ROBOTICS EXAM #1

(All work must be shown to receive full credit. May be submitted by a 2 student team)

Question #1

A differential drive robot is characterized by the following kinematics equations:

$$\dot{x} = v\cos(\theta)$$
$$\dot{y} = v\sin(\theta)$$
$$\dot{\theta} = \omega$$

We know that;

Rotational speed of the right wheel is 60rpm

Rotational speed of the left wheel is 40rpm

Radius of the wheels is 5cm

Distance between the wheels is 20cm

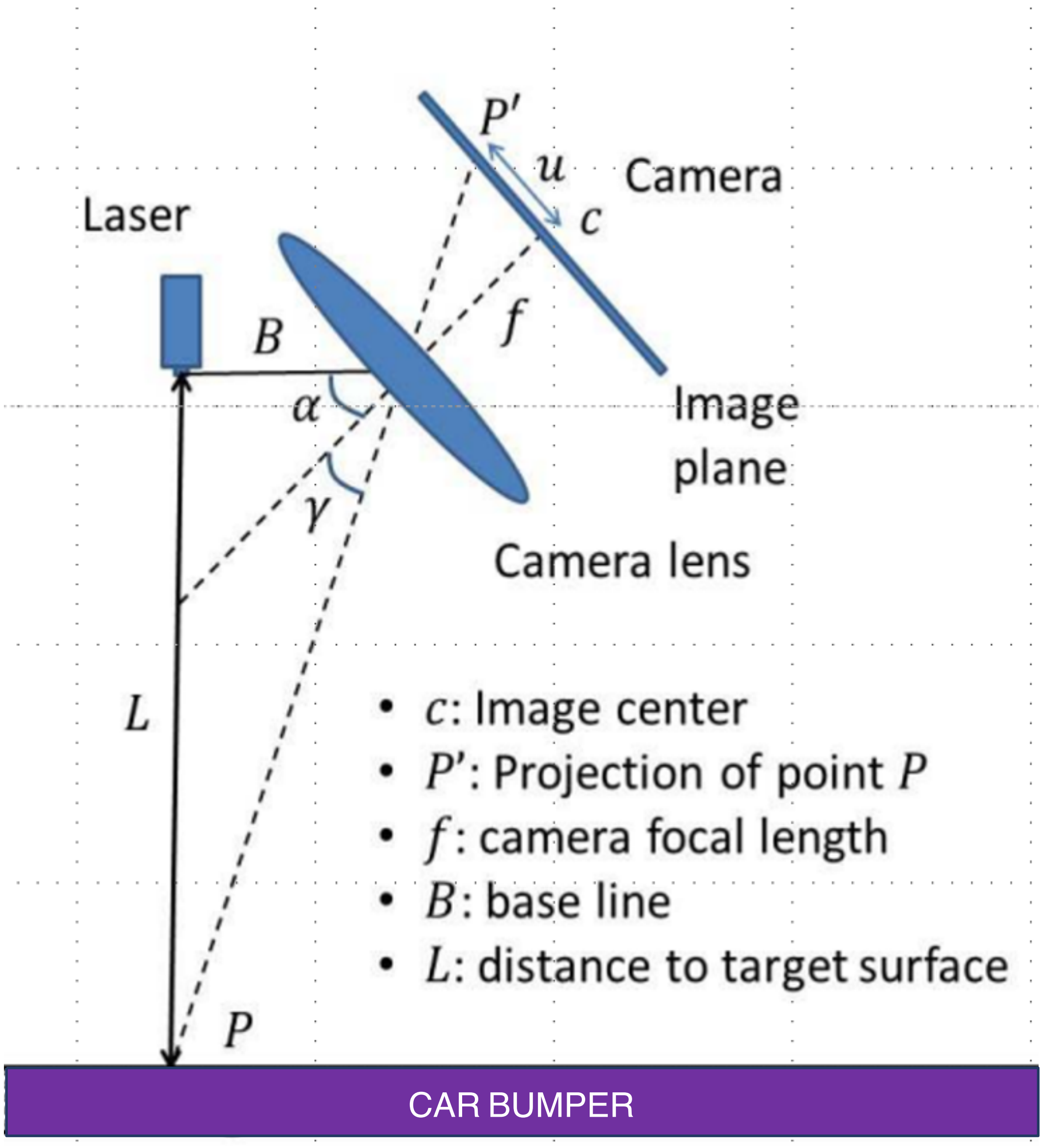
Assuming that at time =0sec the robot is at the origin and traveling in the positive X direction, plot the complete circular path of the differential drive robot in 1 sec intervals. Label each point on the circular path with its position (x,y) coordinates and its velocity (Vx,Vy) components. Its ok to use matlab or other software.

