Chapter 4

Techniques of Circuit Analysis

Text: *Electric Circuits*, by J. Nilsson and S. Riedel
Prentice Hall

Engr 17 Introductory Circuit Analysis
Instructor: Russ Tatro

Chapter 4 Overview

We will now start the bread and butter of circuit analysis – how to use organized methods to solve circuits.

Node-Voltage Method – writing equations for currents in a circuit.

Mesh-Current Method – writing equations for voltages in a circuit.

Source transformations – make a circuit simpler.

Thévenin Equivalent – voltage source with a load.

Norton Equivalent – current source with a load.

Section 4.1
Circuit Terminology

The Vocabulary

We need to use some terms with specific meanings:

Node – a point where circuit elements join

Node definition used to write current equations at the point in the circuit.

Reference Node – a node that is *assumed* to be at the lowest potential in the circuit. All voltages from this node are voltage "rises".

The Vocabulary

Loop – a path through the circuit that begins and ends at the same node.

Mesh – a loop that does not enclose any other loops.

Mesh definition used to write voltage equations around a desired loop.

Simultaneous Equations

How many equations are needed to solve a circuit?

The number of equations must equal of the number of independent circuit unknowns.

You will need to an equation each to solve for the voltage across all elements and the currents through all elements in the circuit.

In real circuits, the number of required equations is often a huge number which leads to either simplifying the circuit or the use of circuit simulation tools.

Section 4.2
Node-Voltage Method

Node-Voltage Method

Using nodes – write a sufficient number of independent equations in order to solve for all the unknown circuit values.

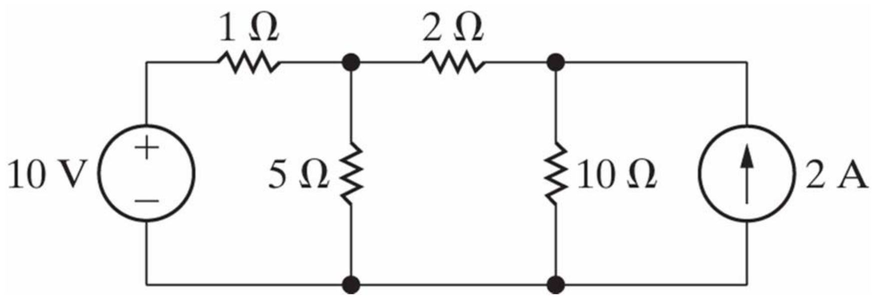
You are writing equations for currents – usually in terms of voltage and other circuit element values.

Remember that by *conservation of charge* all the currents at a node must sum to zero.

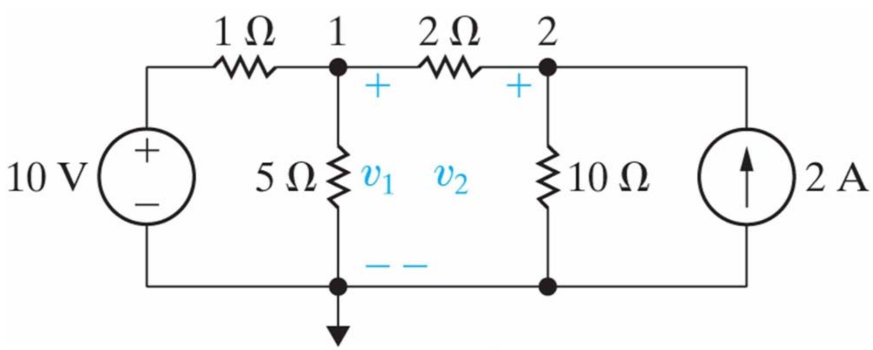
Your first task is to pick your reference node from which you calculate voltage changes (drops or rises).

Node-Voltage Method

We want to know the voltages and currents in the following circuit.

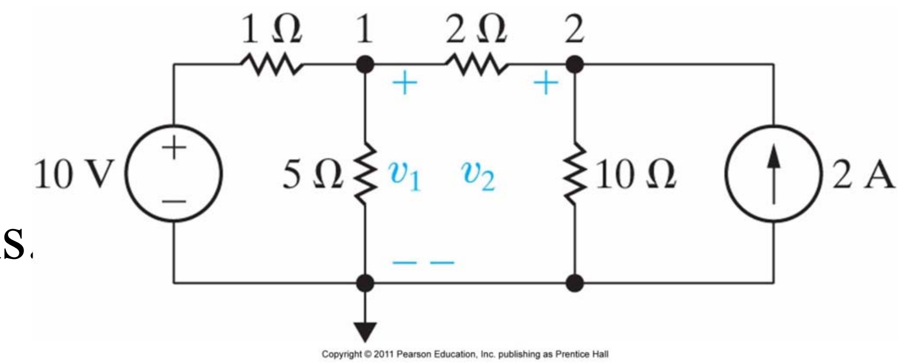


Since four circuit elements connect at the “bottom” node, I will choose that node as the reference node.



Note that we only needed to define two voltages! With these two defined voltages and the two independent sources, we can fully describe the circuit.

Node-Voltage Method



Now we can write the node equations.

At node 1:

$$\frac{v_1 - 10V}{1\Omega} + \frac{v_1}{5\Omega} + \frac{v_1 - v_2}{2\Omega} = 0$$

At node 2:

$$\frac{v_2 - v_1}{2\Omega} + \frac{v_2}{10\Omega} - 2A = 0$$

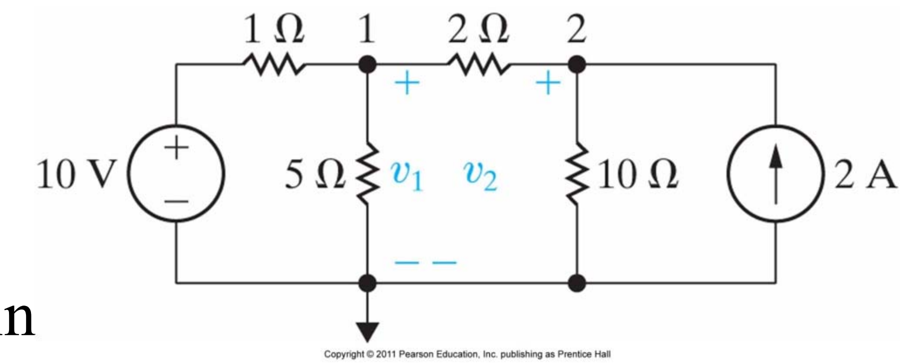
Note that I consistently wrote the equations so that the current leaves the node except for current sources which force a current direction.

$$(+)\frac{v_2 - v_1}{2\Omega}$$

The "+" comes from the passive sign convention.

The above equation assumes the voltage drop is right to left in the circuit and thus the current flow is from node 2 to node 1.

Node-Voltage Method



We now have a set of equations that will solve for all unknown voltages in the circuit.

$$\frac{v_1 - 10V}{1\Omega} + \frac{v_1}{5\Omega} + \frac{v_1 - v_2}{2\Omega} = 0 \qquad \frac{v_2 - v_1}{2\Omega} + \frac{v_2}{10\Omega} - 2A = 0$$

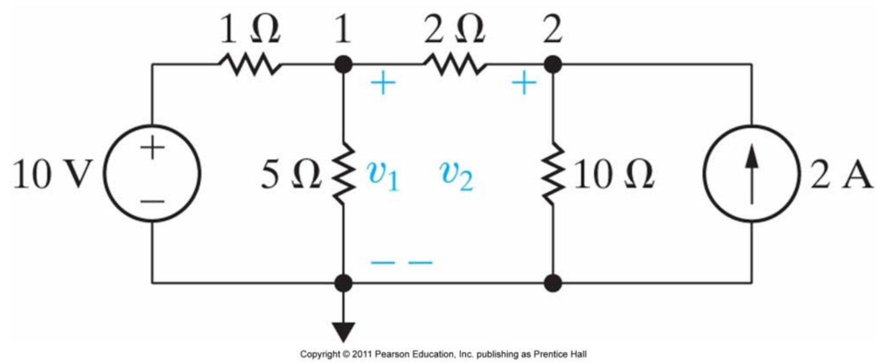
Which we can simplify:

$$v_1\left(\frac{1}{1\Omega} + \frac{1}{5\Omega} + \frac{1}{2\Omega}\right) - \frac{v_2}{2\Omega} = 10A \qquad v_1\left(\frac{-1}{2\Omega}\right) + v_2\left(\frac{1}{2\Omega} + \frac{1}{10\Omega}\right) = 2A$$

Clear the denominator by multiplying both sides by 10 Ω.

$$17v_1 - 5v_2 = 100V \qquad -5v_1 + 6v_2 = 20V$$

Node-Voltage Method



We have two equations and two unknown voltages and can thus solve for the unknowns.

$$17v_1 - 5v_2 = 100V$$

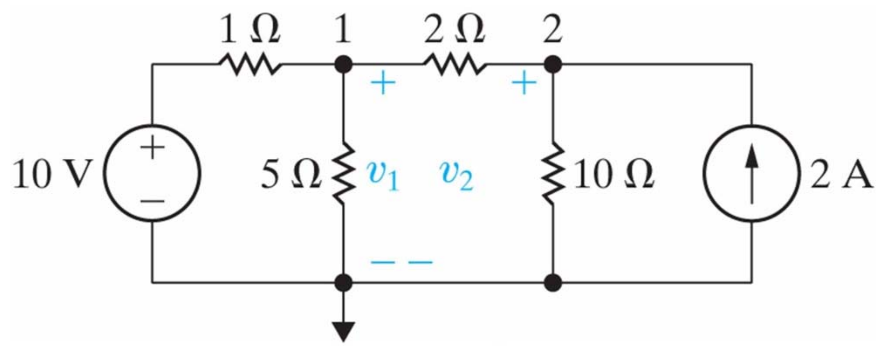
$$-5v_1 + 6v_2 = 20V$$

$$v_2 = \frac{20V + 5v_1}{6}$$

Thus

$$17v_1 - 5\left(\frac{20V + 5v_1}{6}\right) = 100V \quad \Rightarrow v_1\left(17 - \frac{25}{6}\right) = 100 + \frac{100}{6}V$$

Node-Voltage Method



$$v_1(12.8333) = 116.67V$$

Thus

$$v_1 = 9.09V$$

Now use $v_2 = \dfrac{20V + 5v_1}{6}$ to solve for v_2.

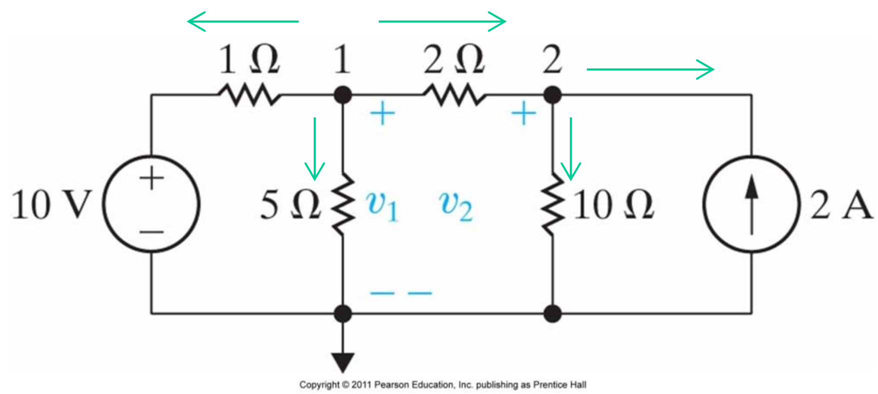
$$v_2 = \frac{20V + 5(9.09)}{6} = 10.909V$$

It is now straight forward to solve for every unknown current – a task I leave to you.

