Find the general solution to the following differential equations:

1. $y' + 2y = 0$

2. $y'' - 9y = 0.$

3. $y'' + 4y' - 5y = 0.$

4. $y'' - y' - 11y = 0.$

5. $y'' - 4y' + 7y = 0.$

6. $y'' + 7y = 0.$

7. Solve the initial value problem: $y'' + y' = 0$, $y(0) = 2$, $y'(0) = 1$.

8. Solve the initial value problem: $y'' - 4y' + 4y = 0$, $y(1) = 1$, $y'(1) = 1$.

9. Solve the initial value problem: $y'' + 9y = 0$, $y(0) = 1$, $y'(0) = 1$.

10. Find the general solution to the differential equation $y''' - y'' + y' + 3y = 0$.

11. Consider the initial value problem: $y'' + y' - 6y = 0$, $y(0) = a$, $y'(0) = 1$. For what values of a does the solution go to infinity as t goes to infinity? For what values of a does the solution go to zero as t goes to infinity? For what values of a does the solution go to negative infinity as t goes to infinity?

12. (Bonus problem) For two function $y_1(t), y_2(t)$ we define the Wronskian as $W[y_1, y_2](t) = \det \begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix}$.

    (a) If the Wronskian is nonzero at a point t_0, what does that tell you about the system of equations: $\begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$?

    (b) Explain why if the Wronskian is nonzero at a point t_0 then y_1 and y_2 are linearly independent. (From this we see that if the Wronskian is nonzero at a point the two functions are linealy independent.)

    (c) Suppose y_1 and y_2 solve the differential equation $y'' + ay' + by = 0$. Show that the $y(t) = c_1y_1(t) + c_2y_2(t)$ also satisfies the differential equation.

    (d) Following part (3), show that if the Wronskian is 0 at a point t_0 then $y(t)$ solves the initial value problem: $y'' + ay' + by = 0$, $y(t_0) = 0$, $y'(t_0) = 0$.

    (e) Explain why this means that $y(t)$ is zero everywhere. Explain why this means that the Wronskian is zero everywhere.

    (f) We have shown that if y_1 and y_2 solve the same ODE and the Wronskian is 0 at a point, then we have constants c_1 and c_2 not both zero such that $c_1y_1(t) + c_2y_2(t) = 0$ for all t, thus y_1 and y_2 are linearly dependent.