1. $y'' + y' - 6y = 5t$ try $y_p(t) = At + B$ ← (note $r^2 + r - 6 = 0$ has roots $r = -3, +2$, so set $s = 0$)

→ get $-6At + (A - 6B) = 5t$ so $A = 6B$ and $-6A = 5$

→ so $y_p(t) = -\frac{5}{6}t - \frac{5}{36}$

2. $y'' - 4y' - 12 = 3e^{5t}$ $r^2 - 4r - 12$ has roots $r = -2, 6$

try $y_p(t) = Ae^{5t}$

→ get $-7Ae^{5t} = 3e^{5t}$ → so $y_p(t) = -\frac{3}{7}e^{5t}$

3. $y'' - 4y - 12 = \sin(2t)$ auxiliary equation roots are still $-2, 6$

guess $y_p(t) = A\cos(2t) + B\sin(2t)$

→ end up getting $\left.\begin{aligned} -16A - 8B &= 0 \\ 8A - 16B &= 1 \end{aligned}\right\} \to y_p(t) = \frac{1}{40}\cos(2t) - \frac{1}{20}\sin(2t)$

4. $y'' - 4y - 12 = 2t^3 - t + 3$ again, roots aren't conflicting (eg $s = 0$)

so guess $y_p(t) = At^3 + Bt^2 + Ct + D$

→ get $-12A = 2$

$$\left.\begin{aligned} -12A - 12B &= 0 \\ 6A - 8B - 12C &= -1 \\ 2B - 4C - 12D &= 3 \end{aligned}\right\} \to y_p(t) = -\frac{1}{6}t^3 + \frac{1}{6}t^2 - \frac{1}{9}t - \frac{5}{27}$$

5. $y'' - 4y - 12y = te^{4t}$ → guess $y_p(t) = e^{4t}(At+B)$

→ get $-12A = 1$, $4A - 12B = 0$ → $y_p(t) = -\frac{1}{36}(3t+1)e^{4t}$

6. $y'' - 4t' - 12t = e^{6t}$ recall 6 was a simple root of the auxiliary equation, so guess $y_p(t) = Ate^{6t}$.

→ get $8A = 1$ → $y_p(t) = \frac{t}{8}e^{6t}$

7. $y'' + y' + y/4 = te^{-t/2}$ $r^2 + r + 1/4$ has double root $r = 1/2$

try $y_p(t) = t^2(A+Bt)e^{-t/2}$

→ $\frac{1}{6}e^{-t/2}t^3$

8. (a) $y_p(t) = e^{-7t}(A\cos(10t) + B\sin(10t))$

(b) $y_p(t) = (At^2+Bt+C)\cos t + (Dt^2+Et+F)\sin t$

(c) $y_p(t) = (At+B)e^{-2t}\cos(9t) + (Ct+D)e^{-2t}\sin(9t)$

9. $y_p(t) = -\frac{3}{7}e^{5t} + 2\left(\frac{1}{40}\cos(2t) - \frac{1}{20}\sin(2t)\right) + 4\left(-\frac{1}{36}(3t+1)e^{4t}\right)$

10. $y_p(t) = -\frac{5}{6}t - \frac{5}{36} + \frac{4}{5}te^{2t}$ (solve $y'' + y' - 6y = 4e^{2t}$ and reuse soln to Q1)

11. reuse question 2 to get general soln: $y(t) = C_1e^{-2t} + C_2e^{6t} - \frac{3}{7}e^{5t}$

→ apply initial conditions: $y(t) = 2e^{-2t} + e^{6t} - 3/7\,e^{5t}$

12. get general soln: $y = C_1e^{-3t} + C_2e^{2t} + \frac{5}{14}e^{4t}$ → really gross algebra...

$y = \frac{5}{14}e^{4t} + \left(\frac{12}{35} - \frac{\sqrt{21}}{10}\right)e^{-3t} + \left(\frac{-1}{5} + \frac{\sqrt{21}}{10}\right)e^{2t}$