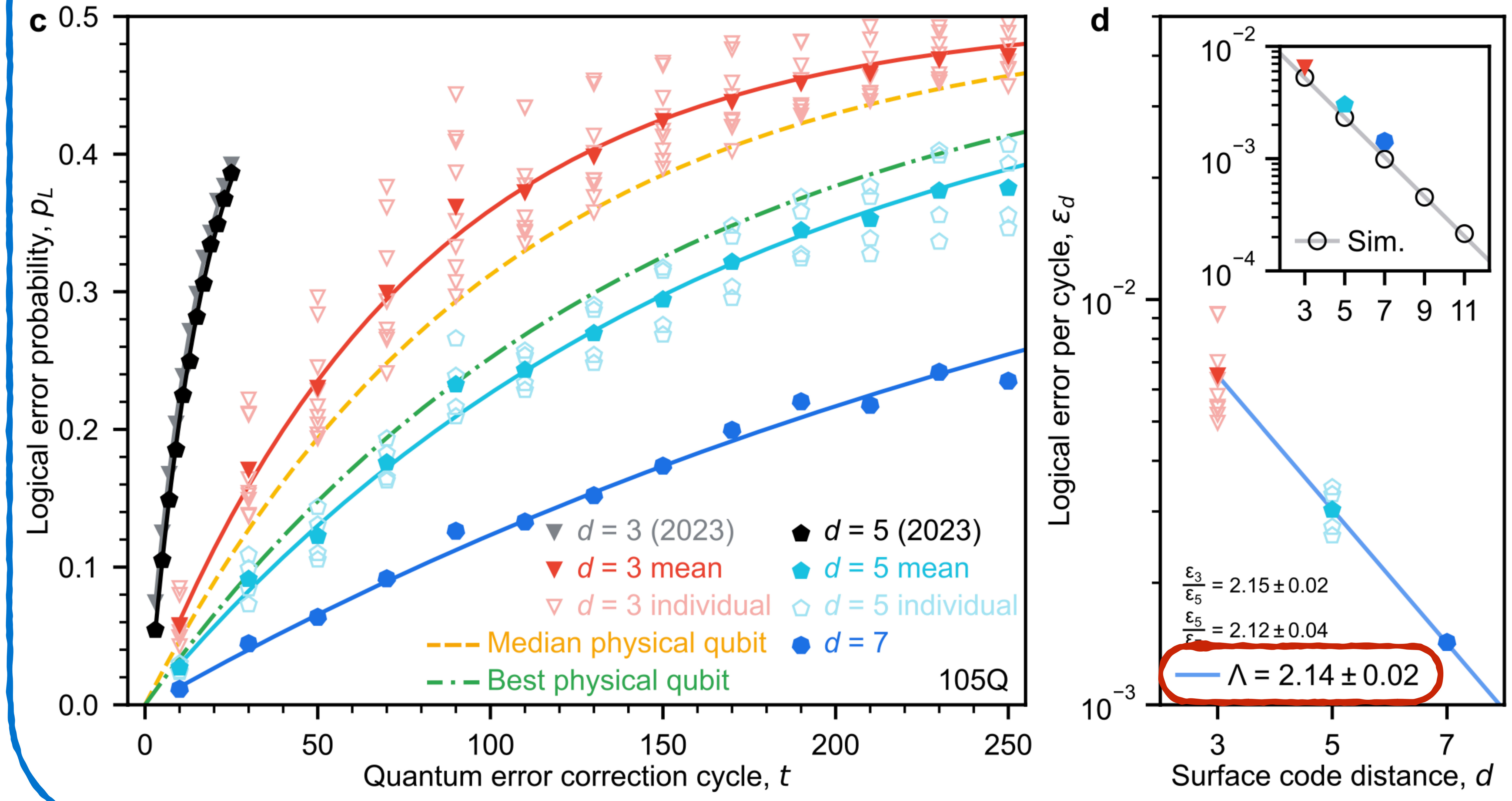
MURI: Quantum Error Correction Under Control

Google Quantum AI and collaborators, Nature 638, 920 (2024)

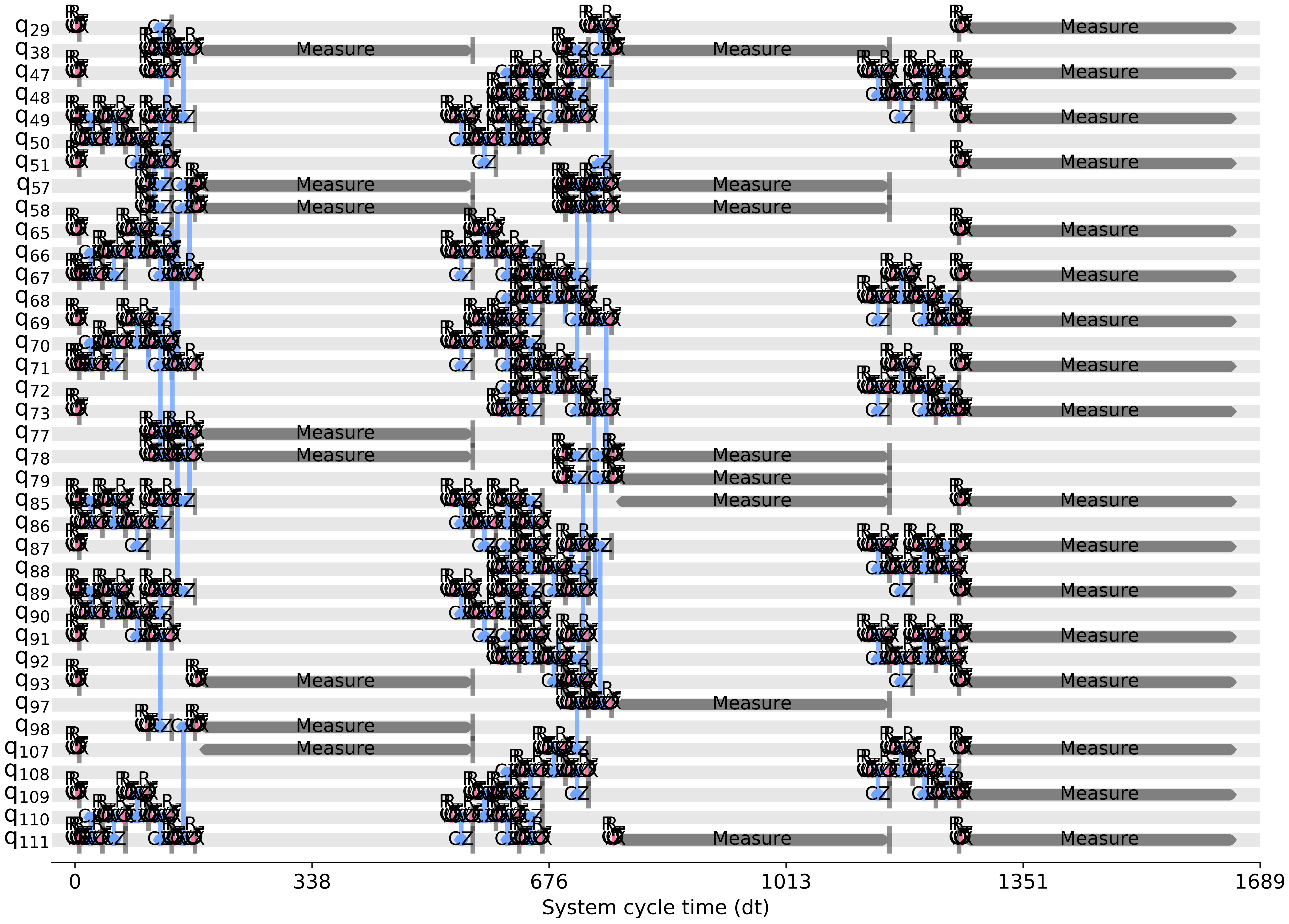


USC Collaboration Results

backend	$(d_x = 3,\ d_z = 5)$		$(d_x = 5,\ d_z = 3)$	
	Λ_ε	$\Lambda_{p_L(r)}$	Λ_ε	$\Lambda_{p_L(r)}$
ibm_aachen	1.0127 ± 0.0002	0.989 ± 0.003	1.2942 ± 0.0004	1.075 ± 0.034
ibm_marrakesh	1.0731 ± 0.0002	1.057 ± 0.014	1.0206 ± 0.0002	1.009 ± 0.025
ibm_fez	0.8525 ± 0.0001	0.922 ± 0.005	0.8653 ± 0.0002	0.957 ± 0.011
ibm_kyiv	0.8766 ± 0.0001	0.968 ± 0.025	0.8451 ± 0.0001	0.953 ± 0.023

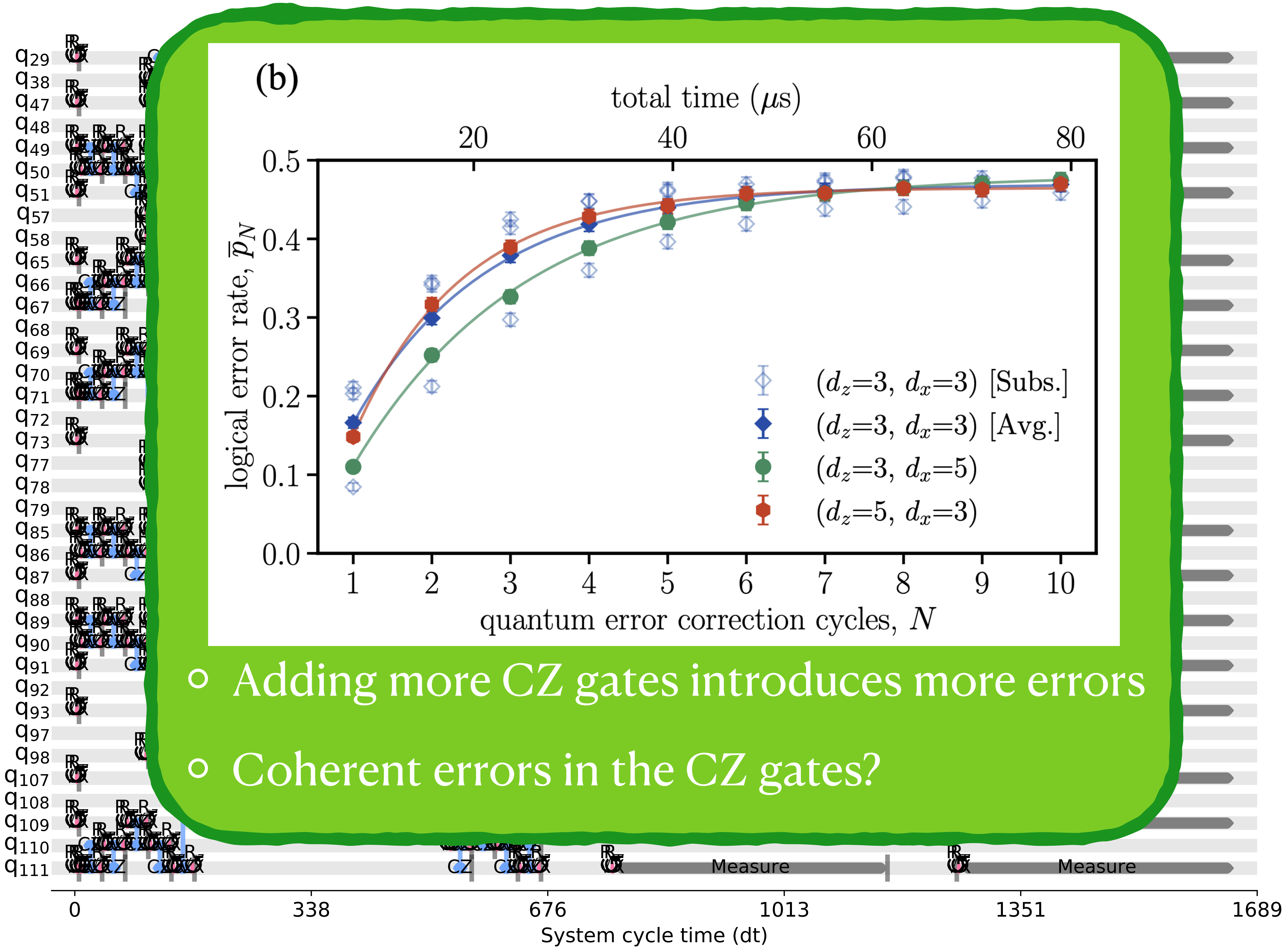
2Qubit gates, the corner stone of QEC codes

A distance-3 surface code on *ibm_acchen*



2Qubit gates, the corner stone of QEC codes

A distance-3 code on *ibm_acchen*



- Adding more CZ gates introduces more errors
- Coherent errors in the CZ gates?

Coherent errors of a single-qubit gate

- The time-dependent system Hamiltonian that generates single-qubit X rotation:

$$H(t) = \varepsilon_{\text{tot}}(t)\frac{\sigma_x}{2} + H_{\text{err}}$$

$$H_{\text{err}} = \varepsilon_{\text{err}}\frac{\sigma_x}{2} + \Delta_{\text{err}}\frac{\sigma_z}{2}$$

- Ideally, $\varepsilon_{\text{err}} = \Delta_{\text{err}} = 0$, but both are present and give rise to rotation errors $\delta\theta$ and phase errors $\delta\varphi$

Coherent errors of a single-qubit gate

- The time-dependent system Hamiltonian that generates single-qubit X rotation:

$$H(t) = \varepsilon_{\text{tot}}(t)\frac{\sigma_x}{2} + H_{\text{err}}$$

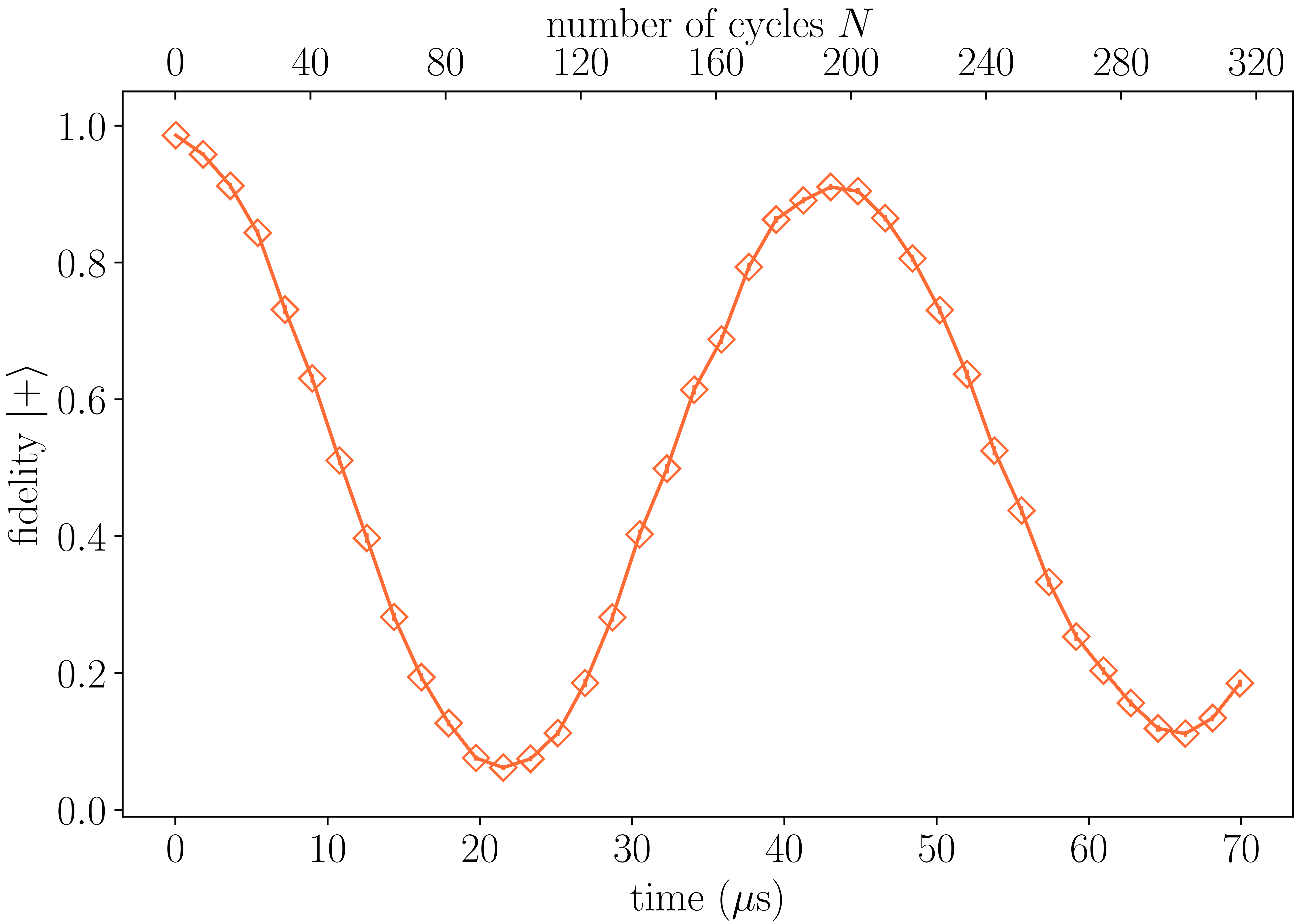
$$H_{\text{err}} = \varepsilon_{\text{err}}\frac{\sigma_x}{2} + \Delta_{\text{err}}\frac{\sigma_z}{2}$$

- Ideally, $\varepsilon_{\text{err}} = \Delta_{\text{err}} = 0$, but both are present and give rise to rotation errors $\delta\theta$ and phase errors $\delta\varphi$
- But how can we *detlrminsticly* detect and correct and these errors?

Amplify the noise through Dynamical decoupling-*like* sequences

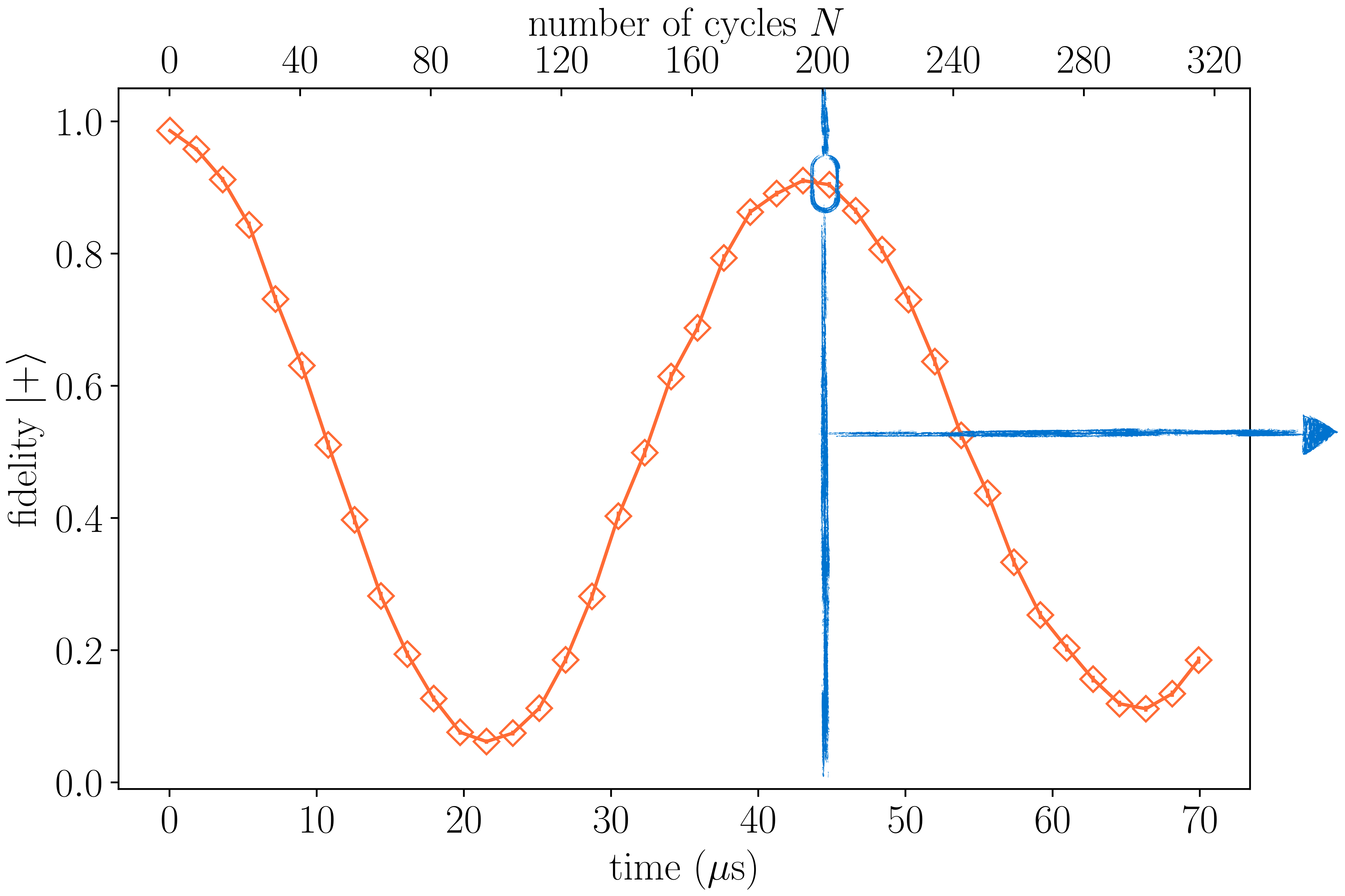
Dynamical decoupling and coherent errors

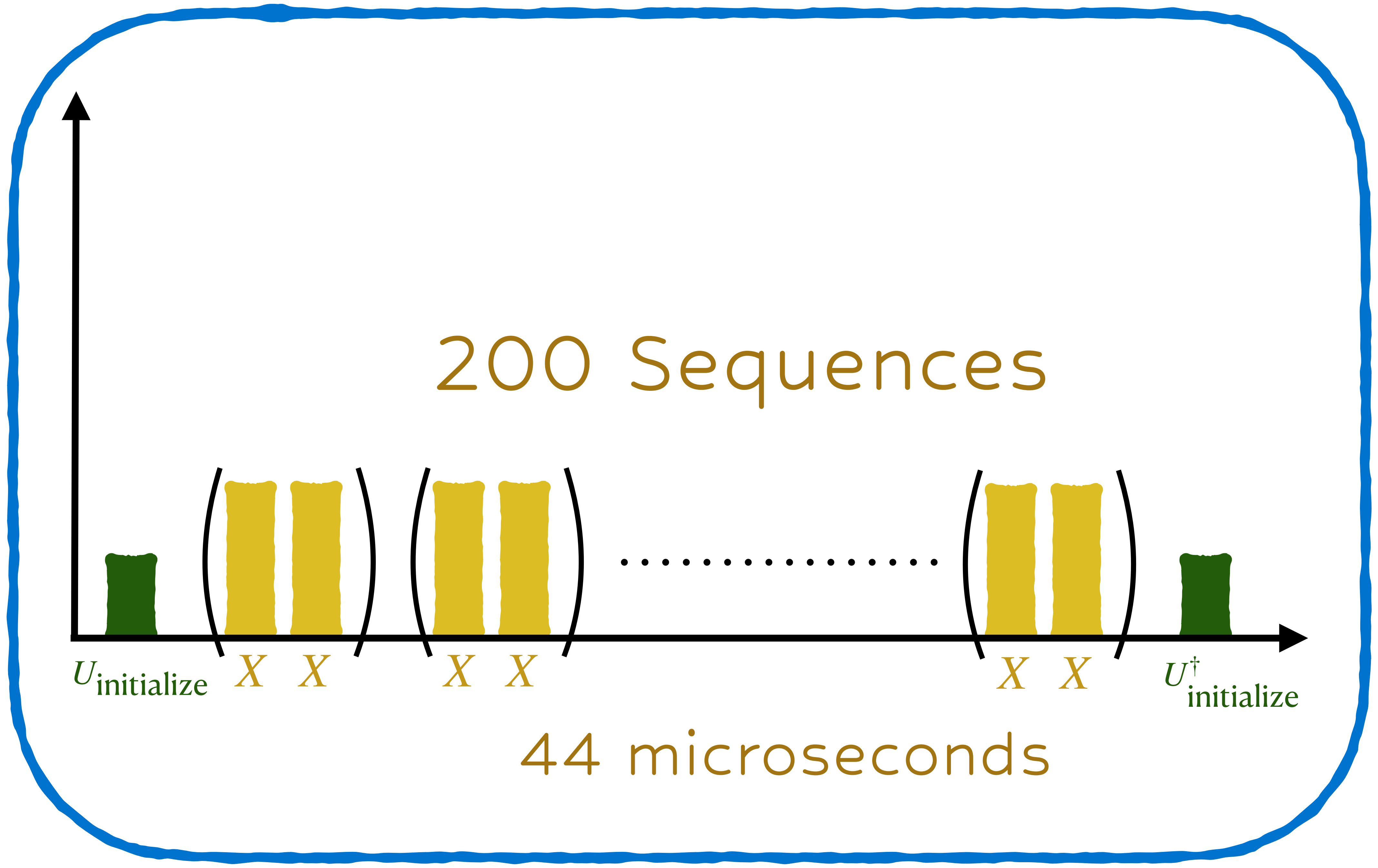
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses



