EEE 117L Laboratory – Network Analysis
Lab #7: Low Pass Filters (Week I)

Lab Day and Time: Wednesday 1:30 pm - 4:10 pm

Group Number: # 03

Group Members: (Last Name, First Name)

Member #1: Algador, Vigomar Kim

Member #2: Chan, Casey

Member #3: Bon, Trinh

Total Score: /100

General Instructions:

1) Chapter 14 in the text on frequency selective circuits may be of use in the hand calculations and the general understanding of low pass filters and the cutoff frequency.

2) You will need more pages in order to complete this worksheet. Make sure to include a title and page number for each section added. Make sure to show all your work and reasoning.

Work Breakdown Structure: It is important that every group member do their share of the work in these labs. Remember that you will receive no credit for the lab worksheet if you did not contribute. Write in the Table provided below, which group member(s) contributed to the solution of each problem in the lab worksheet. Also remember that only on lab worksheet per group will be turned in to Canvas. If there was any group member that did not contribute, then write their name in the space provided below.

Problem Number	Group member(s) that worked on the problem.
Part 1a	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 1b	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 1c	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 1d	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2a	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2b	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2c	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 2d	Algador, Vigomar Kim Chan, Casey Trinh, Bon
Part 3	Algador, Vigomar Kim Chan, Casey Trinh, Bon

Absent member(s):____

1. Unloaded Passive Low Pass Filter Total Score: ____/45

a) Preliminary Measurements: Score: ____/5

This section of the lab involves the circuit shown in Figure 1 below. Before simulating or building the circuit, the values of the resistor, R, and capacitor, C, must be measured using the DMM. Do so and record the values in Table 1.

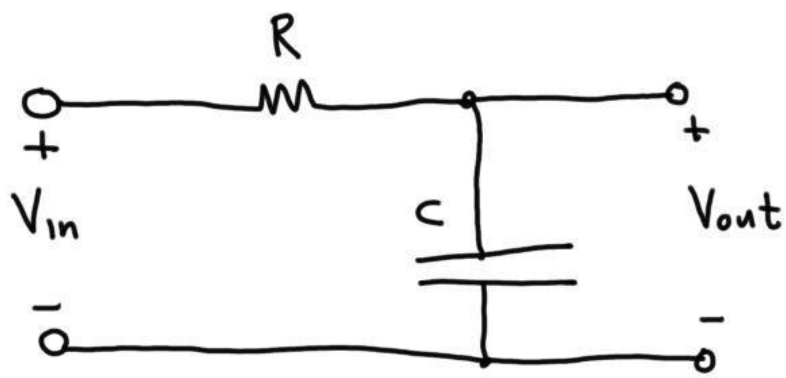


Figure 1. Unloaded Passive Low Pass Filter

	Theoretical Value	Experimental Value
R	1000 Ω	992
C	100 nF	97.5

Table 1. Component Values for the Unloaded Passive Low Pass Filter

b) Hand Calculations: Finding the Cutoff Frequency   Score: ____/10

Using the resulting equation for the transfer function, $H(s)$, from the prelab calculations, follow the steps described below to calculate the cutoff frequency, f_c, for the circuit.

1) Change from Laplace space to frequency space using the relation $s = j\omega$ and find the magnitude of the transfer function, $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right|$, and the angle of the transfer function, $\angle H(j\omega) = \angle(\theta_{out} - \theta_{in})$. These will both be functions of ω.

$$H(s) = \frac{1}{1+sRC}$$

$$H(j\omega) = \frac{1}{1+j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad \angle H(j\omega) = 0 - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= -\tan^{-1}\left(\frac{\omega RC}{1}\right)$$

Answers: $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$

$\angle H(j\omega) = \angle(\theta_{out} - \theta_{in}) = -\tan^{-1}(\omega RC)$

2) Using the magnitude of the transfer function, $|H(j\omega)|$, from the previous calculation, find the maximum value of the transfer function, H_{max}.

let $\omega = 0$:

$$H_{max} = \frac{1}{\sqrt{1+[(0)RC]^2}} = \frac{1}{\sqrt{1+0}} = 1$$

Answer: $H_{max} = 1$

3) Using the results from steps 1) and 2) above use the definition shown below to find the cutoff angular frequency, ω_c.

