CALIFORNIA STATE UNIVERSITY
SACRAMENTO

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EEE 117 Network Analysis

Text: Electric Circuits by J. Nilsson and S. Riedel Prentice Hall

Lecture Set 6: Active Filters, Single & Three Phase System

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Active Filters, Single & Three Phase System

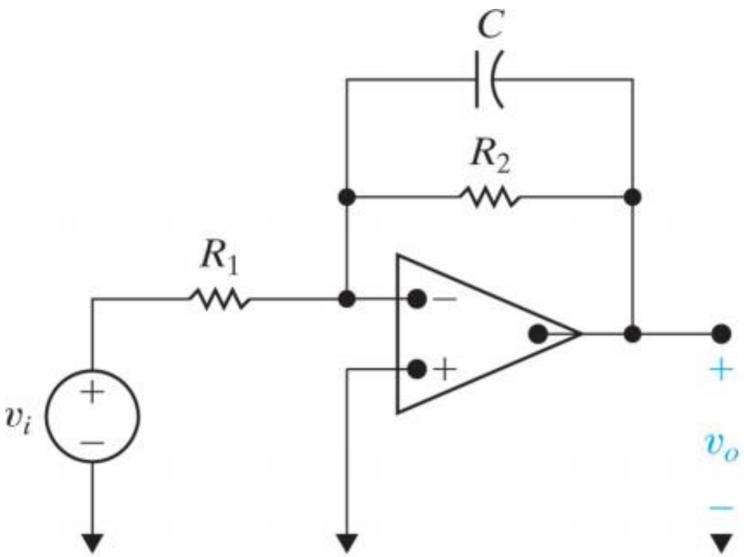
- What is Active Filter
- Active Low-Pass Filter
- Active High-Pass Filter
- Single Phase System
- Generation of Single Phase AC Source
- Introduction of 3-Phase System
- Types of 3-Phase System
- Principles of generation of 3-phase
- Phase difference angle
- Phase Sequence
- Connection in 3-phase system
- Star Connection Analysis
- Delta Connection Analysis
- 4- Scenarios of Source and Load Connection

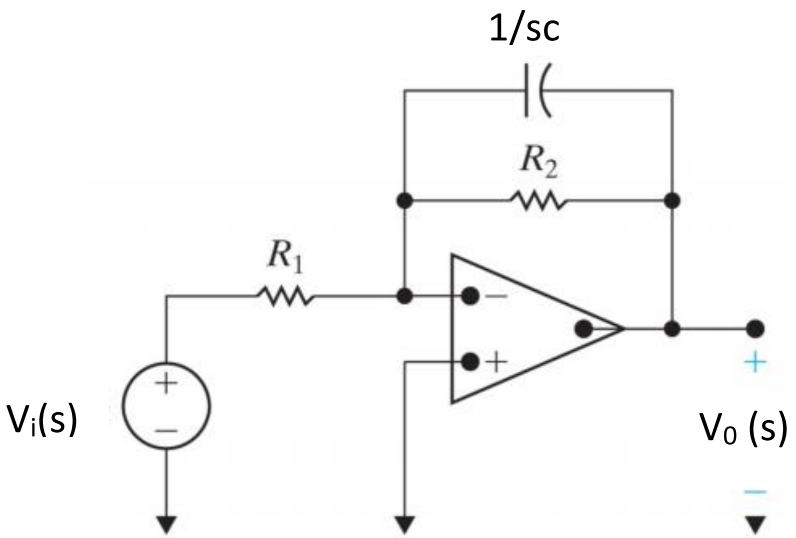
Active Filter Circuit

- In the passive filter circuits, the maximum magnitude of a transfer function was 1.
- In the active filter circuits, the maximum magnitude of a transfer function can be more than 1.
- By the use of Opamps, we can create an active filter circuit.
- Active circuits can be designed for both to amplify and attenuate an input signal by some significant amount.
- In this chapter we will be limited to first-order circuits, that is simple low-pass and high-pass active circuits.

Active Low-Pass Filter

- A common active low pass filter circuit is shown here.





- The node equation at node "n" is

$$\frac{v_n - V_i}{R_1} + i_n + \frac{v_n - V_0}{R_2 \,||\, Z_c} = 0$$

$$\Rightarrow v_n\left(\frac{1}{R_1} + \frac{1}{R_2 \,||\, Z_c}\right) + V_i\left(\frac{-1}{R_1}\right) + V_0\left(\frac{-1}{R_2 \,||\, Z_c}\right) + i_n = 0$$

- For an ideal opamp:

$$v_n = v_p \qquad i_n = i_p = 0 \qquad -V_{CC} \le V_0 \le +V_{CC}$$

where V_{CC} is the power supplies

In this circuit, $v_n = v_p = 0$. Thus

$$\frac{V_0}{R_2 \| Z_c} = V_i\left(\frac{-1}{R_1}\right) \qquad \Rightarrow \frac{V_0}{V_i} = \frac{-\left(R_2 \| Z_c\right)}{R_1}$$

$$H(s) = \frac{V_0}{V_i} = \frac{-\left(R_2 \| Z_c\right)}{R_1} = -\frac{Z_{feedback}}{Z_{input}}$$

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{-\left(R_2 \| Z_c\right)}{R_1} = \frac{-\left(\dfrac{R_2 \dfrac{1}{sC}}{R_2 + \dfrac{1}{sC}}\right)}{R_1} = \left(-\frac{R_2}{R_1}\right)\frac{\dfrac{1}{sC}}{R_2 + \dfrac{1}{sC}}$$

$$\boxed{H(s) = \frac{V_0(s)}{V_i(s)} = \left(-\frac{R_2}{R_1}\right)\frac{1}{sR_2C+1} = \left(-\frac{R_2}{R_1}\right)\frac{\frac{1}{R_2C}}{s + \frac{1}{R_2C}}}$$

$$H(j\omega) = -\frac{R_2}{R_1} * \frac{\frac{1}{R_2C}}{j\omega + \frac{1}{R_2C}}$$

$$H(j\omega) = -\frac{R_2}{R_1} \times \frac{\frac{1}{R_2C}}{\sqrt{\left(\frac{1}{R_2C}\right)^2 + \omega^2} \angle \tan^{-1} \omega R_2C}$$

$$H(j\omega) = \frac{R_2}{R_1} \angle 180 * \frac{\frac{1}{R_2C} \angle 0}{\sqrt{\left(\frac{1}{R_2C}\right)^2 + \omega^2} \angle \tan^{-1} \omega R_2C}$$

Hence

<u>Magnitude:</u>

$$|H(j\omega)| = \frac{R_2}{R_1} \times \frac{\frac{1}{R_2C}}{\sqrt{(1/R_2C)^2 + \omega^2}}$$

<u>Phase Angle:</u>

$$\theta = 180 + 0 - \tan^{-1} \omega R_2C = 180 - \tan^{-1} \omega R_2C$$

FIND Corner frequency:

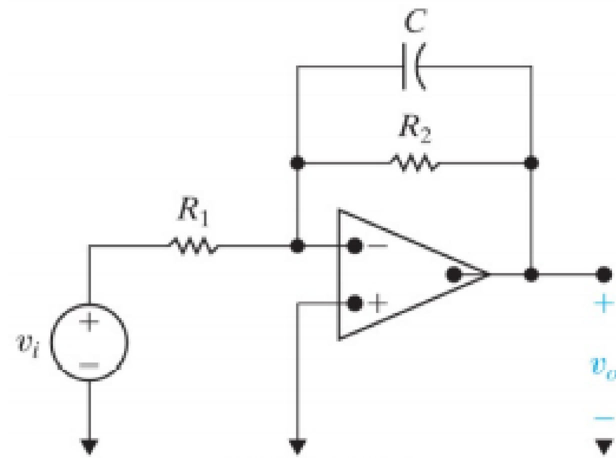
$$\left|H_{max}\right| = \frac{R_2}{R_1}$$ at ω=0 Hence at corner frequency

$$\left|H(j\omega)\right| = \left(\frac{1}{\sqrt{2}} * \frac{R_2}{R_1}\right) = \frac{R_2}{R_1} \left| * \frac{\frac{1}{R_2 C}}{j\omega + \frac{1}{R_2 C}} \right|$$ After solving we get

$$\boxed{\omega_c = \frac{1}{R_2 C} \qquad H(s) = \frac{V_0(s)}{V_i(s)} = (-K)\frac{\omega_c}{s+\omega_c} \qquad \text{gain is } K = \frac{R_2}{R_1}}$$

- The transfer function has a similar form but with a gain.
- The gain in the circuit is associated with phase of 180°.
- Note that the corner frequency can be set independently of the passband gain by adjusting R_2 and/or C.
- The phase angle at corner frequency = 180 - 45 = 135°

Example 1: Design an opamp based low-pass filter with a cutoff frequency of 2500 Hz and a passband gain of 5. Let C = 10 nF



The corner frequency was given in Hz. ω_C in radians is

$$\omega_C = 2\pi f = 2\pi(2,500Hz) = 5,000\pi \tfrac{rad}{sec}$$

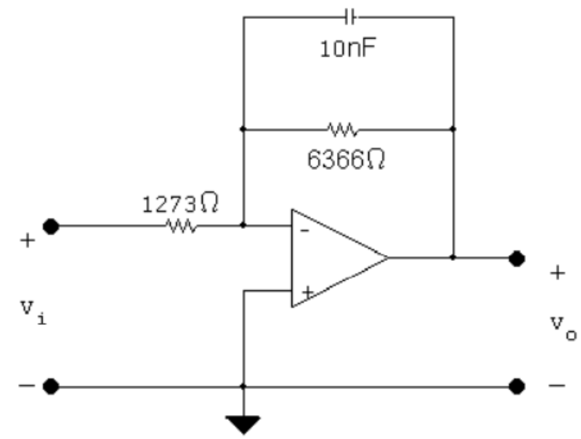
We can now find the feedback resistor R_2.

$$\omega_C = \frac{1}{R_2 C} \Rightarrow R_2 = \frac{1}{\left[5,000\pi \tfrac{rad}{sec}\right](10nF)} = 6,366.198\ \Omega$$

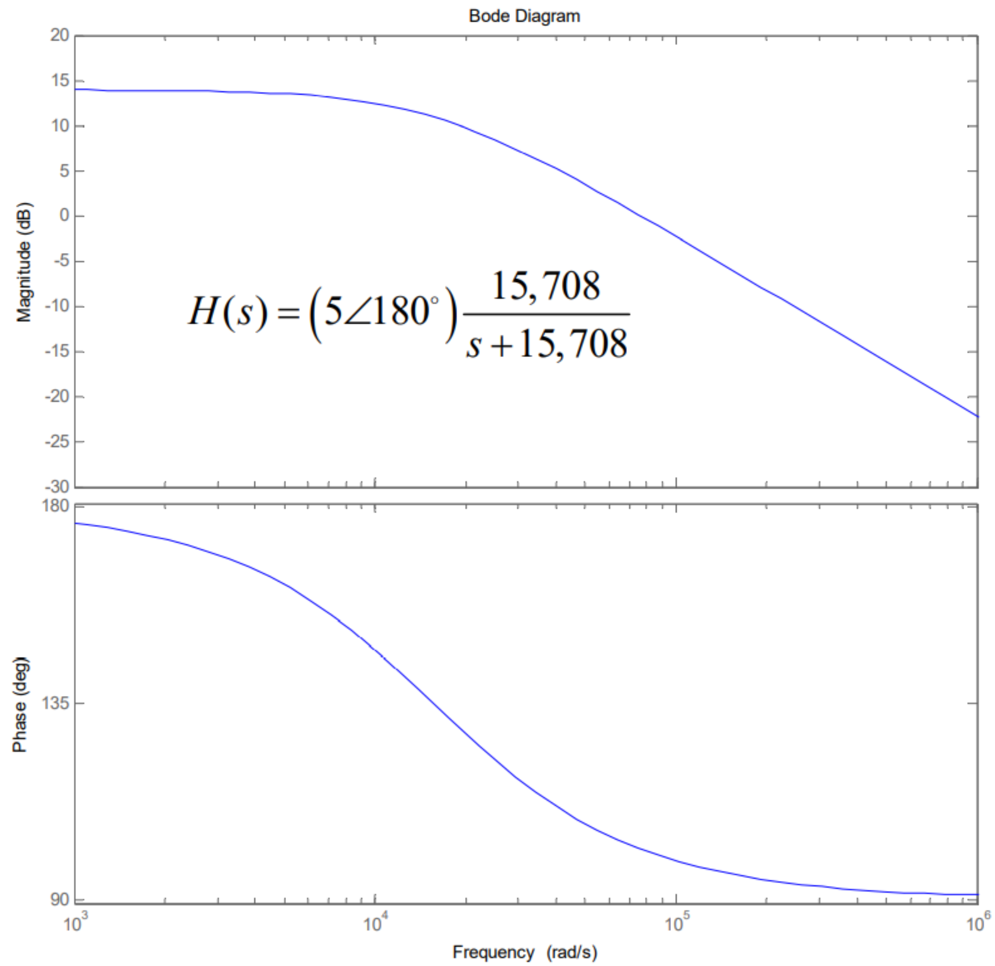
The input resistor is found from the desired gain K.

