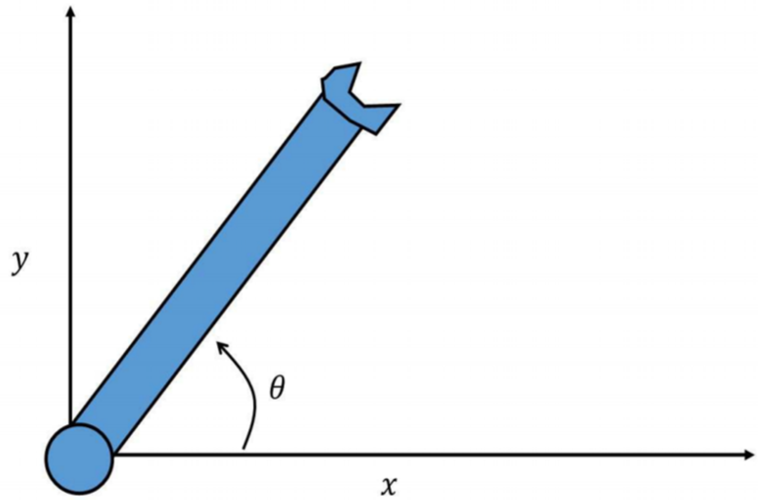
**EXAM # 2**

**All work must be shown to receive full credit. Collaboration with one single friend is permitted but not required. The friend's name must be included in the submitted uploaded document.**

QUESTION #1 (75 points)




Robotic manipulator

We consider the manipulator of figure 3. We want to plan the trajectory of the robot so that

$\Theta_0 = 0$ radians (initial position)

$\Theta_f = 2\pi$ radians (final position after 10 sec)

$\dot{\Theta}_0 = \omega_0 = 0$ radians/sec (initial angular velocity $= d\Theta_0/dt$)

$\dot{\omega}_0 = 0.1$ radians/sec$^2$ (initial angular acceleration$= d\omega_0/dt$)

The initial and final times are $t_0 = 0$ seconds and $t_f = 10$ seconds

1) What is the degree of the polynomial that describes the rotational motion of the manipulator?
2) Write the polynomial equation for the time evolution of $\Theta(t)$.
3) Write the polynomial equation for $d\Theta(t)/dt$
4) Write the polynomial equation for $d\omega(t)/dt$
5) What will be the final angular velocity $= d\Theta_f/dt = \omega_f$ at 10 seconds?

QUESTION #2 (50 points)

Consider the planar manipulator shown below where;

