term is fixed and does not depends on time.
- 2nd and 3rd terms can go up to V*I and also –V*I.
- Resultant can also be a negative value.
- Eq.1 has three terms and can be rewritten as

$$\boxed{p(t) = P + P\,\mathrm{Cos}\,(2\omega t) - Q\,\mathrm{Sin}\,(2\omega t)}$$

- "P" is called average, real, true or active power measured in watts.
- "Q" is called reactive power measured in VAR.
- First term is non oscillating while 2nd and 3rd terms are oscillating.

Average, Real, True or Active power

"P" is called average, real, true or active power measured in watts.

$$\boxed{P_{avg} = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}$$

Take the average of the equation 1.

First term is non oscillating while 2nd and 3rd terms are oscillating.

$P_{avg} = \frac{1}{T} \int_0^T p(t)dt$ Since area under oscillating terms = 0

$$P_{avg} = \frac{1}{T}\int_0^T \left(\frac{VmIm}{2}\right) Cos(\theta v - \theta i) \ \ dt$$

$$= (V_m I_m/2)\ Cos(\Theta_v\text{-}\Theta_i) \times \frac{1}{T}\int_0^T dt$$

$$P_{avg} = (V_m I_m/2)\ Cos(\Theta_v\text{-}\Theta_i) \times \frac{1}{T} \times T$$

$$\boxed{P_{avg} = (V_m I_m/2)\ Cos(\Theta_v\text{-}\Theta_i) = V_{rms} I_{rms}\ Cos(\Theta_v\text{-}\Theta_i)}$$

It is called:

- ***Average power:*** Because it is the average of all the instantaneous values in one time period and is defined as the average power delivered to a circuit.

- ***Real power:*** Because it is the power which cause the energy to flow. This is the power which is generated, transmitted, distributed and consumed.

- ***True power:*** Because this is the power which actually exists in the system.

- ***Active power:*** because this is the power which do the work.

Reactive Power

- In equation 1, "Q" is called Reactive power measured in VAR (volt-ampere-reactive)

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) \quad = V_{rms} I_{rms} \sin(\Theta_v - \Theta_i)$$

- Reactive power comes from apparent and real power.
- Reactive power is just a hypothetical term, an imaginary quantity, does not actually exist and used only for mathematical reason.
- If the consumed reactive power (Q) is positive, then the load is inductive. If the consumed reactive power (Q) is negative, then the load is capacitive.

Complex Power

- Complex power is the complex sum of real and reactive power measured in Volt-Ampere (VA).
- Complex power is a complex number where its real and imaginary parts determine the active and reactive powers consumed by the load, respectively. Complex power is given as

$$\boxed{S = P + jQ}$$

- Real part of the complex power is real power
- $$\mathrm{Re}\{\ S\ \} = P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$
- Imaginary part of the complex power is reactive power.
- $$\mathrm{Im}\{\ S\ \} = Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$
- The magnitude of the complex power is given as:

$$S = |S| = \sqrt{P^2 + Q^2} \qquad \Theta = \tan^{-1}\left(\frac{Q}{P}\right)$$

- Consider the equation of complex power:

  S = P + jQ

$$S = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

$$S = \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

$$S = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} \qquad = V_{eff} e^{j\theta_v} I_{eff} e^{-j\theta_i} \qquad \boxed{S = V_{eff} I^{*}_{eff}}$$

Where * indicates the complex conjugate of the angle.

- In terms of the maximum voltage (V_m) and current (I_m)

$$\boxed{S = \tfrac{1}{2} V I^{*}}$$

- and in terms of the rms voltage (V_{rms}) and current (I_{rms})

$$\boxed{S = V\, I^{*}}$$

Apparent Power

- The magnitude of the complex power is called apparent power

$$\boxed{|S| = \sqrt{P^2 + Q^2} \qquad \Theta = \tan^{-1}\left(\frac{Q}{P}\right)}$$

$$P = |S|\cos\theta$$

$$Q = |S|\sin\theta$$

- It is the power that we expect would exist in the system, measured in volt-ampere (VA) and denoted by |S|.
- BUT note, the power actually exist in the system is always real power.
- Apparent power > real power
- OR real power < apparent power
- It means system is under-utilized.
- This leads us to the concept of power factor.

Power factor and Reactive factor

- Power factor is the fraction of system capability that can be actually utilized.
- Total power of the system is $V_{rms}\, I_{rms}$
- Useful power is $V_{rms}\, I_{rms} \cos(\Theta_v - \Theta_i)$

$$\boxed{pf = \frac{Vrms Irms\ Cos\ (\theta v - \theta i)}{Vrms Irms} = \frac{active\ power}{apparent\ power} = \frac{P}{S} = \cos(\Theta_v - \Theta_i)}$$

- Ideal value of pf = 1 and practically cannot be achieved.
- The phase angle between the voltage and current is called the power factor angle.
- The sinusoidal power factor (pf) is defined as

$$\boxed{pf = \cos(\theta_v - \theta_i)}$$

- Since cosine(+ϴ) = cosine(-ϴ), we need to add the term lagging to the power factor for an inductive load where current lags the voltage.
- We add the term leading to the power factor for a capacitive load where current leads the voltage.

- The sinusoidal reactive factor is defined as

$$\boxed{rf = \sin(\theta_v - \theta_i)}$$

Power in Circuit elements

Power in purely resistive circuit:

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

- For resistors, voltage and currents are in phase.

