Math 54 Worksheet: Review! **#11**

1 Subspaces / Span / Linear Independence

1. Given a subset U of a vector space V, what are the requirements for U to be a subspace?
2. Find a linearly independent subset of the following: $\{[1,1,0,1]^T, [0,2,-1,0]^T, [-4,-2,-1,-4]^T, [0,0,0,1]^T\}$.
3. If S is the subset of linearly independent vectors found in the previous step, is $Span\{S\}$ = Span of the original set of vectors?
4. For what values of t is $[2,3,t]^T$ in $Span\{[1,0,1]^T, [1,2,3]^T, [0,-2,-2]^T\}$

2 Basis / Dimension

1. What are the two requirements of a basis? Is $\{[1,0,0],[0,1,0]\}$ a basis of some vector space?
2. What does dimension have to do with the basis? Give an example of a vector space of dimension 3 other than $\mathbb{R}^3$ (and give a basis for it).
3. Can we extend any set of vectors to a basis? If so, extend $\{[1,0,1],[0,1,0]\}$ to a basis for $\mathbb{R}^3$.

3 Matrix Algebra

1. Give an example of two 2×2 matrices that commute (e.g. A, B such that $AB = BA$). Give an example of two that do not. Prove both.
2. Can two non-zero matrices multiply to zero? If so, give an example with 3×3 matrices.

4 Linear Transformations

1. What are the requirements for a function to be a linear transformation?
2. For each of the following functions, decide if they are linear transformations or not. If they are, find a matrix representation.

(a) $f : \mathbb{R}^3 \to \mathbb{R}$, $f([x, y, z]) = [(x + y)(x - y)]$

(b) $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f([x, y, z]) = [x, y, 0]$

(c) $f : \mathbb{R}^3 \to \mathbb{R}^2$, $f([x, y, z]) = [x + y + z, 1]$

3. Find the matrix for T given that $T([1, 2]) = [3, 4]$ and $T([-1, 1]) = [4, 0]$.

5 Null Space / Column Space / Row Space / L Null Space

Recall we can simultaneously find all 4 subspaces associated with a matrix with a single row reduction. (And if you don't recall, ask me about it or check out worksheet #9). Apply this algorithm to find the 4 subspaces associated with the following matrix:

$$\begin{bmatrix} 1 & 5 & -3 \\ -1 & -4 & 1 \\ -1 & -2 & -3 \\ -2 & -7 & 0 \end{bmatrix}$$

For what values of h is $[-4, 3, 1, h]^T$ in the Column space? Give a set of basis vectors for the Null space and Row space. Is $[1, 1, 1, 1]$ in the L Null space?

6 Rank / Invertibility

What is the relationship between Rank, dim Col A and dim Row A? What is the relationship between Rank and dim Null A? What is the connection between rank and invertibility of a matrix? Invert the following matrix if possible:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 6 \end{bmatrix}$$

7 Change of Basis / Coordinates

Let $\mathcal{B} = \{[7, 5], [-3, -1]\}$ and $\mathcal{C} = \{[1, -5], [-2, 2]\}$. Find the change of basis matrix from $\mathcal{C}$ to $\mathcal{B}$. Use it to write $\mathbf{x} = [1, 1]_{\mathcal{C}}$ in $\mathcal{B}$ coordinates. What is the matrix from $\mathcal{B}$ to $\mathcal{C}$?