Math 54 Worksheet **#2**

Questions

false, this is the coefficient matrix

1. True or False: The augmented matrix for the system

$$\begin{array}{rrrrcr} 2x_1 & -3x_2 & +x_3 & -x_4 & = & 4 \\ 3x_1 & +2x_2 & +x_3 & -3x_4 & = & 1 \\ 5x_1 & +x_2 & -x_3 & +x_4 & = & 3 \\ 2x_1 & -5x_2 & +4x_3 & -6x_4 & = & 6 \end{array} \quad \text{is} \quad \begin{bmatrix} 2 & -3 & 1 & -1 \\ 3 & 2 & 1 & -3 \\ 5 & 1 & -1 & 1 \\ 2 & -5 & 4 & -6 \end{bmatrix}$$

2. (a) Identify the first pivot of the matrix

$$(a) \rightarrow \begin{bmatrix} 1 & -1 & 2 & 1 \\ -2 & 3 & -5 & 0 \\ -1 & 2 & -1 & 0 \\ 1 & 0 & -1 & 3 \end{bmatrix}$$

you are free to choose another value and move it to the pivot position if you like.

   (b) If that pivot in part (a) was not in the first row, interchange rows so that it is.

   (c) Now add suitable multiples of the first row to the other rows to make all other entries in the first column zero.

   (d) Ignoring the first row, find the next pivot and repeat steps (b) and (c) on the second column.

   (e) Continue until all of the rows that contain only zeros are at the bottom of the matrix and each pivot appears to the right of all the pivots above it.

get an echelon form (so not unique!) similar to mine:

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Problems

1. Write down the augmented matrix for the given systems of equations and then reduce to row echelon form.

$$\begin{array}{rrrrcr} x_1 & +2x_2 & & -x_4 & = & -1 \\ -x_1 & -3x_2 & +x_3 & +2x_4 & = & 3 \\ x_1 & -x_2 & +3x_3 & +x_4 & = & 1 \\ 2x_1 & -3x_2 & +7x_3 & +3x_4 & = & 4 \end{array} \qquad \begin{array}{rrrcr} x_1 & +2x_2 & -x_3 & = & 9 \\ 2x_1 & -x_2 & +x_3 & = & 0 \\ 3x_1 & -2x_2 & +4x_3 & = & 0 \\ 4x_1 & -x_2 & +x_3 & = & 4 \end{array}$$

REF:

$$\begin{matrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$\mapsto$ REF

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

2. Suppose you accept a software maintenance job in which you make 80 a day for each day you show up to work, but are penalized 20 per day that you dont

X = days worked
Y = days skipped

$$\left.\begin{array}{l} 80x - 20y = 2200 \\ x + y = 60 \end{array}\right] \rightarrow \begin{array}{l} x = 34 \\ y = 26 \end{array}$$

go to work. After 60 days you find youve earned 2200. How many days have you gone to work? (Assume that you were expected to work during each of the 60 days.) You may wish to set up a system of two linear equations and solve it.

3. Find a linear system in 3 variables, or show that none exists, which:
(a) has the unique solution x = 2, y = 3, z = 4.
(b) has infinitely many solutions, including x = 2, y = 3, z = 4

$$\text{(a)}\ \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix} \qquad \text{(b)}\ \begin{matrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{matrix}$$

4. As you know, two points determine a line. But what does this mean? The equation of a line is $ax + by + c = 0$. Use a linear system to find an equation of the line through the points $(-1, 1)$ and $(2, 0)$. Check your answer. How can two equations determine the three unknowns a, b, and c?

So you can rewrite this $y = -\frac{a}{b}x - \frac{c}{b}$ and solve for the slope $(-a/b)$ and y-intercept $-\frac{c}{b}$ the usual way.
This will give you values for $-a/b$, and $-c/b$, but not a, b or c!
we have a free variable. to get values you can just pick a value for b and then solve for a, c.
Note this means there are really infinitely many equations of this one line.
$y = mx + b$ is just the standard form.

Additional Problems

soln : $\lambda = 4, 2$

1. For which values of λ does the system

$$\begin{aligned} (\lambda - 3)x \quad &+ y &= 0 \\ x \quad &+ (\lambda - 3)y &= 0 \end{aligned}$$

have more than one solution?

There are a couple ways of seeing this either that both lines need to be the same or the bottom row of the REF matrix needs to be 0. → either way get that the bottom row is a times the top row for some a. → solve for λ.

2. Suppose that the system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= 0 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= 0 \end{aligned}$$

has only $x1 = x2 = x3 = 0$ as a solution (the trivial solution). Then consider the system obtained from the given system by replacing the three zeros on the right with three 1s.
(a) Must this new system have a solution or is it possible that the solution set of the new system is empty?
(b) Might the new system have more than one solution?

because the original system had only the trivial solution you know the REF matrix has no free variables. So number of pivots = number of variables = number of columns.
Looks like $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$

If you replaced the right column with 1's and did row reduction you will end up with something like
$\left(\begin{array}{ccc|c} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{array}\right)$ where $*$ may or may not be 0.
→ has a unique solution!