Chapter 3

Simple Resistive Circuits

Text: *Electric Circuits*, 9^{th} Edition, by J. Nilsson and S. Riedel
Prentice Hall

Engr 17 Introductory Circuit Analysis
Instructor: Russ Tatro

Chapter 3 Overview

We will now start the “meat” of circuit analysis – calculating voltages and currents by using various simplifications.

Series and Parallel connections

Series connection – same current through each device in series.

Parallel connection – same voltage across each device in parallel.

Voltage and current “division”

Voltage expressed across parts of the circuit

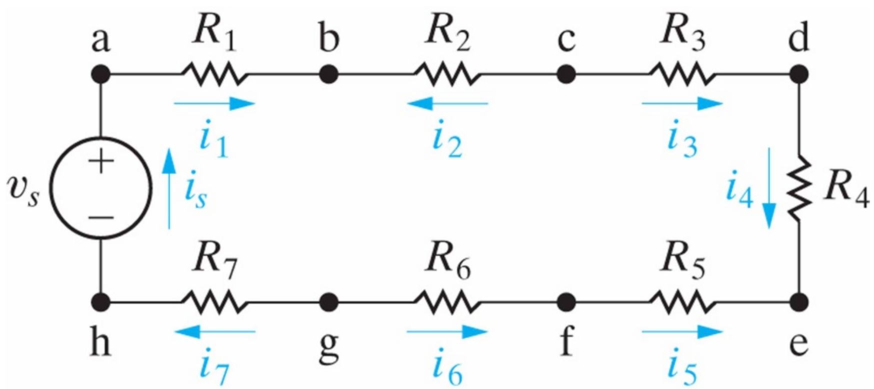
Current splitting into multiple paths in the circuit

Section 3.1
Resistors in Series

Resistors in Series

Two equivalent statements:

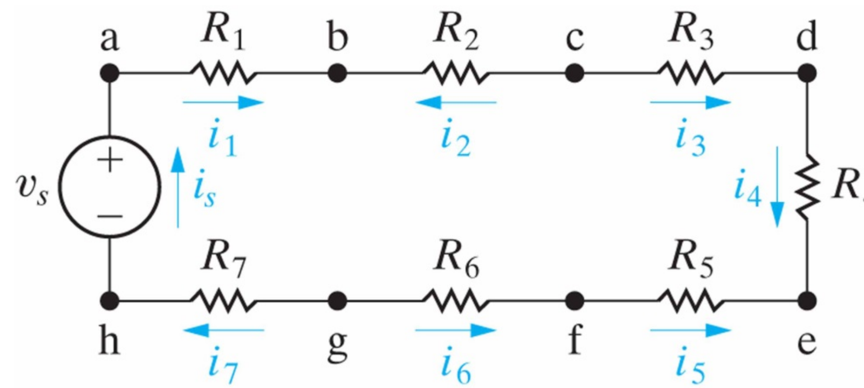
Same current in both elements
Two elements connect at a single node



Notice that each element only connects to one neighbor.

Resistors in Series

The voltages around this circuit are then:



$$-v_s + v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 = 0$$

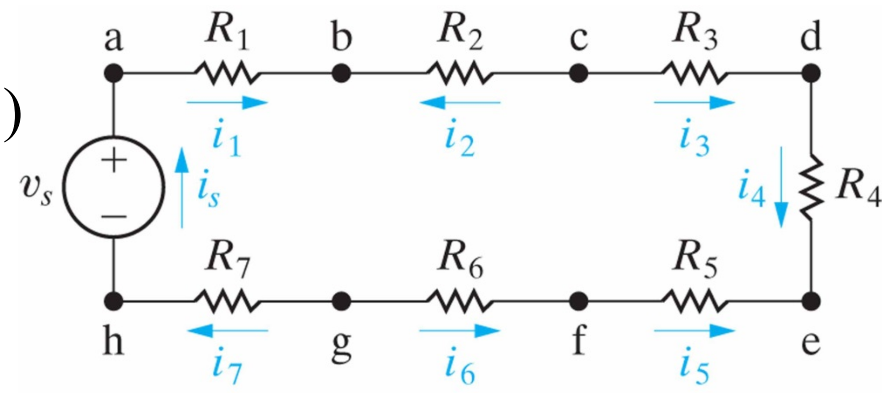
$$v_s = v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7$$