$$\left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}H_{max}$$

$$|H(j\omega_c)| = \frac{1}{\sqrt{1+(\omega_c RC)^2}}$$

$$\frac{1}{\sqrt{2}}H_{max} = \frac{1}{\sqrt{1+(\omega_c RC)^2}}$$

$$1+(\omega_c RC)^2 = \frac{1}{\left[\left(\frac{1}{\sqrt{2}}\right)(1)\right]^2}$$

$$\omega_c RC = \sqrt{2-1} = 1$$

$$\omega_c = \frac{1}{RC}$$

Answer: $\omega_c = \frac{1}{RC}$

4) Using the cutoff angular frequency found in step d) above, evaluate the phase shift at the cutoff angular frequency by calculating $\angle H\left(j\omega_c\right)$. Lastly, use the relation $\omega_c = 2\pi f_c$, calculate the cutoff frequency, f_c.

$$\angle H(j\omega_c) = -\tan^{-1}(\omega_c RC)$$

$$= -\tan^{-1}\left(\frac{RC}{RC}\right) = -\tan^{-1}(1)$$

$$= -45^\circ$$

$$\omega_c = 2\pi f_c$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1/RC}{2\pi} = \frac{1}{2\pi RC}$$

Answers:

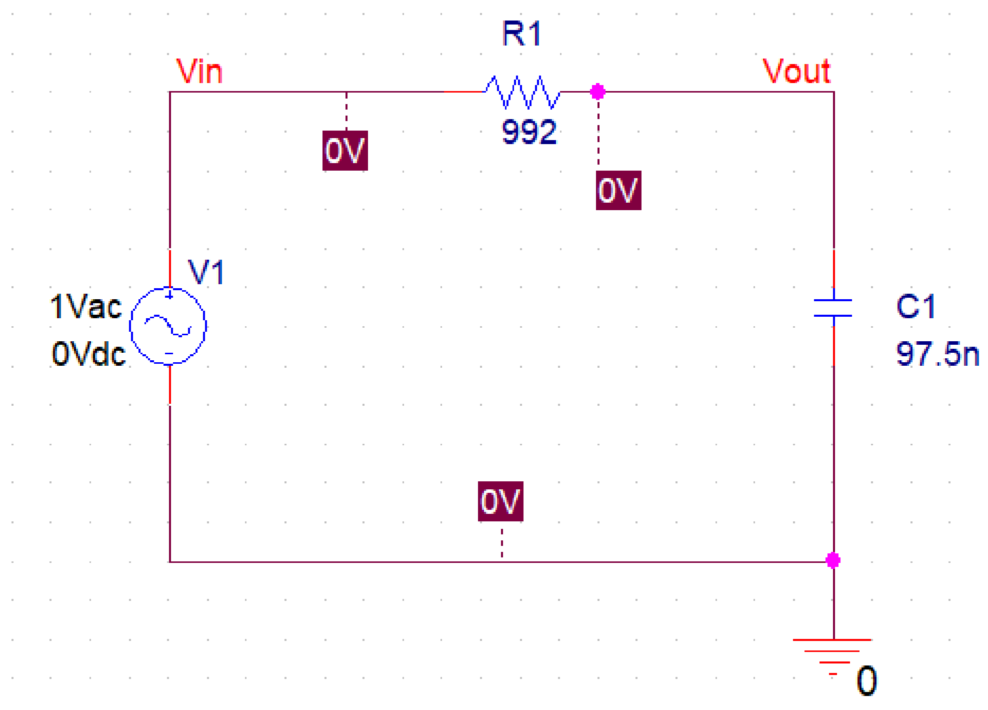
$$\angle H\left(j\omega_c\right) = \angle\left(\theta_{out} - \theta_{in}\right) = -45°$$