  $\theta_v - \theta_i = 0$

- So, for a resistor (which can only absorb power "+"), we have the instantaneous power as

$$p = P_{avg} + P_{avg}\cos(2\omega t)$$

Since total of 2nd part is zero

p(t) = $\frac{V_m I_m}{2}$ = $V_{rms}\, I_{rms}$

Active Power = $\frac{V_m I_m}{2}$ = $V_{rms}\, I_{rms}$

Power in purely Inductive circuit:

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

- For inductor, voltage leads currents by 90°

$\theta_v - \theta_i = +90^\circ$

$$\boxed{p(t) = -\frac{V_m I_m}{2}\sin(2\omega t)}$$

- which is a sinusoidal function with frequency twice of supply.

$$\boxed{p(t) = 0}$$

- So, an ideal inductor does not dissipate real power but only stores or delivers reactive power as required by the circuit.
- Inductors demand (absorb) magnetizing VARs.

Power in purely capacitive circuit:

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

- For capacitor, voltage lags currents by 90°

  $\theta_v - \theta_i = -90^\circ$

$$\boxed{p(t) = -\frac{V_m I_m}{2}\sin(2\omega t)}$$

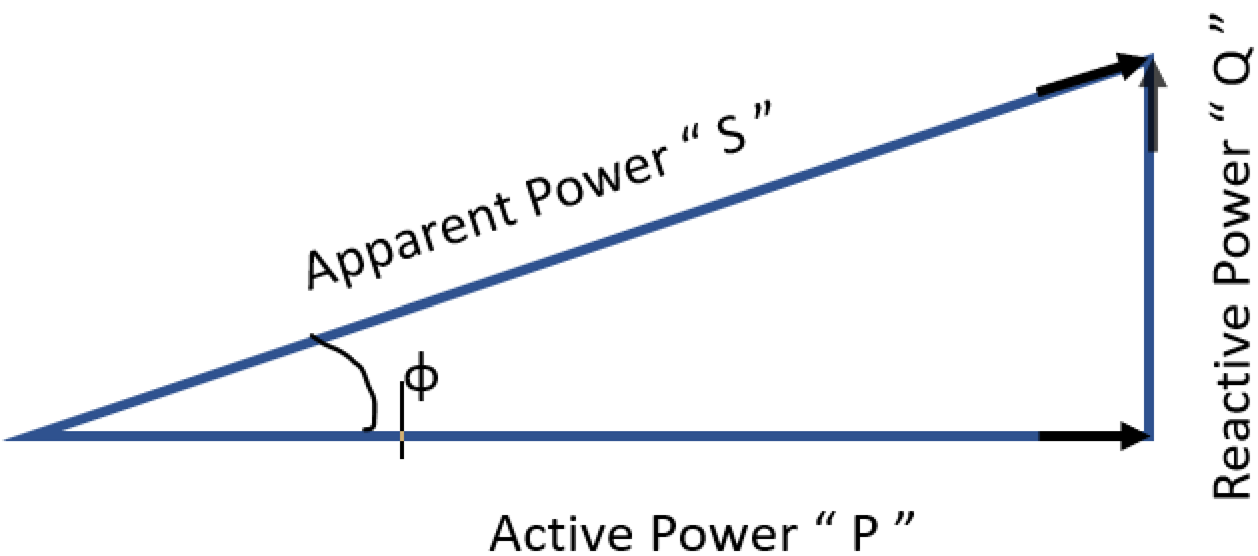
- which is a sinusoidal function with frequency twice of supply.

$$\boxed{p(t) = 0}$$

- So, an ideal capacitor does not dissipate real power but only stores or delivers reactive power as required by the circuit.
- Capacitors furnish (deliver) magnetizing VARs.

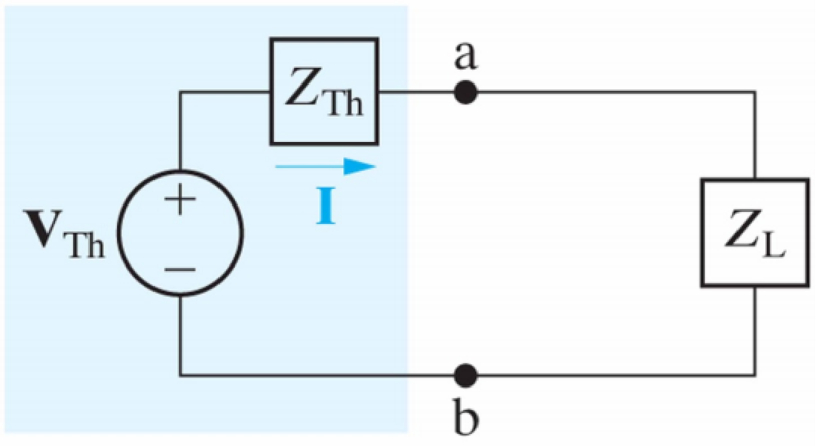
Power Triangle

Power Triangle: Power triangle is a pictorial method to represent and calculate the powers in the system.



Maximum Power Transfer

- Maximum power is transferred to a load when the load resistance equals the Thévenin resistance of the source.



$$\boxed{R_{load} \equiv R_{Th}}$$

- In the complex case, we must “cancel out” the reactive components.

$$\boxed{Z_{load} \equiv Z_{Th}^{*}}$$

- Thus

$$\boxed{P_{\max} = \frac{|V_{Th}|^2 R_L}{4R_L^2} = \frac{|V_{Th}|^2}{4R_L}}$$

- If V_{Th} is expressed in terms of the maximum voltage V_m rather than rms values then

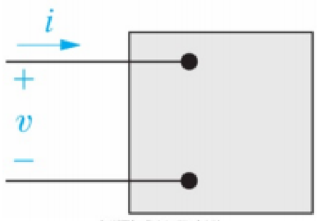
$$\boxed{P_{\max} = \frac{V_m^2}{8R_L}}$$

- What about the case when the load cannot be fully manipulated by varying both R_L and X_L but the magnitude can be adjusted?