Dynamical decoupling and coherent errors

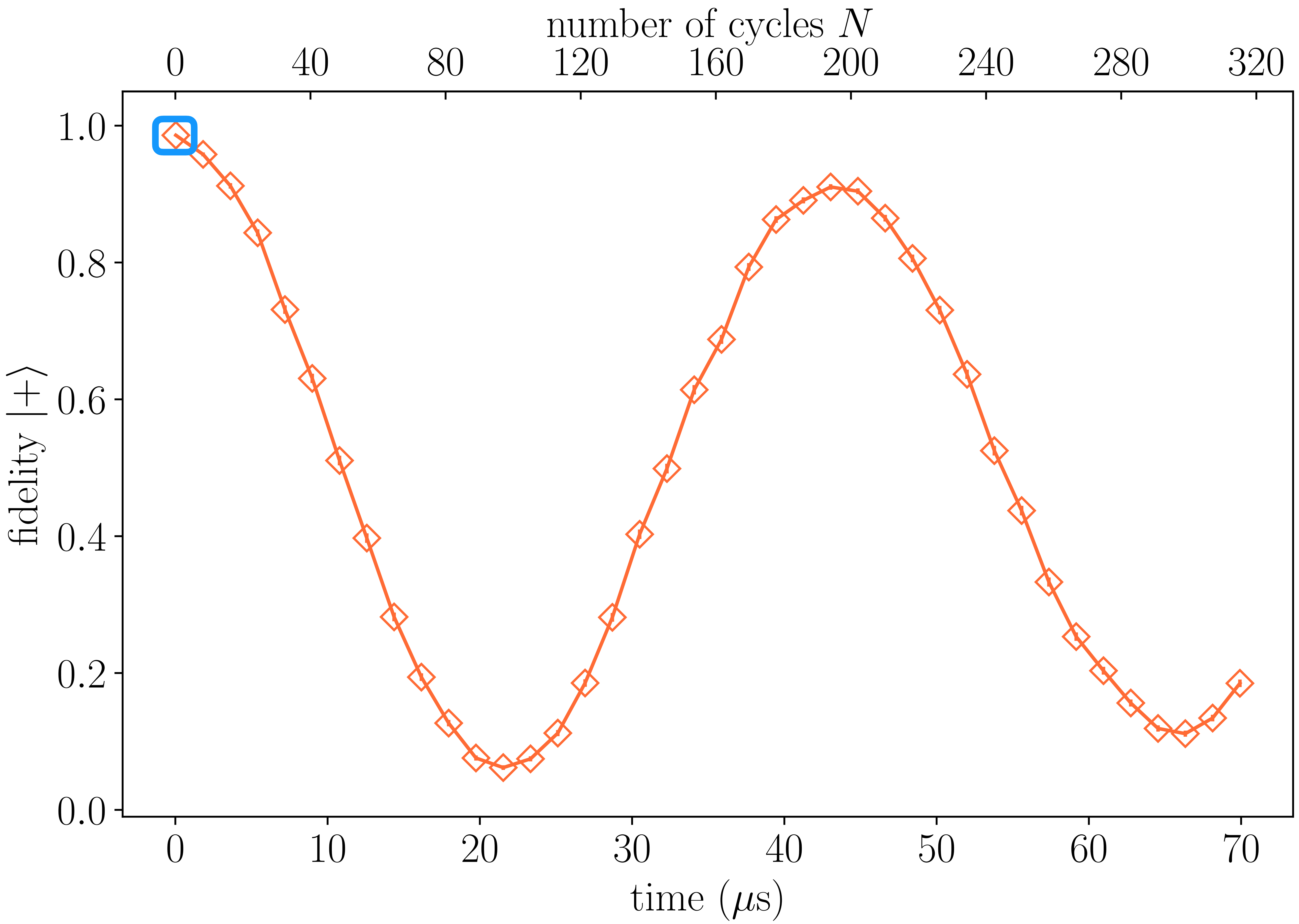
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses

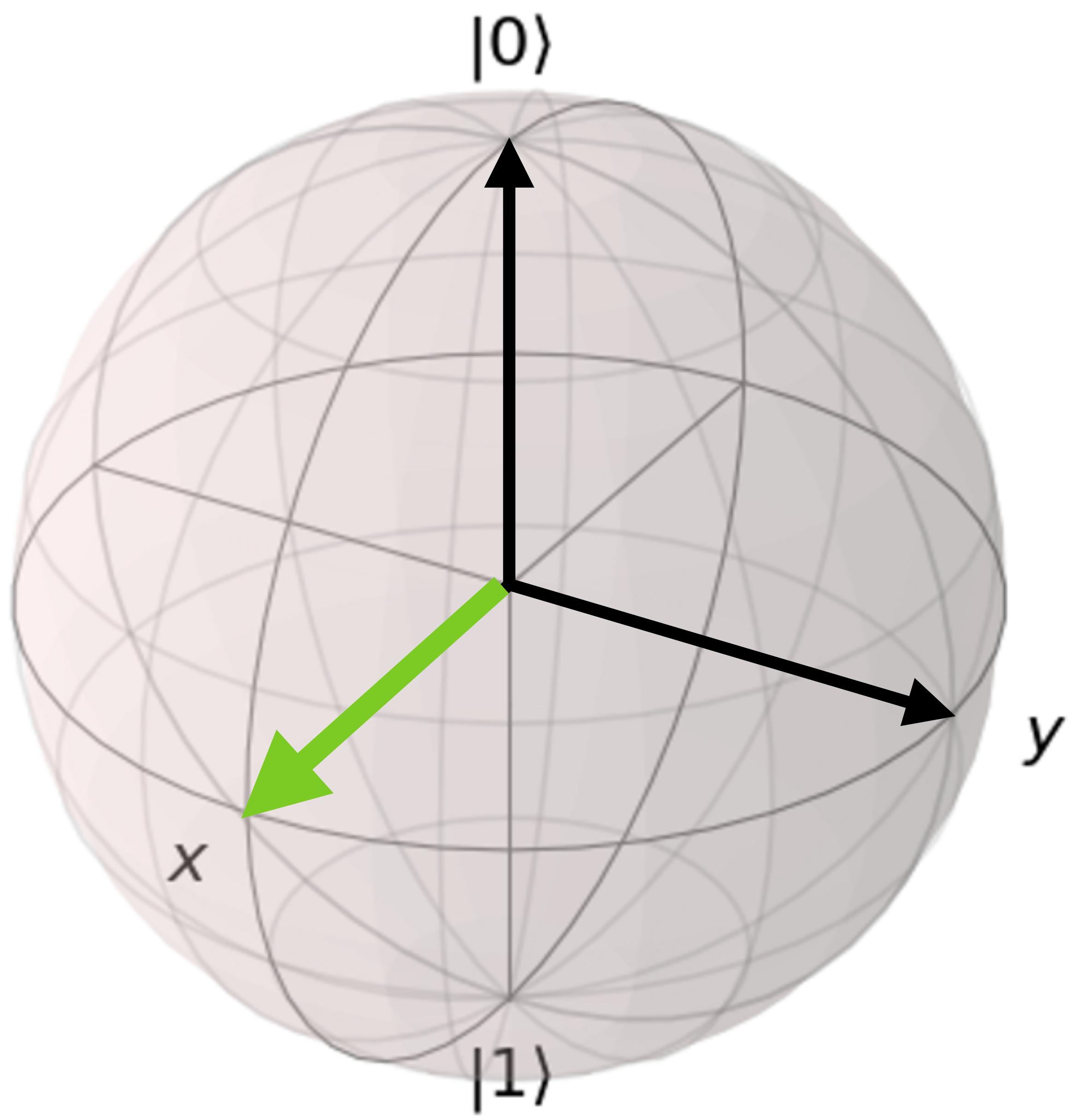




Dynamical decoupling and coherent errors

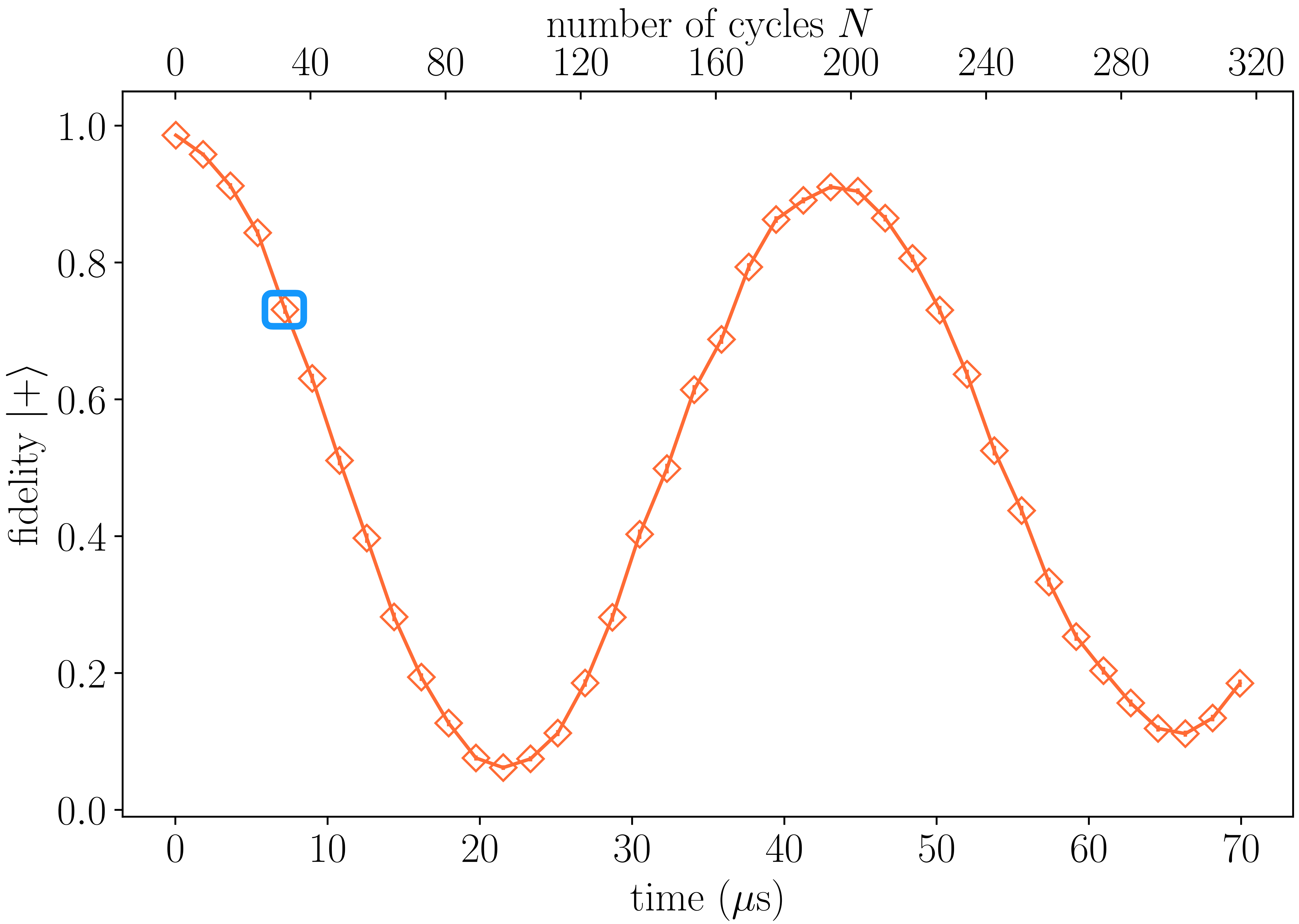
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses

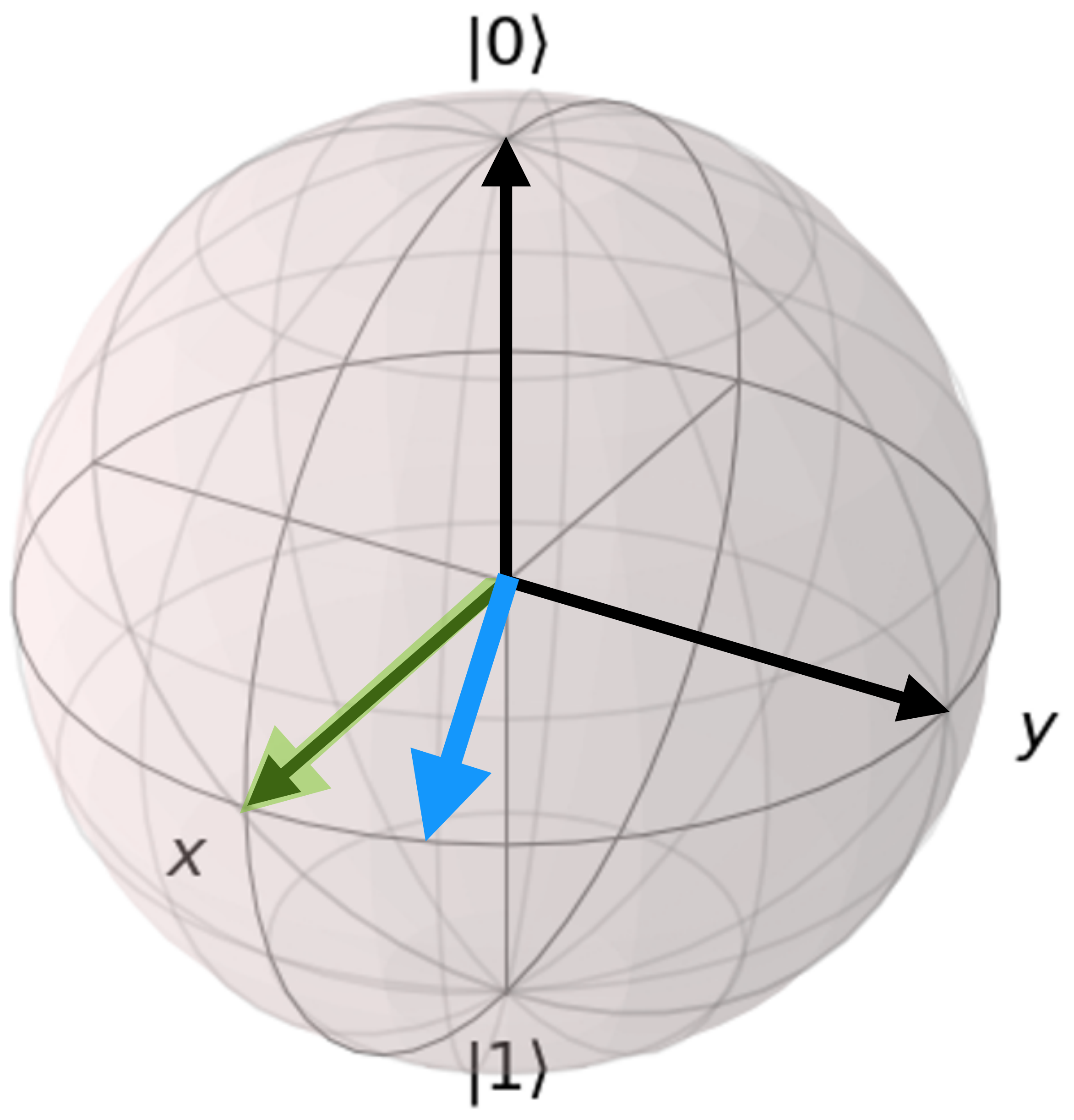




Dynamical decoupling and coherent errors

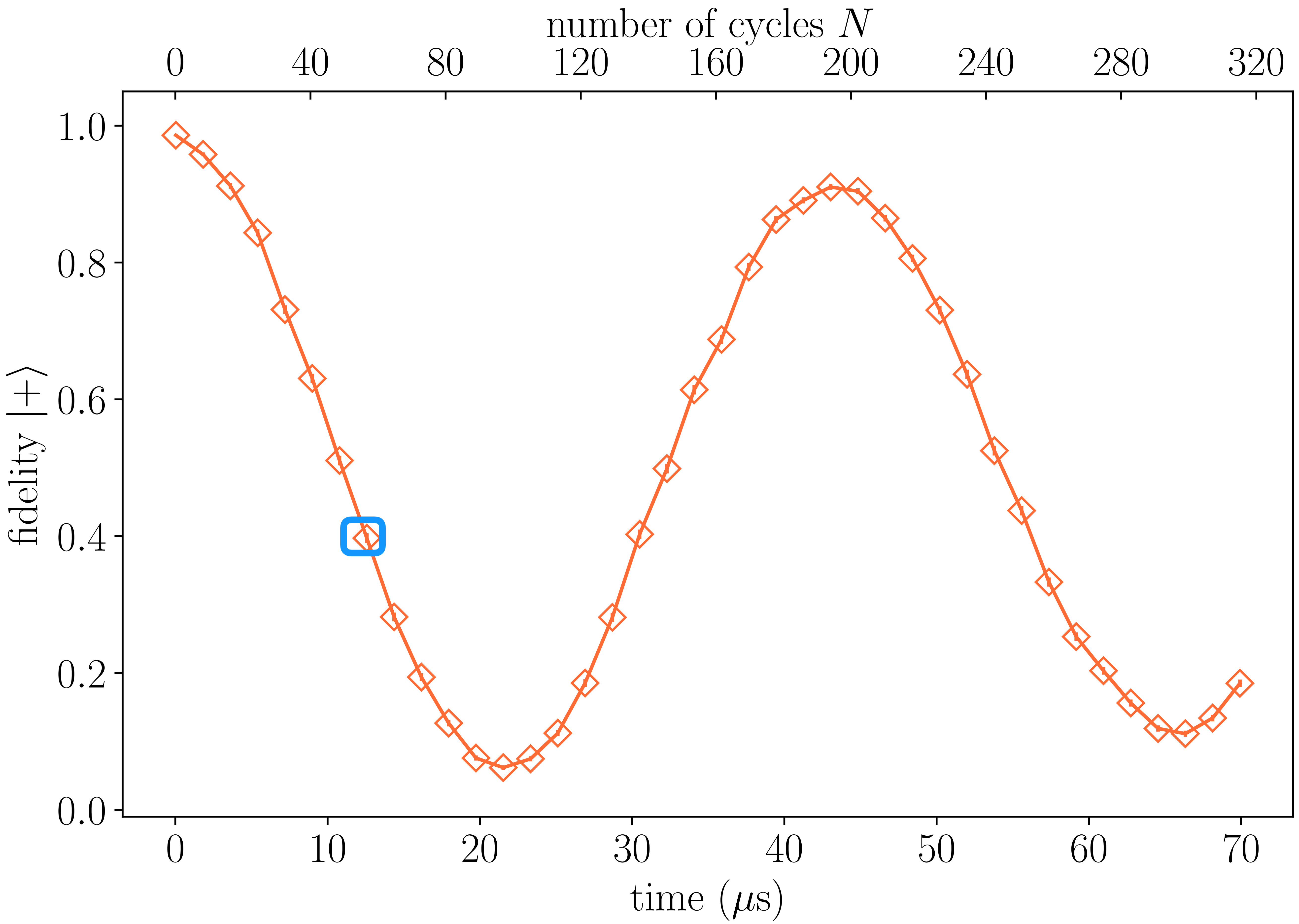
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses

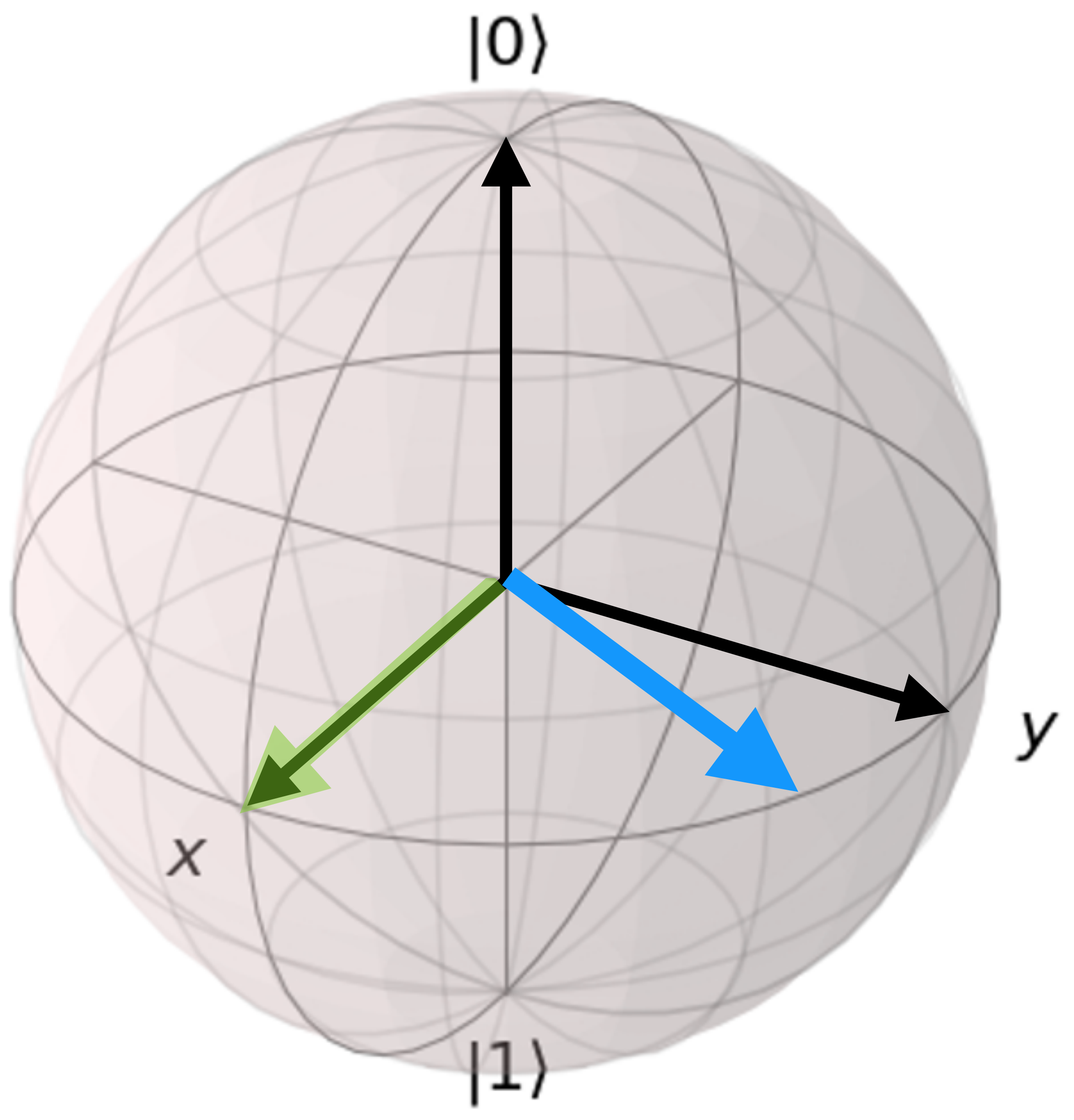




Dynamical decoupling and coherent errors

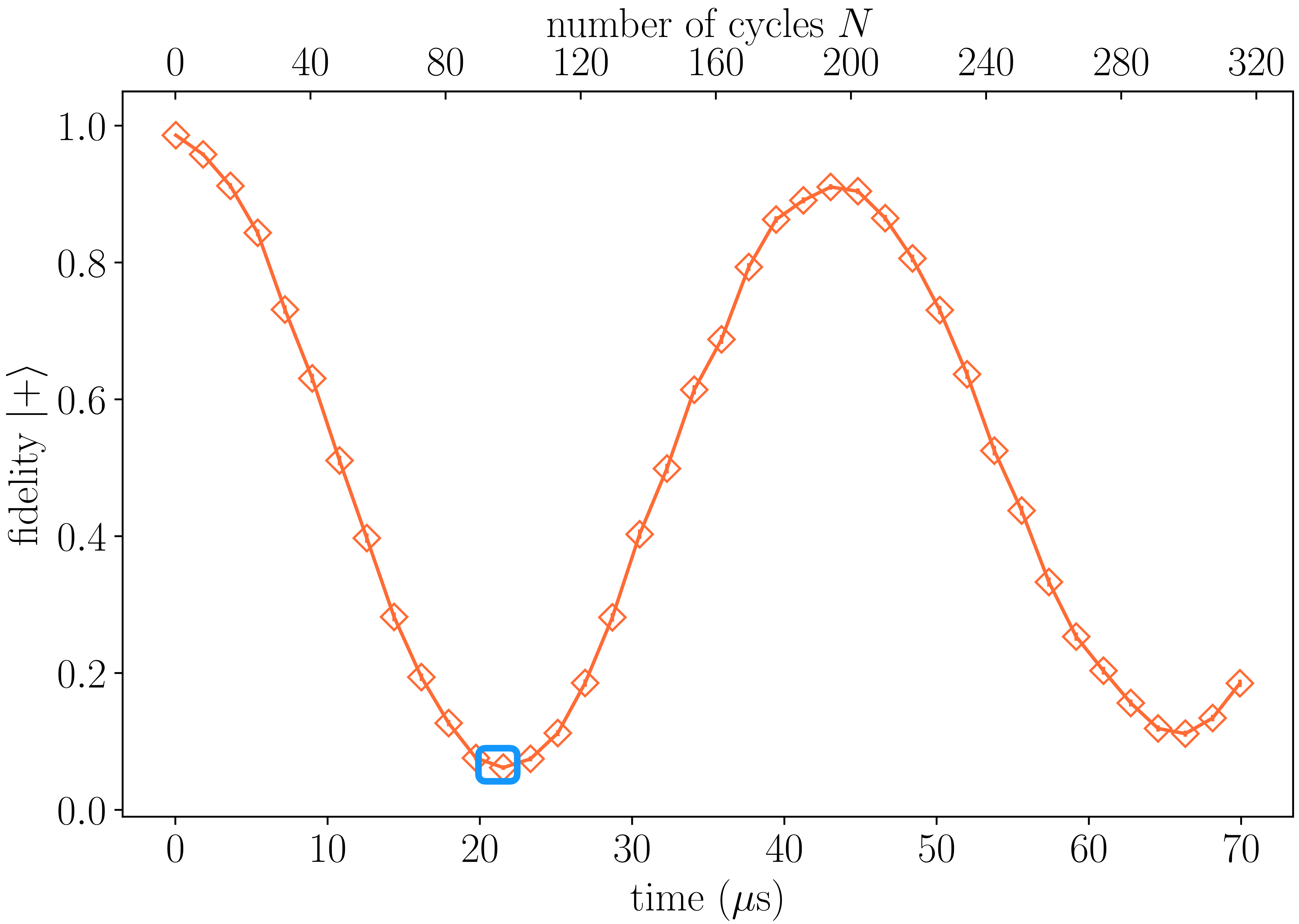
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses

