1. F (unless A is Δ)
2. F (unless you're just adding one row to another)
3. T
4. T
5. F, T if they meant $|\det A|$.
6. F $\det A^T = \det A$
7. T
8. F $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ has $\lambda_1 = 1, \lambda_2 = 0$, $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ has $\lambda = 0$ only.
9. T
10. F (T if A is diagonalizable!)
11. F (T if D is diagonal)
12. F (eg $\begin{smallmatrix} 2 & 1 \\ 0 & 2 \end{smallmatrix}$)
13. F ($\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}$ is diagonalizable)
14. F (T if they are linearly indep)
15. F (eg $\begin{smallmatrix} 2 & 0 \\ 0 & 2 \end{smallmatrix}$)
16. F (P could be the zero matrix)
17. F ($\begin{smallmatrix} 2 & 1 \\ 0 & 2 \end{smallmatrix}$ is invertible b/c $\det \neq 0$ but not diagonalizable)
18. F (P is made up of a basis of eigenvectors, and any other basis of eigenvectors would work too eg scale all the vectors by 2)

Diagonalization

1. A can have 0 to 3 eigenspaces, and their dimensions can be any 3 (or fewer) numbers whose sum is less than or equal to 3.

2. $\mathrm{Det}(A) = 1 \cdot 2 \cdot 3 = 6$, $\mathrm{Tr}(A) = 1 + 2 + 3 = 6$

3. $Ax = \lambda x$ for some x, conjugating: $\bar{A}\bar{x} = \bar{\lambda}\bar{x}$. Assuming A is real, $\bar{A} = A$ so $A\bar{x} = \bar{\lambda}\bar{x}$.
$\rightarrow [1-2i, -1+i, 0]$.

4. (a) $\lambda_1 = 4+i$, $\lambda_2 = 4-i$

$v_1 = \begin{bmatrix}1+i\\1\end{bmatrix}$ $v_2 = \begin{bmatrix}1-i\\1\end{bmatrix}$

(b) $A = \begin{bmatrix}1+i & 1-i\\1 & 1\end{bmatrix}\begin{bmatrix}4+i & 0\\0 & 4-i\end{bmatrix}\begin{bmatrix}-i/2 & (1+i)/2\\ i/2 & (1-i)/2\end{bmatrix}$

$\lambda = \lambda_1$ so $a=4$, $b=-1$

(c) $P = \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ $C = \begin{bmatrix}4 & 1\\-1 & 4\end{bmatrix}$ $P^{-1} = \begin{bmatrix}0 & 1\\1 & -1\end{bmatrix}$

check: $\begin{bmatrix}5 & -2\\1 & 3\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}4 & 1\\-1 & 4\end{bmatrix}$ ✓

Inner Products and Orthogonality

1. (a) T (b) F (they could be orthogonal!)
(c) T (b/c it'll all be squares)

2. $BA = \begin{matrix} b_1\cdot a_1 & b_1\cdot a_2 & b_1\cdot a_3\\ b_2\cdot a_1 & b_2\cdot a_2 & b_2\cdot a_3\\ b_3\cdot a_1 & b_3\cdot a_2 & b_3\cdot a_3\end{matrix}$

3. Consider a vector $\bar{v} = \begin{pmatrix}a\\b\end{pmatrix}$. Pictorally:

by pythagoras $|v|^2 = a^2+b^2$ so
indeed $|v| = \sqrt{a^2+b^2}$

4. $\bar{u}-\bar{v} = (3,4,5)$ so $\|\bar{u}-\bar{v}\| = \sqrt{9+16+25}$

5. $\|(-30,40)\| = 50 \rightarrow \left[-\frac{30}{50}, \frac{40}{50}\right]$

6. $\bar{u}+\bar{v} = (2,1,1)$ so $\|\bar{u}+\bar{v}\| = \sqrt{4+1+1} = \sqrt{6}$
$\|\bar{u}\| = \sqrt{1+4} = \sqrt{5}$, $\|\bar{v}\| = \sqrt{3}$
$\rightarrow$ not equal, but they would be if $\bar{v}\cdot\bar{u} = 0$.

$\bar{u}\cdot\bar{v} = 1-2 = -1$ so $|\bar{u}\cdot\bar{v}| = 1$. $\|u\|\|v\| = \sqrt{15}$.
$\rightarrow$ not equal, but would be if $\bar{v}, \bar{u}$ were parallel.

7. $8\cdot(-2)+(-5)(-3) = -1$ so no.

8. Any vector in the span is of the form $c_1\bar{u}+c_2\bar{v}$ so
$\bar{w}\cdot(c_1\bar{u}+c_2\bar{v}) = c_1\underbrace{\bar{w}\cdot\bar{u}}_{=0} + c_2\underbrace{\bar{w}\cdot\bar{v}}_{=0} = 0$

b/c $\bar{w}$ orthogonal to $\bar{u}, \bar{v}$.

9. Say $\bar{x} \in W$ and $W^\perp$.
b/c $\bar{x} \in W^\perp$, for all $\bar{y} \in W$, $\bar{x}\bar{y} = 0$. In particular, $\bar{x}\bar{x} = 0$. But this can only happen when $\bar{x} = \bar{0}$
(b/c $\bar{x}\bar{x} = x_1^2 + x_2^2 + \dots x_n^2$.)

10. want (x,y,z) such that $x+y+z=0$ and $2x-z=0$ eg something in the null space of

$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix}$. Reduce: $A \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 3/2 \end{pmatrix}$

$\rightarrow x = 1/2 z,\ y = -3/2 z,\ z = z \rightarrow \begin{pmatrix} 1/2 \\ -3/2 \\ 1 \end{pmatrix}$ is a basis for $W^\perp$.

$\leftarrow$ scaled by 2 for neatness

$x\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + y\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + z\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ row reduce

$\underbrace{-\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}}_{W} \underbrace{-\frac{1}{2}\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}}_{W^\perp} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$