Node-Voltage Method

You may have found it difficult to accept writing the equations with all the currents leaving the nodes.

So let's try also defining the current directions "one time" and rework the problem.

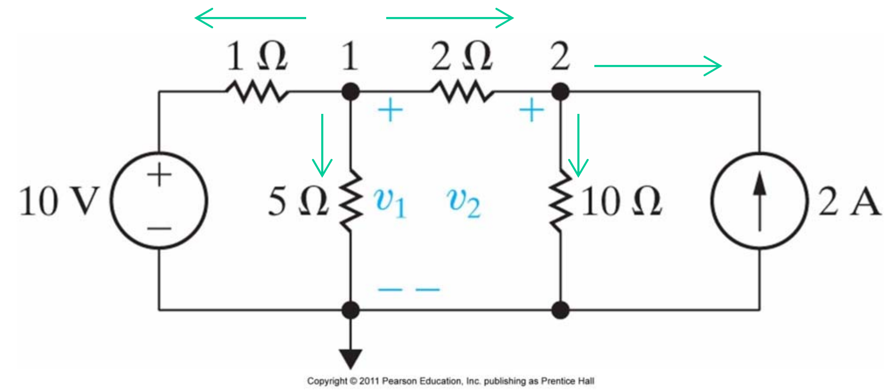


Once again, use KCL to write the equations at nodes 1 and 2 but this time obey the pre-defined current directions.

At node 1:

$$\frac{v_1 - 10V}{1\Omega} + \frac{v_1}{5\Omega} + \frac{v_1 - v_2}{2\Omega} = 0$$

Node-Voltage Method



At node 2:

$$(-)\frac{v_1 - v_2}{2\Omega} + \frac{v_2}{10\Omega} - 2A = 0$$

$$(+)\frac{v_2 - v_1}{2\Omega} + \frac{v_2}{10\Omega} - 2A = 0$$

Assume current "out"

$$\frac{v_1 - 10V}{1\Omega} + \frac{v_1}{5\Omega} + \frac{v_1 - v_2}{2\Omega} = 0$$

$$\frac{v_2 - v_1}{2\Omega} + \frac{v_2}{10\Omega} - 2A = 0$$

Use pre-defined current direction

$$\frac{v_1 - 10V}{1\Omega} + \frac{v_1}{5\Omega} + \frac{v_1 - v_2}{2\Omega} = 0$$

$$(+)\frac{v_2 - v_1}{2\Omega} + \frac{v_2}{10\Omega} - 2A = 0$$

Note that the two sets of equations are identical. Thus I choose to "assume current out" and not bother with pre-defining the current directions!

Node-Voltage Method

The Node-Voltage method can result in a large set of independent equations.

In exams for this course, the largest set will be for three unknown voltages.

Those of you who know linear algebra can see how the equations can be formed as matrices and readily solved for even larger systems.

Section 4.3
Node-Voltage Method
With Dependent Sources

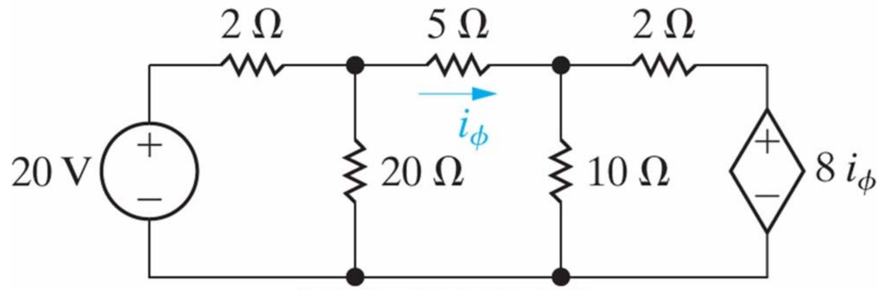
How to handle dependent sources

If the circuit contains dependent sources -

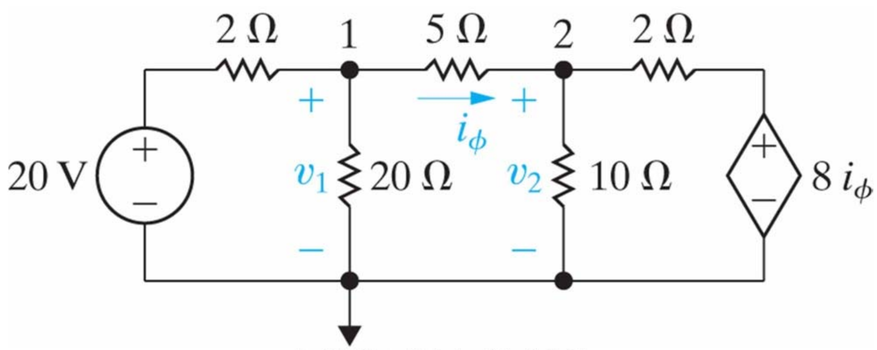
You must write a *constraint equation* for each dependent source.

Example 4.3

Use the node-voltage method.



First define your reference node and write all needed unknown voltages.



At node 1

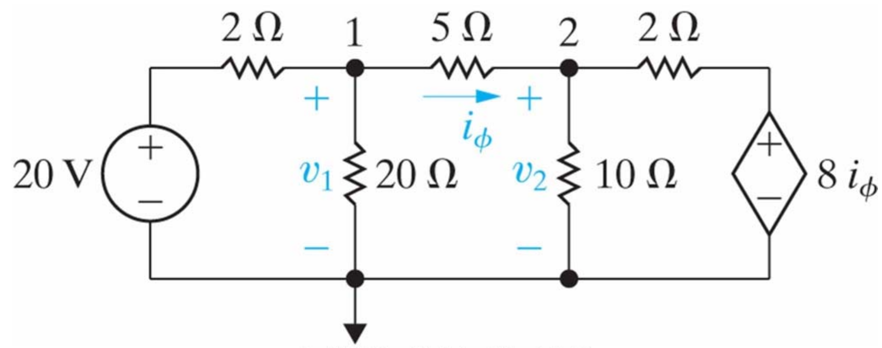
$$\frac{v_1 - 20V}{2\Omega} + \frac{v_1}{20\Omega} + \frac{v_1 - v_2}{5\Omega} = 0$$

At node 2

$$\frac{v_2 - v_1}{5\Omega} + \frac{v_2}{10\Omega} + \frac{v_2 - 8i_\phi}{2\Omega} = 0$$

Example 4.3

We have two equations already, but we have 3 unknowns.



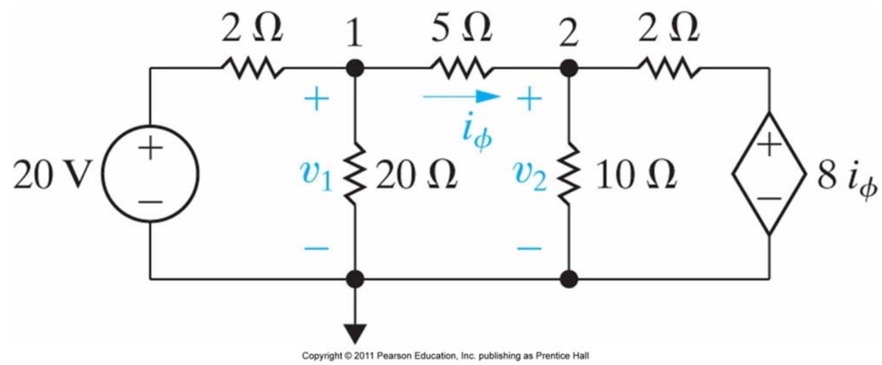
$$\frac{v_1 - 20V}{2\Omega} + \frac{v_1}{20\Omega} + \frac{v_1 - v_2}{5\Omega} = 0 \qquad \frac{v_2 - v_1}{5\Omega} + \frac{v_2}{10\Omega} + \frac{v_2 - 8i_\phi}{2\Omega} = 0$$

So we need another equation – which is the desired *constraint equation.*

Look at the figure. How is i_ϕ defined? $i_\phi = \dfrac{v_1 - v_2}{5\Omega}$

$$i_\phi = \frac{v_1 - v_2}{5\Omega} \Rightarrow v_1 - v_2 - i_\phi(5\Omega) = 0$$

Example 4.3



Write these three equations in *standard form.*

$$v_1(15) + v_2(-4) + i_\phi(0) = 200\text{ V}$$
$$v_1(-2) + v_2(8) + i_\phi(-40) = 0$$
$$v_1(1) + v_2(-1) + i_\phi(-5) = 0$$

Solving in Matlab, the three unknowns are:

$V_1 = 16$ V $\quad V_2 = 10$ V $\quad i_\phi = 1.2$ A

Section 4.5
Mesh-Current Method

Mesh-Current Method

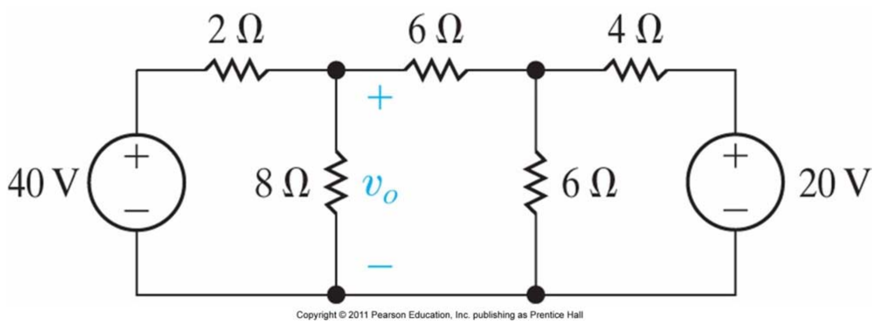
Another way to create the necessary equations to solve a circuit is to write the voltages around a path.

This is a variation of *KVL* where the equations are voltages around a *mesh.*

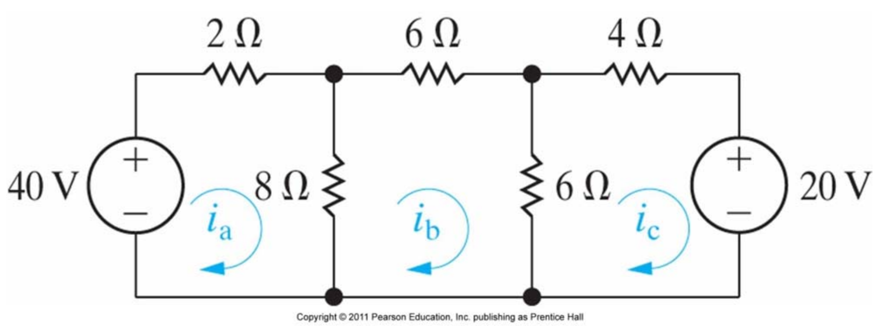
These equations usually written in terms of currents and other circuit values (which we will later call impedances).