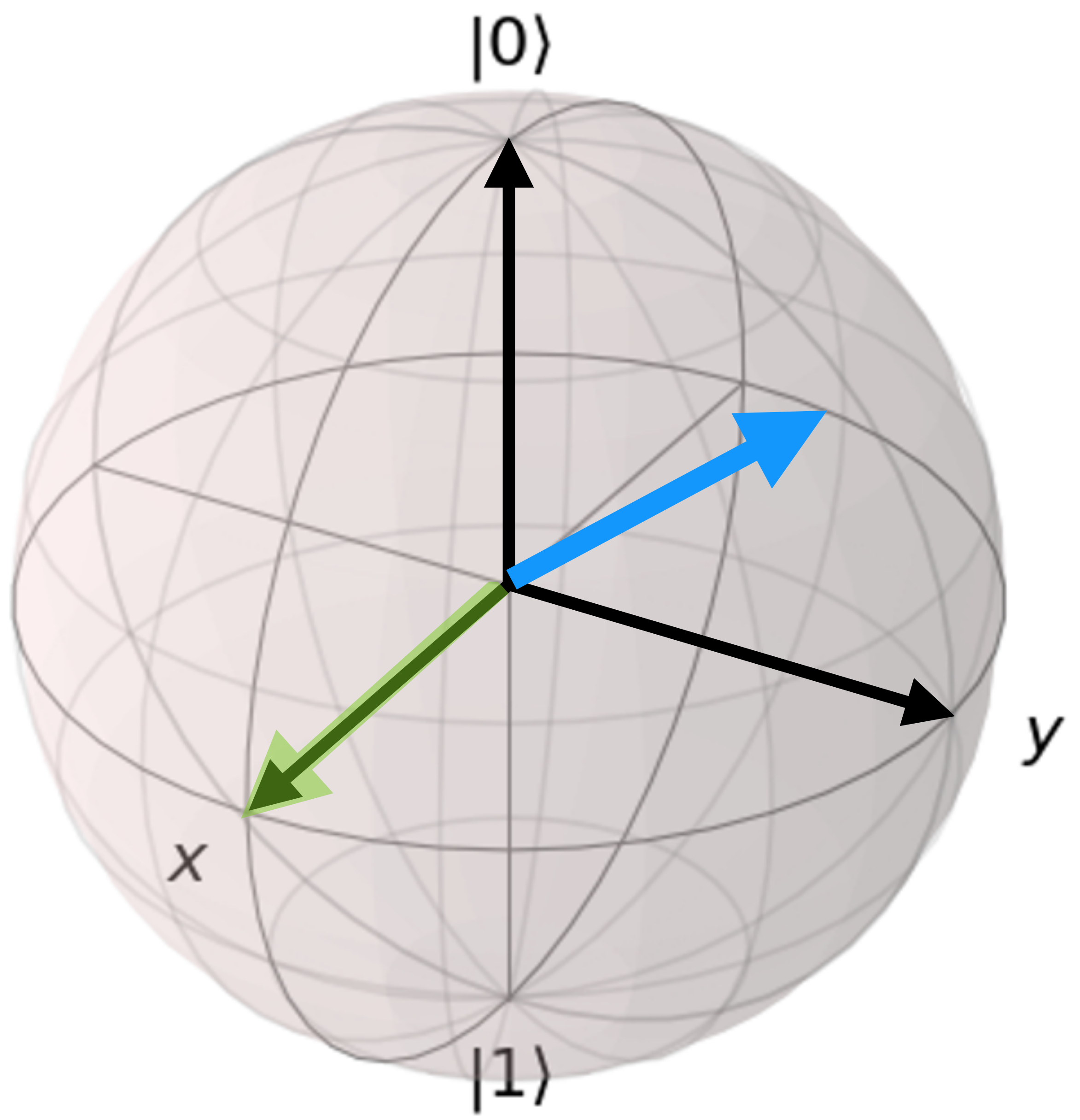




Dynamical decoupling and coherent errors

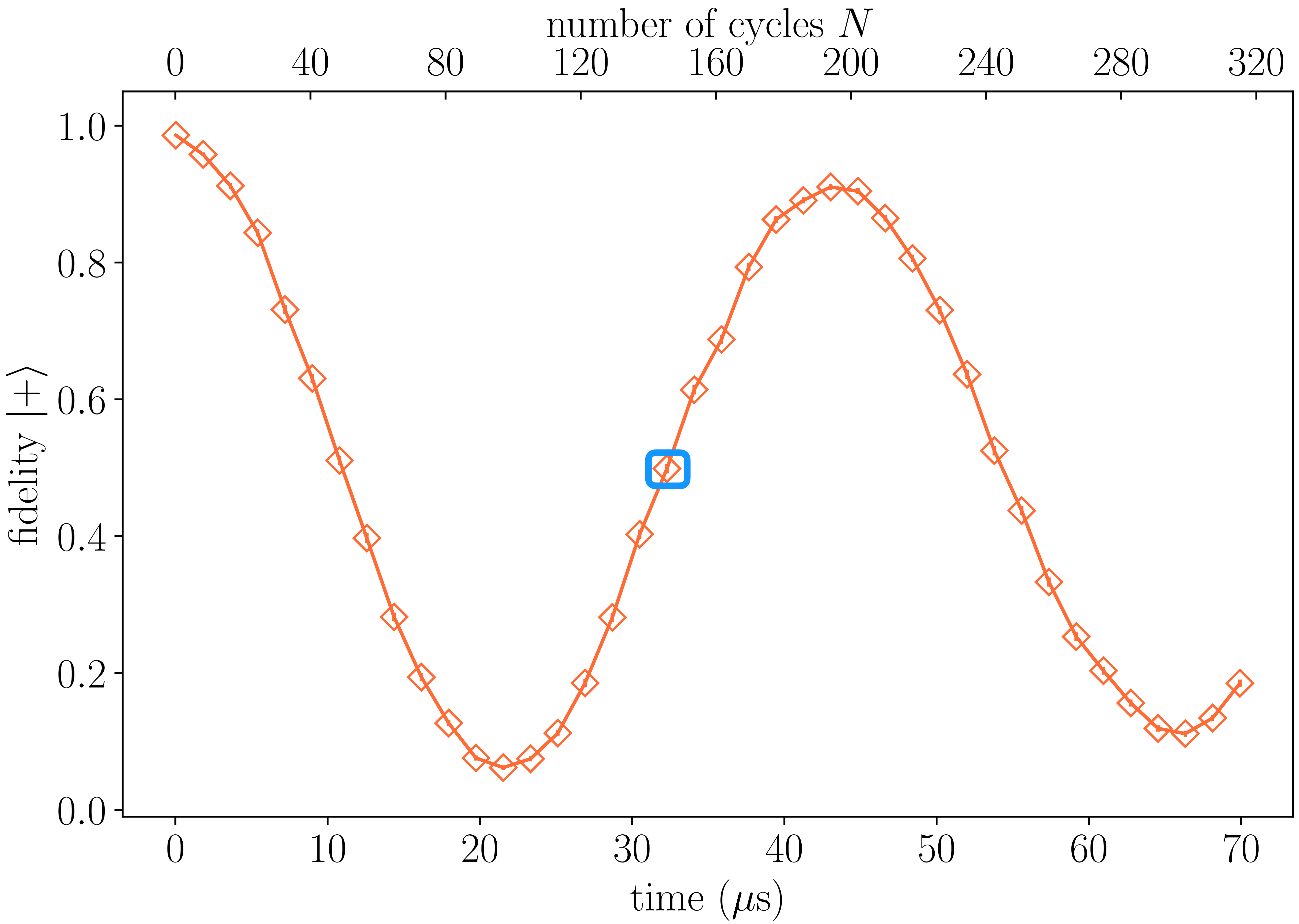
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses

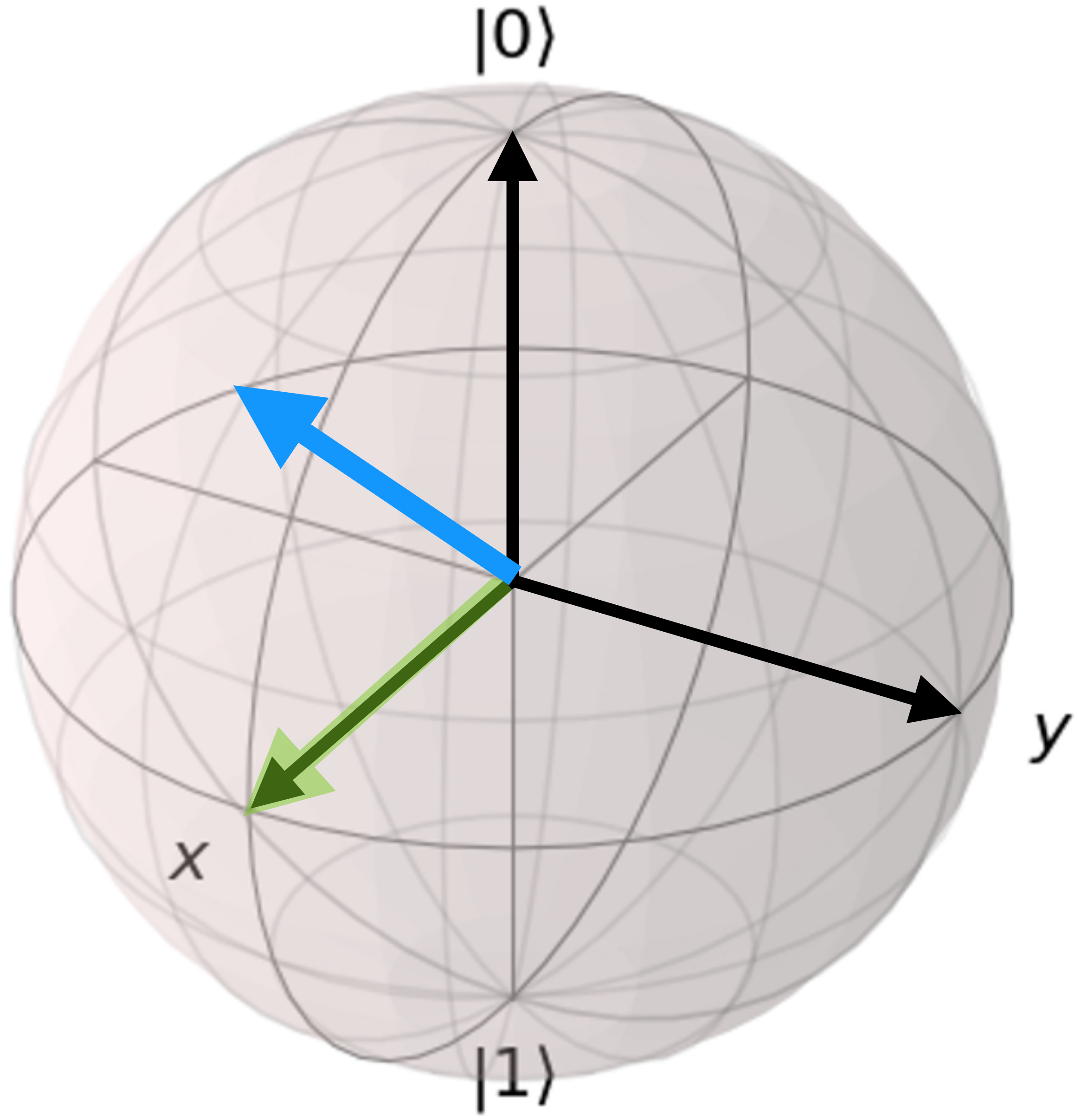




Dynamical decoupling and coherent errors

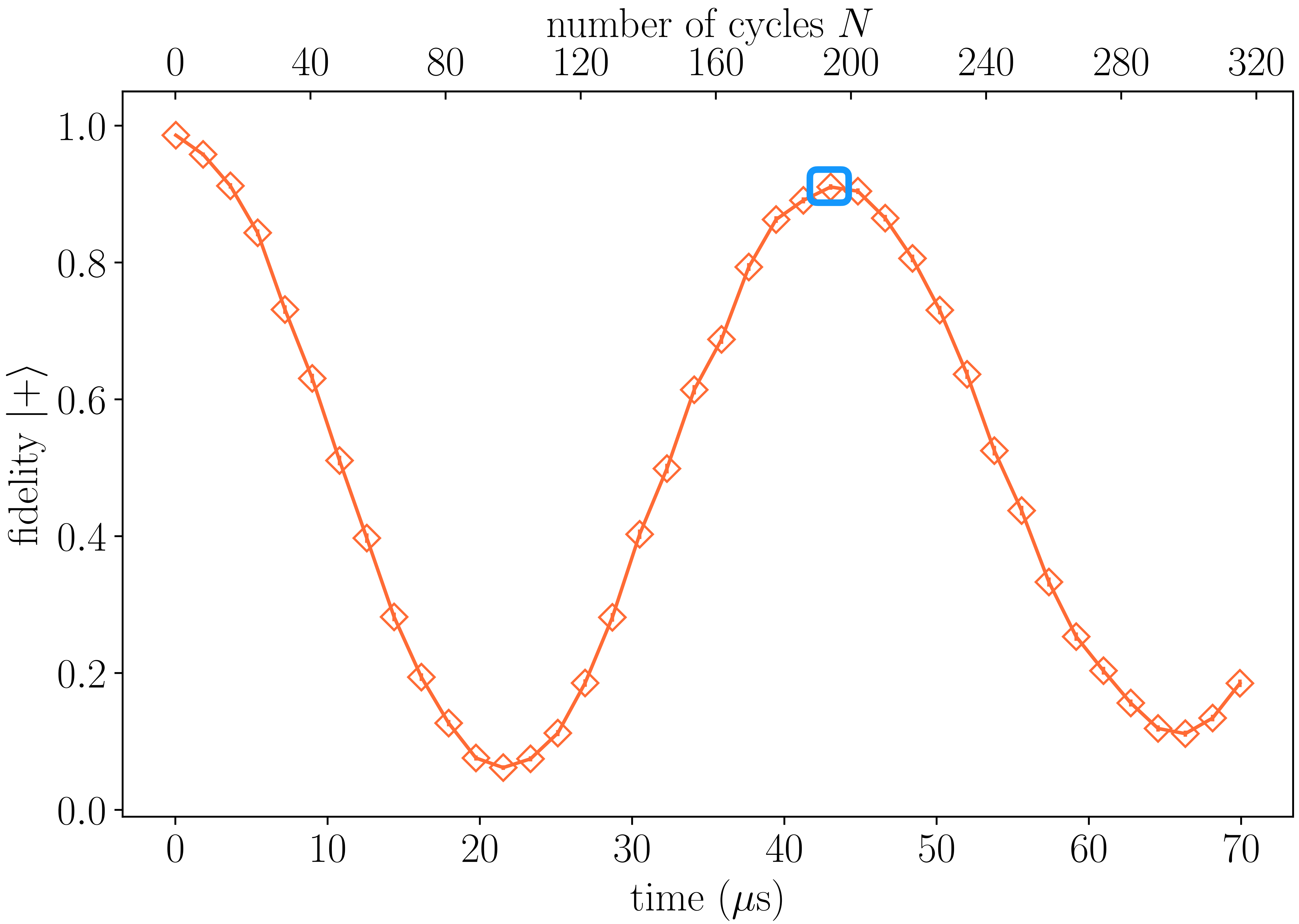
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses

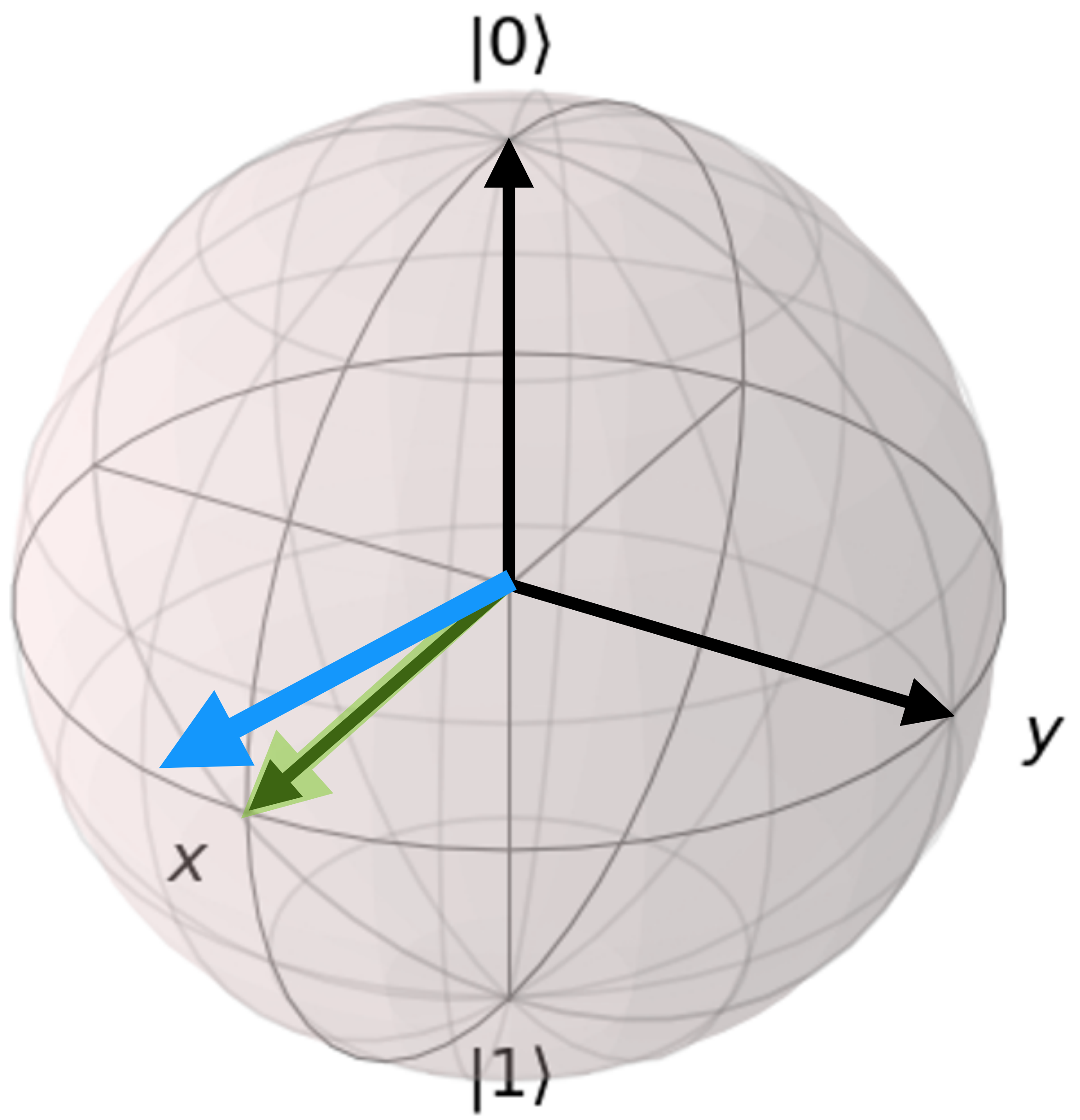




Dynamical decoupling and coherent errors

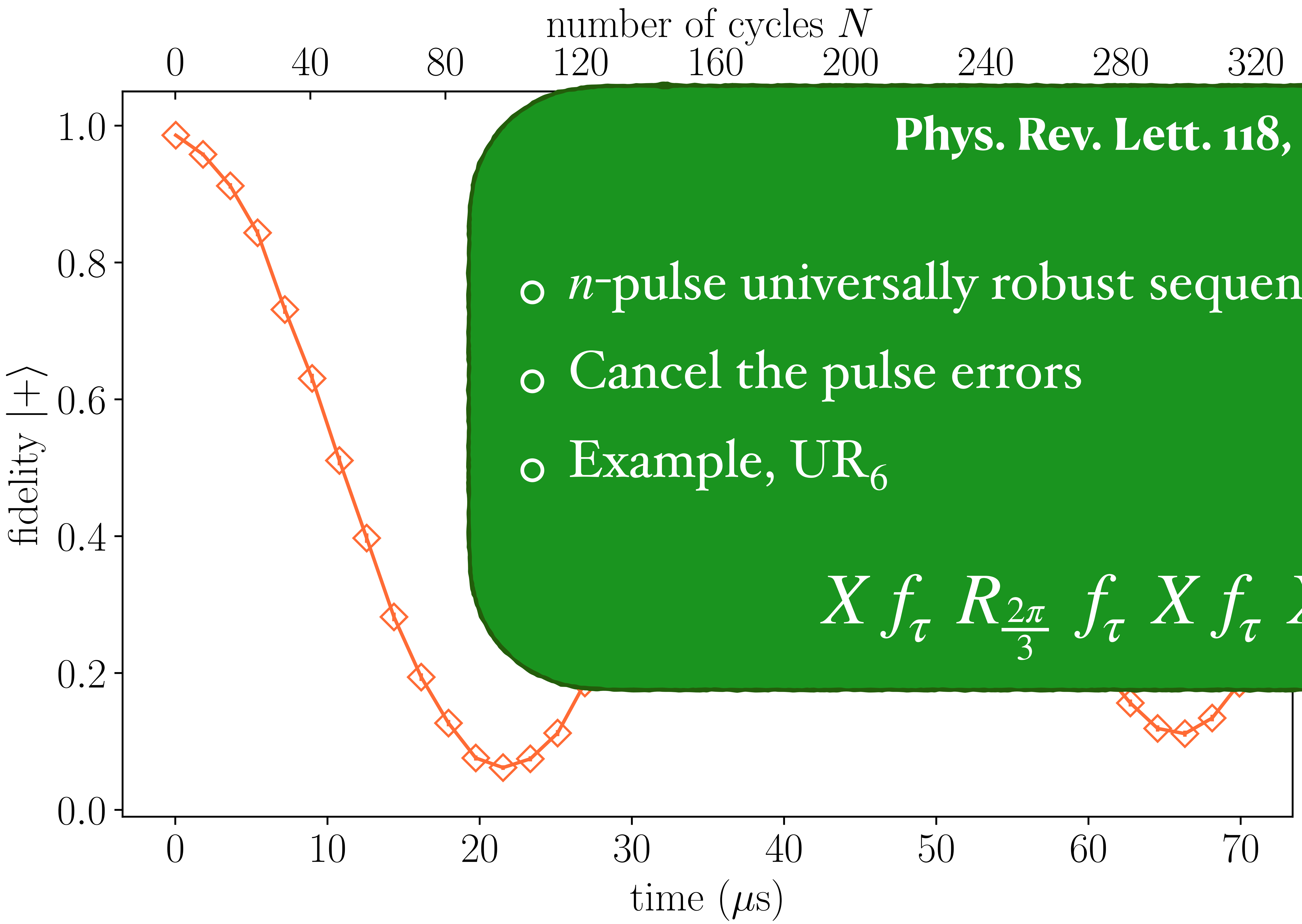
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses





Dynamical decoupling and coherent errors

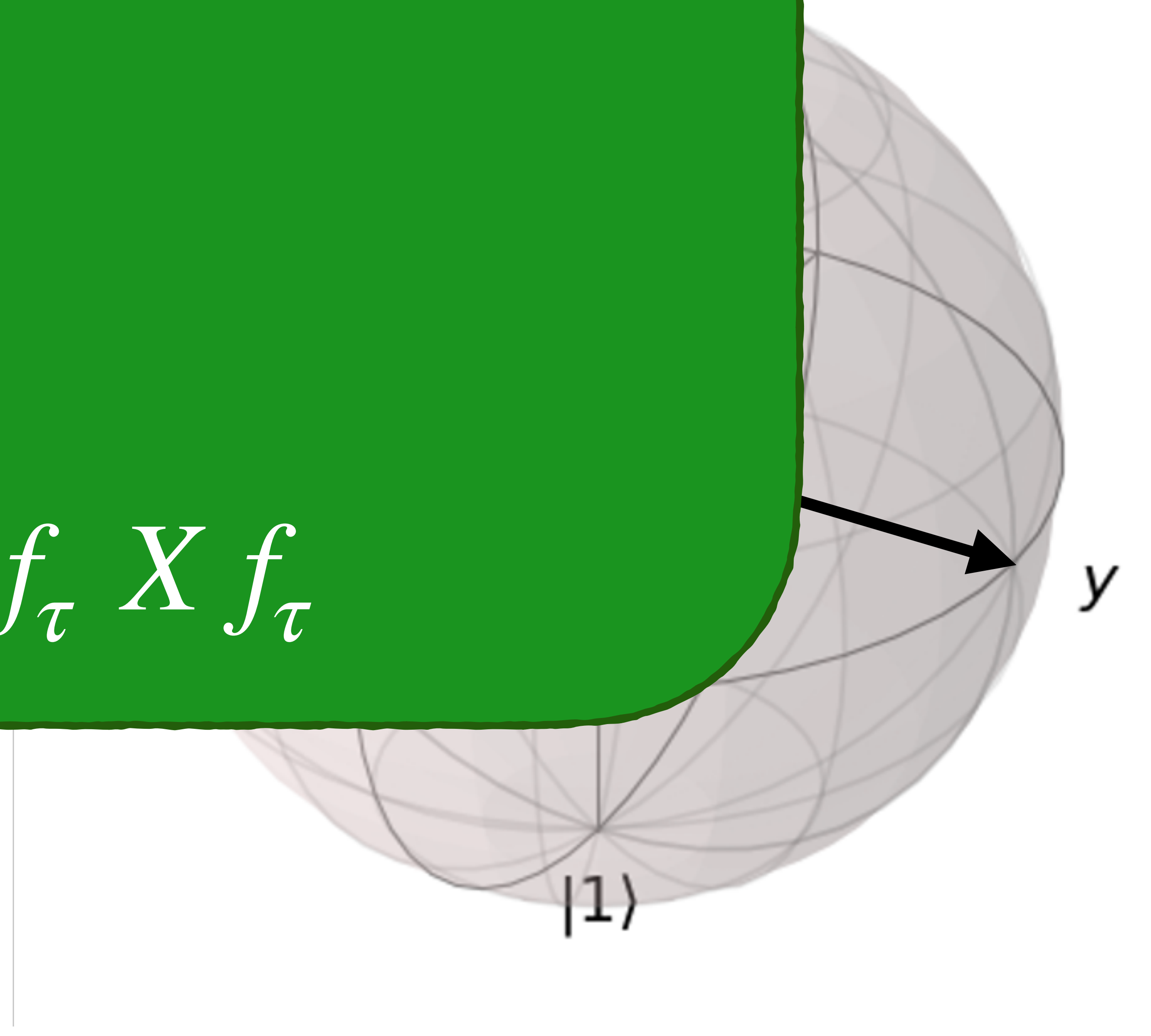
- Coherent errors will lead to oscillation in the fidelity of dynamical decoupling sequences
- Based on these oscillations we can pick up the errors in the pulses



Phys. Rev. Lett. 118, 133202 (2017)

- n-pulse universally robust sequences
- Cancel the pulse errors
- Example, UR_6

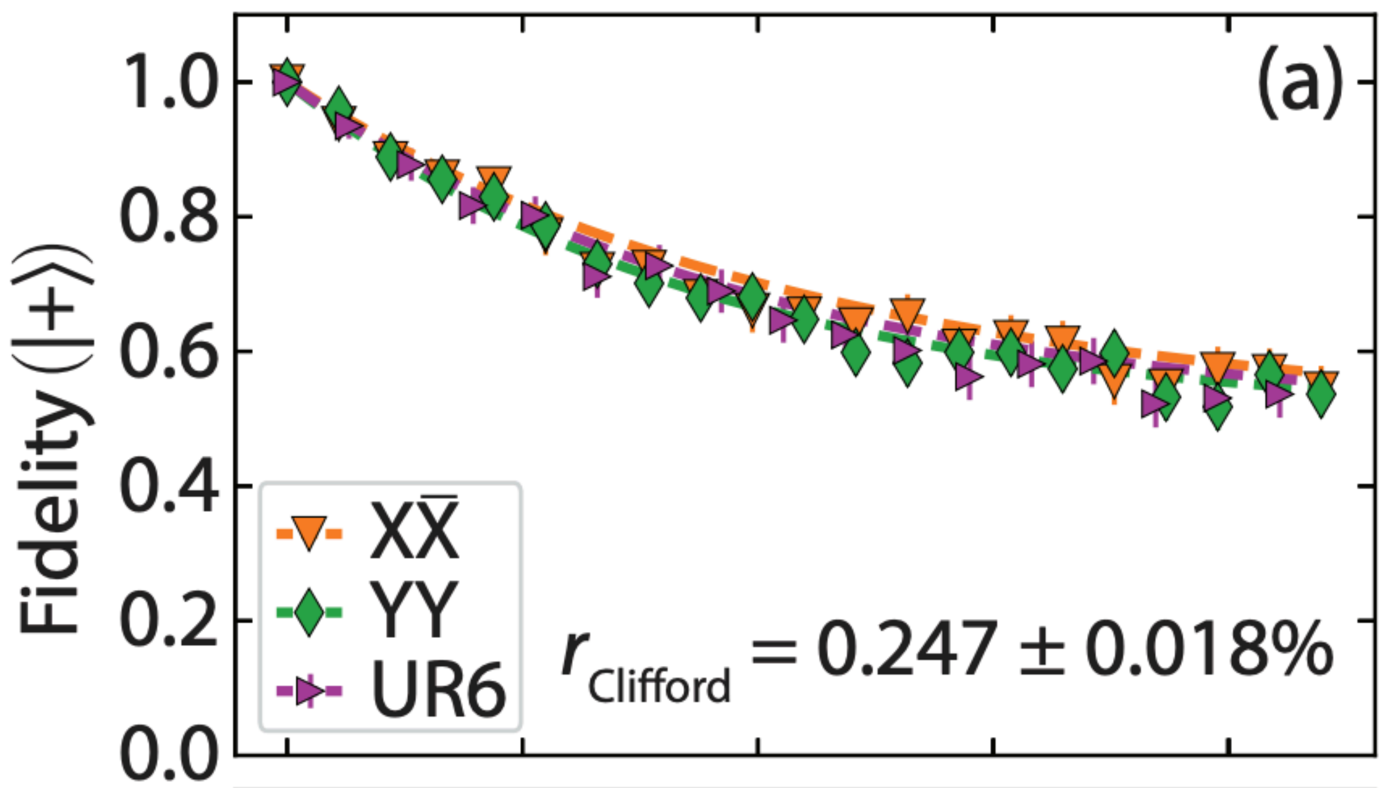
$$X\, f_\tau\, R_{\frac{2\pi}{3}}\, f_\tau\, X\, f_\tau\, X\, f_\tau\, R_{\frac{2\pi}{3}}\, f_\tau\, X\, f_\tau$$



Dynamical decoupling and coherent errors

V. Tripathi, D. Kowsari, K. Saurav, H. Zhang, E. M. Levenson-Falk, and D. A. Lidar,
Benchmarking Quantum Gates and Circuits, **ACS Chemrev 4c00870 (2025).**

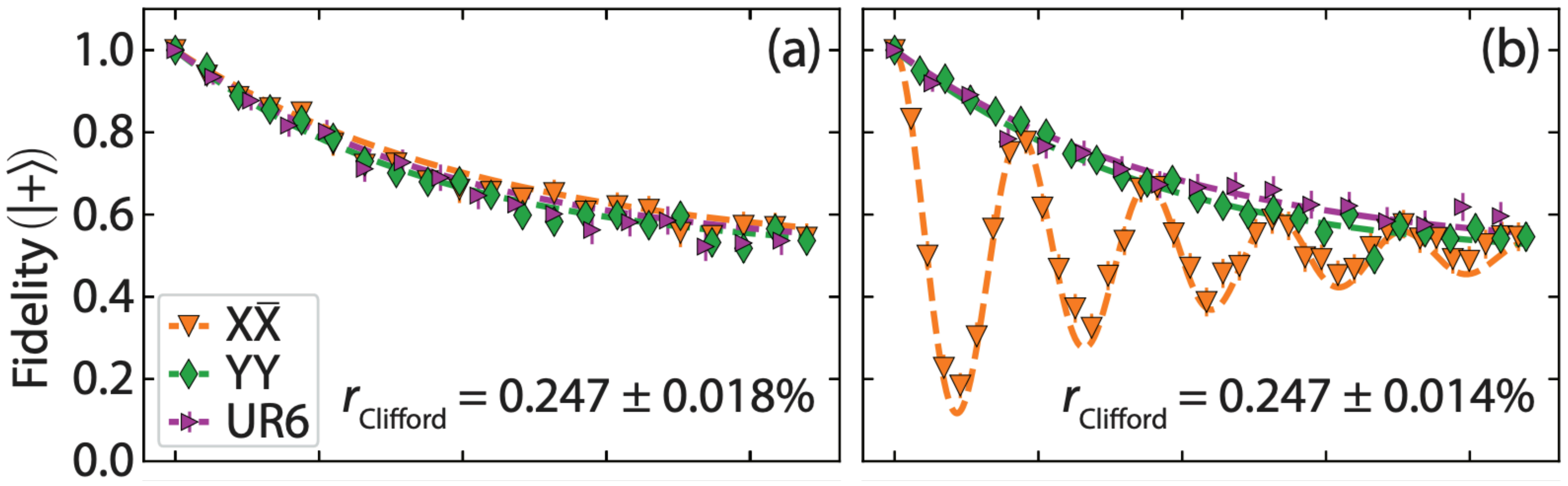
- We can detect phase errors using $X\bar{X}$ and rotation errors using YY sequences



Dynamical decoupling and coherent errors

V. Tripathi, D. Kowsari, K. Saurav, H. Zhang, E. M. Levenson-Falk, and D. A. Lidar, Benchmarking Quantum Gates and Circuits, **ACS Chemrev 4c00870 (2025).**

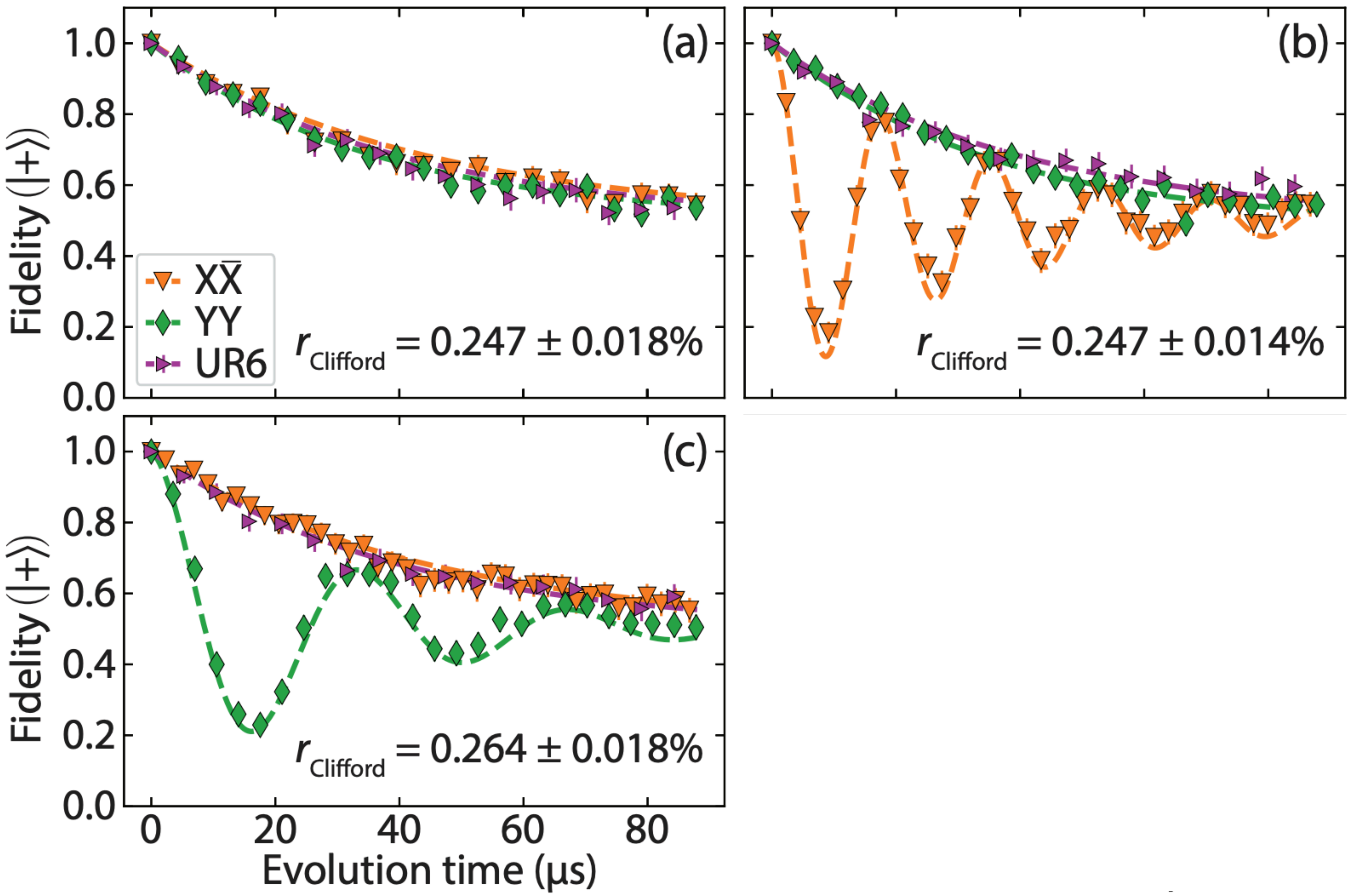
- We can detect phase errors using $X\bar{X}$ and rotation errors using YY sequences



Dynamical decoupling and coherent errors

V. Tripathi, D. Kowsari, K. Saurav, H. Zhang, E. M. Levenson-Falk, and D. A. Lidar,
Benchmarking Quantum Gates and Circuits, **ACS Chemrev 4c00870 (2025).**

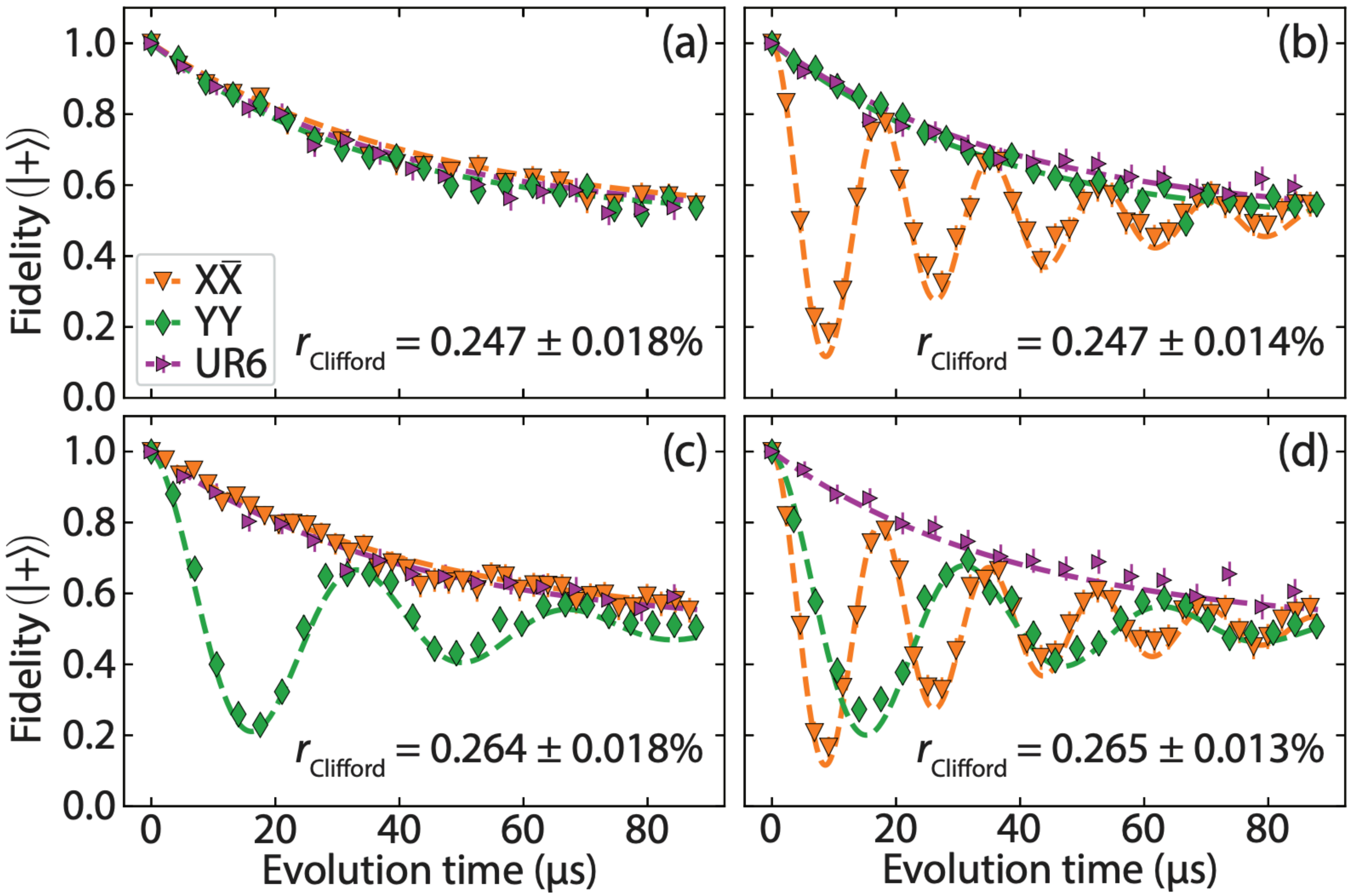
- We can detect phase errors using $X\bar{X}$ and rotation errors using YY sequences



Dynamical decoupling and coherent errors

V. Tripathi, D. Kowsari, K. Saurav, H. Zhang, E. M. Levenson-Falk, and D. A. Lidar,
Benchmarking Quantum Gates and Circuits, **ACS Chemrev 4c00870 (2025).**

- We can detect phase errors using $X\bar{X}$ and rotation errors using YY sequences



Extension to 2Qubit gates

- The ideal Hamiltonian for a CZ gate:

$$H_{\text{CZ}} = \frac{\pi}{4}(Z \otimes Z - I \otimes Z - Z \otimes I)$$

- We consider a CZ gate, up to a single leading coherent error term

$$U_{\text{CZ}} = \exp\{-i(H_{\text{CZ}} + \varepsilon H_{\text{err}})\}$$

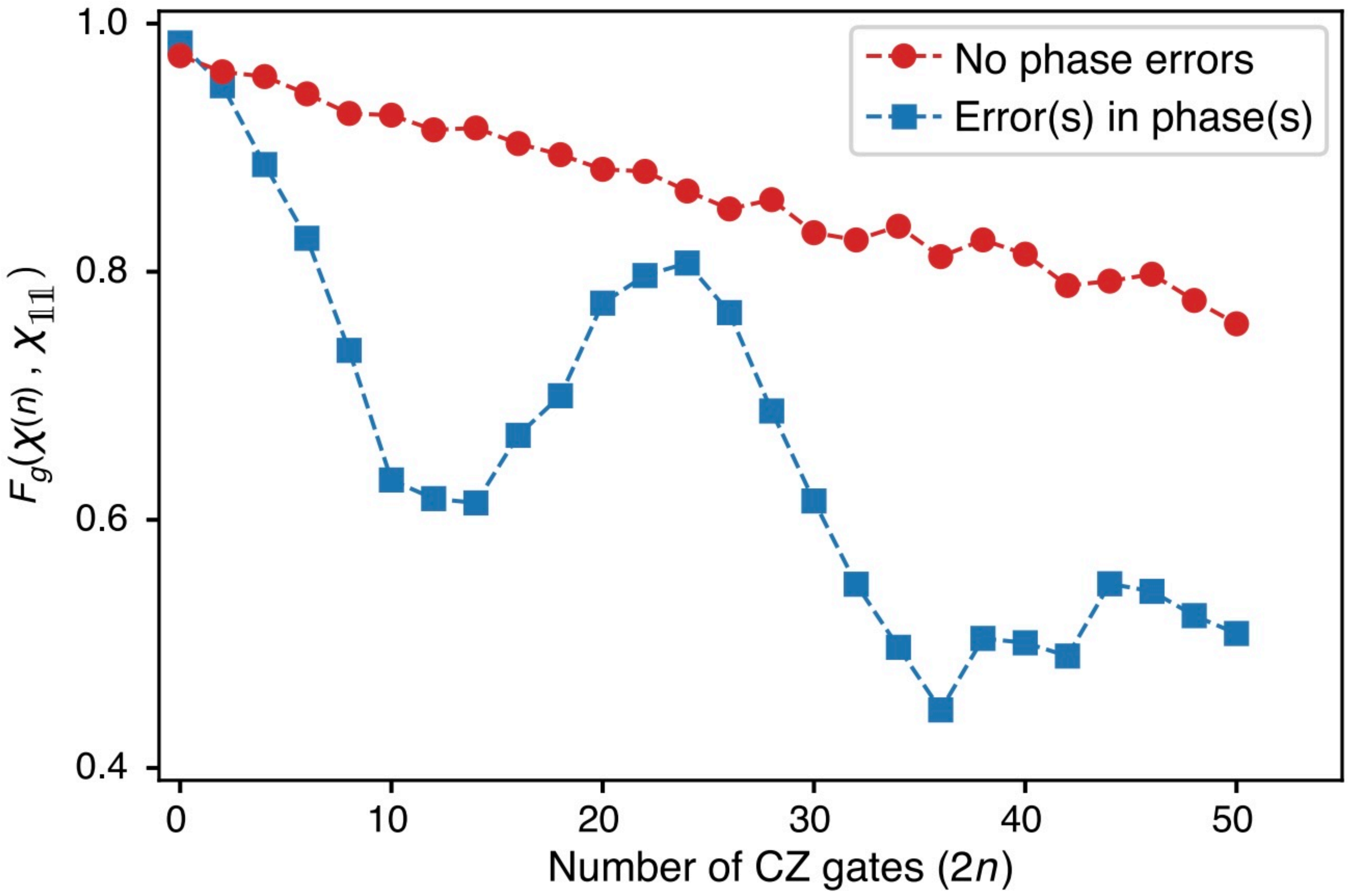
$$H_{\text{err}} \in \{I \otimes Z,\ I \otimes X,\ \cdots, Z \otimes Z\}$$

$$H_{\text{err|likely}} \in \{I \otimes Z,\ Z \otimes I, Z \otimes Z\}$$

CZ Gate Error

Demonstration of Density Matrix Exponentiation Using a Superconducting Quantum Processor

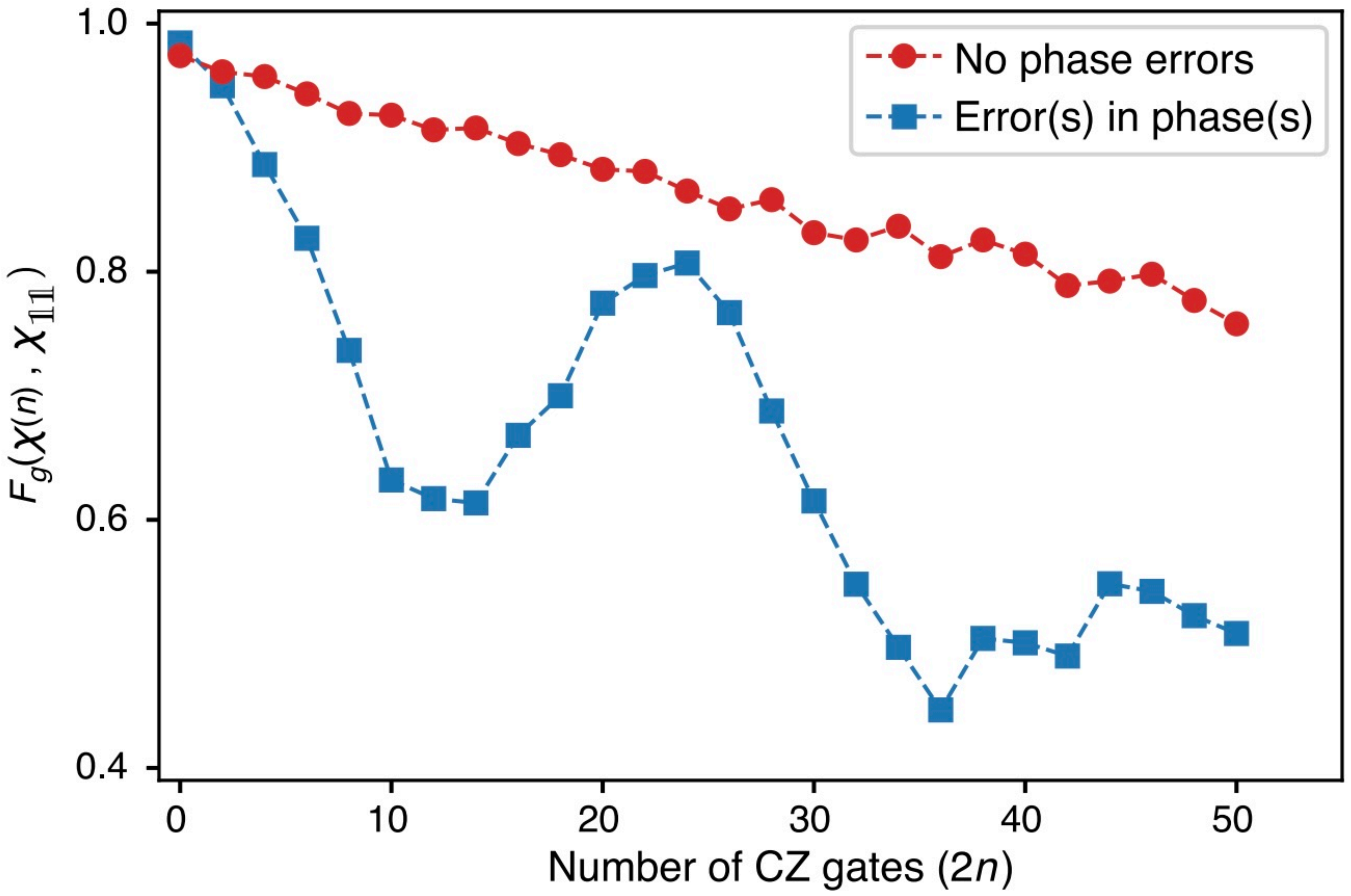
M. Kjaergaard,[1,*,†] M. E. Schwartz,[2,†] A. Greene,[1,3] G. O. Samach,[1,2,3] A. Bengtsson,[1,4] M. O'Keeffe,[2] C. M. McNally,[1,3] J. Braumüller,[1] D. K. Kim,[2] P. Krantz,[1,‡] M. Marvian,[1,5] A. Melville,[2] B. M. Niedzielski,[2] Y. Sung,[1,3] R. Winik,[1] J. Yoder,[2] D. Rosenberg,[2] K. Obenland,[2] S. Lloyd,[1,5] T. P. Orlando,[1,3] I. Marvian,[6] S. Gustavsson,[1] and W. D. Oliver[1,2,7,3]

