1.1) F 1.2) T 1.3) F 1.4) F (true if same eigenvalue)
1.5) T 1.6) F

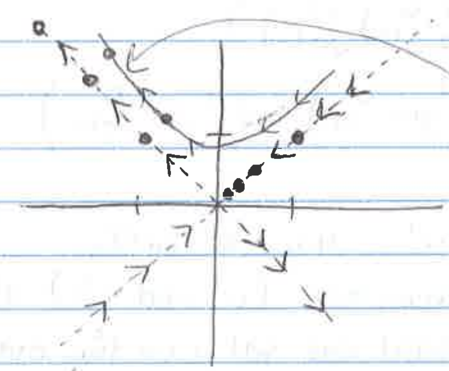
2.1) $\bar{x}_0 = 2.5\begin{bmatrix}1\\1\end{bmatrix} + 0.5\begin{bmatrix}-1\\1\end{bmatrix}$ Thus $\bar{x}_k = A^k\bar{x}_0$

$= (2.5)\lambda_1^k\bar{v}_1 + (0.5)\lambda_2^k\bar{v}_2$

$= (2.5)(0.4)^k v_1 + (0.5)2^k\bar{v}_2.$

2.2) As explained in the text, as $k\to\infty$ $(0.4)^k \to 0$ so overall $\bar{x}_k \approx (0.5)(2)^k\bar{v}_2$.

2.3)



$X_0 = \begin{bmatrix}1\\1\end{bmatrix} = \bar{v}_1 \to \bar{x}_k = \lambda_1^k\bar{v}_1 = (0.4)^k\bar{v}_1$

$X_0 = \begin{bmatrix}-1\\1\end{bmatrix} = \bar{v}_2 \to \bar{x}_k = 2^k\bar{v}_2$

now that we know the behavior of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$ we see the system is a saddle point and so can guess the behavior of all other points b/c we know trajectories on saddle points look like:

note this system is rotated compared to ours.

alternatively can calculate:

$\bar{x}_0 = \begin{bmatrix}0\\0\end{bmatrix} = 0\bar{v}_1 + 0\bar{v}_2$ so $\bar{x}_k = \begin{bmatrix}0\\0\end{bmatrix}$

$\bar{x}_0 = \begin{bmatrix}-0.8\\1.2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} + 0.2\begin{bmatrix}-1\\1\end{bmatrix}$

so $\bar{x}_k = (0.4)^k\begin{bmatrix}1\\1\end{bmatrix} + (0.2)(2)^k\begin{bmatrix}-1\\1\end{bmatrix}$

2.4) saddle point.

3.1) T 3.2) T 3.3) F, eg $\begin{pmatrix}2&1\\0&2\end{pmatrix}$ and $\begin{pmatrix}2&0\\0&2\end{pmatrix}$ are not similar (the first matrix is not even diagonalizable!)

4.1) (a) so $A = PDP^{-1}$ thus $A^k = (PDP^{-1})^k$ but note
$(PDP^{-1})(PDP^{-1})\cdots(PDP^{-1})(PDP^{-1}) = PD^kP^{-1}$
these all cancel

(b) eigenvalues: $\lambda_1 = 1$, $\lambda_2 = 2$
at λ_1, $\begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}$, so $\bar{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ So $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{-1}$
at λ_2, $\begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}$, so $\bar{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ↳ sanity check:
$\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ ✓

thus $A^{10} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^{10}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{-1}$
$= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2^{10} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$
$= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2^{10} & 2^{10} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1-2^{10} & 2^{10} \end{pmatrix}$

4.2) (a) F, $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ is clearly diagonalizable
(b) F, $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ has eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ at $\lambda = 2$
(c) T b/c it'll have at least one value on the diagonal

4.3) Nothing, A could be $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ or $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ (the 1st is diagonalizable, the second is not).

4.4) So we know $A = PDP^{-1}$ for $D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

and $P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 0 & -2 \end{pmatrix}$

Calculate $P^{-1} = \begin{pmatrix} 4/3 & 2/3 & -1 \\ 1/3 & -1/3 & 0 \\ -2/3 & -1/3 & 1 \end{pmatrix}$ → and you can also multiply this out, but finding these 3 matrices is the interesting part.

4.5) (a) $\lambda_1 = 2,\ \lambda_2 = 3$

(b) $\begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(c) rank = 2 so nullity (~~size~~ dimension of the null space) is 1. Thus there is 1 linearly independent eigenvector for λ_1

(d) $(\lambda_2 I - A) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ so rank = 1 thus nullity = 2 and there are 2 linearly independent eigenvectors.

(e) Yes! (b/c we have found a basis of eigenvectors)