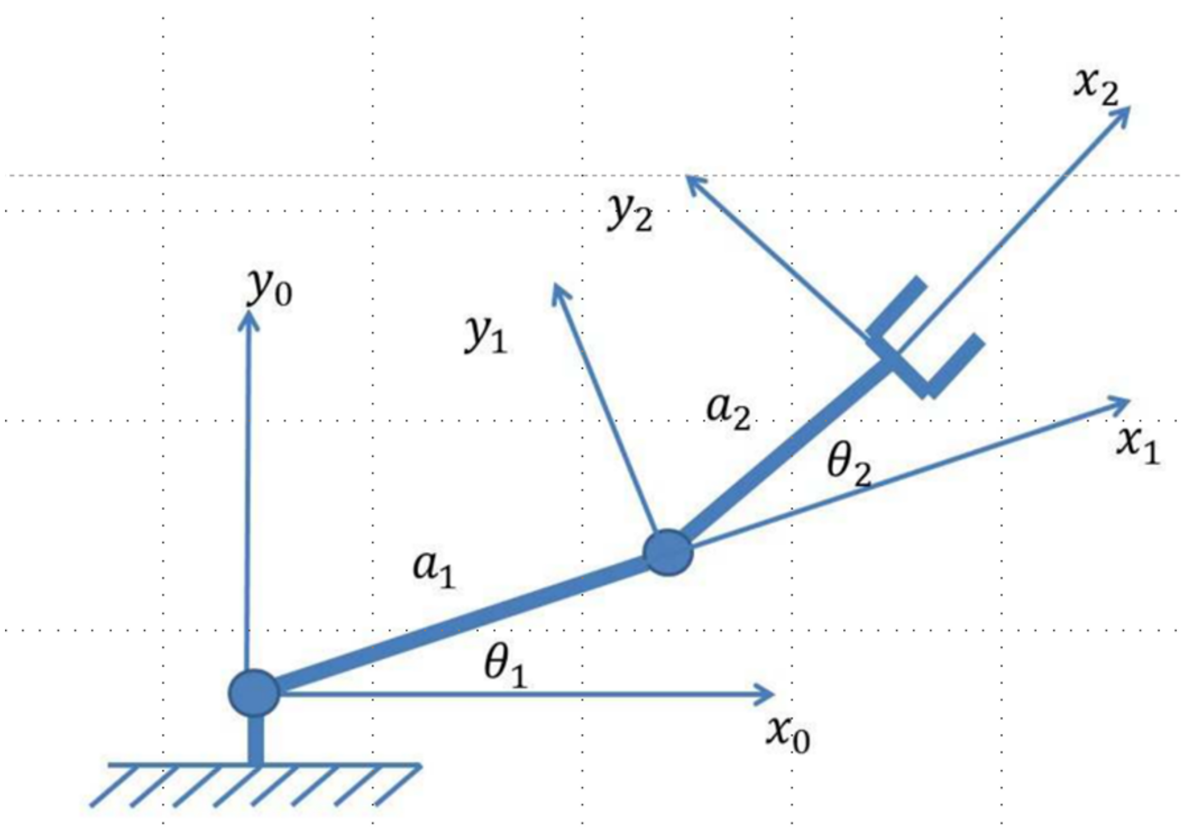
$\theta_1 = \boxed{0.2t^2}$ and $\theta_2 = \boxed{0.1t^2}$ (angles measured in radians, t measured in seconds, moving counter clockwise)

$a_1$=2 meters and $a_2$ = 1 meter

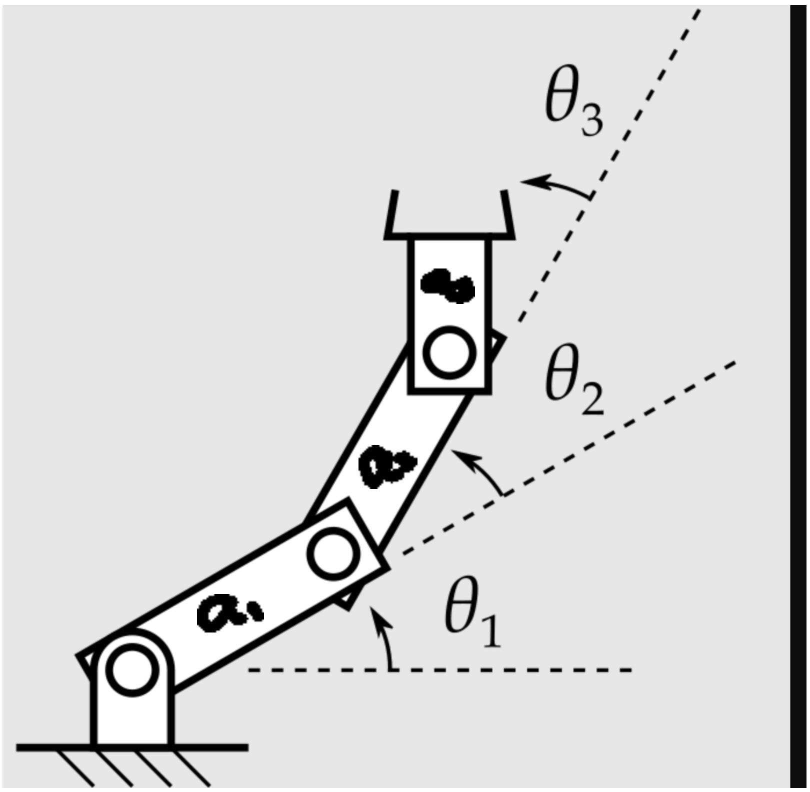
What will be the values of X and Y after 2 seconds?

