Propositional calculus Hw

#1

1. Which of the following is a proposition?
   a. $2 + 5 = 19$
   b. the difference of two primes
   c. for some positive integer n, $n * 17 = 19340$
2. Given the proposition p is false, proposition q is true and the proposition r is false, determine whether each of the following proposition is true or false.
   a. $\neg p \lor \neg(q \land r)$
   b. $(p \lor \neg r) \land \neg((q \lor r) \lor \neg(r \lor p))$
   c. $(p \lor q) \land (\neg p \lor q) \land (p \lor \neg q) \land (\neg p \lor \neg q)$
3. Represent the given propositions symbolically by letting
   p: $5 < 9$, q: $9 < 7$, r : $5 < 7$
   a. it is not the case that ($5 < 9$ and $9 < 7$)
   b. $5 < 9$ or it is not the case that ($9 < 7$ and $5 < 7$)
4. Formulate the following symbolic expression in words using: P: lee takes computer science q: lee takes Mathematics
   a. $\neg p \land \neg q$
   b. $p \land \neg q$
5. Formulate the symbolic expression in words using:
   P: today is Monday
   q: it is raining
   r: it is hot
   a. $(p \land q) \land \neg(r \lor p)$
   b. $(p \land (q \lor r)) \land (r \lor (q \lor p))$
6. Represent the proposition symbolically by letting P: you run 10 laps daily q: you are healthy r: you take multivitamins
   a. you run 10 laps daily , but you are not healthy
   b. you do not run 10 laps daily, you do not take multi-vitamins and you are not healthy
7. State the meaning of each sentence if "or" is interpreted as inclusive-or($\lor$) then state the meaning of each sentence. If "or" is interpreted as exclusive-or($\oplus$)
   a. to enter Utopia you must have a driver's license or a passport
   b. the car comes with a cup holder that heats or cools your drink
   c. do you want fries or salad with your burger
8. Assuming that p and r are false and that q and s are true, find the truth value for each of the propositions:
   a) $p \to q$
   b) $\neg p \to \neg q$

c) $(p \to q) \land (q \to r)$

d) $(s \to (p \land \neg r) \land ((p \to (r \lor p) \land s)$

9. represent the given propositions symbolically

   P: $4 < 2$, q: $7 < 10$, r: $6 < 6$ a.

   a. if $4 < 2$ then $7 < 10$

   b. it is not true that ($6 < 6$ and 7 is not less than 10) then $6 < 6$

10. formulate the symbolic expression in words using P: today is Monday q: it is raining r: it is hot

    a) $\neg(p \lor q) \leftrightarrow r$

    b) $\neg p \to (q \lor r)$

11. Write each conditional proposition symbolically. Write the converse, contrapositive for each proposition symbolically and in words. You need to assign a letter (p, q, r,..) to each proposition to write it symbolically

    a) if $4 < 6$, then $9 > 12$

    b) $141 < 3$ if $-3 < 4 < 3$

12. Formulate the arguments of the following symbolically and determine whether each is valid

    P: I study hard q: I got A's r: I get rich

a. if I study hard, then I get A's if I don't get rich, then I don't get A's

---

∵ I get rich

b. if I study hard, then I get A's, or I get rich

I don't get A's and I don't get rich

∵ I don't study hard

13. Write the given argument in words and determine whether each argument is valid using the rules of inference

P: 4 megabytes is better than no memory at all

q: we will buy more memory r: we will buy a new computer a. $p \to r \lor q$

$r \to \neg q$

∵ $p \to r$

b. $p \to r$

$r \to q$

∵ q

C. $\neg r \rightarrow \neg q$

r

$\therefore q$

14. Determine whether each argument is valid

a. $(p \rightarrow q) \wedge (r \rightarrow s)$

$p \vee r$

$\therefore q \vee s$

b. $p \rightarrow (q \rightarrow r)$

$q \rightarrow (p \rightarrow r)$

$\therefore (p \vee q) \rightarrow r$

15. Given an argument using rules of inference show that conclusion follows the hypothesis

a. Hypothesis: if there is gas in the car, then I will go to the store, then I will get a soda. I do not get a soda.

