1. rearrange to standard form: $y'' + \frac{1}{2t}y' - \frac{3}{2t^2}y = 0$

$\int p(t)dt = \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}\ln t \rightarrow y_2(t) = \frac{1}{t}\int \frac{t^{-1/2}}{(1/t)^2}dt = \frac{1}{t}\int t^{3/2}dt = \frac{1}{t}\frac{2}{5}t^{5/2}$

$\rightarrow y_2(t) = \frac{2}{5}t^{3/2} \sim t^{3/2}$

drop $\frac{2}{5}$ b/c all multiples of this are also solns by linearity.

+C but we can let C=0 b/c we just need *a* soln, not all of them.

2. $y'' + \frac{2}{t}y' - \frac{2}{t^2}y = 0$

$\int p(t)dt = 2\ln t \longrightarrow y_2(t) = t\int \frac{t^{-2}}{(t)^2}dt = \frac{-1}{3}t^{-2} \sim t^{-2}.$

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1. $y'' = 6t \rightarrow r^2 = 0 \rightarrow y_h(t) = c_1 + c_2 t$

and $y_p(t) = t^2(A_1 t + A_0)$

plug in $y_p \rightarrow A_1 = 1,\ A_0 = 0$ so general soln: $y = c_1 + c_2 t + t^3$

apply initial conditions $\rightarrow$ $y = 3 - t + t^3$

2. $y'' + 9y = 27 \rightarrow r^2 + 9 = 0 \rightarrow y_h(t) = c_1\cos(3t) + c_2\sin(3t)$

$\rightarrow y_p(t) = A_0$

plug in $y_p \rightarrow A_0 = 3 \rightarrow$ general soln: $y(t) = 3 + c_1\cos(3t) + c_2\sin(3t)$

apply IV's $\rightarrow$ $y(t) = 3 + \cos(3t) + 2\sin(3t)$

3. $y'' + y' - 12y = e^t + 2^{2t} - 1 \rightarrow r^2 + r - 12 = 0 \rightarrow r = -4, 3$

So $y_h(t) = C_1 e^{-4t} + C_2 e^{3t}$

$y_p(t) = A_0 e^t + B_0 e^{2t} + C_0$ (superposition principle, see ch 4.5)

plug in to find $A_0 = -1/10$, $B_0 = -1/6$, $C_0 = 1/12$

Apply IC's to general solution to find: $C_1 = 1/60$, $C_2 = 7/6$

$$y(t) = -\frac{e^t}{10} - \frac{e^{2t}}{6} + \frac{1}{12} + \frac{e^{-4t}}{60} + \frac{7e^{3t}}{6}$$

4. $y'' + 2y' + y = t^2 + 1 - e^t \rightarrow r^2 + 2r + 1 = 0 \rightarrow r = -1, -1$

$\rightarrow y_h(t) = C_1 e^{-t} + C_2 t e^{-t}$

$y_p(t) = A_2 t^2 + A_1 t + A_0 + B e^t$

↳ take derivatives and plug it in... get $A_2 = 1$, $A_1 = -4A_2 = -4$

$A_0 = 1 - 2A_1 - 2A_2 = 7$, $B_0 = -1/4$

Satisfy I.C.s and get:

$$y(t) = t^2 - 4t + 7 - \frac{e^t}{4} - \frac{27e^{-t}}{4} - \frac{te^{-t}}{2}$$

5. $y'''' - 5y'' + 4y = 10\cos t - 20\sin t$

$r = i$ is not a root of the auxiliary equation (just try plugging it in) so $y_p = A\cos t + B\sin t$

differentiate y_p 4 times and plug in to get $y(t) = \cos t - 2\sin t$.

use: $[\text{inertia}]y'' + [\text{damping}]y' + [\text{stiffness}]y = F_{ext}$.

6. $y'' + 2y' + 5y = 2\sin 3t + 10\cos 3t$ $y(0) = -1$, $y'(0) = 5$

$\rightarrow y_h(t) = (C_1\cos(2t) + C_2\sin(2t))e^{-t}$, $y_p(t) = A_0\cos 3t + B_0 \sin 3t$

↳ plug in: $A_0 = -1$, $B_0 = 1$

→ apply I.C.s

$$y(t) = -\cos(3t) + \sin(3t) + e^{-t}\sin(2t)$$