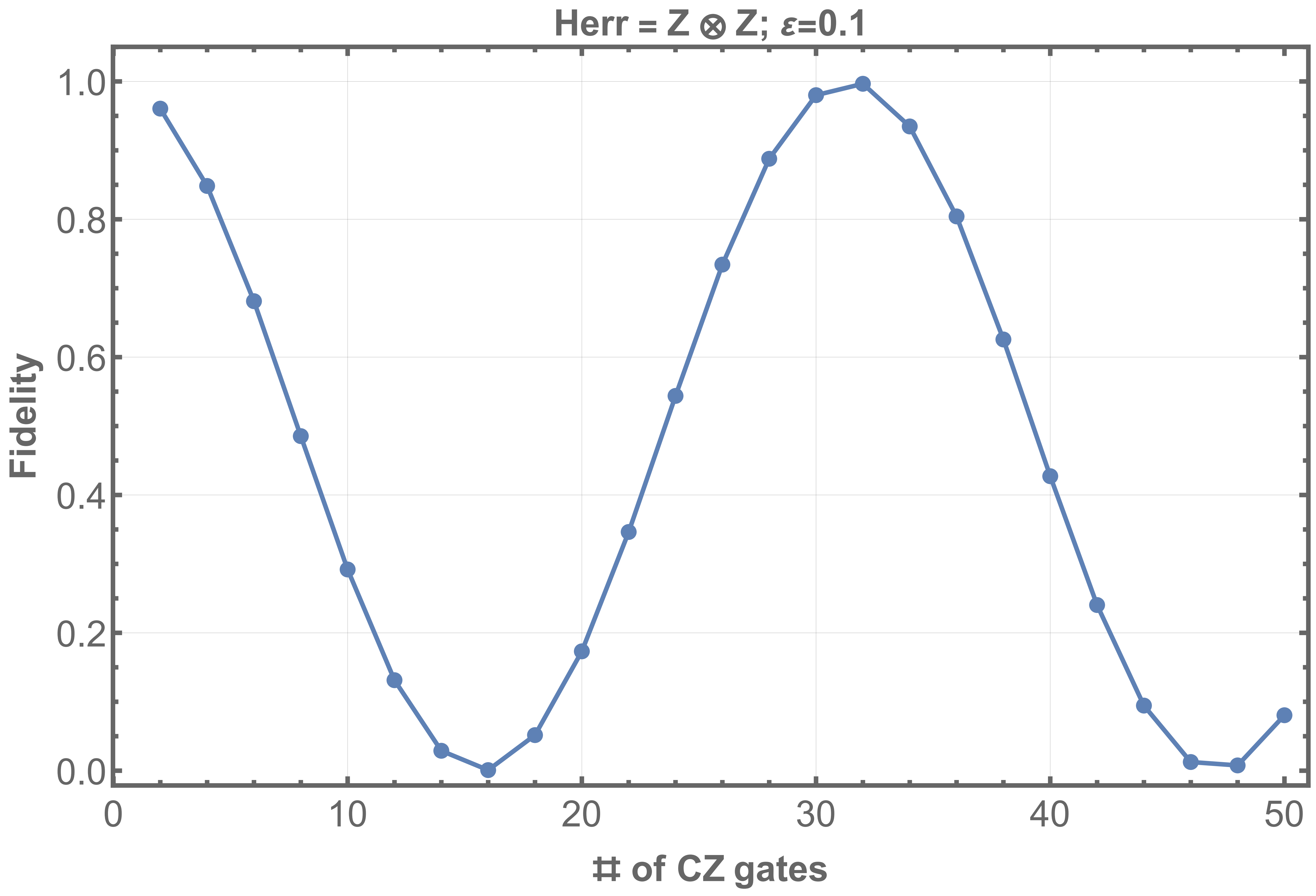


CZ Gate Error

Demonstration of Density Matrix Exponentiation Using a Superconducting Quantum Processor

M. Kjaergaard,[1,*,†] M. E. Schwartz,[2,†] A. Greene,[1,3] G. O. Samach,[1,2,3] A. Bengtsson,[1,4] M. O'Keeffe,[2] C. M. McNally,[1,3] J. Braumüller,[1] D. K. Kim,[2] P. Krantz,[1,‡] M. Marvian,[1,5] A. Melville,[2] B. M. Niedzielski,[2] Y. Sung,[1,3] R. Winik,[1] J. Yoder,[2] D. Rosenberg,[2] K. Obenland,[2] S. Lloyd,[1,5] T. P. Orlando,[1,3] I. Marvian,[6] S. Gustavsson,[1] and W. D. Oliver[1,2,7,3]

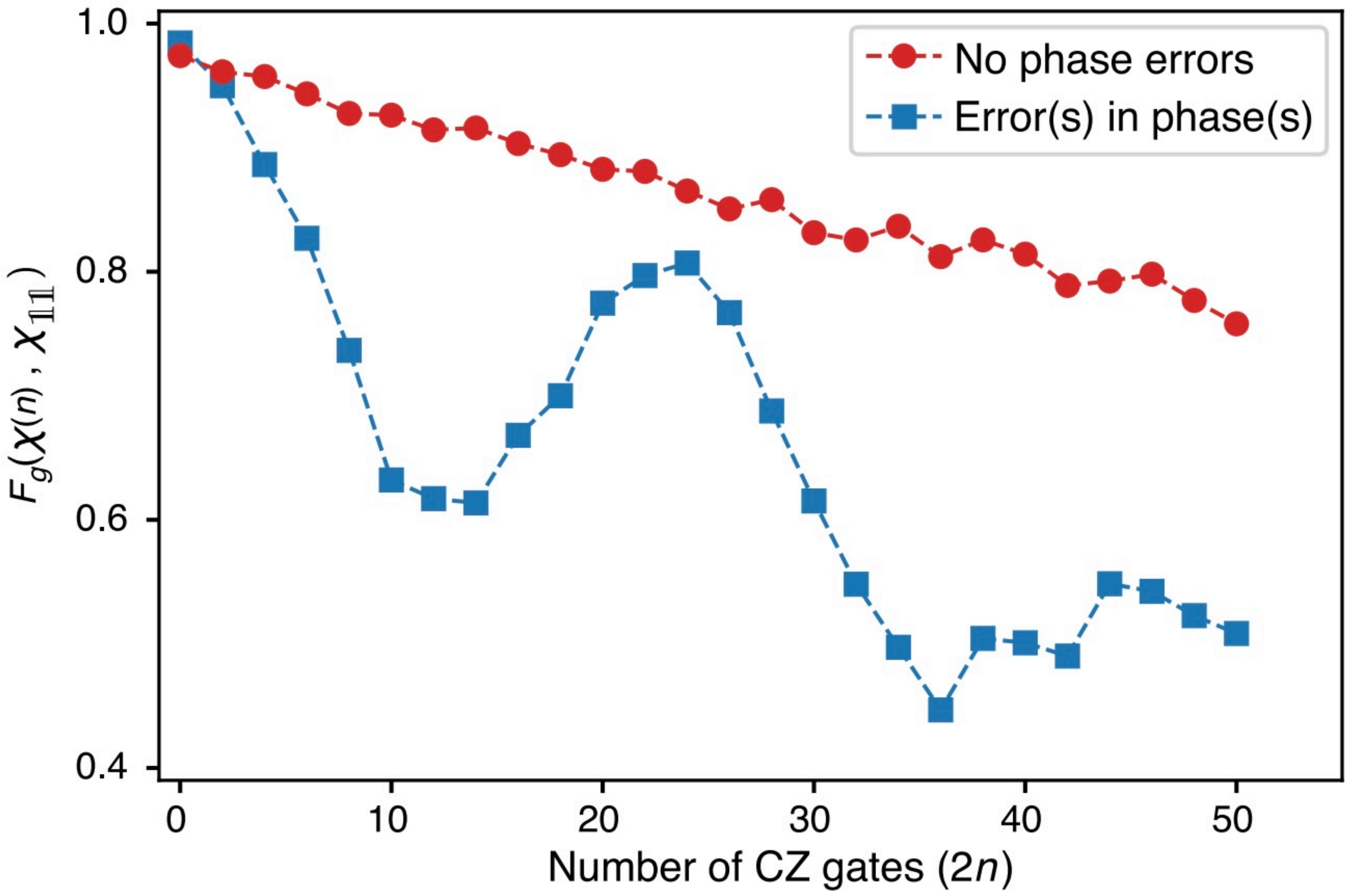


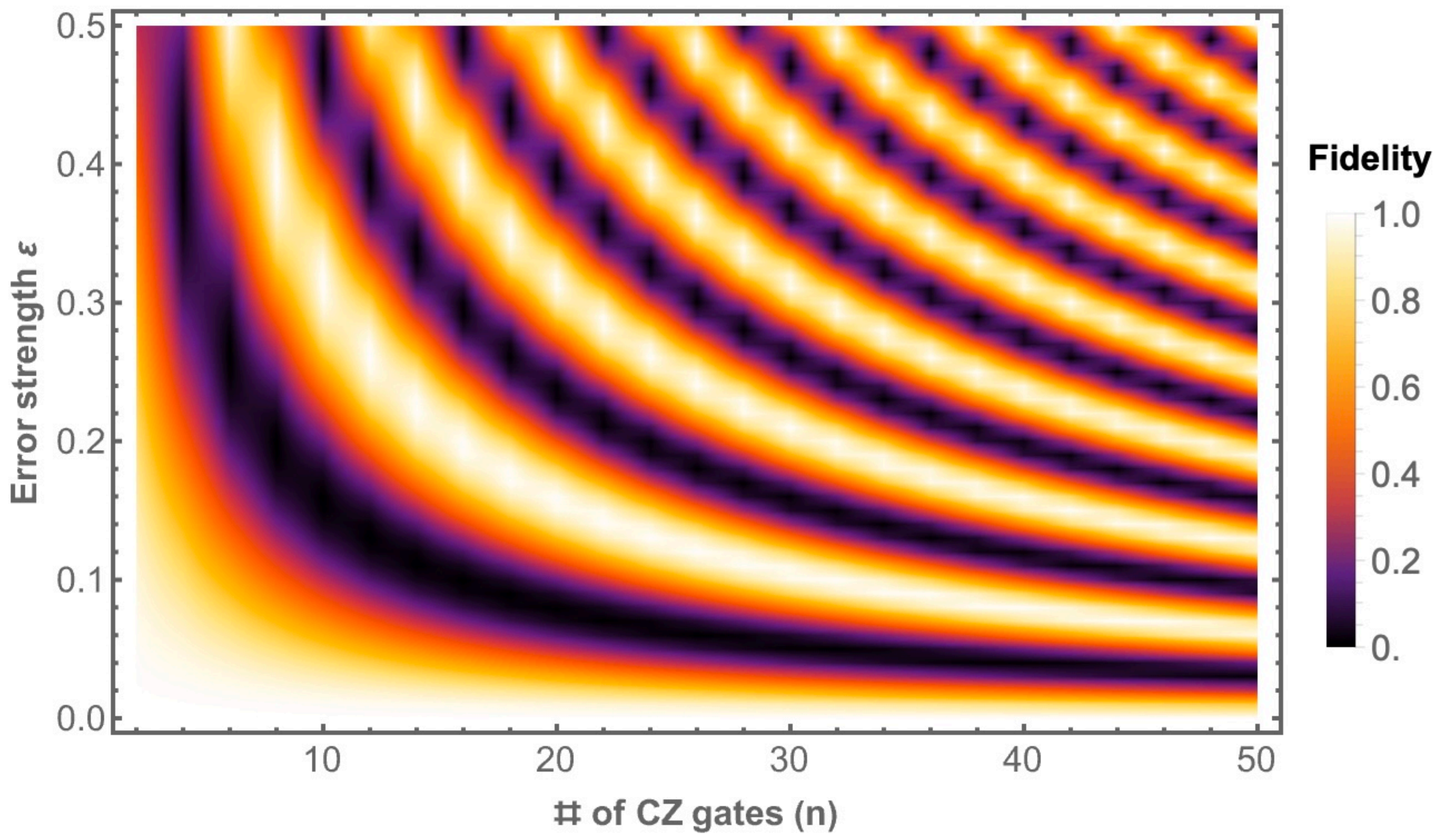


CZ Gate Error

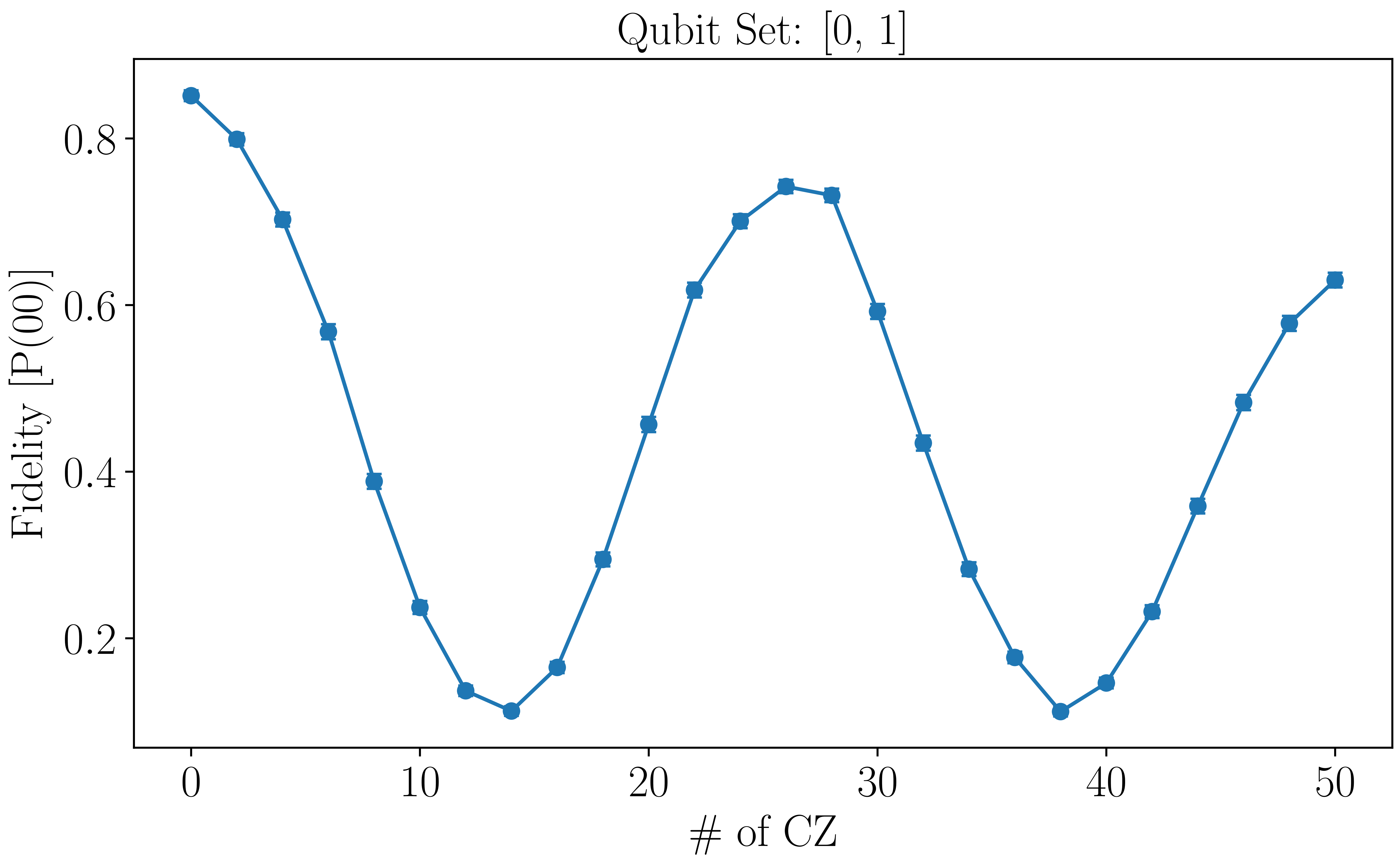
Demonstration of Density Matrix Exponentiation Using a Superconducting Quantum Processor

M. Kjaergaard,[1,*,†] M. E. Schwartz,[2,†] A. Greene,[1,3] G. O. Samach,[1,2,3] A. Bengtsson,[1,4] M. O'Keeffe,[2] C. M. McNally,[1,3] J. Braumüller,[1] D. K. Kim,[2] P. Krantz,[1,‡] M. Marvian,[1,5] A. Melville,[2] B. M. Niedzielski,[2] Y. Sung,[1,3] R. Winik,[1] J. Yoder,[2] D. Rosenberg,[2] K. Obenland,[2] S. Lloyd,[1,5] T. P. Orlando,[1,3] I. Marvian,[6] S. Gustavsson,[1] and W. D. Oliver[1,2,7,3]



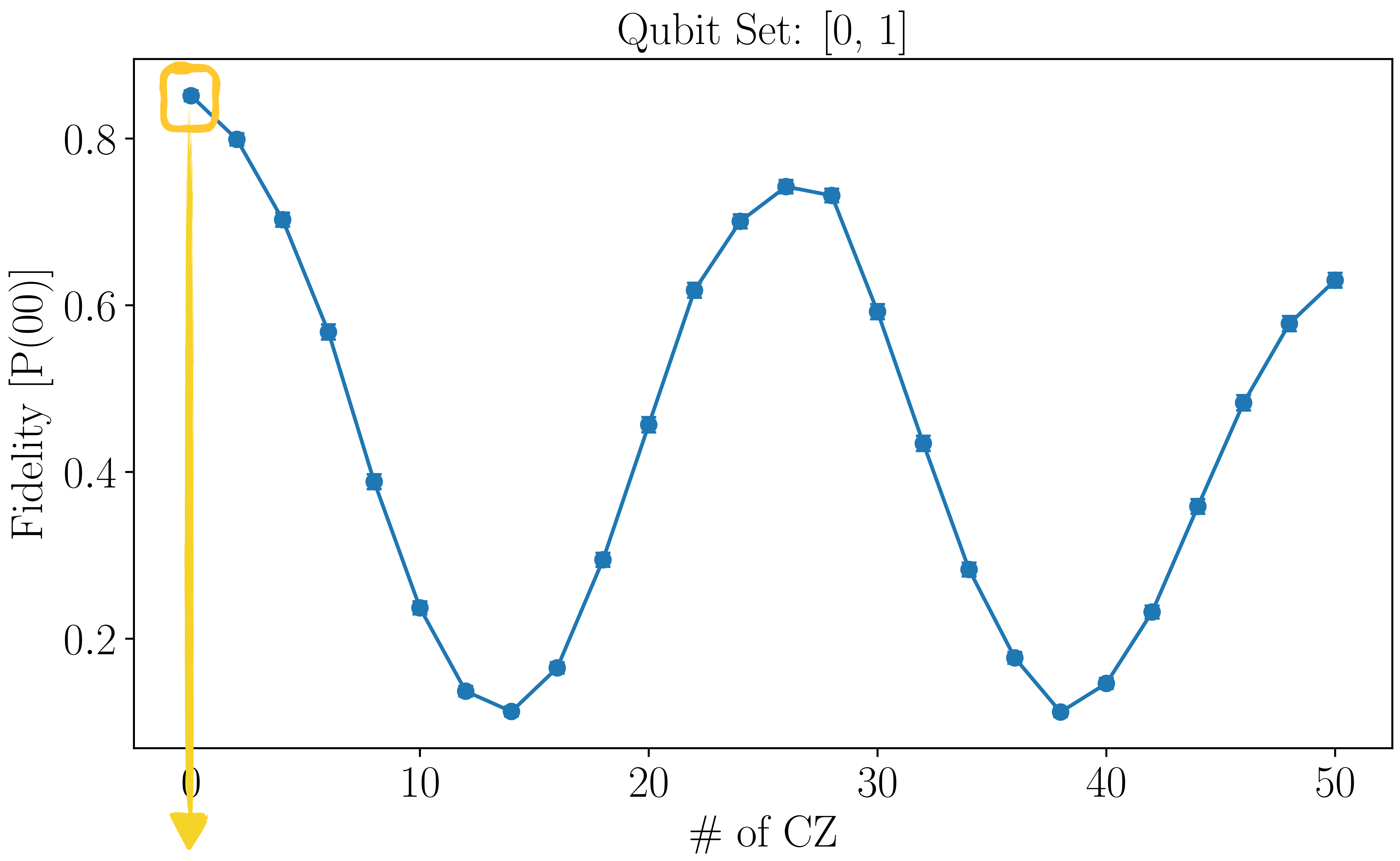


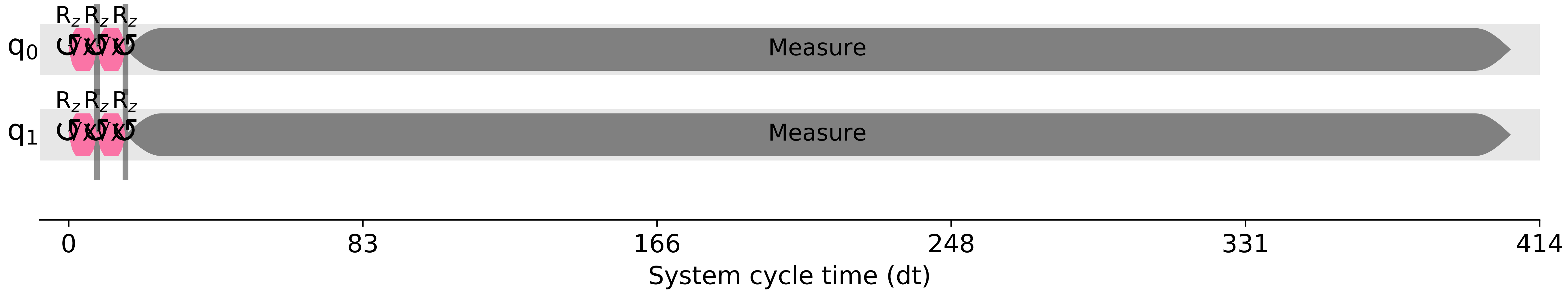
Experimental test on *ibm_torino*



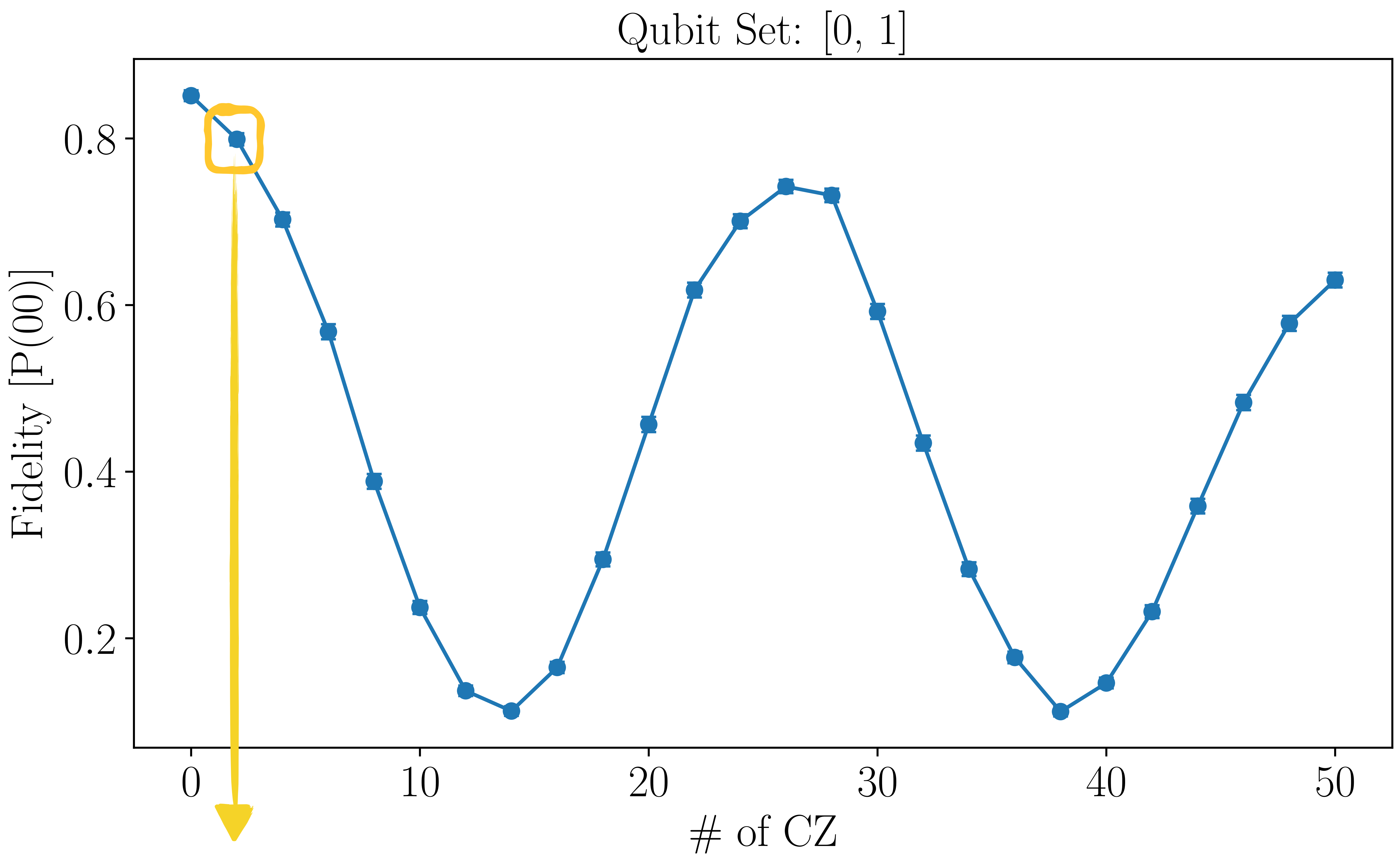
Experiment: encode $U_{|++\rangle}$, apply (CZ.CZ), then decode $U^{\dagger}_{|++\rangle}$ and then read off bitstring 00