Write the Jacobian Matrix for the manipulator

What will be the values of dX/dt and dY/dt after 2 seconds?



QUESTION #3 (50 points)




For a three-link manipulator the equations for the position of the End Effector would be as follows;

$P_x = a_1 COS(\theta_1) + a_2 COS(\theta_1 + \theta_2) + a_3 COS(\theta_1 + \theta_2 + \theta_3)$

$P_y = a_1 SIN(\theta_1) + a_2 SIN(\theta_1 + \theta_2) + a_3 SIN(\theta_1 + \theta_2 + \theta_3)$

If a1=2cm, a2=3cm, a3=2cm;

Find $dP_x/dt$ and $dP_y/dt$ for the manipulator when;

$d\theta_1/dt = d\theta_2/dt = d\theta_3/dt = 6.28$ radians per second.

$\theta_1 = \theta_2 = \theta_3 = pi/6$ radians = 30 degrees

QUESTION #4 (25 points)

POTENTIAL FIELD (values in units of Joules)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **START** | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.5 | 2.5 | 2.5 | 2.5 | 4 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 4 | 5 | 4 | 2.5 | 2.5 | 2.5 | 2.5 | 4 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 4 | 5 | 4 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 4 | 5 | 4 | 3 | 2 | 2 | 2 | 2.5 | 4 | 5 | 4 | 2 | 1 | 1 |
| 5 | 3 | 5 | 5 | 5 | 3 | 1 | 1 | 1 | 2.5 | 5 | 5 | 5 | 2 | 1 | 1 |
| 5 | 3 | 4 | 5 | 4 | 3 | 1 | 1 | 1 | 2.5 | 4 | 5 | 4 | 2 | 4 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2.5 | 2.5 | 2.5 | 2.5 | 4 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | **END** |

Trace all of the paths that a robot can travel from START to END while encountering no instances of negative forces that resist its forward progress.

QUESTION #5 (100 points)

The inverse kinematics problem for our popular two-link planar manipulator has two solutions in general. We want to solve the inverse kinematics problem by finding **both solutions**. You can use the following equations:

$$\cos\theta_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2} \quad (1)$$

and

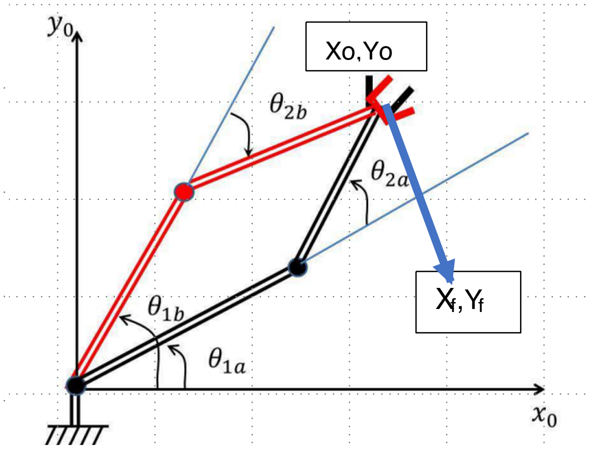
$$\sin\theta_2 = \pm\sqrt{1 - cos^2\theta_2} \quad (2)$$

Now we can write the solution for $\theta_2$:

$$\theta_2 = atan2(\sin\theta_2, \cos\theta_2) \quad (3)$$

The solution for $\theta_1$ is

$$\theta_1 = atan2(p_y, p_x) - atan2(a_2\sin\theta_2, a_1 + a_2\cos\theta_2) \quad (4)$$




