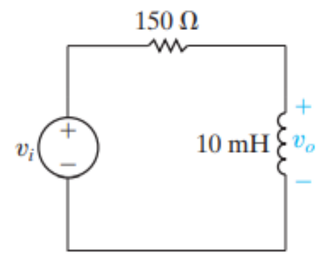
Homework-05 ENGR 117 Due date 04/25/2022

5 Questions 20 points each

Q-1 Consider the circuit shown below.



a) This circuit behaves like what type of filter?

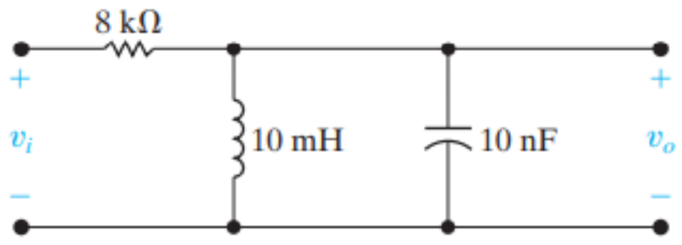
b) What is the transfer function, of this filter?

c) What is the cutoff frequency of this filter?

d) Find the magnitude and phase of the transfer function at $s=j\omega_c$?

Q-2 For the bandpass filter shown. Find:

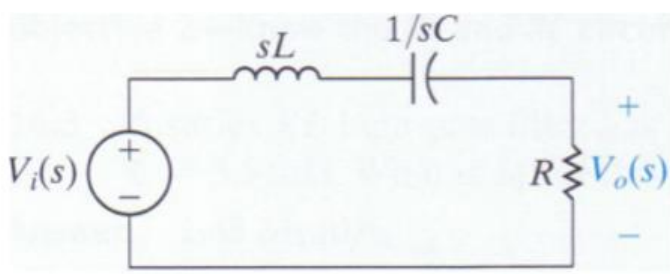
(a) ω_o, (b) f_o, (c) Q, (d) ω_{c1}, (e) f_{c1}, (f) ω_{c2}, (g) f_{c2}, and (h) β.



Q-3 Verify the following for the bandpass filter: (show your work)

$$\omega_{C1} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C2} = \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



Q-4 Use a 5 nF capacitor to design a series RLC bandpass filter. The center frequency of the filter is 8 kHz, and the quality factor is 2. (Show your circuit)

a) Specify the values of R and L.

b) What is the lower cutoff frequency in kilohertz?

c) What is the upper cutoff frequency in kilohertz?

d) What is the bandwidth of the filter in kilohertz?

Q-5 Design the component values for the series RLC band reject filter so that the center frequency is 4 kHz and the quality factor is 5. Use a 500 nF capacitor. (Show your circuit)

a) Specify the values of R and L.

b) Find quality factor Q.

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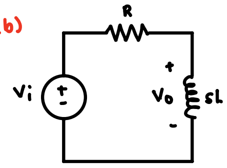
EEE 117

HOMEWORK 05
APRIL 25, 2022

Q-1 (a) High Pass Filter

(b)



$$V_o(s) = \frac{sL}{R+sL} V_i(s)$$

$$H(s) = \frac{s}{s + R/L} = \frac{s}{s + 150/10\times10^{-3}} = \frac{s}{s + 15000}$$

$$\boxed{H(j\omega) = \frac{j\omega}{j\omega + 15000}}$$

(c) $\omega_c = R/L = 150/10\times10^{-3} = \boxed{15\,000 \text{ rad/s}}$

$\theta = 45°$

(d)
$$H(j\omega_c) = \frac{\omega_c \angle 90°}{\sqrt{\omega_c^2 + (15\,000)^2} \angle \tan^{-1}\left(\frac{\omega_c}{15\,000}\right)}$$

$$= \frac{15\,000 \angle 90°}{\sqrt{(15\,000)^2 + (15\,000)^2} \angle \tan^{-1}\left(\frac{15000}{15000}\right)}$$

$$= 0.7071 \angle 90° - 45°$$

Magnitude : $|H(j\omega)| = \boxed{0.7071}$

Phase : $\theta(j\omega) = \boxed{45°}$

Q-2 (a) $\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(10\times10^{-3})(10\times10^{-9})}} = \boxed{100\,000 \text{ rad/s}}$