$$v_s = i_s R_1 + i_s R_2 + i_s R_3 + i_s R_4 + i_s R_5 + i_s R_6 + i_s R_7$$

$$v_s = i_s (R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7) \quad = i_s R_{eq}$$

Resistors in Series

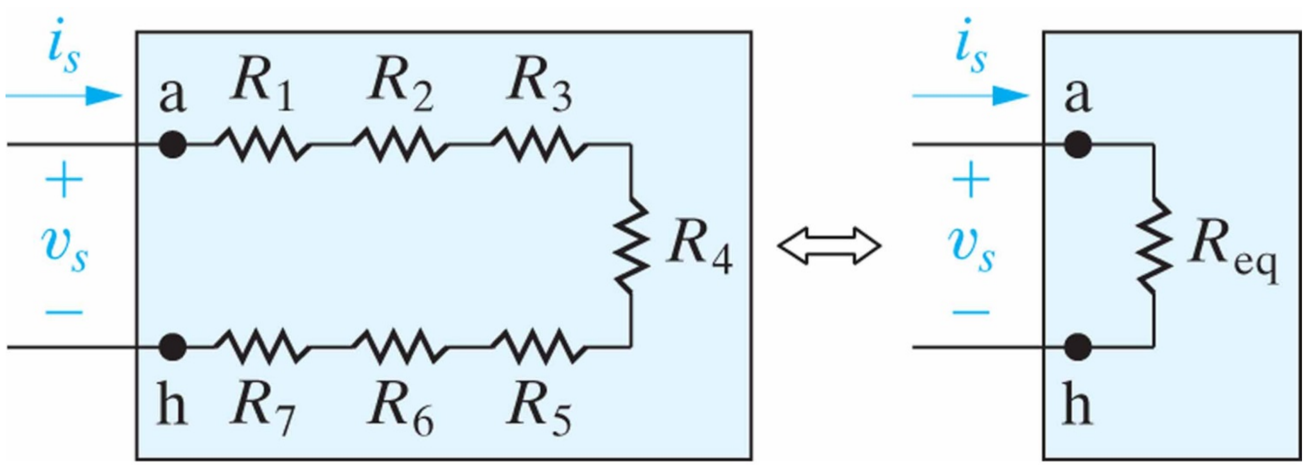
$$i_s R_{eq} = i_s (R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7)$$



Thus for resistors in series we have:

$$R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 = R_{eq}$$

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \ldots + R_k$$



Resistors in Series

Reality check – a very useful simplistic confirmation of a result:

The equivalent resistance of a set of series resistors is always <u>LARGER</u> than the <u>largest single</u> resistor.

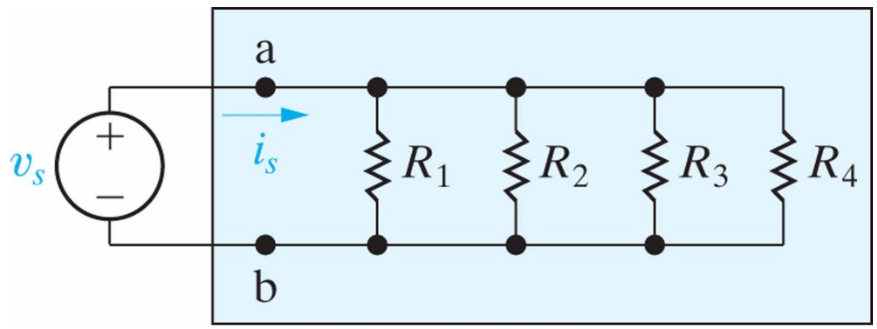
Section 3.2
Resistors in Parallel

Resistors in parallel

Two equivalent statements:

Same voltage across all elements

All elements connect at a single node with a common reference node.

