CALIFORNIA STATE UNIVERSITY

SACRAMENTO

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EEE 117 Network Analysis

Text: Electric Circuits by J. Nilsson and S. Riedel Prentice Hall

Examples Set 4: Laplace Transform in Circuit Analysis

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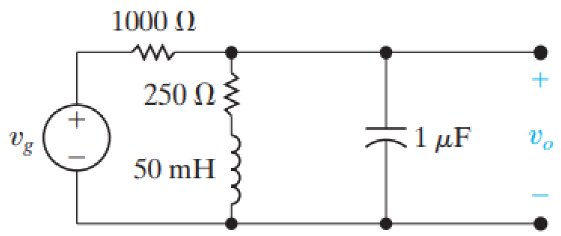
Ex-1: The circuit shown below is driven by a voltage source whose voltage increases linearly with time, namely $v_g = 50\ t$

a) Use the transfer function to find v_o.

b) Identify the transient component of the response.

c) Identify the steady-state component of the response.

d) Sketch v_o versus t for $0 \leq t \leq 1.5$ ms.



$$H(s) = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}.$$

The transform of the driving voltage is $50/s^2$; therefore, the s-domain expression for the output voltage is

$$V_o = \frac{1000(s + 5000)}{(s^2 + 6000s + 25 \times 10^6)} \frac{50}{s^2}.$$

The partial fraction expansion of V_o is

$$V_o = \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000} + \frac{K_2}{s^2} + \frac{K_3}{s}.$$

$$K_1 = 5\sqrt{5} \times 10^{-4} \underline{/79.70^\circ};$$

$$K_1^* = 5\sqrt{5} \times 10^{-4} \underline{/-79.70^\circ},$$

$$K_2 = 10,$$

$$K_3 = -4 \times 10^{-4}.$$

The time-domain expression for v_o is

$$v_o = [10\sqrt{5} \times 10^{-4} e^{-3000t} \cos(4000t + 79.70^\circ)$$

$$+ 10t - 4 \times 10^{-4}]u(t) \text{ V}.$$

b) The transient component of v_o is

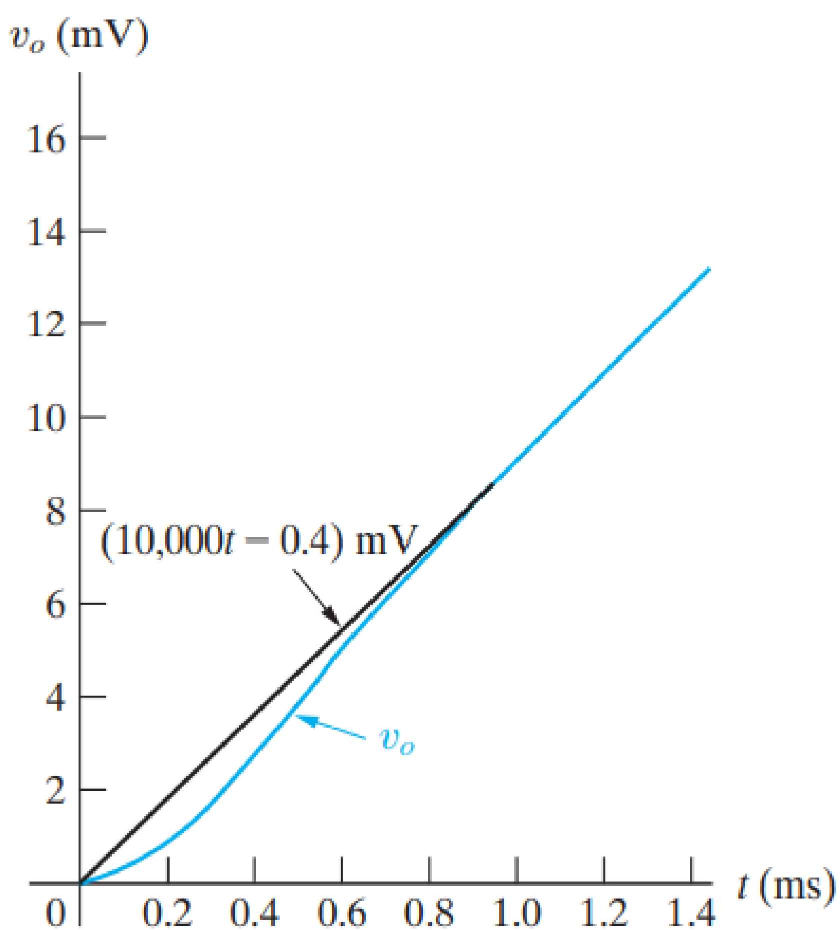
$$10\sqrt{5} \times 10^{-4} e^{-3000t} \cos(4000t + 79.70^\circ).$$

Note that this term is generated by the poles $(-3000 + j4000)$ and $(-3000 - j4000)$ of the transfer function.

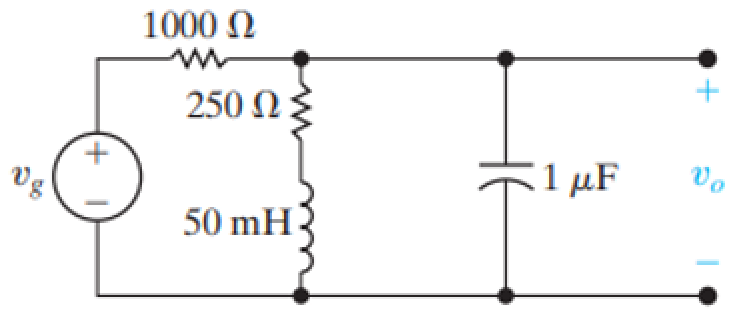
c) The steady-state component of the response is

$$(10t - 4 \times 10^{-4})u(t).$$

These two terms are generated by the second-order pole (K/s^2) of the driving voltage.



Ex-2: For the circuit shown below, the sinusoidal source voltage is $120 \cos(5000t + 30^0)$ V. Find the steady-state expression for v_o.



From previous example, $$H(s) = \frac{1000(s + 5000)}{s^2 + 6000s + 25 * 10^6}$$

The frequency of the voltage source is 5000 rad/s; hence we evaluate $H(s)$ at $H(j5000)$:

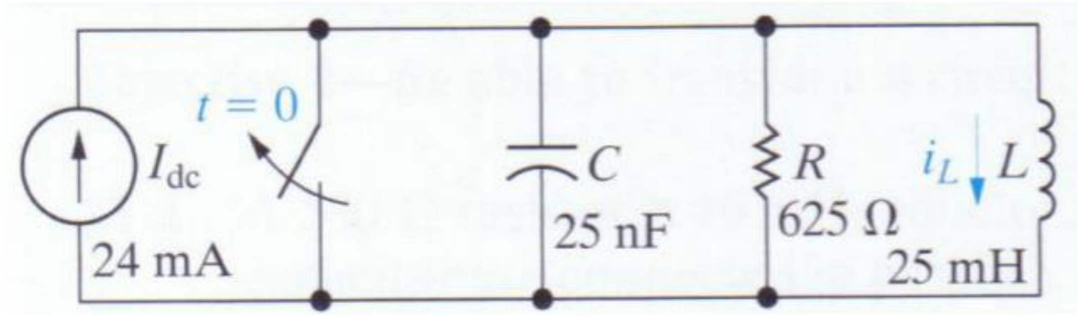
$$H(j5000) = \frac{1000(5000 + j5000)}{-25 * 10^6 + j5000(6000) + 25 \times 10^6}$$

$$= \frac{1 + j1}{j6} = \frac{1 - j1}{6} = \frac{\sqrt{2}}{6}\angle{-45^\circ}.$$

$$v_{o_{ss}} = \frac{(120)\sqrt{2}}{6} \cos(5000t + 30^\circ - 45^\circ)$$

$$= 20\sqrt{2} \cos(5000t - 15^\circ) \text{ V}.$$

Ex-3: Step Response of a Parallel RLC Circuit.