(b) $\omega_0 = 2\pi f_0$

$f_0 = \frac{100\,000}{2\pi} = \boxed{15\,915.49 \text{ Hz}}$

(c) $Q = \sqrt{\frac{R^2 C}{L}} = \sqrt{\frac{(8k)^2(10\times10^{-9})}{10\times10^{-3}}} = \boxed{8}$

(d) $\omega_{c1} = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = \frac{-1}{2(8k)(10\times10^{-9})} + \sqrt{\left[\frac{1}{2(8k)(10\times10^{-9})}\right]^2 + \frac{1}{(10\times10^{-3})(10\times10^{-9})}}$

$= \boxed{93\,945.12 \text{ rad/s}}$

(e) $f_{c1} = \frac{93\,945.12}{2\pi} = \boxed{14\,951.83 \text{ Hz}}$

(f) $\omega_{c2} = \frac{+1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = \frac{+1}{2(8k)(10\times10^{-9})} + \sqrt{\left[\frac{1}{2(8k)(10\times10^{-9})}\right]^2 + \frac{1}{(10\times10^{-3})(10\times10^{-9})}}$

$= \boxed{106\,445.12 \text{ rad/s}}$

(g) $f_{c2} = \frac{106\,445.12}{2\pi} = \boxed{16\,941.27 \text{ Hz}}$

(h) $\beta = \frac{1}{RC} = \frac{1}{(8k)(10\times10^{-9})} = \boxed{12\,500 \text{ Hz}}$

Q-3 $-V_i(s) + I(s)\cdot sL + I(s)\cdot 1/sC + I(s)\cdot R = 0$

$$V_i(s) = I(s)\left[sL + \frac{1}{sC} + R\right]$$

$$= \frac{V_o}{R}\left[\frac{s^2LC + 1 + sCR}{sC}\right]$$

$$\frac{V_i(s)}{V_o(s)} = \frac{1}{H(s)} = \frac{s^2LC + 1 + sCR}{sCR}$$

$$H(s) = \frac{sCR}{s^2LC + 1 + sCR} = \frac{s(R/L)}{s^2 + s(R/L) + (1/LC)}$$

$$H(j\omega) = \frac{R/L\,(j\omega)}{(j\omega)^2LC + (j\omega)CR + 1}$$

$$|H(j\omega)| = \frac{\omega(R/L)}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left[\omega(R/L)\right]^2}}$$

$I(s)\cdot R = V_o$

$I(s) = V_o/R$

$$H_{max} = |H(j\omega_o)|$$

$$= \frac{\omega_o(R/L)}{\sqrt{\left(\frac{1}{LC} - \omega_o^2\right)^2 + \left[\omega_o(R/L)\right]^2}}$$

$(1/\sqrt{2})\,H_{max}$

$$\frac{1}{\sqrt{2}} = \frac{\omega_c(R/L)}{\sqrt{\left(\frac{1}{LC} - \omega_c^2\right)^2 + \left[\omega_c(R/L)\right]^2}}$$

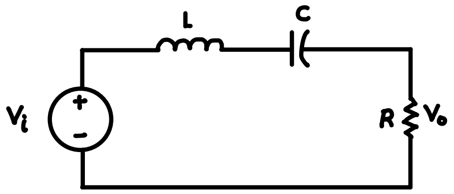
$$= \frac{1}{\sqrt{\left(\frac{\omega_c L}{R} - \frac{1}{\omega_c RC}\right)^2 + 1}}$$

$$\pm 1 = \frac{\omega_c L}{R} - \frac{1}{\omega_c RC}$$

$$\omega_c^2 L \pm \omega_c R - 1/C = 0$$

$$\boxed{\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad \omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