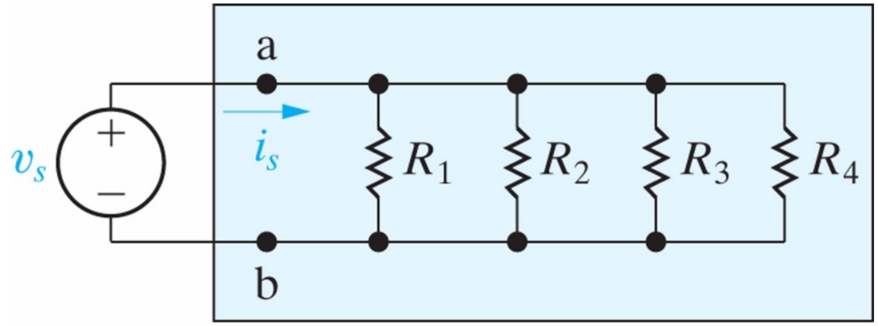


Key point: Voltage is ALWAYS measured with respect to some reference – which I just called the "common reference node".

Very often this common reference node is called *ground*.
But this is not always accurate.

Resistors in parallel

Same voltage across all parallel elements, so what is the current?



At node a:

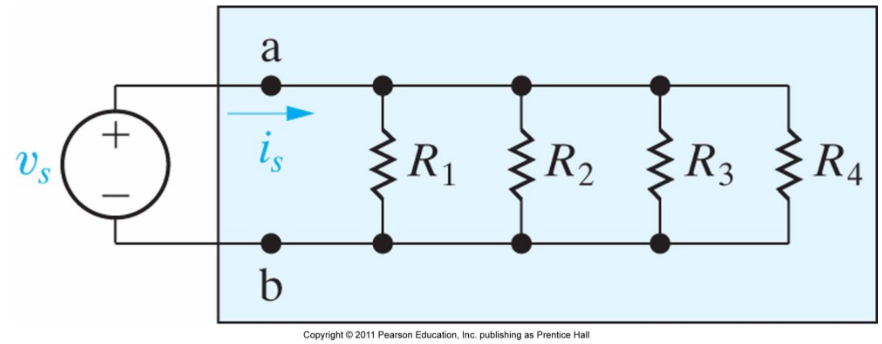
$$-i_s + i_{R_1} + i_{R_2} + i_{R_3} + i_{R_4} = 0$$

$$i_s = i_{R_1} + i_{R_2} + i_{R_3} + i_{R_4}$$

$$\frac{v_s}{R_{eq}} = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3} + \frac{v_s}{R_4}$$

Resistors in parallel

Thus for resistors in parallel we have



$$\frac{v_s}{R_{eq}} = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3} + \frac{v_s}{R_4}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

For k resistors, the summation is

$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i}$$

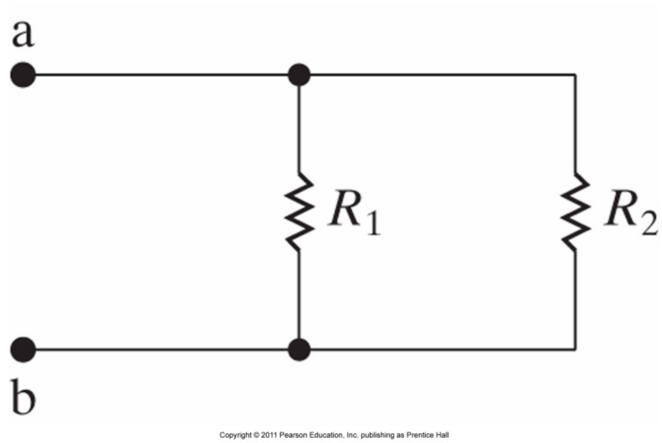
Resistors in Parallel

Reality check

The equivalent resistance of a set of parallel resistors is always SMALLER than the smallest single resistor.