You start by defining a *mesh current* for each loop.

Example 4.4



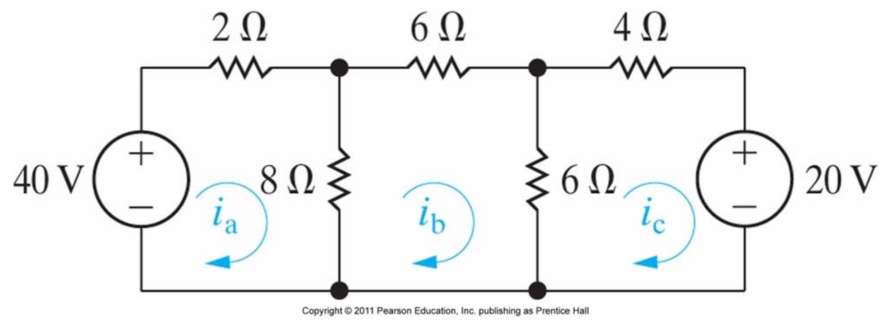
There are three loops in this circuit which meet the mesh definition. One possible choice of mesh currents is



We will now write the three sets of equations.

You MUST take care and obey the reference current direction in each mesh.

Example 4.4



For mesh a:

$$-40V + i_a(2\Omega) + i_a(8\Omega) - i_b(8\Omega) = 0$$

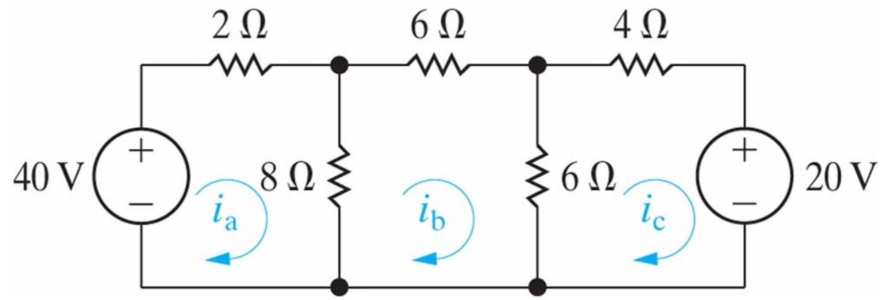
For mesh b:

$$-i_a(8\Omega) + i_b(8\Omega) + i_b(6\Omega) + i_b(6\Omega) - i_c(6\Omega) = 0$$

For mesh c:

$$-i_b(6\Omega) + i_c(6\Omega) + i_c(4\Omega) + 20V = 0$$

Example 4.4



The equations become easier to solve if written in as follows:

$$-40V + i_a(2\Omega) + i_a(8\Omega) - i_b(8\Omega) = 0$$

$$\Rightarrow i_a(10\Omega) + i_b(-8\Omega) + i_c(0\Omega) = 40V$$

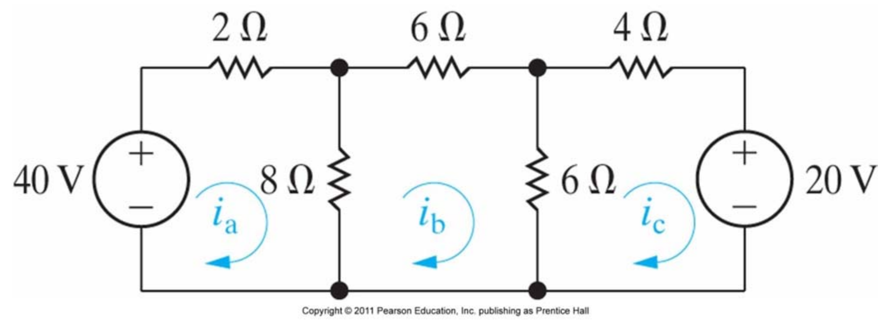
$$-i_a(8\Omega) + i_b(8\Omega) + i_b(6\Omega) + i_b(6\Omega) - i_c(6\Omega) = 0$$

$$\Rightarrow i_a(-8\Omega) + i_b(20\Omega) + i_c(-6\Omega) = 0$$

$$-i_b(6\Omega) + i_c(6\Omega) + i_c(4\Omega) + 20V = 0$$

$$\Rightarrow i_a(0\Omega) + i_b(-6\Omega) + i_c(10\Omega) = -20V$$

Example 4.4



These equations can be solved by algebra and substitution.

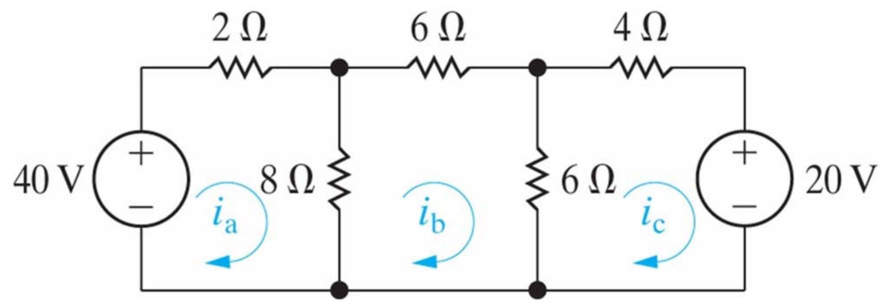
$$\Rightarrow i_a(10\Omega) + i_b(-8\Omega) + i_c(0\Omega) = 40V$$

$$\Rightarrow i_a(-8\Omega) + i_b(20\Omega) + i_c(-6\Omega) = 0$$

$$\Rightarrow i_a(0\Omega) + i_b(-6\Omega) + i_c(10\Omega) = -20V$$

The math process is tedious and error prone but straight forward.

Example 4.4



$$\Rightarrow i_a(10\Omega) + i_b(-8\Omega) + i_c(0\Omega) = 40V$$

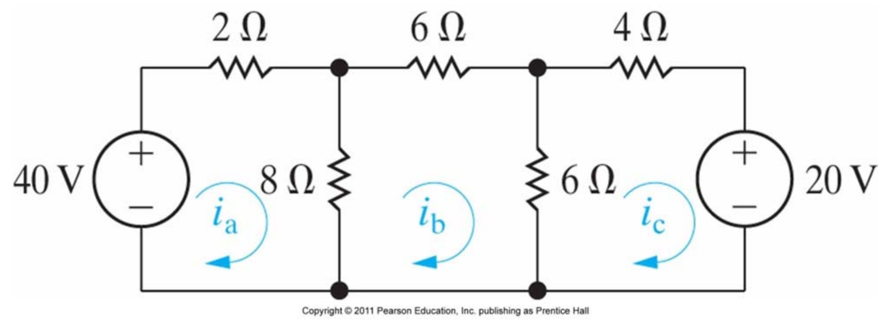
$$\Rightarrow i_a(-8\Omega) + i_b(20\Omega) + i_c(-6\Omega) = 0$$

$$\Rightarrow i_a(0\Omega) + i_b(-6\Omega) + i_c(10\Omega) = -20V$$

These equations can also be written in matrix form:

$$\begin{pmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} 40 \\ 0 \\ -20 \end{pmatrix}$$

Example 4.4



$$\begin{pmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} 40 \\ 0 \\ -20 \end{pmatrix}$$

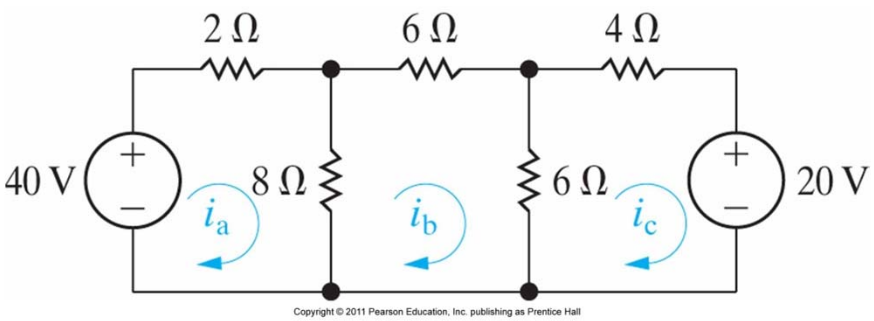
With this form and then using Matlab, the system is in the form A*C = B

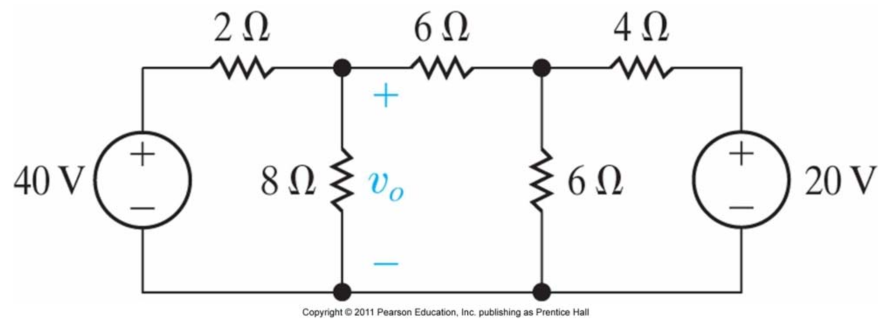
A=[10 -8 0; -8 20 -6; 0 -6 10]
B = [40; 0; -20]
C = A \ B

Where Matlab calculates the solution as

$i_a = 5.6$ A
$i_b = 2.0$ A
$i_c = -0.8$ A

Example 4.4





The mesh current solution is:

$i_a = 5.6$ A

$i_b = 2.0$ A

$i_c = -0.8$ A

Then

$$v_0 = (8\Omega)(i_a - i_b) = (8\Omega)(5.6 - 2) = (8\Omega)(3.6A) = 28.8V$$

$$P_{40V} = (-)(40\,\text{V})i_a = -40V(5.6A) = -224W$$

$$P_{20V} = (+)(20\,\text{V})i_c = 20V(-0.8A) = -16W$$

Mesh-Current Method – mesh versus branch currents

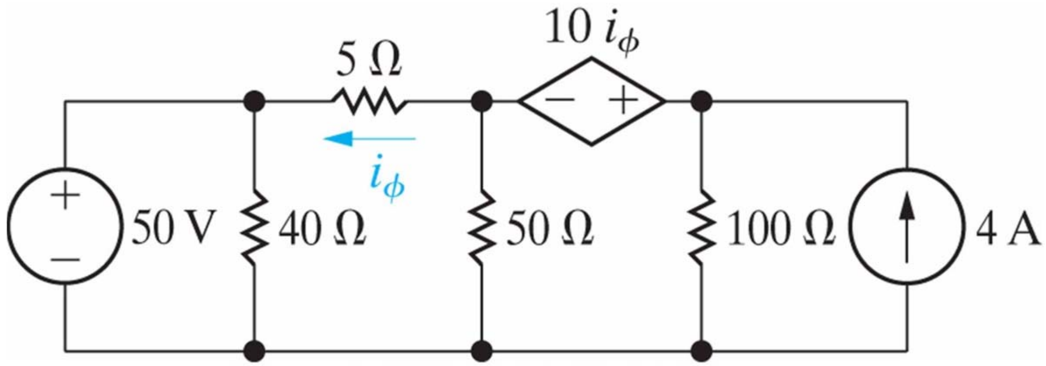
A special case for a mesh current is when a branch contains a current source.

