Math 54 Worksheet **#3**

Notation: Throughout this note matrices will be denoted by capital letters such as A or B, and vectors by lowercase boldface letters like $\mathbf{x}$ and $\mathbf{b}$.

Problems

1. Write the following system as a matrix equation of the form A$\mathbf{x} = \mathbf{b}$.

$$\begin{array}{rrrcr} 6x & +5y & +2z & = & 11 \\ 5x & +4y & +2z & = & 7 \\ -3x & -3y & -z & = & 4 \end{array}$$

2. Multiply out the following equations of the form $A\mathbf{b}$.

$$\text{(a)} \begin{bmatrix} 1 & -1 & 2 & 1 \\ -2 & 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ 1 \\ 0 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 2 & -3 \\ 8 & 0 \\ 1 & 3 \\ 9 & -2 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

3. We're going to start with a linear system, solve it and then put the result into parametric form (basically like you've already been doing in webwork).

   (a) Write down the augmented matrix and solve this system.

$$\begin{array}{rrrrcr} 2x_1 & +4x_2 & & -4x_4 & = & 10 \\ 3x_1 & +6x_2 & +x_3 & +3x_4 & = & 11 \\ x_1 & +2x_2 & +x_3 & +7x_4 & = & 1 \end{array}$$

   (b) Turn the REF matrix you got in part (a) back into a system of equations. Set any free variables equal to 's' or 't'. E.g. if x_4 was free, we would set $x_4 = t$, and then replace all x_4's in the other equations with t's. (Hint at this point you should have 4 equations total).

   (c) Turn your 4 equations into a vector equation of the form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} + s \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} + t \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

4. For each of the following formula, either explain why it is true[1], or give an example where it's false. Here A is a matrix, $\mathbf{b}$, $\mathbf{v}$ are vectors and c is just a number.
   (a) $A(\mathbf{u}+\mathbf{v}) = A\mathbf{u} + A\mathbf{v}$
   (b) $A = \mathbf{u} + c\mathbf{v}$ for some $\mathbf{u}, \mathbf{v}$ and c
   (c) $Ac\mathbf{u} = cA\mathbf{b}$
5. For the following, instead of finding the solution set of a linear system, we're looking for a linear system with a given solution set:
   (a) Construct a 3×3 matrix A such that the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{0}$ (here $\mathbf{0}$ means the vector of all zeros).
   (b) Construct a 3×3 matrix A such that vector $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{0}$.

[1] Ideally to someone sitting next to you