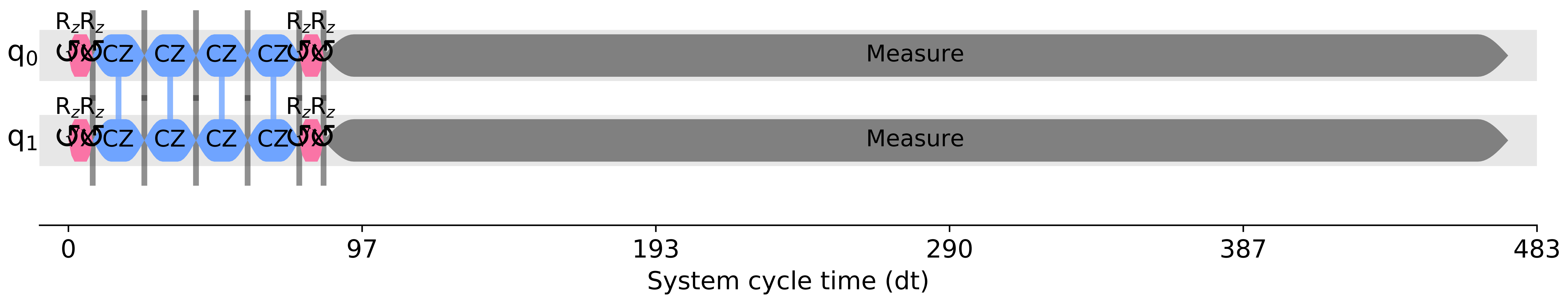
Experimental test on *ibm_torino*



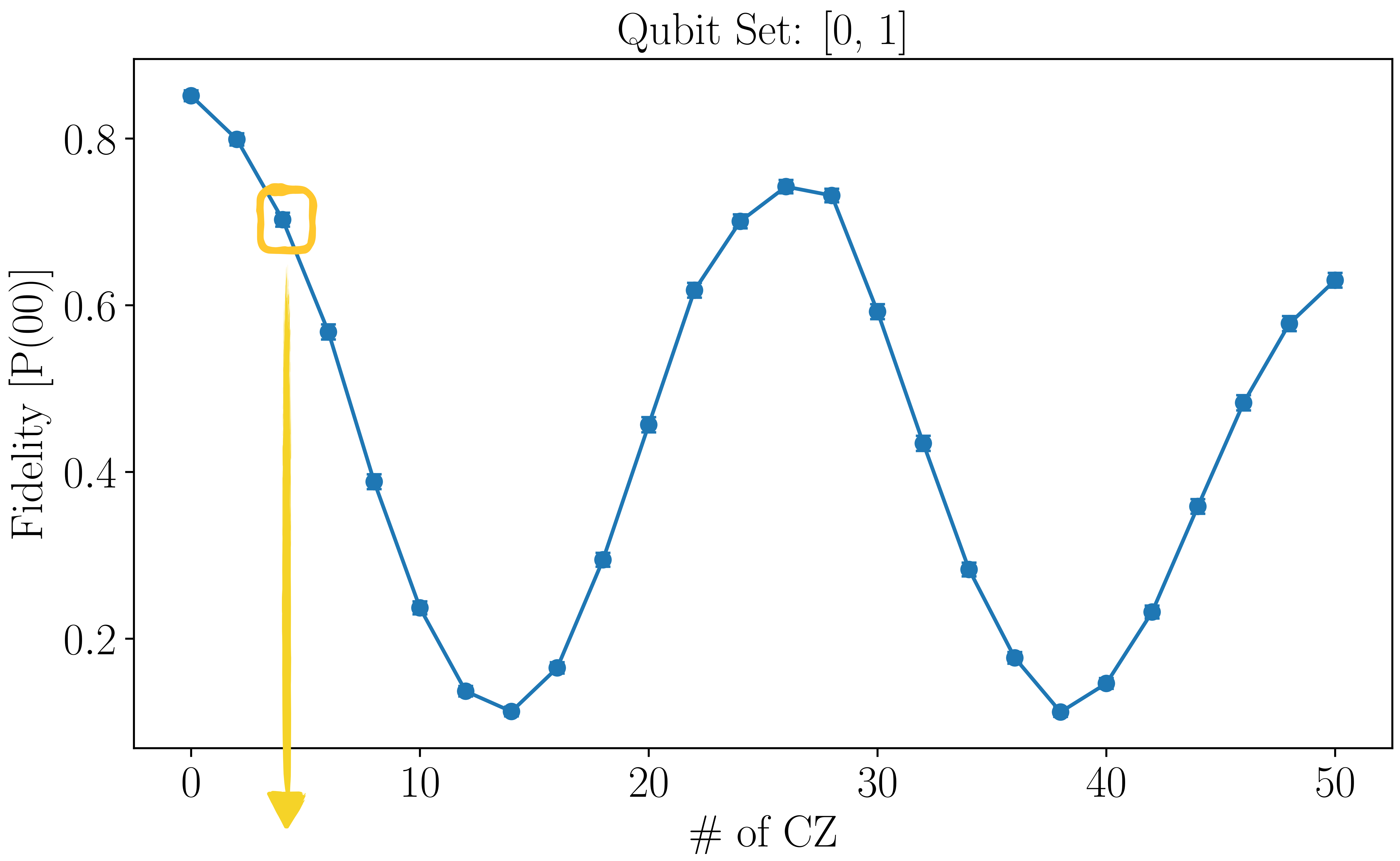


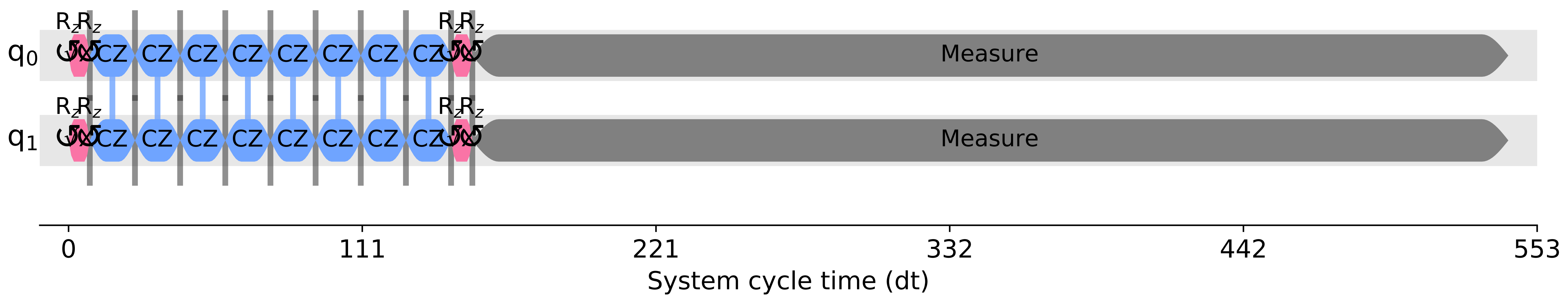
Experimental test on *ibm_torino*



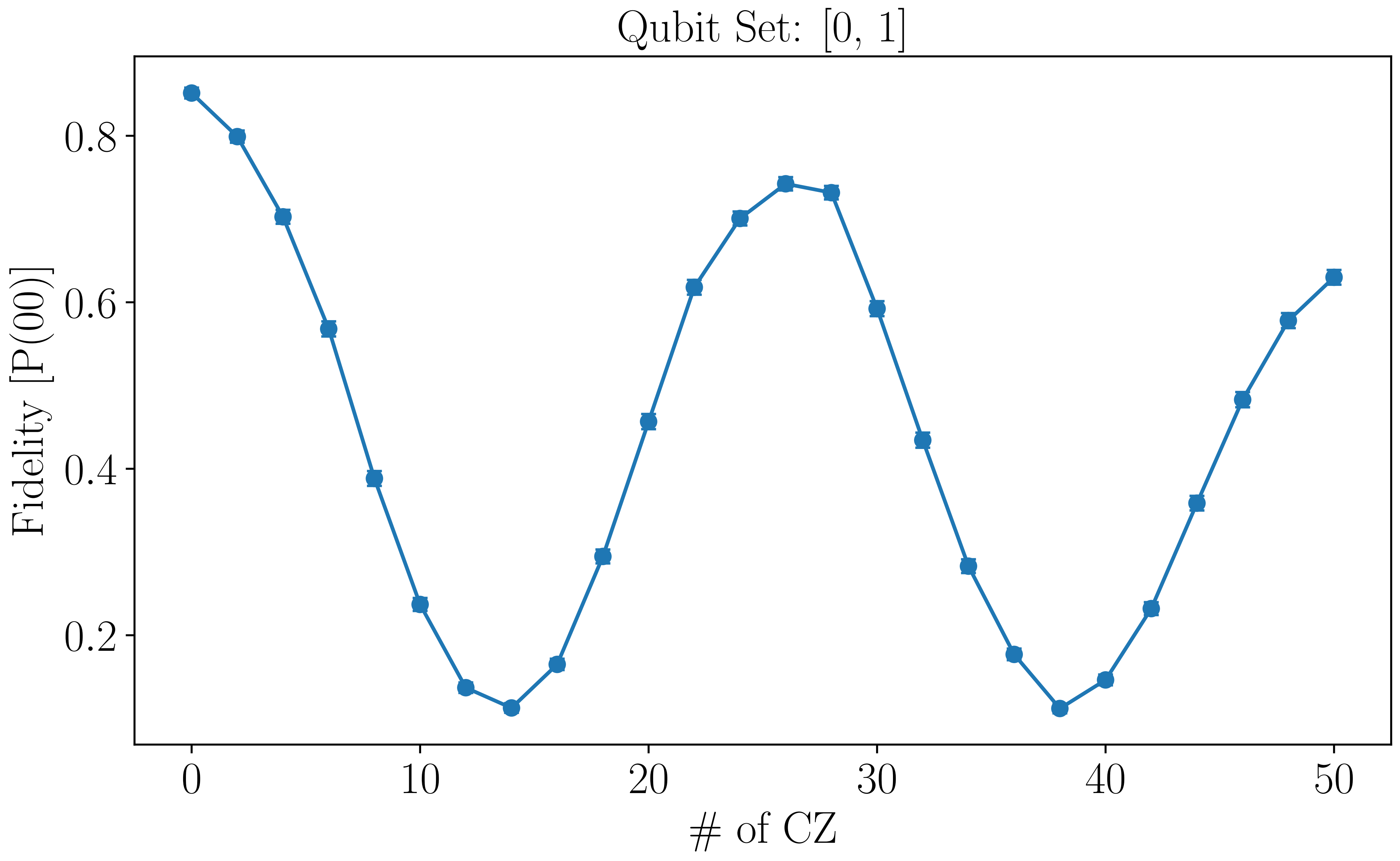


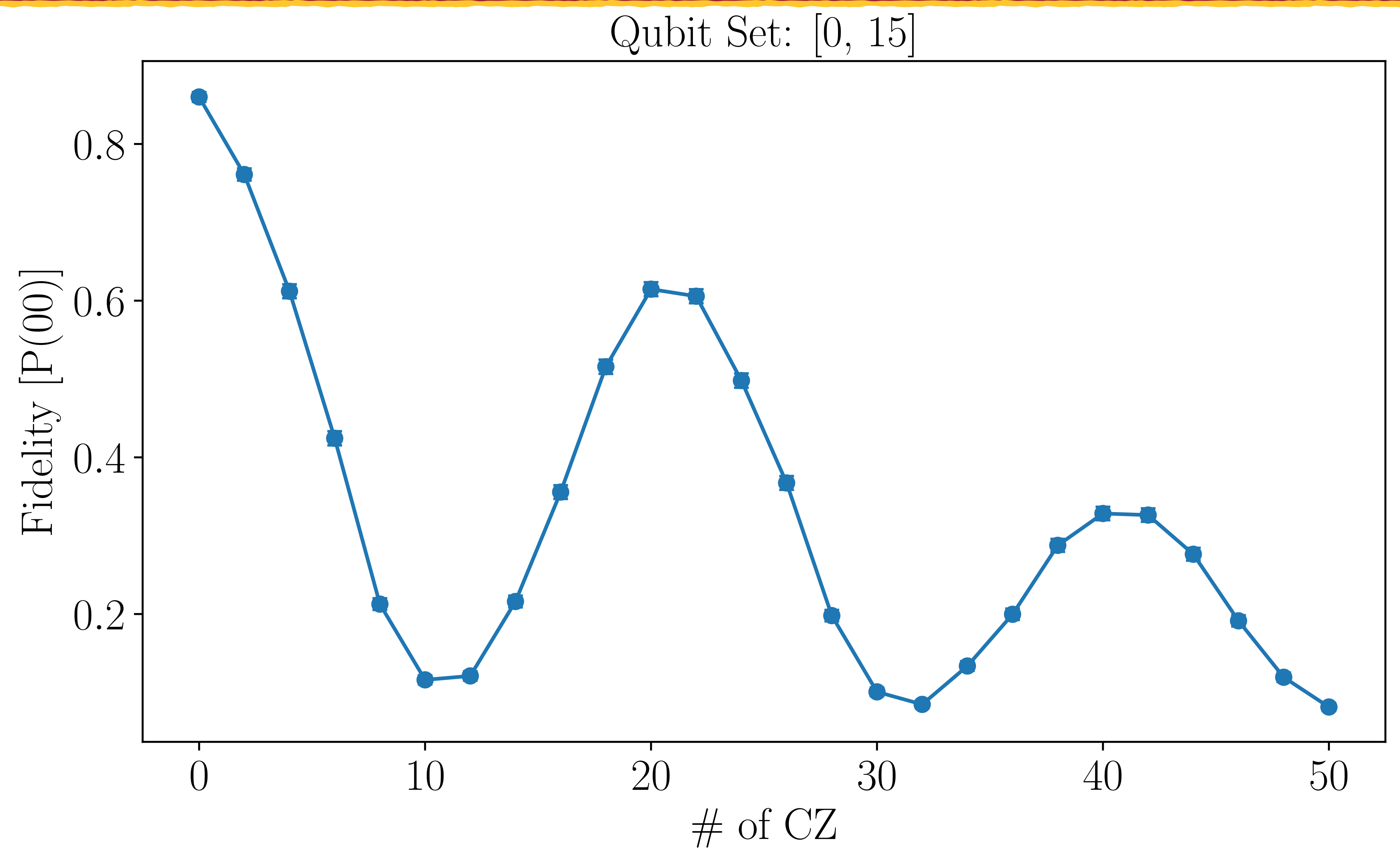
Experimental test on *ibm_torino*

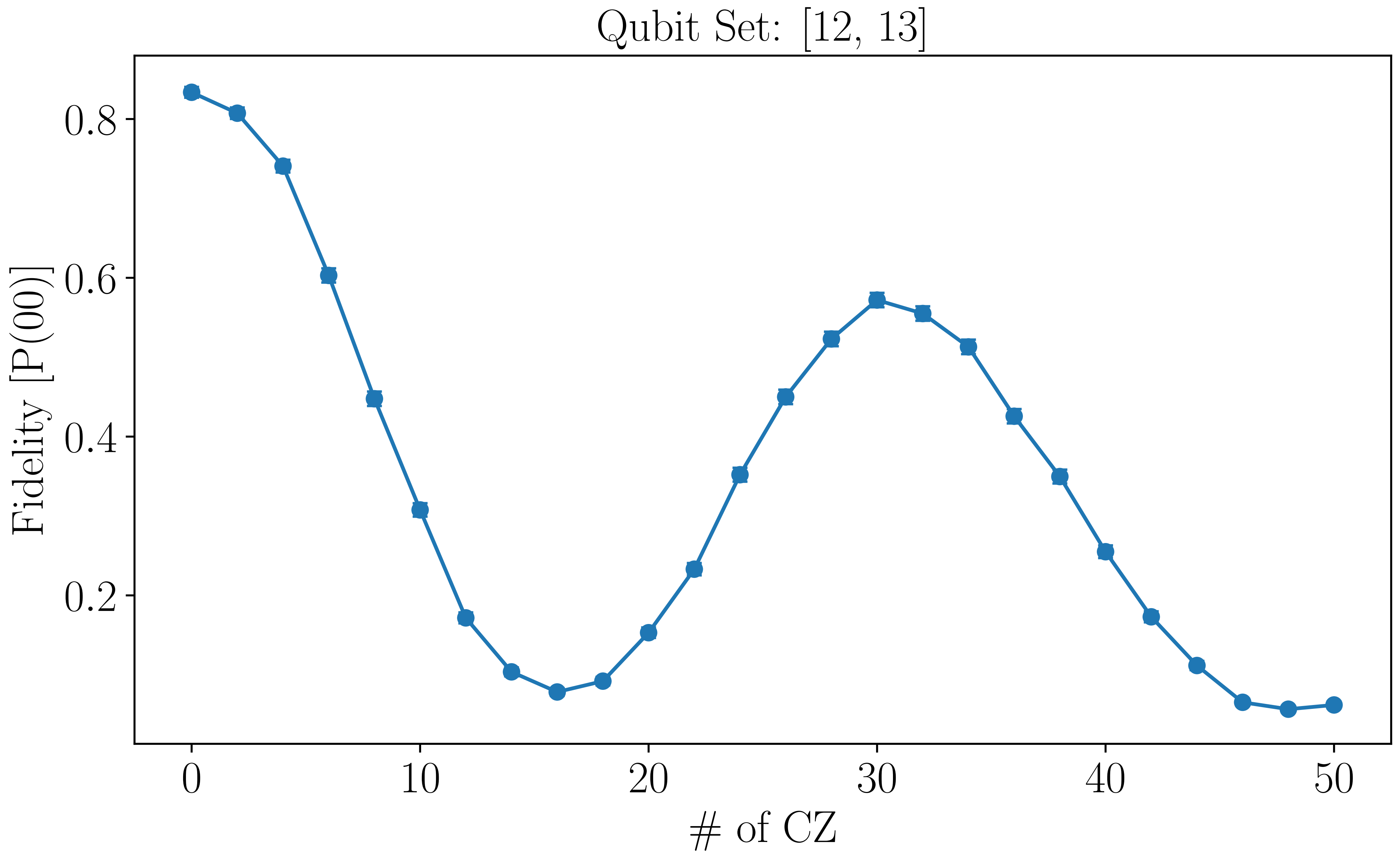


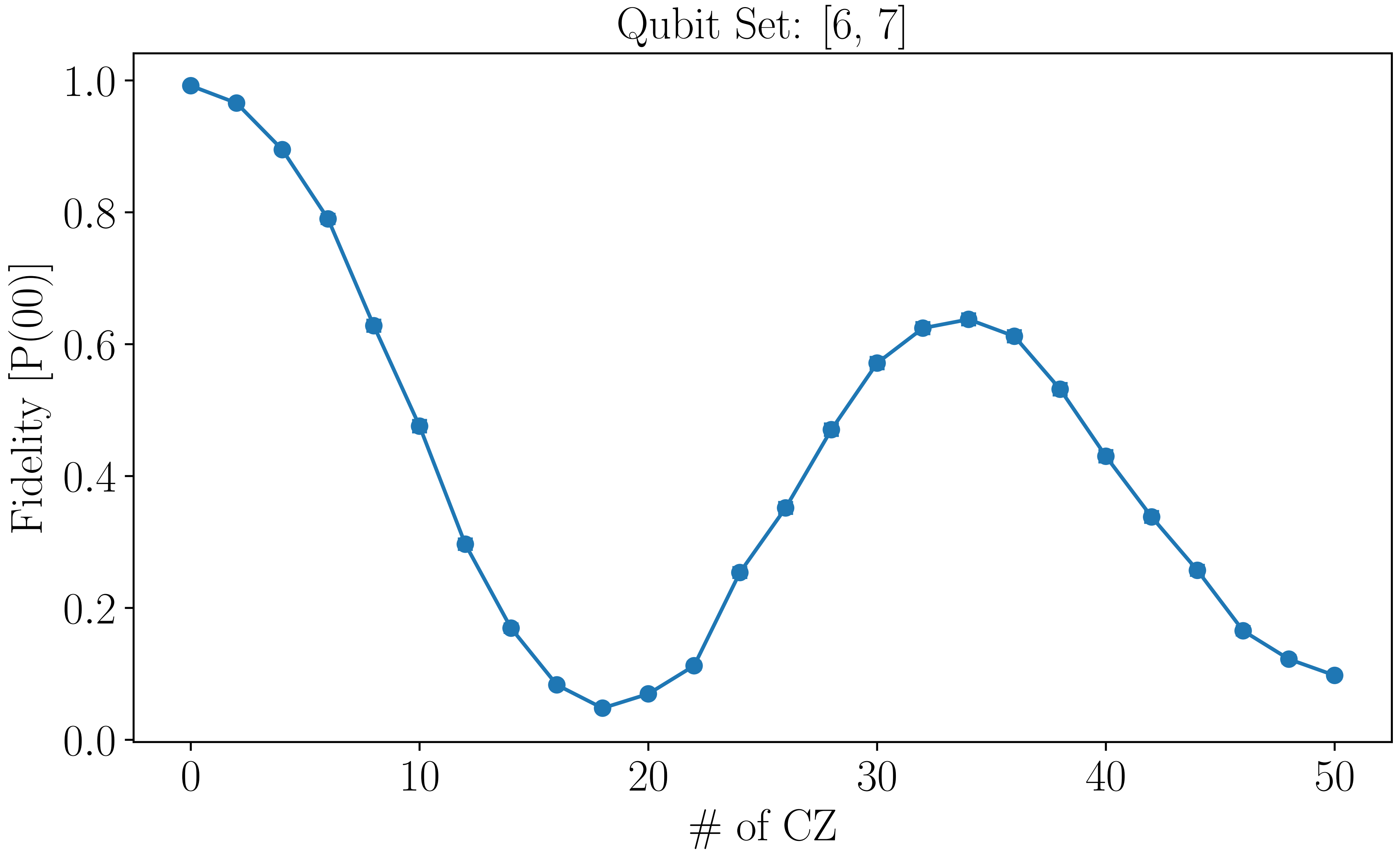


Experimental test on *ibm_torino*

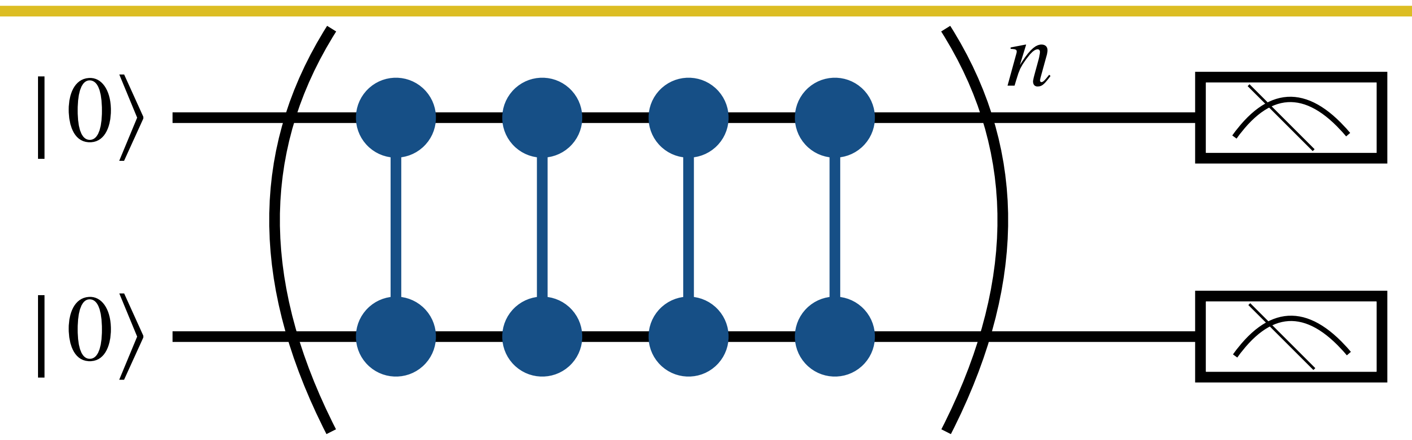






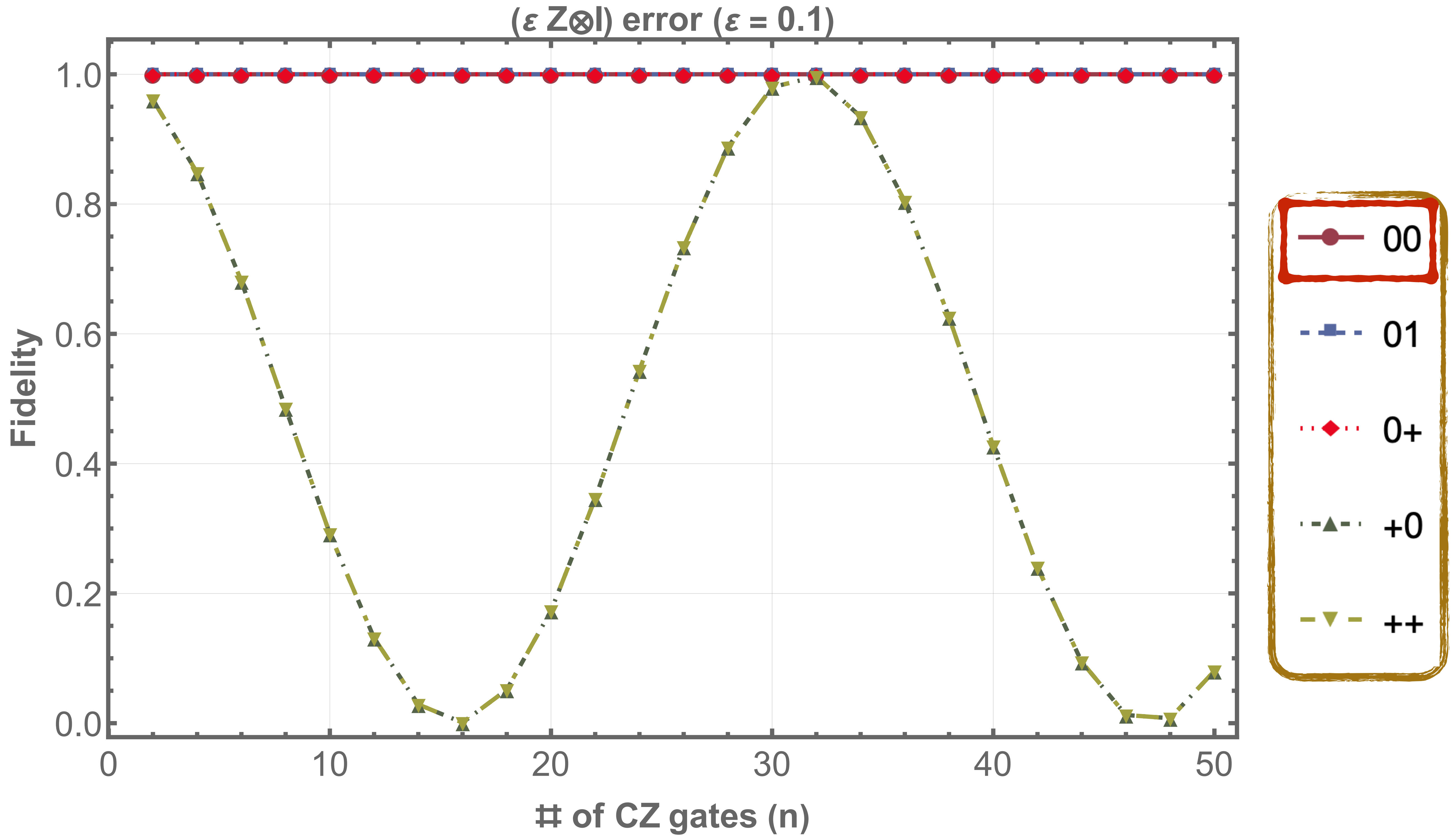


Circuits

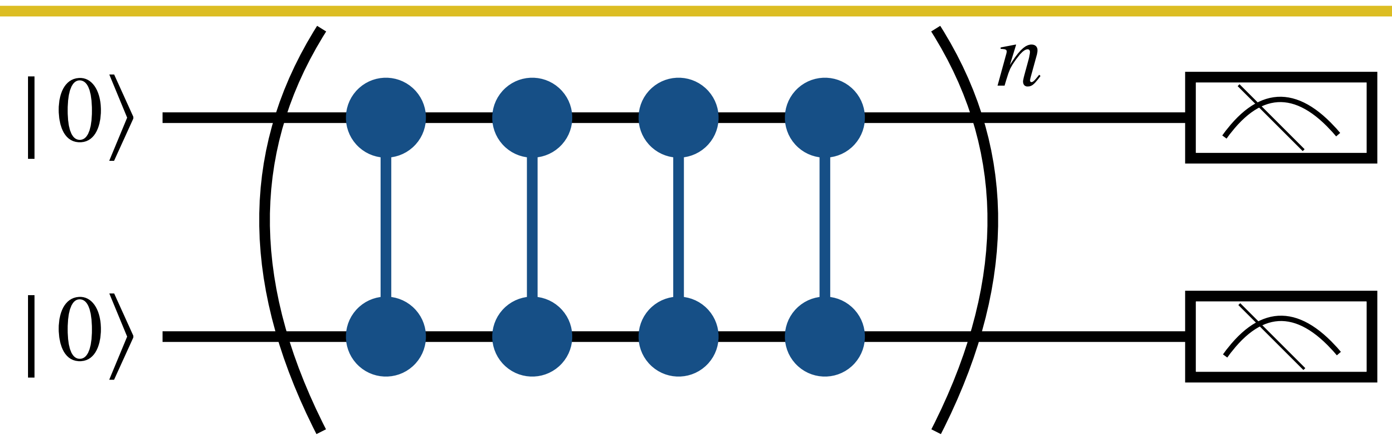


No oscillations reduces the likely errors to

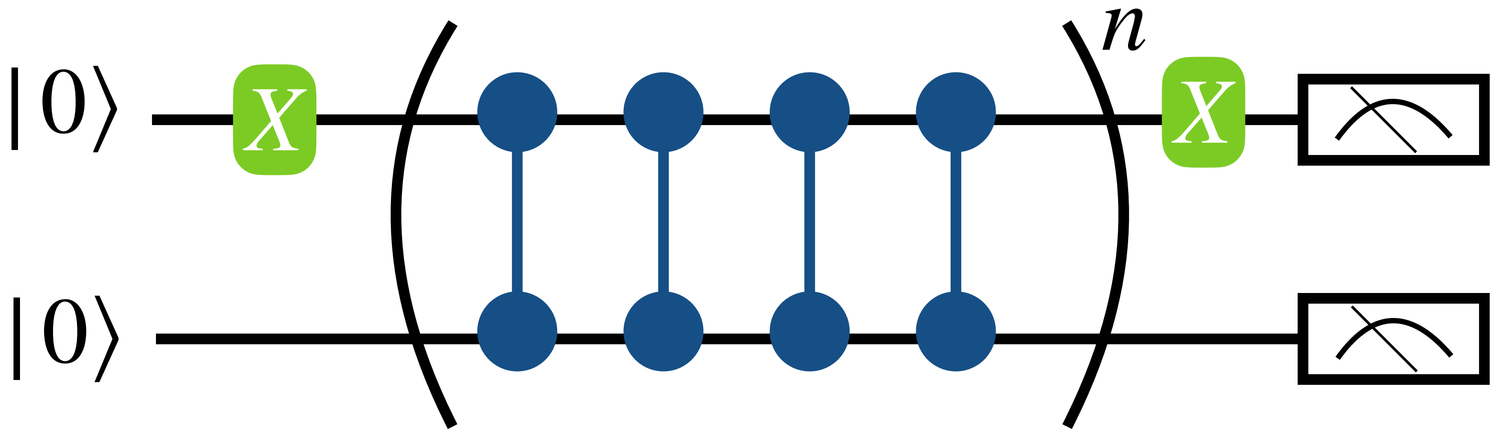
$I \otimes Z, Z \otimes I, Z \otimes Z, X \otimes X, X \otimes Y, Y \otimes X, Y \otimes Y$



