

**California State University, Sacramento**  
**The College of Engineering and Computer Science**

**EEE 180 Signals & Systems**

Midterm 1

Spring 2023

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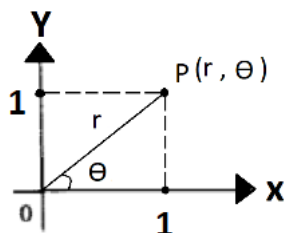
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1. [30 points]. Select one correct answer for each of the following questions.

(1). The multiplication result of two complex numbers  $(3 + 4j) \times (3 - 4j)$  is equal to

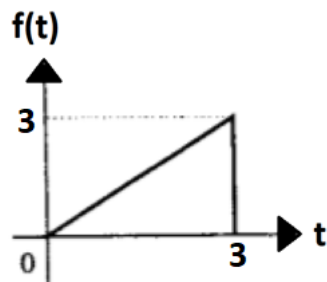
- a. 6
- ☒ b. 25
- c.  $6 + 8j$

(2). For the cartesian coordinates shown in the diagram below, find the polar coordinate  $(r, \theta)$  for the point p.



- ☒ a.  $(r, \theta) = (\sqrt{2}, \frac{\pi}{4})$ .
- b.  $(r, \theta) = (2, \frac{\pi}{6})$ .
- c.  $(r, \theta) = (2, \frac{\pi}{4})$ .

(3). Find the energy of the following signal  $f(t) = t$  when  $t \geq 0$ , and  $t \leq 3$ , also  $f(t) = 0$  when  $t < 0$  or  $t > 3$ .



- ☒ a. 9
- b. 4.5
- c. 1.5

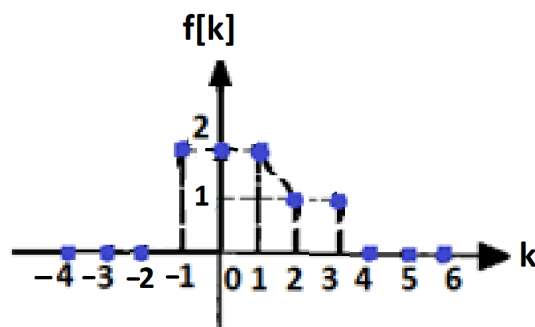
(4).  $u(t)$  is a continuous time unit step function.

For the discrete time signal  $f[k] = (0.5^k) u[k]$ , which statement below is correct?

- a. When  $k=2$   $f[k] = f[2] = 0.5 + 0.5 = 1$ .
- ☒ b. When  $k \rightarrow \infty$ ,  $f[k] \rightarrow \infty$ .
- c. When  $k \rightarrow \infty$ ,  $f[k] \rightarrow 0$ .

-3

(5). Find the signal energy for the discrete time signal shown below.



- a. 5
- b. 8
- ☒ c. 14

(6). A system is said to be linear if \_\_\_\_\_

- a. It satisfies only the additivity property.
- b. It satisfies only the scaling property.
- ☒ c. It satisfies the superposition property.

(7). Is the system  $y[n] = 2x[n]$  linear?

- ☒ a. Yes
- b. No

(8). When are LTI (linear time invariant) systems stable?

- ☒ a. Only when a bounded input produces a bounded output
- b. Only when a bounded input produces an unbounded output
- c. Only when an unbounded input produces an unbounded output

(9). The discrete time cosine signal is always periodic.

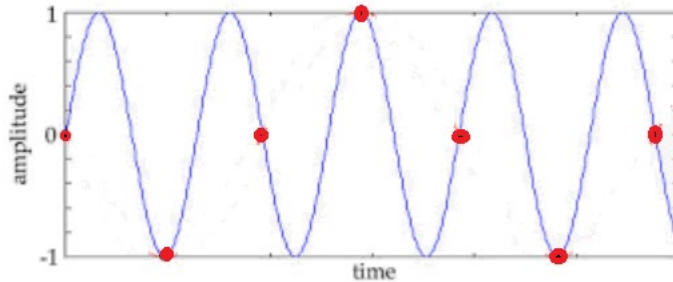
- ☒ a. True
- b. False

(10). A time invariant signal must be linear.

- ☒ a. True
- b. False

2.[32 points].

(1). In the following figure, the continuous time signal is marked in blue and the discrete time samples are marked in red. Does this sampling cause aliasing?



Your Answer: Yes

(2). What is the requirement of the sampling period  $T_s$  for a continuous time signal with a maximum frequency of 1000 Hz?

$$f = 2f_{\max} = 2(1\text{ kHz}) = 2\text{ kHz}$$

$$T = 1/f = 1/2\text{ kHz} = \boxed{0.0005\text{ s}}$$

(3).  $f(t) = 20 \cos(5\pi t + 0.6\pi)$  is sampled with a sampling interval of  $T = 0.2$  second. Find the expression for the resulting discrete-time signal.

$$f[k] = 20 \cos(5\pi kT + 0.6\pi) \quad T = 0.2$$

$$= \boxed{20 \cos(\pi k + 0.6\pi)}$$

(4). The periodic discrete time signal  $\cos(0.4\pi k + 0.5)$  has a minimum period of  $N_0$  samples. You must show your calculation procedure on how to find the value of  $N_0$ .