If this branch current is between two meshes then we would usually form a *supermesh* (not covered in this course).

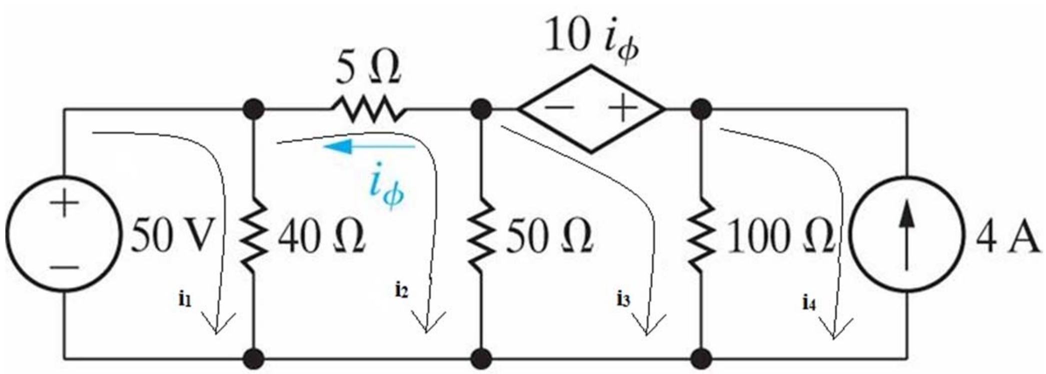
When the branch current is NOT between two meshes (on the "outside" of a mesh loop), then that current *constrains* the mesh current.

This mesh versus branch current constraint is due to the requirement of charge conservation – the current must be continuous for both the branch and the mesh current at that point in the circuit.

Example Figure 4.13_9ed (supernode example in the text – worked here as mesh)

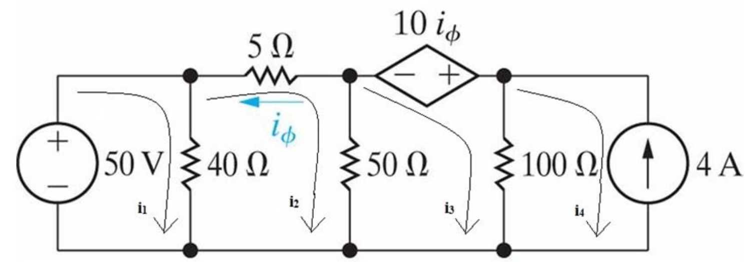


There are four loops in this circuit which meet the mesh definition. One possible choice of mesh currents is



Note that the right hand mesh (mesh 4) has an independent current source on the outside branch that constrains the mesh current!

Example Figure 4.13_9ed



Mesh current i_4 is in the opposite direction of the 4A current source. Thus we constrain the mesh current to

$$i_4 = -4A$$

The set of mesh equations is then

Mesh 1: $i_1(40\Omega) + i_2(-40) + i_3(0) + i_4(0) + i_\phi(0) = 50$

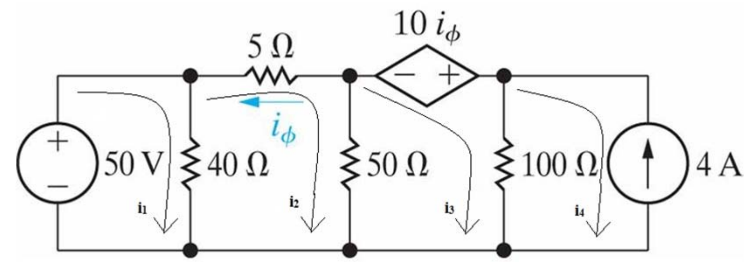
Mesh 2: $i_1(-40\Omega) + i_2(95) + i_3(-50) + i_4(0) + i_\phi(0) = 0$

Mesh 3: $i_1(0) + i_2(-50) + i_3(150) + i_4(-100) + i_\phi(-10) = 0$

Mesh 4: $i_1(0) + i_2(0) + i_3(0) + i_4(1) + i_\phi(0) = -4$

Constraint: $i_1(0) + i_2(1) + i_3(0) + i_4(0) + i_\phi(1) = 0$

Example Figure 4.13_9ed



The Matlab solutions are:

$$i_1 = -0.750A$$

$$i_2 = -2.000A$$

$$i_3 = -3.200A$$

$$i_4 = -4.000A$$

$$i_\phi = 2.000A$$

Section 4.9
Source Transformations

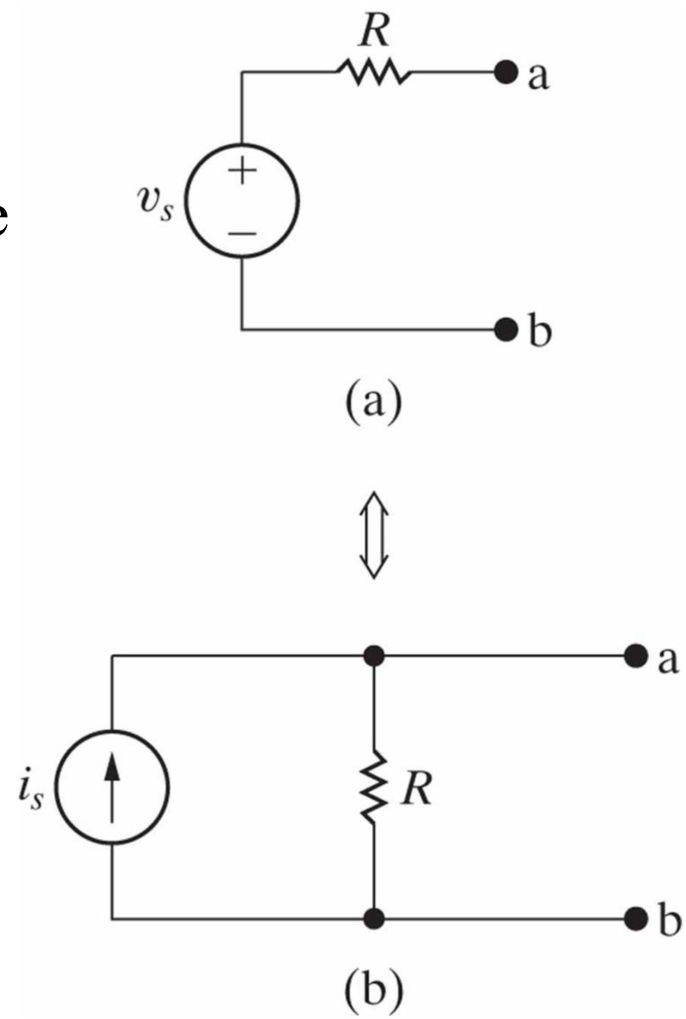
Source transformation

In circuit simplification, it is often useful to transform a series sub-circuit into an equivalent parallel version and vice versa.

A voltage source in series with a resistor can be also represented as a current source with a parallel resistance.

These two circuits appear the same at their terminals ab with the following defining equation.

$$i_s = \frac{v_s}{R}$$



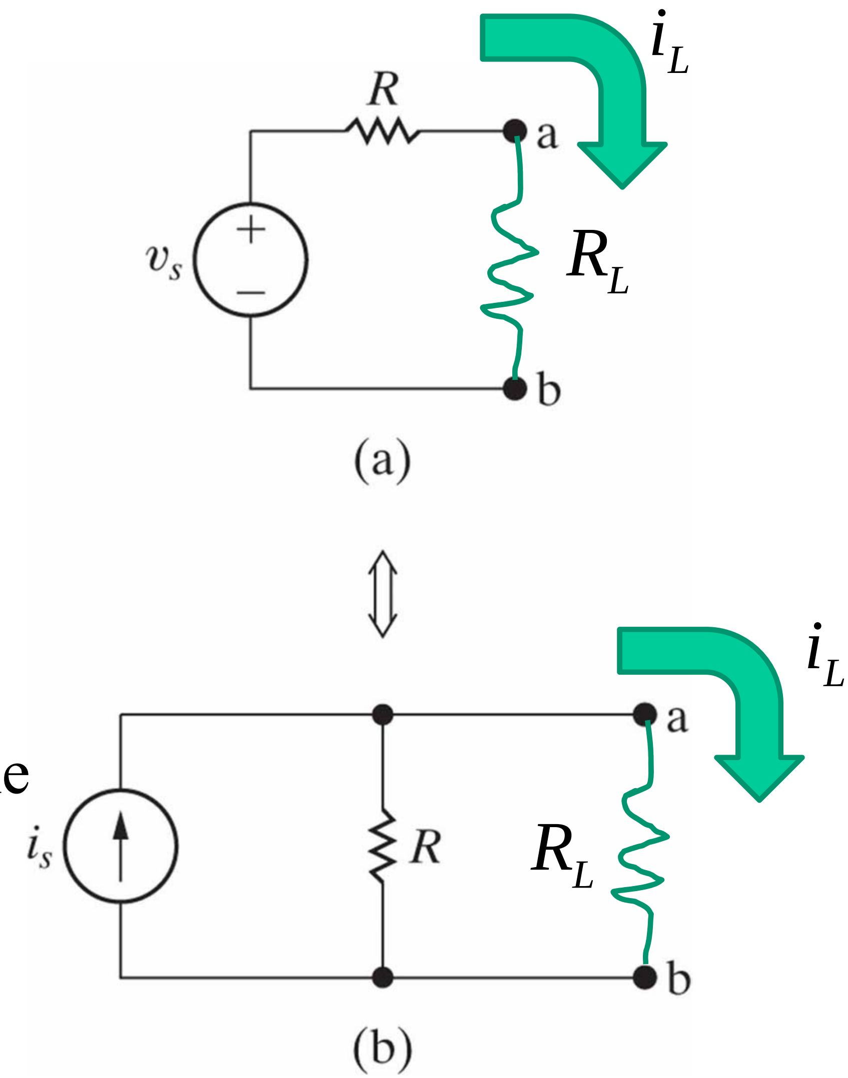
Source transformation

$$i_s = \frac{v_s}{R}$$

Proof: Connect in a resistor across terminals ab in each circuit.

