

CSc 28

HW: combinational circuits

1. Minimize each of the following functions using K-Maps. For each one of the question first write the function using the min-terms and then use the k-map to simplify it.

- a) $f(A,B,C) = \Sigma \min(1,3,4,6)$
- b) $f(A,B,C) = \Sigma \min(0,6)$
- c) $f(A,B,C,D) = \Sigma \min(0,2,5,7,8,10,13,15)$
- d) $f(A,B,C,D) = \Sigma \min(1,4,11,14)$
- e) $f(A,B,C,D) = \Sigma \min(0,3,5,6,9,10,12,15)$

2. Use K-Maps to write a simplified minterm for each of the following functions. Also write each function in the form $\Sigma \min(, , ,)$

- a) $f(A,B,C) = A'B'C + ABC' + AB'C + A'BC' + A'B'C'$
- b) $f(A,B,C) = A'B'C' + A'BC + AB'C' + ABC$
- c) $f(A,B,C) = A'BC' + ABC' + AB'C + A'BC$
- d) $f(A,B) = A'B + AB + A'B' + B'A$

For the following two problems determine an un-normalized expression from the given truth tables and draw the corresponding logic circuit.

3.

x	y	f
0	0	1
0	1	0
1	0	1
1	1	0

4.

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

For the following two problems determine a minimal SOP (sum of product) expression and draw its circuit.

5. Problem 1.a

6. Problem 2.a

7. Given that $f(w, x, y, z) = \sum(0, 2, 5, 8, 9, 11, 12, 13)$. That is f is the sum of the given min-terms.

$$xz + xy\bar{z} + w\bar{x}\bar{y}$$

8. Given $f(w, x, y, z) =$ find a minimal SOP expression for f .

$$a \oplus b$$

9. Design the circuit for $f =$ using only NAND gates

10. show that NOR gate is a complete set

11. show that {OR, NOT} is a complete set

12. show that {AND, NOT} is a complete set

13. design a circuit to add two bits(half adder) discussed in class

14. write the following logical expression in its complete form

$$f(a,b,c,d) = ab + cd' + bd + abc + bcd'$$

15. Use KMap to minimize the following function: $f(x,y,z) = xy + xyz + yz' + x'z' + x'y'z'$

1. (a) $f(A,B,C) = \sum \min(1,3,4,6)$

$$= A'B'C + A'BC + AB'C' + ABC'$$

A \ BC				
	00	01	11	10
0		1	1	
1	1			1

$$f(A,B,C) = A'C + AC'$$

(b) $f(A,B,C) = \sum \min(0,6)$

$$= A'B'C' + ABC'$$

A \ BC				
	00	01	11	10
0	1			
1				1

$$f(A,B,C) = A'B'C' + ABC'$$

(c) $f(A,B,C,D) = \sum \min(0,2,5,7,8,10,13,15)$

$$= A'B'C'D' + A'B'CD' + A'BC'D + A'BCD + AB'C'D' + AB'CD + ABC'D + ABCD$$

AB \ CD				
	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

$$f(A,B,C,D) = B'D' + BD$$

(d) $f(A,B,C,D) = \sum \min(1,4,11,14)$

$$= A'B'C'D + A'BC'D' + AB'CD + ABCD'$$

AB \ CD				
	00	01	11	10
00		1		
01	1			
11				1
10			1	

$$f(A,B,C,D) = A'B'C'D + A'BC'D' + AB'CD + ABCD'$$

(e) $f(A, B, C, D) = \sum \min(0, 3, 5, 6, 9, 10, 12, 15)$

$$= A'B'C'D' + A'B'CD + A'BC'D + A'BCD' + AB'C'D + AB'CD' + ABC'D + ABCD$$

AB \ CD	00	01	11	10
00	1		1	
01		1		1
11	1		1	
10		1		1

$$f(A, B, C, D) = A'B'C'D' + A'B'CD + A'BC'D + A'BCD' + AB'C'D + AB'CD' + ABC'D + ABCD$$

2. (a) $f(A, B, C) = A'B'C + ABC' + AB'C + A'BC' + A'B'C'$
 $= \sum \min(0, 1, 2, 5, 6)$