Find the current i_L through the inductor for t > 0.

Assume that the switch has been closed for "a long time".

What are the initial conditions?

$$v_C(t<0)=0$$

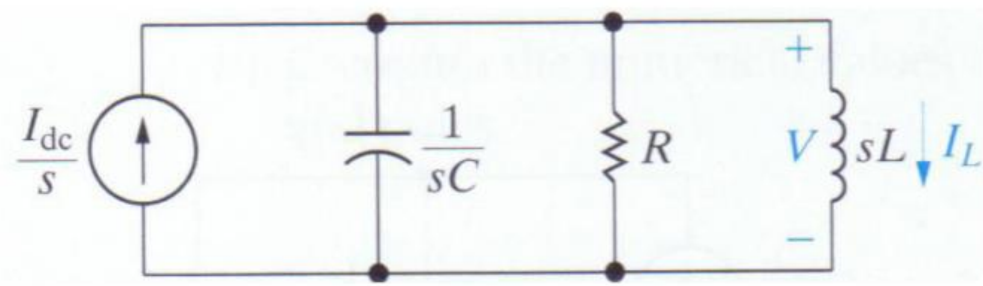
$$i_L(t<0)=0$$

$$v_R(t<0)=0$$

$$I_{DC}u(t) \Leftrightarrow \frac{I_{DC}}{s}$$

$$Z_C = \frac{1}{sC}$$

$$Z_L = sL$$



Apply KCL:

$$-\frac{I_{DC}}{s}+\frac{V}{Z_C}+\frac{V}{R}+\frac{V}{Z_L}=0$$

$$V\left(\frac{1}{\frac{1}{sC}}+\frac{1}{R}+\frac{1}{sL}\right)=\frac{I_{DC}}{s}$$

$$V=\frac{I_{DC}}{s\left(sC+\frac{1}{R}+\frac{1}{sL}\right)} \quad =\frac{\frac{I_{DC}}{C}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}$$

Now we can write the current I_L as

$$I_L=\frac{V}{sL} \quad =\frac{1}{sL}\frac{\frac{I_{DC}}{C}}{s^2+s\frac{1}{RC}+\frac{1}{LC}} \quad =\frac{\frac{I_{DC}}{LC}}{s\left(s^2+s\frac{1}{RC}+\frac{1}{LC}\right)}$$

$$\frac{I_{DC}}{LC}=\frac{24\times10^{-3}}{(25\times10^{-3})(25\times10^{-9})}=384\times10^5$$

$$\frac{1}{RC}=\frac{1}{(625)(25\times10^{-3})}=64,000$$

$$\frac{1}{LC}=\frac{1}{(25\times10^{-3})(25\times10^{-9})}=16\times10^8$$

$$I_L = \frac{384\times10^5}{s\left(s^2+64,000s+16\times10^8\right)}$$

Now find the partial fraction expansion.

$$\frac{384\times10^5}{s\left(s^2+64,000s+16\times10^8\right)} = \frac{A}{s} + \frac{B}{s+32,000-j24,000} + \frac{B^*}{s+32,000+j24,000}$$

$$A = \frac{384\times10^5}{s^2+64,000s+16\times10^8}\Big|_{s=0} \quad = \frac{384\times10^5}{16\times10^8} \quad = 24\times10^{-3}$$

$$B = \frac{384\times10^5}{s\left(s+32,000+j24,000\right)}\Big|_{s=-32,000+j24,000}$$

$$= \frac{384\times10^5}{\left(-32,000+j24,000\right)\left(-32,000+j24,000+32,000+j24,000\right)}$$

$$= \frac{384\times10^5}{\left(-32,000+j24,000\right)\left(j48,000\right)} \quad = \frac{384\times10^5}{-1.152\times10^9-j1.536\times10^9}$$

$$= \frac{384\times10^5}{1.92\times10^9\angle-126.87^\circ} \quad = 20\times10^{-3}\angle126.87^\circ$$

The PFE is

$$I_L = \frac{24 \times 10^{-3}}{s} + \frac{20 \times 10^{-3} \angle 126.87^\circ}{s + 32,000 - j24,000} + \frac{20 \times 10^{-3} \angle -126.87^\circ}{s + 32,000 + j24,000}$$

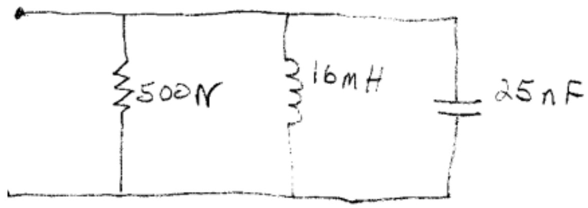
So now write $i_L(t)$ from the inverse Laplace transform. I will write the current in terms of milli Amps.

$$i_L(t) = \left[24 + 2(20e^{-32,000t}\cos(24,000t + 126.87^\circ)\right]u(t)\, mA$$

$$= \left[24 + 40e^{-32,000t}\cos(24,000t + 126.87^\circ\right]u(t)\, mA$$

$$\boxed{\frac{K\langle\theta}{s + \alpha - j\beta} + \frac{K\langle -\theta}{s + \alpha + j\beta} \Leftrightarrow 2|K|e^{-\alpha t}\cos(\beta t + \theta)}$$

<u>Ex-4:</u> A 500 Ω resistor, a 16 mH inductor, and a 25 nF capacitor are connected in parallel. Find the zeros and poles.



Analyze the circuit and find the zeros and poles.

Admittance is given by

$$Y = \frac{1}{Z} \quad = \frac{1}{R} + \frac{1}{sL} + \frac{1}{\frac{1}{sC}} \quad = \frac{1}{R} + \frac{1}{sL} + sC$$

$$= \frac{\frac{s}{R} + \frac{1}{L} + s^2 C}{s} \quad = \frac{C\left(\frac{s}{RC} + \frac{1}{LC} + s^2\right)}{s} \quad = \frac{C\left(s^2 + s\frac{1}{RC} + \frac{1}{LC}\right)}{s}$$

Now find the numeric result.

$$\frac{1}{RC} = \frac{1}{(500\Omega)(25\times10^{-9}F)} \quad = 80,000\,\tfrac{1}{\text{sec}}$$

$$\frac{1}{LC} = \frac{1}{(16\times10^{-3}H)(25\times10^{-9}F)} \quad = 25\times10^{8}\,\tfrac{1}{\text{sec}^2}$$

Thus the numerical admittance is

$$Y = \frac{(25\times10^{-9})\left(s^2 + 80,000s + 25\times10^{8}\right)}{s}$$

Now find the zeros and poles of the admittance Y.

