# Homework-03 ENGR 117 Due date 03/14/2022

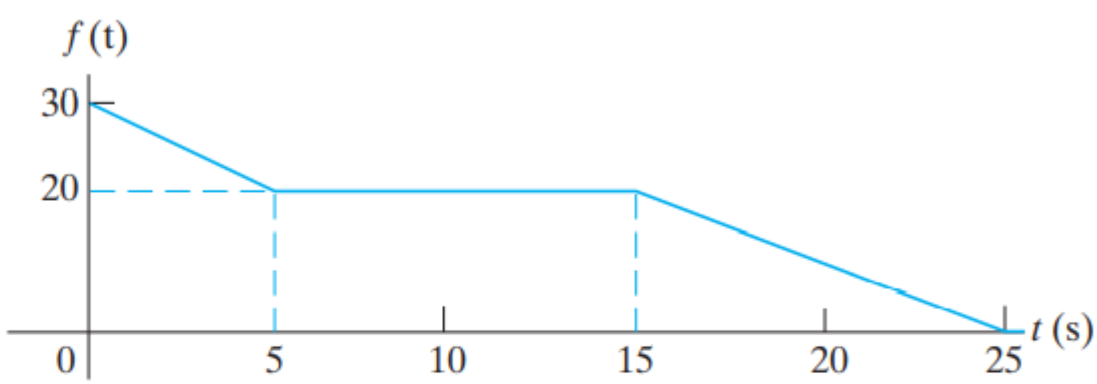
**10 Questions 10 points each**

**Q-1** Write an expression for f(t)

A function $f(t)$ is defined as follows:

$$
\begin{aligned}
f(t) &= 0, && t \le 0 \\
&= 5t, && 0 \le t \le 10\,\text{s} \\
&= -5t + 100, && 10\,\text{s} \le t \le 30\,\text{s} \\
&= -50, && 30\,\text{s} \le t \le 40\,\text{s}; \\
&= 2.5t - 150 && 40\,\text{s} \le t \le 60\,\text{s} \\
&= 0, && 60\,\text{s} \le t < \infty.
\end{aligned}
$$

**Q-2** Write an expression for f(t) for the following:



**Q-3** Use the initial- and final-value theorems to find the initial and final values of f(t) for the following functions.

**a)**

$$F(s) = \frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)}.$$

$$f(t) = (4e^{-3t} + 6e^{-4t} - 3e^{-5t})u(t).$$

**b)**

$$F(s) = \frac{(4s^2 + 7s + 1)}{s(s+1)^2}.$$

$$f(t) = (1 + 2te^{-t} + 3e^{-t})u(t).$$

**Q-4** There is no energy stored in the circuit shown in Fig. at the time the switch is opened.

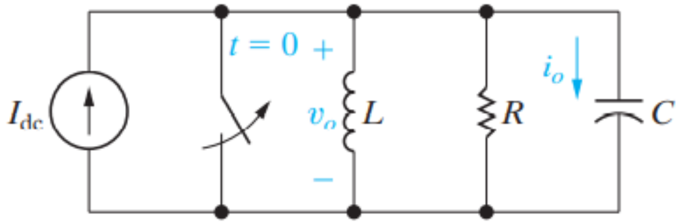
a) Derive the integrodifferential equation that governs the behavior of the voltage $v_o$.

b) Show that

$$V_o(s) = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}.$$

c) Show that

$$I_o(s) = \frac{sI_{dc}}{s^2 + (1/RC)s + (1/LC)}.$$

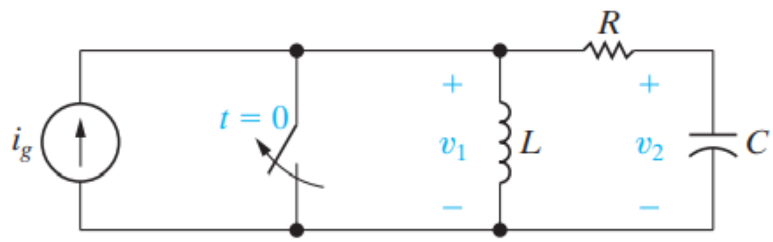


**Q-5** There is no energy stored in the circuit shown in Fig. at the time the switch is opened.

a) Derive the integrodifferential equations that govern the behavior of the node voltages $v_1$ and $v_2$.