$$K = \frac{R_2}{R_1} = 5 \Rightarrow R_1 = \frac{R_2}{5} = \frac{6,366.198\Omega}{5} = 1,273.24\ \Omega$$

The low-pass filter is then



$$H(s) = \frac{V_0}{V_i} = (-5)\frac{15,708}{s+15,708} = \left(5\angle 180^\circ\right)\frac{15,708}{s+15,708}$$



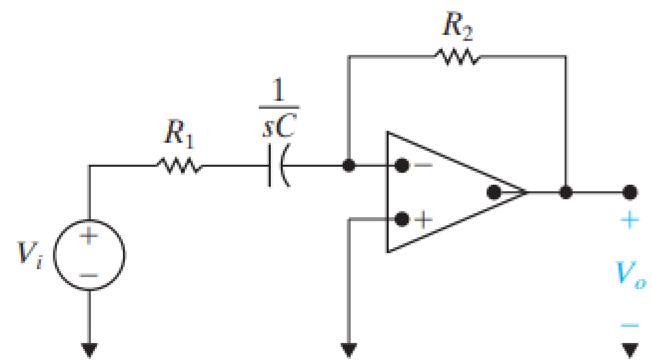
Note phase run from “+”180° rather than “-”180 °

Active High-Pass Filter

- A common active high pass filter circuit is shown here.

$$H(s) = \frac{-Z_f}{Z_i} = \frac{-R_2}{R_1 + \frac{1}{sC}} = -K\frac{s}{s + \omega_c}$$

where $K = \frac{R_2}{R_1}$ and $\omega_c = \frac{1}{R_1 C}$



- Note the difference in corner frequency between active low pass and active high pass filter.
- The phase angle at corner frequency = -135^{o}.
- Explanation is shown below.

$$H(s) = -\frac{R_2}{R_1 + \frac{1}{sC}} = -\frac{R_2}{R_1} * \frac{s}{s + \frac{1}{R_1C}}$$

$$H(j\omega) = -\frac{R_2}{R_1} * \frac{j\omega}{j\omega + \frac{1}{R_1C}}$$

$$H(j\omega) = \frac{R_2}{R_1}\angle 180 * \frac{\omega \angle \tan^{-1}\left(\frac{\omega}{0}\right)}{\sqrt{\left(\frac{1}{R_1C}\right)^2 + \omega^2} \;\angle \tan^{-1} \omega R_1 C}$$

Hence

Magnitude:

$$|H(j\omega)| = \frac{R_2}{R_1} * \frac{\omega}{\sqrt{\left(\frac{1}{R_1C}\right)^2 + \omega^2}}$$

$$|H_{max}| = \frac{R_2}{R_1} \quad @ \quad \omega = \infty$$

Phase:

$$\theta = 180 + 90 - \angle \tan^{-1} \omega R_1 C$$

Expression of Cutoff Frequency:

$$|H_{max}| = \frac{R_2}{R_1}$$

at Corner frequency $|H(j\omega)| = \frac{1}{\sqrt{2}} H_{max}$ so

$$\frac{R_2}{R_1} \times \frac{\omega}{\sqrt{\left(\frac{1}{R_1 C}\right)^2 + \omega^2}} = \frac{1}{\sqrt{2}} \cdot \frac{R_2}{R_1}$$

$$\frac{\omega^2}{\left(\frac{1}{R_1 C}\right)^2 + \omega^2} = \frac{1}{2}$$

$$\omega^2 = \left(\frac{1}{R_1 C}\right)^2 \Rightarrow \omega = \frac{1}{R_1 C}$$

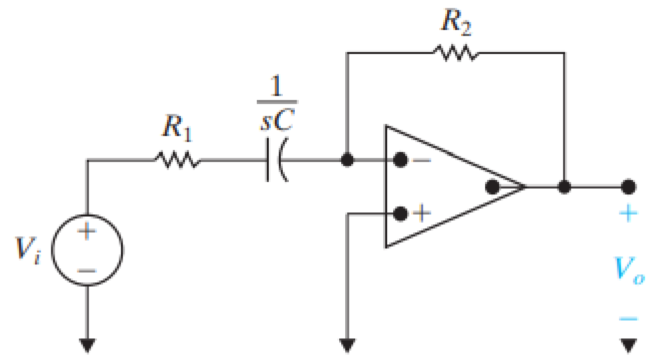
Hence

$$\boxed{\omega_c = \frac{1}{R_1 C}}$$

Phase @ Corner frequency:

$$\theta = 180 + 90 - 45 = 225 = \boxed{-135^\circ}$$

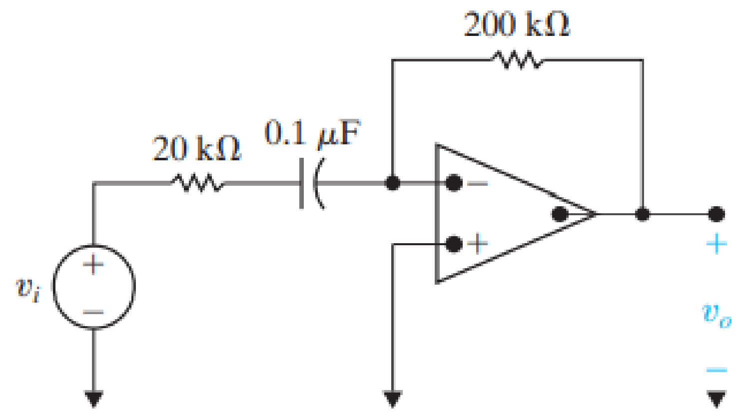
Example 2: Design an active high-pass filter for pass band gain K=10. Use a 0.1 uF capacitor and R1=20 k-Ω resistor. Use the following active high-pass filter circuit.



$R_2 = K*R_1 = 200$ k-Ω

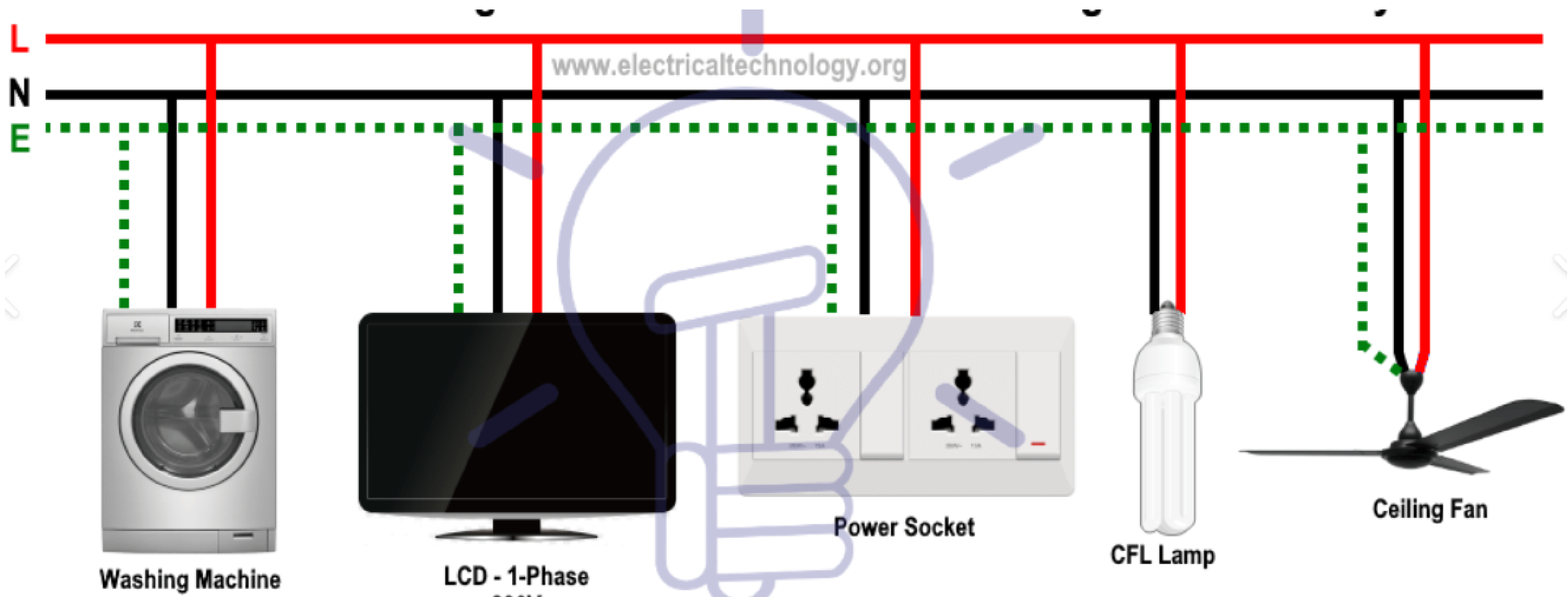
$\omega_c = 1/R_1C = 500$ rad/sec

$$H(s) = \frac{-10s}{s + 500}$$



Single Phase System

- Single phase system consists of one phase and one neutral line.
- It delivered electrical energy from power station to home.
- Single phase is used for small premises.
- In single phase supply the voltage varies sinusoidally with respect to time.

