1 Half review half new true / false

Write T or F next to each (if true explain why, if false think of a counter example)

1. If A is invertible and 1 is an eigenvalue of A then 1 is also an eigenvalue of A^{-1}.
2. The normal equations for a least squares solution of $A\mathbf{x} = \mathbf{b}$ are given by $\widehat{\mathbf{x}} = (A^TA)^{-1}A^T\mathbf{b}$.
3. If A is a row equivalent to the identity matrix then A is diagonalizable.
4. A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is the vector $A\widehat{\mathbf{x}}$ in $Col(A)$ closest to $\mathbf{b}$, so that $||b - A\widehat{\mathbf{x}}|| \leq ||b - A\mathbf{x}||$ for all $\mathbf{x}$.
5. If A contains a row or column of zeros then 0 is an eigenvalue of A.
6. If W is a subspace, then $||proj_W\mathbf{v}||^2 + ||\mathbf{v} - proj_W\mathbf{v}||^2 = ||\mathbf{v}||^2$.
7. Each eigenvalue of A is also an eigenvalue of A^2.
8. If a square matrix has orthonormal columns then it also has orthonormal rows.
9. Each eigenvector of A is also an eigenvector of A^2.
10. A square matrix with orthogonal columns is an orthogonal matrix.
11. Each eigenvector of an invertible matrix A is also an eigenvector of A^{-1}.
12. If a matrix U has orthonormal columns then $UU^T = I$.
13. Eigenvalues and eigenvectors must be non-zero.
14. If W is a subspace of $\mathbb{R}^n$ then W and $W^\perp$ have no vectors in common.
15. Two eigenvectors corresponding to the same eigenvalue will be linearly dependent.
16. The subset of all vectors in $\mathbb{R}^n$ of all vectors orthogonal to one fixed vector is a subspace in $\mathbb{R}^n$.
17. Similar matrices always have exactly the same eigenvalues.

18. If a vector $\mathbf{y}$ coincides its orthogonal projection into a subspace W then $\mathbf{y}$ is in $\mathbf{W}$.
19. Similar matrices always have exactly the same eigenvectors.
20. The orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$ is a scalar multiple of $\mathbf{y}$.
21. The sum of two eigenvectors of a matrix A is also an eigenvector of A.
22. If $||\mathbf{u}+\mathbf{v}||^2 = ||\mathbf{u}||^2 + ||\mathbf{v}||^2$ then $\mathbf{u} \perp \mathbf{v}$.
23. The eigenvalues of an upper triangular matrix A are exactly the non-zero entries on the diagonal of A.
24. If $||\mathbf{u}-\mathbf{v}||^2 = ||\mathbf{u}||^2 - ||\mathbf{v}||^2$ then $\mathbf{u} \perp \mathbf{v}$.
25. The matrices A, A^T have the same eigenvalues, counting multiplicities.
26. If two vectors are orthogonal then they must be linearly independent.
27. If a 5×5 matrix A has fewer than 5 distinct eigenvalues then A is not diagonalizable.
28. If r is any scalar then $||r\mathbf{x}|| = r||\mathbf{x}||$.
29. There exists a 2×2 matrix A with no eigenvectors in $\mathbb{R}^2$.
30. The length of every vector is a positive number.
31. If A is diagonalizable then the columns of A are linearly independent.
32. A nonzero vector cannot correspond to two different eigenvalues of A.
33. A (square) matrix is invertible if and only if there is a coordinate system where the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is represented by a diagonal matrix.
34. If each vector $\mathbf{e}_j$ in the standard basis is an eigenvector of A then A is diagonal.
35. If A is similar to a diagonalizable matrix B then A is diagonalizable.
36. If A, B are invertible $n \times n$ matrices then AB is similar to BA.
37. An $n \times n$ matrix with n linearly independent eigenvectors is invertible.
38. If A is an $n \times n$ diagonalizable matrix then each vector in $\mathbb{R}^n$ can be written as a linear combination of eigenvectors of A.