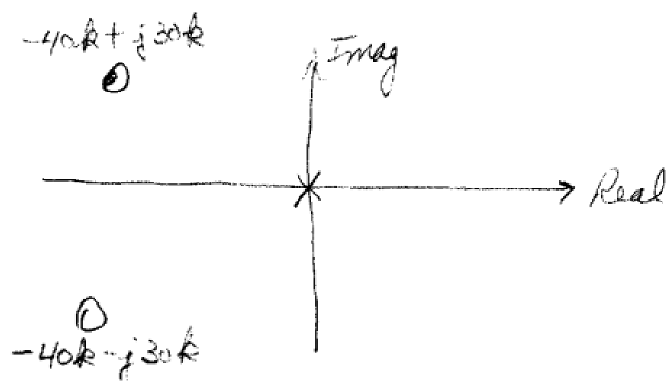
$$Y = \frac{(25\times10^{-9})\left(s^2+80{,}000s+25\times10^8\right)}{s}$$

$$= \frac{(25\times10^{-9})(s+40{,}000+j30{,}000)(s+40{,}000-j30{,}000)}{s}$$

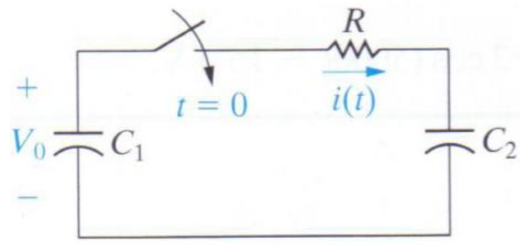
There is a single pole at s = 0.

There is a complex zero.

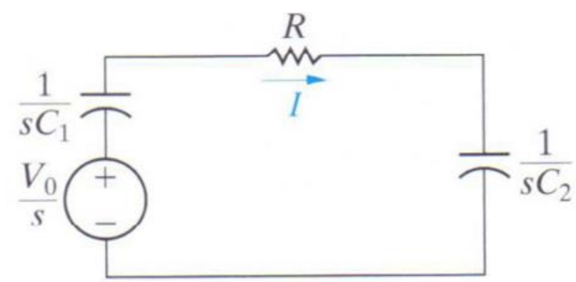
$$s = -40{,}000 \pm j30{,}000$$



Ex-5: In the circuit above, the capacitor C_1 is charged to an initial voltage V_0 prior to closing the switch at t = 0.
The capacitor C_2 is discharged so that V_{C2} = 0 prior to closing the switch at t = 0.
Analyze the circuit and find the current i(t) expression.



The s-domain equivalent circuit for t > 0



Apply KVL:

$$-\frac{V_0}{s}+V_{C1}+IR+V_{C2}=0 \qquad Where\ V_{C1}=I\frac{1}{sC_1} \qquad V_{C2}=I\frac{1}{sC_2}$$

$$\frac{V_0}{s}=I\left(\frac{1}{sC_1}+R+\frac{1}{sC_2}\right) \quad =I\left(R+\frac{1}{sC_1}+\frac{1}{sC_2}\right) \quad =I\left[R+\frac{1}{s}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right]$$

The equivalent capacitance for this series connection is

$$\frac{1}{C_{Equivalent}}=\frac{1}{C_1}+\frac{1}{C_2}$$

$$\frac{1}{sC_1}+\frac{1}{sC_2}=\frac{1}{s}\left(\frac{1}{C_1}+\frac{1}{C_2}\right) \quad =\frac{1}{s}\left(\frac{1}{C_{Equiv}}\right)$$

Thus

$$I\left(R+\frac{1}{sC_{Equiv}}\right)=\frac{V_0}{s}$$

$$I\left(\frac{RsC_{Equiv}+1}{sC_{Equiv}}\right)=\frac{V_0}{s} \quad \Rightarrow I=\frac{V_0}{s}\left(\frac{sC_{Equiv}}{RsC_{Equiv}+1}\right) \quad \Rightarrow I=\frac{V_0}{s}\left(\frac{sC_{Equiv}}{RC_{Equiv}\left[s+\frac{1}{RC_{Equiv}}\right]}\right)$$