Two resistors in parallel

There is a closed form for two resistors in parallel.

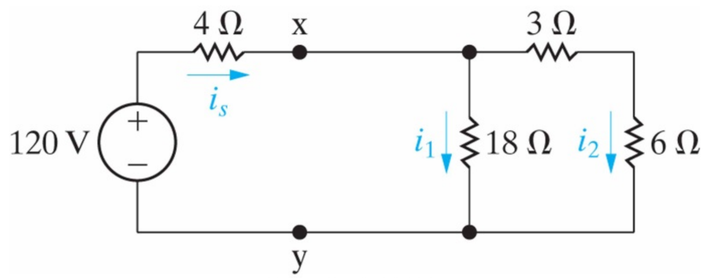


$$\frac{1}{R_{eq}} = \sum_{i=1}^{2} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \left[\frac{1}{R_1} + \frac{1}{R_2} \right]^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

Example 3.1

Solve the following circuit for i_s, i_1, i_2 and R_{eq} at terminals xy.



First solve for i_s by finding R_{total} of the circuit.

$$3+6=9\,\Omega$$

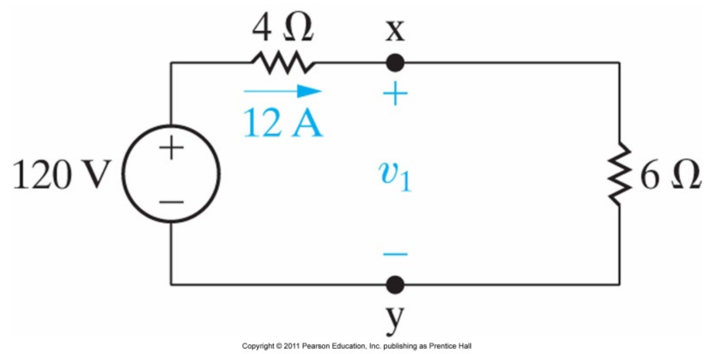
$$18\,||\,9=\frac{1}{\frac{1}{18}+\frac{1}{9}}=\frac{18*9}{18+9}=\frac{162}{27}=6\,\Omega$$ $||\rightarrow$ shorthand for parallel

$$R_{total}=4+6=10\,\Omega$$

$$i_s=\frac{120V}{10\Omega}=12A$$

Example 3.1

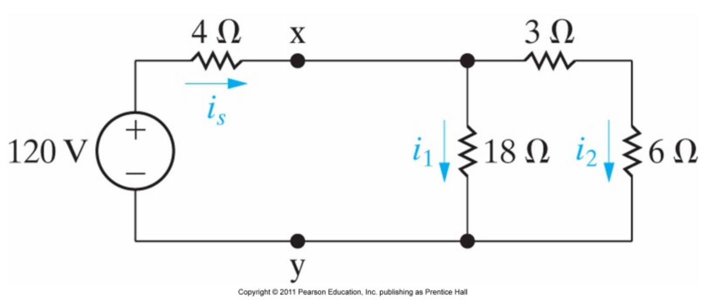
The simplified circuit is



Like many problems, there are several ways to proceed.
One way - we can find v_1 and use that to solve for the remaining currents.

$$v_1 = (12A)6\,\Omega = 72V$$

It is also handy to refer to the circuit diagram to see the relationships.