For the voltage source circuit:

$$i_s = \frac{v_s}{R + R_L} = i_L$$



And for the current source circuit

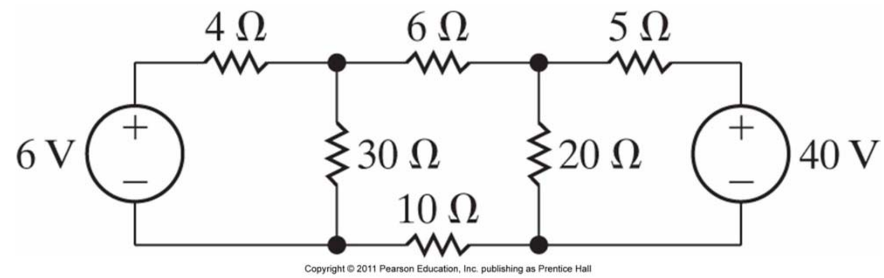
$$i_L = i_s \frac{R}{R + R_L}$$

Equate the two load currents to assert the equivalence!

$$i_s \frac{R}{R + R_L} = \frac{v_s}{R + R_L} \quad \Rightarrow i_s = \frac{v_s}{R}$$

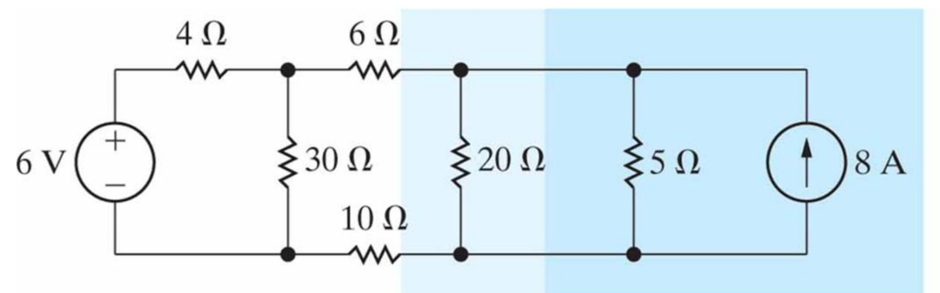
Example 4.8

What is the power associated with the 6V source?



Let's use source transformations from right to left on the circuit to find out.

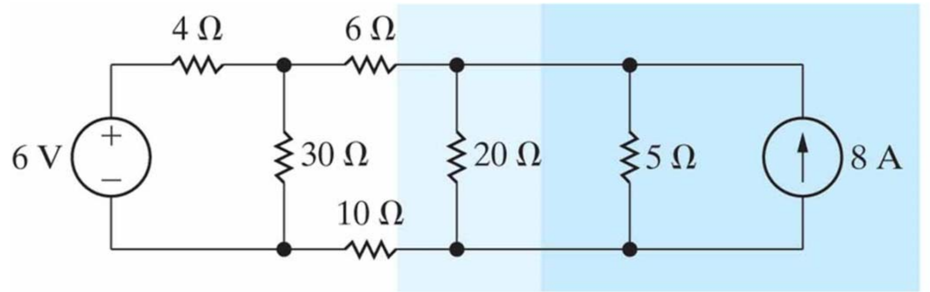
(1) Transform the 40 Volt source and 5 Ω resistor into a current source configuration.



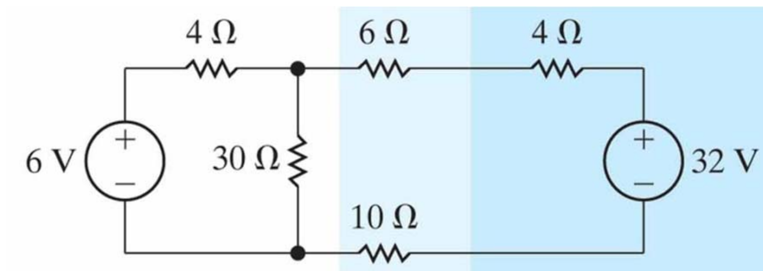
$$i_s = \frac{v_s}{R} = \frac{40V}{5\Omega} = 8A$$

Example 4.8

What is the power associated with the 6V source?



(2) Transform the 8A source in parallel with the 5 Ω and in parallel with the 20 Ω resistor.

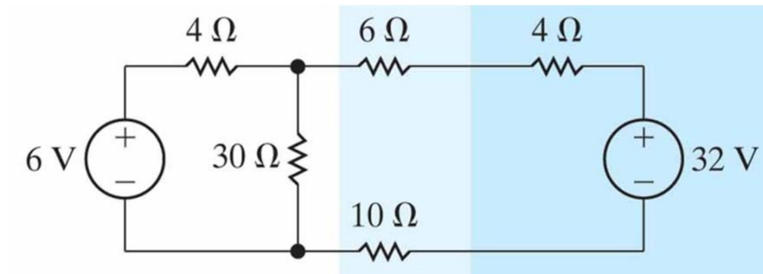


$$5 \,||\, 20 = \frac{5 \cdot 20}{5+20} = 4\Omega$$

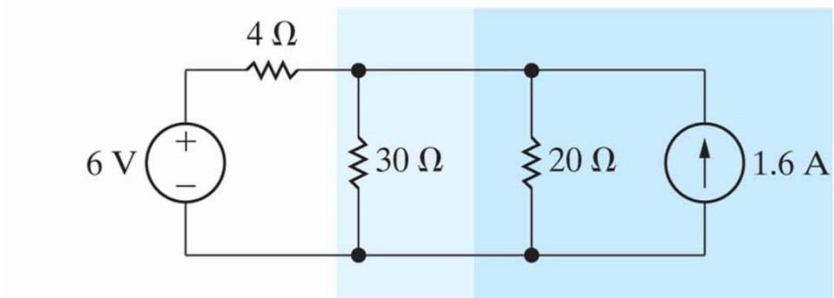
$$v_s = i_s R = (8A)(4\Omega) = 32V$$

Example 4.8

What is the power associated with the 6V source?



(3) Transform the 32V source and 6 Ω, 4 Ω and 10 Ω series resistors into a current source configuration.



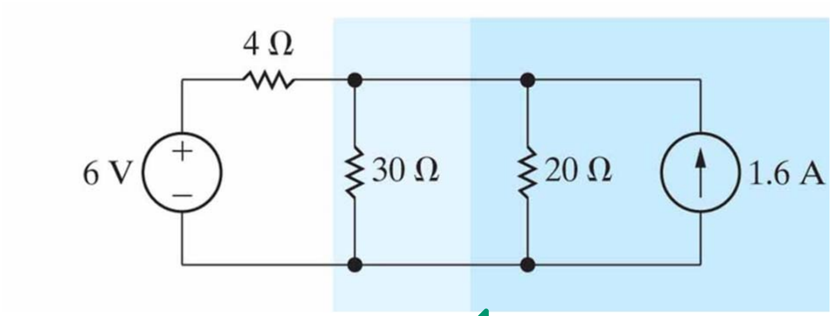
$$6+4+10=20\Omega$$

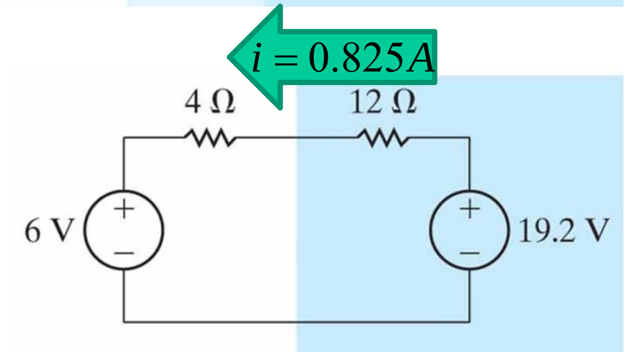
$$i_s=\frac{v_s}{R}=\frac{32V}{20\Omega}=1.6A$$

Remember that a series connection has the same current through each element even if on other sides of a voltage source.

Example 4.8

What is the power associated with the 6V source?





(4) Transform the 1.6A source and 30 Ω, and 20 Ω parallel resistors into a voltage source configuration.

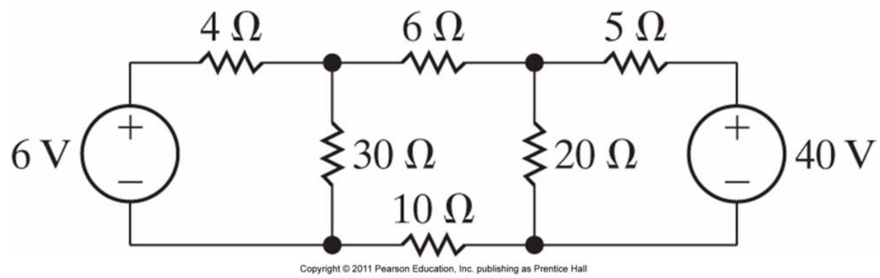
$$v_s = i_s R = (1.6A)(12\Omega) = 19.2V$$

$$30 \,||\, 20 = \left[\tfrac{1}{30} + \tfrac{1}{20}\right]^{-1} = 12\ \Omega$$

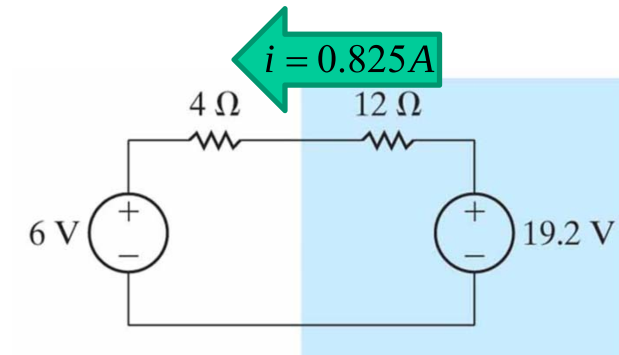
Now we can calculate the current in the circuit.

$$i = \frac{19.2 - 6}{4\Omega + 12\Omega} = \frac{13.2}{16\Omega} = 0.825A$$

Example 4.8



So what is the power associated with the 6V source?



$$P_{6V} = (\pm)vi$$

$$= (+)(6V)(0.825A)$$

$$= 4.95W\ (absorbed)$$

Section 4.10

Thévenin and Norton Equivalents

Terminal Behavior

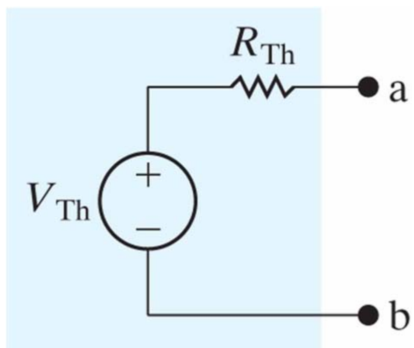
A very useful method is the examination of *terminal behavior* in a circuit.

Thévenin and Norton equivalents of a linear circuit is one way to create this terminal behavior view.

This is the view of the circuit’s total behavior without direct knowledge of the internal workings of the circuit.

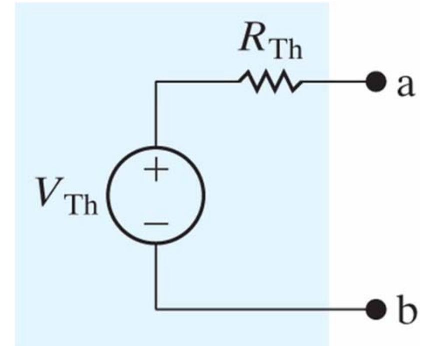
Thévenin Equivalent

If the circuit can be reduced to an independent voltage source and a series resistance – the equivalent circuit is called a Thévenin equivalent circuit.



This circuit is the equivalent of the actual circuit in the sense that a load across terminals ab will result in the same behavior as when the same load attached to the original circuit.

