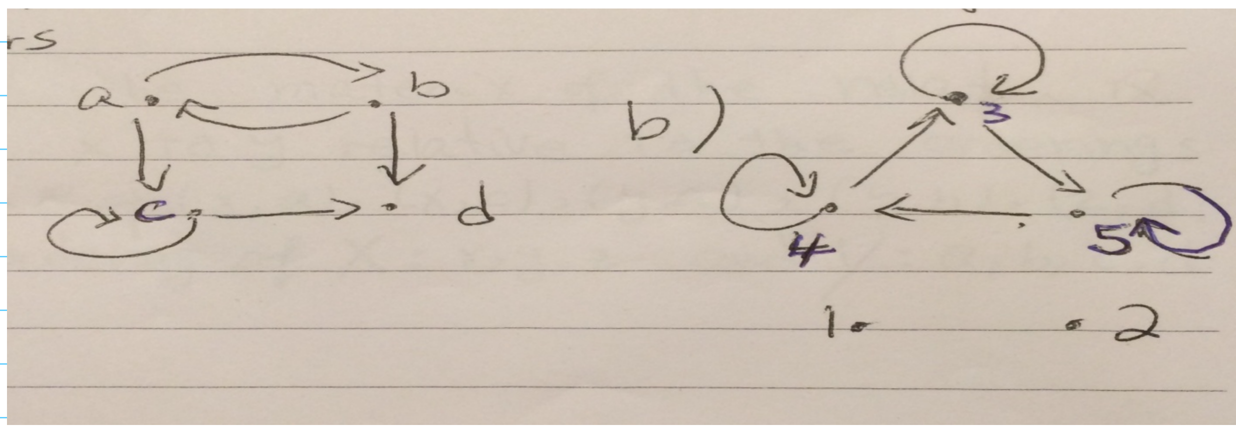
CSC 28

HW Relations

1. Write the following relation using a table diagram and { }
   a. Relation R on {1, 2, 3, 4} defined by (x, y) ϵ R if $x^2 \geq y$
   b. Relation R on {1,2,3,4,5,6,7,8,9} defines by (x, y) ϵ R if x = y^2
   c. (x, y) ϵ R on {1,2,3,4,5} if 3 divides x + 2y.
2. Give examples of relations on {1, 2, 3, 4} having the following properties
   a. Partial order
   b. Anti-symmetric
   c. Reflexive, non-symmetric and not transitive
   d. Equivalence relation
3. Write the following two relations as a set of order pairs.
   a.



4. Let R1 and R2 be the relations as {1, 2, 3 ,4} given by R1 = {(1,1), (1,2), (3,4), (4,2)} and R2 = {(1,1), (2,1), (3,1),(4,4), (2,2)} , list all the elements of R1 O R2 and R2 O R1
5. Determine whether the given relation is an equivalence relation on the set of all the people
   a. {(x,y) | x and y have the same parents
   b. (x,y) ϵ {1,2,3,4,5} , R = {(x,y) | 1 ≤ x <= 5, 1 ≤ y <= 5
6. List the members of the equivalence relation on { 1,2,3,4} by the given partition
   a. A. {{1,2}, {3,4}}
   b. {{1,2,3,4}}
   c. Your own choice of partition
7. Find the matrix of the relation R from x to y relative to the orderings
   a. R = {(x, a), (x, c), (y, a), (y, b), (z, d)} with the ordering of X : x, y, z and Y : a, b, c, d

8. The following relation has been given using a matrix.

i. Write the relation R in pairs

ii. Use matrix to s determine if the relation is reflexive, symmetric, transitive , equivalence

$$\begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

9. Give the matrix of the following relation called A1 and A2. R1 = { (x, y)| x + y ≤ 6} , R2 = {(y, z) | y = z + 1} , X = Y = Z = {1, 2, 3, 4, 5. Use the ordering of x, y, z : 1, 2, 3, 4, 5 to create the matrix

a. Give the matrix product of A1 X A2

b. use the product to find the matrix relation of R2 O R1

10. Given the matrix of a relation from X to y, how can we find the matrix of the inverse relation R^{-1}.

11. Find the inverse of the following matrix which is a relation from {1, 2} to {1, 2, 3, 4}

$$\begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{matrix}$$

12. Suppose the matrix of relation R1 on {1, 2, 3} is

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}$$

and the matrix of the relation R2 on the set [1, 2, 3}

$$\begin{matrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$$

Find the matrix of relation R1∪ R2 relative to the ordering of {1, 2, 3}

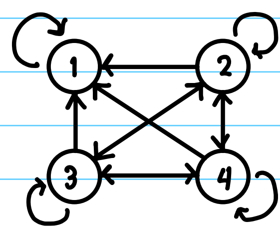
VIGOMAR KIM ALGADOR — HOMEWORK 06

CSC 28 - 01 SU22 — 08 JULY 2022

1. (a) Relation R on $\{1,2,3,4\}$ $(x,y) \in R$ if $x^2 \geq y$

$R = \{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

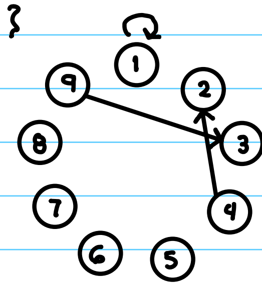
x	y	x	y	x	y
1	1	3	1	4	2
2	1	3	2	4	3
2	2	3	3	4	4
2	3	3	4		
2	4	4	1		