b) Show that

$$V_2(s) = \frac{sI_g(s)}{C[s^2 + (R/L)s + (1/LC)]}.$$




**Q-6** Find f(t) for the following function:

$$F(s) = \frac{6(s + 10)}{(s + 5)(s + 8)}.$$

**Q-7** Find f(t) for the following function:

$$F(s) = \frac{15s^2 + 112s + 228}{(s + 2)(s + 4)(s + 6)}.$$

**Q-8** Find f(t) for the following function:

$$F(s) = \frac{14s^2 + 56s + 152}{(s + 6)(s^2 + 4s + 20)}$$

**Q-9** Find f(t) for the following function:

$$F(s) = \frac{60(s + 5)}{(s + 1)^2(s^2 + 6s + 25)}.$$

**Q-10** Find f(t) for the following function:

$$F(s) = \frac{5s^3 + 20s^2 - 49s - 108}{s^2 + 7s + 10}$$

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HOMEWORK 03
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**Q1.** $f(t) = 5t\,[u(t-0)-u(t-10)] + (-5t+100)[u(t-10)-u(t-30)]$
$\quad - 50\,[u(t-30)-u(t-40)] + (2.5t-150)[u(t-40)-u(t-60)]$

$$= \boxed{5t\,u(t) + (-10t+100)u(t-10) + (5t-150)u(t-30) + (2.5t-100)u(t-40) - (2.5t-150)u(t-60)}$$

**Q2.**

$f(t) = 30 \qquad t \le 0$
$= -2t+30 \qquad 0 \le t \le 5$
$= 20 \qquad 5 \le t \le 15$
$= -2t+50 \qquad 15 \le t \le 25$
$= 0 \qquad 25 \le t \le \infty$

$f(t) = (-2t+30)[u(t-0)-u(t-5)] + 20[u(t-5)-u(t-15)]$
$\quad + (-2t+50)[u(t-15)-u(t-25)]$

$$= \boxed{(-2t+30)\,u(t) + (2t-10)u(t-5) + (-2t+30)u(t-15) + (2t-50)u(t-25)}$$

**Q3. (a)** $F(s) = \dfrac{7s^2+63s+134}{(s+3)(s+4)(s+5)}$ $\qquad f(t) = (4e^{-3t} + 6e^{-4t} - 3e^{-5t})\,u(t)$

INITIAL-VALUE THEOREM:

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s\,\frac{7s^2+63s+134}{(s+3)(s+4)(s+5)}$$

$$= \lim_{s\to\infty} s\,\frac{s^2(7+63/s+134/s^2)}{s^3(1+3/s)(1+4/s)(1+5/s)}$$

$$= \lim_{s\to\infty} \frac{\cancel{s^3}(7+63/s+134/s^2)}{\cancel{s^3}(1+3/s)(1+4/s)(1+5/s)} = \frac{7+0+0}{(1+0)(1+0)(1+0)} = \boxed{7}$$

$$f(t=0^+) = \left[4e^{-3(0^+)} + 6e^{-4(0^+)} - 3e^{-5(0^+)}\right] = 4+6-3 = \boxed{7}$$

FINAL-VALUE THEOREM:

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0} s\,\frac{7s^2+63s+134}{(s+3)(s+4)(s+5)} = \frac{0}{(3)(4)(5)} = \boxed{0}$$

$$f(t\to\infty) = 4e^{-3(\infty)} + 6e^{-4(\infty)} - 3e^{-5(\infty)} = \boxed{0}$$

**(b)** $F(s) = \dfrac{(4s^2+7s+1)}{s(s+1)^2}$ $\qquad f(t) = (1 + 2te^{-t} + 3e^{-t})\,u(t)$

INITIAL-VALUE THEOREM:

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{\cancel{s}(4s^2+7s+1)}{\cancel{s}(s+1)^2} = \lim_{s\to\infty} \frac{\cancel{s^2}(4+7/s+1/s^2)}{\cancel{s^2}(1+1/s)^2} = \frac{4}{1} = \boxed{4}$$

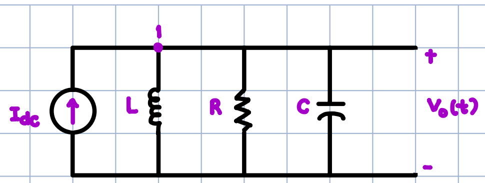
$$f(t=0^+) = 1 + 2(0^+)e^{-(0^+)} + 3e^{-(0^+)} = 1+0+3 = \boxed{4}$$

FINAL-VALUE THEOREM

$$\lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{\cancel{s}(4s^2+7s+1)}{\cancel{s}(s+1)^2} = \frac{0+0+1}{(0+1)^2} = \boxed{1}$$

$$f(t\to\infty) = 1 + 2(0^+)e^{-(\infty)} + 3e^{-(\infty)} = 1+0+0 = \boxed{1}$$

**Q4**




$i_L = {}^1/_L \int_0^t v(x)\,dx$

$i_C = C\,\dfrac{dv(t)}{dt}$

**(a)** NODE 1: $\boxed{I_{DC}\,u(t) = \frac{1}{L}\int_0^t V_o(x)\,dx + \frac{V_o(t)}{R} + C\,\frac{dV_o(t)}{dt}}$

**(b)** $I_{DC}\,({}^1/_s) = \frac{1}{L}\cdot\frac{V_o(s)}{s} + \frac{V_o(s)}{R} + C\left[sV_o(s) - \cancel{V(0^-)}^{\,0}\right]$

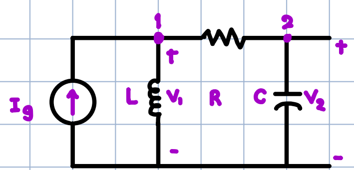
$= V_o(s)\left[\frac{1}{sL} + \frac{1}{R} + sC\right]$

$V_o(s) = \dfrac{I_{DC}\,({}^1/_s)}{\frac{1}{sL} + \frac{1}{R} + sC} = \dfrac{I_{DC}\,({}^1/_s)}{\frac{R + sL + s^2LRC}{sL\cdot R}} = \dfrac{I_{DC}\cdot LR}{R + sL + s^2LRC}\cdot\dfrac{{}^1/_{LRC}}{{}^1/_{LRC}}$

$$\boxed{V_o(s) = \frac{I_{dc}/C}{s^2 + ({}^1/_{RC})s + ({}^1/_{LC})}}$$

**(c)** $i_o = C\,\dfrac{dV_o}{dt} \longrightarrow I_o = sCV_o(s) = sC\left[\dfrac{I_{dc}/C}{s^2 + ({}^1/_{RC})s + ({}^1/_{LC})}\right] = \boxed{\dfrac{sI_{dc}}{s^2 + ({}^1/_{RC})s + ({}^1/_{LC})}}$