Thévenin Equivalent



To find the Thévenin equivalent we must:

Find the *Thévenin voltage* V_{Th}

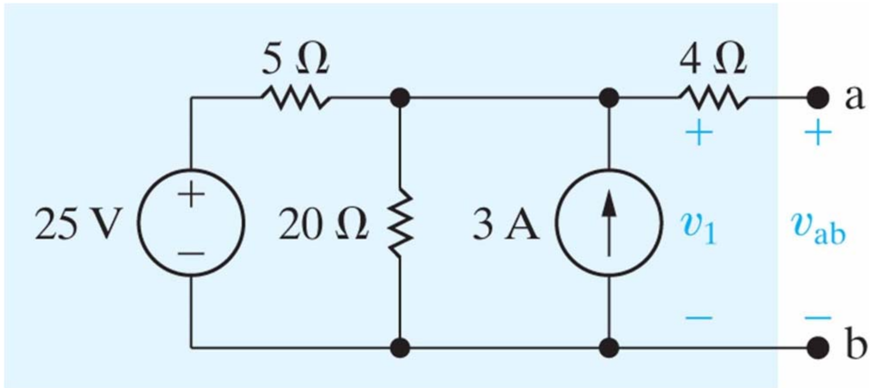
by finding the open circuit voltage at terminals ab

Find the *Thévenin resistance* R_{Th}.

by finding the equivalent resistance of the circuit as seen from terminals ab.

Finding the Thévenin Equivalent

Find the Thévenin equivalent of the following circuit:

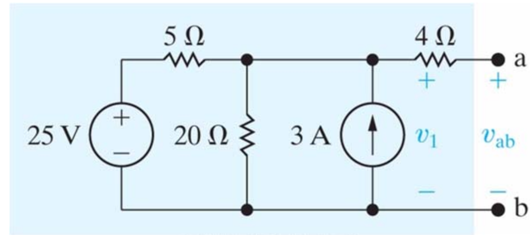


Find open circuit voltage at terminals ab = v_{ab}.

There is no current through the 4 Ω resistor when the terminals ab are open circuited. (Why?)

So now the task is to calculate the voltage across the 3A current source.

Finding the Thévenin Equivalent



By node analysis:

$$\frac{v_{3A} - 25V}{5\Omega} + \frac{v_{3A}}{20\Omega} - 3A = 0$$

$$v_{3A}\left(\frac{1}{5\Omega} + \frac{1}{20\Omega}\right) - \frac{25V}{5\Omega} - 3A = 0$$

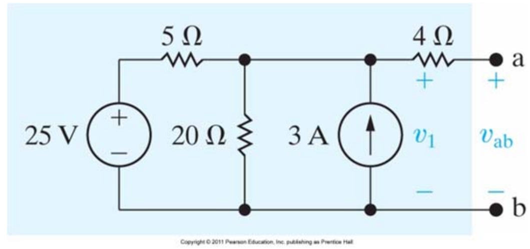
$$v_{3A}\left(\frac{1}{4}\mho\right) = 8A \qquad \Rightarrow v_{3A} = 32V$$

So the Thévenin voltage is

$$V_{Th} = v_{3A} = 32V$$

Finding the Thévenin Equivalent

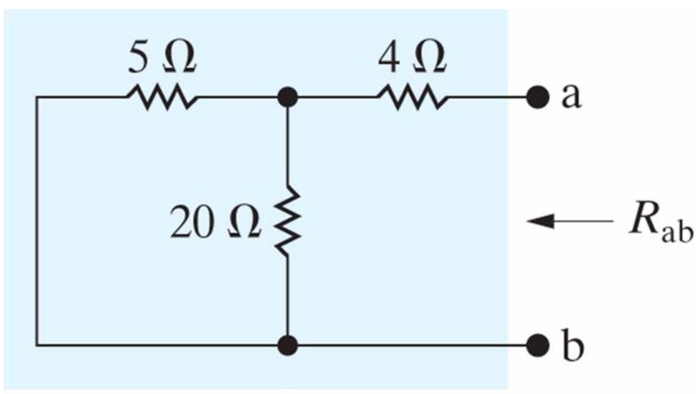
Now we need to find the Thévenin resistance.



One way to find the equivalent resistance of the circuit is to temporarily ignore the voltage and current source – i.e. *deactivate* the independent sources.

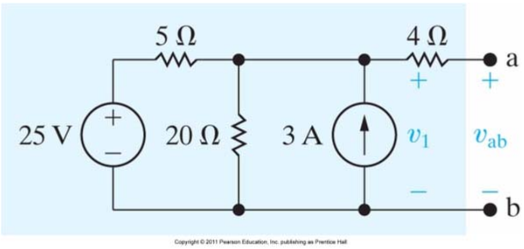
A *short circuited* voltage source equals zero volts which is the voltage goal.

A *open circuited* current source equals zero amps which is the current goal.

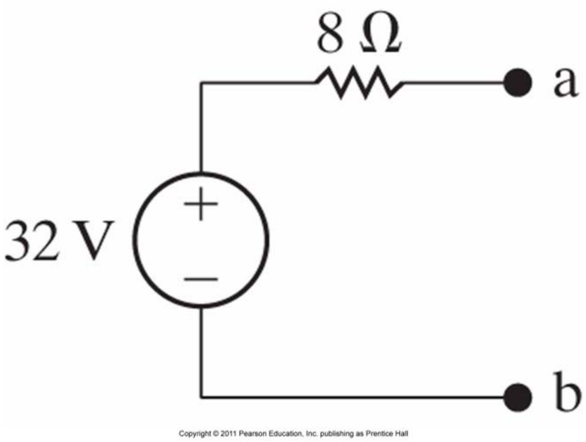


$$R_{Th} = R_{ab} = 5 \,||\, 20 + 4 = 8\Omega$$

Finding the Thévenin Equivalent

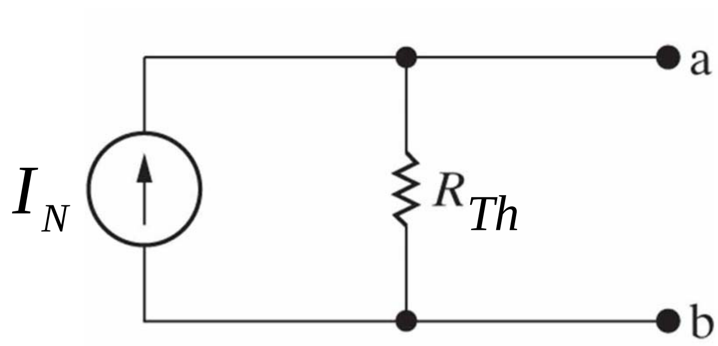


So the Thévenin equivalent is



Norton Equivalent

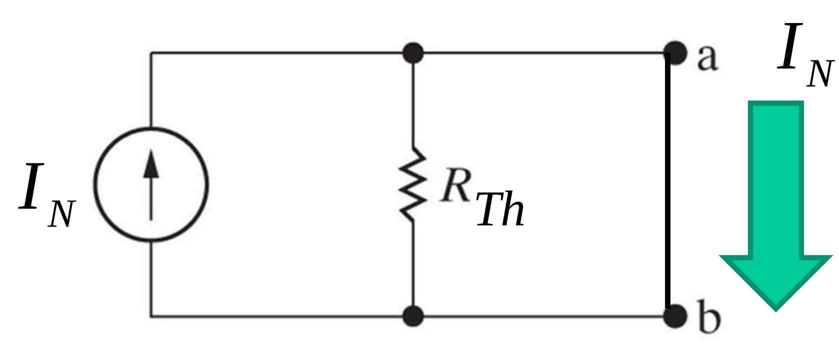
If the circuit can be reduced to an independent current source and a parallel resistance – the equivalent circuit is called a Norton equivalent circuit.



The Norton current is the *short circuit current* at the terminals ab.

$$I_N = \frac{V_{Th}}{R_{Th}}$$

Norton Equivalent



To find the Norton equivalent we must:

Find the *Norton current* I_N

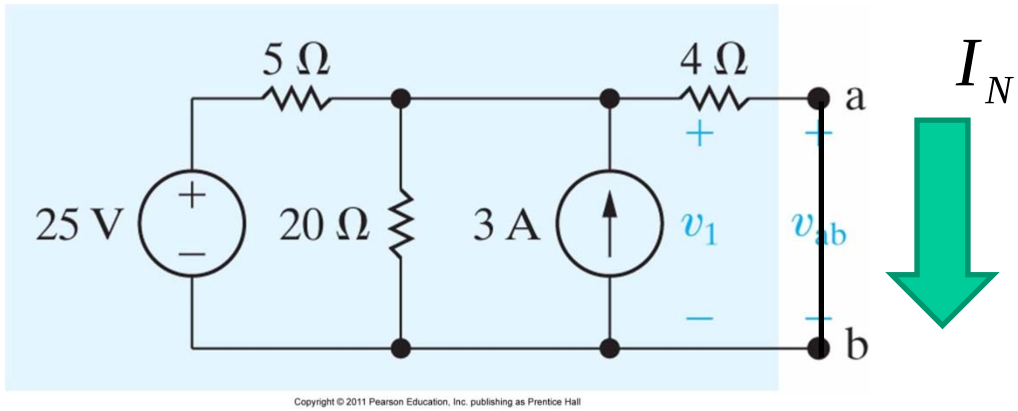
by finding the short circuit current at terminals ab

Find the *Thévenin resistance* R_{Th}.

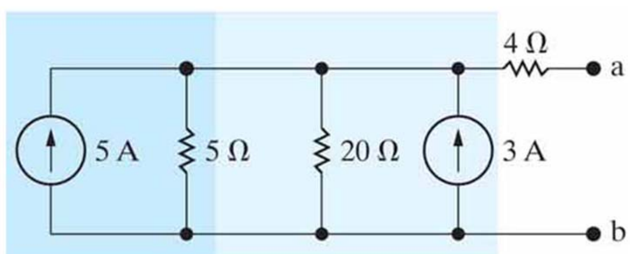
by finding the equivalent resistance of the circuit as seen from terminals ab.

Finding the Norton Equivalent

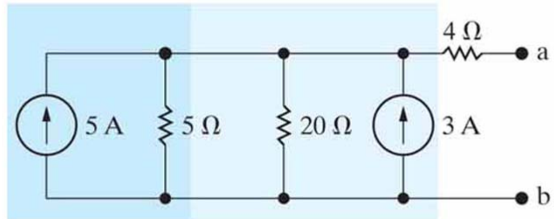
Find the Norton equivalent of the following circuit:



I chose to use a source transformation of the 25V source and the 5 Ω resistor.

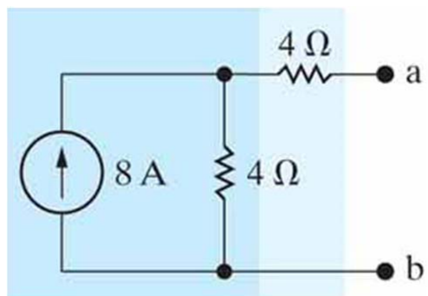


Finding the Norton Equivalent

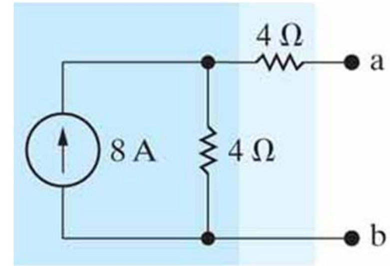


Then recognize that current sources feeding the same node can be summed together.