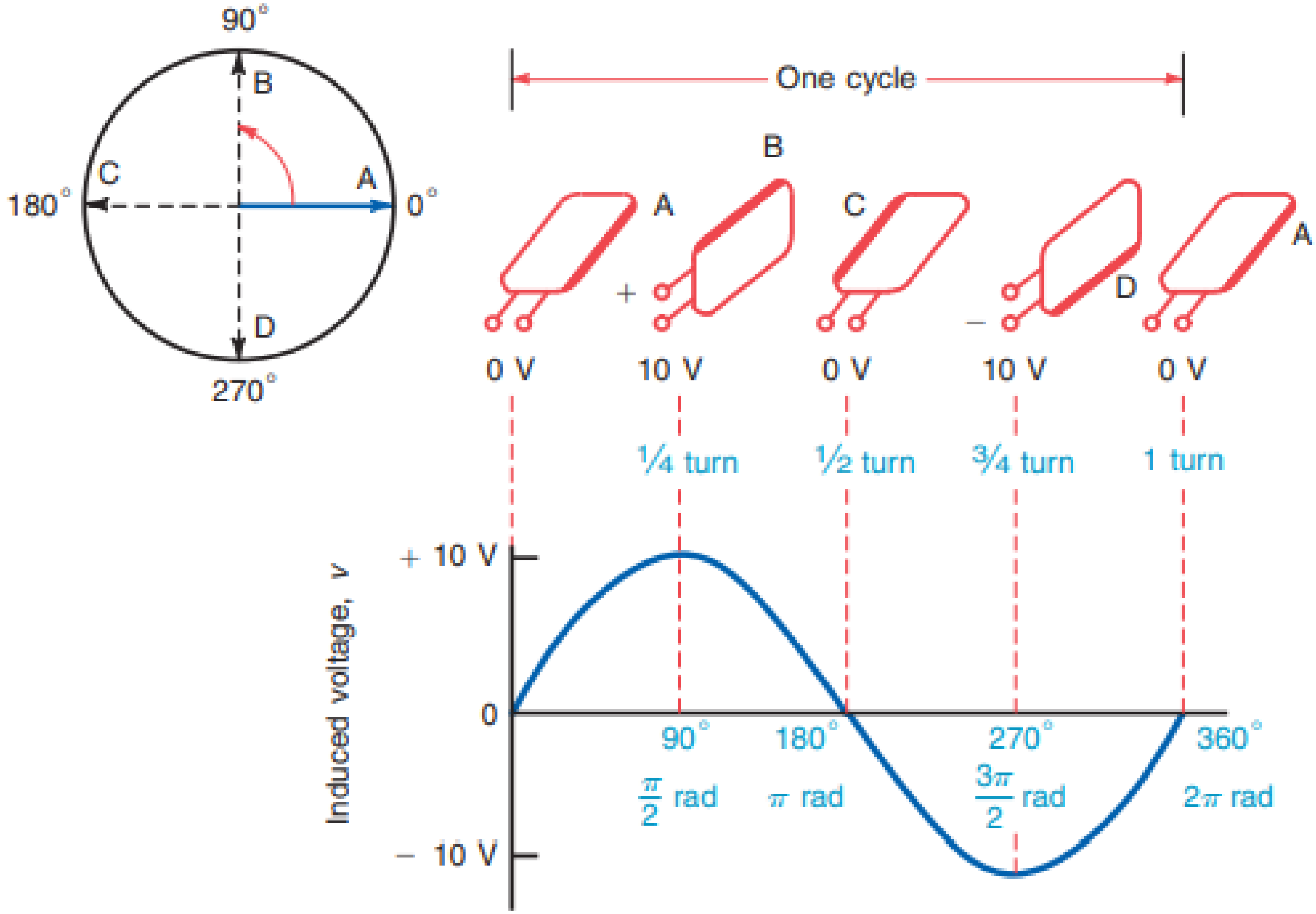


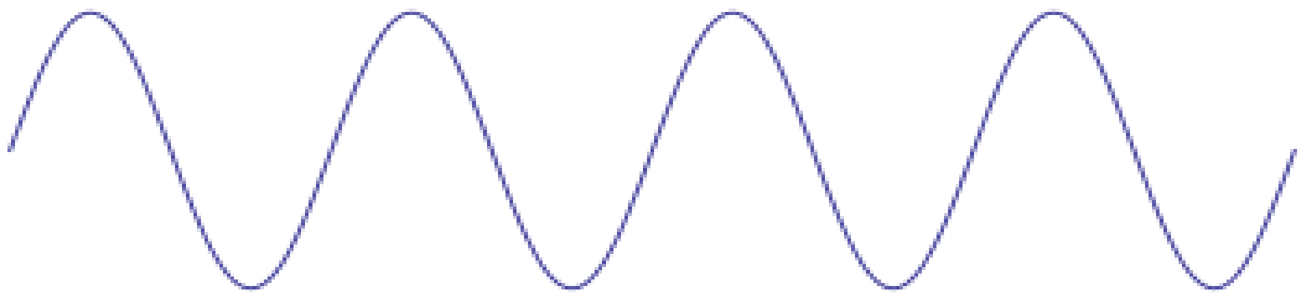
Single Phase AC Source

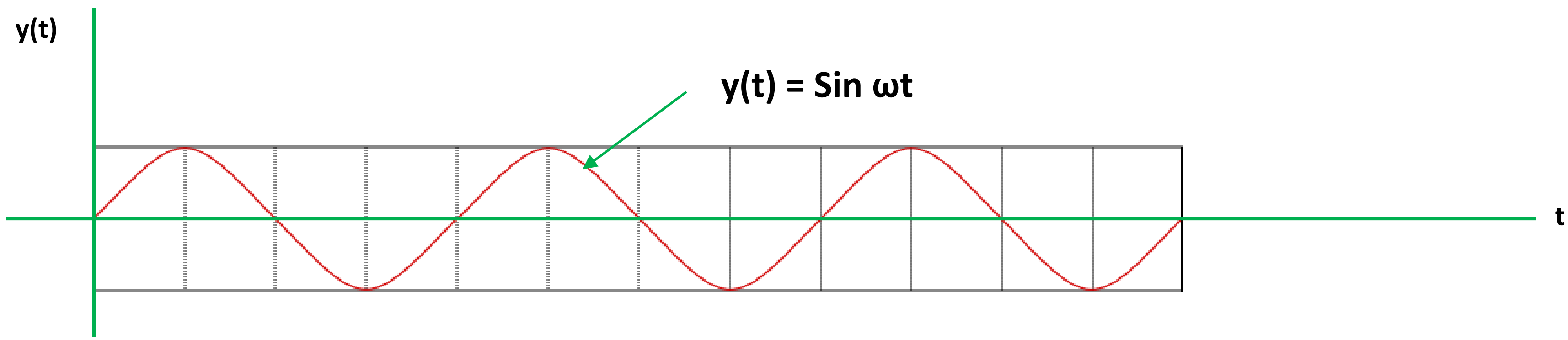
- A sinusoidal voltage or current source produces a voltage or current that varies sinusoidally with respect to time.

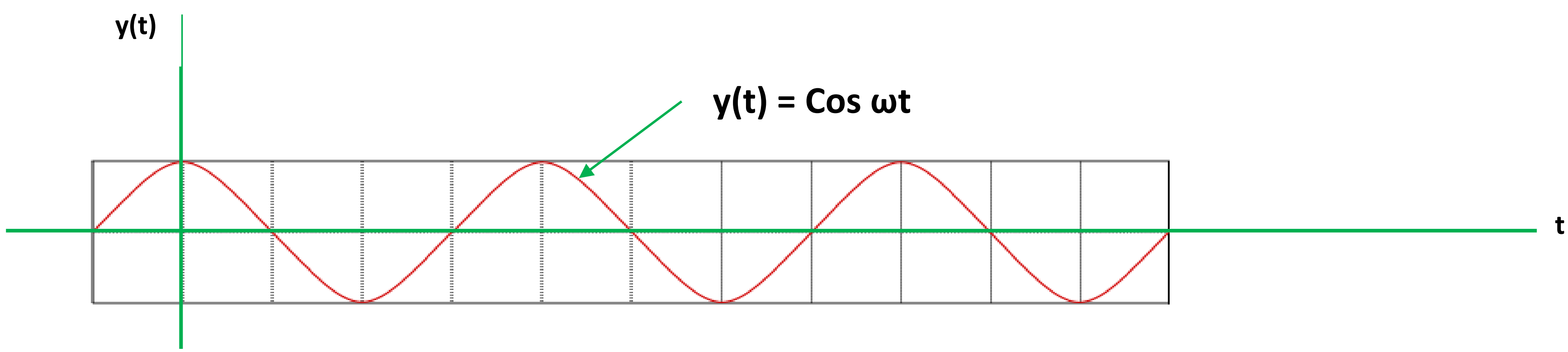
Principle of Generation of Single Phase AC Source:

- A relative motion of magnetic field and a conductor inside the field, induces an emf.
- Induced emf = $-N\, d\Phi_B/dt$, which in turn creates voltage and current.
- A loop rotating in a magnetic field, produce induced voltage "*v*" with alternating polarities.
- Loop conductors moving parallel to magnetic field results in zero voltage
- Loop conductors cutting across magnetic field produce maximum induced voltage.

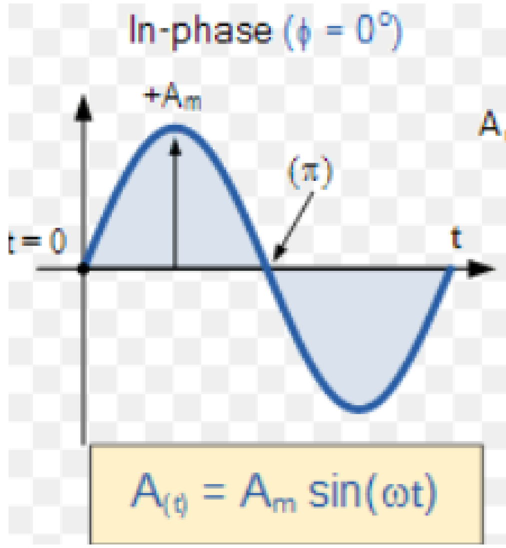

90°
B
C
A
180°
0°
D
270°
One cycle
B
A
C
D
A
+
−
0 V
10 V
0 V
10 V
0 V
1/4 turn
1/2 turn
3/4 turn
1 turn
Induced voltage, v
+ 10 V
0
− 10 V
90°
180°
270°
360°
$\frac{\pi}{2}$ rad
π rad
$\frac{3\pi}{2}$ rad
2π rad




y(t)
y(t) = Sin ωt
t


y(t)
y(t) = Cos ωt
t

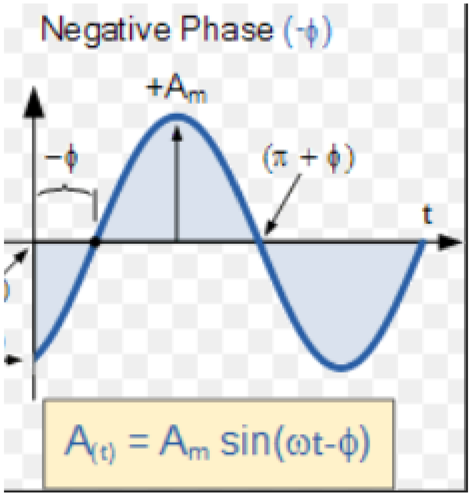
Case 1: (In Phase)



- Cycle starts at t=0, $\omega t = 0$ and emf=0 at t=0.
- emf values are constantly changing at every instant or at every angle from 0 to 2π.
- It attains three zero and two peak values in one complete cycle.
- Angular frequency of rotation = ω rad/s.
- Time period to complete one cycle (T) = $2\pi/\omega$.
- The wave form generated are sinusoidal.

$$\boxed{v(t) = V_m \text{ Sin } \omega t \qquad\qquad i(t) = I_m \text{ Sin } \omega t}$$

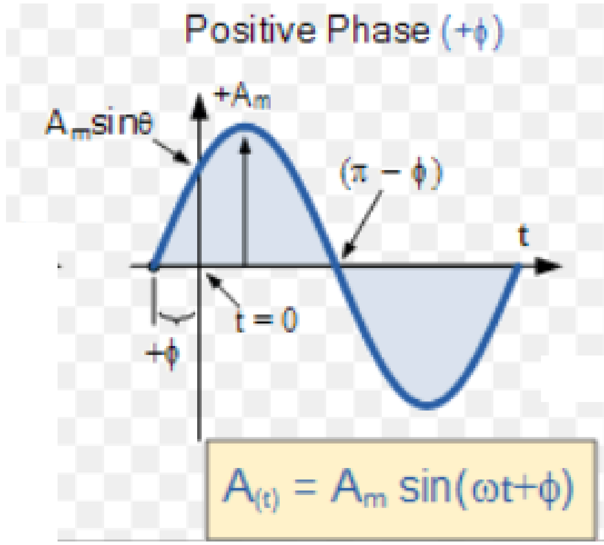
Case 2: (Negative Phase)



- At t=0, ωt = - ϕ and emf is not “0” at t=0.
- Same as case 1, emf values are constantly changing at every instant or at every angle.
- It attains three zero and two max values in one complete cycle.
- Angular frequency of rotation = ω rad/s.
- Time period to complete one cycle (T) = 2π/ω.
- The wave form generated are sinusoidal.

$$v(t) = V_m \text{Sin}(\omega t - \phi) \qquad i(t) = I_m \text{Sin}(\omega t - \phi)$$

Case 3: (Positive Phase)



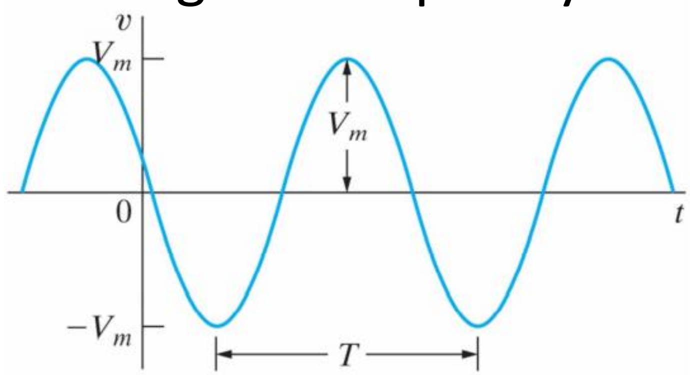
- At t=0, $\omega t = \phi$ and emf is not "0" at t=0.
- Same as case 1 & 2, emf values are constantly changing at every instant or at every angle.
- It attains three zero and two max values in one complete cycle.
- Angular frequency of rotation = ω rad/s.
- Time period to complete one cycle (T) = $2\pi/\omega$.
- The wave form generated are sinusoidal.

$$v(t) = V_m \, \text{Sin}(\omega t + \phi) \qquad i(t) = I_m \, \text{Sin}(\omega t + \phi)$$

- Sinusoidal signal can be represented either by Sine or Cosine function.

$v(t) = V_m \text{Sin}(\omega t \pm \phi)$	OR	$v(t) = V_m \text{Cos}(\omega t \pm \phi)$
$i(t) = I_m \text{Sin}(\omega t \pm \phi)$	OR	$i(t) = I_m \text{Cos}(\omega t \pm \phi)$

- Some prefer Sine function, while other prefer Cosine function to represent sinusoids.
- In this course we will use cosine function to represent the sinusoidal quantities.
- A typical sinusoidal voltage source is shown below:
- The period of the sinusoid function is T in seconds.
- The frequency is f (in Hz).
- The angular frequency is ω in radian/sec.



$$v = V_m \cos(\omega t + \phi)$$

$$f = \frac{1}{T}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Introduction of 3-Phase System

- For a single-phase system, we get zero value three times in one cycle.
- To get an uninterrupted supply, we need 3-phase system.
- We can create 3-phase for almost at the same cost as 1-phase.
- If we have 3-phase, in case of failure we can use other two phases.
- 3-phase is used essentially for commercial purposes where uninterrupted supply is required.
- 3-phase is a part of poly phase system.
- 2 and 4 phases can also be created but are not generated because 180 and 360 degrees phase shifts will nullify the effect.
- 5, 6 and 7 phases can also be created but not generated because it is very uneconomical.
- Practically, 3-phase is widely used in poly phase system.