$$I=\frac{V_0}{R}\left(\frac{1}{s+\frac{1}{RC_{Equiv}}}\right)$$

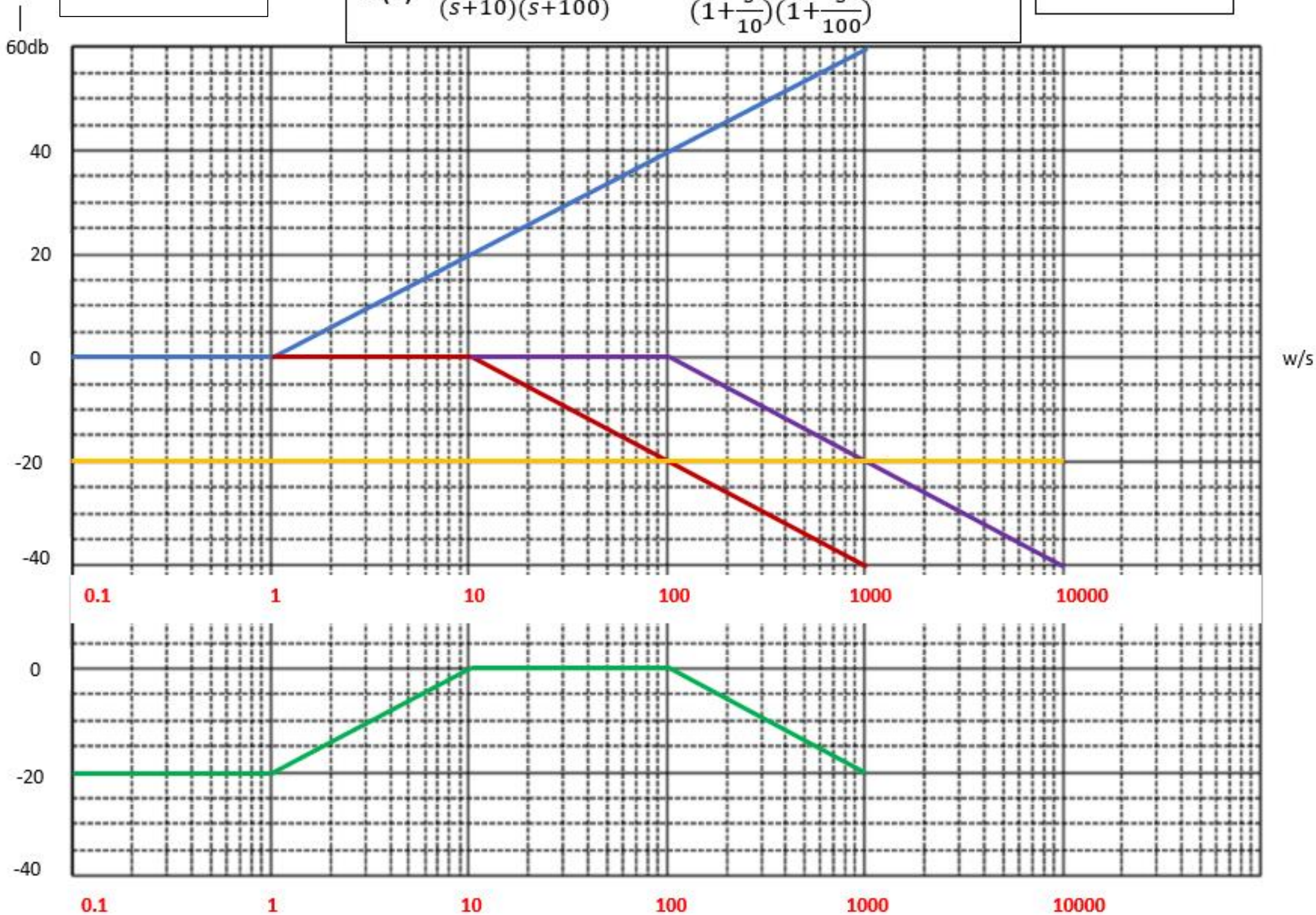
Thus the inverse Laplace transform is

$$i(t)=\frac{V_0}{R}e^{-\frac{t}{RC_{Equiv}}}u(t)$$

Example - 6

$$H(s)=\frac{100(s+1)}{(s+10)(s+100)} = \frac{0.1(1+\frac{s}{1})}{(1+\frac{s}{10})(1+\frac{s}{100})}$$

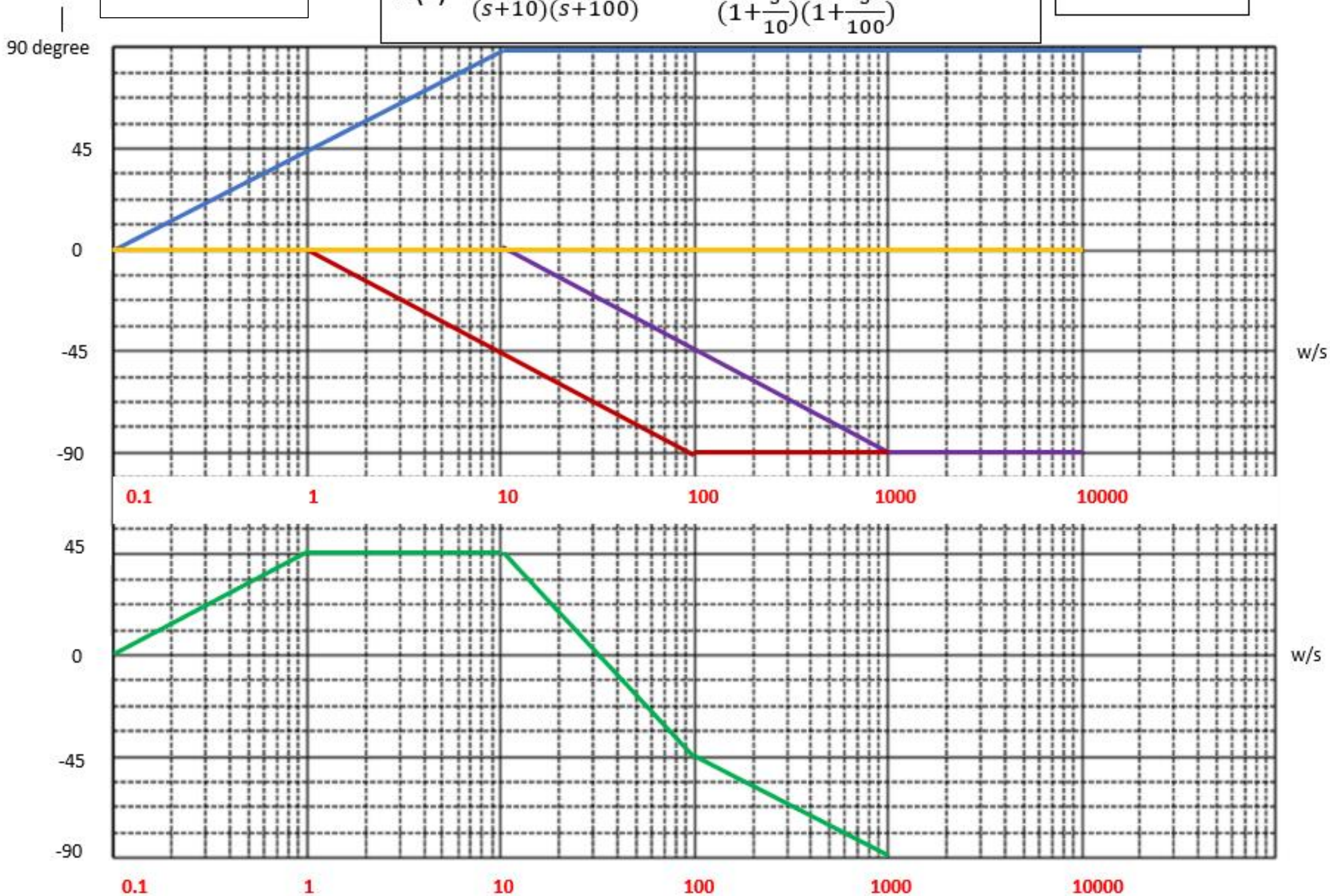
MAGNITUDE



Example - 6

$$H(s) = \frac{100(s+1)}{(s+10)(s+100)} = \frac{0.1(1+\frac{s}{1})}{(1+\frac{s}{10})(1+\frac{s}{100})}$$

