1. T $A\bar{v} = \bar{v}$ so $\bar{v} = A^{-1}\bar{v}$
2. F T if the columns of A are linearly indep so that A^TA is invertible
3. F eg $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ is not diagonalizable (notice that this is always my counter example, it's always a good one to check for T/F questions!)
4. T
5. T then $\det A = 0 \Rightarrow$ non-trivial null space
6. T b/c $\mathrm{proj}_W \bar{v} \perp \bar{v} - \mathrm{proj}_W \bar{v}$
7. F if $A\bar{v} = \lambda\bar{v}$ then $A^2\bar{v} = \lambda^2\bar{v} \rightarrow$ so λ^2 is an eigenvalue
eg consider $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
8. T b/c A has orthonormal columns, $A^TA = I$, b/c A is square and has a left inverse A is actually invertible so $AA^T = I$ too $\rightarrow (A^T)^T(A^T) = I$
so A^T is orthogonal
9. T
10. F the columns need to be normalized too
eg $\begin{smallmatrix} 2 & 0 \\ 0 & 2 \end{smallmatrix}$ is not orthogonal
11. T Start with Av = cv for some eigenvalue c, and multiply by A inverse. Note that c cannot be zero else A wouldn't be invertible.
12. F $U^TU = I$ but UU^T is the projection matrix onto U's columns (provided its columns are orthogonal)
13. F eigenvectors must be non-zero, $\lambda = 0$ is allowed.
14. F $\bar{0}$ is in both (but that's it!)
15. F eg $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
16. T Check the 3 axioms! ($\bar{0}$, closed under addition and scalar mult)
17. T Write $A = PBP^{-1}$ and subtract λI...
18. T this question was trying to be tricky by saying coincides instead of equals...
19. F if $A = PBP^{-1}$ then A's eigenvectors are transformed by P^{-1} into eigenvectors of B.

20. F it's a scalar multiple of u

21. F T if they have the same eigenvalue (why?)

22. T b/c in general $\|u+v\|^2 = \|u\|^2 + \|v\|^2 + 2u \cdot v$ (see ch 6.1 after ex. 5)

23. F this is only false b/c they said non-zero. Don't know why they did that..

24. F $\|u-v\|^2 = \|u\|^2 + \|v\|^2 + 2uv$ so if it equalled $\|u\|^2 - \|v\|^2$ we'd have $\|v\|^2 + 2u \cdot v = -\|v\|^2$
$\rightarrow 2\|v\|^2 + 2u \cdot v = 0 \rightarrow 2v \cdot v + 2u \cdot v = 0$
$\rightarrow 2(v+u) \cdot v = 0 \rightarrow v+u \perp v$, not quite $u \perp v$.

25. T they have the same char. poly. b/c
$\det((A-\lambda I)) = \det((A-\lambda I)^T) = \det(A^T - \lambda I)$.

26. T

27. F multiplicities could save you

28. T $\|r\bar{x}\| = \sqrt{r\bar{x} \cdot r\bar{x}} = r\sqrt{\bar{x} \cdot \bar{x}} = r\|\bar{x}\|$

29. T Just pick a char. poly. that has complex roots.

30. T

31. F eg the zero matrix

32. T

33. F this is the definition of diagonalizability!

34. T then $I = P$ in $A = PDP^{-1}$

35. T $A = SBS^{-1}$, $B = PDP^{-1} \rightarrow A = SPDP^{-1}S^{-1} = (SP)D(SP)^{-1}$

36. T this one's tricky. AB invertible so $AB(AA^{-1}) = AB$ so $A(BA)A^{-1} = AB$ shows $BA \sim AB$.

37. F eg the zero matrix again

38. T diagonalizable = there exists a basis of eigenvectors.