$$i_1 = \frac{72V}{18\Omega} = 4A$$

$$i_2 = \frac{72V}{9\Omega} = 8A$$

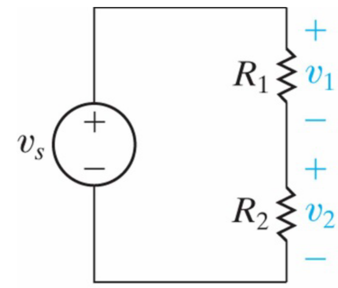
Section 3.3 and 3.4

Voltage-Divider   Current-Divider

Voltage Divider

A (highly inefficient) way to create two voltage levels from one reference voltage is by *voltage division.*

$$-v_s + v_1 + v_2 = 0$$



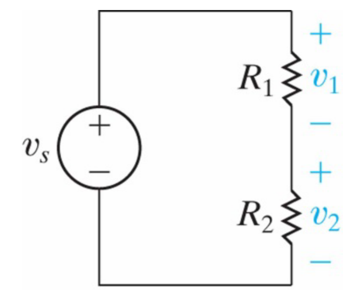
$$v_s = i_s R_1 + i_s R_2$$

$$v_s = i_s (R_1 + R_2) \Rightarrow i_s = \frac{v_s}{R_1 + R_2}$$

Now we can write the voltage v_1 as

$$v_1 = i_s R_1 = \frac{v_s}{R_1 + R_2} R_1 = v_s \frac{R_1}{R_1 + R_2}$$

Voltage Divider



$$v_1 = v_s \frac{R_1}{R_1 + R_2}$$

By a similar derivation, we have v_2 as

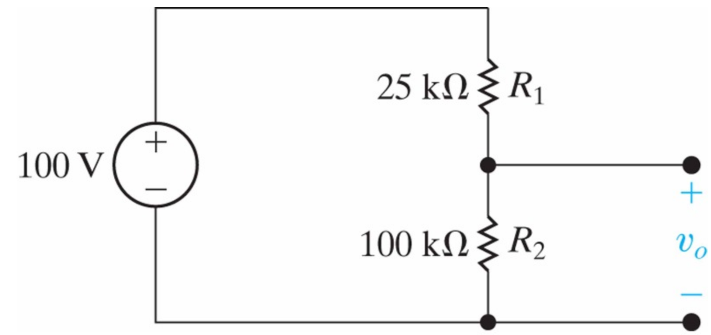
$$v_2 = v_s \frac{R_2}{R_1 + R_2}$$

So you see that by setting the proportion of each resistor, you can control the fraction of voltage across each resistor.

$$v_x = v_s \frac{R_x}{R_{total}}$$

Example 3.2

Find the voltage v_0.

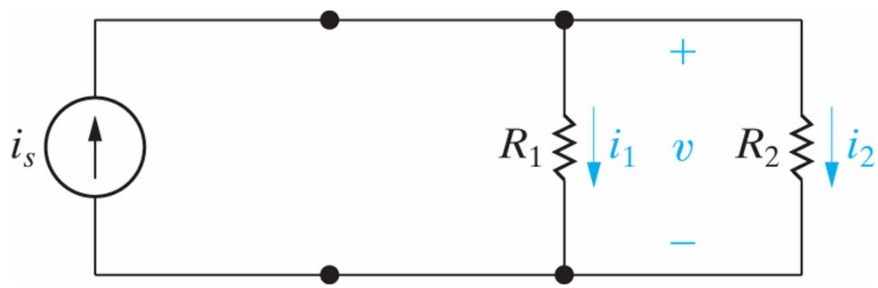


$$v_0 = v_s \frac{R_2}{R_1 + R_2}$$

$$= 100V \frac{100k}{25k + 100k} \quad = 100V \frac{4}{5} \quad = 80V$$

Current Divider

A current will divide between resistors in parallel.