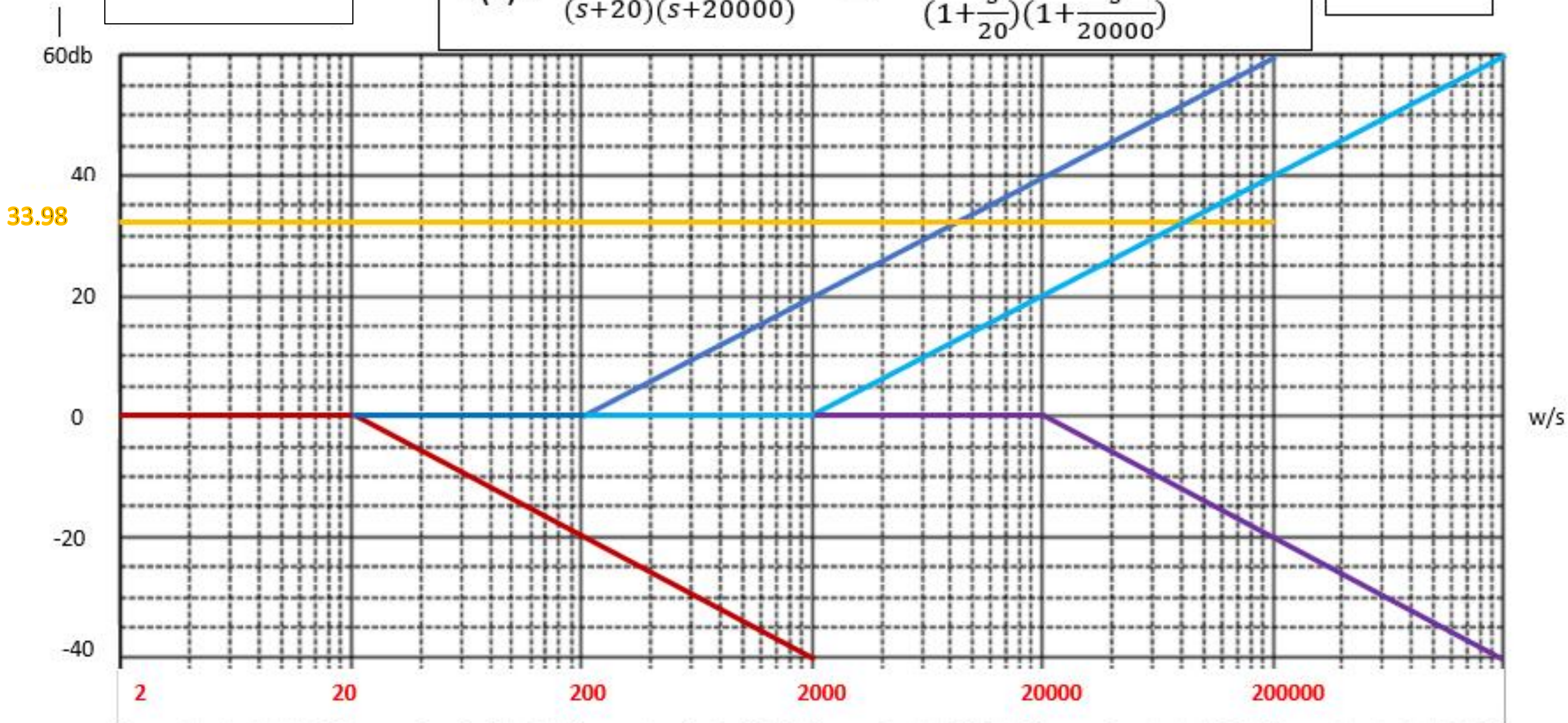
Phase

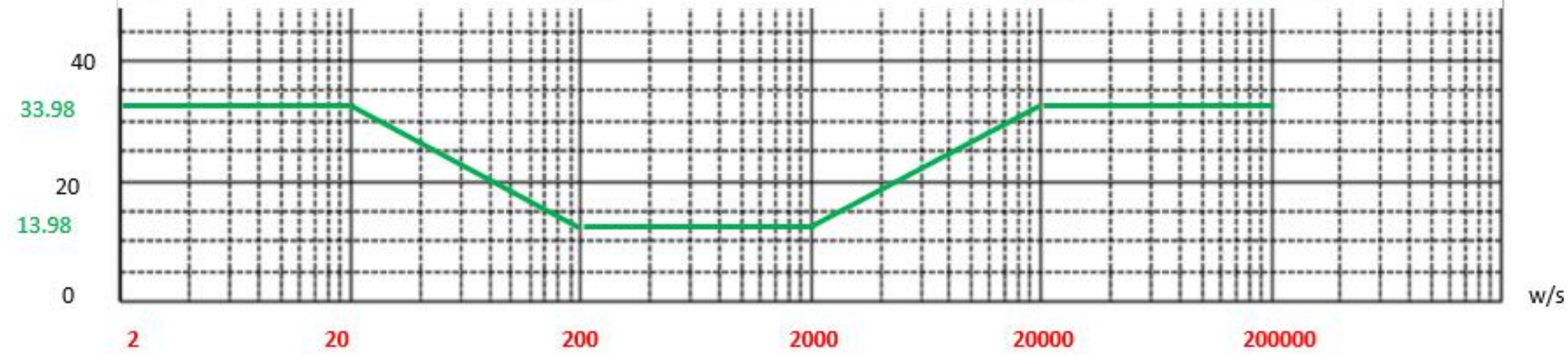


Example - 7

$$H(s)=\frac{50(s+200)(s+2000)}{(s+20)(s+20000)} = \frac{50(1+\frac{s}{200})(1+\frac{s}{2000})}{(1+\frac{s}{20})(1+\frac{s}{20000})}$$

MAGNITUDE

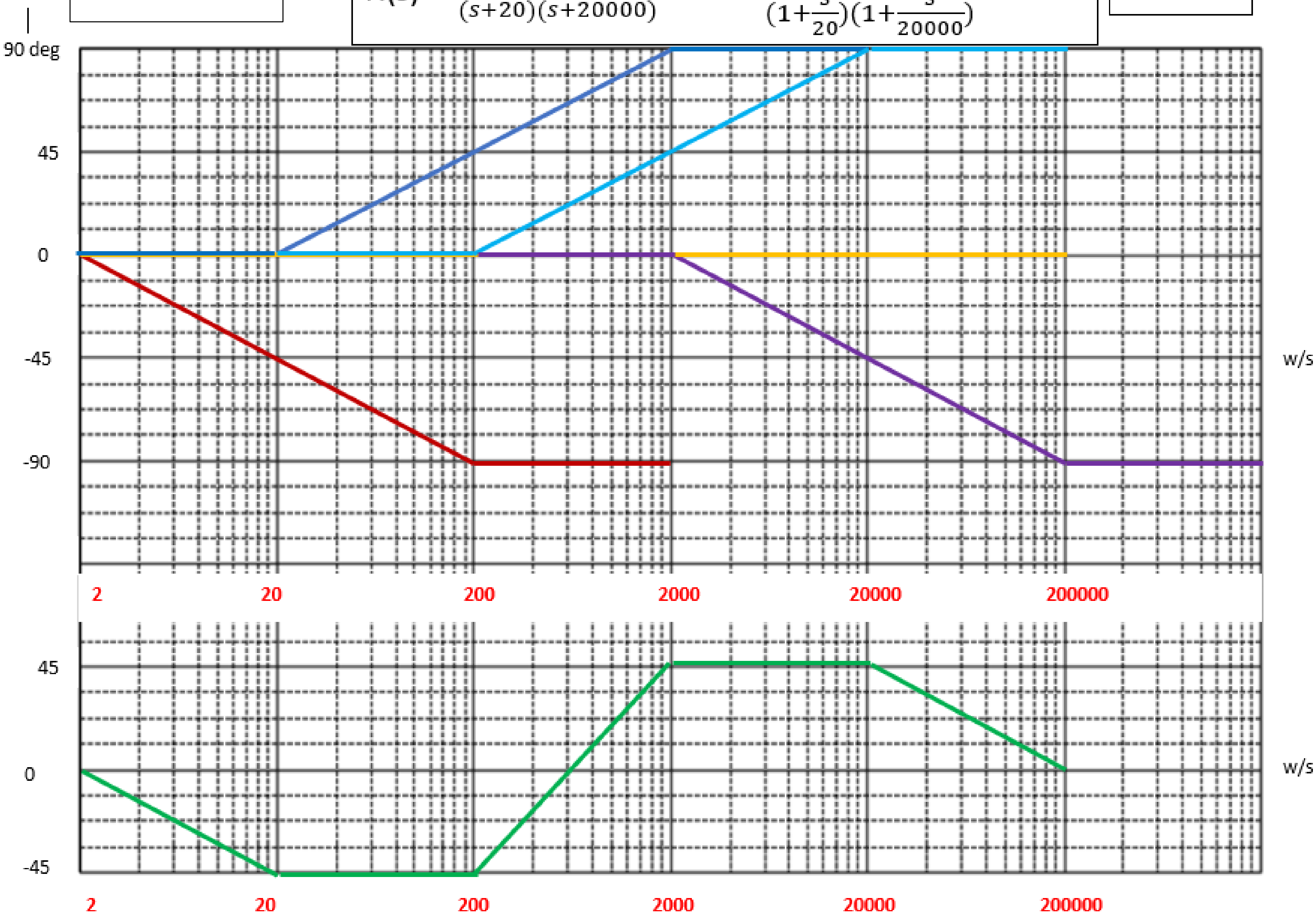




Example - 7

$$H(s)=\frac{50(s+200)(s+2000)}{(s+20)(s+20000)} = \frac{50\left(1+\frac{s}{200}\right)\left(1+\frac{s}{2000}\right)}{\left(1+\frac{s}{20}\right)\left(1+\frac{s}{20000}\right)}$$