$$f_c = \frac{1}{2\pi RC}$$

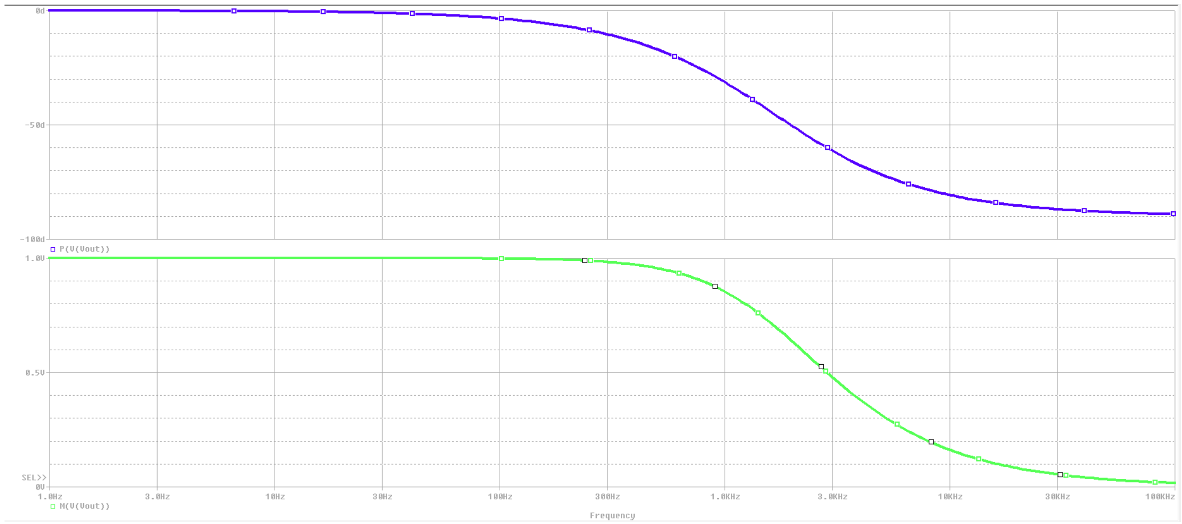
c) Prediction: Simulation Score: ____/15

Using the experimentally found values of the resistor, R, and capacitor, C in part a), run an AC Sweep simulation using PSpice of the circuit shown in Figure 1 in order to find the cutoff frequency. Adjust the source for 1 Volt with 0° phase shift. For sweep type, select decades. The sweep parameters should be 50-points/decade, with a start frequency of 1Hz and an end frequency of 100KHz. For the first trace, use the magnitude of the output voltage. Select M (for magnitude) from the functions and V(VOUT) from the variables. The next trace will be the phase. Since the values of the magnitude and phase are so different, a different plot should be used for the phase. Use ADD new plot (under PLOT on the tool bar.) In the new plot add P[V(VOUT)]. If the simulation is correct this RC circuit should have a magnitude of 0.707 and a phase shift of -45° at $\omega = 1$ krps ($f = 1000/2\pi$ Hz.) Note that PSPICE plots the graphs in Hertz and not radians per second. Include pictures of 1) the circuit schematic and 2) the output graph of the simulation below. What according to your plots, what is the cutoff frequency and what is the phase shift at the cutoff frequency?

1) Circuit Schematic:



2) Output:



Trace Color	Trace Name	Y1	Y2	Y1 - Y2
	X Values	1.6562K	1.0000	1.6552K
CURSOR 1,2	M(V(Vout))	704.818m	1.0000	-295.182m
	P(V(Vout))	-44.948	-34.819m	-44.913

Answers: Cutoff frequency, $f_C = 1.6562$k

Phase shift at cutoff frequency, $\angle H\left(j\omega_c\right) = \angle\left(\theta_{out} - \theta_{in}\right)$ = -44.948°

d) Data Collection and Analysis Score: ____/15

Build the circuit shown in Figure 1 and fill in Tables 2a-c below. Show your work for the calculations of the theoretical values on a separate sheet of paper.

	R (Ω)	C (μF)
Theoretical Values	1000	0.1
Experimental Values	992	0.0975
% Error	0.8%	2.5%

Table 2a. Preliminary Measurements for Unloaded Passive LPF

<u>For Low Frequency Values:</u> $f = 100\ Hz$

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	1	0	- 3.48
Experimental Values	1.065	1.065	0	- 3.87
% Error	6.10%	6.10%	0%	10.07%

Table 2b. Low Frequency Measurements for Unloaded Passive LPF

<u>At the Cutoff Frequency:</u> f_C = **<u>1645.52 Hz</u>**

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	0.7071	0.7071	-45
Experimental Values	1.045	0.755	0.7225	-44.06
% Error	4.5 %	6.77 %	2.18 %	2.09 %

