California State University, Sacramento
The College of Engineering and Computer Science

EEE 180 Signals & Systems

Final Exam

Spring 2023

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1.[24 points] Select one correct answer for Each of the following questions. Each question below has only one correct answer.

(1). The signal y is defined by: $y = \int_0^{\infty} \delta(t)dt$, then which answer below is correct? _______________

A. y=1 B. $y = \infty$ C. y = u(t)

(2). The discrete-time signal y=2 $\cos(1.5\,\pi\, k + \frac{\pi}{4})$ is periodic. Which N value below can be used as the period of y? _______________

A. N=1 B. N = 2 C. N = 4

(3). A bounded-input and bounded-output system is called a ________ system.

A. Causal B. Stable C. Linear

(4). The continuous-time signal y=2 $\cos(6\,\pi\, t + \frac{\pi}{4})$. When the sampling frequency is 10 Hz, will the aliasing problem show up? ____________

A. Yes B. No

(5). Two continuous time signals are: y1 = sin(t) and y2= sin(t) / t.

Are they even or odd signals? _____________

A. y1: even, y2: even
B. y1: even, y2: odd
C. y1: odd, y2: even
D. y1: odd, y2: odd

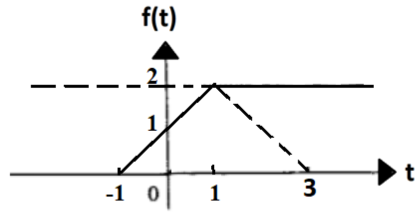
(6). The unilateral Laplace transform of δ(t) is _____________

D. 1 B. 0 C. π/2 D. None of above

(7). The system is governed by the following equation:
dy(t) /dt + 3 y(t) + 2 = x(t) . Is this a linear system? _____________

A. Yes B. No

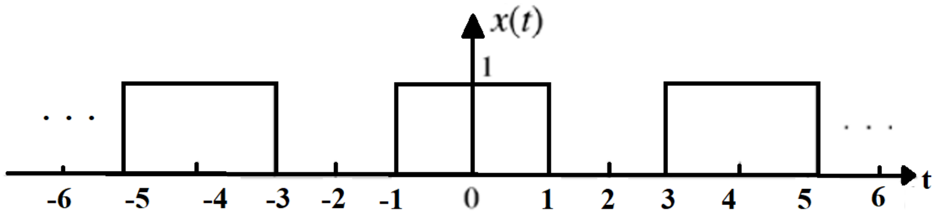
(8). The solid line below shows the waveform for f(t). What is the signal in the dashed line? ____________________



A. f(-t) B. f(-t + 2) C. f(-t – 2)

2.[36 points]

(1). Find the Exponential Fourier Series of the following periodic signal with a period of 4.



$$c_k = \frac{1}{T_0}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}\,dt \qquad T_0 = 4,\ \omega_0 = 2\pi/T_0 = \pi/2$$

$$= \frac{1}{4}\int_{-1}^{1} e^{-jk(\pi/2)t}\,dt = \frac{1}{4}\left[-\frac{1}{jk(\pi/2)}e^{-jk(\pi/2)t}\right]_{-1}^{1}$$

$$= -\frac{1}{j2\pi k}\left[e^{-jk(\pi/2)} - e^{jk(\pi/2)}\right]$$

$$= \frac{1}{\pi k}\left[\frac{e^{jk(\pi/2)} - e^{-jk(\pi/2)}}{2j}\right] = \boxed{\frac{1}{k\pi}[\sin(k\pi/2)]}$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{k\pi}[\sin(k\pi/2)]e^{jk(\pi/2)t}}$$

(2). Find the energy of the following signal:

$$x(t) = e^{-5t}u(t),$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$= \int_{-\infty}^{\infty} |e^{-5t}u(t)|^2\,dt = \int_{0}^{\infty} e^{-10t}\,dt$$

$$= \left[-\frac{e^{-10t}}{10}\right]_0^{\infty} = \boxed{\frac{1}{10}}$$

(3). Find the Fourier Transform of the following signal:

$$x(t) = e^{-5t}u(t)$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-5t} u(t) e^{-j\omega t} dt$$
$$= \int_{0}^{\infty} e^{-5t} e^{-j\omega t} dt$$
$$= \int_{0}^{\infty} e^{-(j\omega+5)t} dt = \left[-\frac{1}{j\omega+5} e^{-(j\omega+5)t} \right]_0^{\infty}$$
$$= \boxed{\frac{1}{j\omega+5}}$$

(4). Find the z-transform of the sequence $x[n] = (0.2)^n u[n]$, and determine the region of convergence.

$$Z\{(0.2)^n u[n]\} = \sum_{n=0}^{\infty} (0.2)^n z^{-n}$$
$$= \sum_{n=0}^{\infty} (5z)^{-n} = \frac{1}{1 - 1/5\, z^{-1}} = \boxed{\frac{z}{z-0.2}}$$
$$\boxed{ROC : |z| > 0.2}$$

3. [40 points]

(1). Suppose the unilateral z-transform of f(t) is F(z), and the ROC is |z| > 20.

$$F(z) = \frac{z}{(z-10)(z-20)}$$. Find the f(t) signal equation.