Conclusion: there is no gas in the car, or the car's transmission is defective

b. Hypothesis: if Jill can sing or Jack can play, then I will buy the compact disk . Jill can sing. I will buy the compact disk player

Conclusion: I will by the compact disk and I will buy the compact disk player

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1. a, c

2. $p = F$, $q = T$, $r = F$

a. $\neg p \vee \neg(q \wedge r)$

$\equiv \sim F \vee \sim(T \wedge F)$

$\equiv T \vee \sim F$

$\equiv T \vee T \equiv \boxed{T}$

b. $(p \vee \neg r) \wedge \neg((q \vee r) \vee \neg(r \vee p))$

$\equiv (F \vee \sim F) \wedge \sim((T \vee F) \vee \sim(F \vee F))$

$T \wedge \sim(T \vee T) \equiv T \wedge F \equiv \boxed{F}$

c. $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$

$\equiv (F \vee T) \wedge (T \vee T) \wedge (F \vee F) \wedge (T \vee F)$

$\equiv T \wedge T \wedge F \wedge T \equiv \boxed{F}$

3. a. $\sim(p \wedge q)$

b. $p \vee \sim(q \wedge r)$

4. a. Lee didn't take computer science and didn't take Mathematics.

b. Lee takes computer science but didn't take Mathematics.

5. a. Today is Monday and it is raining, but it is not hot nor today is Monday.

b. Today is Monday and it is raining or hot, but it is hot or it is raining or today is Monday.

6. a. $p \wedge \sim q$

b. $\sim(p \wedge r \wedge q)$

7. a. inclusive-or ($\vee$) : you can show driver license or a passport or both. It is better to show both.

exclusive-or ($\oplus$) : you can show either driver license or a passport but not both.

b. inclusive-or ($\vee$) : It means the cup holder can heat and cool at once.

exclusive-or ($\oplus$) : It means the cup holder can only perform heat or only cool, but not both

c. inclusive-or ($\vee$): It means that you can have fries or salad or both.
exclusive-or ($\oplus$): It means you can only choose between fries or salad but not both.

8\. a. $p \to q \equiv F \to T \equiv \boxed{T}$

b. $\sim p \to \sim q \equiv T \to F \equiv \boxed{F}$

c. $(p \to q) \wedge (q \to r) \equiv (F \to T) \wedge (T \to F) \equiv \boxed{F}$

d. $(s \to (p \wedge \sim r) \wedge ((p \to (r \vee p) \wedge s) \equiv (T \to (F \wedge T) \wedge ((F \to (F \vee F) \wedge T)$

$\equiv (T \to F \wedge ((F \to T)))$

$\equiv (T \to F \wedge T) \equiv \boxed{F}$

9\. a. $p \to q$

b. $\sim(r \wedge q) \to r$

10\. a. Today is not Monday or not raining if and only if it is hot.

b. If today is Monday, then it is raining or hot.

11\. a. $p: 4<6$, $q: 9>12$

$p \to q$

CONVERSE:
- $q \to p$
- if $9>12$, then $4<6$

CONTRAPOSITIVE:
- $\sim q \to \sim p$
- if $9 \leq 12$, then $4 \geq 6$

b. $r: -3<4<3$, $s: |4|<3$

$r \to s$

CONVERSE:
- $s \to r$
- if $|4|<3$, then $-3<4<3$

CONTRAPOSITIVE:
- $\sim s \to \sim r$
- if $|4| \geq 3$, then $-3 \geq 4 \geq 3$

12\. a. $p \to q$
$\sim r \to \sim q$
$\therefore r$

		CONC	PREMISE	
p	q	r	1	2
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	T

from the highlight, premises are true but conclusion is false therefore $\boxed{\text{invalid.}}$

b. $p \to (q \vee r)$

$\sim q \wedge \sim r$

$\therefore \sim p$

			PREMISE		CONCL.
p	q	r	1	2	~p
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	T	F
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	F	T
F	F	F	T	T	T

STEPS:

① $\sim q \wedge \sim r$

$\therefore \sim r$

$\therefore \sim q$

② $q \vee r$

$\therefore F$

③ $p \to F$

$\therefore \sim p$

The highlight shows that the premises are true and conclusion is true. Therefore, $\boxed{\text{valid}}$ argument.