Types of 3-Phase System

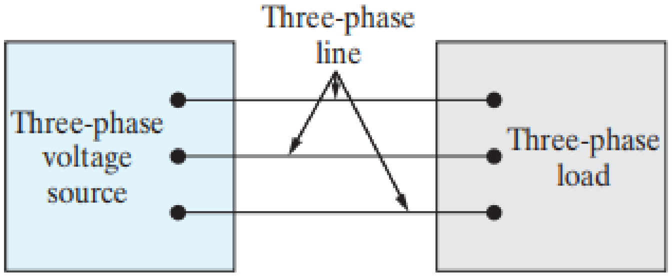
➢ **Symmetrical or Balance System:**

- Number of turns "N", flux, length and cross-sectional area are identical for all three conductors.
- Same phase angle difference among three conductors.
- Hence amplitudes of emf induced are same for three phases.

➢ **Un-Symmetrical or Un-Balance System:**

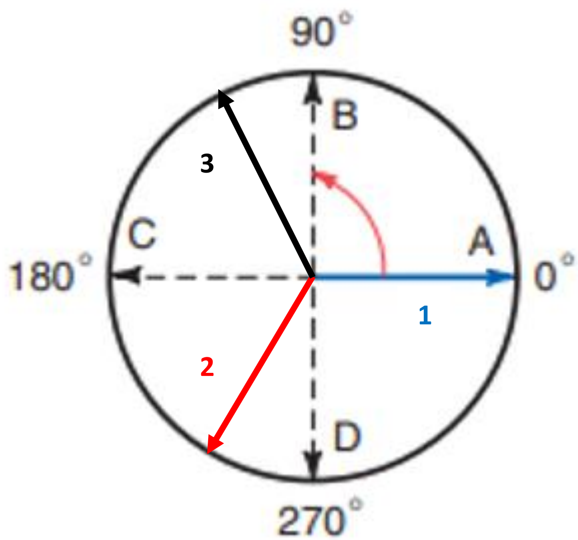
- if any of the conditions for balance system is not met, it is unbalanced.

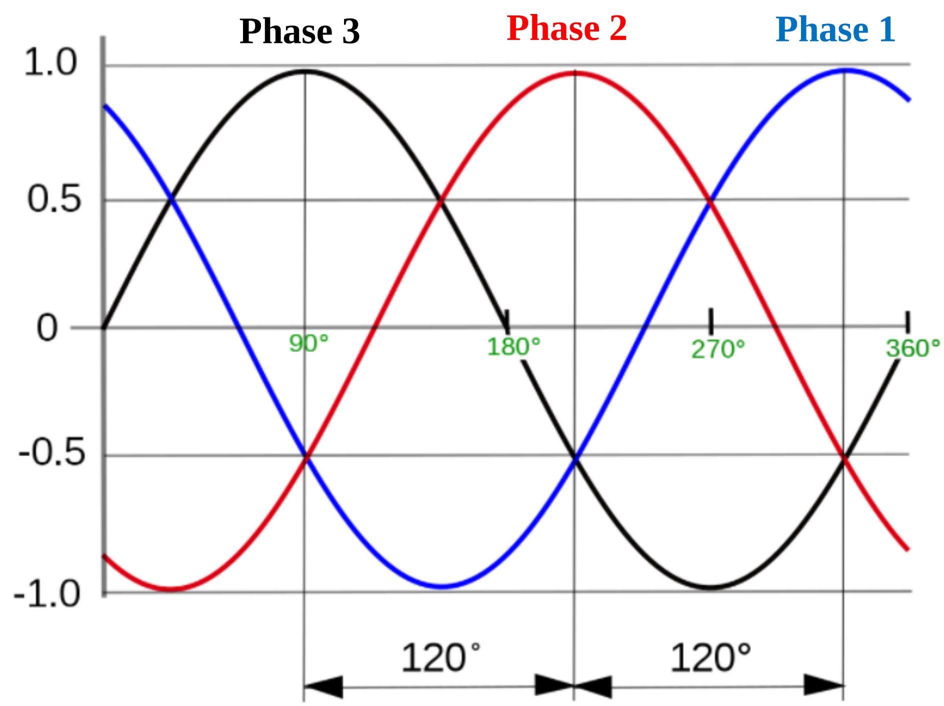
➢ In this course we will only study Symmetrical or Balance three phase system. A basic three-phase circuit is shown below.



Principles of Generation of 3-Phase System

- Principle of generation of 3-phase is same as 1-phase BUT here we have 3 conductors (120^{o} apart) instead of one.



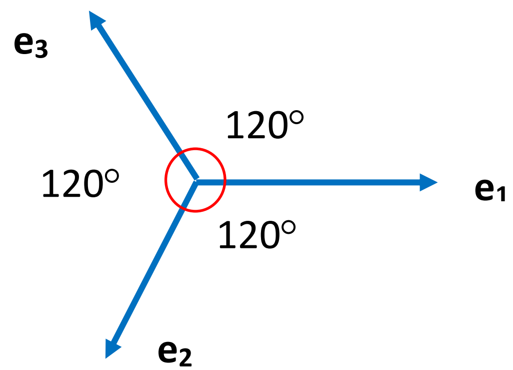


Phase Difference Angle

- For a balanced poly phase system, phase difference angles among conductors are calculated as:

$$\text{Phase difference angles} = \frac{360}{\#\, of\ phases}$$

- Hence, for a 3-φ system, Phase difference angles is 120^o.
- The emf's of a balanced 3-phase system are as follows:
  - $e_1(t) = E_m \, Sin(\omega t)$ or $e_1(t) = E_m \, Cos(\omega t)$
  - $e_2(t) = E_m \, Sin(\omega t\text{-}120)$ or $e_2(t) = E_m \, Cos(\omega t\text{-}120)$
  - $e_3(t) = E_m \, Sin(\omega t+120)$ or $e_3(t) = E_m \, Cos(\omega t+120)$
- They can be represented in phasor form as:



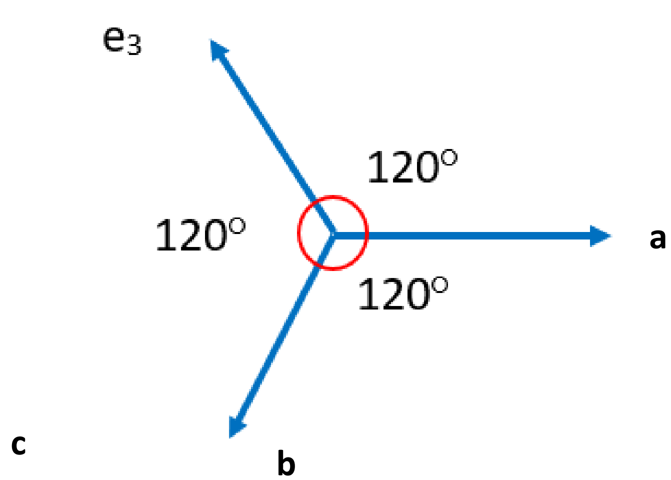
Phase Sequence

- Phase sequence is defined as the order or sequence in which the alternating quantities attain their ***positive peak*** values.
- Sequence can be labelled in any suitable manner. Some sequences are as follows:

First	2^{nd}	3^{rd}
1	2	3
a	b	c
X	Y	Z
R	Y	B

- In this course we will follow sequence a, b, c

- The sequence **a ------> b -------> c** is called positive sequence.
- The sequence **a ------> c -------> b** is called negative sequence.

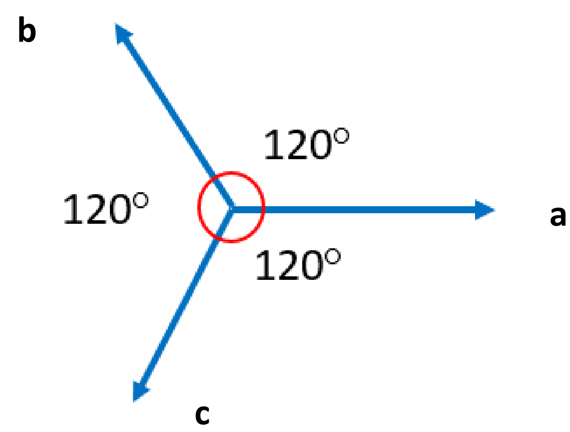


Positive

$$\mathbf{V}_a = V_m \angle 0^\circ,$$

$$\mathbf{V}_b = V_m \angle -120^\circ,$$

$$\mathbf{V}_c = V_m \angle +120^\circ,$$



Negative

$$\mathbf{V}_a = V_m \angle 0^\circ,$$

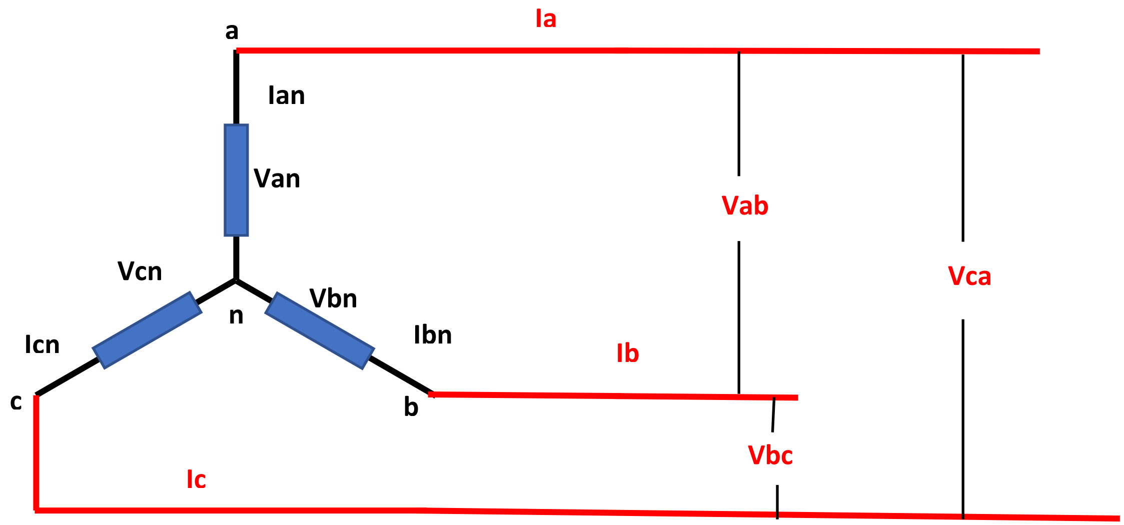
$$\mathbf{V}_b = V_m \angle +120^\circ,$$

$$\mathbf{V}_c = V_m \angle -120^\circ.$$

Connections in a Balanced 3-phase system

- There are two types of connections which are used in a balanced 3-phase system.
  1. Star Connection also known as wye “Y” connection.
  2. Delta Connection

Star (Y) Connection: It is a type of connection where one terminal of each winding is connected to a common point and other three terminals are connected to circuits.



- Current will be going out if we are considering generator convention. For motor convention it will be inward.
- Regardless of the convention, there are two types of quantities:

Phase quantities: These quantities are between nodes "a", "b", "c" and common point "n".

Voltages: V_{an}, V_{bn}, V_{cn}

Currents: I_{an}, I_{bn}, I_{cn}

Line quantities: These quantities are on and between terminals.

