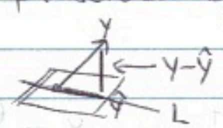
True / False

1. T (by definition)
2. F (the "closest point" definition of the projection explains why this is true)
3. T (by definition)
4. F, $UU^T x$ is the projection of x onto the subspace spanned by the columns of U, but this will not be all of $\mathbb{R}^n$ unless $p=n$.
5. F, the vectors in Row(A) are orthogonal to Null(A), those of Col(A) are orthogonal to LNull(A).
6. T, though note if we had said orthonormal instead this would be false.
7. T
8. T, by construction (hard to explain but easier to see from the examples on pg 356)
9. T (eg $[1,1]$ and $[1,0]$ in $\mathbb{R}^2$)
10. T (b/c Q orthogonal $\Rightarrow Q^TQ = I \Rightarrow Q^TA = Q^TQR = R$)
11. T this is the great thing about orthogonal bases, just project onto them to get the weights
12. F orthogonality is preserved by scaling
13. T (weird question)
14. F, $\|y-\hat{y}\|$ is $\rightarrow$ 

15. T

once you've projected the first time the output is already in the subspace you're projecting into and so is unchanged by further projection.

Problems

1. Note W_2 is given in terms of an orthogonal basis, so it's much easier to compute the projection onto it. We just compute $c_1 = \dfrac{[3,-2,10]\cdot[5,-2,1]}{[5,-2,1]\cdot[5,-2,1]} = \dfrac{29}{30}$

and $C_2 = \dfrac{[3,-2,1]\cdot[1,2,-1]}{[1,2,-1]\cdot[1,2,-1]} = \dfrac{-2}{6}$

so the projection is $\frac{29}{30}[5,-2,1] + \frac{-2}{6}[1,2,-1]$.

2-(a) $-1 = m(-1)+b$
$0 = m(1)+b$
$4 = m(2)+b$

(b) $\underbrace{\begin{pmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}}_{A} \begin{pmatrix} m \\ b \end{pmatrix} = \underbrace{\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}}_{y}$

(c) $\begin{pmatrix} -1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix} = A^TA$

(d) $\frac{1}{14}\begin{pmatrix} 3 & -2 \\ -2 & 6 \end{pmatrix}$ so

$\begin{pmatrix} m \\ b \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 3 & -2 \\ -2 & 6 \end{pmatrix}\begin{pmatrix} -1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

$= \frac{1}{14}\begin{pmatrix} 3 & -2 \\ -2 & 6 \end{pmatrix}\begin{pmatrix} 9 \\ 3 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 21 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 3/2 \\ 0 \end{pmatrix}$

(e) $y = 3/2\, x$

3(a) $v_1 = (3,1,-1,3)^T$, $v_2 = x_2 - \dfrac{(-5,1,5,-7)\cdot(3,1,-1,3)}{(3,1,-1,3)(3,1,-1,3)}(3,1,-1,3)$

$= x_2 - \dfrac{(-40)}{20}(3,1,-1,3) = (1,3,3,-1)$

$v_3 = x_3 - \dfrac{v_2\cdot x_3}{v_2\cdot v_2}v_2 - \dfrac{v_1\cdot x_3}{v_1\cdot v_1}v_1 = (-3,1,1,3)$

(b) $\dfrac{(1,2,3,4)\cdot(3,1,-1,3)}{(3,1,-1,3)\cdot(3,1,-1,3)} = \dfrac{14}{20}$ $\quad \dfrac{(1,2,3,4)\cdot(1,3,3,-1)}{(1,3,3,-1)\cdot(1,3,3,-1)} = \dfrac{12}{20}$

$\dfrac{(1,2,3,4)\cdot(-3,1,1,3)}{(-3,1,1,3)\cdot(-3,1,1,3)} = \dfrac{14}{20}$ $\quad$ so $\bar{x} = \frac{7}{10}\bar{v}_1 + \frac{6}{10}\bar{v}_2 + \frac{7}{10}\bar{v}_3$

3(c) Yes, then we'd probably want to compute the matrix from class $P = A(A^TA)^{-1}A^T$. For the dimensions you can either examine $A(A^TA)^{-1}A^T$ directly, or remember that P sends vectors from $\mathbb{R}^n$ into the column space of A (where n is the length of vectors in the column space of A) so P is $n \times n$.

4. $$\underbrace{\begin{bmatrix} 2 & 1 \\ -3 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 13 \\ -32 \\ 22 \end{bmatrix}}_{b_1}, \quad \underbrace{\begin{bmatrix} 2 & 1 \\ -3 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ -5 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 8 \end{bmatrix}}_{b_2}$$

So if b_i were the least squares solution we'd have $b - b_i$ orthogonal to the column space of A (b/c b_i would be the closest point in the column space to b).

$$b_1 - b = \begin{bmatrix} 8 \\ -36 \\ 18 \end{bmatrix} \text{ dot with } \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix} \text{ to get } 178 \neq 0$$

$$b_2 - b = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \text{ dot with } \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix} \text{ to get } 22 \neq 0$$

→ so no it's not possible.

5. Row reduce to find (a basis for) the row space and null space of A.

$$\begin{bmatrix} 1 & -2 & -4 \\ 2 & -5 & -3 \\ 3 & -7 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -14 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

so the row space has dimension 2, while the null space only has dim = 1 (one free variable)

→ so it's easier to project onto the null space $= \text{Span}\{\underbrace{(14, 5, 1)^T}_{v}\}$

$$\frac{x \cdot v}{v \cdot v}\bar{v} = \frac{214}{222} = \frac{107}{111} \quad \text{also } x - \frac{107}{111}v = w \quad \text{then } \boxed{x = \frac{107}{111}v + w}$$