California State University, Sacramento
The College of Engineering and Computer Science

EEE 180 Signals & Systems

Midterm 2

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Unilateral Laplace Transform Table

	$f(t)$	$F(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$

Unilateral Z-transform Pair Table

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\frac{z}{z-\gamma}$

1.[25 points]

The discrete system equation and initial conditions are given below:

$$y[k+2] + \frac{3}{2}y[k+1] + \frac{1}{2}y[k] = 0, \qquad y[-1] = -3,\ \ y[-2] = 1.$$

Please find the system output response for the above discrete-time system by using the following three steps.

(1). What is the characteristic polynomial equation for the above system?

$\lambda^2 + 3/2\,\lambda + 1/2 = 0$ ✓

(2). What are the values of the two roots of the characteristic polynomial equation for this system?

$(\lambda+1)(\lambda+1/2)=0$

$\lambda = -1, -1/2$ ✓

(3). Find the output system response of this discrete time system.

$y[k] = B_1(-1)^k + B_2(-1/2)^k$ ✓

FOR $y[-1]=-3$:

$B_1(-1)^{-1} + B_2(-1/2)^{-1} = -3$

$-B_1 - 2B_2 = -3$

FOR $y[-2]=1$:

$B_1(-1)^{-2} + B_2(-1/2)^{-2} = 1$

$B_1 + 4B_2 = 1$ ✓

$\left.\begin{aligned}-B_1 - 2B_2 &= -3\\ B_1 + 4B_2 &= 1\end{aligned}\right]$ $\begin{aligned}B_1 &= 5\\ B_2 &= -1\end{aligned}$

$y[k] = 5(-1)^k - (-1/2)^k$ ✓

2. [35 points]

(1). Determine the Inverse Laplace transform of $F(s) = \frac{5}{s+3} + \frac{8}{s-4}$ by using the unilateral Laplace transform table.

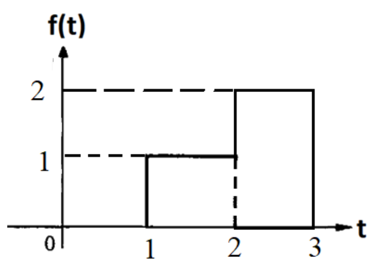
Your solution: f(t) = $(5e^{-3t} + 8e^{4t})u(t)$

(2). Determine the Inverse Laplace transform of $F(s) = 2 + \frac{2}{s^2}$ by using the unilateral Laplace transform table.

Your solution: f(t) = $2\delta(t) + 2t\,u(t)$

(3). Calculate Laplace transform $F(s) = \int_0^{\infty} f(t)\, e^{-st} dt$ of the following signal and find the region of convergence.

$$f(t) = \begin{cases} 1, & 1 \le t < 2 \\ 2, & 2 \le t < 3 \end{cases} .$$



$$\begin{aligned} F(s) &= \int_0^{\infty} f(t) e^{-st} dt \\ &= \int_0^1 0 e^{-st} dt + \int_1^2 1 e^{-st} dt + \int_2^3 2 e^{-st} dt \\ &= 0 + \left[-1/s\,(e^{-st})\right]_1^2 + \left[-2/s\,(e^{-st})\right]_2^3 \\ &= -1/s\,(e^{-2s} - e^{-s}) - 2/s\,(e^{-3s} - e^{-2s}) \\ &= -1/s\,(-e^{-s} - e^{-2s} + 2e^{-3s}) \\ &= 1/s\,(e^{-s} + e^{-2s} - 2e^{-3s}) \end{aligned}$$

Missing ROC x -2

3. [40 points]

(1). The discrete-time system is described by
y[k+1] + 2y[k] = f[k], with $f[k] = u[k]$ and $y[0] = 0$.
Solve the above equation iteratively to determine y[1] and y[2] values.

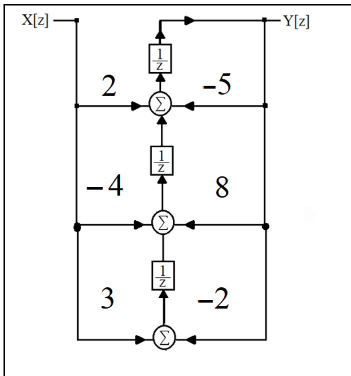
y[k+1] + 2y[k] = f[k]

when k=0:
y[1] + 2y[0] = f[0]
y[1] + 2(0) = u(0)
y[1] = 0 X

when k=1:
y[2] + 2y[1] = f[1]
y[2] + 2(0) = 1
y[2] = 1 X

-2

(2). The transformed direct form II structure is shown below.



The system transfer function is:

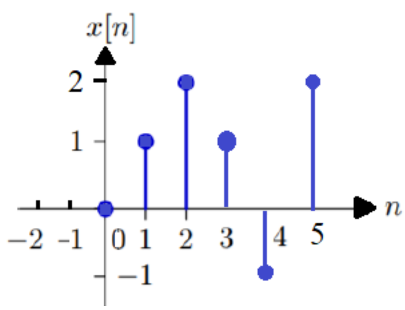
$$H[z] = \frac{Y[z]}{X[z]} = \frac{Az^{-1} + Bz^{-2} + C\,z^{-3}}{D + Ez^{-1} + Fz^{-2} + Gz^{-3}}$$

According to the structure on the left,

A=<u>2</u>, B=<u>-4</u>, C = <u>3</u>, D=<u>1</u>

E=<u>5</u>, F=<u>-8</u>, G=<u>2</u>.

(3). Find the z-transform for the following discrete-time signal.



$$X[z] = 0 + \frac{1}{z} + \frac{2}{z^2} + \frac{1}{z^3} - \frac{1}{z^4} + \frac{2}{z^5}$$
$$= z^{-1} + 2z^{-2} + z^{-3} - z^{-4} + 2z^{-5}$$

(4). Find the inverse z-transform of the following function with ROC: |z| > 4.

$$F[z] = \frac{z(z-3)}{z^2 - 6z + 8}$$

$$\frac{F[z]}{z} = \frac{\cancel{z}(z-3)}{z^2-6z+8} = \frac{A}{z-4} + \frac{B}{z-2}$$ -1

$$z-3 = A(z-2) + B(z-4)$$

when $z=2$: $-1 = A(0) + B(-2) \longrightarrow B = 1/2$

$z=4$: $1 = A(2) + B(0) \longrightarrow A = 1/2$

$$F[z] = \frac{1/2\, z}{z-4} + \frac{1/2\, z}{z-2}$$

$$f[k] = \frac{1}{2}[(4)^k + (2)^k]\,u[k]$$