DESCRIBING THE LINEAR TRAVEL OF THE END EFFECTOR

Using the parametric equations for the end effector displayed below

a. **$X = X_0$**
b. **$Y = Y_0e^{-t}$**
c. **a1=5cm, a2=4cm; $X_0$=7cm at t=0, $Y_0$=6cm at t=0;**

Fill in the following table using the Jacobian methodology and the Inverse Kinematics equations above. (Note: Either of the two possible Θvalues may be used).

| X | Y | $\Theta_1$ (radians) | $\Theta_2$ (radians) | dX/dt | dY/dt | $d\Theta_1/dt$ | $d\Theta_2/dt$ | t (sec) |
|---|---|---|---|---|---|---|---|---|
| 7 | 6 | | | | | | | 0 |
| 7 | | | | | | | | 1 |
| 7 | | | | | | | | 2 |

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## QUESTION 01

1. $\theta = a_0 + a_1 t + a_2 t^2 + a_3 t^3$
$\dot{\theta}_0 = a_1 + 2a_2 t + 3a_3 t^2$
$\dot{\omega}_0 = 2a_2 + 6a_3 t$

$\theta_0(t_0=0) = a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 = 0$
$a_0 = 0$
$\dot{\theta}_0(t_0=0) = a_1 + 2a_2(0) + 3a_3(0)^2 = 0$
$a_1 = 0$
$\dot{\omega}_0(t_0=0) = 2a_2 + 6a_3(0) = 0.1$
$a_2 = 0.05$

$\theta_f(t_f=10) = a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3 = 2\pi$
$0 + 0(10) + 0.05(10)^2 + a_3(10)^3 = 2\pi$
$a_3 = 1.28 \times 10^{-3}$

2. $\theta = 0.05t^2 + 1.28 \times 10^{-3} t^3$
3. $\dot{\theta} = 0.1t + 3.84 \times 10^{-3} t^2$
4. $\dot{\omega} = 0.1 + 7.68 \times 10^{-3} t$
5. $\dot{\theta}_f = 0.1(10) + 3.84 \times 10^{-3}(10)^2 = 1.384$ rad/s

## QUESTION 02

$a_1 = l_1 = 2$ $\quad \theta_1 = 0.2t^2$
$a_2 = l_2 = 1$ $\quad \theta_2 = 0.1t^2$

$x(t) = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$
$= 2\cos(0.2t^2) + \cos(0.2t^2 + 0.1t^2)$
$= 2\cos(0.2t^2) + \cos(0.3t^2)$

$y(t) = a_1 \sin\theta_1 + a_1 \sin(\theta_1 + \theta_2)$
$= 2\sin(0.2t^2) + \sin(0.2t^2 + 0.1t^2)$
$= 2\sin(0.2t^2) + \sin(0.3t^2)$

$t = 2$:
$x(2) = 2\cos[0.2(2)^2] + \cos[0.3(2)^2]$
$= 1.756$ m
$y(2) = 2\sin[0.2(2)^2] + \cos[0.3(2)^2]$
$= 2.367$ m

$$J = \begin{bmatrix} -a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2) & -a_2\sin(\theta_1+\theta_2) \\ a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) & a_2\cos(\theta_1+\theta_2) \end{bmatrix} = \begin{bmatrix} -2\sin(0.8) - \sin(1.2) & -\sin(1.2) \\ 2\cos(0.8) + \cos(1.2) & \cos(1.2) \end{bmatrix}$$

$$= \begin{bmatrix} -2.367 & -0.9320 \\ 1.756 & 0.3623 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -2.367 & -0.9320 \\ 1.756 & 0.3623 \end{bmatrix} \begin{bmatrix} 0.4t \\ 0.2t \end{bmatrix} = \begin{bmatrix} -2.367 & -0.9320 \\ 1.756 & 0.3623 \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} -2.266 \\ 1.530 \end{bmatrix}$$

