Robotics: Homework 6

Problem

The inverse kinematics problem for our popular two-link planar manipulator has two solutions in general. We want to solve the inverse kinematics problem by finding **both solutions**. You can use the following equations:

$$\cos\theta_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2} \tag{1}$$

and

$$\sin\theta_2 = \pm\sqrt{1 - cos^2\theta_2} \tag{2}$$

Now we can write the solution for θ_2:

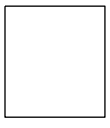
$$\theta_2 = atan2(\sin\theta_2, \cos\theta_2) \tag{3}$$

The solution for θ_1 is

$$\theta_1 = atan2(p_y, p_x) - atan2(a_2\sin\theta_2, a_1 + a_2\cos\theta_2) \tag{4}$$

The numerical values for the desired end effector coordinates are (1.0m,0.5m) , and the length of the links is $a_1 = 0.8m, a_2 = 0.5m$.

1) Use the equations above to find both solutions.
2) Use fsolve or a similar function to confirm your results. Change the initial guess to find both solutions.
3) Sketch or plot both solutions.



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HOMEWORK 06

GIVEN: $(P_x, P_y) = (1.0m, 0.5m)$
$a_1 = 0.8m$, $a_2 = 0.5m$

1. $\cos\theta_2 = \frac{P_x^2 + P_y^2 - a_1^2 - a_2^2}{2a_1a_2} = \frac{(1.0)^2 + (0.5)^2 - (0.8)^2 - (0.5)^2}{2(0.8)(0.5)}$

$= 0.45$

$\theta_2 = \pm 1.104 \text{ rad} \approx \pm 63.26°$

if $\theta_2 = +1.104$

$\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right) - \tan^{-1}\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$

$= \tan^{-1}\left(\frac{0.5}{1.0}\right) - \tan^{-1}\left[\frac{0.5\sin 1.104}{0.8 + 0.5\cos 1.104}\right]$

$= 0.464 - 0.411 = 0.053 \text{ rad} \approx 3.04°$

if $\theta_2 = -1.104$

$\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right) - \tan^{-1}\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$

$= \tan^{-1}\left(\frac{0.5}{1.0}\right) - \tan^{-1}\left[\frac{0.5\sin(-1.104)}{0.8 + 0.5\cos(-1.104)}\right]$

$= 0.464 - (-0.411) = 0.875 \text{ rad} \approx 50.13°$

2. $P_x = a_1\cos\theta_1 + a_2\cos(\theta_1 + \theta_2)$
$P_y = a_1\sin\theta_1 + a_2\sin(\theta_1 + \theta_2)$

$P_x = 0.8\cos(0.053) + 0.5\cos(0.053 + 1.104) = 1.000$
$P_y = 0.8\sin(0.053) + 0.5\sin(0.053 + 1.104) = 0.500$

$P_x = 0.8\cos(0.875) + 0.5\cos(0.875 - 1.104) = 1.000$
$P_y = 0.8\sin(0.875) + 0.5\sin(0.875 - 1.104) = 0.500$

3.