Find the general solution to the following differential equations:

1. 
$$y' + 2y = 0$$

$$r+2=0 - r=-2$$
  $y=ce^{-2t}$ 

2. 
$$y'' - 9y = 0$$
.

$$r^2 = 0 - 9 r = \pm 3$$
  $y = 4 e^{3t} + 6 e^{3t}$ 

3. 
$$y'' + 4y' - 5y = 0$$
.

$$r^{2}+4r-5=0 \rightarrow r=-511$$
  $y=c_{1}e^{-5t}+c_{2}e^{t}$   $(r+5)(r-1)$ 

4. 
$$y'' - y' - 11y = 0$$

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.  
 $r^2 - r - 11 = 0$   $\Rightarrow r = \frac{1}{2} \pm \frac{3\sqrt{5}}{2}$   $y = C_1 e^{(\frac{1}{2} + \frac{3\sqrt{5}}{2})} + C_2 e^{(\frac{1}{2} - \frac{3\sqrt{5}}{2})} + C_3 e^{(\frac{1}{2} - \frac{3\sqrt{5}}{2})} + C_4 e^{(\frac{1}{2} + \frac{3\sqrt{5}}{2})} + C_5 e^{(\frac{1}{2} + \frac{3\sqrt{5}}{2})} + C_6 e^{(\frac{1}{2} + \frac{3\sqrt{5}}{2})$ 

5. 
$$y'' - 4y' + 7y = 0$$
.

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. (2+3i)t (2-3i)t  
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6. 
$$y'' + 7y = 0$$
.

$$y = 0.$$
 $y = c_1 e^{-\sqrt{7}it} + c_2 e^{-\sqrt{7}it} = c_3 \cos(\sqrt{7}t) + c_4 \sin(\sqrt{7}t)$ 

7. Solve the initial value problem: y'' + y' = 0, y(0) = 2, y'(0) = 1.

$$r^{2}+r=0$$
 so general soln:  $y(0) = c_{1}+c_{2}=2$   $y = 3-e^{-t}$   
 $y = c_{1}+c_{2}e^{-t}$   $y'(0) = -c_{2}=1$ 

8. Solve the initial value problem: y'' - 4y' + 4y = 0, y(1) = 1, y'(1) = 1

$$r^{2}$$
 -  $4r + 4 = 0$  | So general soln:  
 $y(1) = (1e^{2} + (2e^{2} = 1))$   
 $y = (1e^{2} + (2e^{2} + 2e^{2} +$ 

9. Solve the initial value problem: 
$$y'' + 9y = 0$$
,  $y(0) = 1$ ,  $y'(0) = 1$ .

$$r^2+9=0$$
 | general soln:  
 $r=\pm i3$  |  $y=C_1\cos(3t)+C_2\sin(3t)$ 

problem: 
$$y'' + 9y = 0$$
,  $y(0) = 1$ ,  $y'(0) = 1$ .

general soln:

 $y = C_1 \cos(3t) + C_2 \sin(3t)$ 
 $y = \frac{1}{3} \sin(3t) + \cos(3t)$ 

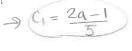
10. Find the general solution to the differential equation 
$$y''' - y'' + y' + 3y = 0$$
. (+ry e<sup>rt</sup> again)

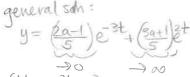
$$r^3 - r^2 + r + 3 = 0$$
  
note  $-1$  is a root  
 $-9$   $r = -1$ ,  $1 \pm i\sqrt{8}$ 

$$r^3 - r^2 + r + 3 = 0$$
 $y = c_1 e^{-t} + c_2 e^{t} \cos(\sqrt{\epsilon}t)$ 
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11. Consider the initial value problem: y'' + y' - 6y = 0, y(0) = a, y'(0) = 1. For what values of a does the solution go to infinity as t goes to infinity? For what values of a does the solution go to zero as t goes to infinity? For what values of a does the solution go to negative infinity as t goes to infinity?

$$y(0) = C_1 + C_2 = a$$
  
 $y'(0) = -3C_1 + 2C_2 = 1$ 





$$5C_2 = 3a + 1$$
 $C_2 = 3a + 1$ 

- general soln:  $y(0) = C_1 + C_2 = a$   $\Rightarrow C_1 = 2a 1$   $\Rightarrow C_2 = 1$   $\Rightarrow C_1 = 2a 1$   $\Rightarrow C_2 = 1$   $\Rightarrow C_1 = 2a 1$   $\Rightarrow C_2 = 1$   $\Rightarrow C_1 = 2a 1$   $\Rightarrow C_2 = 1$   $\Rightarrow C_1 = 2a 1$   $\Rightarrow C_2 = 1$   $\Rightarrow C_2 = 1$   $\Rightarrow C_3 = 2a 1$   $\Rightarrow C_4 = 2a 1$ 
  - (a) If the Wronskian is nonzero at a point  $t_0$ , what does that tell you about the system of equations:

$$\begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}? \quad \text{has ansalution}.$$

- (b) Explain why if the Wronskian is nonzero at a point  $t_0$  then  $y_1$  and  $y_2$  are linearly independent. (From this we see that if the Wronskian is nonzero at a point the two functions are linealy independent.)
- (c) Suppose  $y_1$  and  $y_2$  solve the differential equation y'' + ay' + by = 0. Show that the  $y(t) = c_1y_1(t) + c_2y_2(t)$  also satisfies the differential equation.
- (d) Following part (3), show that if the Wronskian is 0 at a point  $t_0$  then y(t) solves the initial value problem: y'' + ay' + by = 0,  $y(t_0) = 0$ ,  $y'(t_0) = 0$ .
- (e) Explain why this means that y(t) is zero everywhere. Explain why this means that the Wronskian is zero every-
- (f) We have shown that if  $y_1$  and  $y_2$  solve the same ODE and the Wronskian is 0 at a point, then we have constants  $c_1$  and  $c_2$  not both zero such that  $c_1y_1(t) + c_2y_2(t) = 0$  for all t, thus  $y_1$  and  $y_2$  are linearly dependent.

(b) if 
$$y_2(t) = Cy_1(t)$$
 then  $y_2'(t) = Cy_1'(t) \rightarrow W = 0$  everywhere.

(d) I think this question insant that if w=0 then the matrix in (a) has a non-trivial solution, so choosing that circ solves the initial value problem.

(e) Then all higher demuatives are zero too (differentiate the ODE) -> y=0 everywhere