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CSC28 – 01 22SU — 18 JUNE 2022

Q2. $\sim(p \wedge q) \vee (\sim p \wedge q) \equiv \sim p \vee \sim q$

LHS: $\sim(p \wedge q) \vee (\sim p \wedge q)$

$\equiv (\sim p \vee \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \vee (\sim p \wedge q) \vee \sim q \vee (\sim p \wedge q)$

$\equiv \sim p \vee [(\sim q \vee \sim p) \wedge (\sim q \vee q)]$

$\equiv \sim p \vee (\sim q \vee \sim p \wedge T)$

$\equiv \sim p \vee \sim q \vee \sim p$

$\equiv \sim p \vee \sim q$

Q3. $(p \wedge \sim q) \vee (p \wedge q) \equiv p$

LHS: $(p \wedge \sim q) \vee (p \wedge q)$

$\equiv p \vee (p \wedge q) \wedge \sim q \vee (p \wedge q)$

$\equiv p \wedge [(\sim q \vee p) \wedge (\sim q \vee q)]$

$\equiv p \wedge [(\sim q \vee p) \wedge T]$

$\equiv p \wedge (\sim q \vee p) \equiv p \wedge (p \vee \sim q)$

$\equiv p$

Q4. If his gpa is above 4.0, then he will go to a top university

p : gpa is above 4.0

q : he will go to a top university

- symbolically : $p \longrightarrow q$

CONTRAPOSITIVE :

- words : If he didn't go to a top university, then is below 4.0
- symbolically : $\sim q \longrightarrow \sim p$

INVERSE :

- words : If his gpa is below 4.0, then he didn't go to a top university
- symbolically : $\sim p \longrightarrow \sim q$

CONVERSE:

- words: If he will go to a top university, then his gpa is above 4.0
- symbolically: $q \rightarrow p$

Q5. p: It is sunny

q: I will go to watch a movie

r: I will buy a popcorn

s: there is good movie

Hypothesis: $\sim p \vee q$

$\sim q \vee r$

$\underline{\sim r}$

CONCLUSION: $\because \sim p \vee \sim s$

STEPS:

① $\sim p \vee q \equiv p \rightarrow q$

② $\sim q \vee r \equiv q \rightarrow r$

③ $p \rightarrow q$

$\underline{q \rightarrow r}$

$\because p \rightarrow r$

④ $p \rightarrow r \equiv \sim p \vee r$

⑤ $\sim p \vee r$

$\underline{\sim r}$

$\because \sim p$

⑥ $\underline{\sim p}$

$\because \sim p \vee \sim s$

Q6. p: I study

q: I will not fail Mathematics

r: I play basketball

Hyph: $p \rightarrow q$

$\sim r \rightarrow p$

$\underline{\sim q}$

Con: $\because r$

STEPS:

① $\sim r \rightarrow p$

$\underline{p \rightarrow q}$

$\because \sim r \rightarrow q$

② $\sim r \rightarrow q \equiv r \vee q$

③ $r \vee q$

$\underline{\sim q}$

$\because r$

Q7. p: x is a doctor

q: x is rich

(1) All doctors are rich

(a) Domain: All people

$\forall x \in D \quad p(x) \rightarrow q(x)$

(b) Domain: All doctors

$\forall x \; q(x)$

(2) Someone who is rich is a doctor

Domain: All doctors

$\exists x \; p(x) \wedge q(x)$

Q8. (1) $\forall x \forall y\ O(x,y)$

For all x and y siblings, x is older than y.

(2) $\exists x \exists y\ O(x,y)$

For some x and y siblings, x is older than y.

Q9 $P(x)$: x is a Math professor

$q(x)$: x teaches discrete math

Domain : all people

(1) $\forall x\,(p(x) \longrightarrow q(x))$

All Math professor teaches discrete math.

(2) $\exists x\,(p(x) \longrightarrow q(x))$

Some Math professor teaches discrete math.

Q10 $p = F,\ q = T,\ r = F$

(1) $q \wedge r \longrightarrow p \vee q$

$\equiv T \wedge F \longrightarrow F \vee T$

$\equiv F \longrightarrow T$

$\equiv \boxed{T}$

(2) $p \longrightarrow \sim p \wedge (r \vee \sim p)$

$\equiv F \longrightarrow \sim F \wedge (F \vee \sim F)$

$\equiv F \longrightarrow T \wedge T$

$\equiv F \longrightarrow T \quad \equiv \boxed{T}$

Q11. $[p \longrightarrow (q \longrightarrow r)] \longleftrightarrow [(p \wedge q) \longrightarrow r]$

$\equiv [\sim p \vee (\sim q \vee r)] \longleftrightarrow [(\sim p \vee \sim q) \vee r]$

$\equiv (\sim p \vee \sim q \vee r) \longleftrightarrow (\sim p \vee \sim q \vee r)$

$\equiv [(\sim p \vee \sim q \vee r) \longrightarrow (\sim p \vee \sim q \vee r)] \wedge$
$[(\sim p \vee \sim q \vee r) \longrightarrow (\sim p \vee \sim q \vee r)]$

$\equiv [(p \wedge q \wedge \sim r) \vee (\sim p \vee \sim q \vee r)] \wedge$
$[(p \wedge q \wedge \sim r) \vee (\sim p \vee \sim q \vee r)]$

let $x = \sim p \vee \sim q \vee r$

$\equiv x \longleftrightarrow x$

$\equiv (x \longrightarrow x) \wedge (x \longrightarrow x)$

$\equiv (\sim x \vee x) \wedge (\sim x \vee x)$

$\equiv T \wedge T$

$\equiv \boxed{T}$