(b) Relation R on $\{1,2,3,\dots,7,8,9\}$ $(x,y) \in R$ if $x = y^2$

$R = \{(1,1), (4,2), (9,3)\}$

x	y
1	1
4	2
9	3

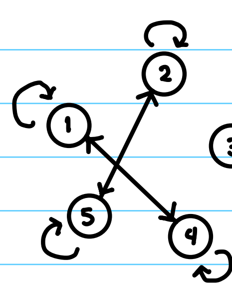


(c) $(x,y) \in R$ on $\{1,2,3,4,5\}$ if 3 divides $x+2y$

$x + 2y = 3z$

$R = \{(1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (4,4), (5,2), (5,5)\}$

x	y	x	y
1	1	4	1
1	4	4	4
2	2	5	2
2	5	5	5
3	3		



2. $\{1,2,3,4\}$

(a) PARTIAL ORDER

$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(2,3)\}$

(b) ANTI-SYMMETRIC

$R = \{(1,2),(2,2)\}$

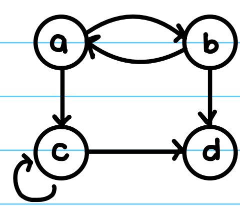
(c) REFLECTIVE, NON-SYMMETRIC, NOT TRANSITIVE

$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,3),(3,4)\}$

(d) EQUIVALENCE RELATION

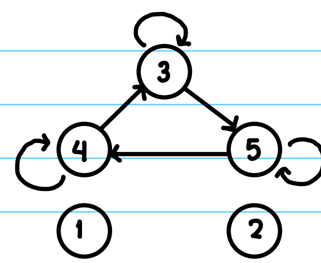
$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),$
$(1,3),(3,1),(1,4),(4,1),(2,3),(3,2),$
$(2,4),(4,2),(3,4),(4,3)\}$

3. (a)



$R = \{(a,b),(a,c),(b,a),(b,d),$
$(c,c),(c,d)\}$

(b)



$R = \{(3,3),(3,5),(4,3),(4,4),$
$(5,4),(5,5)\}$

4. $\{1,2,3,4\}$ $R1 = \{(1,1),(1,2),(3,4),(4,2)\}$

$R2 = \{(1,1),(2,1),(3,1),(4,4),(2,2)\}$

$R1 \circ R2 = \{(1,1),(1,2),(3,4),(4,1),(4,2)\}$

$R2 \circ R1 = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(4,2)\}$

5. (a) $\{(x,y) \mid x$ and y have the same parents$\}$

- Reflective: x related to x TRUE
- Symmetric: x related to y $\longrightarrow$ y related to x TRUE
- Transitive: if x related to y and y related to z, then x related to z TRUE

(b) $(x,y) \in \{1,2,3,4,5\}$, $R = \{(x,y) \mid 1 \le x \le 5,\ 1 \le y \le 5\}$

- Reflective: (1,1), (2,2), (3,3), (4,4), (5,5) TRUE
- Symmetric: (1,2), (2,1) TRUE
- Transitive: (1,2), (2,3), (1,3) TRUE

6. $\{1,2,3,4\}$

(a) $\{\{1,2\},\{3,4\}\}$

$R = \{(1,1),(1,2),(2,1),(2,2),(3,3),(3,4),(4,3),(4,4)\}$

(b) $\{\{1,2,3,4\}\}$

$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),$
$(1,3),(3,1),(1,4),(4,1),(2,3),(3,2),$
$(2,4),(4,2),(3,4),(4,3)\}$

(c) $\{\{1\},\{2\},\{3\},\{4\}\}$

$R = \{(1,1),(2,2),(3,3),(4,4)\}$

7. $R = \{(x,a),(x,c),(y,a),(y,b),(z,d)\}$

with ordering of $X: x,y,z$ and $Y: a,b,c,d$

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

8.

$$\begin{array}{c c} & \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{array}{c} w \\ x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(a) $R = \{(w,w),(w,y),(y,w),(y,y),(z,z)\}$

(b) • Reflective :

$$\begin{array}{c c} & \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{array}{c} w \\ x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

not reflective

• Symmetric :

$$\begin{array}{c c} & \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{array}{c} w \\ x \\ y \\ z \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

symmetric

• Transitive:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transitive

• Equivalence:

since the given relation is not reflective, therefore, it is not equivalence relation

9. $R1 = \{(x,y) \mid x+y \le 6\}$, $R2 = \{(y,z) \mid y = z+1\}$

$X = Y = Z = \{1,2,3,4,5\}$

(a) Matrix product of $A1 \times A2$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \\ A1 \end{array} \underset{\times}{} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\ A2 \end{array} = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \end{array}$$

(b) $R_2 \circ R_1 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

10. $R = X \times Y \longrightarrow R^{-1} = Y \times X$ (rows and columns are switched)

11. $R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ $\quad R^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

12. $$\underset{R1}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}} \cup \underset{R2}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$