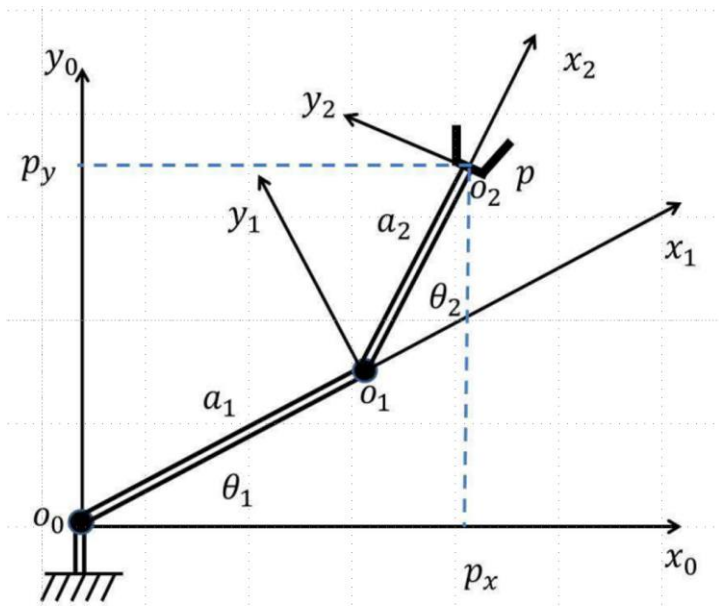


## Robotics Homework #7



Find the rotational velocity of the joints that will move the end effector as described

- Case 1: Horizontally with speed 2 m/s at configuration  $(\theta_1, \theta_2) = (45^\circ, 45^\circ)$
- Case 2: Vertically with speed 4 m/s at configuration  $(\theta_1, \theta_2) = (45^\circ, 90^\circ)$

Note:  $a_1=7\text{m}$   $a_2=10\text{m}$

## CASE 1:

$$\begin{aligned}
 J &= \begin{bmatrix} -a_1 \sin \theta_1 - a_2 \sin(\theta_1 + \theta_2) & -a_2 \sin(\theta_1 + \theta_2) \\ a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) & a_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \\
 &= \begin{bmatrix} -7 \sin 45^\circ - 10 \sin(45^\circ + 45^\circ) & -10 \sin(45^\circ + 45^\circ) \\ 7 \cos 45^\circ + 10 \cos(45^\circ + 45^\circ) & 10 \cos(45^\circ + 45^\circ) \end{bmatrix} \\
 &= \begin{bmatrix} -14.950 & -10 \\ 4.950 & 0 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 J^{-1} &= \frac{1}{-14.950(0) - (-10)(4.950)} \begin{bmatrix} 0 & 10 \\ -4.950 & -14.950 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0.202 \\ -0.1 & -0.302 \end{bmatrix}
 \end{aligned}$$

CHECK:

$$\begin{aligned}
 &\begin{bmatrix} -14.950 & -10 \\ 4.950 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0.202 \\ -0.1 & -0.302 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark
 \end{aligned}$$

$$J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \longrightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J^{-1} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0.202 \\ -0.1 & -0.302 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.2 \end{bmatrix} \approx \begin{bmatrix} 0^\circ \\ -11.46^\circ \end{bmatrix}$$

## CASE 2:

$$\begin{aligned}
 J &= \begin{bmatrix} -7 \sin 45^\circ - 10 \sin(45^\circ + 90^\circ) & -10 \sin(45^\circ + 90^\circ) \\ 7 \cos 45^\circ + 10 \cos(45^\circ + 90^\circ) & 10 \cos(45^\circ + 90^\circ) \end{bmatrix} \\
 &= \begin{bmatrix} -12.021 & -7.071 \\ -2.121 & -7.071 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 J^{-1} &= \frac{1}{(-12.021)(-7.071) - (-7.071)(-2.121)} \begin{bmatrix} -7.071 & 7.071 \\ 2.121 & -12.021 \end{bmatrix} \\
 &= \begin{bmatrix} -0.101 & 0.101 \\ 0.030 & -0.172 \end{bmatrix}
 \end{aligned}$$

CHECK:

$$\begin{bmatrix} -12.021 & -7.071 \\ -2.121 & -7.071 \end{bmatrix} \begin{bmatrix} -0.101 & 0.101 \\ 0.030 & -0.172 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J^{-1} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -0.101 & 0.101 \\ 0.030 & -0.172 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.404 \\ -0.688 \end{bmatrix} \approx \begin{bmatrix} 23.15^\circ \\ -39.42^\circ \end{bmatrix}$$