- Then the closest approach to maximum power is when the magnitudes of the load and the Thévenin equivalent are made equal.

$$\boxed{|Z_L| = |Z_{Th}|}$$

Examples



Ex-1: If $v = 100 \cos(\omega t + 15^o)$ V and

$i = 4 \sin(\omega t - 15^o)$ A then

a) Calculate the power factor.
b) Calculate the active power.
c) Calculate the reactive power.

$$\boldsymbol{v} = 100 \cos(\omega t+15^o) = 100 \angle 15^o \text{ V}$$

$$\boldsymbol{i} = 4 \sin(\omega t-15^o) = 4 \cos(\omega t-15^o-90^o) \text{ A}$$

$$\boldsymbol{i} = 4 \cos(\omega t-105^o) = 4 \angle -105^o \text{ A}$$

a) Power factor (pf)= cos [15-(-105)] = -0.5

b) $P = \frac{1}{2} * 100*2* \cos[15-(-105)] = -100$ W (-ve, Delivering)

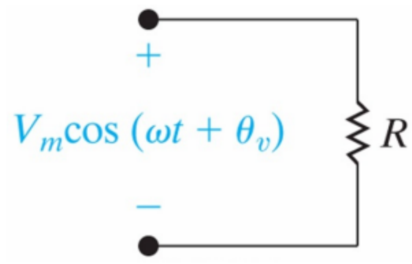
c) $Q = \frac{1}{2} * 100*2* \sin[15-(-105)] = 173.21$ VAR (+ve, Absorbing)

-100 mean its delivering the active power instead of absorbing.

Ex-2: If $v = 625 \cos(\omega t)$ V and

$R = 50\ \Omega$ then

Find the average power delivered to the resistor.



$$\boldsymbol{v} = 625 \cos(\omega t) = 625 \angle 0^{o}\ \text{V}$$

$$\boldsymbol{R} = 50 \angle 0^{o}$$

$$\boldsymbol{i} = \frac{v}{R} = \frac{625\angle 0}{50\angle 0} = 12.5 \angle 0^{o}\ \text{A}$$

$$\boxed{P = \frac{1}{2} * 625*12.5* \cos[0-0] = 3906.25\ \text{W}}$$

Ex-3: An electrical load operates at 240 V_{rms}. The load absorbs an average power of 8 kW at a lagging power factor of 0.8

- Calculate the complex power of the load.

pf = 0.8 lagging ==➔ so load is inductive.

pf = Cos ϴ = 0.8 ===➔ ϴ = 36.87° ==➔ Sinϴ = 0.6

P = 8 kW = 8000 W

We know: P= |S| Cos ϴ = |S| * pf So

$|S| = \frac{P}{pf} = \frac{8000}{0.8} = 10000 \text{ VA} = 10 \text{ kV}$

Q = |S| sin ϴ = 10000 * 0.6 = 6000 VAR = 6 KVAR

$\boxed{\bar{S} = 8000 + j6000 = 10 \angle 36.87^\circ \text{ VA}}$

Ex-4: If $v = 100 \cos(\omega t + 15^o)$ V and

$i = 4 \cos(\omega t - 105^o)$ A then

- Calculate the complex power using phasor form.

$$\boldsymbol{v} = 100 \cos(\omega t+15^o) = 100 \angle 15^o \text{ V}$$

$$\boldsymbol{i} = 4 \cos(\omega t-105^o) = 4 \angle -105^o \text{ A}$$

$$\boldsymbol{i^*} = 4 \angle 105^o \text{ A}$$

$$\bar{S} = ½\, VI^* = ½\,(100 \angle 15^o)(4 \angle -105^o) \text{ VA}$$

$$\boxed{\bar{S} = 200 \angle 120^o = -100 + j173.2 \text{ VA}}$$

Ex-5: A single-phase load with an impedance of $Z = 1.25\angle 60^\circ\ \Omega$ is connected to a sinusoidal source with $v(t) = 100\cos(\omega t)$. Determine:

a) The instantaneous current $i(t)$ and
b) The instantaneous power $p(t)$
c) The power for $\omega t = 0$ and $\omega t = 2\pi$

$$\boldsymbol{v(t)} = 100\cos(\omega t) = 100\ \angle\ 0^o\ \text{V}$$

$$\boldsymbol{Z} = 1.25\ \angle\ 60^o\ \Omega$$

$$\boxed{\boldsymbol{i(t)} = \frac{v}{z} = \frac{100\angle 0}{1.25\angle 60} = 80\ \angle\ \text{-}60^o = 80\cos(\omega t - 60^o)}$$

$$\boldsymbol{p(t)} = v(t)\, i(t) = [100\cos(\omega t)] \times [80\cos(\omega t - 60^o)]$$

$$\boxed{\boldsymbol{p(t)} = [2000(1+\cos 2\omega t)] + [2000\sqrt{3}\ \text{Sin}(2\omega t)]}$$

$$\boxed{\boldsymbol{p(\omega=0)} = 4000\ \text{VA} \qquad \boldsymbol{p(\omega=2\pi)} = 4000\ \text{VA}}$$

