1 Second Order Variable Coefficient Equations

We've previously seen how to solve second order constant coefficient equations ($ay'' + by' + cy = g(t)$) and first order variable coefficient equations ($y' + u(t)y = w(t)$). But what if we wanted to experience the joys of both at once? Enter: second order variable coefficient equations. I.e. equations of the form:

$$y'' + p(t)y' + q(t)y = 0.$$

Turns out, these equations are so hard there isn't even an algorithm to solve them (unlike the previous two cases above). However if we have one solution (call it $y_1(t)$) there's a formula to find a second, $y_2(t)$:

$$y_2(t) = y_1(t) \int \frac{e^{-\int p(t)dt}}{y_1(t)^2} dt.$$

Let's do some examples. Given $y_1(t)$, use the above formula to find $y_2(t)$. (hints: divide through by anything on the y'' if you have to, to get your equation into the right form. Also you can make all the $+C$'s that you get from integration zero because we're just looking for *a* second solution, not all second solutions).

1. $2t^2y'' + ty' - 3y = 0$, $y_1(t) = 1/t$.

2. $t^2y'' + 2ty' - 2y = 0$, $y_1(t) = t$.

2 Miscellaneous ODE Review

1. Solve the initial value problem: $y'' = 6t$, $y(0) = 3$, $y'(0) = -1$.

2. Solve the initial value problem: $y'' + 9y = 27$, $y(0) = 4$, $y'(0) = 6$.

3. Solve the initial value problem: $y'' + y' - 12y = e^t + e^{2t} - 1$, $y(0) = 1$, $y'(0) = 3$.

4. Solve the initial value problem: $y'' + 2y' + y = t^2 + 1 - e^t$, $y(0) = 0$, $y'(0) = 2$.

5. Find a particular solution to the following higher-order equation: $y'''' - 5y'' + 4y = 10\cos t - 20\sin t$.

6. A mass spring system is driven by the external force $g(t) = 2\sin(3t) + 10\cos(3t)$. The mass equals 1, the spring constant equals 5 and the dampening coefficient equals 2. If the mass is initially located at $y(0) = -1$, with initial velocity $y'(0) = 5$, find its equation of motion.