Math 54 Worksheet #6

Definitions and Notation

1. if $A = (a_{ij})$ is the matrix $\left(\begin{smallmatrix} 1 & 3 \\ 12 & -3 \end{smallmatrix}\right)$ then what is the value $a_{1,2}$?
2. What is $\begin{bmatrix} 13 & 4 \\ 0 & -2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 7 \\ -3 & -1 \\ 1 & 1 \end{bmatrix}$? How about $\begin{bmatrix} 13 & 4 \\ 0 & -2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 7 & -9 \\ -3 & -1 & 2 \end{bmatrix}$?
3. Give an example of a diagonal matrix. Why is multiplying diagonal matrices easy?
4. Is it true that $A + B = B + A$ for $n \times n$ matrices A and B? Is it true that $AB = BA$? If $AC = BC$, it it always true that $A = B$?

Problems

1. Suppose that Math 54 is being taught by two different professors. Prof. As lecture is more popular than Prof. Bs lecture. In fact, each week 90% of As students remain in the lecture, while only 10% switch into Bs lecture. On the other hand, 20% of Bs students switch into As lecture, with 80% remaining in Bs section.

   This situation is described in the following table:

	from A	from B
into A	90%	20%
into B	10%	80%

   which can be represented by the matrix $\begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix}$.

   Supposing that at the start of the semester each professor had 200 students, use matrix multiplication to answer the following:
   (a) How many students are there in each professors section after the 1^{st} week? (Hint: represent the number of students in each section by a 2×1 column matrix.)
   (b) How many students are there in each professors section after the second week of classes?

2. Let $A = \begin{bmatrix} -2 & 1 & -2 \\ 1 & 0 & 2 \\ 3 & -3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \\ -1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$ Which of the following matrix multiplications are defined? Compute those which are defined.
   (a) AB (b) BC (c) CA (d) ABC

3. Let A and B be $n \times n$ matrices. Under what conditions is it true that $(A+B)(A-B) = A^2 - B^2$?

4. (a) What special property does the matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ possess?
   (b) Given a 2×2 matrix A, can you always find another matrix B so that $AB = I$?
   (c) Given two 2×2 matrices A and B such that $AB = I$, is there anything noteworthy about BA?

5. Compute the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

6. Without doing any row reduction, determine if the following matrices are invertible: (a) $\begin{bmatrix} 2 & 1 & -3 & 1 \\ 0 & 5 & 4 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 1 & 4 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

7. There are many equivalent conditions for when a matrix is invertible. One you have probably already seen is that a matrix is invertible whenever you can row reduce it to the identity matrix. But there are others. Give a condition for when an $n \times n$ matrix A is invertible in terms of: (a) the pivots of A, (b) the linear transformation (e.g. is it onto?), (c) the columns of A.

8. Compute the inverse of the following matrices, if they exist.

   (a) $\begin{bmatrix} 12 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 & 1 & 1 \\ 20 & 5 & 4 & 1 \\ -7 & -2 & -1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & -1 & -4 \\ 5 & 2 & 10 \\ 3 & 1 & 6 \end{bmatrix}$