Table 2c. Measurements at the Cutoff Frequency for Unloaded Passive LPF

CALCULATIONS:

$$\% \text{ Error} = \frac{|\text{Experimental} - \text{Theoretical}|}{\text{Theoretical}} \times 100$$

RESISTOR ; $\% \text{ Error} = \dfrac{|992\Omega - 1000\Omega|}{1000\Omega} \times 100 = 0.8\,\%$

CAPACITOR ; $\% \text{ Error} = \dfrac{|0.0975 - 0.1|}{0.1} \times 100 = 2.5\,\%$

FOR LOW FREQUENCY VALUES : $f = 100\text{ Hz}$

$$\omega = 2\pi f = 2\pi(100) = 200\pi$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+[(200\pi)(992)(97.5\times10^{-9})]^2}}$$

$$\left|\frac{V_{out}}{1}\right| = 0.99816$$

$$V_{out} = 0.99816\text{ V} \approx 1\text{ V}$$

GAIN: $\dfrac{V_{out}}{V_{in}} = \dfrac{1}{1} = 0$

PHASE SHIFT: $\angle H(j\omega) = -\tan^{-1}(\omega RC)$

$$= -\tan^{-1}[(200\pi)(992)(97.5\times10^{-9})]$$

$$= -3.48^\circ$$

AT THE CUTOFF FREQUENCY :

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(992)(97.5\times10^{-9})} = 1645.52\text{ Hz}$$

$$\omega_c = \frac{1}{RC} = \frac{1}{(992)(97.5\times10^{-9})} = 10\,339.12\text{ rad/s}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+[10339.12)(992)(97.5\times10^{-9})]^2}} = 0.7071$$

$$\left|\frac{V_{out}}{1}\right| = 0.7071$$

$$V_{out} = 0.7071\text{ V}$$

GAIN: $\dfrac{V_{out}}{V_{in}} = \dfrac{0.7071}{1} = 0.7071$

PHASE SHIFT : -45

Experimental output: Include an image both channels of the oscilloscope at the cutoff frequency

Question: How does this compare to the simulated results?

The simulation and the experimental output showed very similar results. The simulated values are also similar to the theoretical values.

2. Loaded Passive Low Pass Filter Total Score: ____/45

a) Preliminary Measurements: Score: ____/5

This section of the lab involves the circuit shown in Figure 2 below. Before simulating or building the circuit, the values of the resistor, R, R_L, and capacitor, C, must be measured using the DMM. Do so and record the values in Table 3.

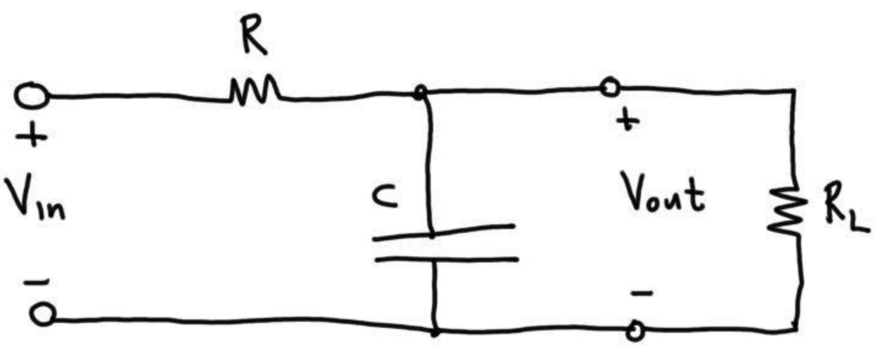


Figure 2. Loaded Passive Low Pass Filter

	Theoretical Value	Experimental Value
R	1000 Ω	992
R_L	1000 Ω	994
C	100 nF	97.5

Table 3. Component Values for the Loaded Passive Low Pass Filter

b) Hand Calculations: Finding the Cutoff Frequency — Score: ____/10

Using the resulting equation for the transfer function, $H(s)$, from the prelab calculations, follow the steps described below to calculate the cutoff frequency, f_c, for the circuit.