And we can find the parallel combination of the 5 Ω and 20 Ω resistors.

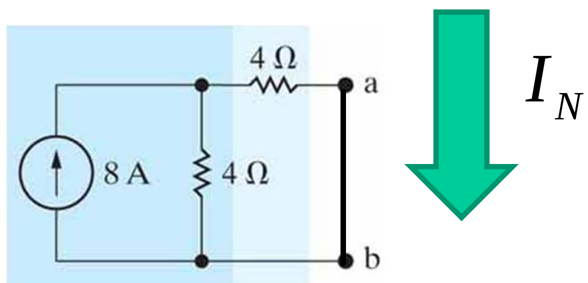


Finding the Norton Equivalent

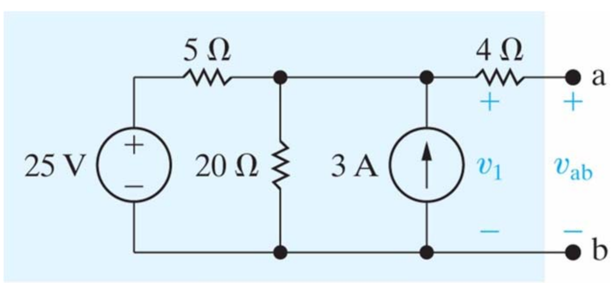


Finally we see the 8A current source feeds two identical valued resistors.

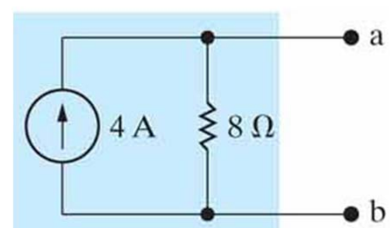
Thus $I_N = 4A$.



$R_{Th} = 8\ \Omega$ has already been found (in the previous problem)



Norton Equiv



Section 4.11
Finding the Thévenin Equivalent
Using a Test Source

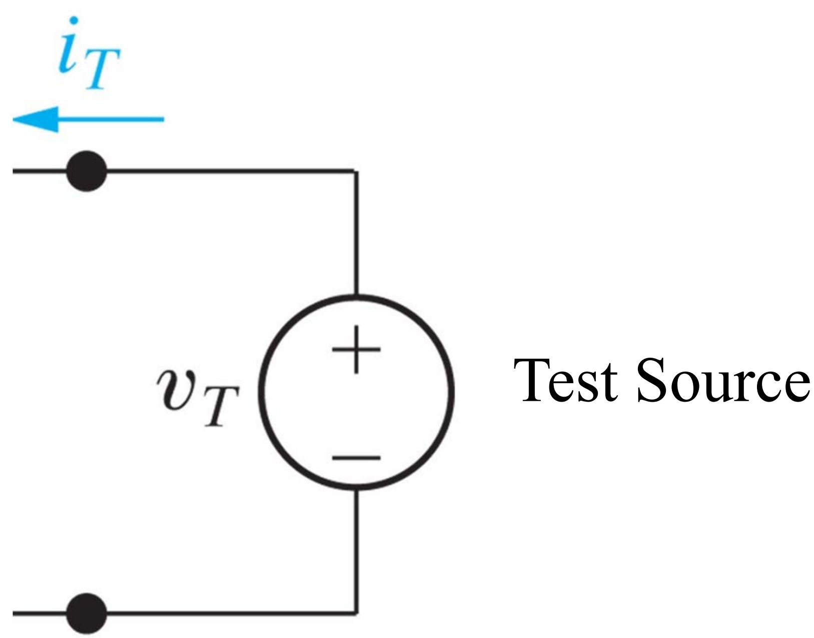
Finding the Thévenin Equivalent Using a Test Source

The Thévenin equivalent can be found from Ohm's Law.

$$R_{Th} = \frac{V_{Th}}{I_N}$$

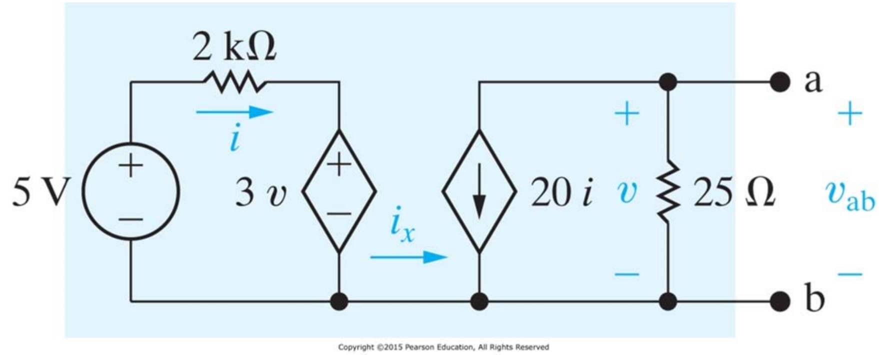
Both finding both V_{Th} and I_N can be a lot of work.

An alternate technique is to "apply" a Test Source to the terminals of the circuit.



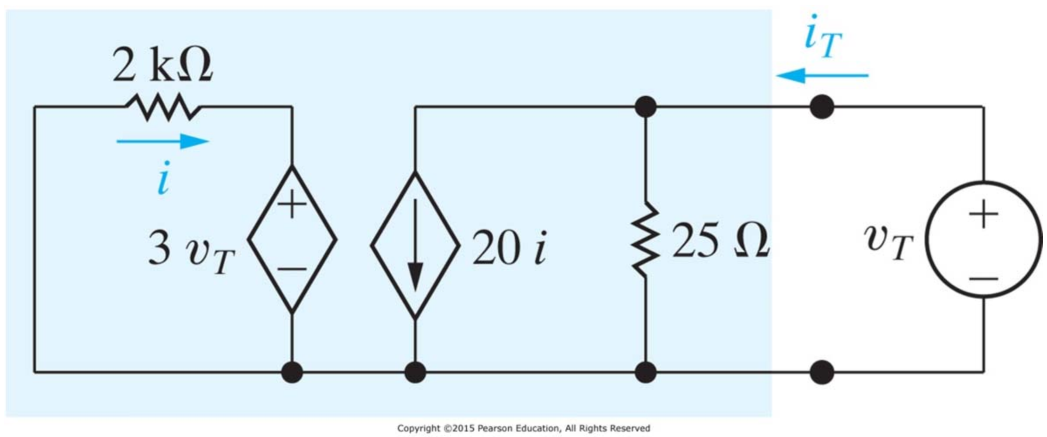
Example: Finding the Thévenin Equivalent Using a Test Source

Find the Thévenin equivalent of this circuit.



This example will employ the Test Source method.

First *deactivate* all independent sources and apply the test source.

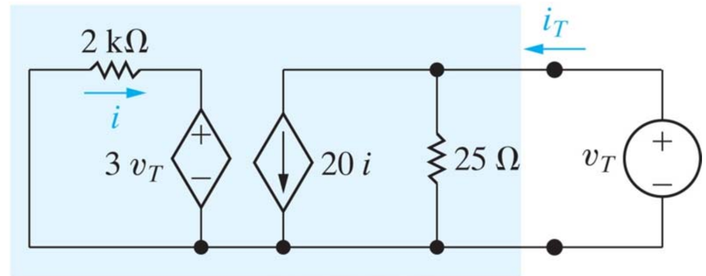


Note how the dependent voltage source is defined as the voltage across the 25 Ω resistance.

Example: Finding the Thévenin Equivalent Using a Test Source

The Thévenin equivalent resistance is

$$R_{Th} = \frac{v_T}{i_T}$$



KCL at the right hand side yields:

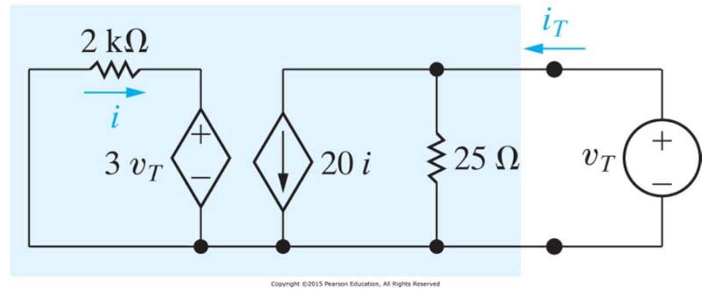
$$-i_T + \frac{v_T}{25\Omega} + 20i = 0$$

The current i from the left hand side is (carefully with defined current direction and defined voltage polarity).

$$i = \frac{-3v_T}{2k\Omega} = \frac{-3v_T}{2} mA$$

Example: Finding the Thévenin Equivalent Using a Test Source

Now solve for v_T/i_T



$$-i_T + \frac{v_T}{25\Omega} + 20i = 0 \qquad i = \frac{-3v_T}{2k\Omega}$$

$$-i_T + \frac{v_T}{25\Omega} + 20\left(\frac{-3v_T}{2k\Omega}\right) = 0 \qquad \Rightarrow v_T\left[\frac{1}{25\Omega} - \frac{60}{2k\Omega}\right] = i_T$$

$$\frac{v_T}{i_T} = \frac{1}{\dfrac{1}{25} - \dfrac{60}{2,000}} = \frac{1}{0.04 - 0.03} = \frac{1}{0.01} = 100\Omega = R_{Th}$$

Section 4.12

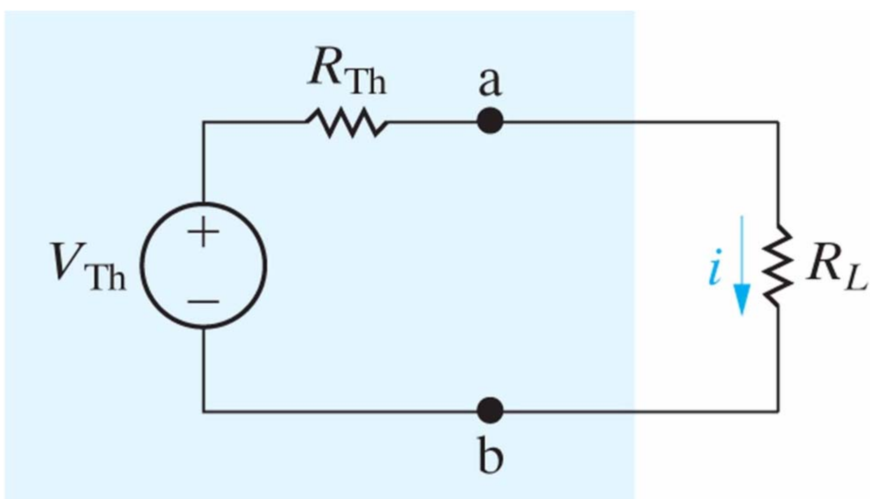
Maximum Power Transfer

Max Power Transfer - Derivation

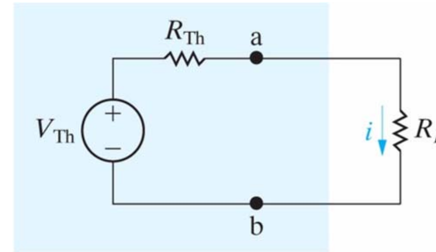
Often we want to match *impedances* (which is the more general form rather than being restricted to resistances) in a circuit so that we get the most possible power transferred to the load.