## QUESTION 03

$$dP_x = -a_1\sin\theta_1(d\theta_1/dt) - a_2\sin(\theta_1+\theta_2)(d\theta_1/dt + d\theta_2/dt)$$
$$-a_3\sin(\theta_1+\theta_2+\theta_3)(d\theta_1/dt + d\theta_2/dt + d\theta_3/dt)$$
$$= -2\sin(\pi/6)(6.28) - 3\sin(\pi/6+\pi/6)(6.28+6.28)$$
$$-2\sin(\pi/6+\pi/6+\pi/6)(6.28+6.28+6.28)$$
$$= -6.28 - 32.63 - 37.68 = \underline{-76.59 \text{ cm}\cdot\text{rad/s}}$$

$$dP_y = a_1\cos\theta_1(d\theta_1/dt) + a_2\cos(\theta_1+\theta_2)(d\theta_1/dt + d\theta_2/dt)$$
$$+a_3\cos(\theta_1+\theta_2+\theta_3)(d\theta_1/dt + d\theta_2/dt + d\theta_3/dt)$$
$$= 2\cos(\pi/6)(6.28) + 3\cos(\pi/6+\pi/6)(6.28+6.28)$$
$$+2\cos(\pi/6+\pi/6+\pi/6)(6.28+6.28+6.28)$$
$$= 10.88 + 18.84 + 0 = \underline{29.72 \text{ cm}\cdot\text{rad/s}}$$

## QUESTION 04

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| START | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.5 | 2.5 | 2.5 | 2.5 | 4 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 4 | 5 | 4 | 2.5 | 2.5 | 2.5 | 2.5 | 4 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 4 | 5 | 4 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 4 | 5 | 4 | 3 | 2 | 2 | 2 | 2.5 | 4 | 5 | 4 | 2 | 1 | 1 |
| 5 | 3 | 5 | 5 | 5 | 3 | 1 | 1 | 1 | 2.5 | 5 | 5 | 5 | 2 | 1 | 1 |
| 5 | 3 | 4 | 5 | 4 | 3 | 1 | 1 | 1 | 2.5 | 4 | 5 | 4 | 2 | 4 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2.5 | 2.5 | 2.5 | 2.5 | 4 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | 1 |
| 5 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 2.5 | 2.5 | 2.5 | 2.5 | 2 | 1 | END |

## QUESTION 05

| x | y | $\theta_1$ | $\theta_2$ | dx/dt | dy/dt | $d\theta_1/dt$ | $d\theta_2/dt$ | t |
|---|---|---|---|---|---|---|---|---|
| 7 | 6 | 0.3109 | 0.7954 | 0 | $-Y_0e^{-t}$ | $-1.5024e^{-t}$ | $2.0166e^{-t}$ | 0 |
| 7 | $6e^{-1} \approx 2.207$ | -0.4412 | 1.493 | 0 | $-Y_0e^{-t}$ | $-1.0452e^{-t}$ | $0.5316e^{-t}$ | 1 |
| 7 | $6e^{-2} \approx 0.812$ | -0.6734 | 1.578 | 0 | $-Y_0e^{-t}$ | $-0.9432\ e^{-t}$ | $0.1951\ e^{-t}$ | 2 |

$$\cos\theta_2 = \frac{(7)^2 + (6)^2 - (5)^2 - (5)^2}{2(5)(5)}$$

$$\theta_2 = 0.7954$$

$$\theta_1 = \tan^{-1}(6/7) - \tan^{-1}[5\sin(0.7954)\ /\ 5+5\cos(0.7954)]$$

$$= 0.3109$$

$$J = \begin{bmatrix} -a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2) & -a_2\sin(\theta_1+\theta_2) \\ a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) & a_2\cos(\theta_1+\theta_2) \end{bmatrix}$$

