Chapter 6

Inductance and Capacitance

Text: *Electric Circuits* by J. Nilsson and S. Riedel
Prentice Hall

Engr 17 Introductory Circuit Analysis
Instructor: Russ Tatro

Chapter 6 Overview

The separation of charge used to define *Voltage* which is also used to define the concept of capacitance.

$$i_c = C\frac{dv}{dt}$$

The movement of charge is used to define *Current* which is also used to define the concept of inductance.

$$v_L = L\frac{di}{dt}$$

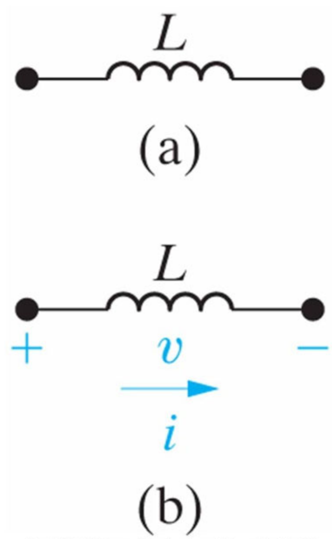
Sections 6.4 and 6.5 will be deferred until the discussion of ideal transformers in Chapter 9.

Section 6.1
The Inductor

The Inductor

The inductance – symbol L - is the consequence of a conductor carrying a current which is linked to a magnetic field.

$$v_L = L\frac{di}{dt}$$



$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L \, dt + i_L(t_0)$$

Note that the voltage across an inductor is written in accordance with the passive sign convention!

The unit of inductance is the henry H.

The Inductor

The behavior of the inductor's voltage/current relationship is thus different from the resistor.

It is the slope of a constant <u>change</u> in current that creates a constant voltage across the inductor.

$$v_L = L\frac{di}{dt}$$

If the current through an inductor is constant, then the voltage across the inductor is ZERO.

Power and Energy in the Inductor

Recall that power is given by: $p = (\pm)vi$

And the voltage across an inductor is $v_L = L\dfrac{di}{dt}$

Then the power stored in an inductor's magnetic field is

$$p_L = (\pm)vi = (\pm)\left(L\frac{di}{dt}\right)i = (\pm)Li\frac{di}{dt}$$

The energy w that an inductor can deliver to a load is (see text for derivation)

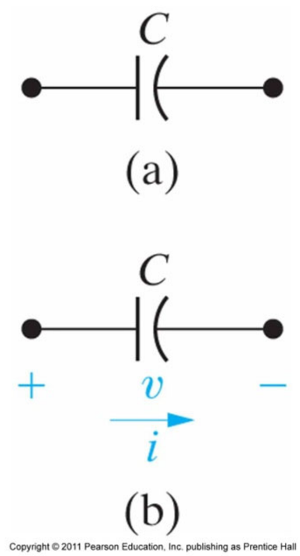
$$w = \frac{1}{2}Li^2$$

Section 6.2
The Capacitor

The Capacitor

The capacitor – symbol C – is the consequence charge separated by a *dielectric* (insulator) which is linked to an electric field.

$$i_C = C\frac{dv}{dt}$$



$$v_C(t) = \frac{1}{C}\int_{t_0}^{t} i_C \, dt + v(t_0)$$

Note that the charge separation is due to a *displacement current* due to a time varying electric field.

Note that the voltage across a capacitor is written in accordance with the passive sign convention!

The unit of capacitance is the farad F.

The Capacitor

The capacitor acts like the opposite twin of the inductor.

It is the slope of a constant <u>change</u> in voltage that creates a constant displacement current around the capacitor!

$$i_C = C\frac{dv}{dt}$$

If the voltage across a capacitor is constant then the change in current displaced around a capacitor is ZERO.

Power and Energy in the Capacitor

Recall that power is given by: $p = (\pm)vi$

And the displacement current around a capacitor is $i_C = C\dfrac{dv}{dt}$

Then the power stored in a capacitor's electric field is

$$p_C = (\pm)vi = (\pm)v\left(C\frac{dv}{dt}\right) = (\pm)Cv\frac{dv}{dt}$$

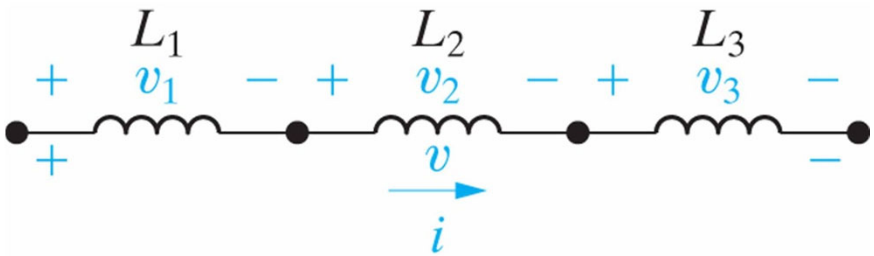
The energy w that a capacitor can deliver to a load is (see text for derivation)

$$w = \frac{1}{2}Cv^2$$

Section 6.3
Series-Parallel Combinations of Inductance and Capacitance

Series Inductors

Recall: a series connection has the same current through the series elements.