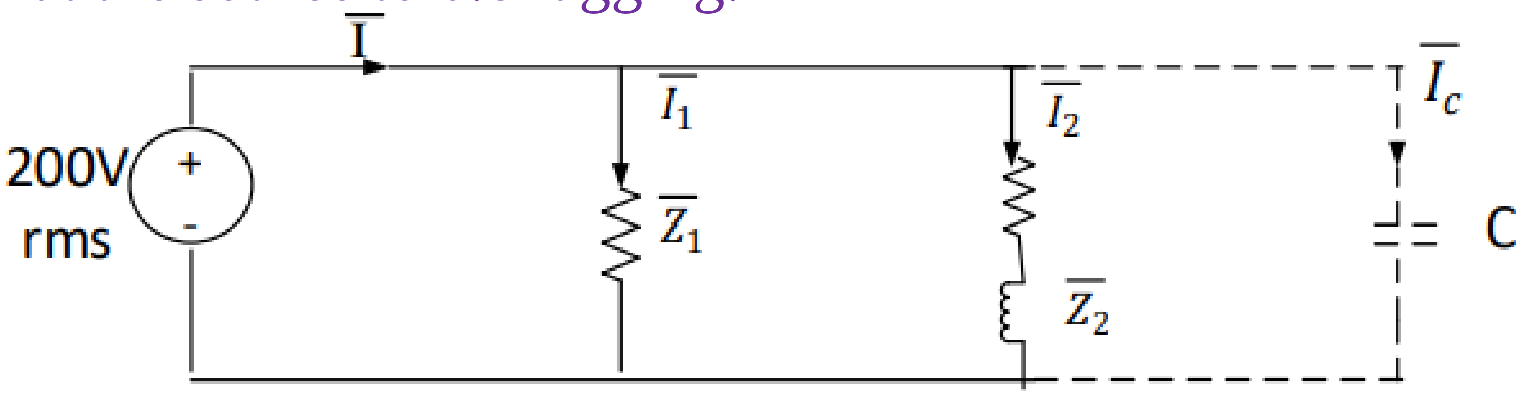
Ex-6: The following figure shows two parallel single-phase loads connected to a source. The first load has a resistance of 100 Ω and the second load has an impedance of 10 + j20 Ω. Determine:

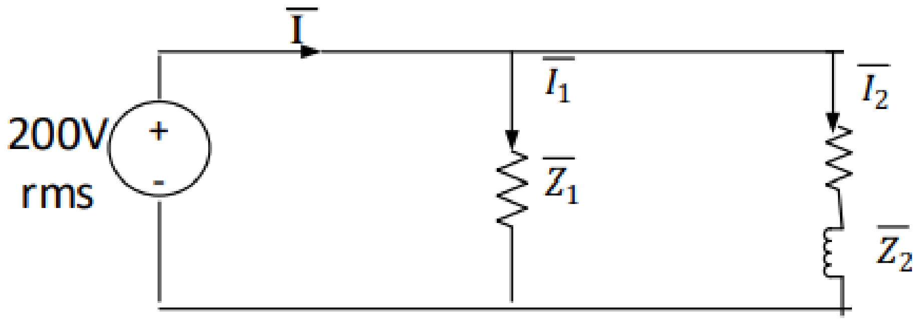
a) The total active and reactive power generated by the source.

b) The power factor at the source.

c) The total current I.

d) Assuming the same active power consumption found in (a), derive the amount of capacitance C needed to improve the power factor at the source to 0.8 lagging.





Vrms = 200 $\angle$0

$\boldsymbol{Z_1}$ = 100 + j0 = 100 $\angle$ 0° Ω

$\boldsymbol{Z_2}$ = 10 + j20 = 22.36 $\angle$ 63.43° Ω

$$I_1 = \frac{v}{z1} = \frac{200\angle 0}{100\angle 0} = 2\angle 0 = 2+j0$$

$$I_1^* = 2\angle 0 = 2-j0$$

$$I_2 = \frac{v}{z2} = \frac{200\angle 0}{22.36\angle 63.43} = 8.944\ \angle{-63.43} = 4 - j8$$

$$I_2^* = 8.944\ \angle\ 63.43 = 4 + j8$$

$S_1 = VI_1^* = (200\ L0)\ (2L0) = 400\ L0 = 400+j0$ VA

$S_2 = VI_2^* = (200\ L0)\ (8.944L63.43) = 1788.8\ L63.43 = 800-j1600$ VA

$S = S_1 + S_2 = (400+j0) + (800-j1600)$ VA

$S = 1200+j1600 = 2000\ L53.13$ VA

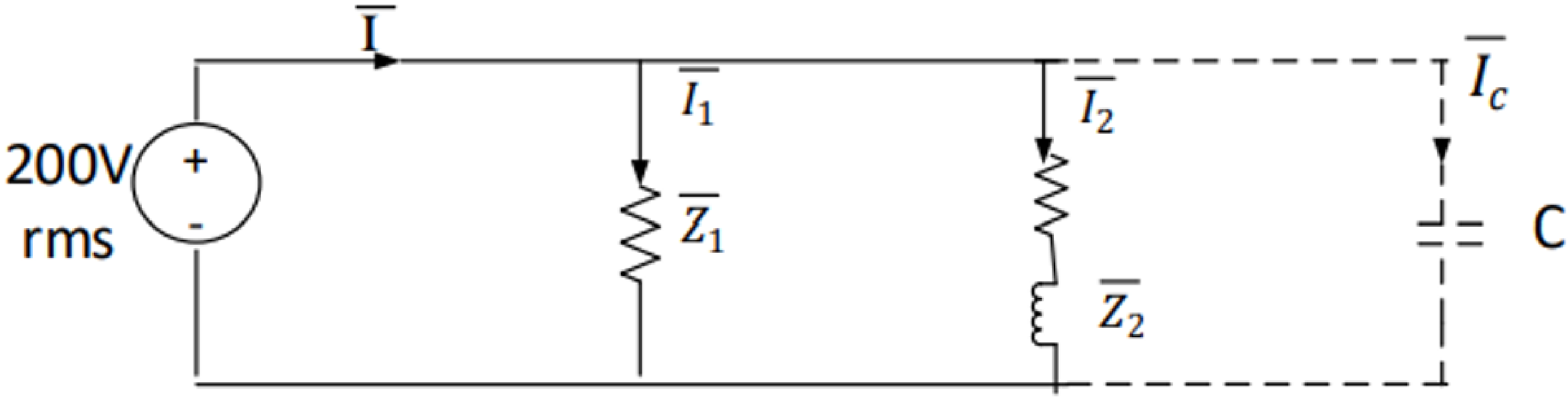
a) So, P = 1200 att and Q = 1600 VAR

b) Power factor (pf) = Cos ϴ = Cos (53.13) = 0.6

$I = I_1 + I_2 = (2+j0) + (4-j8)$ VA

c) $I = 6 - j8 = 10\ L -53.13$ VA

d)