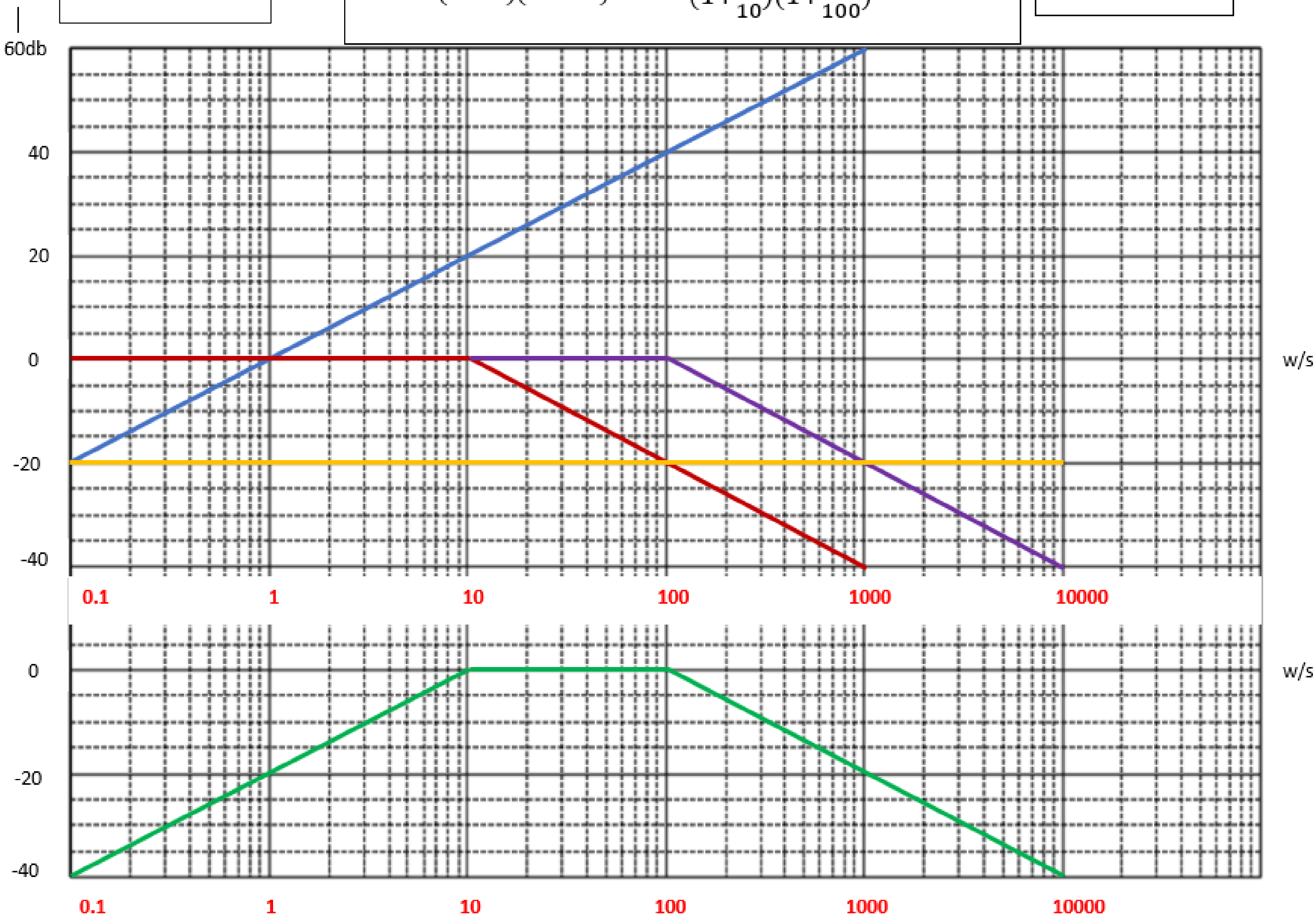
PHASE



Example - 8

$$H(s)=\frac{100s}{(s+10)(s+100)} = \frac{0.1s}{(1+\frac{s}{10})(1+\frac{s}{100})}$$