$$v_1 = L_1 \frac{di}{dt} \qquad v_2 = L_2 \frac{di}{dt} \qquad v_3 = L_3 \frac{di}{dt}$$

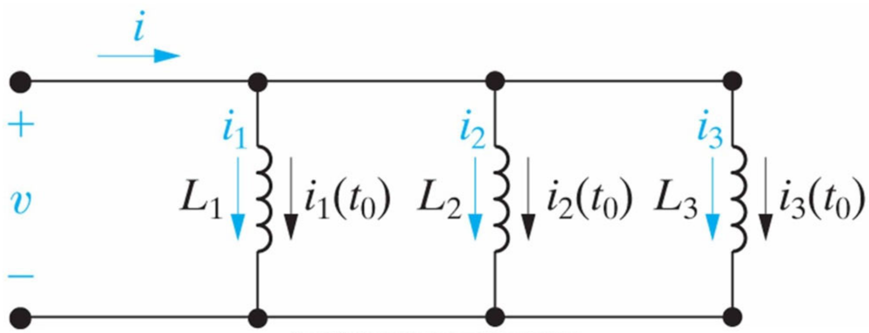
$$v = v_1 + v_2 + v_3 \quad = \left(L_1 + L_2 + L_3\right)\frac{di}{dt} \quad = L_{Eq}\frac{di}{dt}$$

Thus we see that the series inductors behave the "resistor in series" rule.

$$Series \quad L_{Eq} = L_1 + L_2 + L_3$$

Parallel Inductors

Recall: a parallel connection has the same voltage across the parallel elements.



$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\, d\tau + i_1(t_0)$$

$$i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\, d\tau + i_2(t_0)$$

$$i_3 = \frac{1}{L_3}\int_{t_0}^{t} v\, d\tau + i_3(t_0)$$

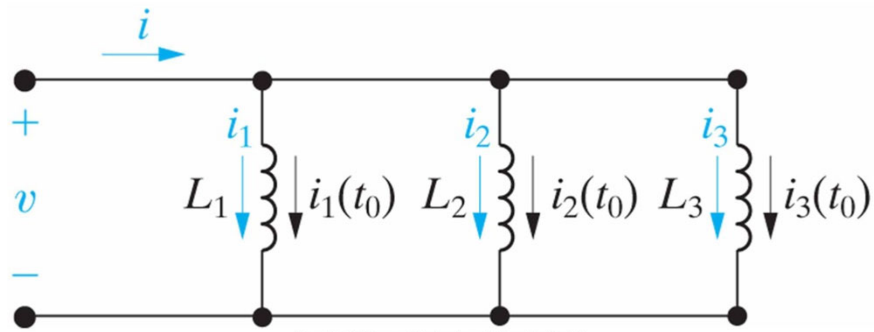
$$i = i_1 + i_2 + i_3$$

$$= \left[\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v\, d\tau\right] + \underbrace{i_1(t_0) + i_2(t_0) + i_3(t_0)}_{\text{initial conditions}}$$

$$= \left[\frac{1}{L_{Eq}}\int_{t_0}^{t} v\, d\tau\right] + i_1(t_0) + i_2(t_0) + i_3(t_0)$$

Parallel Inductors

$$i = \frac{1}{L_{Eq}} \int_{t_0}^{t} v \, d\tau + \left[i_1(t_0) + i_2(t_0) + i_3(t_0) \right]$$



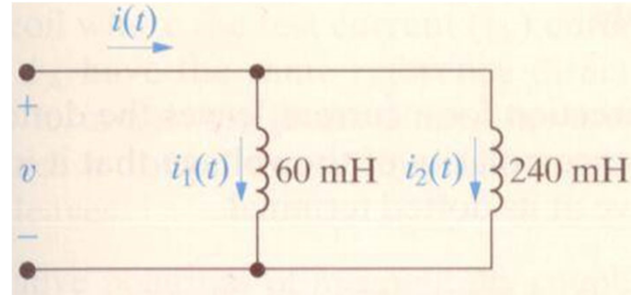
Thus we see that the parallel inductors behave the "resistors in parallel" rule.

$$Parallel \quad \frac{1}{L_{Eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