For instance in an audio amplifier matching the amplifier impedance to the load speaker's impedance results in the most sound out of the speaker and the least heat in the amplifier.

For our derivation we will use the following circuit:



Max Power Transfer - Derivation



The power delivered to R_L is

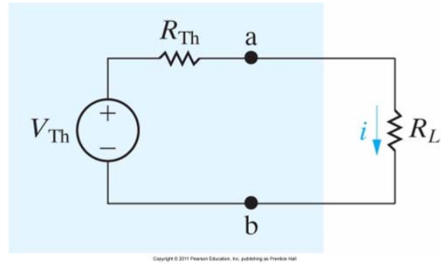
$$p = i^2 R_L \quad = \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 R_L$$

For a given circuit, V_{Th} and R_{Th} are fixed (i.e. a constant)

So we can find the maximum of the function by taking the derivative w.r.t. R_L and then set the derivative to zero.

$$\frac{dp}{dR_L} = V_{Th}^2 \frac{d}{dR_L} \left[\frac{R_L}{\left(R_{Th} + R_L\right)^2} \right]$$

Max Power Transfer - Derivation



$$\frac{dp}{dR_L} = V_{Th}^2 \frac{d}{dR_L} \frac{R_L}{\left(R_{Th} + R_L\right)^2}$$

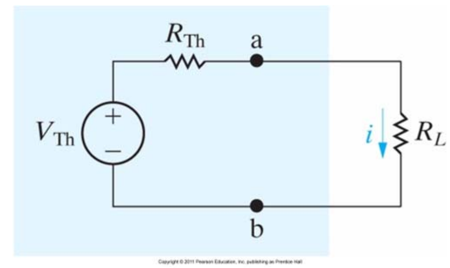
The derivative is (using the quotient rule)

$$= V_{Th}^2 \left[\frac{\left(R_{Th} + R_L\right)^2 - R_L \cdot 2\left(R_{Th} + R_L\right)}{\left(R_{Th} + R_L\right)^4} \right]$$

$$= V_{Th}^2 \left[\frac{\left(R_{Th} + R_L\right) - 2R_L}{\left(R_{Th} + R_L\right)^3} \right]$$

The derivative will be zero only when the numerator = zero, thus

Max Power Transfer - Derivation



Set the numerator = zero

$$\left(R_{Th} + R_L\right) - 2R_L = 0$$

$$R_{Th} + R_L - 2R_L = 0$$

$$R_{Th} - R_L = 0$$

$$R_{Th} = R_L$$ The max power occurs when $R_{Th} = R_L$!

Use the last result to find the maximum power delivered to R_L.

$$p = i^2 R_L = \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 R_L = \left[\frac{V_{Th}}{2R_L}\right]^2 R_L = \frac{V_{Th}^2}{4R_L}$$

Section 4.13
Superposition

Superposition

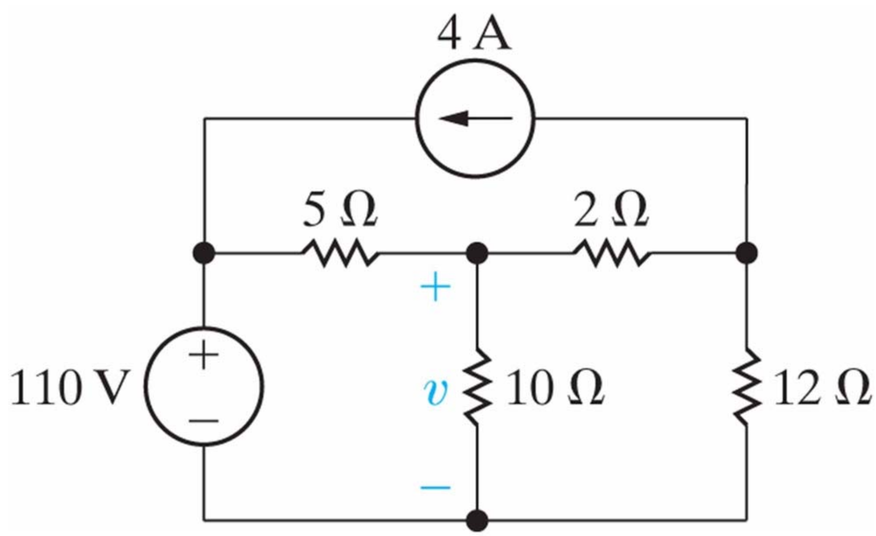
When a system is *linear* we can use the principal of *superposition*.

Superposition – the total response of a system is the sum of all of the responses from each independent source applied individually.

This is a very powerful technique, particularly when the circuit is driven by sources of different *frequency*.

Superposition example – P4.91

Use superposition to find v in the figure.

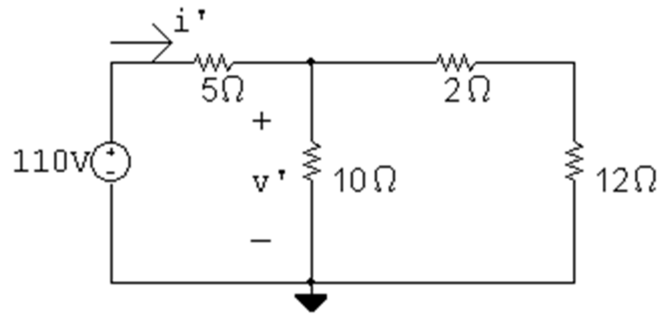


We will start by deactivating the current source and find the response due to the 110V voltage source.

Remember – a current source that is *open circuited* has zero current flow – i.e. deactivated.

Superposition example – P4.91

The new circuit is



The right side equivalent resistance is 10Ω ‖ 2+12Ω = 5.83 Ω.

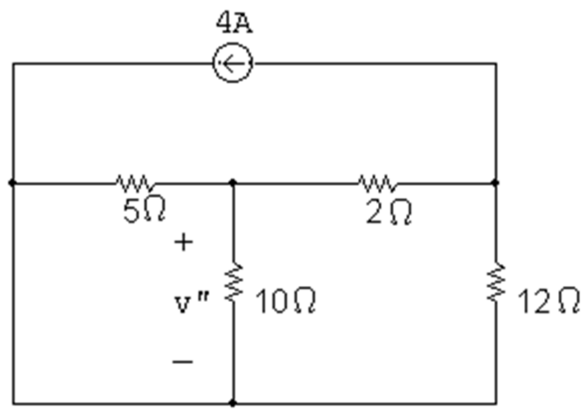
The voltage v′ can be found by voltage division:

$$v' = 110V \frac{5.83}{5+5.83} = 59.23V$$

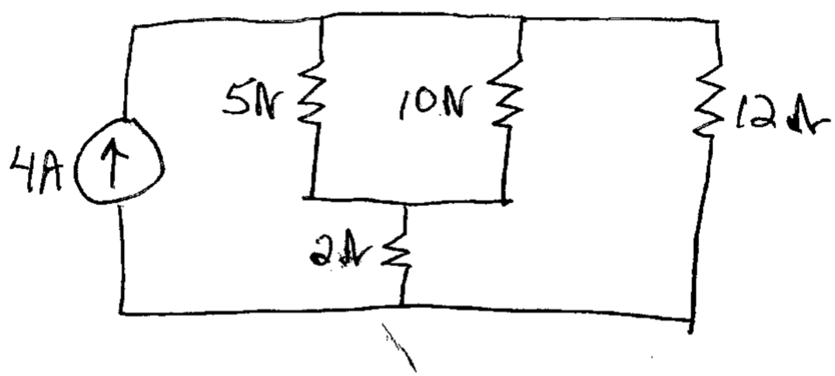
Now we will find the response due to the current source.

Superposition example – P4.91

The new circuit is

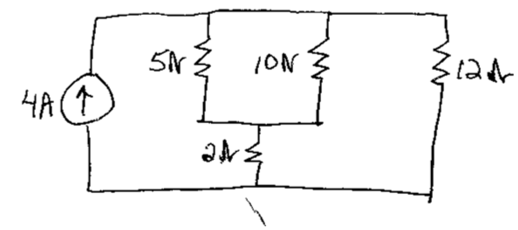


You might want to redraw the circuit to see the topology better.



We need to find the current through the 10 Ω resistor in order to find the voltage across it.

Superposition example – P4.91



The "left side" resistance is

$$R' = 2 + 5 \| 10 = 2 + 3.33 = 5.33\Omega$$

The current through this "left side" can be found by current division:

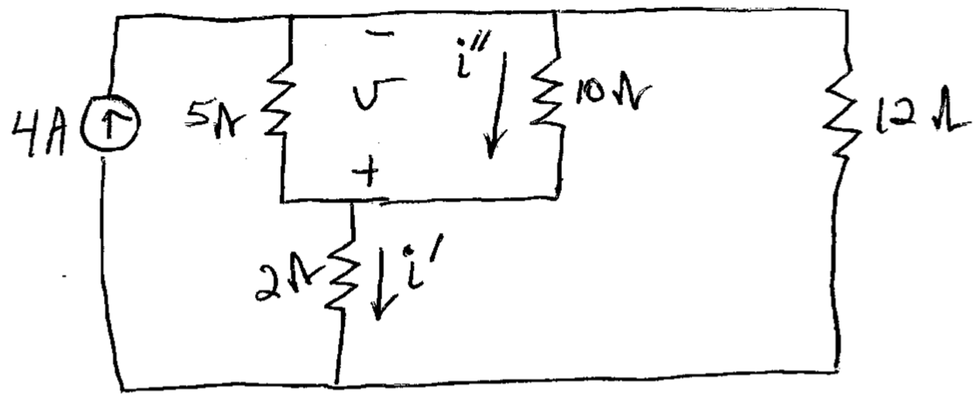
$$i' = 4A\left(\frac{12}{12 + 5.33}\right) = 2.77A$$

This is the current through the 2 Ω resistor – work backwards to find the current through the 10 Ω resistor (another current divider).

$$i'' = 2.77A\left(\frac{5}{5 + 10}\right) = 0.923A$$

Superposition example – P4.91

Now we need to check current direction versus the original defined voltage.

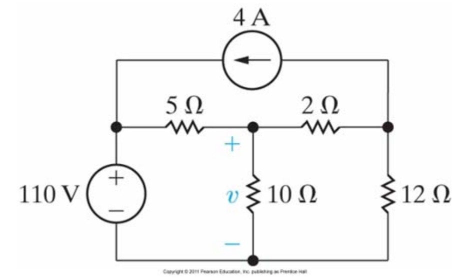


The voltage created by this current source is negative w.r.t. to the original voltage definition.

$$v'' = -i'' \cdot 10\Omega$$

$$= (-0.923A) \cdot 10\Omega = -9.23V$$

Superposition example – P4.91



The voltage v is the sum of the two responses:

$$v = v' + v''$$

$$= 59.23 - 9.23V = 50V$$

This is not necessarily the quickest way to find the total response in a circuit with only dc sources and resistive circuits.

Chapter 4

Techniques of Circuit Analysis

Text: *Electric Circuits*, 9^{th} Edition, by J. Nilsson and S. Riedel
Prentice Hall

Engr 17 Introductory Circuit Analysis
Instructor: Russ Tatro