1) Change from Laplace space to frequency space using the relation $s = j\omega$ and find the magnitude of the transfer function, $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right|$, and the angle of the transfer function, $\angle H(j\omega) = \angle\left(\theta_{out} - \theta_{in}\right)$. These will both be functions of ω.

$$H(s) = \frac{R_L}{sCRR_L + R + R_L} = \frac{1}{sCR + \frac{R}{R_L} + 1} = \frac{1}{sCR + \frac{R+R_L}{R_L}}$$

$$H(j\omega) = \frac{1}{j\omega CR + \frac{R+R_L}{R_L}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(\omega CR)^2 + \left(\frac{R+R_L}{R_L}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega CR}{\frac{R+R_L}{R_L}}\right) = -\tan^{-1}\left(\frac{\omega CRR_L}{R+R_L}\right)$$

Answers: $|H(j\omega)| = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{(\omega CR)^2 + (\frac{R+R_L}{R_L})^2}}$

$\angle H(j\omega) = \angle\left(\theta_{out} - \theta_{in}\right) = -\tan^{-1}\left(\frac{\omega CRR_L}{R+R_L}\right)$

2) Using the magnitude of the transfer function, $|H(j\omega)|$, from the previous calculation, find the maximum value of the transfer function, H_{max}.

let $\omega = 0$:

$$H_{max} = \frac{1}{\sqrt{(0 \cdot CR)^2 + \left(\frac{R+R_L}{R_L}\right)^2}} = \frac{1}{\frac{R+R_L}{R_L}} = \frac{R_L}{R+R_L}$$

Answer: $H_{max} = \frac{R_L}{R+R_L}$

3) Using the results from steps 1) and 2) above use the definition shown below to find the cutoff angular frequency, ω_c.

$$\left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}H_{max}$$

$$|H(j\omega_c)| = \frac{1}{\sqrt{(\omega_c CR)^2 + (\frac{R+R_L}{R_L})^2}}$$

$$\frac{1}{\sqrt{2}}H_{max} = \frac{1}{\sqrt{(\omega_c CR)^2 + (\frac{R+R_L}{R_L})^2}}$$

$$\frac{1}{2}H_{max}^2 = \frac{1}{(\omega_c CR)^2 + (\frac{R+R_L}{R_L})^2}$$

$$(\omega_c CR)^2 = \frac{2}{H_{max}^2} - (\frac{R+R_L}{R_L})^2$$

$$= \frac{2}{(\frac{R_L}{R+R_L})^2} - (\frac{R+R_L}{R_L})^2$$

$$(\omega_c CR)^2 = (\frac{R+R_L}{R})^2$$

$$\omega_c = \frac{R+R_L}{CR^2}$$

Answer: $\omega_c = \frac{R+R_L}{CR^2}$

4) Using the cutoff angular frequency found in step d) above, evaluate the phase shift at the cutoff angular frequency by calculating $\angle H(j\omega_c)$. Lastly, use the relation $\omega_c = 2\pi f_c$, calculate the cutoff frequency, f_c.

$$\angle H(j\omega_c) = -\tan^{-1}(\frac{\omega_c CRR_L}{R+R_L})$$

$$= -\tan^{-1}\left[(\frac{R+R_L}{CR^2})(\frac{CRR_L}{R+R_L})\right]$$

$$= -\tan^{-1}(R_L/R)$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}(\frac{R+R_L}{CR^2})$$

$$= \frac{R+R_L}{2\pi CR^2}$$

Answers:

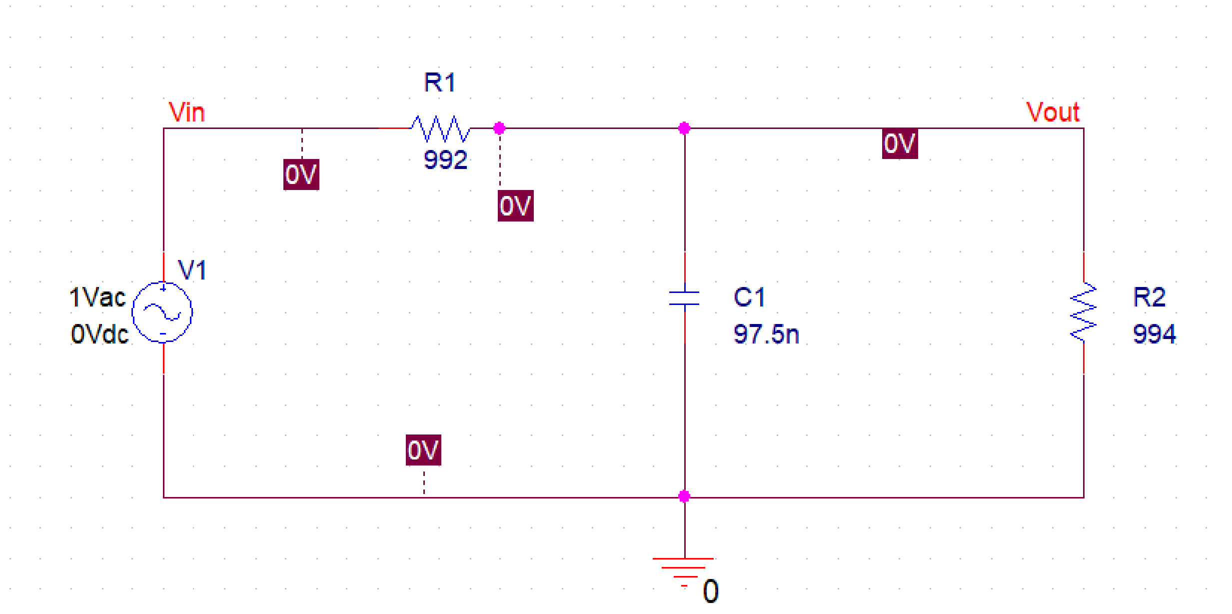
$$\angle H(j\omega_c) = \angle(\theta_{out} - \theta_{in}) = -\tan^{-1}(\frac{R_L}{R})$$

$$f_c = \frac{R+R_L}{2\pi CR^2}$$

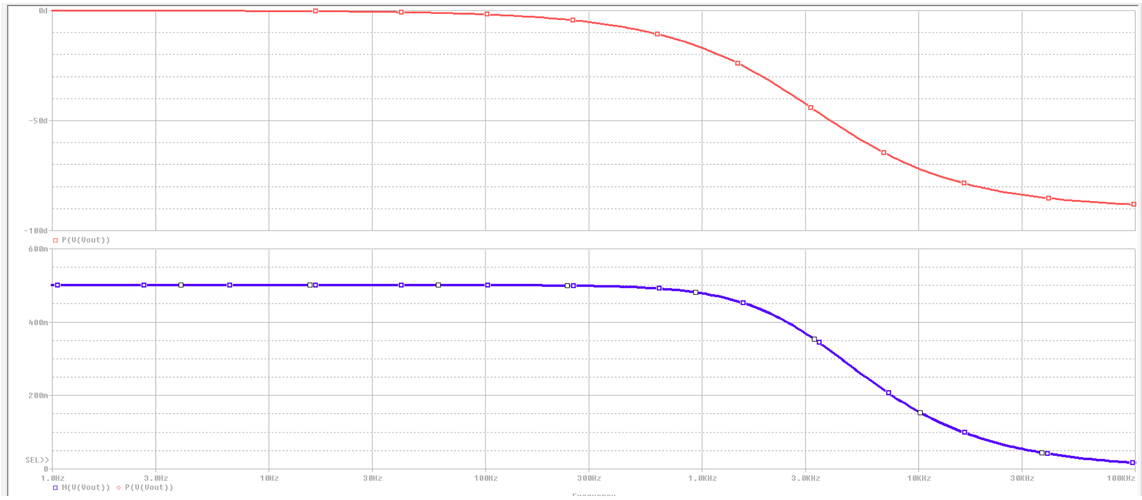
c) Prediction: Simulation Score: ____/15

Using the experimentally found values of the resistors, R and R_L, and capacitor, C in part a), run an AC Sweep simulation using PSpice of the circuit shown in Figure 2 in order to find the cutoff frequency. Adjust the source for 1 Volt with 0° phase shift. For sweep type, select decades. The sweep parameters should be 50-points/decade, with a start frequency of 1Hz and an end frequency of 100KHz. For the first trace, use the magnitude of the output voltage. Select M (for magnitude) from the functions and V(VOUT) from the variables. The next trace will be the phase. Since the values of the magnitude and phase are so different, a different plot should be used for the phase. Use ADD new plot (under PLOT on the tool bar.) In the new plot add P[V(VOUT)]. If the simulation is correct this RC circuit should have a magnitude of 0.707 and a phase shift of -45° at ω = 1 krps (f = 1000/2π Hz.) Note that PSPICE plots the graphs in Hertz and not radians per second. Include pictures of 1) the circuit schematic and 2) the output graph of the simulation below. What according to your plots, what is the cutoff frequency and what is the phase shift at the cutoff frequency?