AP6.4 Inductors in Parallel

Given: $i_1(t_0) = 3A$ $\qquad$ $i_2(t_0) = -5A$

$V = -30e^{-5t}$ mV for $t \geq 0$

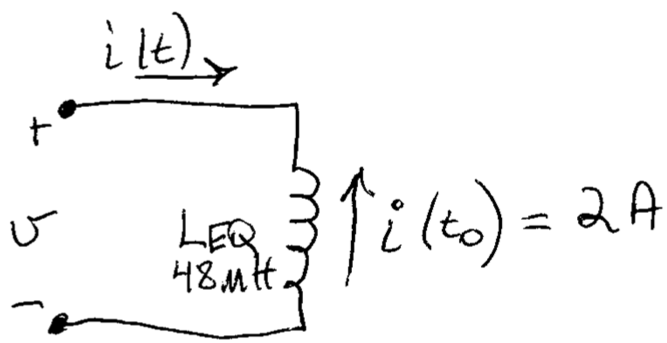


a) What is the L_{Eq}?

$$L_{Eq} = \left[\frac{1}{L_1} + \frac{1}{L_2}\right]^{-1} = \left[\frac{1}{60mH} + \frac{1}{240mH}\right]^{-1} = 48mH$$

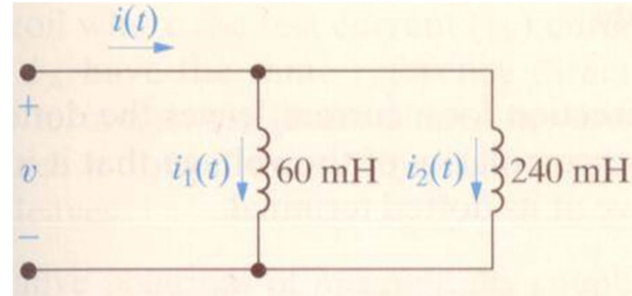
b) What is the initial current and its reference direction in the equivalent inductor?

$$i(t_0) = 3 - 5 = -2A$$



AP6.4 Inductors in Parallel

Given: V = $-30e^{-5t}$ mV for $t \ge 0$



c) Find i(t).

$$i(t) = \left[\frac{1}{L_{Eq}} \int_{0^+}^{t} \left(-.03e^{-5x} \right) dx \right] - 2A$$

$$use \int e^{au} du = \frac{1}{a} e^{au} + C$$

$$= \left[\frac{-.03}{.048} \int_{0^+}^{t} e^{-5x} dx \right] - 2A \quad = \left[-0.625 \left(\frac{-1}{5} e^{-5x} \Big|_{0}^{t} \right) \right] - 2A$$

$$= \left[\frac{-0.625}{-5} \left(e^{-5t} - e^{0} \right) \right] - 2A \quad = \left[0.125e^{-5t} - 0.125(1) \right] - 2A$$

$$= 0.125e^{-5t} - 2.125A \; for \; t \ge 0$$

Note i(t = 0) = -2A!

$$i(t = 0) = 0.125e^{0} - 2.125 = -2A$$

Answer checks at t = 0

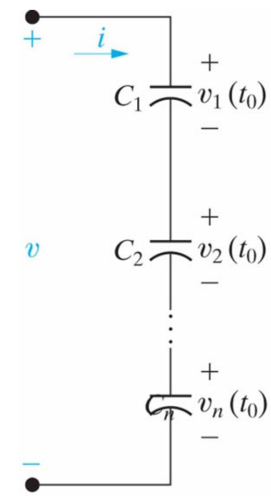
Series Capacitors

$$v = v_1 + v_2 + v_3$$

$$= \frac{1}{C_1}\int_{t_0}^{t} i\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i\,d\tau + v_2(t_0) + \frac{1}{C_3}\int_{t_0}^{t} i\,d\tau + v_3(t_0)$$

$$= \left[\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\int_{t_0}^{t} i\,d\tau\right] + \underbrace{v_1(t_0) + v_2(t_0) + v_3(t_0)}_{\text{initial conditions}}$$

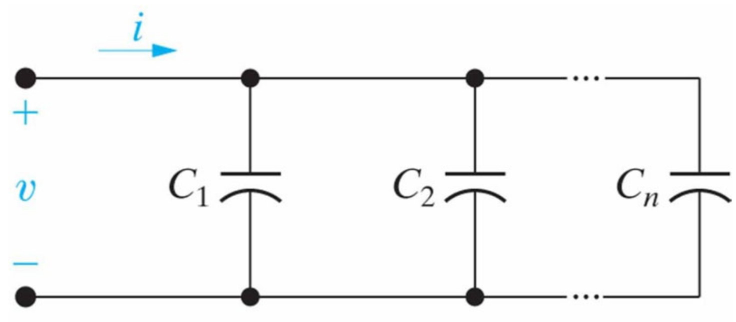
$$= \left[\frac{1}{C_{Eq}}\int_{t_0}^{t} i\,d\tau\right] + v_1(t_0) + v_2(t_0) + v_3(t_0)$$



