

HW 2

For questions 2-4 refer to your textbook. Recall that the textbook uses the algebraic format for unions and intersections, i.e. $(A \cup B) \Rightarrow (A+B)$, and $(A \cap B) \Rightarrow (AB)$. You can use any format for this homework.

- 1) A missile can be accidentally launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.01 and 0.03 respectively. It is also known that B is more likely to fail (probability 0.06) if A has failed.
 - a. What is the probability of an accidental missile launch?
 - b. What is the probability that A will fail if B has failed?
 - c. Are the events " A fails" and " B fails" statistically independent?
- 2) Problem 2.9.
- 3) Problem 2.10.
- 4) Problem 2.12.

$$\begin{aligned}
 1. \quad a. \quad P(\text{launch}) &= P(A_{\text{fail}} \cap B_{\text{fail}}) \\
 &= P(B_{\text{fail}} | A_{\text{fail}}) P(A_{\text{fail}}) \\
 &= (0.06)(0.1) \\
 &= 0.0006
 \end{aligned}$$

$$\begin{aligned}
 b. \quad P(A_{\text{fail}} | B_{\text{fail}}) &= P(A_{\text{fail}} \cap B_{\text{fail}}) / P(B_{\text{fail}}) \\
 &= 0.0006 / 0.03 \\
 &= 0.02
 \end{aligned}$$

$$\begin{aligned}
 c. \quad P(A_{\text{fail}})P(B_{\text{fail}}) &\stackrel{?}{=} P(A_{\text{fail}} \cap B_{\text{fail}}) \\
 0.01(0.03) &\stackrel{?}{=} 0.0006 \\
 0.0003 &\neq 0.0006 \quad \therefore \text{They are not independent}
 \end{aligned}$$

$$2. \quad a. \quad \Pr[A + B] = \Pr[A] + \Pr[B] = 1/4 + 1/3 = 7/12$$

$$\begin{aligned}
 b. \quad \Pr[A + B] &= \Pr[A] + \Pr[B] - \Pr[A] \Pr[B] \\
 &= 1/4 + 1/3 - (1/4)(1/3) = 6/12 = 1/2
 \end{aligned}$$

c. It is not possible that all A, B, C, and D are mutually exclusive since it should be $\Pr[A + B + C + D] > 1$ which is not possible

$$\begin{aligned}
 3. \quad \Pr[\text{odd}] &= x & \sum \Pr[x] &= 1 \rightarrow 6(x) + 6(2x) = 1 \\
 \Pr[\text{Even}] &= 2x & 18x &= 1 \rightarrow x = 1/18
 \end{aligned}$$

$$a. \quad \Pr[A] = 6 \text{ odd} = 6x = 6(1/18) = 1/3$$

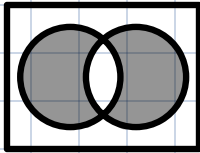
$$b. \quad \Pr[B] = 2 \text{ odd} + 3 \text{ even} = 2(x) + 3(2x) = 8x = 8(1/18) = 4/9$$

$$c. \quad \Pr[AB] = P(5) + P(7) = x + x = 2x = 2(1/18) = 1/9$$

4. let A = Hard drive fail
 B = Memory fail

$$\Pr[A] = 0.3 \quad \Pr[A \cap B] = 0.1$$
$$\Pr[B] = 0.2$$

(a.)



$$\Pr[A \cup B] - \Pr[A \cap B]$$
$$\Pr[A] + \Pr[B] - \Pr[A \cap B] - \Pr[A \cap B]$$
$$= 0.3 + 0.2 - 2(0.1)$$
$$= 0.3$$

(b) NO FAILURE = $1 - \Pr[A \cup B]$

$$= 1 - (\Pr[A] + \Pr[B] - \Pr[A \cap B])$$
$$= 1 - (0.3 + 0.2 - 0.1)$$
$$= 1 - 0.4$$
$$= 0.6$$