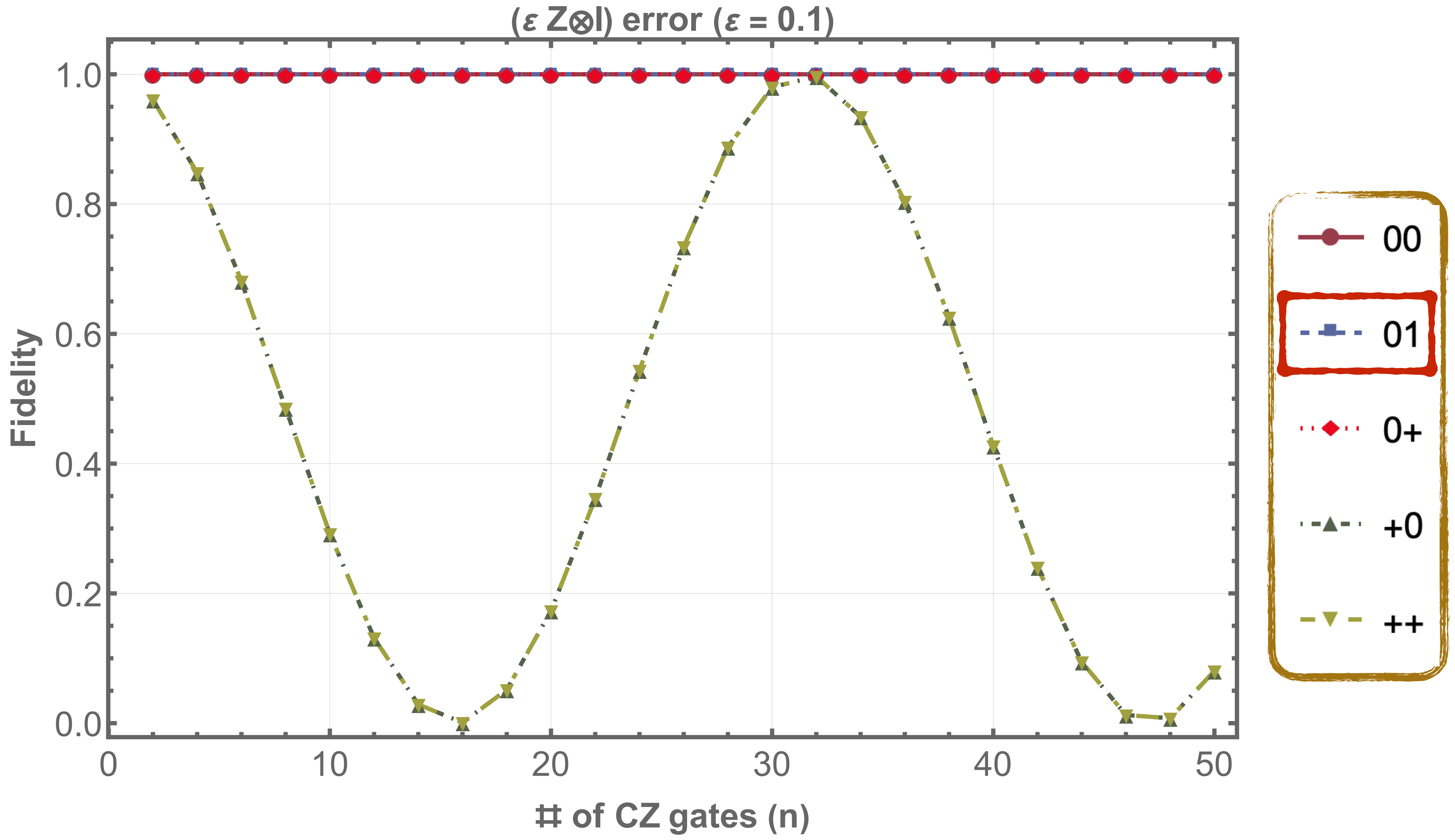
Circuits



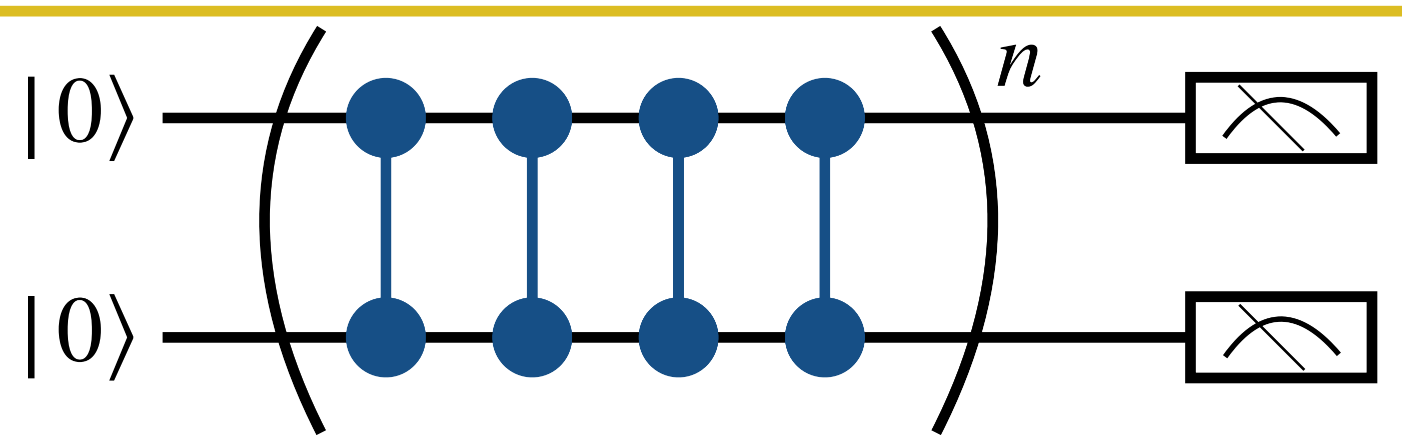
No oscillations reduces the likely errors to
$I \otimes Z, Z \otimes I, Z \otimes Z, X \otimes X, X \otimes Y, Y \otimes X, Y \otimes Y$



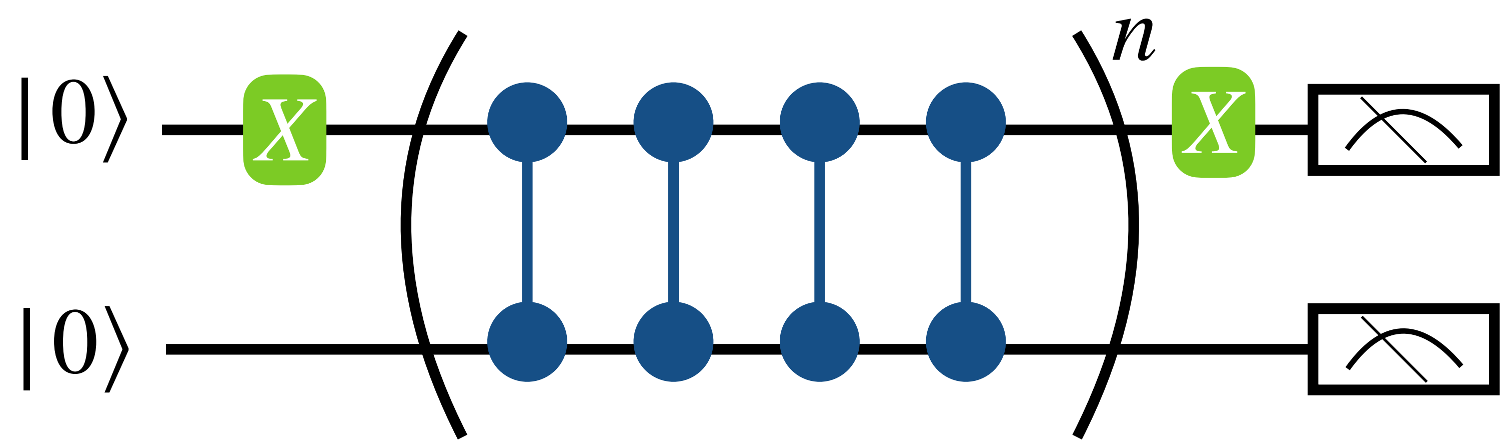
No oscillations reduces the likely errors to
$I \otimes Z, Z \otimes I, Z \otimes Z$



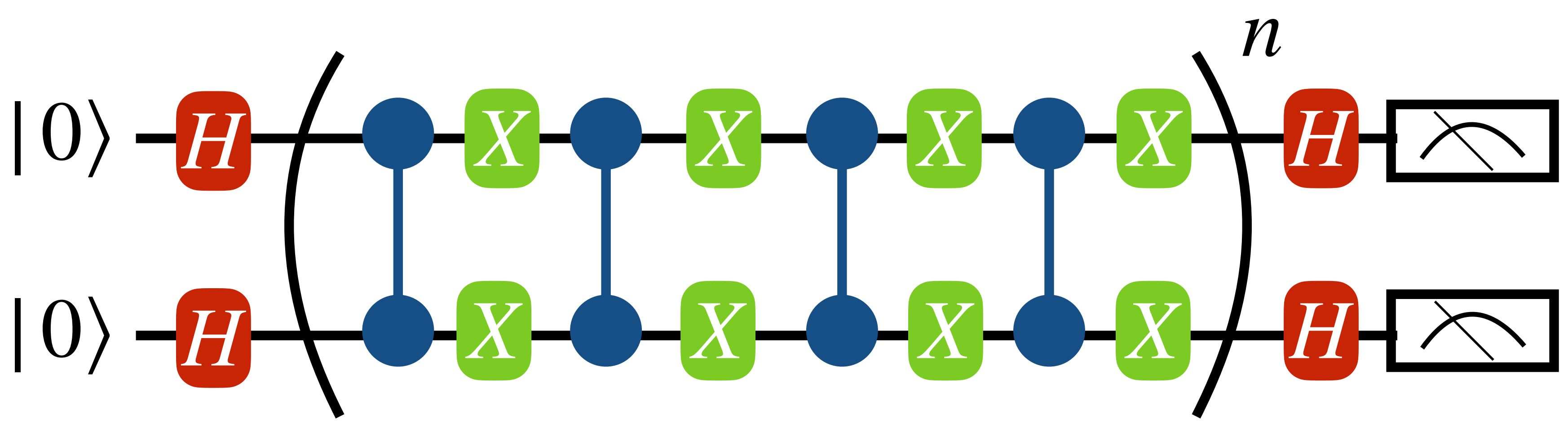
Circuits [With DD]



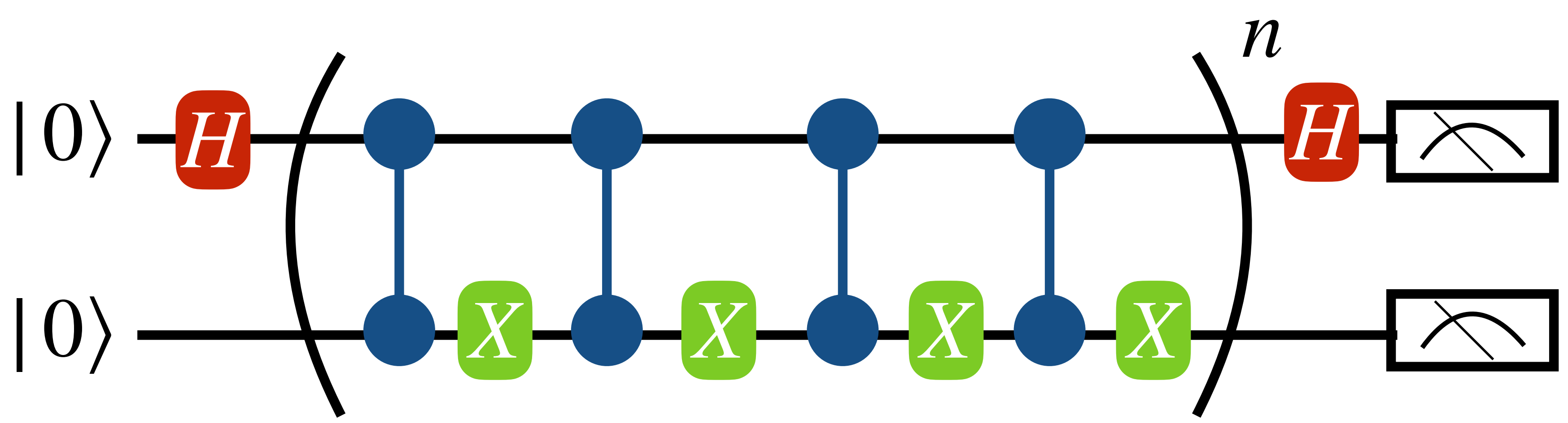
No oscillations reduces the likely errors to
$I \otimes Z, Z \otimes I, Z \otimes Z, X \otimes X, X \otimes Y, Y \otimes X, Y \otimes Y$



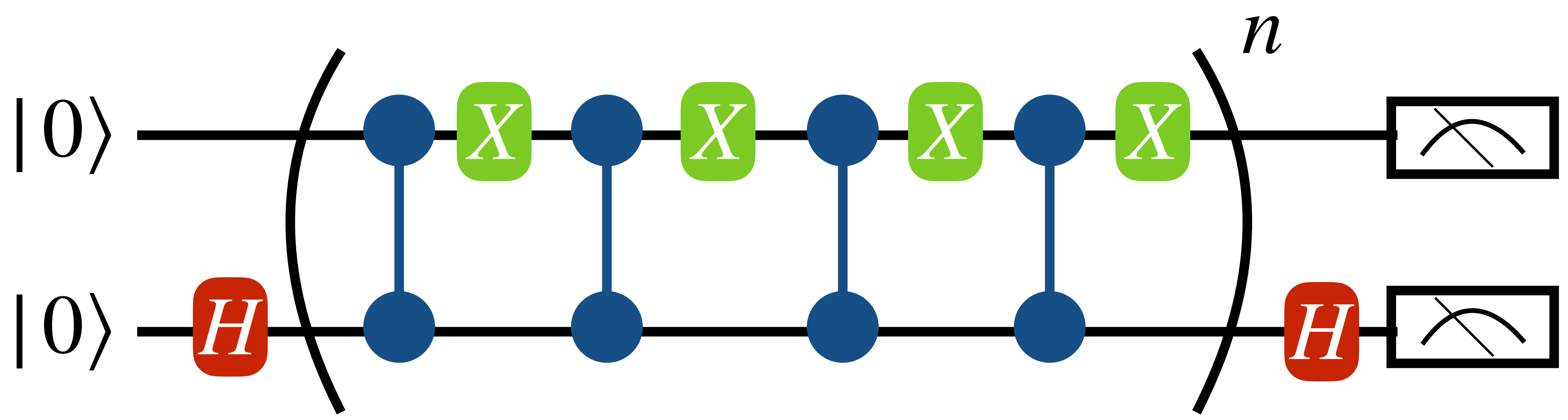
No oscillations reduces the likely errors to
$I \otimes Z, Z \otimes I, Z \otimes Z$