The unilateral z-transform pair table is given below.

Unilateral z-transform Pair Table

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$

$$\frac{F[z]}{z} = \frac{z}{(z-10)(z-20)} = \frac{A}{z-10} + \frac{B}{z-20}$$

$$1 = A(z-20) + B(z-10)$$

when $z=20$: $1 = B(10) \longrightarrow B = 1/10$

when $z=10$: $1 = A(-10) \longrightarrow A = -1/10$

$$\frac{F[z]}{z} = \frac{-1/10}{z-10} + \frac{1/10}{z-20}$$

$$F[z] = -\frac{1}{10}\left(\frac{z}{z-10}\right) + \frac{1}{10}\left(\frac{z}{z-20}\right)$$

$$f[k] = \left[-1/10\,(10)^k + 1/10\,(20)^k\right]u(k)$$

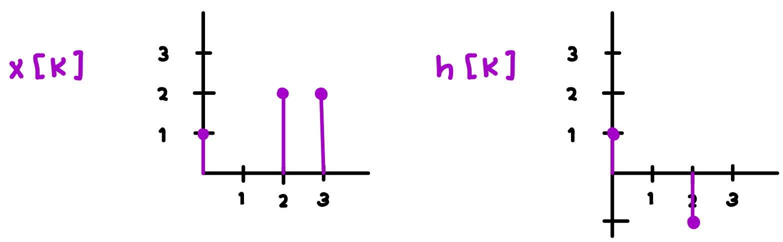
$$= \boxed{\frac{1}{10}\left[20^k - 10^k\right]u(k)}$$

(2). The discrete time input signal $x[k] = \delta[k] + 2\delta[k-2] + 2\,\delta[k-3]$

The discrete time signal system impulse response signal

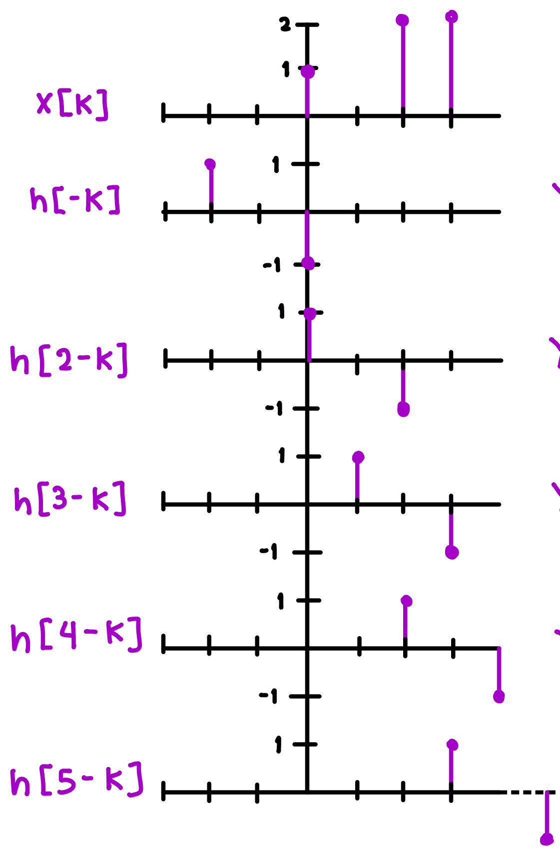
$$h[\mathrm{k}] = \mathrm{u}[\mathrm{k}] - u[\mathrm{k}-2]$$

Draw the waveforms of x[k] and h[k].



(3). For the above question (3), the system output response signal is defined as the convolution result of y[k] = x[k] * h[k].
Find the values of y[k].

$$y[k] = x[k] \cdot h[k] = \sum_{k=-\infty}^{\infty} x[n]h[k-n]$$



$y[0] = 1 \times -1 = -1$

$y[2] = (1 \times 1) + (2 \times -1) = -1$

$y[3] = 2 \times -1 = -2$

$y[4] = 2 \times 1 = 2$

$y[5] = 2 \times 1 = 2$

(4). For an LTIC system described by the transfer function

$$H(s) = \frac{s + 0.5}{s + 1}$$

Find the steady-state system response y(t) to the input signal of $f(t) = 2\,u(t)$.

$$H(j\omega) = \frac{j\omega + 0.5}{j\omega + 1}$$

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + 0.25}}{\sqrt{\omega^2 + 1}}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{0.5}\right) - \tan^{-1}\left(\frac{\omega}{1}\right)$$

$f(t) = 2u(t)$, $\omega = 0$

$|H(j\omega)| = 1/2$

$\angle H(j\omega) = 0$

Output response $y(t) = 2 \cdot 1/2 \cdot u(t)$

$= 1\,u(t)$

$\boxed{= u(t)}$