$$v = i_1 R_1 = i_2 R_2$$

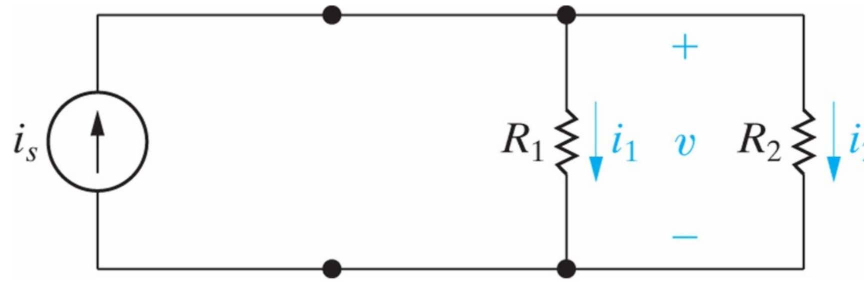
We can use the equivalent resistance and write v as

$$v = \frac{R_1 R_2}{R_1 + R_2} i_s$$

Current Divider

Our results so far are

$$v = i_1 R_1 = i_2 R_2$$



$$v = \frac{R_1 R_2}{R_1 + R_2} i_s$$

Now we can solve for i_1.

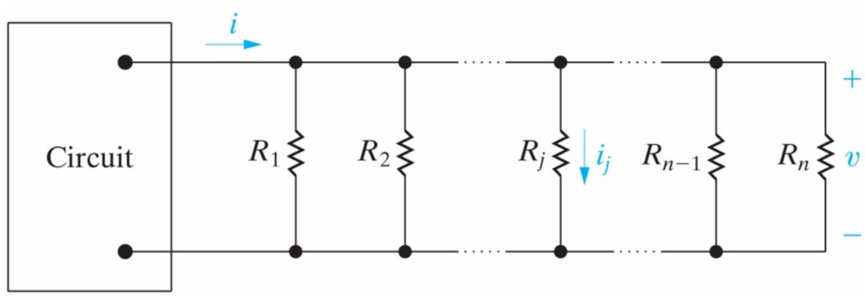
$$i_1 R_1 = \frac{R_1 R_2}{R_1 + R_2} i_s \qquad \Rightarrow i_1 = \frac{R_2}{R_1 + R_2} i_s$$

And for i_2.

$$i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s \qquad \Rightarrow i_2 = \frac{R_1}{R_1 + R_2} i_s$$

Current Divider

We can extend our results to a system of n parallel resistors as follows



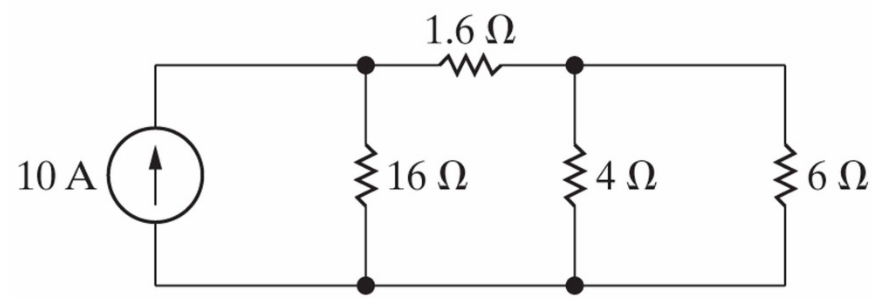
$$i_j = \frac{v}{R_j} \qquad\qquad v = R_{eq} i_s$$

Combine these two equations and we have the general current divider form as

$$i_j = \frac{R_{eq}}{R_j} i_s$$

Example 3.3

Find the current in the 4 Ω resistor.



There are many ways to approach this problem. Since we are focusing on current division here, I will see how to divide the 10A source current.