**Q5**




**(a)** NODE 1: $\boxed{I_g = \frac{1}{L}\int_0^t V_1(x)\,dx + \frac{V_1 - V_2}{R}}$

NODE 2: $\boxed{\frac{V_2 - V_1}{R} + C\,\frac{dV_2}{dt} = 0}$

**(b)** $I_g = \frac{1}{L}\int_0^t V_1(x)\,dx + \frac{V_1 - V_2}{R}$

$I_g(s) = \dfrac{V_1(s)}{sL} + \dfrac{V_1(s) - V_2(s)}{R}$

$sLR\,I_g(s) = RV_1(s) + sL\,V_1(s) - sL\,V_2(s)$

$V_2(s) = \dfrac{-sLR\,I_g(s) + V_1(s)(R + sL)}{sL}$

$V_2(s) = \left(\dfrac{R + sL}{sL}\right)V_1(s) - R\,I_g(s)$

$V_2(s) = \left(\dfrac{R + sL}{sL}\right)(1 + sCR)\,V_2(s) - R\,I_g(s)$

$= \dfrac{R + sCR^2 + sL + s^2CLR}{sL}\,V_2(s) - R\,I_g(s)$

$V_2(s) = \dfrac{R\,I_g(s)\cdot(sL)}{R + sCR^2 + \cancel{sL} + s^2CLR - \cancel{sL}} = \dfrac{s\,I_g(s)}{{}^1/_L + sCR/_L + s^2C} = \boxed{\dfrac{s\,I_g(s)}{C\left[s^2 + s(R/L) + ({}^1/_{LC})\right]}}$

$\dfrac{V_2 - V_1}{R} + C\,\dfrac{dV_2}{dt} = 0$

$\dfrac{V_2(s) - V_1(s)}{R} + sCV_2(s) = 0$

$\left(\frac{1}{R} + sC\right)V_2(s) = \dfrac{V_1(s)}{R}$

$V_1(s) = (1 + sCR)\,V_2(s)$

**Q6.** $F(s) = \dfrac{6(s+10)}{(s+5)(s+8)} = \dfrac{A}{s+5} + \dfrac{B}{s+8}$

$6(s+10) = A(s+8) + B(s+5)$

$s = -8:\ 12 = A(0) + B(-3) \longrightarrow B = -4$

$s = -5:\ 30 = A(3) + B(0) \longrightarrow A = 10$

$F(s) = \dfrac{10}{s+5} - \dfrac{4}{s+8}$

$$\boxed{f(t) = \left[10e^{-5t} - 4e^{-8t}\right]u(t)}$$

**Q7.** $F(s) = \dfrac{15s^2 + 112s + 228}{(s+2)(s+4)(s+6)} = \dfrac{A}{s+2} + \dfrac{B}{s+4} + \dfrac{C}{s+6}$

$15s^2 + 112s + 228 = A(s+4)(s+6) + B(s+2)(s+6) + C(s+2)(s+4)$

$s = -2:\ 64 = A(8) + B(0) + C(0) \longrightarrow A = 8$

$s = -4:\ 20 = A(0) + B(-4) + C(0) \longrightarrow B = -5$

$s = -6:\ 96 = A(0) + B(0) + C(8) \longrightarrow C = 12$

$F(s) = \dfrac{8}{s+2} - \dfrac{5}{s+4} + \dfrac{12}{s+6}$

$$\boxed{f(t) = \left[8e^{-2t} - 5e^{-4t} + 12e^{-6t}\right]u(t)}$$

**Q8**

$$F(s)=\frac{14s^2+56s+152}{(s+6)(s^2+4s+20)}=\frac{14s^2+56s+152}{(s+6)(s^2+4s+4+16)}=\frac{14s^2+56s+152}{(s+6)[(s+2)^2+16]}$$

$$=\frac{14s^2+56s+152}{(s+6)(s+2-j4)(s+2+j4)}=\frac{A}{s+6}+\frac{B}{s+2+j4}+\frac{C}{s+2-j4}$$

$$14s^2+56s+152=A(s+2-j4)(s+2+j4)+B(s+6)(s+2-j4)+C(s+6)(s+2+j4)$$

$s=-6:\ 320=A(-4-j4)(-4+j4)+B(0)+C(0)\longrightarrow A=10$

$s=-2-j4:\ -128=A(0)+B(4-j4)(-j8)+C(0)\longrightarrow B=2-j2$

$s=-2+j4:\ -128=A(0)+B(0)+C(4+j4)(j8)\longrightarrow C=2+j2$

$$F(s)=\frac{10}{s+6}+\frac{2-j2}{s+2+j4}+\frac{2+j2}{s+2-j4}=\frac{10}{s+6}+\frac{2\sqrt{2}\angle -45^\circ}{s+2+j4}+\frac{2\sqrt{2}\angle 45^\circ}{s+2+j4}$$

$$\frac{K\angle\theta}{s+\alpha-j\beta}+\frac{K\angle-\theta}{s+\alpha+j\beta}\longrightarrow 2|K|e^{-\alpha t}\cos(\beta t+\theta)$$

