Notation: Throughout this note matrices will be denoted by capital letters such as A or B, and vectors by lowercase boldface letters like $\mathbf{x}$ and $\mathbf{b}$.

Problems

1. Write the following system as a matrix equation of the form $A\mathbf{x} = \mathbf{b}$.

$$\begin{array}{rrrcr} 6x & +5y & +2z & = & 11 \\ 5x & +4y & +2z & = & 7 \\ -3x & -3y & -z & = & 4 \end{array} \rightarrow \begin{pmatrix} 6 & 5 & 2 \\ 5 & 4 & 2 \\ -3 & -3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 7 \\ 4 \end{pmatrix}$$

2. Multiply out the following equations of the form $A\mathbf{b}$.

$$\begin{pmatrix} 2+6+2 \\ -4-18-5 \end{pmatrix} = \begin{pmatrix} 10 \\ -27 \end{pmatrix} \leftarrow \text{(a)} \begin{bmatrix} 1 & -1 & 2 & 1 \\ -2 & 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -6 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{(b)} \begin{bmatrix} 2 & -3 \\ 8 & 0 \\ 1 & 3 \\ 9 & -2 \end{bmatrix}\begin{bmatrix} -4 \\ 5 \end{bmatrix} \rightarrow \begin{pmatrix} -8-15 \\ -32 \\ -4+15 \\ -36-10 \end{pmatrix} = \begin{pmatrix} -23 \\ -32 \\ 11 \\ -46 \end{pmatrix}$$

3. We're going to start with a linear system, solve it and then put the result into parametric form (basically like you've already been doing in webwork).

   (a) Write down the augmented matrix and solve this system.

$$\begin{array}{rrrrcr} 2x_1 & +4x_2 & & -4x_4 & = & 10 \\ 3x_1 & +6x_2 & +x_3 & +3x_4 & = & 11 \\ x_1 & +2x_2 & +x_3 & +7x_4 & = & 1 \end{array} \rightarrow \begin{pmatrix} 2 & 4 & 0 & -4 & 10 \\ 3 & 6 & 1 & 3 & 11 \\ 1 & 2 & 1 & 7 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & -2 & 5 \\ 0 & 0 & 1 & 9 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

   (b) Turn the REF matrix you got in part (a) back into a system of equations. Set any free variables equal to 's' or 't'. E.g. if x_4 was free, we would set $x_4 = t$, and then replace all x_4's in the other equations with t's. (Hint at this point you should have 4 equations total).

$x_1 + 2x_2 - 2x_4 = 5$

$x_3 + 9x_4 = -4$

so → $x_4 = t$

$x_2 = s$

   (c) Turn your 4 equations into a vector equation of the form:

writing them out clearly (you don't have to do this step :)

$x_1 = 5 - s2 + t2$

$x_2 = s$

$x_3 = -4 - t9$

$x_4 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -4 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ 0 \\ -9 \\ 1 \end{bmatrix}$$

← these are your 4 equations

$$\left\{ \begin{array}{l} x_1 + 2s - 2t = 5 \\ x_3 + 9t = -4 \\ x_4 = t \\ x_2 = s \end{array} \right.$$

4. For each of the following formula, either explain why it is true[1], or give an example where it's false. Here A is a matrix, $\mathbf{b}$, $\mathbf{v}$ are vectors and c is just a number.

   there is a proof on page 39 of the text but after doing an example you kinda get the intuition.

   T (a) $A(\mathbf{u}+\mathbf{v}) = A\mathbf{u} + A\mathbf{v}$

   F (b) $A = \mathbf{u} + c\mathbf{v}$ for some $\mathbf{u}, \mathbf{v}$ and c

   false because $\bar{u} + c\bar{v}$ will be a vector and have only 1 column

   T (c) $Ac\mathbf{u} = cA\mathbf{b}$

5. For the following, instead of finding the solution set of a linear system, we're looking for a linear system with a given solution set:

   (a) Construct a 3×3 matrix A such that the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{0}$ (here $\mathbf{0}$ means the vector of all zeros).

   (b) Construct a 3×3 matrix A such that vector $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{0}$.

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 5 & -5 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

[1] Ideally to someone sitting next to you