Question #2

Use the Least Squares Fit technique to approximate the second order equation that describes the following sensor data.

X	Y
2	28
4	52
6	92
8	148
10	220

Question #3



a) A car uses a rear facing camera and a laser that are configured for optical triangulation to measure the distance to an adjacent car's bumper. The angle between the camera's optical axis and the laser beam source is 15 degrees. The focal length of the camera is 10cm and the baseline (B) is 15 cm. The image of the laser hitting the adjacent car's bumper appears 10 cm from the camera's optical axis. How far away is the car's bumper?

b) If the speed of sound is 343 m/sec and a sonar pulse can travel the distance to a tree and back in 0.15 sec, how far away is the tree?

Question #4

c) Using 8 black/white sensors, into how many equally sized sectors can a circle be divided? How many degrees will be contained within each equal sized sector?
d) What will be the angular size of each sector in degrees?

Question #5

Defining the Psuedo Range (distance)of each of four satellites from a user as

PSR_i = = c(apparent measured radio signal transit time)=c(time on satellite clock – time on user clock) = SQRT($(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2$)+ $c\Delta T_0$

i = 1, ..., 4

For the GPS satellites listed below, the actual distances from each satellite to the user = SQRT($(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2$)

where c = 299792458 m/sec

ΔT_0 = Time Delay or Error at Receiver

(x_u, y_u, z_u) = Coordinates of the user

	Actual Distance to Each Satellite	Time Delay or Error at Receiver	PSR
Satellite #1	23197900.2	.01 sec	?
Satellite #2	23197900.05	.01 sec	?

Satellite #3	23197900.04	.01 sec	?
Satellite #4	23197900.17	.01 sec	?

What is the PSR to each satellite from the user?

Question #6

What are the rotation matrices $R^0{}_1$ and $R^1{}_0$ for 90 degree rotations about the

a) X-axis
b) Y-axis
c) Z-axis

Question #7

A vector v with coordinates $v_0 = (1, 0, 1)$ is rotated by 90 degrees about the X_0 axis. What are the coordinates of the resulting vector?

Question #8

Do a Similarity Transformation that will represent the

001

Transformation Matrix = 010

-100

within a reference frame that has been rotated by 180 degrees around the Z axis.

Question #9

a) Write the Homogeneous Transformation Matrix for a 15 cm coordinate translation along Y-axis.

b) Write the Homogeneous Transformation Matrix for a 60 degree rotation with respect to the Y-axis

c) Write the Homogeneous Transformation Matrix for a 15 cm Y-axis translation together with a 60 degree rotation with respect to the Y-axis.

Question #10

Calculate the four Homogeneous Transformation Matrices and the total Transformation Matrix using the following DH parameters (MatLab permitted).

Link	a_i	α_i	d_i	θ_i
1	5 cm	0 degrees	0	45 degrees

2	3 cm	90 degrees	0	45 degrees
3	3	90	2 cm	45 degrees
4	0	0	2 cm	0

.

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EXAM01

QUESTION 01

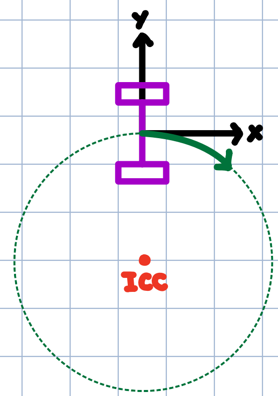
GIVEN: $n_r = 60\text{ rpm}$ $\quad t = 1s$
$n_l = 40\text{ rpm}$
$r = 5\text{cm}$
$L = 20\text{cm}$

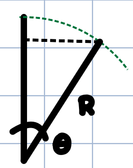
$$U_r = \frac{60\text{ rpm}}{60} \cdot 2\pi = 2\pi \text{ rad/s}$$

$$U_l = \frac{40\text{ rpm}}{60} \cdot 2\pi = \frac{4}{3}\pi \text{ rad/s}$$

$$\omega = \frac{r}{L}(U_r - U_l) = \frac{5}{20}\left(2\pi - \frac{4}{3}\pi\right) = \frac{\pi}{6} \text{ rad/s}$$