13. a. • If 4 megabytes is better than no memory at all, then we will buy a new computer or more memory.

• If we will buy a new computer, then we will not buy more memory.

$\therefore$ If 4 megabytes is better than no memory at all, then we will buy a new computer.

$(p \to r) \vee (p \to q)$

$r \to \sim q$

$\therefore p \to r$

CONTRAPOSITIVE:

$r \to \sim q$

$\equiv q \to \sim r$

SYLLOGISM:

$p \to q$

$q \to \sim r$

$\therefore p \to \sim r$

$\boxed{\text{invalid argument}}$

b. • If 4 megabytes is better than no memory at all, then we will buy a new computer.

• If we will buy a new computer, then we will buy more memory.

$\therefore$ we will buy more memory.

SYLLOGISM:

$p \to r$

$r \to q$

$\therefore p \to q$

MODUS PONENS:

$p \to q$

$\therefore q$

$\boxed{\text{invalid argument}}$

c. · If we will not buy a new computer, then we will not buy more memory.

· we will buy a new computer.

∴ we will buy more memory

CONTRAPOSITIVE : $\sim r \rightarrow \sim q \equiv q \rightarrow r$

modus ponens :

$$\begin{array}{l} q \rightarrow r \\ \underline{r} \\ \therefore q \end{array}$$

$\boxed{\text{valid argument}}$

14. a.

$$\begin{array}{l} (p \rightarrow q) \wedge (r \rightarrow s) \\ \underline{p \vee r} \\ \therefore q \vee s \end{array}$$

				PREMISE		
p	q	r	s	1	2	CONCL.
T	T	T	T	~~T~~	~~T~~	~~T~~
T	T	T	F	F	T	T
T	T	F	T	~~T~~	~~T~~	~~T~~
T	T	F	F	~~T~~	~~T~~	~~T~~
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	~~T~~	~~T~~	~~T~~
F	T	T	F	F	T	T
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	~~T~~	~~T~~	~~T~~
F	F	T	F	F	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	F

$\boxed{\text{Valid argument}}$

① $\dfrac{(p \rightarrow q) \wedge (r \rightarrow s)}{\therefore p \rightarrow q}$

② $\dfrac{(p \rightarrow q) \wedge (r \rightarrow s)}{\therefore r \rightarrow s}$

③ $p \vee r \equiv \sim p \rightarrow r$

④ $\begin{array}{l} \sim p \rightarrow r \\ \underline{r \rightarrow s} \end{array}$

⑤ $\therefore \sim p \rightarrow s \equiv \sim s \rightarrow p$

⑥ $\begin{array}{l} \sim s \rightarrow p \\ \underline{p \rightarrow q} \end{array}$

⑦ $\sim s \rightarrow q \equiv s \vee q \equiv q \vee s$

b. $p \to (q \to r)$

$\underline{q \to (p \to r)}$

$\because (p \vee q) \to r$

p	q	r	PREMISE 1	PREMISE 2	CONCL.
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	T

$\boxed{\text{invalid argument}}$

15 a. p : There is gas in the car

q : I will go to store

r : I will get a soda

s : the car's transmission is defective.

$p \to q$

$q \to r$

$\underline{\sim r}$

$\because \sim p \vee s$

SYLLOGISM:

$p \to q$

$\underline{q \to r}$

$\because p \to r$

MODUS TOLLENS:

$p \to r$

$\underline{\sim r}$

$\because \sim p$

ADDITION:

$\underline{\sim p}$

$\because \sim p \vee s$

$\boxed{\text{Argument : valid}}$

b. p : Jill can sing

q : Jack can play

r : I will buy the compact disk

s : I will buy the compact disk player

$(p \vee q) \to r$

p

$\underline{s}$

$\because r \wedge s$

ADDITION:

$\underline{p}$

$\because p \vee q$

MODUS PONENS:

$\underline{p \to r}$

$\because r$

CONJUNCTION:

r

$\underline{s}$

$\because r \wedge s$

$\boxed{\text{Argument : valid}}$

16. a. If he studies, he will pass the exam

p : he studies

q : he will pass the exam