Voltages: V_{ab}, V_{bc}, V_{ca}

Currents: I_a, I_b, I_c

Phase and Line Quantities in Star (Y) Connection:

CURRENTS: From the figure above, it is clear that

$$\overline{I_a} = \overline{I_{an}}$$

$$\overline{I_b} = \overline{I_{bn}}$$

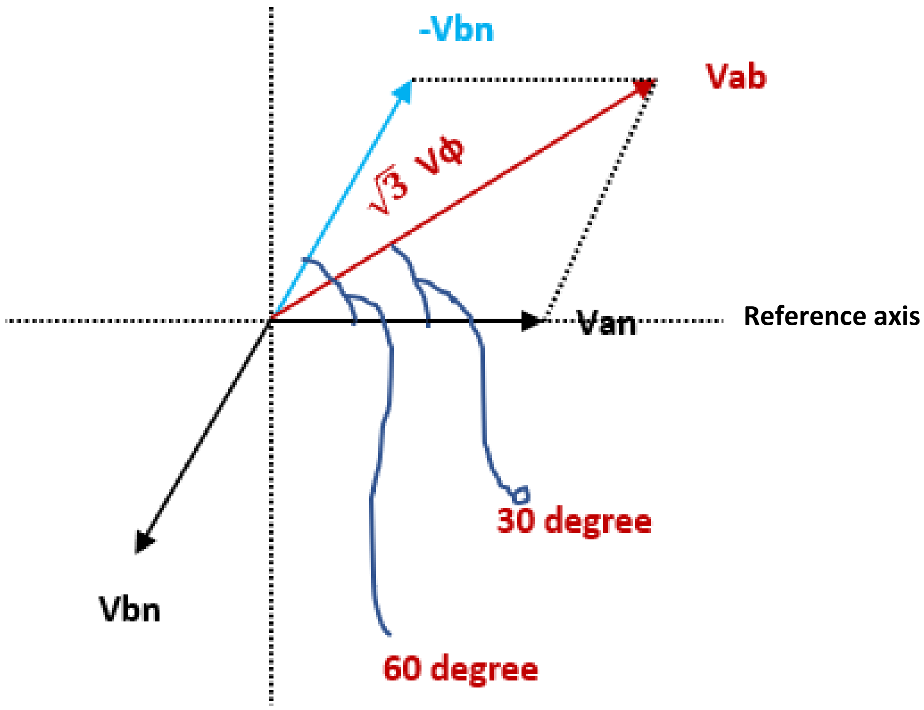
$$\overline{I_c} = \overline{I_{cn}}$$

VOLTAGES: From the figure above, it is clear that by applying KVL we get:

$$\overline{V_{ab}} = \overline{V_{an}} - \overline{V_{bn}}$$

$$\overline{V_{bc}} = \overline{V_{bn}} - \overline{V_{cn}}$$

$$\overline{V_{ca}} = \overline{V_{cn}} - \overline{V_{an}}$$



Consider only first term

$$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn} \quad ===\rightarrow \quad \overline{V}_{ab} = \overline{V}_{an} + (\overline{- V_{bn}})$$

<u>Magnitudes can be expressed as:</u>

$$(|V_{ab}|)^2 = (|V_{an}|)^2 + (|V_{bn}|)^2 + 2\,(|V_{an}|)\,(|V_{bn}|)\,\mathrm{Cos}\,60^\circ$$

Since for a balanced system $|V_{an}| = |V_{bn}| = |V_{cn}| = V_\varphi$

$$(|V_{ab}|)^2 = (V_\varphi)^2 + (V_\varphi)^2 + 2\,(V_\varphi)\,(V_\varphi)\,\mathrm{Cos}\,60^\circ$$

$|V_{ab}| = \sqrt{3}\ V_\varphi$ and the angle is $+30^o$ from reference axis.

$V_{ab} = \sqrt{3}\ V_\varphi$ angle $+30^o$ from reference axis and $+30^o$ from V_{an}.

In the same way we can prove that:

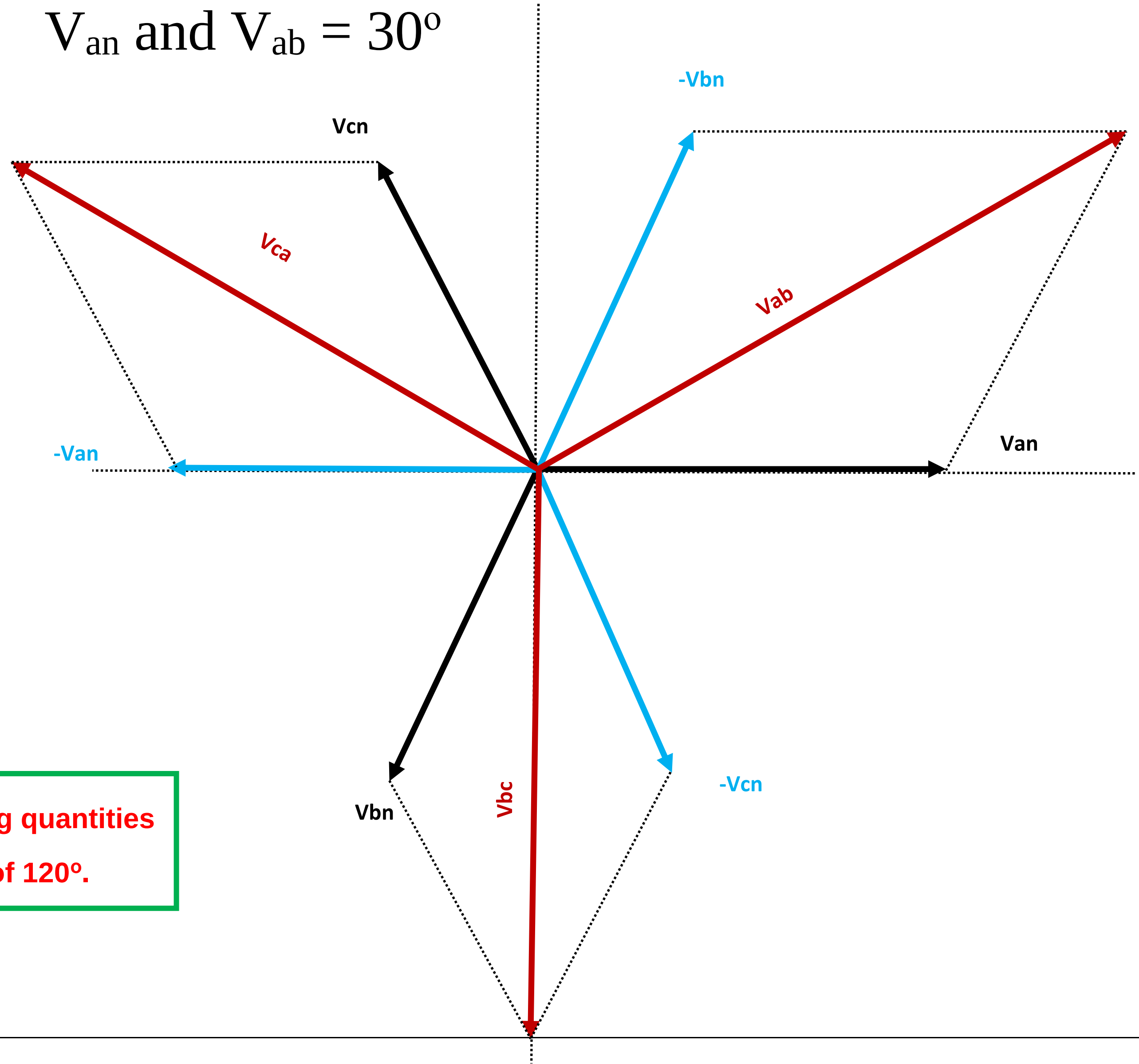
$V_{bc} = \sqrt{3}\ V_\varphi$ angle -90^o from reference axis and $+30^o$ from V_{bn}.

$V_{ca} = \sqrt{3}\ V_\varphi$ angle $+150^o$ from reference axis and $+30^o$ from V_{cn}.

Phase angle between any two quantities can be found out easily.

For example: V_{bc} and $V_{ca} = 120^o$

V_{an} and $V_{ab} = 30^o$



Note: All corresponding quantities have angle difference of 120°.

Power for 3-Phase balance Star (Y) Connected System:

- Total instantaneous power for a 3-phase system is the sum of the powers in each part.

$$p_T = p_{an} + p_{bn} + p_{cn}$$

- Hence power in star (Y) connected system can be summarized as.

Magnitude	In terms of Phase Qty	In terms of Line Qty
Active Power (P) (Watts)	$P = 3\ V_\varphi\ I_\varphi\ \mathrm{Cos}\ (\Theta_v - \Theta_i)$	$P = \sqrt{3}\ V_L\ I_L\ \mathrm{Cos}\ (\Theta_v - \Theta_i)$
Reactive Power (Q) (VAR)	$Q = 3\ V_\varphi\ I_\varphi\ \mathrm{Sin}\ (\Theta_v - \Theta_i)$	$Q = \sqrt{3}\ V_L\ I_L\ \mathrm{Sin}\ (\Theta_v - \Theta_i)$
Apparent Power (S) (VA)	$S = 3\ V_\varphi\ I_\varphi$	$S = \sqrt{3}\ V_L\ I_L$

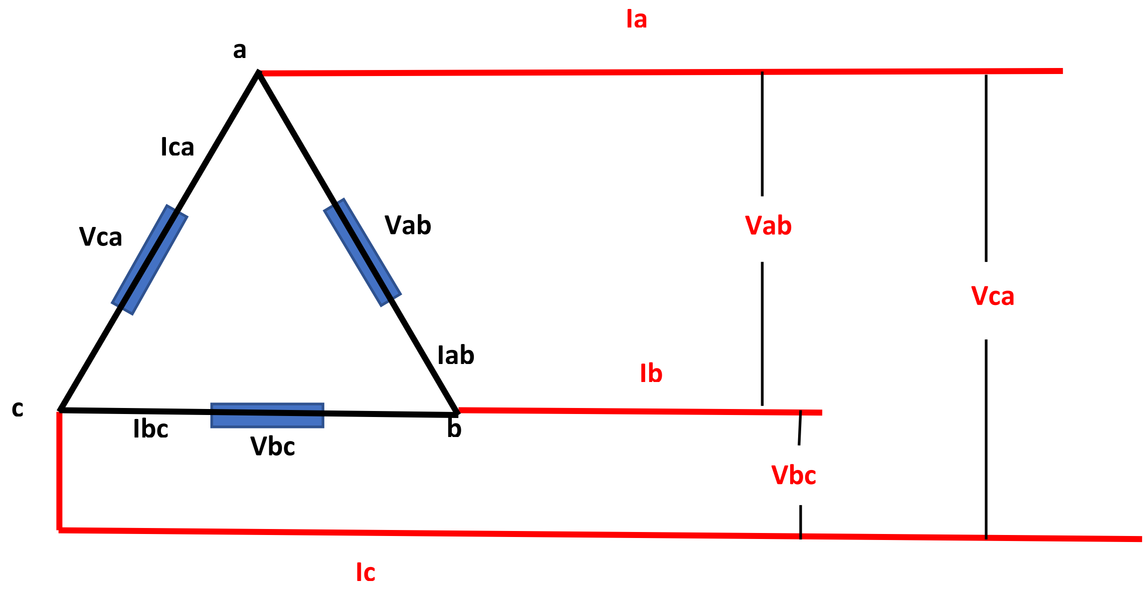
Note: V_φ, I_φ, V_L and I_L are RMS values

In Star (Y) Connection:

Magnitude of $V_L = \sqrt{3}\ V_\varphi$

Magnitude of $I_L = I_\varphi$

Delta Connection: It is a type of connection where all the coils are connected in a back-to-back arrangement.



- Current will be going out if we are considering generator convention. For motor convention it will be inward.

- Regardless of the convention, there are two types of quantities:

  Phase quantities: These quantities are between the nodes “a”, “b”, “c”.

  Voltages: V_{ab}, V_{bc}, V_{ca}

  Currents: I_{ab}, I_{bc}, I_{ca}

  Line Quantities: These quantities are on and between terminals.

  Voltages: V_{ab}, V_{bc}, V_{ca}

  Currents: I_a, I_b, I_c

Phase and Line Quantities in Delta Connection:

VOLTAGES: From the figure above, it is clear that

$$\overline{V}_{ab} = \overline{V}_{ab}$$

$$\overline{V}_{bc} = \overline{V}_{bc}$$

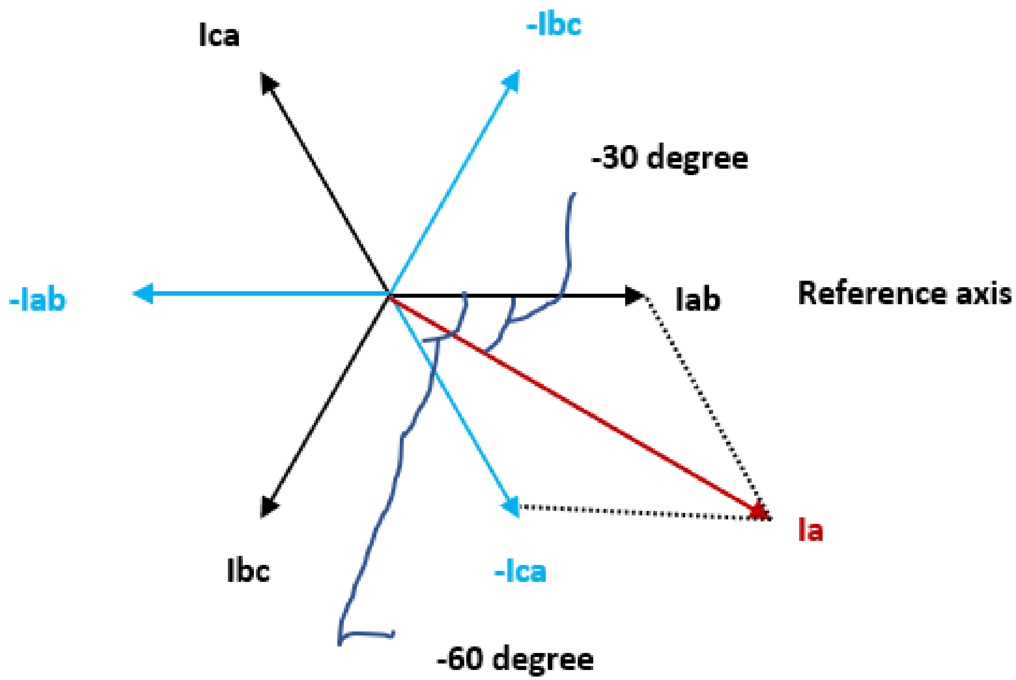
$$\overline{V}_{ca} = \overline{V}_{ca}$$

CURRENTS: By applying KCL in the figure above:

$$\overline{I}_{a} = \overline{I}_{ab} - \overline{I}_{ca}$$

$$\overline{I}_{b} = \overline{I}_{bc} - \overline{I}_{ab}$$

$$\overline{I}_{c} = \overline{I}_{ca} - \overline{I}_{bc}$$



Consider only first term

$$\bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = \bar{I}_{ab} + (-\bar{I}_{ca})$$

<u>Magnitudes can be expressed as:</u>

$(|I_a|)^2 = (|I_{ab}|)^2 + (|I_{ca}|)^2 + 2\,(|I_{ab}|)\,(|I_{ca}|)\ \mathrm{Cos}\ 60^o$

since for a balanced system $|I_{ab}| = |I_{bc}| = |I_{ca}| = I_\varphi$

$(|I_a|)^2 = (I_\varphi)^2 + (I_\varphi)^2 + 2\,(I_\varphi)\,(I_\varphi)\ \mathrm{Cos}\ 60^o$

$|I_a| = \sqrt{3}\ I_\varphi$ and the angle is -30^o from reference axis.