1) Circuit Schematic:



2) Output:



	Trace Color	Trace Name	Y1	Y2	Y1 - Y2
		X Values	3.3087K	1.0000	3.3077K
		M(V(Vout))	352.779m	500.504m	-147.725m
	CURSOR 1,2	P(V(Vout))	-45.183	-17.427m	-45.165

Answer: Cutoff frequency, $f_C = 3.3087\text{k}$

Phase shift at cutoff frequency, $\angle H(j\omega_c) = \angle(\theta_{out} - \theta_{in}) = \text{-}45.183$

d) Data Collection and Analysis Score: ____/15

Build the circuit shown in Figure 2 and fill in Tables 4a-c below. Show your work for the calculations of the theoretical values on a separate sheet of paper.

	R (Ω)	R_L (Ω)	C (μF)
Theoretical Values	1000	1000	0.1
Experimental Values	992	994	0.0975
% Error	0.8 %	0.6 %	2.5 %

Table 4a. Preliminary Measurements for Loaded Passive LPF

For Low Frequency Values: $f = 100\ Hz$

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	0.5	0.5	-1.74
Experimental Values	1.045	0.545	0.52	-1.52
% Error	4.31%	8.26%	3.85%	12.64%

Table 4b. Low Frequency Measurements for Loaded Passive LPF

At the Cutoff Frequency: f_C = **3294.36 Hz**

	V_{in} (V)	V_{out} (V)	Gain	Phase shift(degrees)
Theoretical Values	1	0.3536	0.3536	-45.06
Experimental Values	1.035	0.390	0.3768	-45.43
% Error	3.38%	9.33%	6.16%	0.81%

Table 4c. Measurements at the Cutoff Frequency for Loaded Passive LPF

CALCULATIONS:

$$\% \text{ Error} = \frac{|\text{Experimental} - \text{Theoretical}|}{\text{Theoretical}} \times 100$$

RESISTOR: R $\quad \% \text{ Error} = \dfrac{|992\Omega - 1000\Omega|}{1000\Omega} \times 100 = 0.8\%$

R_L $\quad \% \text{ Error} = \dfrac{|994\Omega - 1000\Omega|}{1000\Omega} \times 100 = 0.6\%$

CAPACITOR: $\% \text{ Error} = \dfrac{|0.0975 - 0.1|}{0.1} \times 100 = 2.5\%$

FOR LOW FREQUENCY VALUES: $f = 100\text{ Hz}$

$$\omega = 2\pi f = 2\pi(100) = 200\pi$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{(\omega RC)^2 + \left(\frac{R+R_L}{R_L}\right)^2}} = \frac{1}{\sqrt{[(200\pi)(992)(97.5\times10^{-9})]^2 + \left(\frac{992+994}{994}\right)^2}}$$

$$\left|\frac{V_{out}}{1}\right| = 0.50$$

$$V_{out} = 0.50\text{ V}$$

GAIN: $\dfrac{V_{out}}{V_{in}} = \dfrac{0.50}{1} = 0.50$

PHASE SHIFT:
$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega C R R_L}{R+R_L}\right) = -\tan^{-1}\left[\frac{(200\pi)(97.5\times10^{-9})(992)(994)}{992+994}\right] = -1.74^\circ$$

AT THE CUTOFF FREQUENCY:

$$f_c = \frac{R+R_L}{2\pi C R^2} = \frac{992+994}{2\pi(97.5\times10^{-9})(992)^2} = 3294.36\text{ Hz}$$

$$\omega_c = \frac{R+R_L}{CR^2} = \frac{992+994}{(97.5\times10^{-9})(992)^2} = 20699.09\text{ rad/s}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{(\omega_c RC)^2 + \left(\frac{R+R_L}{R_L}\right)^2}} = \frac{1}{\sqrt{[(20699.09)(992)(97.5\times10^{-9})]^2 + \left(\frac{992+994}{994}\right)^2}} = 0.3536$$

