VIGOMAR KIM ALGADOR | EXAM 02

CSC28 – 01 22SU | 29 JUNE 2022

Q2. $f(x,y,z,t) = x'yz + x'yz't + y'zt' + xyzt' + xy'z't'$

$= x'yz(t+t') + x'yz't + (x+x')y'zt' + xyzt' + xy'z't'$

$= x'yzt + x'yzt' + x'yz't + xy'zt' + x'y'zt' + xyzt' + xy'z't'$

0111 0110 0101 1010 0010 1110 1000

xy \ zt	00	01	11	10
00				1
01		1	1	1
11				1
10	1			1

$f(x,y,z,t) = xy't' + x'yt + zt'$

Q3. $f(x,y,z) = \Sigma \min(7,3,2,0,1)$

$= (0,1,2,3,7)$

$= (000, 001, 010, 011, 111)$

x \ yz	00	01	11	10
0	1	1	1	1
1			1	

$f(x,y,z) = x' + yz$

Q4. (1) $f(x,y,z) = x'y'z' + x'y'z + xy'z' + xyz' + xyz$

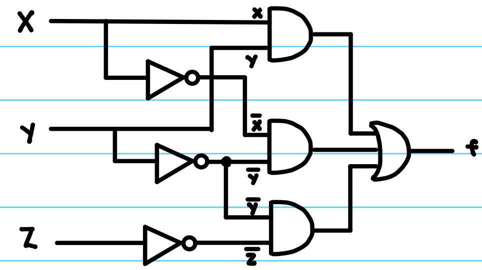
$= \Sigma(0,1,4,6,7)$

(2)

x \ yz	00	01	11	10
0	1	1		
1	1		1	1

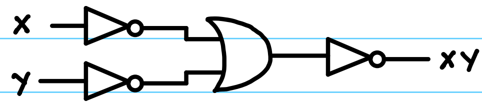
$f(x,y,z) = x'y' + y'z' + xy$

(3)



Q5. {OR, NOT} is a complete set

AND gate $\overline{\overline{x}+\overline{y}} = \overline{\overline{x}}\cdot\overline{\overline{y}} = xy$



Q6. Prove by induction:

$$P(n) = \frac{1}{(1\cdot 3)} + \frac{1}{(3\cdot 5)} + \frac{1}{(5\cdot 7)} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

STEP 1: $p(1)$: $\frac{1}{1\cdot 3} = \frac{1}{2(1)+1} \implies \frac{1}{3} = \frac{1}{3}$

STEP 2: $p(k)$: $\frac{1}{(1\cdot 3)} + \frac{1}{(3\cdot 5)} + \frac{1}{(5\cdot 7)} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$

STEP 3: $p(k+1)$: $\frac{1}{(1\cdot 3)} + \frac{1}{(3\cdot 5)} + \frac{1}{(5\cdot 7)} + \dots + \frac{1}{(2k-1)(2k+1)}$

$$+ \frac{1}{[2(k+1)-1][2(k+1)+1]} = \frac{k+1}{2(k+1)+1}$$

LHS: $\frac{1}{(1\cdot 3)} + \frac{1}{(3\cdot 5)} + \frac{1}{(5\cdot 7)} + \dots + \frac{1}{(2k-1)(2k+1)}$

$$+ \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$= \frac{k}{2k+1} + \frac{1}{[2(k+1)-1][2(k+1)+1]} = \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)} = \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)} = \frac{k+1}{2k+3} = \frac{k+1}{2k+2+1}$$

$$= \frac{k+1}{2(k+1)+1} \checkmark$$

Q7 Prove by contrapositive:

let $p : x^3$ is irrational

$q : x$ is irrational

$\sim q \longrightarrow \sim p$

x is rational $\longrightarrow x^3$ is rational

$$x = \frac{q}{b} \Rightarrow x^3 = \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3} = \frac{c}{d}$$

$x^3 = c/d$

$\sim q \longrightarrow \sim p$ is true

therefore $p \longrightarrow q$ is also true.

Q8 Proof by cases:

$\max(x,y) = (x+y+|x-y|)/2$ for all x and y in R.

case 1: $x > y$: $|x-y| = x-y$ $\qquad (x+y+x-y)/2 = x$

case 2: $x < y$: $|x-y| = -x+y$ $\qquad (x+y-x+y)/2 = y$

case 3: $x = y$: 0 $\qquad (x+y+0)/2$

$$\frac{x+x}{2} = x \qquad \frac{y+y}{2} = y$$