$f(t)=10e^{-6t}+2|2\sqrt{2}|e^{-2t}\cos(4t+45^\circ)$

$$\boxed{=[10e^{-6t}+4\sqrt{2}e^{-2t}\cos(4t+45^\circ)]u(t)}$$

**Q9**

$$F(s)=\frac{60(s+5)}{(s+1)^2(s^2+6s+25)}=\frac{60(s+5)}{(s+1)^2(s^2+6s+9+16)}=\frac{60(s+5)}{(s+1)^2(s+3+j4)(s+3-j4)}$$

$$=\frac{A}{(s+1)^2}+\frac{B}{s+1}+\frac{C}{s+3+j4}+\frac{D}{s+3-j4}$$

$$60(s+5)=A(s+3+j4)(s+3-j4)+B(s+1)(s+3+j4)(s+3-j4)+C(s+1)^2(s+3-j4)$$
$$+D(s+1)^2(s+3+j4)$$

$s=-1:\quad 240=A(20)+B(0)+C(0)+D(0)\longrightarrow A=12$

$s=-3-j4:\ 60(2-j4)=A(0)+B(0)+C(128+j96)+D(0)\longrightarrow C=-3/10-j\,33/20$

$s=-3+j4:\ 60(2+j4)=A(0)+B(0)+C(0)+D(128-j96)\longrightarrow D=-3/10+j\,33/20$

$s=-5:\quad 0=A(20)+B(-80)+C(-32-j64)+D(-32+j64)$

$0=12(20)+B(-80)+(-3/10-j33/20)(-32-j64)+(-3/10+j33/20)(-32+j64)$

$=240-80B+(-96+j72)+(-96-j72)\longrightarrow B=3/5$

$$F(s)=\frac{12}{(s+1)^2}+\frac{3/5}{s+1}+\frac{-3/10-j33/20}{s+3+j4}+\frac{-3/10+j33/20}{s+3-j4}$$

$$=\frac{12}{(s+1)^2}+\frac{3/5}{s+1}+\frac{1.68\angle-100.30^\circ}{s+3+j4}+\frac{1.68\angle 100.30^\circ}{s+3-j4}$$

$=12te^{-t}+0.6e^{-t}+2|1.68|e^{-3t}\cos(4t+100.30^\circ)$

$$\boxed{=[12te^{-t}+0.6e^{-t}+3.36e^{-3t}\cos(4t+100.30^\circ)]u(t)}$$

**Q10**

$$F(s)=\frac{5s^3+20s^2-49s-108}{s^2+7s+10}=5s-15+\frac{6s+42}{(s+2)(s+5)}$$

$$\begin{array}{r l}
 & 5s-15 \\
s^2+7s+10 \,\big) & 5s^3+20s^2-49s-108 \\
(-) & \underline{5s^3+35s^2+50s} \\
 & -15s^2-49s-108 \\
(-) & \underline{-15s^2-105s-150} \\
 & 6s+42
\end{array}$$

$$\frac{6s+42}{(s+2)(s+5)}=\frac{A}{s+2}+\frac{B}{s+5}$$

$6s+42=A(s+5)+B(s+2)$

$s=-2:\ 30=A(3)+B(0)\longrightarrow A=10$

$s=-5:\ 12=A(0)+B(-3)\longrightarrow B=-4$

$$\frac{6s+42}{(s+2)(s+5)}=\frac{10}{s+2}+\frac{-4}{s+5}$$

$$F(s)=5s-15+\frac{10}{s+2}-\frac{4}{s+5}$$

$$\boxed{f(t)=5\frac{d\delta(t)}{dt}-15\delta(t)+[10e^{-2t}-4e^{-5t}]u(t)}$$