pf = Cos Ɵ = 0.8 ===➔ Ɵ = 36.87° ==➔ SinƟ = 0.6

P= 1200

We know: P= |S| Cos Ɵ = |S| x pf So

$|S| = \frac{P}{pf} = \frac{1200}{0.8} = 1500$ VA

Q = |S| sin Ɵ = 1500 * 0.6 = 900 VAR

Note Q has been reduced from 1600 to 900 VAR.

so

Compensated Qc = 1600 – 900 = 700 VAR

$Xc = V^2 / Qc = (200)^2/700 = 57.14\ \Omega$

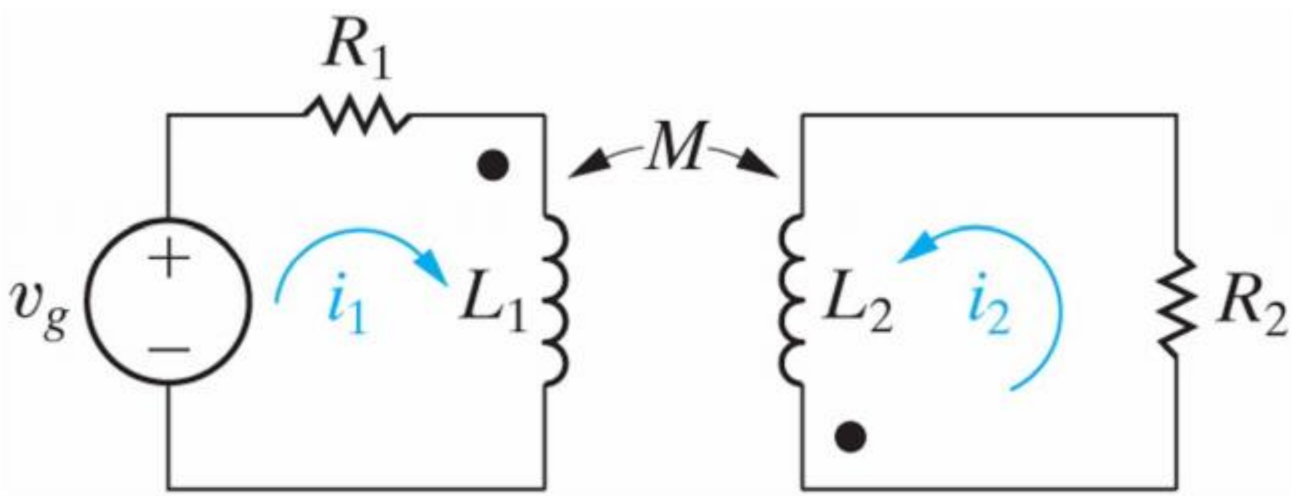
$\frac{1}{2\pi f C} = 57.14$ =====➔ $\boxed{C = \frac{1}{2\pi(60)(57.14)} = 46.42\ uF}$

Note: Assuming f=60 Hz

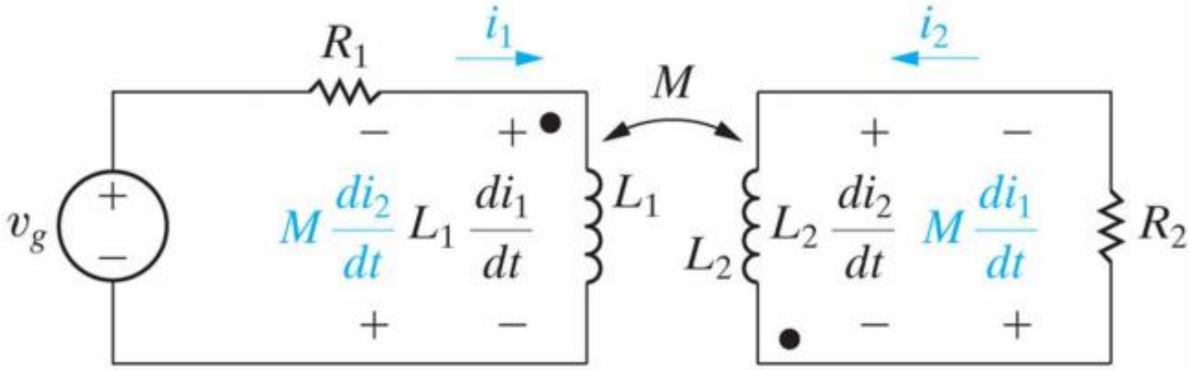
Mutual Inductance

- We previously looked at the effect of a moving charge (current) creating a magnetic field which is called inductance and since the effect was the current in a single circuit, it should be properly called **self-inductance**.
- Here in this section, we will investigate the situation where the magnetic field created by a time varying current in one circuit is linked with the other circuit and induce voltage. This phenomenon is known as **mutual inductance**.
- The polarity of mutually induced voltage depends on the way the coils are wound in relation to the reference direction of coil currents.
- Dots are placed on the terminals to carry the polarity information schematically (rather than knowing righty or lefty directions).
- Hence for mutual inductance, we also rely on the dot convention. we will discuss it later in following section.


