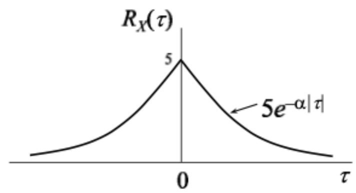
HW 8

1) Problem 9.1

2) Problem 9.3

3) Problem 9.5

4) The auto-correlation function $R_X(\tau)$ of a random process *X(t)* is shown in the figure below. Determine the average power of the signal *X(t)*.



5) The autocorrelation function for a certain random signal is given by

$$R_X(\tau) = e^{-2|\tau|}$$

a) Determine the power spectral density $S_X(f)$ W/Hz
b) What's the average power of this signal.

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ENGR120 - 01 SU
HOMEWORK 08

1. PROBLEM 9.1

(a) THE MEAN OF $X(t)$.

$$E\{x(t)\} = E[A + \cos(\omega t + \phi)]$$
$$= A + E[\cos(\omega t + \phi)]$$
$$= A = m_x$$

(b) THE AUTOCORRELATION FUNCTION OF $X(t)$.

$$R_x(\tau) = E[x(t_1)x(t_2)]$$
$$= E[(A + \cos(\omega t_1 + \phi))(A + \cos(\omega t_2 + \phi))]$$
$$= E[A^2 + A\cos(\omega t_2 + \phi) + A\cos(\omega t_1 + \phi) + \cos(\omega t_1 + \phi) \cdot \cos(\omega t_2 + \phi)]$$
$$= A^2 + 0 + 0 + E[A\cos(\omega t_1 + \phi)\cos(\omega t_2 + \phi)]$$
$$= A^2 + 1/2\, E[\cos(\omega t_1 + \omega t_2 + 2\phi) + \cos(\omega(t_2 - t_1))]$$
$$= A^2 + 1/2\, E[\cos(\omega t_1 + \omega t_2 + 2\phi)] + 1/2\, E[\cos(\omega(t_2 - t_1))]$$
$$= A^2 + 0 + 1/2 \cos(\omega(t_2 - t_1))$$
$$= A^2 + 1/2 \cos \tau \quad \text{where } \tau = t_2 - t_1$$

(c) THE AUTOCOVARIANCE FUNCTION OF $X(t)$.

$$C_x(\tau) = R_x(\tau) - |m_x|^2$$
$$= A^2 + 1/2 \cos \omega\tau - A^2$$
$$= 1/2 \cos \omega\tau$$

2. PROBLEM 9.3

$$X(t) = A\cos \omega t + B$$

(a) FIND THE MEAN OF $X(t)$

$$E[x(t)] = E[A\cos \omega t + B]$$
$$= E[A]\cos \omega t + E[B]$$
$$= 0(\cos \omega t) + 4$$
$$= 4$$

(b) DETERMINE THE AUTOCORRELATION FUNCTION OF $X(t)$.

$$R_x(\tau) = E[x(t_1)x(t_2)]$$
$$= E[(A\cos \omega t_1 + B)(A\cos \omega t_2 + B)]$$
$$= E[A^2]\cos \omega t_1 \cos \omega t_2 + E[A]E[B](\cos \omega t_1 + \cos \omega t_2) + E[B^2]$$
$$= 2\cos \omega t_1 \cos \omega t_2 + 25$$

$E[A^2] = \sigma_A^2 + m_A^2 = 2$

$E[A]E[B] = 0$

$E[B^2] = \sigma_B^2 + m_B^2 = 3^2 + 4^2 = 25$

(c) COMPUTE THE FIRST MOMENT AND THE SECOND MOMENT OF $X(2)$

FIRST MOMENT : $E[X(2)] = 4$

SECOND MOMENT: $R_X(2,2) = 2\cos 2\omega \cos 2\omega + 25$ $\qquad \omega = 0.10\pi$

$= 2\cos^2 2\omega + 25$

$= 26.31$

3. PROBLEM 9.5

$X(t) = A\cos(\omega t + \phi)$

(a) THE MEAN OF $X(t)$.

$m_X(t) = E[A\cos(\omega t + \phi)]$ $\qquad E[A] = \int_2^5 a f_A(a)\,da = \int_2^5 1/3\, a\,da$

$= 7/2 \cos(\omega t + \phi)$ $\qquad = 1/3\,[a^2/2]_2^5 = 7/2$

(b) THE AUTOCORRELATION FUNCTION OF $X(t)$

$R_X(\tau) = E[X(t_1)X(t_2)]$

$= E[(A\cos(\omega t_1 + \phi))(A\cos(\omega t_2 + \phi))]$

$= E[A^2\cos(\omega t_1 + \phi)\cos(\omega t_2 + \phi)]$ $\qquad E[A^2] = \int_2^5 a^2 f_A(a)\,da = \int_2^5 1/3\, a^2\,da$

$= 13\cos(\omega t_1 + \phi)\cos(\omega t_2 + \phi)$ $\qquad = 1/3\,[a^3/3]_2^5 = 13$

(c) IS $X(t)$ WIDE SENSE STATIONARY? PROVIDE A BRIEF EXPLANATION.

NO, because $X(t)$ mean is not 0 and autocorrelation is a function of both t and τ.

4. $R_X(0) = 5$

5. (a) $S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f\tau}\,d\tau$

$= \int_0^{\infty} e^{-2\tau} e^{-j2\pi f\tau}\,d\tau + \int_{-\infty}^{0} e^{-2\tau} e^{-j2\pi f\tau}\,d\tau$

$= \frac{1}{2+j2\pi f} + \frac{1}{2-j2\pi f} = \frac{4}{4+4\pi^2 f^2} = \frac{1}{1+\pi^2 f^2}$

(b) $R_X(0) = 1$