$$= \begin{bmatrix} -5\sin(0.3109) - 5\sin(1.1063) & -5\sin(1.1063) \\ 5\cos(0.3109) + 5\cos(1.1063) & 5\cos(1.1063) \end{bmatrix} = \begin{bmatrix} -6 & -4.470 \\ 7 & 2.240 \end{bmatrix}$$

$$J^{-1} = \frac{1}{(-6)(2.240) - (-4.470)(7)} \begin{bmatrix} 2.240 & 4.470 \\ -7 & -6 \end{bmatrix}$$

$$\begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \end{pmatrix} = J^{-1}\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \frac{1}{17.85}\begin{bmatrix} 2.240 & 4.470 \\ -7 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ -Y_0e^{-t} \end{bmatrix} = \begin{pmatrix} -0.2504Y_0e^{-t} \\ 0.3361\ Y_0e^{-t} \end{pmatrix} = \begin{pmatrix} -1.5024e^{-t} \\ 2.0166\ e^{-t} \end{pmatrix}$$

$$\cos\theta_2 = \frac{(7)^2 + (6e^{-1})^2 - (5)^2 - (5)^2}{2(5)(5)}$$

$$\theta_2 = 1.493$$

$$\theta_1 = \tan^{-1}(6e^{-1}/7) - \tan^{-1}[5\sin(1.493)/5+5\cos(1.493)]$$

$$= 0.3055 - 0.7465 = -0.4410$$

$$J = \begin{bmatrix} -5\sin(-0.441) - 5\sin(1.052) & -5\sin(1.052) \\ 5\cos(-0.441) + 5\cos(1.052) & 5\cos(1.052) \end{bmatrix} = \begin{bmatrix} -2.207 & -4.342 \\ 7 & 2.479 \end{bmatrix}$$

$$J^{-1} = \frac{1}{(-2.207)(2.479) - (7)(-4.342)} \begin{bmatrix} 2.479 & 4.342 \\ -7 & -2.207 \end{bmatrix}$$

$$\begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \end{pmatrix} = \frac{1}{24.92}\begin{bmatrix} 2.479 & 4.342 \\ -7 & -2.207 \end{bmatrix}\begin{bmatrix} 0 \\ -Y_0e^{-t} \end{bmatrix} = \begin{pmatrix} -0.1742Y_0e^{-t} \\ 0.0886Y_0e^{-t} \end{pmatrix} = \begin{pmatrix} -1.0452e^{-t} \\ 0.5316\ e^{-t} \end{pmatrix}$$

$$\cos\theta_2 = \frac{(7)^2 + (6e^{-2})^2 - (5)^2 - (5)^2}{2(5)(5)}$$

$$\theta_2 = 1.578$$

$$\theta_1 = \tan^{-1}(6e^{-2}/7) - \tan^{-1}[5\sin(1.578)/5+5\cos(1.578)]$$

$$= 0.1156 - 0.789 = -0.6734$$

$$J = \begin{bmatrix} -5\sin(-0.6734) - 5\sin(0.9046) & -5\sin(0.9046) \\ 5\cos(-0.6734) + 5\cos(0.9046) & 5\cos(0.9046) \end{bmatrix} = \begin{bmatrix} -0.8127 & -3.931 \\ 7 & 3.090 \end{bmatrix}$$

$$J^{-1} = \frac{1}{(-0.8127)(3.090) - (7)(-3.931)} \begin{bmatrix} 3.090 & 3.931 \\ -7 & -0.8127 \end{bmatrix}$$

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} = \frac{1}{25} \begin{bmatrix} 3.090 & 3.931 \\ -7 & -0.8127 \end{bmatrix} \begin{bmatrix} 0 \\ -Y_0 e^{-t} \end{bmatrix} = \begin{pmatrix} -0.1572\ Y_0 e^{-t} \\ 0.03251\ Y_0 e^{-t} \end{pmatrix} = \begin{pmatrix} -0.9432\ e^{-t} \\ 0.1951\ e^{-t} \end{pmatrix}$$