R_1
v_g
+
−
i_1
L_1
L_2
R_2


R_1
M
v_g
+
−
i_1
L_1
L_2
i_2
R_2

- Let's analyze the following mutual inductance circuit.



KVL on the left hand side gives us

$$-v_g + i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

KVL on the right hand side gives us

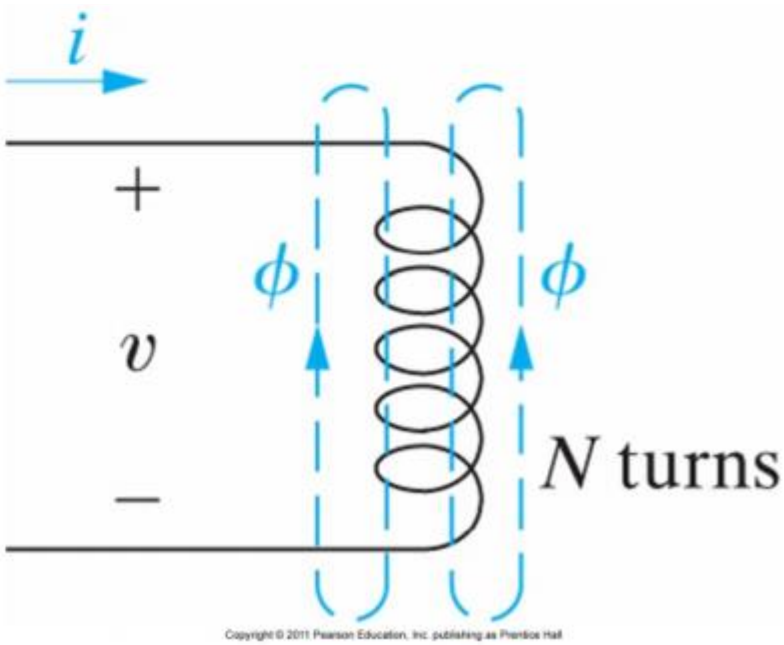
$$-L_2\frac{di_2}{dt} - i_2R_2 + M\frac{di_1}{dt} = 0$$

Or equally (multiply last result by -1)

$$L_2\frac{di_2}{dt} + i_2R_2 - M\frac{di_1}{dt} = 0$$

Note: M12 = M21

The voltage induced by the magnetic field surrounding a current carrying conductor is described by Faraday's Law:



$$v = \frac{d\lambda}{dt} = \frac{d(N\phi)}{dt}$$

$$= N\frac{d\phi}{dt} = N\frac{d}{dt}(\mathcal{P}Ni)$$

$$= N^2\mathcal{P}\frac{di}{dt} = L\frac{di}{dt},$$

- Where λ is called the magnetic flux linkage – measured in weber-turns.
- The flux linkage λ is the product of the magnetic field (ϕ) and the number of turns linked (N).

$$\lambda = N\phi$$

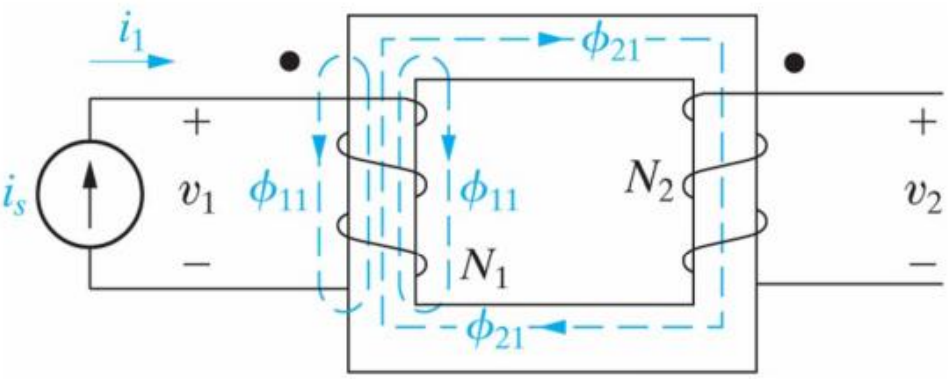
- The magnetic field strength per unit volume depends on the material in that space - described by the permeance of the material.
- The text has more details, but the important message is the magnetic coupling M may not be perfect.
- Thus, we introduce a coefficient of coupling k which can be used to better model a specific physical circumstance.
- The value of k depends on what fraction of flux is linking to the other coil.

$$M = k\sqrt{L_1 L_2}$$

Where $0 \le k \le 1$.

In general, k is found by experimental measurement.

- We thus have the following view of a mutually coupled circuit:



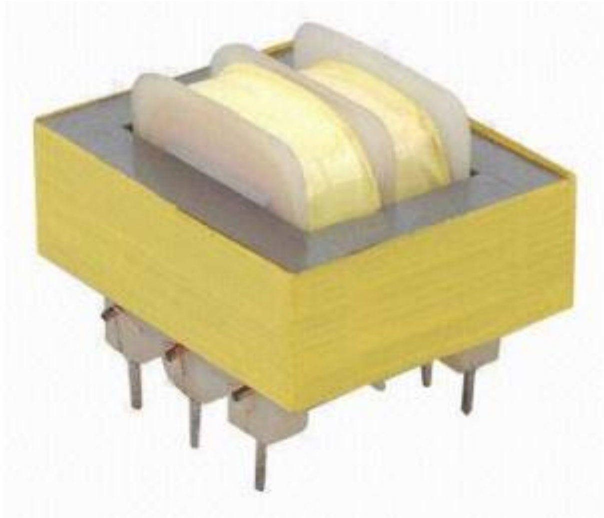
$$M = k\sqrt{L_1 L_2}$$

The energy stored in these two coils (self-inductance) and the energy stored in the coupled magnetic fields (mutual inductance) has the form:

$$w(t) = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

Transformer

- A transformer is a device that is based on magnetic coupling.
- In communications, a transformer can be used to match impedance between two circuits and to eliminate dc signals.
- A very common use of transformers is in power systems to raise or lower voltage levels.