Thus we see that the series capacitors behave the "resistor in parallel" rule.

$$Series\ \ \frac{1}{C_{Eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Parallel Capacitors



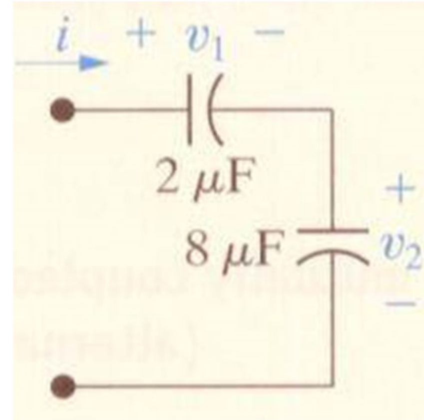
$$i = i_1 + i_2 + i_3 \quad = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt}$$

$$= \left(C_1 + C_2 + C_3\right) \frac{dv}{dt} = C_{Eq} \frac{dv}{dt}$$

Thus we see that the parallel capacitors follow the "resistors in series" rule.

$$C_{Eq} = C_1 + C_2 + C_3$$

AP6.5 Capacitors in Series



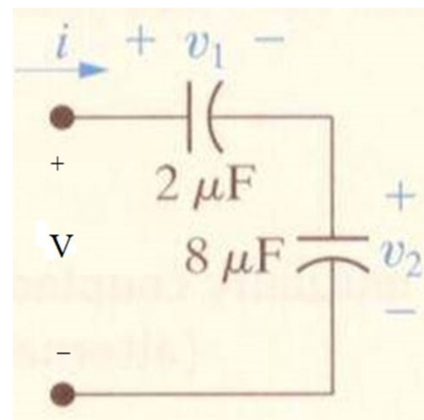
Given: $v_1(t_0) = -10V$ $v_2(t_0) = -5V$

$i = 240e^{-10t}\ \mu A$ for $t \geq 0$

a) Find C_{Eq}.

$$C_{Eq} = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1} = \left[\frac{1}{2\mu F} + \frac{1}{8\mu F}\right]^{-1} = 1.6\mu F$$

b) What is the initial voltage across C_{eq}?



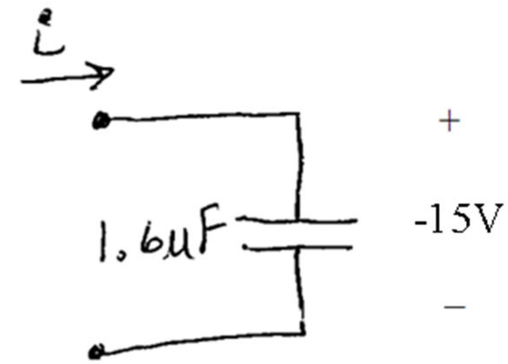
First define the polarity across the capacitors.

Then write a KVL equation:

$$-V - 10 - 5 = 0$$

$$V = -10 - 5 = -15V$$

AP6.5 Capacitors in Series



Given: $i = 240e^{-10t}\ \mu A$ for $t \geq 0$

$$v(t)=\left[\frac{1}{C_{Eq}}\int_{t_0}^{t} i\,d\tau\right]+v(t_0)=\left[\frac{1}{1.6\mu F}\int_{t_0}^{t} i\,d\tau\right]-15V$$

c) What is the voltage across C_{Eq} as $t \rightarrow \infty$?

$$v(t\rightarrow\infty)=\left[\frac{1}{1.6\times10^{-6}F}\int_{0^+}^{\infty}\left(240\times10^{-6})e^{-10x}\right)dx\right]-15V$$

$$=\left[\frac{240\times10^{-6}}{1.6\times10^{-6}}\int_{0^+}^{\infty}e^{-10x}dx\right]-15V \quad =\left[150\int_{0^+}^{\infty}e^{-10x}dx\right]-15V$$

$$use\ \int e^{au}du=\frac{1}{a}e^{au}+C$$

$$=\left[\frac{150}{-10}\left(e^{-10x}\Big|_0^{\infty}\right)\right]-15V \quad =\left[-15\left(e^{-\infty}-e^{0})\Big|\right)\right]-15V$$

$$=\left[-15\left(0-1\right)\right]-15V \quad =15V-15V=0V$$

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