1. Use the method of undetermined coefficients to find a general solution to the system $\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{f}(t)$ for the following:

   (a) $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} -t-1 \\ -4t-2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} -4\cos t \\ -\sin t \end{bmatrix}$

2. Find the solution to $\mathbf{x}'(t) = \begin{bmatrix} 0 & 2 \\ 4 & -2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 4t \\ -4t-2 \end{bmatrix}$ satisfying each of the following initial conditions:

   (a) $\mathbf{x}(0) = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$

   (b) $\mathbf{x}(2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3. If $A^2 = 0$, prove that $e^A = I + A$.

4. Let $A = \begin{bmatrix} 5 & 1 \\ -2 & 2 \end{bmatrix}$. Find the eigenvalues and vectors of A and use them to compute e^{At}.

5. Find the general solution to the following systems using matrix exponentiation.

   (a) $\mathbf{x}'(t) = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \mathbf{x}(t)$

   (b) $\mathbf{x}'(t) = \begin{bmatrix} 5 & -3 \\ 1 & 1 \end{bmatrix} \mathbf{x}(t)$

6. Bonus: Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Show by direct computation that $e^{A+B} \neq e^A e^B$.