Ideal Transformer:

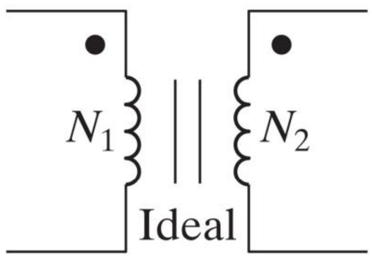
- An ideal transformer is an imaginary transformer, consists of two magnetically coupled coils having:
- N1 turns around the coil on the primary side.
- N2 turns around the coil on the secondary side.
- Coefficient of coupling k = 1. Perfect coupling.
- The self-inductance of each coil is infinite. $L_1 = L_2 = \infty$
- The resistance of each coil is negligible. Parasitic resistance → zero.
- There are no copper losses (no winding resistance).
- There is no iron loss in core.
- There is no leakage flux.
- an ideal transformer gives output power exactly equal to the input power.
- Efficiency of an idea transformer is 100%.
- It is impossible to have such a transformer in practice, but ideal transformer model makes problems easier.

Dot Convention:

Dot convention is a method to define the polarity of voltage and current in a mutually coupled circuit and can be summarized as:

- If the coil voltages V_1 and V_2 are both positive or both negative at the dot-marked terminal, then use a plus sign. Otherwise, use a minus sign.
- If the coil currents I_1 and I_2 are both directed into or both directed out of the dot-marked terminal, then use a minus sign. Otherwise, use a plus sign.

Let's consider the following four cases of an ideal transformer to determine the correct polarity of voltages and currents.

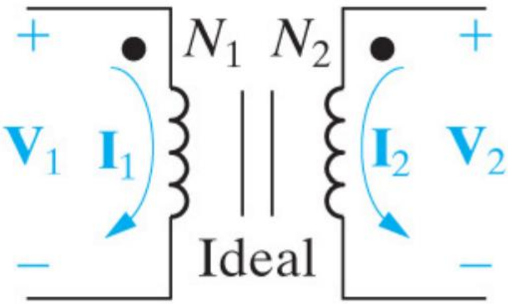


$$\frac{V_1}{N_1} = (\pm)\frac{V_2}{N_2}$$

$$I_1 N_1 = (\pm) I_2 N_2$$

Case 1: Both voltages are positive at the dot-marked terminal.

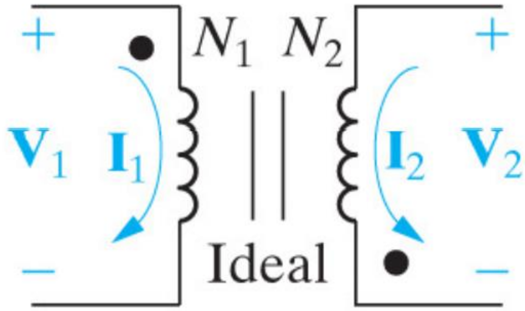
Both currents are into the dot-marked terminal.



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

Case 1



$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

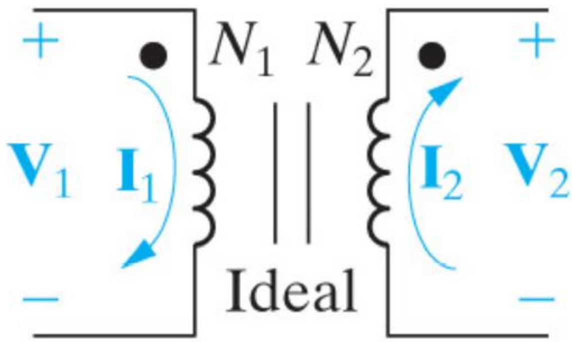
$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

Case 2

Case 2: Only one voltage is positive at the dot-marked terminal.

Only one current is into the dot-marked terminal.

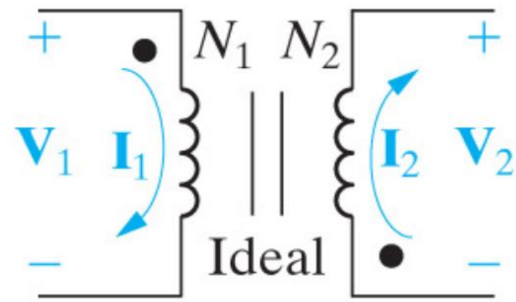
Case 3: Both voltages are positive at the dot-marked terminal.
Only one current is into the dot-marked terminal.



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

Case 3



$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

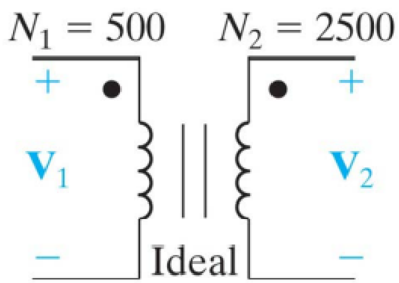
Case 4

Case 4: Only one voltage is positive at the dot-marked terminal.
Both currents are into the dot-marked terminal.