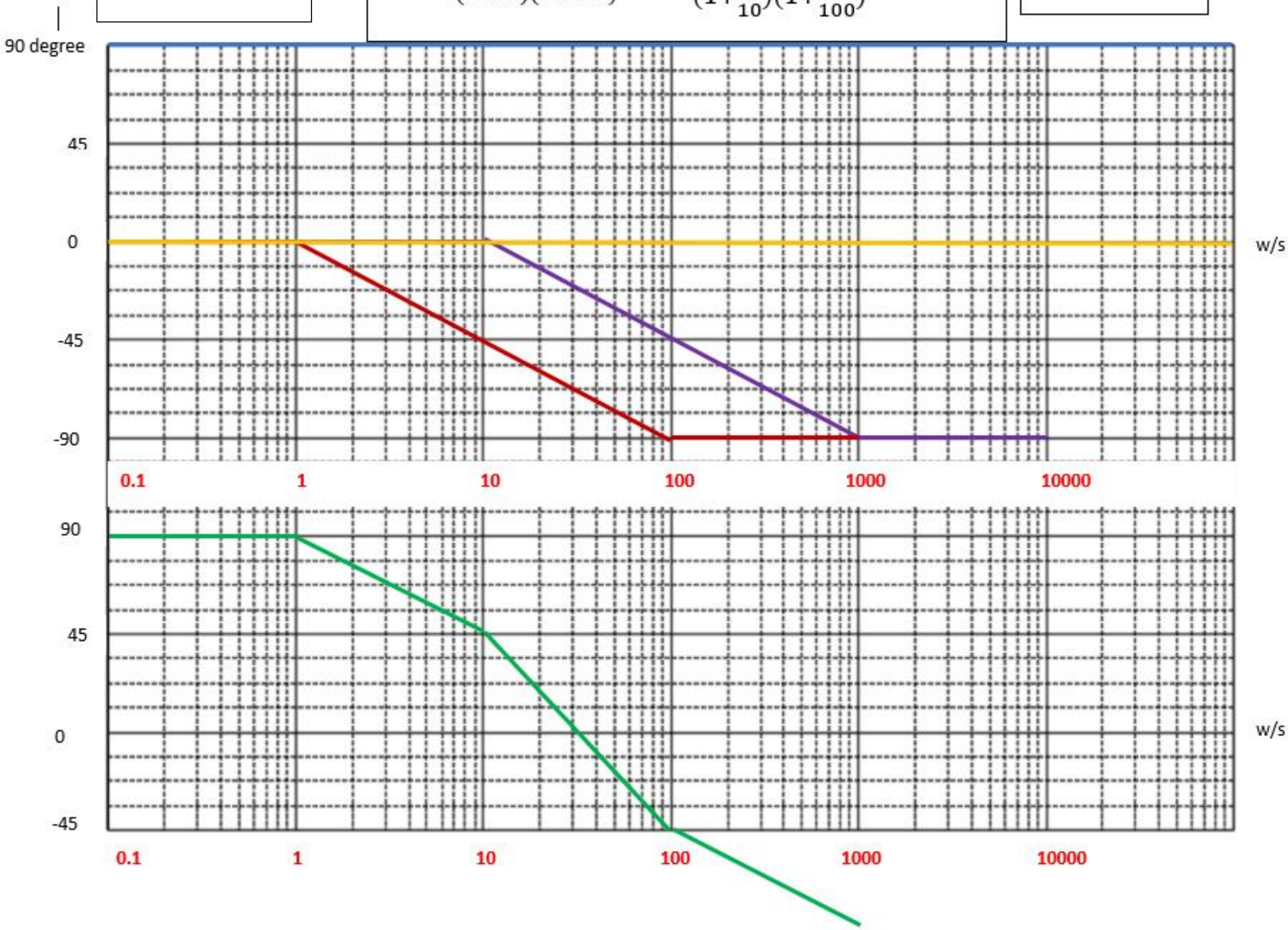
MAGNITUDE



Example - 8

$$H(s)=\frac{100s}{(s+10)(s+100)} = \frac{0.1\,s}{(1+\frac{s}{10})(1+\frac{s}{100})}$$

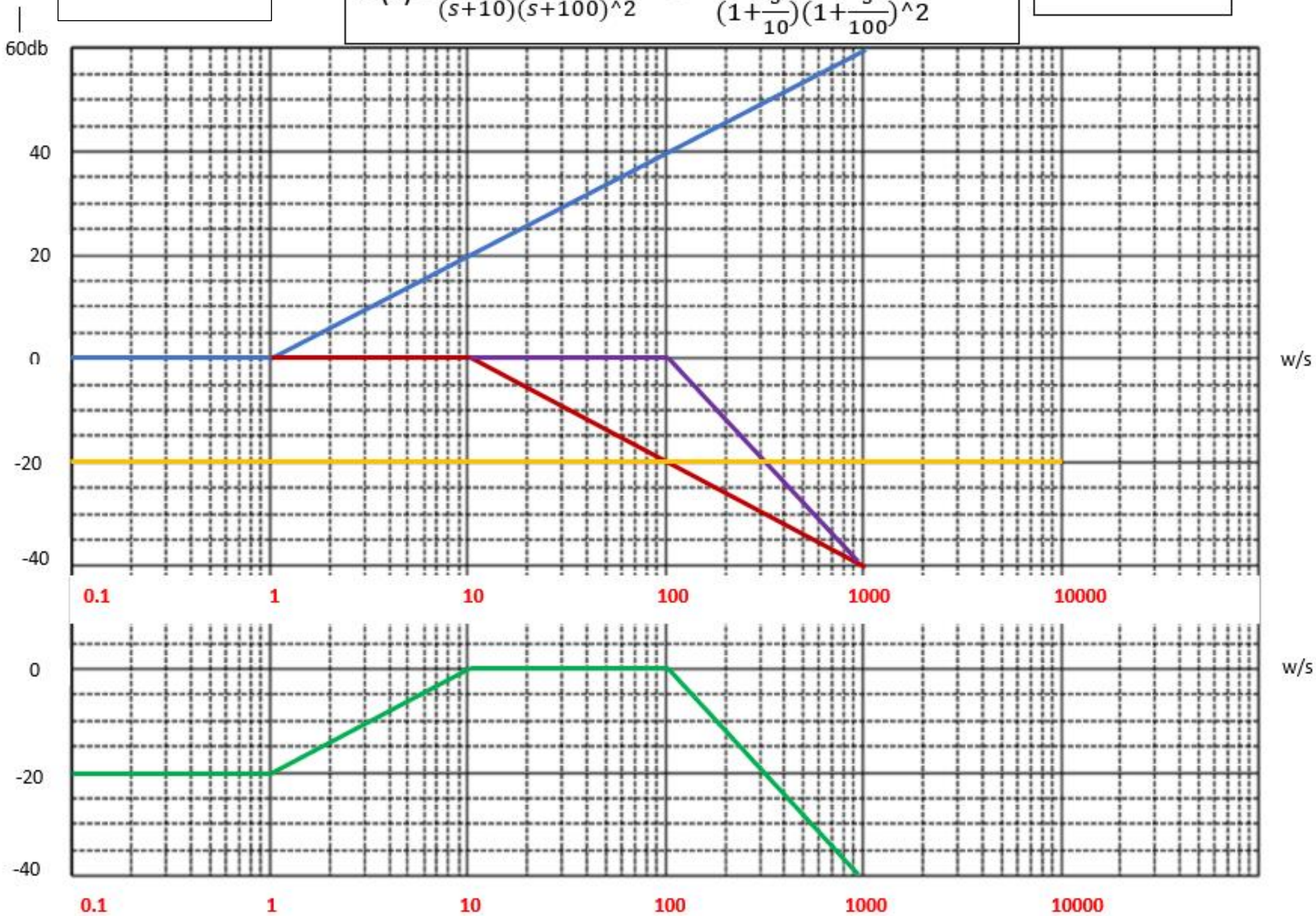
Phase



Example - 9

$$H(s)=\frac{10000(s+1)}{(s+10)(s+100)^2} = \frac{0.1(1+\frac{s}{1})}{(1+\frac{s}{10})(1+\frac{s}{100})^2}$$