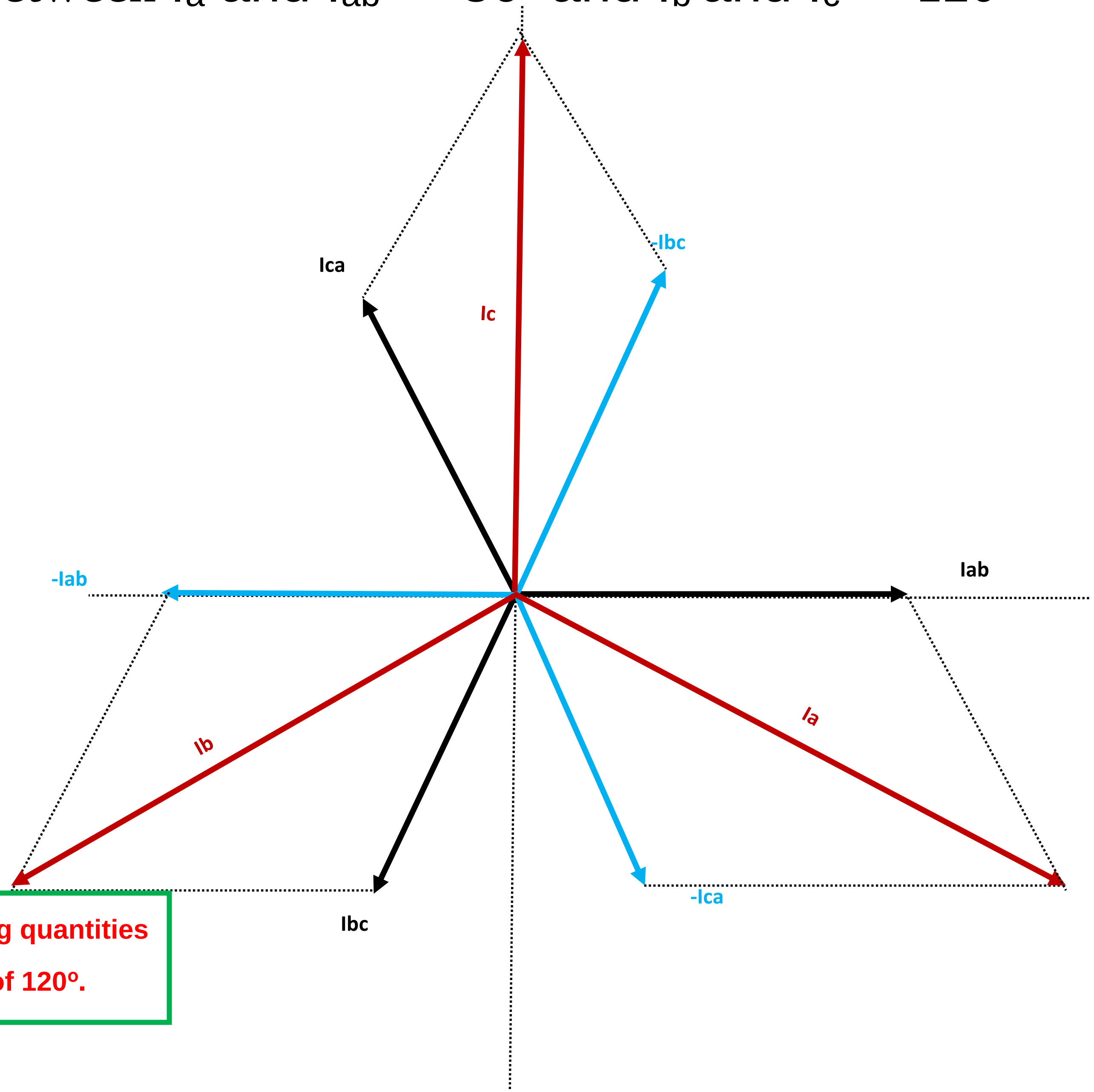
$\boxed{I_a = \sqrt{3}\ I_\varphi \text{ angle } -30^o \text{ from reference axis and } -30^o \text{ from } I_{ab}.}$

In the same way we can prove that:

$I_b = \sqrt{3}\ I_\varphi$ angle -150^o from reference axis and -30^o from I_{bc}.

$I_c = \sqrt{3}\ I_\varphi$ angle $+90^o$ from reference axis and -30^o from I_{ca}.

Phase angle between I_a and I_{ab} = -30° and I_b and I_c = -120°



Note: All corresponding quantities have angle difference of 120°.

Power for 3-Phase balance Delta Connected System:

- Total instantaneous power for a 3-phase system is the sum of the powers in each part.

$$p_T = p_{ab} + p_{bc} + p_{ca}$$

- Hence power in Delta connected system can be summarized as.

Magnitude	In terms of Phase Qty	In terms of Line Qty
Active Power (P) (Watts)	$P = 3\ V_\varphi\ I_\varphi\ \mathrm{Cos}\ (\Theta_v\text{-}\Theta_i)$	$P = \sqrt{3}\ V_L\ I_L\ \mathrm{Cos}\ (\Theta_v\text{-}\Theta_i)$
Reactive Power (Q) (VAR)	$Q = 3\ V_\varphi\ I_\varphi\ \mathrm{Sin}\ (\Theta_v\text{-}\Theta_i)$	$Q = \sqrt{3}\ V_L\ I_L\ \mathrm{Sin}\ (\Theta_v\text{-}\Theta_i)$
Apparent Power (S) (VA)	$S = 3\ V_\varphi\ I_\varphi$	$S = \sqrt{3}\ V_L\ I_L$

Note: V_φ, I_φ, V_L and I_L are RMS values

In Delta Connection: Magnitude of $I_L = \sqrt{3}\ I_\varphi$ $\qquad V_L = V_\varphi$

CONCLUSIONS

All Quantities are Magnitudes

Quantities	STAR	DELTA
Voltages	$V_L = \sqrt{3}\ V\varphi$	$V_L = V\varphi$
Currents	$I_L = I\varphi$	$I_L = \sqrt{3}\ I\varphi$
Power is same regardless the type of connection		
Active Power (P) (Watts)	$P = \sqrt{3}\ V_L\ I_L \text{ Cos } (\Theta_v - \Theta_i)$	$P = \sqrt{3}\ V_L\ I_L \text{ Cos } (\Theta_v - \Theta_i)$
Reactive Power (Q) (VAR)	$Q = \sqrt{3}\ V_L\ I_L \text{ Sin } (\Theta_v - \Theta_i)$	$Q = \sqrt{3}\ V_L\ I_L \text{ Sin } (\Theta_v - \Theta_i)$
Apparent Power (S) (VA)	$S = \sqrt{3}\ V_L\ I_L$	$S = \sqrt{3}\ V_L\ I_L$

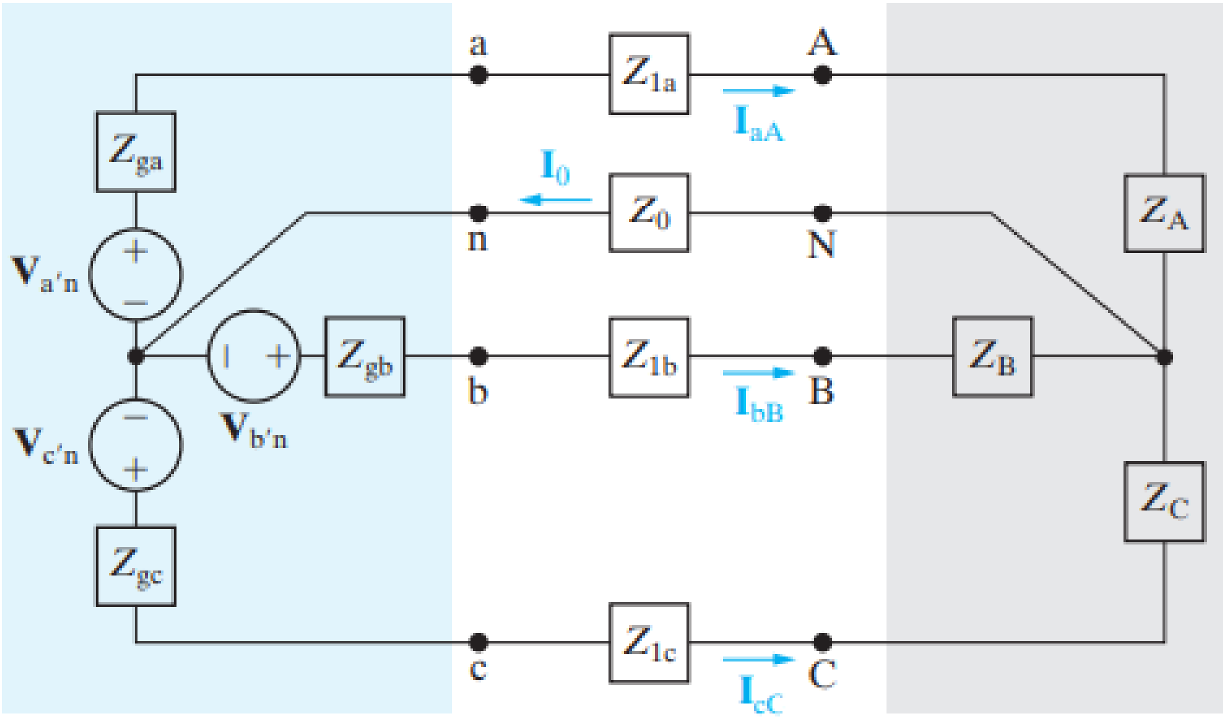
4- Scinerios of Source and Load Connection

- Three-phase sources and loads can be either Y-connected or Δ-connected, the basic circuit can have four different configurations.

Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

- Analyze the Y-Y circuit.
- The remaining three arrangements can be reduced to a Y-Y equivalent circuit.
- Hence analysis of the Y-Y circuit is the key to solving all balanced three-phase arrangements.

- A three-phase Y-Y system is shown below.



- Formal definition of a balanced three-phase circuit states:
  - The voltage sources form a set of balanced three-phase voltages $\mathbf{V}_{a'n}$, $\mathbf{V}_{b'n}$, and $\mathbf{V}_{c'n}$ are same.

- The impedance of each phase of the voltage source is the same.

$$Z_{ga} = Z_{gb} = Z_{gc}.$$

- The impedance of each line (or phase) conductor is the same.

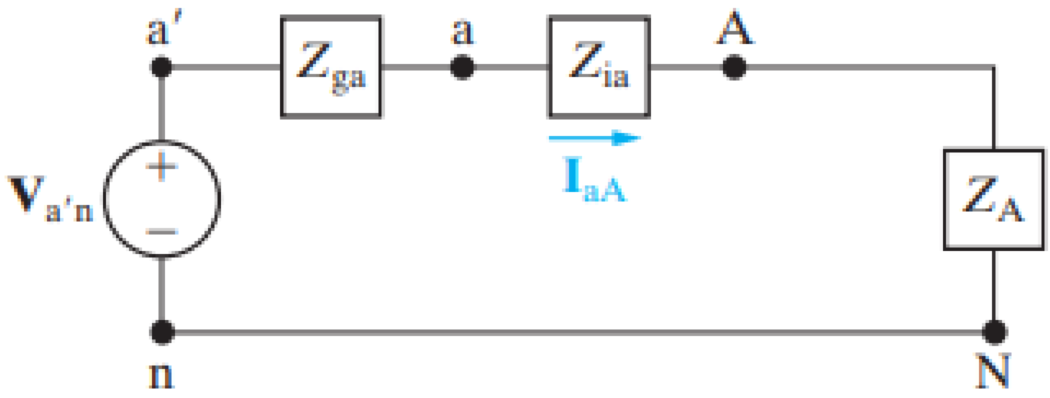
$$Z_{1a} = Z_{1b} = Z_{1c}.$$

- The impedance of each phase of the load is the same.

$$Z_A = Z_B = Z_C.$$

➢ So, we can analyze the system for one phase **ONLY** and deduce the quantities for the other phases.