A \ BC	00	01	11	10
0	1	1		1
1		1		1

$$f(A, B, C) = A'B' + B'C + BC'$$

(b) $f(A, B, C) = A'B'C' + A'BC + AB'C' + ABC$
 $= \sum \min(0, 3, 4, 7)$

A \ BC	00	01	11	10
0	1		1	
1	1		1	

$$f(A, B, C) = B'C' + BC$$

(c) $f(A, B, C) = A'BC' + ABC' + AB'C + A'BC$
 $= \sum \min(2, 3, 5, 6)$

A \ BC	00	01	11	10
0			1	1
1		1		1

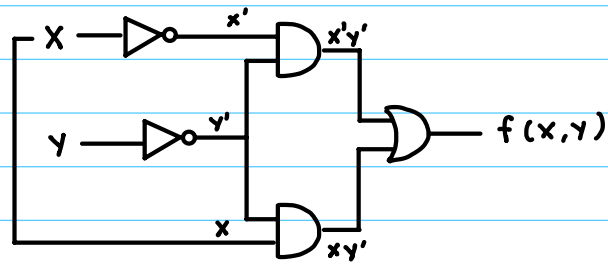
$$f(A, B, C) = AB'C + A'B + BC'$$

(d) $f(A, B) = A'B + AB + A'B' + B'A$
 $= \sum \min(0, 1, 2, 3)$

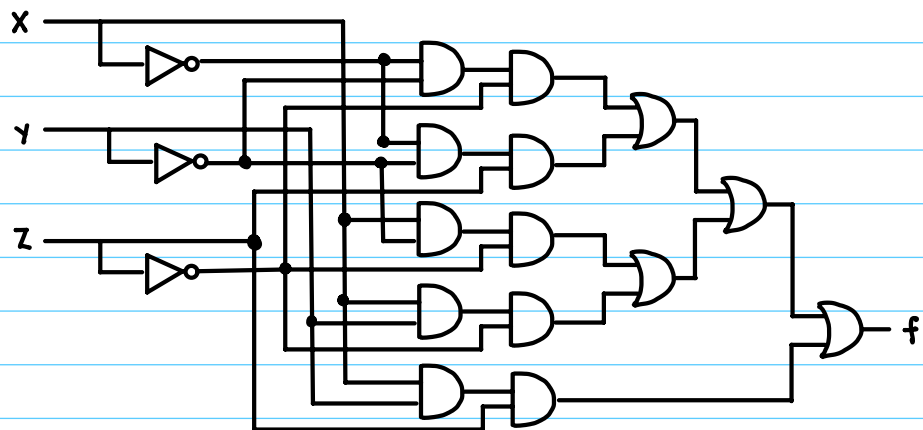
A \ B	0	1
0	1	1
1	1	1

$$f(A, B) = 1$$

3. $f(x,y) = x'y' + xy'$



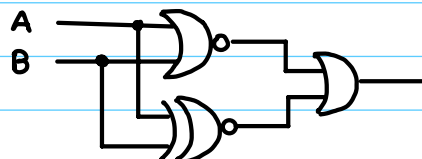
4. $f(x,y,z) = x'y'z' + x'y'z + xy'z' + xyz' + xyz$



5. $f(A,B,C) = \sum \min(1,3,4,6)$
 $= A'C + AC' = A \oplus C$



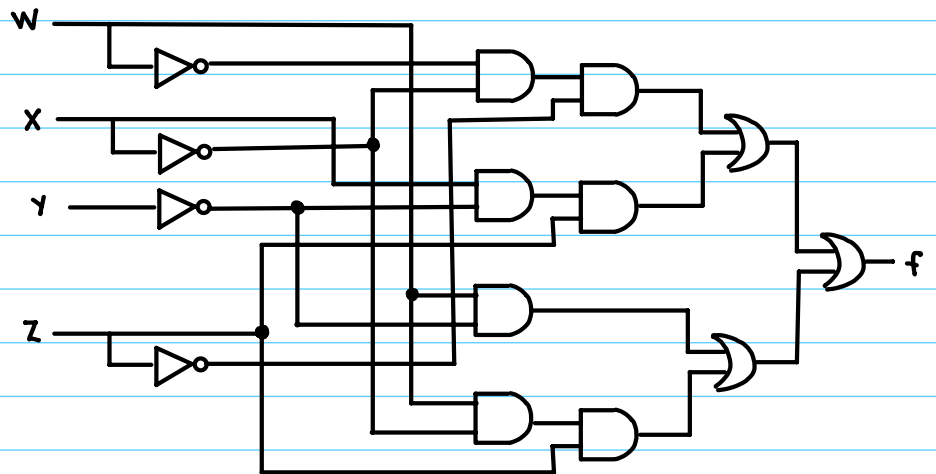
6. $f(A,B,C) = A'B'C + ABC' + AB'C + A'BC' + A'B'C'$
 $= A'B' + B'C + BC'$
 $= (A+B)' + (A \oplus B)$