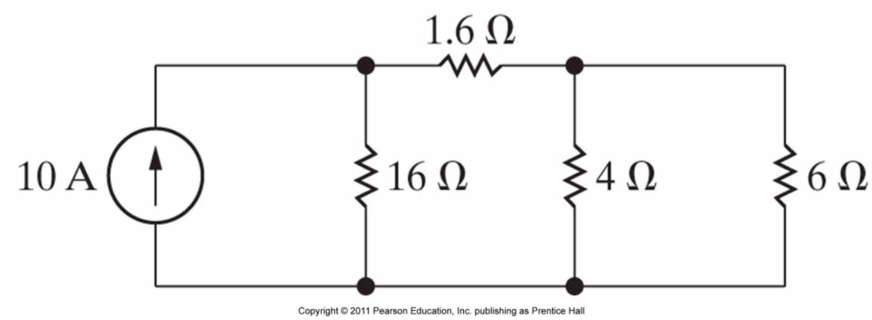
R_p on the right hand side (1.6 Ω in series with 4//6 Ω)

$$R_p = 1.6 + \frac{4*6}{4+6} = 1.6 + \frac{24}{10} = 1.6 + 2.4 = 4\,\Omega$$

$$i_p = \frac{16}{16+4}(10A) = \frac{4}{5}(10A) = 8A$$

Example 3.3

Find the current in the 4 Ω resistor.



$$i_p = 8A$$

Thus

$$i_{4\Omega} = \frac{6}{4+6}(8A) = 4.8A \qquad v_{4\Omega} = 4(4.8A) = 19.2V$$

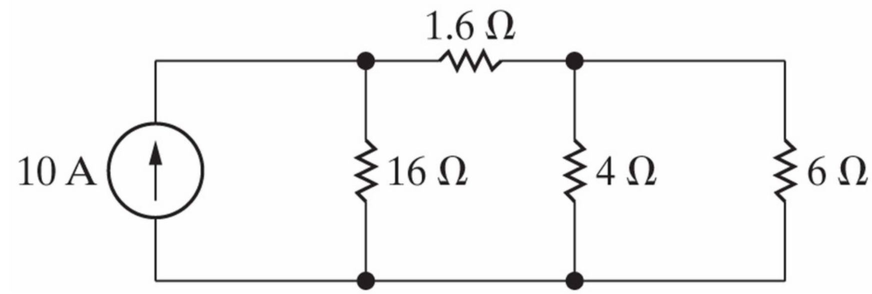
Is our result correct? Let's check.

$$i_{6\Omega} = 8 - 4.8 = 3.2A \qquad v_{6\Omega} = 6(3.2A) = 19.2V$$

Our results agree with for the two right side parallel resistors. But let's keep checking.

Example 3.3

What is the voltage v_p?



$$v_p = 1.6(8A) + 19.2V = 12.8 + 19.2 = 32V$$

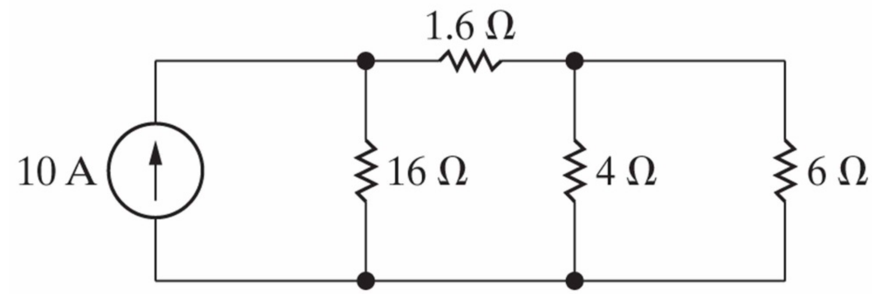
What about the voltage across the 16 Ω resistor?

$$i_{16\Omega} = 10 - 8 = 2A \qquad v_{16\Omega} = 16(2A) = 32V$$

So far so good, one last check.

Example 3.3

What is R_{eq} for the circuit?



$$R_{eq} = \frac{16*4}{16+4} = 3.2\,\Omega$$

So the voltage across the current source is

$$v = 3.2(10A) = 32V$$

So our results is consistent with all our checks!

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