Single-phase equivalent circuit:



- Considering only phase "a"

$$I_{aA} = \frac{Van}{(Zga+Z1a+ZA)}$$

- Let's do one example to understand the Procedure.

EX - 1: A balanced three-phase Y-connected generator with positive sequence has an impedance of 0.2 + j0.5 Ω/φ and an internal voltage of 120 V/φ. The generator feeds a balanced three-phase Y-connected load having an impedance of 39 + j28 Ω/φ. The impedance of the line connecting the generator to the load is 0.8 + j1.5 Ω/φ. The a-phase internal voltage of the generator is specified as the reference phasor.

a) Construct the a-phase equivalent circuit of the system.

b) Find the three-line currents I_{aA}, I_{bB} and I_{cC}

c) Find the three phase voltages at the load V_{AN}, V_{BN} and V_{CN}

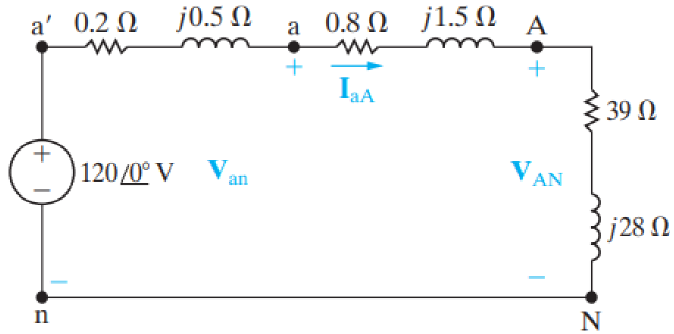
d) Find the line voltages V_{AB}, V_{BC} and V_{CA} at the terminals of the load.

e) Find the phase voltages at the terminals of the generator V_{an}, V_{bn} and V_{cn}.

f) Find the line voltages V_{ab}, V_{bc} and V_{ca} at the terminals of the generator.

g) Repeat (a)–(f) for a negative phase sequence

[a]



[b] The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120\,\angle 0^\circ}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)}$$

$$= \frac{120\,\angle 0^\circ}{40 + j30}$$

$$= 2.4\,\angle -36.87^\circ \text{ A.}$$

For a positive phase sequence,

$$\mathbf{I}_{bB} = 2.4\,\angle -156.87^\circ \text{ A,}$$

$$\mathbf{I}_{cC} = 2.4\,\angle 83.13^\circ \text{ A.}$$

[c] The phase voltage at the A terminal of the load is

$$\mathbf{V}_{AN} = (39 + j28)(2.4\,\angle -36.87^\circ)$$

$$= 115.22\,\angle -1.19^\circ \text{ V.}$$

For a positive phase sequence,

$$\mathbf{V}_{BN} = 115.22\,\angle -121.19^\circ \text{ V,}$$

$$\mathbf{V}_{CN} = 115.22\,\angle 118.81^\circ \text{ V.}$$

[d] For a positive phase sequence, the line voltages lead the phase voltages by 30°; thus

$$\mathbf{V}_{AB} = (\sqrt{3}\,\angle 30^\circ)\mathbf{V}_{AN}$$

$$= 199.58\,\angle 28.81^\circ \text{ V,}$$

$$\mathbf{V}_{BC} = 199.58\,\angle -91.19^\circ \text{ V,}$$

$$\mathbf{V}_{CA} = 199.58\,\angle 148.81^\circ \text{ V.}$$

[e] The phase voltage at the a terminal of the source is

$$\mathbf{V}_{an} = 120 - (0.2 + j0.5)(2.4 \angle -36.87^\circ)$$
$$= 120 - 1.29 \angle 31.33^\circ$$
$$= 118.90 - j0.67$$
$$= 118.90 \angle -0.32^\circ \text{ V.}$$

For a positive phase sequence,

$$\mathbf{V}_{bn} = 118.90 \angle -120.32^\circ \text{ V},$$
$$\mathbf{V}_{cn} = 118.90 \angle 119.68^\circ \text{ V.}$$

[f] The line voltages at the source terminals are

$$\mathbf{V}_{ab} = (\sqrt{3} \angle 30^\circ)\mathbf{V}_{an}$$
$$= 205.94 \angle 29.68^\circ \text{ V},$$

$$\mathbf{V}_{bc} = 205.94 \angle -90.32^\circ \text{ V},$$
$$\mathbf{V}_{ca} = 205.94 \angle 149.68^\circ \text{ V.}$$

Source-- Y

Ian	=	2.4	L	-36.8699	A/φ
Ibn	=	2.4	L	-156.8699	A/φ
Icn	=	2.4	L	83.130102	A/φ

Van	=	118.8979	L	-0.323832	V/φ
Vbn	=	118.8979	L	-120.3238	V/φ
Vbn	=	118.8979	L	119.67617	V/φ

Line

IaA	=	2.4	L	-36.8699	A/φ
IbB	=	2.4	L	-156.87	A/φ
IcC	=	2.4	L	83.1301	A/φ

Vab	=	205.937	L	29.676	V
Vbc	=	205.937	L	-90.32	V
Vca	=	205.937	L	149.68	V

VAB	=	199.58	L	28.80651	V
VBC	=	199.58	L	-91.1935	V
VCA	=	199.58	L	148.8065	V

Load-- Y

IAN	=	2.4	L	-36.8699	A/φ
IBN	=	2.4	L	-156.87	A/φ
ICN	=	2.4	L	83.1301	A/φ

VAN	=	115.225	L	-1.19349	V/φ
VBN	=	115.225	L	-121.193	V/φ
VCN	=	115.225	L	118.8065	V/φ

[g] Changing the phase sequence has no effect on the single-phase equivalent circuit. The three line currents are

$$\mathbf{I}_{aA} = 2.4\,\angle{-36.87^\circ}\text{ A},$$

$$\mathbf{I}_{bB} = 2.4\,\angle{83.13^\circ}\text{ A},$$

$$\mathbf{I}_{cC} = 2.4\,\angle{-156.87^\circ}\text{ A}.$$

The phase voltages at the load are

$$\mathbf{V}_{AN} = 115.22\,\angle{-1.19^\circ}\text{ V},$$

$$\mathbf{V}_{BN} = 115.22\,\angle{118.81^\circ}\text{ V},$$

$$\mathbf{V}_{CN} = 115.22\,\angle{-121.19^\circ}\text{ V}.$$

For a negative phase sequence, the line voltages lag the phase voltages by 30°:

$$\mathbf{V}_{AB} = (\sqrt{3}\,\angle{-30^\circ})\mathbf{V}_{AN}$$

$$= 199.58\,\angle{-31.19^\circ}\text{ V},$$

$$\mathbf{V}_{BC} = 199.58\,\angle{88.81^\circ}\text{ V},$$

$$\mathbf{V}_{CA} = 199.58\,\angle{-151.19^\circ}\text{ V}.$$

The phase voltages at the terminals of the generator are

$$\mathbf{V}_{an} = 118.90\,\angle{-0.32^\circ}\text{ V},$$

$$\mathbf{V}_{bn} = 118.90\,\angle{119.68^\circ}\text{ V},$$

$$\mathbf{V}_{cn} = 118.90\,\angle{-120.32^\circ}\text{ V}.$$

The line voltages at the terminals of the generator are

$$\mathbf{V}_{ab} = (\sqrt{3}\,\angle{-30^\circ})\mathbf{V}_{an}$$

$$= 205.94\,\angle{-30.32^\circ}\text{ V},$$

$$\mathbf{V}_{bc} = 205.94\,\angle{89.68^\circ}\text{ V},$$

$$\mathbf{V}_{ca} = 205.94\,\angle{-150.32^\circ}\text{ V}.$$

- To analyze a balanced three-phase Y-Δ, Δ-Y or Δ-Δ system, Change the impedances of Δ connection into equivalent Y connection impedances and analyze it as Y-Y system.

$$Z_Y = \frac{Z_\Delta}{3}$$

- Let’s do one example to understand the Procedure.

<u>EX-2:</u> A balanced three-phase Y-connected generator with positive sequence has an impedance of $0.2 + j0.5\ \Omega/\varphi$ and an internal voltage of $120\ V/\varphi$, feeds a Δ-connected load through a distribution line having an impedance of $0.3 + j0.9\ \Omega/\varphi$. The load impedance is $118.5 + j85.8\ \Omega/\varphi$. Use the a-phase internal voltage of the generator as the reference.

a) Construct a single-phase equivalent circuit of the three-phase system.

b) Calculate the line currents.

c) Calculate the phase voltages at the load terminals.

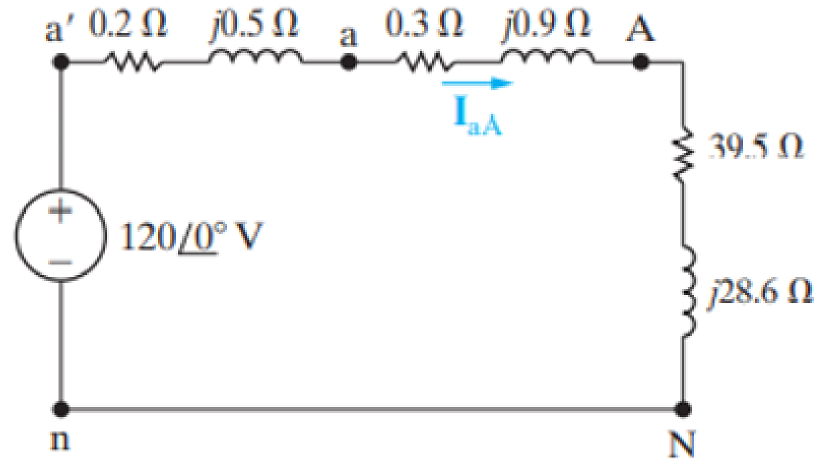
d) Calculate the phase currents of the load.

e) Calculate the line voltages at the source terminals

Solution

a) Figure shows the single-phase equivalent circuit. The load impedance of the Y equivalent is

$$\frac{118.5 + j85.8}{3} = 39.5 + j28.6\ \Omega/\phi.$$



b) The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6)} = \frac{120\angle 0^\circ}{40 + j30} = 2.4\angle -36.87^\circ \text{ A}.$$

$$\mathbf{I}_{bB} = 2.4\angle -156.87^\circ \text{ A},$$

$$\mathbf{I}_{cC} = 2.4\angle 83.13^\circ \text{ A}.$$

c) To calculate the line voltage, first calculate phase voltages

$V_{AB\ (phase)} = I_{AB\ (Phase)} *(118.5 + j85.8)=(1.39L\text{-}6.87^o)\ (146.30L35.91^o)$

$V_{AB\ (phase)} = 202.72\ \boldsymbol{L}\ 29.04^o$ V

For delta connection $V_{AB\ (phase)} = V_{AB\ (Line)}$ So

$V_{AB\ (Line)} = 202.72\ \boldsymbol{L}\ 29.04^o$ V

$V_{BC\ (Line)} = 202.72\ \boldsymbol{L}\ \text{-}90.96^o$ V

$V_{CA\ (Line)} = 202.72\ \boldsymbol{L}\ 149.04^o$ V

d) The phase currents of the load may be calculated directly from the line currents:

$$\mathbf{I}_{AB} = \left(\frac{1}{\sqrt{3}}\angle 30^\circ\right)\mathbf{I}_{aA}$$

$$= 1.39\angle -6.87^\circ\ \text{A}.$$

$$\mathbf{I}_{BC} = 1.39\angle -126.87^\circ\ \text{A},$$

$$\mathbf{I}_{CA} = 1.39\angle 113.13^\circ\ \text{A}.$$

e) To calculate the line voltage at the terminals of the source, we first calculate $\mathbf{V}_{an}$.

$$\mathbf{V}_{an} = (39.8 + j29.5)(2.4 \underline{/-36.87^\circ})$$
$$= 118.90 \underline{/-0.32^\circ} \text{ V}.$$

The line voltage $\mathbf{V}_{ab}$ is

$$\mathbf{V}_{ab} = (\sqrt{3} \underline{/30^\circ})\mathbf{V}_{an},$$

or

$$\mathbf{V}_{ab} = 205.94 \underline{/29.68^\circ} \text{ V}.$$

Therefore

$$\mathbf{V}_{bc} = 205.94 \underline{/-90.32^\circ} \text{ V},$$
$$\mathbf{V}_{ca} = 205.94 \underline{/149.68^\circ} \text{ V}.$$