7. $f(w, x, y, z) = \sum(0, 2, 5, 8, 9, 11, 12, 13)$

w \ x \ yz	yz			
	00	01	11	10
00	1			1
01		1		
11	1	1		
10	1	1	1	

$$f(w, x, y, z) = \bar{w}\bar{x}\bar{z} + x\bar{y}z + w\bar{y} + w\bar{x}z$$



8. $f(w, x, y, z) = xz + xy\bar{z} + w\bar{x}\bar{y}$

w \ x \ yz	yz			
	00	01	11	10
00				
01		1	1	1
11		1	1	1
10	1	1		

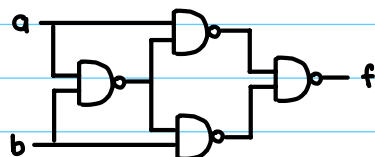
$$f(w, x, y, z) = xz + xy\bar{z} + w\bar{x}\bar{y}$$

9. $f = a \oplus b$

$$= (\overline{\overline{a}b})a \cdot (\overline{\overline{a}b})b = (\overline{\overline{a}b})a + (\overline{\overline{a}b})b$$

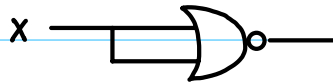
$$= (\bar{a} + \bar{b})a + (\bar{a} + \bar{b})b$$

$$= \bar{a}a + a\bar{b} + \bar{a}b + \bar{b}b = a\bar{b} + \bar{a}b = a \oplus b$$



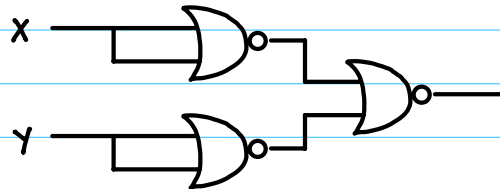
10. NOR gate is a complete set of every gate

• NOT gate



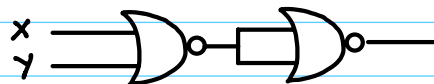
$$\overline{(x+x)} = \bar{x} \cdot \bar{x} = \bar{x}$$

• AND gate



$$\begin{aligned} & \overline{(\overline{x+x}) + (\overline{y+y})} \\ &= (\overline{x+x})(\overline{y+y}) \\ &= (x+x)(y+y) = xy \end{aligned}$$

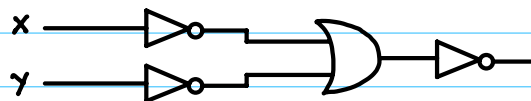
• OR gate



$$\begin{aligned} & \overline{(\overline{x+y}) + (\overline{x+y})} \\ &= (\overline{x+y}) \cdot (\overline{x+y}) \\ &= (x+y) \cdot (x+y) = x+y \end{aligned}$$

11. {OR, NOT} is a complete set

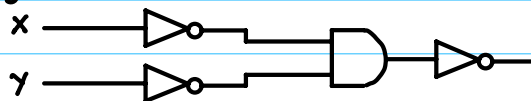
AND gate



$$\overline{\overline{x} + \overline{y}} = \overline{\overline{x}} \cdot \overline{\overline{y}} = xy$$

12. {AND, NOT} is a complete set

OR gate



$$\overline{\overline{x} \cdot \overline{y}} = \overline{\overline{x}} + \overline{\overline{y}} = x+y$$

13. Half adder

TRUTH TABLE

x	y	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

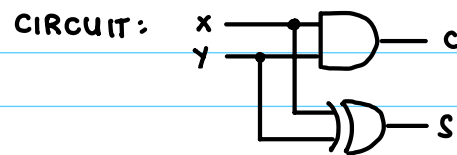
x \ y	0	1
0	0	1
1	1	0

$$s = \bar{x}y + x\bar{y}$$

$$= x \oplus y$$

x \ y	0	1
0	0	0
1	0	1

$$c = xy$$



14. $f(a,b,c,d) = ab + cd' + bd + abc + bcd'$

$$= ab(c+c')(d+d') + cd'(a+a')(b+b') + bd(a+a')(c+c')$$

$$+ abc(d+d') + (a+a')bcd'$$

$$= abcd + abcd' + abc'd + abc'd' + abcd' + ab'cd'$$

$$+ a'bcd' + a'b'cd' + abcd + abc'd + a'bcd + a'bc'd$$

$$+ abcd + abcd' + abcd' + a'bcd'$$

$$= a'b'cd' + a'bc'd + a'bcd' + a'bcd + ab'cd'$$

$$+ abc'd' + abc'd + abcd' + abcd$$

15. $f(x,y,z) = xy + xyz + yz' + x'z' + x'y'z'$

x \ yz	00	01	11	10
0	1			1
1			1	1

$$f(x,y,z) = \bar{x}\bar{z} + xy$$