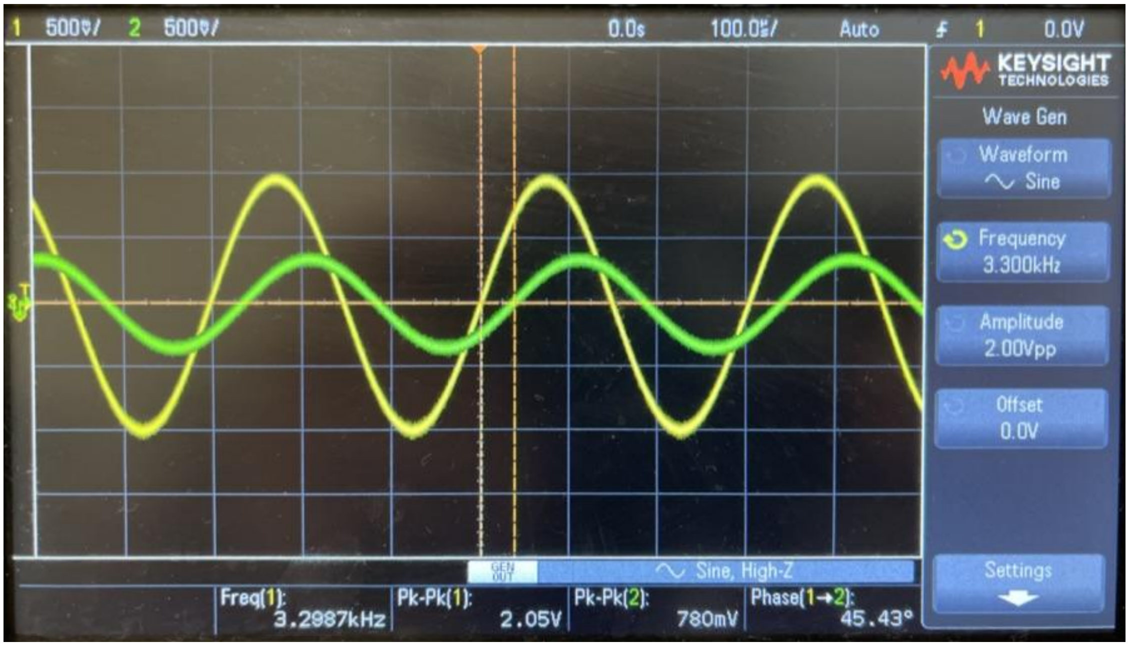
$$\left|\frac{V_{out}}{1}\right| = 0.3536$$

$$V_{out} = 0.3536\text{ V}$$

GAIN: $\dfrac{V_{out}}{V_{in}} = \dfrac{0.3536}{1} = 0.3536$

PHASE SHIFT:
$$\angle H(j\omega_c) = -\tan^{-1}(R_L/R) = -\tan^{-1}(994/992) = -45.06^\circ$$

Experimental output: Include an image both channels of the oscilloscope at the cutoff frequency



Question: How does this compare to the simulated results?

The experimental output is very similar to the simulated results. The experimental values are also similar to the theoretical values.

3. Conclusions Total Score: /10

Explain in a few paragraphs the purpose of the lab, the experimental set up and methodology, and central results of the lab and these experiments. **You should be quantitative** in this summary. Include any important equations used and explain their significance. Write the conclusion as if you were writing an English essay. This is an important portion of the lab, so make sure to do a good and thorough job.

In this lab, we had to do hand calculations to find the cutoff frequency. First we had to use the transfer function based on the prelab. Once we found the magnitude, we had to find the maximum value of the transfer function H_{max}. Using the results from the previous equations, we plugged it into the formula, $\left|H(j\omega_c)\right| = \frac{1}{\sqrt{2}}H_{max}$, to find the cutoff angular frequency. From the answer we got from the cutoff frequency, we can now find the phase shift at the cutoff frequency. Simulating the circuits and building it on the breadboard we were already familiar with since we have done it for a lot of the labs. The hardest part this time for us was doing the hand calculations.