Oscillations frequency determines the error in $Z \otimes Z$

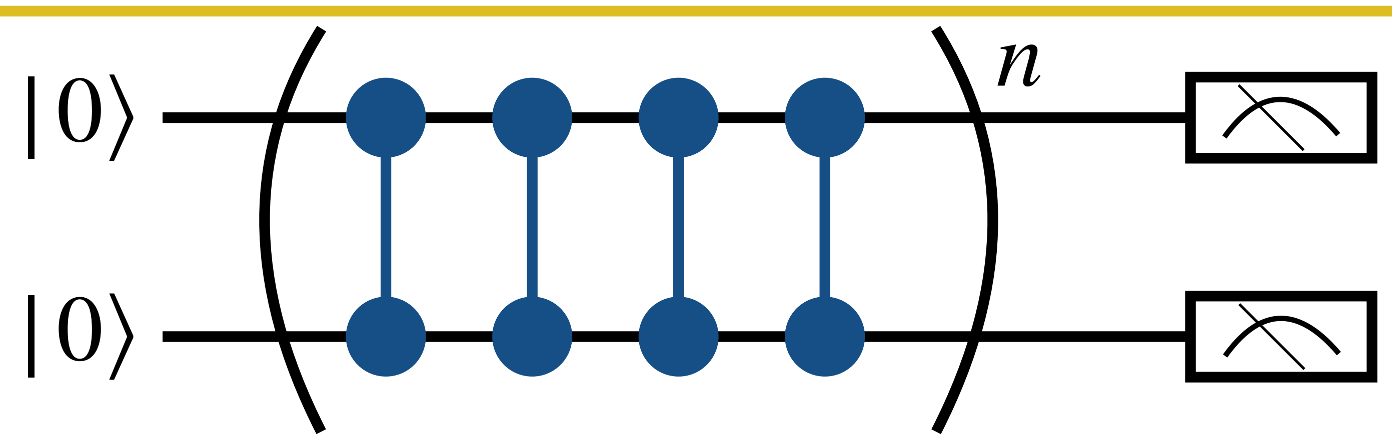


Oscillations frequency determines the error in $Z \otimes I$

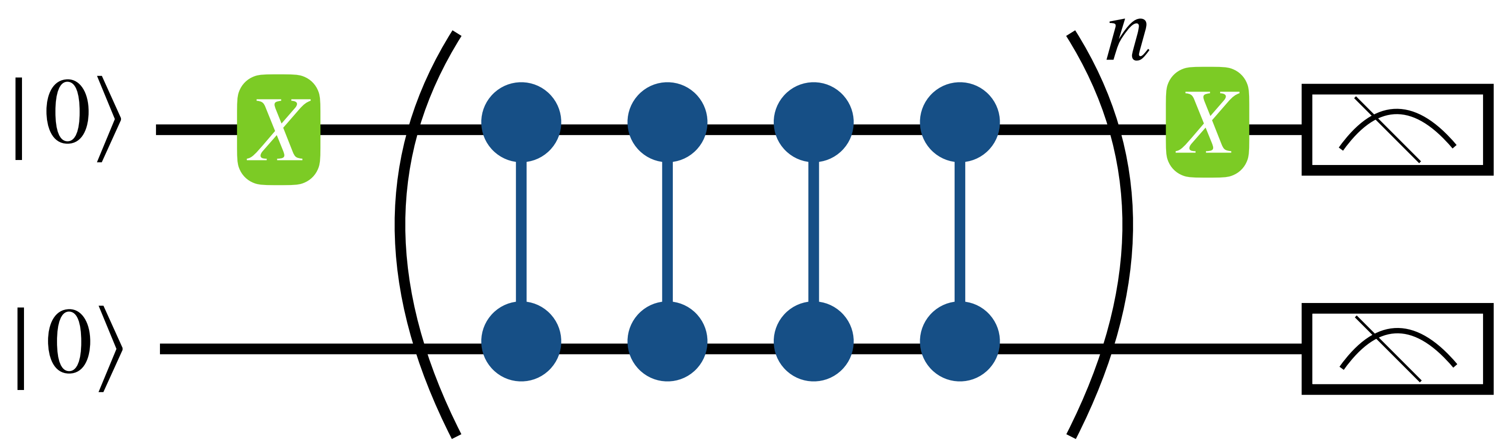


Oscillations frequency determines the error in $I \otimes Z$

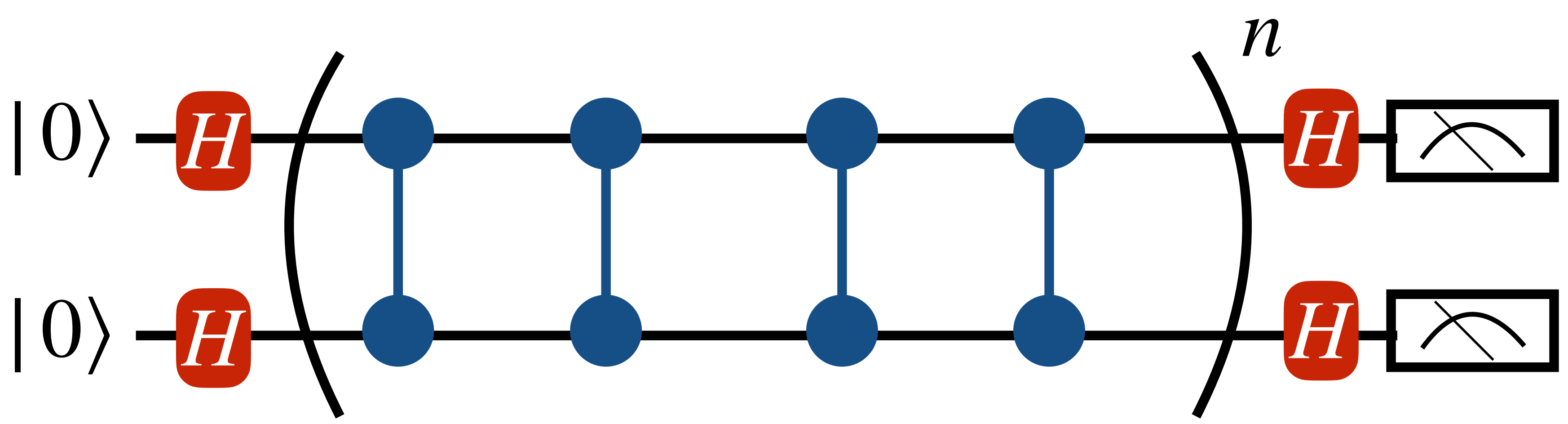
Circuits [No DD]



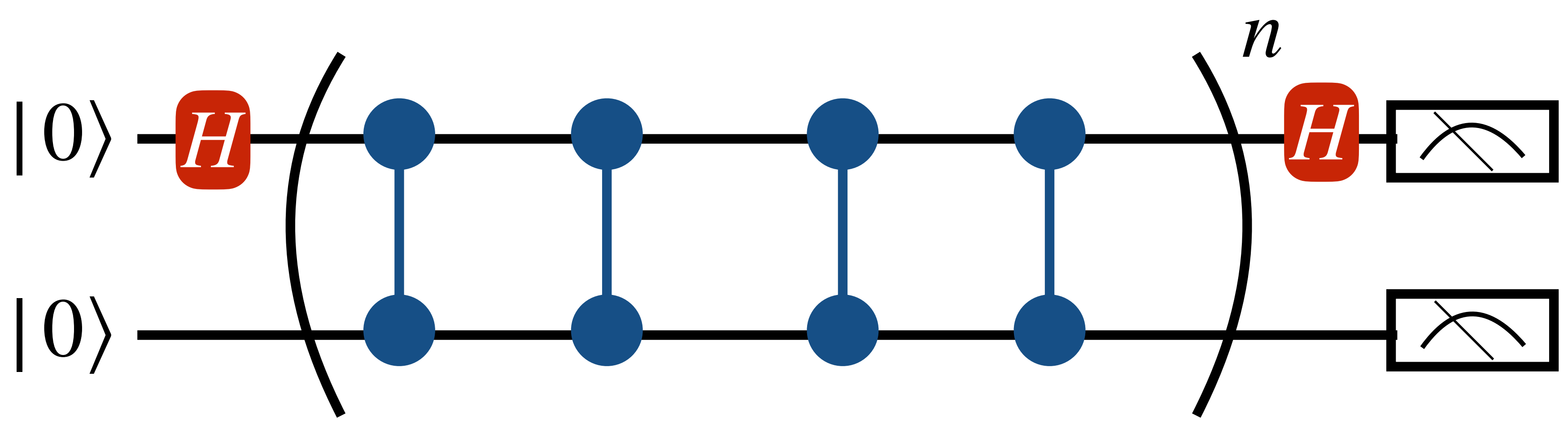
No oscillations reduces the likely errors to
$I \otimes Z, Z \otimes I, Z \otimes Z, X \otimes X, X \otimes Y, Y \otimes X, Y \otimes Y$



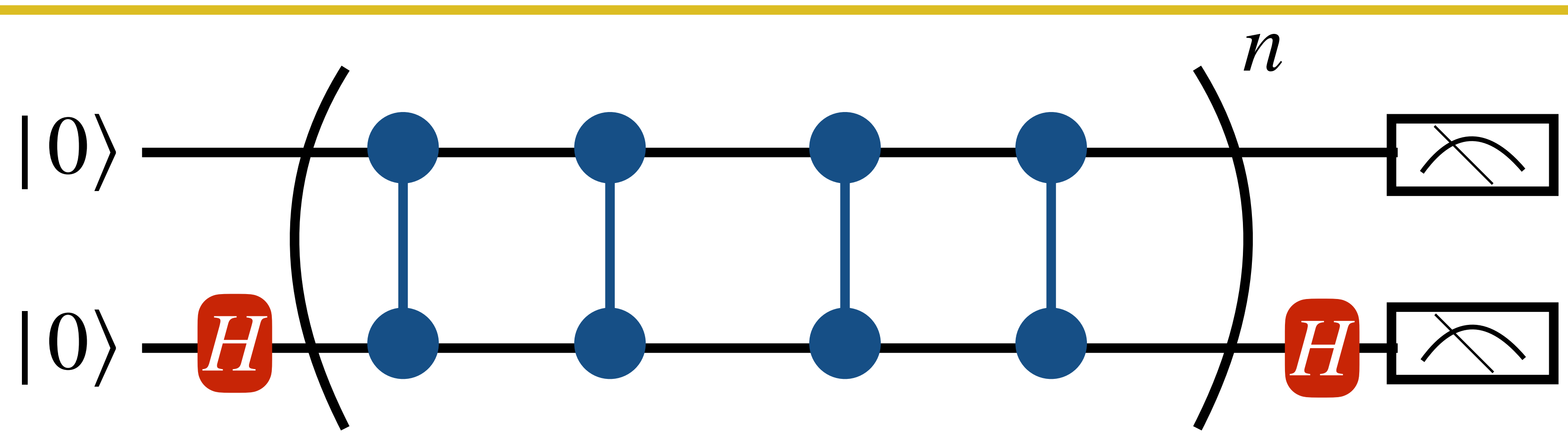
No oscillations reduces the likely errors to
$I \otimes Z, Z \otimes I, Z \otimes Z$



Oscillations frequency determines the error in $Z \otimes Z$



Oscillations frequency determines the error in $Z \otimes I$



Oscillations frequency determines the error in $I \otimes Z$

Analytical expressions for closed systems

Using Circuits [No DD] we can still derive an expression for the fill Hamiltonian. Consider,

$$H_{\mathrm{CZ}} = \frac{\pi}{4}(Z \otimes Z - I \otimes Z - Z \otimes I) + H_{\mathrm{err}},$$

$$H_{\mathrm{err}} = \varepsilon\, Z \otimes I + \eta\, Z \otimes Z + \kappa\, I \otimes Z$$

After even k repetition,

$$U^k = \mathrm{diag}\left(e^{-ik(\varepsilon+\eta+\kappa)}, e^{-ik(\varepsilon-\eta-\kappa)}, e^{+ik(\varepsilon+\eta-\kappa)}, e^{+ik(\varepsilon-\eta+\kappa)}\right)$$

Fidelity of encoding $|++\rangle$, applying U^k, then decoding and reading of $|00\rangle$:

$$A_k = \langle 00 | U_{++}^{\dagger}\; U^k U_{++} | 00\rangle = \frac{1}{4}\left[e^{-ik(\varepsilon+\eta+\kappa)} + e^{-ik(\varepsilon-\eta-\kappa)} + e^{+ik(\varepsilon+\eta-\kappa)} + e^{+ik(\varepsilon-\eta+\kappa)}\right]$$

$$A_k = \cos(k\varepsilon)\cos(k\eta)\cos(k\kappa) + i\sin(k\varepsilon)\sin(k\eta)\sin(k\kappa)$$

$$\rightarrow \mathscr{F} = |A_k|^2 = \cos^2(k\varepsilon)\;\cos^2(k\eta)\;\cos^2(k\kappa) + \sin^2(k\eta)\;\sin^2(k\eta)\;\sin^2(k\kappa)$$

Dynamical Decopuling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$

- The following sequence cancels the effect of the bath:

$$\hat{X} f_\tau \hat{X} f_\tau$$

Dynamical Decopuling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$
- The following sequence cancels the effect of the bath:

$$\hat{X} f_\tau \hat{X} f_\tau = X e^{-i\tau H_{SB}} X e^{-i\tau H_{SB}}$$

Dynamical Decopuling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$

- The following sequence cancels the effect of the bath:

$$\hat{X} f_\tau \hat{X} f_\tau = \boxed{X e^{-i\tau H_{SB}} X} e^{-i\tau H_{SB}}$$

$$U e^{A} U^\dagger = e^{UAU^\dagger}$$

Dynamical Decoupling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$

- The following sequence cancels the effect of the bath:

$$\hat{X} f_\tau \hat{X} f_\tau = \boxed{X e^{-i\tau H_{SB}} X} e^{-i\tau H_{SB}} = \boxed{e^{-i\tau X H_{SB} X}} e^{-i\tau H_{SB}}$$

$$U e^{A} U^\dagger = e^{U A U^\dagger}$$

Dynamical Decoupling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$
- The following sequence cancels the effect of the bath:

$$\hat{X} f_\tau \hat{X} f_\tau = X e^{-i\tau H_{SB}} X e^{-i\tau H_{SB}} = \boxed{e^{-i\tau X H_{SB} X}} e^{-i\tau H_{SB}}$$

$$X\, H_{SB}\, X = X\, Z\, X \otimes B$$

Dynamical Decoupling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$

- The following sequence cancels the effect of the bath:

$$\hat{X}f_\tau\hat{X}f_\tau = Xe^{-i\tau H_{SB}}Xe^{-i\tau H_{SB}} = \boxed{e^{-i\tau XH_{SB}X}}e^{-i\tau H_{SB}}$$

$$X\ H_{SB}\ X = X\ Z\ X \otimes B$$

$$\boxed{\{X, Z\} = 0}$$

Dynamical Decoupling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$

- The following sequence cancels the effect of the bath:

$$\hat{X} f_\tau \hat{X} f_\tau = X e^{-i\tau H_{SB}} X e^{-i\tau H_{SB}} = \boxed{e^{-i\tau X H_{SB} X}} e^{-i\tau H_{SB}}$$

$$X\, H_{SB}\, X = -Z \otimes B$$

$$\boxed{\{X, Z\} = 0}$$

Dynamical Decopuling

- Consider a qubit coupled to a bath, $H = H_Q + H_{SB}$, where $H_{SB} = Z \otimes B$.
- For a given time τ, the system evolves freely under the bath influence as $f_\tau \equiv e^{-i\tau H_{SB}}$
- We can act on the system using $\hat{X} = e^{-i\pi/2\sigma_x} \otimes I_B = -iX \otimes I_B$

- The following sequence cancels the effect of the bath:

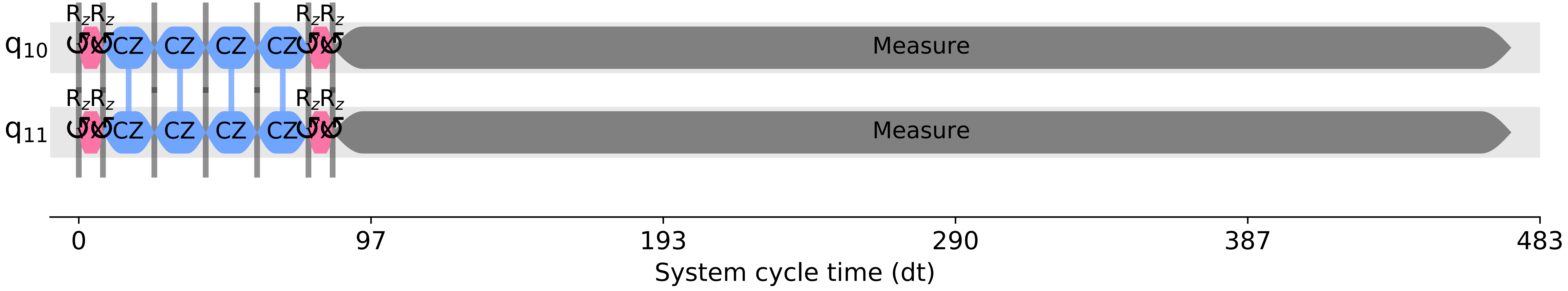
$$\begin{aligned}\hat{X}f_\tau\hat{X}f_\tau = Xe^{-i\tau H_{SB}}Xe^{-i\tau H_{SB}} &= e^{-i\tau XH_{SB}X}e^{-i\tau H_{SB}} \\ &= e^{+i\tau H_{SB}}e^{-i\tau H_{SB}} \\ &= I\end{aligned}$$

Combining with dynamical decoupling

$$H = H_{Q_1} + H_{Q_2} + H_{CZ} + H_{SB}$$

$$H_{CZ} = \frac{\pi}{4}(Z \otimes Z - I \otimes Z - Z \otimes I) + (\varepsilon\, Z \otimes Z - \kappa\, I \otimes Z - \eta\, Z \otimes I)$$

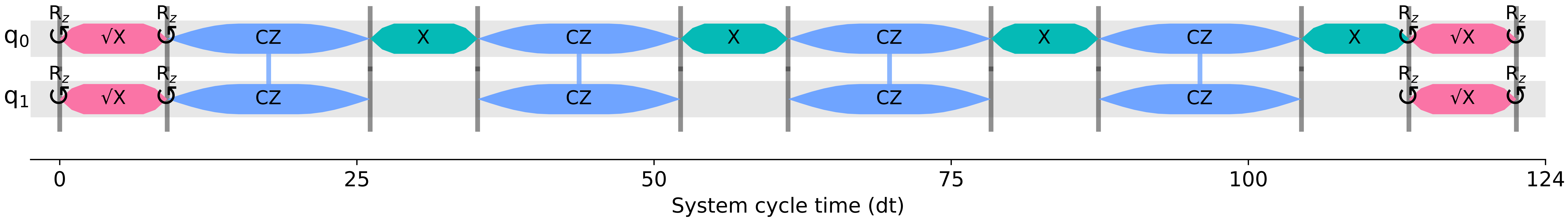
No DD keeps all the terms



Combining with dynamical decoupling

$$H = \frac{\pi}{4}\left(\cancel{Z \otimes Z} - \boxed{I \otimes Z} - \cancel{Z \otimes I}\right) + \left(\varepsilon\, \cancel{Z \otimes Z} - \boxed{\kappa\, I \otimes Z} - \eta\, \cancel{Z \otimes I}\right)$$

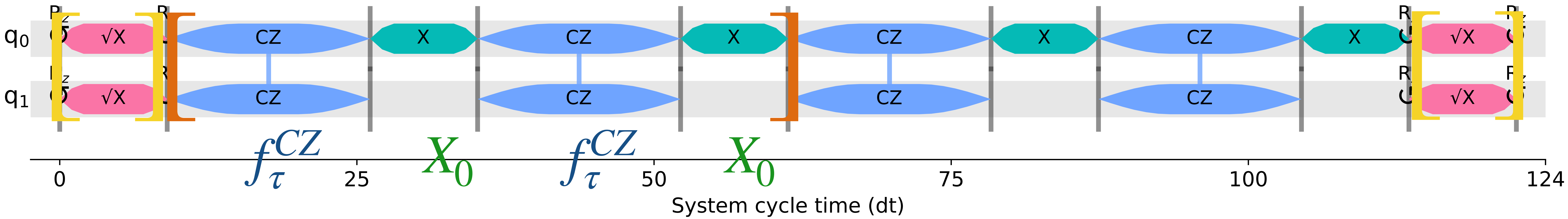
DD on the first qubit



Combining with dynamical decoupling

$$H = \frac{\pi}{4}\left(\cancel{Z \otimes Z} - \boxed{I \otimes Z} - \cancel{Z \otimes I}\right) + \left(\varepsilon\, \cancel{Z \otimes Z} - \boxed{\kappa\, I \otimes Z} - \eta\, \cancel{Z \otimes I}\right)$$

DD on the first qubit

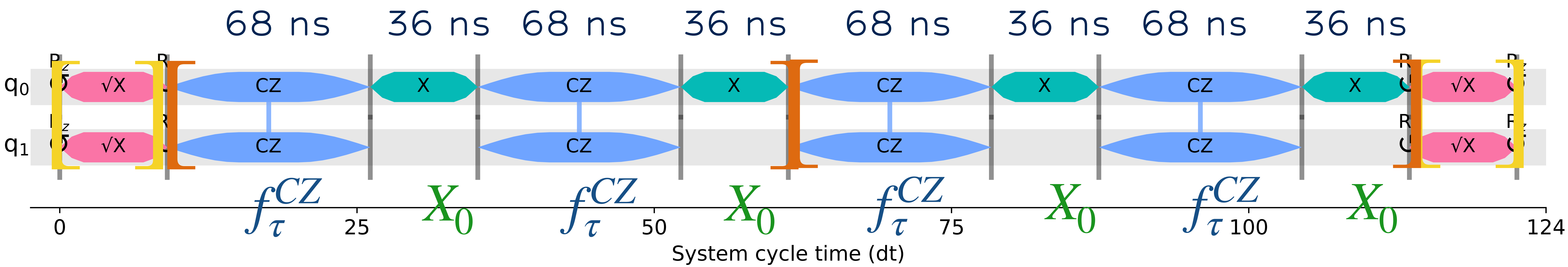


$$f_\tau^{CZ} X_0 f_\tau^{CZ} X_0 = e^{-i\frac{\pi}{4}(-I\otimes Z + \eta\, I \otimes Z)} + \mathcal{O}(\tau^2)$$

Combining with dynamical decoupling

$$H = \frac{\pi}{4}(Z\otimes Z - I\otimes Z - Z\otimes I) + (\varepsilon\, Z\otimes Z - \kappa\, I\otimes Z - \eta\, Z\otimes I)$$

DD on the first qubit



$$f_\tau^{CZ} X_0 f_\tau^{CZ} X_0 = e^{-i\frac{\pi}{4}(-I\otimes Z + \eta\, I\otimes Z)} + \mathcal{O}(\tau^2) \qquad f_\tau^{CZ} X_0 f_\tau^{CZ} X_0 = e^{-i\frac{\pi}{4}(-I\otimes Z + \eta\, I\otimes Z)} + \mathcal{O}(\tau^2)$$

Assumption $\tau^x \ll \tau^{CZ} \ll \tau^{SB}$

Combining with dynamical decoupling

$$H = \frac{\pi}{4}\left(\cancel{Z \otimes Z} - \boxed{I \otimes Z} - \cancel{Z \otimes I}\right) + \left(\varepsilon\, \cancel{Z \otimes Z} - \boxed{\kappa\, I \otimes Z} - \eta\, \cancel{Z \otimes I}\right)$$

DD on the first qubit

$$H = \frac{\pi}{4}\left(\cancel{Z \otimes Z} - \cancel{I \otimes Z} - \boxed{Z \otimes I}\right) + \left(\varepsilon\, \cancel{Z \otimes Z} - \cancel{\kappa\, I \otimes Z} - \boxed{\eta\, Z \otimes I}\right)$$

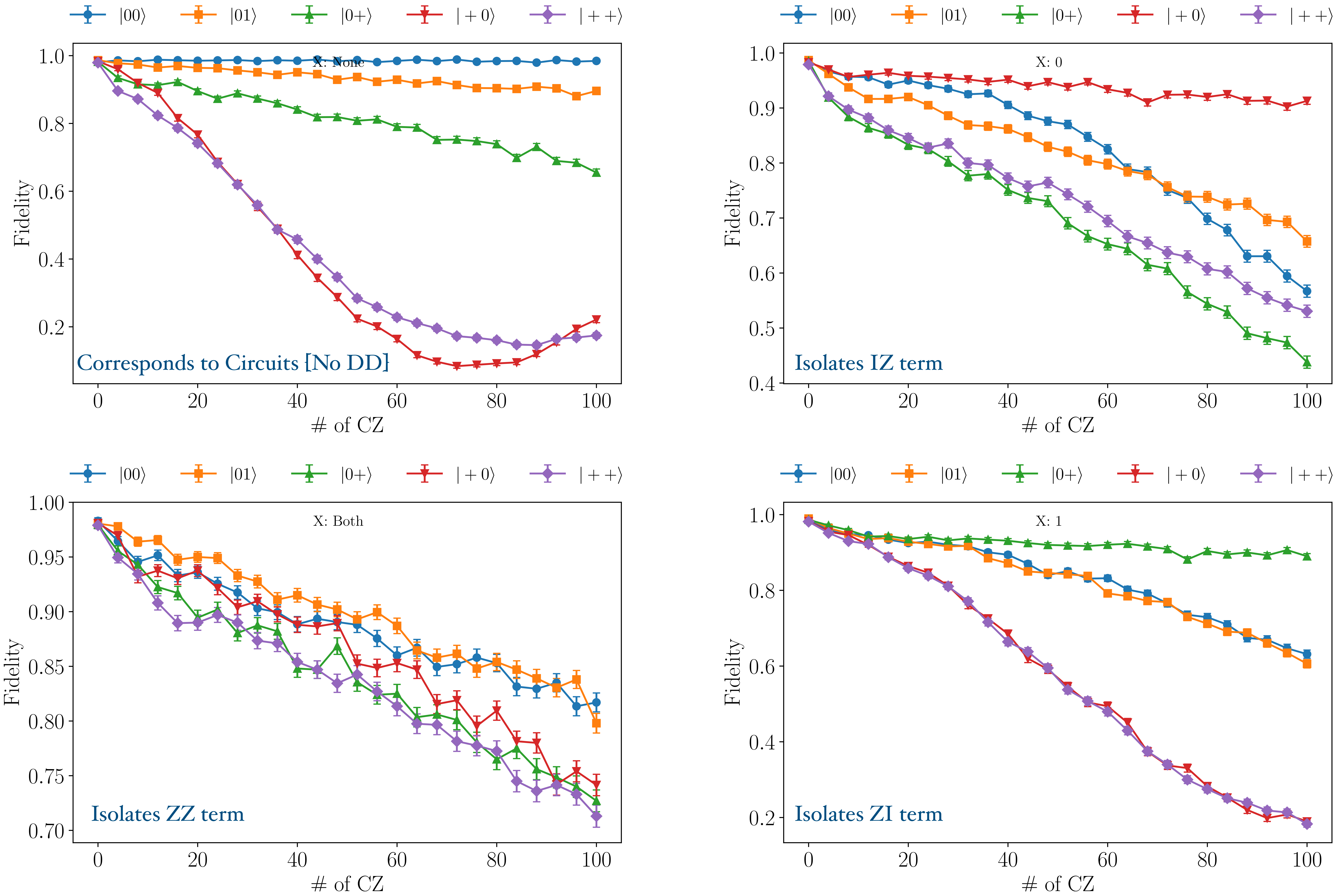
DD on the second qubit

$$H = \frac{\pi}{4}\left(\boxed{Z \otimes Z} - \cancel{I \otimes Z} - \cancel{Z \otimes I}\right) + \left(\boxed{\varepsilon\, Z \otimes Z} - \cancel{\kappa\, I \otimes Z} - \eta\, \cancel{Z \otimes I}\right)$$

Simultaneous DD on both slots

Simultaneous DD on both qubits

Marrakesh Qubits [21, 22]



Single qubit DB