CSUS College of Engineering and Computer Science
Electrical & Electronic Engineering
ENGR 120 Probability and Random Signals

Final Exam (100 points, 120 min.)

Name:

Question 1 [25 points]

(a) Answer the following questions (True or False).

(i) If *A* and *B* are both nonempty events of a sample space S and *A* and *B* are mutually exclusive, then *A* and *B* are dependent. **T/F**

(ii) For the random variables *X* and $Y = 3X+9$, $\text{Var}(Y) = 9\,\text{Var}(X) + 9$. **T/F**

(iii) The Markov inequity bound only applies to nonnegative random variables. **T/F**

(iv) The power spectral density of a WSS random process is defined as The Fourier transform of the auto-correlation function of the random process. **T/F**

(v) The central limit theorem always holds true, regardless of the sample size **T/F**

(b) Two 6-sided dice are rolled. What is the probability that their sum is at most 3?

(c) For the following experiment, *S* is the sample space and *A*, *B*, and *C* are events.
$S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$. $A = \{s_2\}$. $B = \{s_3, s_5, s_5\}$. $C = \{s_2, s_3, s_6\}$. $D = \{s_1, s_2\}$.

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	1/12	1/6	1/8	1/8	1/6	1/3

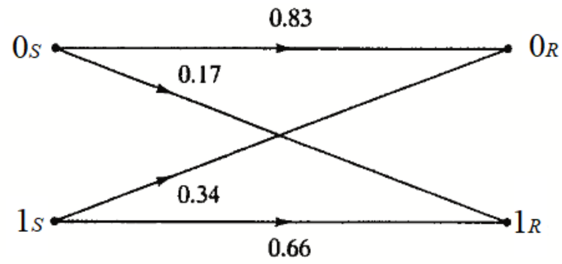
Sketch the Venn diagram of events and find the following probabilities.

(i) Pr(*A*).

(ii) $\Pr(A^c \cap C)$.

(iii) Which pair of events, *A,B,C*, and *D,* (if any) are mutually exclusive?

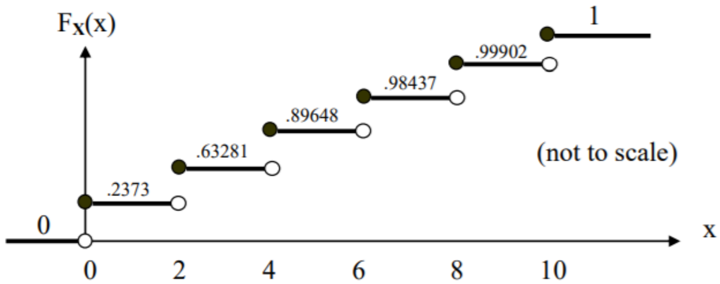
Question 2 [25 points]
Consider the binary communication channel depicted below. Given that the a "1" is observed at the receiver, calculate the probability that a "1" was transmitted, i.e. $\Pr(1_S \mid 1_R)$. Assume $\Pr(0_S) = \Pr(1_S) = 0.5$.



b) Consider the random variable $Y = X + k$, where k is a constant and X is a random variable that is always strictly larger than -10 and has an expected value of $E(X) = -6$. Choose a suitable value for the constant k and determine an upper bound for the probability $\Pr(Y > 8)$. Justify your choice for the constant k.

Question 3 [25 points]

a) The distribution (CDF) of a discrete random variable is shown in the figure below.



i) Find the probability Pr(X≤2).

ii) Calculate the variance of X, ie VAR[X].

iii) Sketch the corresponding probability mass function PMF.

b) The PDF of a random variable X is given by:

$$f_X(x) = \begin{cases} 0.4 + kx, & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

i) Find the value k that makes f_X a valid PDF.
ii) Find P(X>1/2).
iii) Find the CDF, $F_X(x)$.

Question 4 [25 points]

a) Let the joint density function $f_{X,Y}$ *(x,y)* be

$$f_{X,Y}(x,y) = \frac{xy}{9} \qquad ;\, 0 \leq x \leq 2,\ 0 \leq y \leq 3$$

Determine:

i) The marginal PDFs $f_X(x)$ and $f_y(y)$.

ii) The expected values of X and Y.

iii) Are X and Y statistically independent?

iv) Are X and Y uncorrelated? Justify your answer. ***Hint: There is an easier way to solve this question without working-out the double integral***

b) Consider the random process

$$Y(t) = A,$$

where A~$N(0,1)$ is a standard Gaussian random variable. Is this process WSS? Justify your answer.