Q-4



$C = 5\,nF$

$Q = 2$

$f_o = 8\,kHz \longrightarrow \omega_o = 16k\pi$

(a) $\omega_o = \sqrt{\frac{1}{LC}} \longrightarrow L = \frac{1}{\omega_o^2 C} = \frac{1}{(16k\pi)^2(5\times10^{-9})} = \boxed{0.07916\ H}$

$Q = \sqrt{\frac{L}{CR^2}} \longrightarrow R = \sqrt{\frac{L}{CQ^2}} = \sqrt{\frac{0.07916}{(5\times10^{-9})(2)^2}} = \boxed{1989.47\ \Omega}$

(b) $\omega_{c1} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$

$$= \frac{-1989.47}{2(0.07916)} + \sqrt{\left[\frac{1989.47}{2(0.07916)}\right]^2 + \left[\frac{1}{(0.07196)(5\times10^{-9})}\right]}$$

$= 39245.41$ rad/s

$f_{c1} = \frac{39245.41}{2\pi} = \boxed{6246.10\ Hz}$

(c) $\omega_{c_2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$

$$= \frac{1989.47}{2(0.07916)} + \sqrt{\left[\frac{1989.47}{2(0.07916)}\right]^2 + \left[\frac{1}{(0.07196)(5\times10^{-9})}\right]}$$

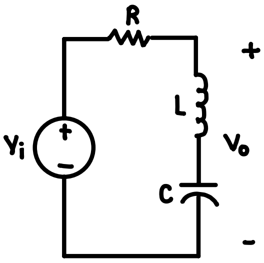
$= 64377.68$ rad/s

$f_{c_2} = \frac{64377.68}{2\pi} = \boxed{10\,246.03\ Hz}$

(d) $\beta = \omega_{c2} - \omega_{c1} = 10\,246.03\ Hz - 6246.10\ Hz$

$= 3999.93\ Hz \approx \boxed{4\ kHz}$

Q-5



$C = 500\,nF$

$f_0 = 4\,kHz \longrightarrow \omega_0 = 8000\pi$ rad/s

$Q = 5$

(a) $\omega_0 = \sqrt{\dfrac{1}{LC}} \longrightarrow L = \dfrac{1}{\omega_0^2 C} = \dfrac{1}{(8k\pi)^2(500\times10^{-9})} = \boxed{3.166\ mH}$

$Q = \sqrt{\dfrac{L}{CR^2}} \longrightarrow R = \sqrt{\dfrac{L}{CQ^2}} = \sqrt{\dfrac{3.166\times10^{-3}}{(500\times10^{-9})(5)^2}} = \boxed{15.915\ \Omega}$

(b) $\boxed{Q = 5}$

KVL: $-V_{in} + iR + iZ_c = 0$

$V_{out} = iZ_c \longrightarrow i = \frac{V_{out}}{Z_c}$

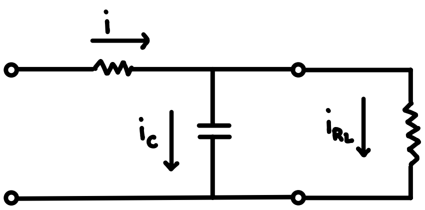
$V_{in} = i(R + Z_c)$

$V_{in} = \frac{V_{out}}{Z_c}(R + Z_c)$

$Z_c = 1/sC$

$$\frac{V_{out}}{V_{in}} = \frac{Z_c}{R + Z_c} = \frac{1/sC}{R + 1/sC}$$

$$\boxed{H(s) = \frac{1}{1 + sRC}}$$



KVL: $-V_{in} + iR + V_{out} = 0$

$i = i_c + i_{R_L}$

$V_{in} = (V_{out}\, sC + \frac{V_{out}}{R_L})R + V_{out}$

$i_c = \frac{V_{out}}{1/sC} = V_{out}\, sC$

$i_{R_L} = \frac{V_{out}}{R_L}$

$V_{in} = V_{out}\left(sCR + \frac{R}{R_L} + 1\right)$

$$\frac{V_{out}}{V_{in}} = \frac{R_L}{sCRR_L + R + R_L}$$

$$\boxed{H(s) = \frac{R_L}{sCRR_L + R + R_L}}$$