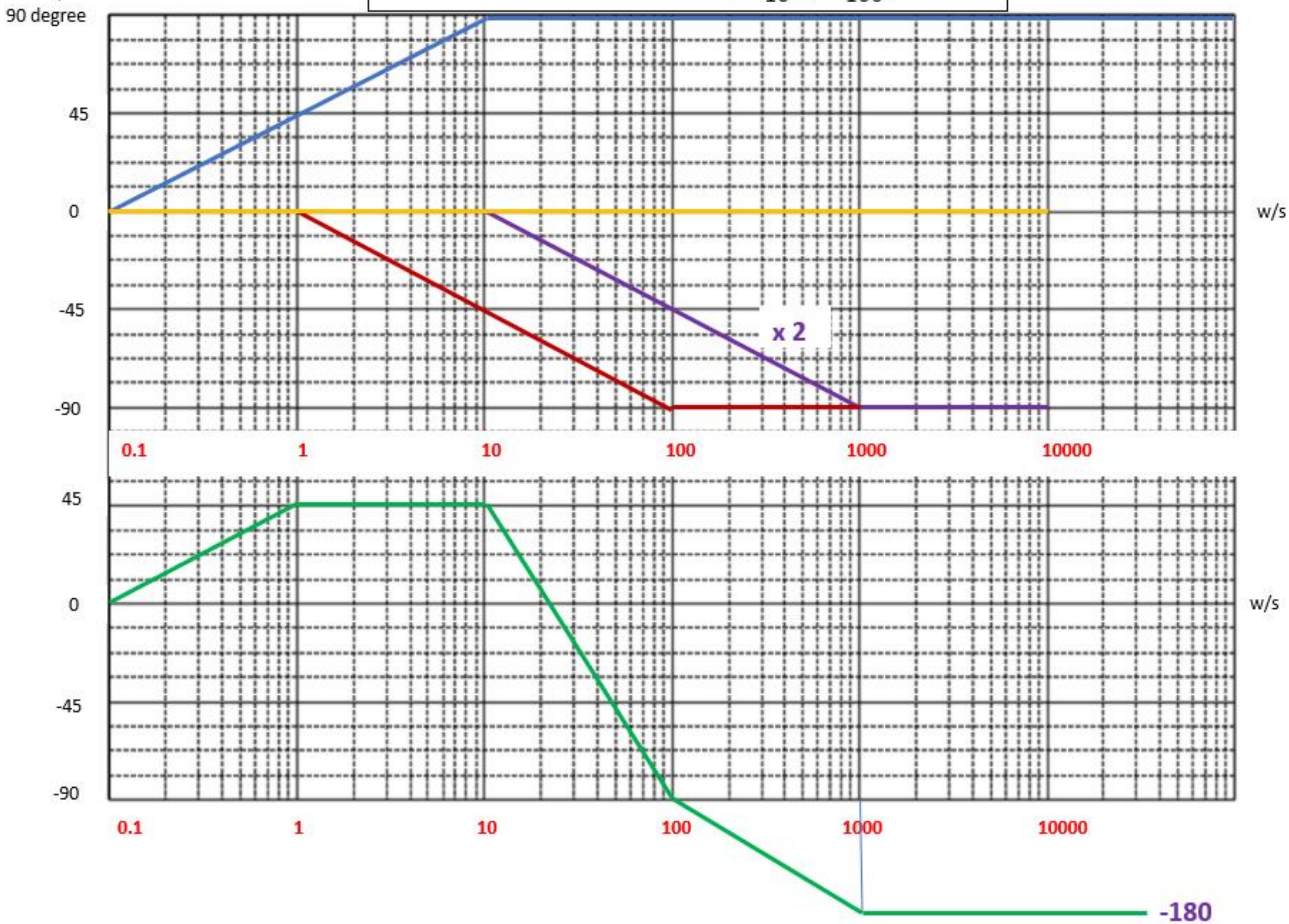
MAGNITUDE



Example - 9

$$H(s)=\frac{10000(s+1)}{(s+10)(s+100)^2} = \frac{0.1(1+\frac{s}{1})}{(1+\frac{s}{10})(1+\frac{s}{100})^2}$$

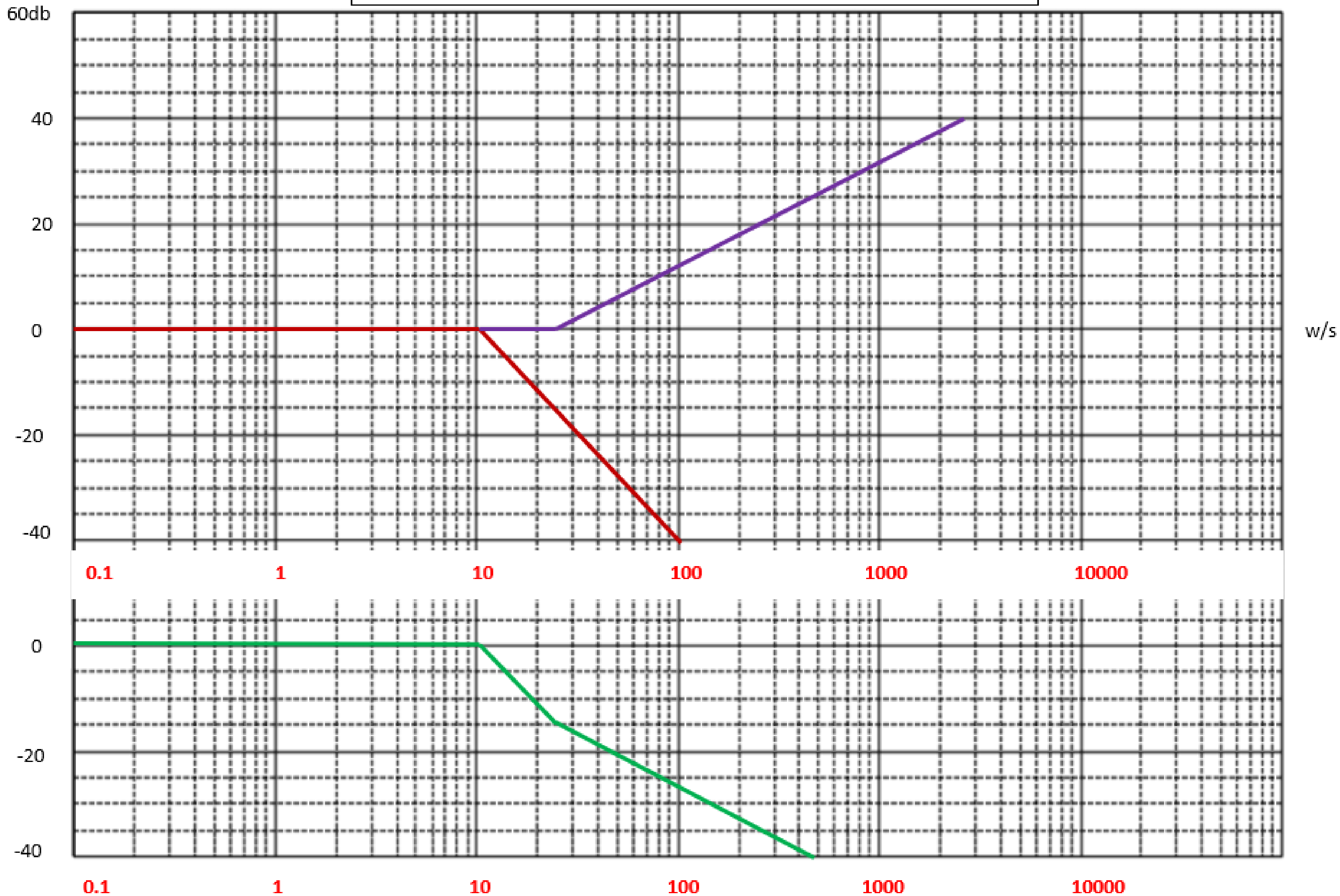
Phase



Example - 10

$$H(s)=\frac{4(s+25)}{(s^2+4s+100)} = \frac{(1+\frac{s}{25})}{1+\left(\frac{s}{10}\right)^2+0.4(\frac{s}{10})}$$

MAGNITUDE



Example - 10

$$H(s)=\frac{4(s+25)}{(s^2+4s+100)} = \frac{(1+\frac{s}{25})}{1+\left(\frac{s}{10}\right)^2+0.4(\frac{s}{10})}$$

Phase