

1. T $A\bar{v} = \bar{v}$ so $\bar{v} = A^{-1}\bar{v}$
2. F T if the columns of A are linearly indep so that $A^T A$ is invertible
3. F eg $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ is not diagonalizable (notice that this is always my counter example, it's always a good one to check for T/F questions!)
4. T
5. T then $\det A = 0 \Rightarrow$ non-trivial null space
6. T blc $\text{proj}_{W^\perp} \bar{v} \perp \bar{v} - \text{proj}_{W^\perp} \bar{v}$
7. F if $A\bar{v} = \lambda\bar{v}$ then $A^2\bar{v} = \lambda^2\bar{v} \rightarrow$ so λ^2 is an eigenvalue
eg consider $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
8. T blc A has orthonormal columns, $A^T A = I$, blc
A is square and has a left inverse A is actually
invertible so $A A^T = I$ too $\rightarrow (A^T)^T (A^T) = I$
so A^T is orthogonal
9. T
10. F the columns need to be normalized too
eg $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ is not orthogonal.
11. T Start with $Av = cv$ for some eigenvalue c, and multiply by A inverse.
Note that c cannot be zero else A wouldn't be invertible.
12. F $U^T U = I$ but $U U^T$ is the projection matrix onto U's columns (provided its columns are orthogonal)
13. F eigenvalues must be non-zero, $\lambda = 0$ is allowed.
14. F $\vec{0}$ is in both (but that's it!)
15. F eg $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
16. T Check the 3 axioms! ($\vec{0}$, closed under addition and scalar mult)
17. T Write $A = PBP^{-1}$ and subtract λI ...
18. T this question was trying to be tricky by saying coincides instead of equals...
if $A = PBP^{-1}$ then A's eigenvectors are transformed by P^{-1} into eigenvectors of B.
19. F

20. F it's a scalar multiple of u
21. F T if they have the same eigenvalue (why?)
22. T b/c in general $\|u+v\|^2 = \|u\|^2 + \|v\|^2 + 2u \cdot v$
(see ch 6.1 after ex. 5)
23. F this is only false b/c they said non-zero. Don't know why they did that.
24. F $\|u-v\|^2 = \|u\|^2 + \|v\|^2 + 2uv$ so if it equalled $\|u\|^2 - \|v\|^2$ we'd have $\|v\|^2 + 2u \cdot v = -\|v\|^2$
 $\rightarrow 2\|v\|^2 + 2u \cdot v = 0 \rightarrow 2v \cdot v + 2u \cdot v = 0$
 $\rightarrow 2(v+u) \cdot v = 0 \rightarrow v+u \perp v$, not quite $u \perp v$.
25. T they have the same char. poly. b/c $\det((A-\lambda I)) = \det((A-\lambda I)^T) = \det(A^T - \lambda I)$.
26. T
27. F multiplicities could save you
28. F True if it had said $|r| \|x\|$ (thanks Lucy!)
29. T Just pick a char. poly. that has complex roots.
30. T
31. F eg the zero matrix
32. T
33. F this is the definition of diagonalizability!
34. T then $I = P$ in $A = PDP^{-1}$
35. T $A = SBS^{-1}$, $B = PDP^{-1} \rightarrow A = SPD P^{-1}S^{-1} = (SP)D(SP)^{-1}$
36. T this one's tricky. AB invertible so $AB(AB^{-1}) = AB$
so $A(BA)A^{-1} = AB$ shows $BA \sim AB$.
37. F eg the zero matrix again
38. T diagonalizable = there exists a basis of eigenvectors.