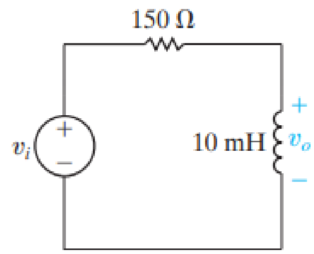
Solution of Homework-05 ENGR 117

5 Questions 20 points each

Q-1 Consider the circuit shown below.



a) This circuit behaves like what type of filter?

b) What is the transfer function, of this filter?

c) What is the cutoff frequency of this filter?

d) Find the magnitude and phase of the transfer function at $s=j\omega_c$?

[a] For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.
For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.
Thus, the circuit is a high-pass filter.

[b] $$H(s) = \frac{sL}{R + sL} = \frac{s}{s + R/L} = \frac{s}{s + 15{,}000}$$

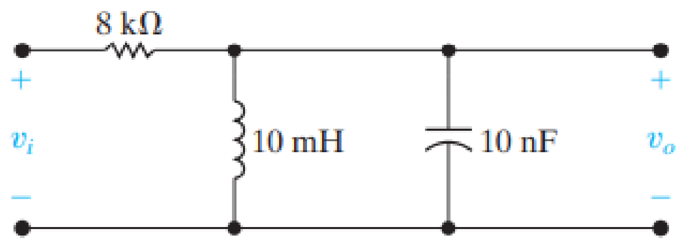
[c] $$\omega_c = \frac{R}{L} = 15{,}000 \text{ rad/s}$$

[d] $$|H(jR/L)| = \left|\frac{jR/L}{jR/L + R/L}\right| = \left|\frac{j}{j+1}\right| = \frac{1}{\sqrt{2}}$$

ϴ = 90 – 45 = +45 degree

Q-2 For the bandpass filter shown. Find:

(a) ω_o, (b) f_o, (c) Q, (d) ω_{c1}, (e) f_{c1}, (f) ω_{c2}, (g) f_{c2}, and (h) β.



[a] $$\omega_o^2 = \frac{1}{LC} = \frac{1}{(10\times 10^{-3})(10\times 10^{-9})} = 10^{10}$$

$$\omega_o = 10^5 \text{ rad/s} = 100\,\text{krad/s}$$

[b] $$f_o = \frac{\omega_o}{2\pi} = \frac{10^5}{2\pi} = 15.9\,\text{kHz}$$

[c] $$Q = \omega_o RC = (100\times 10^3)(8000)(10\times 10^{-9}) = 8$$

[d] $$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1+\left(\frac{1}{2Q}\right)^2}\right] = 10^5\left[-\frac{1}{16} + \sqrt{1+\frac{1}{256}}\right] = 93.95\,\text{krad/s}$$

[e] $$\therefore\quad f_{c1} = \frac{\omega_{c1}}{2\pi} = 14.95\,\text{kHz}$$

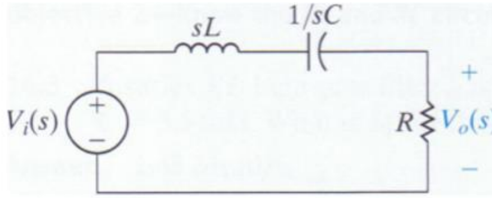
[f] $$\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1+\left(\frac{1}{2Q}\right)^2}\right] = 10^5\left[\frac{1}{16} + \sqrt{1+\frac{1}{256}}\right] = 106.45\,\text{krad/s}$$

[g] $$\therefore\quad f_{c2} = \frac{\omega_{c2}}{2\pi} = 16.94\,\text{kHz}$$

[h] $$\beta = \frac{\omega_o}{Q} = \frac{10^5}{8} = 12.5\,\text{krad/s or } 1.99\,\text{kHz}$$

Q-3 Verify the following for the bandpass filter: (show your work)

$$\omega_{C1} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



$$\omega_{C2} = \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

At Corner frequency

$$|H(j\omega)| = \frac{\frac{R}{L}\omega}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2 + \left(\frac{R}{L}\omega\right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$2\left(\frac{R}{L}\omega\right)^2 = \left(\frac{1}{LC}-\omega^2\right)^2 + \left(\frac{R}{L}\omega\right)^2$$

$$\left(\frac{R}{L}\omega\right)^2 = \left(\frac{1}{LC}-\omega^2\right)^2$$

$$\pm\left(\frac{R}{L}\omega\right) = \pm\left(\frac{1}{LC}-\omega^2\right)$$ ⇒ This gives 4 Combinations

$$\omega^2 + \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega^2 - \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega^2 - \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega^2 + \frac{R}{L}\omega - \frac{1}{LC} = 0$$

These 4 reduce to 2 equations.

$$\omega^2 + \frac{R}{L}\omega - \frac{1}{LC} = 0 \quad \text{— (1)}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$ ← +ve

$$\omega_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$ ← −ve ← ignore

$$\omega^2 - \frac{R}{L}\omega - \frac{1}{LC} = 0 \quad \text{— (2)}$$

$$\omega_1 = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$ ← +ve

$$\omega_2 = +\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$ ← −ve ← ignore

So

$$\boxed{\begin{aligned} \omega_{C_1} &= \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_{C_2} &= \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned}}$$ ← Proved

Q-4 Use a 5 nF capacitor to design a series RLC bandpass filter. The center frequency of the filter is 8 kHz, and the quality factor is 2. (Show your circuit)

a) Specify the values of R and L.

b) What is the lower cutoff frequency in kilohertz?

c) What is the upper cutoff frequency in kilohertz?

d) What is the bandwidth of the filter in kilohertz?

[a] $\omega_o^2 = \dfrac{1}{LC}$ so $L = \dfrac{1}{[8000(2\pi)]^2(5 \times 10^{-9})} = 79.16\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{8000(2\pi)(79.16 \times 10^{-3})}{2} = 1.99\,\text{k}\Omega$$

[b] $f_{c1} = 8000\left[-\dfrac{1}{4} + \sqrt{1 + \dfrac{1}{16}}\right] = 6.25\,\text{kHz}$

[c] $f_{c2} = 8000\left[\dfrac{1}{4} + \sqrt{1 + \dfrac{1}{16}}\right] = 10.25\,\text{kHz}$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{8000}{2} = 4\,\text{kHz}$$

Q-5 Design the component values for the series RLC band reject filter so that the center frequency is 4 kHz and the quality factor is 500. Use a 500 nF capacitor. (Show your circuit)

a) Find the values of R and L.

$$\omega_o = 8000\pi \text{ rad/s}$$

$$C = 500\,\text{nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R}$$

$$\therefore \quad R = \frac{1}{\omega_o C Q} = \frac{1}{(8000\pi)(500)(5 \times 10^{-9})} = 15.92\,\Omega$$

[a] R=15.92 Ω L= 3.17 mH