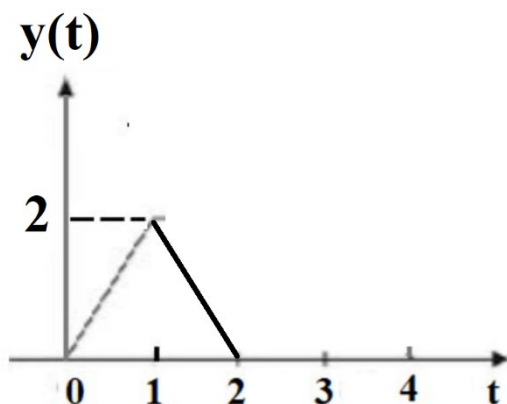
$$N_0 = m \left( \frac{2\pi}{\Omega} \right) \quad \Omega = 0.4\pi$$

$$N_0 = m \left( \frac{2\pi}{0.4\pi} \right) = m(5)$$

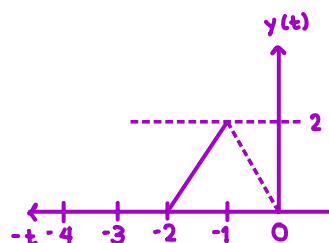
$$\boxed{m = 1}$$

$$\boxed{N_0 = 5}$$

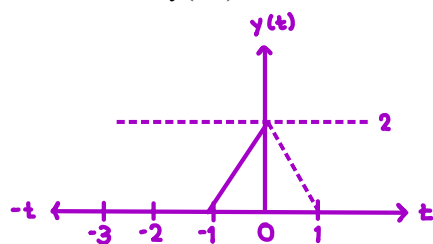
3. [38 points] (1). The waveform of the signal  $y(t)$  is given below:



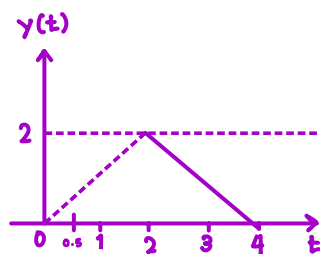
a. Draw the waveform of  $y(-t)$  below.



b. Draw the waveform of  $y(1-t)$  below.



c. Draw the waveform of  $y(t/2)$  below.



(2).

Suppose  $f(t)$  is the input signal, and  $h(t)$  is the system unit impulse response signal.

The continuous time domain convolution equation is defined below:

$$y(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(x) h(t-x) dx$$

$$\text{where } f(t) = (6t^3 - 2)u(t), \quad h(t) = tu(t),$$

and  $u(t)$  is the continuous time unit step function.

**Please use the above integration formula to calculate the system response  $y(t)$  signal. You must show the detailed calculation procedure to get credit.**

$$f(t) = (6t^3 - 2)u(t)$$

$$h(t) = tu(t)$$

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} f(x) h(t-x) dx \\ &= \int_{-\infty}^{\infty} (6x^3 - 2)u(x) (t-x)u(t-x) dx \\ &= \int_0^t (6x^3 - 2)(t-x) dx \\ &= \int_0^t (6tx^3 - 6x^4 - 2t + 2x) dx \\ &= \left[ \frac{6t}{4} x^4 - \frac{6}{5} x^5 - 2tx + x^2 \right]_0^t = \frac{3}{2} t^5 - \frac{6}{5} t^5 - 2t^2 + t^2 \end{aligned}$$

$$y(t) = \left( \frac{3}{10} t^5 - t^2 \right) u(t)$$

(3). Find  $y_0(t)$ , the zero-input component of the response for an LTI system described by the following differential equation:  $(D^2 + 5D + 4)y(t) = Df(t)$ , when the initial conditions are  $y_0(0) = 0$ ,  $y_0'(0) = -3$ .

**You must show the detailed calculation procedure to get credit.**

$$(D^2 + 5D + 4)y_0(t) = 0$$

Characteristic polynomials:

$$\lambda^2 + 5\lambda + 4 = 0$$

$$(\lambda + 4)(\lambda + 1) = 0$$

roots:  $\lambda = -4, -1$

zero-input system response:

$$y_0(t) = c_1 e^{-4t} + c_2 e^{-t}$$

$$y_0'(t) = -4c_1 e^{-4t} - c_2 e^{-t}$$

$$\begin{aligned} \text{if } y_0(0) = 0 : c_1 e^{-4(0)} + c_2 e^{-1(0)} = 0 &\longrightarrow c_1 + c_2 = 0 \\ y_0'(0) = -3 : -4c_1 e^{-4(0)} - c_2 e^{-1(0)} = -3 &\longrightarrow -4c_1 - c_2 = -3 \end{aligned} \quad \left. \begin{array}{l} c_1 + c_2 = 0 \\ -4c_1 - c_2 = -3 \end{array} \right\} \begin{array}{l} c_1 = 1 \\ c_2 = -1 \end{array}$$

zero-input component of the response:

$$y_0 = e^{-4t} - e^{-t}$$