SOLUTION:





Location of ICC:

$$R = \frac{L}{2} \cdot \frac{U_r + U_l}{U_r - U_l}$$

$$= \frac{0.20}{2} \cdot \frac{2\pi + 4/3\pi}{2\pi - 4/3\pi} = 0.5 \text{ m}$$

$$\theta = \omega t = (\pi/6)(1) = \pi/6$$

$$x = R\sin\theta = 0.5\sin(\pi/6) = 0.25 \text{ m}$$

$$y = R - R\cos\theta = 0.5 - 0.5\cos(\pi/6) = 0.067 \text{ m}$$

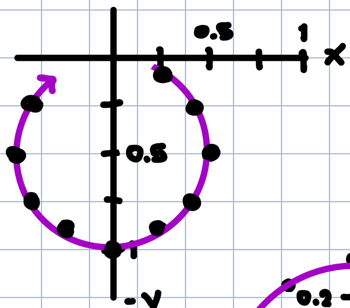
$$\boxed{\text{position}: (0.25\text{ m}, -0.067\text{ m}, \pi/6)}$$

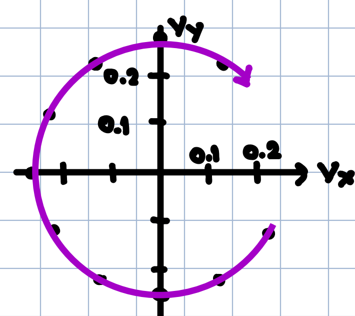
$$V_x = r/2\,(U_r + U_l)\cos\theta = 0.05/2\,(2\pi + 4\pi/3)\cos(-\pi/6) = 0.227$$

$$V_y = r/2\,(U_r + U_l)\sin\theta = 0.05/2\,(2\pi + 4\pi/3)\sin(-\pi/6) = -0.131$$

$$\boxed{\text{velocity}: (0.227, -0.1331)}$$

t	θ	x	y	Vx	Vy
1	30°	0.25	-0.067	0.227	-0.131
2	60°	0.433	-0.25	0.131	-0.227
3	90°	0.5	-0.5	0	-0.262
4	120°	0.433	-0.75	-0.131	-0.227
5	150°	0.25	-0.933	-0.227	-0.131
6	180°	0	-1	-0.262	0
7	210°	-0.25	-0.933	-0.227	0.131
8	240°	-0.433	-0.75	-0.131	0.227
9	270°	-0.5	-0.5	0	0.262
10	300°	-0.433	-0.25	0.131	0.227





QUESTION 02

$$\begin{bmatrix} 5 & 30 & 220 \\ 30 & 220 & 1800 \\ 220 & 1800 & 15664 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 540 \\ 4200 \\ 35728 \end{bmatrix}$$

$$M_0 = \begin{bmatrix} 540 & 30 & 220 \\ 4200 & 220 & 1800 \\ 35728 & 1800 & 15664 \end{bmatrix} \quad M_2 = \begin{bmatrix} 5 & 30 & 540 \\ 30 & 220 & 4200 \\ 220 & 1800 & 35728 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 5 & 540 & 220 \\ 30 & 4200 & 1800 \\ 220 & 35728 & 15664 \end{bmatrix} \quad M = \begin{bmatrix} 5 & 30 & 220 \\ 30 & 220 & 1800 \\ 220 & 1800 & 15664 \end{bmatrix}$$

$$a_0 = \frac{\det(M_0)}{\det(M)} = \frac{896000}{44800} = 20$$

$$a_1 = \frac{\det(M_1)}{\det(M)} = \frac{0}{44800} = 0$$

$$a_2 = \frac{\det(M_2)}{\det(M)} = \frac{89600}{44800} = 2$$

$$\boxed{y = 2x^2 + 20}$$

QUESTION 03

(a) GIVEN: $\alpha = 15°$
$f = 10cm$
$B = 15cm$
$u = 10cm$

$\gamma = \arctan u/f$
$= \arctan(10/10) = 45°$

$L = B\tan(\alpha + \gamma)$
$= 15\tan(15° + 45°)$
$\boxed{= 25.98 \text{ cm}}$

(b) GIVEN: $V_s = 343$ m/s
$t_0 = 0.15$ s

$r_0 = V_s t_0/2$
$= 343(0.15)/2$
$\boxed{= 25.725 \text{ m}}$

QUESTION 04:

(c) $2^n = 2^8 = \boxed{256 \text{ sectors}}$
$360°/256 = 1.40625° \approx \boxed{1.406°}$

(d) $360°/256 = 1.40625° \approx \boxed{1.406°}$

QUESTION 05:

Satellite #1 : PSR = $23197900.2 + (0.01)(299792458)$
$= \boxed{26195824.78 \text{ m}}$

Satellite #2 : PSR = $23197900.05 + (0.01)(299792458)$
$= \boxed{26195824.63 \text{ m}}$

Satellite #3 : PSR = $23197900.04 + (0.01)(299792458)$
$= \boxed{26195824.62 \text{ m}}$

Satellite #4 : PSR = $23197900.17 + (0.01)(299792458)$
$= \boxed{26195824.75 \text{ m}}$

QUESTION 06:

(a) x-axis

$$R_1^0 = R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_0^1 = (R_1^0)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

c) z-axis

$$R_1^0 = R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^1 = (R_1^0)^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) y-axis

$$R_1^0 = R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R_0^1 = (R_1^0)^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

QUESTION 07:

$$v_1^0 = R_{x,\pi/2}\, v^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

coordinates of resulting vector : (1,-1,0)

QUESTION 08:

$$R_{z,\theta} = R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = (R_1^0)^{-1} A R_1^0 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} R_1^0 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

QUESTION 09:

(a)

$$\text{Trans}_{y,15} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H = \text{Trans}_{y,5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

$$\text{Rot}_{y,60^\circ} = \begin{bmatrix} c_\beta & 0 & s_\beta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\beta & 0 & c_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H = \text{Rot}_{y,60^\circ} = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) $H = \text{Trans}_{y,15}\ \text{Rot}_{y,60^\circ}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 1 & 0 & 15 \\ -\sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

QUESTION 10:

$$H_1 = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \cos 0^\circ & \sin 45^\circ \sin 0^\circ & 5\cos 45^\circ \\ \sin 45^\circ & \cos 45^\circ \cos 0^\circ & -\cos 45^\circ \sin 0^\circ & 5\sin 45^\circ \\ 0 & \sin 0^\circ & \cos 0^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 5\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 5\sqrt{2}/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \cos 90^\circ & \sin 45^\circ \sin 90^\circ & 3\cos 45^\circ \\ \sin 45^\circ & \cos 45^\circ \cos 90^\circ & -\cos 45^\circ \sin 90^\circ & 3\sin 45^\circ \\ 0 & \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 & 3\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 3\sqrt{2}/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \cos 90^\circ & \sin 45^\circ \sin 90^\circ & 3\cos 45^\circ \\ \sin 45^\circ & \cos 45^\circ \cos 90^\circ & -\cos 45^\circ \sin 90^\circ & 3\sin 45^\circ \\ 0 & \sin 90^\circ & \cos 90^\circ & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 & 3\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 3\sqrt{2}/2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} \cos 0^\circ & -\sin 0^\circ \cos 0^\circ & \sin 0^\circ \sin 0^\circ & 0 \cdot \cos 0^\circ \\ \sin 0^\circ & \cos 0^\circ \cos 0^\circ & -\cos 0^\circ \sin 0^\circ & 0 \cdot \sin 0^\circ \\ 0 & \sin 0^\circ & \cos 0^\circ & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T = H_1 \cdot H_2 \cdot H_3 \cdot H_4$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 5\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 5\sqrt{2}/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 & 3\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 3\sqrt{2}/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 & 3\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 3\sqrt{2}/2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 5\sqrt{2}/2 \\ 1 & 0 & 0 & 3 + 5\sqrt{2}/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 & 3\sqrt{2}/2 + \sqrt{2} \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & 3\sqrt{2}/2 - \sqrt{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 2 + 5\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 & 3 + 5\sqrt{2} \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 & \sqrt{2}/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 5.536 \\ 0.707 & 0 & 0.707 & 10.071 \\ 0.707 & 0 & -0.707 & 0.707 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$