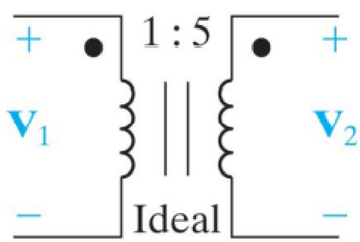
The ratio of turns on the two windings may be defined as either

N_1/N_2 or as N_2/N_1 Here we will use $a = \dfrac{N_2}{N_1}$

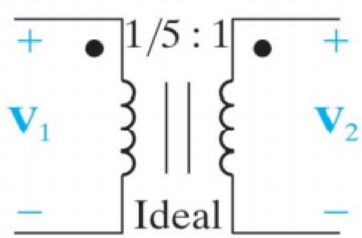
Equivalent ways to show the turns ratio.



$$a = \frac{N_2}{N_1} = \frac{2,500}{500} = 5$$



$$1 : a$$

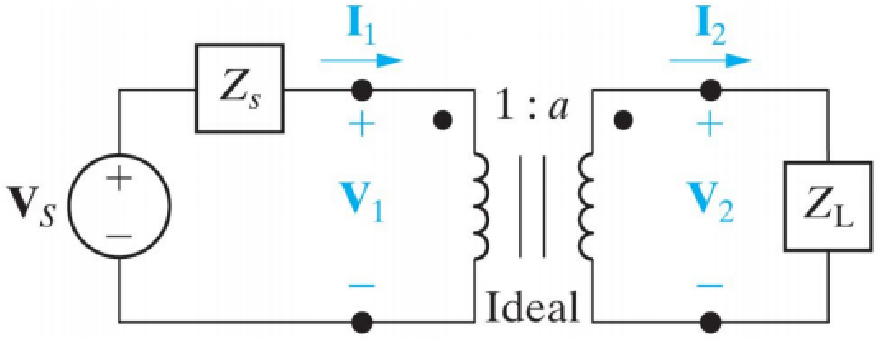


$$\frac{1}{a} : 1$$

Impedance Matching:

- Ideal transformers can be used to raise or lower the impedance of load as seen by the source.

$$\frac{V_1}{1} = (+)\frac{V_2}{a}$$



$$I_1(1) = (+)I_2(a)$$

Thus the input impedance is (leave out the source impedance Z_s)

$$Z_{in} \equiv \frac{V_1}{I_1} \qquad Z_L \equiv \frac{V_2}{I_2}$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{\frac{V_2}{a}}{aI_2} = \frac{1}{a^2}\frac{V_2}{I_2} = \frac{1}{a^2}Z_L$$

- Thus, the ideal transformer’s secondary coil reflects the load impedance back to the primary coil with a scaling factor.
- The magnitude is scaled, but the phase induced by the load is not altered!
- If the phase of Z_L cannot be changed, the greatest power is delivered to the load is when the magnitudes of the Thévenin and load impedances are equal.

$$\left|Z_{TH}\right| = \left|Z_L\right|$$

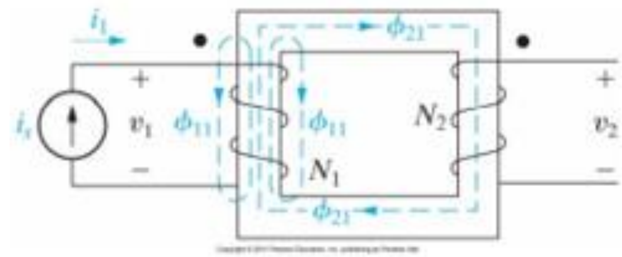
- Impedance matching with ideal transformers allow us to create the above condition.

Ex-7: Two magnetically coupled coils have:

L_1=60 mH $\quad$ L_2 = 9.6 mH $\quad$ M= 22.8 mH

What is the coefficient of coupling?

What is the largest value of M?



Find K:

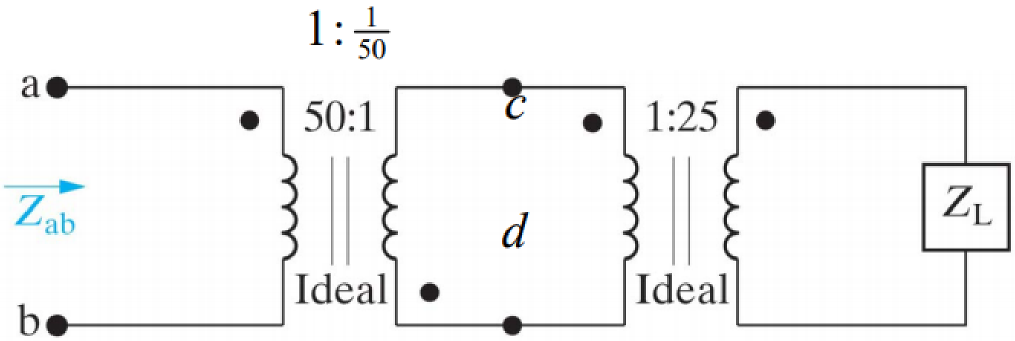
$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{22.8 \times 10^{-3}}{\sqrt{60 \times 10^{-3} \times 9.6 \times 10^{-3}}} = 0.95$$

Find largest value of M:

M is max^m when K = 1.

$$M = 1\sqrt{60 \times 10^{-3} \times 9.6 \times 10^{-3}} = 24.0 \text{ mH}$$

Ex-8: Find the impedance Zab?



$$Z_L = 200 + j150\,\Omega$$

$$Z_{cd} = \left(\frac{N_{cd}}{N_L}\right)^2 \cdot Z_L = \left(\frac{1}{25}\right)^2 \cdot (200 + j150).$$

$$Z_{ab} = \left(\frac{N_{ab}}{N_{cd}}\right)^2 \cdot Z_{cd} = \left(\frac{50}{1}\right)^2 \left[\frac{1}{25^2}\left(200 + j150\right)\right]$$

$$\boxed{Z_{ab} = 800 + j600 = 1000\angle 36.87^\circ} \quad \Omega$$