Source-- Y

Ian	=	2.4	L	-36.8699	A/φ
Ibn	=	2.4	L	-156.8699	A/φ
Icn	=	2.4	L	83.130102	A/φ

Van	=	118.8979	L	-0.323832	V/φ
Vbn	=	118.8979	L	-120.3238	V/φ
Vbn	=	118.8979	L	119.67617	V/φ

Line

Vab	=	205.937	L	29.676	V
Vbc	=	205.937	L	-90.32	V
Vca	=	205.937	L	149.68	V

IaA	=	2.4	L	-36.8699	A/φ
IbB	=	2.4	L	-156.87	A/φ
IcC	=	2.4	L	83.1301	A/φ

VAB	=	202.72	L	29.03654	V
VBC	=	202.72	L	-90.9635	V
VCA	=	202.72	L	149.0365	V

Load --- Δ

IAB(ph)	=	1.385641	L	-6.8699	A/φ
IBC(ph)	=	1.385641	L	-126.87	A/φ
ICA(ph)	=	1.385641	L	113.1301	A/φ

VAB(ph)	=	202.7202	L	29.03654	V/φ
VBC(ph)	=	202.7202	L	-90.9635	V/φ
VCA(ph)	=	202.7202	L	149.0365	V/φ

EX-3: For EX-1, calculate:

a) The average power per phase delivered to the Y-connected load.

b) The total average power delivered to the load.

c) The total average power lost in the line.

d) The total average power lost in the generator.

e) The total number of magnetizing vars absorbed by the load.

f) The total complex power delivered by the source

a) From Example 1 $V_\phi = 115.22$ V, $I_\phi = 2.4$ A, and $\theta_\phi = -1.19 - (-36.87) = 35.68^\circ$. Therefore

$$P_\phi = (115.22)(2.4)\cos 35.68^\circ$$

$$= 224.64 \text{ W}.$$

The power per phase may also be calculated from $I_\phi^2 R_\phi$, or

$$P_\phi = (2.4)^2(39) = 224.64 \text{ W}.$$

b) The total average power delivered to the load is

$$P_T = 3P_\phi = 673.92 \text{ W.}$$

We calculated the line voltage in EX-1, so we may also use those values to find the total average power.

$$P_T = \sqrt{3}(199.58)(2.4)\cos 35.68^\circ$$

$$= 673.92 \text{ W.}$$

c) The total power lost in the line is

$$P_{\text{line}} = 3(2.4)^2(0.8) = 13.824 \text{ W.}$$

d) The total internal power lost in the generator is

$$P_{\text{gen}} = 3(2.4)^2(0.2) = 3.456 \text{ W.}$$

e) The total number of magnetizing vars absorbed by the load is

$$Q_T = \sqrt{3}(199.58)(2.4)\sin 35.68^\circ$$

$$= 483.84 \text{ VAR.}$$

f) The total complex power associated with the source is

$$S_T = 3S_\phi = -3(120)(2.4)\,\underline{/36.87^\circ}$$
$$= -691.20 - j518.40 \text{ VA}.$$

The minus sign indicates that the internal power and magnetizing reactive power are being delivered to the circuit. We check this result by calculating the total and reactive power absorbed by the circuit:

$$P = 673.92 + 13.824 + 3.456$$
$$= 691.20 \text{ W (check)},$$
$$Q = 483.84 + 3(2.4)^2(1.5) + 3(2.4)^2(0.5)$$
$$= 483.84 + 25.92 + 8.64$$
$$= 518.40 \text{ VAR(check)}.$$

EX-4: For example, 2, calculate:

a) Calculate the total complex power delivered to the Δ-connected load.

b) What percentage of the average power at the sending end of the line is delivered to the load?

Solution

a) Using the a-phase values from the solution of Example

$$\mathbf{V}_\phi = \mathbf{V}_{AB} = 202.72 \angle 29.04^\circ \text{ V},$$

$$\mathbf{I}_\phi = \mathbf{I}_{AB} = 1.39 \angle -6.87^\circ \text{ A}.$$

$$S_T = 3(202.72 \angle 29.04^\circ)(1.39 \angle 6.87^\circ)$$

$$= 682.56 + j494.21 \text{ VA}.$$

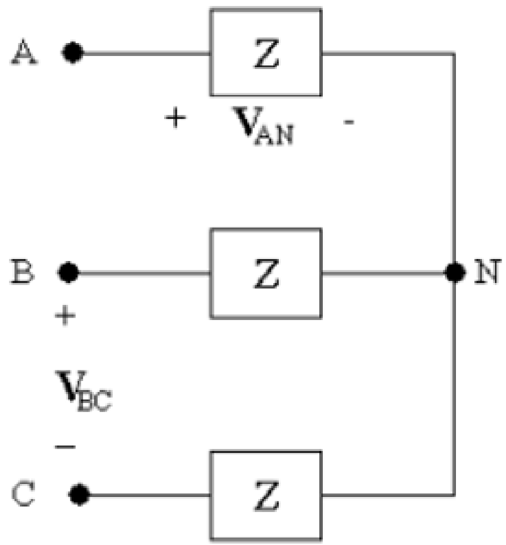
b) The total power at the sending end of the distribution line equals the total power delivered to the load plus the total power lost in the line; therefore

$$P_{\text{input}} = 682.56 + 3(2.4)^2(0.3)$$

$$= 687.74 \text{ W}.$$

The percentage of the average power reaching the load is 682.56/687.74, or 99.25%. Nearly 100% of the average power at the input is delivered to the load because the impedance of the line is quite small compared to the load impedance.

EX-5: The voltage from A to N in a balanced three phase circuit is 240 ***L*** -30. If the phase sequence is positive, calculate V_{BC}?

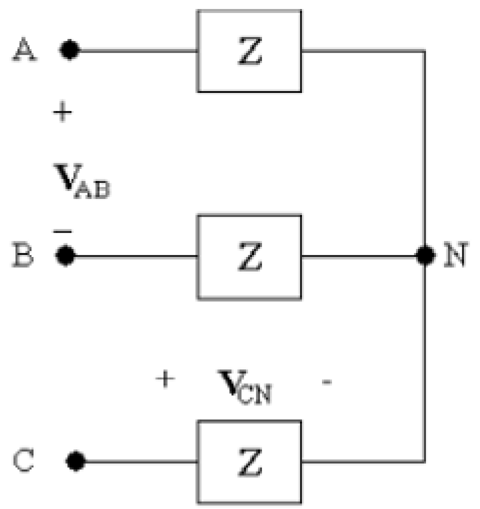


$V_{AN} = 240\ \angle\ -30^o$ ----------- It is star connection so

$V_{AB} = \sqrt{3}\ \angle +30^o\ (V_{AN})$

$V_{AB} = 415.69\ \angle 0^o$ =====➔ $V_{BC} = 415.69\ \angle -120^o$

EX-6: The c-phase voltage of a balanced three-phase Y-connected system is 450 ***L*** -25º. If the phase sequence is negative, what is the value of V_{AB}?



$V_{CN} = 450\ \angle\ -25^{o}$ ----------- It is star (Y) connection so

$V_{AN} = 450\angle -25^{o} +120 = 450\angle +95^{o}$

$V_{AB} = \sqrt{3}\ \angle -30^{o}\ (V_{AN})$ =====➔ $V_{AB} = 779.42\angle +65^{o}$

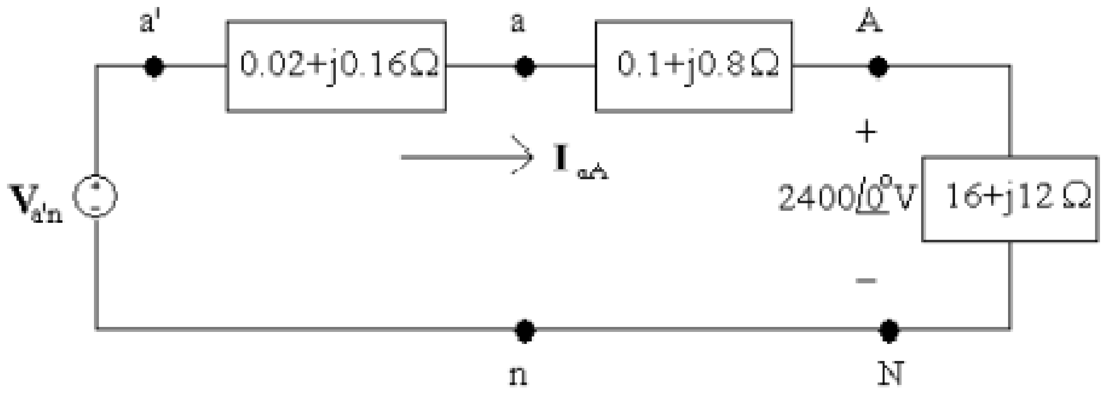
EX-7: The phase voltage at the terminals of a balanced three-phase Y-connected load is 2400 V. The load has an impedance of 16 + j12 ohm/phase and is fed from a line having an impedance of 0.10 + j0.80 ohm/phase. The Y-connected source at the sending end of the line has a phase sequence of acb and an internal impedance of 0.02 + j0.16 ohm/phase. Use the a-phase voltage at the load as the reference and calculate:

(a) The line currents IaA, IbB and IcC

(b) The line voltages at the source, Vab, Vbc and Vca

(c) The internal phase-to-neutral voltages at the source, van, vbn and vcn.

Sketch the a-phase circuit:



Ian	=	120	L	-36.8699	A/φ
Ibn	=	120	L	83.130102	A/φ
Icn	=	120	L	-156.8699	A/φ

IaA	=	120	L	-36.8699	A/φ
IbB	=	120	L	83.1301	A/φ
IcC	=	120	L	-156.87	A/φ

IAN	=	120	L	-36.8699	A/φ
IBN	=	120	L	83.1301	A/φ
ICN	=	120	L	-156.87	A/φ

Van	=	2468.1815	L	1.6158921	V/φ
Vbn	=	2468.1815	L	121.61589	V/φ
Vcn	=	2468.1815	L	-118.3841	V/φ

Vab	=	4275.02	L	-28.384	V
Vbc	=	4275.02	L	91.6159	V
Vca	=	4275.02	L	-148.38	V

Vdrop	=	167.5709	L	16.00509	V

VAB	=	4156.9	L	-30	V
VBC	=	4156.9	L	90	V
VCA	=	4156.9	L	-150	V

VAN	=	2400	L	0	V/φ
VBN	=	2400	L	120	V/φ
VCN	=	2400	L	-120	V/φ

Va'n	=	2482.0456	L	1.9283477	V/φ
Vb'n	=	2482.0456	L	121.92835	V/φ
Vc'n	=	2482.0456	L	-118.0717	V/φ

[a]

$$\mathbf{I}_{\text{aA}} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\text{ A}$$

With an acb phase sequence,

$$\mathbf{I}_{\text{bB}} = 120\underline{/83.13^\circ}\text{ A}$$

$$\mathbf{I}_{\text{cC}} = 120\underline{/-156.87^\circ}\text{ A}$$

[b]

$$\mathbf{V}_{\text{ab}} = \mathbf{V}_{\text{an}}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\text{ V}$$

With an acb phase sequence,

$$\mathbf{V}_{\text{bc}} = 4275.02\underline{/91.62^\circ}\text{ V}$$

$$\mathbf{V}_{\text{ca}} = 4275.02\underline{/-148.38^\circ}\text{ V}$$

[c]

$$\begin{aligned}\mathbf{V}_{\text{a}'\text{n}} &= (0.2 + j0.16)\mathbf{I}_{\text{aA}} + \mathbf{V}_{\text{an}} \\ &= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9) \\ &= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\text{ V}\end{aligned}$$

$$\mathbf{V}_{\text{b}'\text{n}} = 2482.05\underline{/121.93^\circ}\text{ V}$$

$$\mathbf{V}_{\text{c}'\text{n}} = 2482.05\underline{/-118.07^\circ}\text{ V}$$

EX-8: The RMS line-line voltage magnitude across a balanced three-phase load is 120 V and its three-phase complex power is 10 kW+j10 kVAr. What is the equivalent impedance of each leg of the load?

a) Assuming a Y-connected load.

b) Assuming a Δ-connected load.

[a] $V_{LL} = 120$ V ====➔ $Vph = 120/\sqrt{3}$ V

$S = 10000 + j10000 = 14142.1\ \angle\ 45^o$ VA

$S/\phi = \frac{S}{3} = 4714.03\ \angle\ 45^o$

$S/\phi = V_{ph}I_{ph}^{*} = V.\frac{V*}{ZY} = \frac{V(ph)\hat{}2}{ZY}$ =====➔ $Z_Y = \frac{V(ph)\hat{}2}{S}$

$Z_Y/\phi = \frac{120*120}{3*4714.03\angle 45} = 1.0182\ \angle\ -45^o = 0.72\text{-}j0.72\ \Omega$

[b] $Z_\Delta/\phi = 3\ Z_Y = 3.0546\ \angle\ -45^o = 2.16 - j2.16\ \Omega$